education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATH.1

MATHEMATICS P1

FEBRUARY/MARCH 2010

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 3 diagram sheets and an information sheet.

MORNING SESSION

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers. Answers only will not necessarily be awarded full marks.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. THREE diagram sheets for answering QUESTION 5.3, QUESTION 7.2, QUESTION 8.1, QUESTION 13.2 and QUESTION 13.4 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(x+5)=9$ (4)

1.1.2 $2x^2-2\leq 3x$ (4)

1.2 Solve simultaneously for x and y:

$2+y=-2x$
$-2x^2+8xy+42=y$ (7)

1.3 If $f(x)=\sqrt{4x}$ and $g(x)=x^2$, determine $f(g(9))$. (3)

1.4 If $\dfrac{14}{\sqrt{63}-\sqrt{28}}=a\sqrt{b}$, determine, without using a calculator, the value(s) of a and b if a and b are integers. (4)

[22]

QUESTION 2

Consider the following sequence: 399 ; 360 ; 323 ; 288 ; 255 ; 224 ; …

2.1 Determine the n^{th} term T_n in terms of n. (6)

2.2 Determine which term (or terms) has a value of 0. (3)

2.3 Which term in the sequence will have the lowest value? (1)

[10]

QUESTION 3

3.1 Prove that: $a+ar+ar^2+\ldots$ (to n terms) $=\dfrac{a(r^n-1)}{r-1}$, $r\neq 1$ (4)

3.2 Given the geometric series: $3+1+\dfrac{1}{3}+\ldots$

Calculate the sum to infinity. (3)

[7]

QUESTION 4

Matli's annual salary is R120 000 and his expenses total R90 000. His salary increases by R12 000 each year while his expenses increase by R15 000 each year. Each year he saves the excess of his income.

4.1 Represent his total savings as a series. (4)

4.2 If Matli continues to manage his finances this way, after how many years will he have nothing left to save? (3)

4.3 Matli calculates that if his expenses increase by x rand every year (instead of R15 000 each year), he will spend as much as he earns in the 25^{th} year. Determine x. (2)

[9]

QUESTION 5

Given: $f(x) = \dfrac{2}{x-3} + 1$

5.1 Write down the equations of the asymptotes of f. (2)

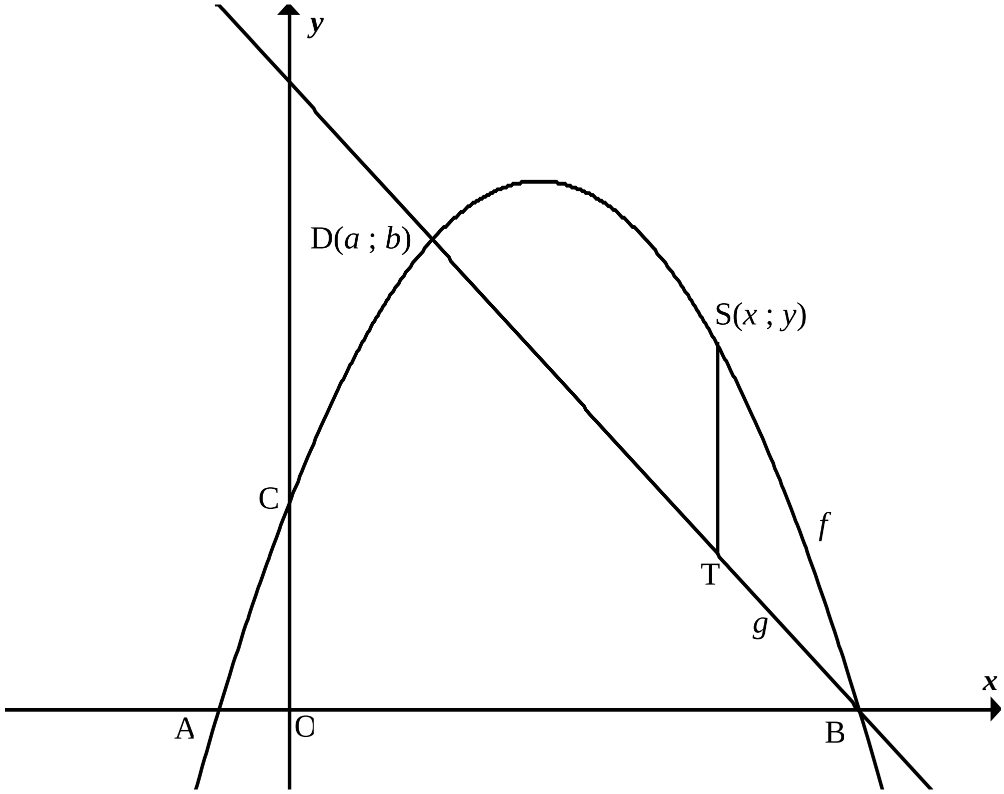
5.2 Calculate the coordinates of the x- and y-intercepts of f. (3)

5.3 Sketch f on the grid provided on DIAGRAM SHEET 1. Show all intercepts with the axes and the asymptotes. (3)

[8]

QUESTION 6

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$ are sketched below.
f and *g* intersect in D and B. A and B are the *x*-intercepts of f.



6.1 Determine the coordinates of A and B. (4)

6.2 Calculate a, the *x*-coordinate of D. (4)

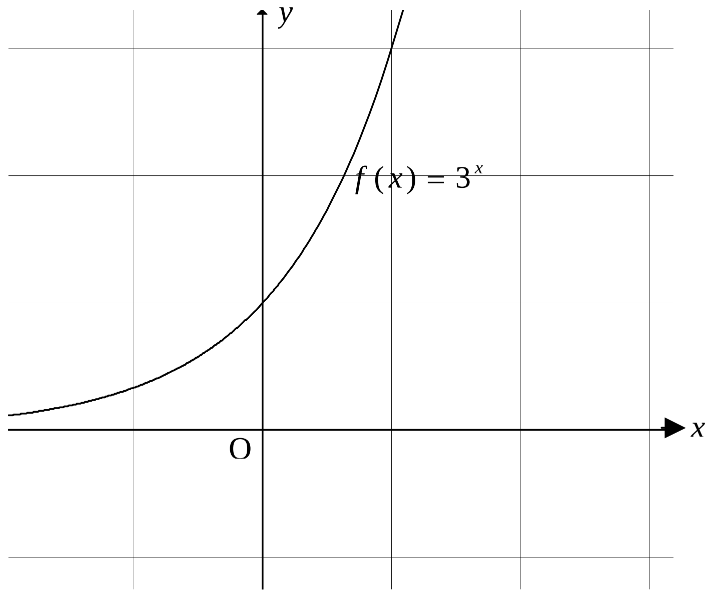
6.3 S(x ; y) is a point on the graph of *f*, where $a \le x \le 8$. ST is drawn parallel to the *y*-axis with T on the graph of *g*. Determine ST in terms of *x*. (2)

6.4 Calculate the maximum length of ST. (2)

[12]

QUESTION 7

The graph of $f(x) = 3^x$ is drawn below.



7.1 Write f^{-1} in the form $y = \ldots$ (1)

7.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x-2)$ on the grid provided in DIAGRAM SHEET 2. (4)

7.3 Use your graphs to solve for x if $\log_3(x-2) < 1$. (2)
[7]

QUESTION 8

Given: $f(x) = \tan(x - 30°)$

8.1 Sketch the graph of $y = f(x)$ for $-90° \le x \le 90°$ on the grid provided in DIAGRAM SHEET 2. (3)

8.2 Write down the equation of an asymptote of *f*. (1)

8.3 Describe in words the transformation of *f* to *g* if $g(x) = \tan(30° - x)$. (2)
[6]

QUESTION 9

9.1 Andrew wants to borrow money to buy a motorbike that costs R55 000,00 and plans to repay the full amount over a period of 4 years in monthly instalments. He is presented with TWO options:

Option 1: *The bank calculates what Andrew would owe if he borrows R55 000,00 for 4 years at a simple interest rate of 12,75% p.a., and then pays that amount back in equal monthly instalments over 4 years.*

Option 2: *He borrows R55 000,00 from the bank. He pays the bank back in equal instalments over 4 years, the first payment being made at the end of the first month. Compound interest at 20% p.a. is charged on the reducing balance.*

9.1.1 If Andrew chooses Option 1, what will his monthly instalment be? (4)

9.1.2 Which option is the better option for Andrew? Justify your answer with appropriate calculations. (4)

9.1.3 What interest rate should replace 12,75% p.a. in Option 1 so that there is no difference between the two options? (3)

9.2 Lindiwe receives a bursary of R80 000,00 for her studies at university. She invests the money at a rate of 13,75% p.a., compounded yearly. She decides to withdraw R25 000,00 at the end of each year for her studies, starting at the end of the first year.

Determine for how many full years will this investment finance her studies. (4)

9.3 Given: $A = P(1 + n\,i)$ where *P* and *i* are positive constants

9.3.1 State whether the graph of *A*, as a function of *n*, is linear, quadratic, exponential or none of these. (1)

9.3.2 Draw a possible graph of *A*, as a function of *n*. (2)

9.3.3 If *n* increases by 1, then determine the increase in *A*. (1)

[19]

QUESTION 10

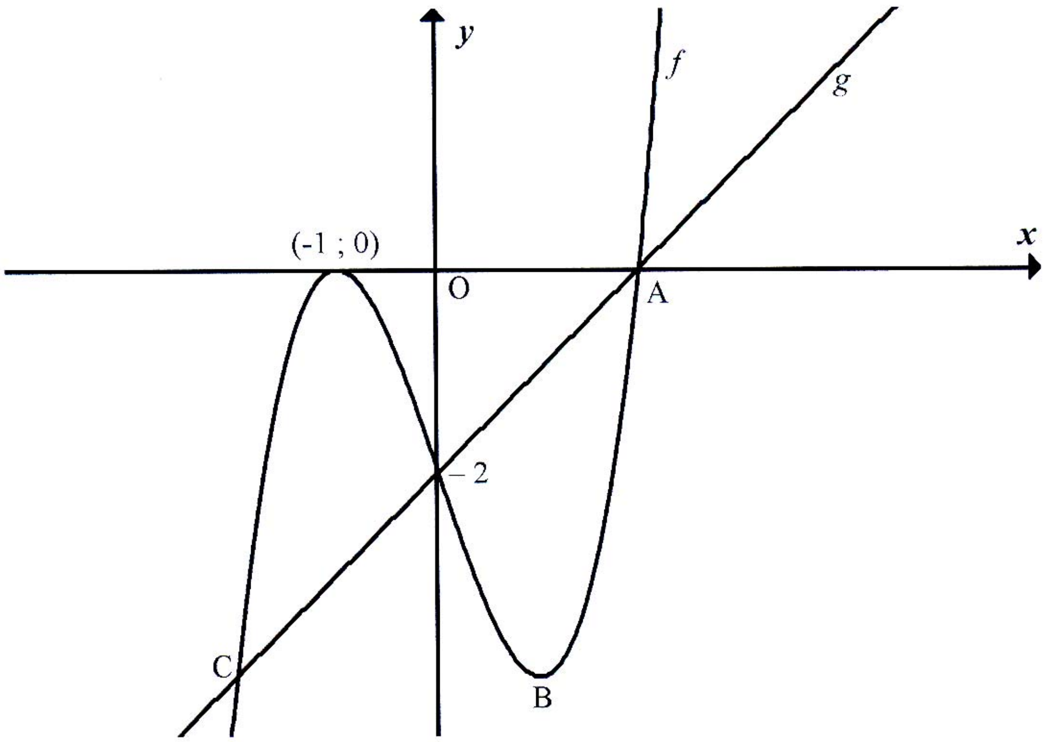
10.1 Differentiate *f* from first principles: $f(x) = \dfrac{1}{x}$ (4)

10.2 Use the rules of differentiation to determine $\dfrac{dy}{dx}$ if $y = (2 - 5x)^2$ (3)

[7]

QUESTION 11

The graph below represents the functions f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$.
A and (– 1 ; 0) are the x-intercepts of f. The graphs of f and g intersect at A and C.



11.1	Determine the coordinates of A.	(1)
11.2	Show by calculation that $a = 1$ and $c = -3$.	(4)
11.3	Determine the coordinates of B, a turning point of f.	(3)
11.4	Show that the line BC is parallel to the x-axis.	(7)
11.5	Find the x-coordinate of the point of inflection of f.	(2)
11.6	Write down the values of k for which $f(x) = k$ will have only ONE root.	(3)
11.7	Write down the values of x for which $f'(x) < 0$.	(2) **[22]**

QUESTION 12

A wire, 4 metres long, is cut into two pieces. One is bent into the shape of a square and the other into the shape of a circle.

12.1 If the length of wire used to make the circle is x metres, write in terms of x the length of the sides of the square in metres. (1)

12.2 Show that the sum of the areas of the circle and the square is given by $f(x) = \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2 - \frac{x}{2} + 1$ square metres. (4)

12.3 How should the wire be cut so that the sum of the areas of the circle and the square is a minimum? (3)
[8]

QUESTION 13

Two computer courses, one an introductory course and the other an advanced course, are taught to Grade 10 and Grade 11 learners by a certain company.

- For the introductory course, each Grade 10 learner requires 4 hours of lessons and each Grade 11 learner requires 2 hours of lessons. The company has a maximum of 32 hours available for this course.
- For the advanced course, each Grade 10 learner requires 2 hours of lessons and each Grade 11 learner requires 4 hours of lessons. The company has a maximum of 36 hours available for this training.
- The total number of learners that can be trained at one time is 10.

Let x represent the number of Grade 10 learners to be trained.
Let y represent the number of Grade 11 learners to be trained.
x and y are positive integers.

13.1 Write down the equations of the constraints. (3)

13.2 Draw these constraints on DIAGRAM SHEET 3 and clearly indicate the feasible region by shading it. (4)

13.3 If the company makes a profit of R60 per hour to train a Grade 10 learner and R80 per hour to train a Grade 11 learner, write down an expression that will represent the hourly profit (P) of this training. (1)

13.4 Using a search line on your graph, determine the number of Grade 10 and Grade 11 learners who must be trained to give the company a maximum profit per hour. Indicate the search line on the graph. (3)

13.5 If the profit function for the company is P = 80x + 60y, per hour, would there be any difference in the optimal solution? If there is, determine the new maximum profit per hour. (2)
[13]

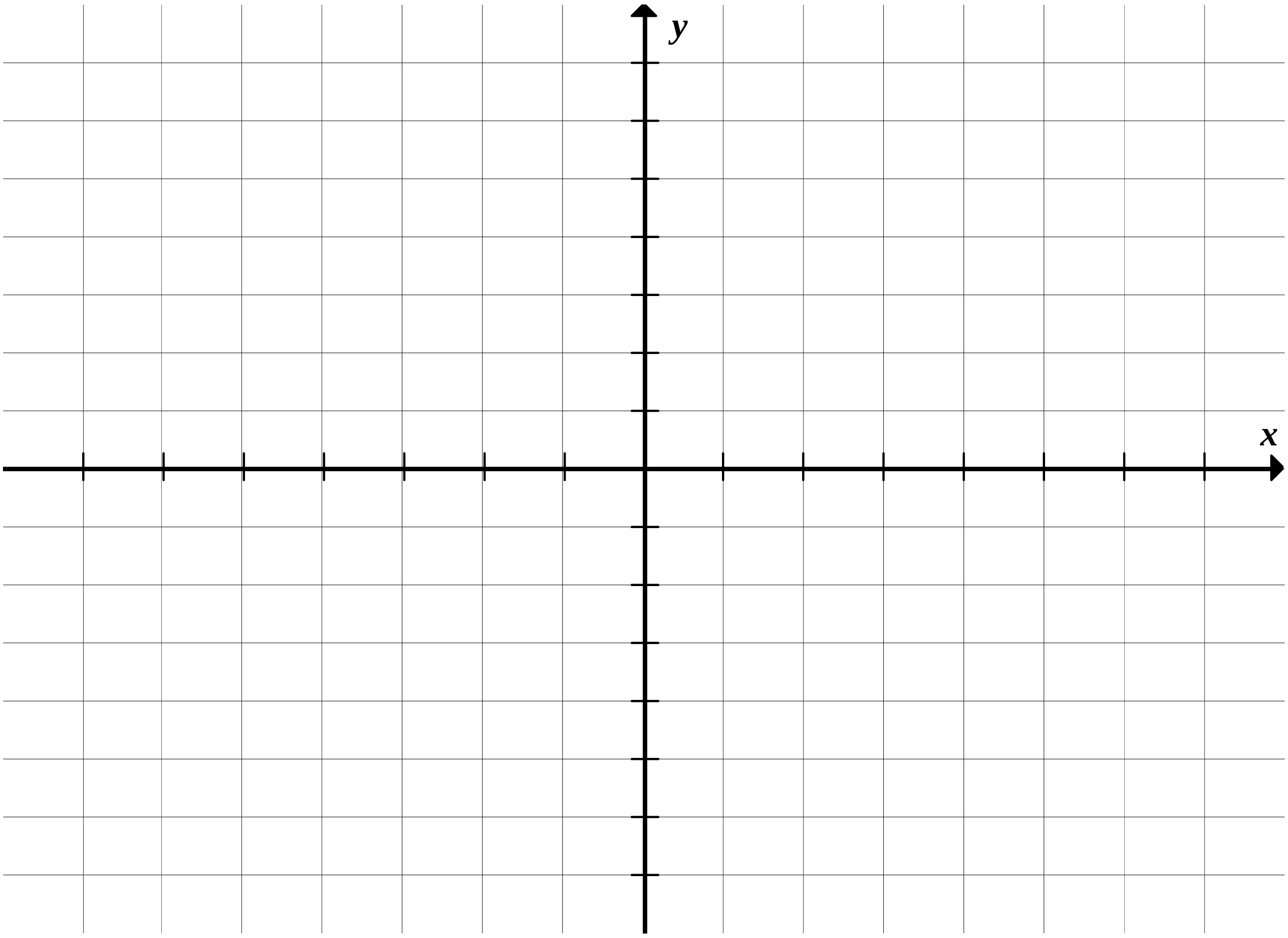
TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 5.3



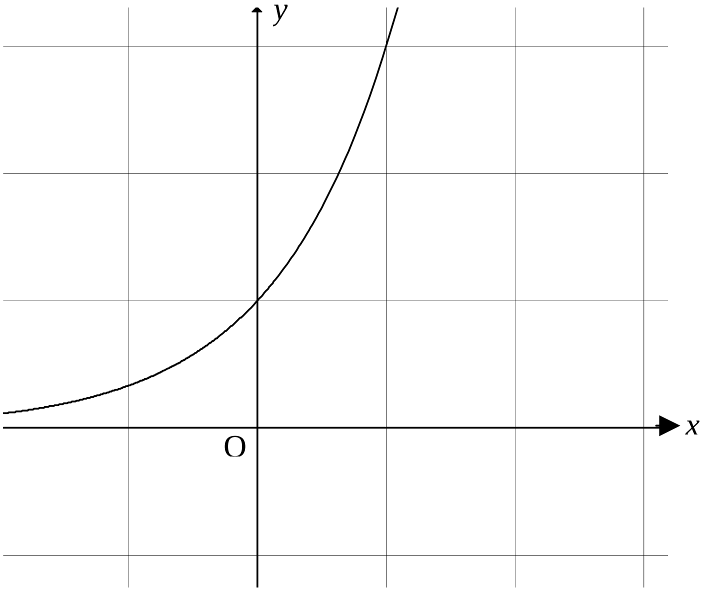
NB.: PLEASE HAND IN TOGETHER WITH YOUR ANSWER BOOK.

CENTRE NUMBER:

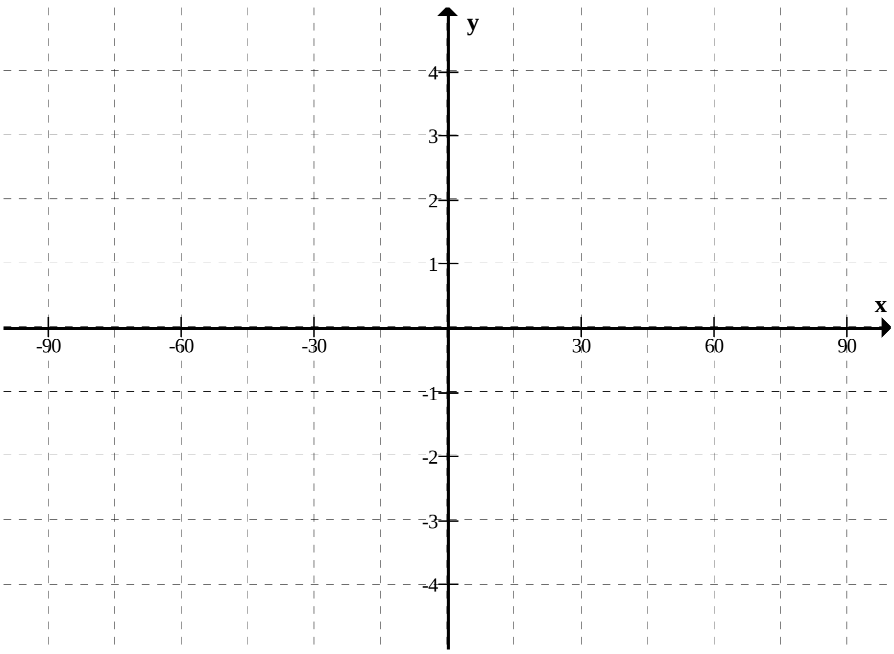
EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 7.2



QUESTION 8.1

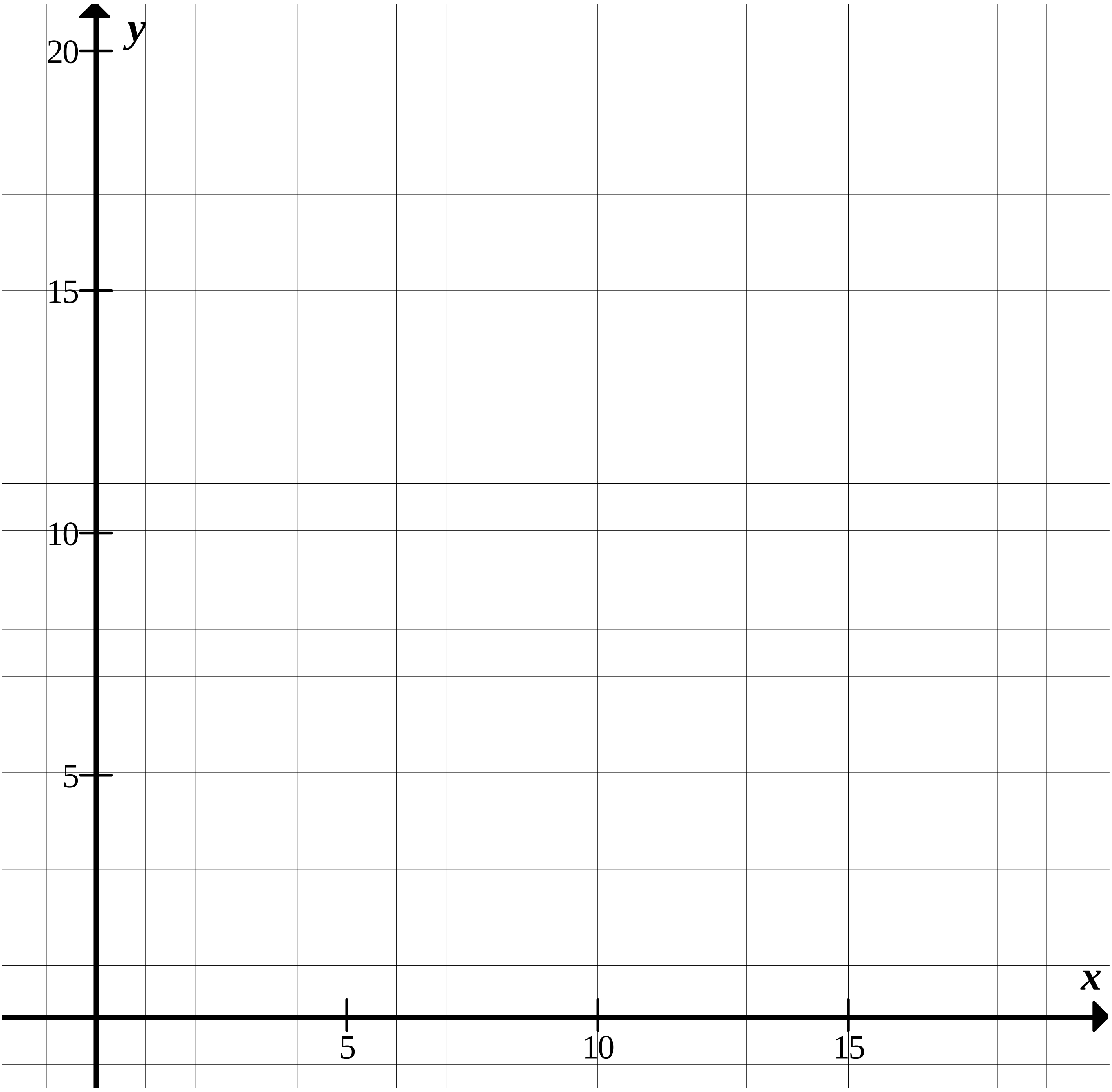


CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 3

QUESTION 13.2 AND 13.4



INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\,;\,y) \to (x\cos\theta + y\sin\theta\,;\,y\cos\theta - x\sin\theta)$ $\quad (x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\,y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$