The University of British Columbia
MECH 479

Module 1 Example Problems

September 1, 2022

Example 1

A CFD code is used to solve a two-dimensional (x and y), incompressible, laminar flow without free surfaces. The fluid is Newtonian and appropriate boundary conditions are considered. List the variables (unknowns) in the problem, and describe the corresponding equations to be solved by the computer.

Hint: There are only three unknowns in this problem, u, v, and p. Thus, we require three equations: continuity, x momentum (or x component of Navier-Stokes), and y momentum (or y component of Navier-Stokes). These equations, when combined with the appropriate boundary conditions, are sufficient to solve the problem.

Example 2

You have been contacted by the B.C. Center of Disease Control to perform a CFD analysis of the Covid-19 spread in a classroom. Sketch a computational domain to study ventilation in a room. You can assume a rectangular 2D room with a velocity inlet in the ceiling to model the supply of air flow, and a pressure outlet in the ceiling to model the return air.

Example 3

Using Gauss's theorem, transform the integral form of mass conservation

$$\underbrace{\frac{\partial}{\partial t}\int_V \rho dV}_{\text{Rate of change of mass}} = -\underbrace{\oint_S \rho \mathbf{u}\cdot\mathbf{ndS}}_{\text{Net inflow of mass}} \tag{1}$$

to the differential form

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0 \tag{2}$$

Express the mass conservation into convective form:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} = 0 \tag{3}$$

Example 4

Consider isothermal incompressible Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \frac{\nabla p}{\rho} = v\Delta\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = \mathbf{0}$$

Explain how these equations can be recovered in two forms using appropriate assumptions

Hyperbolic : $\dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \dfrac{\nabla p}{\rho} = \mathbf{0}$

Parabolic: $\dfrac{\partial \mathbf{u}}{\partial t} = v\Delta\mathbf{u}$