§10.3 - Series

After completing this section, students should be able to:

- Determine if a geometric series converges or diverges.
- Recognize a telescoping series and use its partial sums to determine if it converges or diverges.
- Determine if sums and scalar multiples of series converge or diverge based on the convergence status of their component series.

Definition. A geometric **sequence** is a sequence of the form ... {ar^n}infinity n=0

Definition. A geometric **series** is a series of the form ... Σ(infinity, n=0) ar^n

Example. Is $\sum_{i=2}^{\infty} \frac{5(-2)^i}{3^{2i-3}}$ a geometric series? If so, what is the first term and what is the common ratio?

Σ(infinity, i=2) 5(-2)^i/3^2i 3^-3
Σ(infinity, i=2) 5(-2)^i/(3^2)^i 3^-3
Σ(infinity, i=2) 5*27 ((-2)/3^2)^i
Σ(infinity, i=2) 135 (-2/9)^i

Σ(infinity, i=0) 135 (-2/9)^i+2
Σ(infinity, i=0) 135 (-2/9)^i (-2/9)^2
Σ(infinity, i=0) 20/3 (-2/9)^i

Fact. A geometric **sequence** $\{ar^n\}_{n=0}^{\infty}$ converges to 0 when $\underline{\quad |r|<1 \quad}$, converges to $\underline{\quad a \quad}$ when $\underline{\quad r=1 \quad}$ and diverges when $\underline{\quad |r|>1 \quad}$

r=-1

Question. For what values of r does the geometric **series** $\sum_{n=0}^{\infty} ar^n$ converge?

|r|<1

Stragegy:

(a) Find a formula for the Nth partial sum $sum_{k=0}^{N} a \cdot r^k$.

(b) Take the limit of the partial sums.

Conclusion: The geometric series $\sum_{n=0}^{\infty} ar^n$ converges to a/1-r when |r|<1 .

The geometric series $\sum_{n=0}^{\infty} ar^n$ diverges when |r|>=1 .

You don't need to reindex Σ but a = first term of series

Example. Does $\sum_{i=2}^{\infty} \frac{5(-2)^i}{3^{2i-3}}$ converge or diverge?

Σ(i=2, infinity) 5(-2)^i/3^-3 - 3^2i
Σ(i=2, infinity) 5(-2)^i/3^-3 - 9^i
Σ(i=2, infinity) 5/3^-3 (-2/9)^i

r = -2/9, a=5/3^-3 (-2/9)^2

sum = a/1-r = 5/3^-3 (-2/9)^2 / 1-(-2/9)
= 20/3 / 11/9

Tricks for determining when series converge:

Trick 1: Recognize geometric series.

Review. A geometric series is a series of the form:

Σ (n=0, infinity) ar^n

Review. For what values of r does a geometric **series** converge?

|r|<1

Example. For what values of x does the series $\sum_{n=2}^{\infty} \frac{3x^{n-1}}{2^n}$ converge? What does it converge to (in terms of x)?

Σ(n=2, infinity) 3 * x^n * x^-1 / 2^n
Σ(n=2, infinity) 1/3x * (x/2)^n

r = x/2
|x/2| < 1 -> |x| < 2
-2 < x < 2

a = 3x^-1 (x/2)^2   sum = 3x^-1(x/2)^2 / 1-x/2

Trick 2: Recognize telescoping series.

Example. $\sum_{k=2}^{\infty} \ln\left(\frac{k}{k+1}\right)$

Series whose partial sums eventually have a fixed number of terms after cancelling

**form: (something-something)

Step 1: Get in formΣ(n=2, infinity) lnk - ln(k+1)

Step 2: Plug in n values to get partial sums:
(ln2 -ln3)+(ln3-ln4)+(ln4+ln5)+.......+(ln(n-1)-ln(n))+(ln(n)-ln(n+1))

Step 3: Look for cancellations:
ln2 - ln(n+1)

Step 4: lim n->infinity ln2 - ln(n+1)
Goes to -infinity, Diverges

Example. $\sum_{n=2}^{\infty} \frac{3}{n^2-1}$

3Σ(n=2, infinity) 1/(n+1)(n-1)

A/n+1 + B/n-1

1= A(n-1)+B(n+1) -> A = 1/2
1= A(n-1)+B(n+1) -> B = -1/2

3Σ(n=2, infinity) 1/2 / n-1 - 1/2/n+1
3[(DNE - 1) + (1/2 - 3/2) + (1-2) + (3/2, 5/2)+ .... + (1/2/n-2 - 1/2/n) + (1/2/n-1) - 1/2 / n+1]

-1/2 / n - 1/2 /n+1

Trick 3: Use Limit Laws.

Fact. If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$, then

$$\sum_{n=1}^{\infty} a_n + b_n =$$

$$\sum_{n=1}^{\infty} a_n - b_n =$$

$$\sum_{n=1}^{\infty} c \cdot a_n =$$

where c is a constant.

Example. Does the series converge or diverge? If it converges, to what?

$$\sum_{n=1}^{\infty} \frac{4 \cdot 5^n - 5 \cdot 4^n}{6^n}$$

Σ(n=1, infinity) 4 * 5^n / 6^n - Σ(n=1, infinity) 5 * 4^n / 6^n

= 4 (5/6)^n - 5 (4/6)^n

r = 5/6, 4/6

a = 20/6, 20/6

((20/6) / 1- 5/6) - ((20/6) / 1-4/6)

Question. True or False: If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty}(a_n + b_n)$ diverges.

True

c+c = c
c+d = d
c-c = C

Question. True or False: If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty}(a_n + b_n)$ diverges.

Inconclusive

False

Question. True or False: If $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=5}^{\infty} a_n$.

False

Question. True or False: If $\sum_{n=5}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

True

Σ(n=1, infinity) an + Σ(n=5, infinity) an
B+A

a1 + a2 + a3 + a4 + A

Question. True or False: If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$, then $\sum_{n=1}^{\infty} a_n \cdot b_n = A \cdot B$

False

Question. True or False: If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$, then $\sum_{n=1}^{\infty} \frac{a_n}{b_n} = \frac{A}{B}$.

False