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Research article

Measuring conditional correlation between financial markets' inefficiency

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Abstract: Assuming that stock prices follow a multi-fractional Brownian motion, we estimated a time-varying Hurst exponent (h_t). The Hurst value can be considered a relative volatility measure and has been recently used to estimate market inefficiency. Therefore, the Hurst exponent offers a level of comparison between theoretical and empirical market efficiency. Starting from this point of view, we adopted a multivariate conditional heteroskedastic approach for modeling inefficiency dynamics in various financial markets during the 2007 financial crisis, the COVID-19 pandemic and the Russo-Ukranian war. To empirically validate the analysis, we compared different stock markets in terms of conditional and unconditional correlations of dynamic inefficiency and investigated the predicted power of inefficiency measures through the Granger causality test.

Keywords: Hurst exponent; efficient market hypothesis; dynamic conditional correlation; multivariate GARCH; Hurst-based GARCH

JEL Codes: G15, G170, C58

1. Introduction

The efficient market hypothesis (EMH) is based on many unrealistic assumptions such as serial independence, normality of returns, homoskedasticity and the absence of long memory. The so-called stylized facts (Cont, 2001) suggest a strong deviation from market efficiency. From the mathematical point of view, the EMH implies that stock prices follow standard Brownian motions. In terms of the auto-covariance function, the standard Brownian motion assumes that the Hurst exponent takes a constant value of $h = 0.5$ (Bianchi, 2005; Bianchi and Pianese, 2007). Recently, scholars highlighedt that the EMH can be considered as a particular case of a more general theory, called the fractal market hypothesis (FMH) (Di Sciorio, 2020). Under the FMH, we have that $h \in [0, 1]$. When $h = 0.5$, the FMH coincides with the EMH, and the market is efficient. Moreover, according to the adaptive market hypothesis (AMH) (Lo, 2004), the market inefficiency is a dynamic rather than static concept. Indeed, AMH states that market efficiency adapts to changing environments so that, while it is efficient on average (e.g., when $h = 0.5$) it admits deviation from the efficiency over time.

Proposed by Peters (1994) on a Mandelbrot's previous work (Mandelbrot and Van Ness, 1968), the FMH provides a new framework for modeling turbulence in financial markets. FMH is based on the concept of market liquidity and how information is interpreted by different agents. Accordingly, the market is stable when there are investors covering a large number of investment horizons so that the market is endowed with ample liquidity. Under FMH, the market collapses when traders on a given horizon dominate the market and place many sell orders that the rest of the agents cannot absorb.

Market inefficiency has been studied from many different points of views. The most common approach adopted in many empirical studies is based on the returns' autocorrelation (Ito et al., 2014, 2016; Noda, 2016; Le Tran and Leirvik, 2019). Recently, new methodologies have been proposed by physicists and mathematicians working in econophysics (Sánchez-Granero et al., 2020; Di Matteo et al., 2005; Puertas et al., 2023, 2021). The use of multi-fractional Brownian motion (mBm) with the time-varying Hurst exponent, recently explored by Mattera et al. (2022), belongs to this new class and provides a suitable modeling framework consistent with both AMH and FMH. Therefore, in this paper we consider this approach for measuring dynamic inefficiencies of different stock markets. In this respect, a first contribution of this paper is the proposal of a framework for the measurement of conditional volatility in the markets' inefficiency time series. Next, we use this framework for studying markets' correlation from the inefficiency viewpoint.

Measuring the correlation between the efficiency levels of different stock markets has its practical relevance. First, it can help investors and policymakers understand how different financial markets may be influenced by global macroeconomic fluctuations and react to common shocks. Second, it can be used to identify potential opportunities for diversification from a portfolio perspective. Indeed, by understanding the nature of markets' correlations, traders can adapt their trading strategies to gain on interdependencies among markets in a given period. For instance, they could fine-tune their approach based on the evolving correlations to better position themselves in better market conditions and potentially mitigate risk.

Overall, the measurement of correlation between the efficiency levels is a valuable tool for understanding the behavior of global financial systems and making informed policy decisions. It is well known that, at times of high uncertainty or volatility, certain countries have opted for the urgent implementation of certain measures to restrict normal market operations. These measures, which can be considered strategic, are designed to protect some companies when their market price can be dragged down by a situation of high uncertainty in another company in the same sector from a different market. An example is the prohibition of short positions, such as the "anti-takeover law" launched by the Spanish government in the midst of the COVID-19 crisis. Understanding the existing correlations between the different markets can help in policy development and even in defending the existing ones, it is always a highly controversial issue in the financial literature.

For measuring the dynamic correlation between the inefficiency of different stock markets we develop an approach that builds upon – and combines – both the dynamic conditional correlation (DCC) model of Engle (2002) and the mBm. In particular, for each stock market we first estimate a dynamic Hurst exponent following Cannon et al. (1997) and construct a market inefficiency measure as proposed in Mattera et al. (2022). Second, assuming that the inefficiency indices are heteroskedastic zero-mean process, we model the conditional volatility of market inefficiency by means of Hurst-based Generalized Autoregressive Conditional Heteroskedasticity (H-GARCH) processes. Third, conditional correlations are estimated by means of Hurst-based DCC models.

For the empirical analysis, we studied the conditional correlations in terms of inefficiency of four stock markets: China, US, UK and Italy. Given the relevance in the literature (Choudhry and Jayasekera, 2014; Yamani, 2021; Mishra, 2009), we first studied the conditional correlations during the 2007 subprime crisis. Then, following the recent studies by Boungou (2022) , Okorie and Lin (2021) and Wang and Wang (2021) we analyzed the conditional correlations during the war between Russia and Ukraine, as well as the COVID-19 pandemic. The empirical results demonstrate that stock markets are correlated not only in terms of returns, but also in terms of their inefficiency levels. Therefore, we find that when inefficiency in a given market changes, during turbolent times it spills to different markets.

The paper is structured as follows. First, we briefly introduce mBm and FMH. Second, we describe the estimation of the Hurst index and the computation of inefficient measure in section 2. Section 3 describes the data adopted for the empirical study and the obtained results. The empirical experiment involves the choice of the best fitting distribution for the inefficiency indices, the modeling of the process equations and the estimation of the multivariate Hurst-based DCC models. Lastly, Granger causality tests are used to assess the significance of the relationships across different markets, and section 4 concludes with final remarks.

2. Modeling conditional correlations in market inefficiencies

2.1. Measuring market inefficiency with the Hurst exponent

We considered a suitable measure of market efficiency based on FMH. Exploring of dynamic inefficiency requires the estimation of the Hurst exponent. Nowadays, Hurst-based measures of inefficiency are well established (Kristoufek and Vosvrda, 2013, 2016; Sensoy and Tabak, 2015; Kristoufek and Vosvrda, 2019; Sánchez-Granero et al., 2020). By following this approach, EMH can be considered as a particular case of FMH, whenever the Hurst exponent at a given point in time t is $h_t = 0.5$.

The fractional Brownian motion (fBm) proposed by Mandelbrot and Van Ness (1968), captured many important stylized facts, such as fat tails and the slow decay of the auto-correlation function (ACF) of the volatility proxies, as well as ensured a more realistic representation of asset dynamics (Bianchi et al., 2013). The fBm admits the following representation (Couillard and Davison, 2005):

$$W_h(t) = \frac{1}{\Gamma(h + \frac{1}{2})} \int_0^t (t - \tau)^{h-\frac{1}{2}} dW_\tau \tag{1}$$

where W_t is the Brownian motion and h is the so-called Hurst exponent. The covariance of the fBm process is:

$$\mathbb{E}[W_h(t)W_h(s)] = \frac{k^2}{2}(|t|^{2h} + |s|^{2h} - |t - s|^{2h}) \tag{2}$$

where $k = \mathbb{V}ar[W_h(1)]$ and h is the Hurst exponent within the interval $[0, 1]$. The value of h determines the type of fBm process under study. Summarizing, we have that:

- if $h = \frac{1}{2}$ then the process is a Brownian motion;
- if $h > \frac{1}{2}$ then the increments of the process are positively correlated;
- if $h < \frac{1}{2}$ then the increments of the process are negatively correlated.

The above properties make the fBm a natural candidate for financial modeling. However, in the fBm the Hurst exponent is constant (Bianchi, 2005). Although the fBm is a useful model, the fact that most of its properties are governed by a constant exponent h limits its applicability in practice. Considering a constant Hurst exponent, indeed, means overlooking changes in the investors' behavior over time and the occurrence of macroeconomic shocks affecting the stock market.

A very useful generalization of the fBm is represented by mBm. The main idea underlying mBm is to replace the real value h with a function of time, i.e. $h(t)$ between $[0, 1]$. Hence, the values of $h(t)$ provide the regularity function of the mBm. The variability of $h(t)$ allows us to reconcile the mathematical approach with the financial theory. In fact, when the Hurst exponent moves away from $h = 0.5$, it can explain irrational market behaviors.

Initially proposed by Péltier and Lévy Véhel (1995) and studied also by other authors (Bianchi, 2005; Lebovits and Lévy Vehel, 2014; Mattera and Sciorio, 2021; Cerqueti and Mattera, 2023), mBm allows the Hurst exponent to vary over time. The mBm can be represented by the following equation (Péltier and Lévy Véhel, 1995):

$$W_{h(t)}(t) = \int_R (t-s)_+^{h(t)-\frac{1}{2}} - (-s)_+^{h(t)-\frac{1}{2}} dW_s \tag{3}$$

where $(s)_+$ denotes the positive part of the function. On the other hand, the covariance of the mBm is Bianchi and Pianese (2018):

$$\mathbb{E}[W_{h(t)}(t)W_{h(s)}(s)] = K^2 D(h(t), h(s))(t^{h(t)} + s^{h(s)} - (t-s)^{h(t)+h(s)}), \tag{4}$$

with:

$$D(h(t), h(s)) = \frac{\sqrt{\Gamma(2h(t)+1)\Gamma(2h(s)+1) sin(\pi h(t)) sin(\pi h(s))}}{2\Gamma(2h(t)+h(s)+1) sin[\pi(h(t)+h(s))/2]}. \tag{5}$$

Nevertheless, there are some relevant issues regarding to the use of the Hurst exponent in finance. First of all, the approach adopted for its computation is crucial. Indeed, even if different approaches to compute the Hurst exponent had been proposed in the last decades (see Gómez-Águila et al., 2022, for an interesting review of the Hurst exponent estimation methods), several authors (Lo, 1991; Granero et al., 2008; Weron and Weron, 2000) claim that the Hurst exponent estimation using classical methodologies presented a lack of precision, especially when the length of the series was not large enough. In addition, Mercik et al. (2003), Fernandez-Martinez et al. (2013) and Sánchez et al. (2015) demonstrated that most of the classical algorithms used for the Hurst exponent computation were valid only for fBms and do not work properly for other types of distributions such as stable ones.

In this paper, we propose to estimate h_t with the variance method, which builds on Cannon et al. (1997). Considering the self-similarity property of fBm:

$$W(h)_{at} = |at|^h W(h)_t \tag{6}$$

with a being a scaling factor, and applying this property to the volatility or (variation of mBm), we obtain:

$$\sigma_{at} = |at|^h * \sigma_t. \tag{7}$$

By computing the log forms, we get:

$$\log(\sigma_{at}) - \log(\sigma_t) = log(at) * h, \tag{8}$$

then, we obtain h by a linear model where the Hurst index h is the slope in a log-log regression. Following Zanin and Marra (2012), we applied a rolling window ($k = 10$ days) regression to obtain time-varying Hurst exponent (h_t). On one hand, small windows k did not capture self-similarity attributes of a correlated signal. Moreover, with small windows, standard deviations computed using only a few points, so the estimates were less reilable. On the other hand, wide windows k may lead to greater variability of the standard deviation. According to the source, ignoring the number of these large window sizes in the log-log regression calculation reduces the variance of the h estimates.

In the end, once the time-varying Hurst exponent h_t has been estimated for a given i-th market, we use a time-varying measure of market inefficiency, or 'relative volatility measure' $I_{i,t}$, called Hurst-based efficiency index (HEI) (Mattera et al., 2022). This measure is given by the deviation of the empirical, time-varying, Hurst exponent h_t from its theoretical value of 0.5 under the hypothesis of efficient markets, that is:

$$\mathrm{I}_{i,t} = 0.5 - h_{i,t}. \tag{9}$$

Therefore, when $\mathrm{I}_{i,t} = 0$ the i-th market is efficient, while deviations from this value capture shocks in the financial market that lead to inefficiency.

2.2. *Modeling conditional correlations with Hurst-based models*

Suitable approaches for volatility and correlation between financial markets are the multivariate GARCH models (MGARCH). Although very useful, the number of parameters to be estimated in the MGARCH is usually large and grows exponentially with the increase in the number of time series. To overcome this issue, Bollerslev (1990) introduced the constant conditional correlation (CCC) model, which is a more parsimonious procedure as it assumes that the conditional correlations are constant. However, such a constraint was not realistic enough and made the model perform poorly in periods of market instability. To solve this problem, Engle (2002) introduced the dynamic donditional dorrelation (DCC) model that allowed for time-varying correlations and variances. In the DCC model the number of parameters to be estimated increases linearly rather than exponentially, just like the case of the older MGARCH, thus solving the problem of dimensionality.

Therefore, suitable frameworks for modeling conditional variances and correlations in the inefficiency are the GARCH and DCC models. Since our aim is to estimate the DCC of the inefficiency of the various market indices, a natural approach would be the use of DCC computed on a measure of market inefficiency (9). From the statistical point of view, we can write the correlation between two market inefficiencies at time t as follows:

$$\rho_{12,t} = \frac{E\left[I'_{1,t} I_{2,t}\right]}{\sqrt{E\left[I^2_{1,t}\right] E\left[I^2_{2,t}\right]}}, \tag{10}$$

which holds if we assume that the market is efficient unconditionally, that is $E\left[I_{i,t}\right] = 0.5 - E\left[h_{i,t}\right] = 0$. Considering the multivariate case of N stock markets, an empirical estimate of the conditional covariance is simply provided by the cross product $\mathcal{H}_t = I'_t I_t$, with I_t being the matrix associated with the N stock market inefficiency measures. The conditional covariance matrix H_t can be then decomposed as usual:

$$\mathcal{H}_t = \mathcal{D}_t \mathcal{R}_t \mathcal{D}_t, \quad \mathcal{D}_t = \text{diag}\left(\sqrt{h_{i,t}}\right) \tag{11}$$

with $\mathcal{D}_t$ being a diagonal matrix with conditional volatilities and $\mathcal{R}_t$ being the correlation matrix. Assuming that the inefficiency indices are heteroskedastic zero-mean processes, we can write each entry of the diagonal matrix $\mathcal{D}_t$ as follows:

$$h_{i,t} = \omega + \alpha I^2_{i,t} + h_{i,t-1}, \tag{12}$$

which is a standard GARCH process. Consequently, if we scale the inefficiency index for its volatility $\nu_t = I_{i,t}/\sqrt{h_{i,t}}$, we get that the correlation dynamics can be modeled as GARCH-type processes (Engle, 2002):

$$q_{ij,t} = \bar{\rho}_{i,j} + \alpha\left(\nu'_{i,t}\nu_{j,t} - \bar{\rho}_{i,j}\right) + \beta\left(q_{ij,t-1} - \bar{\rho}_{i,j}\right), \tag{13}$$

with $\bar{\rho}_{i,j}$ being the unconditional correlation.

In sum, under the assumption that the inefficiency measures conditional to an information set $\mathcal{F}_t$ follow zero-mean multivariate normal distribution with covariance $\mathcal{H}_t$:

$$I_t \mid \mathcal{F}_t \sim \mathcal{N}\left(0, \mathcal{H}_t\right), \tag{14}$$

we can write the conditional variances equation as follows:

$$D^2_t = \text{diag}\left(\omega_i\right) + \text{diag}\left(\alpha_i\right) \circ I'_t I_t + \text{diag}\left(\beta_i\right) \circ D^2_{t-1}, \tag{15}$$

and, under standardization, we can write the conditional correlation equations as:

$$Q_t = \bar{R}\left(\iota'\iota - A - B\right) + A \circ \nu'_t \nu_t + B \circ Q_{t-1}, \tag{16}$$

given that $\bar{R}$ is the sample unconditional correlation matrix, ι is a vector of ones and $\circ$ denotes the Hadarmand product. In the end, to ensure that in the conditional correlation matrix holds $\text{diag}\left(\mathcal{R}_t\right) = \iota$, we perform the following adjustment (Engle, 2002):

$$\mathcal{R}_t = \text{diag}\,(Q_t)^{-0.5}\, Q_t \text{diag}\,(Q_t)^{-0.5}\,. \tag{17}$$

Inference on model parameters is conducted with the maximum likelihood estimation (MLE) approach (Engle, 2002) and in-sample predictions are used to estimate conditional correlations across market inefficiencies.

3. Empirical analysis

For the empirical analysis, we considered the relationship in terms of inefficiency of four stock markets, FTSE100 (London), SSEC (China), FSTEMIB (Italy) and S&P500 (USA), during the time period between the 2007 crisis (from 07-01-2007 to 09-01-2009), the COVID-19 pandemic (from 12-31-2020 to 12-31-2021) and the war between Russia and Ukraine (from 02-24-2022 to 02-24-2023). We considered daily log return time series computed on the adjusted prices. The software used for the analysis was RStudio version 4.01. In the first step of our empirical analysis, we estimated the dynamic Hurst exponent time series h_t through the variance method derived from self-similarity of fBm (Gripenberg and Norros, 1996). Next, we obtained the resulting inefficiency measure (9) and estimated conditional correlations, and then Granger causality tests were used for inspecting the relationships between financial markets' inefficiency during turbulent periods.

3.1. *Financial crisis (2007–2009)*

First, we analyzed the relationships across the considered stock markets during the 2007–2009 financial crisis. The time series of the inefficiency measure computed for the four considered markets are shown in Figure 1.

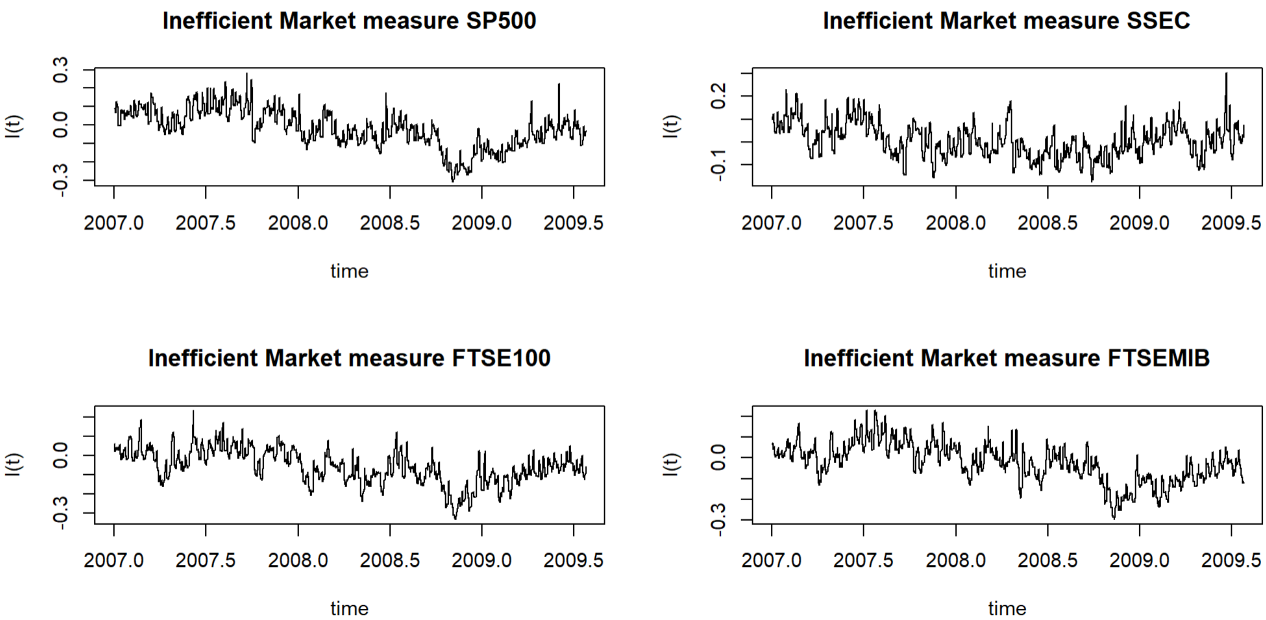

Figure 1. Time series Inefficient Measures I_t during the financial crisis (2007–2009).

We investigated the time series stationarity with the Augmented Dickey-Fuller test. All time series for the three events considered were stationary. The tests' results are shown in Table 1. According to the results, we reject the null hypothesis of non-stationary for all the inefficiency measures.

Table 1. Dickey-Fuller Test: results.

Market Index	Dickey-Fuller	Lag Order	p-value
SP500	−3.6462	9	0.02824
SSEC	−5.0859	9	0.01
FTSEMIB	−4.5159	9	0.01
FTSE100	−4.2157	9	0.01

Before estimating the DCC model as discussed in section 2.2, we first have to asses whether the Gaussian distribution assumption holds. For this aim, we consider the procedure proposed in Laure and Dutang (2015). By analyzing the kurtosis and the squared of skewness, we chose the best distribution parameters with MLE. Figure 2 shows the test results.

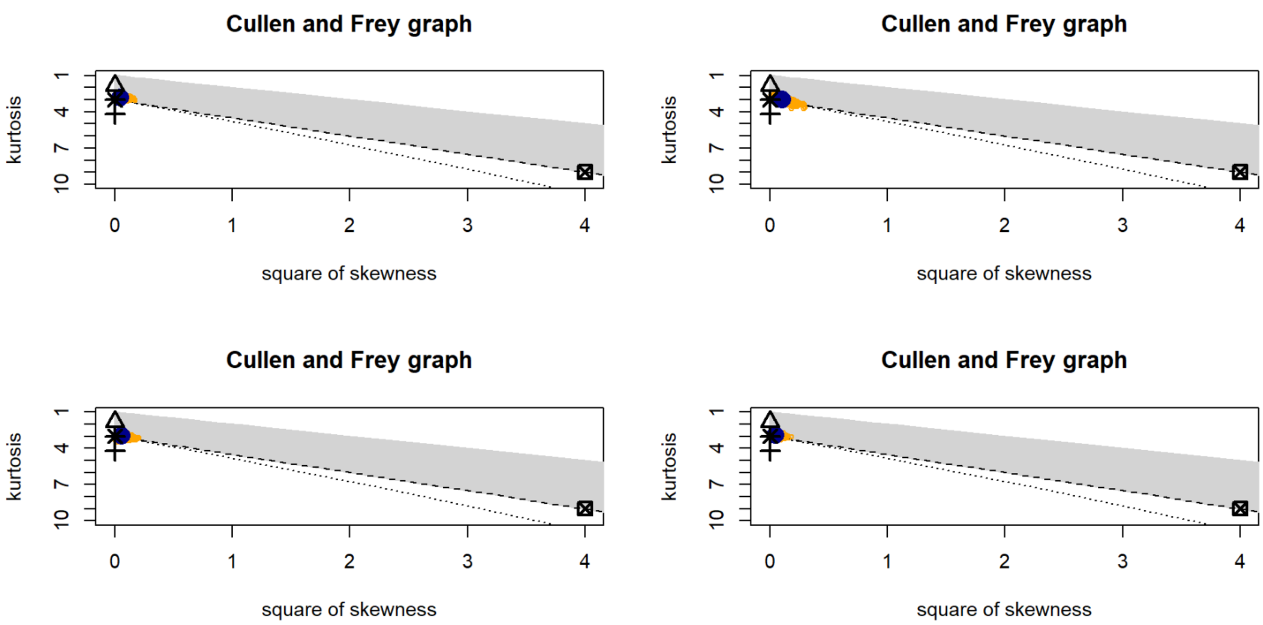

Figure 2. Fitting Distribution for the Inefficiency measures.

The blue points in Figure 2 represent the empirical values, so we noted that the empirical distribution approximates the normal in term of skewness and kurtosis. Therefore, the levels of empirical skewness and kurtosis show that the assumption of Gaussian distribution holds for all the considered time series.

Next, we estimated different GARCH models whose equations are characterized by the presence of a conditional mean $\mu_{i,t}$ that takes the form of an autoregressive moving average (ARMA) process. In doing so, we let the inefficiency indices to fluctuate around their zero-mean values. To determine the best model for the mean equation, we adopted the automatic procedure proposed by Hyndman and Khandakar (2008). Before applying an Hurst-based GARCH modeling for the estimation of conditional volatilities and correlations, we first tested whether the inefficiency indices were likely to be heteroskedastic. The results of the ARCH Lagrange Multiplier (ARCH LM) test are reported in Table 2.

Table 2. ARCH LM test: results.

Market Index	Chi-squared	df	p-value
SP500	727.21	12	2.2e-16
SSEC	522.32	12	2.2e-16
FTSEMIB	762.99	12	2.2e-16
FTSE100	812.46	12	2.2e-16

From Table 2, we get evidence that all the series are affected by heteroskedasticity, and consequently introduce DCC modeling. In particular, we fit a DCC(1,1) model with Gaussian innovations. Given the resulting estimates, we computed the $\mathcal{R}_t$ matrix. The estimated conditional correlations are shown in Figure 3.

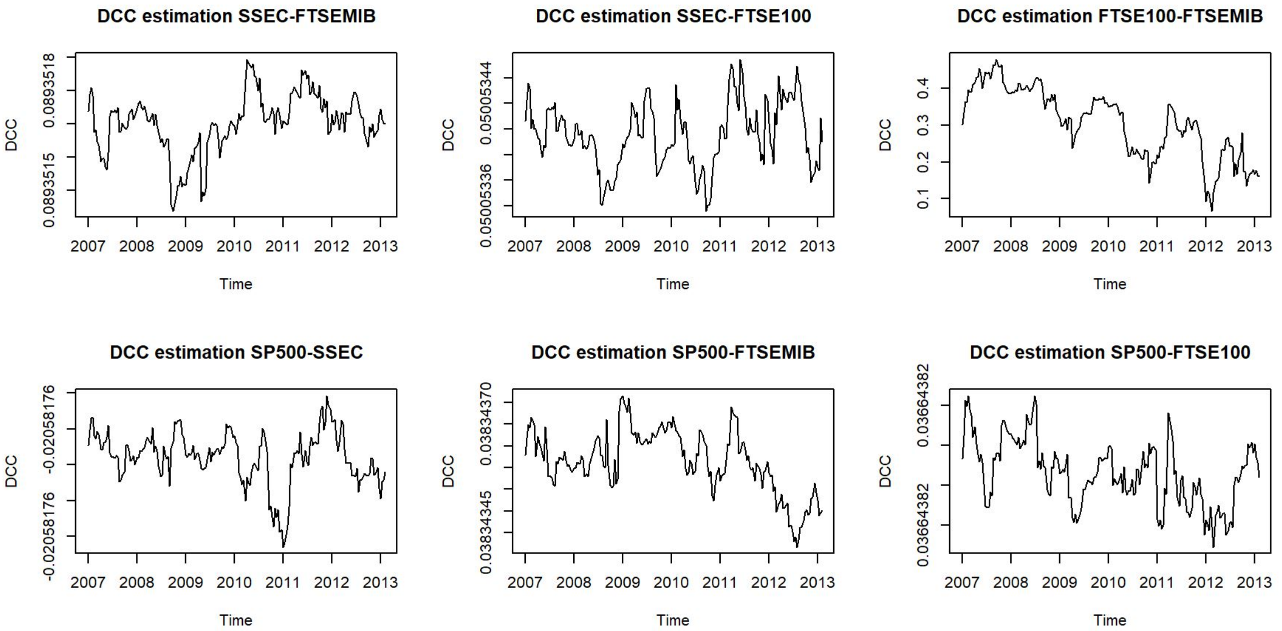

Figure 3. Estimated conditional correlations for the different stock market indices.

The results indicate that the conditional correlation in the market inefficiency between all the considered stock market indexes is quite low. The only exception is given by the FTSE100 and FTSEMIB indices. This result is particularly interesting in light that FTSEMIB and FSTE100 – the Italian and UK stock markets, respectively – belonged to EU during the considered time period. Finally, we noticed that during the financial crisis the stock inefficiency levels of the stock markets were instead quite highly correlated unconditionally (i.e. in the long-run), while scarcely correlated during the short-run fluctuations.

Starting from the result obtained with DCC modeling, we built a Granger causality test to investigate whether the inefficiency in a market helps in predicting the inefficiency of another one (Durcheva, 2021; Chu and Qiu, 2016). In particular, we used Granger causality to analyze the dynamic between FTSEMIB and FTSE 100, which are the two markets with the highest conditional correlations, we estimated the following bivariate vector autoregressive (VAR) model:

$$\begin{bmatrix} \text{FSTE100}_t \\ \text{FSTEMIB}_t \end{bmatrix} = \mathbf{c} + \sum_{p=1}^{P} \mathbf{\Pi}_p \begin{bmatrix} \text{FSTE100}_{t-p} \\ \text{FSTEMIB}_{t-p} \end{bmatrix} + \boldsymbol{\varepsilon}_t \tag{18}$$

where $\mathbf{c}$ is a vector of constant, $\mathbf{\Pi}_p$ is a matrix of parameters related to the p-th autoregressive component and $\boldsymbol{\varepsilon}_t$ is a vector containing the shocks. According to information criteria, we selected a one-order process. Table 3 shows the results.

Table 3. VAR Estimation Results FTSEMIB.

FSTEMIB equation				
	Estimate	Std. Error	t value	p-value
$I(FTSEMIB)_{t-1}$	0.138	0.08	1.830	0.07*
$I(SP500)_{t-1}$	0.173	0.06	2.696	0.01 **
$I(FTSE100)_{t-1}$	−0.019	0.07	−0.277	0.78
$I(SSEC)_{t-1}$	0.004	0.07	0.064	0.95
const	−0.033	0.006	−5.60	$7.86e^{-08}$ ***
FSTE100 equation				
	Estimate	Std. Error	t-value	p-value
$I(FTSEMIB)_{t-1}$	0.18	0.08	2.19	0.03**
$I(SP500)_{t-1}$	−0.04	0.07	−0.52	0.6
$I(FTSE100)_{t-1}$	0.04	0.08	0.52	0.61
$I(SSEC)_{t-1}$	0.019	0.07	0.26	0.79
const	−0.04	0.007	−5.80	$2.82e^{-08}$ ***

Next we proceed with the Granger causality test, with results shown in Table 4.

Table 4. Granger Multivariate Wald causality test.

	F	p-val
FTSE100 $\Leftarrow$ FTSEMIB	0.25	0.957
FTSEMIB $\Leftarrow$ FTSE100	19.86	0.001 ***

Table 4 shows that the relationship between the two stock indices is not bidirectional, since the FSTEMIB Granger caused the FSTEMIB but not vice-versa. Therefore, we can conclude that knowing the values of FTSE100 inefficiency is a valuable information that can be used for forecasting the future values of FTSEMIB inefficiency.

The results, in line with those of the estimated conditional correlations between the two markets' inefficieny levels, show a significant correlation between the FTSEMIB and FTSE100 indices. The Granger test confirms the presence of causality in the measure of inefficiency in the London market compared to the Italian one, while the results for the other considered stock market indices failed in showing the same evidence of these considered European markets.

3.2. *COVID-19 pandemic (2019–2020)*

Next, we analyzed the period of the COVID-19 pandemic. The methodology adopted here followed the previous empirical experiment. For readers' convenience, we only reported the analysis of the estimated conditional correlations among the market inefficiency measures. As in the previous experiment, we determined the optimal ARMA processes according to the corrected akaike information criterion (AICc) following the Hyndman and Khandakar (2008) procedure, and estimated the corresponding DCC process under Gaussian innovations. Figure 4 shows the estimated conditional correlations.

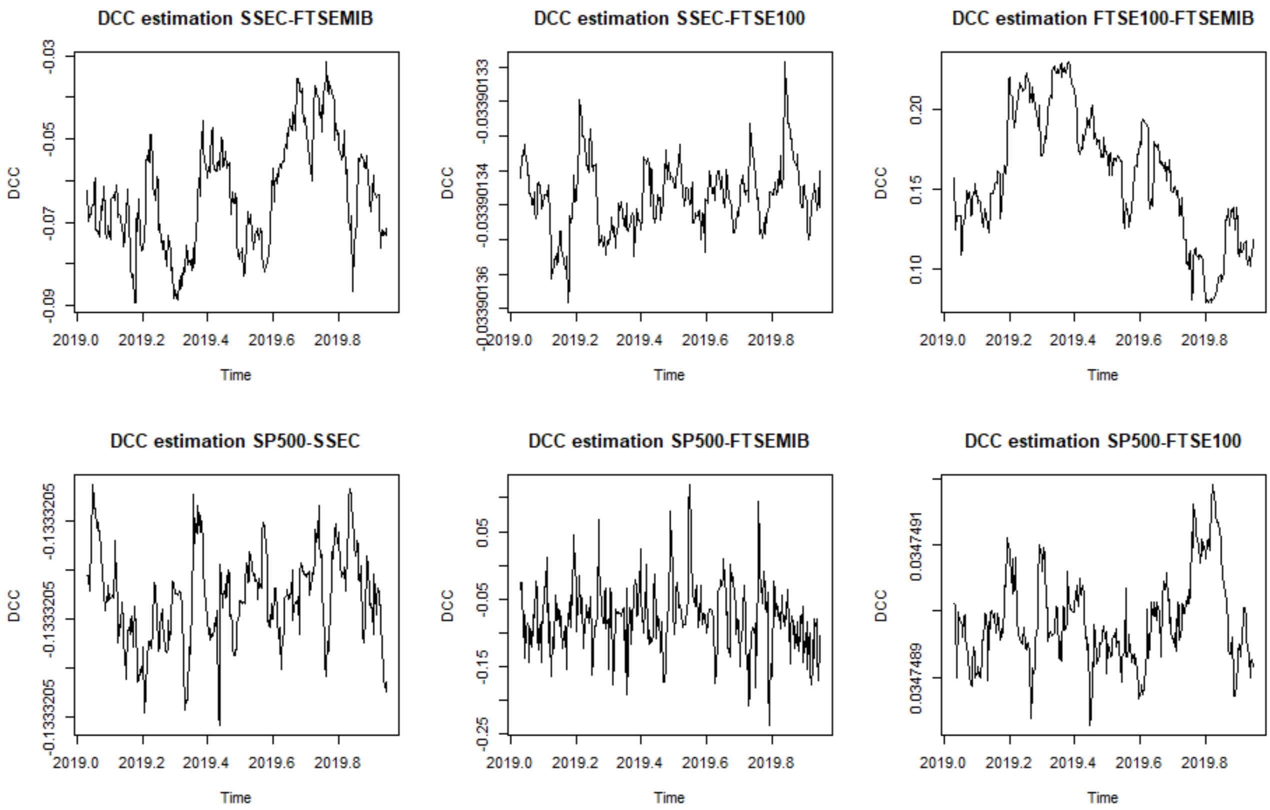

Figure 4. Estimated conditional correlations for the different stock market indices: COVID-19.

The correlations across different markets show larger magnitudes compared with those in Figure 3, suggesting larger connectedness across markets in terms of inefficiency levels during the last years. However, the correlation patterns show peculiar dynamics in the different markets. For example, the correlation between FSTE100 and FSTEMIB decreased during the pandemic, while FSTE100 and SSEC experiments had an increasing correlation. Moreover, we conducted the Granger causality tests. In accordance with the information criteria, we estimate a VAR(1). We noticed, however, that the analysis carried out in the first years of the COVID-19 pandemic reported no significant relationship as the parameters estimated by the VAR model were not statistically significant. Therefore, there were no causal effects among the market inefficiencies. Table 5 shows the results with the F-statistics and the associated p-values.

Table 5. Granger Multivariate Wald causality test: COVID-19.

	F	p-val
SP500 ⇐ SSEC	0.13	0.716
SP500 ⇐ FSTE100	0.27	0.605
SP500 ⇐ FTSEMIB	1.01	0.316
SSEC ⇐ SP500	0.06	0.808
SSEC ⇐ FSTE100	0.59	0.443
SSEC ⇐ FTSEMIB	2.17	0.142
FSTE100 ⇐ USA	0.56	0.456
FSTE100 ⇐ SSEC	0.98	0.323
FSTE100 ⇐ FTSEMIB	0.19	0.664
FTSEMIB ⇐ USA	0.06	0.802
FTSEMIB ⇐ SSEC	1.27	0.261
FTSEMIB ⇐ FSTE100	0.00	0.968

3.3. Russo-Ukraian War (2020–2023)

Finally, the last period under analysis was related to the Russo-Ukraian War. The adopted methodology followed the two previous empirical experiments. For readers' convenience, we also reported the analysis of the estimated conditional correlations among market inefficiency measures only. Figure 5 shows the estimated conditional correlations.

Focusing on the war period, we noticed that inefficiencies' correlations of most of the markets with SSEC decreased at the beginning of the war, and then reverted to pre-war values some months after. Then, the correlation across the other markets overall increased in the considered time period. Thus, consistent with previous evidence, we found that stock markets become more integrated during this turbulent period. We then investigated the causality between the inefficiencies by means of a VAR(1) process, estimated in accordance with information criteria. The results are shown in Table (6).

Table 6. Granger Multivariate Wald causality test: Russo-Ukraian War.

	F	p-val
SP500 ⇐ SSEC	2.20	0.140
SSEC ⇐ SP500	3.37	0.068*
FTSE100 ⇐ FTSEMIB	1.00	0.319
FTSEMIB ⇐ FTSE100	4.18	0.042*

From Table 6 we found a consistent relationship between FSTEMIB and FTSE100, where the FTSE100 Granger caused the FSTEMIB and not vice versa. This result was interesting because, different from the 2007–2009 financial crisis, the UK does not belong to EU. Moreover, we also found that the S&P500 inefficiency predicted the inefficiency of the SSEC, but not vice-versa. Therefore, we found significant and asymmetric relationships between market inefficiency levels during the war, and we confirmed the existence of significant relationships between the inefficiency of markets in turbulent periods.

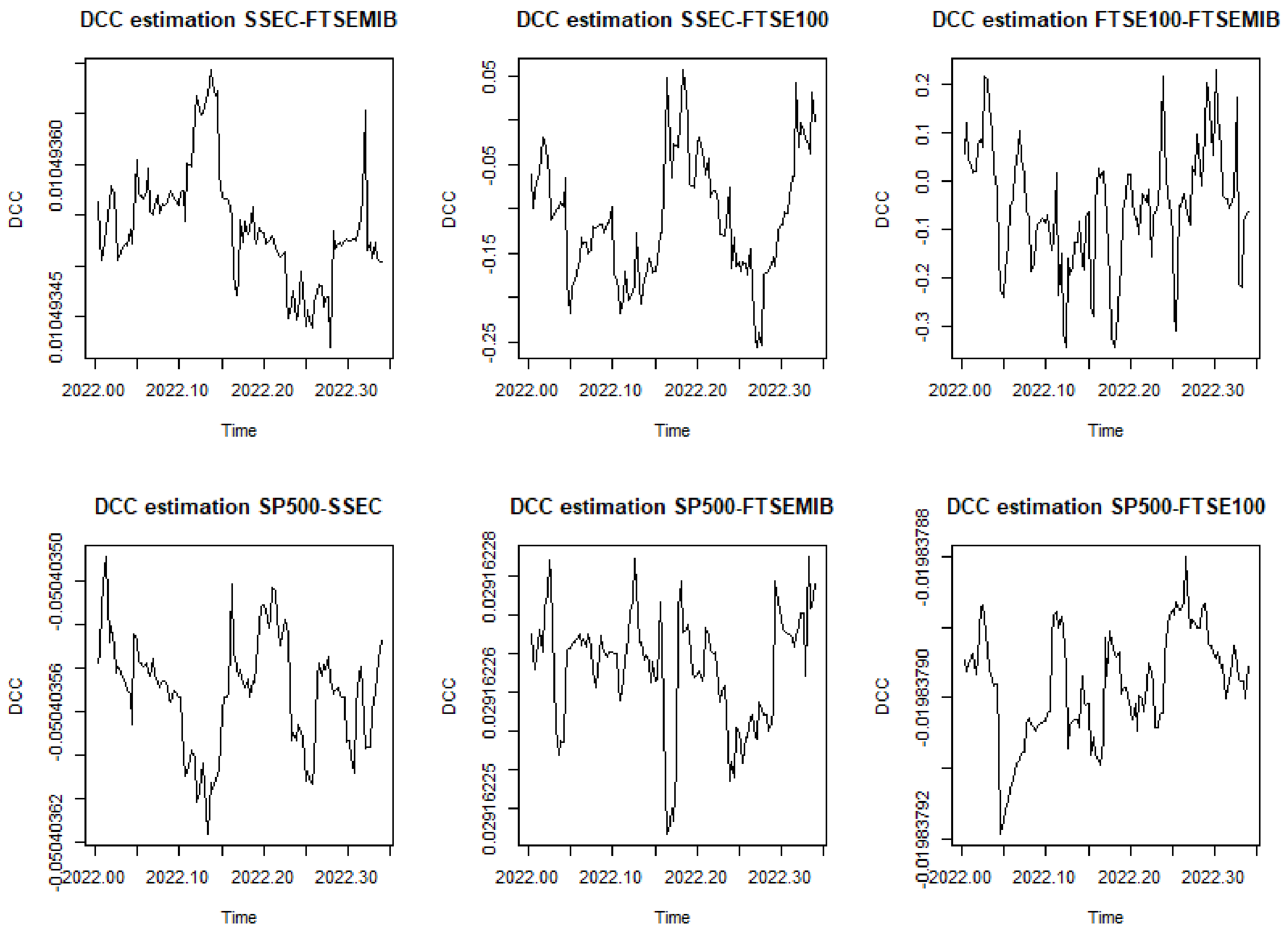

Figure 5. Dynamic Conditional Correlations for the different stock market indices: Ukranian war.

4. Conclusions

In this paper, we verified the existence of a dynamic correlation between the inefficiencies of the main market indices. For this aim, Hurst-based GARCH and DCC models have been proposed in the paper. The decision of modeling the Hurst exponent as a measure of relative volatility (with respect to the efficient level) with the DCC model derives from the heteroskedastic properties that we found empirically for all the analyzed time series. We have noticed that some markets show a significant dynamic correlation and we have investigated the nature of this relationship through Granger's causality test.

The empirical results are based on the analysis of different turbulent periods, which are the 2007–2009 financial crisis and the first year of the COVID-19 pandemic and the Russo-Ukraian War. The results of the pandemic tests did not report a significant causal relationship between the inefficiency of the analyzed market indices, while we obtained statistically significant results for the other two considered time periods. In this respect, it is very likely there are other factors not considered in the analysis of the COVID-19 period. As far as the Russo-Ukraian War is concerned, the results obtained were very similar to those of the 2007 crisis, and we noted a significant causal relationship between the Italian and English markets.

In terms of the relationship between specific markets, our findings shed light on a significant association between the inefficiencies of the UK and Italian markets. This connection remained consistent even when we expanded our analysis to encompass distinct historical periods, namely the initial year of the COVID-19 pandemic and the Russo-Ukraian War. Moreover, another interesting relationship highlighted during the war period was between US and Chinese stock markets.

Overall, the existence of such relationships is important for several reasons, mainly related to the understanding of financial markets, risk management, and portfolio diversification. For example, the dynamic correlation between inefficiencies across different markets can impact the effectiveness of portfolio diversification strategies. If the inefficiencies are correlated, it suggests that certain market events or factors are affecting multiple indices simultaneously. This means that diversifying across different indices might not provide as much risk reduction as expected, as downturns in one index could coincide with downturns in others. Moreover, understanding the interplay of inefficiencies and their correlations can improve risk management strategies. If inefficiencies are correlated across indices, it indicates that there is a higher likelihood of systemic risk, where a market-wide event can trigger simultaneous inefficiencies and negative impacts across multiple indices. Risk managers need to be aware of these correlations to properly assess potential losses and develop effective risk mitigation strategies. In the end, as shown by Granger causality tests, the presence of conditional correlations suggests the presence of inefficiency predictability. This allows traders to make more informed decisions.

Further research directions include the study of predictive capacity of inefficiency in other contexts, analyzing the dependence with the implied volatility of different market indices and investigating the presence of spillovers in inefficiency volatilty across different markets.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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