4.0 Integer Programming

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Module 4 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

Today's Topics:

- Integer Programming
- Binary Variables

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Integer Programming

A mathematical program with linear objective function and linear constraints.

Some decision variables (or at least some dummy variables) cannot assume fractional values

Includes Binary Integer Programming (BIP) that uses the very useful binary variable (0-1) to model yes-no decisions.

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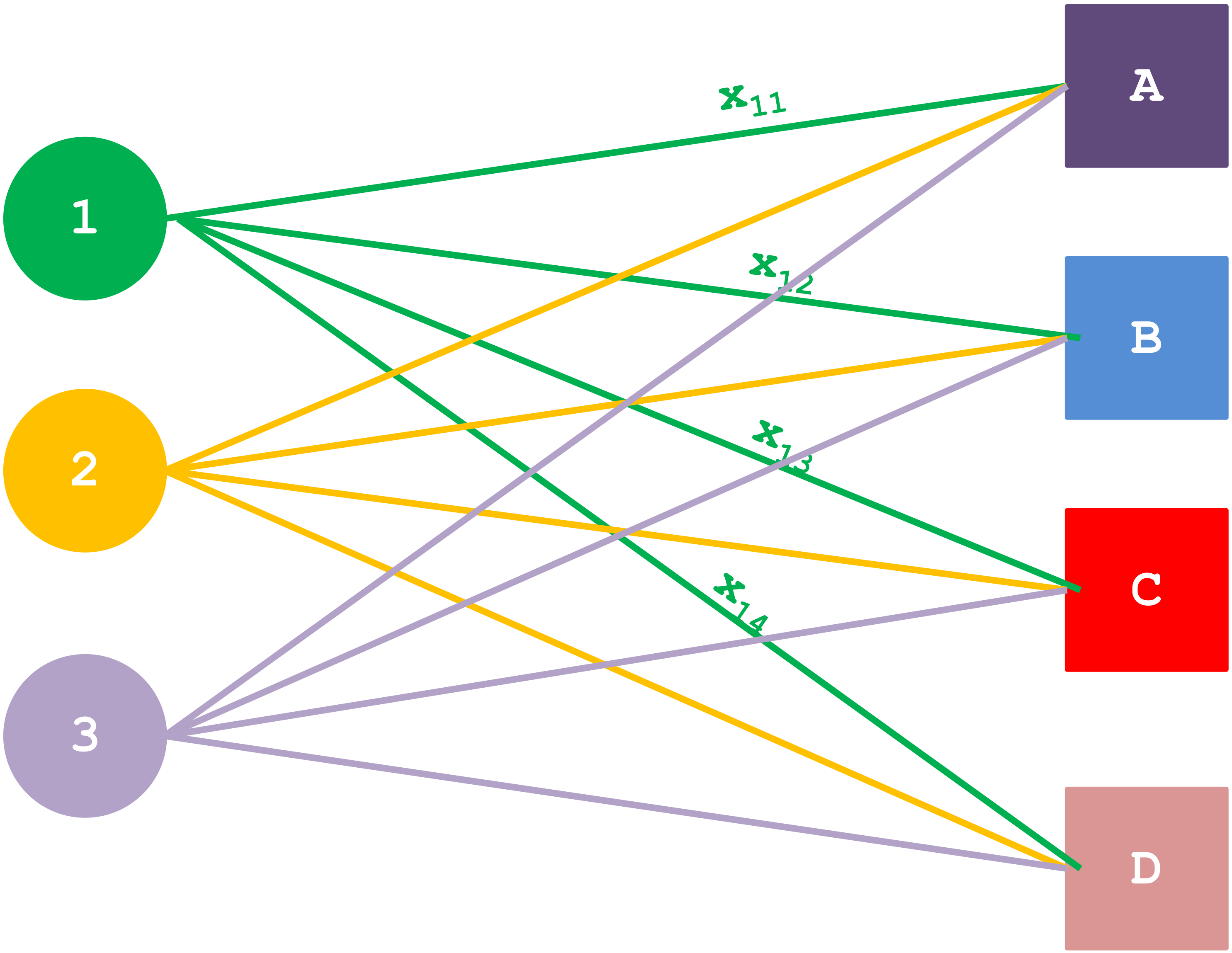
[1] wikipedia.com [2] Winston [3] Hillier and Lieberman

Transportation Model

Concerned with transferring of goods from a source to a destination

Transportation Model



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Transportation Model

A firm producing a single product has three plants and four customers. The three plants will produce 3,000, 5,000 and 5,000 units, respectively, during the next time period. The first has made the commitment to sell at least 4,000, 3,000, 3,000 and 2,000 to Clients 1, 2, 3 and 4, respectively. The cost (in $) associated with shipping a unit from supplier **i** to customer **j** is given below.

		Customer			
		Client 1	Client 2	Client 3	Client 4
Supplier	Plant 1	65	63	62	64
	Plant 2	68	67	65	62
	Plant 3	63	60	59	60

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Source: Example 4.1-2, Taha 8th ed.

Transportation Model

X_{ij} = Number of units from plant i to client j

$$\text{Min } z = 65X_{11} + 63X_{12} + 62X_{13} + 64X_{14} + 68X_{21} + 67X_{22} + 65X_{23} + 62X_{24} + 63X_{31} + 60X_{32} + 59X_{33} + 60X_{34}$$

Subject to

$$\left.\begin{aligned} X_{11} + X_{12} + X_{13} + X_{14} &\le 3000 \\ X_{21} + X_{22} + X_{23} + X_{24} &\le 5000 \\ X_{31} + X_{32} + X_{33} + X_{34} &\le 5000 \end{aligned}\right\} \text{Capacity Constraints}$$

$$\left.\begin{aligned} X_{11} + X_{21} + X_{31} &\ge 4000 \\ X_{12} + X_{22} + X_{32} &\ge 3000 \\ X_{13} + X_{23} + X_{33} &\ge 3000 \\ X_{14} + X_{24} + X_{34} &\ge 2000 \end{aligned}\right\} \text{Demand Constraints}$$

$X_{ij} \ge 0$, for all ij

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Nowjuice, Inc. produces bottled juice. A planner has developed an aggregate forecast for demand (in cases) for the next six months. Develop a least-cost aggregate plan using the following information. Assume zero (0) beginning inventory.

Setup the relevant transportation tableau for this problem.

Mode of Production	Mo Capacity (in cases)	Production Cost per Case ($)
Regular	5,000	10
Overtime	500	16
Subcontracting	n/a	20

Holding Cost Per Month	$1

Month	May	June	July	Aug	Sep	Oct
Forecast	4,000	4,800	5,600	7,200	6,400	5,000

BINARY VARIABLES

$$x_i = 0 \text{ or } 1$$

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Assignment Problem

Concerned with assignment of a certain set A to another set B.

Assignment Problem

The manager of a department has three tasks to assign to three workers. Each worker must only perform one task, and a task must only be done by one worker. The cost of assigning a task to a worker is shown in the table below. Determine the optimal assignment of task to the workers

	Task 1	**Task 2**	**Task 3**
Worker A	5	7	9
Worker B	14	10	12
Worker C	15	13	16

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Constructing the LP

Let $\mathbf{x_{ij}}$ = 1 if worker i is assigned to task j, 0 otherwise

$$\text{Min } z = 5x_{A1} + 7x_{A2} + 9x_{A3} + 14x_{B1} + 10x_{B2} + 12x_{B3} + 15x_{C1} + 13x_{C2} + 16x_{C3}$$

$$x_{A1} + x_{A2} + x_{A3} = 1$$
$$x_{B1} + x_{B2} + x_{B3} = 1$$
$$x_{C1} + x_{C2} + x_{C3} = 1$$

$$x_{A1} + x_{B1} + x_{C1} = 1$$
$$x_{A2} + x_{B2} + x_{C2} = 1$$
$$x_{A3} + x_{B3} + x_{C3} = 1$$

$$x_{ij} \geq 0$$

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The Gotham City police have just received three calls for police. Five cars are available. The distance (in city blocks) of each car from each call is given below. Gotham City wants to minimize the total distance cars must travel to respond to the three police calls.

	Distance (in city blocks)		
Car	Call 1	Call 2	Call 3
1	10	11	18
2	6	7	7
3	7	8	5
4	5	6	4
5	9	4	7

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Fixed Cost Problems

Incurs cost or reward if activity A is undertaken

A company is considering to opening warehouses in four cities: NY, LA, Chicago, and Atlanta. The company wants to minimize total cost of meeting the weekly demand.

		Cost of Transpo/unit to Demand Centers (Regions)			
Possible Location	FC/week (if open)	Region 1	Region 2	Region 3	Weekly Capacity
NY	$400	$ 10	$ 11	$ 18	100
LA	$500	$ 6	$ 7	$ 7	100
Chicago	$300	$ 7	$ 8	$ 5	100
Atlanta	$150	$ 5	$ 6	$ 4	100
Weekly Demand		80	70	40	

EITHER-OR CONSTRAINTS

Either Plant A or Plant B must be opened

X_i = 1 if Plant i is opened

$$X_A + X_B = 1$$

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AT MOST CONSTRAINTS

At most two plants (of A, B, C, D) must be opened

$$X_i = 1 \text{ if Plant } i \text{ is opened}$$

$$X_A + X_B + X_C + X_D \leq 2$$

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IF-THEN CONSTRAINTS

If A is opened B must be opened

X_i = 1 if Plant i is opened

$$X_A \leq X_B$$

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A company is considering to opening warehouses in four cities: NY, LA, Chicago, and Atlanta. The company wants to minimize total cost of meeting the weekly demand.

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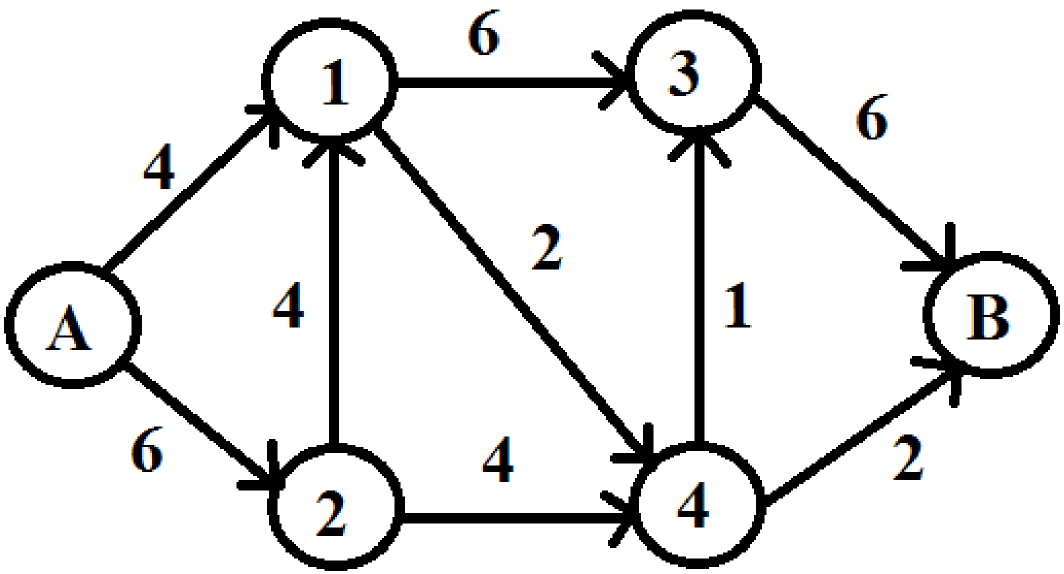
Revise the original ILP to accommodate the following (independent) additional restrictions.

- If the NY warehouse is opened, the LA warehouse must be opened.
- At most two warehouses can be opened.
- Either the Atlanta or the LA warehouse must be opened.

Network Models

Networks are collections of <u>nodes</u> and <u>arcs</u>.
Concerned with the shortest or longest routes

Determine the shortest route from A to B.



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