Assignment 00

AERO 455 - CFD for Aerospace Applications

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Given: 15th January 2022
Due: 5pm EST on 28th January 2022

1 The Momentum Equations

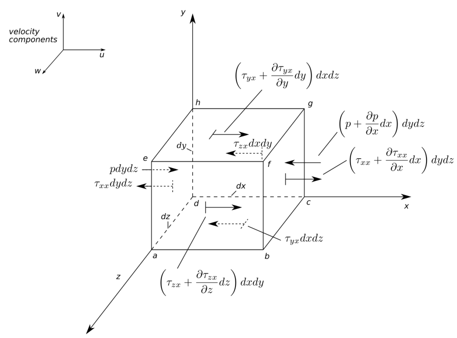

Figure 1.1: Infinitesimal fluid element with forces in the x-direction.

1.1 The x-momentum equation

The infinitesimal fluid element shown in figure 1.1 was used in the fluid dynamics review of lecture 03 to derive the differential x-momentum equation in non-conservation form for a moving viscous fluid element.

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_{b,x}. \tag{1.1}$$

1. Derive the differential x-momentum equation in *conservation* form starting from equation 1.1 by expanding the substantial derivative and using divergence vector identities to make the continuity equation appear so that it can be eliminated. Comment all your steps. (5 points)

2. Assuming a Newtonian fluid expand the force terms from the previous part using the dynamic viscosity and rate of strain tensor and explain what each term means. (10 points)

3. Derive the *integral* x-momentum equation in *conservation* form by integrating the differential x-momentum equation in conservation form derived in the previous part. Use the divergence theorem and comment all your steps. (5 points)

4. What is the difference between the substantial and local time derivatives and when should you use them? (2.5 points)

5. Which form of the momentum equations results directly from using a *fixed finite control volume*? (2.5 points)

1.2 The y-momentum equation in non-conservation form

1. Draw the infinitesimal fluid element shown in figure 1.1 for pressure and stress forces in the y-direction using the same sign convention from lecture 03. Explain what each force term means. (10 points)

2. Assuming a Newtonian fluid expand the force terms from the previous part using the dynamic viscosity and rate of strain tensor. (5 points)

3. Derive the y-momentum equation in *non-conservation* form based on the infinitesimal fluid element drawn in the previous part and following the procedure from lecture 03. Comment all your steps. (10 points)

1.3 The z-momentum equation in non-conservation form

1. Draw the infinitesimal fluid element shown in figure 1.1 for pressure and stress forces in the z-direction using the same sign convention from lecture 03. Explain what each force term means. (10 points)

2. Assuming a Newtonian fluid expand the force terms from the previous part using the dynamic viscosity and rate of strain tensor. (5 points)

3. Derive the z-momentum equation in *non-conservation* form based on the infinitesimal fluid element drawn in the previous part and following the procedure from lecture 03. Comment all your steps. (10 points)

2 The Energy Equation

In the lecture 03 of the fluid dynamics review we derived the total energy equation 2.1, with the total energy being the sum of the internal energy e and the mechanical energy $\frac{|\mathbf{v}|^2}{2}$.

$$
\begin{aligned}
\rho \frac{D}{Dt}\left(e + \frac{|\mathbf{v}|^2}{2}\right) = \rho \dot{q} &+ \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) \\
&- \frac{\partial (up)}{\partial x} - \frac{\partial (vp)}{\partial y} - \frac{\partial (wp)}{\partial z} \\
&+ \frac{\partial (u\tau_{xx})}{\partial x} + \frac{\partial (u\tau_{yx})}{\partial y} + \frac{\partial (u\tau_{zx})}{\partial z} \\
&+ \frac{\partial (v\tau_{xy})}{\partial x} + \frac{\partial (v\tau_{yy})}{\partial y} + \frac{\partial (v\tau_{zy})}{\partial z} \\
&+ \frac{\partial (w\tau_{xz})}{\partial x} + \frac{\partial (w\tau_{yz})}{\partial y} + \frac{\partial (w\tau_{zz})}{\partial z} \\
&+ \rho \mathbf{f} \cdot \mathbf{v}.
\end{aligned} \tag{2.1}
$$

1. Explain what each term in equation 2.1 means. (5 points)

2. Derive the differential *mechanical energy* equation in *non-conservation* form by multiplying the differential x-, y- and z-momentum equations in non-conservation form with the corresponding x-, y- and z-components of velocity respectively (u, v, w). You will need rewrite the time derivative terms in the multiplied momentum equations so as to obtain the substantial derivatives of $\frac{u^2}{2}$, $\frac{v^2}{2}$ and $\frac{w^2}{2}$. Keep in mind that $|\mathbf{v}|^2 = u^2 + v^2 + w^2$. Comment all your steps. (10 points)

3. Derive the differential *internal energy* equation in *non-conservation* form by substracting the differential mechanical energy equation obtained in the previous part from equation 2.1. Comment all your steps. (5 points)

4. Derive the differential *internal energy* equation in *conservation* form by expanding the substantial derivative and using divergence vector identities to make the continuity equation appear so it can be eliminated. Comment all your steps. (5 points)