education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2010

MEMORANDUM

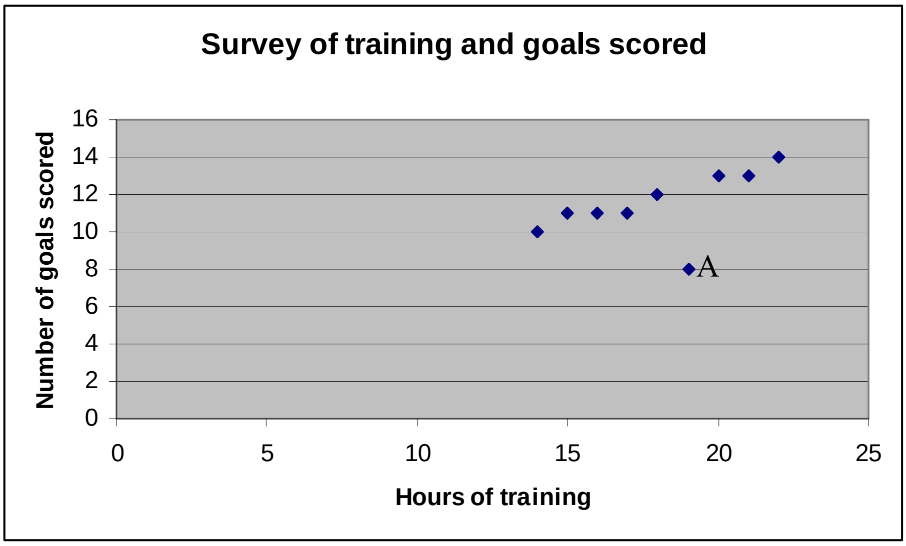
MARKS: 150

This memorandum consists of 14 pages.

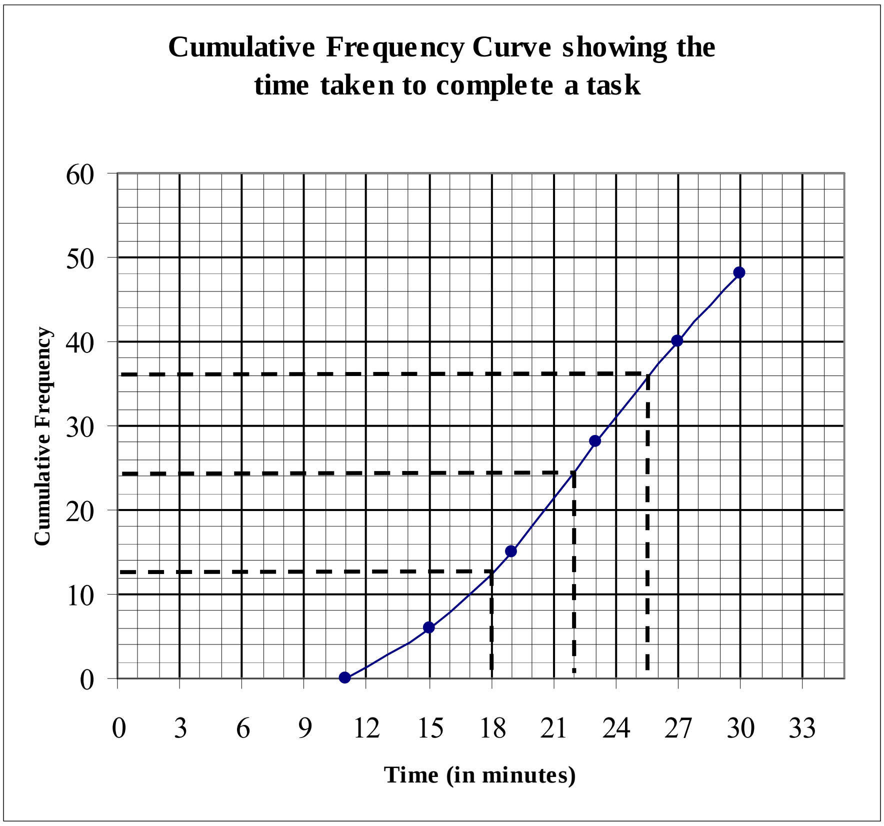
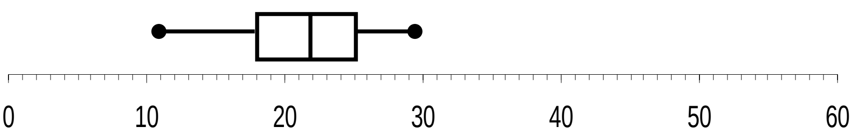
QUESTION 1

1.1	Range = 26 – 4 =22	✓maximum and minimum values ✓answer ANSWER ONLY: Full Marks (2)
1.2	Mean $= \frac{4+5+8+13+19+22+25+26+23+17+14+7}{12}$ $= \frac{183}{12}$ = 15,25	✓ method ✓183 ✓answer (3)
1.3	Standard deviation = 7,6 (7,59522…..)	✓✓answer (2)
1.4.1	Increase in mean $= \frac{(3\times 5)+(9\times 1)}{12}$ = 2°C per month.	✓✓answer (2)
1.4.2	The maximum value increases by 1°C and the minimum value increases by 5°C. This implies that the range of the range of the data will now decrease. This will result in the standard deviation getting smaller. (new SD = 6,27…..)	✓decrease in range ✓decrease in standard deviation (2) **[11]**

QUESTION 2

2.1.1		✓✓✓plotting the points All 9 point correct – 3 marks 5 or 7 points correct – 2 marks 1 or 2 points correct – 1 mark 0 points correct – 0 marks (3)
2.1.2	A(indicated on the graph)	✓answer (1)
2.1.3	8 Goals	✓✓answer (2)
2.2	Let the mean time for all 560 learners be x. Then the mean time for the learners living in neighbourhood C is also x. $x = \dfrac{(135 \times 24) + (225 \times 32) + (200 \times x)}{560}$ $560x = 3240 + 7200 + 200x$ $360x = 10440$ $x = 29$	✓equal mean times ✓mean × number ✓simplification ✓answer (4) **[10]**

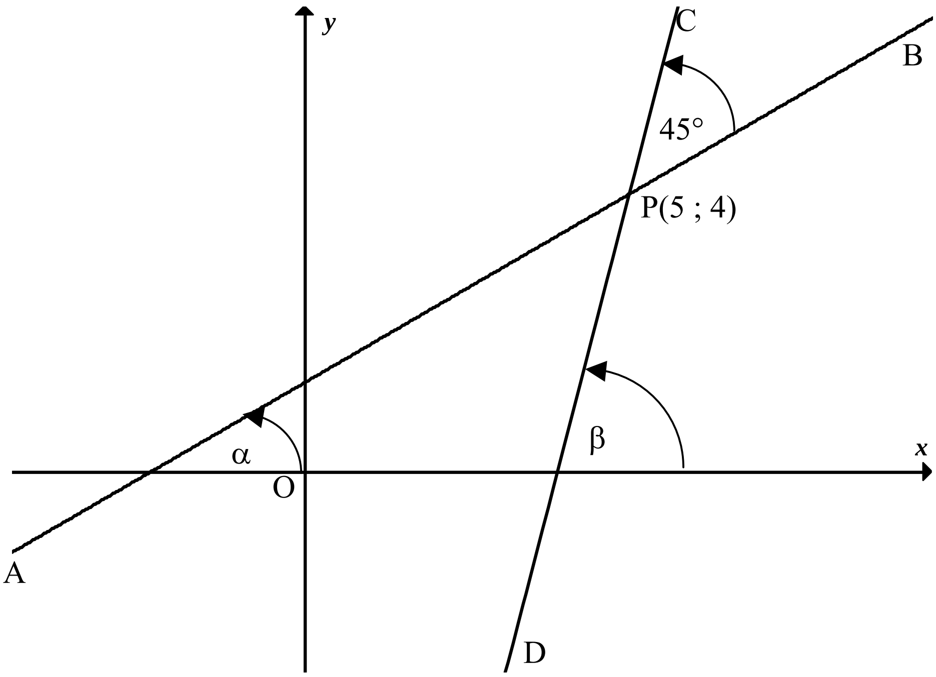
QUESTION 3

3.1	<table><tr><td>Time (in minutes)</td><td>$11 \le t < 15$</td><td>$15 \le t < 19$</td><td>$19 \le t < 23$</td><td>$23 \le t < 27$</td><td>$27 \le t < 30$</td></tr><tr><td>Frequency</td><td>6</td><td>9</td><td>13</td><td>12</td><td>8</td></tr><tr><td>Cumulative Frequency</td><td>6</td><td>15</td><td>28</td><td>40</td><td>48</td></tr></table>	✓cumulative frequency totals (1)
3.2		✓✓✓plotting points at upper limits 6 correct – 3 marks 3 to 5 correct – 2 marks 1 or 2 correct – 1 mark 0 correct – 0 marks ✓curve (4)
3.3	Median value at position 24. Reading off the ogive gives Median ≈ 22 minutes LQ value at position 12. Lower quartile ≈ 18 minutes (from ogive) UQ value at position 36. Upper quartile ≈ 25,5 minutes (from ogive) **NOTE**: Allow margin of error for reading off the graph.	✓ median ✓ lower quartile ✓ upper quartile (3)
3.4		✓ box ✓ whiskers (2)
3.5	The times are skewed to the right. A small number of people finished this task very quickly whilst others took more time.	✓ skewed to the right (1) **[11]**

QUESTION 4

4.1	$m_{PQ} = \dfrac{2-0}{0-4} = -\dfrac{1}{2}$	✓ substitution (1)
4.2	A: $\left(\dfrac{0+4}{2};\dfrac{2+0}{2}\right)$ A (2 ; 1)	✓ x-coordinate ✓ y-coordinate (2)
4.3	$m_{AB}.m_{PQ} = -1$ $m_{AB}.(-1/2) = -1$, $\therefore m_{AB} = 2$ Equation of AB is $y = 2x + c$ $\therefore\ 1 = 2(2) + c$ $c = -3$ Equation of AB is $y = 2x - 3$. **OR** $m_{AB}.m_{PQ} = -1$ $m_{AB}.(-1/2) = -1$, $\therefore m_{AB} = 2$ $y - 1 = 2(x - 2)$ $y - 1 = 2x - 4$ $y = 2x - 3$	✓ $m_{AB}.m_{PQ} = -1$ ✓ $m_{AB} = 2$ ✓ equation of AB ✓ $y = 2x - 3$ ✓ $c = -3$ (5) ✓ $m_{AB}.m_{PQ} = -1$ ✓ $m_{AB} = 2$ ✓ gradient of AB ✓ substitution into formula ✓ equation of AB (5)
4.4	B is the point (0 ; –3) BQ = $\sqrt{(0-4)^2 + (-3-0)^2}$ $= 5$	✓ coordinates of B ✓ substitution ✓ answer (3)
4.5	BP = $\sqrt{(0-0)^2 + (-3-2)^2}$ $= 5$ BP = BQ $\therefore \Delta$BPQ is isosceles. **OR** BP = 2 + 3 = 5 BP = BQ $\therefore \Delta$BPQ is isosceles	✓ BP = 5 ✓ BP = BQ (2) ✓ BP = 5 ✓ BP = BQ (2)
4.6	If PBQR is a rhombus then A is the midpoint of BR. Let the coordinates of R be (x ; y) $\dfrac{x+0}{2} = 2$ and $\dfrac{y-3}{2} = 1$ $x = 4$ $y = 5$ $\therefore$R(4 ; 5) **OR** RQ || PB so $x_R = 4$ RQ = PB = 5, so $y_R = 5$ $\therefore$R(4 ; 5)	✓ A is the midpoint of BR ✓ x coordinate ✓ y coordinate (3) ✓ RQ || PB ✓ x coordinate ✓ y coordinate (3) **[16]**

QUESTION 5



5.1	AB is defined as $5y-3x-5=0$ which can be written as $y=\frac{3}{5}x+1$ $m_{AB}=\frac{3}{5}$ Let α be the inclination of AB. $\tan\alpha=\frac{3}{5}$ $\alpha=30.96°$. Let β be the inclination of CD β = 45° + 30,96° = 75,96° Gradient of CD = tan 75,96° = 4. **OR** $\tan\beta=\tan(\alpha+45°)$ $=\frac{\tan\alpha+\tan 45°}{1-\tan\alpha.\tan 45°}$ $=\frac{\frac{3}{5}+1}{1-\frac{3}{5}\times 1}$ $=4$ $m_{CD}=\tan\beta$ $m_{CD}=4$	✓ $m_{AB}=\frac{3}{5}$ ✓ $\tan\alpha=\frac{3}{5}$ ✓ $\alpha=30.96°$ ✓ β = 75,96° ✓ gradient of CD (5) ✓ expansion ✓ tan 45° = 1 ✓ tan α = $\frac{3}{5}$ ✓ substitution ✓ answer (5)

5.2	Equation of CD is $y = 4x + c$ $\therefore\ 4 = 4(5) + c$ $c = -16$ Equation of CD is $y = 4x - 16$. **OR** $y - 4 = 4(x - 5)$ $y - 4 = 4x - 20$ $y = 4x - 16$	✓ *y*- intercept ✓ equation of CD (2) ✓ substitution ✓ equation of CD (2) **[7]**

QUESTION 6

6.1	$x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + 8x + 16 + y^2 + 4y + 4 = 16 + 4 + 38$ $(x + 4)^2 + (y + 2)^2 = 58$ Centre is (–4 ; –2) and the radius is $\sqrt{58}$	✓completing the square (both or one) ✓ factor form ✓ centre ✓ radius (4)
6.2	Centre of second circle is (4 ; 6) Distance between centres is $\sqrt{(4+4)^2 + (6+2)} = \sqrt{128} = 11{,}31$	✓centre ✓distance (2)
6.3	Sum of radii = $\sqrt{58} + \sqrt{26} = 12{,}71$ Distance between centres is 11,31. sum of the radii > distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.	✓✓ sum of radii ✓ conclusion (3)
6.4	Equation of second circle: $(x - 4)^2 + (y - 6)^2 = 26$ $x^2 - 8x + 16 + y^2 - 12y + 36 = 26$ $x^2 - 8x + y^2 - 12y + 26 = 0$ Let (*x* ; *y*) be either of the two points on intersection. Then $x^2 + y^2 + 8x + 4y - 38 = 0$ and $x^2 + y^2 - 8x - 12y + 26 = 0$ Subtract $16y + 16x - 64 = 0$ $y = -x + 4$ Both points of intersection lie on this line. $\therefore$ $y = -x + 4$ is the equation of the common chord. **OR**	✓ equation of circle in form = 0 ✓ statement – two points of intersection ✓ subtracting ✓ simplification (4)

	Check that the line $y = -x + 4$ cuts the two circles at the same points: $(x-4)^2 + (-x-2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3$ or $x = -1$	✓ substitution ✓ answer ✓ substitution ✓ answer (4) **[13]**

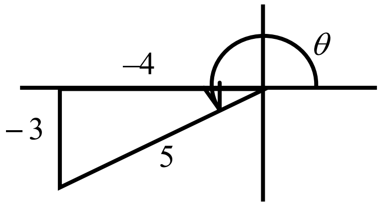
QUESTION 7

7.1.1	$P^{/}(5;-2)$	✓ answer (1)
7.1.2	$P^{/}(5;2)$	✓ x coordinate ✓ y coordinate (2)
7.2.1	$K \to K'' : (14;4) \to (2;2)$ $U \to U'' : (18;6) \to (3;9)$ $H \to H'' : (16;8) \to (4;8)$ $L \to L'' : (18;10) \to (5;9)$ $E \to E'' : (14;12) \to (6;7)$ So "halve" and 'interchange" or 'interchange" and "halve". Reflection across $y = x$ followed by contraction by $\frac{1}{2}$ OR Contraction by $\frac{1}{2}$ followed by reflection across $y = x$.	✓ reflected ✓ the line $y = x$ ✓ enlarged ✓ scale factor of $\frac{1}{2}$ (4)
7.2.2	$H^{/} = \frac{1}{2}(16;8) = (8;4)$ OR $H^{/}(8;16)$	✓ (8 ; 4) ✓ (8 ; 16) (2)
7.2.3	Area KUHLE : Area $K^{//}U^{//}H^{//}L^{//}E^{//} = \left(\frac{2}{1}\right)^2 = 4:1$	✓ ✓answer (2) **[11]**

QUESTION 8

8.1	For anti-clockwise rotation: $x^{/} = x\cos\theta - y\sin\theta$ $= 3\cos 120° - 2\sin 120°$ $= 3(-\cos 60°) - 2\sin 60°$ $= 3\left(-\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{-3-2\sqrt{3}}{2}$ $y^{/} = x\sin\theta + y\cos\theta$ $= 3\sin 120° + 2\cos 120°$ $= 3\sin 60° + 2(-\cos 60°)$ $= 3\left(\frac{\sqrt{3}}{2}\right) + 2\left(-\frac{1}{2}\right)$ $= \frac{3\sqrt{3}-2}{2}$ $P^{/}\left(\frac{-3-2\sqrt{3}}{2}\ ;\ \frac{3\sqrt{3}-2}{2}\right)$	✓ formula ✓ simplification ✓ substitution ✓ answer ✓ simplification ✓ answer (6)
8.2	$-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ $-4 = -x - \sqrt{3}y$ …… equation 1 $0 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(-\frac{1}{2}\right)$ $0 = \sqrt{3}x + y$ $y = -\sqrt{3}x$ …… equation 2 Substitute equation 2 into equation 1 $-4 = -x - \sqrt{3}\left(-\sqrt{3}x\right)$ $-4 = -x + 3x$ $-4 = 2x$ $x = -2$ $y = 2\sqrt{3}$ Q $\left(-2\ ;\ 2\sqrt{3}\right)$	✓ $-4 = -x - \sqrt{3}y$ ✓ $y = -\sqrt{3}x$ ✓ x-coordinate ✓ y-coordinate (4) **[10]**

QUESTION 9

9.1.1	$\sin\theta = -\frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$ $\sin\theta + \cos\theta = -\frac{7}{5}$  	✓ correct quadrant and values. ✓ $\sin\theta = -\frac{3}{5}$ ✓ $\cos\theta = -\frac{4}{5}$ ✓answer (4)
9.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ $= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$ **OR** $\tan 2\theta$ $= \frac{2\tan\theta}{1-\tan^2\theta}$ $= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$ $= \frac{24}{7}$	✓ $\frac{\sin 2\theta}{\cos 2\theta}$ ✓ sin 2θ = 2sinθ.cosθ ✓ cos 2θ = $\cos^2\theta$ – $\sin^2\theta$ ✓ substitution ✓ answer (5) ✓✓ expansion ✓✓ substitution ✓ answer (5)
9.2.1	$\frac{\cos(360° - x).\tan^2 x}{\sin(x - 180°).\cos(90° + x)}$ $= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $= (\cos x)\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{1}{\sin^2 x}\right)$ $= \frac{1}{\cos x}$	✓ $\cos x$ ✓ $-\sin x$ ✓ $-\sin x$ ✓ $\frac{\sin^2 x}{\cos^2 x}$ ✓ answer (5)
9.2.2	$x = 30°$ $\frac{1}{\cos 30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$	✓ $x = 30°$ ✓ answer (2) **[16]**

QUESTION 10

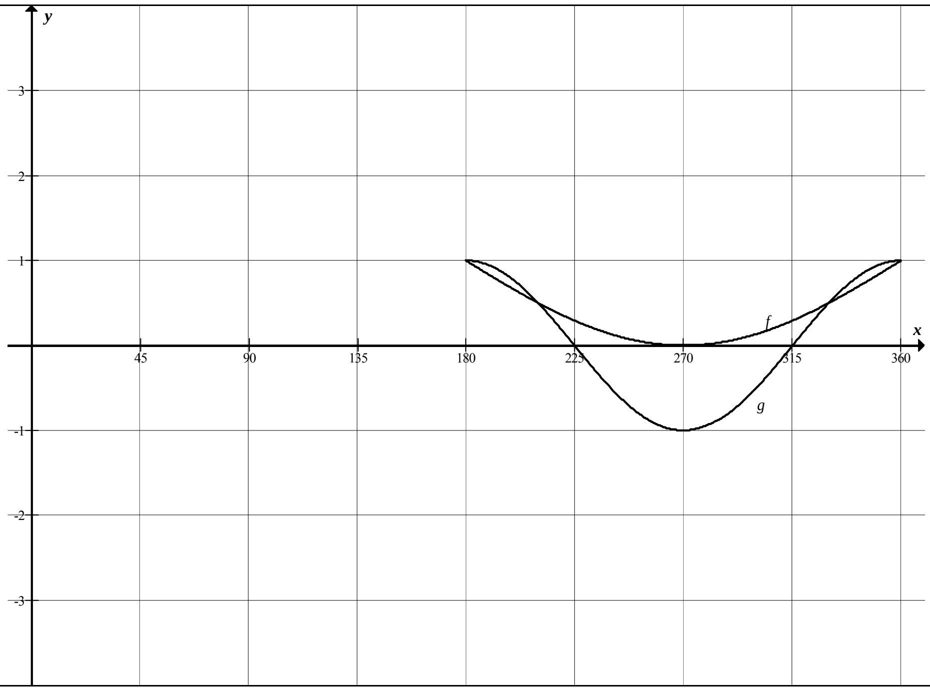
10.1.1	$\sin 48° = \sin(36° + 12°)$ $= \sin 36° \cos 12° + \cos 36° \sin 12°$ $= p + q$	✓writing 48° in terms of 36° and 12° ✓ expansion ✓ answer (3)
10.1.2	$\sin 24° = \sin(36° - 12°)$ $= \sin 36° \cos 12° - \cos 36° \sin 12°$ $= p - q$ **OR** $\sin 24° = \sin(36° - 12°)$ $= \sin 36° \cos 12° - \cos 36° \sin 12°$ $= p - q$	✓writing 24° in terms of 36° and 12° ✓ expansion ✓ $\sin 24° = p - q$ (3) ✓writing 24° in terms of 36° and 12° ✓ expansion ✓ $\sin 24° = p - q$ (3)
10.1.3	$\sin 48° = 2\sin 24° \cos 24°$ $\therefore\ p + q = 2(p - q)\cos 24°$ $\therefore \cos 24° = \dfrac{p+q}{2(p-q)}$ **OR** $\cos 48° = 2\cos^2 24° - 1$ $\therefore \cos 24° = \sqrt{\dfrac{1+\cos 48°}{2}} = \sqrt{\dfrac{1}{2}\left(1 + \sqrt{1 - \sin^2 48°}\right)}$ $= \sqrt{\dfrac{1}{2}\left(1 + \sqrt{1 - (p+q)^2}\right)}$ **OR** $\cos^2 24° = 1 - \sin^2 24°$ $\cos^2 24° = 1 - (p - q)^2$ $\cos 24° = \sqrt{1 - (p-q)^2}$	✓ $\cos 48° = 2\cos^2 24° - 1$ ✓ $\sin 48° = p + q$ ✓ answer (3) ✓ $\cos 48° = 2\cos^2 24° - 1$ ✓ $\sin 24° = p - q$ ✓ answer (3) ✓ $\cos^2 24° = 1 - \sin^2 24°$ ✓ $\sin 24° = p - q$ ✓ answer (3)

10.2	$\sin^2 20° + \sin^2 40° + \sin^2 80°$ $= \sin^2 20° + (\sin(60° - 20°))^2 + (\sin(60° + 20°))^2$ $= \sin^2 20° + (\sin 60° \cos 20° - \cos 60° \sin 20°)^2 + (\sin 60° \cos 20° + \cos 60° \sin 20°)^2$ $= \sin^2 20° + \left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)^2 + \left(\frac{\sqrt{3}}{2}\cos 20° + \frac{1}{2}\sin 20°\right)^2$ $= \sin^2 20° + \frac{3}{4}\cos^2 20° - \frac{\sqrt{3}}{2}\cos 20° \sin 20° + \frac{1}{4}\sin^2 20° + \frac{3}{4}\cos^2 20°$ $+ \frac{\sqrt{3}}{2}\cos 20° \sin 20° + \frac{1}{4}\sin^2 20°$ $= \sin^2 20° + \frac{3}{2}\cos^2 20° + \frac{1}{2}\sin^2 20°$ $= \frac{3}{2}(\sin^2 20° + \cos^2 20°)$ $= \frac{3}{2}$ **OR** Use $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ *LHS* $= \frac{3}{2} - \frac{1}{2}\{(\cos 40° + \cos 80°) + \cos 160°\}$ $= \frac{3}{2} - \frac{1}{2}\{(\cos 60°.\cos 40° + \sin 60° \sin 40° + \cos 60°.\cos 40° - \sin 60° \sin 40°) + \cos 160°\}$ $= \frac{3}{2} - \frac{1}{2}\{(2\cos 60° \cos 20°) - \cos 20°\}$ $= \frac{3}{2} - \frac{1}{2}\left\{\left(2 \times \frac{1}{2}\cos 20°\right) - \cos 20°\right\}$ $= \frac{3}{2} - 0$ $= \frac{3}{2}$	✓40°= 60° - 20° ✓80°= 60° + 20° ✓ ✓expansions ✓ substitution ✓ simplification ✓ factorisation (7) ✓40°= 60° – 20° ✓80°= 60° + 20° ✓ expansion of cos 40° ✓ expansion of cos 60° ✓ simplification ✓ simplification ✓ answer for bracket (7)

10.3.1	$\frac{\sin^4 x + \sin^2 x \cos^2 x}{1+\cos x}$ $= \frac{\sin^2 x(\sin^2 x + \cos^2 x)}{1+\cos x}$ $= \frac{\sin^2 x}{1+\cos x}$ $= \frac{1-\cos^2 x}{1+\cos x}$ $= \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)}$ $= 1-\cos x$	✓ factorisation ✓ $\sin^2 x + \cos^2 x = 1$ ✓identity ✓ factorisation (4)
10.3.2	$1+\cos x = 0$ $\cos x = -1$ $x = 180° + k.360°;\ k \in Z$ Undefined for $x = 180° + k.360°;\ k \in Z$.	✓$1+\cos x = 0$ ✓$180° + k.360°$ (2) **[22]**

QUESTION 11

11.1	$1+\sin x = \cos 2x$ $1+\sin x = 1-2\sin^2 x$ $\sin x + 2\sin^2 x = 0$ $\sin x(1+2\sin x) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = k.180$ or $x = -30° + k.360$ $k \in Z$ $x = 210° + k.360$ $x \in \{180°; 210; 330°; 360°\}$ **OR** $1+\sin x = \cos 2x$ $1+\sin x = \cos^2 x - \sin^2 x$ $1+\sin x = 1-\sin^2 x - \sin^2 x$ $\sin x + 2\sin^2 x = 0$ $\sin x(1+2\sin x) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = k.180$ or $x = -30° + k.360$ $k \in Z$ $x = 210° + k.360$ $x \in \{180°; 210; 330°; 360°\}$	✓ expansion ✓ factorisation ✓ equations ✓ $x = k.180$ ✓ solution for $\sin x = -\frac{1}{2}$ ✓✓answers (7) ✓ expansion ✓ factorisation ✓ equations ✓ $x = k.180$ ✓ solution for $\sin x = -\frac{1}{2}$ ✓✓answers (7)

11.2		$1+\sin x$ ✓ max and min values ✓ shape $\cos 2x$ ✓ amplitude ✓ intercepts (4)
11.3	$180° \leq x \leq 210°$ or $330° \leq x \leq 360°$	✓✓✓ answer (3) **[14]**

QUESTION 12

12.1	$\dfrac{b}{\sin[180° - (\alpha + \beta)} = \dfrac{\text{BC}}{\sin\alpha}$ $\text{BC}\sin(\alpha + \beta) = b\sin\alpha$ $\text{BC} = \dfrac{b\sin\alpha}{\sin(\alpha + \beta)}$ but $\text{BC} = \text{DF}$ $\therefore \text{DF} = \dfrac{b\sin\alpha}{\sin(\alpha + \beta)}$ $\cos\theta = \dfrac{\text{DF}}{\text{DE}}$ $\therefore \text{DE} = \dfrac{\text{DF}}{\cos\theta}$ $\therefore \text{DE} = \dfrac{b\sin\alpha}{\sin(\alpha + \beta)\cos\theta}$	✓ sine rule ✓ $A\hat{B}C = 180° - (\alpha + \beta)$ ✓ BC = … ✓ BC = DF ✓ manipulation ✓ DE = … (6)
12.2	$\text{DE} = \dfrac{2000\sin 43°}{\sin 79°.\cos 27°}$ $= 1559{,}50 \text{ m}$	✓substitution numerator ✓substitution denominator ✓answer (3) **[9]**

TOTAL: 150