education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

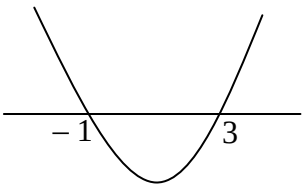
FEBRUARY/MARCH 2009

MEMORANDUM

MARKS: 150

This memorandum consists of 15 pages.

QUESTION 1

1.1.1	$3x+\frac{1}{x}=4$ $3x^2-4x+1=0$ $(3x-1)(x-1)=0$ $x=\frac{1}{3}$ or $x=1$	✓ standard form ✓ factors ✓✓ answers (4)
1.1.2	$5x(x-3)=2$ $5x^2-15x-2=0$ $x=\frac{-(-15)\pm\sqrt{(-15)^2-4(5)(-2)}}{2(5)}$ $x=\frac{15\pm\sqrt{265}}{10}$ $x = 3{,}13$ or $x = -0{,}13$	✓ standard form ✓ substitution into formula ✓ 265 ✓✓ answers (5)
1.1.3	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ + 0 − 0 + (−1, 3) OR  $x<-1$ or $x>3$	✓ standard form ✓ critical values ✓✓ answers (4)
1.2	$x-3y=1$ $x=1+3y$ $(1+3y)^2-2(1+3y)y+9y^2=17$ $1+6y+9y^2-2y-6y^2+9y^2-17=0$ $12y^2+4y-16=0$ $3y^2+y-4=0$ $(3y+4)(y-1)=0$ $y=-\frac{4}{3}$ or $y=1$ $x=-3$ or $x=4$	✓ $x=1+3y$ ✓ substitution ✓ simplification ✓ factors ✓ answers ✓ ✓answers (7

	OR $x-3y=1$ $y=\frac{x-1}{3}$ $x^2-2x\left(\frac{x-1}{3}\right)+9\left(\frac{x-1}{3}\right)^2=17$ $x^2-\frac{2x^2-2x}{3}+9\left(\frac{x^2-2x+1}{9}\right)=17$ $x^2-\frac{2x^2-2x}{3}+x^2-2x+1=17$ $3x^2-2x^2+2x+3x^2-6x+3=51$ $4x^2-4x-48=0$ $x^2-x-12=0$ $(x+3)(x-4)=0$ $x=-3$ or $x=4$ $y=-\frac{4}{3}$ or $y=1$	
1.3	let 1234567893 = n Then $1234567893\times1234567894-1234567895\times1234567892$ $=n(n+1)-(n+2)(n-1)$ $=n^2+n-n^2-n+2$ $=2$ **OR** Full marks for $(3\times4)-(5\times2)=2$	✓ method ✓ simplification ✓ answer (3) **[23]**

QUESTION 2

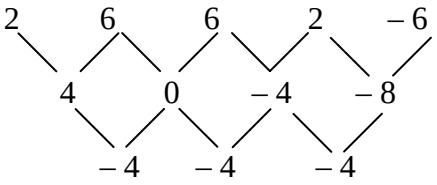
2.1.1	$S_2 = \frac{1}{1\times 2} + \frac{1}{2\times 3}$ $= \frac{1}{2} + \frac{1}{6}$ $= \frac{2}{3}$	✓ answer (1)
2.1.2	$S_3 = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4}$ $= \frac{2}{3} + \frac{1}{12}$ $= \frac{9}{12}$ $= \frac{3}{4}$	✓ answer (1)
2.1.3	$S_4 = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5}$ $= \frac{3}{4} + \frac{1}{20}$ $= \frac{16}{20}$ $= \frac{4}{5}$	✓ answer (1)
2.2	If n represents the number of terms then the sum will be n divided by $n + 1$. **OR** $S_n = \frac{n}{n+1}$	✓ ✓ answer (2)
2.3	$S_n = \frac{2008}{2009}$	✓ answer (1) **[6]**

QUESTION 3

3.1	$(2p-3)-(1-p)=(p+5)-(2p-3)$ $2p-3-1+p=p+5-2p+3$ $3p-4=-p+8$ $4p=12$ $p=3$	✓ equating differences ✓ simplification ✓ answer (3)
3.2.1	First term = $1-p=1-3=-2$	✓ -2 (1)
3.2.2	-2 ; 3 ; 8 Common difference = 5 **OR** $3p-4=3(3)-4=5$	✓ answer (1)
3.3	After the first term -2, all the other terms end in either a 3 or an 8. Perfect squares never end in a 3 or an 8.	✓✓ explanation (2) **[7]**

QUESTION 4

4.1	6 ; 6 ; 2 ; – 6 ; – 18 ;… First difference: 0, – 4, – 8, – 12, … Therefore the next term is: – 18 – 16 = – 34	✓ – 34 (1)
4.2	6 6 2 – 6 0 – 4 – 8 – 4 – 4 $2a=-4$ $a=-2$ $T_n=-2n^2+bn+c$ $6=-2+b+c$ $8=b+c$ ...(*i*) $6=-8+2b+c$ $14=2b+c$ …(ii) (ii) – (i): $6=b$ $\therefore c=2$ $T_n=-2n^2+6n+2$ **OR**	✓ $a=-2$ ✓ solving simultaneously ✓ $b=6$ ✓ $c=2$ ✓ general term (5)



$2a = -4$

$a = -2$

$T_0 = c = 2$

$T_1 = -2 + b + 2 = 6$

$b = 6$

$T_n = -2n^2 + 6n + 2$

✓ $a = -2$

✓ $c = 2$
✓ substitution
✓ $b = 6$
✓ general term

(5)

OR

✓obtaining differences

$T_2 - T_1 = 0$

$T_3 - T_2 = -4$

$T_4 - T_3 = -8$

$\vdots$

$T_n - T_{n-1} = -4(n-2) = 8 - 4n$

✓ adding both sides

Adding both sides

$T_n - 6 = -4[1 + 2 + 3 + ...(n-2)]$

$T_n = -4\left[\frac{(n-2)(n-1)}{2}\right] + 6$

✓ substituting $T_1 = 6$
✓ expression for sum of terms

✓simplification

(5)

$T_n = -2n^2 + 6n + 2$

4.3

$-6838 = -2n^2 + 6n + 2$

$-2n^2 + 6n + 6840 = 0$

$n^2 - 3n - 3420 = 0$

✓ standard form

$n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3420)}}{2}$

✓ substitution

$n = \frac{3 \pm 117}{2}$

n = 60 or n = - 57
not possible

✓ values of n

$\therefore T_{60} = -6838$

✓ answer

(4)
[10]

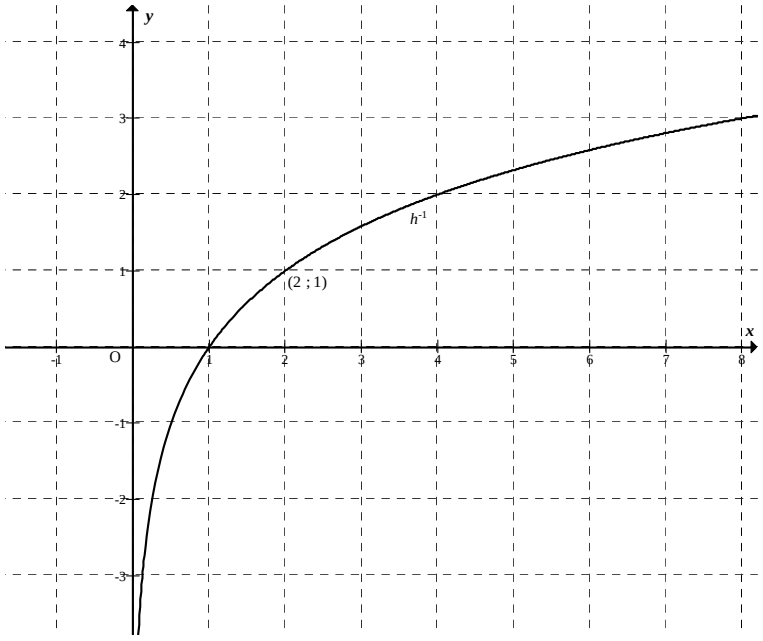
QUESTION 5

5.1	Area of unshaded square = 1- $\frac{1}{16}$ = $\frac{15}{16}$	✓1 ✓ $\frac{1}{16}$ ✓ answer (2)
5.2	Sum of the unshaded areas of the first seven squares $=(1-1)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{4^2}\right)+\cdots+\left(1-\frac{1}{4^6}\right)$ $=7-\left(1+\frac{1}{4}+\frac{1}{4^2}+\cdots+\frac{1}{4^6}\right)$ $=7-\left(\frac{1\left(1-\left(\frac{1}{4}\right)^7\right)}{1-\frac{1}{4}}\right)$ $=7-1{,}333251953...$ $=5{,}666748047...$ $=5{,}67$ **OR** Sum of the unshaded areas of the first seven squares $=(1-1)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{4^2}\right)+\cdots+\left(1-\frac{1}{4^6}\right)$ $=6-\left(\frac{1}{4}+\frac{1}{4^2}+\cdots+\frac{1}{4^6}\right)$ $=6-\frac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^6\right)}{1-\frac{1}{4}}$ $=6-0{,}332519531...$ $=5{,}666748047...$ $=5{,}67$	✓expression for sum of unshaded areas ✓ simplification ✓ answer (5) **[7]**

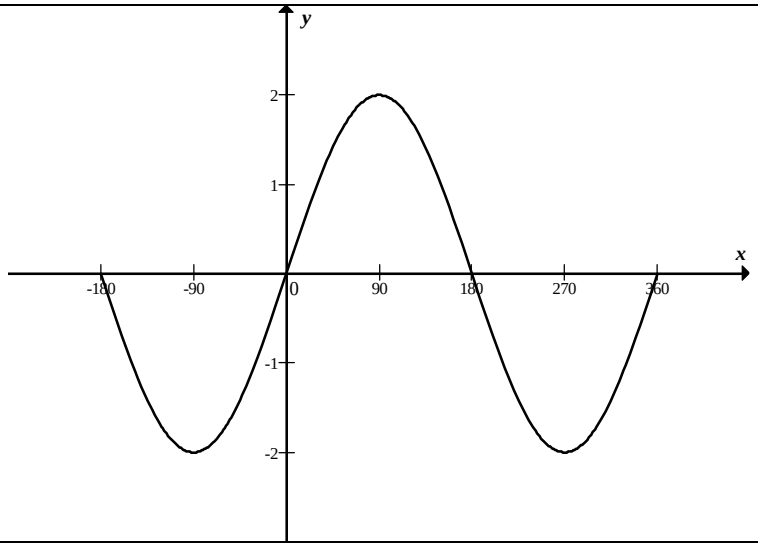
QUESTION 6

6.1	$y = \frac{a}{x-1} + 2$ $0 = \frac{a}{0-1} + 2$ $0 = -a + 2$ $a = 2$ $y = \frac{2}{x-1} + 2$ **OR** $(x-1)(y-2) = k$ But g passes through the origin $\therefore (-1)(-2) = k = 2$ $\therefore (x-1)(y-2) = 2$	✓ $b = 1$ ✓ $c = 2$ ✓ substitution of (0 ; 0) ✓ $a = 2$ (4) ✓ equation ✓ through origin ✓ substitution ✓ equation (4)
6.2	$g(x) = (x-1)^2 + q$ $0 = (2{,}5-1)^2 + q$ $q = -\frac{9}{4}$ Turning point $\left(1 ; -\frac{9}{4}\right)$	✓ $p = 1$ ✓ substitution of (2,5 ; 0) ✓ $q = -\frac{9}{4}$ ✓ answer (4)
6.3	$y = 2$ $x = 2$	✓ answer ✓ answer (2)
6.4	$h(x) = -(x-1)^2 + \frac{9}{4}$	✓ answer (1) **[11]**

QUESTION 7

7.1	Any base raised to the power 0 is 1 which means the *y*-intercept of the graph $h(x) = a^x$ will be (0 ; 1) therefore Q(0 ; 1) **OR** $h(0) = a^0 = 1$ $\therefore$ Q(0 ; 1)	✓ *y*-intercept ✓ any base raised to power 0 is 1 (2)
7.2	$a^{-1} = \frac{1}{2}$ $\frac{1}{a} = \frac{1}{2}$ $a = 2$	✓ substitution ✓ answer (2)
7.3	$2^y = x$ $y = \log_2 x$	✓ interchanging *x* and *y* ✓ answer (2)
7.4		✓ point (0,5 ; – 1) or any other valid point ✓ point (1 ; 0) ✓ shape (3)
7.5	$x > 0{,}5$	✓ reading off from graph ✓ answer (2)
7.6	$\therefore 2 + x\log 3 = x\log 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$ **OR** $\left(\frac{2}{3}\right)^x = 100$ $\therefore x \log\left(\frac{2}{3}\right) = 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$	✓ equating ✓ logs both sides ✓ answer (3) **[14]**

QUESTION 8

8.1		✓ shape ✓ amplitude (2)
8.2	$y \in [-4\,;\,4]$	✓✓ answer (2)
8.3	720°	✓✓ answer (2)
8.4	Shift f 90° to the left to get $2\cos x$. $\therefore \theta = 90° + n.360°$	✓✓ answer Accept 90° (2) **[8]**

QUESTION 9

9.1	$2000\left(1+\frac{i}{12}\right)^{18} = 2860$ $\left(1+\frac{i}{12}\right)^{18} = 1{,}43$ $1+\frac{i}{12} = 1{,}020069541...$ $\frac{i}{12} = 0{,}020069541...$ $i = 0{,}24083...$ $i = 24{,}08\%$	✓ substitution ✓ $\left(1+\frac{i}{12}\right)^{18} = 1{,}43$ ✓ $i = 0{,}24083\ldots$ ✓ answer (4)
9.2	$F_v = \frac{100\left[\left(1+\frac{0{,}08}{12}\right)^{12}-1\right]}{\frac{0{,}08}{12}}$ $= R1\,244{,}99$ The accumulated amount is less than R1 300 required to buy the bike. Farouk will not be able to buy the bike on 1 January 2009.	✓ formula ✓ substitution ✓✓ answer ✓ conclusion (5) **[9]**

QUESTION 10

10.1	Loan = $125000-\frac{15}{100}\times 125000$ Loan = R 106 250 OR Loan = 0,85 × 125 000 Loan = R 106 250	✓ answer (1)
10.2	$106250 = x\left[\frac{1-\left(1+\frac{0{,}125}{12}\right)^{-6\times 12}}{\frac{0{,}125}{12}}\right]$ $1106{,}770833 = x\left(1-\left(1+\frac{0{,}125}{12}\right)^{-6\times 12}\right)$ $x =$ R 2104,94	✓ substitution ✓ – 72 ✓ $\left(\frac{0{,}125}{12}\right)$ ✓ simplification ✓ answer (5) **[6]**

QUESTION 11

11.1	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{(x+h)^2-2(x+h)-x^2+2x}{h}$ $= \lim_{h\to 0}\frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}$ $= \lim_{h\to 0}\frac{2xh+h^2-2h}{h}$ $= \lim_{h\to 0}\frac{h(2x+h-2)}{h}$ $= \lim_{h\to 0}(2x-2+h)$ $= 2x-2$	✓ method ✓ substitution ✓ simplification ✓ factorising ✓ answer (5)
11.2.1	$D_x[(x^3-3)^2]$ $= D_x\left[x^6-6x^3+9\right]$ $= 6x^5-18x^2$	✓ simplification ✓✓ answer (3)
11.2.2	$y = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ $y = 4x^{-\frac{1}{2}} - \frac{1}{9}x^3$ $\frac{dy}{dx} = -2x^{-\frac{3}{2}} - \frac{x^2}{3}$	✓ power form ✓✓ answer (3) **[11]**

QUESTION 12

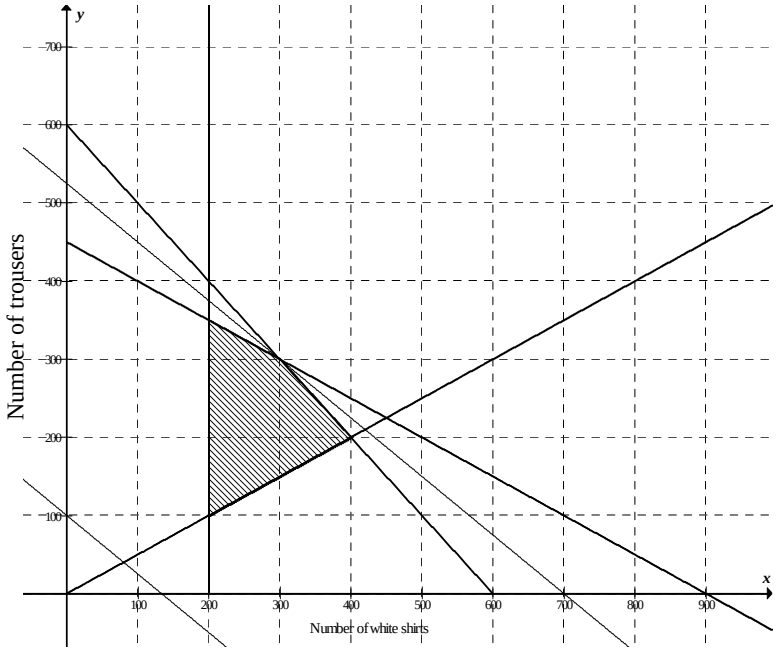
12.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \ldots$ (i) $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \ldots$ (ii) (ii) + (i): $6a = 9$ $a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$ **OR** $h(-1) = -(-1)^3 + a(-1)^2 + b(-1) = \frac{-7}{2}$ $\therefore a - b = \frac{-9}{2}$ $2a - 2b = -9 \quad \cdots$(i) $h(2) = -(2)^3 + a(2)^2 + b(2) = 10$ $4a + 2b = 18 \quad \cdots$(ii) (i) + (ii): $6a = 9$ $a = \frac{3}{2}$ $\left(\frac{3}{2}\right) - b = \frac{-9}{2}$ $b = 6$	✓ $h'(x)$ ✓ substitution of $x = -1$ ✓ $h'(x) = 0$ ✓ simplification ✓ substitution ✓ solving simultaneously (6) ✓ substitution of $x = -1$ ✓ $h(-1) = \frac{-7}{2}$ ✓ simplification ✓ substitution of $x = 2$ and $h(2) = 10$ ✓ simplification ✓ solving simultaneously (6)
12.2	Average Gradient $= \frac{10 - (-3{,}5)}{2 - (-1)}$ $= \frac{13{,}5}{3}$ $= \frac{9}{2}$	✓ substitution ✓ answer (2)

12.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\ ;\ 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	✓ $h'(x)$ ✓ substitution ✓ gradient ✓ point ✓ answer (5)
12.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ **OR** $x = \frac{-1+2}{2}$ $x = \frac{1}{2}$	✓ second derivative ✓ = 0 ✓ answer (3)
12.5	$p > 3{,}5$ or $p < -10$	✓✓ answer (2) **[19]**

QUESTION 13

13.1	$AB^2 = (a^2 - 0)^2 + (a - 3)^2$ $AB^2 = a^4 + a^2 - 6a + 9$	✓substitution ✓ simplification (2)
13.2	$\frac{d}{da}AB^2 = 4a^3 + 2a - 6$ $0 = 4a^3 + 2a - 6$ $0 = 2a^3 + a - 3$ $0 = (a-1)(2a^2 + 2a + 3)$ $a = 1$ There is no real solution for $2a^2 + 2a + 3 = 0$	✓ $\frac{d}{da}AB^2$ ✓ $\frac{d}{da}AB^2 = 0$ ✓ simplification ✓ factorisation ✓ answer (5) **[7]**

QUESTION 14

14.1	$x \geq 200$ $x + y \leq 600$ $50x + 100y \leq 45000$	✓ answer ✓ answer ✓ answer (3)
14.2 and 14.3		✓ 600 ✓450 ✓ 200 ✓ 600 ✓ 900 (5) ✓ feasible region (1)
14.4	P = 30x + 40y	✓✓ answer (2)
14.5	Maximum at (300 ; 300)	✓ search line ✓ answer (2) **[13]**

TOTAL: 150