NATIONAL ENGINEERING CENTER
University of the Philippines
Diliman, Quezon City

Forecasting and Time Series Analysis with R

Day 3

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Module 5 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

Outline for This Training

1. Introduction to Forecasting in Business Intelligence
2. Demand Forecasting Techniques
   – Qualitative
   – Quantitative
3. Accuracy of Forecasts
4. Monitoring of Forecasts
5. Forecasting with R
6. Introduction to Time Series Data Mining
7. Advanced Time Series

Outline for This Training

1. Introduction to Forecasting in Business Intelligence
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   – Qualitative
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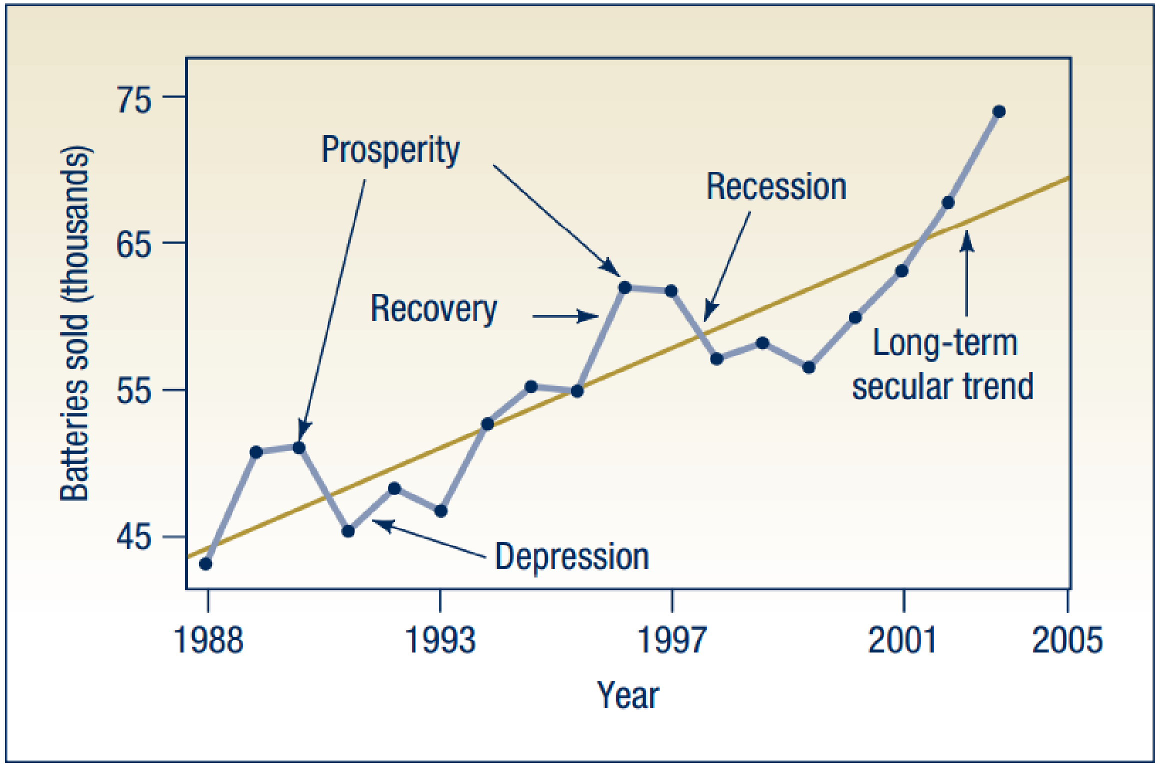
5. **Forecasting with R**
6. Introduction to Time Series Data Mining
7. Advanced Time Series

Quick Review

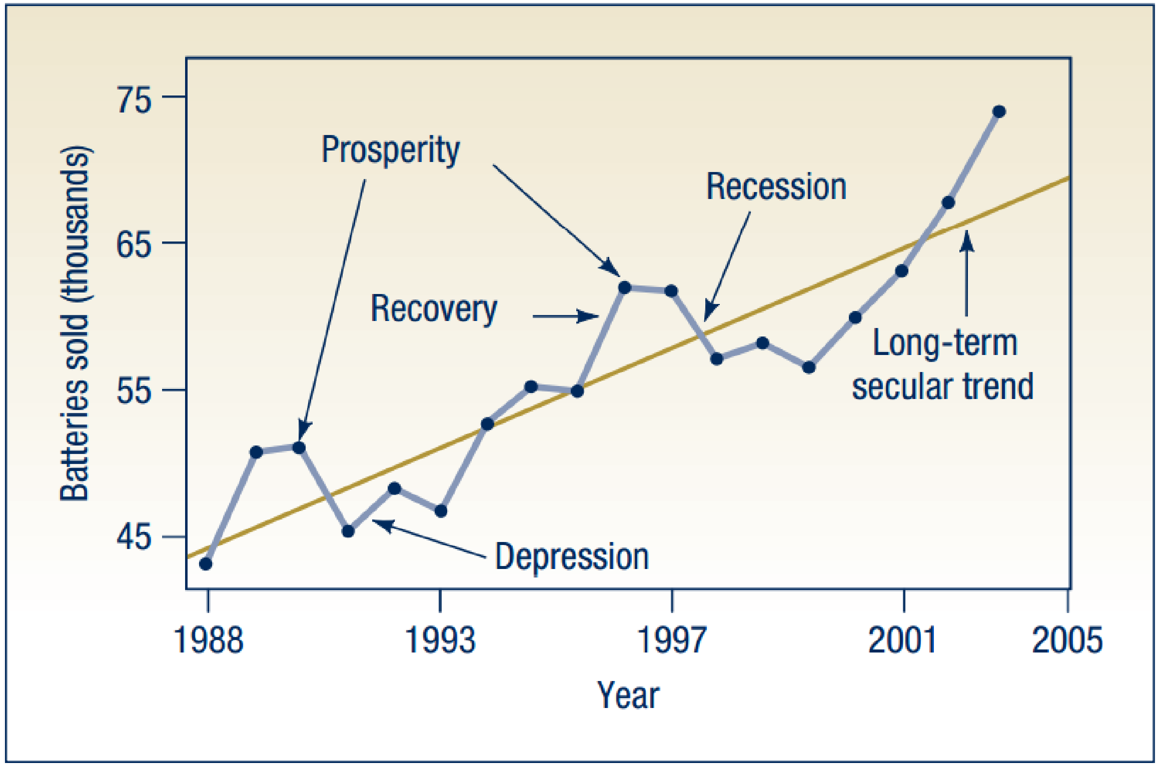
- A time series is a collection of observations made sequentially in time.



- Battery Sales by National Battery Sales, Inc., 1988–2005

Short Review

- Forecasting is making a statement about the future



- Forecast for 2006?
- How about for 2007?
- For 2008?
- Then for 2017?

- Battery Sales by National Battery Sales, Inc., 1988–2005

General Types of Forecasting

- Qualitative
  - Delphi
  - Sales Force
  - Executive Opinion
  - Consumer Market Survey
- Quantitative
  - Time Series
  - Causal / Associative

Qualitative

- Delphi Method
  - Panel of experts questioned on their perceptions on future events
- Executive Opinion
  - Views of executives or experts from specified departments
- Consumer Surveys
  - Interviews
- Sales Force Polling
  - Insights from sales people

Quantitative

- Associative Forecasting
- Time-series Forecasting Methods
  - Naïve Method
  - Simple Moving Average
  - Weighted Moving Average
  - Exponential Smoothing
  - Trend Adjusted Exponential Smoothing
  - Linear Trend / Linear Regression
  - Decomposition Method

Quantitative

- Time-series Forecasts
  - Uses historical data assuming that the future is simply a repetition of the past

y

x = t = time period

  - Let y_{t+1} be the forecast using the k past periods
  - Y = revenues, sales, expenses, losses, costs, employees hired, etc.

Time Series

- Time-series Examples
  - Annual rainfall
  - GDP each quarter
  - Daily stock market index
  - Number of students recorded per year
  - Population index per census

Quantitative

- The pattern or behavior of the data in a time series has several components.
- Theoretically, any time series can be decomposed into:
  - Trend
  - Cyclical
  - Seasonal
  - Irregular
- However, this decomposition is often not straight-forward because these factors interact.

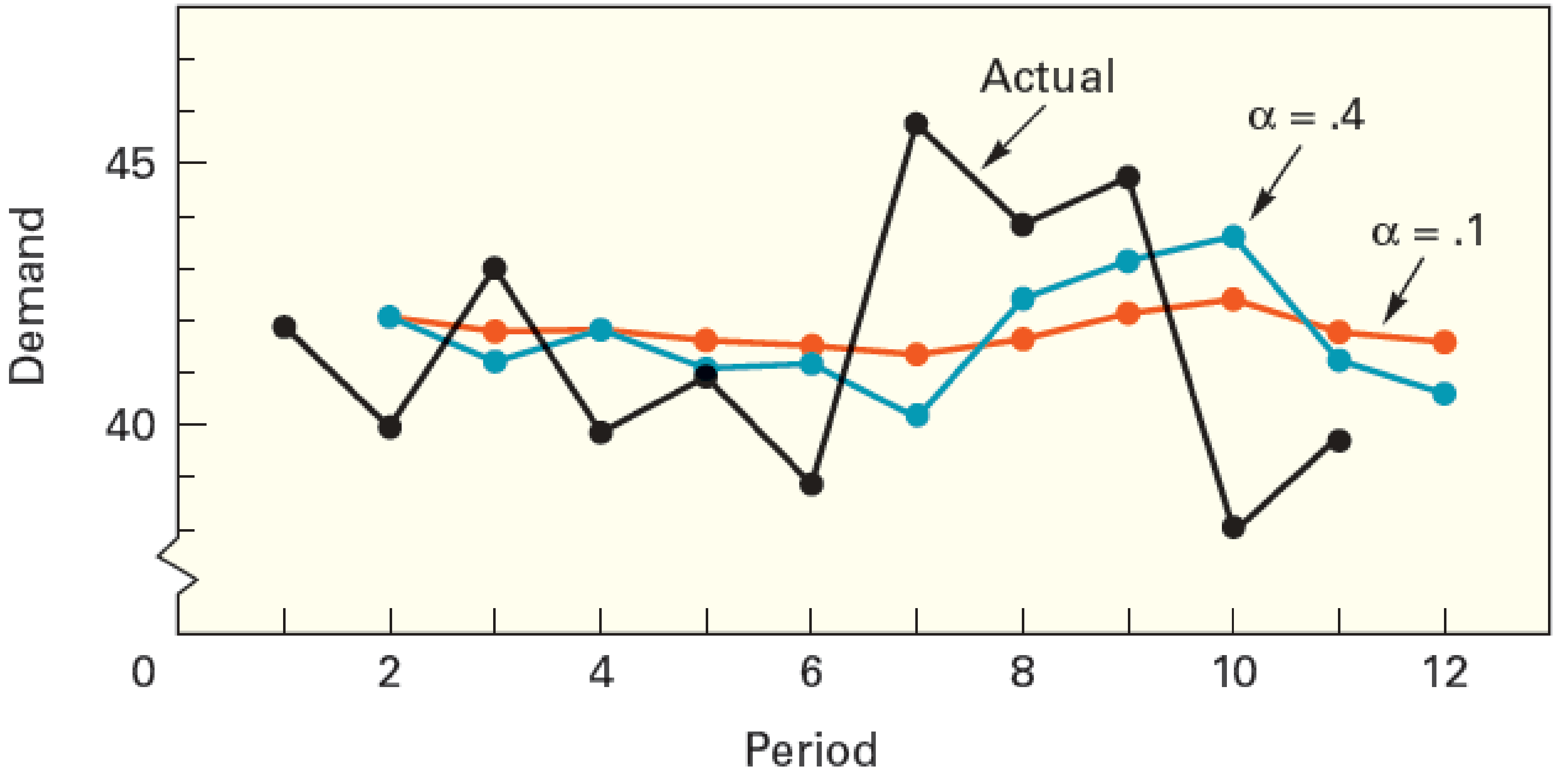
The Effect of the Smoothing Factor

Period (t)	Actual Demand	$\alpha = .10$ Forecast	$\alpha = .40$ Forecast
1	42 starting forecast	—	—
2	40	42	42
3	43	41.8	41.2
4	40	41.92	41.92
5	41	41.73	41.15
6	39	41.66	41.09
7	46	41.39	40.25
8	44	41.85	42.55
9	45	42.07	43.13
10	38	42.35	43.88
11	40	41.92	41.53
12		41.73	40.92

The Effect of the Smoothing Factor

What does the Smoothing Factor tells us?

The Effect of the Smoothing Factor



Methods of Smoothing Time Series

- Smoothing a time series: to eliminate some of short-term fluctuations or effect due to random variation
- Smoothing also can be done to remove seasonal fluctuations, i.e., to de-seasonalize a time series.
  - Arithmetic Moving Average
  - Exponential Smoothing Methods
  - Holt-Winters method for Exponential Smoothing
- These models are deterministic in that no reference is made to the sources or nature of the underlying randomness in the series.
- The models involves extrapolation techniques.

Averaging Methods

- Simple Averages - quick, inexpensive
- Moving Average method consists of computing an average of the most recent n data values for the series and using this average for forecasting the value of the time series for the next period.
- Recall Simple Moving Average

$$\hat{y}_{t+1} = \frac{y_t + y_{t-1} + \cdots + y_{t-n}}{n}$$

Averaging Methods

- Centered Moving Average
  - Useful in getting overall idea of trends
- Centered Moving Average (n is odd)

$$\hat{y}_t = \frac{y_{t+\frac{n-1}{2}} + y_{t+\frac{n-1}{2}-1} + \cdots + y_t + \cdots + y_{t-\frac{n-1}{2}}}{n}$$

- Centered Moving Average (n is even)

$$\hat{y}_t = \frac{y_{t+\frac{n}{2}} + y_{t+\frac{n}{2}-1} + \cdots + y_{t-\frac{n}{2}+1}}{2n} + \frac{y_{t+\frac{n}{2}-1} + y_{t+\frac{n}{2}-2} + \cdots + y_{t-\frac{n}{2}}}{2n}$$

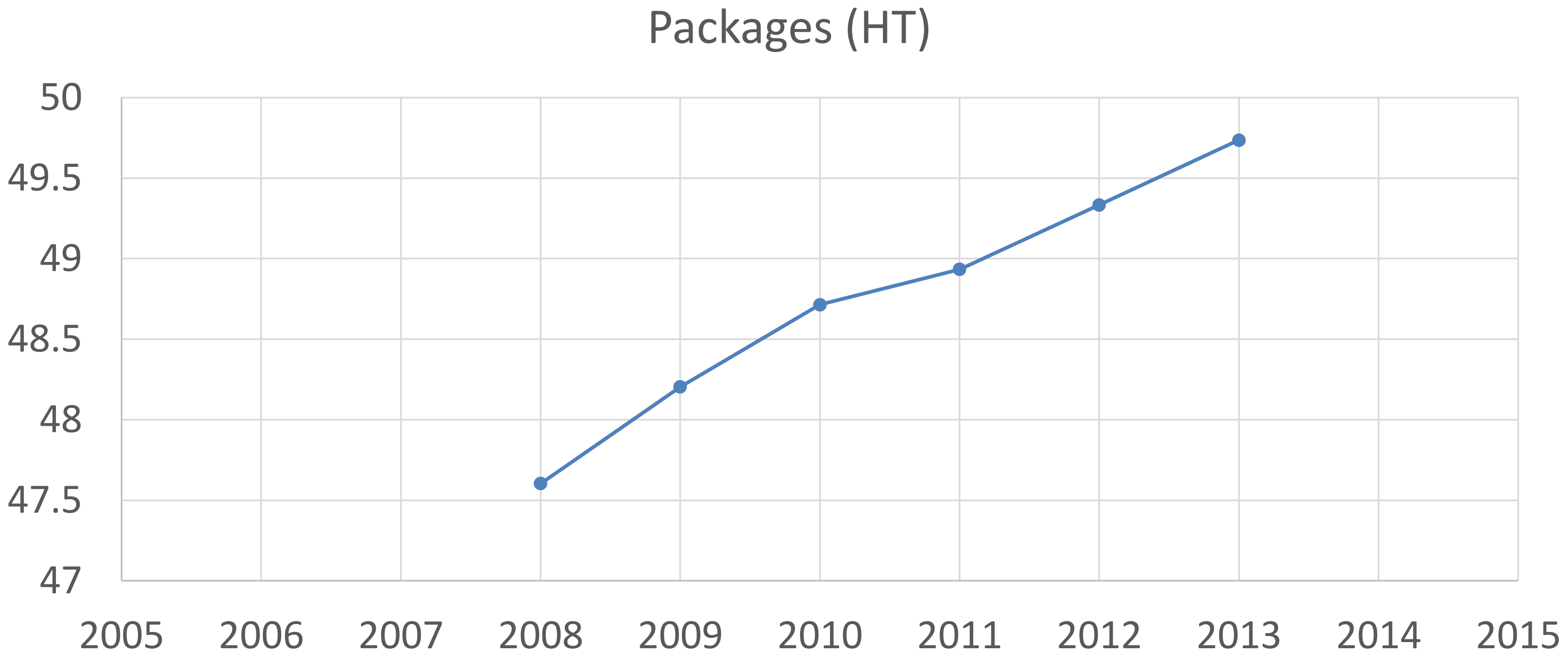
Averaging Methods

- Example, n = 5

Year	# of Small Packages (Hundred Thousands)	5 - CMA
2006	46.16	
2007	46.99	
2008	47.81	
2009	48.31	
2010	48.75	
2011	49.16	
2012	49.54	
2013	48.91	
2014	50.31	
2015	50.76	

Averaging Methods

- Example, n = 5



Moving Averages

- Moving averages are useful if one can assume item to be forecast will stay steady over time.
- Series of arithmetic means – used only for smoothing, provides overall impression of data over time
  - The smaller the number, the more weight given to recent periods. This is desirable when there are sudden shifts in the level of the series.
  - The greater the number, less weight is given to more recent periods and the greater the smoothing effect.

Moving Averages

- Weighted Moving Average - place more weight on recent observations. Sum of the weights needs to equal 1.
- Used when trend is present
  - Older data usually less important

$$\hat{y}_{t+1} = \frac{w_1 y_t + w_2 y_{t-1} + \cdots + w_n y_{t-n}}{n}$$

Births

- An example is a data set of the number of births per month in New York city, from January 1946 to December 1959
- Forecast using Simple Moving Average with periods 2, 5 and 10

$$\hat{y}_{t+1} = \frac{w_1 y_t + w_2 y_{t-1} + \cdots + w_n y_{t-n}}{n}$$

Preliminary: ts

- Class ts
- Used to create “time series objects”
- Represents data which has been sampled at equally spaced points in time
- By default, frequencies can be 4, 7, and 12
- A weekly, monthly and quarterly series

Preliminary: ts

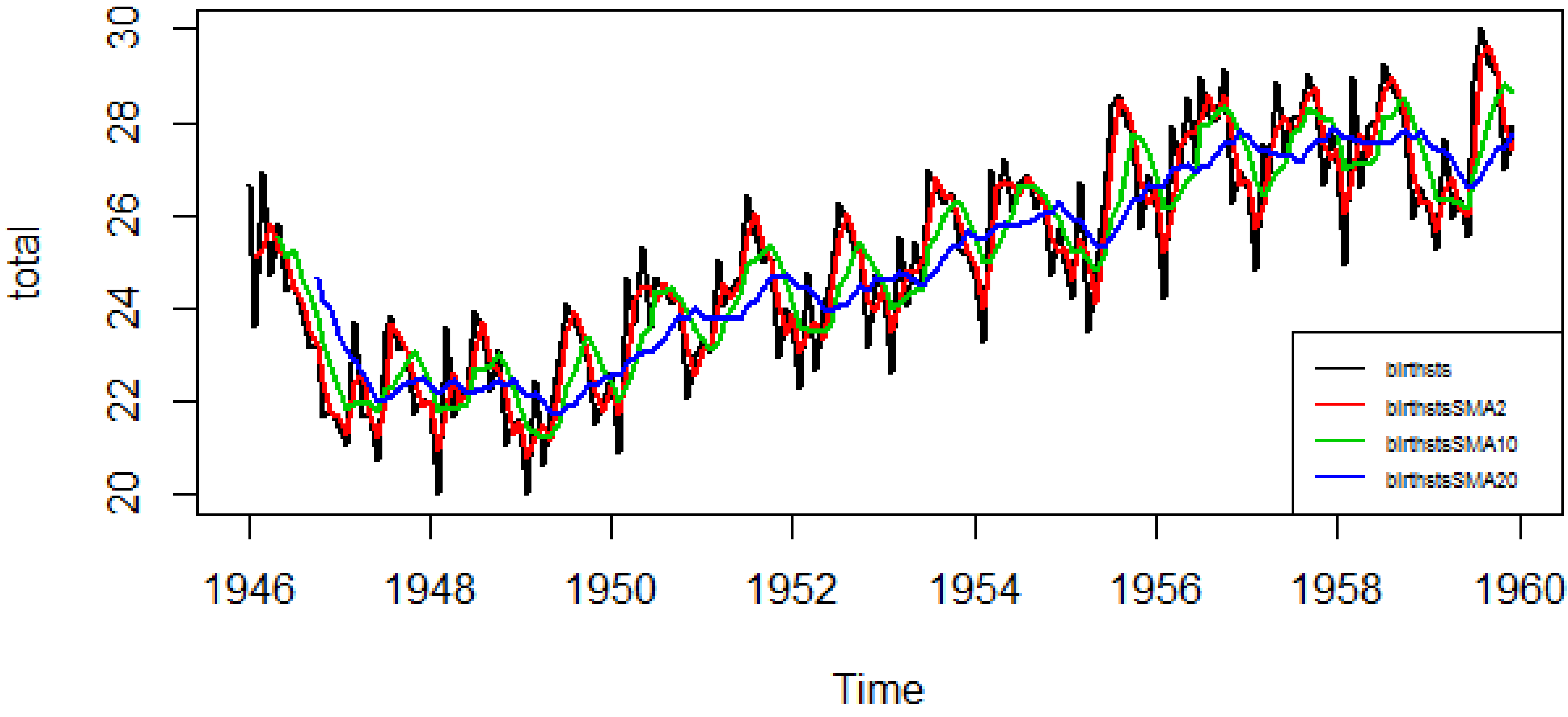
```
install.packages("TTR")
install.packages("forecast")

a <- ts(1:20, frequency=12, start=c(2011,3))
print(a)
str(a)
attributes(a)
```

R Code

```
• library("TTR")
• births = read.csv("births.csv")
• birthsts = ts(births[,2], frequency=12, start=c(1946,1))
• birthstsSMA2 = SMA(birthsts,n=2)
• birthstsSMA5 = SMA(birthsts,n=5)
• birthstsSMA10 = SMA(birthsts,n=10)
• total =
  cbind(birthsts,birthstsSMA2,birthstsSMA5,birthstsSMA10)
• plot(total, plot.type="single",col = 1:ncol(total), lwd = c(2,
  2, 2,2))
• legend("bottomright", colnames(total), col=1:ncol(total), lty
  = c(1, 1, 1,1), cex=.5, y.intersp = 1)
```

Births Dataset



R Code: Forecast Accuracy

- `accuracy(birthstsSMA2, birthsts)`

```
> accuracy(birthstsSMA2, birthsts)
                  ME      RMSE       MAE         MPE     MAPE       ACF1 Theil's U
Test set 0.003694611 0.7526019 0.5976826 -0.07872341 2.399529 -0.5282228       0.5
```

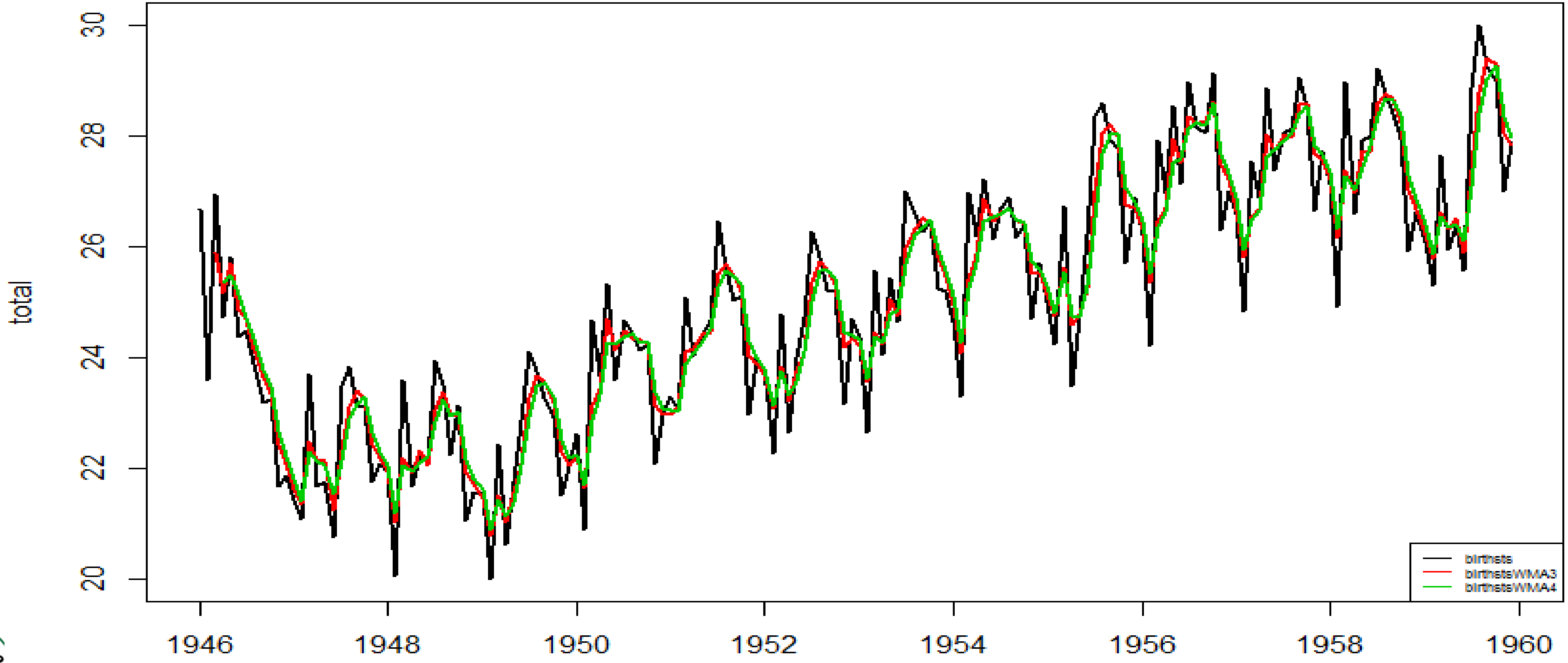
Weighted Moving Average

- Over the years, technicians have found problems with the simple moving average.
- One problem lies in the time frame of the moving average (MA). Analysts believe in assigning more weights to the more recent data.
- A weighted average is any average that has multiplying factors to give different weights to data at different positions in the sample window.

R Code: WMA

```
• xx <- c(.2,.3,.5)
• birthstsWMA3 = WMA(birthsts, n=3, wts=xx)
• birthstsWMA3
• yy <- c(.1,.2,.3,.4)
• birthstsWMA4 = WMA(birthsts, n=4, wts=yy)
• birthstsWMA4
• total = cbind(birthsts,birthstsWMA3,birthstsWMA4)
• plot(total, plot.type="single",col = 1:ncol(total),
  lwd = c(2, 2, 2))
• legend("bottomright", colnames(total),
  col=1:ncol(total), lty = c(1, 1, 1), cex=.5,
  y.intersp = 1)
```

WMA Plot


total
30
28
26
24
22
20
1946
1948
1950
1952
1954
1956
1958
1960
birthsts
birthstsWMA3
birthstsWMA4

Extra on Linear Weighting

- WMA is not really using a manually set weighting syste,
- Weights = 1,2,3,4,5
- 5/15, 4/15, 3/15, 2/15, 1/15
- Therefore WMA is 83(5/15) + 81(4/15) + 79(3/15) + 79(2/15) + 77(1/15) = 80.7

Day	Price
1	77
2	79
3	79
4	81
5 (Current)	83

Notes on Moving Averages

- MA models do not provide information about forecast confidence.
- We can not calculate standard errors.
- We can not explain the stochastic component of the time series. This stochastic component creates the error in our forecast.
- Lag factor – the longer moving average, the more the lag
  - Tells us behavioral change of the time series data (e.g., delay)

Exponential Smoothing Methods

- Single Exponential Smoothing (Averaging)
  - Used for a series without a trend and a seasonal component.
- Double Exponential Smoothing
  - Double Exponential Smoothing is for a series with a trend
  - but without a seasonal component.
- Winter's Model
  - Winter's model is for a series with a trend and seasonal component.

Single Exponential Smoothing

- Averaging (smoothing) past values of a series in a decreasing (exponential) manner.
- The observations are weighted with more weight being given to the more recent observations

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t$$

- New forecast = $\alpha \times$ (old observation) + (1- α) $\times$ old forecast.
- The equation can be rewritten as:

$$\hat{y}_{t+1} = \hat{y}_t + \alpha(y_t - \hat{y}_t)$$

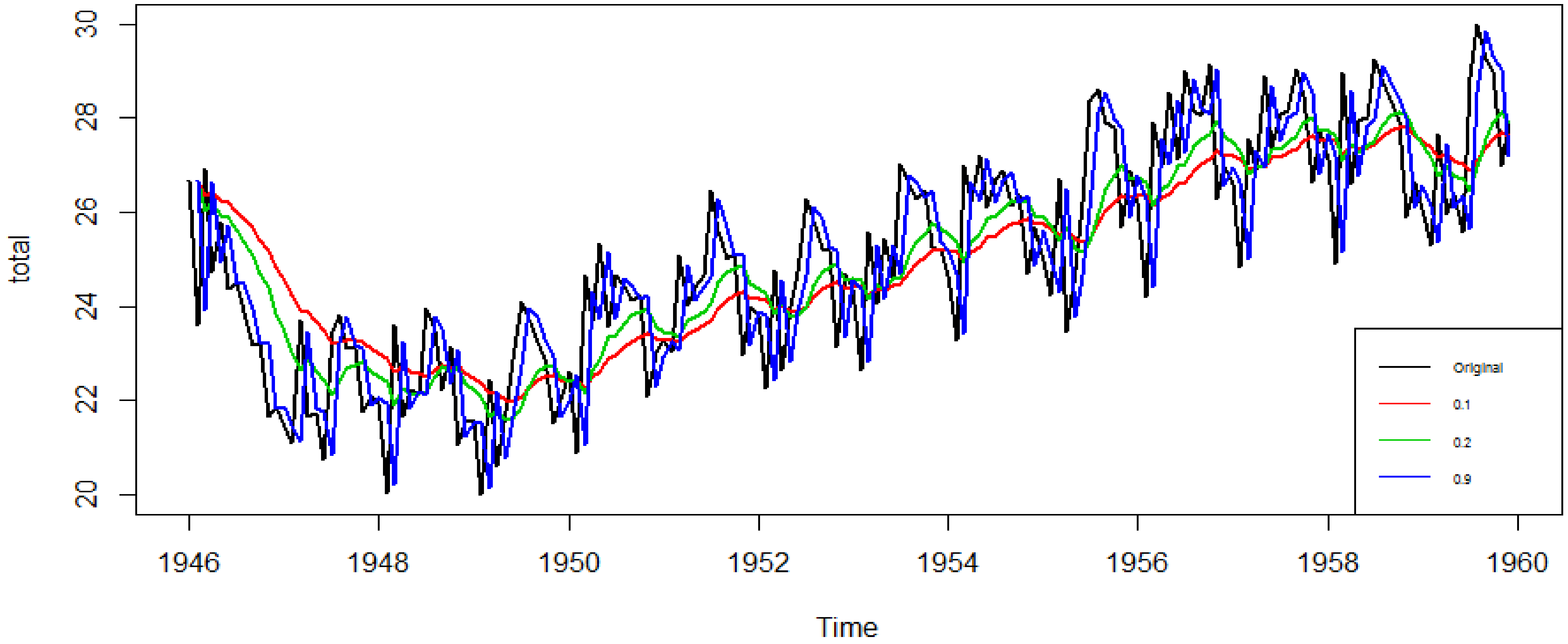
Single Exponential Smoothing

- We need a smoothing constant α, an initial forecast, and an actual value.
- The smoothing constant serves as the weighting factor.
  - When α is close to 1, the new forecast will include a substantial adjustment for any error that occurred in the preceding forecast.
  - When α is close to 0, the new forecast is very similar to the old forecast.
- The smoothing constant α is not an arbitrary choice but generally falls between 0.1 and 0.4.

Single Exponential Smoothing: R Code

- `birthstssesa01 =  HoltWinters(birthsts,alpha=0.1, beta=FALSE, gamma=FALSE)`
- `birthstssesa02 =  HoltWinters(birthsts,alpha=0.2, beta=FALSE, gamma=FALSE)`
- `birthstssesa09 =  HoltWinters(birthsts,alpha=0.9, beta=FALSE, gamma=FALSE)`
- `total = cbind(birthsts,birthstssesa01$fitted[,1],birthstssesa02$fitted[,1],birthstssesa09$fitted[,1])`
- `plot(total, plot.type="single", col = 1:ncol(total), lwd = c(2, 2, 2,2))`
- `legend("bottomright", c("Original","0.1","0.2","0.9"), col=1:ncol(total), lty = c(1, 1), cex=.5, y.intersp = 1)`

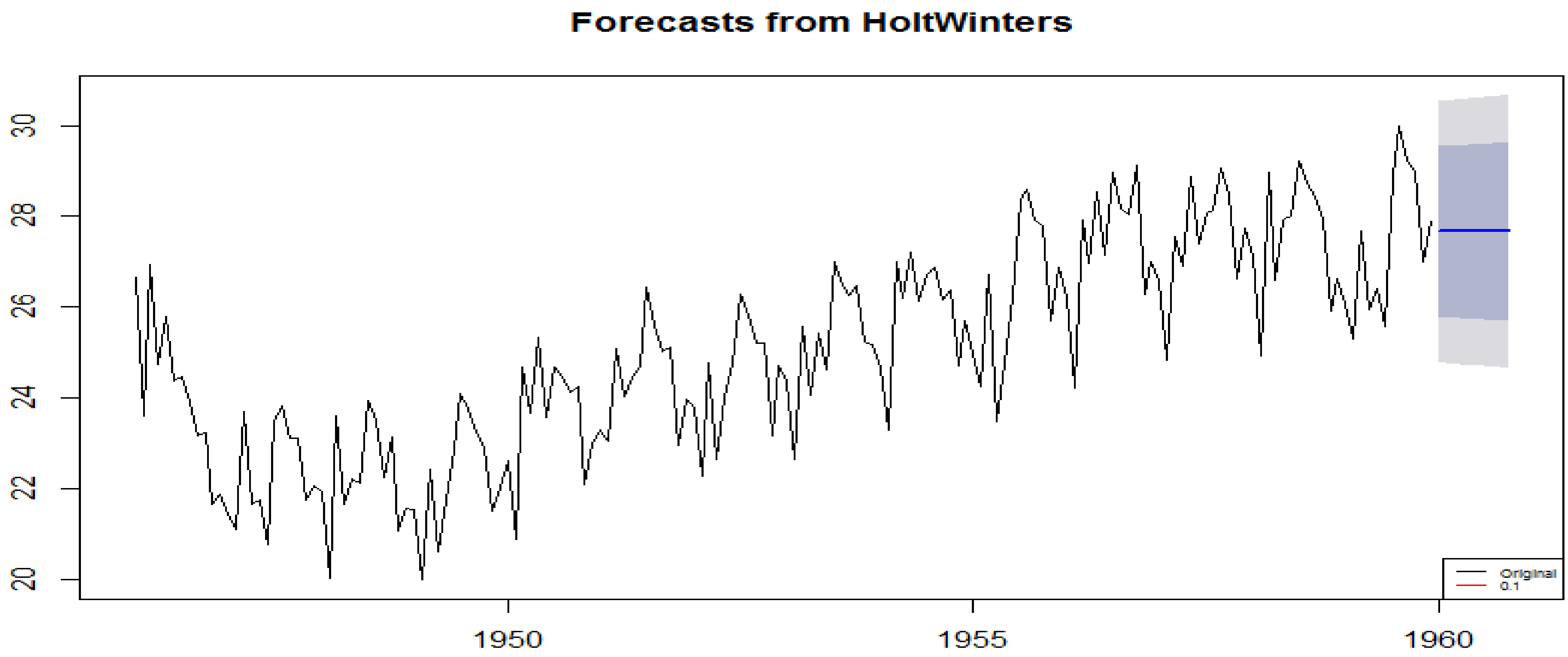
Choice of Alpha, α



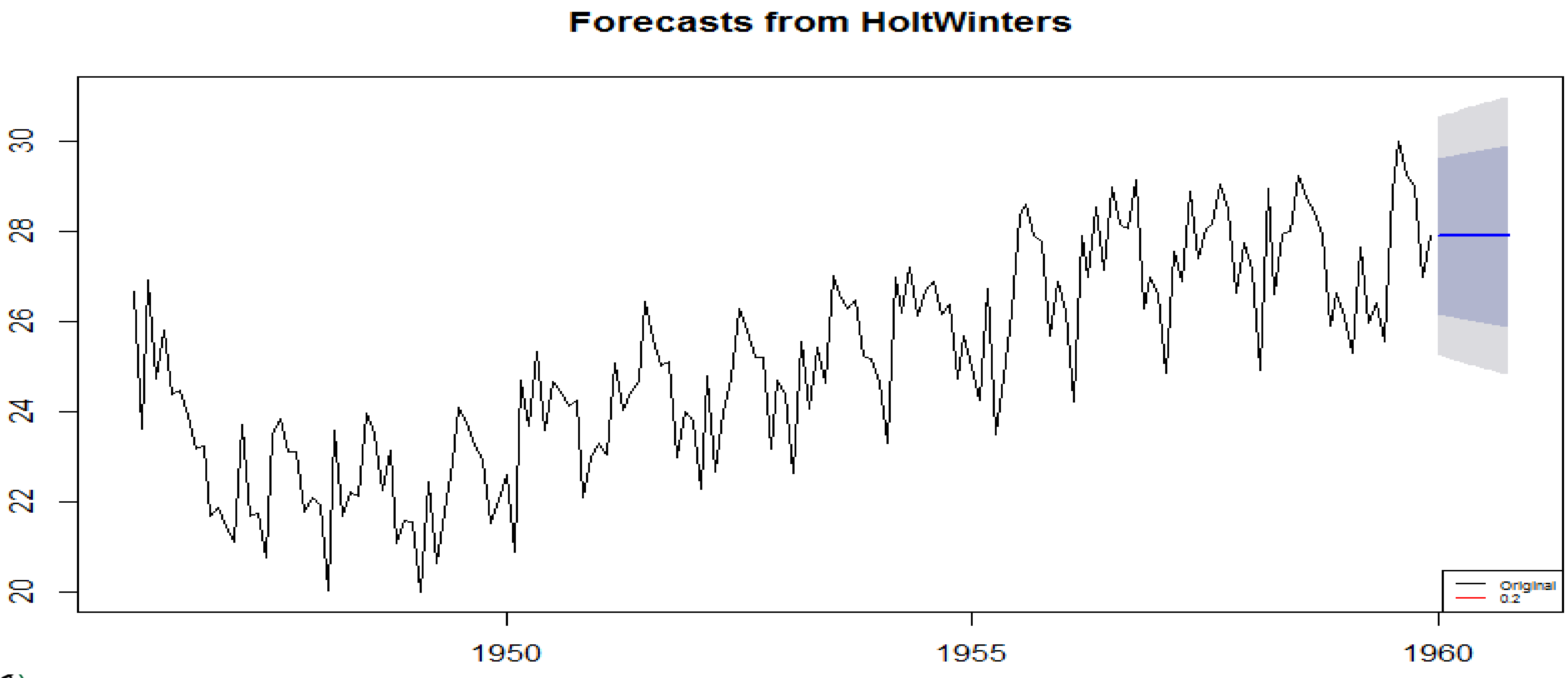
Single Exponential Smoothing Forecast: R Code

- `birthstssesforecast01 = forecast.HoltWinters(birthstssesa01 , h=10)`
- `birthstssesforecast02 = forecast.HoltWinters(birthstssesa02 , h=10)`
- `birthstssesforecast09 = forecast.HoltWinters(birthstssesa09 , h=10)`
- `plot.forecast(birthstssesforecast01)`
- `plot.forecast(birthstssesforecast02)`
- `plot.forecast(birthstssesforecast09)`

Forecast

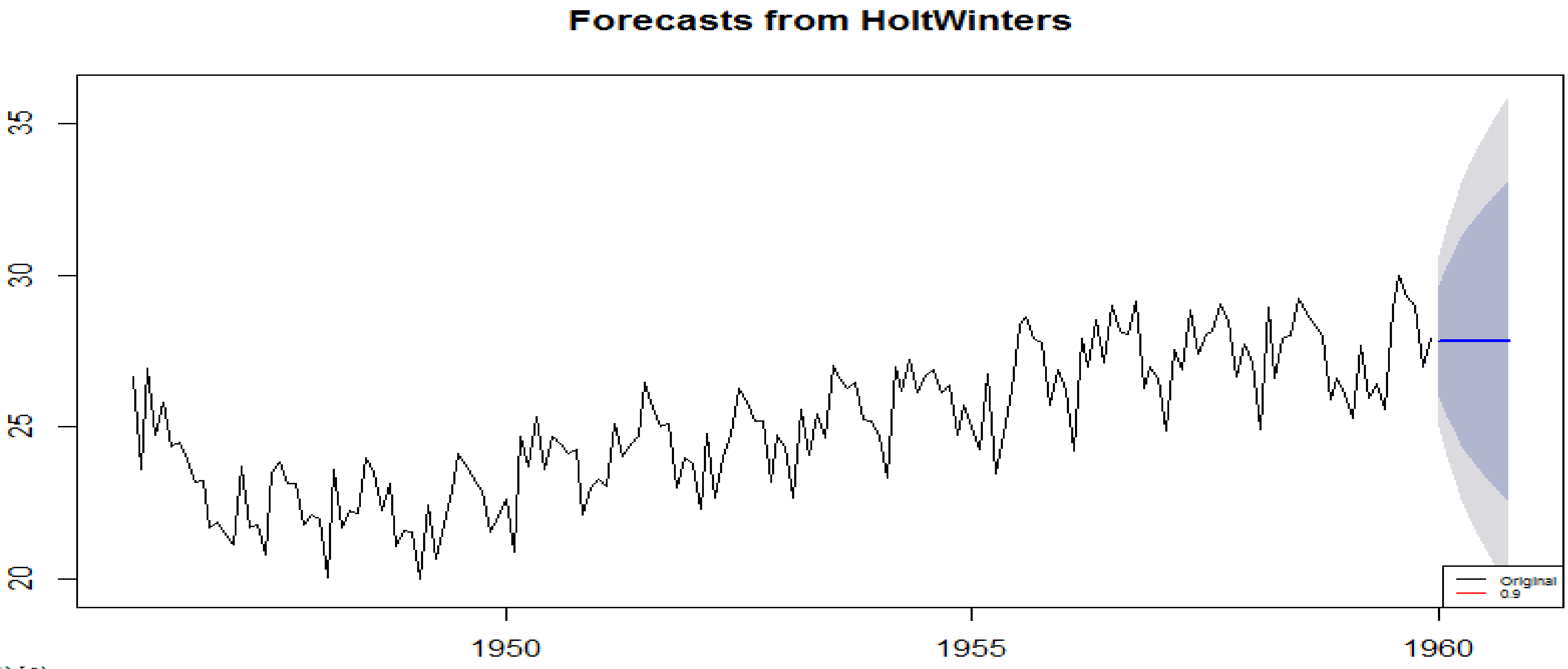

Forecasts from HoltWinters
30
28
26
24
22
20
1950
1955
1960
Original
0.1

Forecast



Forecasts from HoltWinters
30
28
26
24
22
20
1950
1955
1960
Original
0.2

Forecast



Forecasts from HoltWinters
35
30
25
20
1950
1955
1960
Original
0.9

Single Exponential Smoothing Forecast: R Code

- `accuracy(birthstssesforecast01)`
- `accuracy(birthstssesforecast02)`
- `accuracy(birthstssesforecast09)`

Single Exponential Smoothing Forecast: R Code

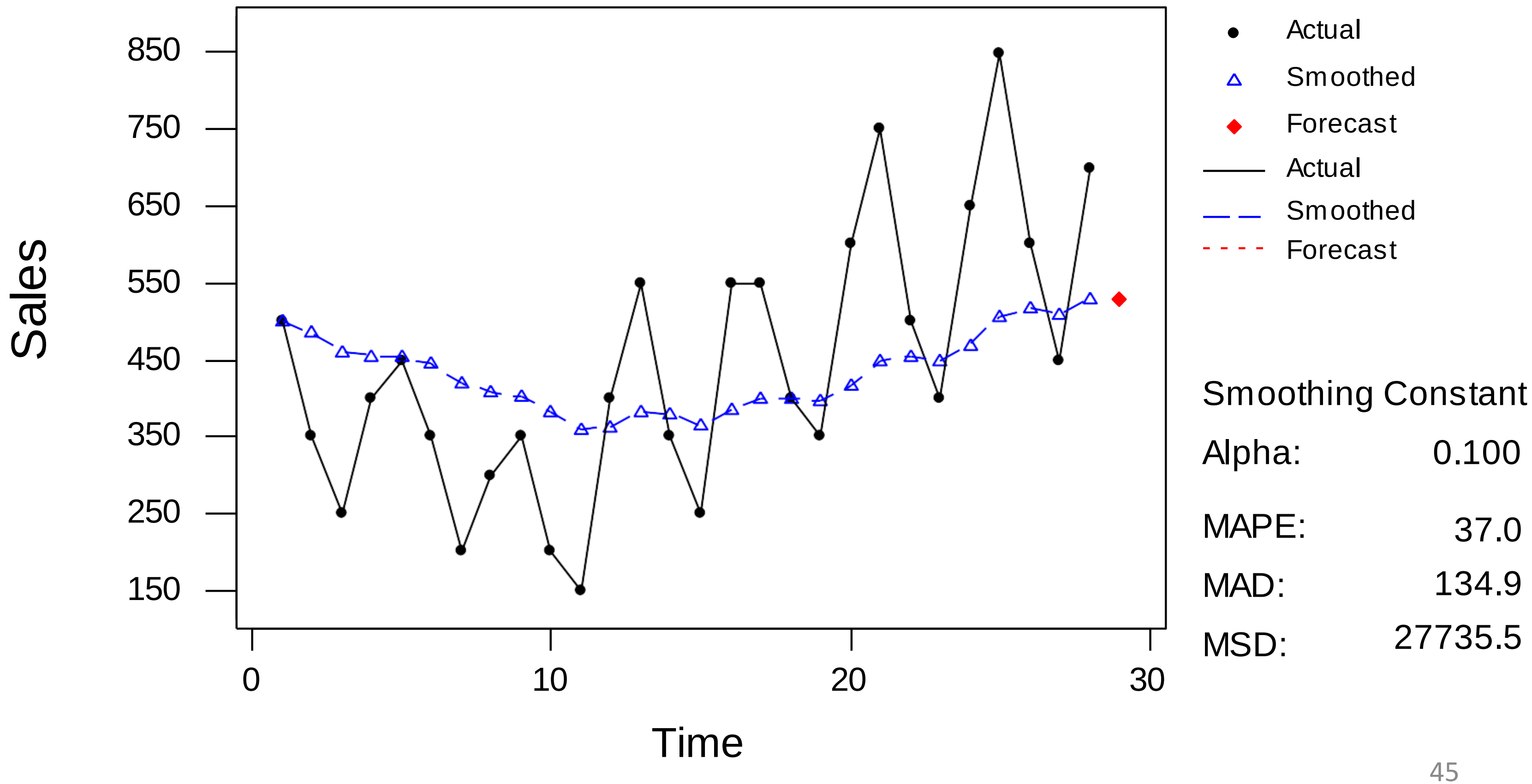
```
> accuracy(birthstssesforecast01)
                     ME    RMSE      MAE       MPE     MAPE     MASE      ACF1
Training set 0.05971923 1.46493 1.219158 0.1068986 4.907687 1.291966 0.4175016
> accuracy(birthstssesforecast02)
                     ME     RMSE      MAE       MPE     MAPE     MASE     ACF1
Training set 0.03688771 1.346141 1.118519 0.1116814 4.506428 1.185318 0.242625
> accuracy(birthstssesforecast09)
                      ME     RMSE      MAE       MPE     MAPE     MASE       ACF1
Training set 0.007744566 1.434287 1.124833 0.1983202 4.568417 1.192009 -0.4568299
```

Monitoring of Forecasts: Tracking

- Use a tracking signal (measure of errors over time) and setting limits.
- For example, if we forecast $\boldsymbol{n}$ periods, count the number of negative and positive errors.
  - If the number of positive errors is substantially less or greater than $\boldsymbol{n}/2$, then the process is out of control
- Or use the running sum of forecast error

Sample Data

Sales data Single Exponential Smoothing alpha = 0.1



Tracking Using a Confidence Interval

- Can also use 95% *prediction interval*

$$\hat{y} \pm z_{\frac{\alpha}{2}} \sqrt{MSE}$$

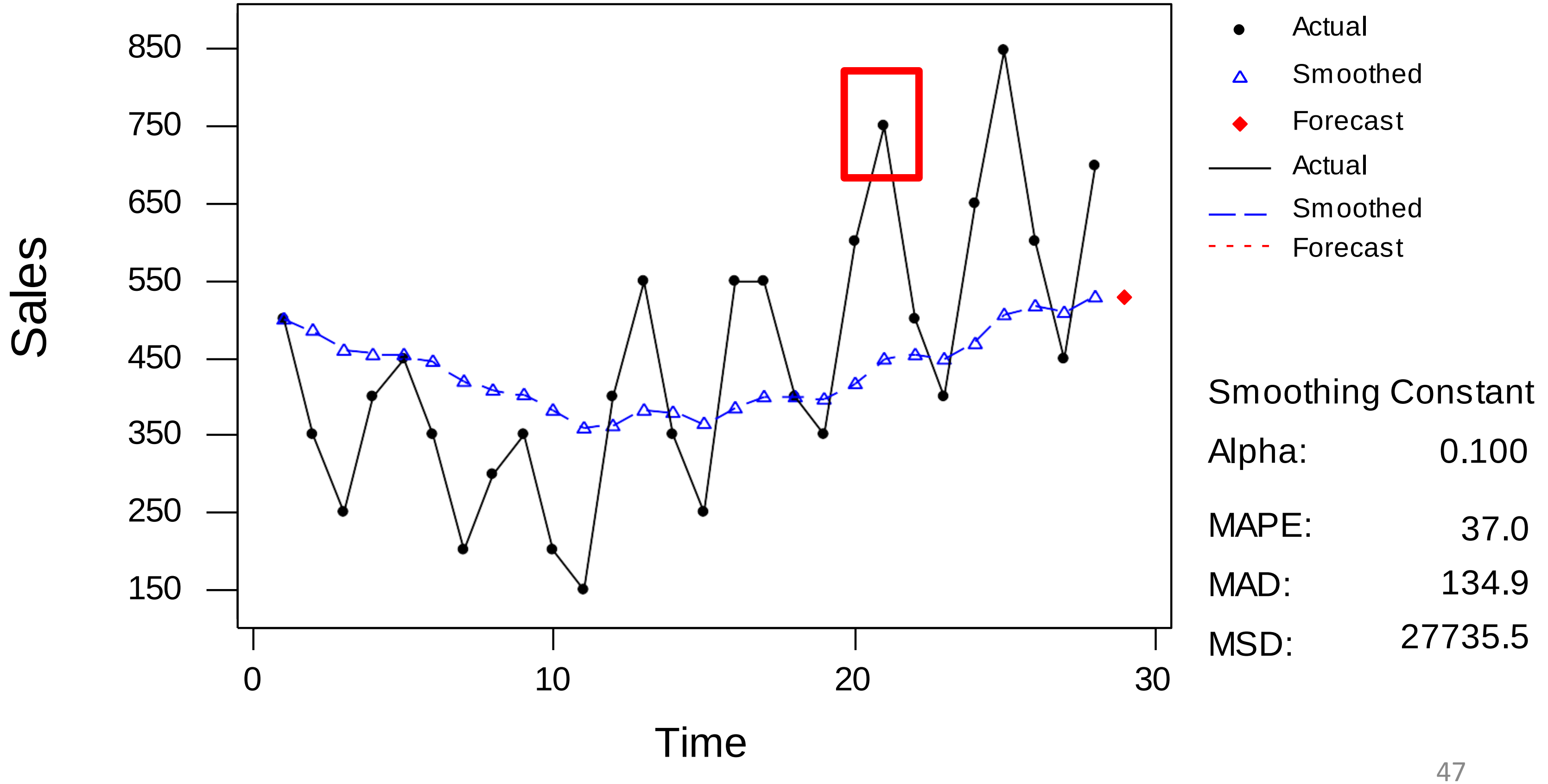
- If the forecast error is outside of the interval, use a new optimal α.
- Looking back at the 0.1 single exponential smoothing:

$$\hat{y} \pm 1.96 * \sqrt{27735.5} = \hat{y} \pm 326.4$$

- – Observation #21 is out-of-control. We need to re-evaluate alpha level because this technique is biased.

Sample Data

Sales data Single Exponential Smoothing alpha = 0.1



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Holt's Method: Double Exponential Smoothing

- In some situations, the observed data are trending and contain information that allows the anticipation of future upward (or downward) movement.
- In that case, a linear trend forecast function is needed.
- Holt's smoothing method allows for evolving local linear trend in a time series and can be used to forecast.
- When there is a trend, an estimate of the current slope and the current level is required.

Holt's Method: Double Exponential Smoothing

- Holt's method uses two coefficients.
  - α is the smoothing constant for the *level*
  - β is the *trend* smoothing constant - used to remove random error.
- Advantage of Holt's method: it provides flexibility in selecting the rates at which the level and trend are tracked.

Equations in Holt's Method

- The exponentially smoothed series, or the current level estimate

$$\hat{y}_t = \alpha y_t + (1 - \alpha)(\hat{y}_{t-1} + T_{t-1})$$

- The trend estimate:

$$T_t = \beta(\hat{y}_t - \hat{y}_{t-1}) + (1 - \beta)T_{t-1}$$

- Forecast *m* periods into the future:

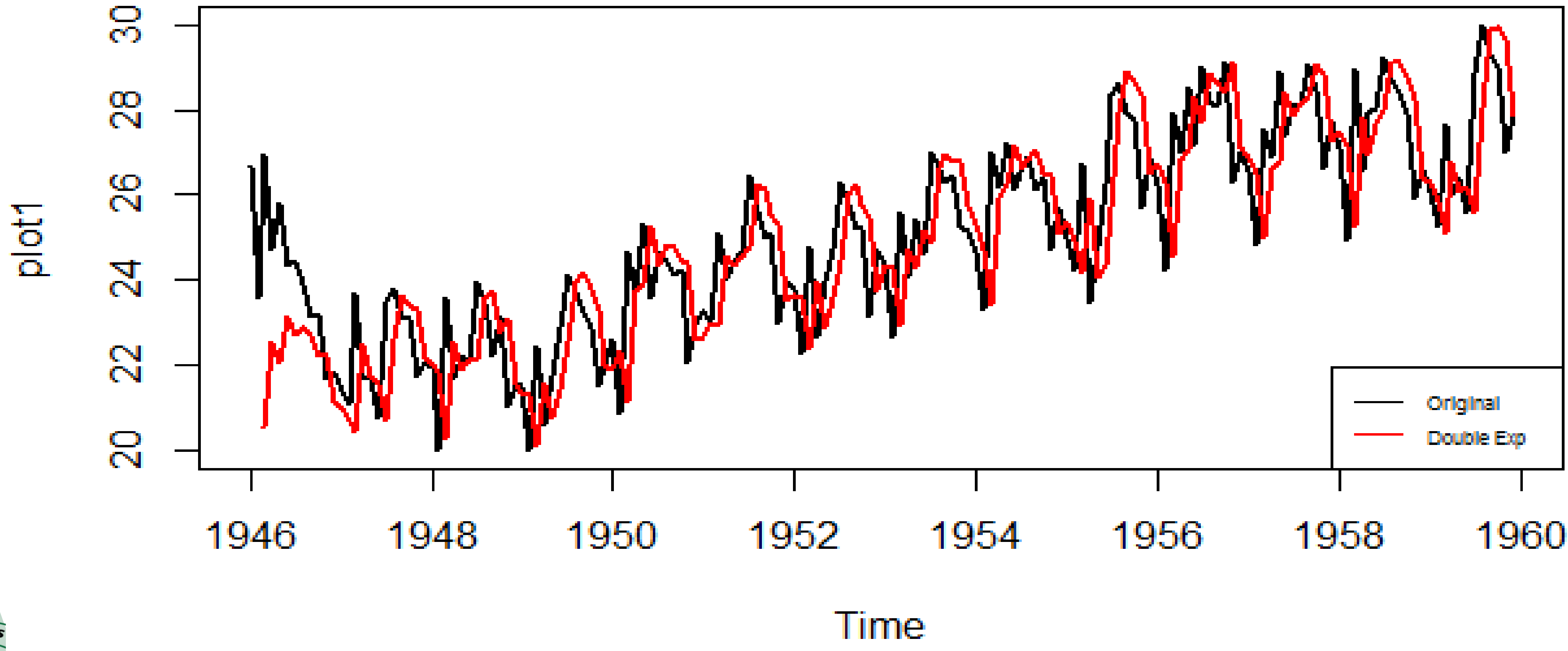
$$\hat{y}_{t+m} = \hat{y}_t + mT_t$$

- Where:
  - T_t = trend estimate
  - y_{t+m} = forecast for *p* periods into the future.
  - α = smoothing constant for the level
  - β = smoothing constant for trend estimate

R Code: Holt's Method - Double Exponential Smoothing

- `birthstsdep = HoltWinters(birthsts, gamma=FALSE)`
- `total = cbind(birthsts,birthstsdep$fitted[,1])`
- `plot(total,plot.type="single", col = 1:ncol(total), lwd = c(2, 2))`
- `legend("bottomright", c("Original","Double Exp"), col=1:ncol(total), lty = c(1, 1), cex=.5,y.intersp = 1)`

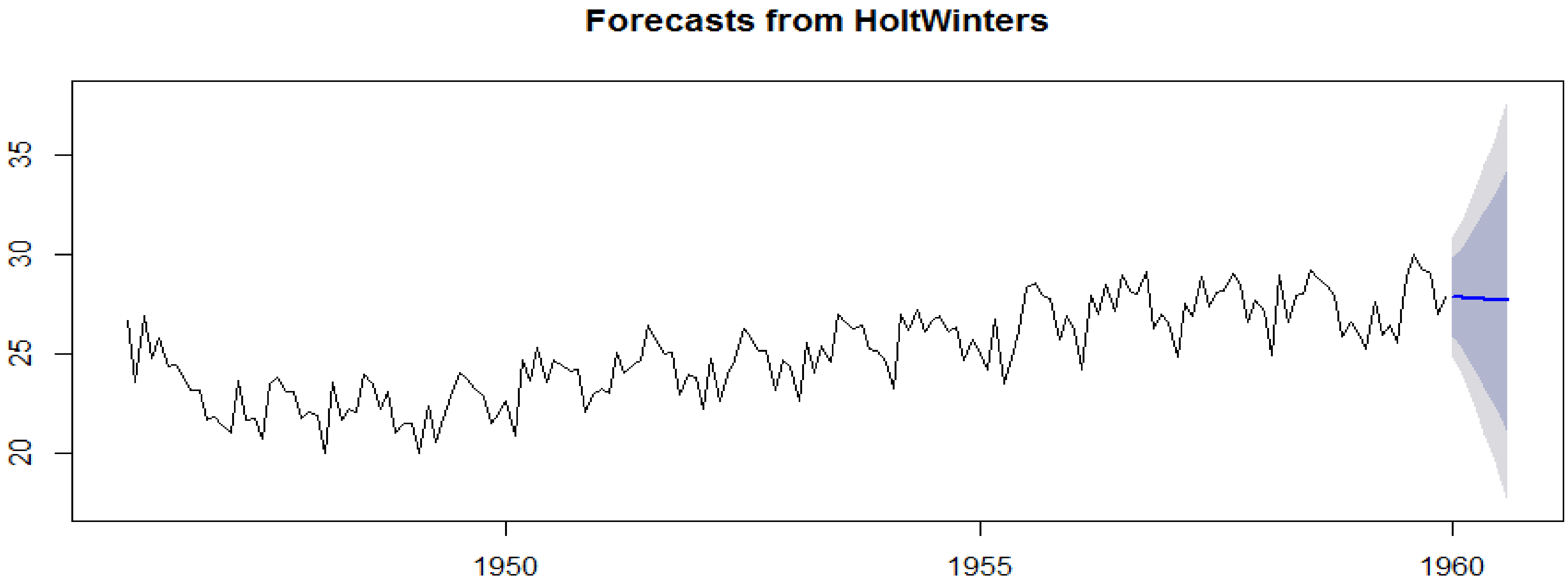
Holt’s Method



R Code: Forecast - Holt's Method

- `birthstsdepforecast = forecast.HoltWinters(birthstsdep , h=8)`
- `plot.forecast(birthstsdepforecast)`

Forecast: Holt's Method



Winter’s Method

- Winters’ method is an easy way to account for seasonality when data have a seasonal pattern.
- It extends Holt’s Method to include an estimate for seasonality.
  - α is the smoothing constant for the level
  - β is the trend smoothing constant - used to remove random error.
  - γ smoothing constant for seasonality
- This formula removes seasonal effects. The forecast is modified by multiplying by a seasonal index.

Equations in Winter's Method

- The exponentially smoothed series, or the current level estimate:
  - $\hat{y}_t = \dfrac{\alpha y_t}{S_{t-s}} + (1 - \alpha)(\hat{y}_{t-1} + T_{t-1})$
- The trend estimate:
  - $T_t = \beta(\hat{y}_t - \hat{y}_{t-1}) + (1 - \beta)T_{t-1}$
- The seasonality estimate:
  - $St = \dfrac{\gamma y_t}{\hat{y}_t} + (1 - \gamma)S_{t-s}$
- Forecast m periods into the future:
  - $\hat{y}_{t+m} = (\hat{y}_t + mTt)\, S_{t-s+m}$

γ = smoothing constant for seasonality estimate

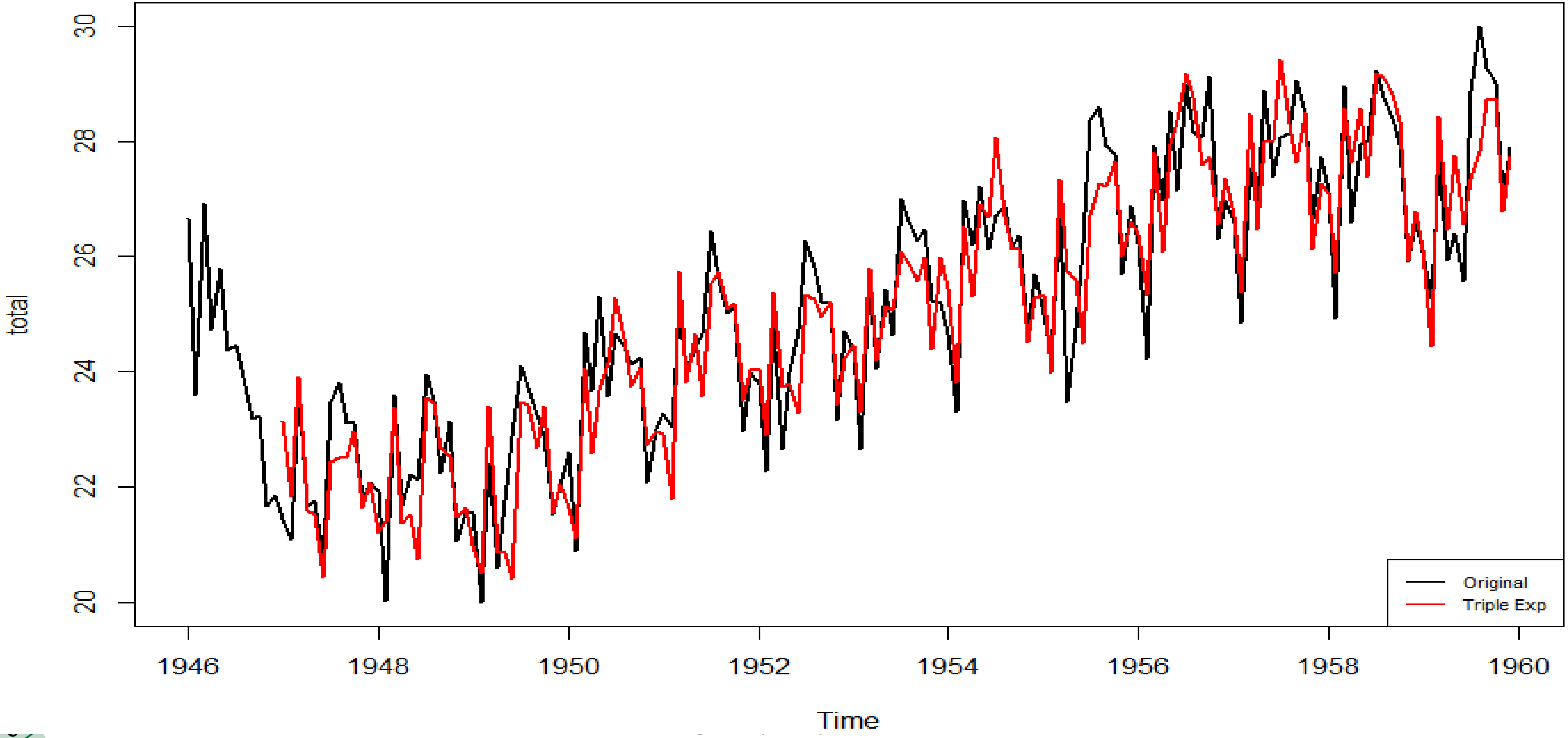
m = periods to be forecast into the future

s = length of season

R Code: Winter’s Method

- `birthststes= HoltWinters(birthsts)`
- `total = cbind(birthsts,birthststes$fitted[,1])`
- `plot(total, plot.type="single", col = 1:ncol(total), lwd= c(2, 2))`
- `legend("bottomright", c("Original","TripleExp"), col=1:ncol(total), lty= c(1, 1), cex=.7, y.intersp= 1)`

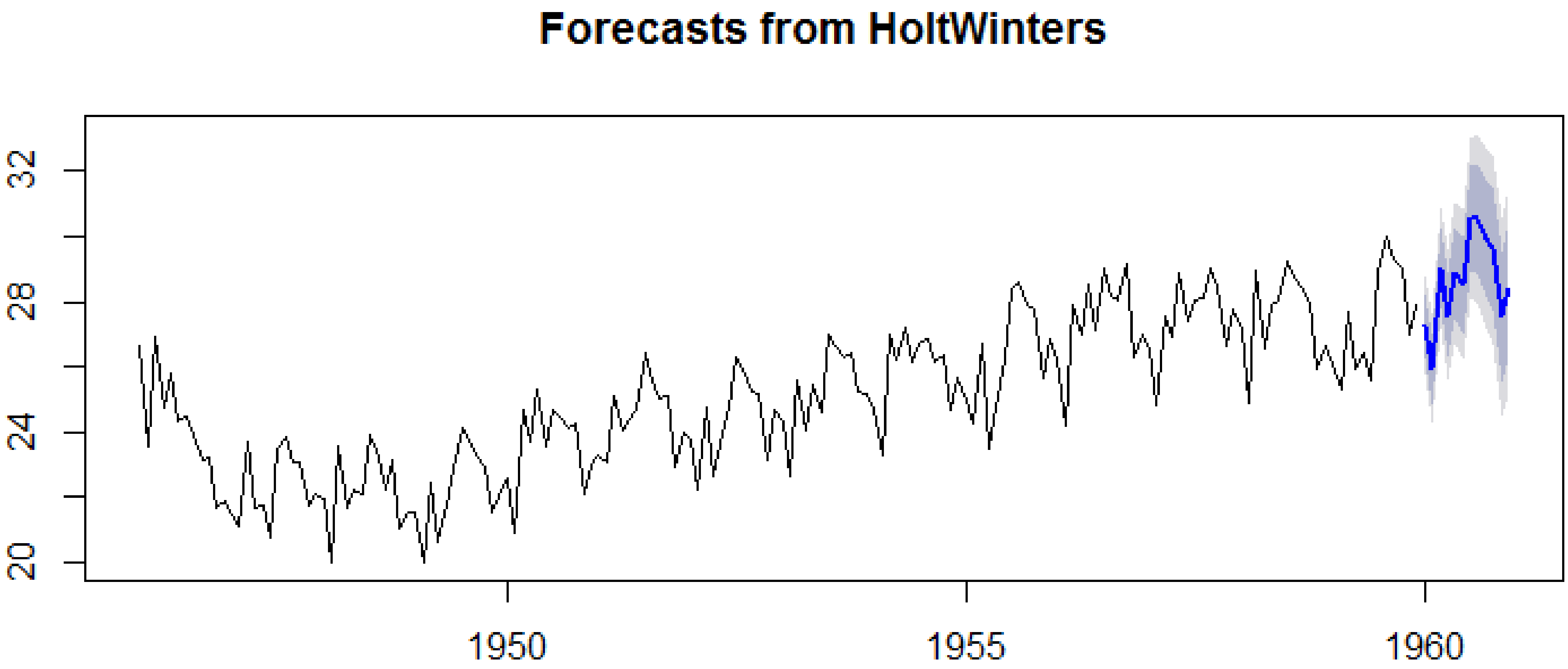
Forecast: Winter's Method



Forecast: Winter's Method

- `library("forecast")`
- `birthststes= HoltWinters(birthsts)`
- `birthstesforecast = forecast.HoltWinters(birthststes , h=12)`
- `plot.forecast(birthstesforecast)`

Forecast: Winter's Method



Decomposition

- Decomposition is a procedure to identify the component factors of a time series.
- How the components relate to the original series: a model that expresses the time series variable Y in terms of the components T (trend), C (cycle), S (seasonal) and I (irregular).
- It is difficult to deal with cyclical component of a time series. To keep things simple we assume that any cycle in the data is part of the trend.

Decomposition

- Two models: Additive components model and multiplicative components model
- Additive model: $y_t = T_t + S_t + I_t + Ct$
- Multiplicative model: $y_t = T_t \times S_t \times I_t \times C_t$
- Decomposition finds the estimates of these four components

Decomposition

- There are a lot of different ways to decompose your data
  - A lot of different functions in R
  - Several manual procedures
  - Use of moving averages (e.g., centered MA)
  - De-trend or de-seasonalizing comes first
  - Use of seasonal relatives/indices

Additive and Multiplicative Models

- The additive model works best when the time series has roughly the same variability through the length of the series.
  - All the values of the series fall within a band with constant width centered on the trend.
  - Appropriate if the magnitude of the seasonal fluctuation does not vary with the level of the series

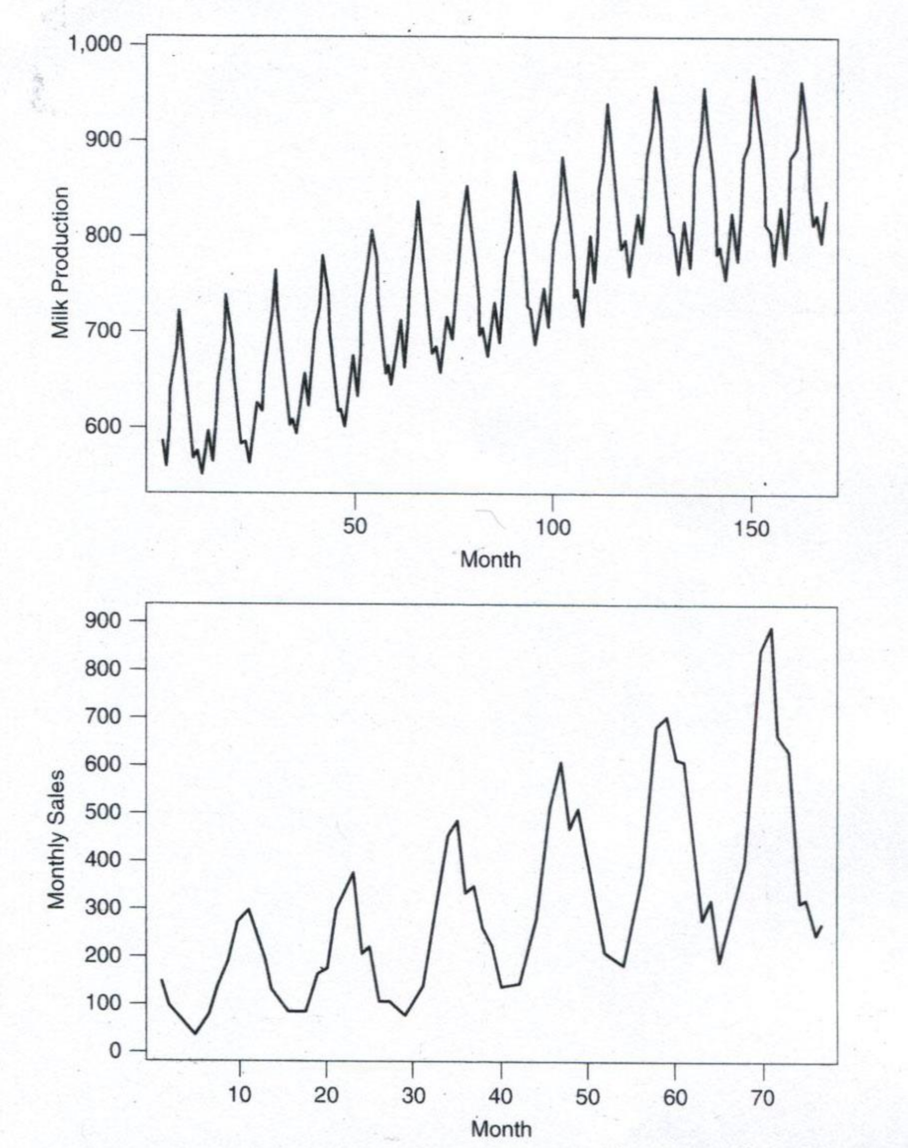
Additive and Multiplicative Models

- The multiplicative model works best when the variability of the time series increased with the level.
  - Variance of the series varies as the trend progresses
  - More prevalent with economic series since most seasonal variation increases with the level of the series

Note on Cycles

- Cycles are often difficult to identify with a short time series
- Thus, in this module, cycle components shall be assumed to be combined with trend components
- Models will be reduced to the following:
  - Additive model: $y_t = TC_t + S_t + I_t$
  - Multiplicative model: $y_t = TC_t \times S_t \times I_t$

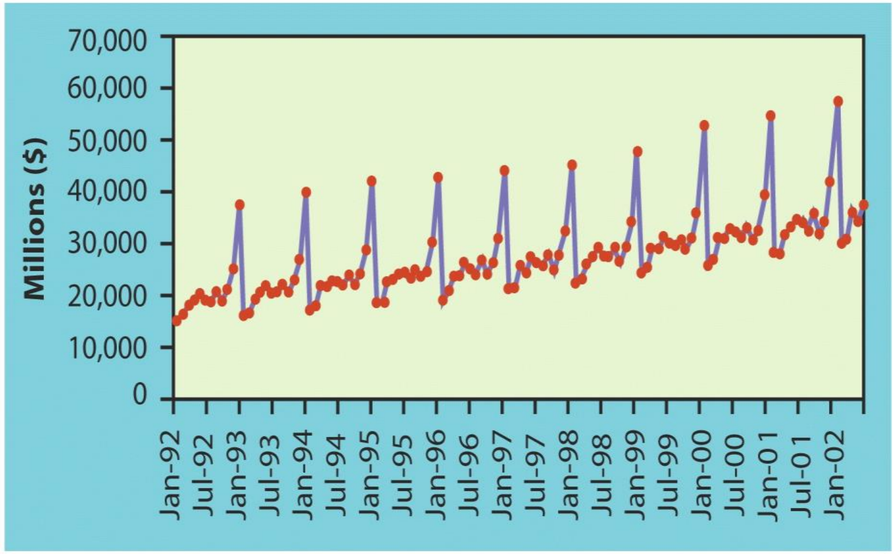
Additive and Multiplicative Models



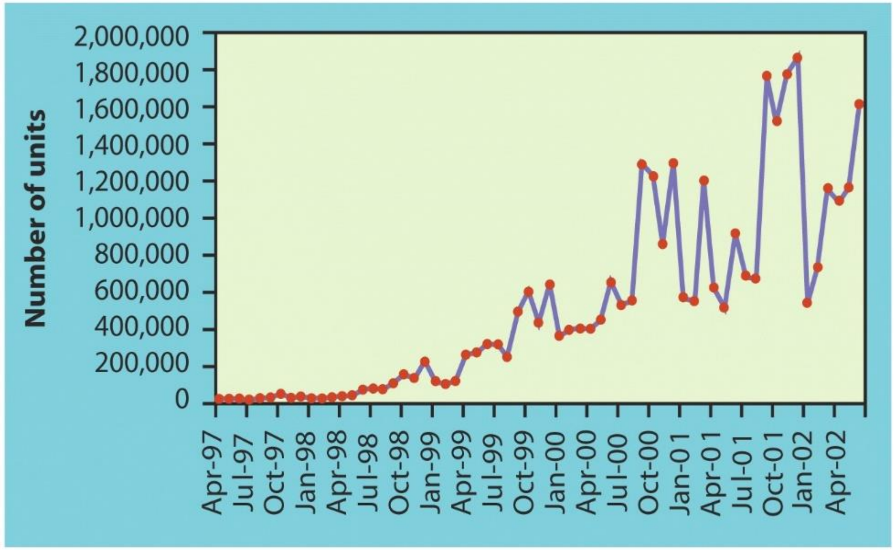
(a) A time series with constant variability

(b) A time series with variability increasing with level

Additive and Multiplicative Models

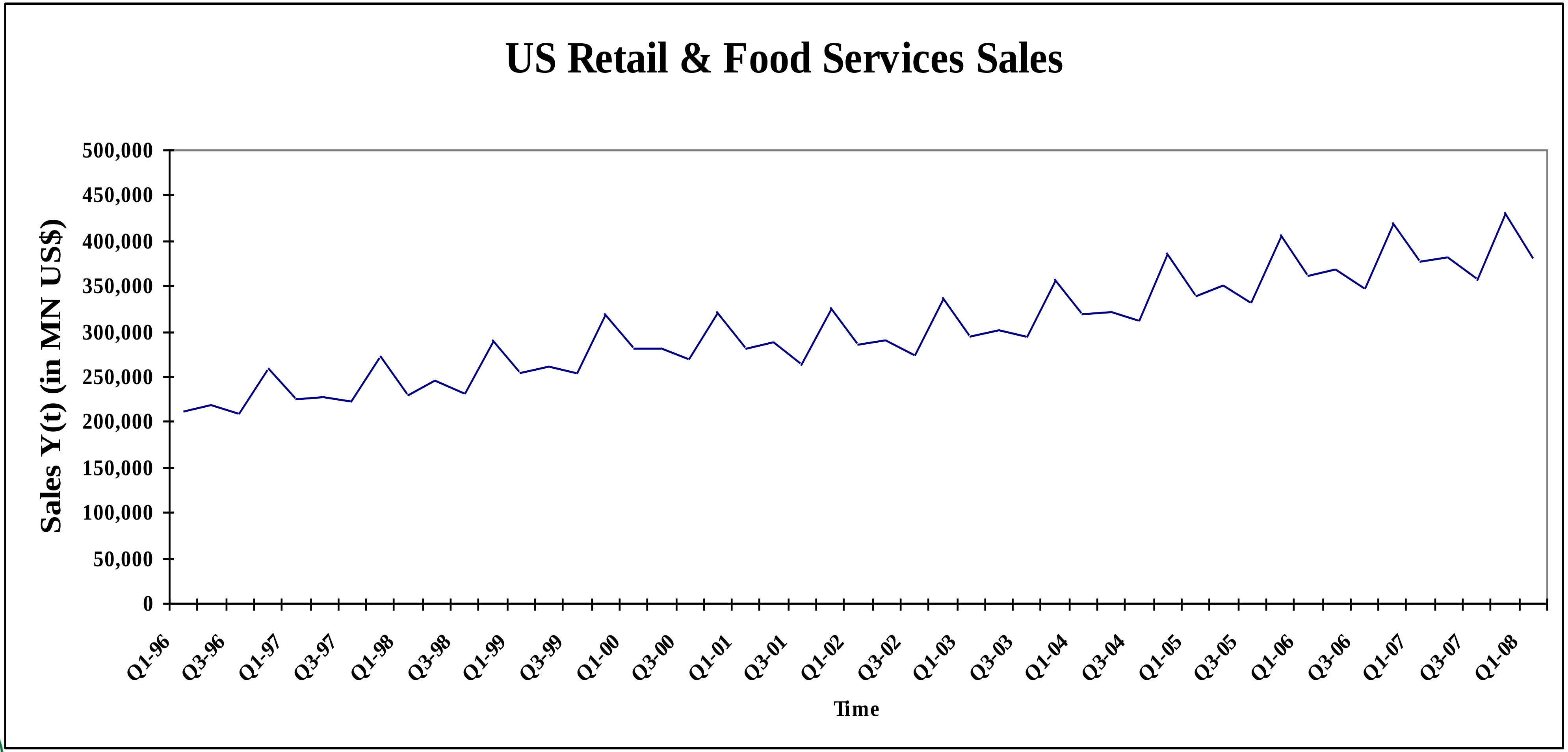


(a) A time series with constant variability

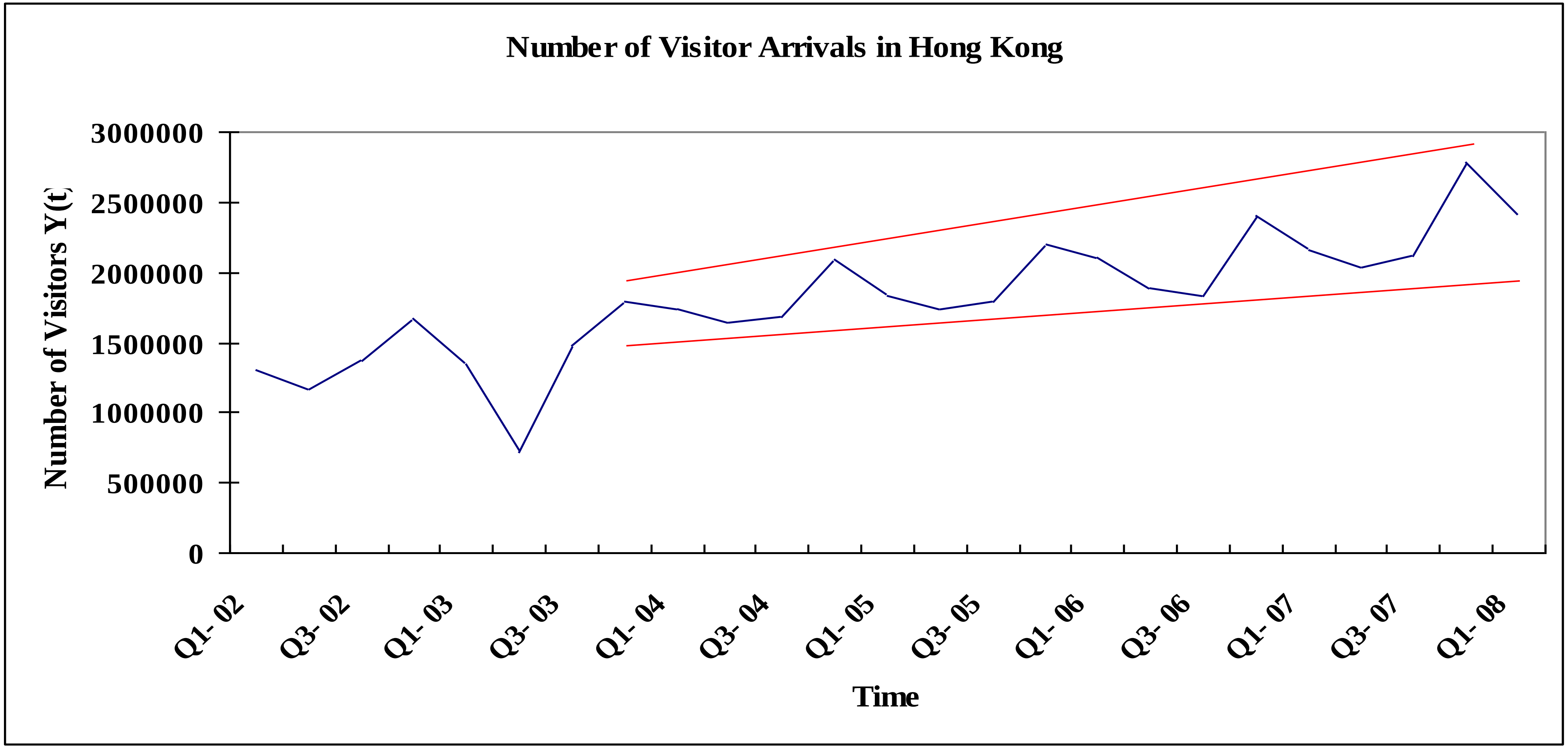


(b) A time series with variability increasing with level

US Retail and Food Services Sales from 1996 Q1 to 2008 Q1



Quarterly Number of Visitor Arrivals in Hong Kong from 2002 Q1 to 2008 Q1



Seasonality

- Several methods for measuring seasonal variation
- The basic idea:
  - First, estimate and remove the trend from the original series and then smooth out the irregular component. This leaves data containing only seasonal variation.
  - The seasonal values are collected and summarized to produce a number for each observed interval of the year (week, month, quarter, and so on)

Identification of Seasonal Components

- The identification of seasonal component in a time series differs from trend analysis in two ways:
  - The trend is determined directly from the original data, but the seasonal component is determined indirectly after eliminating the other components from the data.
  - The trend is represented by one best-fitting curve, but a separate seasonal value has to be computed for each observed interval.

Seasonal Indices

- The seasonal indices measure the seasonal variation in the series. Seasonal indices are percentages that show changes over time.
  - With monthly data, a seasonal index of 1.0 for a particular month means the expected value for that month is equal to the average of the year
  - An index of 1.25 for a different month implies the observation for that month is expected to be 25% more than the average of the year
  - A monthly index of 0.80 indicates that the expected level of that month is 20% less than the average of the year

Seasonal Indices

- The centered moving average is another method to determine seasonal indices
- The centered moving averages represent the de-seasonalized data
- The degree of seasonality, called seasonal factor (SF), is the ratio of the actual value to the de-seasonalized value

$$SF_t = \frac{Y_t}{CMA_t}$$

Seasonal Indices: Example

Time	QRT	Y	CMA(4)	CMA(2)	SRi
1	1	10			
2	2	18	15		
3	3	20	15.5	15.25	1.31
4	4	12	16	15.75	0.76
5	1	12	17	16.5	0.73
6	2	20	17.25	17.125	1.17
7	3	24	17.75	17.5	1.37
8	4	13	18.25	18	0.72
9	1	14	19.25	18.75	0.75
10	2	22	20	19.625	1.12
11	3	28			
12	4	16			

Seasonal Indices: Example

- For Quarter 3

$$SF_3 = \frac{1.31+1.37}{2} = 1.34$$

- For Quarter 4

$$SF_4 = \frac{0.76 + 0.72}{2} = 0.74$$

- For Quarter 1

$$SF_1 = 0.73$$

- For Quarter 2

$$SF_2 = 1.14$$

Seasonal Adjustment

- After the seasonal component has been isolated, it can be used to calculate seasonally adjusted data.
- Seasonal adjustment techniques are ad hoc methods of computing seasonal indices
- Use these indices to de-seasonalize the series by removing those seasonal variation
- Additive Model: de-seasonalized data = raw data-seasonal index
- Multiplicative Model: de-seasonalized data = raw data/seasonal index

Seasonal Adjustment

- A useful by-product of decomposition is that it provides an easy way to calculate seasonally adjusted data.
- For additive decomposition, the seasonally adjusted data are computed by subtracting the seasonal component.

$$y_t - S_t = TC_t + I_t$$

- The average of the detrended value for a given month (for monthly data) or given quarter (for quarterly data) will be the seasonal index for the corresponding month or quarter.

Seasonal Adjustment

- For Multiplicative decomposition, the seasonally adjusted data are computed by dividing the original observation by the seasonal component.

$$\frac{y_t}{S_t} = TC_t \times I_t$$

De-seasonalizing Data

- The process of de-seasonalizing the data has useful results:
  - The seasonalized data allow us to see better the underlying pattern in the data.
  - It provides us with measures of the extent of seasonality in the form of seasonal indexes.
  - It provides us with a tool in projecting what one quarter's (or month's) observation may portend for the entire year.

Computing Seasonal Indices for Each Quarter

- Example Seasonal Indices for an Additive Model
  - Quarter 1: -8.645
  - Quarter 2: 1.783
  - Quarter 3: -1.785
  - Quarter 4: 8.645

Identification of Seasonal Components

- If an additive decomposition is employed, estimates of the trend, seasonal components are added together to produce the original series.
- If an multiplicative decomposition is employed, estimates of individual components must be multiplied together to produce the original series

Trend Component

- Trend-Cycle Estimation
  - Can be done by using smoothing methods or moving averages
  - Idea is that observations which are nearby in time are also likely to be close in value
  - The long-term trend is estimated from the de-seasonalized data for the variable to be forecasted

Trend Component

- Trend-Cycle Estimation
  - The average of the points near an observation will provide a reasonable estimate of the trend-cycle at that observation
  - The average eliminate some of the randomness in the data, and leaves a smooth trend-cycle component
  - Use of method of least squares

Trend Equations Using Simple Linear Regression

- Trend can be described by a moving average trend
- Local regression is a way of fitting a much more flexible trend-cycle curve to the data
- Simple Linear trend:

$$T_t = \beta_0 + \beta_1 t$$

- Can be extended to quadratic models:

$$T_t = \beta_0 + \beta_1 t + \beta_2 t^2$$

- Or Exponential:

$$T_t = \beta_0 \beta_1{}^t$$

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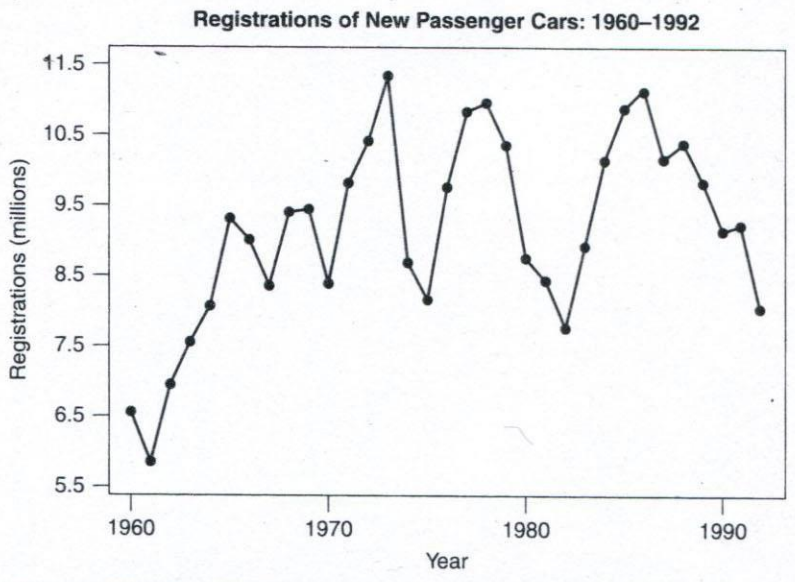
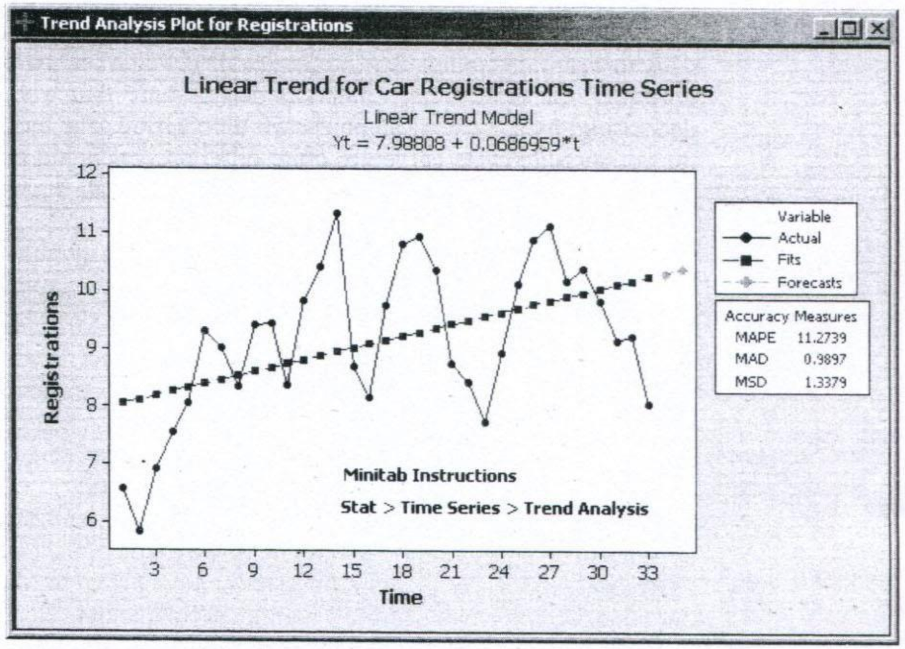
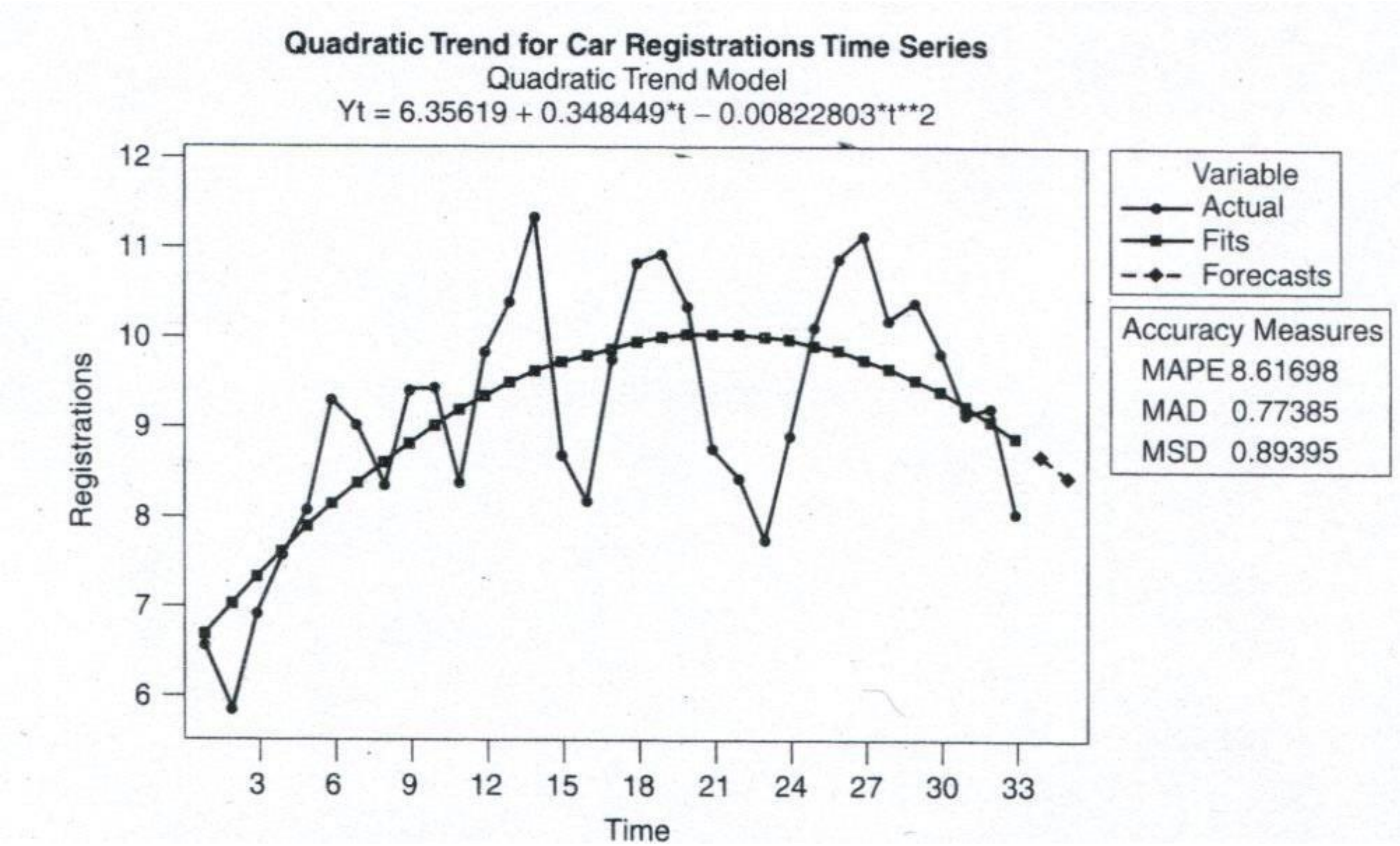


FIGURE 5-2 Car Registrations Time Series for Example 5.1



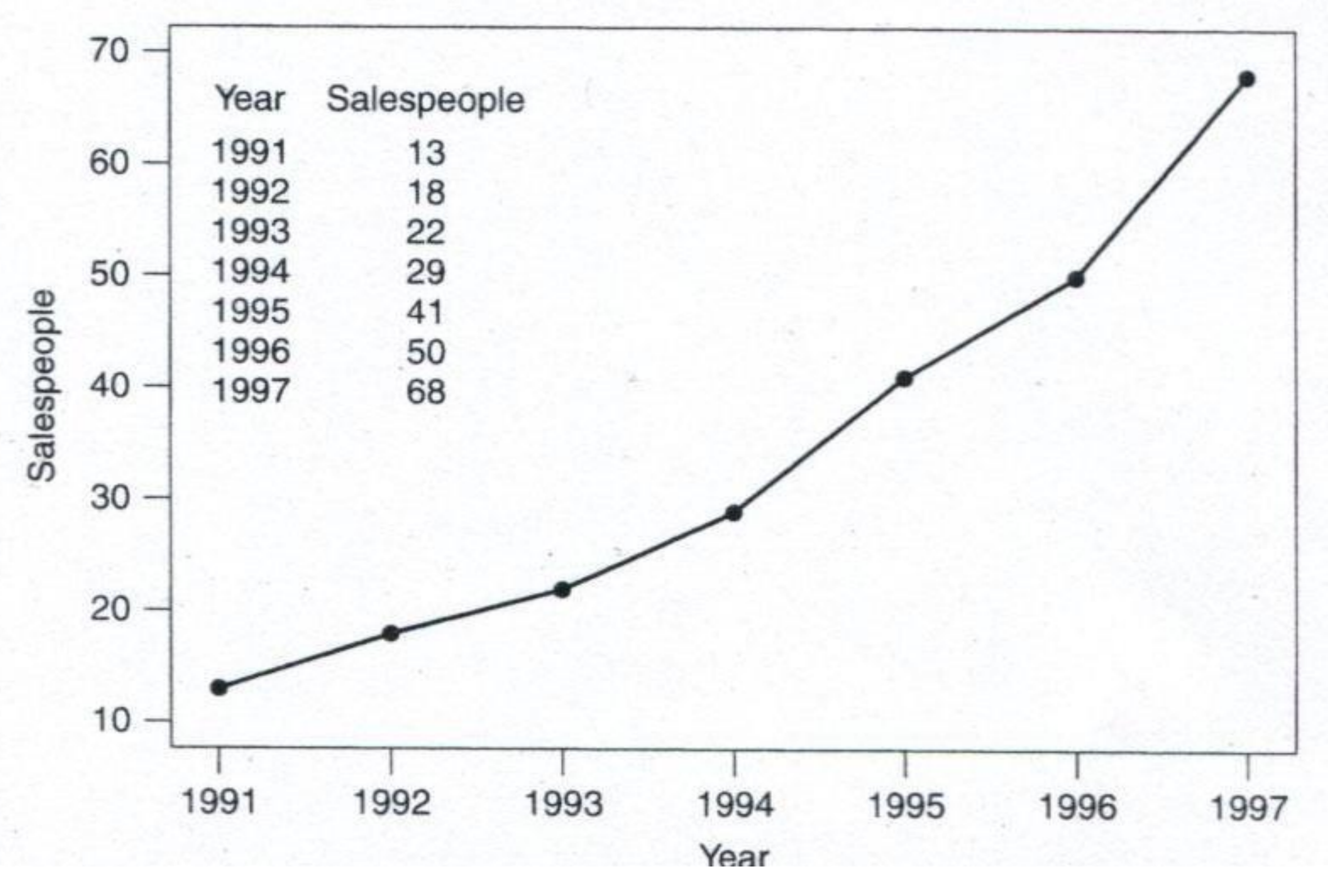
Trend Equations Using Simple Linear Regression: Trend Line for the Car Registrations Time Series

Trend Equations Using Quadratic Regression: Trend Line for the Car Registrations Time Series

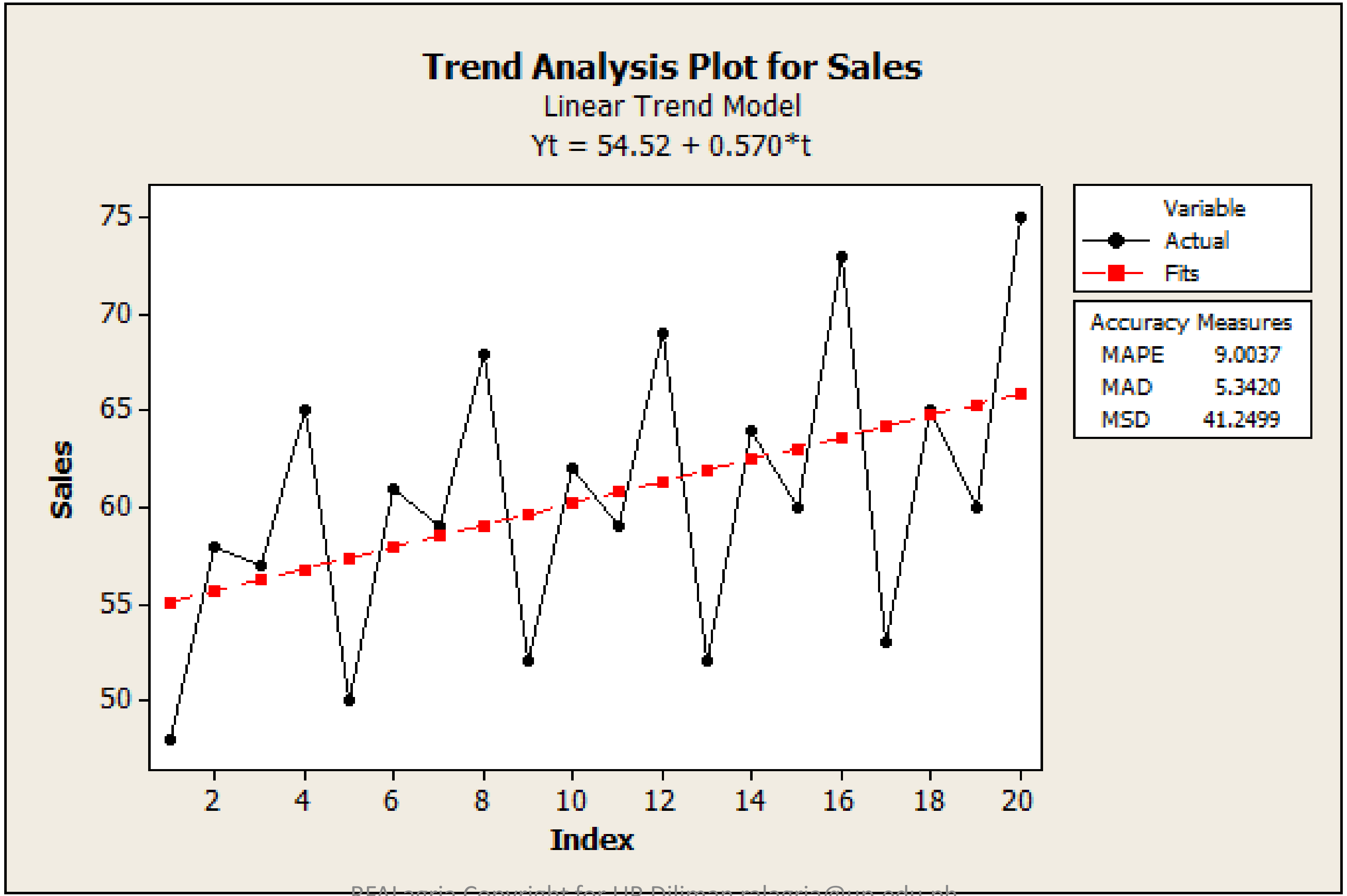


Exponential Trend

- The increase in the number of salespeople is not constant. It appears as if increasingly larger numbers of people are being added in the later years.
- An exponential trend curve fit to the sales people data has the equation:
  - $T'_t = 10.016(1.313)^t$

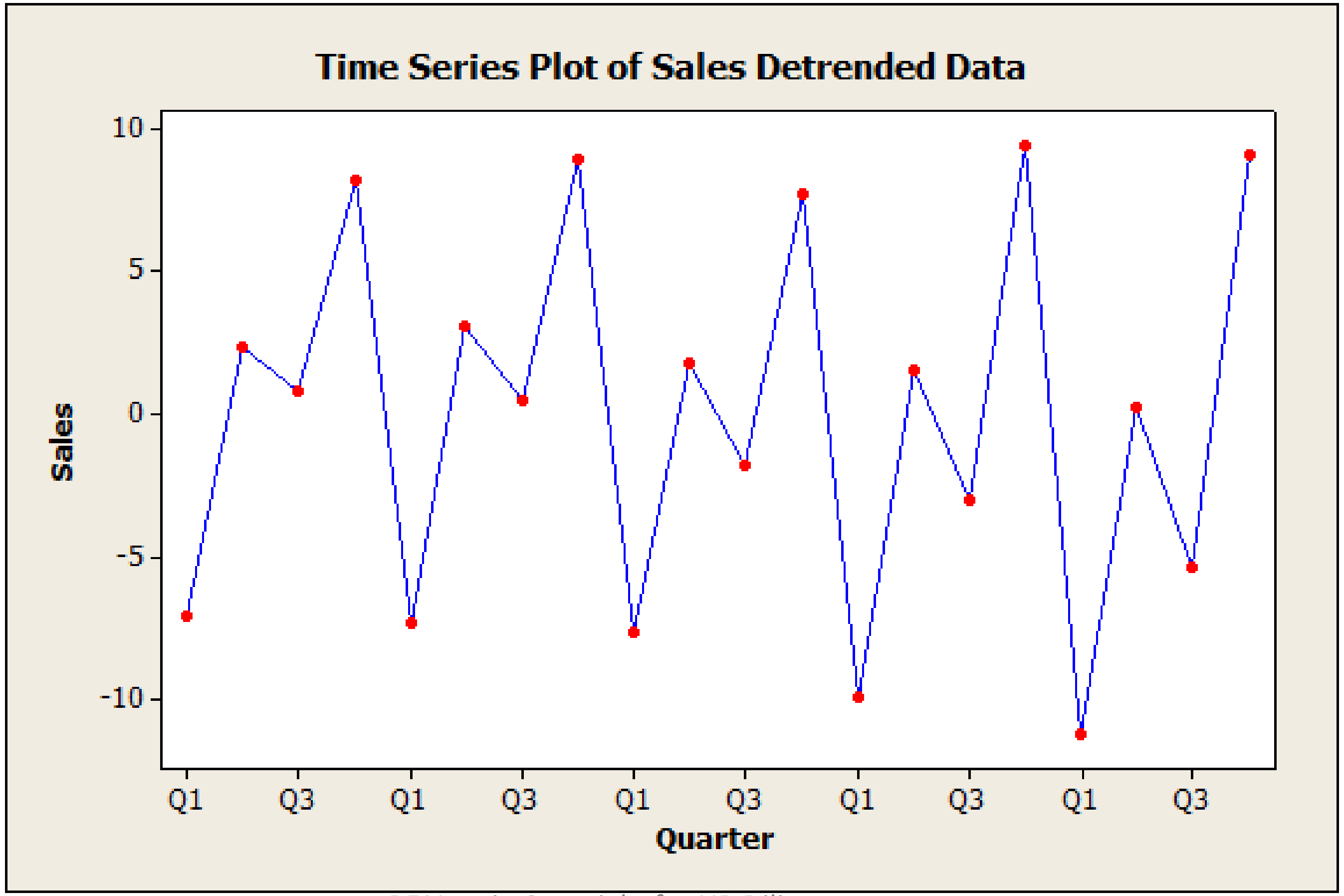


Trend Analysis: Minitab Output



De-trended Sales Data (Additive Model)

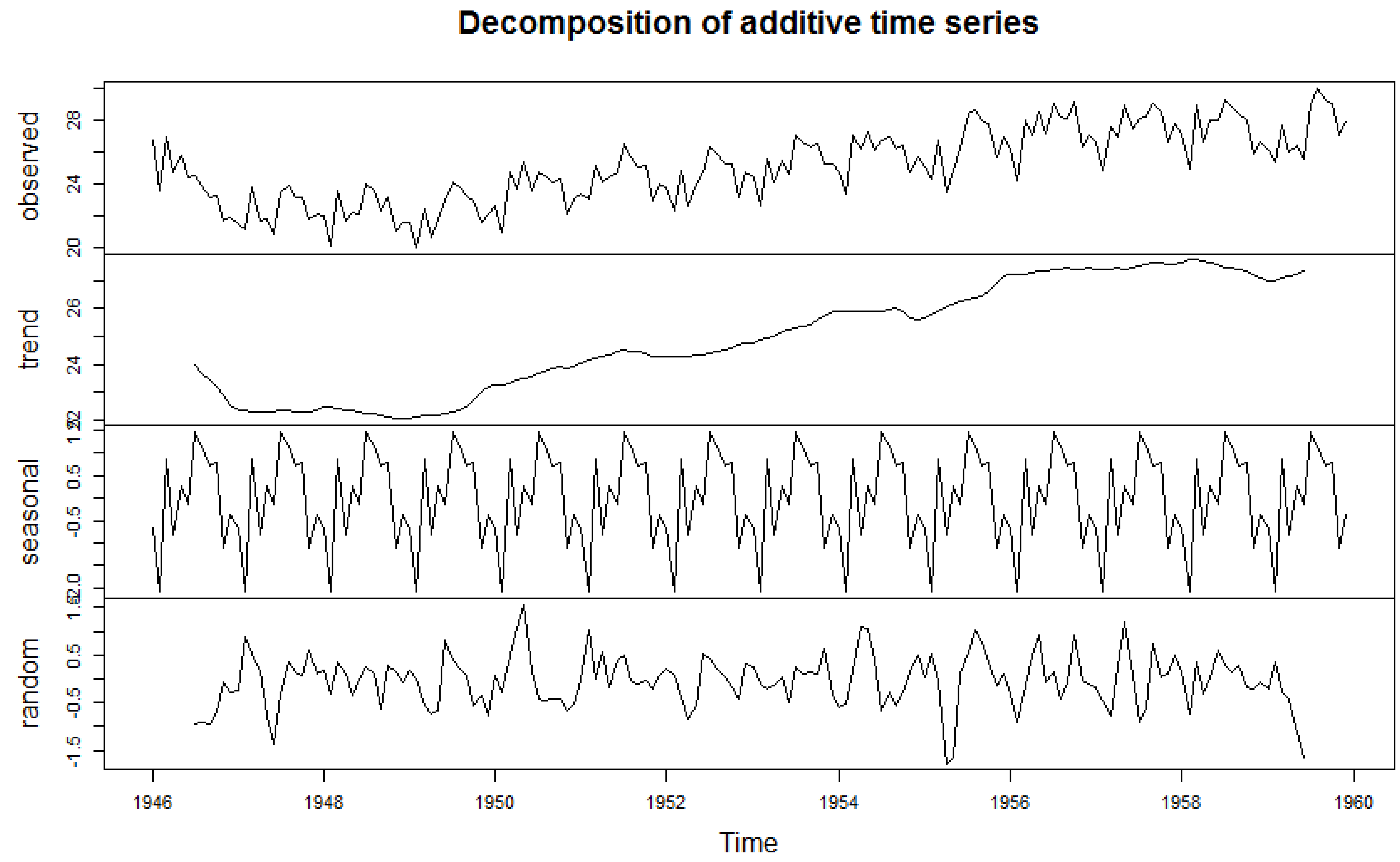
- $y_t - TC_t = S_t + I_t$



R Code: Decomposition 1

- `birthstimeseriescomponents <- decompose(birthsts)`
- `plot(birthstimeseriescomponents)`
- `birthstimeseriescomponents`

R Code: Decomposition 1



R Code: Decomposition 1

```
$seasonal
            Jan        Feb        Mar        Apr        May        Jun        Jul        Aug        Sep        Oct
1946 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1947 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1948 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1949 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1950 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1951 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1952 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1953 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1954 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1955 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1956 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1957 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1958 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444
1959 -0.6771947 -2.0829607  0.8625232 -0.8016787  0.2516514 -0.1532556  1.4560457  1.1645938  0.6916162  0.7752444

$trend
          Jan      Feb      Mar      Apr      May      Jun      Jul      Aug      Sep      Oct      Nov      Dec
1946       NA       NA       NA       NA       NA       NA 23.98433 23.66213 23.42333 23.16112 22.86425 22.54521
1947 22.35350 22.30871 22.30258 22.29479 22.29354 22.30562 22.33483 22.31167 22.26279 22.25796 22.27767 22.35400
1948 22.43038 22.43667 22.38721 22.35242 22.32458 22.27458 22.23754 22.21988 22.16983 22.07721 22.01396 22.02604
1949 22.06375 22.08033 22.13317 22.16604 22.17542 22.21342 22.27625 22.35750 22.48862 22.70992 22.98563 23.16346
1950 23.21663 23.26967 23.33492 23.42679 23.50638 23.57017 23.63888 23.75713 23.86354 23.89533 23.87342 23.88150
1951 24.00083 24.12350 24.20917 24.28208 24.35450 24.43242 24.49496 24.48379 24.43879 24.36829 24.29192 24.27642
1952 24.27204 24.27300 24.28942 24.30129 24.31325 24.35175 24.40558 24.44475 24.49325 24.58517 24.70429 24.76017
1953 24.78646 24.84992 24.92692 25.02362 25.16308 25.26963 25.30154 25.34125 25.42779 25.57588 25.73904 25.87513
1954 25.92446 25.92317 25.92967 25.92137 25.89567 25.89458 25.92963 25.98246 26.01054 25.88617 25.67087 25.57312
1955 25.64612 25.78679 25.93192 26.06388 26.16329 26.25388 26.35471 26.40496 26.45379 26.64933 26.95183 27.14683
1956 27.21104 27.21900 27.20700 27.26925 27.35050 27.37983 27.39975 27.44150 27.45229 27.43354 27.44488 27.46996
1957 27.44221 27.40283 27.44300 27.45717 27.44429 27.48975 27.54354 27.56933 27.63167 27.67804 27.62579 27.61212
1958 27.68642 27.76067 27.75963 27.71037 27.65783 27.58125 27.49075 27.46183 27.42262 27.34175 27.25129 27.08558
1959 26.96858 27.00512 27.09250 27.17263 27.26208 27.36033       NA       NA       NA       NA       NA       NA

$random
              Jan          Feb          Mar          Apr          May          Jun          Jul          Aug          Sep
1946           NA           NA           NA           NA           NA           NA -0.963379006 -0.925718750 -0.939949519
1947 -0.237305288  0.863252404  0.543893429  0.175887019 -0.793193109 -1.391369391 -0.311879006  0.347739583  0.150592147
1948  0.183819712 -0.318705929  0.340268429  0.121262019 -0.354234776  0.001672276  0.256412660  0.119531250 -0.623449519
```

R Code: Decomposition 1

- `birthstimeseriescomponents <- decompose(birthsts)`
- `plot(birthstimeseriescomponents)`
- `birthstimeseriescomponents`

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Seasonal Adjustment in R

- Seasonal time series that can be described using an additive model, adjust by estimating the seasonal component and subtracting it from the original time series
- Thus, removing the seasonal variation from the original series containing only trend and irregular components

R Code: Seasonal Adjustment Decomposition 1

- `birthstimeseriescomponents <- decompose(birthsts)`
- `birthstimeseriesseasonallyadjusted <- birthsts - birthstimeseriescomponents$seasonal`
- `birthstimeseriesseasonallyadjusted`
- `plot(birthstimeseriesseasonallyadjusted)`

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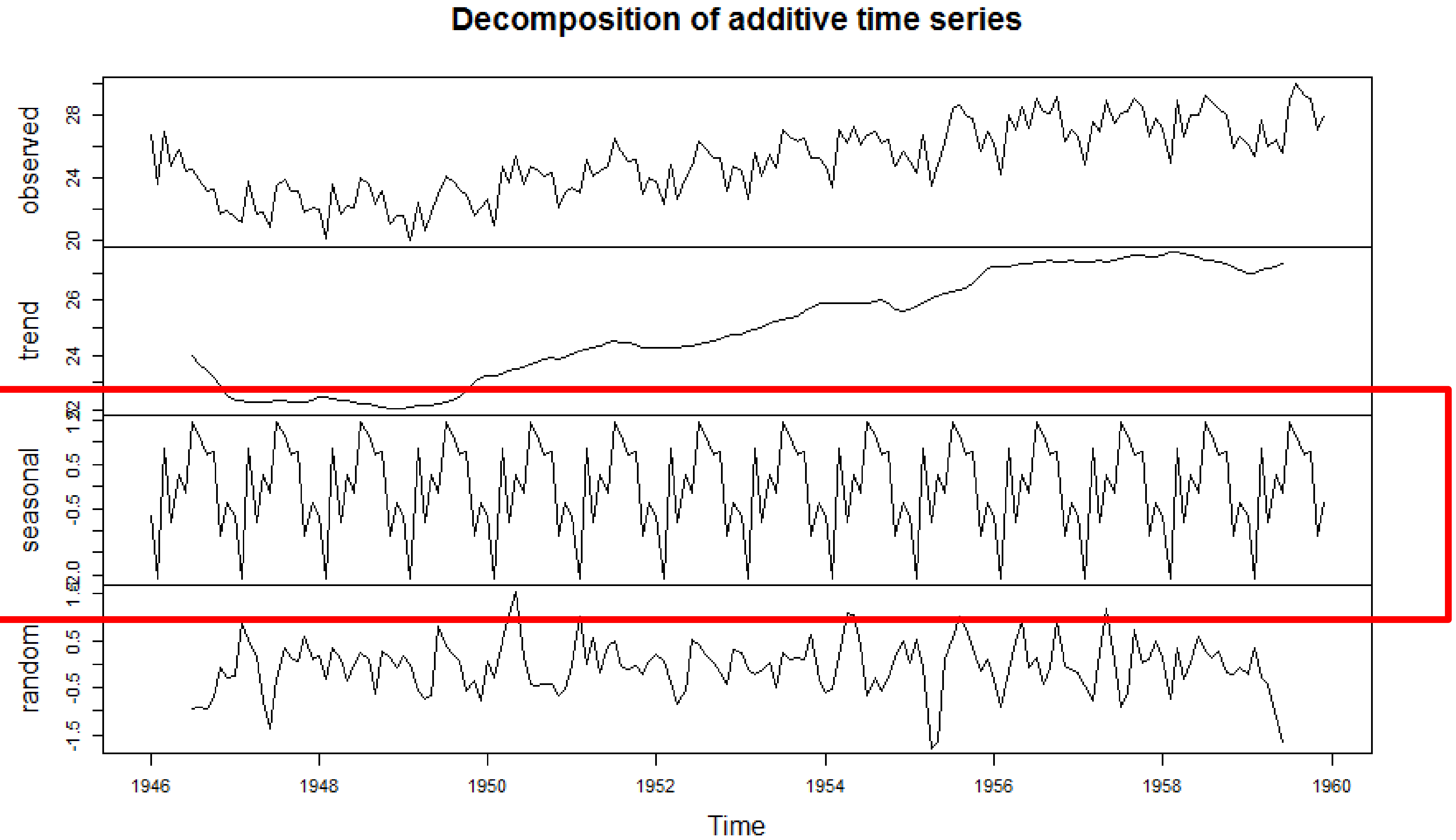
Notes on Seasonality

- There maybe different ways to get the seasonal components in a decomposition method
- However, values will converge in similar plots

Notes on Seasonality

- Decomposing using decompose in R



Notes on Seasonality

- Decomposing using “decompose” in R

$seasonal

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct
1946	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1947	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1948	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1949	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1950	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1951	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1952	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1953	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1954	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1955	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1956	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1957	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1958	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444
1959	-0.6771947	-2.0829607	0.8625232	-0.8016787	0.2516514	-0.1532556	1.4560457	1.1645938	0.6916162	0.7752444

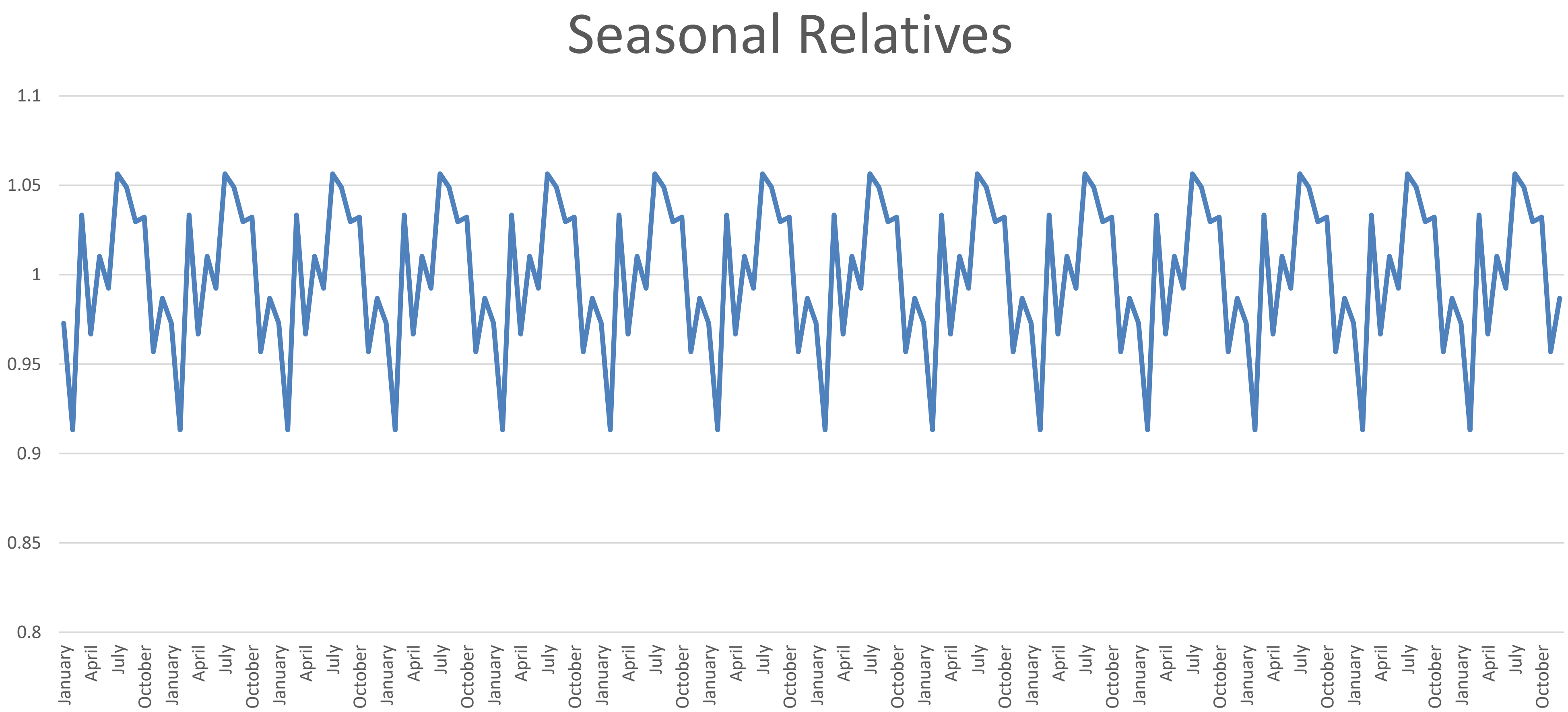
Notes on Seasonality

- Decomposing using seasonal relatives from yesterday

Month	SR
January	0.972838
February	0.913256
March	1.033349
April	0.966846
May	1.010235
June	0.992468
July	1.056422
August	1.048989
September	1.029643
October	1.032263
November	0.95687
December	0.986822

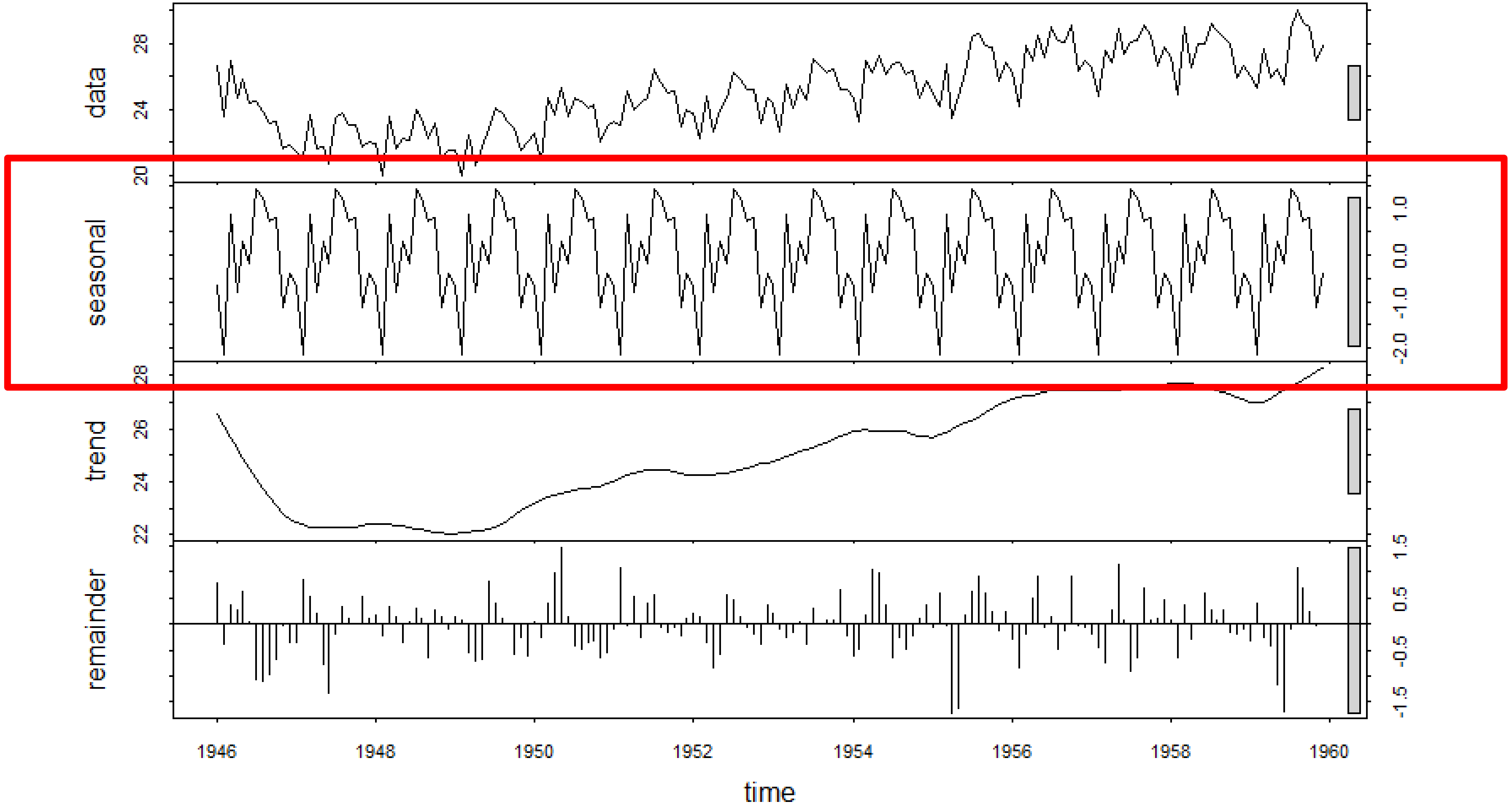
Notes on Seasonality

- Decomposing using seasonal relatives



Notes on Seasonality

- Decomposing using “stl” (Seasonal Trend using Loess)



Notes on Seasonality

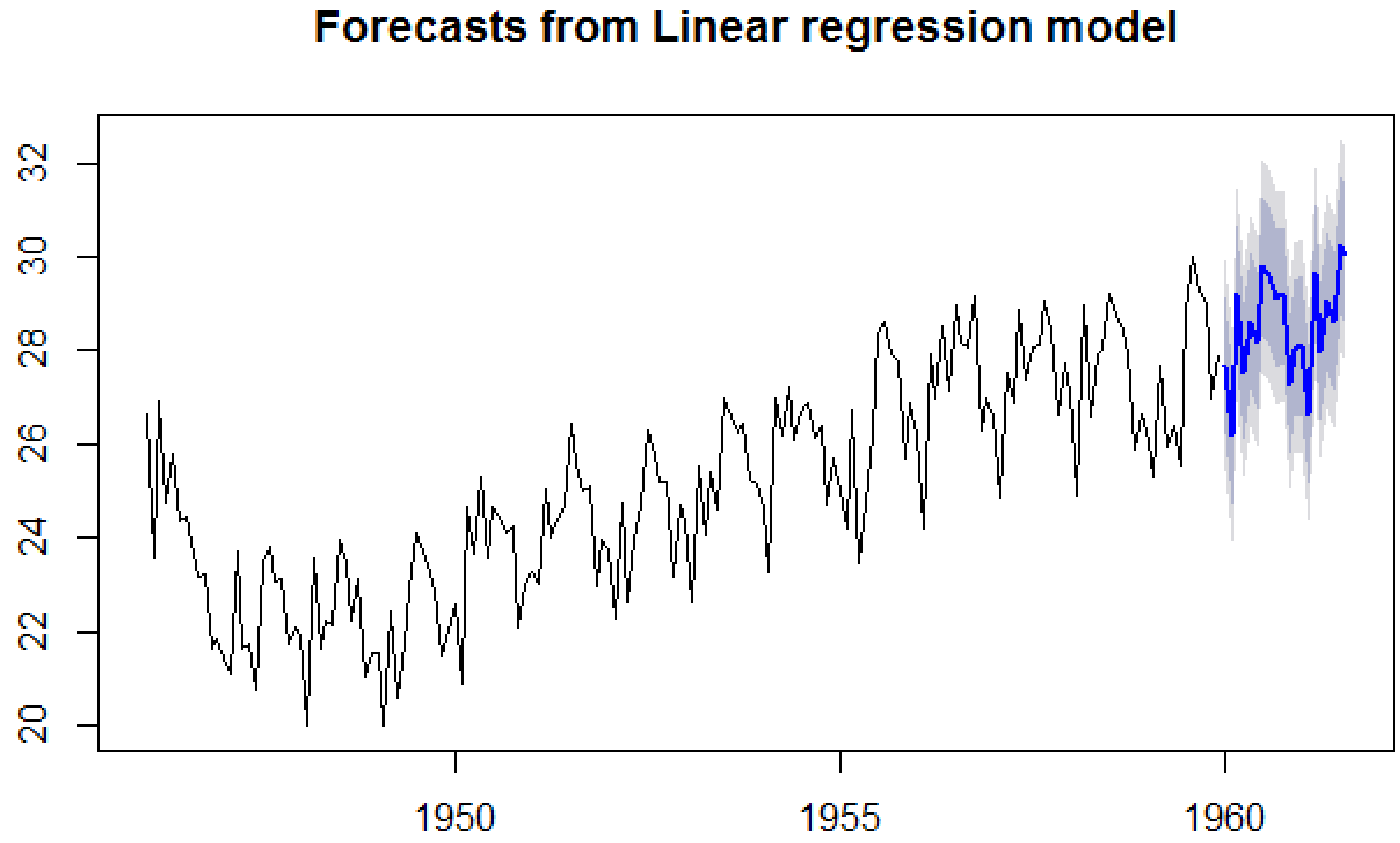
- Decomposing using stl (Seasonal Trend using Loess)

```
Components
            seasonal    trend     remainder
Jan 1946 -0.6434274 26.53282  0.7736051527
Feb 1946 -2.1371410 26.11345 -0.3783113276
Mar 1946  0.8716451 25.69408  0.3652725397
Apr 1946 -0.8054689 25.29487  0.2505942245
May 1946  0.2712028 24.89567  0.6391302125
Jun 1946 -0.1817308 24.51802  0.0277121005
Jul 1946  1.4131922 24.14037 -1.0765626353
Aug 1946  1.2202346 23.77470 -1.0939302553
Sep 1946  0.7287774 23.40902 -0.9627982002
Oct 1946  0.7814986 23.10814 -0.6626395368
Nov 1946 -1.1207089 22.80726 -0.0145520992
Dec 1946 -0.3980735 22.62753 -0.3594580681
Jan 1947 -0.6434274 22.44780 -0.3653745927
Feb 1947 -2.1371410 22.37464  0.8515005042
Mar 1947  0.8716451 22.30148  0.5358759486
Apr 1947 -0.8054689 22.28716  0.1873104341
May 1947  0.2712028 22.27284 -0.7920407771
Jun 1947 -0.1817308 22.27073 -1.3280029842
Jul 1947  1.4131922 22.26863 -0.2028218152
Aug 1947  1.2202346 22.27383  0.3299339394
Sep 1947  0.7287774 22.27903  0.0971893690
```

R Code: Decomposition 2

- `fit = tslm(birthsts~trend + season)`
- `summary(fit)`
- `plot(forecast(fit, h=20))`

Trend Analysis using Regression Model

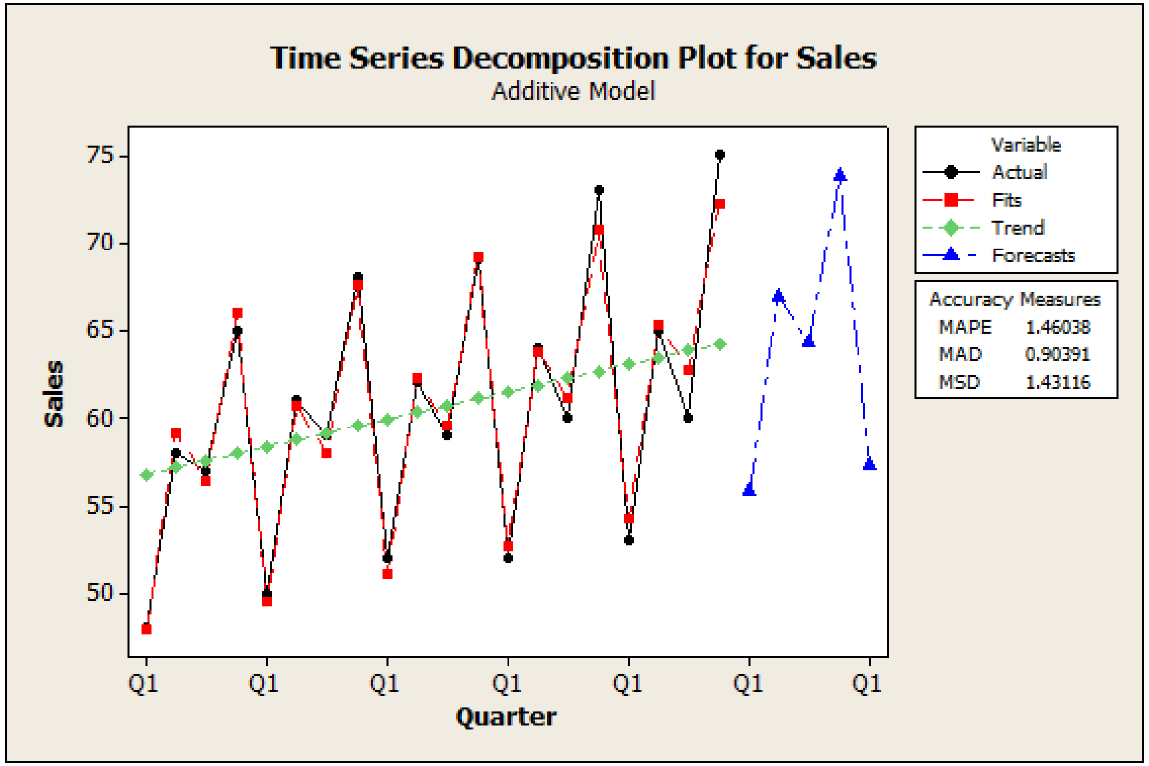


Model with Seasonal Effects

```
Call:
lm(formula = formula, data = "birthsts", na.action = na.exclude)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.1819 -0.5458 -0.1180  0.4999  5.1607 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 21.465397   0.323663  66.320  < 2e-16 ***
trend        0.036877   0.001747  21.108  < 2e-16 ***
season2     -1.529948   0.414028  -3.695 0.000304 ***
season3      1.442604   0.414039   3.484 0.000642 ***
season4     -0.260772   0.414058  -0.630 0.529755    
season5      0.789637   0.414084   1.907 0.058377 .  
season6      0.307546   0.414117   0.743 0.458815    
season7      1.873312   0.414157   4.523  1.2e-05 ***
season8      1.650150   0.414205   3.984 0.000104 ***
season9      1.128488   0.414260   2.724 0.007188 ** 
season10     1.157254   0.414323   2.793 0.005879 ** 
season11    -0.768908   0.414393  -1.856 0.065423 .  
season12    -0.055213   0.414470  -0.133 0.894197    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.095 on 155 degrees of freedom
Multiple R-squared:  0.7929,	Adjusted R-squared:  0.7768 
F-statistic: 49.44 on 12 and 155 DF,  p-value: < 2.2e-16
```

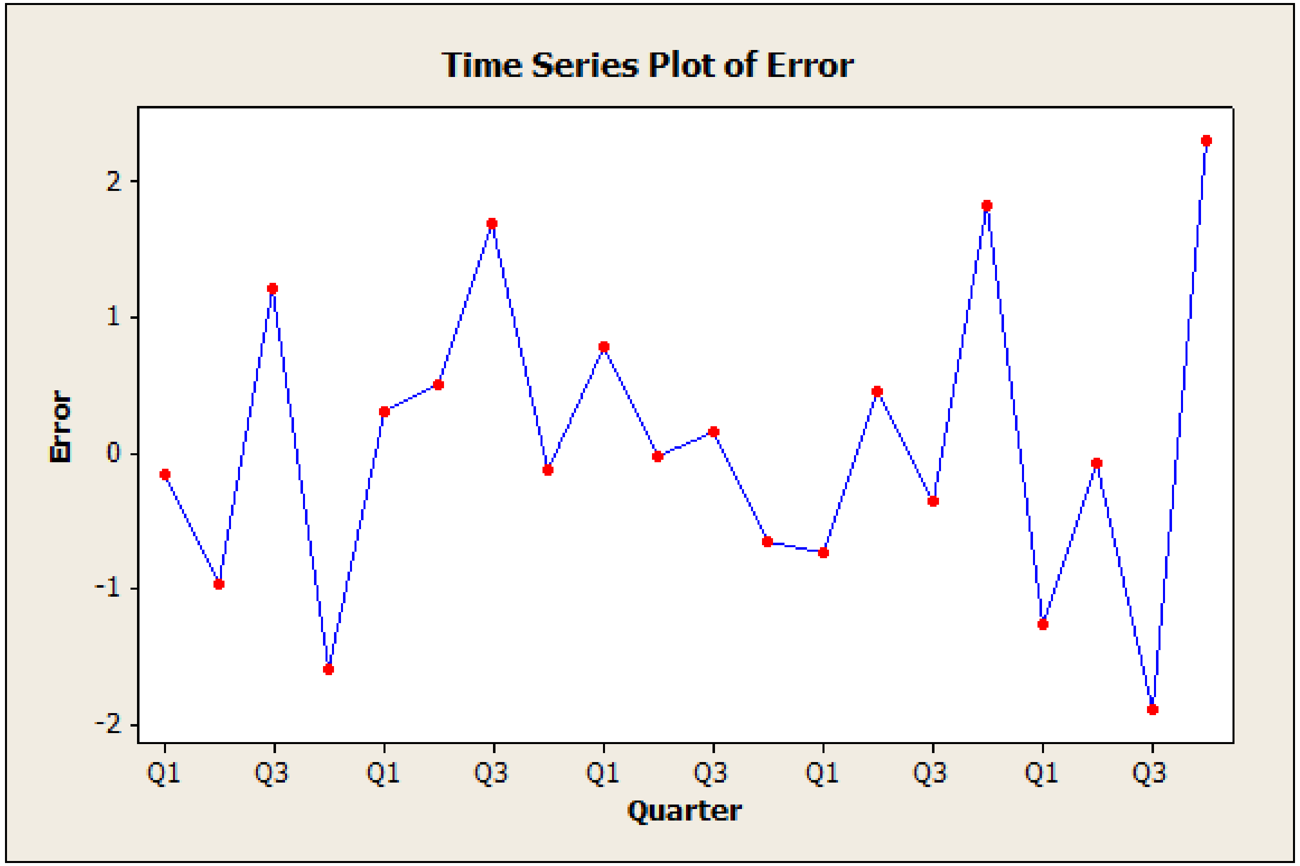
Forecast Predictions With Final Additive Model

$$\hat{y}_t = 54.52 + 0.570t + \begin{cases} -8.644 & if\ Q = 1 \\ 1.785 & if\ Q = 2 \\ -1.75 & if\ Q = 3 \\ 8.645 & if\ Q = 4 \end{cases}$$



Error Component Computation (Additive Model)

- $y_t - Tt - S_t = I_t$



How to Choose?

Forecasting Method	Amount of Historical Data	Data Pattern	Forecast Horizon
Linear Regression	10 to 20 observations, ≤5 observations/season	Stationary, trend and seasonality	Short to Medium
SMA	6 to 12 months, weekly data usually	Data should be stationary (no trend, no season)	Short
WMA, Single Exponential Smoothing	5 to 10 observations to start	Data should be stationary	Short
Double Exponential Smoothing	5 to 10 observations to start	Stationary and with trend	Short

Case Study 1 & 2

- Given M&L.csv data set.
- Data provided is the weekly demand for two products from order records of the previous 14 weeks.

Case Study in R

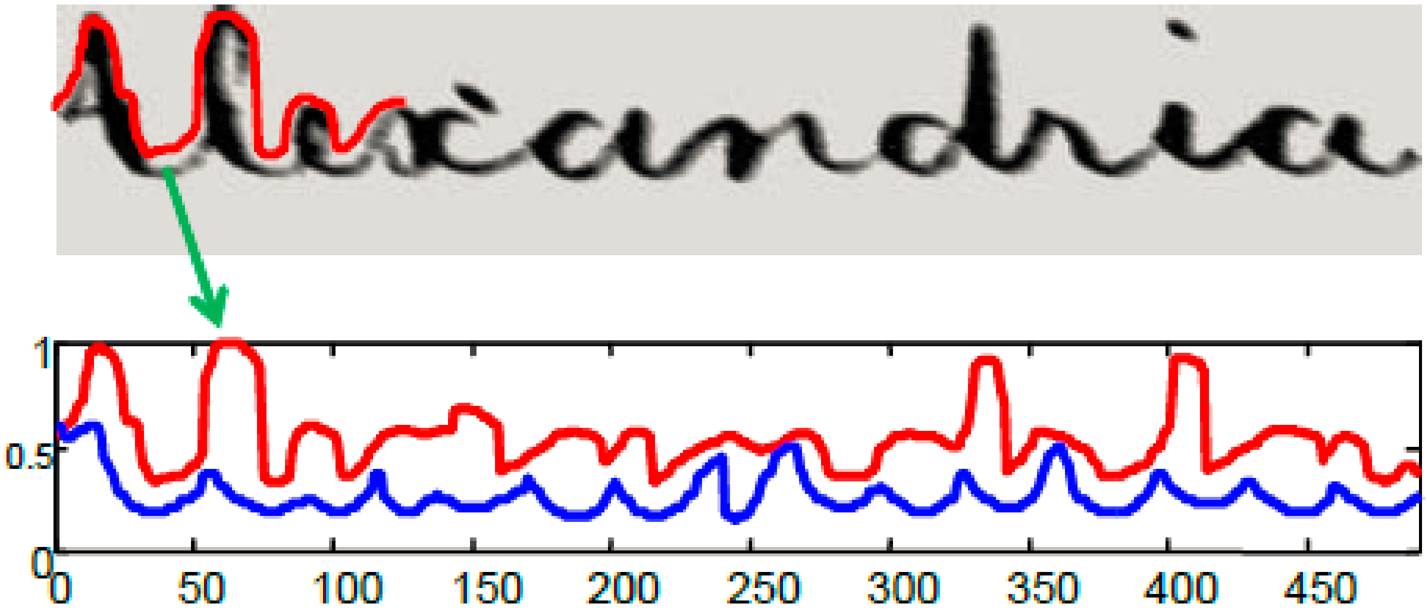
- Given Chocolates.csv and Airlines.csv dataset.
- Data provided is the quarterly production of chocolates (in metric tons) in Australia from 1957 to 1994.

Outline for This Training

1. Introduction to Forecasting in Business Intelligence
2. Demand Forecasting Techniques
   - Qualitative
   - Quantitative
3. Accuracy of Forecasts
4. Monitoring of Forecasts
5. Forecasting with R
6. **Introduction to Time Series Data Mining**
7. Advanced Time Series

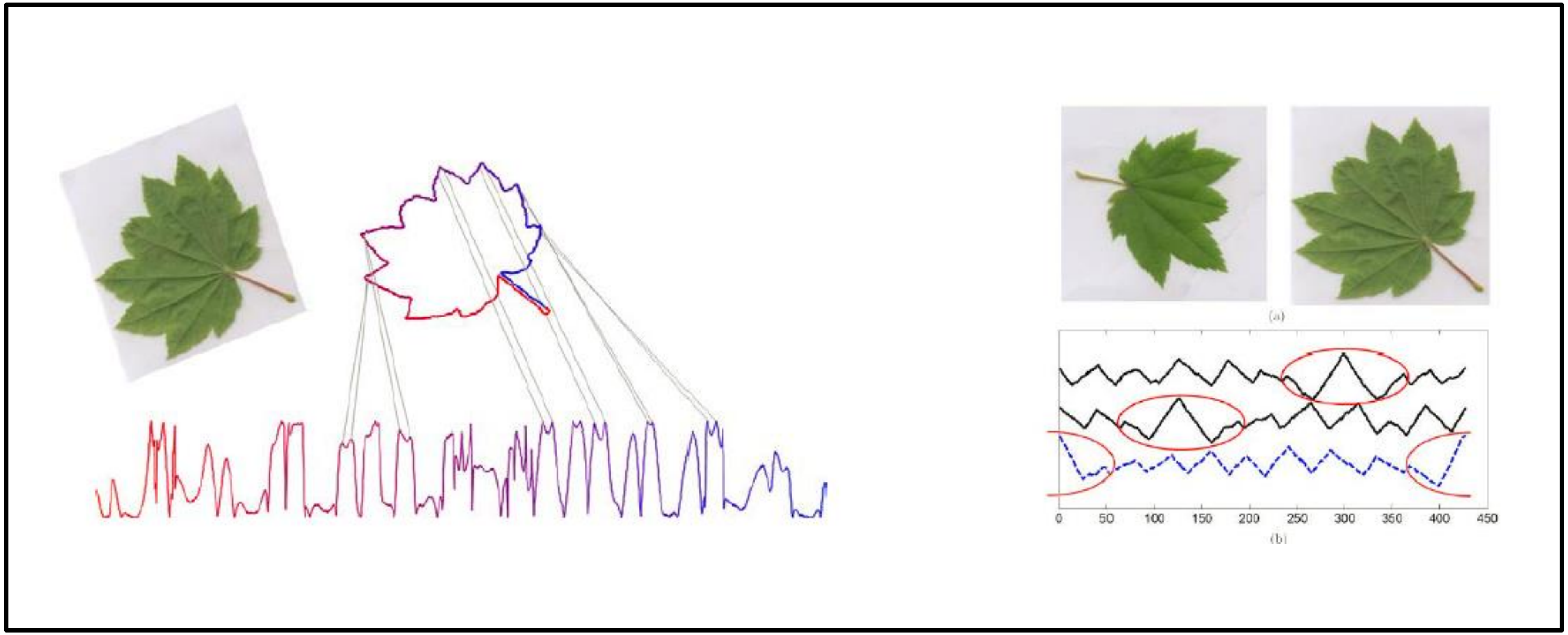
Time Series Analysis

- A word can be represented by two time series created by moving over and under the word



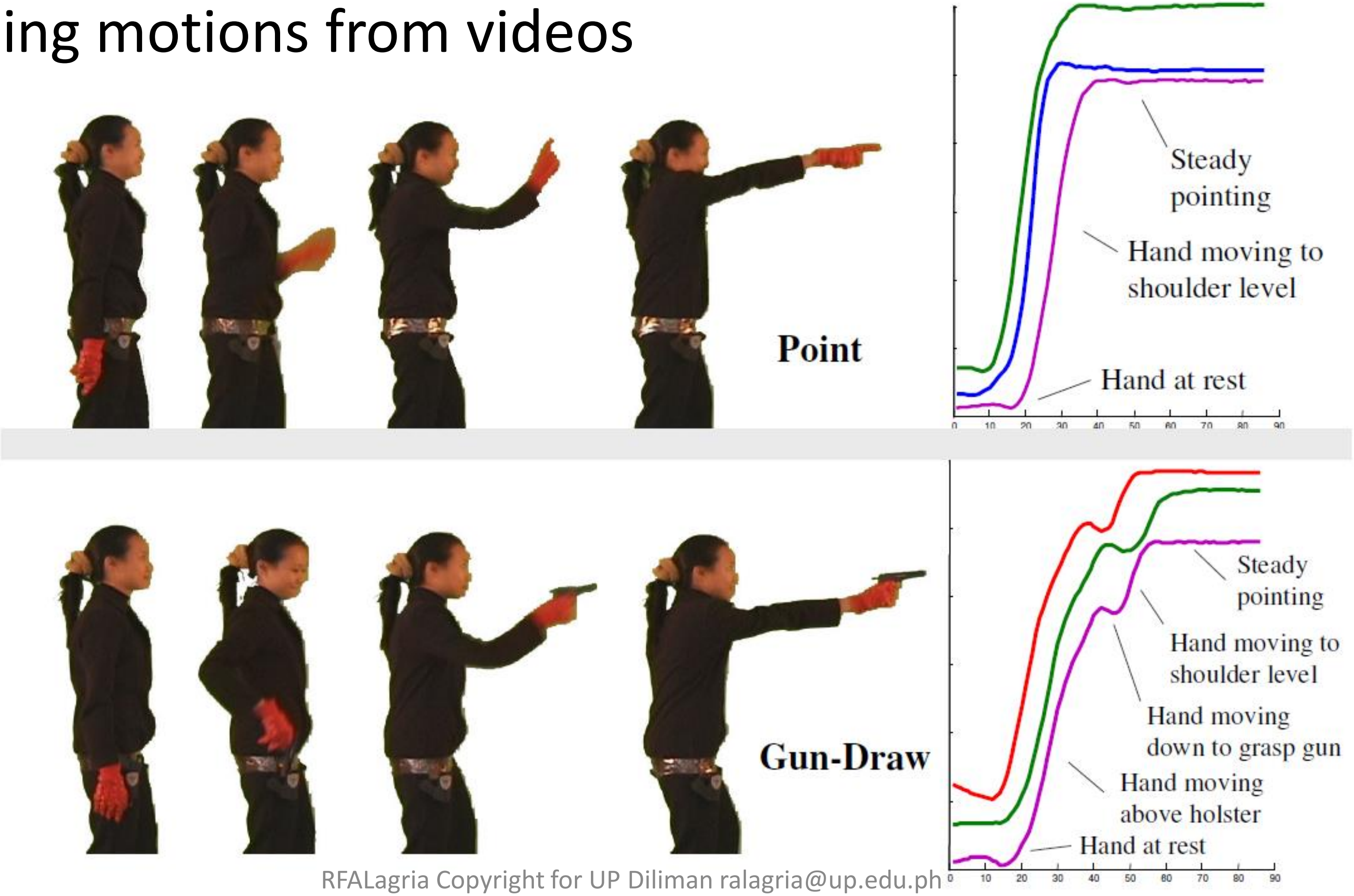
Time Series Analysis

- Recognizing trees from the leaf images



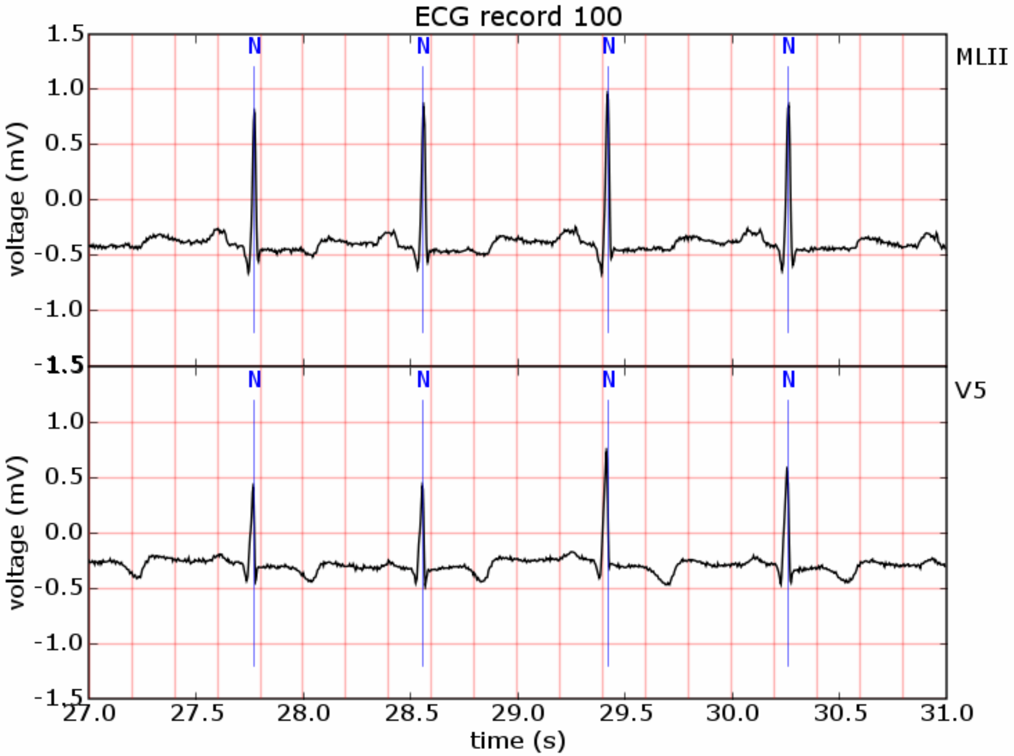
Time Series Analysis

- Recognizing motions from videos



Time Series Mining

- You go to the doctor because of chest pains
- Your ECG looks strange
- The doctor wants to search a database to find similar ECGs to find clues to your condition

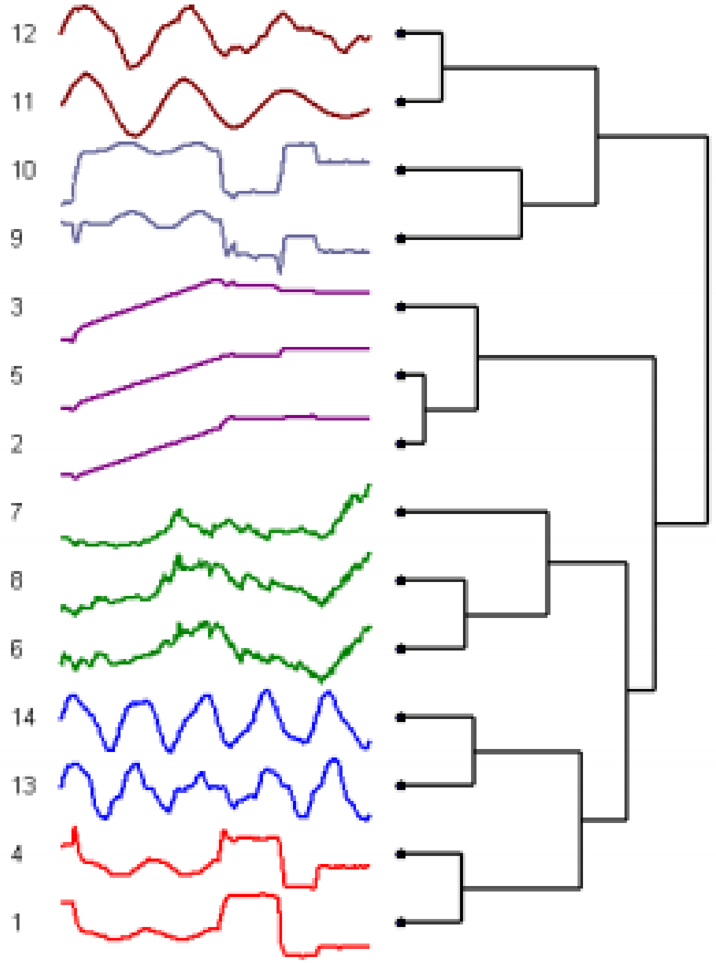


Time Series Analysis

- Some Time Series Data Mining Tasks
  - Clustering
  - Classification
  - Rule Discovery
  - Anomaly Detection
  - Visualization
  - Query by Content

Time Series Clustering

- Identify which time series are similar to each other



Time Series Clustering

- To partition time series data into groups based on similarity or distance
  - Time series data in the same cluster are the same
- First step is to work out an appropriate distance/similarity metric
- Second step is to use existing clustering techniques
  - Hierarchical clustering
  - K-means

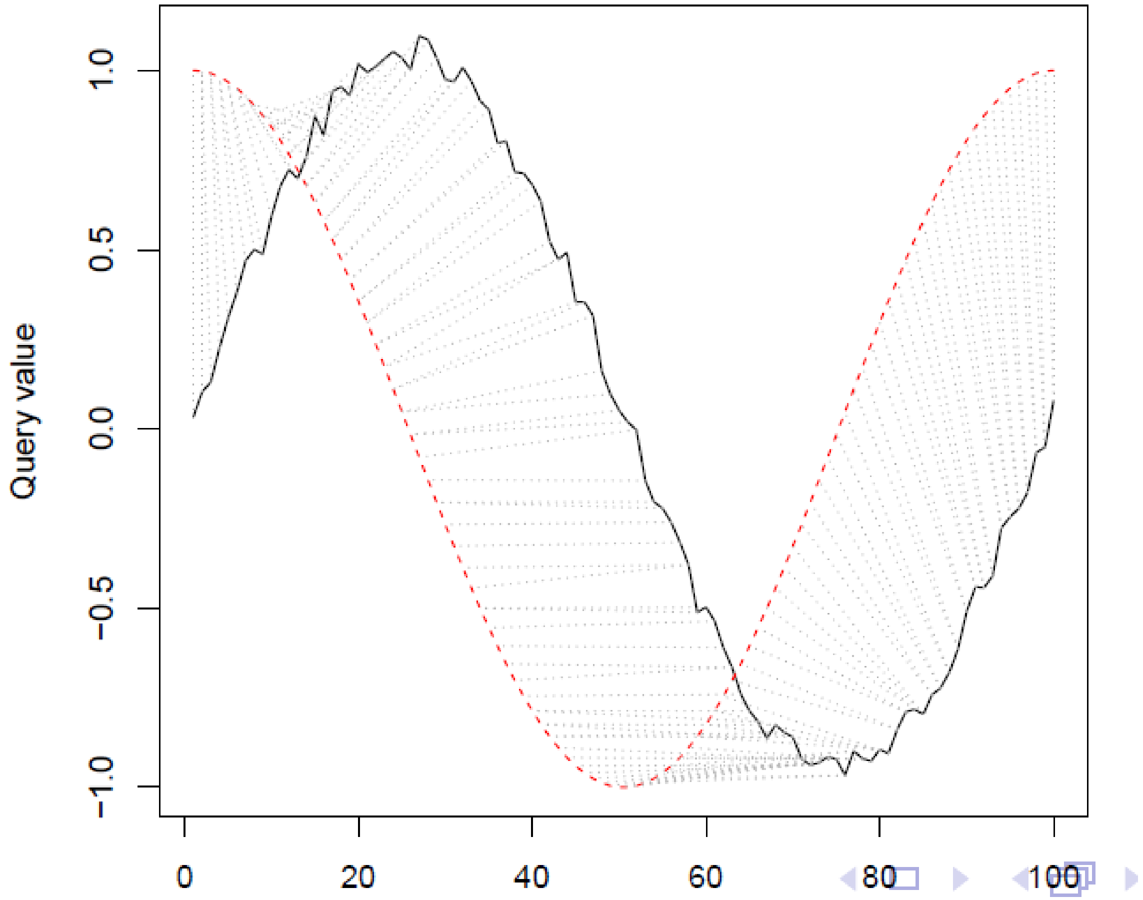
Time Series Clustering

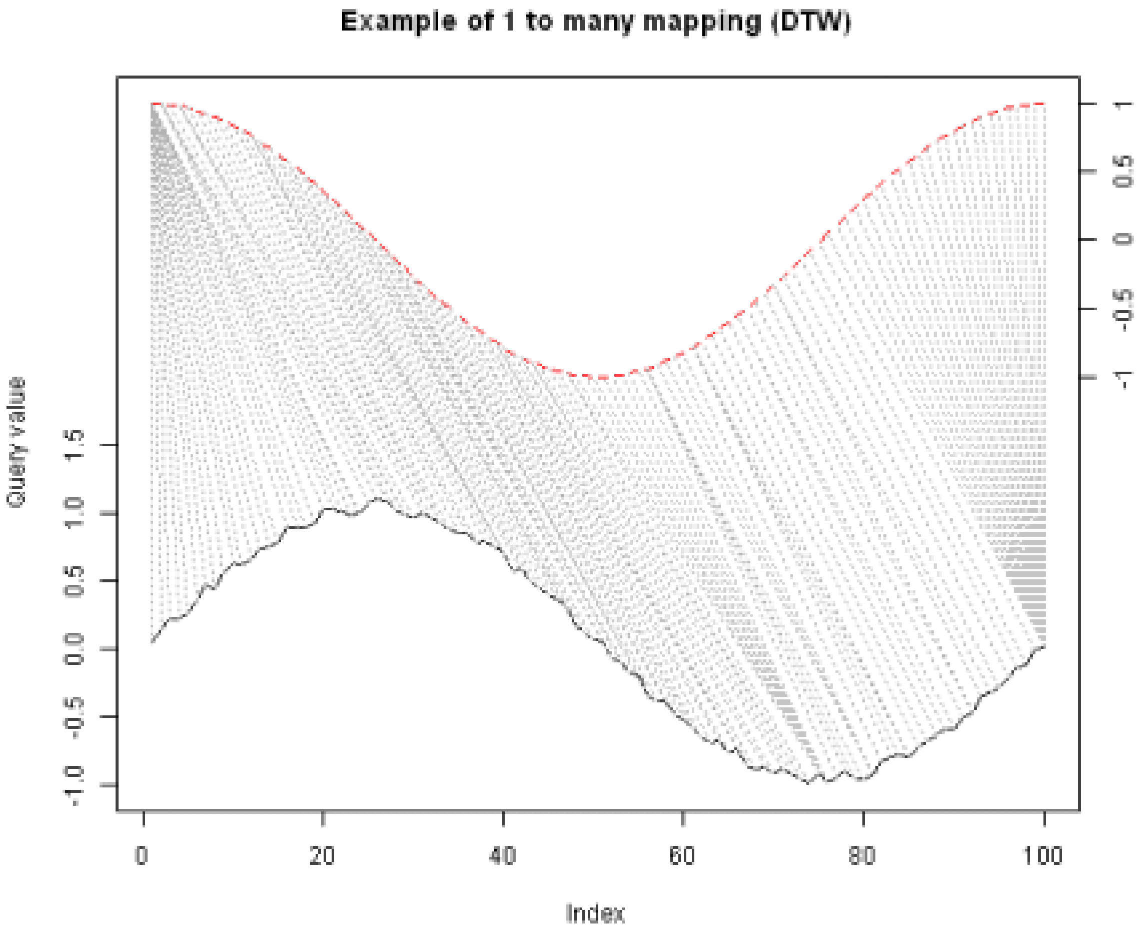
- Measure of distance/similarity
  - Euclidean distance
  - Manhattan distance
  - Maximum norm
  - Hamming distance
  - Inner product
  - Dynamic Time Warping (DTW) distance

Dynamic Time Warping (DTW)

- Finds the optimal non-linear alignment between two time series
- A way to map one time series to another time series
- Looks for the minimum distance mapping between query and reference

Dynamic Time Warping (DTW)

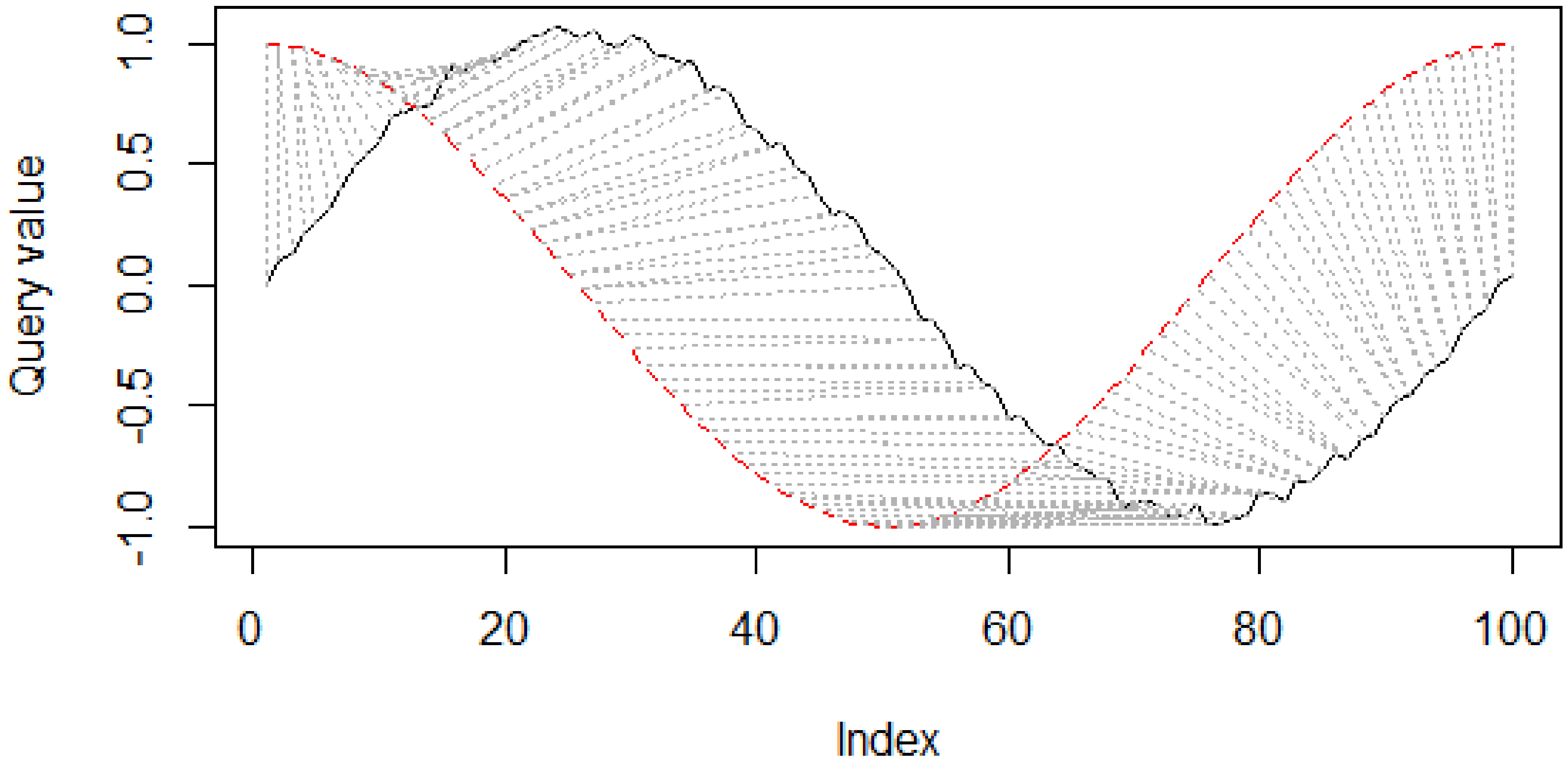




Dynamic Time Warping (DTW): An Illustration

- `library('dtw')`
- `idx <- seq(0, 2*pi, len=100)`
- `aa <- sin(idx) + runif(100)/10`
- `bb <- cos(idx)`
- `align <- dtw(aa, bb, step = asymmetricP1, keep = T)`
- `dtwPlotTwoWay(align)`

Dynamic Time Warping (DTW)



Example

- Synthetic Control Chart Time Series
- Dataset contains 600 examples of control charts synthetically generated by the process in Alcock and Manolopoulos (1999)

Example

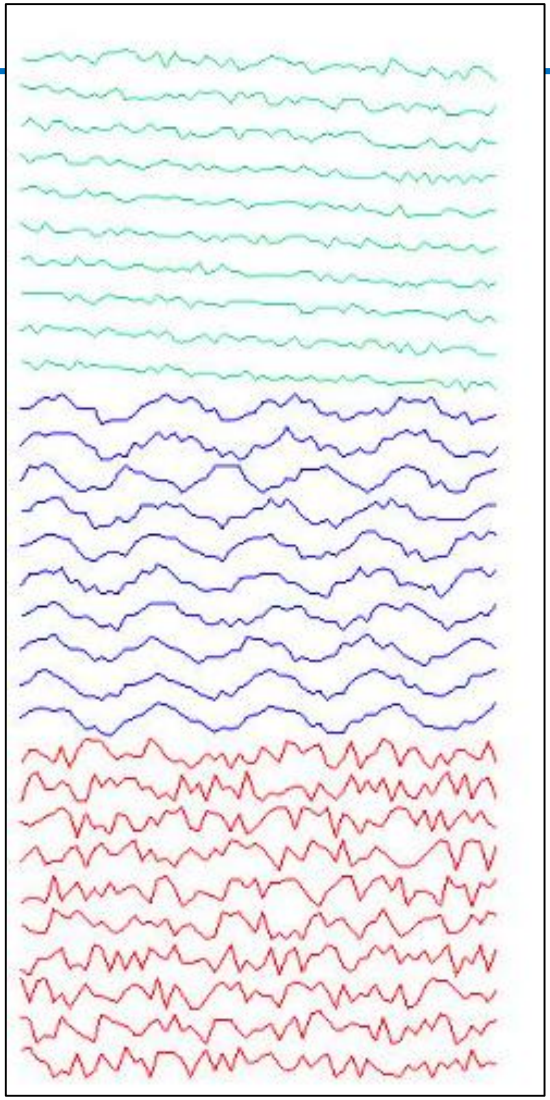
- 600 control charts that are time series in nature with 60 values each
- Six classes are known:
  - 1 – 100 Normal
  - 101 – 200 Cyclic
  - 201 – 300 Increasing Trend
  - 301 – 400 Decreasing Trend
  - 401 – 500 Upward Shift
  - 501 – 600 Downward Shift

Example

A

B

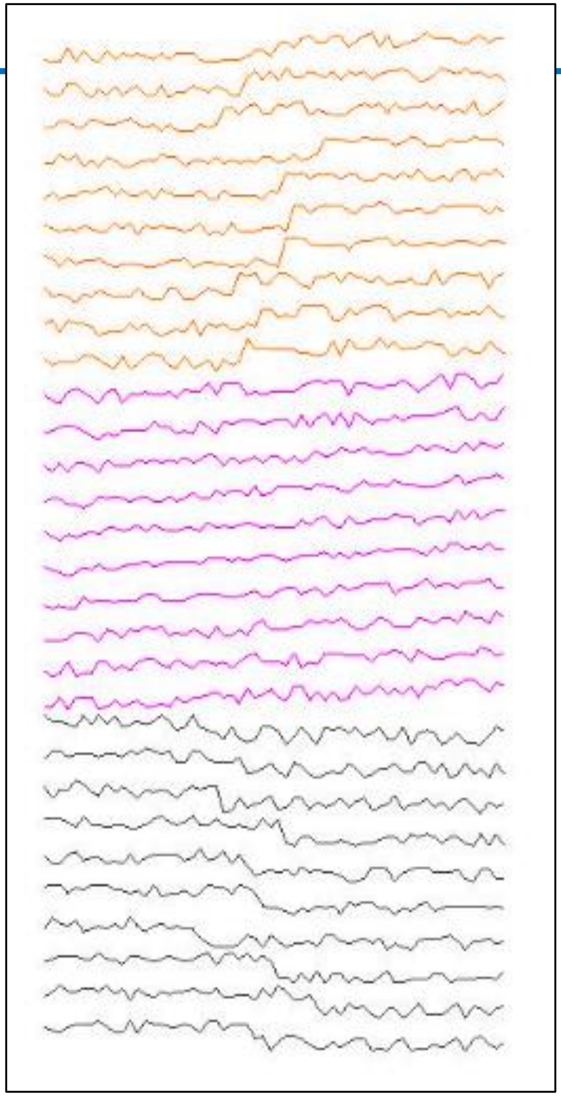
C



D

E

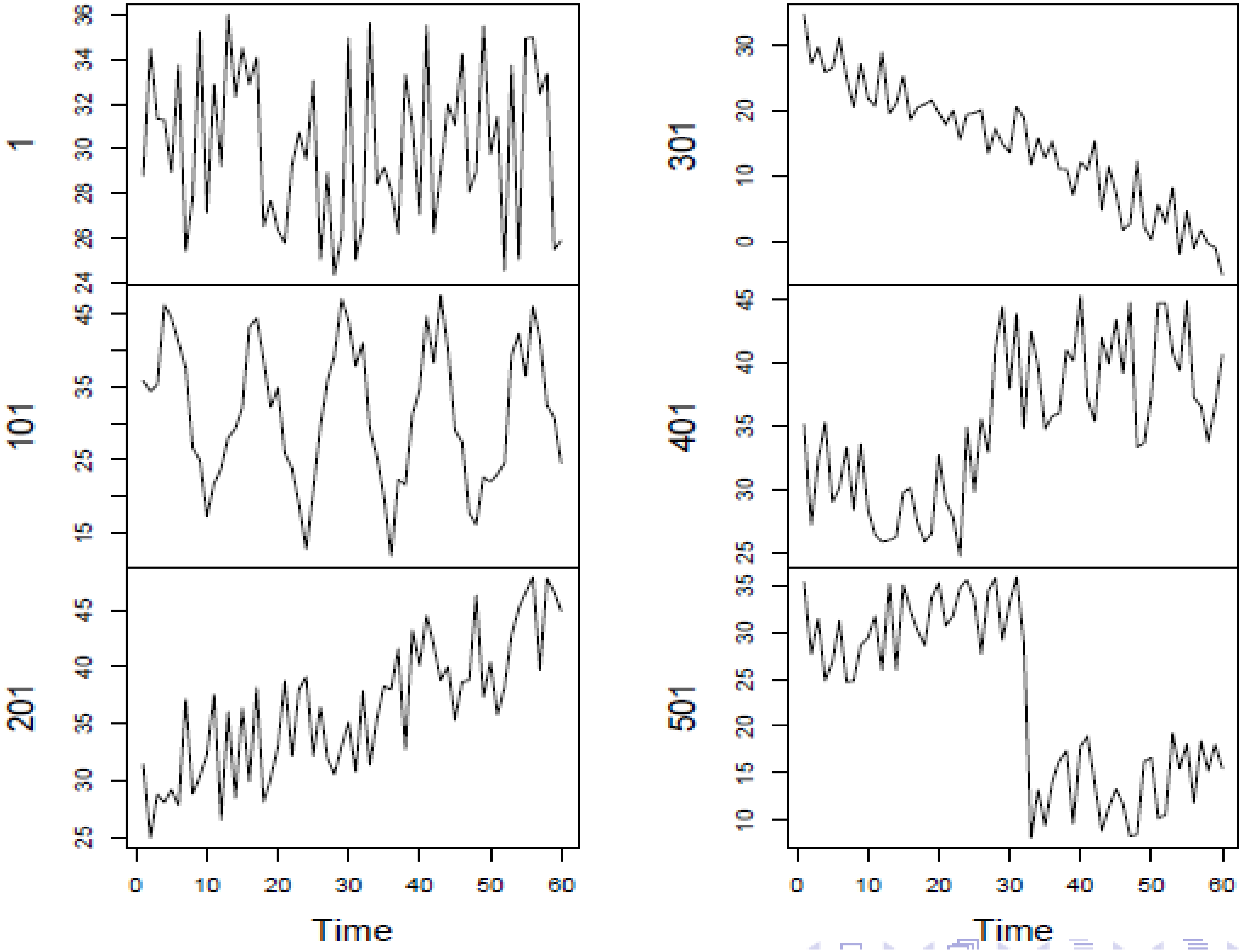
F



Synthetic Control Chart: R Code

- #Read Data into R
- sc <- read.table("synthetic_control.data", header=F, sep="")
- #Show one sample from each class
- idx <- c(1,101,201,301,401,501)
- sample1 <- t(sc[idx,])
- sample1
- plot.ts(sample1, main="")

Example



Synthetic Control Chart: R Code

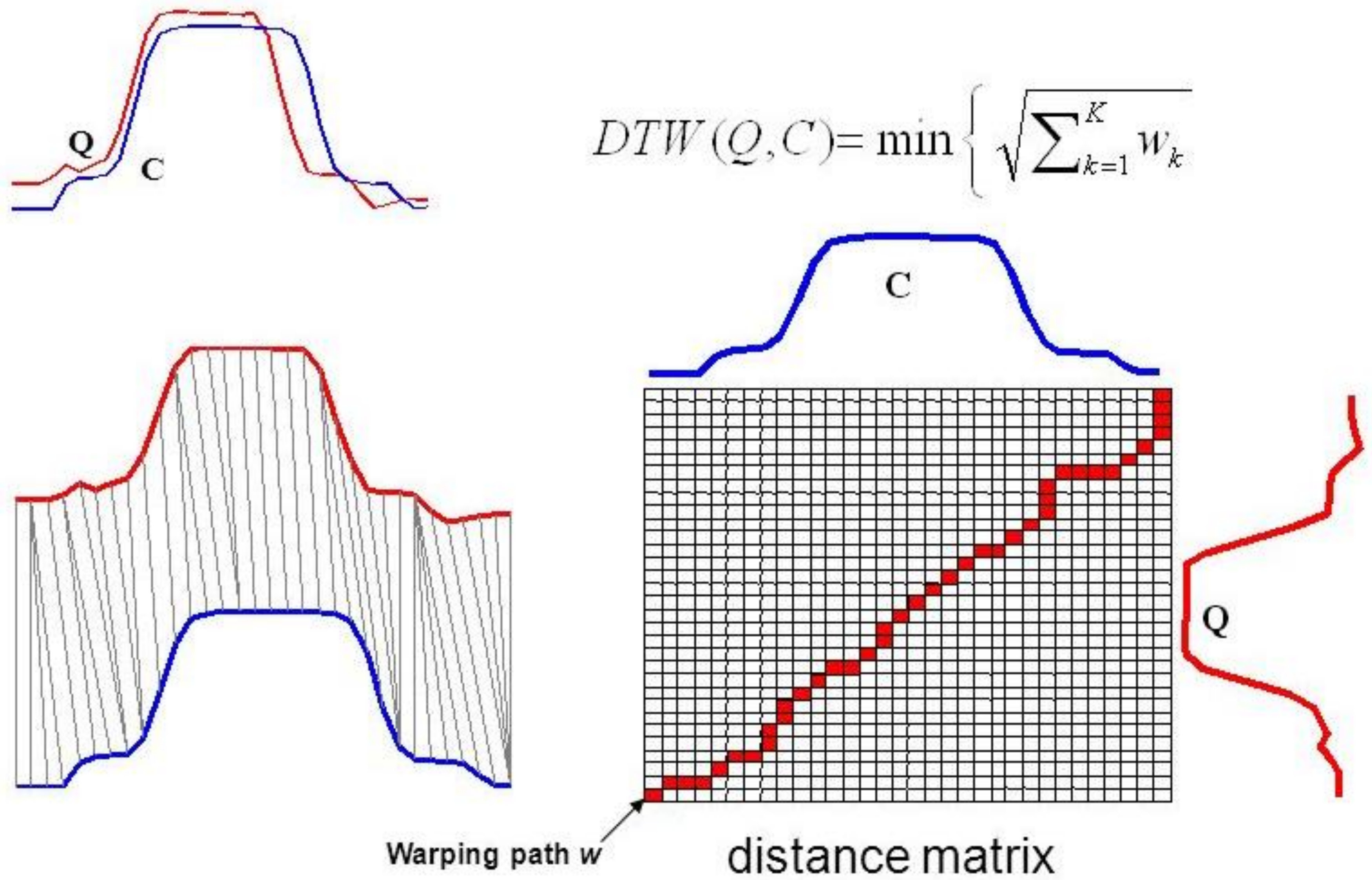
- `#Sample n cases from every class`
- `n <- 10`
- `s <- sample (1:100, n)`
- `idx <- c(s, 100+s, 200+s, 300+s, 400+s, 500+s)`
- `sample2 <- sc[idx,]`
- `observedLabels <- c(rep(1,n),rep(2,n),rep(3,n),rep(4,n),rep(5,n),rep(6,n))`

Synthetic Control Chart: R Code

- `#Compute DTW distances and create a distance matrix`
- `library(dtw)`
- `distMatrix <- dist(sample2, method="DTW")`
- `distMatrix`

The Distance Matrix: DTW

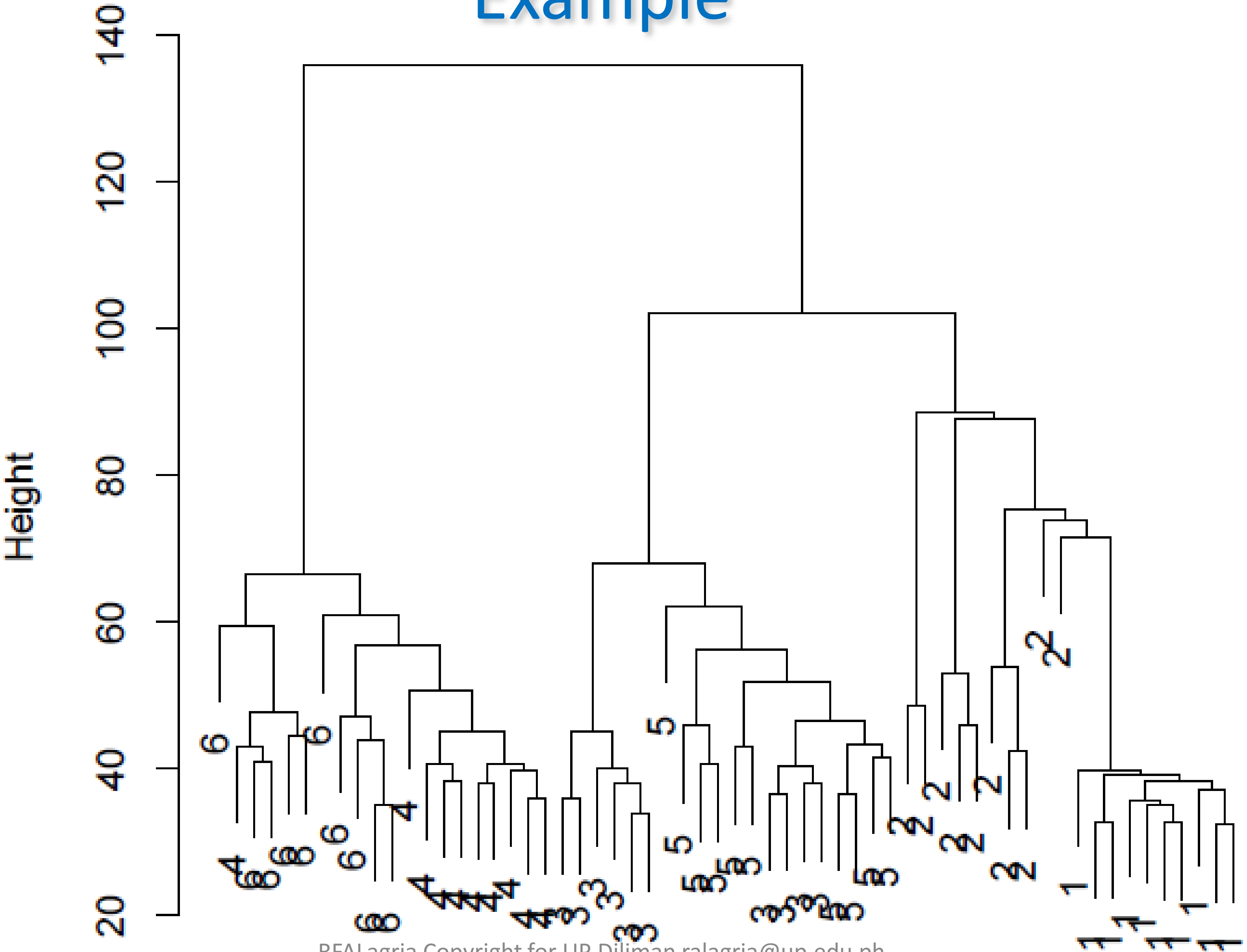
$$DTW(Q,C) = \min\left\{ \sqrt{\sum_{k=1}^{K} w_k} \right.$$



Synthetic Control Chart: R Code

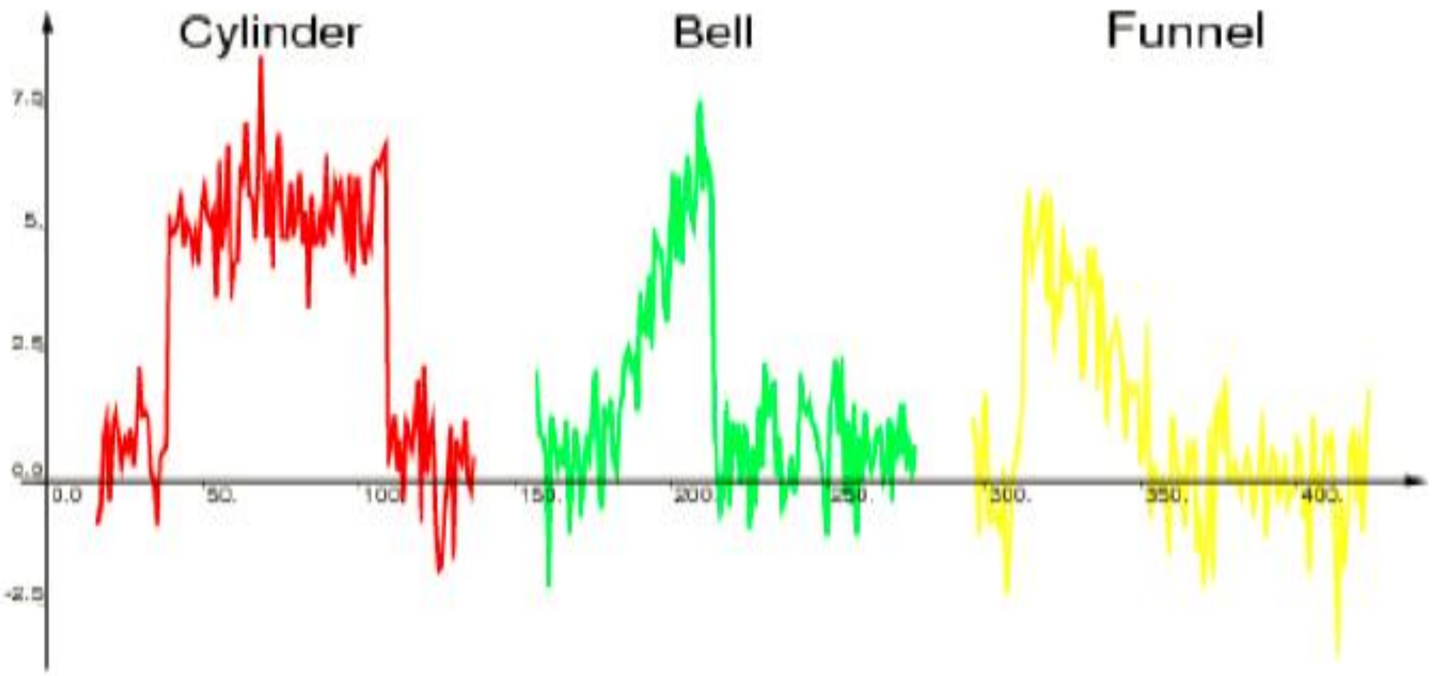
- `#Hierarchical Clustering`
- `hc <- hclust(distMatrix, method="ave")`
- `plot(hc, labels=observedLabels, main ="")`

Example



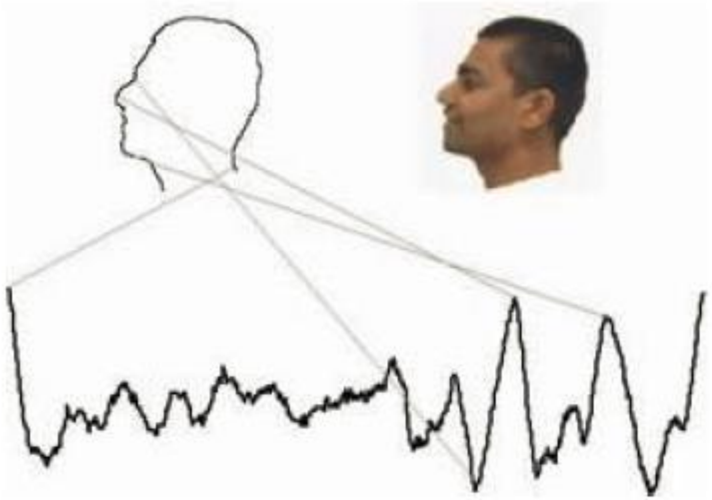
Time Series Classification

- A supervised learning problem aimed at labeling temporally structured univariate (or multivariate) sequences of certain (or variable) length



Time Series Classification: Example

- Task: Classify grad students based on their faces images transformed into "time series"
- Why is difficult?
  - Variation of head angle and expression.
  - Some have glasses/no glasses versions



Time Series Classification: Example

- Build a classification model based on labelled time series
- Use this model to predict unlabeled time series (future)
- The way for time series classification with R is to extract and build features from time series data
- Steps include feature extraction and then apply existing classification techniques
  - SVM
  - K-NN
  - Decision trees

Time Series Classification

- Feature Extraction:
  - Singular Value Decomposition
  - Discrete Fourier Transform (DFT)
  - Discrete Wavelet Transform (DWT)
  - Piecewise Aggregate Approximation (PAA)
  - Perpetually Important Points (PIP)
  - Piecewise Linear Representation
  - Symbolic Representation

R Code: Discrete Wavelet Transform (DWT)

```
• #Extracting DWT coefficients (with Haar filter)
• library(wavelets)
• wtData <- NULL
• for (i in 1:nrow(sc)) {
      a <- t(sc[i,])
      wt <- dwt(a, filter="haar", boundary =
       "periodic")
      wtData <- rbind(wtData,
       unlist(c(wt@W,wt@V[[wt@level]])))}
• wtData <- as.data.frame(wtData)
• wtData
```

R Code: Discrete Wavelet Transform (DWT)

```
#Set class labels into categorical values
classId <- c(rep("1",100), rep("2",100), rep("3",100),rep("4",100), rep("5",100), rep("6",100))
wtSc <- data.frame(cbind(classId, wtData))
wtSc
```

R Code: Discrete Wavelet Transform (DWT)

- `# Build a Decision Tree with ctree() in package party`
- `ct <- ctree(classId ~ ., data=wtSc)`
- `J48M <- J48(classId ~ ., data=wtSc)`
- `J48M`
- `pClassId <- predict(J48M)`
- `pClassId <- predict(ct)`
- `pClassId`

R Code: Discrete Wavelet Transform (DWT)

```
Model formula:
classId ~ W11 + W12 + W13 + W14 + W15 + W16 + W17 + W18 + W19 +
    W110 + W111 + W112 + W113 + W114 + W115 + W116 + W117 + W118 +
    W119 + W120 + W121 + W122 + W123 + W124 + W125 + W126 + W127 +
    W128 + W129 + W130 + W21 + W22 + W23 + W24 + W25 + W26 +
    W27 + W28 + W29 + W210 + W211 + W212 + W213 + W214 + W215 +
    W31 + W32 + W33 + W34 + W35 + W36 + W37 + W41 + W42 + W43 +
    W5 + V57
```

R Code: Discrete Wavelet Transform (DWT)

```
Fitted party:
[1] root
|   [2] V57 <= 116.92152
|   |   [3] W43 <= -3.9333
|   |   |   [4] W5 <= -8.25063: 4 (n = 67, err = 3.0%)
|   |   |   [5] W5 > -8.25063: 6 (n = 7, err = 28.6%)
|   |   [6] W43 > -3.9333
|   |   |   [7] W31 <= -5.69811: 4 (n = 9, err = 11.1%)
|   |   |   [8] W31 > -5.69811: 6 (n = 86, err = 26.7%)
|   [9] V57 > 116.92152
|   |   [10] V57 <= 140.30128: 6 (n = 31, err = 6.5%)
|   |   [11] V57 > 140.30128
|   |   |   [12] V57 <= 177.87899
|   |   |   |   [13] W22 <= -6.3472: 2 (n = 80, err = 2.5%)
|   |   |   |   [14] W22 > -6.3472
|   |   |   |   |   [15] W31 <= -12.68737: 2 (n = 9, err = 0.0%)
|   |   |   |   |   [16] W31 > -12.68737
|   |   |   |   |   |   [17] W36 <= -4.27022: 1 (n = 7, err = 14.3%)
|   |   |   |   |   |   [18] W36 > -4.27022: 1 (n = 92, err = 0.0%)
|   |   |   [19] V57 > 177.87899
|   |   |   |   [20] W31 <= -14.59748: 2 (n = 12, err = 0.0%)
|   |   |   |   [21] W31 > -14.59748
|   |   |   |   |   [22] W43 <= 3.34525
|   |   |   |   |   |   [23] W32 <= 6.14667: 5 (n = 91, err = 11.0%)
|   |   |   |   |   |   [24] W32 > 6.14667: 3 (n = 12, err = 25.0%)
|   |   |   |   |   [25] W43 > 3.34525
|   |   |   |   |   |   [26] W123 <= -3.701: 5 (n = 17, err = 47.1%)
|   |   |   |   |   |   [27] W123 > -3.701
|   |   |   |   |   |   |   [28] W210 <= 6.33655: 3 (n = 73, err = 5.5%)
|   |   |   |   |   |   |   [29] W210 > 6.33655: 3 (n = 7, err = 42.9%)

Number of inner nodes:    14
Number of terminal nodes: 15
```

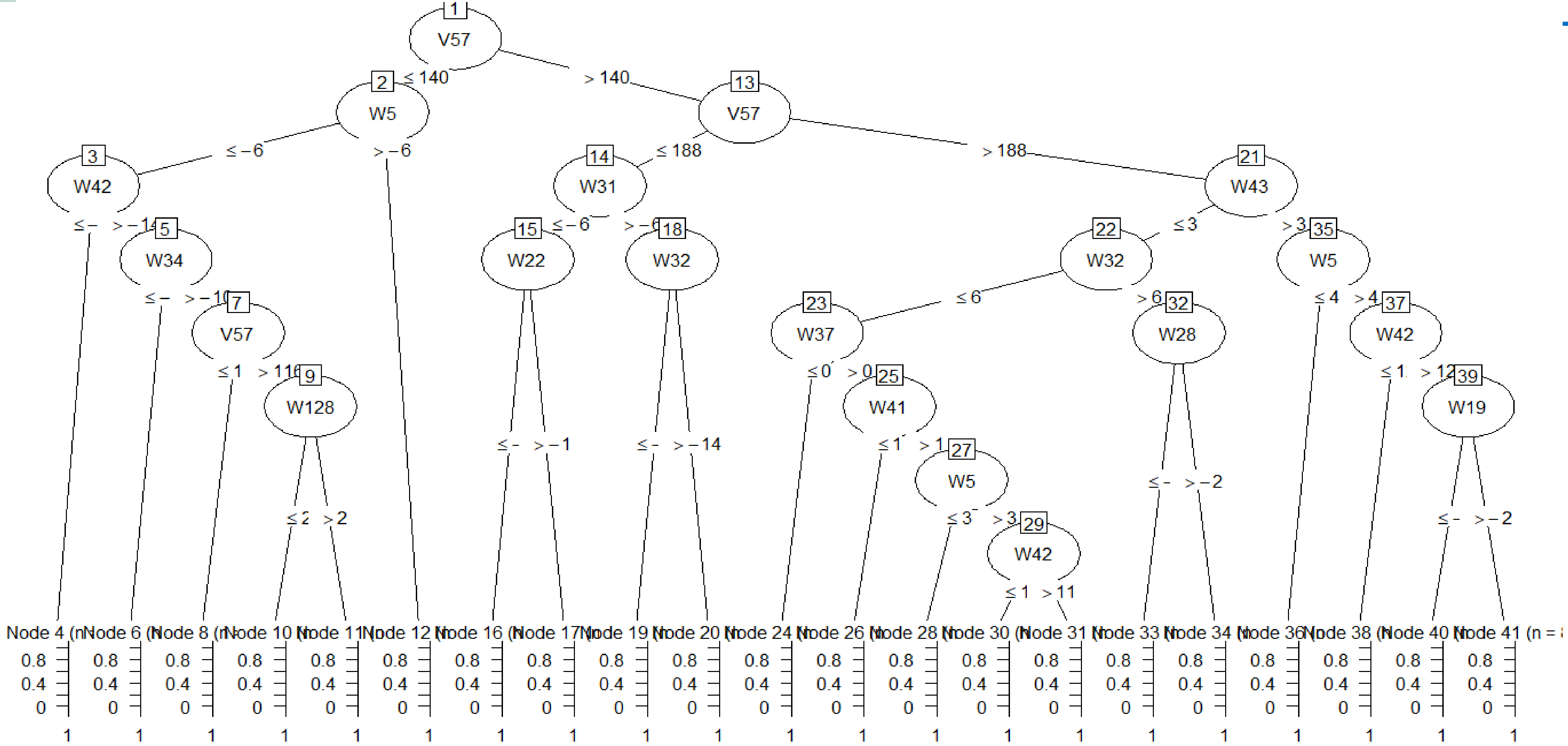
R Code: Discrete Wavelet Transform (DWT)

- `# Check predicted classes against original class labels`
- `table(classId, pClassId)`

- `# Accuracy`
- `(sum(classId==pClassId)) / nrow(wtSc)`

- `#Plot Tree`
- `plot(ct, ip_args=list(pval=FALSE), ep_args=list(digits=0))`

R Code: Discrete Wavelet Transform (DWT)



Final Note

1. Forecasting makes a statement about the future but it does not tell us the actual future.
2. Forecasting needs regular monitoring and adjustments.
3. Forecasting should help the organization, the business, and the team decide on strategic decisions.
4. Forecasting assists in decision making.
5. Forecasting is just a subset of Business Analytics.

References

- Notes and Datasets from Montgomery, Peck and Vining, Introduction to Linear Regression Analysis 4th Ed. Wiley
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- Trevor Hastie, Rob Tibshirani, Friedman: Elements of Statistical Learning (2nd Ed.) 2009
- Time Series Data Library: Australian Bureau of Statistics
- http://datamarket.com/data/list/?q=provider:tsdl
- For R: http://robjhyndman.com/hyndsight/r/
- Time Series Data in R - http://www.rdatamining.com

References

- Duong Tuan Anh, Faculty of Computer Science and Engineering, September 2011
- http://faculty.wiu.edu/F-Dehkordi/DS-533/Lectures/Multiple%20Regression.ppt
- http://faculty.wiu.edu/F-Dehkordi/DS-533/Lectures/The%20Box-Jenkins%20Methodology%20for%20RIMA%20Models.ppt
- www.cse.hcmut.edu.vn/~dtanh/download/TS_PartI_new.ppt

R

- A free software environment for statistical computing and graphics
- http://cran.r-project.org/web/views/TimeSeries.html

R

- R: http://cran.r-project.org/bin/windows/base/R-3.1.3-win.exe
- R Studio: http://download1.rstudio.org/RStudio-0.98.1103.exe
- Make sure to install R first before R Studio which is a GUI of R.
- R is the most widely used Data Mining tool since it is fast, comprehensive and free.