education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2009(1)

MEMORANDUM

MARKS: 150

This memorandum consists of 15 pages.

QUESTION 1

<table>
<tr><td>1.1</td><td>Mean = $\frac{522,5}{12} = 43,54$

ANSWER ONLY: Full marks</td><td>✓ 522,5
✓ answer
(2)</td></tr>
<tr><td>1.2</td><td>Ordered Data
9,3 14,9 15 23,6 26,1 28 32,5 60,9
65,7 71,9 76,4 98,2

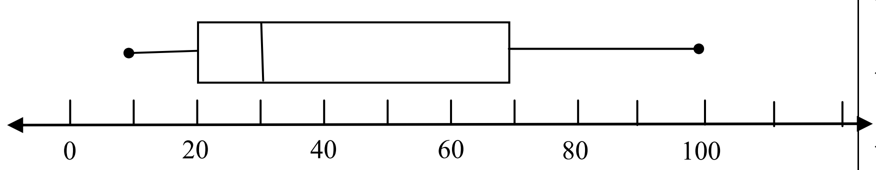
Median = $\frac{28+32,5}{2} = 30,25$

Lower quartile = $\frac{15+23,6}{2} = 19,3$

Upper quartile = $\frac{65,7+71,9}{2} = 68,8$

The five number summary is (9,3 ; 19,3 ; 30,25 ; 68,8 ; 98,2)</td><td>✓ arranging in ascending order

✓ median
✓ lower quartile
✓ upper quartile

✓ five number summary
(5)</td></tr>
<tr><td>1.3</td><td>
</td><td>✓ minimum and maximum values
✓ quartiles and median
✓ whiskers
(3)</td></tr>
<tr><td>1.4</td><td>The data is skewed to the right. This suggests that there was a large difference between the median and the maximum rainfall (some months had exceptionally high rainfall in that year).</td><td>✓ reference to box and whisker
✓ comment about rainfall.
(2)</td></tr>
<tr><td>1.5</td><td>By using the calculator, $\sigma = 28,19$. (28,19058256)

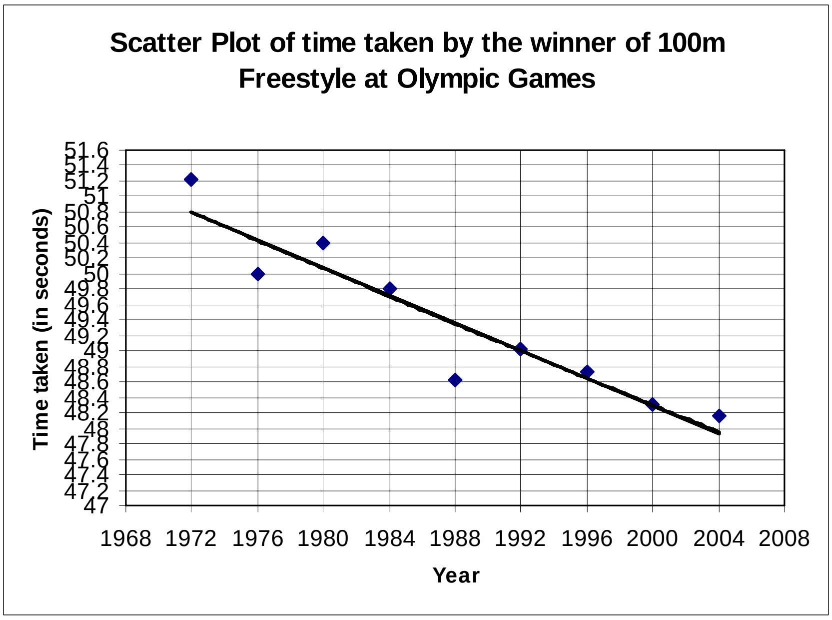
OR Pen and Paper method (not recommended)
Mean = 43,54 (43,54166667)
<table>
<tr><th>x</th><th>$x-\bar{x}$</th><th>$(x-\bar{x})^2$</th></tr>
<tr><td>60,9</td><td>17,36</td><td>301,3696</td></tr>
<tr><td>14,9</td><td>-28,64</td><td>820,2496</td></tr>
<tr><td>9,3</td><td>-32,24</td><td>1172,378</td></tr>
<tr><td>28,0</td><td>-15,54</td><td>241,4916</td></tr>
<tr><td>71,9</td><td>28,36</td><td>804,2896</td></tr>
<tr><td>76,4</td><td>32,86</td><td>1079,78</td></tr>
<tr><td>98,2</td><td>54,66</td><td>2987,716</td></tr>
<tr><td>65,7</td><td>22,16</td><td>491,0656</td></tr>
<tr><td>26,1</td><td>-17,44</td><td>304,1536</td></tr>
<tr><td>32,5</td><td>-11,04</td><td>121,8816</td></tr>
<tr><td>23,6</td><td>-19,94</td><td>397,6036</td></tr>
<tr><td>15,0</td><td>-28,54</td><td>814,5316</td></tr>
<tr><td colspan="2">Sum</td><td>9536,509</td></tr>
</table>
$\sigma = \sqrt{\frac{9536,509}{12}} = 28,19$</td><td>✓✓✓answer
(3)

✓✓sum of the squares of the mean deviations

✓ answer
(3)
[15]</td></tr>
</table>

	(28,19059…..)	

QUESTION 2

2.1	Linear or Exponential	✓ answer (1)
2.2		✓ ✓line of best fit (2)
2.3	The scatter plot shows an overall decrease in the time taken by the winner since 1972.	✓ decrease (1)
2.4	The top athletes of the world have turned professional. This allows them to train at the best facilities and receive the best coaching available. Also, equipment manufacturers are in competition with each other. In this case, manufacturers are designing swimsuits that assist swimmers	✓ any acceptable reason relating to the trend (1)
2.5	In the context of the times around these two observations, one can consider the efforts of 1976 and 1988 to be outliers. This shows that these athletes were exceptionally good swimmers at the time.	✓outliers / not in keeping with trend ✓good swimmers (2)
2.6	Winning time of 2008 is expected to be about 47,6 seconds. (Accept answer in the interval 47,4 to 47,8)	✓47,6 (1) **[8]**

QUESTION 3

3.1	50	✓ answer (1)
3.2	Cut–off mark of 56% (37 students)or 58% (38 students)	✓ answer read off from ogive (1)
3.3	<table><tr><th>Marks (out of 100)</th><th>Frequency (f)</th></tr><tr><td>0 ≤ marks <10</td><td>1</td></tr><tr><td>10 ≤ marks <20</td><td>3</td></tr><tr><td>20 ≤ marks <30</td><td>4</td></tr><tr><td>30 ≤ marks <40</td><td>11</td></tr><tr><td>40 ≤ marks <50</td><td>12</td></tr><tr><td>50 ≤ marks <60</td><td>9</td></tr><tr><td>60 ≤ marks <70</td><td>5</td></tr><tr><td>70 ≤ marks <80</td><td>4</td></tr><tr><td>80 ≤ marks <90</td><td>1</td></tr><tr><td>90 ≤ marks <100</td><td>0</td></tr></table>	✓ class intervals ✓✓ frequencies for each class (3) **[5]**

QUESTION 4

4.1	$\tan 45° = m_{AB}$ $= 1$	✓ $\tan 45° = m_{AB}$ ✓ answer (2)
4.2	$\dfrac{3-0}{1-t} = 1$ $1 - t = 3$ $t = -2$	✓use of gradient ✓value (2)
4.3	$\sqrt{(1-p)^2 + (3+4)^2} = \sqrt{50}$ $(1-p)^2 + (3+4)^2 = 50$ $1 - 2p + p^2 + 49 = 50$ $p^2 - 2p = 0$ $p(p-2) = 0$ $p \neq 0$ or $p = 2$	✓ substitution into distance formula ✓ expansion ✓factors ✓answer (4)

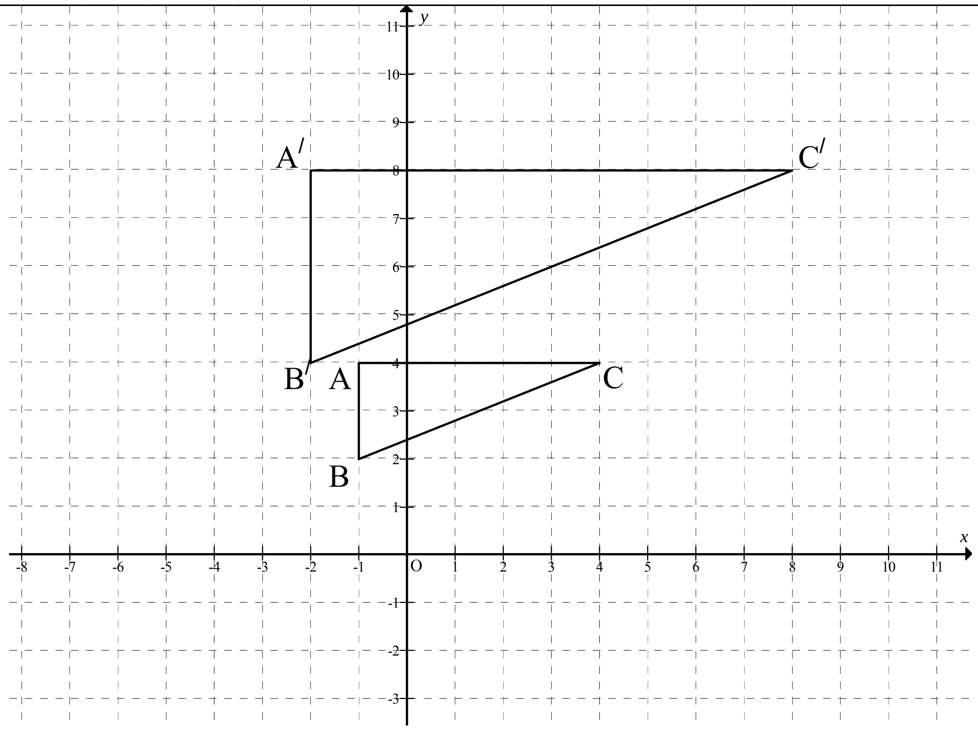
	OR $(1-p)^2+(3+4)^2=50$ $(1-p)^2=50-49$ $(1-p)^2=1$ $1-p=1$ or $1-p=-1$ $p\neq 0$ $p=2$	
4.4	$\text{midpoint of BC}=\left(\frac{-2+2}{2};\frac{0-4}{2}\right)$ $\text{midpoint of BC}=(0\,;-2)$	✓ substitution into midpoint formula ✓midpoint (2)
4.5	Gradient of line = $m_{AB}=1$ Equation of line is: $y+4=1(x-2)$ $y=x-6$	✓ gradients are equal ✓substitution ✓equation (3) **[13]**

QUESTION 5

5.1	Midpoint BD $\left(\frac{0-2}{2};\frac{8-6}{2}\right)$ $=(-1;\ 1)$	✓x-coordinate ✓y-coordinate (2)
5.2	$y=7(-8)+58$ $=2$ $\therefore$ E also on the line.	✓substitution (1)
5.3	$m_{line}=7 \qquad m_{AM}=\frac{2-1}{-8-(-1)}=-\frac{1}{7}$ $m_{line}\times m_{AM}=7\times-\frac{1}{7}=-1$ $\therefore$ AM $\perp$ to the line $\therefore$ The line $y=7x+58$ is a tangent to the circle at A.	✓relationship between line and MA ✓gradient MA ✓product ✓conclusion ✓is a tangent (5)
5.4	$AD=\sqrt{(8-2)^2+(0+8)^2}$ $=\sqrt{36+64}$ $=10$ $AB=\sqrt{(2+6)^2+(-8+2)^2}$ $=\sqrt{64+36}$ $=10$	✓ substitution ✓ answer ✓ substitution ✓ answer (4)
5.5	$m_{AD}=\frac{8-(2)}{0-(-8)}$ $m_{AD}=-\frac{3}{4}$ $m_{AB}=\frac{2-(-6)}{-8-(-2)}$ $=\frac{4}{3}$ $m_{AB}.m_{AD}=\frac{4}{3}\times-\frac{3}{4}$ $=-1$ $D\hat{A}B=90°$ **OR** $BD^2=(8+6)^2+(0+2)^2$ $=200$ $=AD^2+AB^2$ $\therefore D\hat{A}B=90°$	✓ gradient of AD ✓ gradient of AB ✓ conclusion (3) ✓ distance formula ✓ Pythagoras ✓ conclusion (3)
5.6	$\theta=45°$	✓ answer (1)

5.7	Let the radius of circle TNM be r NB = BM (properties of a kite) AN = TZ = r (TZNA is a square) NB = $10 - r$ BD = 2MB $\sqrt{(8-(-6))^2 + (0-(-2))^2} = 2(10-r)$ $\sqrt{200} = 2(10-r)$ $10\sqrt{2} = 2(10-r)$ $r = 10 - 5\sqrt{2}$ $= 2{,}93$ **OR** $Z\hat{M}B = 90°$ $MB = \frac{1}{2}\sqrt{200}$ $= 7{,}07$ $\frac{ZM}{MB} = \tan 22{,}5°$ $ZM = 7{,}07 \tan 22{,}5°$ $= 2{,}93$	✓ NB = BM ✓ AN = TZ = r ✓ NB = $10 - r$ ✓ BD = 2MB ✓BD = $\sqrt{200}$ ✓answer (6) ✓✓tan radius theorem ✓MB ✓✓tan 22,5° ✓answer (6) **[22]**

QUESTION 6

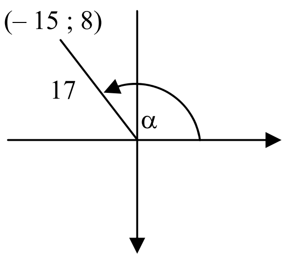
6.1.1	$4 \times 5 = 20$	✓✓answer (2)
6.1.2	$(x\,;\,y) \rightarrow (2x\,;\,2y)$	✓✓ $(2x\,;\,2y)$ (2)
6.1.3		✓ coordinates $A^{/}$ ✓ coordinates $B^{/}$ ✓ coordinates $C^{/}$ (3)
6.1.4	Not rigid. The shape remains the same, whilst the size changes	✓same shape ✓ different size (2)
6.2	Reflection about the line $y = x$: $(x\,;\,y) \rightarrow (y\,;\,x)$ Rotate clockwise about the origin: $(y\,;\,x) \rightarrow (x\,;\,-y)$ Translate 2 left and 3 down: $(x\,;\,-y) \rightarrow (x-2\,;\,-y-3)$ General rule: $(x\,;\,y) \rightarrow (x-2\,;\,-y-3)$	✓✓ reflection ✓✓rotation ✓✓translation (6) **[15]**

QUESTION 7

7.1	$T^{/}\ (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$	✓ x coordinate ✓ y coordinate (2)
7.2	$A^{/}\ (p\cos 135° - q\sin 135°\,;\, q\cos 135° + p\sin 135°)$	✓ x coordinate ✓ y coordinate (2)
7.3	$x' = p\cos(135°) - q\sin(135°)$ $-1-\sqrt{2} = -p\cos 45° - q\sin 45°$ $-1-\sqrt{2} = -p\left(\frac{\sqrt{2}}{2}\right) - q\left(\frac{\sqrt{2}}{2}\right)$ $-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}q$.................(1) and $y' = y\cos(135°) + p\sin(135°)$ $1-\sqrt{2} = -q\cos 45° + p\sin 45°$ $1-\sqrt{2} = q\left(-\frac{\sqrt{2}}{2}\right) + p\left(\frac{\sqrt{2}}{2}\right)$ $1-\sqrt{2} = -\frac{\sqrt{2}}{2}q + \frac{\sqrt{2}}{2}p$..................(2) (1) + (2): $-2\sqrt{2} = -\sqrt{2}q$ $q = 2$ Substitute $q = 2$ into …..(1) $-1-\sqrt{2} = -\frac{\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}(2)$ $-1 = -\frac{\sqrt{2}}{2}p$ $p = \sqrt{2}$ $\therefore A = (\sqrt{2}\,;2)$	✓ reduction ✓ substitution ✓ reduction ✓ substitution ✓ solving simultaneously ✓ answer for y ✓ answer for x (7) **[11]**

	OR $-\frac{\sqrt{2}}{2}(p+q)=-1-\sqrt{2}$ $p+q=-\frac{2}{\sqrt{2}}(-1-\sqrt{2})$ $p+q=\sqrt{2}+2$ and $\frac{1}{\sqrt{2}}(p-q)=1-\sqrt{2}$ $p-q=\sqrt{2}-2$ $p+q=\sqrt{2}+2$ $2p=2\sqrt{2}$ $p=\sqrt{2}$ $q=2$	

QUESTION 8

8.1	$\sin\alpha=\frac{8}{17}$  $\sin\alpha>0$ $\therefore$ in second quadrant $y_\alpha=8$ $r_\alpha=17$ $x_\alpha=-15$ (Pythagoras) $\tan\alpha=-\frac{8}{15}$	✓ sketch ✓ x value ✓ answer (3)
8.2	$\sin(90°+\alpha)=\cos\alpha$ $=-\frac{15}{17}$	✓ reduction ✓ answer (2)
8.3	$\cos 2\alpha=1-2\sin^2\alpha$ $=1-2\left(\frac{8}{17}\right)^2$ $=\frac{161}{289}$	✓ expansion ✓ substitution ✓ answer (3) **[8]**

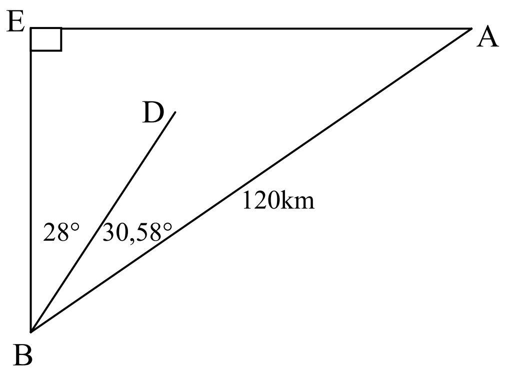
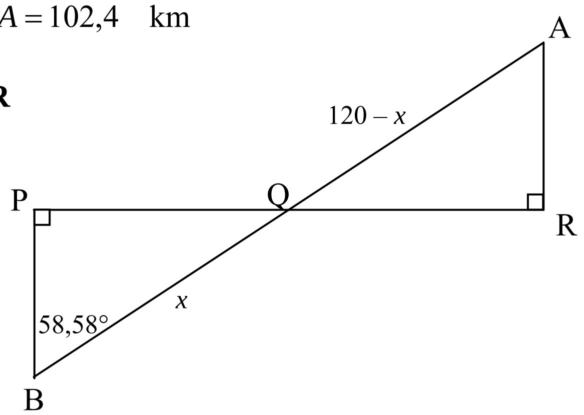
QUESTION 9

9.1	$\sin(90°-x).\cos(180°-x)+\tan x.\cos(-x).\sin(180°+x)$ $=\cos x(-\cos x)+\tan x(\cos x)(-\sin x)$ $=-\cos^2 x-\dfrac{\sin x}{\cos x}\cos x\sin x$ $=-\cos^2 x-\sin^2 x$ $=-(\cos^2 x+\sin^2 x)$ $=-1$	✓ $\sin(90°-x)=\cos x$ ✓ $\cos(180°-x)=-\cos x$ ✓ $\cos(-x)=\cos x$ ✓ $\sin(180°+x)=-\sin x$ ✓ $\tan x=\dfrac{\sin x}{\cos x}$ ✓ simplification ✓ answer (7)
9.2	$\dfrac{\sin 190°\cos 225°\tan 390°}{\cos 100°\sin 135°}$ $=\dfrac{-\sin 10°(-\cos 45°)\tan 30°}{-\sin 10°\sin 45°}$ $=\dfrac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}}$ $=-\dfrac{1}{\sqrt{3}}$	✓ sin 190° = – sin 10° ✓ cos 225° = – cos 45° ✓ tan 390° = tan 30° ✓ cos 100° = – sin 10° ✓ sin 135° = sin 45° ✓ substitution ✓ answer (6)
9.3	$\sin x+2\cos^2 x=1$ $\sin x+2(1-\sin^2 x)=1$ $-2\sin^2 x+\sin x+1=0$ $2\sin^2 x-\sin x-1=0$ $(2\sin x+1)(\sin x-1)=0$ $\sin x=1$ or $\sin x=-\dfrac{1}{2}$ $x=90°+k.360°;k\in Z$ or $x=210°+k.360°;k\in Z$ or $x=330°+k.360°;k\in Z$ OR $x=-150°+k.360°;k\in Z$ or $x=-30°+k.360°;k\in Z$	✓ substitution of identity ✓ standard form ✓ factorisation ✓ answers ✓✓✓ answers (7) **[20]**

QUESTION 10

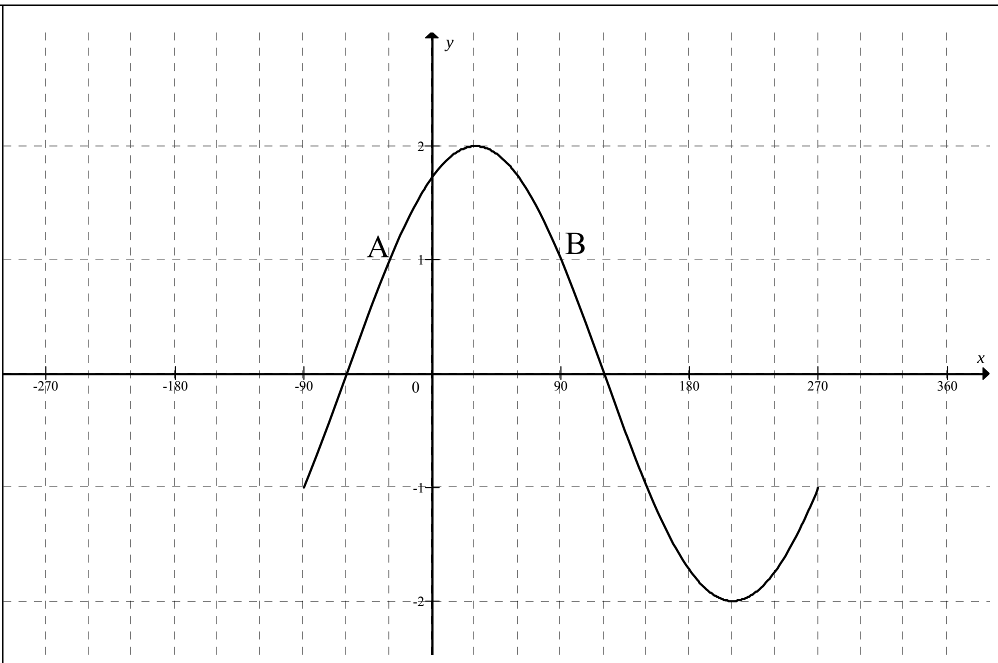
10.1	$\dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{\sin A.\cos B + \cos A.\cos B}{\cos A.\cos B - \sin A.\sin B}$ $= \dfrac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B - \sin A.\sin B} \times \dfrac{\dfrac{1}{\cos A.\cos B}}{\dfrac{1}{\cos A.\cos B}}$ $= \dfrac{\dfrac{\sin A.\cos B}{\cos A.\cos B} + \dfrac{\cos A.\sin B}{\cos A.\cos B}}{\dfrac{\cos A.\cos B}{\cos A.\cos B} - \dfrac{\sin A.\sin B}{\cos A.\cos B}}$ $= \dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$	✓ expansions ✓ divisions ✓ tanA and tanB (3)
10.2	$\tan C = \tan(180° - (A+B))$ $\tan C = -\tan(A+B)$ $\tan C = -\left(\dfrac{\tan A + \tan B}{1 - \tan A.\tan B}\right)$ $\tan C(1 - \tan A.\tan B) = -(\tan A + \tan B)$ $\tan C - \tan A.\tan B.\tan C = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$ OR $\hat{C} = 180° - (\hat{A} + \hat{B})$ (angles in a triangle) $\tan C = \tan(180° - (A+B))$ $\tan C = \tan((180° - A) + (-B))$ $\tan C = \dfrac{\tan(180° - A) + \tan(-B)}{1 - \tan(180° - A).\tan(-B)}$ $\tan C(1 - \tan(180° - A).\tan(-B)) = \tan(180° - A) + \tan(-B)$ $\tan C - \tan C \tan A \tan B = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$	✓ C ✓ rearrange angle ✓ substitution into formula ✓ expansion (4) **[7]**

QUESTION 11

11.1.1	$B\hat{D}A = 208° - 67°$ $= 141°$ $\dfrac{\sin D\hat{B}A}{97} = \dfrac{\sin 141°}{120}$ $\sin D\hat{B}A = 0{,}5087006494...$ $D\hat{B}A = 30{,}58°$ $\therefore$ Bearing of Ship A from Ship B $= 180° - (360° - 208°) + 30{,}58°$ $= 58{,}58°$	✓ $B\hat{D}C = 141°$ ✓ sine rule ✓ substitution ✓ $\hat{B} = 30{,}58°$ ✓ method ✓ answer (6)
11.1.2	$\hat{B} = 30{,}58°$  $\dfrac{EA}{120} = \sin(28° + 30{,}58°)$ $EA = 120\sin(28° + 30{,}58°)$ $EA = 102{,}4$ km **OR**  Let BQ = x, then AQ = $120 - x$ $\sin 58{,}58° = \dfrac{PQ}{x}$ $\quad\sin 58{,}58° = \dfrac{QR}{120 - x}$ $PQ = x.\sin 58{,}58°$ $\quad QR = (120 - x)\sin 58{,}58°$ $PQ + QR = x.\sin 58{,}58° + (120 - x)\sin 58{,}58°$ $= 120\sin 58{,}58°$ $= 102{,}4$	✓ definition ✓ substitution ✓ answer (3) ✓ trigonometeric ratios ✓ sum ✓ answer (3)

11.2	$AB = BC = a = c$ $b^2 = a^2 + c^2 - 2ac \times \cos B$ $b^2 = a^2 + a^2 - 2a \times a \times \cos B$ $b^2 = 2a^2 - 2a^2 \cos B$ $b^2 = 2a^2(1 - \cos B)$ $\frac{b^2}{2a^2} = 1 - \cos B$ $\cos B = 1 - \frac{b^2}{2a^2}$ OR $\sin \frac{B}{2} = \frac{b}{2a}$ $\cos B = 1 - 2\sin^2 \frac{B}{2}$ $= 1 - 2\left(\frac{b}{2a}\right)^2$ $= 1 - \frac{b^2}{2a^2}$	✓ equal sides ✓ cos rule ✓ substitution ✓ simplification (4) ✓ $\sin \frac{B}{2} = \frac{b}{2a}$ ✓ formula ✓ substitution ✓ answer (4) **[13]**

QUESTION 12

12.1		✓ x-intercepts ✓ maximum/minimums (2)
12.2	$\cos(x-30°)=\frac{1}{2}$ $2\cos(x-30°)=1$ See points A and B on the graph	✓manipulation ✓ answer (2)
12.3	$\cos(x-30°)=0{,}5$ $x-30°=60°$ OR $x-30°=-60°$ $x=90°$ $x=-30°$	✓ 60° ✓ 90° ✓ – 30° (3)
12.4	$g'(x)=0$ is at maximum and minimum values of graph x = 30°; 210°	✓✓ answers (2)
12.5	$x\in[-90°\,;-60°)\cup(120°\,;270°]$ OR $-90°\le x<-60°$ or $120°<x\le 270°$	✓ notation ✓✓ critical values (3) **[12]**

TOTAL: 150