Assignment 2: Unstructured Meshes

Mech 588

Due Date: March 6

1. In class, we discussed the general principles involved in setting up an unstructured mesh discretization. In this problem, you will construct a second-order accurate finite-volume discretization for the wave equation on an unstructured triangular mesh, step-by-step. You'll do this in the context of calculating the flux across a single control volume interface, because that calculation, repeated for each face, will give you the complete flux integral.

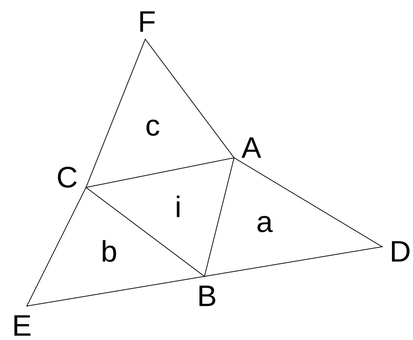
   The wave equation, in control volume form, looks like this, of course:

   $$A_i \frac{d\overline{T}_i}{dt} = -\oint_i \vec{u}T \cdot \hat{n}\, ds$$

   (a) (3 marks) To achieve second-order accuracy, you need to have a second-order accurate value of the flux at the control volume boundary, so you need a second-order accurate approximation to the solution, which implies that you need to know the gradient of the solution in each CV.

   Analytically find the least-squares estimate of the gradient in control volume i in terms of the solution values in i, a, b, and c. (*Hint:* when you reach the point where you have a non-square system of equations to solve, multiply both sides of your equation by the transpose of the 2×3 matrix on the LHS to get a 2×2 system.) Find the numerical value of the gradient if $\bar{T}_i = 100$, $\bar{T}_a = 102$, $\bar{T}_b = 97$, and $\bar{T}_c = 101$.

   (b) (2 marks) Now assume that you've calculated the gradient in every cell, so that you know not only the average value of the solution $\bar{T}$, but also the gradient of the solution ∇T in each control volume. Find the values of T on both sides of interface AB at its midpoint.



Vertex	x	y
A	3.25	1.25
B	3	0.25
C	2	1
D	4.5	0.5
E	1.5	0
F	2.5	2.25

Figure 1: Unstructured mesh fragment, including vertex coordinates.

(c) (2 marks) Given $T_{AB,\text{left}}$ and $T_{AB,\text{right}}$, you need to calculate a flux at the midpoint; for conservation, that flux must be the same regardless of whether you're calculating the flux integral for CV i or a. The simplest approach, as we discussed in class, is to compute the flux once for each interface. For a given velocity field $\vec{u}(x,y)$, how would you determine the upwind flux at the midpoint of AB?

(d) (1 marks) Now that you have the flux at the midpoint of AB, how would you estimate the total flux across that interface, and how would that flux contribute to the flux integrals in control volumes i and a?

And that, as they say, is that. What you've just outlined is all the calculation needed to successfully set up a finite-volume solver for the wave equation on a unstructured mesh! Well, okay, this assignment doesn't talk about time advance, but for the wave equation you could simply choose something explicit, like Runge-Kutta. The assignment also glosses over issues of mesh connectivity information, which is "only" bookkeeping, although admittedly a fairly large amount of bookkeeping.