Module 4:
Optimization

6.0 Queuing Theory

Ramon Miguel C. Panis
Instructor
Department Industrial Engineering and Operations Research
University of the Philippines – Diliman

Module 4 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

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Instructor, UP Diliman

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QUEUE

"DO YOU WANT A WORLD WITHOUT QUEUES?"

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QUEUE

any system wherein customers arrive looking for service and depart once service is provided

"PILA"

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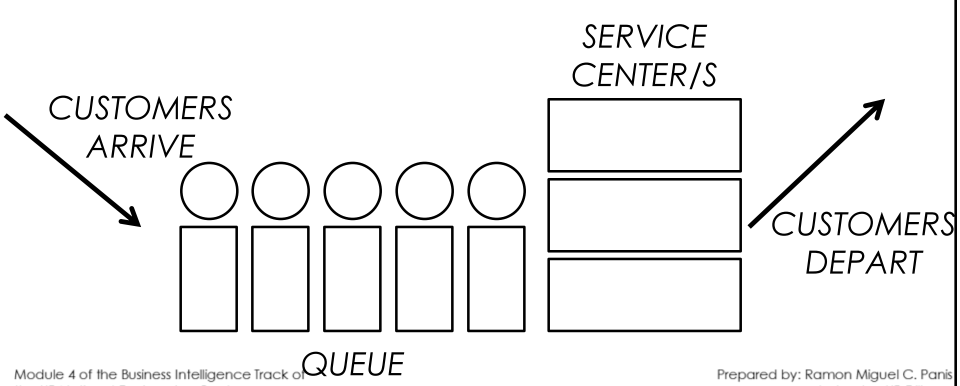
QUEUING THEORY

the mathematical study of waiting lines



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KENDALL-LEE NOTATION

There are six parameters noted in a particular queuing system.

OUTPUT/SERVICE PROCESS:
M – Markovian
E – Erlang
D – Degenerate
G – General

QUEUING DISCIPLINE:
FCFS – First Come First Served
LCFS – Last Come First Served
SIRO – Strictly in Random Order
GD – General Distribution

CALLING POPULATION:
Any counting number up to infinity

$$M / M / 1 \ / FCFS / \infty / \infty$$

INPUT/ARRIVAL PROCESS:
M – Markovian
E – Erlang
D – Degenerate
G – General

SERVERS:
Any counting number

SYSTEM CAPACITY:
Any counting number up to infinity

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KENDALL-LEE NOTATION EXAMPLE

A bank has three tellers on duty, servicing the customer who holds the next prioritized number. When a customer enters the bank and the tellers are busy, he/she is given a number and waits in line for his/her number to be called.

$$M / M / 3 \ / FCFS / \infty / \infty$$

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KENDALL-LEE NOTATION EXAMPLE

A restaurant has three counters, which means customers form three queues in all. If a queue has five people waiting in line (not counting the one being served), the arriving customer goes to the rival restaurant.

$$3 \quad M / M / 1 \ / FCFS / \ 6 \ / \infty$$

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KENDALL-LEE NOTATION EXAMPLE

A carwash station has three car spaces. Due to mismanagement, the carwashers clean arriving cars at a random order. If the car spaces are taken, no cars are allowed to get inside. Assume that the time it takes for a carwasher to clean a car follows a degenerate distribution.

$$M / D / 3 \ / SIRO / \ 3 \ / \infty$$

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KENDALL-LEE NOTATION EXAMPLE

Describe the following queuing model and give an example:

$M / M / 1 / FCFS / 1 / \infty$

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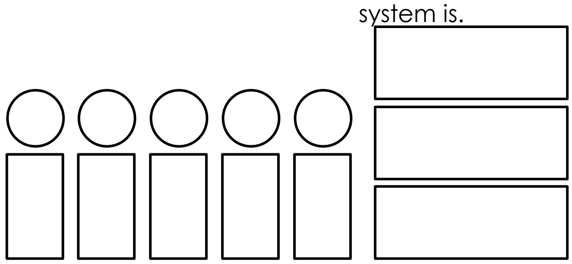
QUEUING PERFORMANCE MEASURES

These quantities are used to evaluate how good or bad a queuing system is.



L_Q = average number of customers in queue

L_S = average number of customers being served

$L = L_Q + L_S$

W_Q = average time a customer spends in queue

W_S = average time a customer is being served

$W = W_Q + W_S$

P_n = probability there are n customers in the system

% utilization

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QUEUING PERFORMANCE MEASURES

Here are the formulas for the queuing performance measures:

(M/M/1/∞/∞/FCFS)

$$\rho = \frac{\lambda}{\mu} < 1$$

$$P_0 = 1 - \rho$$

$$P_n = \rho^n P_0$$

$$L = \frac{\rho}{1-\rho}$$

$$L_q = \frac{\rho^2}{1-\rho}$$

$$W = \frac{1}{\mu - \lambda}$$

$$W_q = \frac{\rho}{\mu - \lambda}$$

(M/M/s/∞/∞/FCFS)

$$\rho = \frac{\lambda}{s\mu} < 1$$

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \frac{1}{1 - \lambda/(s\mu)}}$$

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & \text{if } 0 \le n \le s \\ \frac{(\lambda/\mu)^n}{s!\, s^{n-s}} P_0 & \text{if } n > s \end{cases}$$

$$L_q = \frac{P_0 (\lambda/\mu)^s \rho}{s!\,(1-\rho)^2}$$

(M/M/1/K/∞/FCFS)

$$\rho = \frac{\lambda}{\mu}$$

$$P_n = \frac{1-\rho}{1-\rho^{K+1}} \rho^n, \quad \text{for } \rho \ne 1$$

$$P_n = \frac{1}{K+1}, \quad \text{for } \rho = 1$$

$$L = \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}}, \quad \text{for } \rho \ne 1$$

$$L = \frac{K}{2}, \quad \text{for } \rho = 1$$

$$\bar{\lambda} = \lambda(1 - P_K)$$

(M/M/s/K/∞/FCFS)

$$\rho = \frac{\lambda}{s\mu}, \quad s \le K$$

$$P_0 = \frac{1}{\sum_{n=0}^{s} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \sum_{n=s+1}^{K} \left(\frac{\lambda}{s\mu}\right)^{n-s}}$$

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & \text{if } n = 1, 2, \dots, s \\ \frac{(\lambda/\mu)^n}{s!\, s^{n-s}} P_0 & \text{if } n = s+1, \dots, K \\ 0 & \text{if } n = K+1, \dots \end{cases}$$

$$L_q = \frac{P_0 (\lambda/\mu)^s \rho}{s!\,(1-\rho)^2} [1 - \rho^{K-s} - (K-s)\rho^{K-s}(1-\rho)]$$

$$L = L_q + \sum_{n=0}^{s-1} n P_n + s\left(1 - \sum_{n=0}^{s-1} P_n\right)$$

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QUEUING PERFORMANCE MEASURES

Here are the formulas for the queuing performance measures:

(M/M/1/N/N/FCFS)

$$P_0 = \frac{1}{\sum_{n=0}^{N} \left[\frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n\right]}$$

$$P_n = \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n P_0, \quad \text{if } n = 1, 2, \dots, N;$$

$$L_q = N - \frac{\lambda + \mu}{\lambda}(1 - P_0)$$

$$L = N - \frac{\mu}{\lambda}(1 - P_0)$$

$$\bar{\lambda} = \lambda(N - L)$$

(M/M/s/N/N/FCFS)

$$P_0 = \frac{1}{\sum_{n=0}^{s-1} \left[\frac{N!}{(N-n)!\,n!} \left(\frac{\lambda}{\mu}\right)^n\right] + \sum_{n=s}^{N} \left[\frac{N!}{(N-n)!\,s!\,s^{n-s}} \left(\frac{\lambda}{\mu}\right)^n\right]}$$

$$P_n = \begin{cases} \frac{N!}{(N-n)!\,n!} \left(\frac{\lambda}{\mu}\right)^n P_0 & \text{if } 0 \le n \le s \\ \frac{N!}{(N-n)!\,s!\,s^{n-s}} \left(\frac{\lambda}{\mu}\right)^n P_0 & \text{if } s < n \le N \\ 0 & \text{if } n > N \end{cases}$$

$$L_q = \sum_{n=s}^{N} (n-s) P_n$$

$$L = \sum_{n=0}^{s-1} n P_n + L_q + s\left(1 - \sum_{n=0}^{s-1} P_n\right)$$

$$\bar{\lambda} = \lambda(N - L)$$

(M/G/1/∞/∞/FCFS)

$$\rho = \frac{\lambda}{\mu} < 1$$

$$P_0 = 1 - \rho$$

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

(M/E_k/1/∞/∞/FCFS)

$$\rho = \frac{\lambda}{\mu} < 1$$

$$P_0 = 1 - \rho$$

$$L_q = \frac{1+k}{k} \cdot \frac{\rho^2}{2(1-\rho)}$$

(M/D/1/∞/∞/FCFS)

$$\rho = \frac{\lambda}{\mu} < 1$$

$$P_0 = 1 - \rho$$

$$L_q = \frac{\rho^2}{2(1-\rho)}$$

Of course, we won't use those formulas. However, there is QTP (Queuing ToolPak), an add-on in MS Excel that easily solves that. But I won't require you to learn that nor the use of the formulas. IE 3 students just need to appreciate them :)

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QUEUING PERFORMANCE MEASURES

L = average people in the system
L_q = average people in queue
W = average waiting time in system
W_q = Average waiting time in queue
P_i = probability in state i

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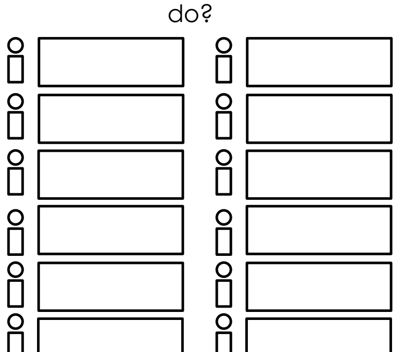
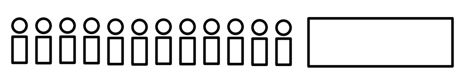
THE CONTRADICTION IN QUEUES

If we want to have no queues, what can we do?



HIGH SERVICE COST

If we want to have minimum service cost, what can we do?



HIGH WAITING COST

We want to optimize the system so we minimize total WAITING COST + SERVICE COST by determining just the right number of service centers

Our IE 142 project: Determining the number of call center agents and their shifts to achieve a certain service level given the daily demand distribution of callers using Linear Programming (Assignment Problem) and Queuing Theory.

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CUSTOMER ATTITUDES

BALKING — When an arriving customer chooses not to enter a queue because it is already too long.

RENEGING — When a customer already in queue gives up and exits without being serviced.

JOCKEYING — When a customer switches between alternate queues in an effort to reduce waiting time.

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HOW TO MITIGATE EFFECTS OF LONG QUEUES

Use the customer as resource

Example: Patients filling out medical history form while waiting for a physician.

Distracting the customers while waiting

Example: Complementary food/drinks while waiting at a restaurant, airports, etc.
Example: Installing of mirrors beside elevators
Example: Have friendly/entertaining servers

Provide pessimistic estimates of waiting time

Example: Fastfood cashiers telling you to wait for the worst case time

But of course, we want to altogether improve the queuing system via this lesson.

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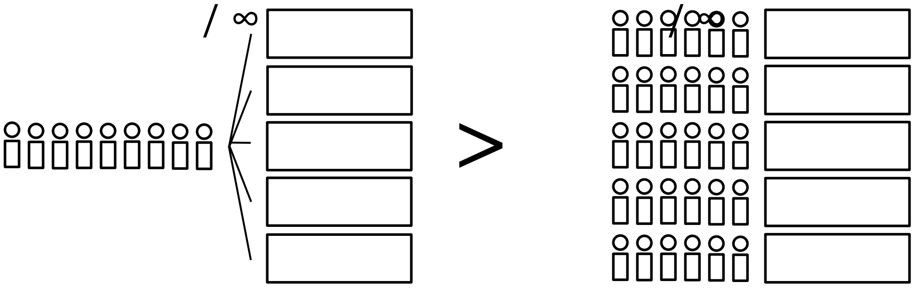
THE MOST IMPORTANT LESSON

1 M / M / 5 / FCFS / ∞ vs 5 M / M / 1 / FCFS / ∞
/ ∞ / ∞



From the formulas, the former always has better performance measures than the latter.

That's what IE Dept does in the enrolment processes.

As well as some grocery cashier counters.

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Fin.