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ROLLING REGRESSION VERSUS TIME-VARYING COEFFICIENT MODELLING: AN EMPIRICAL INVESTIGATION OF THE OKUN'S LAW IN SOME EURO AREA COUNTRIES

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ABSTRACT

During the last decade, economists have shown that the inverse relationship between economic growth and unemployment rate varies over time. Rolling regression has been the main tool used to quantify such a relationship. This methodology suffers from several well-known problems which lead to spurious non-linear patterns in the Okun's coefficient behaviour over time. Here, we take a penalized regression spline approach to estimate the Okun's time-varying effects. As a result, spurious non-linearities are suppressed and hence important time-varying coefficient features revealed. Our empirical results show that the inverse relationship in some Euro area countries is spatially heterogeneous and time-varying. The findings are complemented by the calculation of the rate of output growth needed for a stable unemployment rate, as proposed by Knotek.

Keywords: Okun's coefficient, penalized regression spline, rolling regression, time-varying coefficient model

JEL classification numbers: C14, C50, E24

I. INTRODUCTION

Unemployment rate and gross domestic product (GDP) are relevant to economists and policymakers to measure and monitor the state of health of a country. Unemployment rate is internationally defined as the percentage of those people in the labour force who are unemployed. The International Labour Organization defines unemployed workers as those people who currently do not work but who are willing to work, hence looking for a job. According to economic theory, if production increases then unemployment is expected to decrease. However, this is not always the case due to factors such as population growth (determined, for example,

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by migration phenomena) which may lead to a greater labour force availability, labour market structure (flexible[1] or rigid), tax policies, labour productivity (which is linked to technological factors), job specialization (Gavrel and Lebon, 2009) and labour demand and supply issues.[2] The economic growth model of Solow (1956) identifies in labour, capital and technological progress the combination of factors that contribute to the growth of an economy. The real GDP of the 16 Euro countries[3] for the period 2007–09 has fallen on average by 0.3 percent, whereas unemployment rate has increased by 0.2 percent. This inverse relationship is known as Okun's Law. Arthur Okun, whose seminal contribution has been the object of many scientific papers in recent years, studied the empirical relationship between unemployment rate and economic growth, using quarterly U.S. data for the 1954–62 postwar period (Okun, 1962). He suggested two approaches to study this relationship: the first difference model and the 'gap' model. In the first approach, the model is defined as

$$\Delta y_t = \alpha + \beta(\Delta x_t / x_{t-1}) + \varepsilon_t, \quad t = 2, \ldots, T \tag{1}$$

where Δ is a difference operator, y_t represents the unemployment rate of a country, x_t its real GDP, α is the intercept of the model and the ε_t are $i.i.d.N(0, \sigma^2)$ random variables. The parameter β is usually referred to as Okun's coefficient. Equation (1) models, in percentage terms, the contemporary relation between changes in the unemployment rate and real GDP. The second approach is based on the 'gap' equation

$$y_t - y_t^g = \beta\left(x_t - x_t^g\right) + \varepsilon_t \tag{2}$$

where y_t^g and x_t^g represent the natural rate of unemployment and potential output, respectively. Notice that y_t^g and x_t^g are not observable, hence have to be determined by economists.

The Okun's coefficient has been estimated using several econometric approaches and model specifications. Some economists estimated the coefficient employing models that use the cyclical components of the variables described above (e.g., Weber, 1995; Moosa, 1997; Lee, 2000; Cuaresma, 2003; Fouquau, 2008). Filtering techniques such as the Hodrick–Prescott (HP) filter, the Beveridge–Nelson decomposition, the Kalman filter based on the NAIRU framework, the Harvey's model, and the Baxter and King filter, were used to extract the cycle. Their findings showed that the effect of economic growth on unemployment rate is asymmetric and higher during recession than expansion. Other economists used error-correction models in order to account for the possible presence of co-integrating relations between the variables (Attfield and Silverstone, 1998; Lee, 2000; Harris and Silverstone, 2001; Viren, 2001; Zagler, 2003). Instead, Perman and Tavera (2005) employed an autoregressive-distributed lag (ADL) model which was estimated using the seemingly unrelated regressions technique to capture the common economic shocks of the countries considered. The hypothesis of structural change in the Okun's relationship was also tested (Lee, 2000; Moosa, 1997; Sogner and Stiassny, 2002; Weber, 1995). Other authors used rolling regression to estimate the change in the Okun's coefficient over time (Moosa, 1997; Perman and Tavera, 2005; Knotek, 2007). Notice that the use of models based

[1] From the second half of the 1990s, many European countries have begun a labour market reform process by introducing and/or extending flexible forms of contracts, with the aim to increase the level of employment (Mortensen, 2005). This flexibility can support employment growth, and must be accompanied by investment and income growth. It is known that the last two variables are positively correlated with employment stability, and that the latter one is linked to higher investment in human capital. Schooling, on-the-job training, medical care, etc., represent some ways to invest in human capital (Gary, 1962).

[2] Education level, professional experience, worker mobility propensity as well as many other factors can affect and/or hinder the search for a job. In order to tackle the long-term unemployment issue and promote job growth, European institutions are supporting training and retraining of young and mature workers.

[3] Euro area countries: Austria, Belgium, Cyprus, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands, Portugal, Slovak Republic, Slovenia and Spain.

on the cyclical unobservable component of unemployment rate and real GDP has an impact on the magnitude of the Okun's parameter estimates. Hence, these estimates can not be compared with those obtained using models based on observed time series. As an example, Cuaresma (2003) showed that the Okun's coefficient estimated using models based on two cycle extraction techniques (the HP filter and Harvey's model) and U.S. quarterly data (1965–1991) was smaller than the estimate obtained by Okun (1962). Also, a larger coefficient was found by Moosa (1997) who, using U.S. annual data (1960–95), extracted the cyclical components employing the Harvey's model. As pointed out by Lee (2000), the Okun's parameter estimate is sensitive to the model choice, sample period considered and frequency of the data.

In this paper, we analyse some Euro area countries by using a first difference model and modelling approach which allows us to reliably estimate the time-varying structure of the Okun's coefficient. As mentioned earlier, some economists found that the Okun's coefficient changes over time using classical OLS rolling regression (Moosa, 1997; Perman and Tavera, 2005; Knotek, 2007). This methodology suffers from several well-known problems, which include the problematic window size choice that typically results in unstable parameter estimates. Here, we employ a penalized regression spline approach to estimate time-varying coefficients because it does do not require any arbitrary window size choice and can yield reliable and smooth parameter estimates. Our results show that rolling regression leads to unreliable results in terms of spurious non-linear coefficient patterns. Instead, our penalized spline model yields smoother curve estimates, eliminating spurious non-linearities. This permits the reliable determination of the Okun's parameter trends, and of the rates of output growth needed for a stable unemployment rate (Knotek, 2007), hence facilitating the interpretation of empirical findings. The use of our approach is also supported by the study of Huang and Lin (2008) who found strong evidence in favour of smooth-time-varying Okun's coefficients using a Bayesian spline methodology.

The remainder of the paper is organized as follows. In Section II, we describe the model specification and fitting procedure employed for the econometric analysis. In Section III, we present the empirical results. A framework of discussion is provided in Section IV. The main findings are summarized in the last section.

II. ECONOMETRIC STRATEGY

The model of reference is (1). As suggested by many empirical works, it is assumed that the Okun's coefficient is spatially heterogeneous and time-varying. The model specification and statistical approach employed for each selected country are described in the next sections.

II.1 Model specification

Several model specifications are possible for this study. Given a country, the simplest model is as follows:

$$\Delta y_t = \alpha + \beta(\Delta x_t / x_{t-1}) + \varepsilon_t, \quad t = 2, \dots, T \tag{3}$$

where T represents the number of time point observations. Autoregressive terms are not needed because annual data will be used for the empirical analysis (Section III). Model (3) assumes that the Okun's coefficient is constant over time. This assumption is clearly tenuous since, as pointed out in Section I, the Okun's coefficients for different time points are expected to be dissimilar. To accommodate time-varying features, the model can be specified as follows:

$$\Delta y_t = \alpha + \beta_t(\Delta x_t / x_{t-1}) + \varepsilon_t \tag{4}$$

where the effect of real GDP growth on year t is represented by β_t.

The sign of the Okun's coefficient is expected to be negative. According to this, an increase in real GDP has to result in a decrease in unemployment rate, and vice versa. But, is this inverse relationship always empirically valid? Knotek (2007) suggested to calculate the rate of output growth needed for a stable unemployment rate as follows:

$$\text{Rate} = -\left(\frac{\alpha}{-\beta}\right) \tag{5}$$

This ratio indicates how much the economy of a country must grow to maintain a stable level of unemployment rate. In rate (5), β is replaced with β_t when using model (4).

II.2 Rolling regression

The idea behind this methodology is to estimate the parameter of interest by using model (3) across different sampling periods. The periods have identical temporal dimensions (or window sizes). A window size, here denoted with k, has to be chosen. It determines the number of observations used for each rolling regression. If a relationship is stable over time, then the estimated coefficients are fairly similar. When the estimated parameters are judged to be different to each other, the coefficient of interest can be considered as a time-varying parameter and the resulting varying estimates used to interpret findings. Window size choice is one of the main drawbacks of rolling regression because it can heavily affect the behaviour of the estimates over time. Also, the presence of a window size does not allow researchers to obtain parameter estimates for the whole period of observation.

It should be pointed out that, although rolling regression is typically employed to produce time-varying coefficients, the estimates within the chosen samples are constant. Strictly speaking, this means that in principle such an approach can not produce reliable time-varying parameter estimates.

II.3 Time-varying coefficient models

The Okun's coefficient can be reliably estimated by using the following time-varying model

$$\Delta y_t = \alpha + \beta_t(\Delta x_t/x_{t-1}) + \varepsilon_t, \quad \beta_t = s(t;\boldsymbol{\delta}) \tag{6}$$

where $s(t;\boldsymbol{\delta})$ is an unknown smooth function with parameter vector $\boldsymbol{\delta}$, subject to centering constraints. Here, the vector of real GDP growth effects, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_T)_{T\times 1}$, is modelled as $s(t;\boldsymbol{\delta})$. The advantage of this approach is that $\boldsymbol{\beta}$ is completely smooth, with its shape determined from the data and not from the parametric form specified by the investigator. Notice that a time-varying coefficient model is a special case of a varying coefficient model (Hastie and Tibshirani, 1993), for which the effect modifier is time.

It is important to point out that the use of a smooth term in (6) is crucial since the functional shape of any relationship is not typically known *a priori,* hence it does not make sense to impose any structure on it (e.g., linear or quadratic) but rather we should let the data determine whether this relationship is either linear or non-linear and for which countries. The following sections show how this can be achieved.

II.3.1 Regression spline representation. A smooth function can be represented using regression splines. The regression spline of the time variable t is made up of a linear combination of q known basis functions, $b_k(t)$, and unknown regression parameters, δ_k,

$$s(t;\boldsymbol{\delta}) = \sum_{k=1}^{q} \delta_k b_k(t) \tag{7}$$

where q represents the number of basis functions and regression parameters. As an example, if the generic $s(t;\boldsymbol{\delta})$ is believed to be a 3^{th} order polynomial, then a basis for this space is $b_1(t)=1$, $b_2(t)=t$, $b_3(t)=t^2$ and $b_4(t)=t^3$. Therefore (7) becomes

$$s(t;\boldsymbol{\delta})=\delta_1+\delta_2 t+\delta_3 t^2+\delta_4 t^3$$

which can be easily estimated using standard regression techniques, since the replacement of $s(t;\boldsymbol{\delta})$ with its expression yields a linear model. The number of basis functions determines the maximum possible flexibility allowed for a smooth term; the higher the q is, the 'wigglier' the estimated smooth function will be. Although quite illustrative, polynomial bases are not very useful in practice since they may lead to highly correlated parameter estimators, causing inaccuracies in parameter estimates as well as numerical deficiencies. Hence, the use of polynomial terms does not represent a valid solution to capture non-linearities (e.g., Marra and Radice, 2010). To overcome these issues, thin plate regression splines, which have convenient mathematical properties, good numerical stability and avoid having to choose knot locations, are employed. The mathematical expression for this spline basis can be found in Wood (2006).

II.3.2 Some model fitting details. As mentioned earlier, model (6) can be estimated using standard regression techniques. However, overfitting is likely to occur if the basis dimension is too large. To this end, a penalized approach is adopted where by keeping the number of basis functions fixed at a reasonable level, usually $q=10$, it is possible to control the trade off between goodness of fit and roughness of the smooth term by a smoothing parameter, λ. Here, model (6) is fitted by minimizing

$$\|\mathbf{y}-\mathbf{X}\boldsymbol{\delta}\|^2+\lambda\int\{s^d(t;\boldsymbol{\delta})\}^2\,\mathrm{d}t$$

where $\|\cdot\|$ denotes the Euclidean norm, $\mathbf{y}$ contains the annual changes in the unemployment rate, $\mathbf{X}$ is the model matrix containing the basis functions for the time-varying components interacted with their corresponding real GDP growth, $\boldsymbol{\delta}$ is the spline parameter vector, the integral measures the roughness of the smooth component, and d, which is usually set to 2, indicates the order of the derivative for the smooth term to be used in the fitting process. Because regression splines are linear in their model parameters, $\lambda\int[s^d(t;\boldsymbol{\delta})]^2\mathrm{d}t$ can be written as $\lambda\boldsymbol{\delta}^\top\mathbf{S}\boldsymbol{\delta}$, where $\mathbf{S}$ is a known coefficient penalty matrix. The expression and full mathematical details for the penalty used here can be found in Wood (2006). It follows that the penalized regression spline fitting problem is

$$\text{minimize } \|\mathbf{y}-\mathbf{X}\boldsymbol{\delta}\|^2+\lambda\boldsymbol{\delta}^\mathsf{T}\mathbf{S}\boldsymbol{\delta} \text{ w.r.t. } \boldsymbol{\delta}$$

It can be readily shown that the penalized least squares estimator of $\boldsymbol{\delta}$ is

$$\hat{\boldsymbol{\delta}}=(\mathbf{X}^\mathsf{T}\mathbf{X}+\lambda\mathbf{S})^{-1}\mathbf{X}^\mathsf{T}\mathbf{y}$$

If λ is too high then the smooth function will be over smoothed, and if it is too low then the component will be under smoothed. Both cases are not ideal, and it would be good to choose the smoothing parameter so that the estimated smooth is as close as possible to the true function. As discussed in Wood (2006), λ can be effectively estimated by minimization of a prediction error estimate such as the generalized cross validation score (GCV; Craven and Wahba, 1979)

$$\mathrm{GCV}(\lambda)=\frac{n\|\mathbf{y}-\hat{\boldsymbol{\mu}}\|^2}{\{n-\mathrm{tr}(\mathbf{A})\}^2}$$

where λ enter the GCV score through the hat matrix $\mathbf{A}=\mathbf{X}(\mathbf{X}^\mathsf{T}\mathbf{X}+\lambda\mathbf{S})^{-1}\mathbf{X}^\mathsf{T}$, and $\hat{\boldsymbol{\mu}}=\mathbf{A}\mathbf{y}$. The amount of smoothing to choose for the smooth term in (6) has to be settled in practice, and the fast computational procedure for automatic smoothing parameter estimation of Wood (2006) represents a good solution to this problem. Specifically, once q and d have been set, the numerical procedure will select the smoothing parameter so that the part of smooth term complexity which has no support from the data will be suppressed. This means that this approach can produce smooth and reliable curve estimates. See Marra and Radice (2010) for an overview.

II.3.3 Some inferential results. From the normality of $\mathbf{y}$ it follows that

$$\hat{\boldsymbol{\delta}} \backsim N(\mathbb{E}(\hat{\boldsymbol{\beta}}), \mathbf{V}_{\hat{\delta}}) \tag{8}$$

where the frequentist covariance matrix of $\hat{\boldsymbol{\delta}}$ is

$$\mathbf{V}_{\hat{\delta}} = (\mathbf{X}^{\top}\mathbf{X} + \mathbf{S})^{-1}\mathbf{X}^{\top}\mathbf{X}(\mathbf{X}^{\top}\mathbf{X} + \mathbf{S})^{-1}\sigma^2$$

and σ^2 can be estimated by the usual residual sum of squares divided by the residual degrees of freedom, $\hat{\sigma}^2 = \|\mathbf{y} - \hat{\boldsymbol{\mu}}\|^2/\{n - \text{tr}(\mathbf{A})\}$. The trace of $\mathbf{A}$ represents the estimated degrees of freedom (edf) or number of parameters of the fitted model.

Within this framework $\mathbb{E}(\hat{\boldsymbol{\delta}}) \neq \boldsymbol{\delta}$, hence result (8) cannot be used for hypothesis testing. However, if we are interested in testing smooth terms for equality to zero, then (8) can still be used (Wood, 2006). Specifically, if $\boldsymbol{\delta} = \mathbf{0}$ then $E(\hat{\boldsymbol{\delta}}) = \mathbf{0}$ and p-value calculations can be based on the result

$$\frac{\hat{\boldsymbol{\delta}}^{\top}\mathbf{V}_{\hat{\delta}}^{r-}\hat{\boldsymbol{\delta}}/r}{\hat{\sigma}^2/\sigma^2} = \hat{\boldsymbol{\delta}}^{\top}\hat{\mathbf{V}}_{\hat{\delta}}^{r-}\hat{\boldsymbol{\delta}}/r \backsim F_{r,n-\text{edf}},$$

where $\hat{\boldsymbol{\delta}}$ contains the coefficients for the smooth term, r denotes the rank of the covariance matrix of $\hat{\boldsymbol{\delta}}$, and $\mathbf{V}_{\hat{\delta}}^{r-}$ is the rank r pseudoinverse of $\mathbf{V}_{\hat{\delta}}$, which has to be employed to overcome possible matrix rank deficiencies due to the fact that the smoothing penalty may suppress some dimensions of the parameter space.

III. DATA AND EMPIRICAL RESULTS

As mentioned in Section I, several studies have supported the negative correlation between economic growth and the change in the unemployment rate of many countries, and the hypothesis that the Okun's coefficient varies over time and space. The main aim of this section is to quantify reliably the magnitude of this coefficient.

III.1 Data

At the end of World War II, six European countries (Belgium, France, Germany, Italy, Luxembourg and the Netherlands) started a process of unification. The signing of the Treaty of Maastricht, in 1992, officially led to the European Union, which initially included twelve countries. Over the years, other countries have joined the EU (27 nations in 2009), each of them with different economic and social features. 2002 represents an important year for 12 European countries which adopted the Euro as the official currency. In 2009, the number of countries that adopted the Euro was 16 (see Note 3).

Here, we use annual data for the period 1960–2009 for some member states of the Euro area. The data source is the European Commission. Our investigation considered the following countries: Austria, Finland, France, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. The remaining countries of the Euro area were excluded from the analysis because the corresponding time series are not available starting from 1960. For Germany, there is not an official

TABLE 1
Okun's coefficient estimates for some Euro area countries

Country	*Real GDP growth*	*UN rate*	*Intercept*	*Okun's coefficient*	*Rate* (5)
Austria	2.9	2.9	0.33	−0.10(0.02)	3.4
Finland	3.1	6.3	1.15	−0.32(0.04)	3.6
France	3.0	6.8	0.53	−0.12(0.04)	4.4
Greece	3.9	6.5	0.38	−0.07(0.02)	5.0
Ireland	4.6	8.9	1.54	−0.31(0.04)	5.0
Italy	2.8	7.5	0.18	−0.05(0.03)	3.8
The Netherlands	3.0	4.4	0.62	−0.19(0.06)	3.3
Portugal	3.6	5.4	0.67	−0.14(0.03)	4.8
Spain	3.8	10.0	1.64	−0.34(0.07)	4.8

Notes: Within parentheses are standard errors. The estimates were obtained using OLS for the period 1961–2009. Some diagnostic tests are available in Table A.2 in the Appendix. Columns 2 and 3 show annual averages (1961–2009) expressed in percentage terms. UN rate stands for unemployment rate.

reconstruction of the data from 1960 until the reunification in 1990 (Viren, 2001). The variables considered are GDP at constant prices (base year 2000 in euros) and unemployment rate. Notice that unemployment rate is calculated on the basis of a continuous survey of workforce, hence subject to sampling error, while national account variables are subject to periodic reviews carried out by national statistical offices. We use annual time series, as opposed to quarterly data, in order to reduce the variability and autocorrelation in the data. This allowed us to avoid having to include autoregressive terms (whose number is likely to change from country to country) and lagged components of the independent variable, hence keeping the model specification simpler and more interpretable. Annual data were also used, for example, in the studies of Moosa (1997) and Viren (2001).

III.2 Empirical results

Table 1 shows the estimated Okun's coefficients for the period 1961–2009 when using model specification (3), and the rate of output growth needed for a stable unemployment rate (5). Classic OLS was employed. The sign of the estimated parameters are consistent and statistically significant. Italy is the only country whose Okun's coefficient is not significant at the 5 percent level. Spain, Finland and Ireland exhibit the highest Okun's coefficient, whereas Italy the lowest. According to our results, the economy of the Netherlands and Austria has to grow at a lower rate as compared to that of the other countries to maintain a stable level of unemployment rate. As an example, the Netherlands' GDP has to grow by 3.3 percent in order to maintain a stable level of unemployment equal to about 4.4 percent, while Ireland's economy has to grow by 5 percent to maintain a level equal to 8.9 percent. In other words, as mentioned in Section II.1, we would expect unemployment rate to decrease only when the economic growth rate is higher than rate (5). It is reasonable to assume that these estimates are inconsistent with the recent economic dynamics, given the long historical period considered in the estimation process. In fact, over the years, events of economic, political and social nature, may have resulted in a change in economic relationships.

The next sections provide estimates of the Okun's coefficient over time and show that the use of a penalized spline approach should be preferred over rolling regression.

III.2.1 Rolling regression results. On the basis of Equation (3) and using the `R` environment, parameter estimates were obtained employing OLS within the rolling regression framework for the period 1961–2009. k was set to 14. This is in line with the duration of almost one business cycle[4] and is in agreement with the choice of Moosa (1997).

Figure 1 shows that the coefficients exhibit the correct (negative) sign, and that important changes in the Okun's relationship of some countries have occurred over time. As an example, Spain exhibits a coefficient whose increase in magnitude is more regular over time than that of the other countries. The Okun's coefficient of Finland shows a downward pattern until the period 1992–2005 (where the highest magnitude of the parameter, equal to -0.75, is registered), which then turns in an upward direction quickly. Also the Netherlands shows a decrease in the coefficient's magnitude, from -0.63 (1980–93) to -0.22 (1996–2009). This suggests that the Okun's relationship varies over time. This is because some socio-economic exogenous factors (observable and unobservable) may have influenced the dynamics of the variables considered here, and ultimately the behaviour of the estimates over time. Notice that, as pointed out in Section I, the coefficient's pattern is also typically influenced by the econometric approach employed for the analysis. We now describe at what level of economic growth, unemployment rate decreases. Figure 2 may answer this question. It shows, for example, that in Austria and on the basis of the period 1961–74, unemployment rate decreases if real GDP grows by 4.1 percent. In 1996–2009, a decrease in the unemployment rate is observed if real GDP grows by more than 2.4 percent. In the interval of the sample periods 1961–74 and 1986–99, Italy shows very high rate values needed to maintain a stable level of unemployment rate (annual average about 8 percent, with a peak equal to 12.9 percent in the period 1971–84[5]).

Figure 1 highlights important features but also indicates that the estimated Okun's coefficients are quite variable over time. For France and Italy, these results are similar to those found by Moosa (1997).[6] The presence of spurious non-linear patterns can be quite problematic for the correct interpretation of the Okun's coefficient and rate of output growth dynamics. This issue can be overcome employing a penalized spline approach.

III.2.2 Time-varying coefficient models results. Rolling regression indicates that the Okun's coefficient changes over time. However, because this approach suffers from several shortcomings (Section II.2), the resulting estimates are not reliable and easy to interpret. This suggests to employ a methodology which can make use of the whole period of observation, and that does not require any window size choice. A time-varying coefficient model fitted by using a penalized thin plate regression spline represents a good alternative.

Computations were performed using the `mgcv` package in `R` with default settings (Section II.3.2). The results obtained when using different values of q were almost indistinguishable. This is because, regardless of how large the value of q is, once q has been set, a penalized approach can select the smoothing parameter so that the part of smooth term complexity which has no

[4] In the economic cycle literature, there are several proposals regarding the duration of a typical business cycle. All of these support the idea of regular periodic cycles. In summary, the main different cycles are: the Kitchin inventory cycle with duration from 3 to 5 years (proposed by Joseph Kitchin); the Juglar fixed investment cycle from 7 to 11 years with an average duration of 8 years (also known as 'the business cycle' and proposed by Clement Juglar); the Kuznets infrastructural investment cycle from 15 to 25 years (proposed by Simon Kuznets); the Kondratieff wave or long technological cycle from 45 to 60 years (proposed by Nikolai Kondratieff).

[5] From 1971 to 1984, the Italian real GDP and unemployment rate grew on annual average by about 3 and 0.2 percent, respectively. Despite the important economic growth, unemployment rate did not decrease. This was probably due to the rigid labour market structure of those years as well as to data collection problems on the Italian labour market and non-regular labour issues; little could be done to decrease unemployment. Also, the economic growth rate required to decrease unemployment (12.9 percent) can be viewed as a very ambitious or unreasonable target to achieve for those years.

[6] For a consistent comparison of the results, recall that Moosa (1997) employed filtering methodologies to extract the business cycles of the data used to estimate the Okun's coefficients.

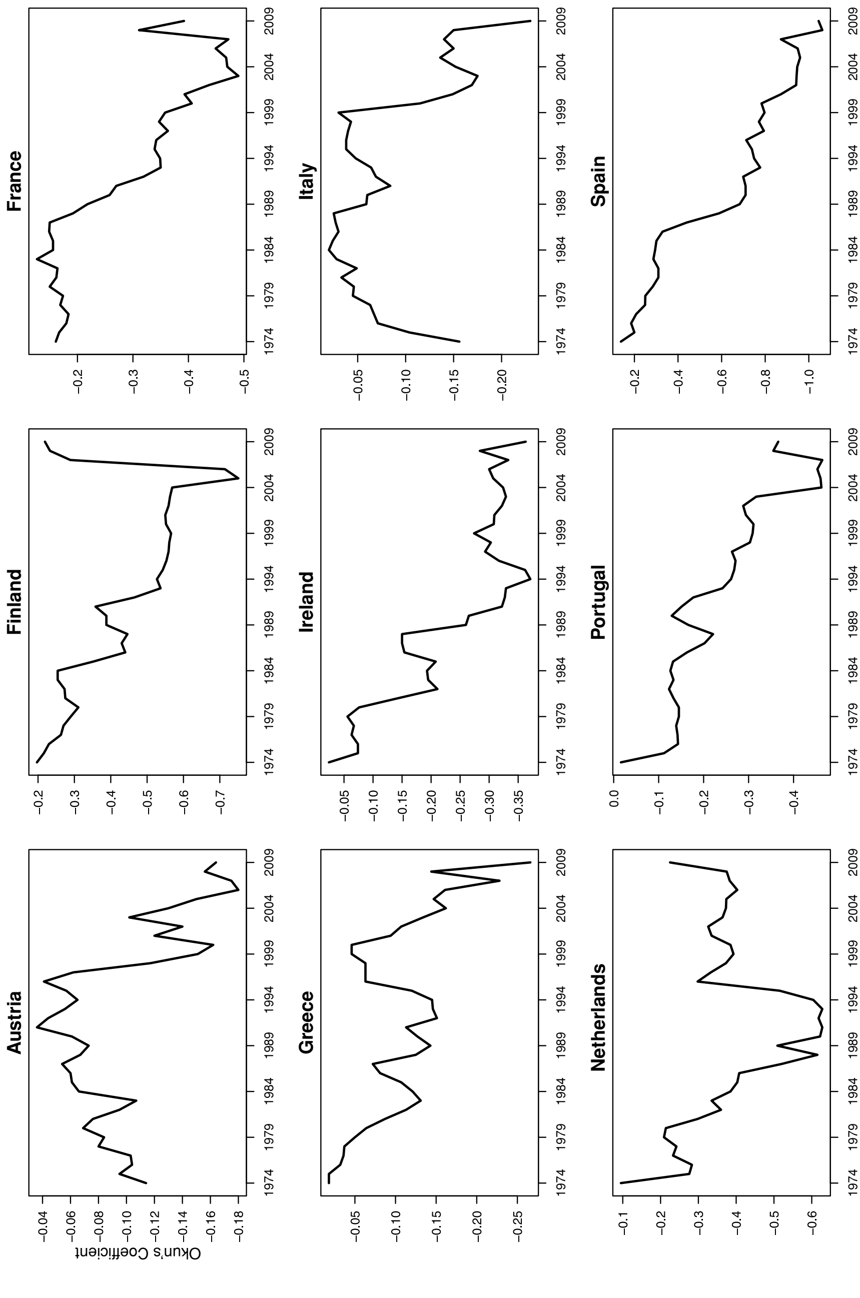

Fig. 1. Okun's coefficients estimated via rolling regression. Data along the *x*-axis denote the last year of each rolling regression. Details are given in Sections II.2 and III.2.1.

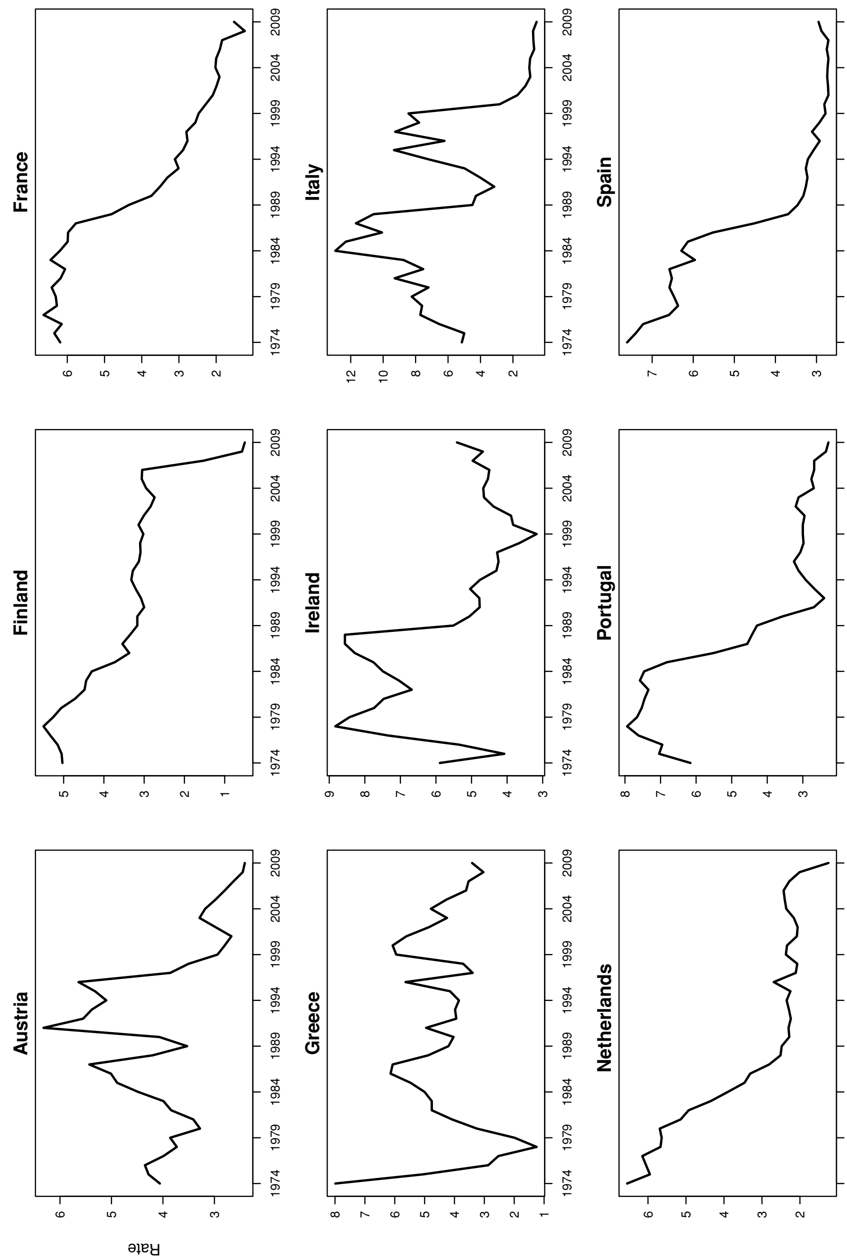

Fig. 2. Rate of output growth (5) based on the rolling regression results of Figure 1.

support from the data is suppressed (e.g., Marra and Radice, 2010). p-Values suggested that all estimated Okun's coefficients are significant at the 5 percent level and time-varying. Some diagnostic tests on the resulting residuals (see Table A.1 in the Appendix) show an improvement over the results obtained when using classic OLS (Section III.2). They also suggest that, overall, the classic model assumptions are not violated.

Figures 3 and 4, which report the results for the period of observation 1961–2009, confirm that the Okun's coefficient of the countries investigated here is time-varing. Importantly, they show that rolling regression may lead to unreliable results in terms of spurious non-linear patterns. According to rolling regression, Greece exhibits a coefficient that is quite variable over time. The use of a pemalized spline time-varying coefficient model allowed us to eliminate spurious non-linear patterns and to reveal the time-varying coefficient features of Greece, which goes from -0.05 in 1961 to -0.20 in 2009. The coefficients for Finland and the Netherlands show an increase in magnitude of the Okun's coefficients until the mid-90s, after which turns in an upward direction. During the last years, Spain, France, Portugal, Ireland and Italy exhibited the higher Okun's coefficient as compared to that of the other countries. Austria, France and Italy show a similar behaviour of the coefficients over time, but with a different magnitude. Figure 4 shows that spurious non-linear patterns are also eliminated. As a result, interpretation of rate (5) is straightforward. Finland and the Netherlands are the only countries which show an increase of the rate from the second half of the 1990s. The lowest rate was registered in Italy during recent years (under one percentage point). Higher levels are found around 1980 (about 2.5 percent), however, these are lower than those observed in Figure 2. In 1980, the Italian real GDP increased by 3.4 percent. According to the Okun's coefficient estimated using model (6), the expected decrease of unemployment for a given increase of real GDP was weak (-0.01 percent). In that year, the Italian unemployment rate decreased by -0.1 percent. Despite an economic growth over 2.5 percent, the decrease in the unemployment rate was very low because of the weak elasticity of the Okun's coefficient. In the following year (1981), real GDP increased by 0.8 percent. The unemployment rate was expected to increase by 0.1 percent. The value observed was 0.3 percent. From this simple example, it is clear that the higher rates observed in Figure 2 are not reliable.

These results show that the use of a penalized time-varying regression spline approach allowed us to estimate reliably the relationship of interest, hence gaining more understanding on the Okun's Law dynamics as opposed to the use of rolling regression. One might argue on the validity of these results, namely that the use of such a methodology would not leave many degrees of freedom to estimate the time-varying coefficients. It should be pointed out that the use of a penalized approach, with automatic smoothness selection, can suppress that part of smooth term complexity which has no support from the data. This means that the effective number of parameters or edf of the fitted model will be smaller than the number of regression spline coefficients actually used during the model fitting process, hence avoiding the common drawbacks of overparameterization (e.g., Marra and Radice, 2010).

IV. A FRAMEWORK OF DISCUSSION

In this section, we present some arguments which may support the presence of a link between the estimated time-varying coefficients and labour market reforms in the countries of the Euro area.

Historically, many European countries have shown a more rigid labour market and a high unemployment level as compared to the situation of the last years (Nickell, 1997). Reforms towards more flexible labour markets have been necessary to absorb the external shock

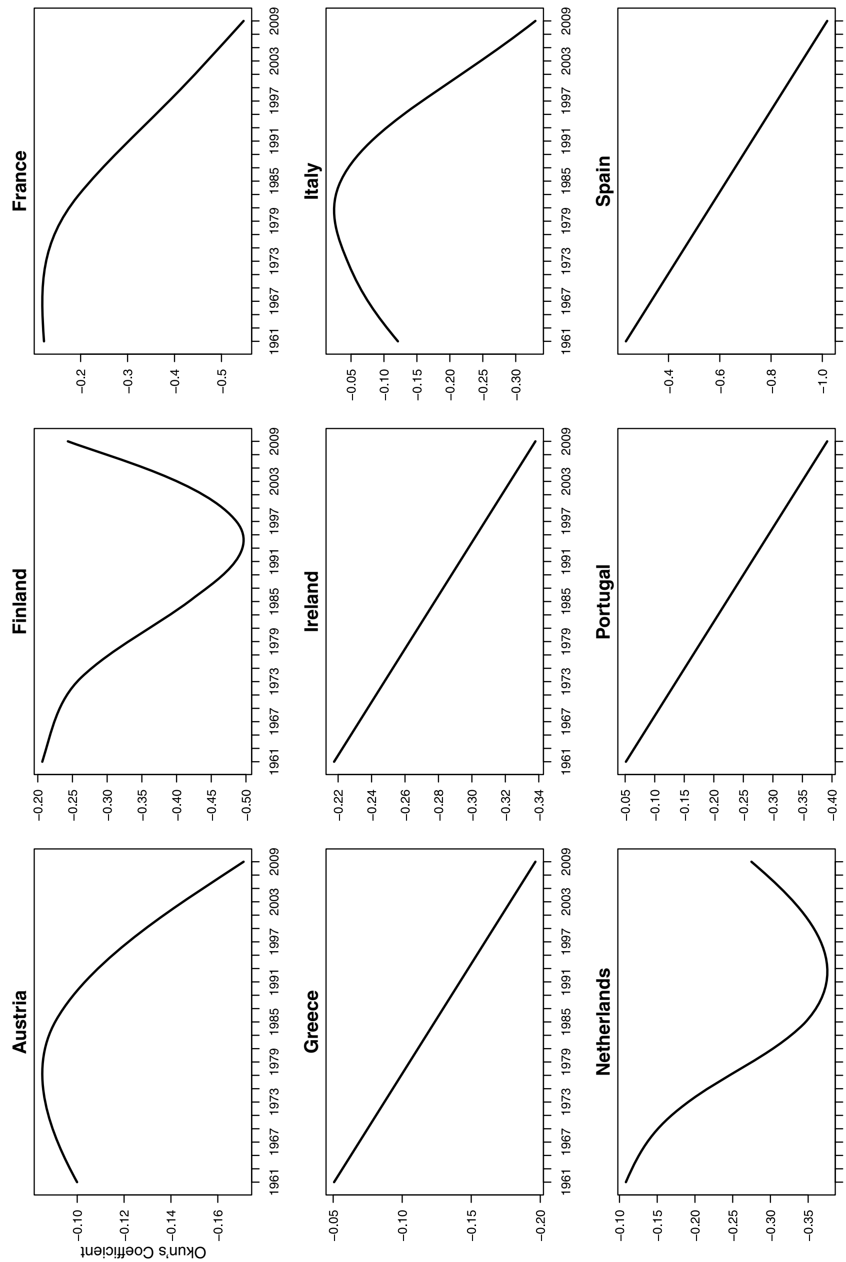

Fig. 3. Okun's coefficients estimated using time-varying coefficient models as described in Sections II.3 and III.2.2. The edf of the curves are in the range 1–4.24. The *p*-values associated with the estimated smooth functions are all smaller than 0.024, supporting the hypothesis that the curves are statistically time-varying at the 5 percent level.

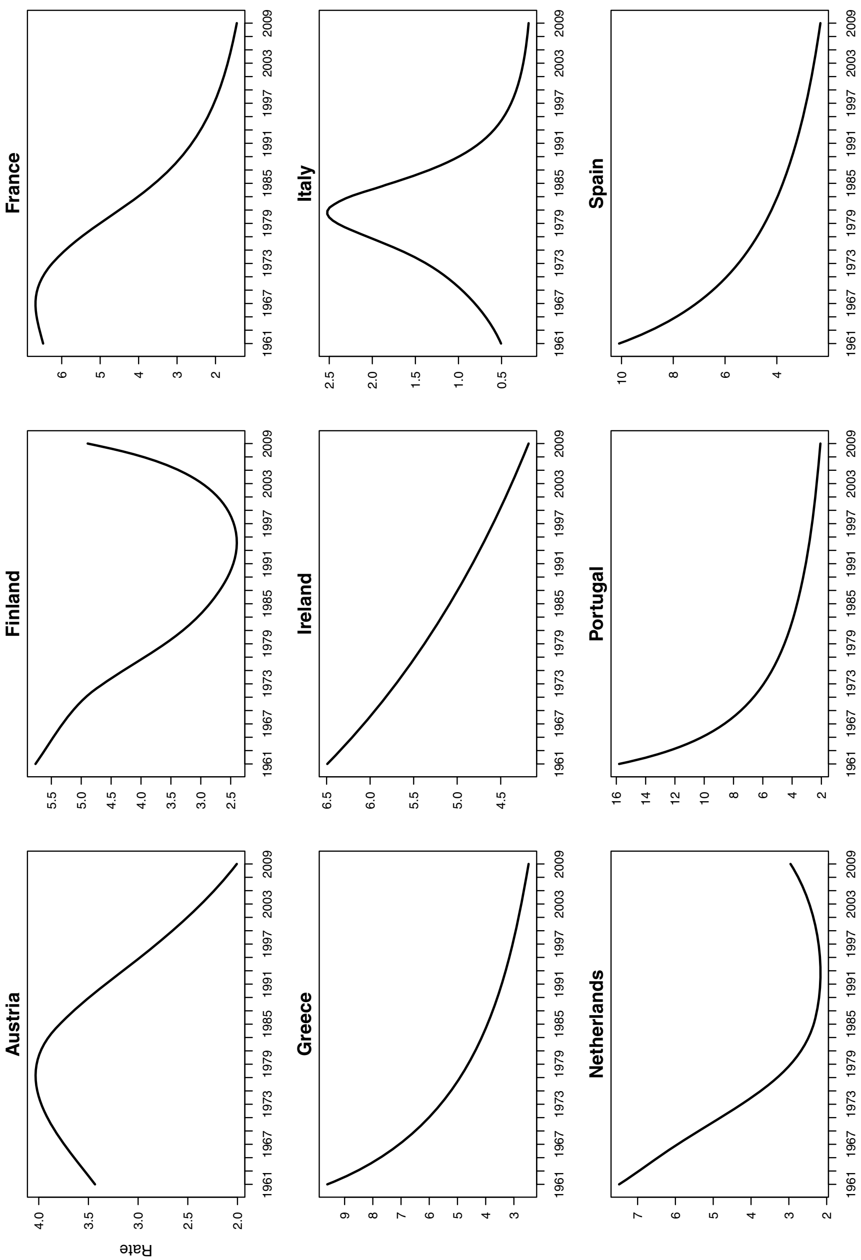

Fig. 4. Rate of output growth (5) based on the time-varying coefficient model results of Figure 3.

associated with the globalization process, increase of competitiveness, and employment growth. A traditional example of consolidated flexible labour market, also characterized by a low protective welfare system, is given by the United States (Swenson, 2002). Some studies have supported the idea that job flexibility results in job insecurity, even though it helps to promote the employment of young, women and older workers (Klein Hesselink and Van Vuuren, 1999; Sverke and Hellgren, 2002).

During the 1990s, institutions and social partners of many European countries have discussed and initiated the process of labour market reform. The main aim was to increase employment and, as a result, to reduce the level of unemployment. Specifically, the target was to decrease the long-term unemployment through a greater flexibility in working time arrangements, and the introduction of atypical forms of employment such as fixed-term employment, temporary agency work, trial periods and part-time jobs, to name a few (European Commission, 2008). The choice of policies and timing of planning have not been uniform across countries. We present some examples of market flexibility introduced via a labour reform process. The part-time work regulation was introduced in France (1998–2002), Italy (towards the end of the 1990s) and the Netherlands (mid-90s). In 2000, Italy reviewed the reform, and introduced new forms of atypical contracts. France introduced the reduction in weekly working hours (1998–2002, from 39 to 35 hours).

Countries such as Austria, France, Ireland, the Netherlands, Portugal and Spain have also introduced lower tax rates. Since 1984, Spain has deregulated fixed-term contracts which in a few years have led to a substantial increase of temporary contracts (Martin, 2002). Long-term unemployment has shown low levels, even though accompanied by a significant degree of income uncertainty in the labour-force (Dolado *et al.*, 2002). Maza and Moral-Arce (2006) discuss the presence of a high degree of rigidity in regional wages. Here, they mention two main reasons: '(a) the regional wage growth shows a lower variability than productivity and unemployment growths, and (b) national and regional wage-setting are closely linked'. In recent years, labour market reforms have led to positive effects in terms of progressive decrease in the unemployment rate of many countries of the Euro area (Moral and Vacas, 2008). However, job insecurity represented a major drawback. To this end, 'flexicurity' policies, which indeed combine flexibility and job security, were proposed. Because the full description of these policies is beyond the scope of this paper, we refer the reader to Keller and Seifert (2005), Neacsu and Baldan (2008) and Origo and Pagani (2008).

We now discuss whether the Okun's coefficient has been influenced by labour market reforms. Based on the results in Figure 3, several answers, which are not conclusive but that seek to explain the Okun's time-varying parameter behaviour, are possible. We only consider the results for some countries. In Italy, the magnitude of the Okun's coefficient elasticity has increased substantially over time. This may be linked to a greater labour market flexibility during the last years. Regarding the whole period of observation, the magnitude of the coefficient may also have been influenced by technical adjustments between economic growth rate and the change in the unemployment rate,[7] and by factors that depend on the state of business cycle. Ireland has exhibited the highest annual economic growth rate and an important decrease in the unemployment rate level from 1998 on. Here, it should be noticed that, as underlined by O'Connell (1999), the increase of part-time work (from 6.7 percent in 1983 to 12.3 percent in

[7] Calmfors and Holmlund (2000) suggested that some of the aspects that may influence the relationship between economic growth and unemployment are: '(i) Exogenous changes of the rate of growth can affect unemployment; (ii) Exogenous changes of the type of growth can affect unemployment; (iii) Changes in labour-market institutions can affect the growth rate indirectly via changes in unemployment; (iv) Changes in labour-market institutions can affect both unemployment and growth directly, but through different mechanisms'.

1997), which has enabled many women to be part of the labour market, may have played an important role. Regarding the Okun's coefficient magnitude, a progressive increase has been observed most likely due to the technical adjustments discussed earlier. The time-varying and magnitude of the Okun's parameters for Spain seem to reflect the degree of flexible labour market, but not only. Bande and Karanassou (2009) showed that the labour market flexibility changes between high and low unemployment regions, and that investment has an influence on the unemployment trajectory. Moreover, Villaverde and Maza (2009) found a spatial heterogeneity in the Spanish regions where a relative low (high) increase in productivity is accompanied by a high (low) Okun's coefficient. Because Spain has been characterized by a substantial unemployment rate as compared to that of the other European countries, unemployment reduction is one of the main goals to achieve. In this respect, a policy has been suggested by the two authors. According to our empirical analysis, it is difficult to separate clearly the effects of labour market reforms from other aspects which may have influenced the coefficient's behaviour over time. Several factors have to be taken into account during the country specific analysis. As described in this section, the empirical findings have to be interpreted in the light of a mix of components which include the economic growth of a country, its demographic structure, labour market policies, policy implementation timing, and spread of policies. These have to be regarded as only some aspects that may influence the estimation results of the complex relationship between economic growth and the change in the unemployment rate. For this reason, each country requires a detailed case study, which is beyond the scope of this paper.

V. CONCLUSIONS

This paper confirms two main results that are in agreement with that which can be found in the economic literature. First, the Okun's coefficient is statistically significant and exhibits a negative sign. Secondly, it changes over time and space.

A time-varying coefficient specification and two fitting procedures were used for the analysis. For each country and using OLS, we estimated the Okun's coefficient for the whole sample period (1961–2009) and for a set of sampling periods in a rolling regression fashion. We then employed a penalized regression spline approach to estimate the time-varying coefficients. The results showed that the use of such an approach can yield parsimonious and smooth results, hence helping to reveal time-varying coefficient features that rolling regression can hardly detect due to spurious non-linear patterns in the estimates. Overall, our main findings confirmed that the coefficient's magnitude increased over time, especially for Spain, France, Portugal, Ireland and Italy. The coefficients for Finland and the Netherlands exhibited a change of direction during the last years. These results have been analysed in the light of the rate of output growth needed for a stable unemployment rate proposed by Knotek (2007). For example, in Italy, France, Portugal and Austria, the Okun's inverse relationship is empirically verified for lower levels of economic growth as compared to the growth needed in past years. Because of the recent global economic slowdown, many countries have experienced an important phase of recessionary economic cycle which is weakening domestic and external demand. This has a negative impact on enterprises which either close down or reduce the number of employees and production levels. These factors are contributing to a general and significant increase in the unemployment rate. Some signals of global economic recovery were registered from the end of 2009. For the next years, according to recent macroeconomic projections (e.g., International Monetary Fund and European Commission), we expect a gradual return to economic growth and an unemployment rate which will take some time to decrease. As shown in our empirical analysis, the rate proposed by Knotek can help to interpret findings in a more consistent manner. We have also provided some arguments to link recent labour market reforms to the estimated time-varying Okun's coefficients. These reforms have most likely influenced the Okun's coefficient magnitude over time, even though many other factors may have had an impact on it. The model specification used here cannot however directly answer these questions, and future work includes this extension.

Finally, we would like to underline that previous studies have estimated the time-varying Okun's coefficients using mainly rolling regression. This approach suffers from several drawbacks which lead to spurious non-linear patterns of the coefficient. Here, we proposed to estimate the time-varying parameters using a penalized regression spline approach. The main advantages of the employed methodology can be summarized as follows: the use of a penalized regression spline model with automatic smoothing parameter selection permitted us to reliably estimate the time-varying Okun's coefficients, hence overcoming the problem of spurious non-linear patterns (which affects rolling regression), hence facilitating the interpretation of findings. It also allowed us not to make any *a priori* assumptions on the functional shape of the relationship of interest but letting the data determine whether it was either linear or non-linear and for which countries; the employed approach avoided having to choose a window size as required in rolling regression and taking a penalized regression spline approach, time-varying coefficients could be obtained for the whole period of observation. This was not possible using rolling regression.

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APPENDIX

TABLE A.1
Some diagnostic tests for the time-varing coefficient models of Section III.2.2 using annual data for the period 1961–2009

Country	*adj-*R^2	$BG = (order\ 1)$	$BG = (order\ 2)$	*GQ*
Austria	0.35	0.927	0.411	0.428
Finland	0.64	0.045	0.047	0.041
France	0.58	0.452	0.645	0.128
Greece	0.26	0.114	0.284	0.298
Ireland	0.57	0.465	0.175	0.889
Italy	0.24	0.058	0.057	0.674
The Netherlands	0.33	0.377	0.639	0.997
Portugal	0.50	0.051	0.076	0.821
Spain	0.77	0.048	0.060	0.819

Notes: See the caption of Table A.2 for further details.

TABLE A.2
Some diagnostic tests for the OLS models of Section 3.2 estimated using annual data for the period 1961–2009

Country	*adj-*R^2	$BG = (order\ 1)$	$BG = (order\ 2)$	*GQ*
Austria	0.27	0.778	0.443	0.372
Finland	0.51	0.000	0.007	0.006
France	0.17	0.003	0.009	0.030
Greece	0.18	0.061	0.172	0.343
Ireland	0.54	0.469	0.175	0.913
Italy	0.03	0.001	0.003	0.221
The Netherlands	0.17	0.029	0.089	0.998
Portugal	0.29	0.003	0.011	0.808
Spain	0.35	0.000	0.001	0.831

Notes: p-Value results are reported. adj-R^2 is the adjusted R^2 measuring the model goodness of fit. BG is the Breush–Godfrey test for serial correlation in the residuals (e.g., Johnston, 1984). GQ is the Goldfeld–Quandt test for heteroskedasticity in the residuals (Goldfeld and Quandt, 1965). The statistical tests were performed using the package `lmtest` in `R`.