§8.2 - Integration by Parts

After completing this section, students should be able to:

- Use integration by parts to compute an integral that is a product of two factors, like $\int xe^x\,dx$
- Identify factors that are good candidates for u vs dv
- Use integration by parts more than one time if necessary.
- Use integration by parts to compute integrals like $\int arctan(x)\,dx$, by using $1dx$ as du
- Use integration by parts to compute integrals like $\int \sin(x)e^x\,dx$, in which the integrands cycle around, and it possible to solve for the integral without ever fully computing it.

Recall: the Product Rule says:

```
d/dx (f(x)*g(x)) = f(x)*g prime(x)+f prime(x)*g(x)
```

Rearranging and integrating both sides gives the formula:

Note. This formula allows us to rewrite something that is difficult to integrate in terms of something that is hopefully easier to integrate. Integrating using this method is called:

```
Integration by parts: ∫udv = uv − ∫vdu
```

Example. Find $\int xe^x \, dx$.

```
product -> integration by parts
u = x
dv = e^x

du = dx
v = e^x

which is easier to take the derivative?
is dv easily integrated

x(e^x) - ∫e^x(dx)
xe^x - e^x + c
```

Review.

$$\int u\, dv =$$

uv − ∫vdu

Example. Integrate $\int t \sec^2(2t)\, dt$ using integration by parts. What is a good choice for u and what is a good choice for dv?

```
u = t
dv = sec^2(2t)

du = dt
v = 1/2(tan(2t))

= t(1/2(tan(2t))) − ∫1/2(tan(2t))dt
= 1/2(t)(tan(2t)) − 1/2(ln|(sec(2t)/2)| + c
= 1/2(t)(tan(2t)) − 1/4ln|sec(2t))| + c
```

Example. Find $\int x(\ln x)^2 dx$

```
u = (lnx)^2
dv = x dx

du = 2lnx(x) ∗ 1/x dx
v = (x^2)/2

(x^2)/2 ∗ (lnx)^2 − ∫(x^2)/2 ∗ 2lnx ∗ 1/x dx
= 1/2(x^2)(lnx)^2− ∫xlnx dx

u = lnx
dv = x dx

du = 1/x
v = (x^2)/2

= 1/2(x^2)(lnx)^2 − ∫xlnx dx
1/2(x^2)(lnx)^2 + lnx(x^/2)/2 − ∫(x^2)/2 ∗ 1/x
```

Example. Integrate $\int_1^2 \arctan(x)dx$.

```
u = tan^−1(x)
dv = 1(dx)

du = 1/(1+x^2)
v = x

tan^−1(x)(x) − ∫x(1/(1+x^2))


u = 1+x^2
du = 2x dx
1/2du = x dx

(x)tan^−1(x) − 1/2∫1/u (du)

1/2(ln|u|)
1/2(ln|1+x^2|)

xtan^−1(x) − 1/2(ln|1+x^2|) (2, 1)
```

Example. Find $\int e^{2x} \cos(x) dx$.

```
u = e^2x
dv = cos(x)

du = 2e^2x
v = sin(x)

e^2x(sin(x)) − ∫2e^2x(sin(x))

u = 2e^2x
dv = sinx

du = 4e^2x
v = −cosx

∫e^2x(cosx)dx = e^2x(sinx) − 2e^2x(−cosx) − ∫−cosx(4e^2x)
∫e^2x(cosx)dx = e^2x(sinx) + 2e^2x(cosx) − 4∫cosx(e^2x)
5∫e^2x(cosx)dx = e^2x(sinx) + 2e^2x(cosx)
∫e^2x(cosx)dx = 1/5(e^2x)sinx + 2/5(e^2x)cosx + c
```

Question. How do we decide what to call u and what to call dv?

Question. Which of these integrals is a good candidate for integration by parts? (More than one answer is correct.)

A. $\int x^3 \, dx$

B. $\int \ln(x) \, dx$

C. $\int x^2 e^x \, dx$

D. $\int x e^{x^2} \, dx$

E. $\displaystyle\int \frac{\ln y}{\sqrt{y}} \, dy$