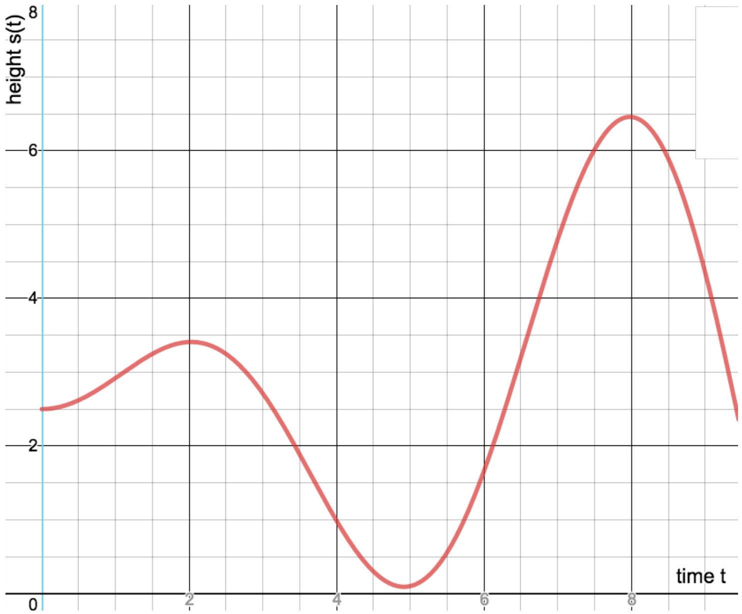
§6.1 - Velocity and Net Change

After completing this section, students should be able to:

- Explain the difference between displacement and distance traveled.
- Estimate displacement and distance traveled from a graph of position over time, or from a graph of velocity over time.
- Compute displacement and distance traveled from an equation of position as a function of time, or from an equation of velocity over time.
- Explain how to calculate the net change of a quantity from the rate of change of that quantity over time.
- Find an equation for velocity and position from an equation for acceleration plus initial conditions.
- Find an equation for the amount of a quantity from an equation for its rate of change plus an initial condition.

Example. A squirrel is running up and down a tree. The height of the squirrel from the ground over time is given by the function $s(t)$ graphed below, where t is in seconds and $s(t)$ is height in meters.



A. After 5 seconds, how far is the squirrel from its original position?

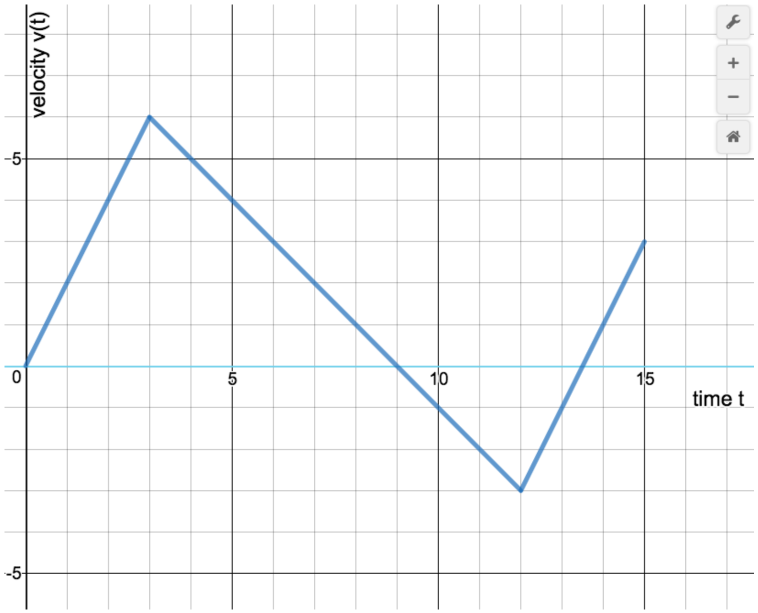
B. How far has the squirrel run in the first 5 seconds?

Definition. Displacement means ...

Definition. Distance traveled means ...

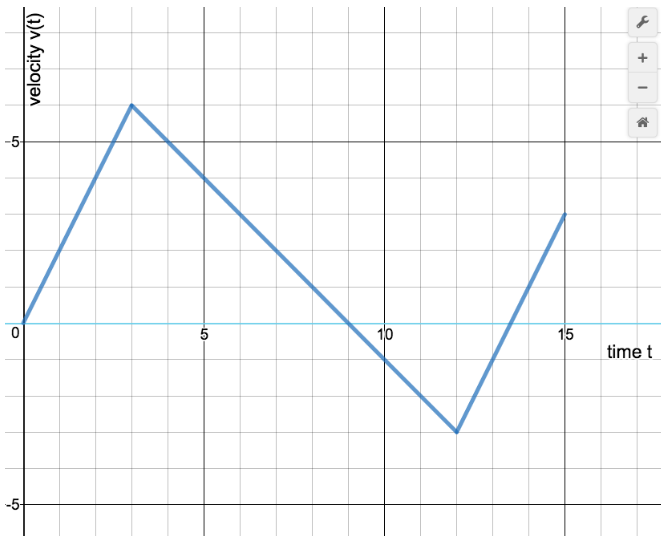
Example. If I get in a 25 meter long pool on the shallow end, and swim 5 laps, what is my displacement and what is my distance traveled?

Example. A swimmer is swimming left and right in a long narrow pool. Her *velocity* over time is given by the following graph, where velocity $v(t)$ is in meters per second and time t is in seconds.



Here, distance is measured from the left end of the pool, so a positive velocity means ________ and a negative velocity means ________ .

A. Describe the swim. Was the swimmer swimming at a constant speed? When was the swimmer swimming left vs. right? At what time(s) did the swimmer turn around?



B. What is the displacement of the swimmer between time 0 and time 12?

Displacement = integral(b-a) v(t)dt = integral (9-0) v(t)dt = 1/2(9)(6) - 1/2(3)(3) = 27-4.5 = 22.5

C. How far did the swimmer swim in the first 3 seconds?

total distance traveled = integral(3-0) abs(v(t))dt = 1/2(3)(6) = 9m

D. the first 9 seconds?

= integral(9-0) abs(v(t))dt = 1/2(9)(6) = 27m

E. the first 12 seconds?

= integral(12-0) abs(v(t))dt = 27 + 1/2(3)(3) = 31.5m

Note. Suppose $f(t)$ represents the velocity of an object.

- The *displacement* of the object between time $t = a$ and time $t = b$ is given by ...

- The *distance traveled* by the object between time $t = a$ and time $t = b$ is given by ...

Example. The velocity function for a particle moving left and right is given by $v(t) = t^2 - 2t - 3$, where $v(t)$ is in meters per second and t is in seconds.

1. When does the particle turn around?

    0 = t^2-2t-3
    0 = (t-3)
    (t+1)
    t = 3, -1

2. Find the displacement of the particle between time $t = 1$ and $t = 4$.

    integral(4-1) v(t)dt
    integral(4-1) (t^2-2t-3)dt
    (t^3)/3 - t^2 - 3t (4-1)
    (64-16-12) - (1/3-1-3)
    -3

3. Find the total distance traveled between $t = 1$ and $t = 4$.

    integral(4-1) v(t)dt
    integral(4-1) abs(t^2-2t-3)dt
    integral(3-1) (t^2-2t-3)dt + integral(4-3) (t^2-2t-3)dt
    - ((t^3)/3 - t^2 - 3t) (3-1) + (t^3)/3 - t^2 - 3t (4-3)
    23/3

4. If the particle starts at position 2, give a formula for the position of the particle at time t.

    Future position = s(0) +displacement

Example. Suppose $f(t)$ represents the rate of change of a quantity over time (e.g. the rate of water flowing out of a resevoir). Then

- $\int_a^b f(t)\,dt$ represents ... displacement
  net change

- If $F(0)$ is the amount of the quantity at time 0, then $F(0) + \int_a^b f(t)\,dt$ represents ...
  Future position
  Future Quantity

- $\int_a^b |f(t)|\,dt$ represents ...

  Total Quantity Used

Example. The population of bacteria is changing at a rate of $f(t) = e^{-t} - 1/e$. What is the net change in population between time $t = 0$ and time $t = 2$?

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net change = integral(b-a) (rate of change formula)dt (ie velocity)
net change = integral(2-0) e^-t - (1/e) dt
net change = -e^-t - (1/e)t (2-0)
net change = (-e^-2 - (1/e)2) - (-e^-0 - (1/e)0)
net change = -1/e^2 - (2/e) + 1
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Extra Example. The acceleration of a particle moving up and down is given by $a(t) = 3\pi \sin(\pi t)$, where $a(t)$ is given in m/s^2 and t is given in seconds. Suppose that $v(0) = 2$ and $s(0) = -1$. Find the velocity and position functions. What is its displacement in the first 2 seconds? How much total distance did it travel in the first 2 seconds.