Forecasting

Day 2

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Module 5 of the Business Intelligence and Analytics Track of UP NEC and the UP Center of Business Intelligence

Our Learning Methodology

1. Understanding of Concepts

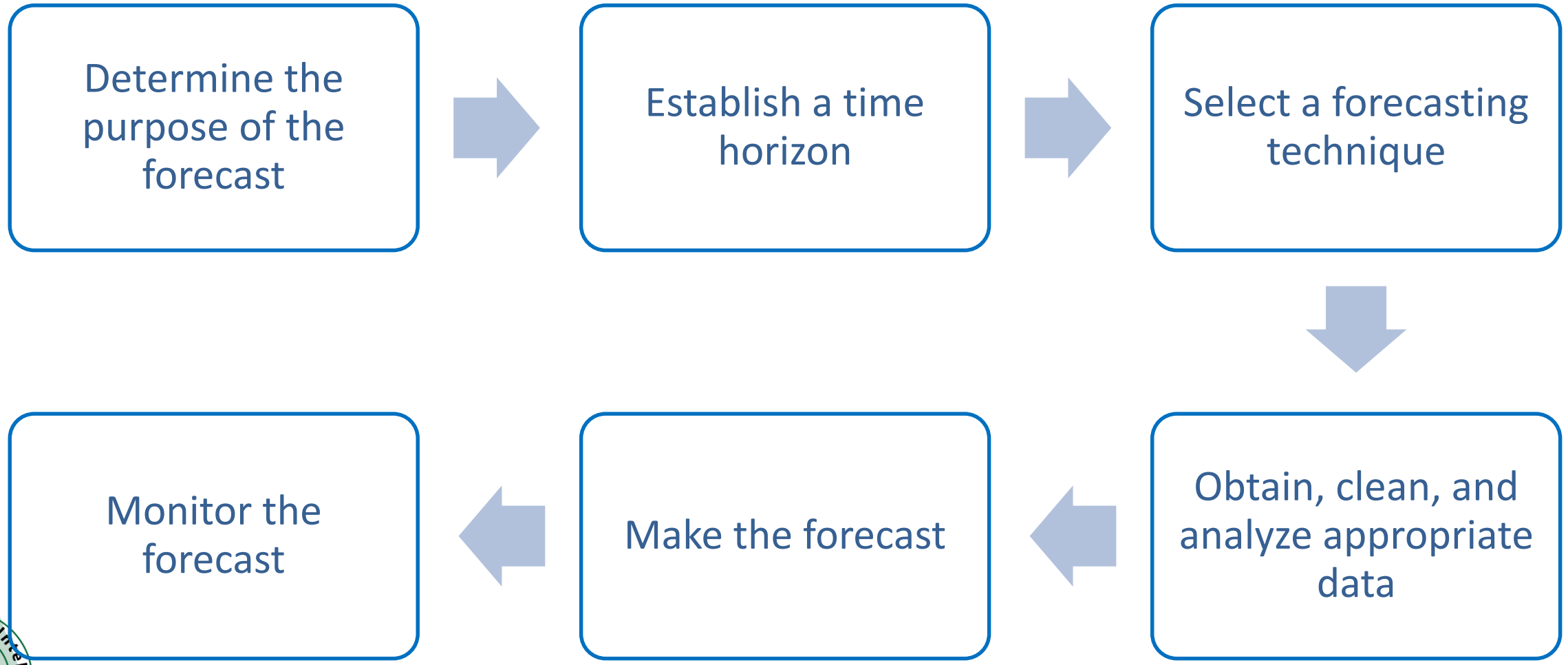
   Discussions with Examples
2. Solving Practice Problem and Exercises

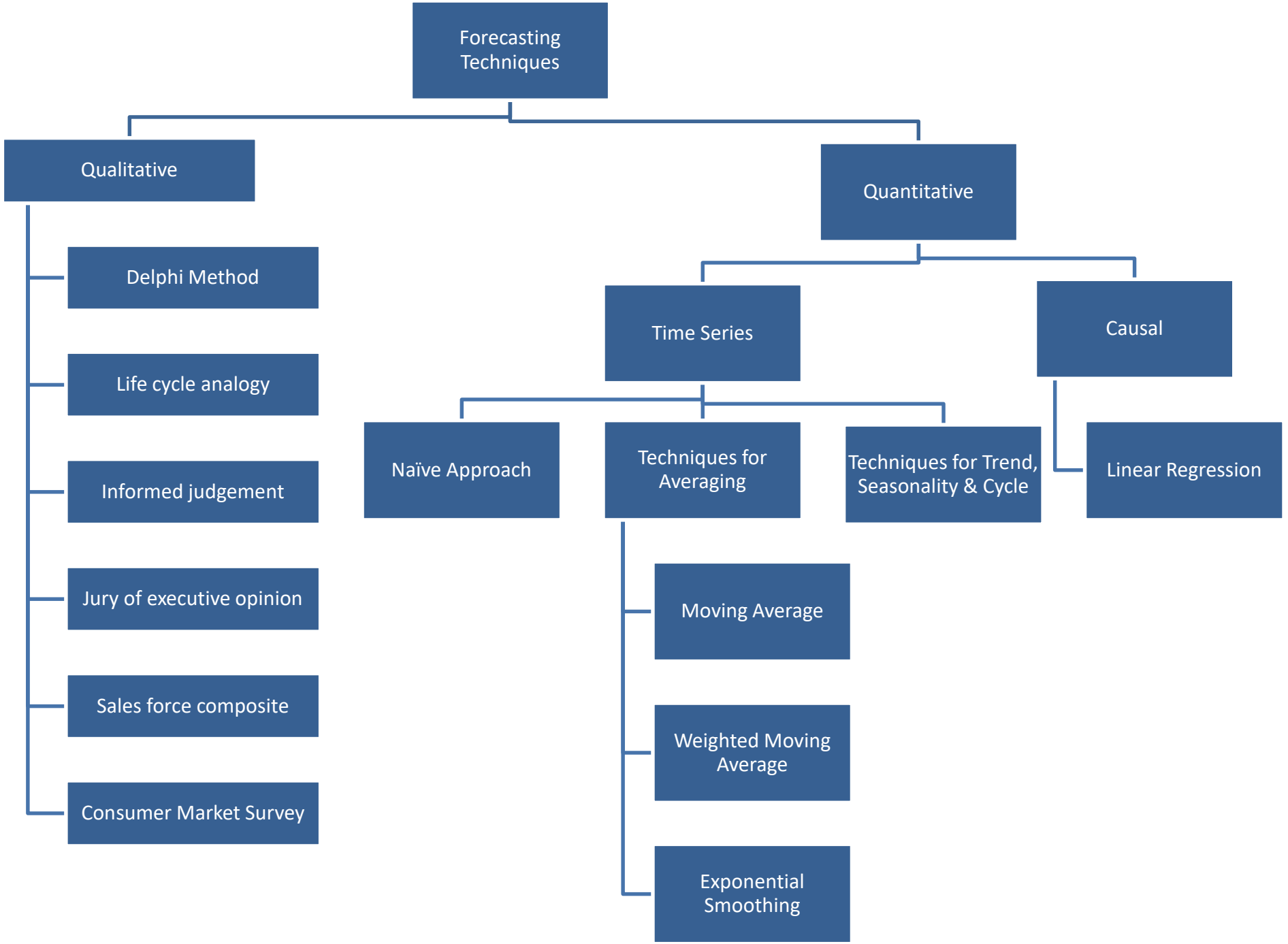
   (Open notes and may discuss with co-participants)
3. Answering Random Concept Tests

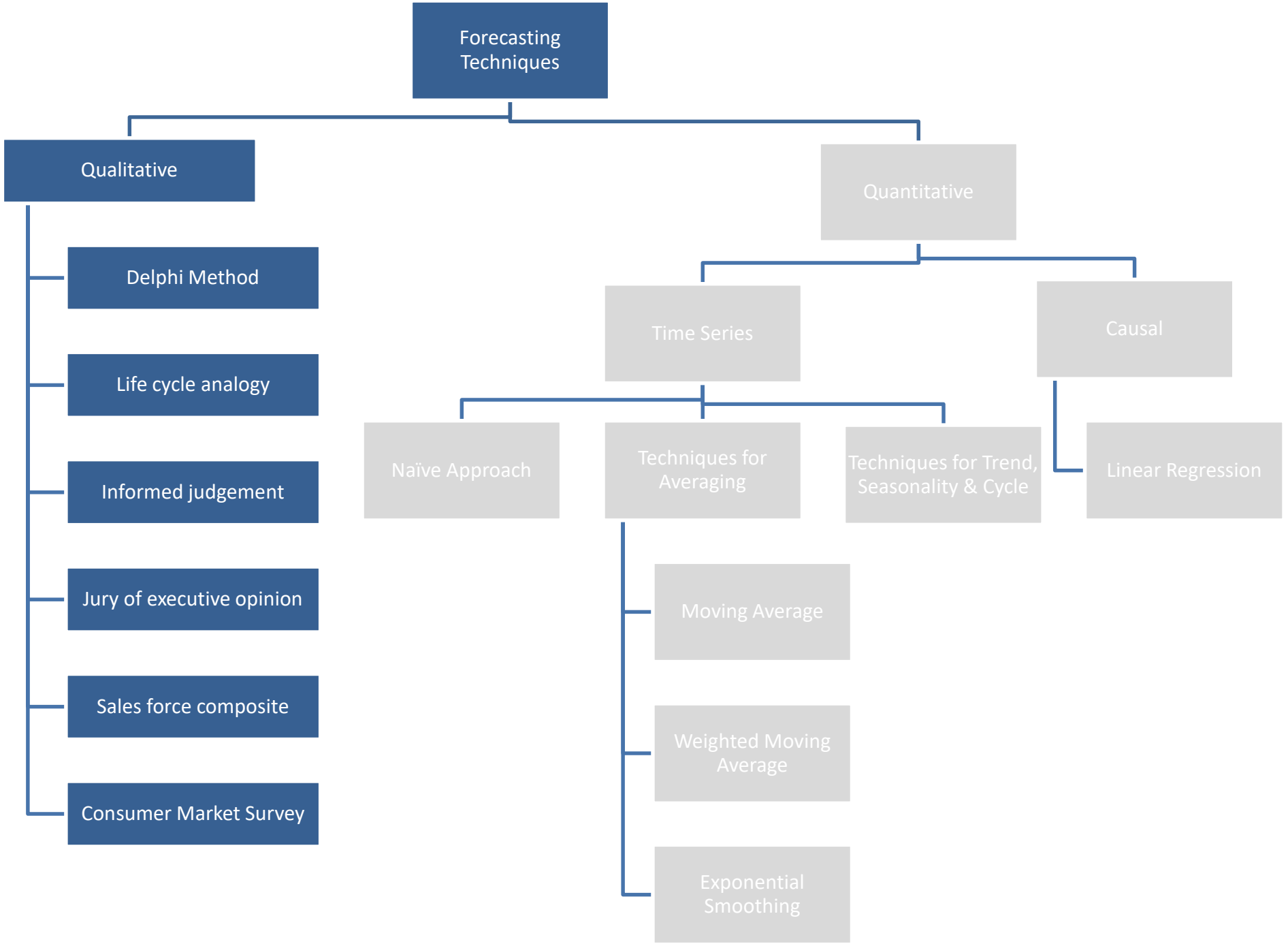
   (to ensure retention)
3. Evaluation/Tests (Closed notes)

(Questions may be asked anytime!)

Overview of the Forecasting Process




Forecasting Techniques
Qualitative
Quantitative
Delphi Method
Life cycle analogy
Informed judgement
Jury of executive opinion
Sales force composite
Consumer Market Survey
Time Series
Causal
Naïve Approach
Techniques for Averaging
Techniques for Trend, Seasonality & Cycle
Linear Regression
Moving Average
Weighted Moving Average
Exponential Smoothing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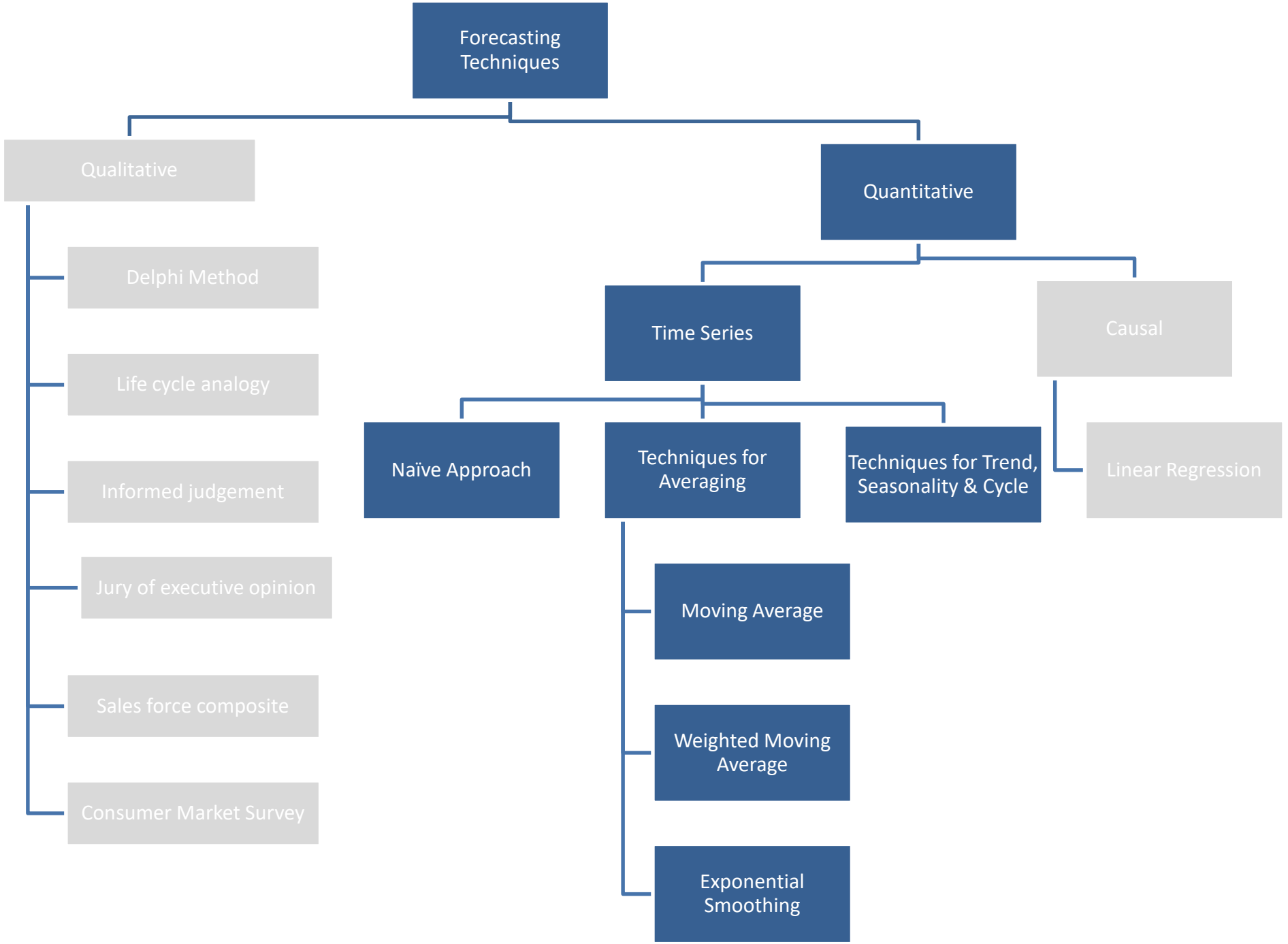
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Review Exercise 1 (Polis 1)

The Polish General's Pizza Parlor is a small restaurant catering to patrons with a taste for European Pizza. One of its specialties is Polis Prize pizza. The manager must forecast weekly demand for these special pizzas so that he can order pizza crusts weekly. Recently , demand has been as follows:

Week of	Pizzas	Week of	Pizzas
June 2	50	June 23	56
June 9	65	June 30	55
June 16	52	July 7	60

Week of	Forecasted	Actual	Error
June 2		50	
June 9		65	
June 16		52	
June 23		56	0.33
June 30		55	- 2.67
July 7		60	5.67
July 14			

Forecast the demand for pizza for June 23 to July 14 by using the simple moving average method with n=3.

Review Exercise 2 (Polis 2)

The Polish General's Pizza Parlor is a small restaurant catering to patrons with a taste for European Pizza. One of its specialties is Polis Prize pizza. The manager must forecast weekly demand for these special pizzas so that he can order pizza crusts weekly. Recently , demand has been as follows:

Week of	Pizzas	Week of	Pizzas
June 2	50	June 23	56
June 9	65	June 30	55
June 16	52	July 7	60

Forecast the demand for pizza for June 23 to July 14 by using weighted moving average method with weights 0.50, 0.30, and 0.20.

Week of	Forecasted	Actual	Error
June 2		50	
June 9		65	
June 16		52	
June 23		56	0.50
June 30		55	- 1.60
July 7		60	5.30
July 14			

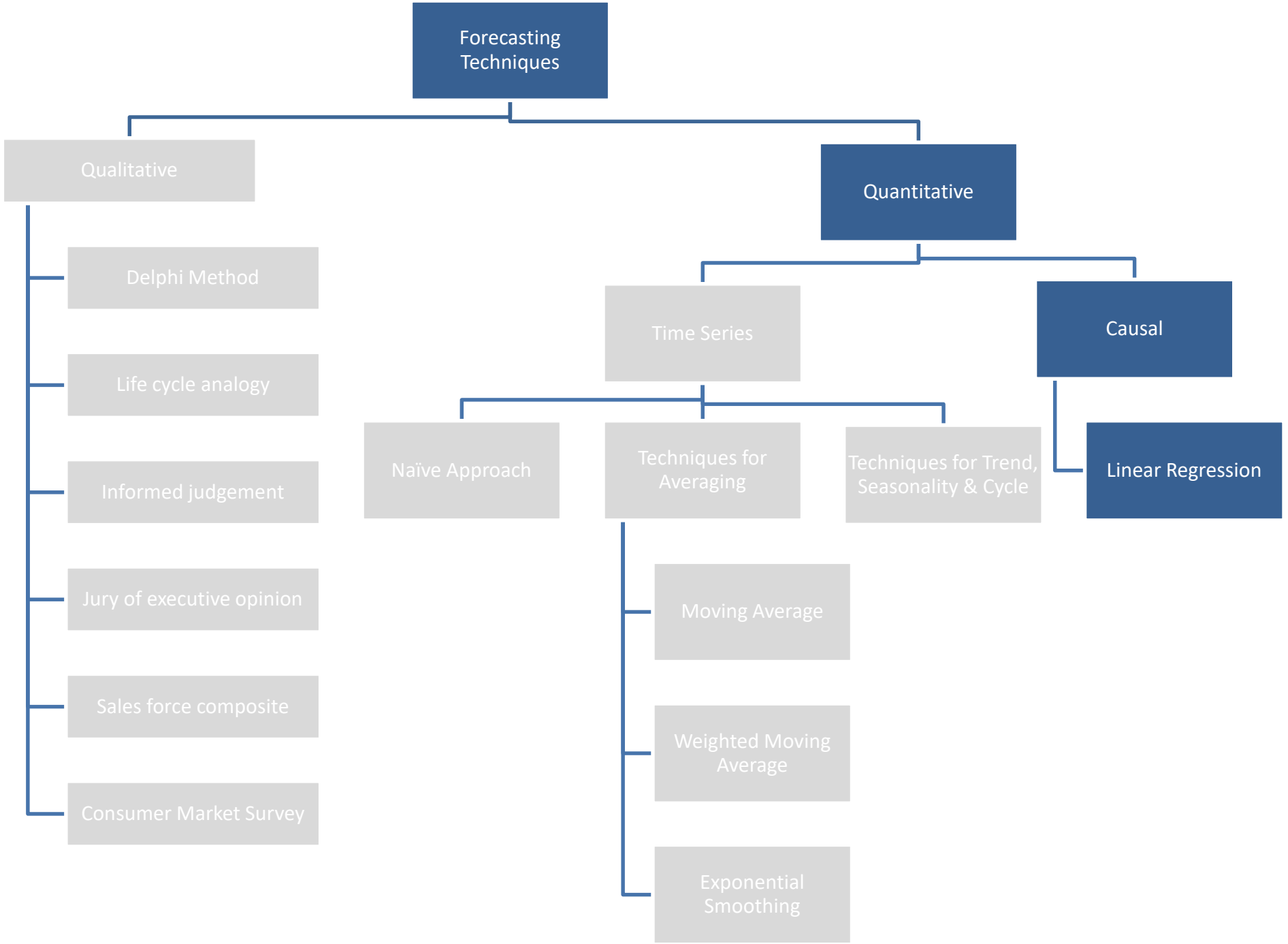
Review Exercise 3 (Acme)

The monthly demand for units manufactured by the Acme Rocket Company has been as follows:

Month	Units	Month	Units
May	100	September	105
June	80	October	110
July	110	November	125
August	115	December	120

Use the exponential smoothing method to forecast the number of units for June – January. The initial forecast for May was 105 units, $\alpha=0.2$.

Week of	Forecasted	Actual	Error
May	105.00	100	-5.00
June		80	-24.00
July		110	10.80
August		115	13.64
September		105	0.91
October		110	5.73
November		125	19.58
December		120	10.67
January			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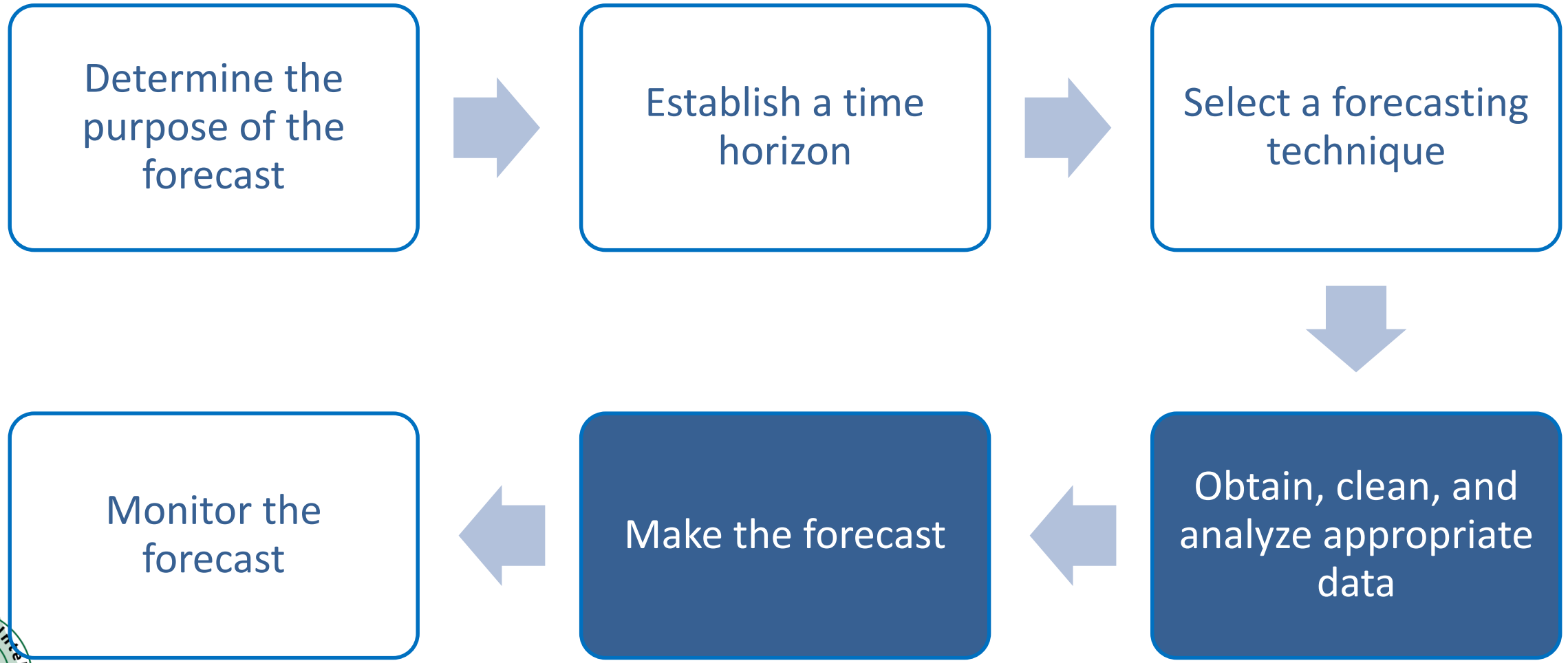
Outline for this Session

1. Associative / Causal Forecasting
   - Linear Regression

Overview of the Forecasting Process



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2. Accuracy of Forecasts
   - Mean Absolute Deviation
   - Mean Squared Error
   - Mean Absolute Percentage Error

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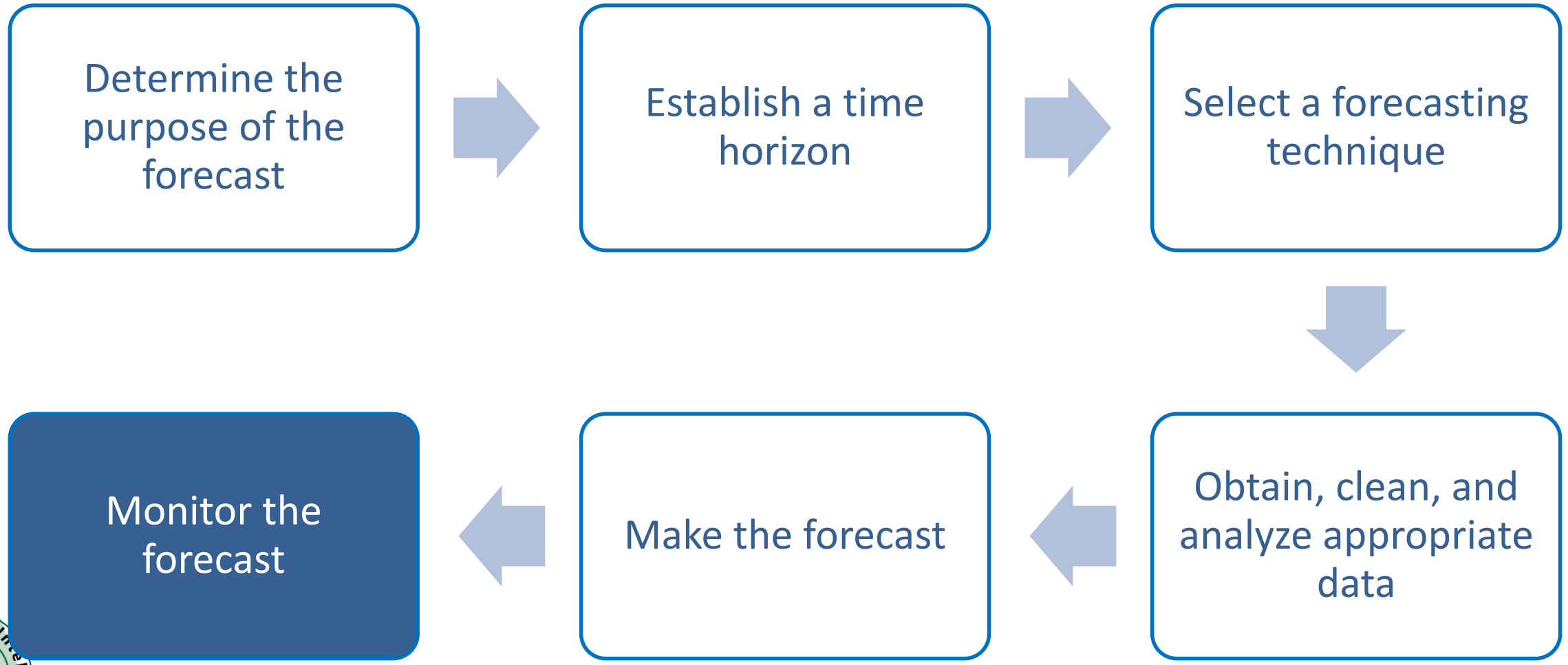

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3. Forecast Monitoring
   - Control Limits
   - Tracking Signals

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Objectives of this Session

At the end of this session, you should be able to:

- Recall the recommended process for business forecasting
- Forecast using Naïve, SMA, WMA, Simple Exponential Method, Trend-adjusted Exponential Smoothing, Linear Trend Equation and using Seasonal Index, and Simple Linear Regression.
- Explain the need to determine accuracy of selected forecasting technique
- Calculate forecasting accuracy using MAD, MSE, and MAPE
- Monitor forecasting performance using control limits and tracking signals

Outline for this Session

1. **Associative / Causal Forecasting**
   - **Linear Regression**
2. Accuracy of Forecasts
   - Mean Absolute Deviation
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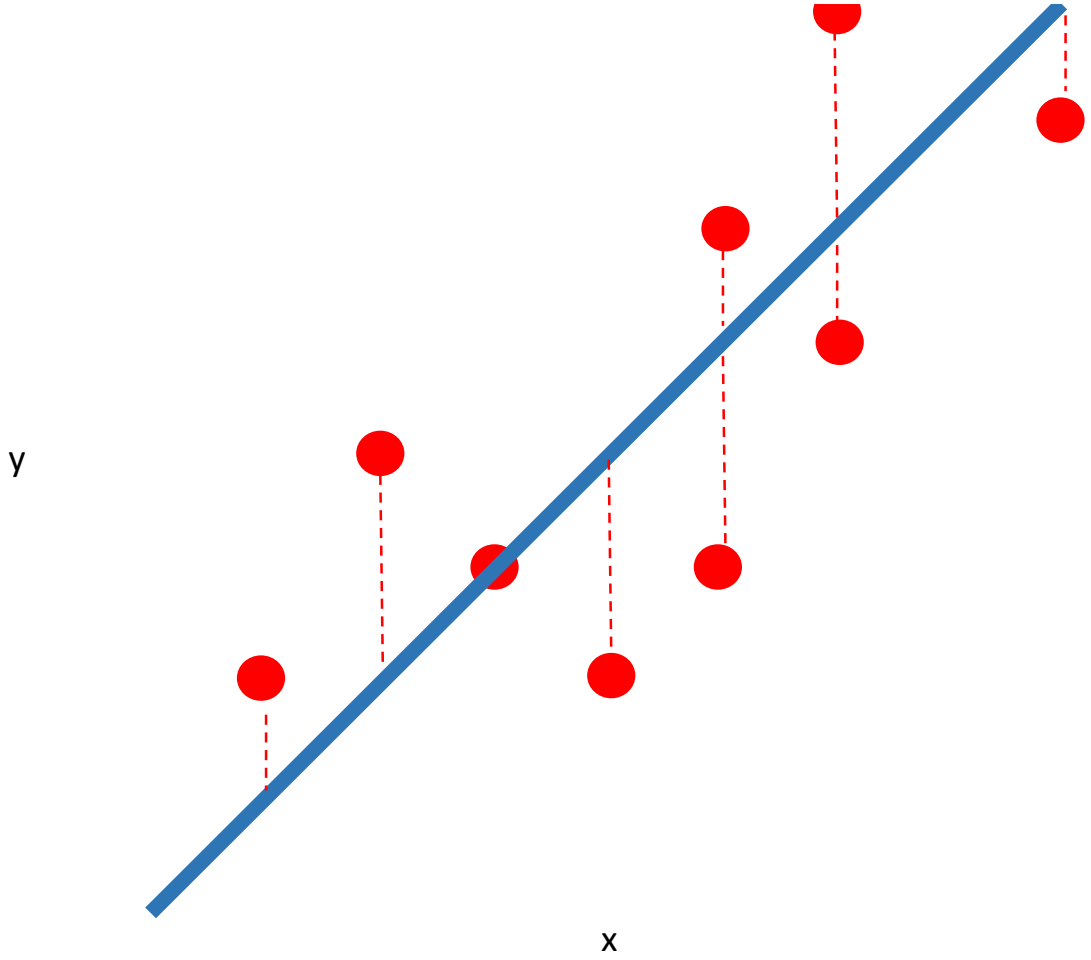
Associative Forecasts

Regression analysis is a method for building a statistical model that defines a relationship between a single dependent variable and one or more independent variables, all of which are numerical.

- $Y_t = a + bx$ ***x is t for trend equation***

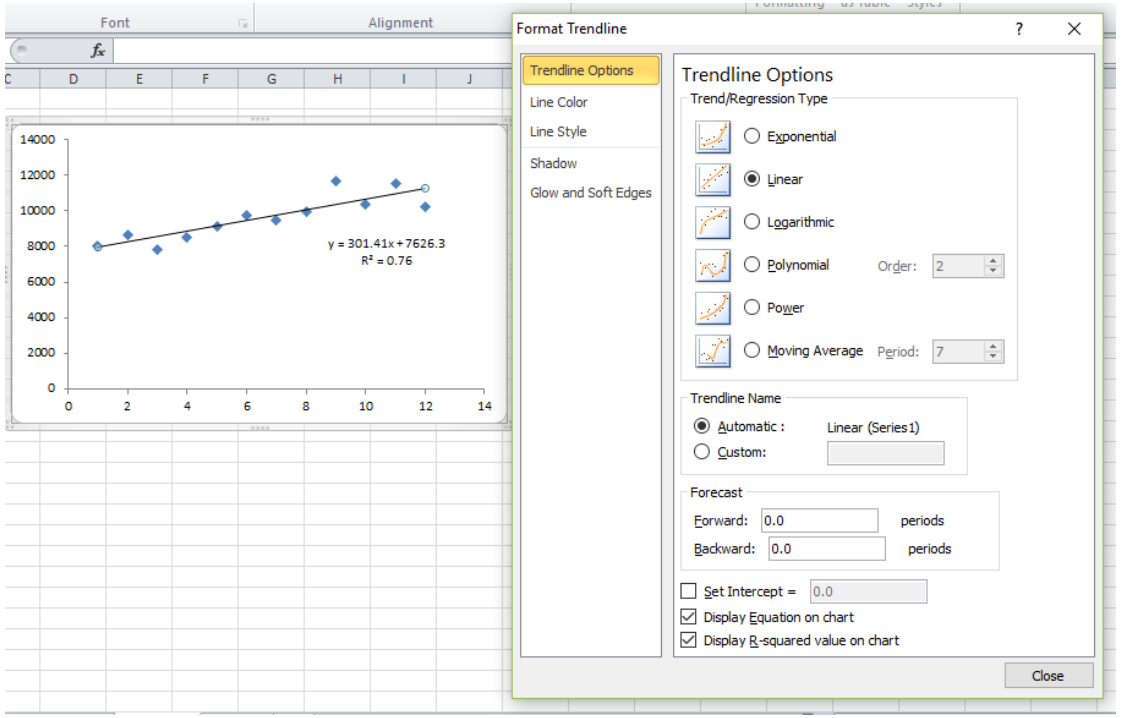
Associative Forecasts

Simple linear regression finds the best values of *a* and *b* using the *method of least squares*.



Associative Forecasts

Excel provides a very simple tool to find the best-fitting regression model for a time series by selecting the **Add Trendline** option from the Chart menu.



Exercise 1 (Energy)

Determine the trend line equation given the following data:

Year	Energy Costs ($)	Year	Energy Costs ($)
1	15,355.38	9	17,461.89
2	15,412.91	10	17,846.76
3	15,926.64	11	18,187.76
4	16,614.18	12	18,782.19
5	16,918.69	13	18,863.18
6	16,837.14	14	18,914.00
7	16,812.51	15	19,319.15
8	17,102.45		

Forecast the energy cost on the 16th and 20th year.

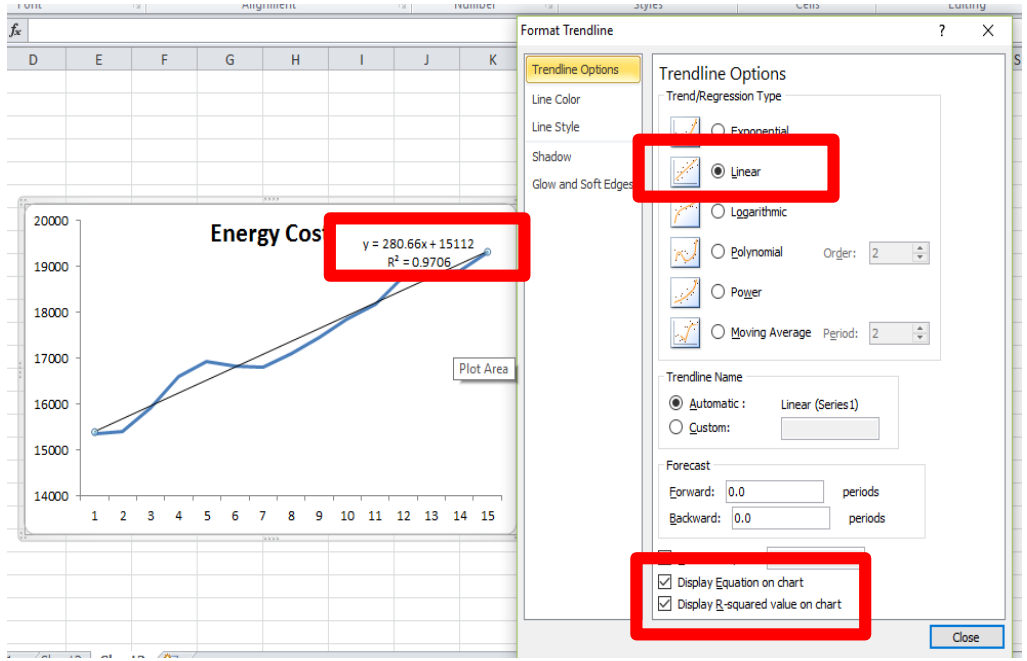
Step 1: Select the Energy Cost table.

Step 2: In the menu, click **Insert** >> **Insert Scatter Chart**

Step 3: Right-click on any of the points on the generated scatter chart, then click **Add Trendline**.

Step 4: Select Linear on the **Trendline Options**.

Step 5: Check the **Display Equation on Chart** and **Display R-squared value on chart**.



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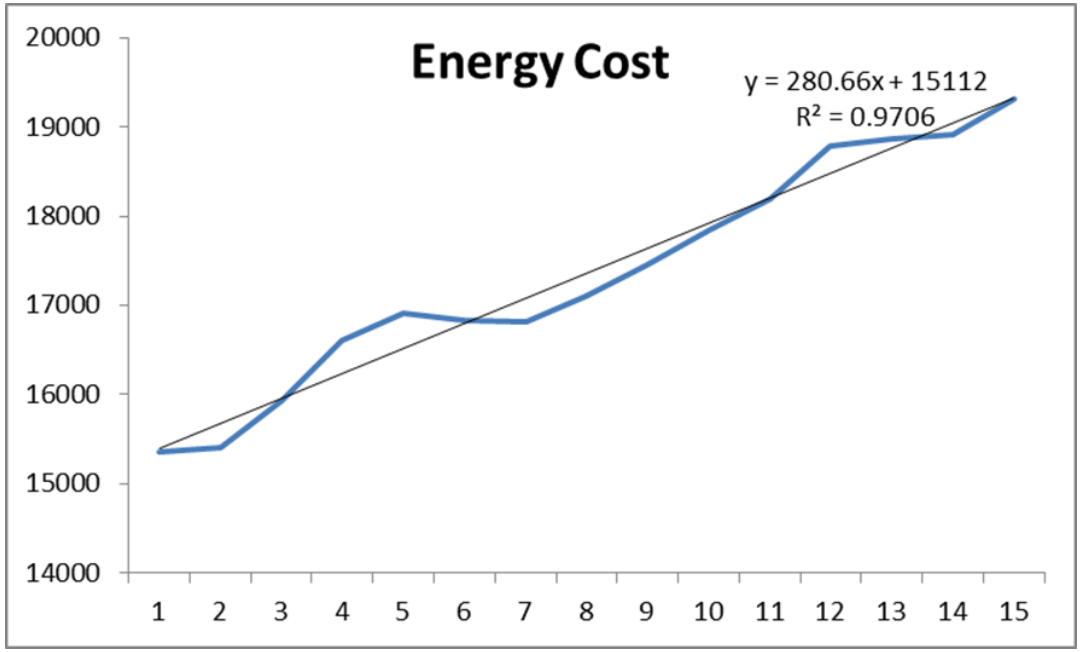

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8	17,102.45		

Forecast the energy cost on the 16th and 20th year.

The trend line equation is:

Forecasted energy cost for Year = 16.

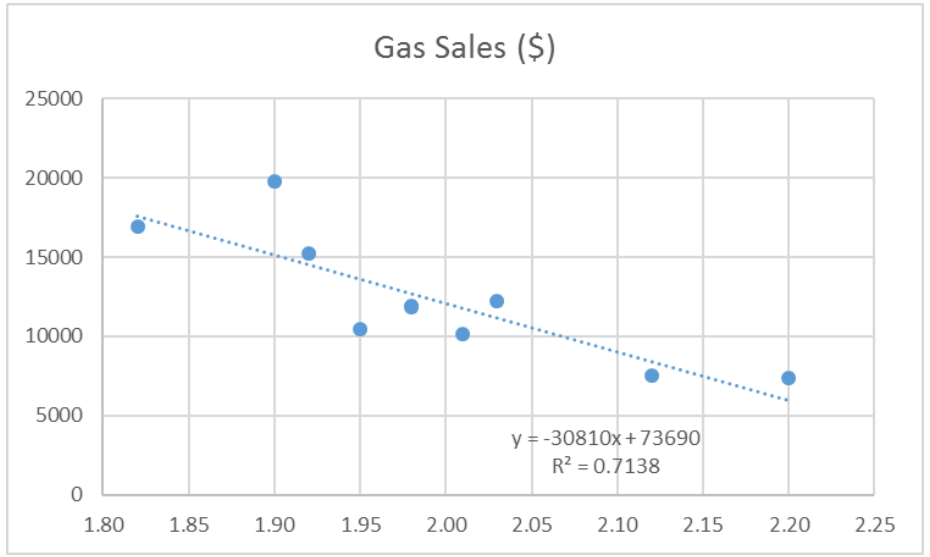
Forecasted energy cost for Year = 20.

Exercise 2 (Gas Sales)

Develop a linear regression line that would relate gas sales (GS) and price per gallon (ppg).

Year	Gas Sales ($)	Price per Gallon ($)	Year	Gas Sales ($)	Price per Gallon ($)
1	10420	1.95	6	15240	1.92
2	7388	2.2	7	12246	2.03
3	7529	2.12	8	11852	1.98
4	11932	1.98	9	16967	1.82
5	10125	2.01	10	19782	1.9

Forecast the gas sales if the price per gallon is $1.80.



The trend line equation is:

Forecasted gas sales for ppg= $1.80.

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Forecasts should have at least

Best estimate of demand

(mean, median or mode)

Forecasting error

(std. deviation, abs. deviation, or range)

Checking Forecast Accuracy

- There are three measures of accuracy of the fitted models: MAPE, MAD and MSD for each of the sample forecasting and smoothing methods.
- For all three measures, the smaller the value, the better the fit of the model.
- Use these statistics to compare the fit of the different methods.

Checking Forecast Accuracy

- Mean Absolute Deviation (MAD)
- Mean Squared Error (MSE)
- Mean Absolute Percentage Error (MAPE)

Definition of Errors

- Given the observations y_t (actual, A_t) of a time series and the corresponding forecasts $\hat{y}_t$ (forecast, F_t) using the k past periods, the prediction error at time t

$$e_t = y_t - \hat{y}_t$$

- The percentage prediction error at time t is

$$e_t^p = \frac{y_t - \hat{y}_t}{y_t} \times 100\%$$

MAPE, MAD, and MSD

- Mean Absolute Deviation:
  - $MAD = \frac{\sum_{t=1}^{n} |e_t|}{n} = \frac{\sum_{t=1}^{n} |A_t - Ft|}{n}$
- Mean Squared Deviation/Error:
  - $MSD = \frac{\sum_{t=1}^{n} e_t^2}{n-1} = \frac{\sum_{t=1}^{n} (A_t - Ft)^2}{n-1}$
- Mean Absolute Percentage Error:
  - $MAPE = \frac{\sum_{t=1}^{n} |e_t^p|}{n} = \frac{\frac{\sum_{t=1}^{n} |A_t - Ft|}{A_t} * 100\%}{n}$

Mean Absolute Deviation

- MAD
  - Expresses accuracy in the same units as the data, which helps conceptualize the amount of error.
  - $MAD = \frac{\sum_{t=1}^{n} |e_t|}{n}$ $= \frac{\sum_{t=1}^{n} |A_t - Ft|}{n}$
  - Average of the absolute errors
  - "Mean Absolute Error"
  - Good statistic if used for single items or single data
  - Caution on aggregation over multiple items

Mean Absolute Deviation

$$MAD = \frac{\sum_{t=1}^{n} |e_t|}{n} = \frac{\sum_{t=1}^{n} |A_t - Ft|}{n}$$

where

t = period number

A_t = demand in period t

F_t = forecast for period t

n = total number of periods

| | = absolute value

Mean Absolute Deviation

PERIOD	DEMAND, D_t	F_t (α =0.3)	$(A_t - F_t)$	$\lvert A_t - F_t \rvert$
1	37	37.00	–	–
2	40	37.00	3.00	3.00
3	41	37.90	3.10	3.10
4	37	38.83	-1.83	1.83
5	45	38.28	6.72	6.72
6	50	40.29	9.69	9.69
7	43	43.20	-0.20	0.20
8	47	43.14	3.86	3.86
9	56	44.30	11.70	11.70
10	52	47.81	4.19	4.19
11	55	49.06	5.94	5.94
12	54	50.84	3.15	3.15
	557		49.31	53.39

$$\text{MAD} = \frac{\Sigma \lvert D_t - F_t \rvert}{n}$$

$$= \frac{53.39}{11}$$

$$= 4.85$$

Exercise 3 (MAD)

Two independent methods of forecasting based on judgment and experience have been prepared each month for the past 10 months. The forecasts and actual sales are as follows.

Compute for the MAD of each forecast. Which forecast is better based on MAD?

Month	Sales	Forecast 1	Forecast 2
1	770	771	769
2	789	785	787
3	794	790	792
4	780	784	798
5	768	770	774
6	772	768	770
7	760	761	759
8	775	771	775
9	786	784	788
10	790	788	788

Mean Squared Deviation

- MSD / MSE
  - A commonly-used measure of accuracy of fitted time series values. This is differentiable hence a minimum can be obtained.
  - $MSD = \frac{\sum_{t=1}^{n} e_t^2}{n-1}$ $= \frac{\sum_{t=1}^{n} (A_t - Ft)^2}{n-1}$
  - Measures average of the squares of errors
  - Squaring is done so negative values do not cancel out positive values

Mean Squared Deviation/Error

$$MSE = \frac{\sum_{t=1}^{n} e_t^2}{n-1} = \frac{\sum_{t=1}^{n} (A_t - Ft)^2}{n-1}$$

where

t = period number

A_t = demand in period t

F_t = forecast for period t

n = total number of periods

Mean Squared Error (MSE)

PERIOD	ACTUAL DEMAND, A_t	FORECAST, F_t	$(A_t - F_t)$	$(A_t - F_t)^2$
1	37	37.00	–	–
2	40	37.00	3.00	9.00
3	41	37.90	3.10	9.61
4	37	38.83	-1.83	3.35
5	45	38.28	6.72	45.16
6	50	40.29	9.69	93.90
7	43	43.20	-0.20	0.04
8	47	43.14	3.86	14.90
9	56	44.30	11.70	136.89
10	52	47.81	4.19	17.56
11	55	49.06	5.94	35.28
12	54	50.84	3.15	9.92
	557		49.31	375.61

$$\text{MSE} = \frac{\Sigma(A_t - F_t)^2}{n-1}$$

$$= \frac{375.61}{10}$$

$$= 3.76$$

Exercise 3 (Accuracy 1.2)

Two independent methods of forecasting based on judgment and experience have been prepared each month for the past 10 months. The forecasts and actual sales are as follows.

Compute for the MSE of each forecast. Which forecast is better based on MSE?

Month	Sales	Forecast 1	Forecast 2
1	770	771	769
2	789	785	787
3	794	790	792
4	780	784	798
5	768	770	774
6	772	768	770
7	760	761	759
8	775	771	775
9	786	784	788
10	790	788	788

Mean Absolute Percentage Error

- MAPE
  - Expresses accuracy as a percentage of the error. For example, if the MAPE is 0.05, on average, the forecast is off by 5%.
  - $MAPE = \frac{\sum_{t=1}^{n} |e_t^p|}{n} = \frac{\frac{\sum_{t=1}^{n} |A_t - Ft|}{A_t} * 100\%}{n}$
  - $MAPE = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{A_t - Ft}{A_t} \right|$
  - Most organizations uses MAPE due to the measure
  - "We were off by less than 4%" is more meaningful
  - Limited if there are zero values (e.g., demand data) and low volume data

Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{\sum_{t=1}^{n} \frac{|A_t - Ft|}{A_t} * 100\%}{n}$$

shows the size of the error with respect to the size of the unit being forecast

Mean Absolute Percentage Error (MAPE)

PERIOD	ACTUAL DEMAND, A_t	FORECAST, F_t $(A_t - F_t)$	$\lvert A_t - F_t \rvert$	$\lvert A_t - F_t \rvert / A_t$
1	37	37.00	–	–
2	40	37.00	3.00	0.08
3	41	37.90	3.10	0.08
4	37	38.83	-1.83	0.05
5	45	38.28	6.72	0.15
6	50	40.29	9.69	0.19
7	43	43.20	-0.20	0.00
8	47	43.14	3.86	0.08
9	56	44.30	11.70	0.21
10	52	47.81	4.19	0.08
11	55	49.06	5.94	0.11
12	54	50.84	3.15	0.06
	557		49.31	1.09

$$\text{MAPE} = \frac{\Sigma \lvert A_t - F_t \rvert / A_t * 100\%}{n}$$

$$= \frac{1.0858 * 100\%}{11}$$

$$= 9.87\%$$

Discussion of MAPE

How much accuracy can we expect from a forecasting system?

In general,

- MAPE of 10% is very good
- MAPE of 20% - 30% or even higher is quite common.

Other Discussions about MAD, MSE and MAPE

It is commonly believed that:

1. MAD is a better criterion than MSE.
2. MSE emphasizes larger errors.
3. MAPE shows the size of the error relative to the unit being forecast.

Discussion:
Comparison of Forecasts

FORECAST	MAD	MAPE
Exponential smoothing (α = 0.30)	4.85	9.6%
Exponential smoothing (α = 0.50)	4.04	8.5%
Adjusted exponential smoothing (α = 0.50, β = 0.30)	3.81	7.5%
Linear trend line	2.29	4.9%

Exercise 4 (Accuracy 2)

Compare the actual and forecast figures using MSE, MAD and MAPE.

Which forecast is better?

Actual Sales	Forecasted Sales	
	Model 1	Model 2
56	54	50
43	44	40
22	20	22
24	19	20
55	50	49

Less Common Measures

4. Cumulative Forecast Error (CFE)
5. Mean Error (ME)
6. Root Mean Squared Error (RSME)

Cumulative Forecast Error (CFE)

- This sums up the total cumulative error at time t.

$$CFE = \sum A_t - Ft$$

Mean Error (ME)

- This is just the average error.

$$ME = \frac{\sum A_t - Ft}{n}$$

Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{MSE} = s$$

Parameter used in control charts for forecast control

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   - Mean Square Error
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3. **Forecast Monitoring**
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Monitoring the Forecast

Forecasts are made at regular intervals (e.g., weekly, monthly, etc.). Tracking of forecast errors and analyzing them provides useful insight on whether forecasts are performing satisfactorily.

Possible forecast errors:

- Model maybe inadequate
- Irregular variations
- Technique may have been used incorrectly
- Random variations

Monitoring the Forecast

A forecast is generally deemed to perform adequately when errors exhibit only random variations.

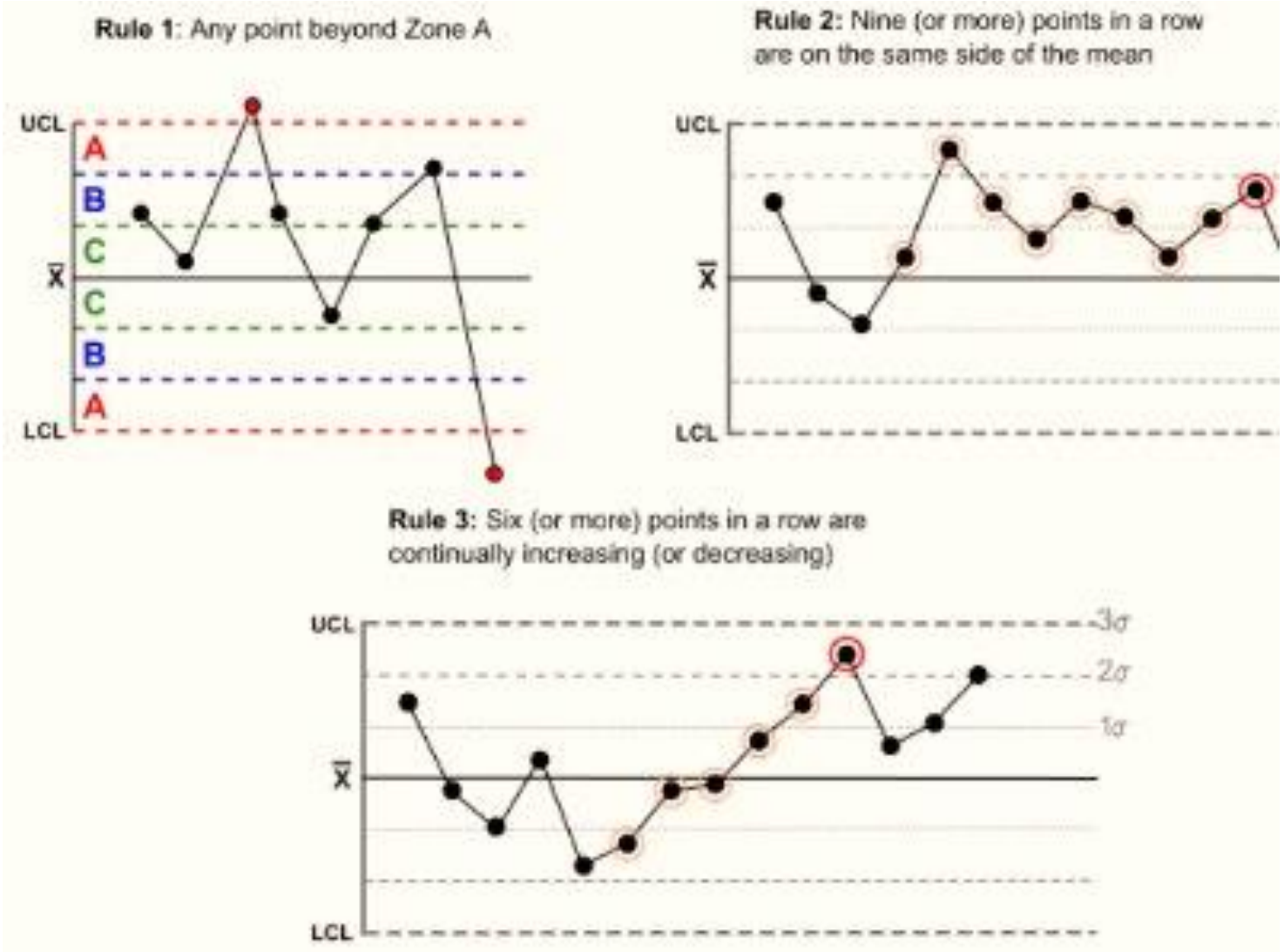
There are two tools used for detecting non-randomness:

1. Control Limits
2. Tracking Signal

Control Charts

- Forecast errors to be judged “in control” and can be judged by inspection:
  - Errors are within the control limits
  - No patterns (e.g., trends, cycles) are present

Control Charts



Control Charts

- To construct a control chart, compute the MSE
- When errors are random, they are assumed to be distributed according to a normal distribution around a mean of zero
- Control Limits (Control Charts)

$$UCL, LCL = 0 \pm z\sqrt{MSE}, \text{ where MSE}$$

z of 95.5 percent (2 standard deviations),

z of 99.7 percent (3 standard deviations)

Control Limits

$$\sigma = \sqrt{MSE} = \sqrt{\frac{\sum(A_t - Ft)^2}{n-1}}$$

Using σ we can calculate statistical control limits for the forecast error.
Control limits are typically set at ± 3σ

Exercise 5 (Control Limits 1)

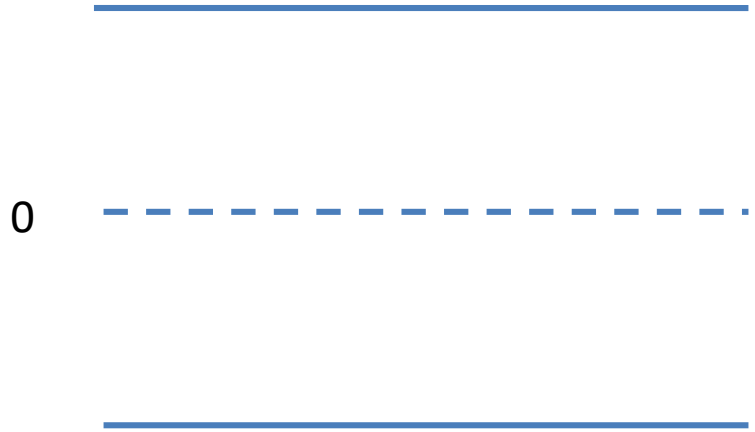
Compute 2s control limits for forecast errors when the MSE is 2.0.

What is z?

What is s?

What is UCL?

What is LCL?



Exercise 6 (Control Limits 2)

Compute for the control limits to be used for the control chart @ z of 95.5%.

Year	Sales	Forecasts
1	320	
2	326	320
3	310	326
4	317	310
5	315	317
6	318	315
7	310	318
8	316	310
9	314	316
10	317	314

What is z?

What is s?

What is UCL?

What is LCL?

Tracking Signal

- Tracking signal
  - Older and less informative technique
  - Relates the cumulative forecast error to the MAD
  - monitors the forecast to see if it is biased high or low (sequence of errors to be positive or negative)

$$TS = \frac{\sum(At - Ft)}{MAD_t} = \frac{e_t}{MAD_t}$$

Tracking Signal

$$Tracking\ Signal = \frac{RSFE}{MAD}$$

RSFE = running sum of the forecast error

MAD = mean absolute deviation

What happens when TS is positive?

What happens when TS is negative?

Tracking Signal

PERIOD	DEMAND A_t	FORECAST, F_t	ERROR $A_t - F_t$	$\sum e = \sum(A_t - F_t)$	MAD
1	37	37.00	–	–	–
2	40	37.00	3.00	3.00	3.00
3	41	37.90	3.10	6.10	3.05
4	37	38.83	-1.83	4.27	2.64
5	45	38.28	6.72	10.99	3.66
6	50	40.29	9.69	20.68	4.87
7	43	43.20	-0.20	20.48	4.09
8	47	43.14	3.86	24.34	4.06
9	56	44.30	11.70	36.04	5.01
10	52	47.81	4.19	40.23	4.92
11	55	49.06	5.94	46.17	5.02
12	54	50.84	3.15	49.32	4.85

Tracking signal for period 3

$$TS_3 = \frac{6.10}{3.05} = 2.00$$

Exercise 7 (Tracking Signal)

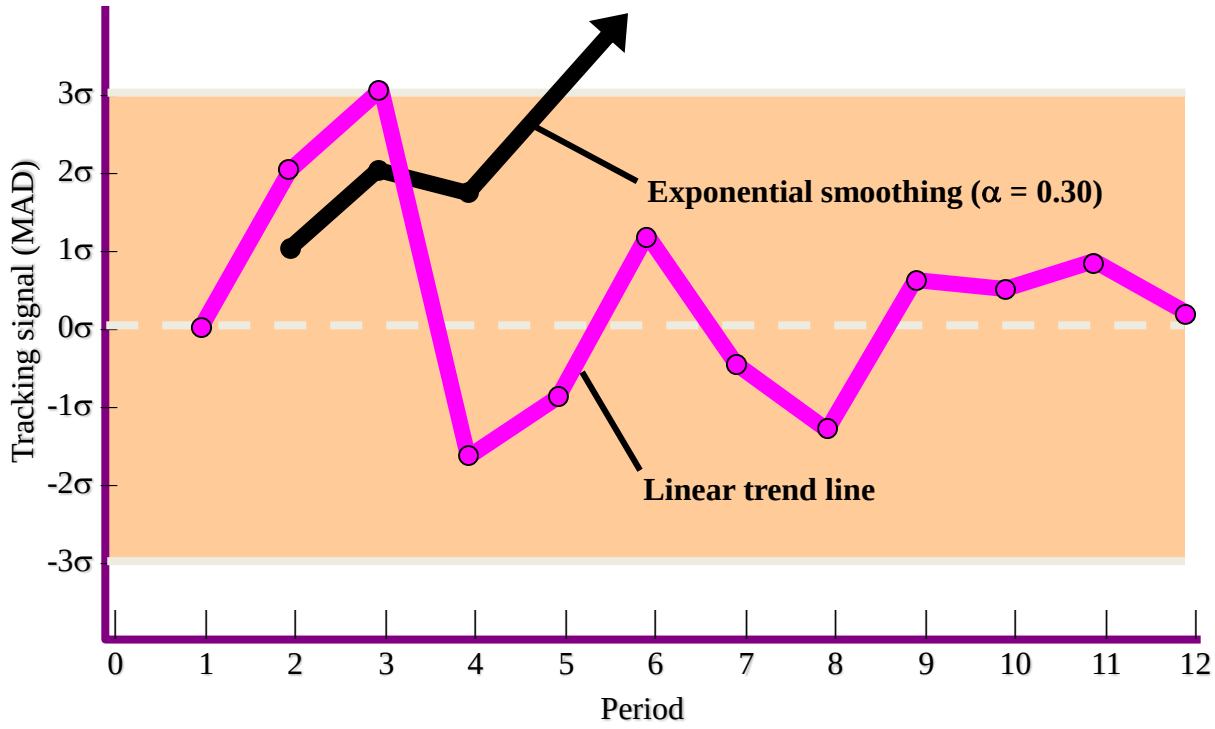
Complete the table and compute for tracking signal

Quarter No.	Forecast Demand	Actual Demand	Error	RSFE	|Forecast Error|	Cum. |Forecast Error|	Cum. MAD	Tracking Signal
1	100	90						
2	100	95						
3	100	115						
4	110	100						
5	110	125						
6	110	140						

Tracking Signal

- Tracking signal
  - monitors the forecast to see if it is biased high or low
  - We commonly use ±4 TS as our limits (-4, +4)

(Visual Inspection) Tracking Signal



Outline for this Session

1. Associative / Causal Forecasting
   - Linear Regression
2. Accuracy of Forecasts
   - Mean Absolute Deviation
   - Mean Square Error
   - Mean Absolute Percentage Error
3. Forecast Monitoring
   - Control Limits
   - Tracking Signals
4. **Workshop**

Workshop

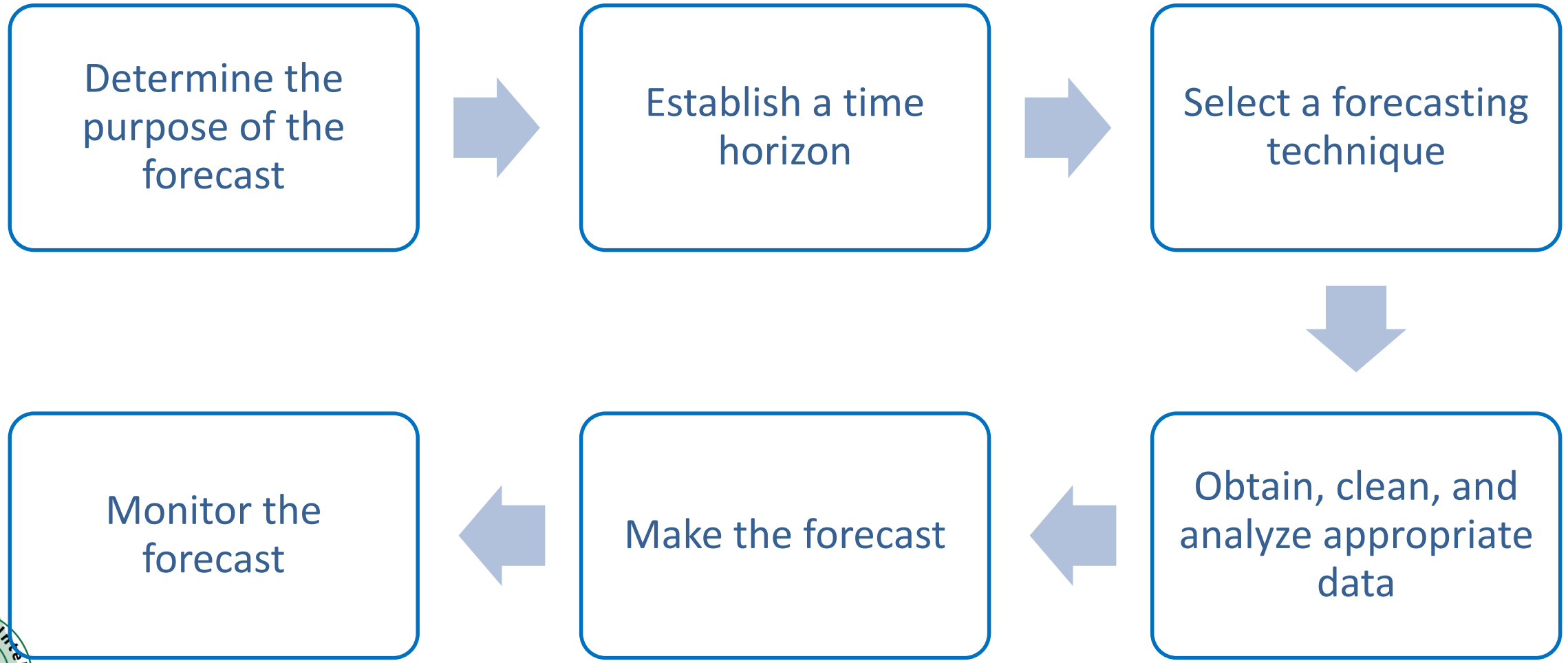
- See Day 2 (Quiz).pdf
- See Day 2(Workshop.pdf)
  - See Dataset – Day 2 (Workshop)

Objectives of this Session

At the end of this session, you should be able to:

- Recall the recommended process for business forecasting
- Forecast using Naïve, SMA, WMA, Simple Exponential Method, Trend-adjusted Exponential Smoothing, Linear Trend Equation and using Seasonal Index, and Simple Linear Regression.
- Explain the need to determine accuracy of selected forecasting technique
- Calculate forecasting accuracy using MAD, MSE, and MAPE
- Monitor forecasting performance using control limits and tracking signals

Recap: Overview of the Forecasting Process



Tomorrow: Forecasting using R

Thank you for listening!

References

- Notes and Datasets from Montgomery, Peck and Vining, Introduction to Linear Regression Analysis 4th Ed. Wiley
- Notes from G. Runger, ASU IEE 578
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- http://datamarket.com/data/list/?q=provider:tsdl
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