3.0 Sensitivity Analysis

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Module 4 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

Today's Topics:

Sensitivity Analysis

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Operations Research Methodology

According to Taha

1. Define the Problem
2. Construct the Model
3. Solve the Model
4. Validate the Model
5. Implement the Solution

According to Winston

1. Formulate the Problem
2. Observe the System
3. Formulate a Mathematical Model
4. Verify the Model
5. Select a Suitable Alternative
6. Present the Results
7. Implement the Solution

3. Solve the Model

5. Select a Suitable Alternative

6. Present the Results

- Use optimization algorithms
- If the model is too big you may use software
- Sensitivity Analysis is crucial

Sensitivity Analysis

- Change in RHS
- Change in Objective Function Coefficient
- Change in Technological Coefficient Constraint
- Addition of New Variable
- Addition of New Constraint

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Problem

TOYCO assembles three types of toys: trains, trucks and cars using three operations. Available assembly times for the three operations are 430, 460 and 420 minutes per day, respectively , and the revenues per toy train, truck and car are $3, $2, and $5, respectively. The corresponding times per train per truck and per car are (1, 3, 1), (2, 0, 4) and (1, 2, 0) minutes (a zero time indicates that the operation is not used)

Source: Example 4.3-2, Taha 8th Edition

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Problem

Let $\mathbf{x_i}$ be the number of unit **i** assembled
(i: 1 train, 2 truck, 3 car)

$$\text{Maximize } z = 3x_1 + 2x_2 + 5x_3$$

$$x_1 + 2x_2 + x_3 \le 430 \quad \text{(operation 1)}$$
$$3x_1 + 2x_3 \le 460 \quad \text{(operation 2)}$$
$$x_1 + 4x_2 \le 420 \quad \text{(operation 3)}$$

$$x_1,\ x_2,\ x_3 \ge 0$$

It is known that the basic variables at the optimal are x_2, x_3 & s_3
solve the problem using revised simplex method

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Change in RHS

- When the value of your resources has changed
- Affects Feasibility

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Change in RHS Value

Shadow or Dual Price

The shadow or dual price of the i^{th} constraint in an LP is the amount by which the optimal value improves (increases for max problem or decrease for min problem) when the Right Hand Side of the i^{th} constraint is increased by 1

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Change in RHS Value

Where can you see the shadow or dual price?

$$\mathbf{C_b B^{-1} b_{new}} = \mathbf{C_b B^{-1} b_{old}} + \mathbf{C_b B^{-1} \Delta b}$$

$C_b B^{-1} A - C$	$C_b B^{-1}$	$C_b B^{-1} b$
$B^{-1} A$	B^{-1}	$B^{-1} b$

	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	1	4	0	0	1	2	0	1350
x_2	0	-1/4	1	0	1/2	-1/4	0	100
x_3	0	3/2	0	1	0	1/2	0	230
s_3	0	2	0	0	-2	1	1	20

Change in RHS Value

	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	1	4	0	0	$\boxed{1}$	$\boxed{2}$	$\boxed{0}$	1350
x_2	0	-1/4	1	0	1/2	-1/4	0	100
x_3	0	3/2	0	1	0	1/2	0	230
s_3	0	2	0	0	-2	1	1	20

Interpretation:

An increase of 1 minute in operation 1 will lead to an increase of \$1 profit

An increase of 1 minute in operation 2 will lead to an increase of \$2 profit

An increase of 1 minute in operation 3 will not lead to an increase in profit

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Change in RHS Value

	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	1	4	0	0	1	2	0	1350
x_2	0	-1/4	1	0	1/2	-1/4	0	100
x_3	0	3/2	0	1	0	1/2	0	230
s_3	0	2	0	0	-2	1	1	20

An extra 60 minutes is available for operation 1 will you be willing to pay $55 for it? $65?

An extra 40 minutes is available for operation 2 will you be willing to pay $40 for it?

An extra 100 minutes is available for operation 3 will you be willing to pay $1 for it?

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Change in Objective Function Coefficient

Occurs when the contribution to the goal is altered

Affects Optimality

Reduced Cost

The amount by which the objective function coefficient of a non-basic variable must be improved to enter the solution

If a non-basic variable is forced to take up some value, the z value worsens by the reduced cost per unit of the non-basic variable

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Change in Technological Coefficients

- Happens When the rate of consumption changes
- Will affect both feasibility and optimality

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Addition of a new variable

Happens when a new product or decision has to be made

Will affect both feasibility and optimality

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Addition of a New constraint

- When a new limiting factor arises
- Will affect both feasibility and optimality

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Suppose that a fourth operation is made available such that the operation times for items are 1 minute each and has a maximum of 400 minutes available. What will happen to the optimal solution?

$$x_1 + x_2 + x_3 \leq 400$$

Check first if the current solution satisfies the new constraint

$$x_1 + x_2 + x_3 \leq 400$$

$$0 + 100 + 230 \leq 400$$

$$330 \leq 400$$

Since the new constraint is satisfied by the current basic solution, the additional constraint is said to be **REDUNDANT** and thus will not enter the solution

Addition of New constraint

Suppose that a fourth operation is made available such that the operation times for items are 2 minutes each and has a maximum of 590 minutes available. What will happen to the optimal solution?

$$2x_1 + 2x_2 + 2x_3 \leq 590$$

Check first if the current solution satisfies the new constraint

$$2x_1 + 2x_2 + 2x_3 \leq 590$$

$$0 + 2(100) + 2(230) \leq 590$$

$$660 \leq 590$$

Since the new constraint is not satisfied by the current basic solution, the constraint will enter the solution

Next Session

Solutions Using a Software

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Source: Taha

Fin.

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