

Assignment 1

Reading Assignment:

1. Chapter 1: Mathematical Review;
2. Chapter 2: Combinatorics.

Problems:

1. (a) How many different 7-place license plates are possible if the first 2 places are for letters and the other 5 for numbers?
(b) Repeat part (a) under the assumption that no letter or number can be repeated in a single license plate.
2. How many outcome sequences are possible when a die is rolled four times, where we say, for instance, that the outcome is 3, 4, 3, 1 if the first roll landed on 3, the second on 4, the third on 3, and the fourth on 1?
3. Fix two sets A and C . If $C \subset A$, show that for every set B ,

$$(A \cap B) \cup C = A \cap (B \cup C). \quad (1)$$

Also show that if (1) holds for some set B , then $C \subset A$ (and thus (1) holds for all sets B).

4. Consider rolling a six-sided die. Let A be the set of outcomes where the roll is an even number. Let B be the set of outcomes where the roll is greater than 3. Calculate and compare the sets on both sides of De Morgan's laws

$$(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c.$$

5. We deal from a well-shuffled 52-card deck. Calculate the probability that the 13th card is the first king to be dealt.
6. Five separate awards (best scholarship, best leadership qualities, ...) are to be presented to selected students from a class of 30. How many different outcomes are possible if
 - (a) a student can receive any number of awards;
 - (b) each student can receive at most 1 award?

Programming Challenges (optional):

1. Write a short C/C++ program that, upon request, generates a random number between 1 and 10.

Optional Problems:

1. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if
 - (a) the books can be arranged in any order;
 - (b) the mathematics books must be together and the novels must be together;
 - (c) the novels must be together but the other books can be arranged in any order?
2. A total of 7 different gifts are to be distributed among 10 children. How many distinct results are possible if no child is to receive more than one gift?