Assignment 02

AERO 455 - CFD for Aerospace Applications

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Given: 27th February 2022
Due: 5pm EST on 12th March 2022
No extensions

1 Finite Difference Schemes (45 points)

In lecture 05, slides 10 to 28, you have seen how to derive a finite difference schemes for the first and second derivatives for arbitrary orders of accuracy and different stencils. The procedure was also generalized.

1.1 Second-order first derivative schemes

1.1.1 The forward three-point scheme

(10 points)
Following the same procedure given in the lecture derive a *second-order forward* finite difference scheme for the first derivative using a three-point stencil. You will need to set up Taylor expansions for the solution variable q at points x_{i+1} and x_{i+2}.

1.1.2 The backward three-point scheme

(10 points)
Repeat the same procedure and derive a *second-order backward* finite difference scheme for the first derivative using a three-point stencil. You will need to set up Taylor expansions for the solution variable q at points x_{i-1} and x_{i-2}.

1.2 First-order second derivative schemes

1.2.1 The forward three-point scheme

(10 points)
Following the same procedure derive a *first-order forward* finite difference scheme for the second derivative using a three-point stencil. You will need to set up Taylor expansions for the solution variable q at points x_{i+1} and x_{i+2}.

1.2.2 The backward three-point scheme

(10 points)
Repeat the same procedure and derive a *first-order backward* finite difference scheme for the second derivative using a three-point stencil. You will need to set up Taylor expansions for the solution variable q at points x_{i-1} and x_{i-2}.

1.3 Scheme validity

(5 points)
How do you know whether your schemes are correct? Is there a quick check you can perform to find out? If so, show how for all four schemes derived.

2 Advanced Concepts (55 points)

All questions below relate to lecture 07.

2.1 Accuracy vs. Stability

Answer the following questions.

1. What is the typical trade-off between accuracy and stability for finite difference schemes? (2 points)

2. What is the Godunov theorem and which scheme introduced in the Godunov finite-volume method does it embody? (5 points)

3. What is the Iserles barrier theorem and which natural conclusion can be drawn from it for upwind schemes? (5 points)

4. What is the total variation of a variable q? Provide mathematical expressions of total variation of a continuous variable $q(x,t)$ and its discrete version q_i^t. (5 points)

5. What is a total variation diminishing scheme? Give an example of a scheme that is total variation diminishing, and an example of a scheme that is not. (5 points)

6. What is the MUSCL scheme, what is the motivation behind it and what is the order of accuracy of its original version as introduced by van Leer? (5 points)

2.2 Error analysis

Answer the following questions.

1. What is well-posedness of an initial value problem? Provide a mathematical expression of it. (3 points)
2. What is consistency of a finite difference scheme? Provide a mathematical expression of it. (3 points)
3. What does the convergence property of a finite difference scheme tell us? (2 points)
4. What are the sources of numerical error? (2 points)
5. What is the roundoff error? (2 points)

2.3 Stability analysis and time stepping

Answer the following questions.

1. Is the Neumann stability analysis linear or non-linear? (2 points)
2. What is a transient explicit scheme? (2 points)
3. What is a transient implicit scheme? (2 points)
4. What is the physical meaning of the Courant number? Give a couple of examples using linear advection and linear diffusion. (10 points)