§8.6 Integration Strategies

After completing this section, students should be able to:

- Choose an appropriate integration strategy for a given integral.
- Express the limitations of the integration techniques that we have learned, as well as the limitations of all integration techniques known to humankind.

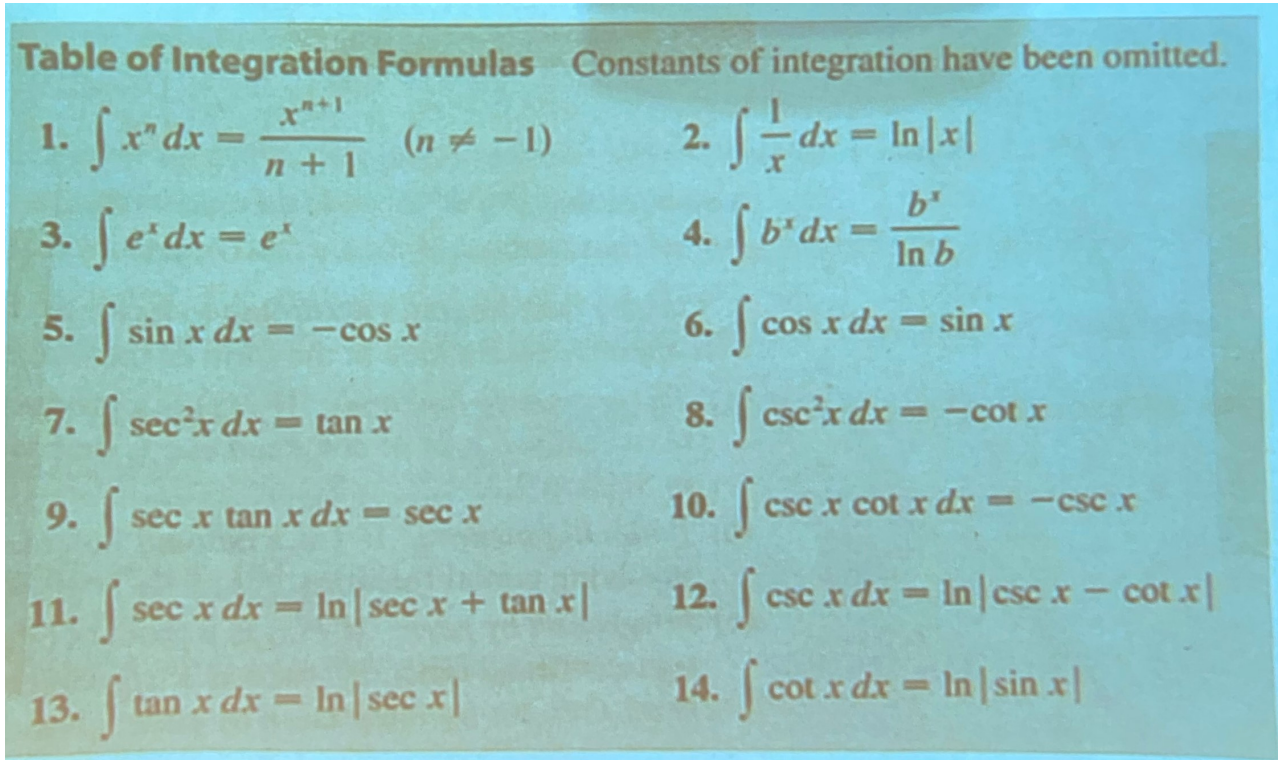

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \dfrac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \dfrac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$

Strategies:

1. Simplify the Integrand
2. Look for Obvious Substitutions
3. Classify the Integrand According to its Form
   - Trigonometric Function - Product of Powers
   - Rational Function - Partial Fractions
   - Product of Polynomial with a Trig, Log, or Exp - Integration by Parts
   - Radicals - sum or differences of squares inside of a square root - Trigonometric Substitution
   - Radicals - Substitution
4. Try Again
   - Non-Obvious Substitution
   - Integration by Parts but with single functions
   - Manipulate the integrand - Tri Ids, Rationalizing
   - Multiple Methods

Integration By Parts....selecting u

ILATE Rule

I: Inverse trigonometric functions

L: Logarithmic functions

A: Algebraic functions

T: Trigonometric functions

E: Exponential functions

For each integral, indicate what technique you might use to approach it and give the first step. You do not need to finish any of the problems.

1. $\int x^3 \ln x \, dx$
2. $\int \cos^2(x) \, dx$
3. $\int \frac{dx}{x \ln(x)}$
4. $\int \arcsin(x) \, dx$
5. $\int \frac{x^2+1}{\sqrt{x}} \, dx$
6. $\int \frac{\sin(x)}{3+\sin^2(x)} \, dx$
7. $\int \frac{x^3}{25-x^2} \, dx$
8. $\int \frac{x^3}{\sqrt{25-x^2}} \, dx$
9. $\int e^{\sqrt{x}} \, dx$

1. ∫udv = uv − ∫vdu
u = lnx, dv = x^3, du = 1/x, v = x^4/4
x^4/4 ln|x| − x^4/16 + c

2. Trig identities
∫cos^2(x) dx= ∫(1 + cos2x)/2 dx
1/2∫1+cos2x dx
1/2(∫1dx + ∫cos2x dx)
1/2(x + sin2x/2) + c
x/2 + sin2x/4 + c

3. U−sub
∫1/x ∗ 1/lnx (dx)
u = lnx, du = 1/x dx
∫1/u (du)
lnu + c
ln|ln|x|| + c

6.
−x^2 + 25sqrt(x^3+0−0+0)
−x^3+0−25x
25x+0
∫−x + 25x/25−x^2 dx
25x/(5−x)(5+x) = A/5−x + B/5+x
25x = A(5+x) + B(5−x)\
25x = 5A + Ax + 5B − Bx
25 = A−B ∗ (5)
0 = 5A + 5B

125 = 5A − 5B
0 = 5A + 5B

125= 10A
A = 125/10
A = 25/2

25 = A−B
25 = 25/2 − B
50/2 − 25/2 = −B
25/2 = −B
B = −25/2

∫−x + (25/2)/5−x + (−25/2)/5+x
= −x^2/2 (−25ln|5−x|/2)

Philosophy about Integration

Definition. (Informal Definition) An *elementary function* is a function that can be built up from familiar functions, like

- polynomials
- trig functions
- exponential and logarithmic functions

using familiar operations:

- addition
- subtraction
- multiplication
- division
- composition

Example. Give an example an elementary function. Make it as crazy as you can.

Question. Is it always true that the derivative of an elementary function is an elementary function?

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f(x) = elementary function
dy/dx f(x) = elementary function

True
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Question. Is it always true that the integral of an elementary function is an elementary function?

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∫f(x)dx exists but we might not be able to integrate it with our integration tech

∫e^x^2 dx infinite series
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Techniques of integration ... and their limitations.