NATIONAL ENGINEERING CENTER
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8.0 Optimization for BI

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Module 1 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

Module 1 Outline

1. Intro to Business Intelligence
   – Case Study on Selecting BI Projects
2. Data Warehousing
   – Case Study on Data Extraction and Report Generation
3. Descriptive Analytics
   – Case Study on Data Analysis
4. Visualization
   – Case Study on Dashboard Design
5. Classification Analysis
   – Case Study on Classification Analysis
6. Regression and Time Series Analysis
   – Case Study on Regression and Time Series Analysis
7. Unsupervised Learning and Modern Data Mining
   – Case Study on Text Mining
8. **Optimization for BI**

Outline for This Session

- What is OR?
- Model Building Process
- Mathematical Modeling
- Graphical Solution
- Types of Mathematical Models
- Modeling Examples
- Other OR Models

What is OR?

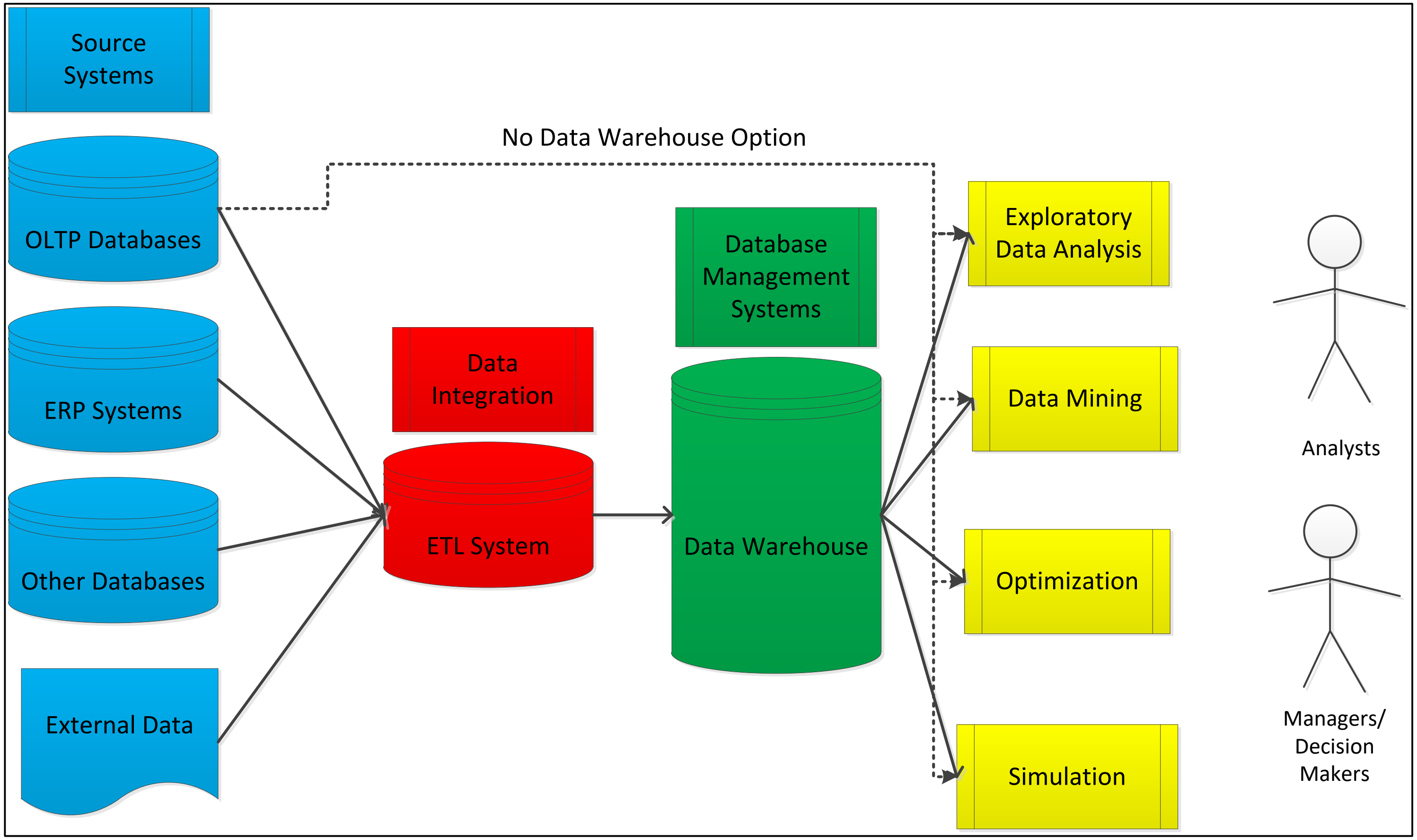

Figure 8.1: BA Framework

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What is OR?

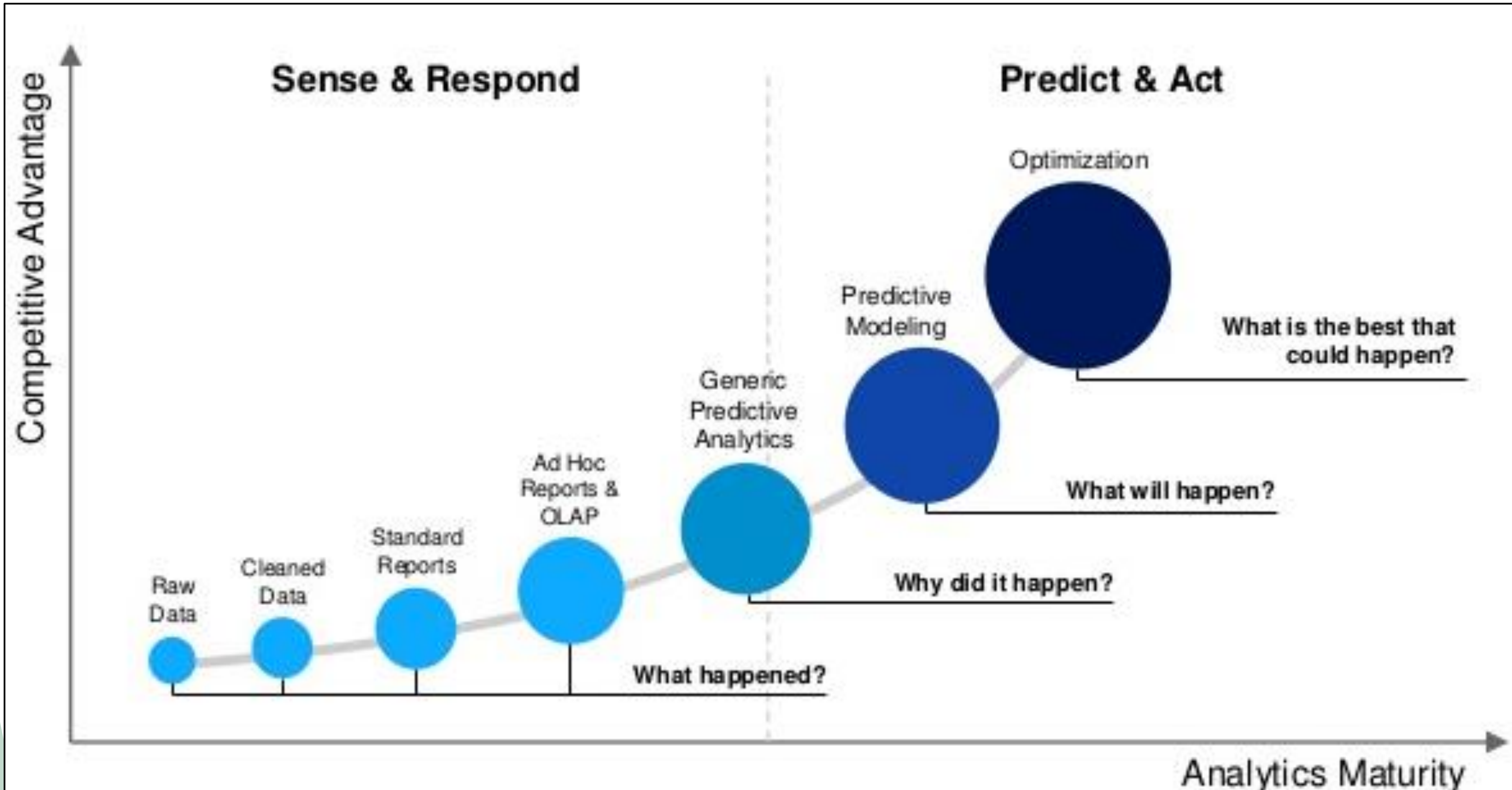

Figure 8.2: Types of BA According to Sophistication

What is OR?

Definition 8.1: Operations Research

- Operations Research (management science) is a scientific approach to decision making that seeks to best design and operate a system
- Usually under conditions requiring the allocation of scarce resources.
- Formal Definition
  - "the use of mathematical models in providing guidelines to managers for making effective decisions within the state of the current information, or in seeking further information if current knowledge is insufficient to reach a proper decision."

What is OR?

- 1947 -1990s
  - Project Scoop (Scientific Computation of Optimum Programs) with George Dantzig and others. Developed the simplex method for solving problems.
  - Lots of excitement, mathematical developments, queuing theory, mathematical programming.
    A.I. in the 1960's
  - More excitement, more development and grand plans. A.I. in the 1980's.
  - Widespread availability of personal computers. Increasingly easy access to data. Widespread willingness of managers to use models.

What is OR?

- 2000's
  - LOTS of opportunities for OR as a field
  - Data, data, data
  - E-business data (click stream, purchases, other transactional data, E-mail and more)
  - Need for more automated decision making
  - Need for increased coordination for efficient use of resources (Supply chain management)

What is OR?

- Success Stories of OR
  - Optimal crew scheduling saves American Airlines $20 million/yr.
  - Improved shipment routing saves Yellow Freight over $17.3 million/yr.
  - Improved truck dispatching at Reynolds Metals improves on-time delivery and reduces freight cost by $7 million/yr.
  - GTE local capacity expansion saves $30 million/yr.
  - Optimizing global supply chains saves Digital Equipment over $300 million.
  - Restructuring North America Operations, Proctor and Gamble reduces plants by 20%, saving $200 million/yr

What is OR?

- Optimal traffic control of Hanshin Expressway in Osaka saves 17 million driver hours/yr.
- Improved production planning at Sadia (Brazil) saves $50 million over three years.
- Production Optimization at Harris Corporation improves on-time deliveries from 75% to 90%.
- Tata Steel (India) optimizes response to power shortage contributing $73 million.
- Optimizing police patrol officer scheduling saves police department $11 million/yr.
- Gasoline blending at Texaco results in saving of over $30 million/yr.

Outline for This Session

- What is OR?
- **Model Building Process**
- Mathematical Modeling
- Graphical Solution
- Types of Mathematical Models
- Modeling Examples
- Other OR Models

Model-Building Process

- A seven-step model building procedure is used to solve OR problems:
  1. Formulate the Problem
     - Define the problem.
     - Specify objectives.
     - Determine parts of the organization to be studied.
  2. Observe the System
     - Determine parameters affecting the problem.
     - Collect data to estimate values of the parameters.
  3. Formulate a Mathematical Model of the Problem
  4. Verify the Model and Use the Model for Prediction
     - Does the model yield results for values of decision variables not used to develop the model?
     - What eventualities might cause the model to become invalid?

Model-Building Process

5. Select a Suitable Alternative
   - Given a model and a set of alternative solutions, determine which solution best meets the organizations objectives.
6. Present the Results and Conclusion(s) of the Study to the Organization
   - Present the results to the decision maker(s)
   - If necessary, prepare several alternative solutions and permit the organization to choose the one that best meets their needs.
   - Any non-approval of the study's recommendations may have stemmed from an incorrect problem definition or failure to involve the decision maker(s) from the start of the project. In such a case, return to step 1, 2, or 3.

Model-Building Process

7. Implement and Evaluate Recommendations
   - Assist in implementing the recommendations.
   - Monitor and dynamically updates the system as the environment and parameters change to ensure that recommendations enable the organization to meet its goals.

Outline for This Session

- What is OR?
- Model Building Process
- **Mathematical Modeling**
- Graphical Solution
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Mathematical Modelling

Definition 8.2: Mathematical Programming

- Mathematical Programming(MP) is a field of management science or operations research that fines most efficient way of using limited resources/ to achieve the objectives of a business.
- Mathematical Modeling is the process of translating the problem into a mathematical program.

Mathematical Modelling

- Mathematical Modeling Process
  1. Understand the problem
  2. Identify the decision variables
  3. State the objective as a linear combination of decision variables
  4. State the constraints as linear combinations of the decision variable
  5. Identify any upper or lower bounds on the decision variable

Mathematical Modelling

Example 8.1: Mathematical Modelling Example

- Blue Ridge Hot Tubs manufactures and sells two models of hot tubs with a pump: Aqua Spa and the Hydro-Lux.
  - Howie Jones, the owner and manager of the company, needs to decide how many of each type of hot tubs to produce
  - 200 pumps available
  - Howie expects to have 1,566 production labor hours and 2,880 feet of tubing available.
  - Aqua-Spa requires 9 hours of labor and 12 feet of tubing
  - Hydro-Lux requires 6 hours of labor and 16 feet of tubing
  - Assume all hot tubs can be sold where Aqua-Spa can be retailed at PhP 350 and Hyrdo-Lux at PhP 300.
  - Income target must be at least PhP 35,000
- To maximize profits, how many Aqua-Spas and Hydro-Luxs should be produced?

Mathematical Modelling

- Step 1: Understand the Problem

Example 8.1 (Cont.): Mathematical Modelling Example

  - To maximize profits, how many Aqua-Spas and Hydro-Luxs should be produced?

Mathematical Modelling

- Step 2: Identify the Decision Variable

Definition 8.3: Decision Variable

- Variables whose values are under our control and influence system performance

Example 8.1 (Cont.): Mathematical Modelling Example

$$Let \begin{cases} x_1 & the\ number\ of\ aqua-spas\ to\ produce \\ x_2 & the\ number\ of\ hydro-luxs\ to\ produce \end{cases}$$

Mathematical Modelling

- Step 3: State the Objective Function

Definition 8.4: Objective Function

- The objective function is a mathematical statement which is a function of decision variables that state a goal of a problem

Example 8.1 (Cont.): Mathematical Modelling Example

- Given the Problem: To maximize profits, how many Aqua-Spas and Hydro-Luxs should be produced?

$$Max\ Profit = 350x_1 + 300x_2$$

Mathematical Modelling

- Step 4: State the constraints

Definition 8.5: Constraints

- Restrictions on the decision variable values

Example 8.1 (Cont.): Mathematical Modelling Example

$$x_1 + x_2 \le 200$$
$$9x_1 + 6x_2 \le 1{,}566$$
$$12x_1 + 16x_2 \le 2{,}880$$
$$350x_1 + 300x_2 \ge 35{,}000$$

$$x_1, x_2 \ge 0$$

Mathematical Modelling

- Putting it all Together: The general form of a Mathematical Program

Let $x_1, x_2, \dots x_n$ be decision variables

$$Max\ (or\ Min)\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

s.t.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$$
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ge b_1$$
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

n variables , m constraints

Mathematical Modelling

Example 8.1 (Cont.): Mathematical Modelling Example

$$Let \begin{cases} x_1 & the\ number\ of\ aqua-spas\ to\ produce \\ x_2 & the\ number\ of\ hydro-luxs\ to\ produce \end{cases}$$

$$Max\ Profit = 350x_1 + 300x_2$$

s.t.

$$x_1 + x_2 \leq 200$$

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$$x_1, x_2 \geq 0$$

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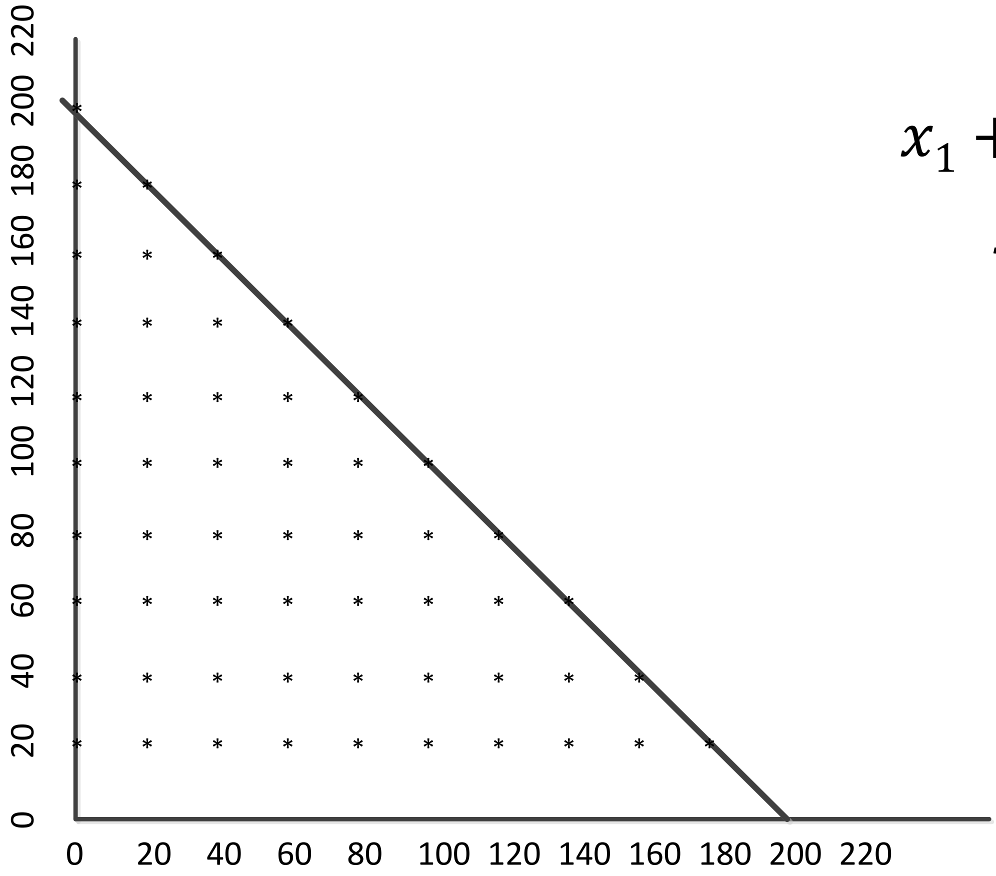
Solving LP Problems

- Graphical Method
- Simplex Method

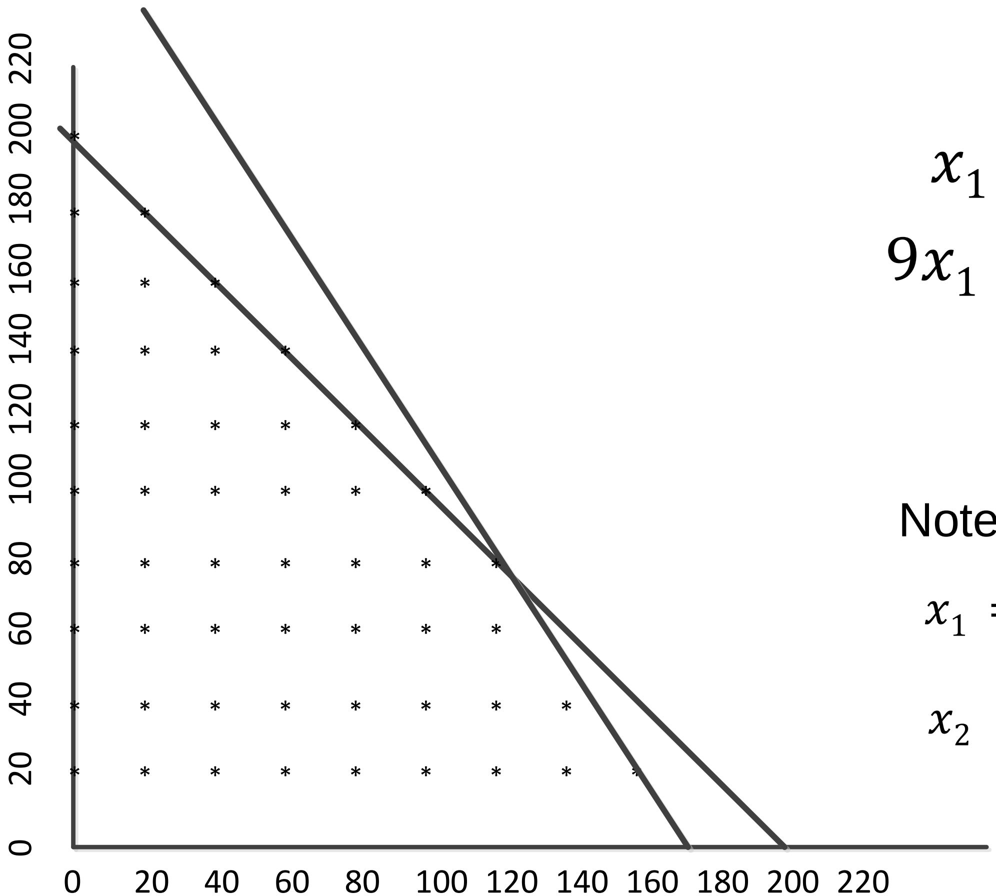
Graphical Method



$$x_1 + x_2 \quad \leq \quad 200 \;\; (1)$$
$$x_1, x_2 \;\geq 0$$

Figure 8.3: Graphical Method

Graphical Method



$$x_1 + x_2 \leq 200 \quad (1)$$

$$9x_1 + 6x_2 \leq 1{,}566 \quad (2)$$

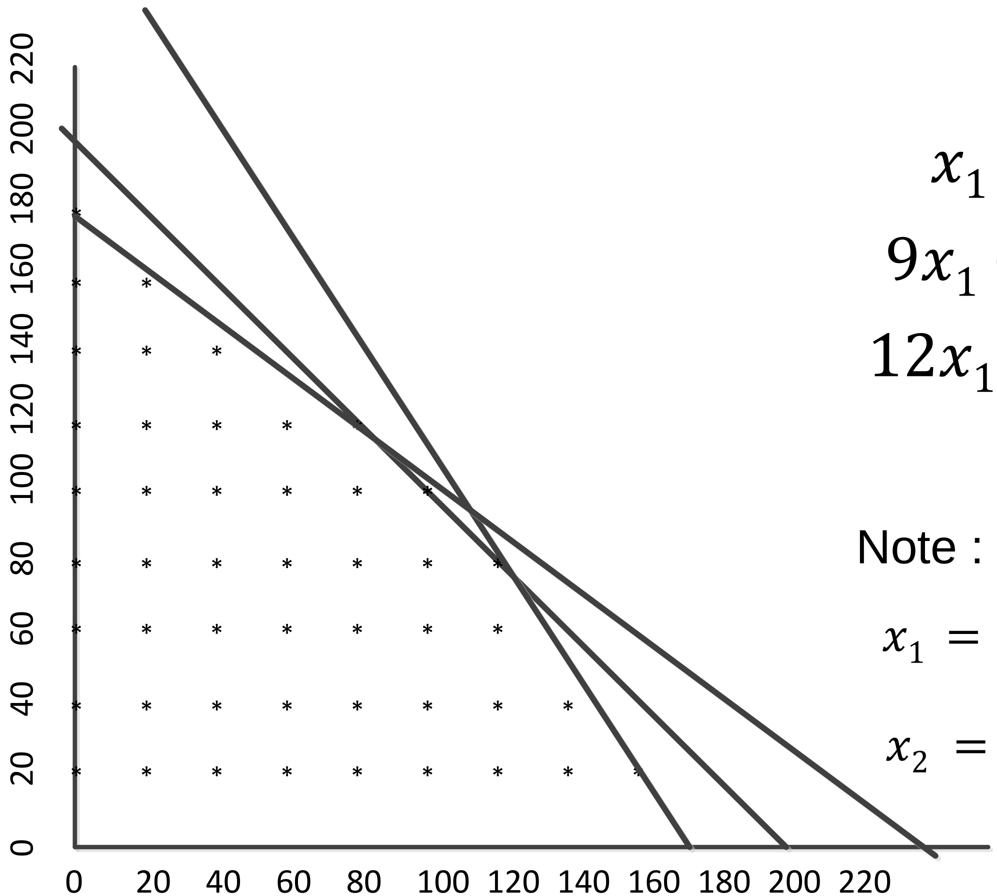
$$x_1, x_2 \geq 0$$

Note :

$$x_1 = 0, x_2 = \frac{1566}{6} = 261$$

$$x_2 = 0, x_1 = \frac{1566}{9} = 174$$

Graphical Method

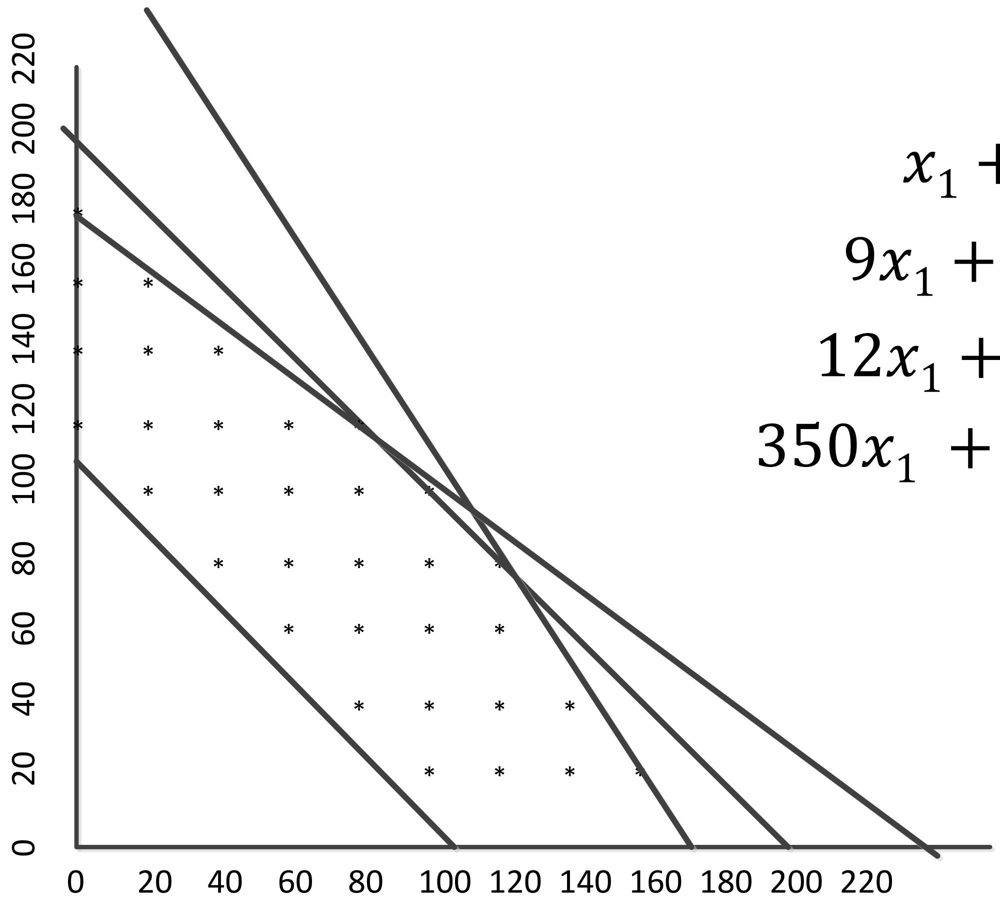


$$x_1 + x_2 \leq 200 \quad (1)$$
$$9x_1 + 6x_2 \leq 1{,}566 \quad (2)$$
$$12x_1 + 16x_2 \leq 2{,}880 \quad (3)$$

Note :

$$x_1 = 0\,, x_2 = \frac{1566}{6} = 261$$
$$x_2 = 0\,, x_1 = \frac{1566}{9} = 174$$

Graphical Method



$$x_1 + x_2 \leq 200 \quad (1)$$

$$9x_1 + 6x_2 \leq 1{,}566 \quad (2)$$

$$12x_1 + 16x_2 \leq 2{,}880 \quad (3)$$

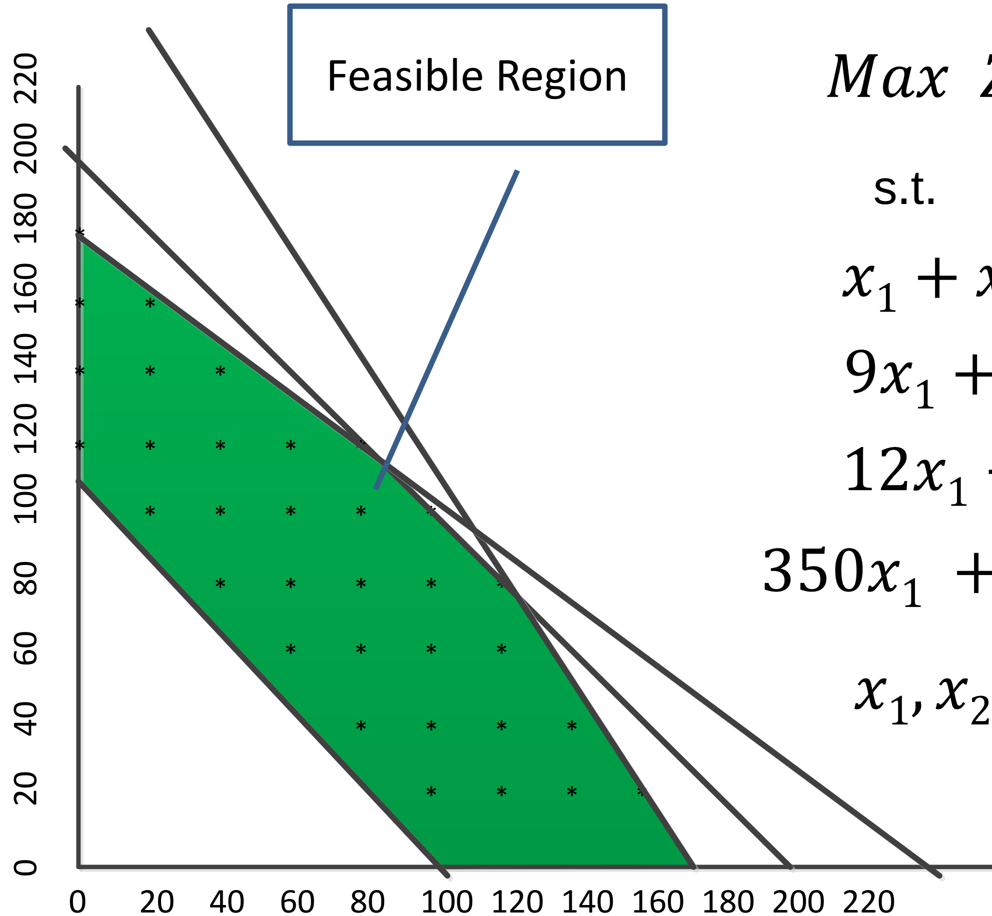
$$350x_1 + 300x_2 \geq 35{,}000 \quad (4)$$

Note :

$x_1 = 100 , x_2 = 0$

$x_2 = 0 , x_1 = 166.67$

Graphical Method



$$Max\ Z = 350x_1 + 300x_2$$

s.t.

$$x_1 + x_2 \le 200 \quad (1)$$

$$9x_1 + 6x_2 \le 1{,}566 \quad (2)$$

$$12x_1 + 16x_2 \le 2{,}880 \quad (3)$$

$$350x_1 + 300x_2 \ge 35{,}000 \quad (4)$$

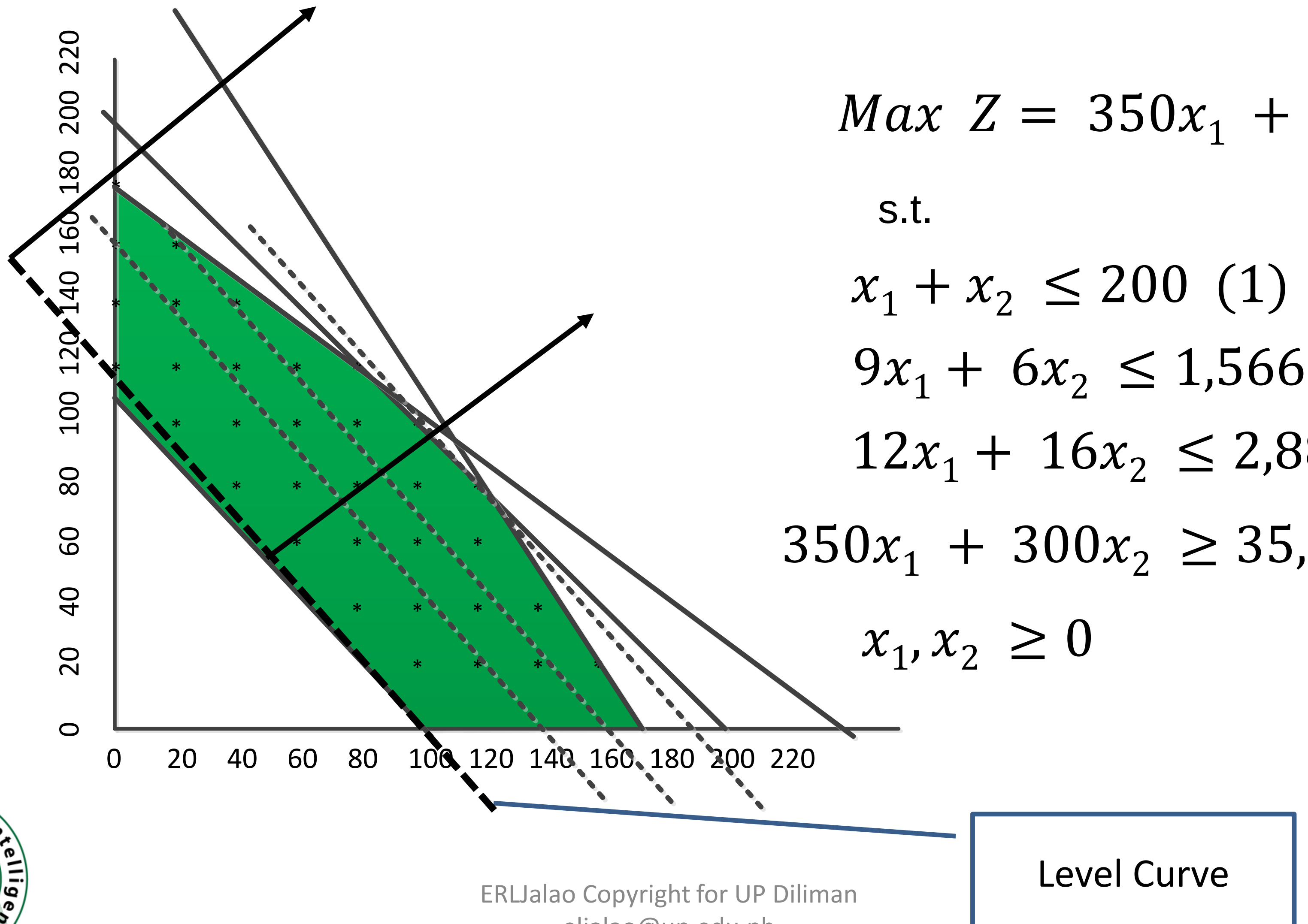
$$x_1, x_2 \ge 0$$

Graphical Method

- Using Level Curves
  - $Max \;\; Z = 350x_1 + 300x_2$

  1. Set an arbitrary value for the objective function
     e.g. $Z = 35{,}000$
  2. Find points (x_1, x_2) which has $Z = 35{,}000$
     $$x_1 = 100, x_2 = 0$$
     $$x_1 = 0, x_2 = 116.67$$
  3. Draw a straight line connecting both points
  4. Move the line where the Z value increases

Graphical Method



$$Max\ Z = 350x_1 + 300x_2$$

s.t.

$$x_1 + x_2 \leq 200 \quad (1)$$

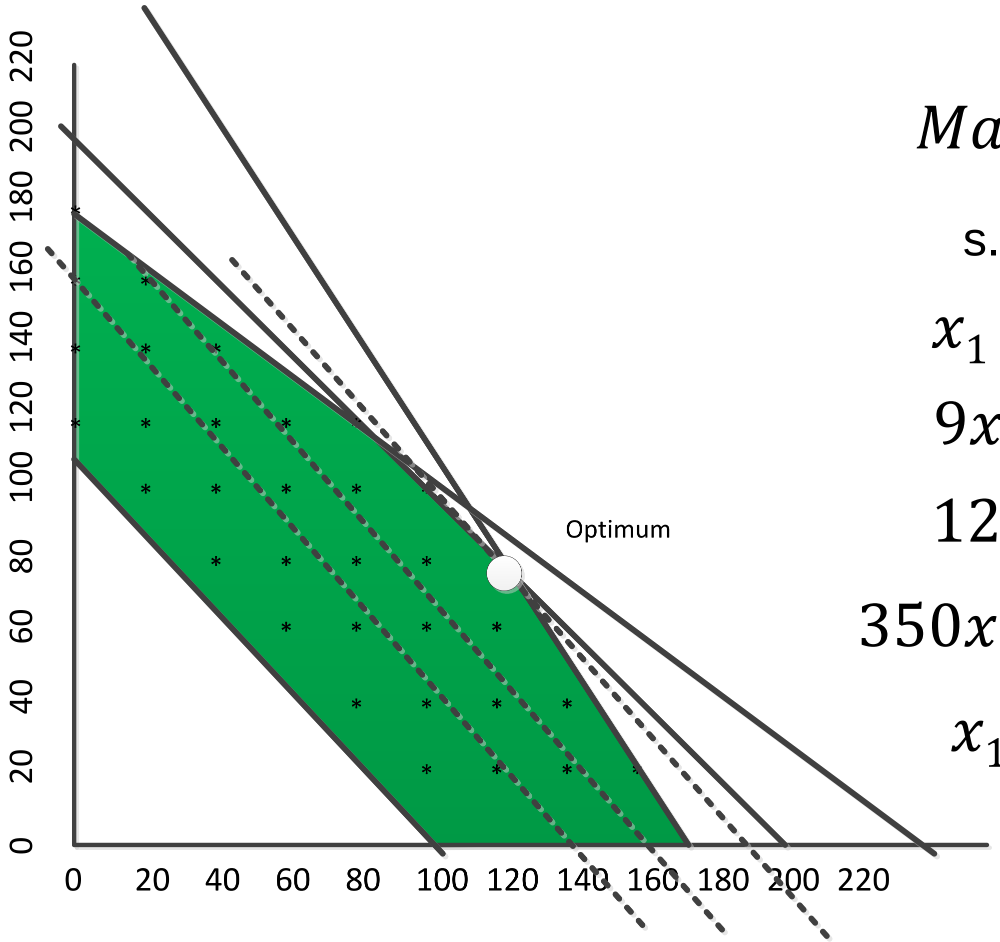
$$9x_1 + 6x_2 \leq 1{,}566 \quad (2)$$

$$12x_1 + 16x_2 \leq 2{,}880 \quad (3)$$

$$350x_1 + 300x_2 \geq 35{,}000 \quad (4)$$

$$x_1, x_2 \geq 0$$

Graphical Method



$$Max\ Z = 350x_1 + 300x_2$$

s.t.

$$x_1 + x_2 \leq 200 \quad (1)$$

$$9x_1 + 6x_2 \leq 1{,}566 \quad (2)$$

$$12x_1 + 16x_2 \leq 2{,}880 \quad (3)$$

$$350x_1 + 300x_2 \geq 35{,}000 \quad (4)$$

$$x_1, x_2 \geq 0$$

Graphical Method

$$x_1 + x_2 = 200 \qquad (1)$$
$$9x_1 + 6x_2 = 1{,}566 \qquad (2)$$

$$9x_1 + 6(200 - x_1) = 1{,}566$$
$$3x_1 + 1{,}200 = 1{,}566$$
$$3x_1 = 366$$

Optimal Solution

$$x_1 = 122$$
$$x_2 = 78$$
$$z = 66{,}100$$

Graphical Method

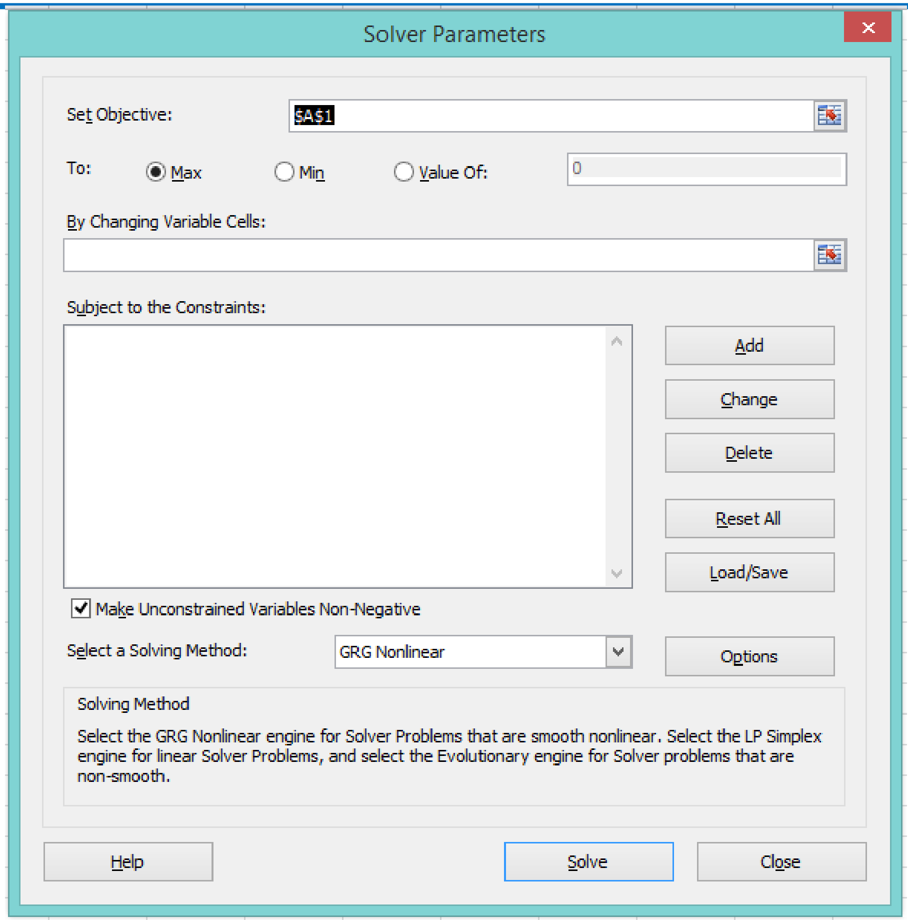


Figure 8.4: Excel Solver

Outline for This Session

- What is OR?
- Model Building Process
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- Graphical Solution
- **Types of Mathematical Models**
- Modeling Examples
- Other OR Models

Types of Mathematical Models

- Static and Dynamic Models
- Linear and Nonlinear models
- Integer and Noninteger Models
- Deterministic and Stochastic Models

Types of Mathematical Models

Definition 8.3: Static and Dynamic Models

- A static model is one in which the decision variables do not involve sequences of decisions over multiple periods.
- A dynamic model is a model in which the decision variables do involve sequences of decisions over multiple periods. In a static model, we solve a one shot problem whose solutions are optimal values of the decision variables at all points in time.

Types of Mathematical Models

Definition 8.4: Linear and Non-Linear Models

- Suppose that when ever decision variables appear in the objective function and in the constraints of an optimization model the decision variables are always multiplied by constants and then added together. Such a model is a linear model.
- $y = mx + b$
- Nonlinear models contain variables have degree that is not equal to one. These models are much harder to solve.
- $y = mx^2 + b$
- $y = mx^{\frac{1}{2}} + b$

Types of Mathematical Models

Definition 8.5: Integer and Non-Integer Models

- If one or more of the decision variables must be integer, then we say that an optimization model is an integer model. If all the decision variables are free to assume fractional values, then an optimization model is a noninteger model.
  - If decision variables in a model represent the number of workers starting during each shift, then clearly we have a integer model.
- Integer models are much harder to solve then noninteger models.

Types of Mathematical Models

Definition 8.6: Stochastic and Deterministic Model

- Suppose that for any value of the decision variables the value of the objective function and whether or not the constraints are satisfied is known with certainty. We then have a deterministic model.
- If this is not the case, then we have a stochastic model.

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Modeling Examples

Example 8.2: Make versus Buy Decisions

- The Electro-Poly corporation received a $750,000 order for various quantities of 3 types of slip rings.
- Each slip ring requires a certain amount of time to wire and hardness.

	Model1	Model2	Model3
Number ordered	3,000	2,000	900
Hours of wiring required per unit	2	1.5	3
Hours of harnessing required per unit	1	2	1

- The company has only 10,000 hours of wiring capacity.
- The company has only 5,000 hours of harnessing capacity.

Modeling Examples

– The company can sub contract to one of its competitors.

Cost($)	Model1	Model2	Model3
Cost to make	50	83	130
Cost to buy	61	97	145

– Determine the number of slip rings to make and the number to buy in order to fill the customer order at the least possible cost

Modeling Examples

- Defining the decision variables

  m_i = number of model i slip rings to make in-house

  b_i = number of model i slip rings to buy from competitor

- Defining the objective function
  - To minimize the total cost

$$Min\ Z = 50m_1 + 83m_2 + 130m_3 + 61b_1 + 97b_2 + 145b_3$$

Subject to:

$2m_1 + 1.5m_2 + 3m_3 \leq 10{,}000$, wiring constraint

$m_1 + 2m_2 + m_3 \leq 5{,}000$, harness constraint

$m_1 + b_1 = 3{,}000$, Required order of model1

$m_2 + b_2 = 2{,}000$, Required order of model2

$m_3 + b_3 = 900$, Required order of model3

$m_1, m_2, m_3, b_1, b_2, b_3, \geq 0$

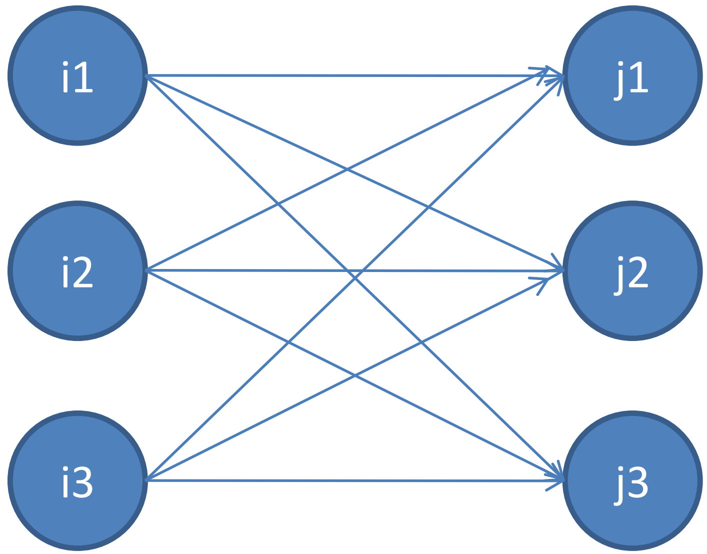
Modeling Examples

Example 8.3: Transportation Problems

- How many product should be shipped from r/m area to the processing plant, the trucking company charges a flat rate for every mile. **Min the total distance ~ Min total cost of transportation**

Modeling Examples

Supply	Warehouse	Processing Plant	Capacity
275,000	i1	j1	200,000
400,000	i2	j2	600,000
300,000	i3	j3	225,000



Distance : (mile)	j1	j2	j3
i1	21	50	40
i2	35	30	22
I3	55	20	25

Modeling Examples

- Defining the decision variable

  x_{ij} = number of R/M to ship from node i to node j

- Defining the objective function

  $$MinZ = 21x_{11} + 50x_{12} + 40x_{13} + 35\,x_{21} + 30x_{22} + 22x_{23} + 55x_{31} + 20x_{32} + 25x_{33}$$

- Defining the constraints

$x_{11} + x_{21} + x_{31}$	$\leq$ 200,000	capacity restriction for j1
$x_{12} + x_{22} + x_{32}$	$\leq$ 600,000	capacity restriction for j2
$x_{13} + x_{23} + x_{33}$	$\leq$ 225,000	capacity restriction for j3
$x_{11} + x_{12} + x_{13}$	$=$ 275,000	supply restriction for i1
$x_{21} + x_{22} + x_{23}$	$=$ 400,000	supply restriction for i2
$x_{31} + x_{32} + x_{33}$	$=$ 300,000	supply restriction for i3

$x_{ij} \geq 0$, for all i and j

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Other OR Models

- Dynamic Programming
- Stochastic Programming
- Stochastic Processes
  - Markov Chains
  - Queuing Theory
- Simulation

Other OR Models

Definition 8.7: Dynamic Programming

- A problem may involve a sequence of decisions that are made over time. Initial decision is followed by a second, the second by a third, and so on.
- Modeling may required multiple states and decisions. Concept of "divide-and-conquer" where states overlap
- Take our problem and somehow break it down into a reasonable number of subproblems in such a way that we can use optimal solutions to the smaller subproblems to give us optimal solutions to the larger ones

Other OR Models

- The Knapsack Problem
  - Load a knapsack for a trip with limited weight and volume capacity
  - There are N alternative items with different weights and sizes.
  - Stage := the kth item to be placed into the knapsack
  - State := the remaining space in the knapsack
  - $V_k(i)$:= the highest total value that can be achieved from item types k through N, assuming that the knapsack has a remaining capacity of i.
  - Recursive Equation

$$V_k(i) = \max_{0 \le x_k \le \lfloor i/w_k \rfloor} [r_k x_k + V_{k+1}(i - w_k x_k)]$$

Other OR Models

- Mathematical programming tools neglect effects of uncertainty and assume results are predictable or deterministic, e.g. effects of weather

Definition 8.8: Stochastic Programming

- Stochastic programming incorporates uncertainty by including random variables into the formulation
- Example

$$Let\ \tilde{x}_i \coloneqq cost\ due\ to\ weather = \begin{cases} 350 & rain\ with\ probability\ 0.4 \\ 300 & sunny\ with\ probability\ 0.6 \end{cases}$$

- Use weighted average cost $= 350 * .4 + 300 * .6 = 320$

Other OR Models

Definition 8.9: Queuing Theory

- The analysis of queues
- The number in line is a random variable that changes with time, the system of customers and servers fits the definition of a stochastic process.
- The basic objective in most queuing models is to achieve a balance between cost of waiting and cost of service

Other OR Models

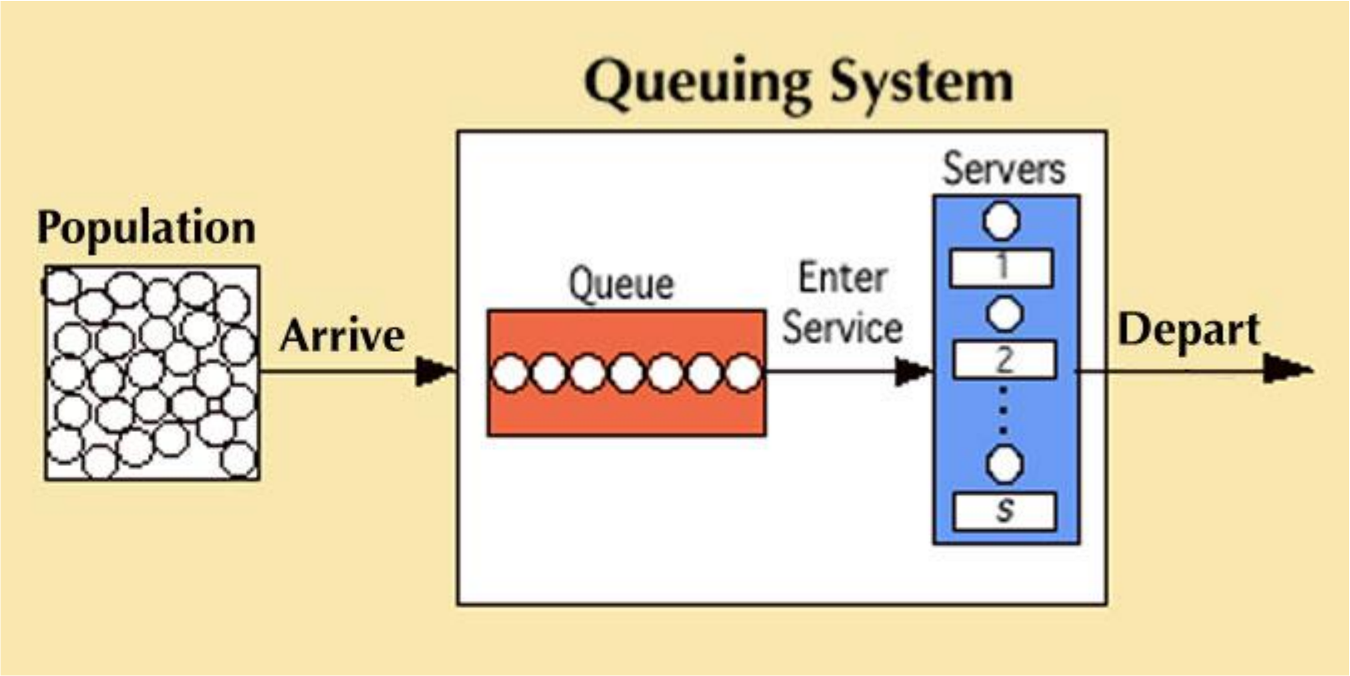

Figure 8.5: Sample Queuing System

Other OR Models

Definition 8.10: Simulation

- Generate a computer model of a system that describes some process as much as possible
- Use historical data to populate the model's parameters
- Perform what-if analysis on the model without disrupting the current system

Other OR Models

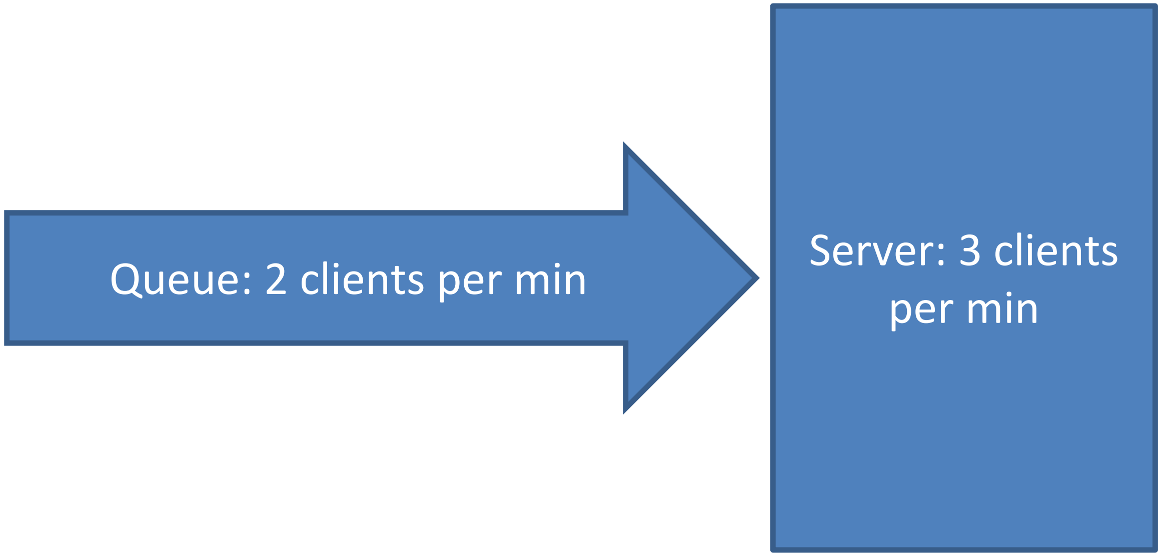

Figure 8.6: Queueing Simulation Example

Outline for This Session

- Definitions
- History of OR
- Model Building Process
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References

- Lieberman, Introduction to Operations Research, 9^{th} Ed. McGraw Hill
- Sheldon Ross, Introduction to Probability Models, 9^{th} Ed. Academic Press
- http://www.me.utexas.edu/~jensen/ORMM/models/
- Slides from Kanjana Thongsanit, Silpakorn University, Introduction to Operations Research and Linear Programming
- Notes from www.engr.sjsu.edu/udlpms/ISE%20265/set1_LP.ppt, Introduction to Model Building