Problem 1) Seven prisoners are given the chance to be set free tomorrow. An executioner will put a hat on each prisoner's head. Each hat can be one of the seven colors of the rainbow and the hat colors are assigned completely at the executioner's discretion. Every prisoner can see the hat colors of the other six prisoners, but not his own. They cannot communicate with others in any form, or else they are immediately executed. Then each prisoner writes down his guess of his own hat color. If at least one prisoner correctly guesses the color of his hat, they all will be set free immediately; otherwise they will be executed. They are given the night to come up with a strategy. Is there a strategy that they can guarantee that they will be set free?

Problem 2) Assuming that temperature is a continuous function, show that at any given time on the earth's equator, there are two points directly opposite points that have the same temperature.

Problem 3) The n consecutive integers from 1 to n are written in a row. When can you place signs + and − in front of them so that the expression obtained is equal to 0?

Problem 4) I'm going to give each of you a hat to wear that is either black or white. You cannot see a hat on your head, but you can see a hat on anyone else's head. Your Goal as a group is to organize yourselves in a line, with all the white-hatted folks on one end, and all the black-hatted folks on the other end. Note that after I give you your hats, you cannot communicate with (nor touch) anyone. However, before I give you the hats you may jointly decide on a strategy. Devise a strategy that will achieve the Goal. Can you find a symmetric strategy, i.e., one that is the same for all players?

Hints:

1. Let's assign the 7 colors rainbow with code 0-6 and x_i be the color code of prisoner i. Then $\sum_{i=1}^{7}$ mod 7 must be must be in $\{0, 1, 2, 3, 4, 5, 6\}$ how many guesses can 7 prisoners make?
2. Use temperature and location on the equator to define a new function whose roots may be important.
3. Experiment with small values of n. Is there a pattern?