Question 1)

x >= 0, y>=0

Recurrence relation:

add(x, y) = x when y = 0
add(x, y) = 1 + add(x, y-1) when y > 0

Base Case:
When y is 0, recurrence relation is add(x,y) = x, which is true
Since when we add integers x, y where y =0, x + y = x
The actual addition result and value given by recurrence relation are same.

Hence, recurrence relation is true for y = 0

Hypothesis: Assuming the recurrence is true for y = k. where k > 0
We assume, when y = k
x+y
= x + k
= 1 + add(x, k - 1)

Inductive Step: Recurrence relation is true for y = k+1
When we add integers x, y where y = k+1
x+y
= x+k+1
= 1+ add(x, d-1) + 1
= 1+ add(x, k)
= 1+ add(x, (k+1) – 1)
= 1+ add(x, y - 1

Hence, recurrence is true for y = k+1
Therefore
Both base case and inductive step are proved true.

Question 2)