Assignment 01

AERO 455 - CFD for Aerospace Applications

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Given: 29th January 2022
Due: 5pm EST on 14th February 2022

1 The Source Panel Method

1.1 Programming a source panel method

Write a 2D source panel code to calculate the solution for the Laplacian equation of the potential function ϕ which describes incompressible, inviscid and irrotational flow,

$$\nabla^2 \phi = 0. \tag{1.1}$$

You need to follow the procedure outlined in the source panel section of lecture 04, from slide 22 to slide 39, for the generic panel geometry in figure 8. The general program flow is the following,

1. Using the Python programming language program a function that programs equation 43, which is required to assemble the normal velocity boundary condition (equation 29) at the panel control points as induced by the potential function of another panel. *(20 points)*

2. This will allow you to assemble this boundary condition for all panels leading to n equations for n λ_i unknows, where λ_i is the source strength per unit length at a panel's control point. *(5 points)*

3. This well-defined system must be solved using the appropriate methods from the Python libraries. *(15 points)*

4. At this point you should program another function for the accuracy check of equation 35. *(5 points)*

5. Then program another function that allows you to assemble the velocity integral in equation 44. *(20 points)*

6. Based on equation 44 and equation 32, program a function that calculates the surface velocity at every panel control point. *(5 points)*

7. Then program the last function to calculate the pressure coefficient at every control point using Bernouilli's law as given by equation 34. *(5 points)*

1.2 Validation of the source panel method code

Validate the code you wrote in section 1.1 by repeating the 2D cylinder example in slides 40 to 44.

1. Use 4, 6, 8 and 10 panels to discretize the cylinder surface and compare the accuracy of each number of panels. *(10 points)*

2. Keep increasing the number of panels. How far can you go and why? Plot the residual against the number of panels. *(5 points)*

3. The analytical solution in polar coordinates for the potential function of a uniform flow around a circular cylinder is,

$$\phi(r,\theta) = u_\infty r \left(1 + \frac{R^2}{r^2}\right) \cos\theta, \tag{1.2}$$

   where u_∞ is the freestream velocity magnitude. From equation 1.2 one can extract the velocity components in polar coordinates,

$$u_r = \frac{\partial \phi}{\partial r} = u_\infty \left(1 - \frac{R^2}{r^2}\right) \cos\theta. \tag{1.3}$$

$$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -u_\infty \left(1 + \frac{R^2}{r^2}\right) \sin\theta. \tag{1.4}$$

   Compare the velocity magnitude obtained from the panel code to the one from the analytical solution for all discretizations. *(5 points)*

4. Plot the pressure coefficient at the surface of the cylinder against the angle θ for all discretizations, and compare to the analytical solution given by,

$$c_p = 2\frac{R^2}{r^2}\cos(2\theta) - \frac{R^4}{r^4} \tag{1.5}$$

(5 points)