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Points 10 Published

Details Questions

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Question 1

1.5 pts

In CFD, we often have to work with sparse matrices. Let us define the sparsity of a matrix as the ratio of zero-valued entries to the total number of entries in the matrix.

Q2_2.png

We attempt to solve the heat equation $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ on a **10-node uniform grid** as shown above.

As discussed in class, let us take the semi-discrete form of the heat equation with a **central-difference spatial scheme** for the second-order derivative

$$\frac{\partial \vec{u}}{\partial t} = \frac{\alpha}{\Delta x^2}\left(\mathbf{A}\vec{u} + \overrightarrow{\mathbf{BC}}\right)$$

where $\vec{u}$ is the solution vector, $\mathbf{A}$ is the finite difference coefficient matrix and $\overrightarrow{\mathbf{BC}}$ is the vector of boundary values.

The sparsity of matrix $\mathbf{A}$ is [numerator]/100.

Show Answers for numerator

swer 72

Question 2

0.5 pts

The leading truncation error terms in a finite difference approximation of the differential operator $\frac{\partial u}{\partial x}$ is

$$-\left(\frac{\partial^2 u}{\partial x^2}\right)_i \frac{\Delta x}{2} - \left(\frac{\partial^3 u}{\partial x^3}\right)_i \frac{(\Delta x)^3}{6}.$$

What is the order of accuracy of the finite difference equation?

swer

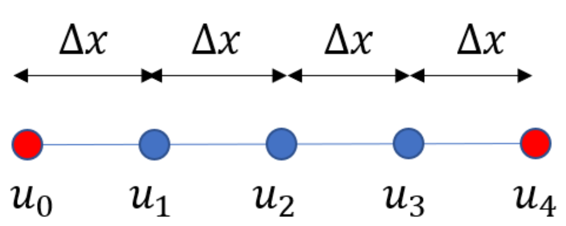
- 1
- 2
- 3
- 0.5

Question 3

3 pts

For the PDE $\frac{\partial^2 u(x)}{\partial x^2} + 2u(x) = 1,$

use the Central Difference Scheme to find the values of the solution variable u at the given points.



Given u_0 = 0, u_4 = 1 and Δx = 0.5

(fill in the integer values of the numerators. Denominators are fixed at 6)

u_1 =[u1]/6

u_2 =[u2]/6

u_3 =[u3]/6

Show Answers for u1

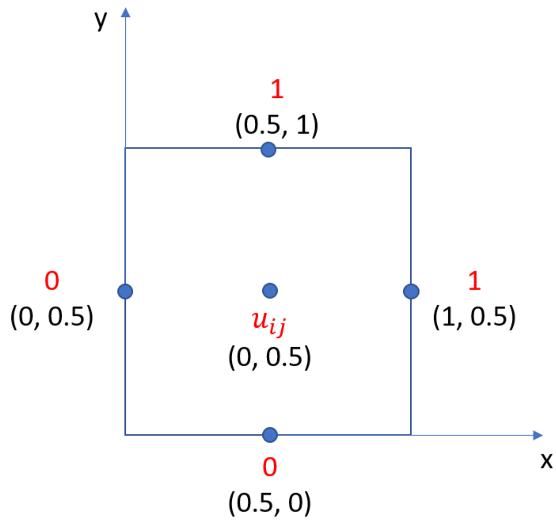
swer

1

Question 4

2 pts

Consider a finite difference solution of the Poisson equation $u_{xx} + u_{yy} = x + y$ on the unit square using the boundary conditions and the mesh shown in the drawing. Use a second-order accurate, centred finite difference scheme to compute the approximate value of the solution at the centre of the square. (correct to **four decimal places**).



Note : The boundary conditions are in red. The tuples below them are the cartesian coordinates of the point. Find the solution value at the centre (u_{ij}).

ıswers 0.4375 (with margin: 0.0005)

Question 5

3 pts

	u_j	u_j'	u_j''	u_j'''	u_j''''
u_j'''	0	0	0	1	0
$\alpha_{-2}u_{j-2}$	α_{-2}	$-2\Delta x\alpha_{-2}$	$\frac{4\Delta x^2}{2}\alpha_{-2}$	$\frac{-8\Delta x^3}{6}\alpha_{-2}$	$\frac{16\Delta x^4}{24}\alpha_{-2}$
$\alpha_{-1}u_{j-1}$	α_{-1}	$-\Delta x\alpha_{-1}$	$\frac{\Delta x^2}{2}\alpha_{-1}$	$\frac{-\Delta x^3}{6}\alpha_{-1}$	$\frac{\Delta x^4}{24}\alpha_{-1}$
$\alpha_0 u_j$	α_0	0	0	0	0
$\alpha_1 u_{j+1}$	α_1	$\Delta x\alpha_1$	$\frac{\Delta x^2}{2}\alpha_1$	$\frac{\Delta x^3}{6}\alpha_1$	$\frac{\Delta x^4}{24}\alpha_1$

Let us approximate the third derivative u_j''' using the solution variables a **4-point stencil** (u_{j-2}, u_{j-1}, u_j and u_{j+1}).

Proceeding as in class, we determine the coefficients α_{-2},α_{-1},α_0 and α_1 by equating the column-wise sums to zero.

$$\begin{array}{l} \text{column 1 (RHS 1)} \longrightarrow \\ \text{column 2 (RHS 2)} \longrightarrow \\ \text{column 3 (RHS 3)} \longrightarrow \\ \text{column 4 (RHS 4)} \longrightarrow \end{array} \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} \alpha_{-2} \\ \alpha_{-1} \\ \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-6}{\Delta x^3} \end{bmatrix}$$

Fill in the missing (**correct to one decimal place**) values of the elements of the matrix A above.

A_{11} = 1; A_{12} = [A12]; A_{13} = [A13]; A_{14} = [A14]

A_{21} = [A21]; A_{22} = -1; A_{23} = [A23]; A_{24} = [A24]

A_{31} = 2; A_{32} = [A32]; A_{33} = [A33]; A_{34} = [A34]

A_{41} = [A41]; A_{42} = [A42]; A_{43} = [A43]; A_{44} = 1

Show Answers for A12

ıswer 1

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