Problem Set 2

MECH 479/587 - Computational Fluid Dynamics
Winter Term 1

Due: October 11, 2022

1 Discretizing the First Order Derivative (15 marks)

Given sufficient continuity of a function and its derivatives, Taylor's series allows for constructing derivatives of any order. Furthermore, by considering sufficient information about the function u, a derivative of k^{th} order can be constructed to an arbitrary order of accuracy. Given the stencil $\{i-2, i-1, i, i+1\}$, and the representation of the first order derivative:

$$\left.\frac{\partial u}{\partial x}\right|_i = \left(\frac{\alpha u_{i+1} + \beta u_i + \gamma u_{i-1} + \delta u_{i-2}}{\Delta x}\right) + O(\Delta x^p), \tag{1}$$

construct the family of second-order schemes and the corresponding truncation errors obtained in terms of the parameter β. Given the construction, answer the followings:

1. What will be the ordered set or parameters $(\alpha, \beta, \gamma, \delta)$ corresponding to the second-order central difference scheme?
2. What will be a second-order accurate scheme by setting $\alpha = 0$ in eq. (1)? Please provide the ordered set (β, γ, δ).
3. For the third-order accurate scheme on the given stencil, what will be the corresponding ordered set $(\alpha, \beta, \gamma, \delta)$?

2 Alternative way to Discretize the First Order Derivative (15 marks)

High gradients in momentum or density can be present in fluid flow problems. There is a need to capture these gradients with a high level of accuracy, either owing to the nature of the flow physics or due to computational resource limitations. Compact high-order finite difference schemes are generally preferred in such situations, where the information about the derivatives at two or more computational points is simultaneously unknown. A general form a compact scheme for the first derivative is given by:

$$\alpha \left.\frac{\partial u}{\partial x}\right|_{i-1} + \left.\frac{\partial u}{\partial x}\right|_i + \alpha \left.\frac{\partial u}{\partial x}\right|_{i+1} = \beta\left(\frac{u_{i+1} - u_{i-1}}{2\Delta x}\right) + \gamma\left(\frac{u_{i+2} - u_{i-2}}{4\Delta x}\right) + O(\Delta x^p). \tag{1}$$

Observe that by setting $(\alpha, \beta, \gamma) = (0, 1, 0)$, we recover the second order central difference scheme for the first order derivative. Given this construction, answer the followings:

1. Let $\alpha = 0$. Obtain (β, γ) such that a fourth-order approximation to the first-order derivative is obtained. Give the truncation error obtained.

2. Let $\gamma = 0$. Obtain (α, β) such that a fourth-order approximation to the first-order derivative is obtained. Give the truncation error obtained.

3 Assessment of Stability Limit (20 marks)

Consider the viscous Burgers' equation (a prototypical equation in 1-D for incompressible Navier-Stokes equations) in a periodic domain of length 1:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \tag{1}$$

$$u(x,0) = 1 + \sin(2\pi x), \tag{2}$$

$$u(1,t) = u(0,t), \tag{3}$$

where $\nu = 0.01$. Consider two different discrete domains with $N = 20$ and $N = 40$ points respectively.

1. Using a forward discretization (Euler explicit) for the time derivatives, and central discretization for the first and second order spatial derivatives, evaluate the solution at $t = 1.0$ using $\Delta t = 5 \times 10^{-3}$.

2. For each of the grids, conduct numerical experiments to determine the maximum value of Δt for which the solution remains stable. Present your results in Tabular form.

Note:

- **A code has been provided as an example for problem 3.**
- **All assignments should be submitted through Canvas.**

$$\left.\frac{\partial u}{\partial x}\right|_i = \left(\frac{\alpha U_{i+1} + \beta U_i + \gamma U_{i-1} + \delta U_{i-2}}{\Delta x}\right) + O(\Delta x^p)$$

$$U_{i+1} = U_i + \Delta x (U_x)_i + \frac{\Delta x^2}{2!}(U_{xx})_i + \frac{\Delta x^3}{3!}(U_{xxx})_i + \dots$$

$$U_{i-1} = U_i - \Delta x (U_x)_i + \frac{\Delta x^2}{2!}(U_{xx})_i - \frac{\Delta x^3}{3!}(U_{xxx})_i + \dots$$

1.1) To isolate $(U_x)_i$, we can perform the following calculation

$$U_{i+1} - U_{i-1} = \left(\cancel{U_i} + \Delta x (U_x)_i + \cancel{\frac{\Delta x^2}{2!}(U_{xx})_i} + \frac{\Delta x^3}{3!}(U_{xxx})_i + \dots\right)$$
$$- \left(\cancel{U_i} - \Delta x (U_x)_i + \cancel{\frac{\Delta x^2}{2!}(U_{xx})_i} - \frac{\Delta x^3}{3!}(U_{xxx})_i + \dots\right)$$

$$U_{i+1} - U_{i-1} = 2\Delta x (U_x)_i + \frac{2\Delta x^3}{3!}(U_{xxx})_i + \dots$$

$$U_{i+1} - U_{i-1} - \frac{2\Delta x^3}{3!}(U_{xxx})_i = 2\Delta x (U_x)_i$$

$$(U_x)_i = \frac{U_{i+1} - U_{i-1}}{2\Delta x} - \frac{\cancel{2}\Delta x^{\cancel{3}}}{(\cancel{2}\cancel{\Delta x})\cdot 3!}(U_{xxx})_i = \frac{U_{i+1} - U_{i-1}}{2\Delta x} - O(\Delta x^2)$$

Compare to $\left.\frac{\partial u}{\partial x}\right|_i = \left(\frac{\alpha U_{i+1} + \beta U_i + \gamma U_{i-1} + \delta U_{i-2}}{\Delta x}\right) + O(\Delta x^p)$

$$\alpha = \frac{1}{2},\ \beta = 0,\ \gamma = -\frac{1}{2},\ \delta = 0$$

1.2) find Taylor Series for U_{i-2}

$$U_{i-2} = U_i - 2\Delta x (U_x)_i + \frac{4\Delta x^2}{2!}(U_{xx})_i - \frac{8\Delta x^3}{3!}(U_{xxx})_i + \dots$$

$$\left.\frac{\partial u}{\partial x}\right|_i = \left(\frac{\alpha u_{i+1} + \beta u_i + \gamma u_{i-1} + \delta u_{i-2}}{\Delta x}\right) + O(\Delta x^p)$$

$$\Delta x \left.\frac{\partial u}{\partial x}\right|_i = \alpha u_{i+1} + \beta u_i + \gamma u_{i-1} + \delta u_{i-2} + O(\Delta x^{p+1})$$

For 1.2, $\alpha = 0$

$$\Delta x \left.\frac{\partial u}{\partial x}\right|_i = \beta u_i + \gamma u_{i-1} + \delta u_{i-2} + O(\Delta x^{p+1})$$

$$\Delta x \left.\frac{\partial u}{\partial x}\right|_i = \beta u_i + \gamma\left(u_i - \Delta x (u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i - \frac{\Delta x^3}{3!}(u_{xxx})_i + \dots\right) + \delta\left(u_i - 2\Delta x (u_x)_i + \frac{4\Delta x^2}{2!}(u_{xx})_i - \frac{8\Delta x^3}{3!}(u_{xxx})_i\right)$$

$$\Delta x (u_x)_i = \beta u_i + \gamma u_i - \gamma \Delta x (u_x)_i + \gamma \frac{\Delta x^2}{2!}(u_{xx})_i - \gamma \frac{\Delta x^3}{3!}(u_{xxx})_i + \dots + \delta u_i - 2\delta \Delta x (u_x)_i + \frac{4}{2!}\delta \Delta x^2 (u_{xx})_i - \frac{8}{3!}\delta \Delta x^3 (u_{xxx})_i + \dots$$

$$\beta u_i + \gamma u_i + \delta u_i = 0 \quad [1]$$

$$-\gamma \Delta x (u_x)_i - 2\delta \Delta x (u_x)_i = \Delta x (u_x)_i \rightarrow$$

$$-\gamma \Delta x (u_x)_i - 2\delta \Delta x (u_{x})_i - \Delta x (u_x)_i = 0 \quad [2]$$

$$\gamma \frac{\Delta x^2}{2!}(u_{xx})_i + \frac{4}{2!}\delta \Delta x^2 (u_{xx})_i = 0 \quad [3]$$

$$\beta u_i + \gamma u_i + \delta u_i = 0 \quad [1]$$

$$-\gamma \Delta x (u_x)_i - 2\delta \Delta x (u_x)_i = \Delta x (u_x)_i \rightarrow$$

$$-\gamma \Delta x (u_x)_i - 2\delta \Delta x (u_x)_i - \Delta x (u_x)_i = 0 \quad [2]$$

$$\gamma \frac{\Delta x^2}{2!} (u_{xx})_i + \frac{4}{2!}\delta \Delta x^2 (u_{xx})_i = 0 \quad [3]$$

$$\begin{array}{cccc} \beta & \gamma & \delta & \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -\Delta x(u_x)_i & -2\Delta x (u_x)_i & \Delta x(u_x)_i \\ 0 & \frac{\Delta x^2}{2!}(u_{xx})_i & \frac{4}{2!}\Delta x^2 (u_{xx})_i & 0 \end{array}\right] & & & \end{array} \underset{\sim}{\xrightarrow{-\frac{1}{\Delta x (u_x)_i} R_2 \to R_2}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & \frac{\Delta x^2}{2!}(u_{xx})_i & \frac{4}{2!}\Delta x^2 (u_{xx})_i & 0 \end{array}\right] \underset{\sim}{\xrightarrow{\frac{2!}{\Delta x^2 (u_{xx})_i} R_3 \to R_3}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 4 & 0 \end{array}\right] \underset{\sim}{\xrightarrow{-R_2 + R_1 \to R_1}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 4 & 0 \end{array}\right] \underset{\sim}{\xrightarrow{-R_2 + R_3 \to R_3}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & -1 \end{array}\right] \underset{\sim}{\xrightarrow{\frac{1}{2} R_3 \to R_3}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \underset{\sim}{\xrightarrow{R_3 + R_1 \to R_1}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1/2 \end{array}\right] \underset{\sim}{\xrightarrow{-2R_3 + R_2 \to R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

$$\beta = 3/2, \quad \gamma = -2, \quad \delta = 1/2$$

1.3) Same as 1.2) but $\alpha \neq 0$

$$u_{i+1} = u_i + \Delta x(u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i + \frac{\Delta x^3}{3!}(u_{xxx})_i + \ldots$$

$$u_{i-1} = u_i - \Delta x(u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i - \frac{\Delta x^3}{3!}(u_{xxx})_i + \ldots$$

$$u_{i-2} = u_i - 2\Delta x(u_x)_i + \frac{4\Delta x^2}{2!}(u_{xx})_i - \frac{8\Delta x^3}{3!}(u_{xxx})_i + \ldots$$

$$\Delta x(u_x)_i = \alpha u_{i+1} + \beta u_i + \gamma u_{i-1} + \delta u_{i-2} + O(\Delta x^{p+1})$$

$$\Delta x(u_x)_i = \alpha\left(u_i + \Delta x(u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i + \frac{\Delta x^3}{3!}(u_{xxx})_i + \ldots\right)$$
$$+ \beta u_i + \gamma\left(u_i - \Delta x(u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i - \frac{\Delta x}{3!}(u_{xxx})_i + \ldots\right)$$
$$+ \delta\left(u_i - 2\Delta x(u_x)_i + \frac{4\Delta x^2}{2!}(u_{xx})_i - \frac{8\Delta x^3}{3!}(u_{xxx})_i + \ldots\right)$$

$$\Delta x(u_x)_i = \alpha u_i + \alpha\Delta x(u_x)_i + \frac{\alpha\Delta x^2}{2!}(u_{xx})_i + \frac{\alpha\Delta x^3}{3!}(u_{xxx})_i + \ldots$$
$$+ \beta u_i + \gamma u_i - \gamma\Delta x(u_x)_i + \frac{\gamma\Delta x^2}{2!}(u_{xx})_i - \frac{\gamma\Delta x^3}{3!}(u_{xxx})_i + \ldots$$
$$+ \delta u_i - 2\delta\Delta x(u_x)_i + \frac{4\delta\Delta x^2}{2!}(u_{xx})_i - \frac{8\delta\Delta x^3}{3!}(u_{xxx})_i + \ldots$$

$$\alpha \cancel{u_i} + \beta \cancel{u_i} + \gamma \cancel{u_i} + \delta \cancel{u_i} = 0 \quad [1]$$

$$\alpha \cancel{\Delta x}(\cancel{u_x})_i - \gamma \cancel{\Delta x}(\cancel{u_x})_i - 2\delta \cancel{\Delta x}(\cancel{u_x})_i = \cancel{\Delta x}(\cancel{u_x})_i \quad [2]$$

$$\alpha\frac{\cancel{\Delta x^2}}{\cancel{2!}}(\cancel{u_{xx}})_i + \frac{\gamma\cancel{\Delta x^2}}{\cancel{2!}}(\cancel{u_{xx}})_i + \frac{4\delta\cancel{\Delta x^2}}{\cancel{2!}}(\cancel{u_{xx}})_i = 0 \quad [3]$$

$$\alpha\frac{\cancel{\Delta x^3}}{\cancel{3!}}(\cancel{u_{xxx}})_i - \gamma\frac{\cancel{\Delta x^3}}{\cancel{3!}}(\cancel{u_{xxx}})_i - \delta\frac{8\cancel{\Delta x^3}}{\cancel{3!}}(\cancel{u_{xxx}})_i = 0 \quad [4]$$

$$\alpha + \beta + \gamma + \delta = 0 \quad [1]$$

$$\alpha - \gamma - 2\delta = 1 \quad [2]$$

$$\alpha + \gamma + 4\delta = 0 \quad [3]$$

$$\alpha - \gamma - 8\delta = 0 \quad [4]$$

$$\begin{array}{cccc} \alpha & \beta & \gamma & \delta \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & -1 & -2 & 1 \\ 1 & 0 & 1 & 4 & 0 \\ 1 & 0 & -1 & -8 & 0 \end{array}\right] \begin{array}{l} -R_1 + R_2 \to R_2 \\ -R_1 + R_3 \to R_3 \\ -R_1 + R_4 \to R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1 \\ 0 & -1 & 0 & 3 & 0 \\ 0 & -1 & -2 & -9 & 0 \end{array}\right] \begin{array}{l} R_2 + R_1 \to R_1 \\ -R_2 + R_3 \to R_3 \\ -R_2 + R_4 \to R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 1 \\ 0 & -1 & -2 & -3 & 1 \\ 0 & 0 & 2 & 6 & -1 \\ 0 & 0 & 0 & -6 & -1 \end{array}\right] \begin{array}{l} R_3 + R_2 \to R_2 \\ \frac{1}{2}R_3 + R_1 \to R_1 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1/2 \\ 0 & -1 & 0 & 3 & 0 \\ 0 & 0 & 2 & 6 & -1 \\ 0 & 0 & 0 & -6 & -1 \end{array}\right] \begin{array}{l} 1/6\, R_4 + R_1 \to R_1 \\ R_4 + R_3 \to R_3 \\ \frac{1}{2}R_4 + R_2 \to R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1/3 \\ 0 & -1 & 0 & 0 & -1/2 \\ 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & -6 & -1 \end{array}\right] \begin{array}{l} -R_2 \to R_2 \\ 1/2\, R_3 \to R_3 \\ -1/6\, R_4 \to R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1/6 \end{array}\right]$$

$$\alpha = 1/3, \quad \beta = 1/2, \quad \gamma = -1, \quad \delta = 1/6$$

2)

$$\alpha (u_x)_{i-1} + (u_x)_i + \alpha (u_x)_{i+1} = \beta\left(\frac{u_{i+1}-u_{i-1}}{2\Delta x}\right) + \gamma\left(\frac{u_{i+2}-u_{i-2}}{4\Delta x}\right) + O(\Delta x^p)$$

2.1) $\alpha = 0$

$$(u_x)_i = \beta\left(\frac{u_{i+1}-u_{i-1}}{2\Delta x}\right) + \gamma\left(\frac{u_{i+2}-u_{i-2}}{4\Delta x}\right) + O(\Delta x^p)$$

$$u_{i\pm1} = u_i \pm \Delta x (u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i \pm \frac{\Delta x^3}{3!}(u_{xxx})_i + \dots$$

$$u_{i\pm2} = u_i \pm 2\Delta x (u_x)_i + \frac{4\Delta x^2}{2!}(u_{xx})_i \pm \frac{8\Delta x^3}{3!}(u_{xxx})_i + \dots$$

$$\Delta x (u_x)_i = \frac{\beta}{2}(u_{i+1}-u_{i-1}) + \frac{\gamma}{4}(u_{i+2}-u_{i-2}) + O(\Delta x^{p+1})$$

$$\Delta x (u_x)_i - \frac{\beta}{2}(u_{i+1}-u_{i-1}) - \frac{\gamma}{4}(u_{i+2}-u_{i-2}) = O(\Delta x^{p+1})$$

$$\Delta x (u_x)_i - \frac{\beta}{2}u_{i+1} + \frac{\beta}{2}u_{i-1} - \frac{\gamma}{4}u_{i+2} + \frac{\gamma}{4}u_{i-2} = O(\Delta x^{p+1})$$

	u_i	$(u_x)_i$	$(u_{xx})_i$	$(u_{xxx})_i$	$(u_4)_i$	$(u_5)_i$
$\Delta x (u_x)_i$	Δx	0	0	0	0	0
$-\frac{\beta}{2}u_{i+1}$	$-\frac{\beta}{2}$	$-\frac{\beta}{2}\Delta x$	$-\frac{\beta}{2}\cdot\frac{\Delta x^2}{2!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^3}{3!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^4}{4!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^5}{5}$
$\frac{\beta}{2}u_{i-1}$	$\frac{\beta}{2}$	$\frac{\beta}{2}\cdot -\Delta x$	$\frac{\beta}{2}\cdot\frac{\Delta x^2}{2!}$	$\frac{\beta}{2}\cdot\frac{-\Delta x^3}{3!}$	$\frac{\beta}{2}\cdot\frac{\Delta x^4}{4!}$	$\frac{\beta}{2}\cdot\frac{-\Delta x^5}{5!}$
$-\frac{\gamma}{4}u_{i+2}$	$-\frac{\gamma}{4}$	$-\frac{\gamma}{4}\cdot 2\Delta x$	$-\frac{\gamma}{4}\cdot\frac{4\Delta x^2}{2!}$	$-\frac{\gamma}{4}\cdot\frac{8\Delta x^3}{3!}$	$-\frac{\gamma}{4}\cdot\frac{16\Delta x^4}{4!}$	$-\frac{\gamma}{4}\cdot\frac{32\Delta x^5}{5!}$
$\frac{\gamma}{4}u_{i-2}$	$\frac{\gamma}{4}$	$\frac{\gamma}{4}\cdot -2\Delta x$	$\frac{\gamma}{4}\cdot\frac{4\Delta x^2}{2!}$	$\frac{\gamma}{4}\cdot\frac{-8\Delta x^3}{3!}$	$\frac{\gamma}{4}\cdot\frac{16\Delta x^4}{4!}$	$\frac{\gamma}{4}\cdot\frac{32\Delta x^5}{5!}$
LHS	RHS1	RHS2	RHS3	RHS4	RHS5	RHS6

	u_i	$(u_x)_i$	$(u_{xx})_i$	$(u_{xxx})_i$	$(u_4)_i$	$(u_5)_i$
$\Delta x (u_x)_i$	0	Δx	0	0	0	0
$-\frac{\beta}{2} u_{i+1}$	$-\frac{\beta}{2}$	$-\frac{\beta}{2} \Delta x$	$-\frac{\beta}{2} \cdot \frac{\Delta x^2}{2!}$	$-\frac{\beta}{2} \cdot \frac{\Delta x^3}{3!}$	$-\frac{\beta}{2} \cdot \frac{\Delta x^4}{4!}$	$-\frac{\beta}{2} \cdot \frac{\Delta x^5}{5}$
$\frac{\beta}{2} u_{i-1}$	$\frac{\beta}{2}$	$\frac{\beta}{2} \cdot -\Delta x$	$\frac{\beta}{2} \cdot \frac{\Delta x^2}{2!}$	$\frac{\beta}{2} \cdot \frac{-\Delta x^3}{3!}$	$\frac{\beta}{2} \cdot \frac{\Delta x^4}{4!}$	$\frac{\beta}{2} \cdot \frac{-\Delta x^5}{5!}$
$-\frac{\gamma}{4} u_{i+2}$	$-\frac{\gamma}{4}$	$-\frac{\gamma}{4} \cdot 2\Delta x$	$-\frac{\gamma}{4} \cdot \frac{4\Delta x^2}{2!}$	$-\frac{\gamma}{4} \cdot \frac{8\Delta x^3}{3!}$	$-\frac{\gamma}{4} \cdot \frac{16\Delta x^4}{4!}$	$-\frac{\gamma}{4} \cdot \frac{32\Delta x^5}{5!}$
$\frac{\gamma}{4} u_{i-2}$	$\frac{\gamma}{4}$	$\frac{\gamma}{4} \cdot -2\Delta x$	$\frac{\gamma}{4} \cdot \frac{4\Delta x^2}{2!}$	$\frac{\gamma}{4} \cdot \frac{-8\Delta x^3}{3!}$	$\frac{\gamma}{4} \cdot \frac{16\Delta x^4}{4!}$	$\frac{\gamma}{4} \cdot \frac{32\Delta x^5}{5!}$
LHS	RHS1	RHS2	RHS3	RHS4	RHS5	RHS6

RHS1: $-\frac{\beta}{2} + \frac{\beta}{2} - \frac{\gamma}{4} + \frac{\gamma}{4} = 0$ ✓

RHS2: $\cancel{\Delta x} - \frac{\beta}{2} \cancel{\Delta x} - \frac{\beta}{2} \cancel{\Delta x} - \frac{\gamma}{\cancel{4}} \cdot \cancel{2} \cancel{\Delta x} - \frac{\gamma}{\cancel{4}} \cdot \cancel{2} \cancel{\Delta x} = 0$

$$1 - \frac{\beta}{2} - \frac{\beta}{2} - \frac{\gamma}{2} - \frac{\gamma}{2} = 0$$

$$1 - \beta - \gamma = 0 \quad [1]$$

RHS3: $-\frac{\beta}{2} \cdot \frac{\Delta x^2}{2!} + \frac{\beta}{2} \frac{\Delta x^2}{2!} - \frac{\gamma}{4} \cdot \frac{4\Delta x^2}{2!} + \frac{\gamma}{4} \cdot \frac{4\Delta x^2}{2!} = 0$ ✓

RHS4: $-\frac{\beta}{2} \cdot \cancel{\frac{\Delta x^3}{3!}} - \frac{\beta}{2} \cdot \cancel{\frac{\Delta x^3}{3!}} - \frac{\gamma}{\cancel{4}} \cdot \cancel{8} \cancel{\frac{\Delta x^3}{3!}} - \frac{\gamma}{\cancel{4}} \cdot \cancel{\frac{8\Delta x^3}{3!}} = 0$

$$-\frac{\beta}{2} - \frac{\beta}{2} - 2\gamma - 2\gamma = 0 \rightarrow -\beta - 4\gamma = 0 \quad [2]$$

RHS5: $-\frac{\beta}{2} \frac{\Delta x^4}{4!} + \frac{\beta}{2} \frac{\Delta x^4}{4!} - \frac{\gamma}{4} \cdot \frac{16\Delta x^4}{4!} + \frac{\gamma}{4} \cdot \frac{16\Delta x^4}{4!} = 0$ ✓

RHS6: $-\frac{\beta}{2} \cdot \frac{\Delta x^5}{5!} - \frac{\beta}{2} \cdot \frac{\Delta x^5}{5!} - \frac{\gamma}{4} \cdot \frac{32\Delta x^5}{5!} - \frac{\gamma}{4} \cdot \frac{32\Delta x^5}{5!} =$

Solve for β & γ

$1-\beta-\gamma=0$ [1], $-\beta-4\gamma=0$ [2], $-\beta-16\gamma=0$ [3]

$$\begin{array}{cc} \beta & \gamma \\ \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & -4 & 0 \end{array}\right] \xrightarrow{R_1+R_2\to R_2} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -3 & 1 \end{array}\right] \xrightarrow{-1/3 R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1/3 \end{array}\right] \xrightarrow{-R_2+R_1\to R_1}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 4/3 \\ 0 & 1 & -1/3 \end{array}\right] \longrightarrow \beta = 4/3, \ \gamma = -1/3$$

find truncation error

$$RHS\ 6: -\frac{\beta}{2}\cdot\frac{\Delta x^5}{5!} - \frac{\beta}{2}\cdot\frac{\Delta x^5}{5!} - \frac{\gamma}{4}\cdot\frac{32\Delta x^5}{5!} - \frac{\gamma}{4}\cdot\frac{32\Delta x^5}{5!}$$

Plug β & γ found below in

$$-\frac{4/3}{2}\cdot\frac{\Delta x^5}{5!} - \frac{4/3}{2}\cdot\frac{\Delta x^5}{5!} - \frac{(-1/3)}{4}\cdot\frac{32\Delta x^5}{5!} - \frac{(-1/3)}{4}\cdot\frac{32\Delta x^5}{5!}$$

$$-\frac{4}{6}\frac{\Delta x^5}{5!} - \frac{4}{6}\frac{\Delta x^2}{5!} + \frac{1}{12}\cdot\frac{32\Delta x^5}{5!} + \frac{1}{12}\cdot\frac{32\Delta x^5}{5!}$$

$$-\frac{4}{3}\frac{\Delta x^5}{5!} + \frac{8}{3}\frac{\Delta x^5}{5!} + \frac{8}{3}\frac{\Delta x^5}{5!} = -\frac{4}{3}\frac{\Delta x^5}{5!} + \frac{16}{3}\frac{\Delta x^5}{5!}$$

$$= \frac{12}{3}\frac{\Delta x^5}{5!} = 4\frac{\Delta x^5}{5!} = \frac{4}{1\cdot 2\cdot 3\cdot \cancel{4}\cdot 5}\Delta x^5 = \frac{\Delta x^5}{30}(u_5)$$

2.2) $\gamma = 0 \rightarrow \alpha \frac{\partial u}{\partial x}\Big|_{i-1} + \frac{\partial u}{\partial x}\Big|_i + \alpha \frac{\partial u}{\partial x}\Big|_{i+1} = \beta\left(\frac{u_{i+1} - u_{i-1}}{2\Delta x}\right) + O(\Delta x^p$

$$\alpha (u_x)_{i-1} + (u_x)_i + \alpha (u_x)_{i+1} = \beta\left(\frac{u_{i+1} - u_{i-1}}{2\Delta x}\right) + O(\Delta x^p)$$

$$\alpha (u_x)_{i-1} + (u_x)_i + \alpha (u_x)_{i+1} - \frac{\beta}{2\Delta x}(u_{i+1} - u_{i-1}) = O(\Delta x^p)$$

$$\alpha (u_x)_{i-1} + (u_x)_i + \alpha (u_x)_{i+1} - \frac{\beta}{2\Delta x} u_{i+1} + \frac{\beta}{2\Delta x} u_{i-1} = O(\Delta x^p)$$

Write Taylor Expansions

$$u_{i\pm1} = u_i \pm \Delta x (u_x)_i + \frac{\Delta x^2}{2!}(u_{xx})_i \pm \frac{\Delta x}{3!}(u_{xxx})_i + \dots$$

$$(u_x)_{i\pm1} = (u_x)_i \pm \Delta x (u_{xx})_i + \frac{\Delta x^2}{2!}(u_{xxx})_i \pm \frac{\Delta x^3}{3!}(u_4)_i + \dots$$

	u_i	$(u_x)_i$	$(u_{xx})_i$	$(u_{xxx})_i$	$(u_4)_i$	$(u_5)_i$
$(u_x)_i$	0	1	0	0	0	0
$\alpha(u_x)_{i-1}$	0	α	$-\alpha\Delta x$	$\alpha\frac{\Delta x^2}{2!}$	$-\frac{\alpha\Delta x^3}{3!}$	$\frac{\alpha\Delta x^4}{4!}$
$\alpha(u_x)_{i+1}$	0	α	$\alpha\Delta x$	$\alpha\frac{\Delta x^2}{2!}$	$\frac{\alpha\Delta x^3}{3!}$	$\frac{\alpha\Delta x^4}{4!}$
$-\frac{\beta}{2\Delta x}u_{i+1}$	$\frac{-\beta}{2\Delta x}$	$\frac{-\beta}{2}$	$\frac{-\beta}{2}\cdot\frac{\Delta x}{2!}$	$\frac{-\beta}{2}\cdot\frac{\Delta x^2}{3!}$	$\frac{-\beta}{2}\cdot\frac{\Delta x^3}{4!}$	$\frac{-\beta}{2}\cdot\frac{\Delta x^4}{5!}$
$\frac{\beta}{2\Delta x}u_{i-1}$	$\frac{\beta}{2\Delta x}$	$\frac{-\beta}{2}$	$\frac{\beta}{2}\frac{\Delta x}{2!}$	$\frac{-\beta}{2}\cdot\frac{\Delta x^2}{3!}$	$\frac{\beta}{2}\cdot\frac{\Delta x^3}{4!}$	$\frac{-\beta}{2}\cdot\frac{\Delta x^5}{5!}$
LHS	RHS1	RHS2	RHS3	RHS4	RHS5	RHS6

RHS1: $\frac{-\beta}{2\Delta x} + \frac{\beta}{2\Delta x} = 0$ ✓

RHS2: $1 + \alpha + \alpha - \frac{\beta}{2} - \frac{\beta}{2} = 0$ [1]

RHS3: $-\alpha\Delta x + \alpha\Delta x - \frac{\beta}{2}\cdot\frac{\Delta x}{2!} + \frac{\beta}{2}\cdot\frac{\Delta x}{2!} = 0$ ✓

	u_i	$(u_x)_i$	$(u_{xx})_i$	$(u_{xxx})_i$	$(u_4)_i$	$(u_5)_i$
$(u_x)_i$	0	1	0	0	0	0
$\alpha(u_x)_{i-1}$	0	α	$-\alpha\Delta x$	$\alpha\frac{\Delta x^2}{2!}$	$-\frac{\alpha\Delta x^3}{3!}$	$\frac{\alpha\Delta x^4}{4!}$
$\alpha(u_x)_{i+1}$	0	α	$\alpha\Delta x$	$\alpha\frac{\Delta x^2}{2!}$	$\frac{\alpha\Delta x^3}{3!}$	$\frac{\alpha\Delta x^4}{4!}$
$-\frac{\beta}{2\Delta x}u_{i+1}$	$\frac{-\beta}{2\Delta x}$	$-\frac{\beta}{2}$	$-\frac{\beta}{2}\cdot\frac{\Delta x}{2!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^2}{3!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^3}{4!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^4}{5!}$
$\frac{\beta}{2\Delta x}u_{i-1}$	$\frac{\beta}{2\Delta x}$	$-\frac{\beta}{2}$	$\frac{\beta}{2}\frac{\Delta x}{2!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^2}{3!}$	$\frac{\beta}{2}\cdot\frac{\Delta x^3}{4!}$	$-\frac{\beta}{2}\cdot\frac{\Delta x^5}{5!}$
LHS	RHS1	RHS2	RHS3	RHS4	RHS5	RHS6

RHS 4: $\alpha\frac{\Delta x^2}{2!} + \alpha\frac{\Delta x^2}{2!} - \frac{\beta}{2}\frac{\Delta x^2}{3!} - \frac{\beta}{2}\frac{\Delta x^2}{3!} = 0$ [2]

RHS 5: $-\alpha\frac{\Delta x^3}{3!} + \alpha\frac{\Delta x^3}{3!} - \frac{\beta}{2}\cdot\frac{\Delta x^3}{4!} + \frac{\beta}{2}\cdot\frac{\Delta x^3}{4!} = 0$ ✓

RHS 6: $\alpha\frac{\Delta x^4}{4!} + \alpha\frac{\Delta x^4}{4!} - \frac{\beta}{2}\frac{\Delta x^4}{5!} - \frac{\beta}{2}\cdot\frac{\Delta x^4}{5!}$ [3]

$1 + \alpha + \alpha - \frac{\beta}{2} - \frac{\beta}{2} = 0$ [1]

$\alpha\frac{\Delta x^2}{2!} + \alpha\frac{\Delta x^2}{2!} - \frac{\beta}{2}\frac{\Delta x^2}{3!} - \frac{\beta}{2}\frac{\Delta x^2}{3!} = 0$ [2]

$\alpha\frac{\Delta x^4}{4!} + \alpha\frac{\Delta x^4}{4!} - \frac{\beta}{2}\frac{\Delta x^4}{5!} - \frac{\beta}{2}\cdot\frac{\Delta x^4}{5!}$ [3]

Use [1] & [2] to solve for α & β

[1] $1 + \alpha + \alpha - \frac{\beta}{2} - \frac{\beta}{2} = 0 \rightarrow 2\alpha - \beta = -1$

[2] $\alpha\frac{\cancel{\Delta x^2}}{2!} + \alpha\frac{\cancel{\Delta x^2}}{2!} - \frac{\beta}{2}\frac{\cancel{\Delta x^2}}{3!} - \frac{\beta}{2}\frac{\cancel{\Delta x^2}}{3!} = 0 \rightarrow \frac{\alpha}{2} + \frac{\alpha}{2} - \frac{\beta}{12} - \frac{\beta}{12} = 0$

$$\frac{\alpha}{2}+\frac{\alpha}{2}-\frac{\beta}{12}-\frac{\beta}{12}=\alpha-\frac{\beta}{6}=0$$

$$\begin{array}{cc} \alpha & \beta \end{array}$$

$$\left[\begin{array}{cc|c} 2 & -1 & -1 \\ 1 & -\frac{1}{6} & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{6} & 0 \end{array}\right] \xrightarrow{-R_1+R_2 \to R_2}$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1/3 & 1/2 \end{array}\right] \xrightarrow{3R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -1/2 & -1/2 \\ 0 & 1 & 3/2 \end{array}\right] \xrightarrow{\frac{1}{2}R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 1/4 \\ 0 & 1 & 3/2 \end{array}\right]$$

$$\alpha = 1/4, \ \beta = 3/2$$

Use 3 to find truncation error

$$\alpha\frac{\Delta x^4}{4!}+\alpha\frac{\Delta x^4}{4!}-\frac{\beta}{2}\frac{\Delta x^4}{5!}-\frac{\beta}{2}\cdot\frac{\Delta x^4}{5!}$$

$$=\frac{2\alpha}{4!}\Delta x^4-\frac{\beta}{5!}\Delta x^4=\Delta x^4\left(\frac{2\alpha}{4!}-\frac{\beta}{5!}\right)$$

$$=\Delta x^4\left(\frac{2}{4!}\cdot\frac{1}{4}-\frac{1}{5!}\cdot\frac{3}{2}\right)=\Delta x^4\left(\frac{2}{1\cdot 2\cdot 3\cdot 4\cdot 4}-\frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\frac{3}{2}\right)$$

$$=\Delta x^4\left(\frac{1}{48}-\frac{1}{80}\right)=\frac{\Delta x^4}{120}(u_5);$$

$$\frac{1}{48}-\frac{1}{80}=\frac{80}{3840}-\frac{48}{3840}=\frac{32}{3840}=\frac{1}{120}$$

3) $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = 1 + \sin(2\pi x)$$

$$u(1,t) = u(0,t)$$

$\nu = 0.01$, consider $N=20$ & $N=40$

For time use forward Euler explicit & central differencing for 1st & 2nd order derivatives

Take Taylor Series using 3 point stencil

$$u_j = u(x_j)$$

$$u_{j+1} = u(x_j + \Delta x) = u_j + \Delta x\, u_j' + \frac{\Delta x^2}{2}u_j'' + \frac{\Delta x^3}{3!}u_j''' + \ldots$$

$$u_{j-1} = u(x_j - \Delta x) = u_j - \Delta x\, u_j' + \frac{\Delta x^2}{2}u_j'' - \frac{\Delta x^3}{3!}u_j''' + \ldots$$

find u_j'

$$u(x_j + \Delta x) - u(x_j - \Delta x) = \left(\cancel{u_j} + \Delta x\, u_j' + \cancel{\frac{\Delta x^2}{2}}u_j'' + \frac{\Delta x^3}{3!}u_j''' + \ldots\right) - \left(\cancel{u_j} - \Delta x\, u_j' + \cancel{\frac{\Delta x^2}{2}}u_j'' - \frac{\Delta x^3}{3!}u_j''' + \ldots\right)$$

$$u(x_j + \Delta x) - u(x_j - \Delta x) = 2\Delta x\, u_j' + \frac{\Delta x^3}{3}u_j'''$$

$$u_j' = \frac{u(x_j + \Delta x) - u(x_j - \Delta x)}{2\Delta x} + O(\Delta x^2)$$

find u_j''

$$u(x_j + \Delta x) + u(x_j - \Delta x) = \left(u_j + \Delta x\, u_j' + \frac{\Delta x^2}{2}u_j'' + \frac{\Delta x^3}{3!}u_j''' + \ldots\right) + \left(u_j - \Delta x\, u_j' + \frac{\Delta x^2}{2}u_j'' - \frac{\Delta x^3}{3!}u_j''' + \ldots\right)$$

$$= 2u_j + \Delta x^2 u_j''$$

$$u'' = \frac{u(x_j + \Delta x) - 2u_j + u(x_j - \Delta x)}{\Delta x^2} + O(\Delta x^2)$$

find time derivative

forward Euler $\rightarrow \dfrac{u_j^{n+1} - u_j^n}{\Delta t}$

Time derivative

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + O(\Delta t)$$

Spatial Derivatives

$$u_j^{n'} = \frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} + O(\Delta x^2)$$

$$u_j^{n''} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2} + O(\Delta x^2$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + u_j^n\left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x}\right) = \nu\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}\right)$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \nu\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}\right) - u_j^n\left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x}\right)$$

$$u_j^{n+1} = \Delta t\left[\nu\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}\right) - u_j^n\left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x}\right)\right] + u_j^n$$

$$u_j^{n+1} = \Delta t\,\nu\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}\right) - \Delta t\,u_j^n\left(\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x}\right) + u_j^n$$

$$u_j^{n+1} = \frac{\Delta t\,\nu}{\Delta x^2}\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right) - \frac{\Delta t}{2\Delta x}u_j^n\left(u_{j+1}^n - u_{j-1}^n\right) + u_j^n$$

Question 3:

Refer to the previous pages to see the discretization of the equations for question 3.

1. Using a forward discretization (Euler explicit) for the time derivatives, and central discretization for the first and second order spatial derivatives, evaluate the solution at t = 1.0 using $\Delta t = 5 \times 10^{-3}$.

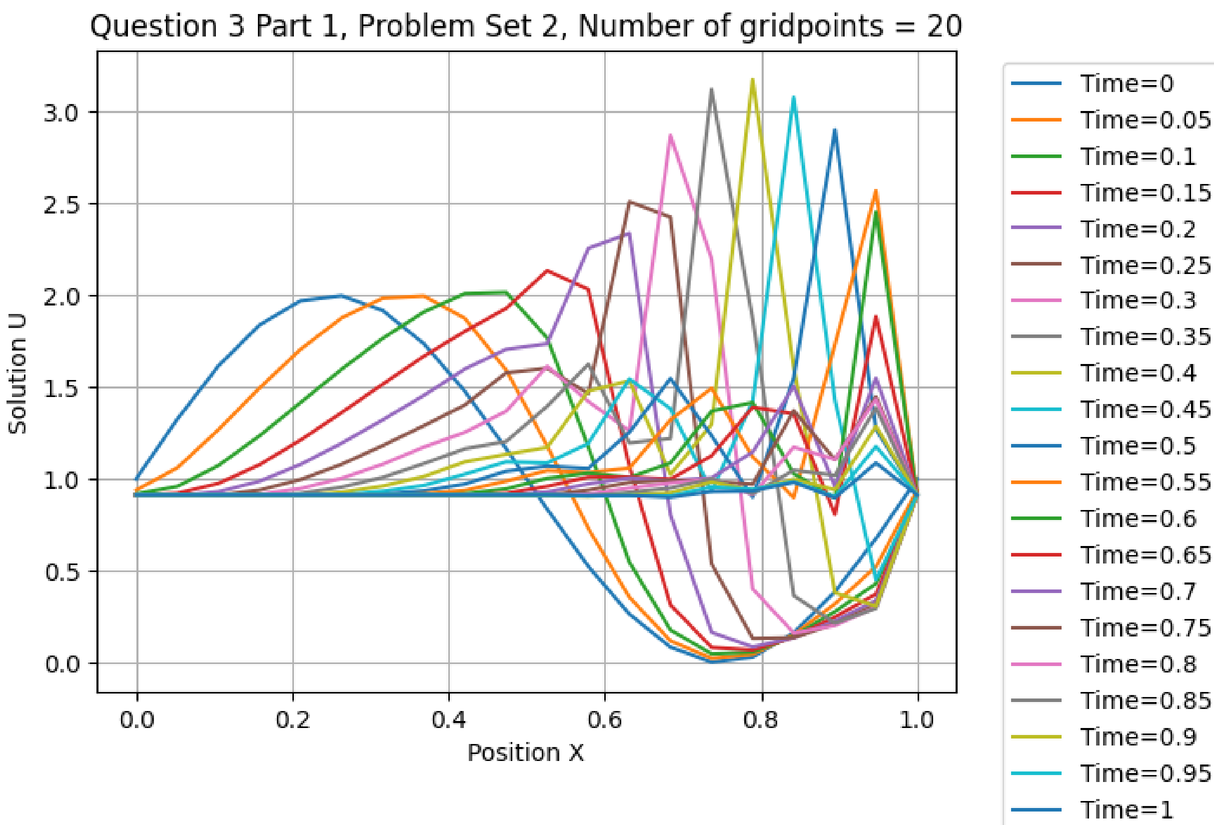

Figure 1: Burgers Equation Discretization using 20 grid points

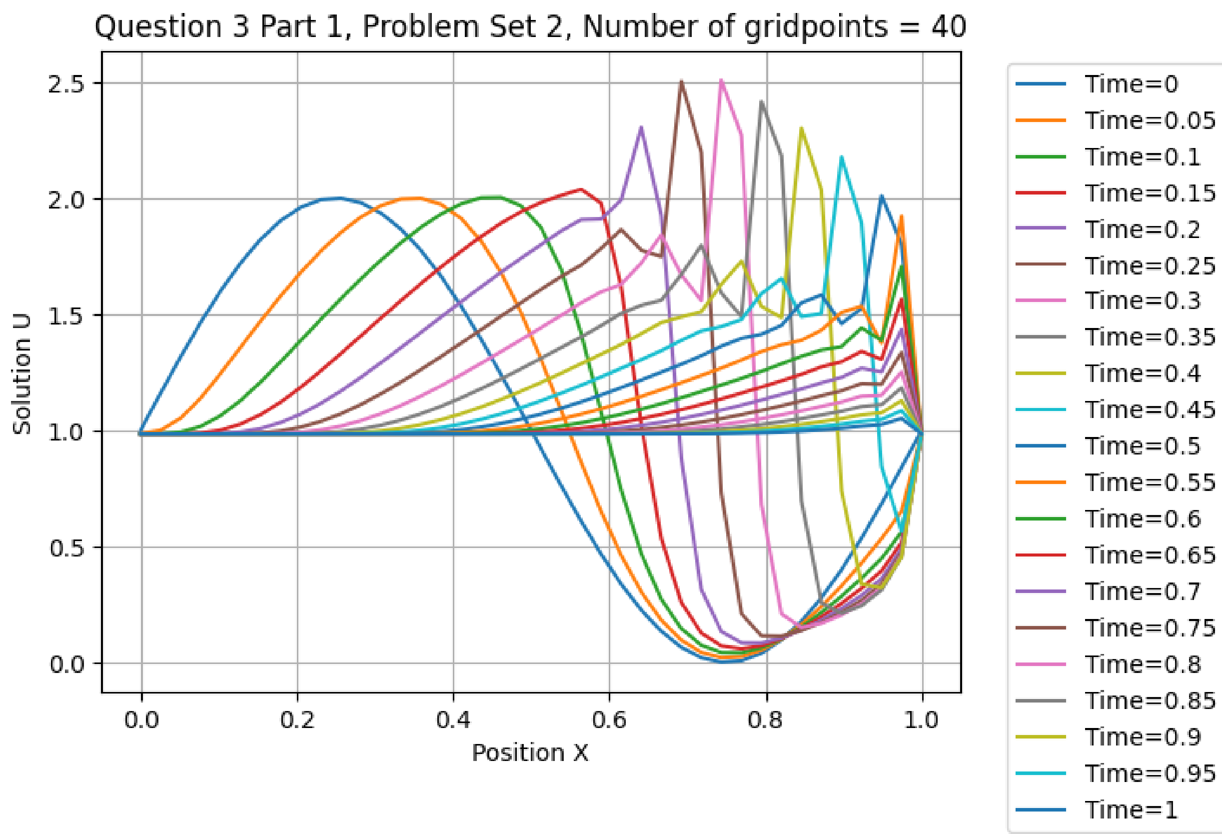

Figure 2: Burgers Equation Discretization using 40 grid points

The code will be included below. In order to calculate the above graph, the periodic boundary condition must be applied. In order to solve the initial point to apply this boundary condition, there are 2 methods. The first would be using the forward differencing method for just the first point (as there are no points before the first point). The second method would be using the last point as a "ghost point", in order to subtract the previous point using central difference method. The second method was chosen in this case. The snippet of code shown below demonstrates how this works.

```
#  Solve initial point for periodic boundary condition
u[0, n + 1] = ((dt * nu) / (dx ** 2)) * (u[1, n] - 2 * u[0, n] + u[-1, n]) -
(dt / (2 * dx)) * u[0, n] * (u[1, n] - u[-1, n]) + u[0, n]
u[-1, n + 1] = u[0, n + 1]
```

2. For each of the grids, conduct numerical experiments to determine the maximum value of Δt for which the solution remains stable. Present your results in Tabular form.

The code was written in the form of a function to allow quick looping through a variety of timesteps. This can be seen below.

```
numx = [20, 40]  # Number of discretization points
dom_len = 1.0  # Domain size
dt = np.arange(5e-3, 4e-2 + 5e-3, 1e-3)
tfinal = 1.0
tinitial = 0.0
for t in dt:
```

```
    for num in numx:
        print(t, num)
        burgers(num, dom_len, tfinal, tinitial, t)
```

This allowed a variety of timesteps between 5e-3 and 4.5e-2 to be tested. Using this method, an overflow in double scalars was encountered for both 40 points and 20 points. A runtime in double scalars means that a number was encountered which is larger than a double scalar can contain (a double scalar contains enough memory to encapsulate values between -1.79769313486e+308 and 1.79769313486e+308). This indicates that at these timesteps the solution blew up to infinity, as the timesteps were too large.

Number of Points	Timestep at which failure occurred
20	0.031
40	0.014

Below the graphs that were created 1 timestep before failure occurred can be seen, and below the subsequent graphs for the smallest timestep used.

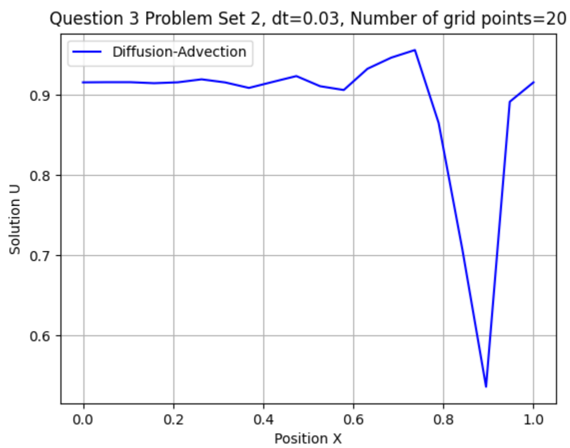

Figure 3: Final timestep before failure occurred with 20 grid points

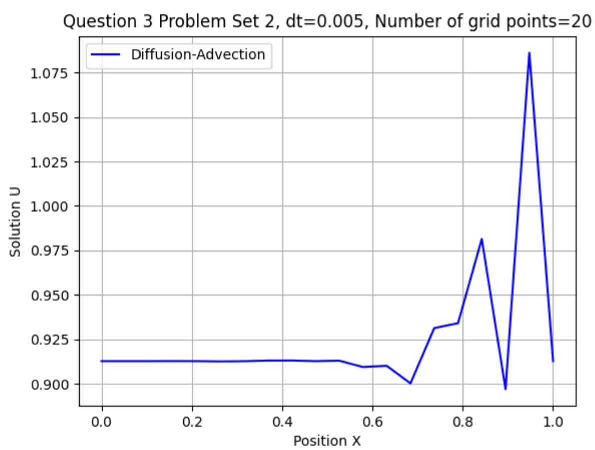

Figure 4: Smallest timestep tested with 20 grid points

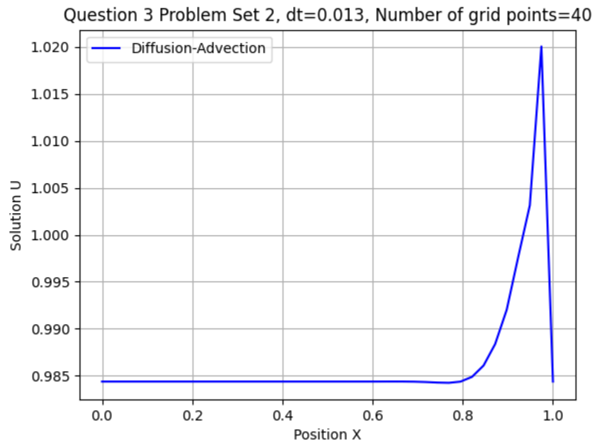

Figure 5: Final timestep before failure occurred with 40 grid points

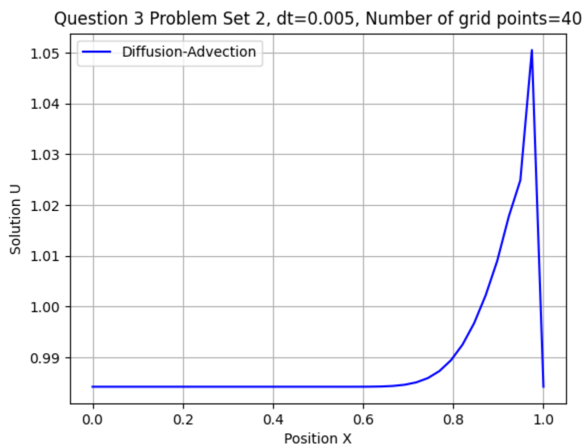

Figure 6: Smallest timestep tested with 40 grid points

Code:

```
# Import packages
import numpy as np
import matplotlib.pyplot as plt
import os
# User defined packages

# Path to save files
current_path = os.getcwd()
plot_folder = current_path + '/plots'

def burgers(numx, dom_len, tfinal, tinitial, dt):
    dx = dom_len / (numx - 1)  # Spatial step size
    x = np.arange(0, dom_len + dx, dx)  # Position vector
    t = np.arange(tinitial, tfinal + dt, dt)  # tfinal + dt is needed to
include tfinal in interval
    u = np.zeros((numx, len(t)))  # Solution vector
    nu = 0.01
    # Initial conditions
```

```
    for j in range(numx):
        u[j, 0] = 1 + np.sin(np.pi * 2 * dx * j)  # Apply initial conditions
    for n in range(len(t) - 1):  # Time loop
        #  Solve initial point for periodic boundary condition
        u[0, n + 1] = ((dt * nu) / (dx ** 2)) * (u[1, n] - 2 * u[0, n] + u[-
1, n]) - (dt / (2 * dx)) * u[0, n] * (u[1, n] - u[-1, n]) + u[0, n]
        u[-1, n + 1] = u[0, n + 1]

        for j in range(1, numx - 1):  # Spatial loop
            #  Solve solution vector
            u[j, n + 1] = ((dt * nu) / (dx ** 2)) * (u[j+1, n] - 2 * u[j, n]
+ u[j - 1, n]) - (dt / (2 * dx)) * u[j, n] * (u[j + 1, n] - u[j - 1, n]) +
u[j, n]
    # For question 1
    # for i in range(len(u[0, :])):
    #     print(i)
    #     if i % 10 == 0:
    #         print("Yes")
    #         plt.plot(x, u[:, i], label='Time=%g' % t[i])
    # plt.grid()
    # plt.legend(loc='upper left', bbox_to_anchor=(1.04, 1))
    # plt.title("Question 3 Part 1, Problem Set 2, Number of gridpoints = %i"
% numx)
    # plt.xlabel('Position X')
    # plt.ylabel("Solution U")
    # save_folder = plot_folder + '_Part 1_Num_X=%i' % numx
    # if not os.path.exists(save_folder):
    #     os.makedirs(save_folder, exist_ok=True)
    # plt.savefig(save_folder + '/Numx=%i' % numx + '.png',
bbox_inches="tight")
    # plt.close()
    # For question 2
    plt.plot(x, u[:, -1], label='Diffusion-Advection', color='b')
    plt.grid()
    plt.legend(loc='best')
    plt.title("Question 3 Problem Set 2, dt=%g, Number of grid points=%i" %
(dt, numx))
    plt.xlabel("Position X")
    plt.ylabel("Solution U")
    save_folder = plot_folder + '_Part 2_Num_X=%i' % numx
    if not os.path.exists(save_folder):
        os.makedirs(save_folder, exist_ok=True)
    plt.savefig(save_folder + '/dt=%g_Numx=%i' % (dt, numx) + '.png')
    # plt.show()
    plt.close()
    # return x, u


numx = [20, 40]  # Number of discretization points
dom_len = 1.0  # Domain size
dt = np.arange(5e-3, 4e-2 + 5e-3, 1e-3)
tfinal = 1.0
tinitial = 0.0
for t in dt:
    for num in numx:
        print(t, num)
        burgers(num, dom_len, tfinal, tinitial, t)
```