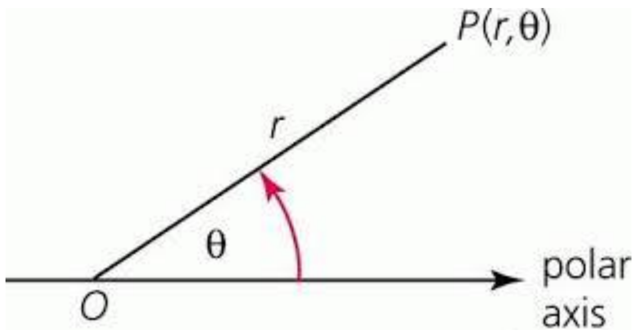
12.2 Polar Equations

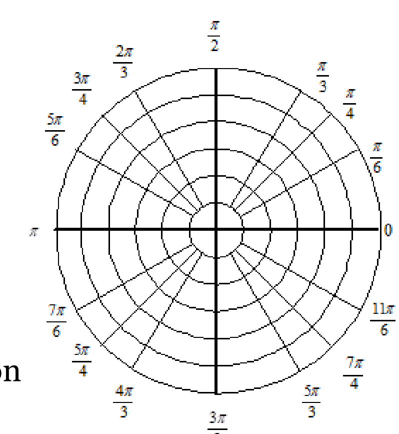
- Polar Coordinates
  - Origin is the pole
  - Polar Axis – axis with the origin as its endpoint
  - (r, θ) where r is the distance from the origin to the point (ie radius) and θ is the measure of any angle determined by the polar axis and the segment from the origin to the point
    - Point P is also denoted as $P(r, \theta)$



- Rectangular and Polar Coordinates
  - $x = r\cos\theta$
  - $y = r\sin\theta$
  - $r^2 = x^2 + y^2$
  - $\tan\theta = \frac{y}{x}$ if $x \neq 0$

Ex 1: Plot the points on the same polar graph grid

A. $\left(3, \frac{3\pi}{2}\right)$
B. $(5,0)$
C. $(0,5)$
D. $\left(2, -\frac{\pi}{2}\right)$
E. $\left(3, \frac{4\pi}{3}\right)$
F. $\left(-3, \frac{4\pi}{3}\right)$
G. $\left(-2, \frac{7\pi}{6}\right)$



4 points between [-2pi, 2pi] that are the same location
(r, theta)
(r, -theta)
(-r, theta)
(-r, -theta)

Ex 2: Find 3 other ways to write the point $\left(-3, \frac{4\pi}{3}\right)$.

(-3, -2pi/3)
(3, 2pi/3)
(3, -4pi/3)

Ex 3: Find the rectangular coordinates of a point with the polar coordinates $\left(5, \frac{7\pi}{4}\right)$. Sketch.

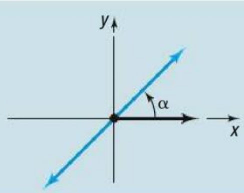
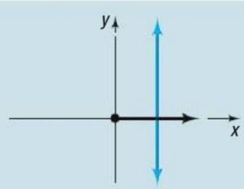
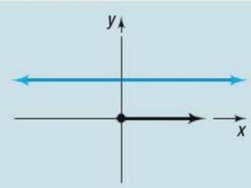
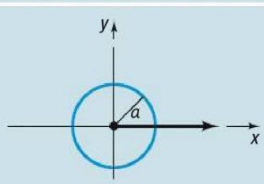
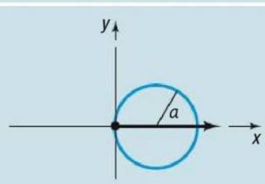
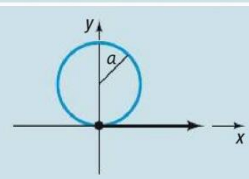
5, 7pi/4
x=5cos(7pi/4)
x = 5sqrt(2)/2

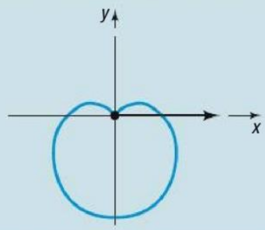
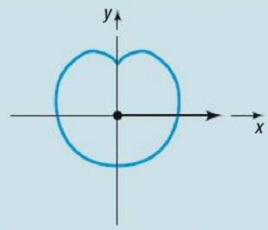
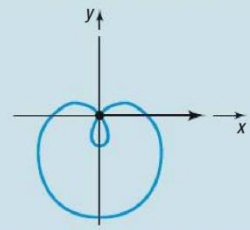
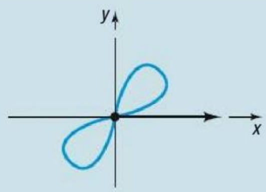
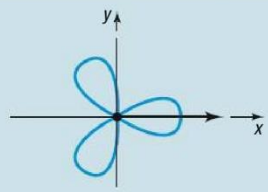
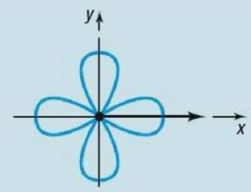
Ex 4: Find the polar coordinates of a point with the rectangular coordinates $\left(-2, -2\sqrt{3}\right)$. Sketch.

r^2=x^2+y^2
tan(theta) = y/x

r^2=(-2)^2 + (-2sqrt(3))^2
r ^2= 16 -> r=+-4

(4, 4pi/3)
tan(theta) = -2sqrt(3)/-2
tan(theta) = sqrt(3)
theta = 4pi/3

Lines			
Description	Line passing through the pole making an angle α with the polar axis	Vertical line	Horizontal line
Rectangular equation	$y = (\tan \alpha) x$	$x = a$	$y = b$
Polar equation	$\theta = \alpha$	$r \cos \theta = a$	$r \sin \theta = b$
Typical graph			
Circles			
Description	Center at the pole, radius a	Passing through the pole, tangent to the line $\theta = \frac{\pi}{2}$, center on the polar axis, radius a	Passing through the pole, tangent to the polar axis, center on the line $\theta = \frac{\pi}{2}$, radius a
Rectangular equation	$x^2 + y^2 = a^2, \quad a > 0$	$x^2 + y^2 = \pm 2ax, \quad a > 0$	$x^2 + y^2 = \pm 2ay, \quad a > 0$
Polar equation	$r = a, \quad a > 0$	$r = \pm 2a \cos \theta, \quad a > 0$	$r = \pm 2a \sin \theta, \quad a > 0$
Typical graph			

Other Equations			
Name	Cardioid	Limaçon without inner loop	Limaçon with inner loop
Polar equations	$r = a \pm a\cos\theta,\quad a > 0$	$r = a \pm b\cos\theta,\quad 0 < b < a$	$r = a \pm b\cos\theta,\quad 0 < a < b$
	$r = a \pm a\sin\theta,\quad a > 0$	$r = a \pm b\sin\theta,\quad 0 < b < a$	$r = a \pm b\sin\theta,\quad 0 < a < b$
Typical graph			
Name	Lemniscate	Rose with three petals	Rose with four petals
Polar equations	$r^2 = a^2\cos(2\theta),\quad a \neq 0$	$r = a\sin(3\theta),\quad a > 0$	$r = a\sin(2\theta),\quad a > 0$
	$r^2 = a^2\sin(2\theta),\quad a \neq 0$	$r = a\cos(3\theta),\quad a > 0$	$r = a\cos(2\theta),\quad a > 0$
Typical graph			

Ex 5: Transform $y^2 = 10x - x^2$ to a polar equation. Solve for r.

x^2+y^2 = 10x
r^2 = 10(rcos(theta))
r = 10cos(theta)

Ex 6: Transform $2x - 3y = 5$ to a polar equation. Solve for r.

2(rcos(theta)) - 3(rsin(theta)) = 5
r(2cos(theta) - 3sin(theta)) = 5
r = 5/(2cos(theta) - 3sin(theta))

Ex 7: Transform $r = \frac{5}{\sin\theta - 8\cos\theta}$ to a rectangular xy-equation.

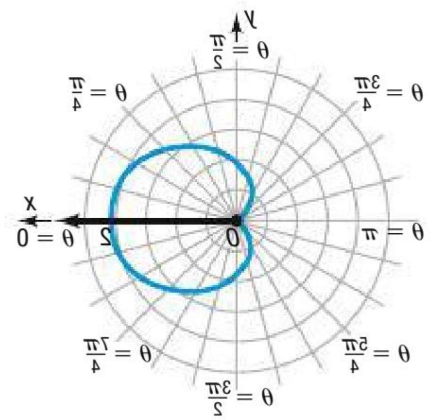
r(sin(theta) - 8cos(theta)) = 5
rsin(theta) - 8rcos(theta) = 5
-8x + y = 5

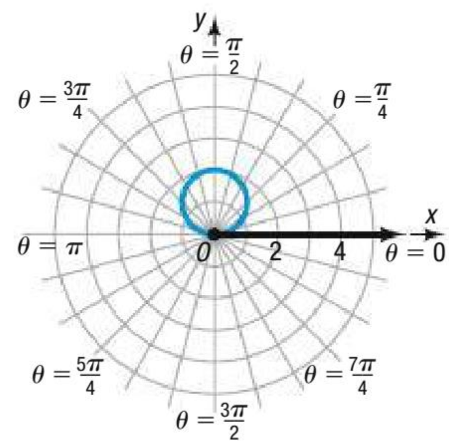
Ex 8: Transform $r = -6\cos\theta$ to a rectangular xy-equation.

Ex 9: Match the graphs:

A. $r = 2\sin\theta$
B. $\theta = \frac{\pi}{4}$
C. $r = 1 + \cos\theta$
D. $r = 1 + 4\sin\theta$

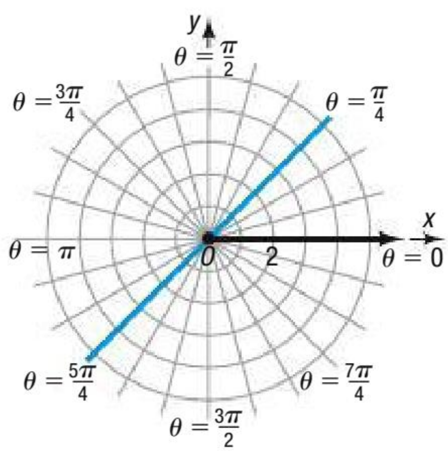
C

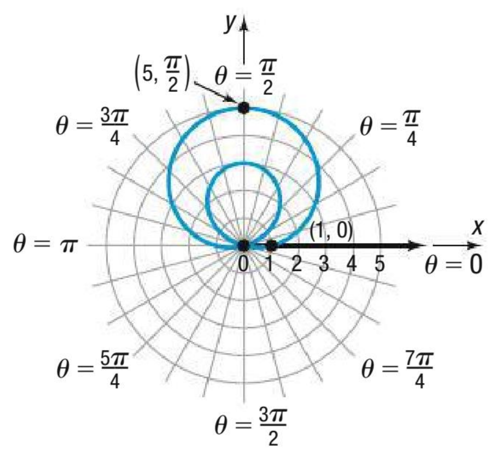




A

B





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