# 1.0 Introduction to Operations Research

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*Module 4 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence*

# Today's Topics:

- Introduction to Operations Research
- The Operations Research Methodology
- Intro to LP Formulation



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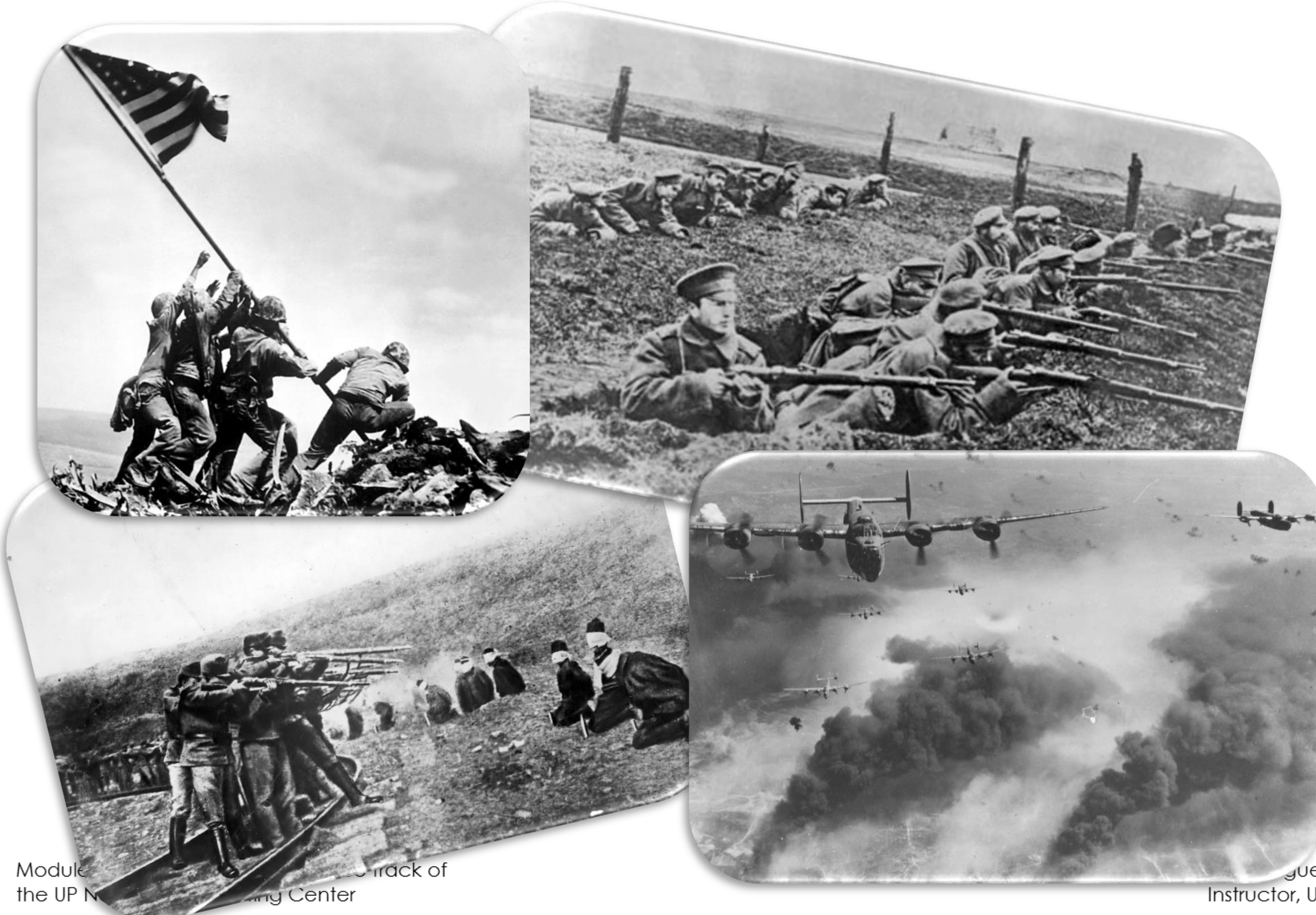
# Brief History



# Operations Research

discipline that deals with the application of advanced analytical methods to help make better decisions[1]

a scientific approach to decision making that seeks to best design and operate a system, usually under conditions requiring the allocation of scarce resources[2]

Multi-disciplinary approach applied to problems that concern how to conduct and coordinate the operations (i.e., the activities) within an organization[3]



[1] wikipedia.com [2] Winston [3] Hillier and Lieberman

## Scientific Discipline

- "Research"
- To a considerable extent, the scientific method is used in Operations Research
- The Operations Research methodology
- System perspective. Deals with the interdependence and interaction of the elements

## Uses Analytical/Mathematical Methods

- Uses a mathematical model
- Means of manipulating abstracted reality

# Mathematical Model

mathematical representation of an actual situation that may be used to make better decisions or simply to understand the actual situation better[1]



[1] Winston

## Used for Decision Making

- Presence of Alternatives
- Appraisal of alternatives
- Concerned with the “best” alternative or the optimal alternative

## Multi-Disciplinary

## "More than just Mathematics" - Taha

Problem: People complaining about long elevator queues in a building

OR Team sees a waiting-line problem and suggests queuing analysis and simulation to solve it

Actual solution turned out to be adding full length mirrors along the entrance to the elevator to keep the people occupied

# Multi-Disciplinary

## "More than just Mathematics" - Taha

Problem: Queues in check-in facility at a UK airport

Part of the solution was to place a sign telling passengers who were within 20 minutes of boarding to go to the front of the line

It was unsuccessful because most of the passengers were British and were reluctant to move ahead of the others

Other Applications of Operations Research

- Flight Scheduling
- Manpower scheduling/Shifting
- Production Lot Size to meet demand given limited supply
- Network models
- Predict Bad Credit Score
- Design of weapons
- Location Problems



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# Scientific Method

principles and procedures for the systematic pursuit of knowledge involving the recognition and formulation of a problem, the collection of data through observation and experiment, and the formulation and testing of hypotheses[1]



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[1] Meriam-Webster Dictionary

# Operations Research Methodology

## According to Taha

1. Define the Problem
2. Construct the Model
3. Solve the Model
4. Validate the Model
5. Implement the Solution

## According to Winston

1. Formulate the Problem
2. Observe the System
3. Formulate a Mathematical Model
4. Verify the Model
5. Select a Suitable Alternative
6. Present the Results
7. Implement the Solution

1. Formulate the Problem

1. Define the Problem

2. Observe the System

- Study the parts of the organization
- Determine the Objectives of the Problem
- Collect data to be used in setting up the parameters for the model

1. Formulate the Problem

1. Define the Problem

2. Observe the System

The three principal elements of the problem

- Description of the Decision Alternatives
- Determination of the Objective of the study
- Specification of the limitations the system operates

## 2. Construct the Model

## 3. Formulate a Mathematical Model

Translate the problem statement into a mathematical model



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2. Construct the Model

3. Formulate a Mathematical Model

# Different kinds of Models

- Linear programming
- Transportation
- Assignment
- Network

Day 1

- Integer Programming
- Dynamic Programming
- Goal Programming
- Game Theory

Day 2

- Queuing
- Simulation

Day 3

4. Validate the Model

4. Verify the Model

- Does the model predict the actual behavior of the system?
- Compare the model results with historical data
- A model may be valid now but not valid tomorrow



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3. Solve the Model

5. Select a Suitable Alternative

6. Present the Results

- Use optimization algorithms
- If the model is too big you may use software
- Sensitivity Analysis is crucial

5. Implement the Solution

7. Implement the Solution

- OR Analysts aid in the implementation and control of the system

Suppose that you have a 5-week business commitment between Manila and Cebu You fly out Manila on Saturdays and return on Mondays. A regular round-trip ticket costs PhP 4000, but the price is PhP 3200 if the dates of the ticket span a weekend. A one-way ticket in either direction costs PhP 3000. How should you buy the tickets for the 5-week period?



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Source: Taha, 9th Edition

# Linear Programming Model

Mathematical tool/model used to solve optimization models

Recall:

The three principal elements of the problem

- Description of the Decision Alternatives
- Determination of the Objective of the study
- Specification of the limitations the system operates



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Objective Function

$$\boxed{\mathbf{Max}\ z = 2x_1 + 3x_2 - x_3}$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

Technological Constraints

$$\boxed{x_1,\ x_2,\ x_3 \geq 0}$$

Non-negativity constraint

$x_1$, $x_2$, $x_3$ are decision variables

$$\text{Max } z = 2x_1 + 3x_2 - x_3$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Is the solution $(x_1, x_2, x_3) = (4, 3, 1)$ feasible?

# Feasible Solution

Set of values of the decision variables that satisfy all of the constraints

$$\text{Max } z = 2x_1 + 3x_2 - x_3$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Is the solution $(x_1, x_2, x_3) = (4, 3, 1)$ feasible?

**NO!** It violates the second constraint

$$\text{Max } z = 2x_1 + 3x_2 - x_3$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Is the solution $(x_1, x_2, x_3) = (2, 2, 2)$ feasible?

**YES!!** It satisfies all constraints

$$\text{Max } z = 2x_1 + 3x_2 - x_3$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Is the solution $(x_1, x_2, x_3) = (2, 2, 2)$ optimal?


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# Optimal Solution

Feasible solution that provides the most desirable value for the objective function

$$\text{Max } z = 2x_1 + 3x_2 - x_3$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Is the solution $(x_1, x_2, x_3) = (2, 2, 2)$ optimal?

We cannot say yet. There are infinitely many feasible points and to determine the optimal (ie. The best) solution we have to compare all of the feasible points.

Do all LP's have an optimal solution?

**NO!** some do not even have a feasible solution

$$\text{Min } z = x_1 + 4x_2 + 2x_3$$

$$x_1 + x_2 \leq 10$$
$$3x_1 + 4x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

## Assumptions of a Linear Programming Model

1. Proportionality
2. Additivity
3. Certainty
4. Divisibility

1. Proportionality

The contribution of your decision variables to the objective function and to the constraints are directly proportional

1. Proportionality

The proportionality assumption is violated if there is a presence of quantity discounts or economies of scale

Example:

- A piece of notebook is Php 15
- If you buy more than 10 the price of each notebook becomes Php 12
- If you buy more than 20 the price of each notebook becomes Php 10



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2. Additivity

The total contribution of each variable in the constraints and objective function is the direct sum of the individual contributions of each variable

Violation of this assumption will make the problem non-linear

3. Certainty

The coefficients of the objective function and the technological constraints are deterministic. This means that they are known constants.

In essence, the coefficients are the average-value approximations of the data. If the standard deviation of the distribution is large, it is a stochastic programming problem.

Sensitivity analysis may also be used to the optimum solution to take into consideration variances

4. Divisibilty

The decision variables are continuous and not discrete

Decision variables may take on decimal values

Violation of the divisiility assumption makes the problem an Integer Linear Program which will be tackled in IE 142

Objective Function

$$\text{Max } z = 2x_1 + 3x_2 - x_3$$

$$x_1 + x_2 + x_3 \leq 10$$
$$3x_1 - x_2 \leq 8$$
$$2x_2 + 3x_3 \leq 12$$

Technological Constraints

$$x_1, x_2, x_3 \geq 0$$

Non-negativity constraint

$x_1, x_2, x_3$ are decision variables

Reddy Mikks produces two kinds of paint, Paint A and Paint B. The two paints share the same raw material, M1 and M2, only in different quantities as shown.

| | Tons of raw mat needed to produce 1 ton of paint | | Maximum available tons daily |
|---|---|---|---|
| | Paint A | Paint B | |
| M1 | 6 | 4 | 24 |
| M2 | 1 | 2 | 6 |

The profit for Paints A and B are 5 & 6 thousand dollars respectively. Also it is projected that daily demand for B cannot exceed that of A by more than 1 ton. B's maximum demand is known to be 2 tons. How many of each paint should the company produce to maximize daily profit?



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Source: Taha

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 /hr

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If Gru has only 65 bananas, how many of each minion must he hire to meet the kid's demand of 40 purple jellies and 30 orange jellies? Gru wishes to lessen the number of minions hired.

A certain company produces shirts and blouses. The company employs 25 workers in the cutting department, 35 in the sewing department and 5 in the packaging department. The table shows the time requirements per process of the garment.

| | Minutes per unit | | |
|---|---|---|---|
| | Cutting | Sewing | Packaging |
| Shirt | 20 | 70 | 12 |
| Blouse | 60 | 60 | 4 |

The company works one 8-hour shift per day, 5 days a week. A shirt has a profit of $8 while a blouse has a profit of $12. Construct the LP that will determine the optimal weekly production schedule of the company


Source: Taha

# Remember!

The three parts of an LP are the Decision Variable, Objective Function and Technological Constraints

Feasibility occurs when all constraints are satisfied

Optimality occurs when it is the best solution to the problem

The assumptions of an LP are proportionality, divisibility, continuity and additivity



Source: Taha

# Next Session

More LP Formulation

The Graphical Solution of an LP



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Source: Taha

Fin.



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