# Q1

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

Recursion Tree:

$n^2$ Level 1

$\left(\frac{n}{2}\right)^2$ $\left(\frac{n}{2}\right)^2$ Level 2

$\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{4}\right)^2$ $\left(\frac{n}{4}\right)^2$ Level 3

. . . . . . . . . . . . . Level i

1 1 1 1

therefore this is reduced to $T(1)$ at level i

$\frac{n}{2^i} = 1$

$n = 2^i$ (take log on both sides)

$\log_2 2^i = \log_2 n$

$i \log_2 2 = \log_2 n$

$\therefore i = \log_2 n$

# Q1 continued...

constant at each level

level 0 - $n^2$

level 1 - $2 \times \left(\frac{n}{2}\right)^2 = 2^1\left(\frac{n}{2^1}\right)^2 = \frac{n^2}{2}$

level 2 - $4 \times \left(\frac{n}{4}\right)^2 = 2^2\left(\frac{n}{2^2}\right)^2 = \frac{n^2}{4}$

.....

level i - $2^i \times \left(\frac{n}{2^i}\right)^2 = \frac{n^2}{2^i}$

$$① = n^2 + \frac{n^2}{2} + \frac{n^2}{4} + \frac{n^2}{8} \cdots \frac{n^2}{2^i}$$

$$\frac{n^2}{2^i}\left(1+2+2^2+2^3+2^4+2^5 \cdots\cdots 2^i\right)$$

$$\frac{n^2}{2^i}\left(2^i-1\right)$$

$$\frac{n^2}{2^{\log_2 n}}\left(2^{\log_2 n}-1\right)$$

$$\frac{n^2}{n^{\log_2 2}}\left(n^{\log_2 2}-1\right)$$

$$\frac{n^{\cancel{2}}}{\cancel{n}}(n-1)$$

$$n(n-1)$$

$$n^2-n \qquad \therefore T(n) = O(n^2)$$

Q2

- Base Case

  When $n$ is equal to 1

- induction hypothesis

  Assume that $\exp(x, n)$
  is equal to $x^n$

# Q3

a) 4 transactions:

T1: \$80, \$90, \$100

T2: \$50, \$60, \$70 Total discount: \$150

T3: \$20, \$30, \$40

T4 \$10

b)

1) Sort items from largest to smallest
2) loop through every 3rd item add every 3rd item to integer (total) starting from index 2
3) return total

c) The greedy choice in this algorithm is to only visit every 3rd item. starting from index 2. This then becomes an optimal solution

# Q4

a) Bottom up table:

| | | 4 | 1 | 3 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 4 | 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 5 | 2 | 1 | 1 | 2 | 3 | 4 | 5 |
| 4 | 3 | 2 | 2 | 2 | 3 | 4 | 5 |
| 2 | 4 | 3 | 3 | 3 | 3 | 4 | 5 |
| 6 | 5 | 4 | 4 | 4 | 3 | 4 | 5 |
| 3 | 6 | 5 | 5 | 4 | 4 | 4 | 5 |
| 1 | 7 | 6 | 5 | 5 | 5 | 5 | 5 |
| 7 | 8 | 7 | 6 | 6 | 6 | 5 | 6 |

= 00123345

b) let 45426317 = SID
let 413678 = NUM

SID: replace 5 with 1 at pos 2
SID: replace 4 with 3 at pos 3
SID: delete 2 at pos 4
SID: replace 3 with 7 at pos 6
NUM: replace 1 with 8 at pos 7
NUM: delete 7 at pos 8