NATIONAL ENGINEERING CENTER
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6.0 Regression and Time Series Analysis

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Module 1 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

Module 1 Outline

1. Intro to Business Intelligence
   - Case Study on Selecting BI Projects
2. Data Warehousing
   - Case Study on Data Extraction and Report Generation
3. Descriptive Analytics
   - Case Study on Data Analysis
4. Classification Analysis
   - Case Study on Classification Analysis
5. **Regression and Time Series Analysis**
   - **Case Study on Regression and Time Series Analysis**
6. Unsupervised Learning and Modern Data Mining
   - Case Study on Text Mining
7. Optimization for BI

Outline for this Session

- Regression and Model Building
- Simple and Multiple Linear Regression
- Model Evaluation
- Indicator Variables
- Alternative Regression Models
- Time Series Analysis
- Components of a Time Series
- Evaluation Methods of Forecast
- Smoothing Methods of Time Series
- Case Study

Regression and Model Building

Definition 6.1: Regression Analysis

- Regression analysis is a statistical technique for investigating and **modeling the relationship between variables.**
- Equation of a straight line (classical)

$$y = mx + b \tag{6.1}$$

- We usually write this as

$$y = \beta_0 + \beta_1 x \tag{6.2}$$

Regression and Model Building

- Input Variables, Regressors, Independent variables or predictor variables (x)
  - Must be **continuous** and have **no missing values**
- Output Variable, Target Variable, Response Variable, or Independent Variable (y)
- Output Variable must be **continuous**

Regression and Model Building

- Not all observations will **fall exactly** on a straight line.

$$y = \beta_0 + \beta_1 x + \varepsilon \tag{6.3}$$

  - where ε represents error
  - it is a variable that accounts for the failure of the model to fit the data exactly.
  - $\varepsilon \sim N(0, \sigma^2)$

Regression and Model Building

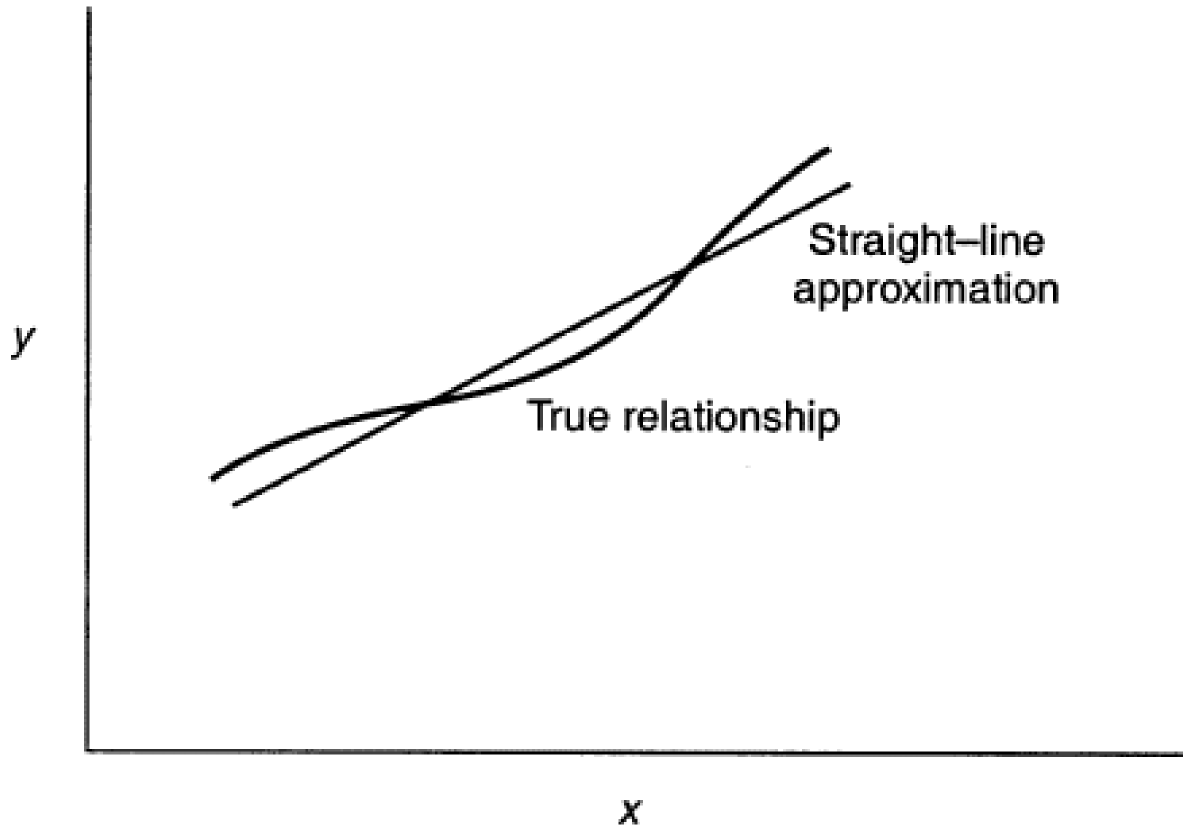

Figure 6.1: Approximation of the True Relationship

Regression and Model Building

- There are many uses of regression, including:
  - Data description
  - Parameter estimation
  - Prediction and estimation
  - Process Control
- Regression analysis is perhaps the **most widely used** statistical technique, and probably the **most widely misused**.

Regression and Model Building

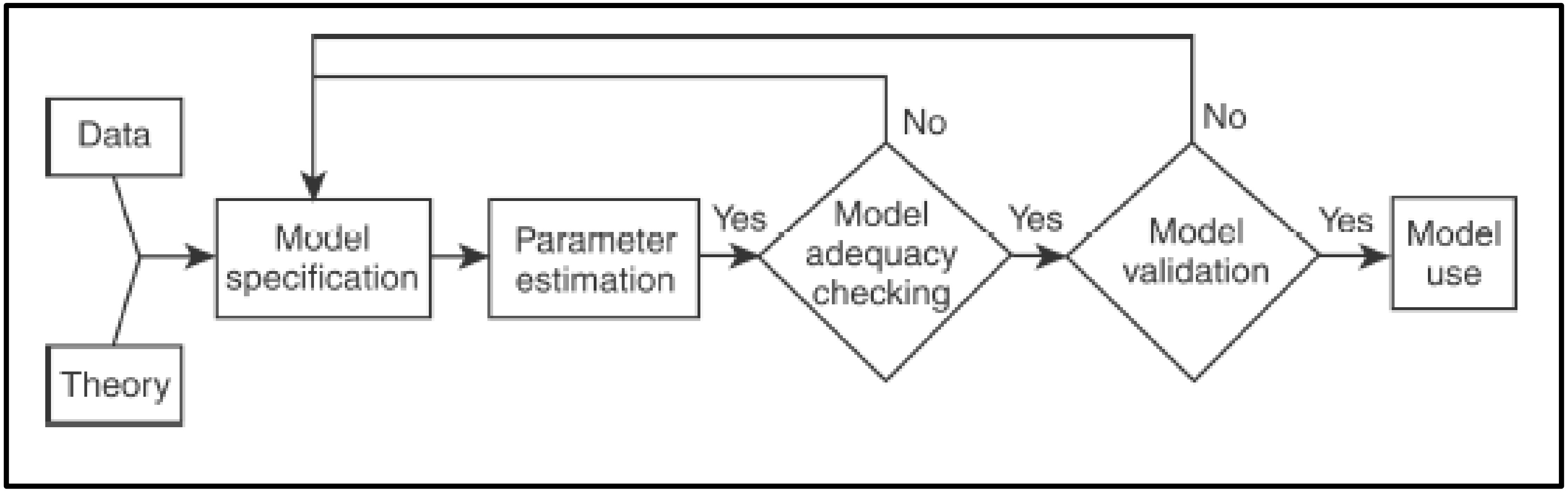

Figure 6.2: Regression Model Building

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Simple and Multiple Linear Regression

- Single predictor, x_1; response, y

$$y = \beta_0 + \beta_1 x + \varepsilon \qquad (6.4)$$

- β_0 – **intercept**: then β_0 is the response y, when $x_1 = 0$
- β_1 – **slope**: change in the mean of the distribution of the response produced by a unit change in x
- ε - random **error**: difference between predicted and actual which is distributed $NID(0, \sigma^2)$

Simple and Multiple Linear Regression

Example 6.1: Simple Linear Regression

- In this study, a random sample of **service call records** for a computer repair operation were examined and the length of each call (in minutes) and the number of components repaired were recorded.
- We would like to model the **relationship** between the number of components repaired to the **total time** it took to repair the computer

Minutes	Units
23	1
29	2
49	3
64	4
74	4
87	5
96	6
97	6
109	7
109	7
119	8
149	9
145	9
154	10

Simple and Multiple Linear Regression

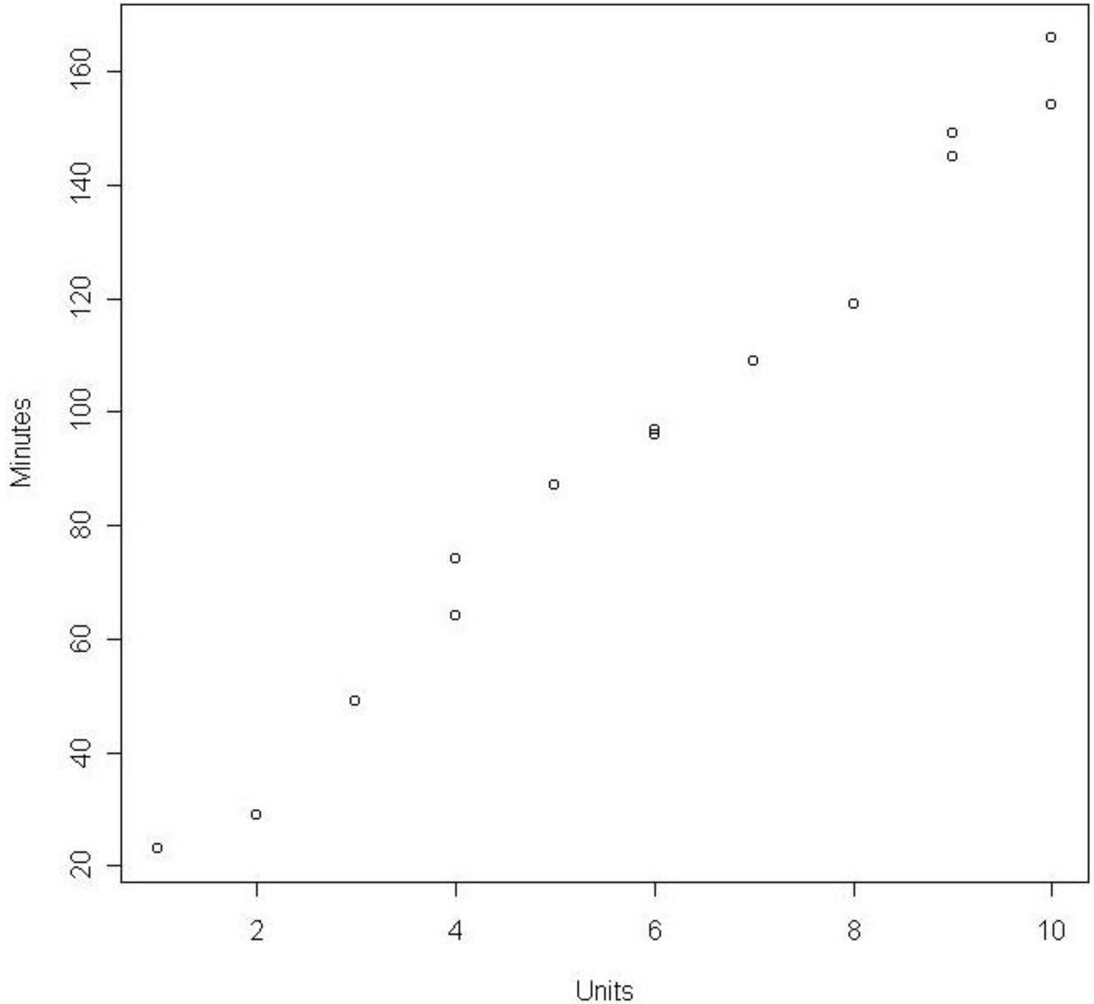
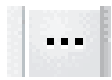


Figure 6.3: Scatter plot of the data

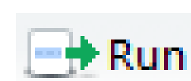
Using R Studio

- Open R Studio from the Programs Menu
- On the file explorer tab click on Files. 

- Click on Explore 

- Go to the Desktop Folder -> BI Training -> 5.0 Regression and Time Series
- Click on More. 

Click on Set as Working Directory.
- Click on File-> New File -> R Script.
- In the new tab script 

, type the following code:
  - `options(scipen=999,digits=2)`
  - `servicecalldata = read.csv("servicecalldata.csv")`
  - `plot(servicecalldata$units,servicecalldata$minutes)`
- Highlight the three lines of code and click on Run 

Simple Regression Using R Studio

- In the new tab script 

, type the following code:
  - `simplelrfit = lm(minutes~units, data=servicecalldata)`
  - `summary(simplelrfit)`
- Highlight the two lines of code and click on Run 


```
Call:
lm(formula = minutes ~ units, data = servicecalldata)

Residuals:
   Min     1Q Median     3Q    Max 
-9.232 -3.341 -0.714  4.777  7.803 

Coefficients:
            Estimate Std. Error t value         Pr(>|t|)    
(Intercept)    4.162      3.355    1.24             0.24    
units         15.509      0.505   30.71 0.00000000000089 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.4 on 12 degrees of freedom
Multiple R-squared:  0.987,	Adjusted R-squared:  0.986 
F-statistic:  943 on 1 and 12 DF,  p-value: 0.000000000000892
```

Simple and Multiple Linear Regression

- Service Call Regression Model:

$$Minutes = 4.162 + 15.509\ Units$$

Simple and Multiple Linear Regression

Definition 6.2: Coefficient of Determination

- R^2 - coefficient of determination

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_{Res}}{SS_T} \qquad (6.5)$$

- **Proportion of variation** explained by the regressor, x
- $R^2 = \rho_{xy}^2$
- For the service call data

$$R^2 = \frac{SS_R}{SS_T} = 0.987$$

Simple and Multiple Linear Regression

Definition 6.3: Multiple Regression Analysis

- The simple regression model can be extended to have ***k* regressors**

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon \qquad (6.6)$$

- Number of regressors (k) must be lesser than the number of rows (n)

Simple and Multiple Linear Regression

Example 6.2: Multiple Regression Example

- **Delivery Time Data**
  - A soft drink bottler is analyzing the vending machine service routes in his distribution system.
  - The analyst thinks that delivery time (y) is affected by the number of cases (x_1) and distance walked by the driver (x_2).

Observation Number	Delivery Time (Minutes) y	Number of Cases x_1	Distance (Feet) x_2
1	16.68	7	560
2	11.50	3	220
3	12.03	3	340
4	14.88	4	80
5	13.75	6	150
6	18.11	7	330
7	8.00	2	110
8	17.83	7	210
9	79.24	30	1460
10	21.50	5	605
11	40.33	16	688
12	21.00	10	215
13	13.50	4	255
14	19.75	6	462
15	24.00	9	448
16	29.00	10	776
17	15.35	6	200
18	19.00	7	132
19	9.50	3	36
20	35.10	17	770
21	17.90	10	140
22	52.32	26	810
23	18.75	9	450
24	19.83	8	635
25	10.75	4	150

R Code to Run

```
› deliverytime = read.csv("deliverytime.csv")
› LRFit=lm(deltime ~ ncases + distance, data= deliverytime)
› summary(LRFit)
```

R Output

```
Call:
lm(formula = deltime ~ ncases + distance, data = deliverytime)

Residuals:
   Min     1Q Median     3Q    Max 
-5.788 -0.663  0.436  1.157  7.420 

Coefficients:
            Estimate Std. Error t value    Pr(>|t|)    
(Intercept)  2.34123    1.09673    2.13     0.04417 *  
ncases       1.61591    0.17073    9.46 0.000000033 ***
distance     0.01438    0.00361    3.98     0.00063 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.3 on 22 degrees of freedom
Multiple R-squared:  0.96,	Adjusted R-squared:  0.956 
F-statistic:  261 on 2 and 22 DF,  p-value: 0.0000000000000469
```

Simple and Multiple Linear Regression

- Delivery Time Data

$$deltime = 2.3412 + 1.61591\, ncases + 0.014\, distance$$

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Model Evaluation

- Testing the Global Significance of Regression
  - To know if the x predictor variables **influences** y we consider the F Statistic from the ANOVA table output from R
  - We usually test for:
    - H_0 : There is no relationship between all x and y.
    - H_a : There is some relationship between some x and y.
  - **p-Value Methodology**
    - If $p < \alpha = 0.05$, Reject H_0

Model Evaluation

```
Call:
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Residuals:
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```

Model Evaluation

- Least-Squares Estimation of the Parameters
  - How well does this equation **fit the data**?
  - Is the model likely to be useful as a **predictor**?

Model Evaluation

- Residuals: $e_i = y_i - \hat{y}_i$

$$e_i = y_i - \hat{y}_i \tag{6.7}$$

- Residuals will be used to determine the **adequacy** of the model

Model Evaluation

- Some issues with R^2

```
Call:
lm(formula = DelTime ~ Ncases + Distance + Gibber, data = DeliveryTime)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.6351 -0.7624  0.5539  1.2116  7.3706 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.579657   1.721687   1.498 0.148930    
Ncases       1.610432   0.177172   9.090    1e-08 ***
Distance     0.014470   0.003725   3.885 0.000855 ***
Gibber      -0.449819   2.464269  -0.183 0.856912    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.334 on 21 degrees of freedom
Multiple R-squared:  0.9597, Adjusted R-squared:  0.9539 
F-statistic: 166.5 on 3 and 21 DF,  p-value: 8.52e-15
```

Model Evaluation

Definition 6.4: Adjusted Coefficient of Determination

- Penalizes for **added terms** to the model that are not significant

$$R_{adj}^2 = 1 - \left(\frac{n-1}{n-p}\right)(1 - R^2) \qquad (6.8)$$

- For the Delivery Time Data

$$R_{adj}^2 = 95.59\%$$

- With Gibberish

$$R_{adj}^2 = 95.39\%$$

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Indicator Variables

Definition 6.5: Indicator Variables

- **Indicator variables** – a variable that assigns levels to the qualitative variable (also known as dummy variables).
- Example Variable:
  - Red
  - Green
  - Blue
- Qualitative variables do not have a **scale of measurement**.
- We **cannot assign** numerical values as follows
  - Red= 1
  - Green=2
  - Blue=3

Indicator Variables

Example 6.3: Tool Life Data

- Relate the effective life of a cutting tool (y) used on a lathe to the lathe speed in revolutions per minute (x_1) and type of cutting tool used.
- Tool type is **qualitative** and can be represented as:

$$x_2 = \begin{cases} 0 & ToolA \\ 1 & ToolB \end{cases}$$

- If a **first-order model** is appropriate:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

Indicator Variables

- If Tool **type A** is used, model becomes:

$$y = \beta_0 + \beta_1 x_1 + \varepsilon$$

- If Tool **type B** is used, model becomes:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 + \varepsilon$$

  - Then:

$$y = (\beta_0 + \beta_2) + \beta_1 x_1 + \varepsilon$$

- Changing from A to B induces a change in the **intercept** (slope is unchanged and identical).
- We assume that the **variance is equal** for all levels of the qualitative variable.

Indicator Variables

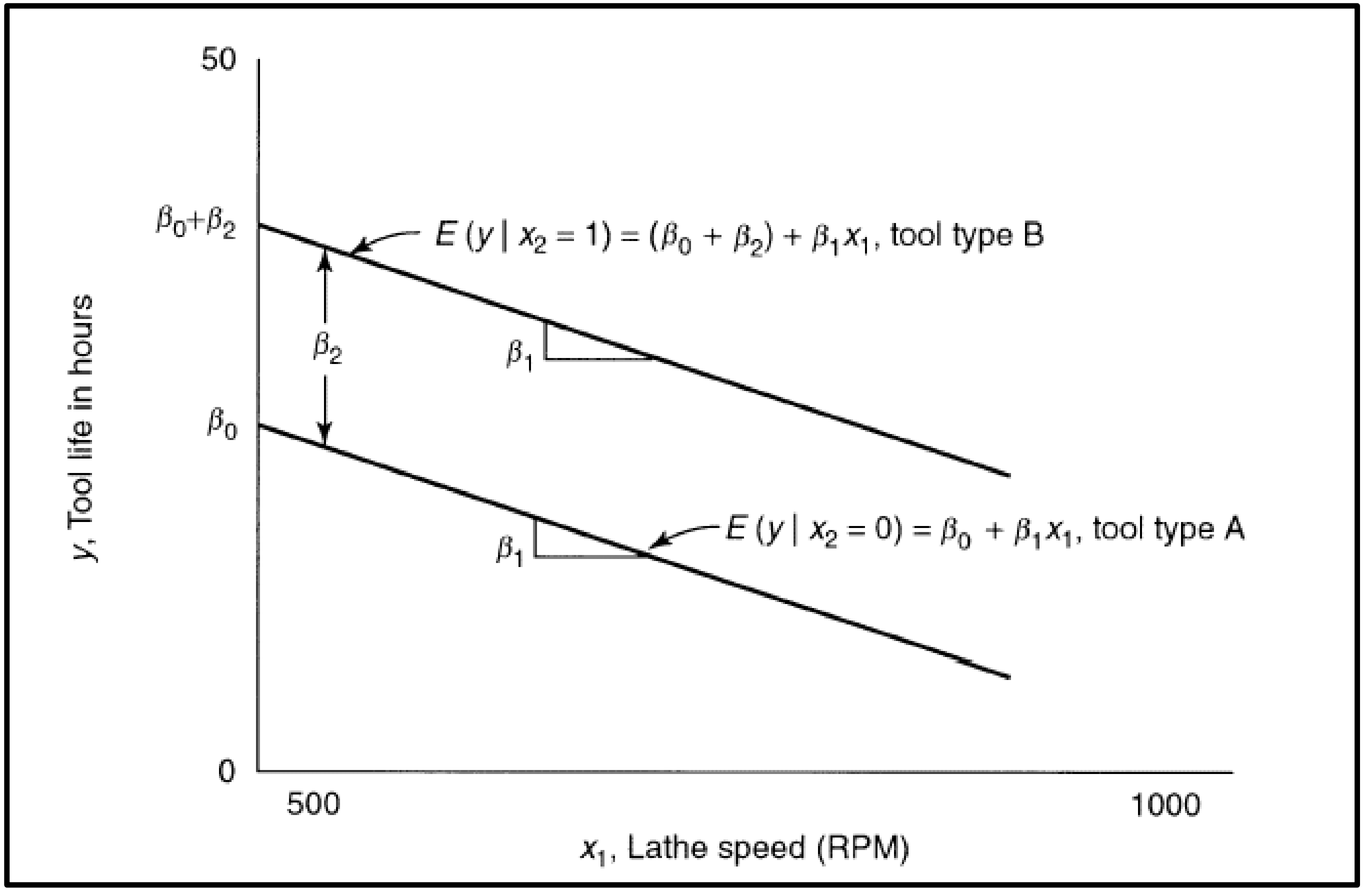

Figure 6.4: Tool Life Data

Indicator Variables

Example 6.3 (Cont.): Tool Life Data

– Twenty observations on tool life and lathe speed are presented and the scatter diagram is shown as follows. Use regression to predict tool life.

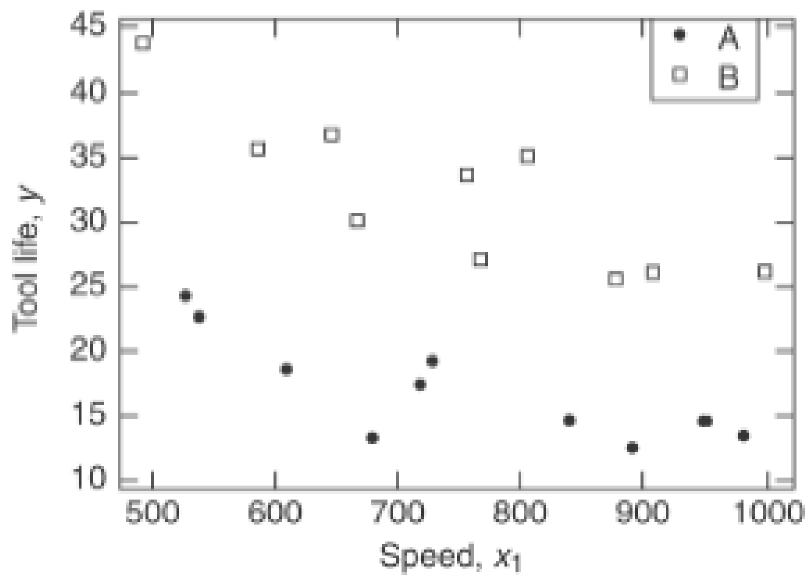

Figure 6.5: Tool Life Data Scatter Plot

Indicator Variables

- toollife = read.csv(“toollife.csv")
- toollifefit=lm(hours~rpm+tooltype,data=toollife)
- summary(toollifefit)

```
Call:
lm(formula = Hours ~ RPM + ToolTypeB, data = ToolLife)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.6255 -1.6308  0.0612  2.2218  5.5044 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 35.208726   3.738882   9.417 3.71e-08 ***
RPM         -0.024557   0.004865  -5.048 9.92e-05 ***
ToolTypeB   15.235474   1.501220  10.149 1.25e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.352 on 17 degrees of freedom
Multiple R-squared:  0.8787, Adjusted R-squared:  0.8645 
F-statistic:  61.6 on 2 and 17 DF,  p-value: 1.627e-08
```

Indicator Variables

- Tool Type Regression Model

$$Hours = 35.208 - 0.024\,RPM + 15.235\,ToolTypeB$$

Indicator Variables

- For qualitative variables with a levels, we would need $\boldsymbol{a-1}$ ***indicator variables*.**
  - For example, say there were three tool types, A, B, and C. Then two indicator variables (called x2 and x3) will be needed:

x_2	x_3	
0	0	if the observation is from tool type A
1	0	if the observation is from tool type B
0	1	if the observation is from tool type C

the regression model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

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Alternative Models of Regression

- Logistic Regression
- Stepwise/Best Subsets Regression

Logistic Regression

Definition 6.6: Logistic Regression

- Logistic regression predicts the **probability** of an outcome that can only have **two values**
- The prediction is based on the use of **one or several predictors** (numerical and categorical).
- Logistic regression produces a **logistic curve**, which is limited to values between 0 and 1.

Logistic Regression

- Logit Function

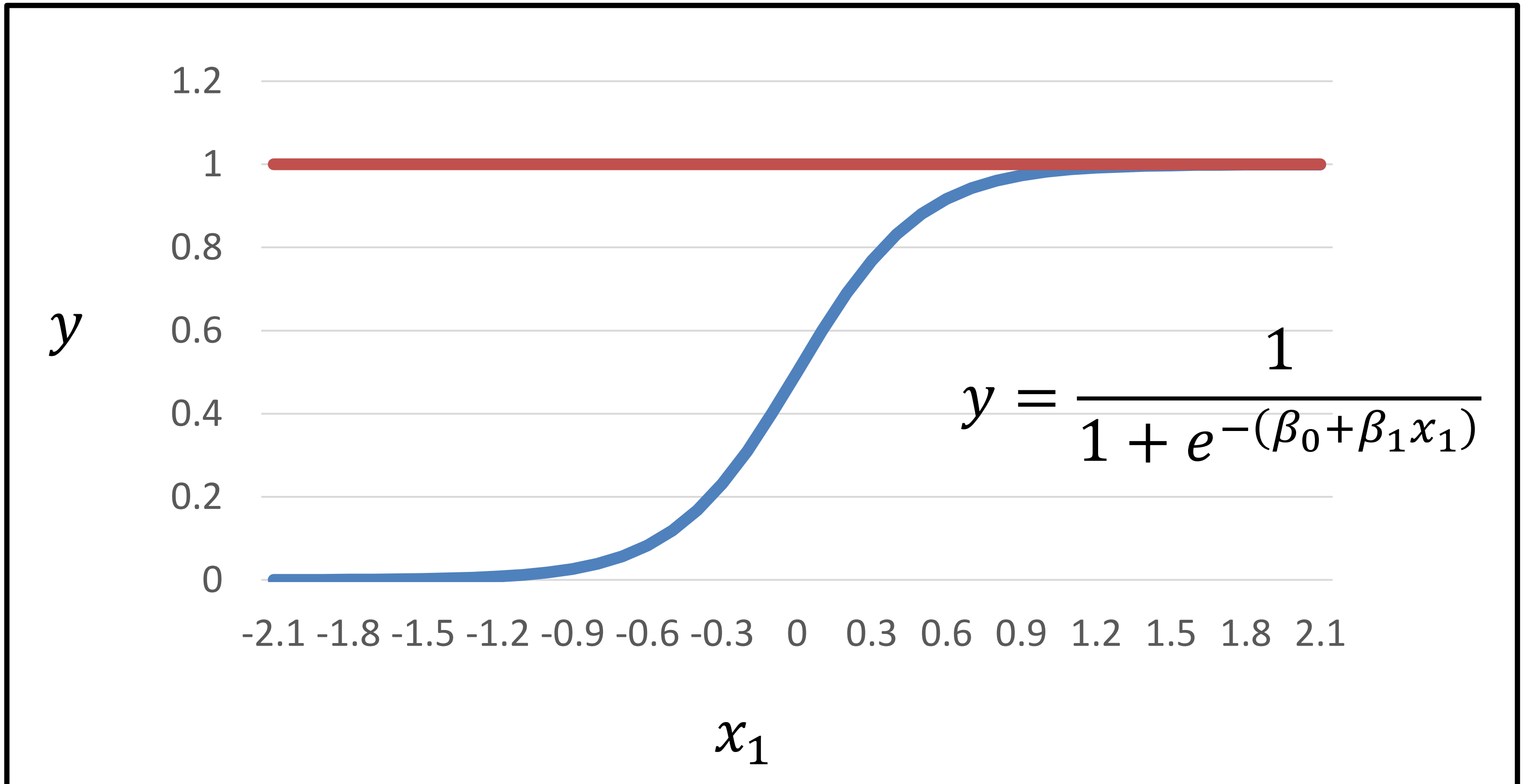

Figure 6.6: Logistic Regression Logit Function

Logistic Regression

Example 6.4 (Cont.): Menarche Data

- Data **contains**:
  - "Age" (average age of age homogeneous groups of girls),
  - "Total" (number of girls in each group),
  - "Menarche" (number of girls in the group who have reached menarche)
- Sources: (Milicer, H. and Szczotka, F., 1966, Age at Menarche in Warsaw girls in 1965, Human Biology, 38, 199-203)

R Code

```
› menarchedata =
  read.csv("menarchedata.csv")
› menarchedata.fit = glm(cbind(menarche,
  total-menarche) ~ age,
  family=binomial(logit), data=menarchedata)
› summary(menarchedata.fit)
› plot(menarche/total ~ age,
  data=menarchedata)
› lines(menarchedata$age,
  menarchedata.fit$fitted, type="l",
  col="red")
```

R Output

```
Call:
glm(formula = cbind(Menarche, Total - Menarche) ~ Age, family = binomial(logit),
    data = menarche)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.0363  -0.9953  -0.4900   0.7780   1.3675

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -21.22639    0.77068  -27.54   <2e-16 ***
Age           1.63197    0.05895   27.68   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3693.884  on 24  degrees of freedom
Residual deviance:   26.703  on 23  degrees of freedom
AIC: 114.76

Number of Fisher Scoring iterations: 4
```

Logistic Regression

$$Probability\ of\ Menarchy = \frac{1}{1 + e^{-(-21+1.61\ Age)}}$$

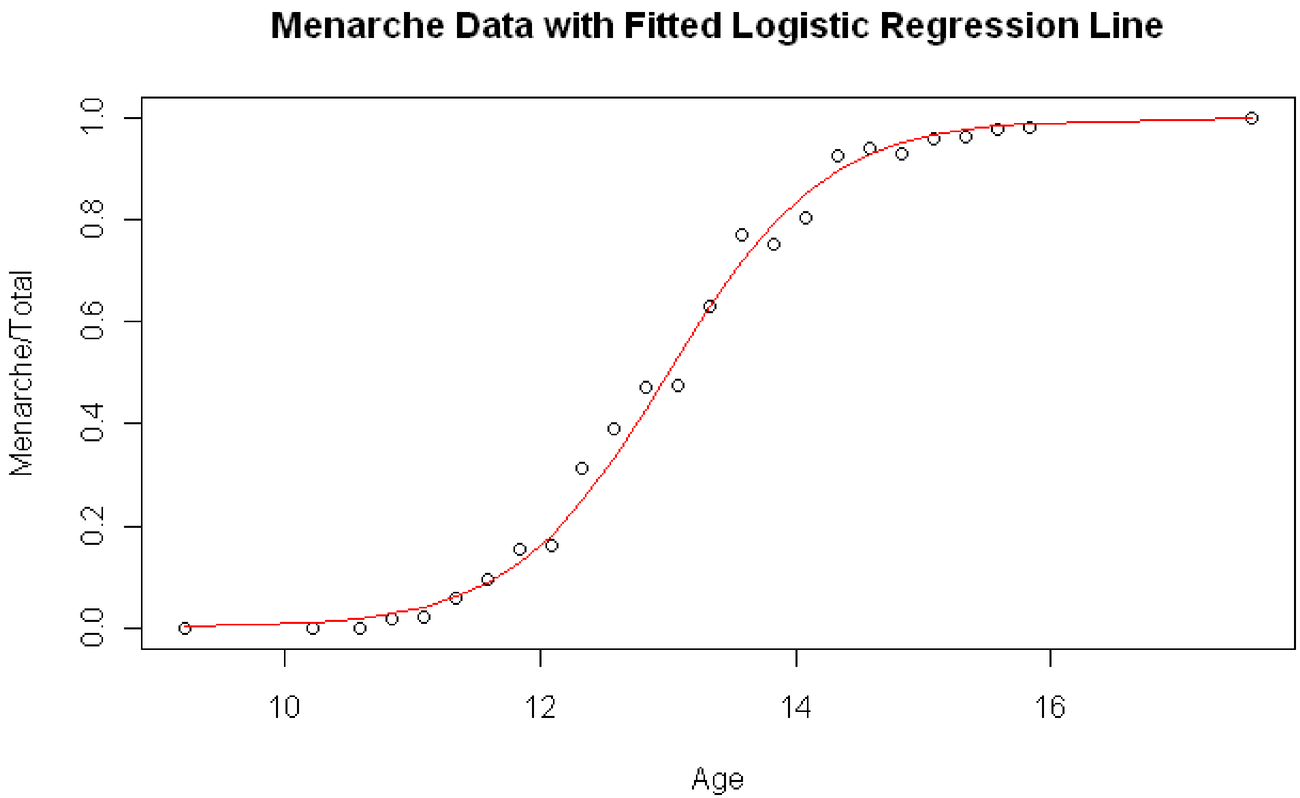

Figure 6.7: Menarche Data

Logistic Regression

- Generated **Model**

$$Probability\ of\ Menarchy = \frac{1}{1 + e^{-(-21+1.61\ Age)}}$$

- The coefficient of "Age" can be **interpreted** as "for every one year increase in age the odds of having reached menarche increase by exp(1.632) = 5.11 times.“
- Prediction for Age = 12

$$Probability\ of\ Menarchy = \frac{1}{1 + e^{-(-21+1.61\ *12)}}$$

$$Probability\ of\ Menarchy = 15.71\%$$

Stepwise Regression

Definition 6.7: Stepwise Regression

- **Stepwise regression**: Enter and remove predictors, in a stepwise manner, until there is no justifiable reason to enter or remove more.
- **Best subsets regression**: Select the subset of predictors that do the best at meeting some well-defined objective criterion.

Stepwise Regression

- Start with no predictors in the "**stepwise model**."
- At each step, enter or remove a predictor based on partial *F*-tests (that is, the *t*-tests).
- Stop when no more predictors can be justifiably entered or removed from the stepwise model.

Stepwise Regression: y versus x1, x2, x3, x4

Alpha-to-Enter: 0.15 **Alpha-to-Remove:** 0.15

Response is **y** on 4 predictors, with N = 13

Step	**1**	**2**	**3**	**4**
Constant	117.57	103.10	71.65	52.58
x4	-0.738	-0.614	-0.237	
T-Value	-4.77	-12.62	-1.37	
P-Value	0.001	0.000	0.205	
x1		1.44	1.45	1.47
T-Value		10.40	12.41	12.10
P-Value		0.000	0.000	0.000
x2			0.416	0.662
T-Value			2.24	14.44
P-Value			0.052	0.000
S	8.96	2.73	2.31	2.41
R-Sq	67.45	97.25	98.23	97.87
R-Sq(adj)	64.50	96.70	97.64	97.44
C-p	138.7	5.5	3.0	2.7

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Time Series Analysis

Definition 6.8: Time Series

- A time series is a collection of observations made **sequentially in time**.

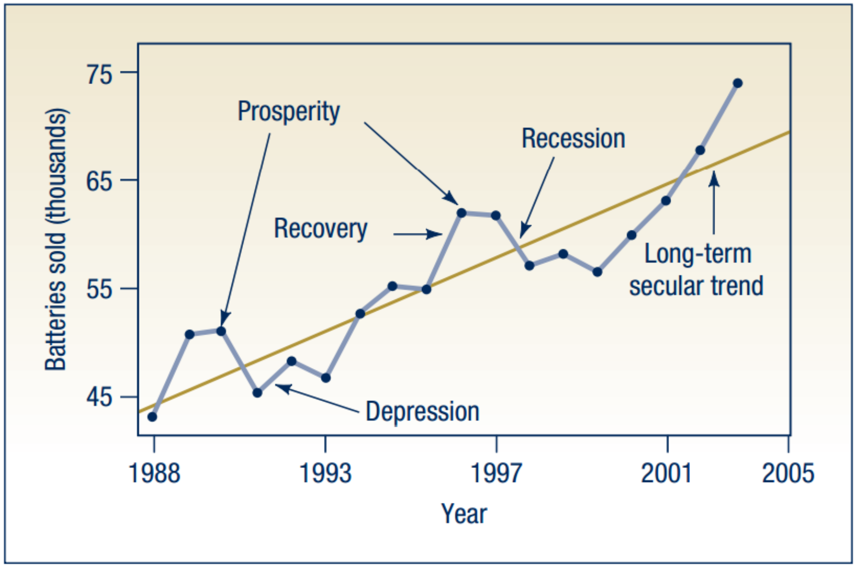

Figure 6.8: Battery Sales by National Battery Sales, Inc., 1988–2005

Time Series Analysis

- A word can be represented by two time series created by moving over and under the word

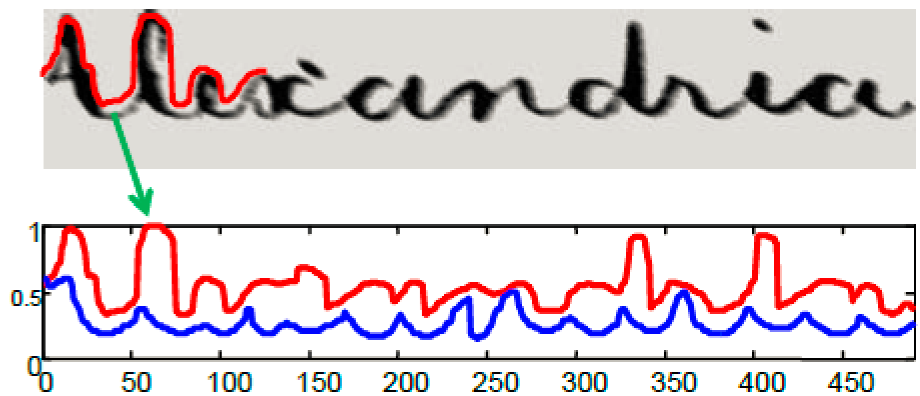

Figure 6.9: Recognizing Words

Time Series Analysis

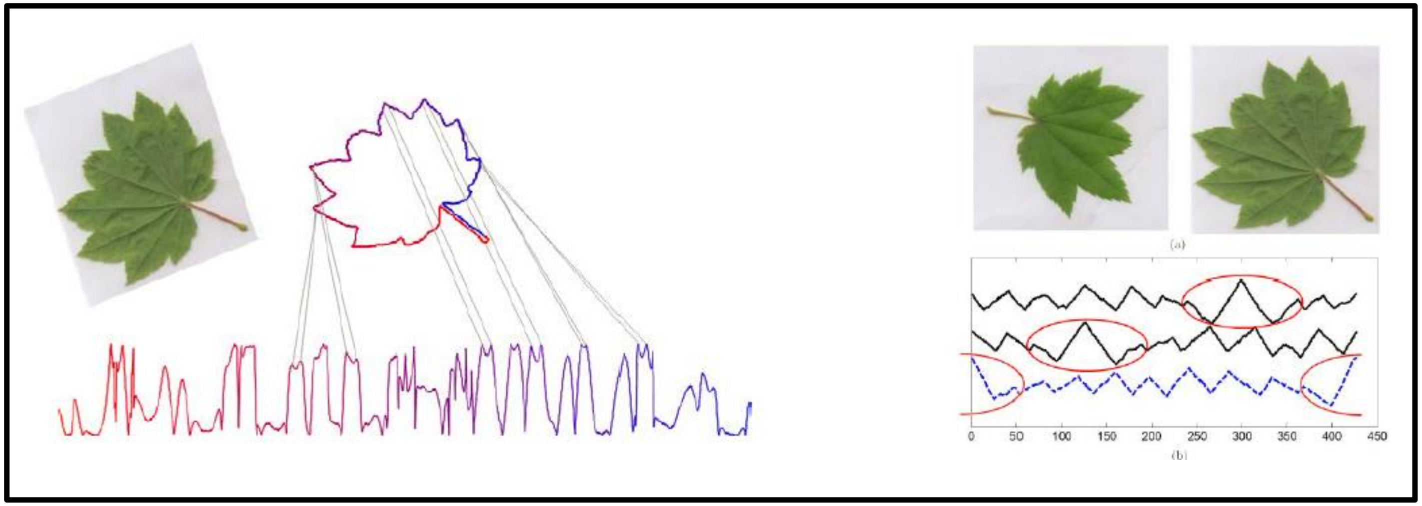

Figure 6.10: Recognizing trees from the leaf images

Time Series Analysis

- Some Time Series Data Mining Tasks
  - Clustering
  - Classification
  - Rule Discovery
  - Anomaly Detection

Time Series Analysis

- Time Series **Clustering**: Identify which time series are similar to each other

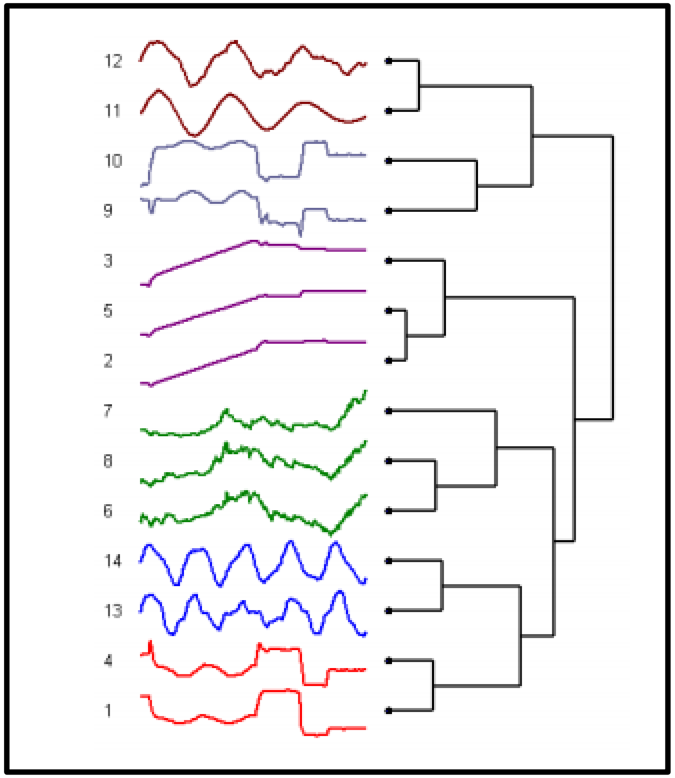

Figure 6.11: Time Series Clustering

Time Series Analysis

- A supervised learning problem aimed at **labeling temporally** structured univariate (or multivariate) sequences of certain (or variable) length.

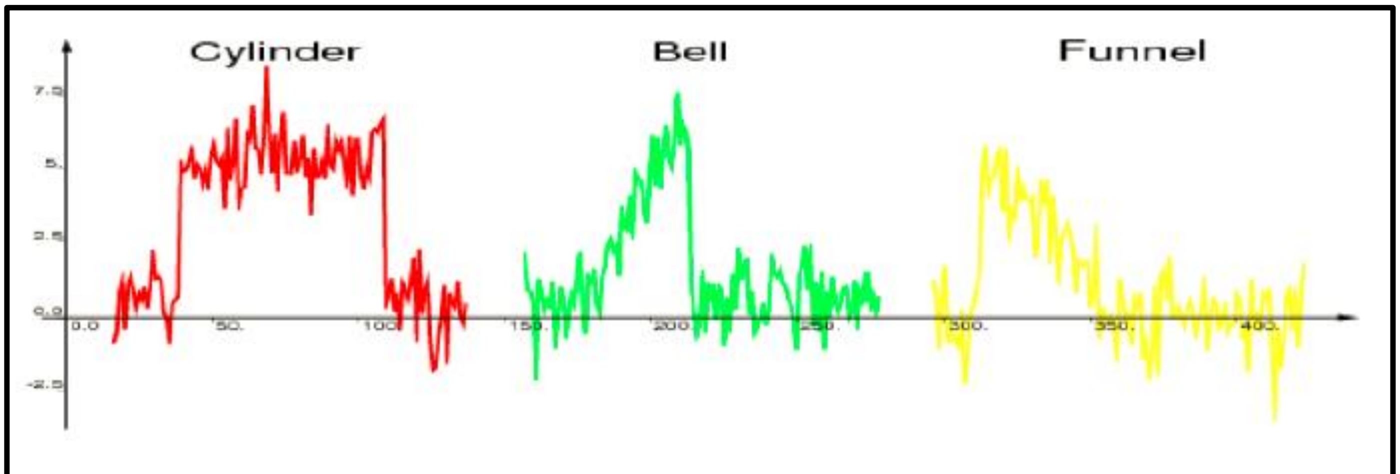

Figure 6.12: Time Series Classification

Time Series Analysis

- Task: Classify grad students based on their faces images transformed into “time series”

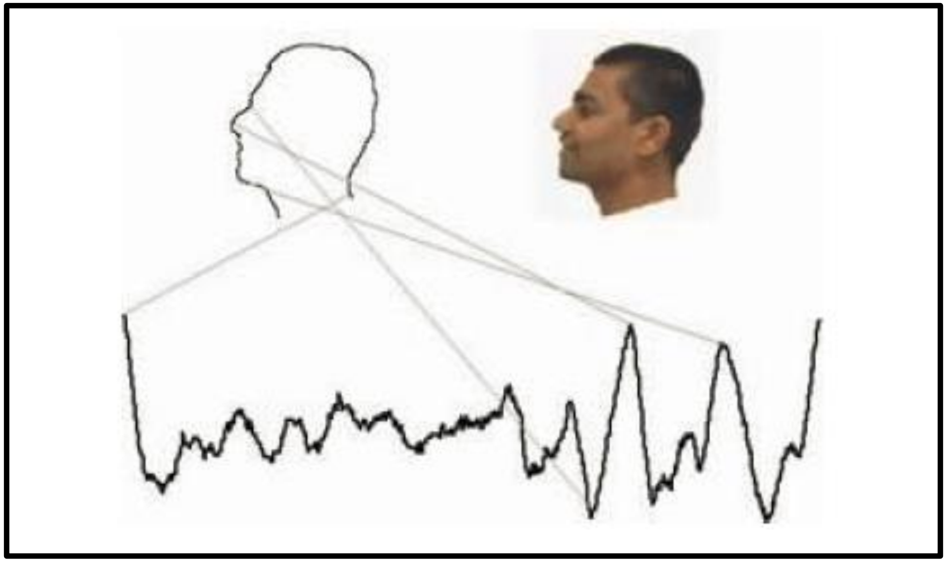

Figure 6.13: Time Series Classification

Time Series Analysis

- Identify sequence of sales:
- - 60% of clients who placed an online order in */company/products/product1.html*, also placed an online order in */company1/products/product4* within 15 days.

Murali K. Kadimi

Time Series Analysis

- Identify anomalous transactions

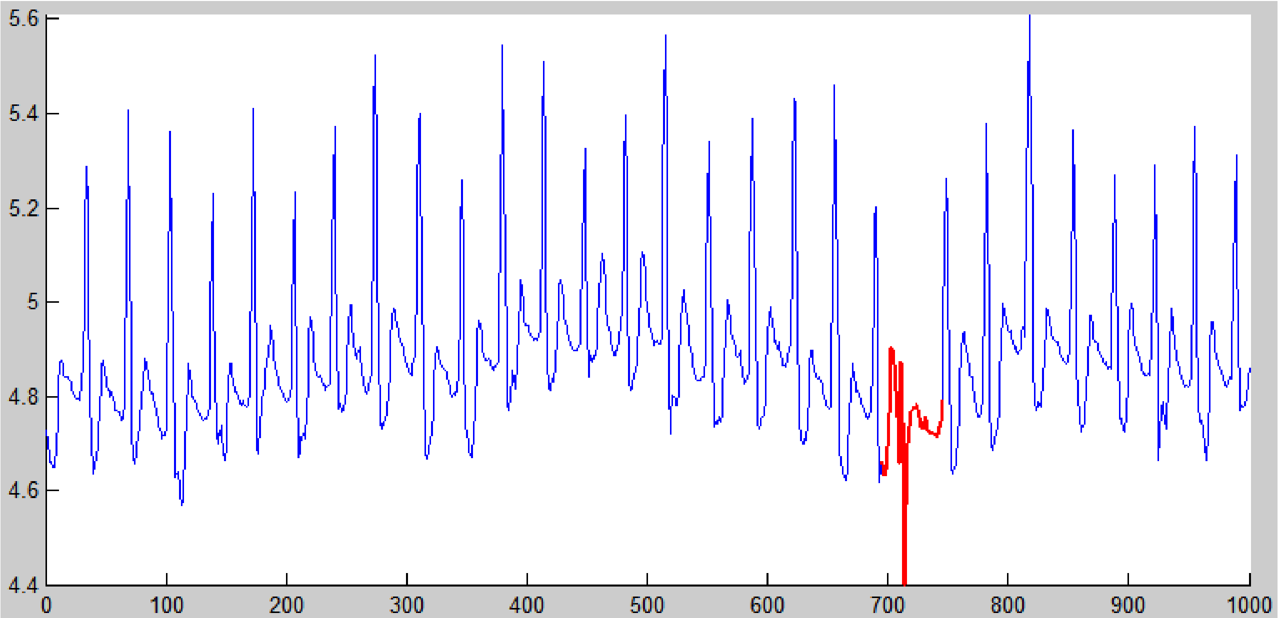

Figure 6.14: Time Series Classification: Anomaly Detection

http://www.anomalydetectionresearch.com/

Time Series Analysis

- Qualitative Modeling
  - Expert Opinion
    - Informed personal insight is always useful.
    - Panel consensus reconciles different views.
    - Delphi method seeks informed consensus.
  - Survey Techniques
    - Random samples give population profile.
    - Stratified samples give detailed profiles of population segments.
- Quantitative Modeling
  - Deterministic modeling
  - Regression modeling
  - Stochastic modeling

Outline for this Session

- Regression and Model Building
- Simple and Multiple Linear Regression
- Model Evaluation
- Indicator Variables
- Alternative Regression Models
- Time Series Analysis
- **Components of a Time Series**
- Evaluation Methods of Forecast
- Smoothing Methods of Time Series
- Case Study

Components of a Time Series

- The pattern or behavior of the data in a time series has **several components.**
- Theoretically, any time series can be decomposed into:
  - Trend
  - Cyclical
  - Seasonal
  - Irregular
- However, this decomposition is often **not straight-forward** because these factors interact.

Components of a Time Series

Definition 6.9: Trend Component

- Accounts for the **gradual shifting** of the time series to relatively higher or lower values over a long period of time.
- Trend is usually the result **of long-term factors** such as changes in the population, demographics, technology, or consumer preferences.

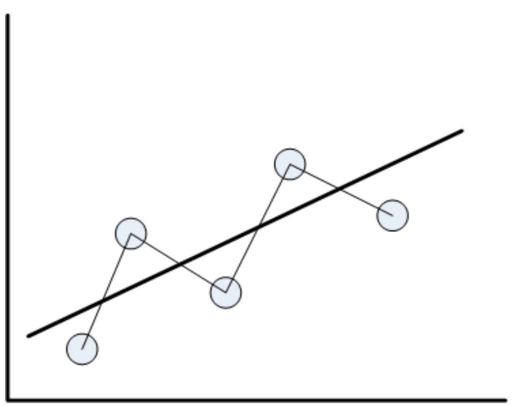

Figure 6.15: Trend Component

Components of a Time Series

Definition 6.10: Seasonal Component

- Accounts for **regular patterns of variability** within certain time periods, such as a year.
- The variability does not always correspond with the seasons of the year (i.e. winter, spring, summer, fall).

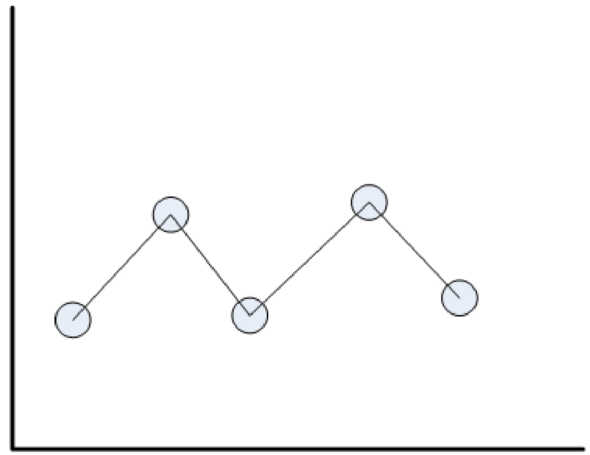

Figure 6.16: Seasonality Component

Components of a Time Series

Definition 6.11: Cyclical Component

- Any regular pattern of sequences of values above and below the trend line lasting more than one year can be attributed to the **cyclical component.**
- Usually, this component is due to multiyear cyclical movements in the economy.

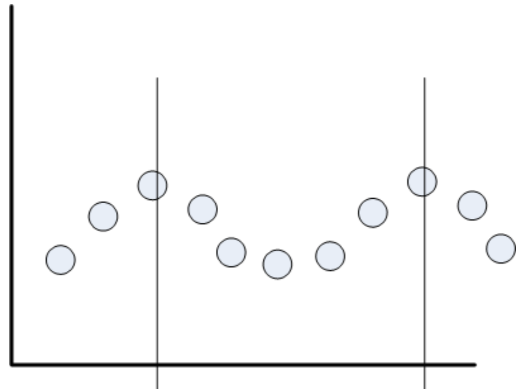

Figure 6.17: Cyclical Component

Components of a Time Series

Definition 6.12: Irregular Component

- Component of a time series **not accounted** for by the other three components
- **Random Error**
- Usually **ignored** in analysis but forms the basis for model evaluation (regression)

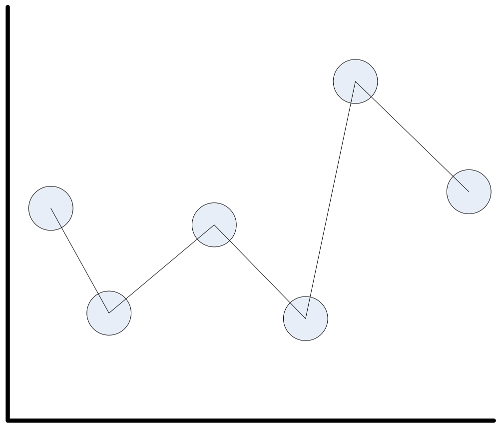

Figure 6.18: Irregular Component

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Evaluation Methods of Forecasts

Definition 6.13: Definition of Errors

- Given the observations y_t of a time series and the corresponding forecasts $\hat{y}_t$ using the k past periods, **the prediction error** at time t

$$e_t = y_t - \hat{y}_t \tag{6.9}$$

- The percentage prediction error at time t is

$$e_t^p = \frac{y_t - \hat{y}_t}{y_t} \times 100\% \tag{6.10}$$

Evaluation Methods of Forecasts

- There are **three measures of accuracy** of the fitted models: MAPE, MAD and MSD for each of the sample forecasting and smoothing methods.
- For all three measures, the smaller the value, the better the fit of the model.
- Use these statistics to compare the **fit of the different methods.**

Evaluation Methods of Forecasts

- Mean Absolute Percentage Error:

$$MAPE = \frac{\sum_{t=1}^{k} \left| e_t^p \right|}{k} \qquad (6.11)$$

- Mean Absolute Deviation:

$$MAD = \frac{\sum_{t=1}^{k} |e_t|}{k} \qquad (6.12)$$

- Mean Squared Deviation:

$$MSD = \frac{\sum_{t=1}^{k} e_t^2}{k} \qquad (6.13)$$

Evaluation Methods of Forecasts

- MAPE
  - Expresses accuracy as a percentage of the error. For example, if the MAPE is 0.05, on average, the forecast is off by 5%.
- MAD
  - Expresses accuracy in the same units as the data, which helps conceptualize the amount of error.
- MSD
  - A commonly-used measure of accuracy of fitted time series values. This is differentiable hence a minimum can be obtained.

Evaluation Methods of Forecasts

Example 6.5 : Forecast Evaluation Example

Actual Sales	Forecasted Sales	
	Model 1	Model 2
56	54	50
43	44	40
22	20	22
24	19	20
55	50	49
MAPE	0.0898	0.0905
MAD	2.6	3.8
MSD	11.8	19.4

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Smoothing Methods for Time Series

- **Smoothing** a time series: to eliminate some of short-term fluctuations.
- Smoothing also can be done to **remove seasonal fluctuations**, i.e., to deseasonalize a time series.
  - Arithmetic Moving Average
  - Exponential Smoothing Methods
  - Holt-Winters method for Exponential Smoothing
- These models are **deterministic**

Smoothing Methods for Time Series

- Simple Averages - **quick, inexpensive**
- Moving Average method
  – Consists of computing an average of the **most recent n data** values for the series and using this average for forecasting the value of the time series for the next period.

$$\hat{y}_{t+1} = \frac{y_t + y_{t-1} + \cdots + y_{t-n}}{n} \qquad (6.14)$$

Smoothing Methods for Time Series

- **Moving averages** are useful if one can assume item to be forecast will stay steady over time.
- **Series of arithmetic means** – used only for smoothing, provides overall impression of data over time
  - The smaller the number, the more weight given to recent periods. This is desirable when there are sudden shifts in the level of the series.
  - The greater the number, less weight is given to more recent periods and the greater the smoothing effect.

Smoothing Methods for Time Series

Example 6.6 : Births Dataset

- An example is a data set of the number of births per month in New York city, from January 1946 to December 1959

R Code

- `library("TTR")`
- `births = read.csv("births.csv")`
- `birthsts = ts(births[,2], frequency=12, start=c(1946,1))`
- `birthstsSMA2 = SMA(birthsts,n=2)`
- `birthstsSMA10 = SMA(birthsts,n=5)`
- `birthstsSMA20 = SMA(birthsts,n=10)`
- `total = cbind(birthsts,birthstsSMA2,birthstsSMA10,birthstsSMA20)`
- `plot(total, plot.type="single", col = 1:ncol(total), lwd = c(2, 2, 2,2))`
- `legend("bottomright", colnames(total), col=1:ncol(total), lty = c(1, 1, 1,1), cex=.5, y.intersp = 1)`

Smoothing Methods for Time Series

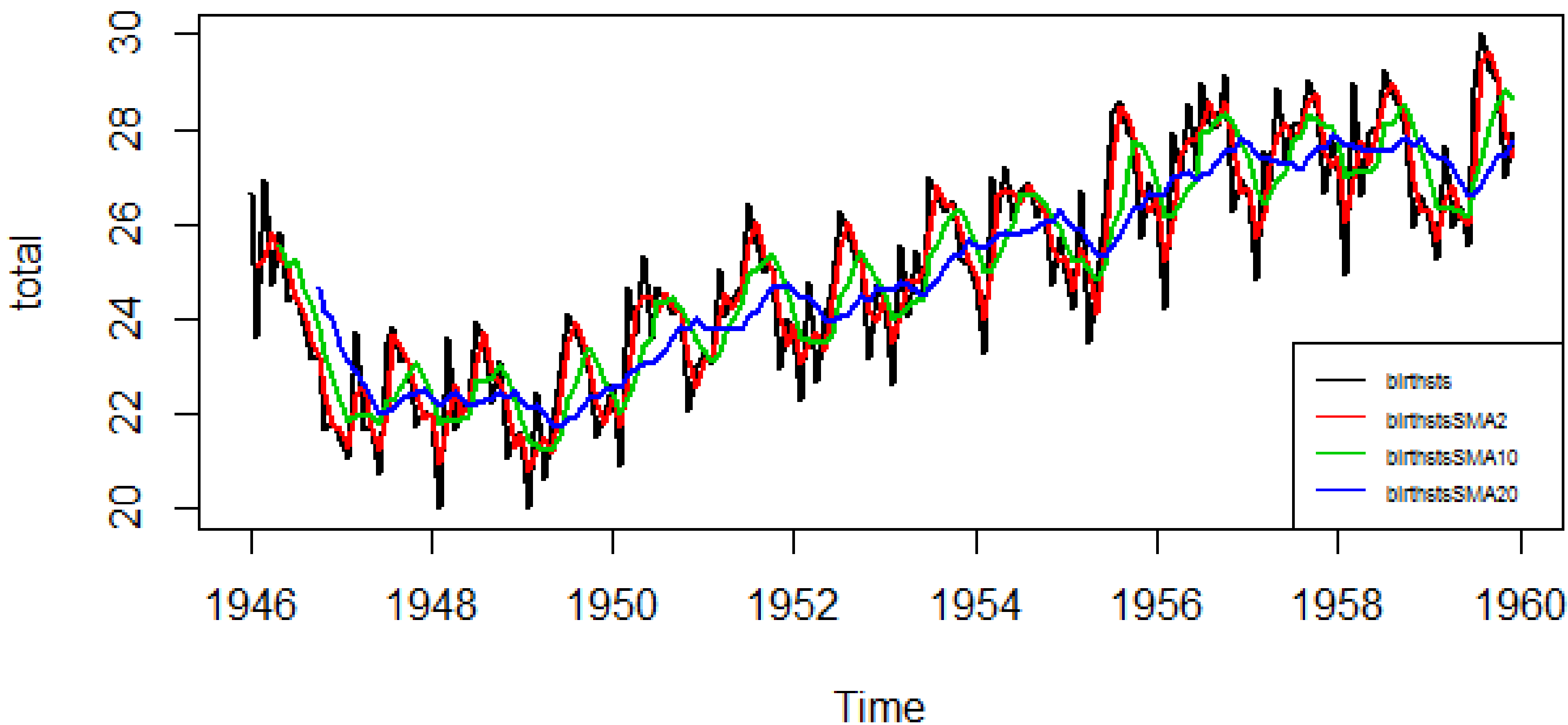

Figure 6.19: Smoothing Methods with Different n

Smoothing Methods for Time Series

- Notes on Moving Averages
  - MA models do not provide information about **forecast confidence**.
  - We can not calculate **standard errors.**
  - We can not explain the stochastic component of the time series. This stochastic component creates the error in our forecast.

Smoothing Methods for Time Series

- Exponential Smoothing Methods
  - **Single Exponential Smoothing (Averaging)**
    - Used for a series without a trend and a seasonal component.
  - **Double Exponential Smoothing**
    - Double Exponential Smoothing is for a series with a trend but without a seasonal component.
  - **Winter's Model.**
    - Winter's model is for a series with a trend and seasonal component.

Smoothing Methods for Time Series

Definition 6.14: Single Exponential Smoothing

- Averaging (smoothing) past values of a **series in a decreasing (exponential) manner.**
- The **observations are weighted** with more weight being given to the more recent observations

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t \qquad (6.16)$$

- New forecast = α × (old observation) + (1- α) × old forecast.
- The equation can be **rewritten as:**

$$\hat{y}_{t+1} = \hat{y}_t + \alpha(y_t - \hat{y}_t) \qquad (6.17)$$

Smoothing Methods for Time Series

- We need a **smoothing constant** α, an initial forecast, and an actual value.
- The smoothing constant serves as the **weighting factor**.
  - When a is close to 1, the new forecast will include a substantial adjustment for any error that occurred in the preceding forecast.
  - When a is close to 0, the new forecast is very similar to the old forecast.
- The smoothing constant α is not an arbitrary choice - but generally falls between 0.1 and 0.4.

R Code

- `birthstssesa01 =  HoltWinters(birthsts,alpha=0.1, beta=FALSE, gamma=FALSE)`
- `birthstssesa02 =  HoltWinters(birthsts,alpha=0.2, beta=FALSE, gamma=FALSE)`
- `birthstssesa09 =  HoltWinters(birthsts,alpha=0.9, beta=FALSE, gamma=FALSE)`
- `total = cbind(birthsts,birthstssesa01$fitted[,1],birthstssesa02$fitted[,1],birthstssesa09$fitted[,1])`
- `plot(total, plot.type="single", col = 1:ncol(total), lwd = c(2, 2,2,2))`
- `legend("bottomright", c("Original","0.1","0.2","0.9"), col=1:ncol(total), lty = c(1, 1), cex=.5, y.intersp = 1)`

Choice of Alpha

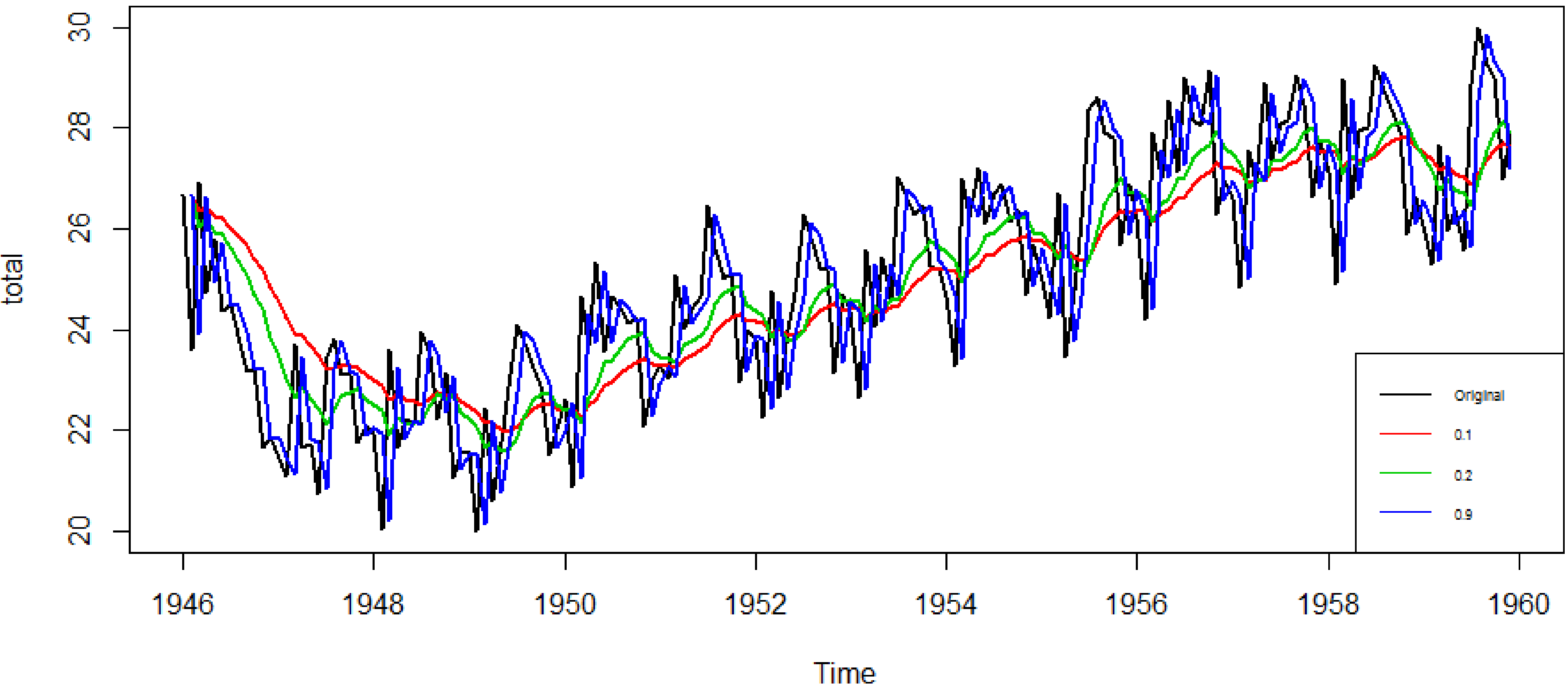

Figure 6.20: Choice of Alpha

Smoothing Methods for Time Series

- Use a **tracking signal** (measure of errors over time) and setting limits.
- For example, if we forecast n periods, count the number of negative and positive errors.
- If the number of positive errors is substantially less or greater than $n/2$, then the process is out of control.

Smoothing Methods for Time Series

Example 6.7 : Use of Tracking Signal

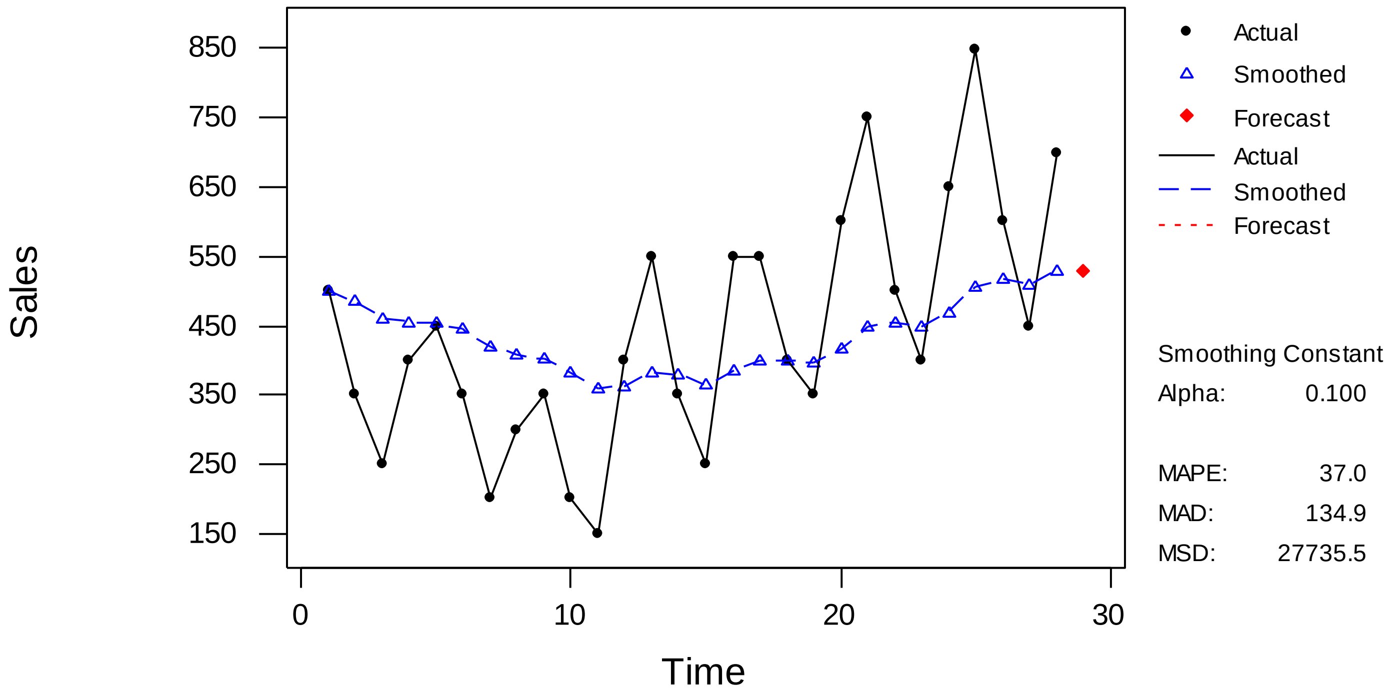

Figure 6.21: Tracking Sales Data

Smoothing Methods for Time Series

- Can also use 95% *prediction interval*

$$\hat{y} \pm z_{\frac{\alpha}{2}} \sqrt{MSD} \tag{6.18}$$

- If the forecast error is **outside of the interval**, use a new optimal α.
- Looking back at the 0.1 single exponential smoothing:

$$\hat{y} \pm 1.96 * \sqrt{27735.5} = \hat{y} \pm 326.4$$

  – Observation #21 is out-of-control. We need to re-evaluate alpha level because this technique is biased.

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Case Study 6

- House Data and Airline Data

References

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