CER Übung 01

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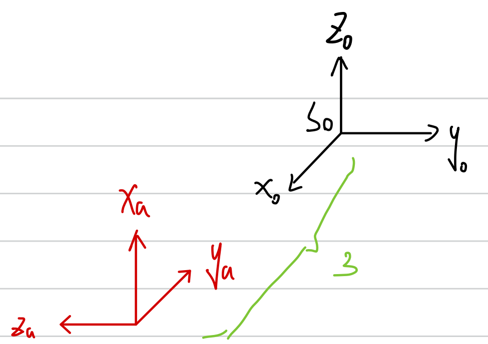
Aufgabe 1:

a):



$${}^0\underline{r}_a = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

$${}^0\underline{\underline{R}}_a = \begin{bmatrix} {}^0\underline{e}_a^x \cdot {}^0\underline{e}_0^x & {}^0\underline{e}_a^y \cdot {}^0\underline{e}_0^x & {}^0\underline{e}_a^z \cdot {}^0\underline{e}_0^x \\ {}^0\underline{e}_a^x \cdot {}^0\underline{e}_0^y & {}^0\underline{e}_a^y \cdot {}^0\underline{e}_0^y & {}^0\underline{e}_a^z \cdot {}^0\underline{e}_0^y \\ {}^0\underline{e}_a^x \cdot {}^0\underline{e}_0^z & {}^0\underline{e}_a^y \cdot {}^0\underline{e}_0^z & {}^0\underline{e}_a^z \cdot {}^0\underline{e}_0^z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Projektion von a nach 0

$$\Rightarrow \quad {}^0\underline{\underline{T}}_a = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b): Projektionsfolge umgekehrt wie Rotationsfolge : (III) → (II) → (I)

$${}^0\underline{\underline{T}}_b = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) $${}^0\underline{\underline{R}}_a^T = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow -{}^0\underline{\underline{R}}_a^T \cdot {}^0\underline{r}_a = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad {}^0\underline{\underline{T}}_a^{-1} = {}^a\underline{\underline{T}}_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $${}^a\underline{\underline{T}}_b = {}^a\underline{\underline{T}}_0 \cdot {}^0\underline{\underline{T}}_b$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e) $\underline{\hat{P}_1} = {}^0\underline{T}_a \cdot \underline{\hat{P}} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 1 \end{pmatrix}$

$\underline{\hat{P}_2} = {}^a\underline{T}_0 \cdot \underline{\hat{P}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}$

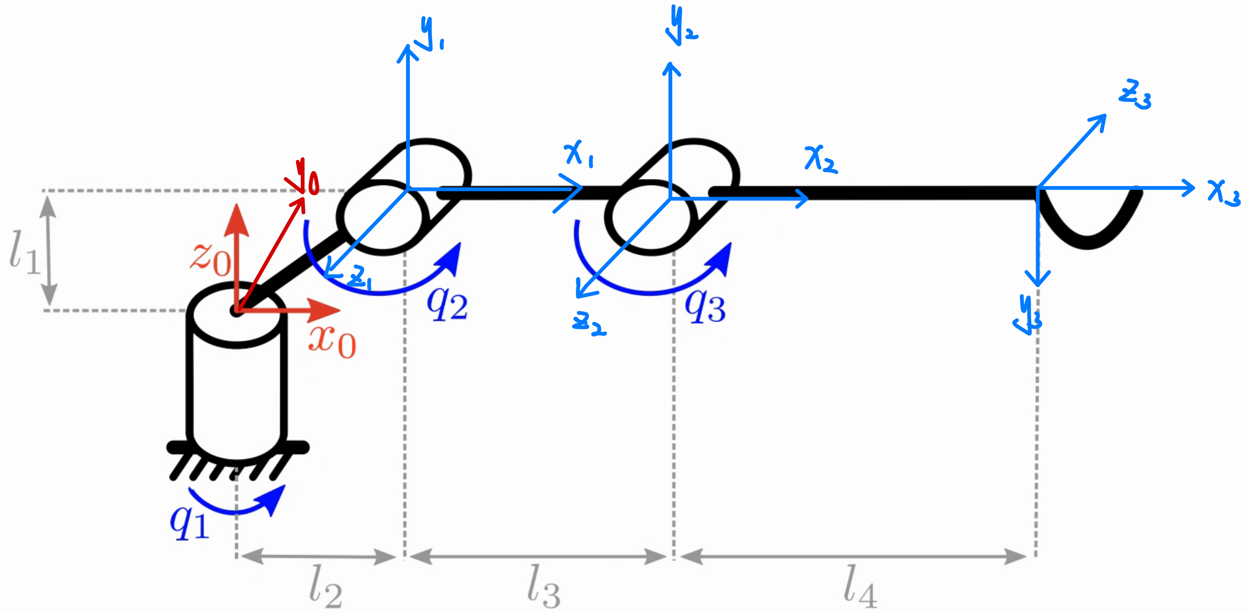
$\because |P_1| \neq |P_2|$

$\therefore$ Punkt bewegt

Vorlage für Aufgabe 2 a)

q um z Achse (Hier als null betrachtet)

a um x Achse



b) ${}^{n-1}\underline{T}_n = \begin{bmatrix} \cos\theta_n & -\sin\theta_n \cos\alpha_n & \sin\theta_n \sin\alpha_n & r_n \cos\theta_n \\ \sin\theta_n & \cos\theta_n \cos\alpha_n & -\cos\theta_n \cdot \sin\alpha_n & r_n \sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$ mit $r_n = a_i$, $d_n = d_i$

① $\Rightarrow {}^0\underline{T}_1 = \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & \cos q_1 \\ \sin q_1 & 0 & -\cos q_1 & \sin q_1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ${}^1\underline{T}_2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 2\cdot\cos q_2 \\ \sin q_2 & \cos q_2 & 0 & 2\cdot\sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

② $\Rightarrow \; {}^0\underline{T}_2 = {}^0\underline{T}_1 \cdot {}^1\underline{T}_2$

$$= \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & \cos q_1 \\ \sin q_1 & 0 & -\cos q_1 & \sin q_1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 2\cdot\cos q_2 \\ \sin q_2 & \cos q_2 & 0 & 2\cdot\sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos q_1\cdot\cos q_2 & -\cos q_1\cdot\sin q_2 & \sin q_1 & \cos q_1\cdot(2\cdot\cos q_2+1) \\ \sin q_1\cdot\cos q_2 & -\sin q_1\cdot\sin q_2 & -\cos q_1 & \sin q_1\,(2\cos q_2+1) \\ \sin q_2 & \cos q_2 & 0 & 2\sin q_2+1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ ${}^0\hat{\underline{r}}_3 = {}^0\underline{T}_2\,{}^2\hat{\underline{r}}_3$ mit ${}^2\underline{r}_3 = {}^2\underline{\underline{R}}_3\,{}^3\underline{r}_3$

$$\Rightarrow {}^2\underline{r}_3 = \begin{bmatrix} \cos q_3 & \sin q_3 & 0 \\ \sin q_3 & -\cos q_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 3\cos q_3 \\ 3\sin q_3 \\ 0 \end{pmatrix}$$

$$\Rightarrow {}^0\hat{\underline{r}}_3 = \begin{bmatrix} \cos q_1\cdot\cos q_2 & -\cos q_1\cdot\sin q_2 & \sin q_1 & \cos q_1\cdot(2\cdot\cos q_2+1) \\ \sin q_1\cdot\cos q_2 & -\sin q_1\cdot\sin q_2 & -\cos q_1 & \sin q_1\,(2\cos q_2+1) \\ \sin q_2 & \cos q_2 & 0 & 2\sin q_2+1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} 3\cos q_3 \\ 3\sin q_3 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos q_1\cdot[3\cos(q_2+q_3)+2\cos q_2+1] \\ \sin q_1\cdot[3\cos(q_2+q_3)+2\cos q_2+1] \\ 3\sin(q_2+q_3)+1 \\ 1 \end{pmatrix} \Rightarrow {}^0\underline{r}_3 = \begin{pmatrix} \cos q_1\cdot[3\cos(q_2+q_3)+2\cos q_2+1] \\ \sin q_1\cdot[3\cos(q_2+q_3)+2\cos q_2+1] \\ 3\sin(q_2+q_3)+1 \end{pmatrix}$$

④ ${}^0\underline{T}_3 = {}^0\underline{T}_2\cdot{}^2\underline{T}_3$

c): $q_1 = 0,\; q_2 = -\frac{\pi}{4},\; q_3 = \frac{\pi}{8} \Rightarrow {}^0\underline{r}_3 = \begin{pmatrix} 5{,}18585 \\ 0 \\ -1{,}14805 \end{pmatrix}$

d): ${}^0\underline{r}_3 = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$

① $\cos q_1 = 0 \quad \sin q_1 = 1 \Rightarrow q_1 = \frac{\pi}{2}$

② $\begin{cases} 3\cos q_3 + 2 + 1 = 6 \\ 3\sin q_3 + 1 = 1 \end{cases} \Rightarrow \begin{cases} \cos q_3 = 1 \\ \sin q_3 = 0 \end{cases} \Rightarrow q_3 = 0$

e) Wenn ${}^{0}r_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} \cos q_1 = 0 \\ \sin(q_2+q_3) = -\frac{1}{3} \end{cases} \Rightarrow q_1 = \frac{\pi}{2}$

für zweite row: $3\cos(q_2+q_3) + 2\cos q_2 + 1 = 1$

$\Rightarrow 3\cos(q_2+q_3) = -2\cos q_2$

mit $\cos(q_2+q_3) = \sqrt{1-\frac{1}{9}} = \pm\frac{2\sqrt{2}}{3}$

$\Rightarrow \cos q_2 = \pm\sqrt{2}$ für Mathmatik unmöglich

da $\cos\theta \in [-1, 1]$