education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2009(1)

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, an information sheet and 4 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. FOUR diagram sheets for answering QUESTION 1.3, QUESTION 2.2, QUESTION 3.3, QUESTION 5.7, QUESTION 6.1.3, QUESTION 11 and QUESTION 12.1 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and place them in the back of your ANSWER BOOK.

6. Diagrams are NOT necessarily drawn to scale.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

The data below shows the total monthly rainfall (in millimetres) at Cape Town International Airport for the year 2002.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
60,9	14,9	9,3	28,0	71,9	76,4	98,2	65,7	26,1	32,5	23,6	15,0

[Source: www.1stweather.com]

1.1 Determine the mean monthly rainfall for 2002. (2)

1.2 Write down the five-number summary for the data. (5)

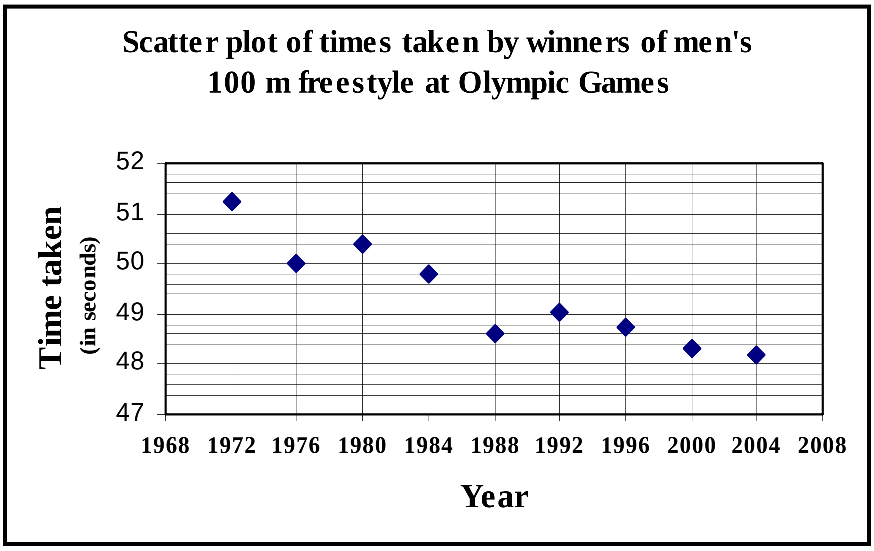
1.3 Draw a box and whisker diagram for the data on DIAGRAM SHEET 1. (3)

1.4 By making reference to the box and whisker diagram, comment on the spread of the rainfall for the year. (2)

1.5 Calculate the standard deviation for the data. (3)
[15]

QUESTION 2

The scatter plot below represents the times taken by the winners of the men's 100 m freestyle swimming event at the Olympic Games from 1972 to 2004. The data was obtained from www.databaseOlympics.com.



2.1 Indicate whether a linear, quadratic or exponential function best fits the data. (1)

2.2 Draw a line of best fit for the data on the graph provided on DIAGRAM SHEET 1. (2)

2.3 Describe the trend that is observed in these times. (1)

2.4 Give ONE reason for this trend. (1)

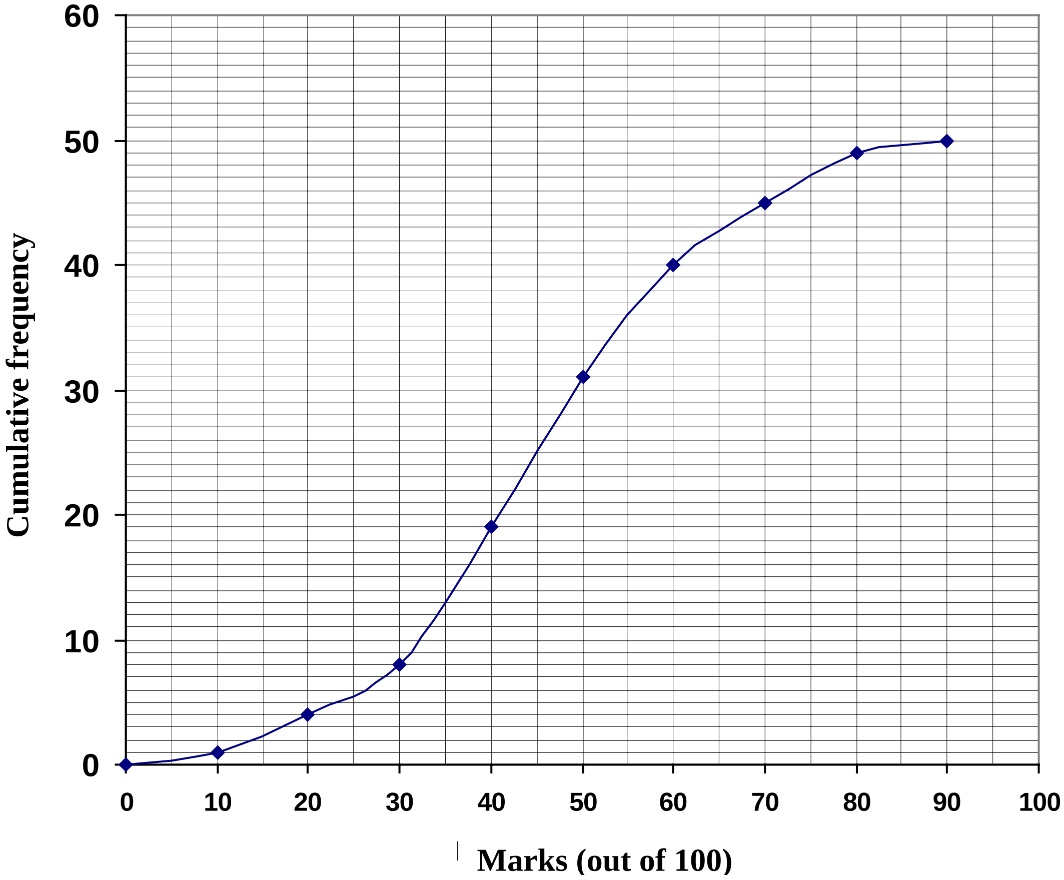
2.5 What can be said about the efforts of the winners in the years 1976 and 1988? (2)

2.6 Use your line of best fit to predict the winning time for 2008. (1)
[8]

QUESTION 3

The ogive (cumulative frequency graph) shows the performance of students who took a test in basic programming skills. The test had a maximum of 100 marks.

Performance in Computer Programming test



3.1 How many students took the test? (1)

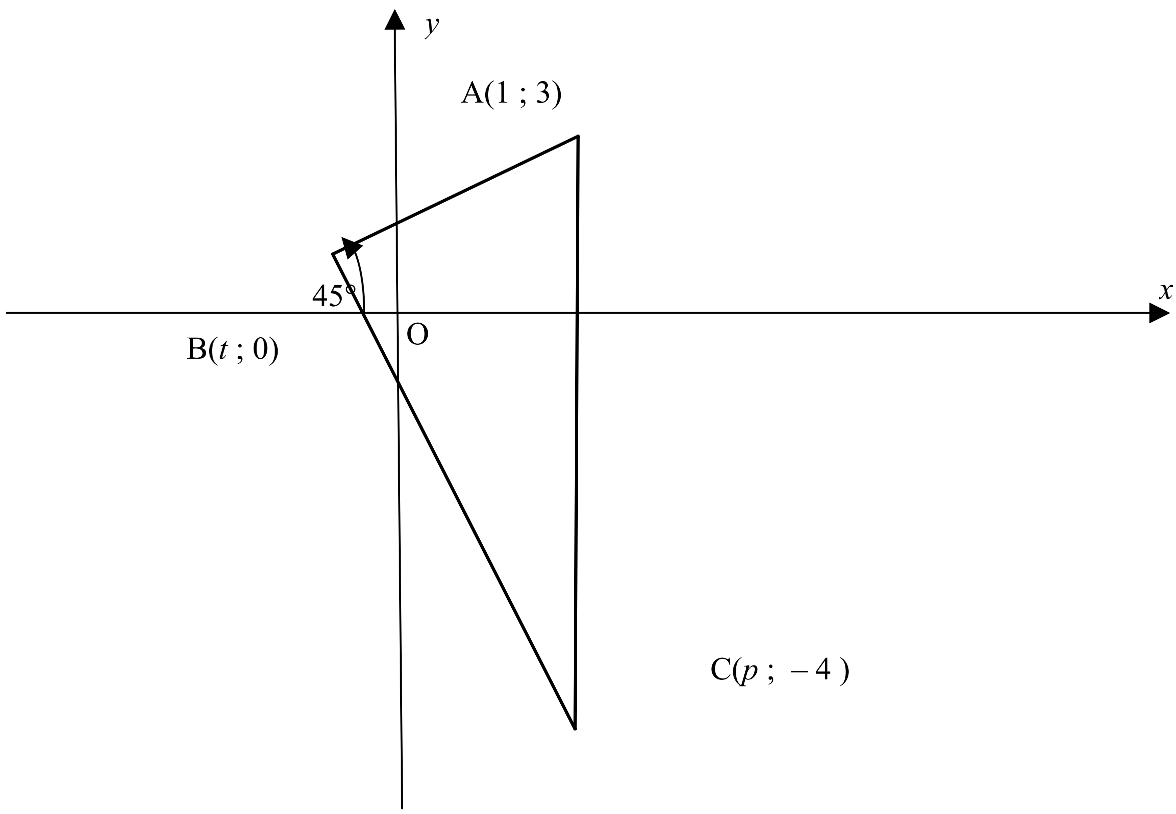
3.2 Only the top 25% of the students are allowed to do an advanced course in programming. Determine the cut-off mark to determine the top 25%. (1)

3.3 Construct a frequency table for the information given in the ogive on DIAGRAM SHEET 2. Complete the table with the information. (3)

[5]

QUESTION 4

ABC is a triangle with vertices A(1 ; 3), B(t ; 0) and C(p ; −4), with $p > 0$, in a Cartesian plane.
AB makes an angle of 45° with the positive x-axis. AC = $\sqrt{50}$.



4.1 Determine the gradient of AB. (2)

4.2 Calculate the value of t. (2)

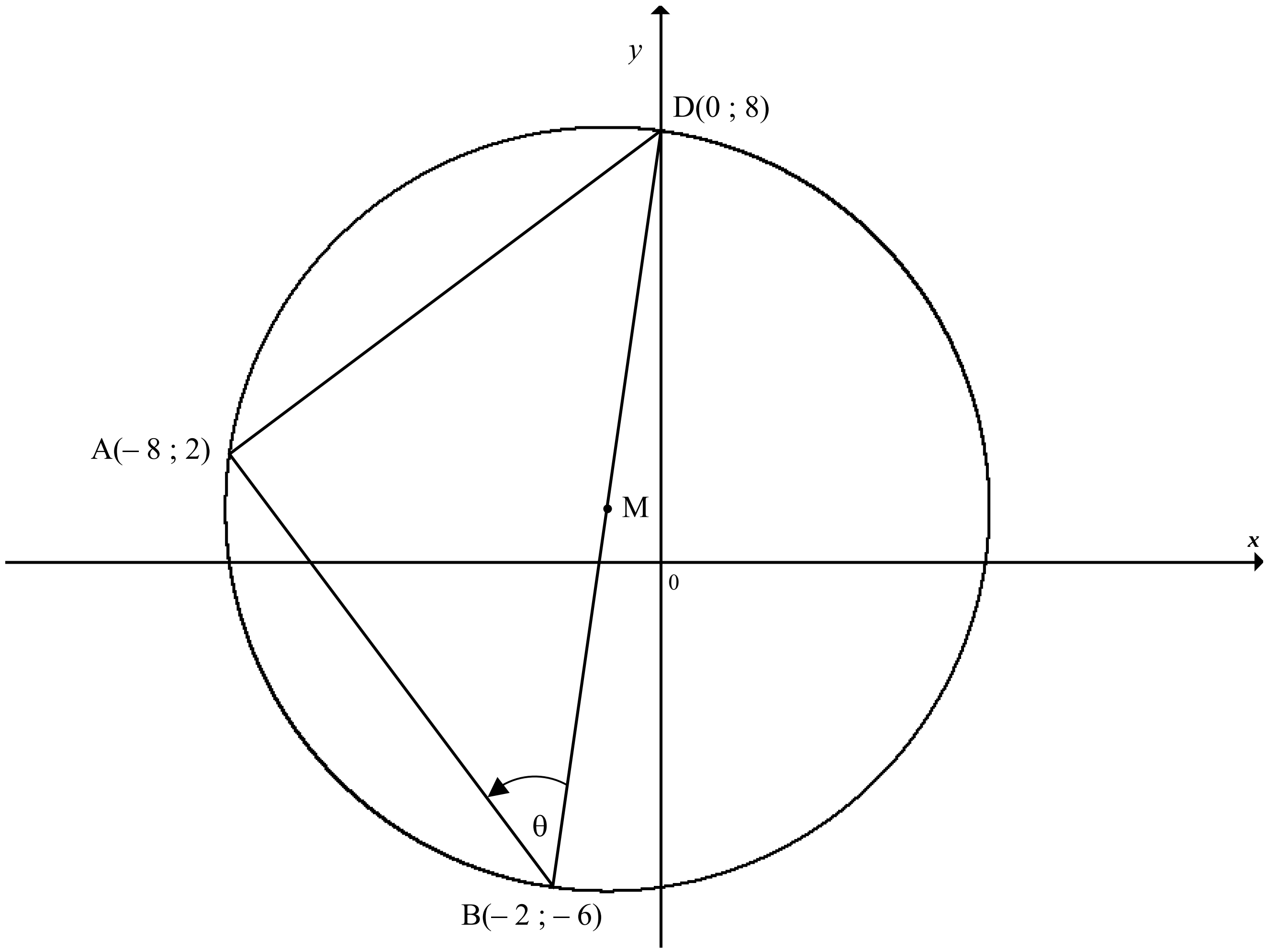
4.3 Calculate p, the x-coordinate of point C. (4)

4.4 Hence, determine the midpoint of BC. (2)

4.5 Determine the equation of the line parallel to AB, passing through point C. (3)
[13]

QUESTION 5

A(−8 ; 2), B(−2 ; −6) and D(0 ; 8) are the vertices of a triangle that lies on the circumference of a circle with diameter BD and centre M, as shown in the figure below.



5.1 Calculate the coordinates of M. (2)

5.2 Show that (–8 ; 2) lies on the line $y = 7x + 58$. (1)

5.3 What is the relationship between the line $y = 7x + 58$ and the circle centred at M? Motivate your answer. (5)

5.4 Calculate the lengths of AD and AB. (4)

5.5 Prove $D\hat{A}B = 90°$. (3)

5.6 Write down the size of angle θ. (1)

5.7 A circle, centred at a point Z inside ΔABD, is drawn to touch sides AB, BD and DA at N, M and T respectively. Given that BMZN is a kite, calculate the radius of this circle. A diagram is provided on DIAGRAM SHEET 2. (6)

[22]

QUESTION 6

6.1 ABC is a triangle that has an area of 5 square units. $\Delta A^{/}B^{/}C^{/}$ is an enlargement of ΔABC through the origin by a scale of 2.

6.1.1 Determine the area of $\Delta A^{/}B^{/}C^{/}$. (2)

6.1.2 Write down the general rule for the transformation from ΔABC to $\Delta A^{/}B^{/}C^{/}$. (2)

6.1.3 The vertices of ΔABC are A(–1 ; 4), B(–1 ; 2) and C(4 ; 4). Use the grid provided on DIAGRAM SHEET 3 to draw $\Delta A^{/}B^{/}C^{/}$. (3)

6.1.4 Comment on the rigidity of the transformation from ΔABC to $\Delta A^{/}B^{/}C^{/}$. (2)

6.2 A quadrilateral EFGH is transformed to its image $E^{///}F^{///}G^{///}H^{///}$ in the following way:

- First, reflect EFGH about the line $y = x$.
- Then, rotate this image through 90° in a clockwise direction about the origin.
- The second image has a translation of 2 units to the left and 3 units down to obtain $E^{///}F^{///}G^{///}H^{///}$.

Write down the general rule of the transformation of EFGH to $E^{///}F^{///}G^{///}H^{///}$. (6)
[15]

QUESTION 7

$A^{/}(-1-\sqrt{2}\ ;\ 1-\sqrt{2})$ is the image of point A(p ; q), after point A has been rotated about the origin in an anti-clockwise direction, through an angle of 135°.

7.1 T(x ; y) is rotated about the origin through an angle of θ in an anti-clockwise direction. Write down a formula to determine the coordinates of $T^{/}$, the image of T. (2)

7.2 Write down the coordinates of $A^{/}$ in terms of p, q and 135°. (2)

7.3 Hence, or otherwise, calculate p and q. (Leave your answer in surd form.) (7)
[11]

QUESTION 8

Given: $\sin\alpha = \frac{8}{17}$ where $90° \leq \alpha \leq 270°$

With the aid of a sketch and without the use of a calculator, calculate:

8.1 $\tan\alpha$ (3)

8.2 $\sin(90° + \alpha)$ (2)

8.3 $\cos 2\alpha$ (3)

[8]

QUESTION 9

9.1 Simplify completely:

$\sin(90° - x)\cos(180° - x) + \tan x.\cos(-x)\sin(180° + x)$ (7)

9.2 Prove, without the use of a calculator, that $\dfrac{\sin 190° \cos 225° \tan 390°}{\cos 100° \sin 135°} = -\dfrac{1}{\sqrt{3}}$ (7)

9.3 Determine the general solution of $\sin x + 2\cos^2 x = 1$. (7)

[21]

QUESTION 10

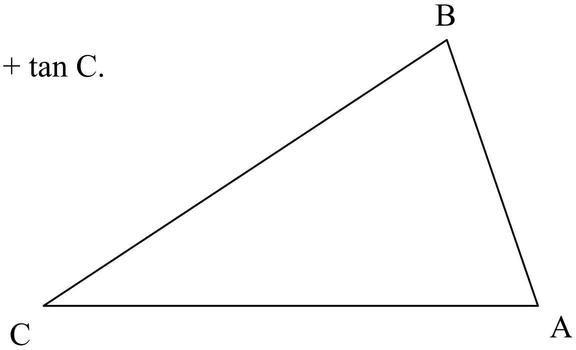
10.1 Using the expansions for sin(A + B) and cos(A + B), prove the identity of:

$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$ (3)

10.2 If $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)}$, prove in any ΔABC that

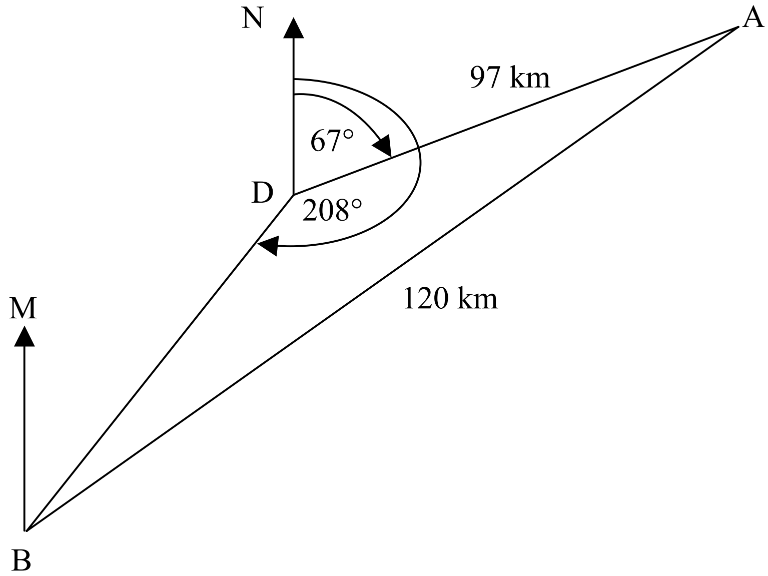
tan A.tan B.tan C = tan A + tan B + tan C. (4)

[7]



QUESTION 11

11.1 Two ships, A and B, are 120 km apart. Ship A is at a bearing of 67° from D and 97 km away from D. DN points due north. Ship B is at a bearing of 208° from D. A diagram is provided on DIAGRAM SHEET 3.

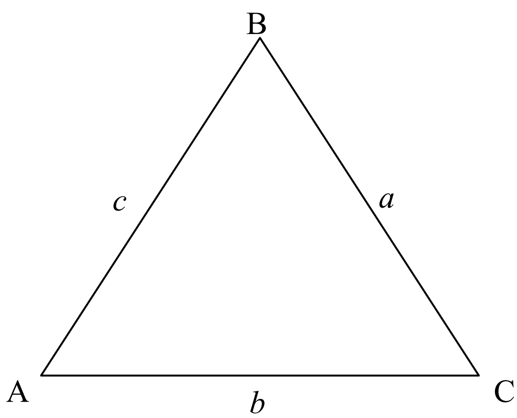


11.1.1 Determine the bearing of Ship A from Ship B, that is $M\hat{B}A$, when BM || DN. (6)

11.1.2 If Ship B travels due north, and Ship A travels due south, then at some instant of time Ship A is due east of Ship B.

Calculate the distance between the two ships at that instant. (3)

11.2 Triangle ABC is isosceles with AB = BC.



Prove that $\cos B = 1 - \dfrac{b^2}{2a^2}$ (4)

[13]

QUESTION 12

Given: $g(x) = 2\cos(x - 30°)$

12.1 Sketch the graph of g for $x \in [-90°\,;270°]$ on DIAGRAM SHEET 4. (2)

12.2 Use the symbols A and B to plot the two points on the graph of g for which $\cos(x - 30°) = 0{,}5$ (2)

12.3 Calculate the x-coordinates of the points A and B. (3)

12.4 Write down the values of x, where $x \in [-90°\,;270°]$ and $g'(x) = 0$. (2)

12.5 Use the graph to solve for x, $x \in [-90°\,;270°]$, where $g(x) < 0$ (3)
[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

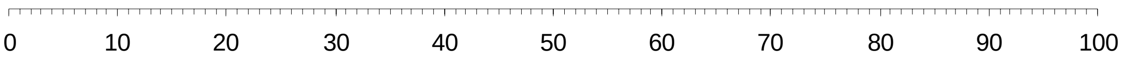
$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

CENTRE NUMBER:

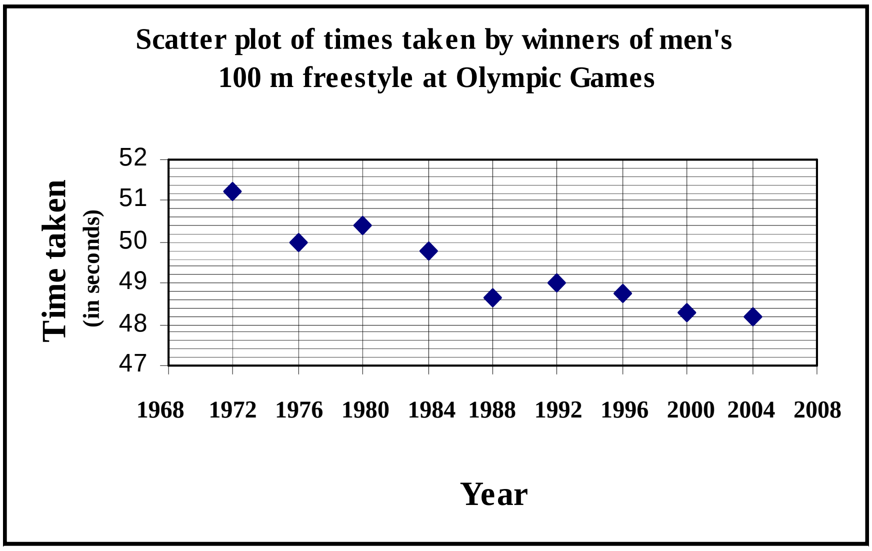
EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 1.3



QUESTION 2.2



CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 3.3

Marks (out of 100)	Frequency (f)

QUESTION 5.7

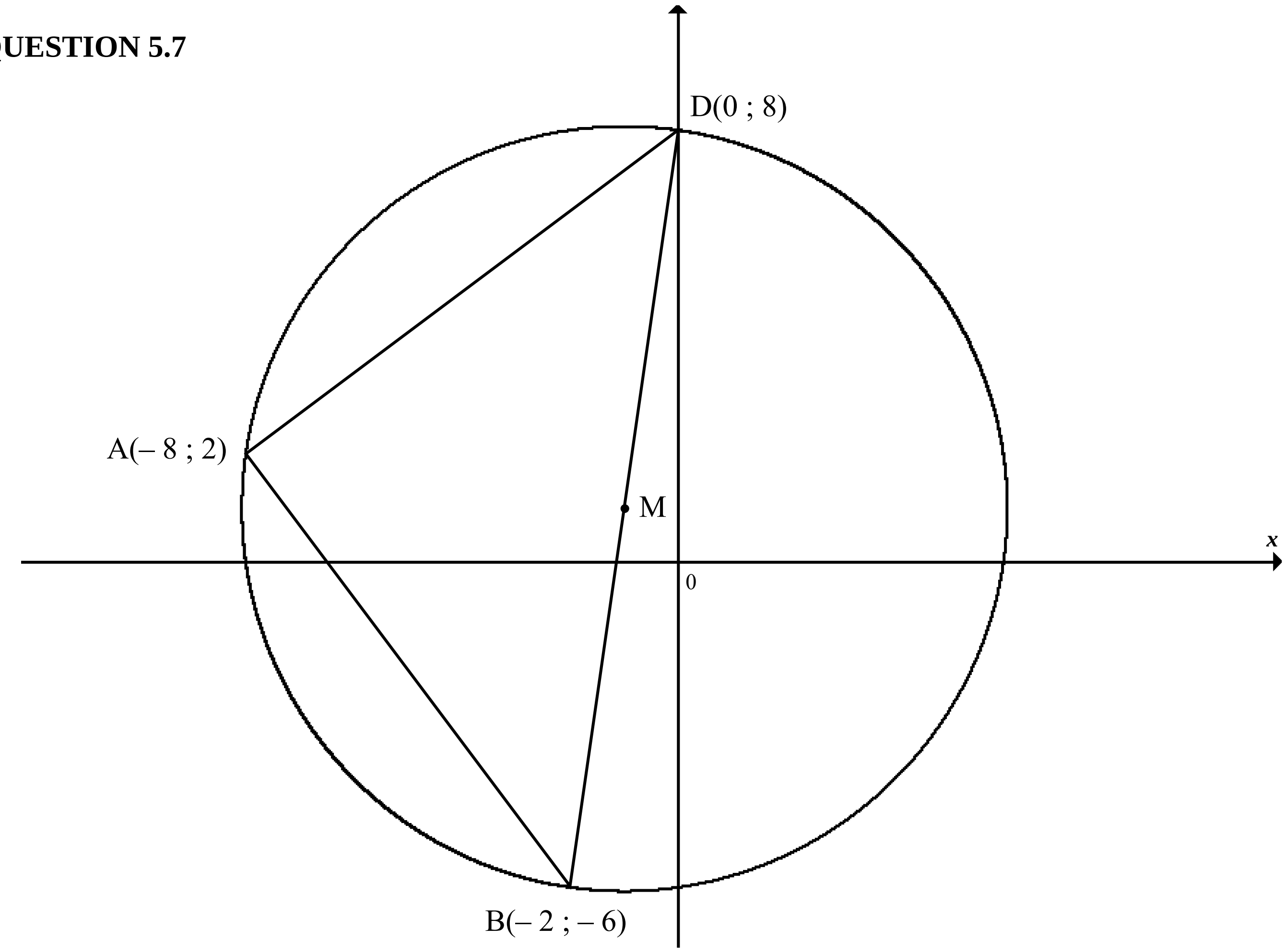


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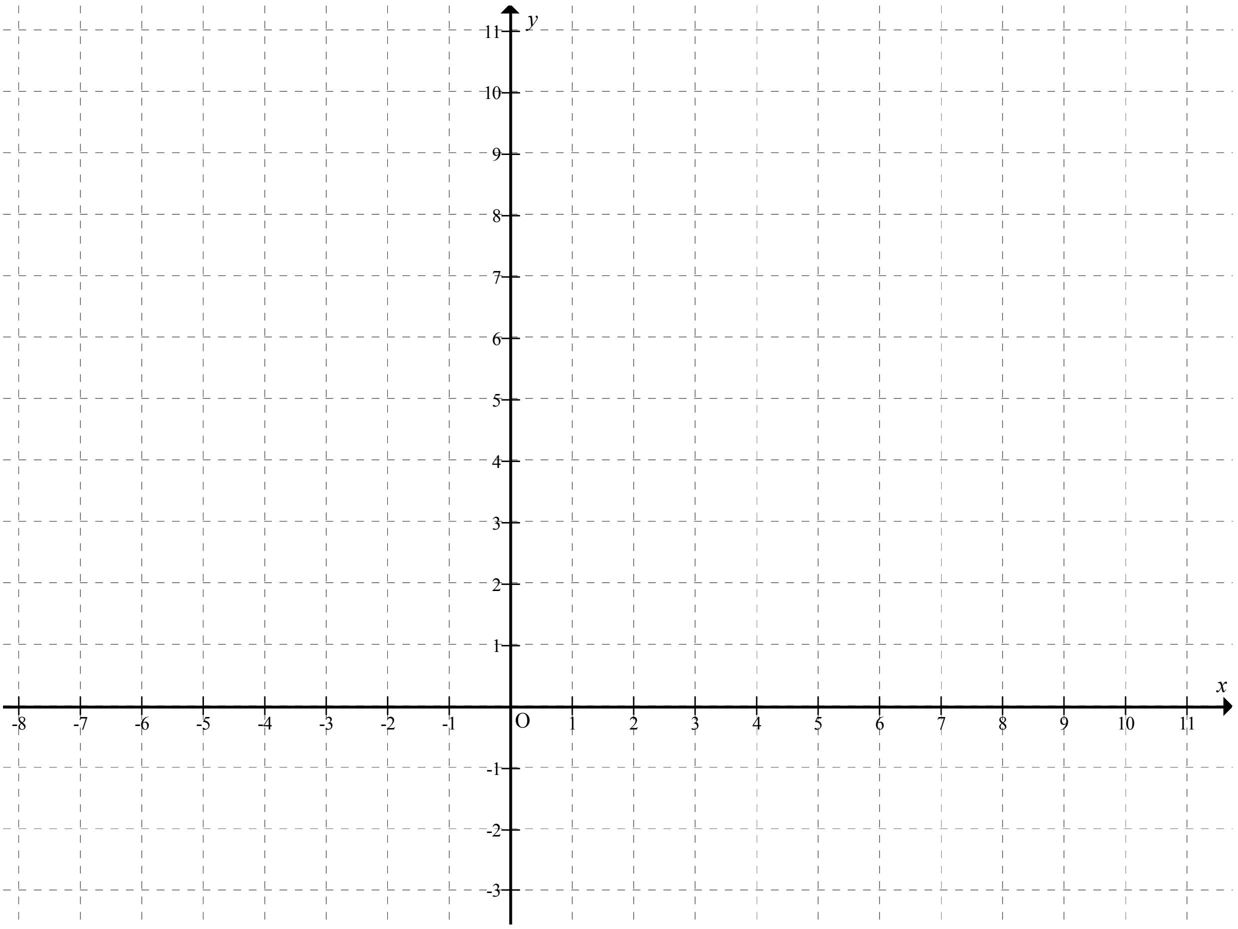
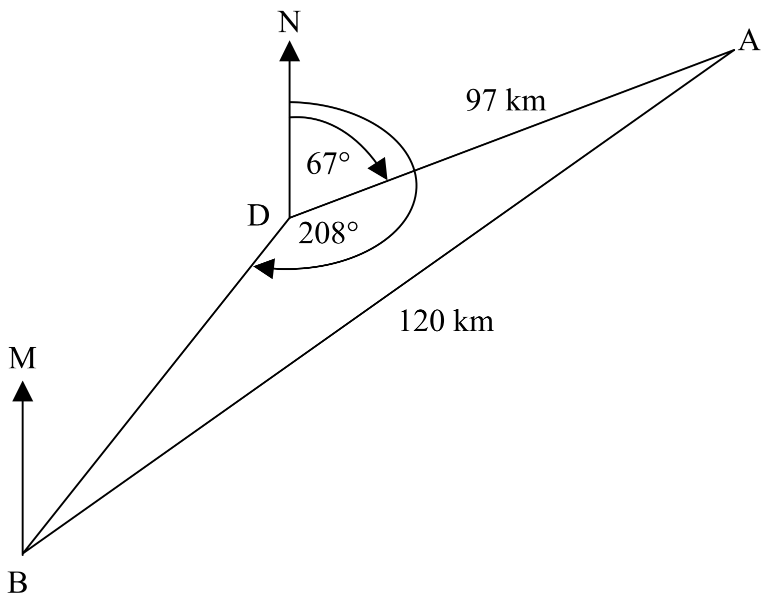
DIAGRAM SHEET 3

QUESTION 6.1.3



QUESTION 11



CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 4

QUESTION 12.1