a) $$x^t(A+zz^t)\,x > 0$$

$x^t A x > 0$ pues A tiene Cholesky

zz^t es sime. pues $(zz^t)^t = zz^t$

y

$$x^t(zz^t)\,x = (z^t x)^t z^t x = \|z x\|_2^2 \geq 0$$

$\therefore$ $$x^t A x + zz^t > 0$$

b) Por un lado

$$\left[\begin{array}{c:c} L & z \end{array}\right]\left[\begin{array}{c} L^t \\ \hdashline z^t \end{array}\right] = \underbrace{L L^t}_{A} + zz^t$$

$$= A + zz^t$$

Por otro

$$\left[\begin{array}{c} L^t \\ \hdashline z^t \end{array}\right]^t = \left(Q\left[\begin{array}{c} R \\ \hdashline 0 \; 0 \cdots 0 \end{array}\right]\right)^t$$

$$\left[\begin{array}{c:c} L & z \end{array}\right] = \left[\begin{array}{c:c} R^t & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array}\right] Q^t$$

lo que puse arriba

$$\Rightarrow \left[\begin{array}{c:c} L & z \end{array}\right]\left[\begin{array}{c} L^t \\ \hdashline z^t \end{array}\right] = \left[\begin{array}{c:c} R^t & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array}\right] \underbrace{Q^t Q}_{=I} \left[\begin{array}{c} R \\ \hdashline 0 \; 0 \cdots 0 \end{array}\right]$$

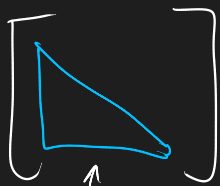
$$A + zz^t = \left[\begin{array}{c|c} R^t & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array}\right] \left[\begin{array}{c} R \\ \hline 0 \;\; 0 \cdots 0 \end{array}\right]$$

$$= R^t \cdot R + 0$$

$$A + zz^t = R^t R$$

Triang. Sup

Como $R =$ 

$\Rightarrow R^t =$  $= \tilde{L}$

Triang. Inf.

$$A + zz^t = \tilde{L}\tilde{L}^t$$

Fact. Cholesky de $A+zz^t$ con

$$\tilde{L}\tilde{L}^t = R^t R$$