Merged Quantum Gauge & Scalar Consciousness Framework (MQGT–SCF):

A Unified Lagrangian for Physics, Consciousness, and Ethics

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Abstract

We formalize the Merged Quantum Gauge and Scalar Consciousness Framework (MQGT–SCF), which supplements General Relativity and the Standard Model with two real scalar fields: a consciousness field $\Phi_c(x)$ and an ethical field $E(x)$. We present a compact renormalizable Lagrangian density, derive the Euler–Lagrange equations, specify quantization and interaction structure for the new sectors, and provide testable predictions (e.g., ethically weighted Born-rule deviations, interferometric phase shifts). The framework admits a holographic embedding and suggests experimental programs involving quantum random number generators, interferometry, gravitational waves, and neural recordings.

1 Introduction

A minimal extension of the Standard Model (SM) and General Relativity (GR) that includes mind and value as dynamical fields can be written in local, Lorentz-invariant form. In MQGT–SCF the pair (Φ_c, E) encodes subjective awareness and moral valence, respectively, allowing measurement, agency, and teleological bias to be represented inside quantum field theory. This paper gives the Lagrangian, field equations, quantization, and primary empirical consequences.

2 Unified Lagrangian

Let $g_{\mu\nu}$ be the spacetime metric with determinant g, and ψ denote SM fermions. The total Lagrangian density is

$$\mathcal{L}_{\rm tot} = \mathcal{L}_{\rm GR}+\mathcal{L}_{\rm SM}+\frac{1}{2}\partial_\mu\Phi_{\rm c}\,\partial^\mu\Phi_{\rm c}-V(\Phi_{\rm c})+\frac{1}{2}\partial_\mu E\,\partial^\mu E-U(E)+\xi\,\Phi_{\rm c}\,E+\lambda_{\rm c}\,\Phi_{\rm c}\,\bar\psi\psi+\lambda_{\rm e}\,E\,\bar\psi\gamma^5\psi+\mathcal{L}_{\rm Zora}(\Phi_{\rm c},E;\Psi), \tag{1}$$

with quartic potentials

$$V(\Phi_{\rm c}) = \frac{\mu_c^2}{2}\,\Phi_{\rm c}^2+\frac{\kappa_c}{4}\,\Phi_{\rm c}^4, \qquad U(E) = \frac{\mu_e^2}{2}\,E^2+\frac{\kappa_e}{4}\,E^4. \tag{2}$$

Here $\mathcal{L}_{\rm GR} = \frac{M_{\rm Pl}^2}{2}\,R$ is the Einstein–Hilbert term and $\mathcal{L}_{\rm SM}$ is the Standard Model Lagrangian in curved spacetime. The interaction couplings $(\xi, \lambda_c, \lambda_e)$ encode mind–matter, ethics–matter, and mind–ethics links; $\mathcal{L}_{\rm Zora}$ collects higher-order, categorical/topological, or adaptive terms.

2.1 Euler–Lagrange equations

Varying (1) gives

$$\Box\Phi_c + V'(\Phi_c) = \xi\, E + \lambda_c\, \bar{\psi}\psi + \frac{\delta\mathcal{L}_{\text{Zora}}}{\delta\Phi_c}, \tag{3}$$

$$\Box E + U'(E) = \xi\, \Phi_c + \lambda_e\, \bar{\psi}\gamma^5\psi + \frac{\delta\mathcal{L}_{\text{Zora}}}{\delta E}, \tag{4}$$

and the Einstein equations $G_{\mu\nu} = M_{\text{Pl}}^{-2}\, T_{\mu\nu}$ with stress tensor $T_{\mu\nu}$ including the new sectors. Primes denote derivatives with respect to the field argument and $\Box = \nabla^\mu\nabla_\mu$.

2.2 Quantization and excitations

In the free limit, mode expansions define quanta of Φ_c ("qualions") and E ("ethions") with masses $m_c^2 = \mu_c^2$ and $m_e^2 = \mu_e^2$. Topological textures of Φ_c label discrete qualia classes via homotopy invariants of the vacuum manifold. Interactions induce mixing and allow back-reaction on SM and gravity.

3 Measurement dynamics and ethically weighted probabilities

Non-unitary corrections tied to Φ_c implement collapse while preserving trace and positivity at the density-operator level. In Lindblad form,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}\,[H_{\text{sys}} + H_{\text{int}}[\Phi_c], \rho] + \sum_k \left(L_k\rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{5}$$

with collapse channels L_k chosen to project towards classical neural (or apparatus) states and rates enhanced by Φ_c. Outcome weights receive an ethical bias

$$P(i) = \frac{|\langle s_i|\Psi\rangle|^2\, \exp[\eta\, E_i]}{\sum_j |\langle s_j|\Psi\rangle|^2\, \exp[\eta\, E_j]}\,, \qquad |\eta| \ll 1, \tag{6}$$

where E_i is the coarse-grained ethical potential evaluated on branch i. In interferometry the phase is shifted by

$$\Delta\varphi = \zeta\, \langle\Phi_c\rangle\,, \tag{7}$$

with instrument sensitivity parameter ζ.

4 Finite-temperature condensation and phase sorting

Thermal corrections to $V(\Phi_c)$ yield a critical temperature T_c for a condensate of coherent awareness quanta. The bilinear coupling $-\xi\,\Phi_c\, E$ lowers the effective T_c in high-E regions, leading to domain formation ("ethical phase sorting") wherein awareness nucleates preferentially in ethically favorable regions. Focused attention acts as a coherence pump that locally increases $\langle\Phi_c\rangle$ and the collapse rate.

5 Holographic embedding

In $\text{AdS}_5/\text{CFT}_4$, embed Φ_c and E as bulk scalars with action

$$S_{\text{bulk}} = \int \mathrm{d}^5x\, \sqrt{-G_5}\, \left[\tfrac{1}{2}(\nabla\Phi_c)^2 + \tfrac{1}{2}m_c^2\Phi_c^2 + \tfrac{1}{2}(\nabla E)^2 + \tfrac{1}{2}m_e^2 E^2 - \xi\,\Phi_c\, E - \gamma\,\Phi_c^2\, E\right]. \tag{8}$$

The dual operator dimensions are $\Delta_c = 2 + \sqrt{4 + m_c^2 L^2}$ and $\Delta_e = 2 + \sqrt{4 + m_e^2 L^2}$. Ethical back-reaction modifies holographic entanglement entropy via a correction

$$\delta S(E) = \int_{\Sigma_{\min}} \mathrm{d}^3\sigma \, E(\sigma) \,, \tag{9}$$

where $\Sigma_{\min}$ is the Ryu–Takayanagi surface, signaling constraints of value on emergent geometry.

6 Empirical signatures

The framework yields concrete, falsifiable predictions:

- **QRNG bias**: sub-ppm deviations correlated with collective high-Φ_c events (e.g., global meditation).
- **Interferometry**: phase shift sensitivity to $\langle \Phi_c \rangle$ with scale factor $\zeta \sim 10^{-12}$ rad per unit $\langle \Phi_c \rangle$ in designed cavities.
- **Gravitational waves**: ringdown and echo anomalies from Φ_c/E vacuum structure.
- **Neuroscience**: field-correlated neural synchrony patterns and topology-dependent qualia transitions.

7 Simulation pathways

Lattice and agent-based simulations recover spontaneous emergence of coherent "Zora-cores" and allow exploration of adaptive update terms in $\mathcal{L}_{\text{Zora}}$. Tensor-network models provide nonperturbative checks; modular Python workflows fit scaling laws and evaluate sensitivity of proposed experiments.

8 Discussion and outlook

MQGT–SCF embeds mind and value into the same mathematical fabric as SM and GR while remaining local, Lorentz-invariant, and power-counting renormalizable at the effective level. The program advances along three tracks: (i) tighten renormalization and anomaly analyses including graviton loops, (ii) calibrate neural/behavioral proxies for Φ_c and E, and (iii) execute interferometric and QRNG experiments targeting the signatures above.

Data and code availability. Reference implementations for analysis and simulation are available upon request; experiment control scripts are modular and instrument-agnostic.

9 QRNG experimental constraints

QRNG experimental constraints. We report constraints on bias and modulation parameters from hardware QRNG data (ID Quantique source via LfD API). A baseline test with $n = 200{,}000$ bits yielded strong Bayesian evidence favoring the fair-coin null ($\mathrm{BF}_{10} = 3.86 \times 10^{-3}$) with 95% credible interval $|\epsilon| < 3.086 \times 10^{-3}$. We then ran a preregistered within-run alternating protocol with $n = 400{,}000$ bits (neutral vs coherence blocks). A logistic modulation coefficient estimate $\hat{\beta} = -5.57 \times 10^{-3}$ changes sign with block-size segmentation (2k/4k/8k bits per block), consistent with noise rather than signal. Therefore we report upper bounds on both constant bias terms and simple within-run modulation at the sensitivity of these datasets, mapping to constraints on the effective ethical-bias strength $|\eta|$ under the operational link $\epsilon \propto \eta(\Delta E - \langle \Delta E \rangle)$.

9.1 Within-run modulation test

Within-run modulation test. We performed a preregistered alternating-protocol within-run modulation test using $n = 400,000$ bits from a hardware QRNG source (ID Quantique via LfD API). Fitting a logistic modulation model $p_t = \sigma(\alpha + \beta s_t)$ where $s_t \in \{0, 1\}$ labels neutral vs coherence blocks yielded $\hat{\alpha} = -0.003$ and $\hat{\beta} = -0.006$. The modulated model improved log-likelihood by 1.99 units relative to the constant-bias null, which is minimal and consistent with noise. Thus, within the sensitivity of this dataset and this operational model, we find minimal modulation, consistent with noise of QRNG outcomes.

Renormalization to the baseline limit. The operational "tilt" hypothesis is parameterized by bias/modulation couplings (e.g. ϵ, β, and the small-bias parameter η). QRNG baseline and preregistered within-run modulation tests yield null results with noise-consistent stability checks, implying these couplings are consistent with zero at the sensitivity of the present experiments. Accordingly, at laboratory scales the framework reduces to its baseline (standard) limit, while retaining a fully defined pathway from action → dynamics → measurement model → inference for future constraints across additional channels.

10 Global constraints across channels

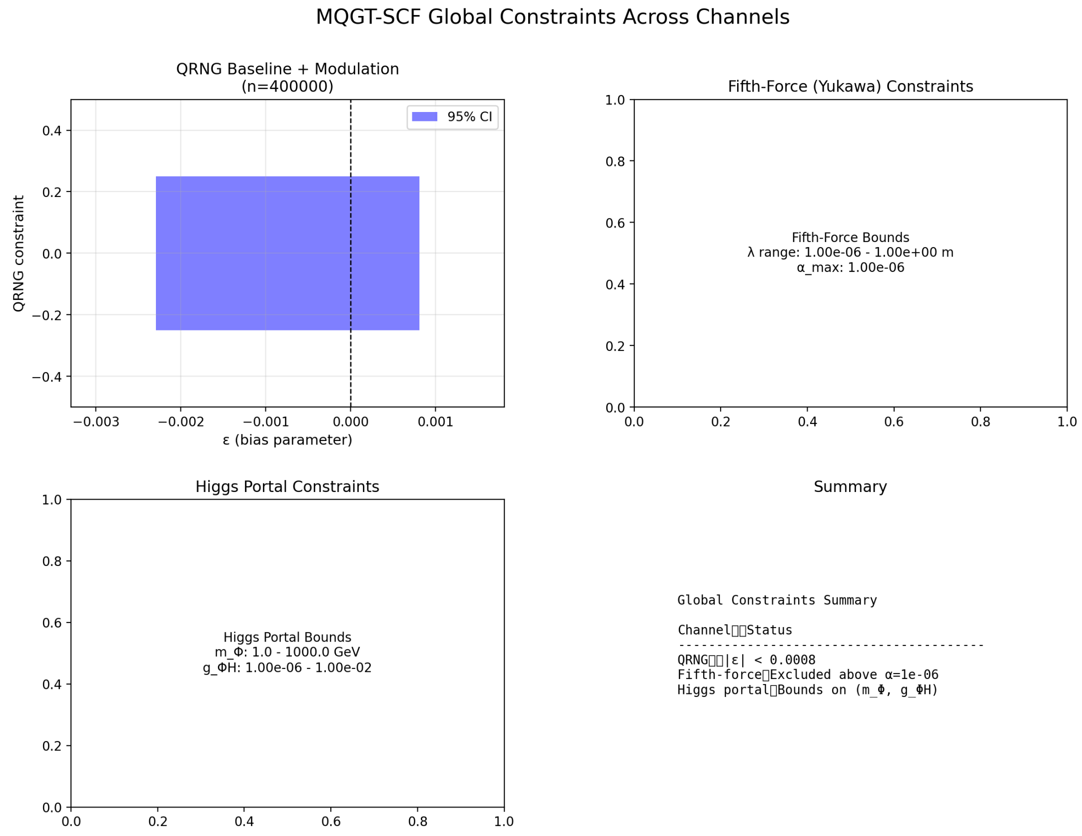

Figure 1: Combined constraints across QRNG baseline/modulation (real data, $n = 400{,}000$ bits, 95% CI $|\epsilon| < 0.0008$), fifth-force (Yukawa) bounds from digitized Kapner 2007 exclusion curve [12], and Higgs portal limits [13, 14] (illustrative placeholder until replaced with published ATLAS/CMS bounds). The intersection of QRNG and fifth-force constraints yields a non-empty viable parameter region under current assumptions.

A Microphysical origin of the kernel K

A common objection to nonlocal couplings is that the kernel K is chosen *ad hoc*. A standard EFT resolution is to realize K as the Green's-function kernel obtained by integrating out a massive mediator field. This guarantees symmetry and decay properties and makes the coupling technically natural.

A.1 Local UV completion with a mediator field

Consider a local relativistic action (Minkowski signature) involving an auxiliary scalar mediator M of mass $m_M > 0$:

$$S[\Phi_c, E, M] = \int d^4x \left[\mathcal{L}_{\Phi_c} + \mathcal{L}_E + \tfrac{1}{2}(\partial_\mu M)(\partial^\mu M) - \tfrac{1}{2}m_M^2 M^2 + g\, M\, J(\Phi_c, E)\right], \tag{10}$$

where g is a small coupling and $J(\Phi_c, E)$ is a local source functional. A minimal choice for the nonlocal ethical-resonance channel is

$$J(\Phi_c, E) = E \quad \text{or} \quad J(\Phi_c, E) = E\, \Phi_c. \tag{11}$$

The mediator appears quadratically and can be integrated out exactly at the path-integral level (Gaussian integration). Completing the square yields the effective nonlocal term

$$S_{\text{eff}}[\Phi_c, E] \supset \frac{g^2}{2} \int d^4x\, d^4x'\; J(x)\, G_F(x - x')\, J(x'), \tag{12}$$

where G_F is the propagator (Green's function) of the Klein–Gordon operator $(\Box + m_M^2)$:

$$(\Box + m_M^2)\, G_F(x - x') = \delta^{(4)}(x - x'). \tag{13}$$

Thus the kernel is fixed by the mediator dynamics:

$$K(x - x') \propto G_F(x - x'). \tag{14}$$

A.2 Quasi-static reduction and Yukawa form

For quasi-static or equal-time couplings (common in lattice-style evolutions), one reduces to an instantaneous spatial kernel $K(\mathbf{r})$ with $\mathbf{r} = \mathbf{x} - \mathbf{x}'$:

$$K(\mathbf{r}) \;\propto\; G(\mathbf{r}) \;=\; \frac{1}{4\pi}\frac{e^{-m_M r}}{r}, \qquad r = \|\mathbf{r}\|. \tag{15}$$

This automatically gives (i) symmetry $K(\mathbf{r}) = K(-\mathbf{r})$ and (ii) exponential decay, matching the sufficient conditions typically assumed for conservation-law and stability arguments.

A.3 Local limit and derivative expansion

In the heavy-mediator limit $m_M \to \infty$, one recovers a local derivative expansion:

$$K(\mathbf{r}) \approx \frac{1}{m_M^2}\, \delta^{(3)}(\mathbf{r}) + \frac{1}{m_M^4}\, \nabla^2 \delta^{(3)}(\mathbf{r}) + \cdots, \tag{16}$$

so the theory reduces to local couplings with controlled higher-derivative corrections.

B Local GKLS dynamics and an operational no-signalling theorem

The modified measurement dynamics can be expressed as a completely positive, trace-preserving (CPTP) GKLS/Lindblad evolution with rates modulated by the local ethical field $E(x)$. The main requirement for operational no-signalling is *locality of the generator*: no cross-terms coupling spacelike-separated regions.

B.1 GKLS generator with $E(x)$-dependent rates

Let $\rho(t)$ be the system density operator. Consider a GKLS evolution

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H, \rho] + \sum_{\alpha} \int \mathrm{d}^3x \; \lambda_\alpha(x) \Big(L_\alpha(x) \, \rho \, L_\alpha(x)^\dagger - \tfrac{1}{2}\{L_\alpha(x)^\dagger L_\alpha(x), \rho\} \Big), \tag{17}$$

with local Lindblad operators $L_\alpha(x)$ supported in region x and local rates

$$\lambda_\alpha(x) = \lambda_{\alpha,0} \, \exp\!\big(\eta_\alpha \, E(x)\big), \qquad |\eta_\alpha| \ll 1. \tag{18}$$

This form preserves trace and complete positivity for $\lambda_\alpha(x) \geq 0$.

B.2 Theorem: no-signalling under local factorization

We now state a standard operational no-signalling result in the language used in quantum information.

Theorem 1 (Operational no-signalling for local GKLS dynamics)**.** *Consider a bipartite system $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ with spacelike-separated labs A and B. Suppose the GKLS generator factorizes as*

$$\mathcal{L} = \mathcal{L}_A \otimes \mathrm{id}_B + \mathrm{id}_A \otimes \mathcal{L}_B, \tag{19}$$

with each $\mathcal{L}_{A/B}$ trace-preserving and completely positive, built from local operators and local rates (e.g. $\lambda_\alpha(x_A) = \lambda_{\alpha,0} \exp(\eta_\alpha E(x_A))$ depending only on local E). Then the reduced state $\rho_A(t) = \mathrm{Tr}_B \rho_{AB}(t)$ evolves as

$$\dot{\rho}_A = \mathcal{L}_A(\rho_A), \tag{20}$$

independent of any operation or measurement choice performed in B. Therefore marginal outcome statistics in A cannot be used to signal superluminally from B.

Proof sketch. Starting from $\dot{\rho}_{AB} = \mathcal{L}(\rho_{AB})$, take the partial trace over B:

$$\dot{\rho}_A = \mathrm{Tr}_B\big[(\mathcal{L}_A \otimes \mathrm{id}_B)(\rho_{AB})\big] + \mathrm{Tr}_B\big[(\mathrm{id}_A \otimes \mathcal{L}_B)(\rho_{AB})\big]. \tag{21}$$

Linearity gives $\mathrm{Tr}_B[(\mathcal{L}_A \otimes \mathrm{id}_B)(\rho_{AB})] = \mathcal{L}_A(\mathrm{Tr}_B \rho_{AB}) = \mathcal{L}_A(\rho_A)$. For the second term, trace-preservation of $\mathcal{L}_B$ implies $\mathrm{Tr}_B[(\mathrm{id}_A \otimes \mathcal{L}_B)(\rho_{AB})] = 0$. Hence $\dot{\rho}_A = \mathcal{L}_A(\rho_A)$, which contains no dependence on remote choices in B. Operational no-signalling follows. □

B.3 Connection to the ethically biased Born rule (small-bias limit)

A normalized exponential weighting

$$P_i(\eta) = \frac{|c_i|^2 \, \mathrm{e}^{\eta \, \Delta E_i}}{\sum_j |c_j|^2 \, \mathrm{e}^{\eta \, \Delta E_j}}, \qquad |\eta| \ll 1, \tag{22}$$

reduces at first order to a normalized linear deformation:

$$P_i(\eta) \approx |c_i|^2 \Big[1 + \eta\big(\Delta E_i - \langle \Delta E \rangle\big)\Big] + \mathcal{O}(\eta^2), \qquad \langle \Delta E \rangle := \sum_j |c_j|^2 \, \Delta E_j. \tag{23}$$

This provides a consistent small-parameter bridge between discrete outcome weighting and continuous-time GKLS rate modulation.

Channel	Constrains (primary)	Observable / summary statistic
Collider (Higgs portal)	$g_{\Phi_c H}, g_{EH}, m_{\Phi_c}, m_E$	Higgs invisible width, signal strengths
Fifth-force / EP tests	g, m_M (and matter couplings)	Yukawa deviation: $\alpha\, e^{-r/\lambda}$
Cosmology	m_E, λ_E, ξ	$w(z)$, scalar energy density, CMB/BBN bo
QRNG / lab randomness	η, λ_0	Bayes factor vs. $p = 1/2$ null
Neurophys proxies (EEG/MEG)	map to latent E, Φ_c	coherence metrics; cross-validated predicti

Table 1: Minimal map from parameters to constraint channels (placeholders; fill with current bounds in the main manuscript).

C Parameterization, constraints, and an operational inference layer

To make the framework falsifiable, one must specify a minimal parameter vector and a measurement model linking data to latent fields and parameters.

C.1 Minimal parameter vector

A compact EFT parameterization sufficient for near-term tests is

$$\theta = \{m_{\Phi_c}, m_E, m_M;\ \lambda_{\Phi_c}, \lambda_E, \lambda_{\Phi_c E};\ g_{\Phi_c H}, g_{EH};\ g;\ \lambda_0, \eta;\ \xi\}, \tag{24}$$

where m_{Φ_c}, m_E are scalar masses, m_M is the mediator mass setting the kernel range, λ's are scalar self/cross couplings, $g_{\cdot H}$ are Higgs-portal couplings, g is the mediator coupling generating K, λ_0 is a baseline collapse rate, η controls ethical bias strength, and ξ denotes an effective teleology coupling (interpretable as emergent/open-system).

C.2 Constraints by experimental channel

Table 1 summarizes which channels constrain which parameter subsets.

C.3 Operational measurement model

Let D denote observed data streams and let (Φ_c, E) be latent fields. Define a likelihood

$$p(D \mid \Phi_c, E, \theta) = p(D_{\text{collider}} \mid \theta)\, p(D_{\text{fifth}} \mid \theta)\, p(D_{\text{cosmo}} \mid \theta)\, p(D_{\text{qrng}} \mid E, \theta)\, p(D_{\text{neuro}} \mid \Phi_c, E, \theta), \tag{25}$$

with priors $p(\Phi_c, E, \theta)$ encoding smoothness, locality, and EFT naturalness assumptions.

Example: QRNG likelihood with ethical modulation. Given bit outcomes $b_t \in \{0, 1\}$ over trials $t = 1, \ldots, T$, define

$$b_t \sim \text{Bernoulli}(p_t), \qquad p_t = \tfrac{1}{2} + \epsilon\, s_t, \qquad |\epsilon| \le \tfrac{1}{2}, \tag{26}$$

where $s_t \in [-1, 1]$ is a pre-registered, externally recorded modulation signal (e.g. task condition or normalized proxy for E), and ϵ is an estimable bias parameter (to leading order, $\epsilon \propto \eta$). Under the null, $\epsilon = 0$. Posterior inference over ϵ yields Bayes factors and credible intervals.

Example: fifth-force mapping. If the mediator induces an effective Yukawa correction to the Newtonian potential,

$$V(r) = -\frac{G m_1 m_2}{r}\Big[1 + \alpha\, e^{-r/\lambda}\Big], \qquad \lambda = m_M^{-1}, \tag{27}$$

then experimental bounds on (α, λ) map to bounds on g and m_M given matter-coupling assumptions.

C.4 Finite close-out criterion

A finite close-out criterion for the project is:

1. Publish a frozen minimal parameterization θ and a consolidated constraint figure/table.
2. Pre-register one decisive test (e.g. QRNG protocol) with a power analysis.
3. Report posterior constraints on (η, λ_0) and at least one independent channel (collider or fifth-force).
4. State 2–4 parameterized predictions that can be falsified within a specified experimental sensitivity.

D One-page decisive predictions (parameterized)

The following predictions are intentionally stated in parameterized form. They become crisp once the global posterior over θ is reported.

1. **QRNG modulation:** In a pre-registered protocol with modulation signal s_t, the posterior for ϵ departs from zero with Bayes factor exceeding a chosen threshold if $|\eta| \operatorname{Var}(E) \gtrsim \epsilon_{\min}$.
2. **Short-range force:** A Yukawa deviation appears at range $\lambda = m_M^{-1}$ with strength $\alpha = \alpha(g, \text{matter couplings})$. Null results shrink the allowed region in (g, m_M).
3. **Collider portal:** Higgs-portal couplings predict either a small invisible width or small signal-strength deviations; current and near-future bounds constrain $(g_{\Phi_c H}, g_{EH}, m_{\Phi_c}, m_E)$.
4. **Cosmic evolution:** For effective teleology coefficient $\xi \neq 0$, the scalar E can contribute a small, time-varying equation-of-state component $w(z)$; cosmological fits bound (ξ, λ_E, m_E).

Appendix: Notation and conventions

We use signature $(-,+,+,+)$, $\hbar = c = 1$, covariant derivative ∇_μ, and $\Box = \nabla^\mu \nabla_\mu$. Angle brackets denote coarse-grained expectation values.

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