education

Department:
Education
REPUBLIC OF SOUTH AFRICA

# NATIONAL SENIOR CERTIFICATE

## GRADE 12

## MATHEMATICS P1

## NOVEMBER 2009(1)

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 10 pages, an information sheet and 1 diagram sheet.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. ONE diagram sheet for answering QUESTION 13.3 is attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and place them in the back of your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

**QUESTION 1**

1.1 Solve for $x$:

1.1.1 $x(x-1)=30$ (3)

1.1.2 $3x^2-5x+1=0$ (Correct to ONE decimal place) (4)

1.1.3 $15x-4<9x^2$ (4)

1.2 Solve simultaneously for $x$ and $y$ in the following set of equations:

$$x-y=3$$
$$x^2-xy-2y^2-7=0$$ (5)

1.3 Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}}-\sqrt{10^{2007}}}$$ (Show ALL calculations.) (3)

1.4 Simplify completely without the use of a calculator:

$$\left(1+\sqrt{2x^2}\right)^2-\sqrt{8x^2}$$ (3)

**[22]**

**QUESTION 2**

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; …
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; …

Write down the $n^{\text{th}}$ term (or the rule in terms of $n$) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)

**[10]**

**QUESTION 3**

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)
**[5]**

**QUESTION 4**

The following sequence of numbers forms a quadratic sequence:

$-3\,;-2\,;-3\,;-6\,;-11\,;\ldots$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the $35^{th}$ and $36^{th}$ terms of the quadratic sequence. (2)

4.3 Determine an expression for the $n^{th}$ term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)
**[11]**

**QUESTION 5**

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

| | First year | Second year | Third year | Fourth year |
|---|---|---|---|---|
| **Tree height (cm)** | 150 | 168 | 184 | $198\frac{2}{9}$ |
| **Growth (cm)** | | 18 | 16 | $14\frac{2}{9}$ |

5.1 Determine the increase in the height of the tree during the seventeenth year. (2)

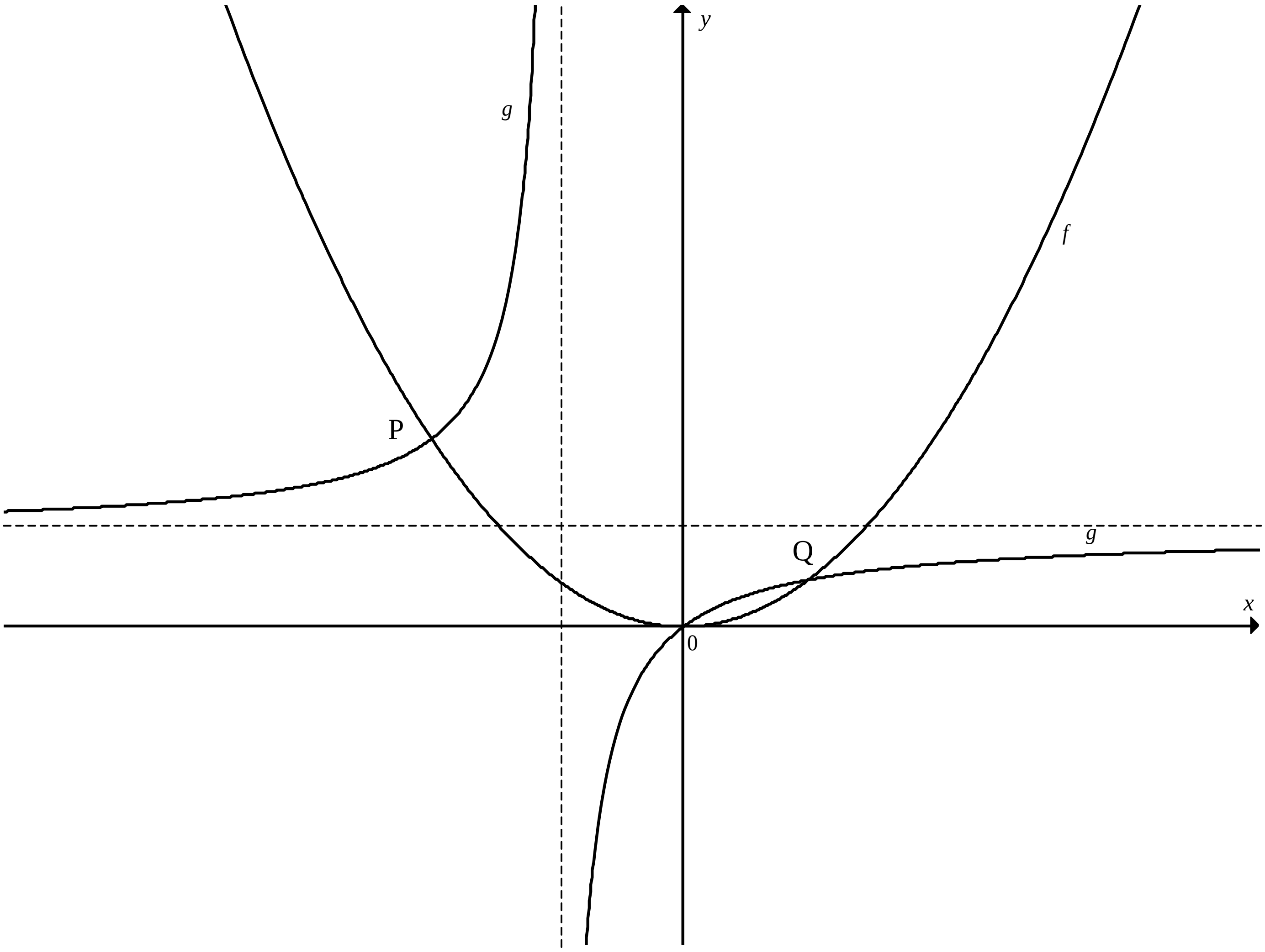
5.2 Calculate the height of the tree after 10 years. (3)

5.3 Show that the tree will never reach a height of more than 312 cm. (3)
**[8]**

## QUESTION 6

Sketched below are the graphs of $f(x)=\frac{1}{2}x^2$ and $g(x)=-\frac{1}{x+1}+1$.

P and Q are the points of intersection of $f$ and $g$.




6.1 Show that the coordinates of P and Q are P(– 2 ; 2) and Q(1 ; $\frac{1}{2}$) respectively. (6)

6.2 An axis of symmetry of the graph of $g$ is a straight line defined as $y = mx + c$, where $m > 0$. Write down the equation of this straight line in the form $y = h(x) = \ldots$ (2)

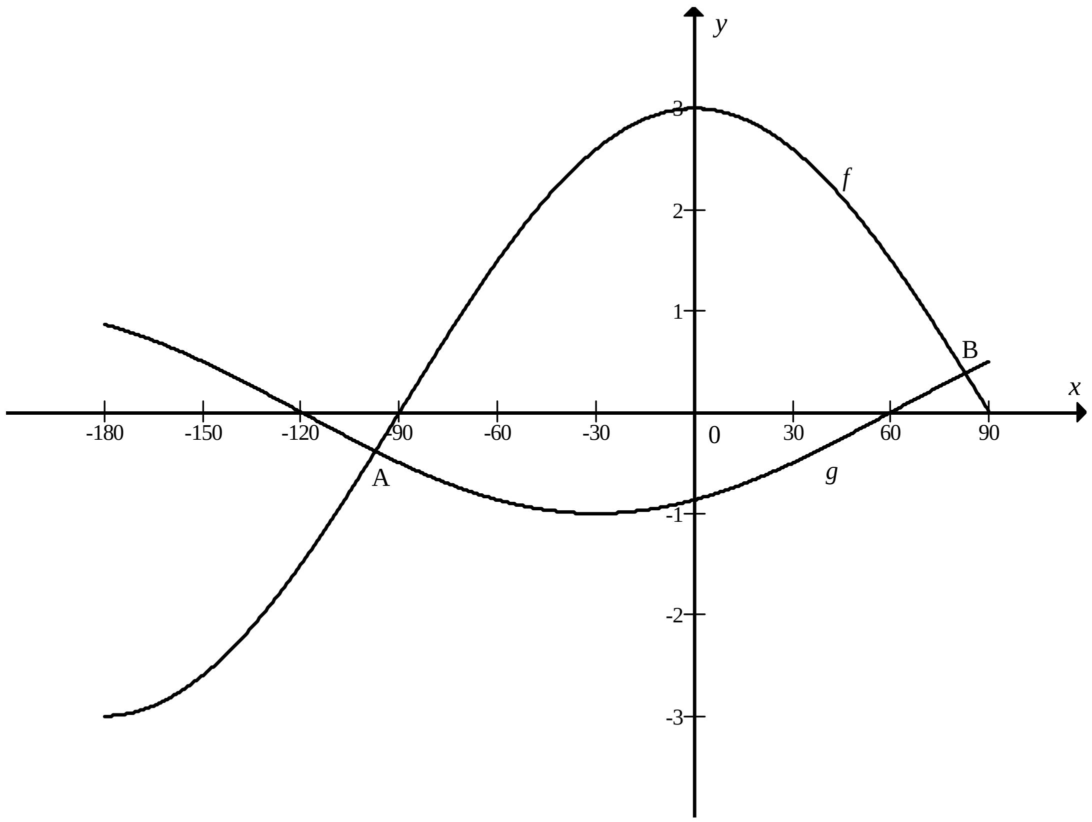
6.3 Determine the equation of $h^{-1}$ in the form $y = \ldots$ (2)

6.4 Show algebraically that $g(x)+g\left(\frac{1}{x}\right)=g(-x).g(x-1)$. ($x \neq 0$ or $x \neq 1$) (3)

**[13]**

## QUESTION 7

The graphs of $f(x) = 3\cos x$ and $g(x) = \sin(x - 60°)$ are sketched below for $x \in [-180° ; 90°]$.




7.1 Write down the range of $f$. (1)

7.2 If A(– 97,37° ; – 0,38), write down the coordinates of B. (3)

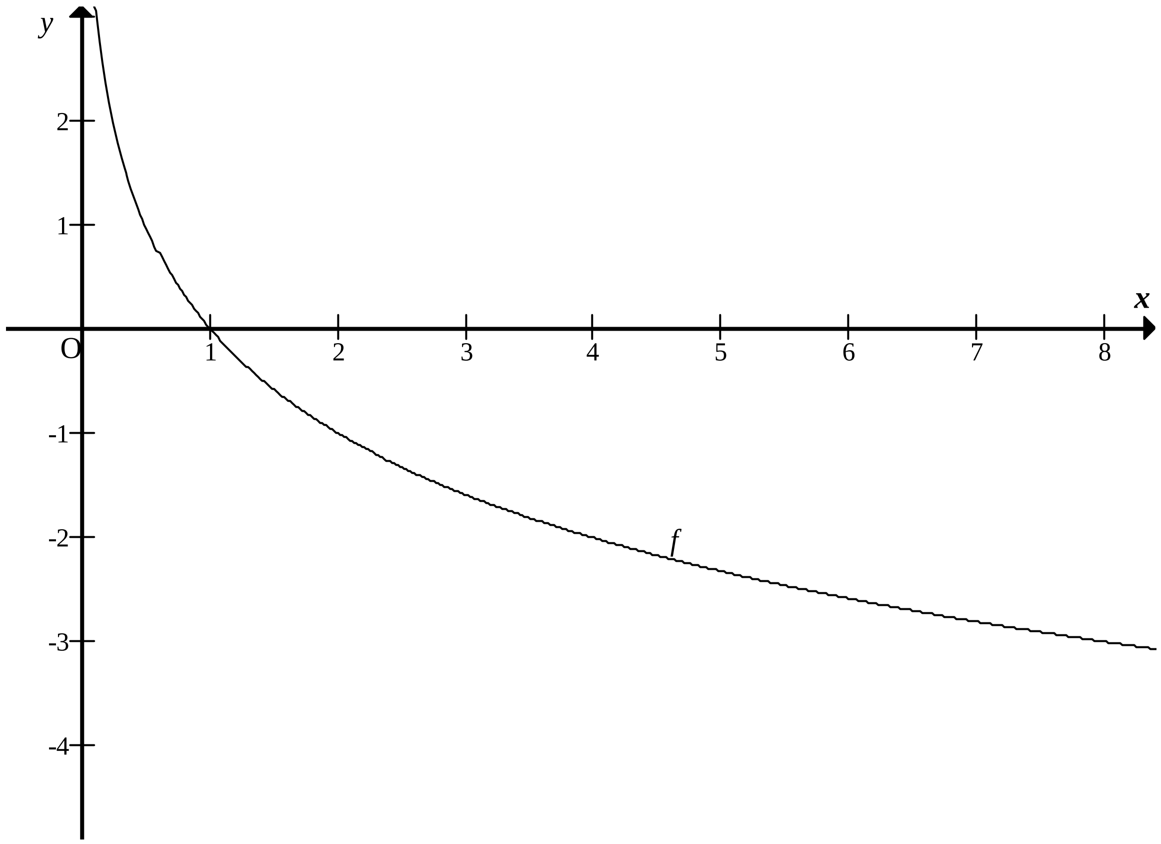
7.3 Write down the period of $g(3x)$. (2)

7.4 Write down a value of $x$ for which $g(x) - f(x)$ is a maximum. (2)

**[8]**

**QUESTION 8**

Sketched below is the graph of $f(x) = -\log_2 x$.




8.1 Write down the domain of $f$. (1)

8.2 Write down the equation of $f^{-1}$ in the form $y = \ldots$ (1)

8.3 Write down the equation of the asymptote of $f^{-1}$. (1)

8.4 Explain how, using the graph of $f$, you would sketch the graphs of:

8.4.1 $g(x) = \log_2 x$ (1)

8.4.2 $h(x) = 2^{-x} - 5$ (3)

8.5 Use the graph of $f$ to solve for $x$ where $\log_2 x < 3$. (3)

**[10]**

## QUESTION 9

9.1 A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000? (4)

9.2 A car that costs R130 000 is advertised in the following way: 'No deposit necessary and first payment due three months after date of purchase.' The interest rate quoted is 18% p.a. compounded monthly.

9.2.1 Calculate the amount owing two months after the purchase date, which is one month before the first monthly payment is due. (3)

9.2.2 Herschel bought this car on 1 March 2009 and made his first payment on 1 June 2009. Thereafter he made another 53 equal payments on the first day of each month.

(a) Calculate his monthly repayments. (3)

(b) Calculate the total of all Herschel's repayments. (1)

9.2.3 Hashim also bought a car for R130 000. He also took out a loan for R130 000, at an interest rate of 18% p.a. compounded monthly. He also made 54 equal payments. However, he started payments one month after the purchase of the car. Calculate the total of all Hashim's repayments. (4)

9.2.4 Calculate the difference between Herschel's and Hashim's total repayments. (1)

**[16]**

## QUESTION 10

10.1 Differentiate $f(x)$ from first principles if $f(x) = -2x^2 + 3$. (5)

10.2 Evaluate: $\frac{dy}{dx}$ if $y = x^2 - \frac{1}{2x^3}$ (2)

**[7]**

## QUESTION 11

Given: $f(x) = -x^3 + x^2 + 8x - 12$

11.1 Calculate the $x$-intercepts of the graph of $f$. (5)

11.2 Calculate the coordinates of the turning points of the graph of $f$. (5)

11.3 Sketch the graph of $f$, showing clearly all the intercepts with the axes and turning points. (3)

11.4 Write down the $x$-coordinate of the point of inflection of $f$. (2)

11.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)

**[17]**

## QUESTION 12

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after $t$ minutes, is given as $s(t) = 5t^3 - 65t^2 + 200t + 100$ metres. The journey lasts 8 minutes.

12.1 How high is the car above sea level when it starts its journey on the mountainous pass? (2)

12.2 Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass. (3)

12.3 Interpret your answer to QUESTION 12.2. (2)

12.4 How many minutes after the journey has started will the rate of change of height with respect to time be a minimum? (3)

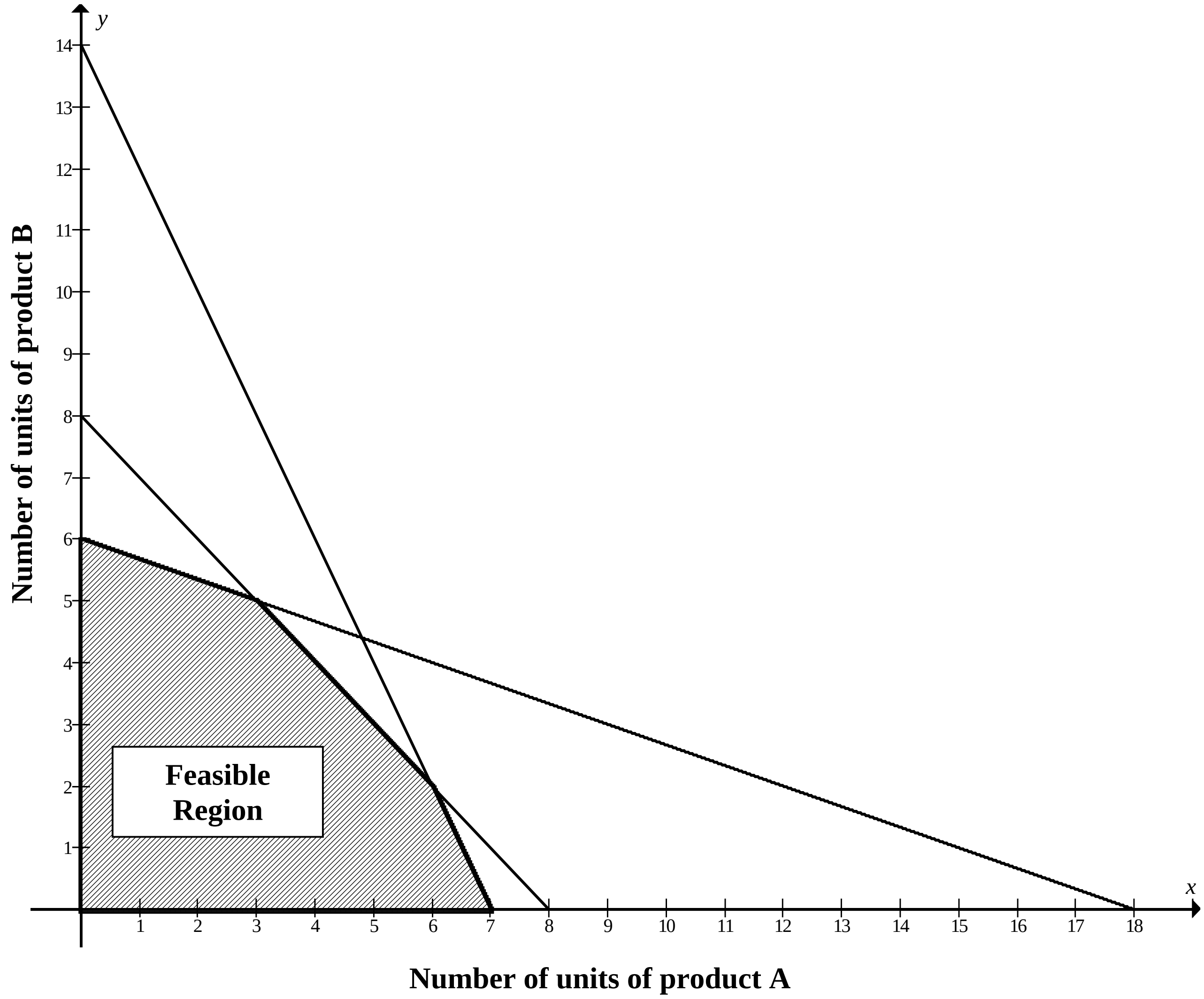
**[10]**

**QUESTION 13**

A steel manufacturer makes two kinds of products, product A and B, having parts that must be cut, assembled and finished. The manufacturer is aware that it can sell as many products as it can produce.

Let $x$ and $y$ be the number of units of product A and product B that are manufactured every day respectively.

The constraints that govern the manufacture of the products are represented below and the feasible region is shaded.




13.1 Write down the constraints in terms of $x$ and $y$ that represent the above information. (7)

13.2 If product A yields a profit of R30 per item and product B yields R40 per item, write down the equation indicating the daily profit in terms of $x$ and $y$. (2)

13.3 Determine the number of units of product A and product B that the manufacturer needs to produce in order to maximise his daily profit. A diagram is provided on DIAGRAM SHEET 1. (2)

13.4 The manufacturer would like the maximum profit to be at (6 ; 2) for the profit equation P = $mx$ + c. Determine the values of $m$ which will satisfy this condition. (2)

**[13]**

**TOTAL: 150**

# INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $\sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1}$ **;** $r \neq 1$ $\quad$ $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$ **;** $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

*In $\Delta ABC$:*

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

**CENTRE NUMBER:**

**EXAMINATION NUMBER:**

**DIAGRAM SHEET 1**

**QUESTION 13.3**