Name:

#### Instructions:

- 1. This is a take home Long Quiz which requires you to use MS Excel and the add-in Queuing Toolpak. Follow the specific instructions below.
- 2. There are three problems in this long quiz. Kindly box your final answers. Give answers up to the 5<sup>th</sup> decimal place if appropriate.
- 3. Print all pages BACK-TO-BACK on A4 size paper. Submissions not in proper format will get a 50% deduction.
- 4. Submit on or before May 8 2015, class hours. Each day late will get a 20% deduction.
- 5. For questions, you may consult with me online or at MH 405 on Wednesday, May 6 2015, class hours. You may also use the computers at the IE/OR Comp Lab that time.
- 6. This is to be answered individually. Answer well. Take your time. Good luck!

### **Specific Instructions:**

Go to http://queueingtoolpak.org/download.shtml. Follow the instructions on how to install Queuing Toolpak 4.o.

Open MS Excel and create something like this (right). If you are not familiar with MS Excel's basic tools, here are some tips:

- Highlight two or more cells then click Merge & Center to merge the cells and center the text
- Highlight all cells then click Borders -> All Borders to put borders on each cell; Borders -> Thick Box Border to put a thick box border surrounding the highlighted cells
- Highlight a cell/s then click on Theme Colors -> (color) to give a cell a color
- Drag the column/row (in its letter or number) to adjust its length/width

The green-shaded cells are inputs. Later when answering specific problems, you will put values there.

- Arrival Rate is arrival rate  $\lambda$  (should have same units as service rate)
- Service Rate is service rate μ (should have same units as arrival rate)
- No. of Servers is the number of servers (should be an integer)
- Queue Capacity is system capacity minus number of servers (if disregarded, this is assumed to be infinite)
- Threshold Time is a required value for computing service level, if applicable (it is the target waiting time for a customer)
- Desired Service Level is the probability of achieving the Threshold Time per customer, if applicable

$\square$	Α	В	С	D
1				
2		INPUT:		L
3		Arrival Rate		L
4		Service Rate		L
5		No. of Servers		L
6		Queue Capacity		L
7		Threshold Time		L
8		Desired Service Level		L
9		оитрит:		
10		% Utilization		
11		L		
12		Lq		
13		W		
14		Wq		
15		p0		
16		p1		
17		p2		
18		р3		
19		p4		
20		p5		
21		Service Level		
22		Minimum Servers		
23				

The pink-shaded cells are outputs. You will put MS Excel functions there (which are present only after installing Queuing Toolpak 4.o. They will then have values automatically depending on the input parameters. For a complete list of functions, go to http://gueueingtoolpak.org/gtp4o/help4o/Functions.htm.

Queuing Quantity	Syntax
Percent Utilization	=QTPMMS_Util(Arrival Rate, Service Rate, No. of Servers, Queue Capacity)
Average Customers in System	=QTPMMS_L(Arrival Rate, Service Rate, No. of Servers, Queue Capacity)
Average Customers in Queue	=QTPMMS_Lq(Arrival Rate, Service Rate, No. of Servers, Queue Capacity)
Average Time in System	=QTPMMS_W(Arrival Rate, Service Rate, No. of Servers, Queue Capacity)
Average Time in Queue	=QTPMMS_Wq(Arrival Rate, Service Rate, No. of Servers, Queue Capacity)
Probability in State n	=QTPMMS_PrState(State, Arrival Rate, Service Rate, No. of Servers, Queue Capacity)
Service Level	=QTPMMS_ServiceLevel(Threshold Time, Arrival Rate, Service Rate, No. of Servers,
	Queue Capacity)
Minimum No. of Servers	=QTPMMS_MinServers(Threshold Time, Service Level, Arrival Rate, Service Rate,
	Queue Capacity)

For example, in the pink-shaded cell beside % Utilization, you will type the syntax =QTPMMS\_Util(Arrival Rate, Service Rate, No. of Servers, Queue Capacity). But of course instead of typing those words, you will locate the cell of those inputs (if applicable) e.g. =QTPMMS\_Util(C3,C4,C5,C6). Do it for all output quantities.

You are now ready to answer problems.

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# PROBLEM 1: THE BARBER SHOP (basic qtp)

In a medium-sized barber shop, there are six barbers. The service time of each barber is exponentially distributed with mean 30 minutes. Arrivals at the barber shop occur at a Poisson process with mean rate 8 per hour. In the waiting area inside, there are six seats for waiting customers. An arriving customer who sees no vacant seats balks and finds another barber shop.

Hint: You may need more output quantities than what is on the example screenshot previously. Add accordingly. Also, some cells may still not be needed. Just leave them blank.

- a) What is the proportion of time all barbers are idle?
- b) What is the proportion of time exactly two barbers are working?
- c) What is the proportion of time the barber shop is full and no one enters?
- d) What is the percent utilization of any given barber?
- e) What is the probability that an arriving customer will have to wait before having service?
- f) What is the average number of customers inside the shop?
- g) What is the average number of customers waiting inside the shop?
- h) What is the average time a customer is inside the shop? (in minutes)
- i) What is the average time a customer waits inside the shop? (in minutes)
- j) What is the average number of barbers working?

```
a) p_0 = 0.01697

b) p_2 = 0.13574

c) p_{12} = 0.00847

d) 1 - [p_0 + (5/6)p_1 + (4/6)p_2 + (3/6)p_3 + (2/6)p_4 + (1/6)p_5] = 0.66102

e) p_{6+} = 0.27264

p_{6-11}|arriving = 0.26642

f) L = 4.39274

g) L_q = 0.42664

h) W = 33.22711 minutes

i) W_q = 3.22711 minutes

j) op_0 + 1p_1 + 2p_2 + 3p_3 + 4p_4 + 5p_5 + 6p_6 + 6p_7 + 6p_8 + 6p_9 + 6p_{10} + 6p_{11} + 6p_{12} = 3.96610
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### PROBLEM 2: THE CALL CENTER (queuing optimization w/ qtp)

A call center is determining the number of agents it should employ per shift. There are three shifts: 8am-4pm, 4pm-12mn, 12mn-8am. The number of customers that arrive per shift is given below. Assume Poisson distribution. The service time of a call center agent is assumed to be exponentially distributed with average 5 minutes per call. Queue capacity is infinity.

Shift	Calls per shift
8am-4pm	640
4pm-12mn	750
12mn-8am	500

Hint: You may need to use the trial-and-error method to determine the optimal number of servers. Or you may copy the formulas many times using different input values to easily determine the optimal number of servers. You may have to use other simple formulas (e.g. adding and multiplying). And you need not compute for all quantities.

- a) What is the minimum number of agents such that the system would not explode during the shift 8am-4pm?
- b) What is the minimum number of agents such that the system would not explode during the shift 4pm-12mn?
- c) What is the minimum number of agents such that the system would not explode during the shift 12mn-8am?

For letters d to g, the criterion for determining number of call center agents is total cost. The waiting cost associated per minute waiting in line is Php 40. The cost of one call center agent per hour is Php 120.

- d) What is the optimal number of call center agents to employ during the shift 8am-4pm?
- e) What is the optimal number of call center agents to employ during the shift 4pm-12mn?
- f) What is the optimal number of call center agents to employ during the shift 12mn-8am?
- g) What is the optimal total cost for one whole day?

For letters h to j, the criterion for determining number of call center agents is service level. The threshold time is 3 minutes and the desired service level is 90%. This means that 90% of customers should only wait for less than 3 minutes.

- h) What is the minimum number of call center agents to attain this service level during the shift 8am-4pm?
- i) What is the minimum number of call center agents to attain this service level during the shift 4pm-12mn?
- j) What is the minimum number of call center agents to attain this service level during the shift 12mn-8am?
- a) 7
- b) 8
- c) 6
- d) 12
- e) 14
- f) 10
- g) 1st shift: waiting cost = 137.35, service cost = 1440, total cost = 1577.35 2nd shift: waiting cost = 100.00, service cost = 1680, total cost = 1780.00 3rd shift: waiting cost = 119.82, service cost = 1200, total cost = 1319.82 Optimal total cost = Php 37,417.34
- h) 9
- i) 10
- j) 8

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### PROBLEM 3: THE MACHINE SHOP (choosing which machine w/ qtp)

You own a machine shop. You are choosing to purchase between five different machines which to install in your machine shop. You can only buy one. The parameters for each machine are found below. The interarrival times of customers in your machine shop was found to be normally distributed with mean 30 minutes and variance 100 minutes<sup>2</sup>. The waiting cost associated per minute waiting in line is Php 200.

Machine Name	Service Time Distribution	Parameters	Service Cost per Hour
Α	Normal	Mean: 21 minutes, Variance: 9 minutes <sup>2</sup>	Php 200
В	Normal	Mean: 18 minutes, Variance: 16 minutes <sup>2</sup>	Php 280
С	Exponential	Poisson Mean: 3 per hour	Php 180
D	Uniform	a = 16, b = 24	Php 320
E	Degenerate	Mean: 25 minutes	Php 360

We are introducing new functions of Queuing Toolpak 4.0 for non-Markovian queuing systems (non-Markovian arrival rate and service rate). The additional inputs needed are standard deviation of interarrival times and standard deviation of service times.

Queuing Quantity	Syntax
Percent Utilization	=QTPGGS_Util(Arrival Rate, Service Rate, No. of Servers, Queue Capacity, Stdev
	Arrivals, Stdev Service)
Average Customers in System	=QTPGGS_L(Arrival Rate, Service Rate, No. of Servers, Queue Capacity, Stdev
	Arrivals, Stdev Service)
Average Customers in Queue	=QTPGGS_Lq(Arrival Rate, Service Rate, No. of Servers, Queue Capacity, Stdev
	Arrivals, Stdev Service)
Average Time in System	=QTPGGS_W(Arrival Rate, Service Rate, No. of Servers, Queue Capacity, Stdev
	Arrivals, Stdev Service)
Average Time in Queue	=QTPGGS_Wq(Arrival Rate, Service Rate, No. of Servers, Queue Capacity, Stdev
	Arrivals, Stdev Service)

Hint: Beware of units. Units must be consistent. If Arrival Rate and Service Rate are in per hour, Stdev Arrivals and Stdev Service should also be in hours.

- a) If the criterion for choosing a machine is average queue length, rank the machines according to preference. Solve for the average gueue length of each machine.
- b) If the criterion for choosing a machine is total cost, rank the machines according to preference. Solve for the total cost per hour of each machine.

```
a) A: L_q = 0.10615
    B: L_q = 0.06855
    C: L_q = 0.60029
    D: L_q = 0.08228
    E: L_0 = 0.23148
    Preference: B > D > A > E > C
```

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b) A: waiting cost = 1273.82, service cost = 200, total cost = 1473.82
    B: waiting cost = 822.62, service cost = 280, total cost = 1102.62
    C: waiting cost = 7203.48, service cost = 180, total cost = 7383.48
    D: waiting cost = 987.38, service cost = 320, total cost = 1307.38
    E: waiting cost = 2777.78, service cost = 360, total cost = 3137.78
```

Preference: B > D > A > E > C