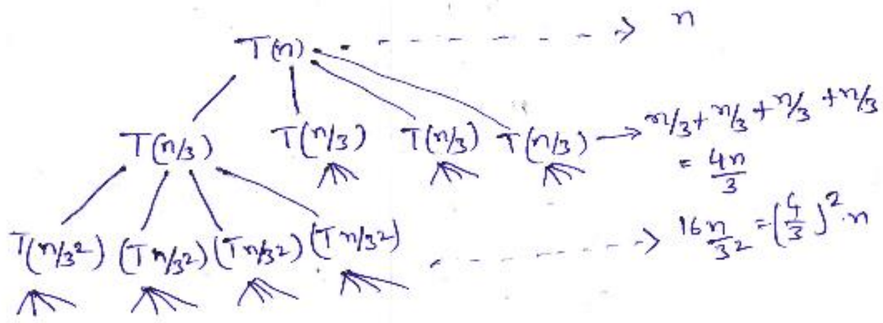
Answer:

(a) The given information is

$$T(n) = 4T(n/3) + n$$



K terms

$$T(n/3^k) \ldots T(n/3^k) \ldots 4^k \text{ terms}$$

Let us Assume $n/3^k = 1$

$$n = 3^k$$

$$\boxed{k = \log_3 n}$$

So, T(n) will be

$$T(n) = 4^k (T(1)) + n + \frac{4}{3} \cdot n + \left(\frac{4}{3}\right)^2 \cdot n + \cdots \left(\frac{4}{3}\right)^k \cdot n$$

Assume $T(1) = \text{Constant } c$

$$T(n) = 4^k \cdot c + n \underbrace{\left(1 + \frac{4}{3} + \left(\frac{4}{3}\right)^2 + \cdots + \left(\frac{4}{3}\right)^k\right)}_{k-1 \text{ terms of G.P}}$$

$$T(n) = 4^k \cdot c + n \left(\frac{1\left(\left(\frac{4}{3}\right)^k - 1\right)}{\frac{4}{3} - 1} \right)$$

$$= 4^k \cdot c + n \cdot 3\left(\left(\frac{4}{3}\right)^k - 1\right)$$

putting the value of 'k' in the above equation.

$$T(n) = c \cdot 4^{\log_3 n} + 3n\left(\frac{4^{\log_3 n}}{3^{\log_3 n}} - 1 \right)$$

We know that $a^{(\log_c b)} = b^{(\log_c a)}$

So, $T(n) = C \cdot n^{(\log_3 4)} + 3n\left(\frac{n^{\log_3 4} - 1}{n}\right)$

$T(n) =$ we can ignore the constant terms

$= C \cdot n^{(\log_3 4)} + 3n^{(\log_3 4)} - 3$

$$\boxed{T(n) = \Theta\, n^{(\log_3 4)}}$$

(b)

$\rightarrow$ Assuming $T(n) \leq C \cdot n^{\log_3 4}$ for any value of 'C'

$T(n) = 4 \cdot T(n/3) + n \longrightarrow (1)$

From Assumptions

$T(n/3) \leq C \cdot (n/3)^{\log_3 4}$

$T(n/3) \leq \frac{C \cdot n^{\log_3 4}}{4} \quad \left(3^{\log_3 2} = 4^{\log_3 3} = 4\right)$

So, the maximum value possible is

$T(n/3) = \frac{C}{4} \cdot n^{\log_3 4}$

putting the above value in equation (1)

$T(n) = 4 \cdot \frac{C}{4} \cdot n^{\log_3 4} + n$

$T(n) = C \cdot n^{\log_3 4} + n$

$$\boxed{T(n) > C \cdot n^{\log_3 4}}$$

So, Here our assumption fails. So, we can't go further to prove the solution.

It can not be solved by this assumption.

As our assumption was wrong.

(c) But, we can see that our assumption goes wrong by a lower order term (lower than $n^{\log_3 4}$)

So, We can make a new assumption.

By subtracting a lower order term in Previous assumption. Which will Componsate the 'n' term in the equation

So, Let's Assume $T(n) \leq (C \cdot n^{\log_3 4} - kn)$

$$T(n/3) \leq \left(\frac{Cn}{4}^{\log_3 4} - \frac{kn}{3}\right)$$

The maximum possible value of $T(n/3)$ is

$$T(n/3) = \left(\frac{c}{4} \cdot n^{\log_3 4} - \frac{kn}{3}\right)$$

putting the value of $T(n/3)$ in the equation(1)

We get

$$T(n) = 4\left(\frac{c}{4} \cdot n^{\log_3 4} - \frac{kn}{3}\right) + n$$

$$= C \cdot n^{\log_3 4} + n - \frac{4kn}{3}$$

$$= C \cdot n^{\log_3 4} + n\left(1 - \frac{4k}{3}\right)$$

So, We can Conclude that

$$\boxed{T(n) \leq C \cdot n^{\log_3 4} - kn} \quad (\text{for } k \geq 3)$$

So, Here Our Assumption was right.

We can say now that.

$$\boxed{T(n) = \Theta\left(n^{\log_3 4}\right)}$$

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