Equations to discretize

$$\frac{\partial y}{\partial t} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \text{ in } (0,1)X(0,1)$$

Taking forward Euler Temporal discreetization

$$\frac{I}{\Delta t}(u^{n+1} - u^n) = A u^n$$

where I is the identity matrix & A is some matrix representing central difference approximation of oure equation

We want to solve for u^{n+1}

$$\frac{I}{\Delta t} u^{n+1} = \underbrace{\left(A + \frac{I}{\Delta t}\right) u^n}_{b}$$

Apply Newton-Raphson iteration

$$\frac{I}{\Delta t}(u_k^{n+1} + \delta u_k^{n+1}) = b$$

where k is the number of iterations

$$\frac{I}{\Delta t}(\delta u_k^{n+1}) = b - \frac{I}{\Delta t} u_k^{n+1}$$

With convergence in 1 iteration, we take our initial guess of solution as the solution of the previous timestep.

$u^{n+1} = u^n$

The true solution after 1 iteration will be

$u_1^{n+1} = u_0^{n+1} + \delta u_0^{n+1} = u^{n+1}$

Substituting

$$\frac{I}{\Delta t}(\delta u_0^{n+1}) = b - \frac{I}{\Delta t} u_0^{n+1}$$

We now get

$$\frac{I}{\Delta t} \delta u^{n+1} = b - \frac{I}{\Delta t} u^n$$

$$u^{n+1} = u^n + \delta u^{n+1}$$

Substitute $b = \left(A + \frac{I}{\Delta t}\right) u^n$

$$\frac{I}{\Delta t} \left(\delta u^{n+1}\right) = \left(A + \frac{I}{\Delta t}\right) u^n - \frac{I}{\Delta t} u^n$$

finally

$$\frac{I}{\Delta t} \left(\delta u^{n+1}\right) = A u^n$$