education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

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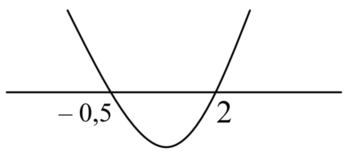
MEMORANDUM

MARKS: 150

This memorandum consists of 19 pages.

QUESTION 1

1.1.1	$(x-3)(x+5)=9$ $x^2+2x-15=9$ $x^2+2x-24=0$ $(x+6)(x-4)=0$ $x=4$ or $x=-6$	✓ expansion ✓ standard form ✓ factorisation ✓ answers (4)
1.1.2	$2x^2-3x-2\le 0$ $(2x+1)(x-2)\le 0$ Critical values : $-\frac{1}{2}$ and 2 + 0 – 0 + – 0,5 2  $-\frac{1}{2}\le x\le 2$	✓ factors ✓ critical values ✓✓ answer (4)
1.2	$y=-2x-2$..................(1) $-2x^2+8xy+42=y$........(2) $-2x^2+8x(-2x-2)+42=-2x-2$ $-2x^2-16x^2-16x+42+2x+2=0$ $-18x^2-14x+44=0$ $9x^2+7x-22=0$ $(9x-11)(x+2)=0$ $\therefore x=\frac{11}{9}$ or $x=-2$ $\therefore y=-2\left(\frac{11}{9}\right)-2$ $\therefore y=-2(-2)-2$ $\therefore y=-\frac{40}{9}$ $\therefore y=2$ **OR**	✓ $y=-2x-2$ ✓ substitution ✓simplification ✓ factors ✓ answers for x ✓✓ answers for y (7)

	$y=-2x-2$...................(1) $-2x^2+8xy+42=y$........(2) $y=-2(x+1)=\dfrac{-2(x^2-21)}{1-8x}$ $\therefore (x+1)(1-8x)=x^2-21$ $x-8x^2+1-8x=x^2-21$ $9x^2+7x-22=0$ $(9x-11)(x+2)=0$ $\therefore x=\dfrac{11}{9}$ or $x=-2$ $\therefore y=-2\left(\dfrac{11}{9}\right)-2$ $\therefore y=-2(-2)-2$ $\therefore y=-\dfrac{40}{9}$ $\therefore y=2$	✓ equating ✓ simplification ✓standard form ✓ factors ✓ answers for x ✓✓ answers for y (7)
	OR $y=-2x-2$...................(1) $-2x^2+8xy+42=y$........(2) $x=\dfrac{(-y-2)}{2}$ $-2\left(\dfrac{(-y-2)}{2}\right)^2+8y\left(\dfrac{(-y-2)}{2}\right)+42-y=0$ $-2\left(\dfrac{y^2+4y+4}{4}\right)+4y(-y-2)+42-y=0$ $y^2+4y+4+8y^2+16y-84+2y=0$ $9y^2+22y-80=0$ $(y-2)(9y+40)=0$ $y=2$ or $y=-\dfrac{40}{9}$ $x=-2$ or $x=\dfrac{11}{9}$	✓ $x=\dfrac{(-y-2)}{2}$ ✓ substitution ✓simplification ✓ factors ✓ answers for y ✓✓ answers for x (7)

1.3	$g(x) = x^2$ $g(9) = 81$ $f(x) = \sqrt{4x}$ $f(g(9)) = f(81) = \sqrt{4(81)}$ $= 2(9)$ $= 18$ **OR** $g(9) = 9^2$ $\therefore f(g(9)) = \sqrt{2^2.9^2} = 18$ **OR** $f(g(x)) = \sqrt{4g(x)}$ $= \sqrt{4x^2}$ $= 2x$ $f(g(9)) = 2(9)$ $= 18$	✓ $g(9) = 81$ ✓ substitution ✓ answer (3) ✓ $g(9) = 9^2$ ✓ substitution ✓ answer (3) ✓ $f(g(x)) = \sqrt{4g(x)}$ ✓ substitution ✓ answer (3)
1.4	$\frac{14}{\sqrt{63}-\sqrt{28}}$ $= \frac{14}{3\sqrt{7}-2\sqrt{7}}$ $= \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ $= 2\sqrt{7}$ $a = 2$ and $b = 7$ *But* $2\sqrt{7} = \sqrt{28}$ So a = 1 and b = 28 is also a solution.	✓ simplification ✓ simplification ✓ rationalising the denominator ✓ answer (4) **[22]**

QUESTION 2

2.1	$399 \; ; \; 360 \; ; \; 323 \; ; \; 288 \; ; \; 255$ $-39 \quad -37 \quad -35 \quad -33$ $2 \quad 2 \quad 2$ Let $T_n = an^2 + bn + c$ Then $2a = 2$ $a = 1$ $T_1 = 399: \; a+b+c = 399; \; b+c = 398$ $T_2 = 360: \; 4a+2b+c = 360; \; 2b+c = 356$ $b = -42$ $c = 440$ $T_n = n^2 - 42n + 440$	✓ 2nd difference constant ✓ $a = 1$ ✓ $b + c = 398$ ✓ $2b + c = 356$ ✓ $b = -42$ ✓ $c = 440$ (6)
	OR $2a = 2$ $a = 1$ $T_2 - T_1 = -39$ $4a + 2b + c - a - b - c = -39$ $3a + b = -39$ $3 + b = -39$ $b = -42$ $a + b + c = 399$ $1 - 42 + c = 399$ $c = 440$ $T_n = n^2 - 42n + 440$	✓ 2nd difference constant ✓ $a = 1$ ✓ $3a + b = -39$ ✓ $b = -42$ ✓ $a + b + c = 399$ ✓ $c = 440$ (6)
	OR $2a = 2$ $a = 1$ $399 - T_0 = -41$ $T_0 = 440.$ $But\ T_0 = c$ $\therefore \; c = 440$ $T_n = n^2 + bn + 440$ $399 = 1^2 + b(1) + 440$ $399 - 441 = b$ $-42 = b$ $T_n = n^2 - 42n + 440$ **OR**	✓ 2nd difference constant ✓ $a = 1$ ✓ $399 - T_0 = -41$ ✓ $c = 440$ ✓ $399 = 1^2 + b(1) + 440$ ✓ $b = -42$ (6)

	The sequence is $20^2 - 1$; $19^2 - 1$; $18^2 - 1$; $17^2 - 1$; ……. $T_1 = 20^2 = (20-0)^2 - 1$ $T_2 = 19^2 = (20-1)^2 - 1$ $T_3 = 18^2 = (20-2)^2 - 1$ $T_n = (20-(n-1))^2 - 1 = (21-n)^2 - 1$	✓✓rewriting terms as squares ✓✓✓ establishing that $T_n = (20-(n-1))^2$ ✓ $T_n = (21-n)^2$ (6)
2.2	$n^2 - 42n + 440 = 0$ $(n-22)(n-20) = 0$ $n = 22$ and $n = 20$ both terms 22 and 20 have values of 1. **OR** $(21-n)^2 - 1 = 0$ $21 - n = 1$ or -1 $n = 20$ or $n = 22$	✓ equation ✓✓ answers (3) ✓ equation ✓✓ answers (3)
2.3	$n = \frac{-(-42)}{2(1)}$ $n = 21$ At the 21st term, the lowest value is obtained. **OR** $2n - 42 = 0$ $2n = 42$ $n = 21$ $\therefore$ At the 21st term, the lowest value is obtained. **OR** $T_n = (21-n)^2 - 1 \therefore$ For $n = 21$, $T_n = (21-n)^2 - 1 = (21-21)^2 - 1 = -1$ For $n = 21$, the lowest value (= –1) is obtained.	✓ answer (1) ✓ answer (1) ✓ answer (1) **[10]**

QUESTION 3

3.1	Let $$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} \quad (1)$$ Then $$r \times S_n = r\left(a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1}\right)$$ $$= ar + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} + ar^n \quad (2)$$ (2) – (1) gives: $$rS_n - S_n = ar^n - a$$ $$S_n(r-1) = a(r^n - 1)$$ $$S_n = \frac{a(r^n-1)}{(r-1)}$$	✓ writing S_n as a series ✓ writing $r.S_n$ as a series ✓ subtracting ✓ removing common factors (4)
3.2	$a = 3\ ;\ r = \frac{1}{3}$ $S_\infty = \frac{a}{1-r}$ $= \frac{3}{1-\frac{1}{3}}$ $= \frac{9}{2}$	✓ $r = \frac{1}{3}$ ✓ substitution ✓ answer (3) **[7]**

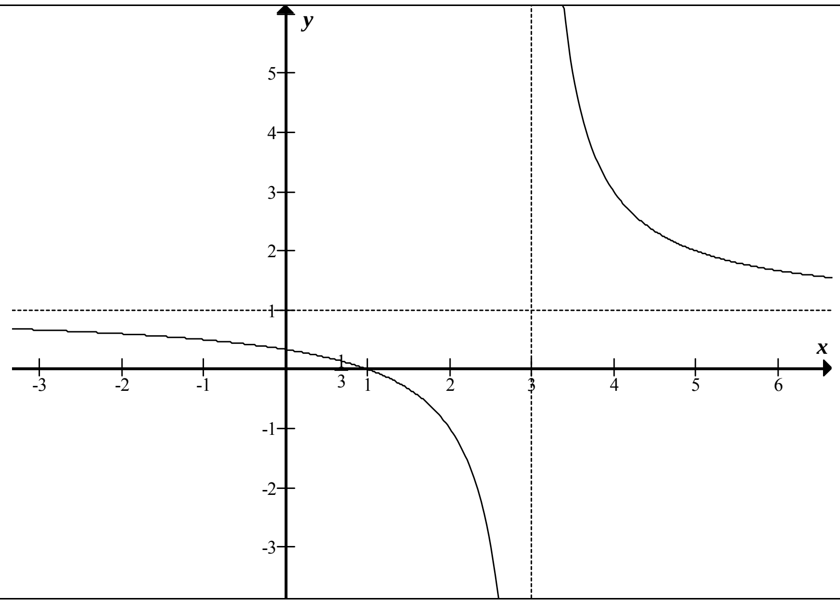
QUESTION 4

4.1	Term | Income | Expenses | Savings 1 | 120 000 | 90 000 | 30 000 2 | 132 000 | 105 000 | 27 000 3 | 144 000 | 120 000 | 24 000 30 000 + 27 000 + 24 000 + …+ 0.	✓ 30 000 ✓ 27 000 ✓ 24 000 ✓ series (4)
4.2	Savings = Income – Expenses Income in year n = 120 000 + 12 000(n – 1) Expenses in year n = 90 000 + 15 000(n – 1) $120000 + 12000(n-1) = 90000 + 15000(n-1)$ $30000 + 12000n - 12000 = 15000n - 15000$ $33000 = 3000n$ $n = 11$ $\therefore$ After 11 years. **OR**	✓✓ equating ✓ answer (3)

	a = 30 000 d = –3000 $T_n = 30000 + (n-1)(-3000)$ $0 = 30000 - 3000n + 3000$ $3000n = 33000$ $\therefore n = 11$ $\therefore$ After 11 years	✓✓ equation ✓ answer (3)
4.3	$120000 + 12000(25-1) = 90000 + x(25-1)$ $x = 13250$	✓equating ✓ answer (2) **[9]**

QUESTION 5

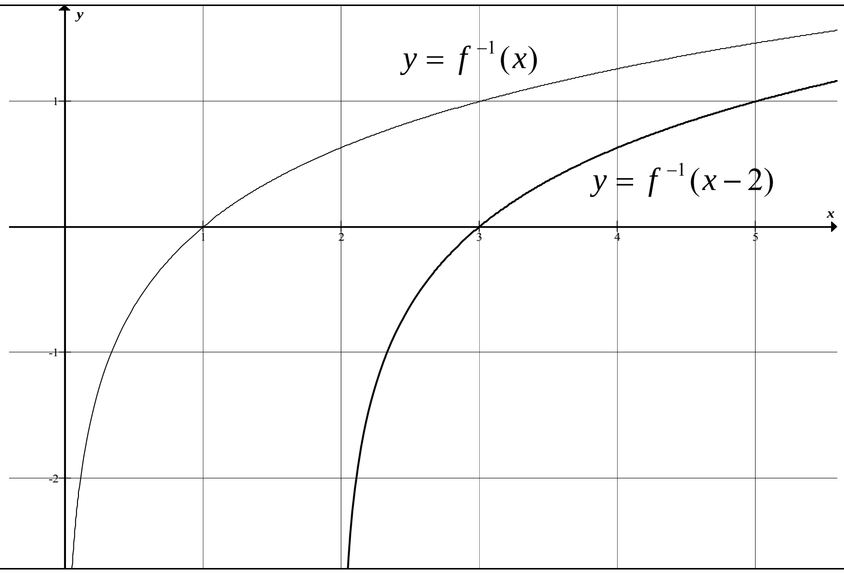
5.1	$y = 1$ $x = 3$	✓ answer ✓ answer (2)
5.2	$\dfrac{2}{0-3} + 1$ **OR** $y = \dfrac{x-1}{x-3}$ $= \dfrac{1}{3}$ $\therefore f(0) = \dfrac{1}{3}$ y-int $\left(0\ ;\ \dfrac{1}{3}\right)$ x-int: $0 = \dfrac{2}{x-3} + 1$ $0 = 2 + (x-3)$ **OR** $f(x) = 0$ $1 = x$ $\Rightarrow x - 1 = 0$ $(1\ ;\ 0)$ $\Rightarrow x = 1$	✓ answer ✓ substitution $y = 0$ ✓ answer (3)

5.3		✓ shape ✓ asymptotes ✓ both intercepts (3) **[8]**

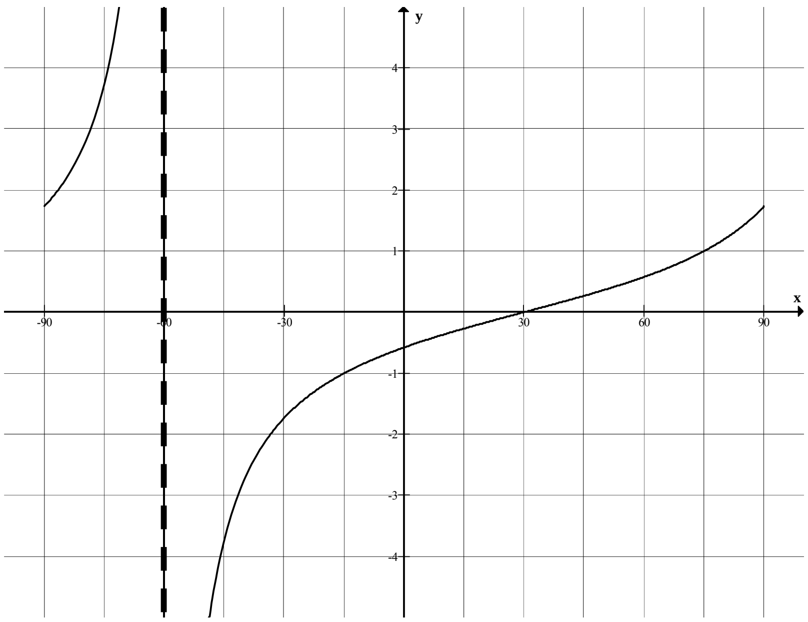
QUESTION 6

6.1	$-x^2+7x+8=0$ $x^2-7x-8=0$ $(x-8)(x+1)=0$ $x=8$ or $x=-1$ A(– 1 ; 0) B(8 ; 0)	✓ = 0 ✓ factors ✓ answer A ✓ answer B (4)
6.2	$-x^2+7x+8=-3x+24$ $-x^2+10x-16=0$ $x^2-10x+16=0$ $(x-8)(x-2)=0$ $x=8$ or $x=2$ x-value of D is 2. (i.e. $a=2$)	✓ equating ✓ standard form ✓ factors ✓ answer (4)
6.3	$\text{ST}=\left(-x^2+7x+8\right)-\left(-3x+24\right)$ $=-x^2+10x-16$	✓ subtraction ✓ answer (2)
6.4	Maximum length of ST is at $x=\dfrac{-10}{2(-1)}=5$. Maximum length of ST is $-5^2+50-16=9$. **OR** Maximum length of ST is $\dfrac{4(-1)(-16)-10^2}{4(-1)}=9$	✓ method ✓ answer (2) ✓ method ✓ answer (2) **[12]**

QUESTION 7

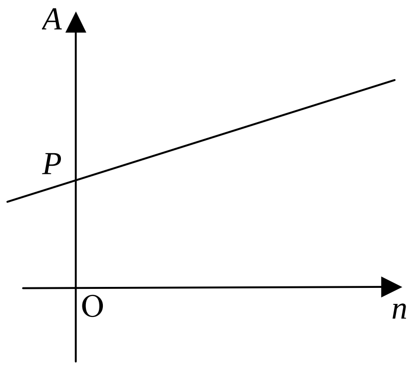
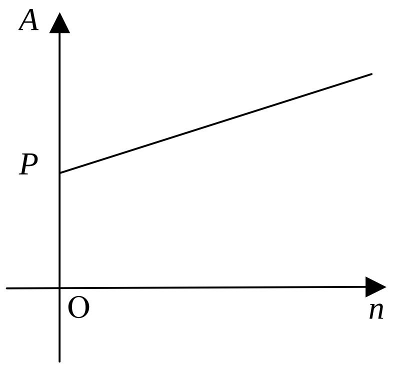
7.1	$y = \log_3 x$	✓ answer (1)
7.2		$y = f^{-1}(x)$ ✓ x-intercept ✓ shape $y = f^{-1}(x-2)$ ✓ x-intercept ✓ shape (4)
7.3	$2 < x < 5$	✓✓ answer (2) **[7]**

QUESTION 8

8.1		✓ shape ✓ x-intercept ✓ vertical asymptote (3)
8.2	$x = -60°$	✓ answer (1)
8.3	$\tan(30° - x) = -\tan(x - 30°)$ Reflection about the x-axis	✓ reflection ✓ x-axis (2) **[6]**

QUESTION 9

9.1.1	Total amount $= P(1 + in)$ $= 55\ 000(1 + 0{,}1275(4))$ $= 83\ 050$ Monthly instalment $= \dfrac{83050}{4 \times 12}$ $= R\ 1730{,}21$	✓ substitution into simple interest formula ✓ answer ✓ $\div 48$ ✓ answer (4)
9.1.2	$55000 = \dfrac{x\left[1-\left(1+\frac{0{,}2}{12}\right)^{-12\times 4}\right]}{\frac{0{,}2}{12}}$ $x = R\ 1\ 673{,}67$ a better option because monthly repayments are less.	✓ substitution into formula ✓ $i = \dfrac{0{,}2}{12}$ ✓ $n = 48$ ✓ answer (4)
9.1.3	$1673{,}67 \times 48$ $= 80336{,}16$ $80336{,}16 = 55000(1 + 4i)$ $1{,}460657455... = 1 + 4i$ $i = 0{,}11516436...$ Rate = 11,52%	✓ 80336,16 ✓ $80336{,}16 = 55000(1 + 4i)$ ✓ answer (3)
9.2	$80000 = \dfrac{25000\left[1-(1+0{,}1375)^{-n}\right]}{0{,}1375}$ $\dfrac{11}{25} = 1-(1+0{,}1375)^{-n}$ $\dfrac{14}{25} = (1+0{,}1375)^{-n}$ $\log\dfrac{14}{25} = -n\log(1{,}1375)$ $n = 4{,}50054779$ The money will last for 4 full years **OR** Candidate guesses 4 years. Then balance available at the end of 4 years (after the 4th withdrawal) is $80000(1+0{,}1375)^4 - 25000\left(\dfrac{(1+0{,}1375)^4-1}{0{,}1375}\right) = R11354{,}86$. At the end of the 5th year cannot have grown to R25000.	✓ substitution into correct formula ✓ simplification ✓ taking log of both sides ✓ answer (4) ✓ guesses 4 years ✓✓ calculates balance at end of 4th year ✓ conclusion about balance. (4)

9.3.1	$A = P + (P\,i)n$ which is a linear function of *n*.	✓ linear (1)
9.3.2	 Accept also: 	✓ $P > 0$ ✓ slope >0 (2)
9.3.3	The slope is Pi . Therefore this is the increase for *A* for each increase of 1 in *n*. **OR** $A(n+1) - A(n) = [P + Pi(n+1)] - [P + Pi\,n]$ $= Pi[(n+1) - n]$ $= Pi$ **OR** $A(1) - A(0) = (P + Pi) - (P + 0) = Pi$	✓ Pi (1) **[19]**

QUESTION 10

10.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$ $=\lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}$ $=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $=\lim_{h\to 0}-\frac{1}{x(x+h)}$ $=-\frac{1}{x^2}$	✓ substitution into correct formula ✓ expansion ✓ simplification ✓ answer (4)
10.2	$y=(2-5x)^2$ $y=4-20x+25x^2$ $\frac{dy}{dx}=-20+50x$ **OR** $y=(2-5x)^2$ By the chain rule $\frac{dy}{dx}=(2)(2-5x)(-5)$ $=-20+50x$	✓ simplification ✓✓ answers (3) ✓ simplification ✓✓ answers (3) **[7]**

QUESTION 11

11.1	$0 = x - 2$ $x = 2$ A(2 ; 0)	✓ answer (1)
11.2	$f(-1) = 0: -a + c = 2$ $f(2) = 0: \quad 8a - 2c = 2$ $a = 1, \quad c = 3$ **OR** $a(x+1)(x+1)(x-2) = 0$ $a(0+1)(0+1)(0-2) = -2$ $-2a = -2$ $a = 1$ $f(x) = (x^2 + 2x + 1)(x - 2)$ $= x^3 - 3x - 2$ $c = -3$	✓ $-a + c = 2$ ✓ $8a - 2c = 2$ ✓ $a = 1$ ✓ $c = 3$ ✓ factors ✓ substitution ✓ a ✓ $c = -3$ (4)
11.3	$f'(x) = 0$ $3x^2 - 3 = 0$ $x^2 - 1 = 0$ $(x+1)(x-1) = 0$ B(1 ; – 4)	✓ $f'(x) = 0$ ✓ $x^2 - 1$ ✓ answer (3)
11.4	$x - 2 = x^3 - 3x - 2$ $0 = x^3 - 4x$ $0 = x(x^2 - 4)$ $0 = x(x-2)(x+2)$ $x_C = -2, \; y_C = (-2)^2 - 3(-2) - 2 = -4$ C(– 2 ; – 4) $m_{BC} = \frac{-4-(-4)}{1-(-2)}$ $= 0$ BC is parallel to the *x*-axis. **OR** Following from C(– 2 ; – 4), B and C have the same y – coordinate, viz. – 4. So BC is parallel to the *x*-axis. **OR**	✓ equating *f* and *g* ✓ standard form ✓ factors ✓ $x_C = -2$ ✓ $y_C = -4$ ✓ $m = 0$ ✓ conclusion (7) (7)

	$(x-2)=(x-2)(x+1)^2$ $\therefore (x+1)^2=1$ for $x \neq 2$ $\therefore x+1=\pm 1$ $\therefore x=0$ *or* $x=-2$ $y=\ -4$	(7)
11.5	$f''(x)=0$ $6x=0$ $x=0$	✓ $f''(x)=0$ ✓ answer (2)
11.6	$k<-4$ or $k>0$	✓✓ answer ✓ or (3)
11.7	$f'(x)<0$ $-1<x<1$ **OR** $3(x^2-1)<0$ if $(x+1)(x-1)<0$ $-1<x<1$	✓✓answer (2) ✓✓answer (2) **[22]**

QUESTION 12

12.1	Length of sides of square = $\dfrac{4-x}{4} = 1 - \dfrac{x}{4}$	✓ answer (1)
12.2	$x = 2\pi r$ $r = \dfrac{x}{2\pi}$ Areas = $\left(\dfrac{4-x}{4}\right)^2 + \pi\left(\dfrac{x}{2\pi}\right)^2$ $= \dfrac{16-8x+x^2}{16} + \dfrac{x^2}{4\pi}$ $= 1 - \dfrac{1}{2}x + \left(\dfrac{1}{16} + \dfrac{1}{4\pi}\right)x^2$ **OR** $x = 2\pi r$ $r = \dfrac{x}{2\pi}$ $\left(1 - \dfrac{x}{4}\right)^2 + \pi\left(\dfrac{x}{2\pi}\right)^2$ $1 - \dfrac{1}{2}x + \dfrac{x^2}{16} + \dfrac{x^2}{4\pi}$ $1 - \dfrac{1}{2}x + \left(\dfrac{\pi+4}{16\pi}\right)x^2$	✓ $r = \dfrac{x}{2\pi}$ ✓ sum of areas ✓ simplification ✓ simplification (4) ✓ r ✓ sum of areas ✓ simplification ✓ simplification (4)
12.3	$x = \dfrac{-b}{2a}$ $= \dfrac{\frac{1}{2}}{2\left(\frac{\pi+4}{16\pi}\right)}$ $= 1{,}76$ meter **OR**	✓✓ substitution ✓ answer (3)

	$f(x)=1-\frac{1}{2}x+\frac{x^2(\pi+4)}{16\pi}$ $f'(x)=0=-\frac{1}{2}+\frac{\pi+4}{8\pi}x$ $4\pi=(\pi+4)x$ $x=\frac{4\pi}{\pi+4}$ x = 1,76 *m* for the circle and 2,24 *m* for the square	✓ $f'(x)=\frac{1}{2}+\frac{\pi+4}{8\pi}x$ ✓ $f'(x)=0$ ✓ answer (3) **[8]**

QUESTION 13

13.1	$4x+2y\le 32 \quad \therefore y\le -2x+16$ $2x+4y\le 36 \quad \therefore y\le -\frac{x}{2}+9$ $x+y\le 10 \quad \therefore y\le -x+10$	✓ answer ✓ answer ✓ answer (3)
13.2	Attached graph	✓ $y=-2x+16$ ✓ $y=-\frac{x}{2}+9$ ✓ $y=-x+10$ ✓ feasible region (4)
13.3	P = 60x + 80y	✓ answer (1)
13.4	$80y=-60x+P$ $y=-\frac{3}{4}x+\frac{P}{480}$ Maximum profit at (2; 8) ∴ Grade 10: 2 learners must be trained to give a maximum profit Grade 11: 8 learners must be trained to give a maximum profit	13.4 ✓ search line ✓✓ (2 ; 8) (3)
13.5	$m=-\frac{4}{3}$ Since the gradient of the new profit function is not equal to the gradient of the initial profit function, the new maximum point is (6 ; 4) that gives an optimal solution.	✓ $m=-\frac{4}{3}$ ✓ (6 ; 4) (2) **[13]**

QUESTION 13.2 & 13.4