education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

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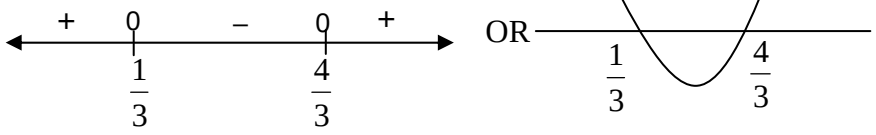
MEMORANDUM

Marks: 150

This memorandum consists of 14 pages.

QUESTION 1

1.1.1	$x(x-1)=30$ $x^2-x=30$ $x^2-x-30=0$ $(x-6)(x+5)=0$ $x=6$ or $x=-5$	✓ simplification ✓ factors ✓answers (3)
1.1.2	$3x^2-5x+1=0$ $x=\frac{-(-5)\pm\sqrt{25-4(3)(1)}}{2(3)}$ $=\frac{5\pm\sqrt{13}}{6}$ $x=1{,}4$ or $x=0{,}2$	✓ substitution ✓ 13 ✓✓ values of x (4)
1.1.3	$-9x^2+15x-4<0$ $9x^2-15x+4>0$ $(3x-4)(3x-1)>0$  $x<\frac{1}{3}$ or $x>\frac{4}{3}$	✓ standard form ✓ factors ✓ test ✓ answer (4)
1.2	Substitute $x = y + 3$ in (2) $(y+3)^2-y(y+3)-2y^2-7=0$ $y^2+6y+9-y^2-3y-2y^2-7=0$ $2y^2-3y-2=0$ $(2y+1)(y-2)=0$ $y=-\frac{1}{2}$ or $y=2$ $x=2\frac{1}{2}$ or $x=5$ **OR**	✓ substitution ✓ standard form ✓ factors ✓ y values ✓ x values (5)

	$y = x - 3$ $x^2 - x(x-3) - 2(x-3)^2 - 7 = 0$ $x^2 - x^2 + 3x - 2(x^2 - 6x + 9) - 7 = 0$ $0 = 2x^2 - 15x + 25$ $0 = (2x-5)(x-5)$ $x = 2\frac{1}{2}$ or $x = 5$ $y = -\frac{1}{2}$ or $y = 2$	✓ substitution ✓ standard form ✓ factors ✓ x values ✓ y values (5)
1.3	$\frac{10^{\frac{2009}{2}}}{10^{\frac{2011}{2}} - 10^{\frac{2007}{2}}}$ $= \frac{10^{\frac{2009}{2}}}{10^{\frac{2007}{2}}(100-1)}$ $= \frac{10}{99}$ **OR** $\frac{10^{1004}\sqrt{10}}{10^{1005}\sqrt{10} - 10^{1003}\sqrt{10}}$ $= \frac{10^{1004}\sqrt{10}}{\sqrt{10}(10^{1005} - 10^{1003})}$ $= \frac{10^{1004}}{10^{1003}(100-1)}$ $= \frac{10}{99}$	✓convert to indices ✓removing $10^{\frac{2007}{2}}$ ✓ answer (3)
1.4	$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$ $= 1 + 2\sqrt{2x^2} + 2x^2 - \sqrt{4}.\sqrt{2x^2}$ $= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2}$ $= 1 + 2x^2$	✓ expansion ✓ simplification ✓ answer (3) – 1 if $\sqrt{x^2} = x$ **[22]**

QUESTION 2

2.1.1	$T_n = 4n+1$ OR $T_n = 5+(n-1)(4)$ $= 4n+1$	✓ ✓✓ Answer only (3) ✓ AP ✓formula and substitution ✓ answer (3)
2.1.2	$T_n = 5(25)^{n-1}$ $= 5^{2n-1}$	✓ GP ✓ answer (2)
2.2	The sequence is $1 ; 1+d ; 1+2d ; 1+3d$; …. (AP) and $1 ; r ; r^2 ; r^3$; …. (GP) $\therefore 1+d = r$ and $d = r-1$ But $1+2d = r^2$ $1+2(r-1) = r^2$ $r^2-2r+1 = 0$ $(r-1)^2 = 0$ $r = 1$ OR $r^2 = 1+2d$ $(1+d)^2 = 1+2d$ $1+2d+d^2 = 1+2d$ $d^2 = 0$ $d = 0$ $r = 1$ $\therefore d = 0$ $\therefore$ the one and only such sequence is 1 ; 1; 1 ; ….. Nomsa is correct.	 ✓ $1+d = r$ ✓ $1+2d = r^2$ ✓ r =1 ✓ $d = 0$ ✓reason (5) **[10]**

QUESTION 3

3.1	–1, 2, 5	✓ all three terms (1)
3.2	$S_n = -1+2+5+8+...\text{to } 100\text{terms}$ $S_n = \frac{n}{2}[2a+(n-1)d]$ $S_{100} = \frac{100}{2}[2(-1)+(100-1)(3)]$ $= 50[-2+297]$ $= 14\,750$ OR	✓ *AP* ✓ n = 100 ✓ substitution ✓ answer (4) **[5]**

	$S_n = -1+2+5+8+...\text{to } 100\text{terms}$ $S_n = \frac{n}{2}[T_1 + T_{99}]$ $S_{100} = \frac{100}{2}[-1+296]$ $= 50[295]$ $= 14\,750$	

QUESTION 4

4.1	The first differences are 1; – 1; – 3; – 5; …. These form a linear pattern $T_n = 1+(n-1)(-2)$ $= 3-2n$	✓ pattern ✓✓ answer (3)
4.2	Between the 35th and 36th terms of the quadratic sequence lies the 35th first difference 35th first difference = 3 – 2(35) = – 67	 ✓ substitution of 35 ✓ answer (2)
4.3	Second difference of terms is -2. $T_n = an^2 + bn + c$ $a = -1$. $3a + b = 1 \Rightarrow -3 + b = 1$ $b = 4$ $a + b + c = -3 \Rightarrow -1 + 4 + c = -3$ $c = -6$ $T_n = -n^2 + 4n - 6$ **OR** Second difference of terms is -2. $T_n = an^2 + bn + c$ $a = -1$. $T_0 = -6 = c$ $T_n = -n^2 + bn - 6$ $-3 = -(1)^2 + (1)b - 6$ $b = 4$ $T_n = -n^2 + 4n - 6$ **OR**	 ✓ $a = -1$ ✓ substitution ✓ $b = 4$ ✓ $c = -6$ (4)

	$2a = -2$ $a = -1$ $3a + b = 1$ $3(-1) + b = 1$ $b = 4$ $a + b + c = -3$ $(-1) + (4) + c = -3$ $c = -6$ $T_n = -n^2 + 4n - 6$	
4.4	Maximum value of T_n is $\frac{4(-1)(-6) - 4^2}{4(-1)} = -2$ The maximum value is negative and hence the sequence can not have any positive terms as the function is maximum valued **OR** $-n^2 + 4n - 6$ $= -(n-2)^2 + 4 - 6$ $= -(n-2)^2 - 2$ The function has a maximum-value of -2 and therefore the pattern will never have positive values.	✓ max value – 2 ✓ explanation (2) ✓ max value – 2 ✓ explanation (2) **[11]**

QUESTION 5

5.1	First year: 150 Second year: 150 + 18 = 168 Third year: $168 + \frac{8}{9}(18) = 184$ Growth = $18\left(\frac{8}{9}\right)^{n-2}$ after n years 17^{th} year growth is $18\left(\frac{8}{9}\right)^{17-2}$ = 3,08 cm	✓ general terms ✓ answer (2)
5.2	Height after 10 years $= 150 + \frac{18\left(1 - \left(\frac{8}{9}\right)^9\right)}{1 - \frac{8}{9}}$ = 150 + 105,8768146 … = 255,88 cm	✓ $n = 9$ ✓ substitution ✓ answer (3)

5.3	Max height = 150 + sum to infinity $= 150 + \dfrac{18}{1-\dfrac{8}{9}}$ = 150 cm + 162 cm = 312 cm The tree will never reach a height of more than 312 cm.	✓ statement ✓ substitution ✓ max height (3) **[8]**

QUESTION 6

6.1	$\frac{1}{2}x^2 = -\frac{1}{x+1} + 1$ $x^2(x+1) = -2 + 2(x+1)$ $x^3 + x^2 = -2 + 2x + 2$ $x^3 + x^2 - 2x = 0$ $x(x^2 + x - 2) = 0$ $x(x+2)(x-1) = 0$ $x = 0 \quad or \quad x = -2 \quad or \quad x = 1$ P(– 2 ; 2) $Q\left(1; \frac{1}{2}\right)$	✓ equating ✓ standard form ✓ factorisation ✓ answer ✓ answer P(– 2 ; 2) ✓ answer $Q\left(1; \frac{1}{2}\right)$ (6)
6.2	For $m > 0$, the equation of the axis of symmetry is $y = x + c$. $1 = (-1) + c$ $c = 2$ Therefore the eqation is $y = h(x) = x + 2$.	✓ gradient ✓ answer (2)
6.3	The equation of the inverse of h is $x = y + 2$ $\therefore y = x - 2$	✓interchange x and y ✓ answer (2)
6.4	$g(x) = -\frac{1}{x+1} + 1 = \frac{-1+x+1}{x+1} = \frac{x}{x+1}$ $LHS = \frac{x}{x+1} + \frac{\frac{1}{x}}{\frac{1}{x}+1} = \frac{x}{x+1} + \frac{1}{x+1} = \frac{x+1}{x+1} = 1$ $RHS = \left(\frac{-x}{1-x}\right)\left(\frac{x-1}{(x-1)+1}\right) = \frac{(1-x)x}{(1-x)x} = 1$ LHS = RHS	✓ simplification of $g(x)$ ✓simplification of LHS ✓simplification of RHS (3) **[13]**

QUESTION 7

7.1	$y \in [-3\,;3]$	✓ answer (1)
7.2	x–value is 7,37° to the left of 90° B(82,63° ; 0,38)	✓ method ✓ x-value ✓ y-value (3)
7.3	Period $= \frac{360°}{3}$ $= 120°$	✓✓ answer (2)
7.4	$x = -180°$	✓ ✓answer (2) **[8]**

QUESTION 8

8.1	$x > 0$ **OR** $x \in (0;\infty)$	✓ answer (1)
8.2	$y = 2^{-x}$	✓ answer (1)
8.3	$y = 0$	✓ answer (1)
8.4.1	Reflect the graph of f over the x-axis	✓ answer (1)
8.4.2	Reflect the graph of f over the line $y = x$. Then shift the graph down 5 units	✓✓ answer ✓ answer (3)
8.5	$\log_2 x < 3$ $-\log_2 x > -3$ $f(x) > -3$ $0 < x < 8$ or $x \in (0\,;8)$	✓ multiplication by -1 ✓✓ answer (3) **[10]**

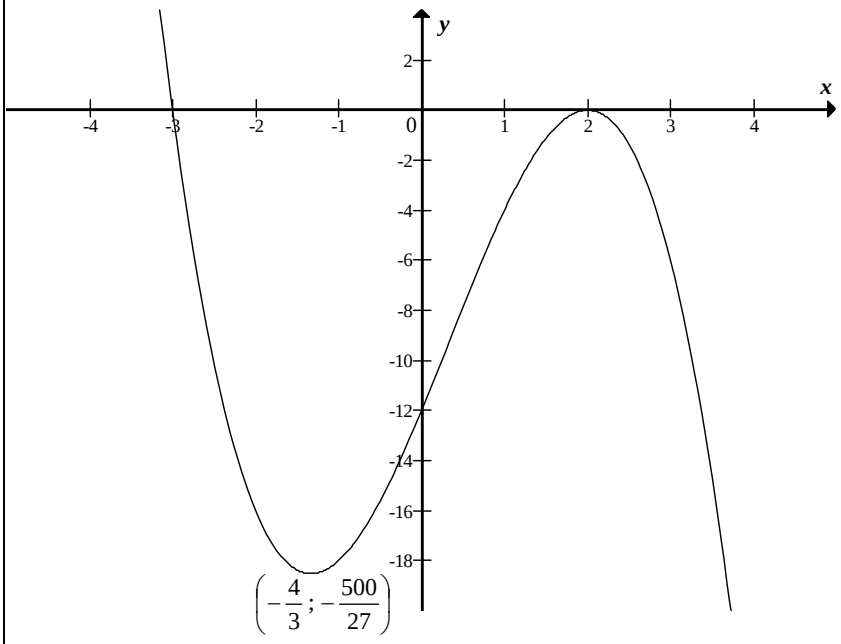
QUESTION 9

9.1	$A = P(1-i)^n$ $15000 = 24000(1-0{,}18)^n$ $0{,}625 = (0{,}82)^n$ $n = \frac{\log 0{,}625}{\log 0{,}82}$ $= 2{,}37$ years	✓ substitution ✓ simplification ✓application of logs ✓ answer (4)
9.2.1	$130\,000\left(1+\frac{0{,}18}{12}\right)^2$ $= 130000\,(1{,}015)^2$ = R 133 929,25	✓✓ substitution ✓ answer (3)
9.2.2a	$133\,929{,}25 = \frac{x[1-(1{,}015)^{-54}]}{0{,}015}$ $2008{,}93875 = x[1-(1{,}015)^{-54}]$ $R3636{,}36 = x$	✓ calculation of A ✓ substitution ✓ answer (3)
9.2.2b	Total = 3636,36 × 54 = R196 363,66	✓answer (1)
9.2.3	$130\,000 = \frac{x[1-(1{,}015)^{-54}]}{0{,}015}$ $1950 = x[1-(1{,}015)^{-54}]$ $R3529{,}68 = x$ Total payments = R 3529,68 × 54 = R 190 602,72	✓ 130 000 ✓substitution ✓ answer ✓ total (4)
9.2.4	R196 363,66 – R190 602,72 =R5 760,96	✓ answer (1) **[16]**

QUESTION 10

10.1	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\frac{-2(x+h)^2+3-(-2x^2+3)}{h}$ $= \lim_{h\to 0}\frac{-2x^2-4xh-2h^2+3+2x^2-3}{h}$ $= \lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $= \lim_{h\to 0}(-4x-2h)$ $= -4x$	✓ method ✓ substitution ✓ simplification ✓ simplification ✓ answer (5)
10.2	$y = x^2 - \frac{1}{2x^3}$ $y = x^2 - \frac{1}{2}x^{-3}$ $\frac{dy}{dx} = 2x + \frac{3}{2}x^{-4}$ $\frac{dy}{dx} = 2x + \frac{3}{2x^4}$	✓✓ answer (2) **[7]**

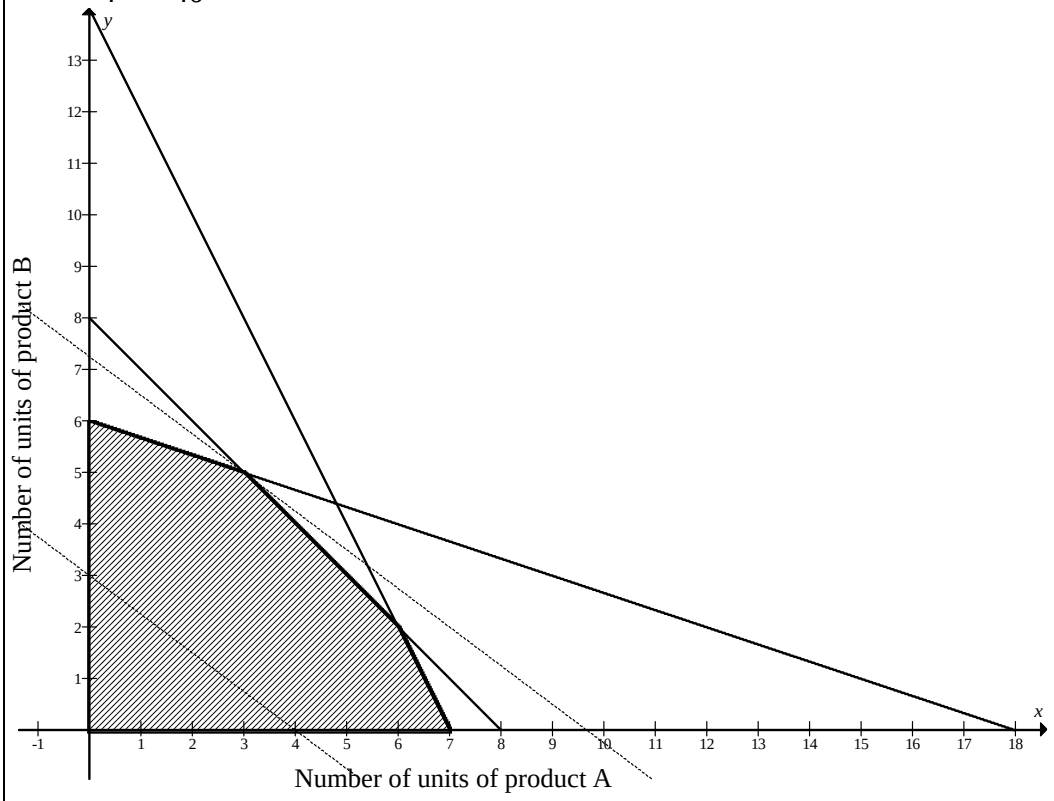
QUESTION 11

11.1	$0 = -x^3 + x^2 + 8x - 12$ $x^3 - x^2 - 8x + 12 = 0$ $(x-2)(x^2 + x - 6) = 0$ $(x-2)(x-2)(x+3) = 0$ $x = 2$ or $x = -3$ intercepts are (2 ; 0) and (– 3 ; 0)	✓ = 0 ✓ factor $(x-2)$ ✓ factor $(x^2 + x - 6)$ ✓ factors ✓ answers (5)
11.2	$f'(x) = -3x^2 + 2x + 8$ $3x^2 - 2x - 8 = 0$ $(x-2)(3x+4) = 0$ $x = 2$ or $x = -\frac{4}{3}$ turning points are (2 ; 0) and $\left(-\frac{4}{3} ; -\frac{500}{27}\right)$ OR (2 ; 0) and (– 1,33 ; – 18,52)	✓ $f'(x) = -3x^2 + 2x + 8$ ✓ $f'(x) = 0$ ✓ factors ✓ answers ✓ answers (5)
11.3		✓ shape ✓ intercepts ✓ turning Pts (3)
11.4	$f''(x) = 0$ $6x - 2 = 0$ $x = \frac{1}{3}$ **OR** $x = \frac{2 - \frac{4}{3}}{2}$ $x = \frac{1}{3}$	✓ method ✓ answer (2)
11.5	(2 ; –3) and $\left(-\frac{4}{3} ; -\frac{581}{27}\right)$ OR (2 ; –3) and $(-1{,}33 ; -21{,}52)$	✓✓ each answer (2) **[17]**

QUESTION 12

12.1	$s(0) = 5(0)^3 - 65(0)^2 + 200(0) + 100$ = 100 metres	✓ $t = 0$ ✓ answer (2)
12.2	$s(t) = 5t^3 - 65t^2 + 200t + 100$ $s'(t) = 15t^2 - 130t + 200$ $s'(4) = 15(4)^2 - 130(4) + 200$ = – 80 metres per minute	✓ $s'(t) = 15t^2 - 130t + 200$ ✓ substitution ✓ answer (3)
12.3	The height car of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change.	✓ speed 80 metres per minute ✓ down (2)
12.4	$s'(t) = 15t^2 - 130t + 200$ $s''(t) = 30t - 130$ $130 = 30t$ $t = 4{,}3\dot{3}$ minutes **OR** $t = -(-130)/2(30)$ $t = 4{,}3\dot{3}$ minutes	✓ $s''(t) = 120t - 520$ ✓ = 0 ✓ answer (3) **[10]**

QUESTION 13

13.1	$x+3y\le 18$ $x+y\le 8$ $2x+y\le 14$ $x,y\ge 0$	✓✓ answer ✓✓ answer ✓✓ answer ✓ answer (7)
13.2	P = 30x + 40y	✓✓ answer (2)
13.3	$y=-\frac{3}{4}x+\frac{P}{40}$  Maximum at (3 ; 5)	✓✓ answer (2)
13.4	$-2<m<-1$	✓✓answer (2) **[13]**

TOTAL: 150