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## *Preface*

In the past decade or so, there have been fascinating developments in multimedia representation and communications. First of all, it has become very clear that all aspects of media are “going digital”; from representation to transmission, from processing to retrieval, from studio to home. Second, there have been significant advances in digital multimedia compression and communication algorithms, which make it possible to deliver high-quality video at relatively low bit rates in today’s networks. Third, the advancement in VLSI technologies has enabled sophisticated software to be implemented in a cost-effective manner. Last but not least, the establishment of half a dozen international standards by ISO/MPEG and ITU-T laid the common groundwork for different vendors and content providers.

At the same time, the explosive growth in wireless and networking technology has profoundly changed the global communications infrastructure. It is the confluence of wireless, multimedia, and networking that will fundamentally change the way people conduct business and communicate with each other. The future computing and communications infrastructure will be empowered by virtually unlimited bandwidth, full connectivity, high mobility, and rich multimedia capability.

As multimedia becomes more pervasive, the boundaries between video, graphics, computer vision, multimedia database, and computer networking start to blur, making video processing an exciting field with input from many disciplines. Today, video processing lies at the core of multimedia. Among the many technologies involved, video coding and its standardization are definitely the key enablers of these developments. This book covers the fundamental theory and techniques for digital video processing, with a focus on video coding and communications. It is intended as a textbook for a graduate-level course on video processing, as well as a reference or self-study text for

researchers and engineers. In selecting the topics to cover, we have tried to achieve a balance between providing a solid theoretical foundation and presenting complex system issues in real video systems.

## SYNOPSIS

Chapter 1 gives a broad overview of video technology, from analog color TV system to digital video. Chapter 2 delineates the analytical framework for video analysis in the frequency domain, and describes characteristics of the human visual system. Chapters 3–12 focus on several very important sub-topics in digital video technology. Chapters 3 and 4 consider how a continuous-space video signal can be sampled to retain the maximum perceivable information within the affordable data rate, and how video can be converted from one format to another. Chapter 5 presents models for the various components involved in forming a video signal, including the camera, the illumination source, the imaged objects and the scene composition. Models for the three-dimensional (3-D) motions of the camera and objects, as well as their projections onto the two-dimensional (2-D) image plane, are discussed at length, because these models are the foundation for developing motion estimation algorithms, which are the subjects of Chapters 6 and 7. Chapter 6 focuses on 2-D motion estimation, which is a critical component in modern video coders. It is also a necessary preprocessing step for 3-D motion estimation. We provide both the fundamental principles governing 2-D motion estimation, and practical algorithms based on different 2-D motion representations. Chapter 7 considers 3-D motion estimation, which is required for various computer vision applications, and can also help improve the efficiency of video coding.

Chapters 8–11 are devoted to the subject of video coding. Chapter 8 introduces the fundamental theory and techniques for source coding, including information theory bounds for both lossless and lossy coding, binary encoding methods, and scalar and vector quantization. Chapter 9 focuses on waveform-based methods (including transform and predictive coding), and introduces the block-based hybrid coding framework, which is the core of all international video coding standards. Chapter 10 discusses content-dependent coding, which has the potential of achieving extremely high compression ratios by making use of knowledge of scene content. Chapter 11 presents scalable coding methods, which are well-suited for video streaming and broadcasting applications, where the intended recipients have varying network connections and computing powers. Chapter 12 introduces stereoscopic and multiview video processing techniques, including disparity estimation and coding of such sequences.

Chapters 13–15 cover system-level issues in video communications. Chapter 13 introduces the H.261, H.263, MPEG-1, MPEG-2, and MPEG-4 standards for video coding, comparing their intended applications and relative performance. These standards integrate many of the coding techniques discussed in Chapters 8–11. The MPEG-7 standard for multimedia content description is also briefly described. Chapter 14 reviews techniques for combating transmission errors in video communication systems, and also describes the requirements of different video applications, and the characteristics

of various networks. As an example of a practical video communication system, we end the text with a chapter devoted to video streaming over the Internet and wireless network. Chapter 15 discusses the requirements and representative solutions for the major subcomponents of a streaming system.

## SUGGESTED USE FOR INSTRUCTION AND SELF-STUDY

As prerequisites, students are assumed to have finished undergraduate courses in signals and systems, communications, probability, and preferably a course in image processing. For a one-semester course focusing on video coding and communications, we recommend covering the two beginning chapters, followed by video modeling (Chapter 5), 2-D motion estimation (Chapter 6), video coding (Chapters 8–11), standards (Chapter 13), error control (Chapter 14) and video streaming systems (Chapter 15). On the other hand, for a course on general video processing, the first nine chapters, including the introduction (Chapter 1), frequency domain analysis (Chapter 2), sampling and sampling rate conversion (Chapters 3 and 4), video modeling (Chapter 5), motion estimation (Chapters 6 and 7), and basic video coding techniques (Chapters 8 and 9), plus selected topics from Chapters 10–13 (content-dependent coding, scalable coding, stereo, and video coding standards) may be appropriate. In either case, Chapter 8 may be skipped or only briefly reviewed if the students have finished a prior course on source coding. Chapters 7 (3-D motion estimation), 10 (content-dependent coding), 11 (scalable coding), 12 (stereo), 14 (error-control), and 15 (video streaming) may also be left for an advanced course in video, after covering the other chapters in a first course in video. In all cases, sections denoted by asterisks (\*) may be skipped or left for further exploration by advanced students.

Problems are provided at the end of Chapters 1–14 for self-study or as homework assignments for classroom use. Appendix D gives answers to selected problems. The website for this book ([www.prenhall.com/wang](http://www.prenhall.com/wang)) provides MATLAB scripts used to generate some of the plots in the figures. Instructors may modify these scripts to generate similar examples. The scripts may also help students to understand the underlying operations. Sample video sequences can be downloaded from the website, so that students can evaluate the performance of different algorithms on real sequences. Some compressed sequences using standard algorithms are also included, to enable instructors to demonstrate coding artifacts at different rates by different techniques.

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## Chapter 1

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# VIDEO FORMATION, PERCEPTION, AND REPRESENTATION

In this first chapter, we describe what is a video signal, how is it captured and perceived, how is it stored/transmitted, and what are the important parameters that determine the quality and bandwidth (which in turn determines the data rate) of a video signal. We first present the underlying physics for color perception and specification (Sec. 1.1). We then describe the principles and typical devices for video capture and display (Sec. 1.2). As will be seen, analog videos are captured/stored/transmitted in a raster scan format, using either progressive or interlaced scans. As an example, we review the analog color television (TV) system (Sec. 1.4), and give insights as to how are certain critical parameters, such as frame rate and line rate, chosen, what is the spectral content of a color TV signal, and how can different components of the signal be multiplexed into a composite signal. Finally, Section 1.5 introduces the ITU-R BT.601 video format (formerly CCIR601), the digitized version of the analog color TV signal. We present some of the considerations that have gone into the selection of various digitization parameters. We also describe several other digital video formats, including high-definition TV (HDTV). The compression standards developed for different applications and their associated video formats are summarized.

The purpose of this chapter is to give the readers background knowledge about analog and digital video, and to provide insights to common video system design problems. As such, the presentation is intentionally made more qualitative than quantitative. In later chapters, we will come back to certain problems mentioned in this chapter and provide more rigorous descriptions/solutions.

### 1.1 Color Perception and Specification

A video signal is a sequence of two dimensional (2D) images projected from a dynamic three dimensional (3D) scene onto the image plane of a video camera. The

color value at any point in a video frame records the emitted or reflected light at a particular 3D point in the observed scene. To understand what does the color value mean physically, we review in this section basics of light physics and describe the attributes that characterize light and its color. We will also describe the principle of human color perception and different ways to specify a color signal.

### 1.1.1 Light and Color

Light is an electromagnetic wave with wavelengths in the range of 380 to 780 nanometer (nm), to which the human eye is sensitive. The energy of light is measured by *flux*, with a unit of watt, which is the rate at which energy is emitted. The *radiant intensity* of a light, which is directly related to the brightness of the light we perceive, is defined as the flux radiated into a unit solid angle in a particular direction, measured in watt/solid-angle. A light source usually can emit energy in a range of wavelengths, and its intensity can be varying in both space and time. In this book, we use  $C(\mathbf{X}, t, \lambda)$  to represent the radiant intensity distribution of a light, which specifies the light intensity at wavelength  $\lambda$ , spatial location  $\mathbf{X} = (X, Y, Z)$  and time  $t$ .

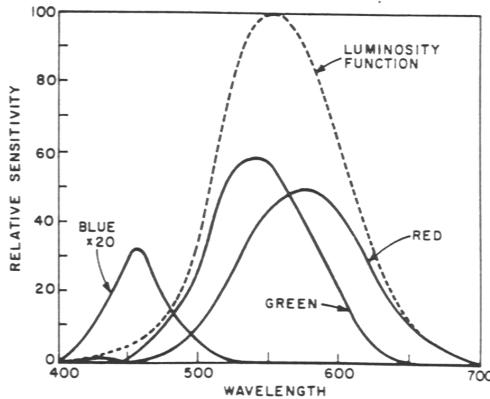
The perceived color of a light depends on its spectral content (i.e. the wavelength composition). For example, a light that has its energy concentrated near 700 nm appears red. A light that has equal energy in the entire visible band appears white. In general, a light that has a very narrow bandwidth is referred to as a *spectral color*. On the other hand, a white light is said to be achromatic.

There are two types of light sources: the *illuminating source*, which emits an electromagnetic wave, and the *reflecting source*, which reflects an incident wave.<sup>1</sup> The illuminating light sources include the sun, light bulbs, the television (TV) monitors, etc. The perceived color of an illuminating light source depends on the wavelength range in which it emits energy. The illuminating light follows an additive rule, i.e. the perceived color of several mixed illuminating light sources depends on the sum of the spectra of all light sources. For example, combining red, green, and blue lights in right proportions creates the white color.

The reflecting light sources are those that reflect an incident light (which could itself be a reflected light). When a light beam hits an object, the energy in a certain wavelength range is absorbed, while the rest is reflected. The color of a reflected light depends on the spectral content of the incident light and the wavelength range that is absorbed. A reflecting light source follows a subtractive rule, i.e. the perceived color of several mixed reflecting light sources depends on the remaining, unabsorbed wavelengths. The most notable reflecting light sources are the color dyes and paints. For example, if the incident light is white, a dye that absorbs the wavelength near 700 nm (red) appears as cyan. In this sense, we say that cyan is the complement of

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<sup>1</sup>The illuminating and reflecting light sources are also referred to as primary and secondary light sources, respectively. We do not use those terms to avoid the confusion with the primary colors associated with light. In other places, illuminating and reflecting lights are also called *additive colors* and *subtractive colors*, respectively.



**Figure 1.1.** Solid line: Frequency responses of the three types of cones on the human retina. The blue response curve is magnified by a factor of 20 in the figure. Dashed Line: The luminous efficiency function. From [10, Fig. 1].

red (or white minus red). Similarly, magenta and yellow are complements of green and blue, respectively. Mixing cyan, magenta, and yellow dyes produces black, which absorbs the entire visible spectrum.

### 1.1.2 Human Perception of Color

The perception of a light in the human being starts with the photo receptors located in the retina (the surface of the rear of the eye ball). There are two types of receptors: cones that function under bright light and can perceive the color tone, and rods that work under low ambient light and can only extract the luminance information. The visual information from the retina is passed via optic nerve fibers to the brain area called the visual cortex, where visual processing and understanding is accomplished. There are three types of cones which have overlapping pass-bands in the visible spectrum with peaks at red (near 570 nm), green (near 535 nm), and blue (near 445 nm) wavelengths, respectively, as shown in Figure 1.1. The responses of these receptors to an incoming light distribution  $C(\lambda)$  can be described by:

$$C_i = \int C(\lambda)a_i(\lambda)d\lambda, \quad i = r, g, b, \quad (1.1.1)$$

where  $a_r(\lambda)$ ,  $a_g(\lambda)$ ,  $a_b(\lambda)$  are referred to as the frequency responses or relative absorption functions of the red, green, and blue cones. The combination of these three types of receptors enables a human being to perceive any color. This implies that the perceived color only depends on three numbers,  $C_r$ ,  $C_g$ ,  $C_b$ , rather than the complete light spectrum  $C(\lambda)$ . This is known as the *tri-receptor theory of color vision*, first discovered by Young [14].

There are two attributes that describe the color sensation of a human being: luminance and chrominance. The term *luminance* refers to the perceived brightness of the light, which is proportional to the total energy in the visible band. The term *chrominance* describes the perceived color tone of a light, which depends on the wavelength composition of the light. Chrominance is in turn characterized by two attributes: hue and saturation. *Hue* specifies the color tone, which depends on the peak wavelength of the light, while *saturation* describes how pure the color is, which depends on the spread or bandwidth of the light spectrum. In this book, we use the word color to refer to both the luminance and chrominance attributes of a light, although it is customary to use the word color to refer to the chrominance aspect of a light only.

Experiments have shown that there exists a secondary processing stage in the human visual system (HVS), which converts the three color values obtained by the cones into one value that is proportional to the luminance and two other values that are responsible for the perception of chrominance. This is known as the *opponent color model* of the HVS [3, 9]. It has been found that the same amount of energy produces different sensations of the brightness at different wavelengths, and this wavelength-dependent variation of the brightness sensation is characterized by a *relative luminous efficiency function*,  $a_y(\lambda)$ , which is also shown (in dashed line) in Fig. 1.1. It is essentially the sum of the frequency responses of all three types of cones. We can see that the green wavelength contributes most to the perceived brightness, the red wavelength the second, and the blue the least. The luminance (often denoted by Y) is related to the incoming light spectrum by:

$$Y = \int C(\lambda) a_y(\lambda) d\lambda. \quad (1.1.2)$$

In the above equations, we have neglected the time and space variables, since we are only concerned with the perceived color or luminance at a fixed spatial and temporal location. We also neglected the scaling factor commonly associated with each equation, which depends on the desired unit for describing the color intensities and luminance.

### 1.1.3 The Trichromatic Theory of Color Mixture

A very important finding in color physics is that most colors can be produced by mixing three properly chosen primary colors. This is known as the *trichromatic theory of color mixture*, first demonstrated by Maxwell in 1855 [9, 13]. Let  $C_k, k = 1, 2, 3$  represent the colors of three primary color sources, and  $C$  a given color. Then the theory essentially says

$$C = \sum_{k=1,2,3} T_k C_k, \quad (1.1.3)$$

where  $T_k$ 's are the amounts of the three primary colors required to match color  $C$ . The  $T_k$ 's are known as *tristimulus values*. In general, some of the  $T_k$ 's can be

negative. Assuming only  $T_1$  is negative, this means that one cannot match color  $C$  by mixing  $C_1, C_2, C_3$ , but one can match color  $C + |T_1|C_1$  with  $T_2C_2 + T_3C_3$ . In practice, the primary colors should be chosen so that most natural colors can be reproduced using positive combinations of primary colors. The most popular primary set for the illuminating light source contains red, green, and blue colors, known as the *RGB primary*. The most common primary set for the reflecting light source contains cyan, magenta, and yellow, known as the *CMY primary*. In fact, RGB and CMY primary sets are complement of each other, in that mixing two colors in one set will produce one color in the other set. For example, mixing red with green will yield yellow. This complementary information is best illustrated by a color wheel, which can be found in many image processing books, e.g., [9, 4].

For a chosen primary set, one way to determine tristimulus values of any color is by first determining the color matching functions,  $m_i(\lambda)$ , for primary colors,  $C_i$ ,  $i=1,2,3$ . These functions describe the tristimulus values of a spectral color with wavelength  $\lambda$ , for various  $\lambda$  in the entire visible band, and can be determined by visual experiments with controlled viewing conditions. Then the tristimulus values for any color with a spectrum  $C(\lambda)$  can be obtained by [9]:

$$T_i = \int C(\lambda) m_i(\lambda) d\lambda, \quad i = 1, 2, 3. \quad (1.1.4)$$

To produce all visible colors with positive mixing, the matching functions associated with the primary colors must be positive.

The above theory forms the basis for color capture and display. To record the color of an incoming light, a camera needs to have three sensors that have frequency responses similar to the color matching functions of a chosen primary set. This can be accomplished by optical or electronic filters with the desired frequency responses. Similarly, to display a color picture, the display device needs to emit three optical beams of the chosen primary colors with appropriate intensities, as specified by the tristimulus values. In practice, electronic beams that strike phosphors with the red, green and blue colors are used. All present display systems use a RGB primary, although the standard spectra specified for the primary colors may be slightly different. Likewise, a color printer can produce different colors by mixing three dyes with the chosen primary colors in appropriate proportions. Most of the color printers use the CMY primary. For a more vivid and wide-range color rendition, some color printers use four primaries, by adding black (K) to the CMY set. This is known as the CMYK primary, which can render the black color more truthfully.

#### 1.1.4 Color Specification by Tristimulus Values

**Tristimulus Values** We have introduced the tristimulus representation of a color in Sec. 1.1.3, which specifies the proportions, *i.e.* the  $T_k$ 's in Eq. (1.1.3), of the three primary colors needed to create the desired color. In order to make the color specification independent of the absolute energy of the primary colors, these values

should be normalized so that  $T_k = 1, k = 1, 2, 3$  for a reference white color (equal energy in all wavelengths) with a unit energy. When we use a RGB primary, the tristimulus values are usually denoted by  $R$ ,  $G$ , and  $B$ .

**Chromaticity Values:** The above tristimulus representation mixes the luminance and chrominance attributes of a color. To measure only the chrominance information (*i.e.* the hue and saturation) of a light, the chromaticity coordinate is defined as:

$$t_k = \frac{T_k}{T_1 + T_2 + T_3}, \quad k = 1, 2, 3. \quad (1.1.5)$$

Since  $t_1 + t_2 + t_3 = 1$ , two chromaticity values are sufficient to specify the chrominance of a color.

Obviously, the color value of an imaged point depends on the primary colors used. To standardize color description and specification, several standard primary color systems have been specified. For example, the CIE,<sup>2</sup> an international body of color scientists, defined a *CIE RGB primary system*, which consists of colors at 700 ( $R_0$ ), 546.1 ( $G_0$ ), and 435.8 ( $B_0$ ) nm.

**Color Coordinate Conversion** One can convert the color values based on one set of primaries to the color values for another set of primaries. Conversion of (R,G,B) coordinate to the (C,M,Y) coordinate is, for example, often required for printing color images stored in the (R,G,B) coordinate. Given the tristimulus representation of one primary set in terms of another primary, one can determine the conversion matrix between the two color coordinates. The principle of color conversion and the derivation of the conversion matrix between two sets of color primaries can be found in [9].

### 1.1.5 Color Specification by Luminance and Chrominance Attributes

The RGB primary commonly used for color display mixes the luminance and chrominance attributes of a light. In many applications, it is desirable to describe a color in terms of its luminance and chrominance content separately, to enable more efficient processing and transmission of color signals. Towards this goal, various three-component color coordinates have been developed, in which one component reflects the luminance and the other two collectively characterize hue and saturation. One such coordinate is the CIE XYZ primary, in which Y directly measures the luminance intensity. The ( $X$ ,  $Y$ ,  $Z$ ) values in this coordinate are related to the ( $R$ ,  $G$ ,  $B$ ) values in the CIE RGB coordinate by [9]:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2.365 & -0.515 & 0.005 \\ -0.897 & 1.426 & -0.014 \\ -0.468 & 0.089 & 1.009 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}. \quad (1.1.6)$$

<sup>2</sup>CIE stands for Commission Internationale de L'Eclairage or, in English, International Commission on Illumination.

In addition to separating the luminance and chrominance information, another advantage of the CIE XYZ system is that almost all visible colors can be specified with non-negative tristimulus values, which is a very desirable feature. The problem is that the X,Y,Z colors so defined are not realizable by actual color stimuli. As such, the XYZ primary is not directly used for color production, rather it is mainly introduced for defining other primaries and for numerical specification of color. As will be seen later, the color coordinates used for transmission of color TV signals, such as YIQ and YUV, are all derived from the XYZ coordinate.

There are other color representations in which the hue and saturation of a color are explicitly specified, in addition to the luminance. One example is the HSI coordinate, where H stands for hue, S for saturation, and I for intensity (equivalent to luminance)<sup>3</sup>. Although this color coordinate clearly separates different attributes of a light, it is nonlinearly related to the tristimulus values and is difficult to compute. The book by Gonzalez has a comprehensive coverage of various color coordinates and their conversions [4].

## 1.2 Video Capture and Display

### 1.2.1 Principle of Color Video Imaging

Having explained what is light and how it is perceived and characterized, we are now in a position to understand the meaning of a video signal. In short, a video records the emitted and/or reflected light intensity, *i.e.*  $C(\mathbf{X}, t, \lambda)$  from the objects in the scene that is observed by a viewing system (a human eye or a camera). In general, this intensity changes both in time and space. Here, we assume that there are some illuminating light sources in the scene. Otherwise, there will be no injected nor reflected light and the image will be totally dark. When observed by a camera, only those wavelengths to which the camera is sensitive are visible. Let the *spectral absorption function* of the camera be denoted by  $a_c(\lambda)$ , then the light intensity distribution in the 3D world that is “visible” to the camera is:

$$\bar{\psi}(\mathbf{X}, t) = \int_0^{\infty} C(\mathbf{X}, t, \lambda) a_c(\lambda) d\lambda. \quad (1.2.1)$$

The image function captured by the camera at any time  $t$  is the projection of the light distribution in the 3D scene onto a 2D image plane. Let  $\mathcal{P}(\cdot)$  represent the camera projection operator so that the projected 2D position of the 3D point  $\mathbf{X}$  is given by  $\mathbf{x} = \mathcal{P}(\mathbf{X})$ . Further more, let  $\mathcal{P}^{-1}(\cdot)$  denote the inverse projection operator, so that  $\mathbf{X} = \mathcal{P}^{-1}(\mathbf{x})$  specifies the 3D position associated with a 2D point  $\mathbf{x}$ . Then the projected image is related to the 3D image by

$$\psi(\mathcal{P}(\mathbf{X}), t) = \bar{\psi}(\mathbf{X}, t) \quad \text{or} \quad \psi(\mathbf{x}, t) = \bar{\psi}(\mathcal{P}^{-1}(\mathbf{x}), t). \quad (1.2.2)$$

The function  $\psi(\mathbf{x}, t)$  is what is known as a video signal. We can see that it describes the radiant intensity at the 3D position  $\mathbf{X}$  that is projected onto  $\mathbf{x}$  in the image

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<sup>3</sup>The HSI coordinate is also known as HSV, where V stands for “value” of the intensity.

plane at time  $t$ . In general the video signal has a finite spatial and temporal range. The spatial range depends on the camera viewing area, while the temporal range depends on the duration in which the video is captured. A point in the image plane is called a *pixel* (meaning picture element) or simply *pel*.<sup>4</sup> For most camera systems, the projection operator  $\mathcal{P}(\cdot)$  can be approximated by a perspective projection. This is discussed in more detail in Sec. 5.1.

If the camera absorption function is the same as the relative luminous efficiency function of the human being, *i.e.*  $a_c(\lambda) = a_y(\lambda)$ , then a luminance image is formed. If the absorption function is non-zero over a narrow band, then a monochrome (or monotone) image is formed. To perceive all visible colors, according to the trichromatic color vision theory (see Sec. 1.1.2), three sensors are needed, each with a frequency response similar to the color matching function for a selected primary color. As described before, most color cameras use the red, green, and blue sensors for color acquisition.

If the camera has only one luminance sensor,  $\psi(\mathbf{x}, t)$  is a scalar function that represents the luminance of the projected light. In this book, we use the word *gray-scale* to refer to such a video. The term *black-and-white* will be used strictly to describe an image that has only two colors: black and white. On the other hand, if the camera has three separate sensors, each tuned to a chosen primary color, the signal is a vector function that contains three color values at every point. Instead of specifying these color values directly, one can use other color coordinates (each consists of three values) to characterize light, as explained in the previous section.

Note that for special purposes, one may use sensors that work in a frequency range that is invisible to the human being. For example, in X-ray imaging, the sensor is sensitive to the spectral range of the X-ray. On the other hand, an infra-red camera is sensitive to the infra-red range, which can function at very low ambient light. These cameras can “see” things that cannot be perceived by the human eye. Yet another example is the range camera, in which the sensor emits a laser beam and measures the time it takes for the beam to reach an object and then be reflected back to the sensor. Because the round trip time is proportional to the distance between the sensor and the object surface, the image intensity at any point in a range image describes the distance or range of its corresponding 3D point from the camera.

### 1.2.2 Video Cameras

All the analog cameras of today capture a video in a frame by frame manner with a certain time spacing between the frames. Some cameras (*e.g.* TV cameras and consumer video camcorders) acquire a frame by scanning consecutive lines with a certain line spacing. Similarly, all the display devices present a video as a consecutive set of frames, and with TV monitors, the scan lines are played back sequentially as separate lines. Such capture and display mechanisms are designed to take advan-

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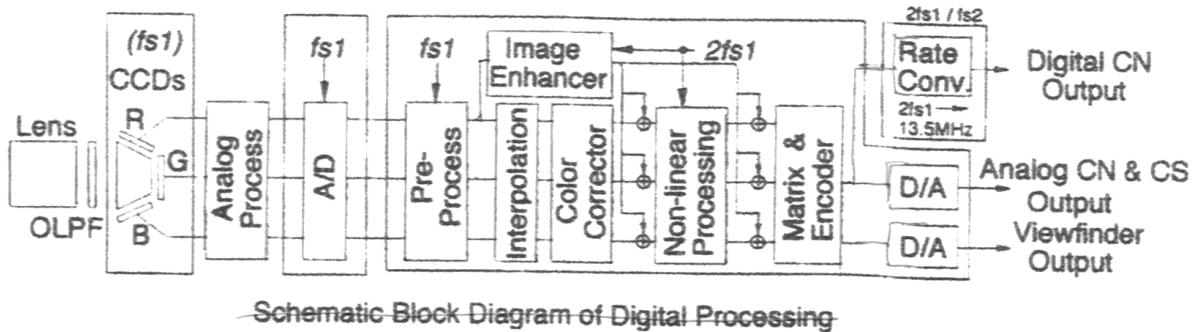
<sup>4</sup>Strictly speaking the notion of pixel or pel is only defined in digital imagery, in which each image or a frame in a video is represented by a finite 2D array of pixels.

tage of the fact that the HVS cannot perceive very high frequency changes in time and space. This property of the HVS will be discussed more extensively in Sec. 2.4.

There are basically two types of video imagers: (1) tube-based imagers such as vidicons, plumbicons, or orthicons, and (2) solid-state sensors such as charge-coupled devices (CCD). The lens of a camera focuses the image of a scene onto a photosensitive surface of the imager of the camera, which converts optical signals into electrical signals. The photosensitive surface of the tube imager is typically scanned line by line (known as raster scan) with an electron beam or other electronic methods, and the scanned lines in each frame are then converted into an electrical signal representing variations of light intensity as variations in voltage. Different lines are therefore captured at slightly different times in a continuous manner. With progressive scan, the electronic beam scans every line continuously; while with interlaced scan, the beam scans every other line in one half of the frame time (a field) and then scans the other half of the lines. We will discuss raster scan in more detail in Sec. 1.3. With a CCD camera, the photosensitive surface is comprised of a 2D array of sensors, each corresponding to one pixel, and the optical signal reaching each sensor is converted to an electronic signal. The sensor values captured in each frame time are first stored in a buffer, which are then read-out sequentially one line at a time to form a raster signal. Unlike the tube based cameras, all the read-out values in the same frame are captured at the same time. With interlaced scan camera, alternate lines are read-out in each field.

To capture color, there are usually three types of photosensitive surfaces or CCD sensors, each with a frequency response that is determined by the color matching function of the chosen primary color, as described previously in Sec. 1.1.3. To reduce the cost, most consumer cameras use a single CCD chip for color imaging. This is accomplished by dividing the sensor area for each pixel into three or four sub-areas, each sensitive to a different primary color. The three captured color signals can be either converted to one luminance signal and two chrominance signal and sent out as a component color video, or multiplexed into a composite signal. This subject is explained further in Sec. 1.2.4.

Many cameras of today are CCD-based because they can be made much smaller and lighter than the tube-based cameras, to acquire the same spatial resolution. Advancement in CCD technology has made it possible to capture in a very small chip size a very high resolution image array. For example, 1/3-in CCD's with 380 K pixels are commonly found in consumer-use camcorders, whereas a 2/3-in CCD with 2 million pixels has been developed for HDTV. The tube-based cameras are more bulky and costly, and are only used in special applications, such as those requiring very high resolution or high sensitivity under low ambient light. In addition to the circuitry for color imaging, most cameras also implement color coordinate conversion (from RGB to luminance and chrominance) and compositing of luminance and chrominance signals. For digital output, analog-to-digital (A/D) conversion is also incorporated. Figure 1.2 shows the typical processings involved in a professional video camera. The camera provides outputs in both digital and analog form, and in the analog case, includes both component and composite formats. To improve the



**Figure 1.2.** Schematic Block Diagram of a Professional Color Video Camera. From [6, Fig. 7(a)].

image quality, digital processing is introduced within the camera. For an excellent exposition of the video camera and display technologies, see [6].

### 1.2.3 Video Display

To display a video, the most common device is the cathode ray tube (CRT). With a CRT monitor, an electron gun emits an electron beam across the screen line by line, exciting phosphors with intensities proportional to the intensity of the video signal at corresponding locations. To display a color image, three beams are emitted by three separate guns, exciting red, green, and blue phosphors with the desired intensity combination at each location. To be more precise, each color pixel consists of three elements arranged in a small triangle, known as a triad.

The CRT can produce an image having a very large dynamic range so that the displayed image can be very bright, sufficient for viewing during day light or from a distance. However, the thickness of a CRT needs to be about the same as the width of the screen, for the electrons to reach the side of the screen. A large screen monitor is thus too bulky, unsuitable for applications requiring thin and portable devices. To circumvent this problem, various flat panel displays have been developed. One popular device is Liquid Crystal Display (LCD). The principle idea behind the LCD is to change the optical properties and consequently the brightness/color of the liquid crystal by an applied electric field. The electric field can be generated/adapted by either an array of transistors, such as in LCD's using active matrix thin-film-transistors (TFT), or by using plasma. The plasma technology eliminates the need for TFT and makes large-screen LCD's possible. There are also new designs for flat CRT's. A more comprehensive description of video display technologies can be found in [6].

The above stated raster scan and display mechanisms only apply to TV cameras and displays. With movie cameras, the color pattern seen by the camera at any

frame instant is completely recorded on the film. For display, consecutive recorded frames are played back using an analog optical projection system.

#### 1.2.4 Composite vs. Component Video

Ideally, a color video should be specified by three functions or signals, each describing one color component, in either a tristimulus color representation, or a luminance-chrominance representation. A video in this format is known as *component video*. Mainly for historical reasons, various composite video formats also exist, wherein the three color signals are multiplexed into a single signal. These composite formats were invented when the color TV system was first developed and there was a need to transmit the color TV signal in a way so that a black-and-white TV set can extract from it the luminance component. The construction of a composite signal relies on the property that the chrominance signals have a significantly smaller bandwidth than the luminance component. By modulating each chrominance component to a frequency that is at the high end of the luminance component, and adding the resulting modulated chrominance signals and the original luminance signal together, one creates a composite signal that contains both luminance and chrominance information. To display a composite video signal on a color monitor, a filter is used to separate the modulated chrominance signals and the luminance signal. The resulting luminance and chrominance components are then converted to red, green, and blue color components. With a gray-scale monitor, the luminance signal alone is extracted and displayed directly.

All present analog TV systems transmit color TV signals in a composite format. The composite format is also used for video storage on some analog tapes (such as the VHS tape). In addition to being compatible with a gray-scale signal, the composite format eliminates the need for synchronizing different color components when processing a color video. A composite signal also has a bandwidth that is significantly lower than the sum of the bandwidth of three component signals, and therefore can be transmitted or stored more efficiently. These benefits are however achieved at the expense of video quality: there often exist noticeable artifacts caused by cross-talks between color and luminance components.

As a compromise between the data rate and video quality, *S-video* was invented, which consists of two components, the luminance component and a single chrominance component which is the multiplex of two original chrominance signals. Many advanced consumer level video cameras and displays enable recording/display of video in S-video format. Component format is used only in professional video equipment.

#### 1.2.5 Gamma Correction

We have said that the video frames captured by a camera reflect the color values of the imaged scene. In reality, the output signals from most cameras are not linearly

related to the actual color values, rather in a non-linear form:<sup>5</sup>

$$v_c = B_c^{-\gamma_c}, \quad (1.2.3)$$

where  $B_c$  represents the actual light brightness, and  $v_c$  the camera output voltage. The value of  $\gamma_c$  range from 1.0 for most CCD cameras to 1.7 for a vidicon camera [7]. Similarly, most of the display devices also suffer from such a non-linear relation between the input voltage value  $v_d$  and the displayed color intensity  $B_d$ , i.e.

$$B_d = v_d^{\gamma_d}. \quad (1.2.4)$$

The CRT displays typically have a  $\gamma_d$  of 2.2 to 2.5 [7]. In order to present true colors, one has to apply an inverse power function on the camera output. Similarly, before sending real image values for display, one needs to pre-compensate the gamma effect of the display device. These processes are known as *gamma correction*.

In TV broadcasting, ideally, at the TV broadcaster side, the RGB values captured by the TV cameras should first be corrected based on the camera gamma and then converted to the color coordinates used for transmission (YIQ for NTSC, and YUV for PAL and SECAM). At the receiver side, the received YIQ or YUV values should first be converted to the RGB values, and then compensated for the monitor gamma values. In reality, however, in order to reduce the processing to be done in the millions of receivers, the broadcast video signals are pre-gamma corrected in the RGB domain. Let  $v_c$  represent the R, G, or B signal captured by the camera, the gamma corrected signal for display,  $v_d$ , is obtained by

$$v_d = v_c^{\gamma_c/\gamma_d}. \quad (1.2.5)$$

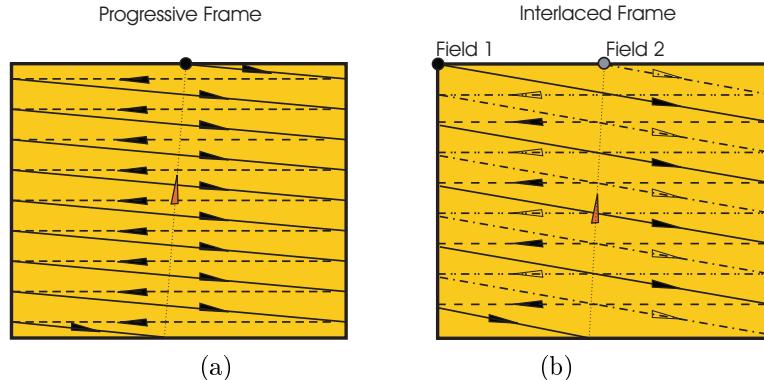
In most of the TV systems, a ratio of  $\gamma_c/\gamma_d = 2.2$  is used. This is based on the assumption that a CCD camera with  $\gamma_c = 1$  and a CRT display with  $\gamma_d = 2.2$  are used [7]. These gamma corrected values are converted to the YIQ or YUV values for transmission. The receiver simply applies a color coordinate conversion to obtain the RGB values for display. Notice that this process applies display gamma correction before the conversion to the YIQ/YUV domain, which is not strictly correct. But the distortion is insignificant and not noticeable by average viewers [7].

### 1.3 Analog Video Raster

As already described, the analog TV systems of today use raster scan for video capture and display. As this is the most popular analog video format, in this section, we describe the mechanism of raster scan in more detail, including both progressive and interlaced scan. As an example, we also explain the video formats used in various analog TV systems.

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<sup>5</sup>A more precise relation is  $B_c = cv_c^{\gamma_c} + B_0$ , where  $c$  is a gain factor, and  $B_0$  is the cut-off level of light intensity. If we assume that the output voltage value is properly shifted and scaled, then the presented equation is valid.



**Figure 1.3.** Progressive (a) and Interlaced (b) Raster Scan Formats.

### 1.3.1 Progressive and Interlaced Scan

**Progressive Scan** In raster scan, a camera captures a video sequence by sampling it in both temporal and vertical directions. The resulting signal is stored in a continuous one dimensional (1D) waveform. As shown in Fig. 1.3(a), the electronic or optic beam of an analog video camera continuously scans the imaged region from the top to bottom and then back to the top. The resulting signal consists of a series of frames separated by a regular frame interval,  $\Delta_t$ , and each frame consists of a consecutive set of horizontal scan lines, separated by a regular vertical spacing. Each scan line is actually slightly tilted downwards. Also, the bottom line is scanned about one frame interval later than the top line of the same frame. However, for analysis purposes, we often assume that all the lines in a frame are sampled at the same time, and each line is perfectly horizontal. The intensity values captured along contiguous scan lines over consecutive frames form a 1D analog waveform, known as a *raster scan*. With a color camera, three 1D rasters are converted into a composite signal, which is a color raster.

**Interlaced Scan** The raster scan format described above is more accurately known as *progressive scan* (also known as sequential or non-interlaced scan), in which the horizontal lines are scanned successively. In the *interlaced scan*, each frame is scanned in two fields and each field contains half the number of lines in a frame. The time interval between two fields, *i.e.*, the field interval, is half of the frame interval, while the line spacing in a field is twice of that desired for a frame. The scan lines in two successive fields are shifted by half of the line spacing in each field. This is illustrated in Fig. 1.3(b). Following the terminology used in the MPEG standard, we call the field containing the first line and following alternating lines in a frame the *top field*, and the field containing the second line and following

alternating lines the *bottom field*.<sup>6</sup> In certain systems, the top field is sampled first, while in other systems, the bottom field is sampled first. It is important to remember that two adjacent lines in a frame are separated in time by the field interval. This fact leads to the infamous zig-zag artifacts in an interlaced video that contains fast moving objects with vertical edges. The motivation for using the interlaced scan is to trade off the vertical resolution for an enhanced temporal resolution, given the total number of lines that can be recorded within a given time. A more thorough comparison of the progressive and interlaced scans in terms of their sampling efficiency is given later in Sec. 3.3.2.

The interlaced scan introduced above should be called 2:1 interlace more precisely. In general, one can divide a frame into  $K \geq 2$  fields, each separated in time by  $\Delta_t/K$ . This is known as K:1 interlace and  $K$  is called *interlace order*. In a digital video where each line is represented by discrete samples, the samples on the same line may also appear in different fields. For example, the samples in a frame may be divided into two fields using a checker-board pattern. The most general definition of the interlace order is the ratio of the number of samples in a frame to the number of samples in each field.

### 1.3.2 Characterization of a Video Raster

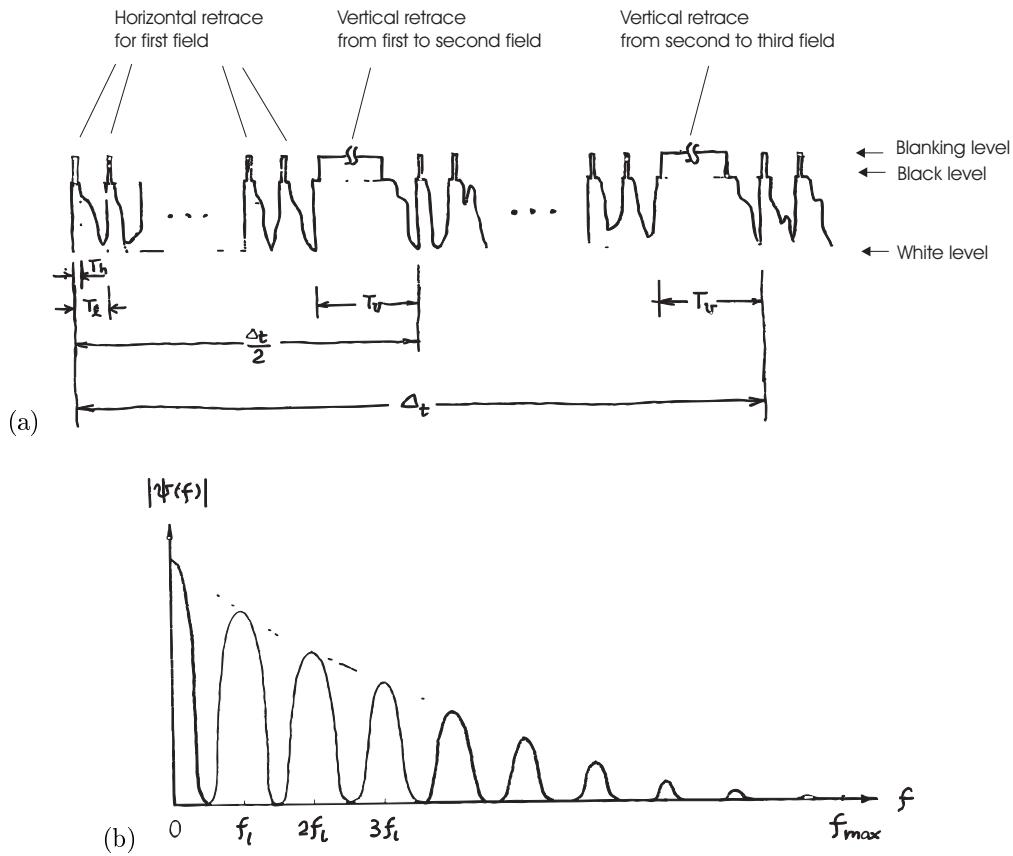
A raster is described by two basic parameters: the frame rate (frames/second or fps or Hz), denoted by  $f_{s,t}$ , and the line number (lines/frame or lines/picture-height), denoted by  $f_{s,y}$ . These two parameters define the temporal and vertical sampling rates of a raster scan. From these parameters, one can derive another important parameter, the line rate (lines/second), denoted by  $f_l = f_{s,t}f_{s,y}$ .<sup>7</sup> We can also derive the temporal sampling interval or frame interval,  $\Delta_t = 1/f_{s,t}$ , the vertical sampling interval or line spacing,  $\Delta_y = \text{picture-height}/f_{s,y}$ , and the line interval,  $T_l = 1/f_l = \Delta_t/f_{s,y}$ , which is the time used to scan one line. Note that the line interval  $T_l$  includes the time for the sensor to move from the end of a line to the beginning of the next line, which is known as the *horizontal retrace time* or just *horizontal retrace*, to be denoted by  $T_h$ . The actual scanning time for a line is  $T'_l = T_l - T_h$ . Similarly, the frame interval  $\Delta_t$  includes the time for the sensor to move from the end of the bottom line in a frame to the beginning of the top line of the next frame, which is called *vertical retrace time* or just *vertical retrace*, to be denoted by  $T_v$ . The number of lines that is actually scanned in a frame time, known as the *active lines*, is  $f'_{s,y} = (\Delta_t - T_v)/T_l = f_{s,y} - T_v/T_l$ . Normally,  $T_v$  is chosen to be an integer multiple of  $T_l$ .

A typical waveform of an interlaced raster signal is shown in Fig. 1.4(a). Notice that a portion of the signal during the horizontal and vertical retrace periods are held at a constant level below the level corresponding to black. These are called

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<sup>6</sup> A more conventional definition is to call the field that contains all even lines the even-field, and the field containing all odd lines the odd-field. This definition depends on whether the first line is numbered 0 or 1, and is therefore ambiguous.

<sup>7</sup> The frame rate and line rate are also known as the vertical sweep frequency and the horizontal sweep frequency, respectively.



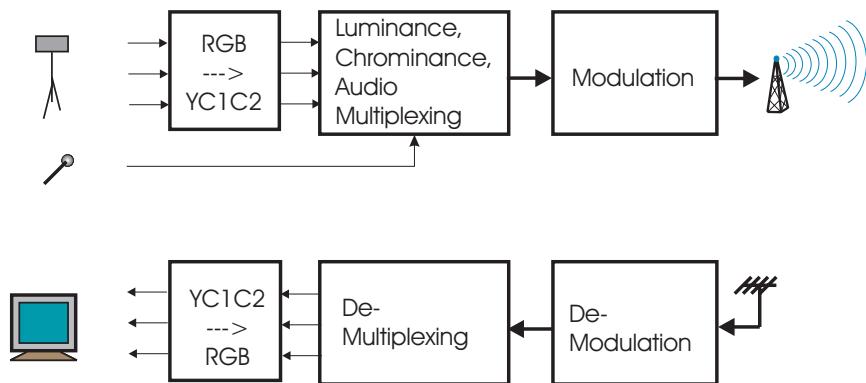
**Figure 1.4.** A Typical Interlaced Raster Scan: (a) Waveform, (b) Spectrum.

sync signals. The display devices start the retrace process upon detecting these sync signals.

Figure 1.4(b) shows the spectrum of a typical raster signal. It can be seen that the spectrum contains peaks at the line rate  $f_l$  and its harmonics. This is because adjacent scan lines are very similar so that the signal is nearly periodic with a period of  $T_l$ . The width of each harmonic lobe is determined by the maximum vertical frequency in a frame. The overall bandwidth of the signal is determined by the maximum horizontal spatial frequency.

The frame rate is one of the most important parameters that determine the quality of a video raster. For example, the TV industry uses an interlaced scan with a frame rate of 25–30 Hz, with an effective temporal refresh rate of 50–60 Hz, while the movie industry uses a frame rate of 24 Hz.<sup>8</sup> On the other hand, in the

<sup>8</sup>To reduce the visibility of flicker, a rotating blade is used to create an illusion of 72 frames/c.



**Figure 1.5.** Analog Color TV systems: Video Production, Transmission, and Reception.

computer industry, 72 Hz has become a de facto standard. The line number used in a raster scan is also a key factor affecting the video quality. In analog TVs, a line number of about 500-600 is used, while for computer display, a much higher line number is used (e.g., the SVGA display has 1024 lines). These frame rates and line numbers are determined based on the visual temporal and spatial thresholds under different viewing environments, as described later in Sec. 2.4. Higher frame rate and line rate are necessary in computer applications to accommodate a significantly shorter viewing distance and higher frequency contents (line graphics and texts) in the displayed material.

The width to height ratio of a video frame is known as the *image aspect ratio* (IAR). For example, an IAR of 4:3 is used in standard-definition TV (SDTV) and computer display, while a higher IAR is used in wide-screen movies (up to 2.2) and HDTVs (IAR=16:9) for a more dramatic visual sensation.

## 1.4 Analog Color Television Systems

In this section, we briefly describe the analog color TV systems, which is a good example of many concepts we have talked about so far. One major constraint in designing the color TV system is that it must be compatible with the previous monochrome TV system. First, the overall bandwidth of a color TV signal has to fit within that allocated for a monochrome TV signal (6 MHz per channel in the U.S.). Secondly, all the color signals must be multiplexed into a single composite signal in a way so that a monochrome TV receiver can extract from it the luminance signal. The successful design of color TV systems that satisfy the above constraints is one of the great technological innovations of the 20th century. Figure 1.5 illustrates the main processing steps involved in color TV signal production, transmission, and reception. We briefly review each of the steps in the following.

There are three different systems worldwide: the NTSC system used in North

America as well as some other parts of Asia, including Japan and Taiwan; the PAL system used in most of Western Europe and Asia including China, and the Middle East countries; and the SECAM system used in the former Soviet Union, Eastern Europe, France, as well as Middle East. We will compare these systems in terms of their spatial and temporal resolution, the color coordinate, as well as the multiplexing mechanism. The materials presented here are mainly from [9, 10]. More complete coverage on color TV systems can be found in [5, 1].

### 1.4.1 Spatial and Temporal Resolutions

All three color TV systems use the 2:1 interlaced scan mechanism described in Sec. 1.3 for capturing as well as displaying video. The NTSC system uses a field rate of 59.94 Hz, and a line number of 525 lines/frame. The PAL and SECAM systems both use a field rate of 50 Hz and a line number of 625 lines/frame. These frame rates are chosen to not to interfere with the standard electric power systems in the involved countries. They also turned out to be a good choice in that they match with the critical flicker fusion frequency of the human visual system, as described later in Sec. 2.4. All systems have an IAR of 4:3. The parameters of the NTSC, PAL, and SECAM video signals are summarized in Table 1.1. For NTSC, the line interval is  $T_l=1/(30*525)=63.5\ \mu s$ . But the horizontal retrace takes  $T_h=10\ \mu s$ , so that the actual time for scanning each line is  $T'_l=53.5\ \mu s$ . The vertical retrace between adjacent fields takes  $T_v=1333\ \mu s$ , which is equivalent to the time for 21 scan lines per field. Therefore, the number of active lines is  $525-42=483$ /frame. The actual vertical retrace only takes the time to scan 9 horizontal lines. The remaining time (12 scan lines) are for broadcasters wishing to transmit additional data in the TV signal (e.g., closed caption, teletext, etc.)<sup>9</sup>

### 1.4.2 Color Coordinate

The color coordinate systems used in the three systems are different. For video capture and display, all three systems use a RGB primary, but with slightly different definitions of the spectra of individual primary colors. For transmission of the video signal, in order to reduce the bandwidth requirement and to be compatible with black and white TV systems, a luminance/chrominance coordinate is employed. In the following, we describe the color coordinates used in these systems.

The color coordinates used in the NTSC, PAL and SECAM systems are all derived from the YUV coordinate used in PAL, which in turn originates from the XYZ coordinate. Based on the relation between the RGB primary and XYZ primary, one can determine the Y value from the RGB value, which forms the luminance component. The two chrominance values, U and V, are proportional to color differences, B-Y and R-Y, respectively, scaled to have desired range. Specifically, the YUV

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<sup>9</sup>The number of active lines cited in different references vary from 480 to 495. This number is calculated from the vertical blanking interval cited in [5].

**Table 1.1.** Parameters of Analog Color TV Systems

Parameters	NTSC	PAL	SECAM
Field Rate	59.94	50	50
Line Number/Frame	525	625	625
Line Rate (Line/s)	15,750	15,625	15,625
Image Aspect Ratio	4:3	4:3	4:3
Color Coordinate	YIQ	YUV	YDbDr
Luminance Bandwidth (MHz)	4.2	5.0, 5.5	6.0
Chrominance Bandwidth (MHz)	1.5(I),0.5(Q)	1.3(U,V)	1.0 (U,V)
Color Subcarrier (MHz)	3.58	4.43	4.25 (Db),4.41 (Dr)
Color modulation	QAM	QAM	FM
Audio Subcarrier (MHz)	4.5	5.5,6.0	6.5
Composite Signal Bandwidth(MHz)	6.0	8.0,8.5	8.0

coordinate is related to the PAL RGB primary values by [9]:

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.100 \end{bmatrix} \begin{bmatrix} \tilde{R} \\ \tilde{G} \\ \tilde{B} \end{bmatrix} \quad (1.4.1)$$

and

$$\begin{bmatrix} \tilde{R} \\ \tilde{G} \\ \tilde{B} \end{bmatrix} = \begin{bmatrix} 1.000 & 0.000 & 1.140 \\ 1.000 & -0.395 & -0.581 \\ 1.000 & 2.032 & 0.001 \end{bmatrix} \begin{bmatrix} Y \\ U \\ V \end{bmatrix}, \quad (1.4.2)$$

where  $\tilde{R}, \tilde{G}, \tilde{B}$  are normalized gamma-corrected values, so that  $(\tilde{R}, \tilde{G}, \tilde{B}) = (1, 1, 1)$  corresponds to the reference white color defined in the PAL/SECAM system.

The NTSC system uses the YIQ coordinate, where the I and Q components are rotated versions (by  $33^\circ$ ) of the U and V components. This rotation serves to make I corresponding to colors in the orange-to-cyan range, whereas Q the green-to-purple range. Because the human eye is less sensitive to the changes in the green-to-purple range than that in the yellow-to-cyan range, the Q component can be transmitted with less bandwidth than the I component [10]. This point will be elaborated more in Sec. 1.4.3. The YIQ values are related to the NTSC RGB system by:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} \tilde{R} \\ \tilde{G} \\ \tilde{B} \end{bmatrix} \quad (1.4.3)$$

and

$$\begin{bmatrix} \tilde{R} \\ \tilde{G} \\ \tilde{B} \end{bmatrix} = \begin{bmatrix} 1.0 & 0.956 & 0.620 \\ 1.0 & -0.272 & -0.647 \\ 1.0 & -1.108 & 1.70 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix} \quad (1.4.4)$$

With the YIQ coordinate,  $\tan^{-1}(Q/I)$  approximates the hue, and  $\sqrt{I^2 + Q^2}/Y$  reflects the saturation. In a NTSC composite video, the I and Q components are multiplexed into one signal, so that the phase of the modulated signal is  $\tan^{-1}(Q/I)$ , whereas the magnitude equals  $\sqrt{I^2 + Q^2}/Y$ . Because transmission errors affect the magnitude more than the phase, the hue information is better retained than saturation in broadcast TV signal. This is desired, as the human eye is more sensitive to the color hue. The name I and Q come from the fact that the I signal is In-phase with the color modulation frequency, whereas the Q signal is in Quadrature (i.e. 1/4 of the way around the circle or 90 degrees out of phase) with the modulation frequency. The color multiplexing scheme is explained later in Sec. 1.4.4.

Note that because the RGB primary set and the reference white color used in the NTSC system are different from those in the PAL/SECAM system, the same set of RGB values corresponds to slightly different colors in these two systems.

The SECAM system uses the YDbDr coordinate, where the Db and Dr values are related to the U and V values by [7]

$$D_b = 3.059U, \quad D_r = -2.169V. \quad (1.4.5)$$

### 1.4.3 Signal Bandwidth

The bandwidth of a video raster can be estimated from its line rate. First of all, the maximum vertical frequency results when the white and black lines alternate in a raster frame, which is equal to  $f'_{s,y}/2$  cycles/picture-height, where  $f'_{s,y}$  represent the number of active lines. The maximum frequency that can be rendered properly by a system is usually lower than this theoretical limit. The attenuation factor is known as the *Kell factor*, denoted by  $K$ , which depends on the camera and display aperture functions. Typical TV cameras have a Kell factor of  $K = 0.7$ . The maximum vertical frequency that can be accommodated is related to the Kell factor by

$$f_{v,\max} = K f'_{s,y}/2 \text{ (cycles/picture-height).} \quad (1.4.6)$$

Assuming that the maximum horizontal frequency is identical to the vertical one for the same spatial distance, then,  $f_{h,\max} = f_{v,\max} \text{IAR}$  (cycles/picture-width). Because each line is scanned in  $T'_l$  seconds, the maximum frequency in the 1D raster signal is

$$f_{\max} = f_{h,\max}/T'_l = \text{IAR} \cdot K f'_{s,y}/2T'_l \text{ Hz.} \quad (1.4.7)$$

For the NTSC video format, we have  $f'_{s,y} = 483, T'_l = 53.5 \mu\text{s}$ . Consequently, the maximum frequency of the luminance component is 4.2 megacycles/second or 4.2

MHz. Although the potential bandwidth of the chrominance signal could be just as high, usually it is significantly lower than the luminance signal. Furthermore, the HVS has been found to have much lower threshold for observing changes in chrominance. Because of the Typically, the two chrominance signals are bandlimited to have much narrower bandwidth. As mentioned previously, the human eye is more sensitive to spatial variations in the orange-to-cyan color range, represented by the I component, than it is for the green-to-purple range, represented by the Q component. Therefore, the I component is bandlimited to 1.5 MHz, and the Q component to 0.5 MHz.<sup>10</sup> Table 1.1 lists the signal bandwidth of different TV systems.

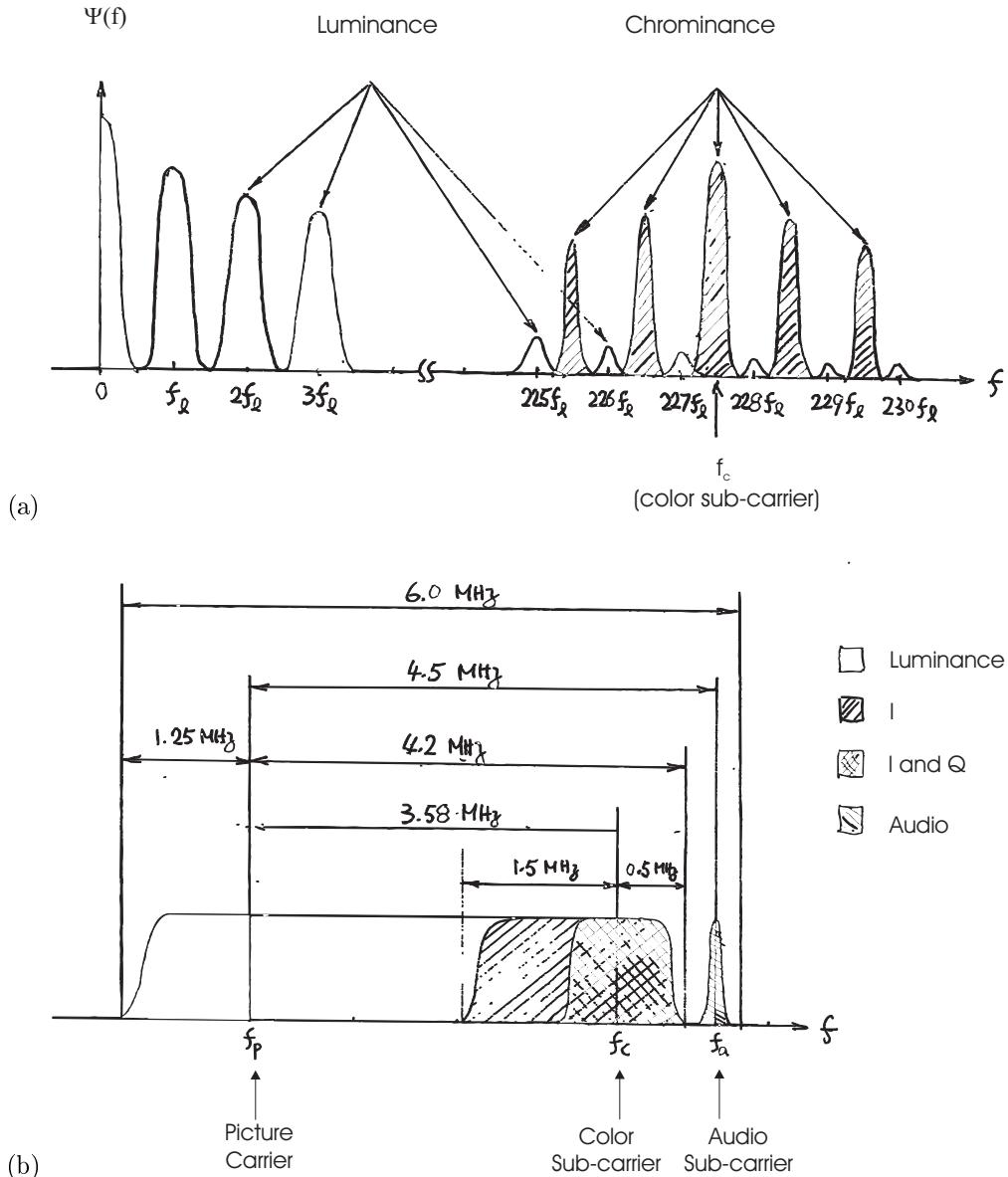
#### 1.4.4 Multiplexing of Luminance, Chrominance, and Audio

In order to make the color TV signal compatible with the monochrome TV system, all three analog TV systems use the composite video format, in which the three color components as well as the audio component are multiplexed into one signal. Here, we briefly describe the mechanism used by NTSC. First, the two chrominance components  $I(t)$  and  $Q(t)$  are combined into a single signal  $C(t)$  using quadrature amplitude modulation (QAM). The *color sub-carrier frequency*  $f_c$  is chosen to be an odd multiple of half of the line rate,  $f_c = 455\frac{f_l}{2} = 3.58$  MHz. This is chosen to satisfy the following criteria: i) It should be high enough where the luminance component has very low energy; ii) It should be midway between two line rate harmonics where the luminance component is strong; and iii) It should be sufficiently far away from the audio sub-carrier, which is set at 4.5 MHz (286  $f_l$ ), the same as in the monochrome TV. Figure 1.6(a) shows how the harmonic peaks of the luminance and chrominance signals interleave with each other. Finally, the audio signal is frequency modulated (FM) using an audio sub-carrier frequency of  $f_a = 4.5$  MHz and added to the composite video signal, to form the final multiplexed signal. Because the I component has a bandwidth of 1.5 MHz, the modulated chrominance signal has a maximum frequency of up 5.08 MHz. In order to avoid the interference with the audio signal, the chrominance signal is bandlimited in the upper sideband to 0.6 MHz. Notice that the lower sideband of the I signal will run into the upper part of the Y signal. For this reason, sometimes the I signal is bandlimited to 0.6 MHz on both sidebands. Finally, the entire composite signal, with a bandwidth of about 4.75 MHz, is modulated onto a picture carrier frequency,  $f_p$ , using vestigial sideband modulation (VSB), so that the lower sideband only extends to 1.25 MHz below  $f_p$  and that the overall signal occupies 6 MHz. This process is the same as in the monochrome TV system. The picture carrier  $f_p$  depends on the broadcasting channel. Figure 1.6(b) illustrates the spectral composition of the NTSC composite signal. The signal bandwidth and modulation methods in the three color TV systems are summarized in Table 1.1.

At a television receiver, the composite signal first has to be demodulated to the baseband, and then the audio and three components of the video signals must

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<sup>10</sup>In [9], the bandwidth of I and Q are cited as 1.3 and 0.6 MHz, respectively.



**Figure 1.6.** Multiplexing of luminance, chrominance, and audio signals in NTSC system. (a) The interleaving between luminance and chrominance harmonics; (b) The overall spectral composition of the NTSC composite signal.

**Table 1.2.** Analog Video Tape Formats

Video Format	Tape Format	Horizontal Lines	Luminance Bandwidth	Applications
composite	VHS, 8mm	240	3 MHz	Consumer
	Umatic SP	330	4 MHz	Professional
S-video	S-VHS, Hi8	400	5.0 MHz	High quality consumer
component	Betacam SP	480	4.5 MHz	Professional

be demultiplexed. To separate the video and audio signals, a low-pass filter can be used. This process is the same in a monochrome TV as that in a color TV. To further separate the chrominance signal from the luminance signal, ideally, a comb filter should be used, to take advantage of the interleaving of the harmonic frequencies in these two signals. Most high-end TV sets implement a digital comb filter with null frequencies at the harmonics corresponding to the chrominance component to accomplish this. Low-end TV sets however use a simple RC circuit to perform low-pass filtering with a cut-off frequency at 3 MHz, which would retain the unwanted I component in the extracted luminance signal, and vice versa. This will lead to cross-color and cross-luminance artifacts. *Cross-color* refers to the spurious colors created by the high frequency luminance signal that is close to the color sub-carrier frequency. *Cross-luminance* refers to false high frequency edge patterns caused by the modulated chrominance information. For a good illustration of the effects of different filters, see [2]. After extracting the chrominance signal, a corresponding color-demodulation method is used to separate the two chrominance components. Finally, the three color components are converted to the RGB coordinate for display.

### 1.4.5 Analog Video Recording

Along with the development of analog TV systems, various video tape recording (VTR) technologies have been developed, to allow professional video production (record and editing) as well as for consumer level recording (home video) and playback (VCR). Table 1.2 summarizes common video tape formats.

## 1.5 Digital Video

### 1.5.1 Notations

A digital video can be obtained either by sampling a raster scan, or directly using a digital video camera. Presently all digital cameras use CCD sensors. As with analog cameras, a digital camera samples the imaged scene as discrete frames. Each frame

comprises of output values from a CCD array, which is by nature discrete both horizontally and vertically. A digital video is defined by the frame rate,  $f_{s,t}$ , the line number,  $f_{s,y}$ , as well as the number of samples per line,  $f_{s,x}$ . From these, one can find the temporal sampling interval or frame interval,  $\Delta_t = 1/f_{s,t}$ , the vertical sampling interval  $\Delta_y = \text{picture-height}/f_{s,y}$ , and the horizontal sampling interval  $\Delta_x = \text{picture-width}/f_{s,x}$ . In this book, we will use  $\psi(m, n, k)$  to represent a digital video, where integer indices  $m$  and  $n$  are the column and row indices, and  $k$  the frame number. The actual spatial and temporal locations corresponding to the integer indices are  $x = m\Delta_x$ ,  $y = n\Delta_y$ , and  $t = k\Delta_t$ . For convenience, we use the notation  $\psi(x, y, t)$  to describe a video signal in a general context, which could be either analog or digital. We will use  $\psi(m, n, k)$  only when specifically addressing digital video.

In addition to the above parameters, another important parameter of a digital video is the number of bits used to represent a pixel value (luminance only or three color values), to be denoted by  $N_b$ . Conventionally, the luminance or each of the three color values is specified with 8 bits or 256 levels. Therefore,  $N_b = 8$  for a monochrome video, while  $N_b = 24$  for a color video. The data rate,  $R$ , of a digital video is determined by  $R = f_{s,t}f_{s,x}f_{s,y}N_b$ , with a unit of bits/second (bps). Usually it is measured in kilobits/second (Kbps) or megabits/second (Mbps). In general, the spatial and temporal sampling rates can be different for the luminance and chrominance components of a digital video. In this case,  $N_b$  should reflect the equivalent number of bits used for each pixel in the luminance sampling resolution. For example, if the horizontal and vertical sampling rates for each chrominance component are both half of that for the luminance, then there are two chrominance samples for every four Y samples. If each sample is represented with 8 bits, the equivalent number of bits per sample in the Y resolution is  $(4*8+2*8)/4=12$  bits.

When displaying the digital video on a monitor, each pixel is rendered as a rectangular region with a constant color that is specified for this pixel. The ratio of the width to the height of this rectangular area is known as the *pixel aspect ratio* (PAR). It is related to the IAR of the display area and the image dimension by

$$\text{PAR} = \text{IAR} \cdot f_{s,y}/f_{s,x}. \quad (1.5.1)$$

For proper display of a digitized video, one must specify either PAR or IAR, along with  $f_{s,x}$  and  $f_{s,y}$ . The display device should conform to the PAR specified for this video (or derived from the specified IAR). Otherwise, the object shape will be distorted. For example, a person will become fatter and shorter, if the display PAR is larger than the PAR specified for this video. In computer industry, a PAR of 1.0 is normally used. On the other hand, in the TV industry, non-square pixels are used because of some historical reasons. The rationale behind this is explained in Sec. 1.5.2.

### 1.5.2 ITU-R BT.601 Digital Video

**Spatial Resolution of the BT.601 Signal** In an attempt to standardize the digital formats used to represent different analog TV video signals with a quality equivalent to broadcast TV, the International Telecommunications Union - Radio Sector (ITU-R) developed the BT.601 recommendation [8]. The standard specifies digital video formats with both 4:3 and 16:9 aspect ratios. Here, we only discuss the version with aspect ratio 4:3.<sup>11</sup> To convert a raster scan to a digital video, one just needs to sample the 1D waveform. If a total number of  $f_{s,x}$  samples are taken per line, the equivalent sampling rate is  $f_s = f_{s,x}f_{s,y}f_{s,t} = f_{s,x}f_l$  samples/second. In the BT.601 standard, the sampling rate  $f_s$  is chosen to satisfy two constraints: i) the horizontal sampling resolution should match with the vertical sampling resolution as closely as possible, *i.e.* let  $\Delta_x \approx \Delta_y$ ; and ii) the same sampling rate should be used for the NTSC and PAL/SECAM systems, and it should be multiples of respective line rates in these systems. The first criterion calls for  $f_{s,x} \approx \text{IAR} \cdot f_{s,y}$ , or  $f_s \approx \text{IAR} \cdot f_{s,y}^2 f_{s,t}$ , which leads to  $f_s \approx 11$  and 13 MHz for the NTSC and PAL/SECAM system. A number that is closest to both numbers and yet satisfies the second criterion is then chosen, which is

$$f_s = 858f_l \text{ (NTSC)} = 864f_l \text{ (PAL/SECAM)} = 13.5\text{MHz.} \quad (1.5.2)$$

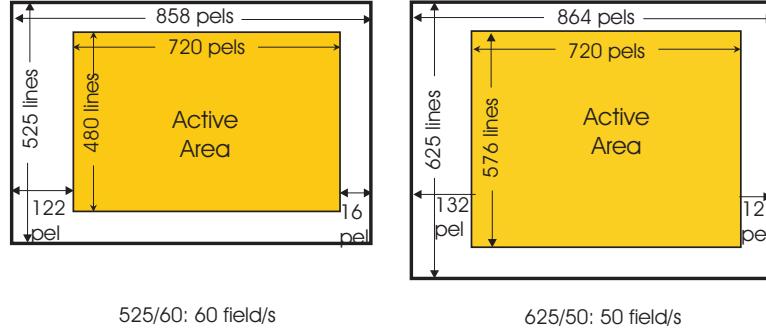
The numbers of pixels per line are  $f_{s,x} = 858$  for NTSC and 864 for PAL/SECAM. These two formats are known as 525/60 and 625/50 signals, respectively, and are illustrated in Fig. 1.7. The numbers of active lines are respectively  $f'_{s,y} = 480$  and 576 in the 525 and 625 line systems, but the number of active pixels/line are the same, both equal to  $f'_{s,x} = 720$  pixels. The rest are samples obtained during the horizontal and vertical retraces, which fall in the non-active area.

With the BT.601 signal, the pixel width to height ratio is not one, *i.e.* the physical area associated with a pixel is not a square. Specifically,  $\text{PAR} = \Delta_x/\Delta_y = \text{IAR} \cdot f'_{s,y}/f'_{s,x} = 8/9$  for 525/60 and 16/15 for 625/50 signals. To display a BT.601 signal, the display device must have a proper PAR, otherwise the image will be distorted. For example, when displayed on a computer screen which has a PAR of 1, a 525/60 signal will appear stretched horizontally, while a 625/50 signal will appear stretched vertically. Ideally, one should resample the original signal so that  $f'_{s,x} = \text{IAR} \cdot f'_{s,y}$ . For example, the 525/60 and 625/50 signals should be resampled to have 640 and 768 active pixels/line, respectively.

**Color Coordinate and Chrominance Subsampling** Along with the image resolution, the BT.601 recommendation also defines a digital color coordinate, known as YCbCr. The Y, Cb, and Cr components are scaled and shifted versions of the analog Y, U, and V components, where the scaling and shifting operations are introduced so that the resulting components take value in the range of (0,255). For a more detailed explanation on the design this color coordinate, the readers are referred

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<sup>11</sup>The ITU-R was formerly known as International Radio Consultative Committee (CCIR) and the 4:3 aspect ratio version of the BT.601 format was called the CCIR601 format.

**Figure 1.7.** BT.601 video formats.

to [9]. Here we only present the transformation matrix for deriving this coordinate from digital RGB coordinate. Assuming that the RGB values are in the range of (0–255), the YCbCr values are related to RGB values by:

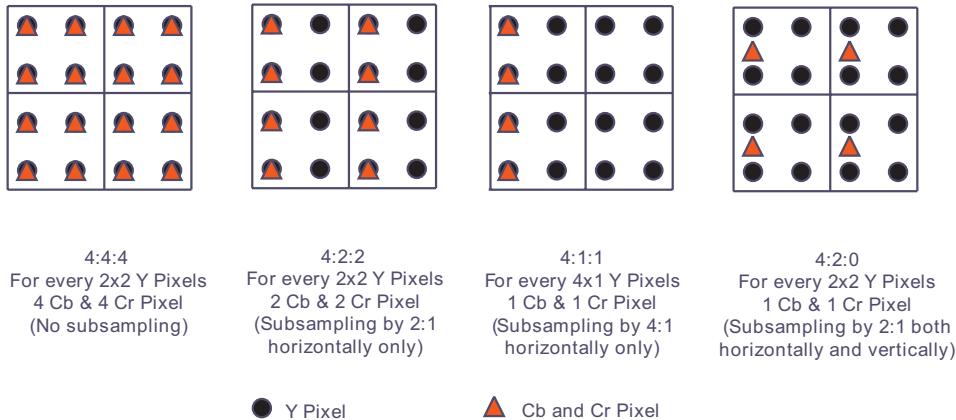
$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.257 & 0.504 & 0.098 \\ -0.148 & -0.291 & 0.439 \\ 0.439 & -0.368 & -0.071 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 16 \\ 128 \\ 128 \end{bmatrix}. \quad (1.5.3)$$

The inverse relation is:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.164 & 0.0 & 1.596 \\ 1.164 & -0.392 & -0.813 \\ 1.164 & 2.017 & 0.0 \end{bmatrix} \begin{bmatrix} Y - 16 \\ C_b - 128 \\ C_r - 128 \end{bmatrix}. \quad (1.5.4)$$

In the above relations,  $R = 255\tilde{R}$ ,  $G = 255\tilde{G}$ ,  $B = 255\tilde{B}$  are the digital equivalent of the normalized RGB primaries,  $\tilde{R}$ ,  $\tilde{G}$ , and  $\tilde{B}$ , as defined either in the NTSC or PAL/SECAM system. In the YCbCr coordinate,  $Y$  reflects the luminance and is scaled to have a range of (16–235),  $C_b$  and  $C_r$  are scaled versions of color differences  $B - Y$  and  $R - Y$ , respectively. The scaling and shifting is designed so that they have a range of (16–240). The maximum value of  $C_r$  corresponds to red ( $C_r = 240$  or  $R = 255, G = B = 0$ ), whereas the minimum value yields cyan ( $C_r = 16$  or  $R = 0, G = B = 255$ ). Similarly, the maximum and minimum values of  $C_b$  correspond to blue ( $C_b = 240$  or  $R = G = 0, B = 255$ ) and yellow ( $C_b = 16$  or  $R = G = 255, B = 0$ ).

The spatial sampling rate introduced previously refers to the luminance component,  $Y$ . For the chrominance components,  $C_b$  and  $C_r$ , usually only half of the sampling rate is used, *i.e.*  $f_{s,c} = f_s/2$ . This leads to half number of pixels in each line, but the same number of lines per frame. This is known as the 4:2:2 format, implying there are 2 Cb samples and 2 Cr samples for every 4 Y samples. To further reduce the required data rate, BT.601 also defined the 4:1:1 format, in which



**Figure 1.8.** BT.601 chrominance subsampling formats. Note that the two adjacent lines in any one component belong to two different fields.

the chrominance components are subsampled along each line by a factor of 4, i.e., there are 1 Cb sample and 1 Cr sample for every 4 Y samples. This sampling method, however, yields very asymmetric resolutions in the horizontal and vertical directions. Another sampling format is therefore developed, which subsamples the Cb and Cr components by half in both the horizontal and vertical directions. In this format, there are also 1 Cb sample and 1 Cr sample for every 4 Y samples. But to avoid the confusion with the previously defined 4:1:1 format, this format is designated as 4:2:0. For applications requiring very high resolutions, the 4:4:4 format is defined, which samples the chrominance components in exactly the same resolution as the luminance components. The relative positions of the luminance and chrominance samples for different formats are shown in Fig. 1.8.<sup>12</sup>

In Chap. 4, we will discuss solutions for converting videos with different spatial/temporal resolutions. The conversion between different color subsampling formats is considered in one of the exercise problems.

The raw data rates of a BT.601 signal depends on the color sub-sampling factor. With the most common 4:2:2 format, there are two chrominance samples per two Y samples, each represented with 8 bits. Therefore, the equivalent bit rate for each Y sample is  $N_b = 16$  bits, and the raw data rate is  $f_s N_b = 216$  Mbps. The raw data rate corresponding to the active area is  $f_{s,t} f'_{s,y} f'_{s,x} N_b = 166$  Mbps. With the 4:2:0 format, there are two chrominance samples per four Y samples, and the equivalent bit rate for each Y sample is  $N_b = 12$  bits. Therefore the raw data rate is 162 Mbps, with 124 Mbps in the active area. For the 4:4:4 format, the equivalent bit rate for each Y sample is  $N_b = 24$  bits, and the raw data rate is 324 Mbps, with 249 Mbps in the active area. The resolutions and data rates of different BT.601 signals are

<sup>12</sup>For the 4:2:0 format, the Cr and Cb samples may also be positioned in the center of the four corresponding Y samples, as shown in Fig. 13.14(a).

summarized in Table 1.3.

The BT.601 formats are used in high-quality digital video applications, with the 4:4:4 and 4:2:2 formats typically used for video production and editing, whereas 4:2:0 for video distribution, e.g., movies on digital video disks (DVD), video-on-demand (VOD), etc. The MPEG2<sup>13</sup> video compression standard was primarily developed for compression of BT.601 4:2:0 signals, although it can also handle videos in lower or higher resolutions. A typical 4:2:0 signal with a raw active data rate of 124 Mbps can be compressed down to about 4-8 Mbps. We will introduce the MPEG2 video coding algorithm in Sec. 13.5.

### 1.5.3 Other Digital Video Formats and Applications

In addition to the BT.601 format, several other standard digital video formats have been defined. Table 1.3 summarizes these video formats, along with their main applications, compression methods, and compressed bit rates. The CIF (Common Intermediate Format) is specified by International Telecommunications Union-Telecommunications Sector (ITU-T), which has about half the resolution of BT.601 4:2:0 in both horizontal and vertical dimensions and is developed for video conferencing applications, and the QCIF, which is a quarter of CIF, used for video phone type applications. Both are non-interlaced. The ITU-T H.261 coding standard was developed to compress videos in either format to  $p \times 64$  Kbps, with  $p = 1, 2, \dots, 30$ , for transport over ISDN (integrated service digital network) lines, which only allow transmission rates in multiples of 64 Kbps. Typically, a CIF signal with a raw data rate of 37.3 Mbps can be compressed down to about 128 to 384 Kbps, with reasonable quality, while a QCIF signal with a raw data rate of 9.3 Mbps can be compressed to 64-128 Kbps. A later standard, H.263, can achieve better quality than H.261, at the same bit rate. For example, it is possible to compress a QCIF picture to about 20 Kbps, while maintaining a quality similar or better than H.261 at 64 Kbps. This enables video phone over a 28.8 Kbps modem line.

In parallel with the effort of ITU-T, the ISO-MPEG also defined a series of digital video formats. The SIF (Source Intermediate Format) is essentially a quarter size of the active area in the BT.601 signal, and is about the same as CIF. This format is targeted for video applications requiring medium quality, such as video games and CD movies. As with BT.601, there are two SIF formats: one with a frame rate of 30 Hz and a line number of 240, and another with a frame rate of 25 and line number of 288, both have 352 pixels/line. There is also a corresponding set of SIF-I format, which is 2:1 interlaced. The MPEG-1 algorithm can compress a typical SIF video with a raw data rate of 30 Mbps to about 1.1 Mbps with a quality similar to the resolutions seen on a VHS VCR, which is lower than broadcast television. The rate of 1.1 Mbps enables the playback of digital movies on CD-ROM's, which have an access rate of 1.5 Mbps. Distribution of MPEG1 movies on video CD's (VCD) marked the entrance of digital video into the consumer market in the early 1990's.

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<sup>13</sup> MPEG standards for Motion Picture Expert Group of the International Standard Organization or ISO.

MPEG2-based DVD's, which started in mid 90's, opened the era of high quality digital video entertainment. MPEG2 technology is also the corner stone of the next generation TV system, which will be fully digital, employing digital compression and transmission technology. Table 1.3 lists the details of the video formats discussed above, along with their main applications, compression methods, and compressed bit rates. More on compression standards will be presented in Chap. 13.

The BT.601 format is the standard picture format for digital TV (DTV). To further enhance the video quality, several HDTV formats have also been standardized by the Society of Motion Picture and Television Engineers (SMPTE), which are also listed in Table 1.3. A distinct feature of HDTV is its wider aspect ratio, 16:9 as opposed to 4:3 in SDTV. The picture resolution is doubled to tripled in both horizontal and vertical dimensions. Furthermore, progressive scan is used to reduce the interlacing artifacts. A high profile has been developed in the MPEG2 video compression standard for compressing HDTV video. Typically it can reduce the data rate to about 20 Mbps while retaining the very high quality required. This video bit rate is chosen so that the combined bit stream with audio, when transmitted using digital modulation techniques, can still fit into a 6 MHz terrestrial channel, which is the assigned channel bandwidth for HDTV broadcast in the U.S.

#### 1.5.4 Digital Video Recording

To store video in digital formats, various digital video tape recorder (DVTR) formats have been developed, which differ in the video format handled and technology for error-correction-coding and storage density. Table 1.4 lists some standard and proprietary tape formats. The D1-D5 formats store a video in its raw, uncompresssed formats, while others pre-compress the video. Only a conservative amount of compression is employed so as not to degrade the video quality beyond that acceptable for the intended application. A good review of digital VTRs can be found in [11]. A extensive coverage on the underlying physics of magnetic recording and operation of DVTRs can be found in the book by Watkinson [12].

In addition to magnetic tape recorders, VCD and DVD are two video storage devices using optical disks. By incorporating MPEG1 and MPEG2 compression technologies, they can store SIF and BT.601 videos, respectively, with sufficient quality. At present, VCD and DVD are read-only, so that they are mainly used for distribution of pre-recorded video, as opposed to as tools for recording video by consumers.

Except video recording systems using magnetic tapes, hard-disk based systems, such as TiVo and ReplayTV, are also on the horizon. These systems enable consumers to record up to 30 hours of live TV programs onto hard-disks in MPEG-2 compressed formats, which can be viewed later with usual VCR features such as fast forward, slow motion, etc. They also allow instant pause of a live program that is being watched, by storing the live video from the time of the pause onto the disk. As the price for hard-disks drops down continuously, hard-disk-based DVTR's

**Table 1.3.** Digital Video Formats for Different Applications

Video Format	Y Size	Color Sampling	Frame Rate	Raw Data (Mbps)
HDTV over air, cable, satellite, MPEG2 video, 20-45 Mbps				
SMPTE 296M	1280x720	4:2:0	24P/30P/60P	265/332/664
SMPTE 295M	1920x1080	4:2:0	24P/30P/60I	597/746/746
Video Production, MPEG2, 15-50 Mbps				
BT.601	720x480/576	4:4:4	60I/50I	249
BT.601	720x480/576	4:2:2	60I/50I	166
High quality video distribution (DVD,SDTV), MPEG2, 4-8 Mbps				
BT.601	720x480/576	4:2:0	60I/50I	124
Intermediate quality video distribution (VCD, WWW), MPEG1, 1.5 Mbps				
SIF	352x240/288	4:2:0	30P/25P	30
Video conferencing over ISDN/Internet, H.261/H.263, 128-384 Kbps				
CIF	352x288	4:2:0	30P	37
Video telephony over wired/wireless modem, H.263, 20-64 Kbps				
QCIF	176x144	4:2:0	30P	9.1

may eventually overtake tape-based systems, which are slower and have less storage capacity.

### 1.5.5 Video Quality Measure

To conduct video processing, it is necessary to define an objective measure that can measure the difference between an original video and the processed one. This is especially important, e.g., in video coding applications where one must measure the distortion caused by compression. Ideally such a measure should correlate well with the perceived difference between two video sequences. Finding such a measure however turns out to be an extremely difficult task. Although various quality measures have been proposed, those that correlate well with visual perception are quite complicated to compute. Most video processing systems of today are designed to minimize the mean square error (MSE) between two video sequences  $\psi_1$  and  $\psi_2$ , which is defined as

$$\text{MSE} = \sigma_e^2 = \frac{1}{N} \sum_k \sum_{m,n} (\psi_1(m, n, k) - \psi_2(m, n, k))^2, \quad (1.5.5)$$

**Table 1.4.** Digital Video Tape Formats

Tape Format	Video Format	Source Rate	Compressed Rate	Compression Method	Intended Application
<b>Uncompressed formats</b>					
SMPTE D1	BT.601 4:2:2	216 Mbps	N/A	N/A	Professional
SMPTE D2	BT.601 composite	114 Mbps	N/A	N/A	Professional
SMPTE D3	BT.601 composite	114 Mbps	N/A	N/A	Professional/ Consumer
SMPTE D5	BT.601 4:2:2 (10 bit)	270 Mbps	N/A	N/A	Professional
<b>Compressed formats</b>					
Digital Betacam	BT.601 4:2:2	166 Mbps	80 Mbps	Frame DCT	Professional
Betacam SX	BT.601 4:2:2	166 Mbps	18 Mbps	MPEG2 (I and B mode only)	Consumer
DVCPRO50	BT.601 4:2:2	166 Mbps	50 Mbps	frame/field DCT	Professional
DVCPRO25 (DV)	BT.601 4:1:1	124 Mbps	25 Mbps	frame/field DCT	Consumer

where  $N$  is the total number of pixels in either sequence. For a color video, the MSE is computed separately for each color component.

Instead of the MSE, the *peak signal to noise ratio* (PSNR) in decibel (dB) is more often used as a quality measure in video coding. The PSNR is defined as

$$\text{PSNR} = 10 \log_{10} \frac{\psi_{\max}^2}{\sigma_e^2} \quad (1.5.6)$$

where  $\psi_{\max}$  is the peak (maximum) intensity value of the video signal. For the most common 8 bit/color video,  $\psi_{\max} = 255$ . Note that for a fixed peak value, PSNR is completely determined by the MSE. The PSNR is more commonly used than the MSE, because people tend to associate the quality of an image with a certain range of PSNR. As a rule of thumb, for the luminance component, a PSNR over 40 dB typically indicates an excellent image (i.e., being very close to the original), between 30 to 40 dB usually means a good image (i.e., the distortion is visible but acceptable), between 20 and 30 dB is quite poor, and finally, a PSNR lower than 20 dB is unacceptable.

It is worth noting that to compute the PSNR between two sequences, it is *incorrect* to calculate the PSNR between every two corresponding frames and then taking the average of the PSNR values obtained over individual frames. Rather one should compute the MSE between corresponding frames, average the resulting MSE values over all frames, and finally convert the MSE value to PSNR.

A measure that is sometimes used in place of the MSE, mainly for reduced computation, is the mean absolute difference (MAD). This is defined as

$$\text{MAD} = \frac{1}{N} \sum_k \sum_{m,n} |\psi_1(m, n, k) - \psi_2(m, n, k)|. \quad (1.5.7)$$

For example, for motion estimation, the MAD is usually used to find the best matching block in another frame for a given block in a current frame.

It is well known that MSE or PSNR does not correlate very well with visual distortion between two imagery. But these measures have been used almost exclusively as objective distortion measures in image/video coding, motion compensated prediction, and image restoration, partly because of their mathematical tractability, and partly because of the lack of better alternatives. Designing objective distortion measures that are easy to compute and yet correlate well with visual distortion is still an open research issue. In this book, we will mostly use MSE or PSNR as the distortion measure.

## 1.6 Summary

### **Color Generation, Perception, and Specification (Sec. 1.1)**

- The color of a light depends on its spectral content. Any color can be created by mixing three primary colors. The most common primary set includes red, green, and blue colors.
- The human eye perceives color by having receptors (cones) in the retina that are tuned to red, green, and blue wavelengths. The color sensation can be described by three attributes: luminance (i.e., brightness), hue (color tone), and saturation (color purity). The human eye is most sensitive to luminance, then to hue, and finally to saturation.
- A color can be specified by three numbers: either those corresponding to the contributions of the three primary colors (i.e., tristimulus values), or a luminance value and two chrominance values.

### **Analog Video (Sec. 1.3)**

- Analog videos used in broadcasting TV, video camcorder, etc., video display, are stored in a raster scan format. The visual quality and bandwidth of a raster scan depends on its frame rate and line number.

- Interlaced scan is a mechanism to trade off vertical resolution for enhanced temporal resolution. But it also leads to interlacing artifacts.

### Analog Color TV Systems (Sec. 1.4)

- There are three analog color TV systems world wide: NTSC, PAL, and SECAM. They all use 2:1 interlace, but differ in frame rate, line number, color coordinate, and luminance and chrominance multiplexing.
- In color TV systems, the luminance and two chrominance components as well as the associated audio signal are multiplexed into a composite signal, using modulation (frequency shifting) techniques. The multiplexing methods are designed so that the color TV system is downward compatible with the monochrome TV system. Furthermore, the modulation frequencies for individual components are chosen to minimize the interference among them.

### Digital Video (Sec. 1.5)

- BT.601 is a digital video format, resulting from sampling the analog color TV signals. The sampling rate is chosen so that the horizontal sampling rate is similar to the vertical sampling rate, and that the data rates for NTSC and PAL/SECAM systems are the same.
- The chrominance components can be sampled at a lower rate than the luminance component. There are different color subsampling formats defined in BT.601.
- Compression is necessary to reduce the raw data rate of a digital video to reduce the storage/transmission cost. Different video compression standards have been developed for videos intended for different applications.

## 1.7 Problems

- 1.1 Describe the mechanism by which the human being perceives color.
- 1.2 What is the perceived color if you have a light that has approximately the same energy at frequencies corresponding to red, green, and blue, and are zero at other frequencies ? What about red and green frequencies only ?
- 1.3 What is the perceived color if you mix red, green, and blue dyes in equal proportion ? What about red and green dyes only ?
- 1.4 For the following colors in the RGB coordinate, determine their values in the YIQ and YUV coordinates, respectively.
  - (a) (1,1,1); (b) (0,1,0); (c) (1,1,0); (d) (0, 1,1).

- 1.5** For the following colors in the digital RGB coordinate, determine their values in the YCbCr coordinate.  
(a) (255,255, 255); (b) (0, 255, 0); (c) (255,255,0); (d) (0, 255, 255).
- 1.6** In Sec. 1.5.2, we say that the maximum value of  $C_r$  corresponds to red, whereas the minimum value yields cyan . Similarly, the maximum and minimum values of  $C_b$  correspond to blue and yellow, respectively. Verify these statements using the YCbCr to RGB coordinate transformation.
- 1.7** In Fig. 1.4, we show the spectrum of a typical raster signal. Why is the spectrum of the video signal nearly periodic? What does the width of harmonic lobes depend on?
- 1.8** What are the pros and cons of progressive vs. interlaced scans? For the same line number per frame, what is the relation between the maximum temporal frequency that a progressive raster can have and that of an interlaced raster which divides each frame into two fields? What about the relation between the maximum vertical frequencies?
- 1.9** In Sec. 1.4.3, we estimated the bandwidth of the NTSC signal based on its scan parameters. Following the same approach, estimate the bandwidth of the PAL and SECAM signals.
- 1.10** Describe the process for forming a composite color video signal. How should you select the color sub-carrier frequency and audio sub-carrier frequency ?
- 1.11** What are the pros and cons of using component vs. composite format?
- 1.12** Project: Using an oscilloscope to i) draw the waveform, and ii) measure the spectrum of a composite video signal output from a TV set or a camcorder.
- 1.13** Project: Digitize a composite video signal using an A/D converter, and using Matlab to determine the spectrum. Also perform filtering to separate the luminance, chrominance and audio signals.

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# FOURIER ANALYSIS OF VIDEO SIGNALS AND PROPERTIES OF THE HUMAN VISUAL SYSTEM

Fourier analysis is an important tool for signal analysis. We assume that the reader is familiar with Fourier transforms for one- and two-dimensional (1D and 2D) spaces as well as signal processing tools using such transforms. In this chapter, we first extend these results to K-dimensions (K-D), where  $K$  can be any positive integer. We then focus on their applications for video signals, which are three-dimensional (3D). We will explore the meaning of spatial and temporal frequencies, and their inter-relationship. Finally, we discuss visual sensitivity to different frequency components.

## 2.1 Multi-dimensional Continuous Space Signals and Systems

Most of the theorems and techniques for multi-dimensional signals and systems are direct extensions of those developed for 1D and 2D signals and systems. In this section, we introduce some important concepts and theorems for signal analysis in the K-D real space,  $\mathcal{R}^K = \{[x_1, x_2, \dots, x_K]^T | x_k \in \mathcal{R}, k \in \mathcal{K}\}$ , where  $\mathcal{R}$  is the set of real numbers, and  $\mathcal{K} = \{1, 2, \dots, K\}$ . We start by defining K-D signals, common operations between K-D signals, and special K-D signals. We then define the Fourier transform representation of K-D signals. Finally, we define K-D systems and properties of the linear and shift invariant systems. This presentation is intentionally kept brief. We also intentionally leave out discussion of the convergence conditions of various integral formulations. For a more substantial treatment of the subject, the reader is referred to the book by Dudgeon and Mersereau [2].

**Definition 2.1:** A K-D continuous space signal  $\psi(\mathbf{x})$  is a function of a K-D continuous variable  $\mathbf{x} = [x_1, x_2, \dots, x_K] \in \mathcal{R}^K$ . The function can take on any values,

real or complex. The function is called a real function if it only takes real values.

Signals can be combined according to vector space operations of addition and scaling. Another important operation between K-D functions is convolution, as defined below.

**Definition 2.2:** The convolution of two signals,  $\psi(\mathbf{x})$  and  $h(\mathbf{x})$ , both defined over  $\mathcal{R}^K$ , is defined as

$$\psi(\mathbf{x}) * h(\mathbf{x}) = \int_{\mathcal{R}^K} \psi(\mathbf{x} - \mathbf{y})h(\mathbf{y})d\mathbf{y}. \quad (2.1.1)$$

Among several special functions, the delta function(also known as the impulse function) plays an important role in characterizing signals and linear systems.

**Definition 2.3:** The delta function in  $\mathcal{R}^K$  satisfies:

$$\delta(\mathbf{x}) = \begin{cases} \infty, & \mathbf{x} = \mathbf{0}, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad \int_{\mathcal{R}^K} \delta(\mathbf{x})d\mathbf{x} = 1. \quad (2.1.2)$$

**Properties of the Delta Function:** The following are several important identities associated with the Delta function, where  $\psi(\cdot)$  is an arbitrary K-D function:

$$\psi(\mathbf{x}) * \delta(\mathbf{x} - \mathbf{x}_0) = \psi(\mathbf{x} - \mathbf{x}_0), \quad (2.1.3)$$

$$\int_{\mathcal{R}^K} \psi(\mathbf{x})\delta(\mathbf{x} - \mathbf{x}_0)d\mathbf{x} = \psi(\mathbf{x}_0), \quad (2.1.4)$$

$$\int_{\mathcal{R}^K} \exp(j2\pi\mathbf{f}_0^T \mathbf{x})d\mathbf{x} = \delta(\mathbf{f}_0). \quad (2.1.5)$$

**Definition 2.4 (Continuous Space Fourier Transform)** The continuous space Fourier transform (CSFT) of a signal  $\psi(\mathbf{x})$  is defined as<sup>1,2</sup>,

$$\Psi_c(\mathbf{f}) = \int_{\mathcal{R}^K} \psi(\mathbf{x}) \exp(-j2\pi\mathbf{f}^T \mathbf{x})d\mathbf{x}, \quad (2.1.6)$$

where  $\mathbf{f} = [f_1, f_2, \dots, f_K]^T \in \mathcal{R}^K$  represents the continuous domain frequency variable.

**Theorem 2.1 (Inverse Continuous Space Fourier Transform)** A function can be obtained from its CSFT by

$$\psi(\mathbf{x}) = \int_{\mathcal{R}^K} \Psi_c(\mathbf{f}) \exp(j2\pi\mathbf{f}^T \mathbf{x})d\mathbf{f}. \quad (2.1.7)$$

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<sup>1</sup>In the 1D case, CSFT is usually called *continuous time Fourier transform* or simply Fourier transform, because the 1D signal is typically a function of time.

<sup>2</sup>In this book, we define CSFT in terms of the frequency variable  $\mathbf{f}$ , rather than the radian frequency  $\mathbf{\Omega} = 2\pi\mathbf{f}$ . With this definition, the normalization by  $2\pi$  is not needed, as each transform basis function in terms of  $f$  has a unit norm.

To prove the above theorem, substituting Eq. (2.1.6) into the right side of Eq. (2.1.7), we obtain

$$\begin{aligned}\int_{\mathcal{R}^K} \Psi_c(\mathbf{f}) \exp(j2\pi\mathbf{f}^T \mathbf{x}) d\mathbf{f} &= \int_{\mathcal{R}^K} \psi(\mathbf{y}) \int_{\mathcal{R}^K} \exp(j2\pi\mathbf{f}^T(\mathbf{x} - \mathbf{y})) d\mathbf{f} d\mathbf{y} \\ &= \int_{\mathcal{R}^K} \psi(\mathbf{y}) \delta(\mathbf{x} - \mathbf{y}) d\mathbf{y} = \psi(\mathbf{x}).\end{aligned}$$

The second equality comes from Eq. (2.1.5), and the third one follows from Eq. (2.1.4).

The inverse CSFT shows that any signal  $\psi(\mathbf{x})$  can be expressed as a linear combination of complex exponential functions with different frequencies. The CSFT at a particular frequency represents the contribution of the corresponding complex exponential basis function.<sup>3</sup>

Most of the properties of the 1D FT can be extended to the K-D case, including the linearity, translation, convolution, etc. Here we only list the convolution theorem, which is very important for signal analysis.

**Theorem 2.2 (The Convolution Theorem)** *The convolution of two K-D signals in the spatial domain is equivalent to the product of the two signals in the frequency domain, i.e.,*

$$\phi(\mathbf{x}) = \psi(\mathbf{x}) * h(\mathbf{x}) \longleftrightarrow \Phi_c(\mathbf{f}) = \Psi_c(\mathbf{f}) H_c(\mathbf{f}). \quad (2.1.8)$$

Conversely, the product of two signals in the spatial domain corresponds to the convolution in the Fourier domain, i.e.,

$$\phi(\mathbf{x}) = \psi(\mathbf{x}) h(\mathbf{x}) \longleftrightarrow \Phi_c(\mathbf{f}) = \Psi_c(\mathbf{f}) * H_c(\mathbf{f}). \quad (2.1.9)$$

**Definition 2.5:** A system with K-D input and output signals is in general described by:

$$\phi(\mathbf{x}) = \mathcal{T}\{\psi(\mathbf{x})\}, \mathbf{x} \in \mathcal{R}^K. \quad (2.1.10)$$

Let  $\phi_1(\mathbf{x})$  and  $\phi_2(\mathbf{x})$  represent the output signals of a system corresponding to input signals  $\psi_1(\mathbf{x})$  and  $\psi_2(\mathbf{x})$  respectively. The system is linear if for any  $\alpha_1, \alpha_2 \in \mathcal{R}$ ,

$$\mathcal{T}\{\alpha_1\psi_1(\mathbf{x}) + \alpha_2\psi_2(\mathbf{x})\} = \alpha_1\phi_1(\mathbf{x}) + \alpha_2\phi_2(\mathbf{x}). \quad (2.1.11)$$

Furthermore, the system is shift-invariant if

$$\mathcal{T}\{\psi(\mathbf{x} + \mathbf{x}_0)\} = \phi(\mathbf{x} + \mathbf{x}_0). \quad (2.1.12)$$

The system is called linear and shift-invariant (LTI) if it is both linear and shift-invariant. The output signal of a LTI system to an impulse function  $\delta(\mathbf{x})$ ,  $\mathcal{T}\{\delta(\mathbf{x})\}$ , is known as the impulse response of the system, usually denoted by  $h(\mathbf{x})$ . Its CSFT is called the frequency response of the system, usually denoted by  $H(\mathbf{f})$ .

<sup>3</sup>The Fourier integral defined in Eq. (2.1.6) does not converge for all functions in  $\mathcal{R}^K$ . But if we allow the Fourier transform to contain delta functions, then the Fourier analysis can be applied to a wide variety of functions. In this book, we choose not to discuss the various convergence conditions of the Fourier transform both for the continuous and discrete time cases. More rigorous definitions of the Fourier transforms can be found in [9, 2].

**Theorem 2.3:** In a LTI system with input and output signals in  $\mathcal{R}^K$ , the output signal  $\phi(\mathbf{x})$  corresponding to any input signal  $\psi(\mathbf{x})$  can be described as a linear convolution of  $\psi(\mathbf{x})$  with the system impulse response function  $h(\mathbf{x})$ , i.e.,

$$\phi(\mathbf{x}) = \psi(\mathbf{x}) * h(\mathbf{x}). \quad (2.1.13)$$

In frequency domain, they are related by

$$\Phi_c(\mathbf{f}) = \Psi_c(\mathbf{f})H_c(\mathbf{f}). \quad (2.1.14)$$

We like to emphasize that the above result is solely the consequence of the linearity and shift-invariance of the system. Obviously, the frequency response  $H(\mathbf{f})$  of the system describes how the input signal will be altered by the system at different frequencies. A LTI system is completely characterized by its impulse response  $h(\mathbf{x})$  or the frequency response  $H(\mathbf{f})$ .

## 2.2 Multi-dimensional Discrete Space Signals and Systems

The previous section considers signals defined over a K-D continuous space,  $\mathcal{R}^K$ . There are also applications where the signal is discrete by nature, i.e., the signal is only defined over a set of discrete points. We can always index each discrete point by an integer vector, so that the signal is defined over a K-D integer space,  $\mathcal{Z}^K = \{[n_1, n_2, \dots, n_K]^T | n_k \in \mathcal{Z}, k \in \mathcal{K}\}$ , where  $\mathcal{Z}$  is the set of all integers. In general, we call such signals K-D discrete signals or sequences. A special case of the discrete signal is the sampled version of a continuous signal. We will discuss the sampling process and properties of sampled signals in the next chapter. In this section, we formally define multi-dimensional discrete signals and systems and their characterization by the discrete space Fourier transform.

**Definition 2.6:** A K-D discrete space signal  $\psi(\mathbf{n})$  is a function with a K-D discrete variable  $\mathbf{n} = [n_1, n_2, \dots, n_K] \in \mathcal{Z}^K$ . The function can take on any values, real or complex. The function is called a real function if it only takes real values.

**Definition 2.7:** The discrete convolution of two discrete space signals  $\psi(\mathbf{n})$  and  $h(\mathbf{n})$  is defined as

$$\psi(\mathbf{n}) * h(\mathbf{n}) = \sum_{\mathbf{m} \in \mathcal{R}^K} \psi(\mathbf{n} - \mathbf{m})h(\mathbf{m}). \quad (2.2.1)$$

**Definition 2.8:** The discrete delta function  $\delta(\mathbf{n})$  is defined as

$$\delta(\mathbf{n}) = \begin{cases} 1, & \mathbf{n} = \mathbf{0}, \\ 0, & \text{otherwise.} \end{cases} \quad (2.2.2)$$

As with the continuous delta function, there are special properties associated with the discrete delta function. These can be obtained by replacing the integral with sum in Eqs. (2.1.3)-(2.1.5).

**Definition 2.9 (Discrete Space Fourier Transform)** *The discrete space Fourier transform (DSFT)<sup>4</sup> of a K-D discrete space signal  $\psi(\mathbf{n})$  is defined as*

$$\Psi_d(\mathbf{f}) = \sum_{\mathbf{n} \in \mathbb{Z}^K} \psi(\mathbf{n}) \exp(-j2\pi\mathbf{f}^T \mathbf{n}) \quad (2.2.3)$$

Here  $\mathbf{f} \in \mathcal{R}^K$  represents the K-D discrete frequency variable.

Notice that because of the complex exponential term in the right hand of Eq. (2.2.3), the DSFT is periodic in each dimension with a period of 1. The fundamental period in the K-D space is the unit hypercube, denoted by  $\mathcal{I}^K = \{\mathbf{f} | f_k \in (-1/2, 1/2), k \in \mathcal{K}\}$ . This fundamental period repeats at all integer points. For this reason, one only needs to specify the DSFT of a signal over the unit hypercube,  $\mathcal{I}^K$ . We will formally define the periodicity of a K-D signal using the concept of lattice in the next chapter (Sec. 3.1).

**Theorem 2.4 (Inverse Discrete Space Fourier Transform)** *A discrete space signal can be reconstructed from its DSFT using:*

$$\psi(\mathbf{n}) = \int_{\mathcal{I}^K} \Psi_d(\mathbf{f}) \exp(j2\pi\mathbf{f}^T \mathbf{n}) d\mathbf{f}. \quad (2.2.4)$$

The above equality is easy to prove by recognizing

$$\int_{\mathcal{I}^K} \exp(j2\pi\mathbf{f}^T \mathbf{n}) d\mathbf{f} = \delta(\mathbf{n}). \quad (2.2.5)$$

Most of the properties of the CSFT can be carried over to DSFT. In particular, the convolution theorem still applies, i.e.

$$\psi(\mathbf{n}) * h(\mathbf{n}) \longleftrightarrow \Psi_d(\mathbf{f}) H_d(\mathbf{f}). \quad (2.2.6)$$

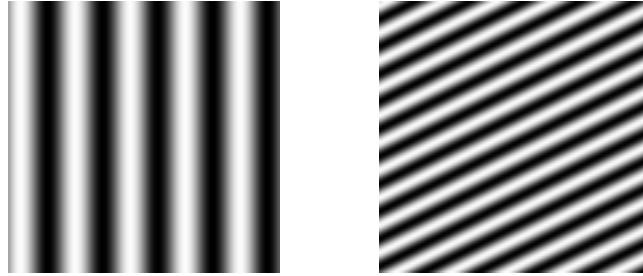
Similar to the continuous space case, we can define a discrete space system with discrete input and output signals. If the system is linear and shift-invariant, then it can be completely characterized by the impulse response,  $h(\mathbf{n}) = \mathcal{T}\{\delta(\mathbf{n})\}$ , the system output to a discrete delta function. The output to any input signal can be described by the convolution of the input signal with the impulse response.

## 2.3 Frequency Domain Characterization of Video Signals

A video is a 3D signal, having two spatial dimensions (horizontal and vertical) and one temporal dimension. One can apply the CSFT and DSFT described in the previous section to an analog and digital video signal, respectively, in a straightforward

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<sup>4</sup>In the 1D case, DSFT is better known as *discrete time Fourier transform* or DTFT, because the 1D signal is typically a function of discrete time indices.



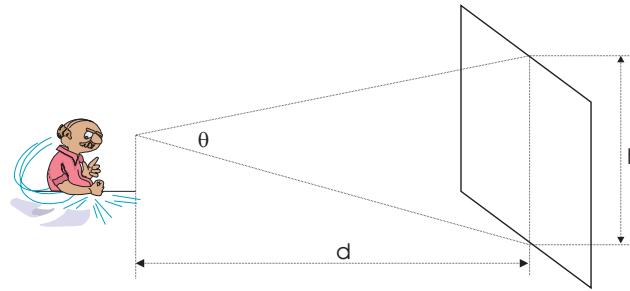
**Figure 2.1.** 2D Sinusoidal signals: (a)  $(f_x, f_y) = (5, 0)$ ; (b)  $(f_x, f_y) = (5, 10)$ . The horizontal and vertical units are the width and height of the image, respectively. Therefore,  $f_x = 5$  means that there are 5 cycles along each row.

manner with  $K = 3$ . One can also apply the various concepts and properties presented for multi-dimensional linear systems to a video processing system. Instead of using the general notation of  $(x_1, x_2, x_3)$  to describe a point in the 3D signal domain, we will use  $(x, y, t)$ , with  $x$  and  $y$  indicating the horizontal and vertical positions and  $t$  the temporal dimensions. Similarly, we will use  $(f_x, f_y, f_t)$  to represent the frequencies associated with these coordinates. A natural question is what do these frequencies mean in the physical world, and how the visual system perceives them. In this section, we attempt to answer the first question. In the next section, we describe the frequency response of the human visual system.

### 2.3.1 Spatial and Temporal Frequencies

**Spatial Frequency** The 2D spatial frequency is a measure of how fast the image intensity or color changes in the 2D image plane. One can measure the spatial frequency along different directions. The spatial frequency in a given direction is measured in terms of cycles per unit length in this direction, which could be 1 meter, the picture-height of a TV monitor, etc. The spatial variation of a 2D pattern can be completely characterized by the frequencies in two orthogonal directions. One can project the frequency in any other direction onto these two. Usually, we characterize the spatial frequency of a 2D image signal by a pair  $(f_x, f_y)$  representing the horizontal and vertical frequencies, respectively. For example, a sinusoidal pattern described by  $\psi(x, y) = \sin(10\pi x)$  has a frequency of  $(5, 0)$ , in that it changes 5 cycles per unit length in the horizontal direction, and it stays constant in the vertical direction. On the other hand, an image  $\psi(x, y) = \sin(10\pi x + 20\pi y)$  has a frequency of  $(5, 10)$ , because there are 5 and 10 cycles per unit length in the horizontal and vertical directions, respectively. This image can also be characterized by a frequency of  $\sqrt{f_x^2 + f_y^2} \approx 11$  cycles per unit length along the direction  $\theta = \arctan(f_y/f_x) \approx 64^\circ$ . These two pure sinusoidal patterns are illustrated in Fig. 2.1.

The above examples are pure sinusoidal patterns that have single frequencies.



**Figure 2.2.** Relation of viewing angle and distance. See Eq. 2.3.2

Using the Fourier transform, an arbitrary signal can be decomposed into many sinusoidal patterns, as described by the inverse CSFT in Eq. (2.1.7).

**Angular Frequency** In the above, we have defined the spatial frequency  $f_s$  along a particular direction in terms of a given unit length. Such a measure is however not very useful as the perceived speed of spatial variation increases with the viewing distance. A more useful measure of the spatial frequency is in terms of cycles per degree of the viewing angle. As illustrated in Fig. 2.2, if a picture has a height of  $h$  and is viewed at a distance of  $d$ , the vertical span of the viewing angle  $\theta$  can be approximated by, if  $h/2 \ll d$ ,

$$\theta = 2 \arctan\left(\frac{h}{2d}\right) \text{(radian)} \approx \frac{h}{d} \text{(radian)} = \frac{180}{\pi} \frac{h}{d} \text{(degree).} \quad (2.3.1)$$

If there are  $f_s$  cycles per picture-height, then the number of cycles per viewing degree is

$$f_\theta = f_s / \theta = \frac{\pi}{180} \frac{d}{h} f_s \text{ (cycle/degree).} \quad (2.3.2)$$

We call  $f_\theta$  the *angular frequency*, which has a unit of cycle/degree or cpd. The above equation shows the relation between the spatial frequency and angular frequency. For the same picture,  $f_\theta$  increases as the viewing distance increases. On the other hand, for a fixed viewing distance, a larger screen size leads to lower angular frequency. These results match well with our intuition: the same picture appears to change more rapidly when viewed farther away, and it changes more slowly if viewed from a larger screen. Although the above relation is determined by evaluating the number of cycles vertically, angular frequency can be defined along the horizontal as well as any other direction. Note that *the angular frequency is not the characterization of the signal itself. It depends on both the spatial frequency in the signal and the viewing condition.*

**Temporal frequency** With a 1D temporal function, the temporal frequency is very clear: the number of cycles per second. With a video signal that consists of

varying 2D frames, the temporal frequency is 2D position dependent. For a fixed 2D position  $(x, y)$ , its temporal frequency is defined as the number of cycles per second, usually denoted by Hz. The maximum temporal frequency of a video refers to the maximum of the temporal frequencies at all points in the imaged area.

### 2.3.2 Temporal Frequencies Caused by Linear Motion

The temporal frequency of a video signal depends on the rate at which the imaged scene varies, which may be due to camera motion, object motion, or both. There are usually many objects in the scene, which may be undergoing different motions. Therefore, it is not easy to relate the motion and the temporal frequency directly. Here, we consider the temporal frequency associated with an object that is undergoing a linear motion (i.e., with a constant velocity). The analysis also applies when the entire scene is undergoing a global linear motion, e.g., due to camera translation. As we will see, *the temporal frequency depends not only on the motion, but also the spatial frequency of the object*.

Let the image pattern of the object at time 0 be described by  $\psi_0(x, y)$  and its velocities in horizontal and vertical directions by  $v_x$  and  $v_y$ . If the scene is under homogeneous ambient illumination, so that the same object point will have the same image intensity at different times, then the image pattern at time  $t$  will be

$$\psi(x, y, t) = \psi_0(x - v_x t, y - v_y t), \quad (2.3.3)$$

because the point  $(x, y)$  at time  $t$  correspond to the point  $(x - v_x t, y - v_y t)$  at time 0 (see Fig. 2.3). This is known as the *constant intensity assumption*. Conditions under which this assumption is valid will be explained further in Sec. 5.2, Eq. (5.2.11). Performing CSFT for the above signal, we obtain

$$\begin{aligned} \Psi(f_x, f_y, f_t) &= \int \int \int \psi(x, y, t) \exp(-j2\pi(f_x x + f_y y + f_t t)) dx dy dt \\ &= \int \int \psi_0(x - v_x t, y - v_y t) \exp(-j2\pi(f_x(x - v_x t) + f_y(y - v_y t))) dx dy \\ &\quad \cdot \int \exp(-j2\pi(f_t + f_x v_x + f_y v_y)t) dt \\ &= \Psi_0(f_x, f_y) \int \exp(-j2\pi(f_t + f_x v_x + f_y v_y)t) dt \\ &= \Psi_0(f_x, f_y) \delta(f_t + f_x v_x + f_y v_y), \end{aligned}$$

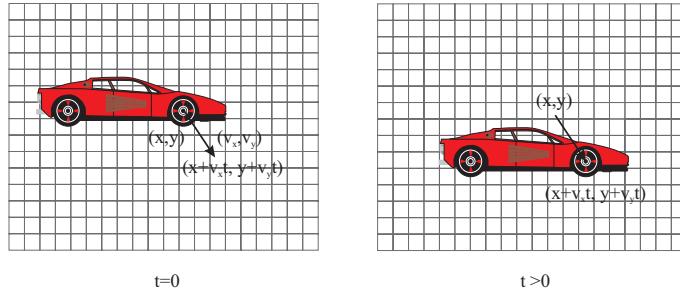
where  $\Psi_0(f_x, f_y)$  represents the 2D CSFT of  $\psi_0(x, y)$ .

The above result shows that  $\Psi(f_x, f_y, f_t)$  is non-zero only on the plane defined by

$$f_t + f_x v_x + f_y v_y = 0. \quad (2.3.4)$$

This means that a spatial pattern characterized by  $(f_x, f_y)$  in the object will lead to a temporal frequency

$$f_t = -f_x v_x - f_y v_y, \quad (2.3.5)$$



**Figure 2.3.** Illustration of the constant intensity assumption under motion. Every point  $(x, y)$  at  $t = 0$  is shifted by  $(v_x t, v_y t)$  to  $(x + v_x t, y + v_y t)$  at time  $t$ , without change in color/intensity. Alternatively, a point  $(x, y)$  at time  $t$  corresponds to a point  $(x - v_x t, y - v_y t)$  at time 0.

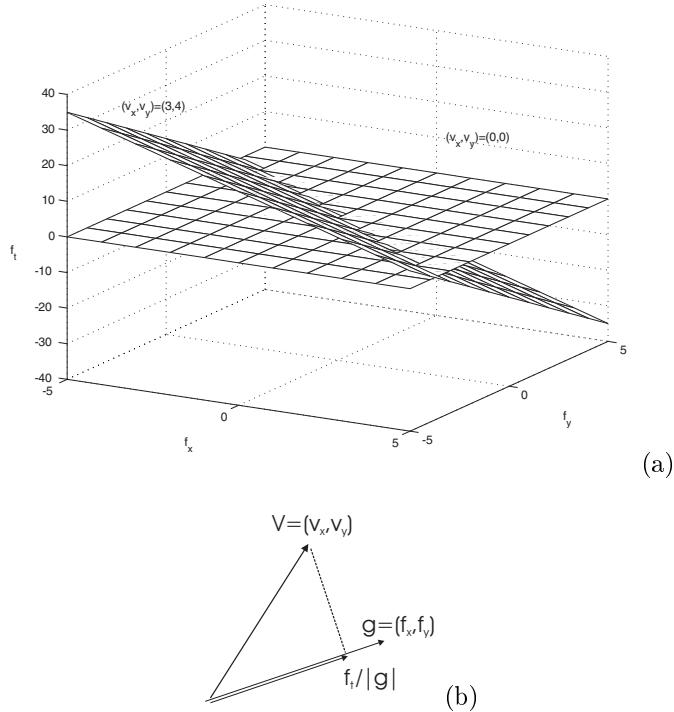
if the object is moving at a velocity of  $(v_x, v_y)$ .<sup>5</sup> Figure 2.4(a) illustrates the relation between  $f_x, f_y$  and  $f_t$  for a given velocity. We can see that the non-zero region of the frequency spectrum in the 3D frequency space is a plane with a normal vector defined by  $(v_x, v_y, 1)$ . If the spatial signal has a finite bandwidth,  $(f_{x,\max}, f_{y,\max})$ , then, the temporal bandwidth is  $f_{t,\max} = v_x f_{x,\max} + v_y f_{y,\max}$ . In Fig. 2.4(a),  $f_{x,\max} = f_{y,\max} = 5$ , therefore,  $f_{t,\max} = 35$  for  $(v_x, v_y) = (3, 4)$ .

From Eq. (2.3.5), *the temporal frequency depends not only on the velocity but also the spatial frequency*. In fact, it is the projection of the velocity vector on the spatial gradient vector, as shown in Fig. 2.4(b). The following observations are immediate:

1. When  $f_x = f_y = 0$ ,  $f_t = 0$  regardless values of  $v_x, v_y$ . This means that if the object has a completely flat pattern (i.e. with a homogeneous brightness or color), then no temporal changes can be observed no matter how fast the object moves along the image plane.
2. The temporal frequency  $f_t = 0$  if the moving direction,  $(v_x, v_y)$ , and spatial frequency direction,  $(f_x, f_y)$ , are orthogonal. The spatial frequency direction  $(f_x, f_y)$  is the direction along which the speed of change is highest. Its orthogonal direction is one along which there is no spatial change. This result indicates that an object moving in the direction where object pattern does not change will not produce any temporal variations. The temporal frequency is largest when the object moves in the direction where the spatial change is greatest.

Consider an infinite plane with a vertical sinusoidal bar pattern, as shown in Fig. 2.1(a). The spatial gradient direction is horizontal in this case. If the plane

<sup>5</sup>Note that with a real signal, the CSFT is symmetric, so that for every frequency component at  $(f_x, f_y)$ , there is also a component at  $(-f_x, -f_y)$  with the same magnitude. The corresponding temporal frequency caused by this other component is  $f_x v_x + f_y v_y$ .



**Figure 2.4.** Relation between spatial and temporal frequency under linear motions. (a) The non-zero frequency plane in the  $(f_x, f_y, f_t)$  space, caused by two different velocity vectors; (b) The temporal frequency is equal to the projection of the velocity onto the spatial gradient.

moves vertically from top to bottom (orthogonal to the spatial frequency direction), then the eye will not perceive any change no matter how fast the plane moves. Once its motion is slightly tilted from the vertical direction, the eye will start to perceive temporal changes. The perceived change is most rapid when the plane moves horizontally from left to right (along the spatial frequency direction).

The above analysis is conducted for an object undergoing a constant speed motion. For a video containing a more complex scene with multiple objects moving in different ways, we can divide the imaged area into small regions so that each region can be considered as undergoing a uniform motion with a constant speed. The local spatial frequency and speed at this region determine the local temporal frequency associated with this region.

## 2.4 Frequency Responses of the Human Visual System

Any video system is ultimately targeted for human viewers. Therefore, it is extremely important to understand how the human visual system (HVS) perceives a video signal. In Sec. 1.1, we described the color perception mechanism, here we focus on the perception of spatial and temporal changes in image luminance. Similar results follow for the perception of chrominance variation, being proportionally lower than for the luminance. As will be shown, the sensitivity of the HVS to a visual pattern depends on its spatial and temporal frequency content. The visual sensitivity is highest at some intermediate spatial and temporal frequencies. It then falls off quickly and diminishes at some cut-off frequencies. Spatial or temporal changes above these frequencies are invisible to the human eye. Knowledge of the visual frequency response is very important in designing a video system. For example, the temporal and spatial cut-off frequencies form the basis for determining the frame rate and line rate in designing a video capture/display system. In the following, we first describe the spatial and temporal frequency responses of the HVS, respectively. We then introduce the joint spatio-temporal frequency response. Finally, we describe how to translate the visual frequency response when the eye is tracking a moving object.

### 2.4.1 Temporal Frequency Response and Flicker Perception

The temporal frequency response of the HVS refers to the visual sensitivity to a temporally varying pattern at different frequencies. A large number of experiments have been conducted to determine the temporal frequency response of the HVS. It has been found that the temporal response of an observer depends on many factors, including viewing distance, display brightness, and ambient lighting. Figure 2.5 shows the results obtained by an experiment conducted by Kelly [5]. In this experiment, the viewer was presented a flat screen whose brightness varied sinusoidally in the form of

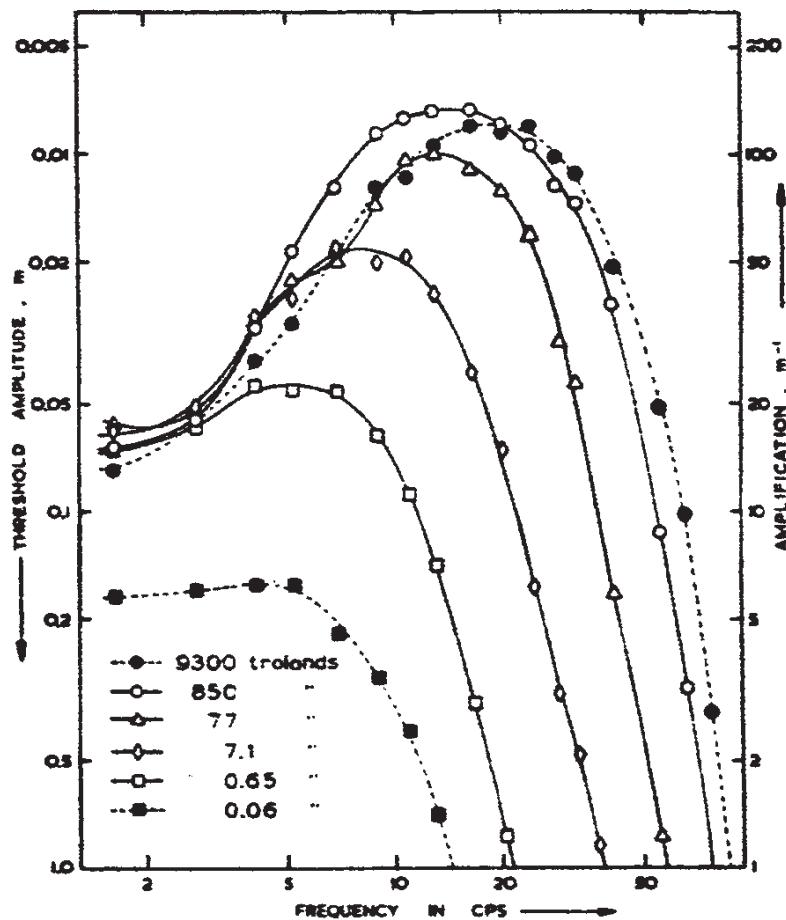
$$\psi(t) = B(1 + m \cos 2\pi ft). \quad (2.4.1)$$

For a fixed mean brightness level  $B$  and frequency  $f$ , the modulation level  $m$  was varied and the viewer was asked to identify the lowest modulation level  $m_{\min}$  at which the temporal variation of the screen brightness (i.e., flicker) became just noticeable. Obviously, the inverse of  $m_{\min}$  represents how sensitive the viewer is to the temporal changes at the given frequency  $f$ . Therefore,  $1/m_{\min}$ , which is also known as *contrast sensitivity*, has been used to describe the visual sensitivity or response.

It can be seen that the temporal response of the HVS is similar to a band-pass filter, with peaks at some intermediate frequencies and the response falls off quickly afterwards, about 4.5 times the peak frequency. The peak increases with the mean brightness of the image. For example, at the mean brightness of 0.65 trolands,<sup>6</sup> the peak response occurs at about 5 Hz, and the cut-off frequency, where

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<sup>6</sup>Troland is the unit used to describe the light entering the retina.



**Figure 2.5.** The temporal frequency response of the HVS obtained by a visual experiment. In this figure, the left vertical axis represents  $m_{\min}$ , and right vertical axis represents  $1/m_{\min}$ . Different curves represent the responses obtained with different mean brightness levels,  $B$ , measured in trolands. The horizontal axis represent the flicker frequency  $f$ , measured in cycle/s or Hz. From [5, Fig. 4].

the response essentially diminishes, is at about 20-25 Hz. On the other hand, at the mean brightness of 850 trolands, the response is highest at about 15-20 Hz and diminishes at about 75 Hz.

One reason that the eye has reduced sensitivity at higher temporal frequencies is because the eye can retain the sensation of an image for a short time interval even when the actual image have been removed. This phenomenon is known as the

*persistency of vision.*<sup>7</sup> On one hand, this causes temporal blurring of the observed pattern, if a pattern changes at a rate faster than the refresh rate of the HVS. The visual response falls off very quickly beyond this frequency. On the other hand, this vision persistency allows the display of a video signal as a consecutive set of frames. As long as the frame interval is shorter than the vision persistency period, then the eye perceives a continuously varying imagery. When the frame interval is longer than this period, the eye will observe *frame flicker*, which refers to the perception of discontinuous frames. The lowest frame rate at which the eye does not perceive flicker is known as the *critical flicker frequency*. By definition, this is equivalent to the temporal cut-off frequency discussed previously. The frame rate used by a video capture/display system should exceed the critical flicker frequency to avoid the perception of flicker.

From Fig. 2.5, the critical flicker frequency ranges from 20 to 80 Hz, depending on the mean brightness of the display. The brighter is the display, the higher is the critical frequency. In a movie theater, the mean brightness is very low; on the other hand, the CRT display is much brighter, close to 9600 trolands [8]. This is why the movie system can use a lower frame rate than the TV system. Presently, the movie industry uses 24 frame/second (fps), while the TV industry uses 50 (in PAL and SECAM systems) and 60 fps (in NTSC system).<sup>8</sup> We can see that these frame rates are close to the cut-off frequencies associated with the mean brightness levels in their intended applications. Computer displays use a much higher rate, 72 fps, because a computer user sits much closer to the screen than does a TV viewer, and at a shorter distance the visual threshold is higher. More discussion on how to determine the spatial-temporal sampling rates of a video system is given in Sec. 3.3.1.

#### 2.4.2 Spatial Frequency Response

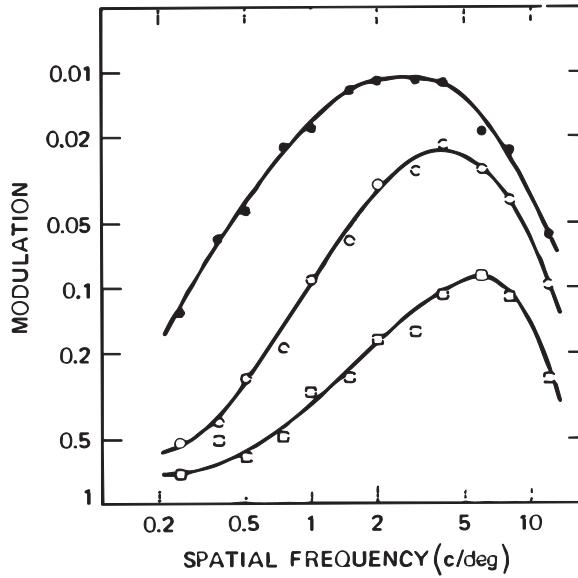
The spatial frequency response of the HVS refers to the visual sensitivity to a stationary spatial pattern with different spatial frequencies. Assuming the visual sensitivity is isotropic with respect to the direction of spatial variation, then the spatial frequency response can be evaluated with respect to an arbitrary spatial direction, usually the horizontal or vertical direction is used.<sup>9</sup> For the response to be independent of the viewing distance, the spatial frequency response is normally expressed as a function of the angular frequency. A large number of experiments have been done to evaluate the spatial frequency response of the HVS. It has been found that the spatial frequency response of the HVS is also similar to a band-pass

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<sup>7</sup>This is caused by the *temporal summation* mechanism of the HVS, which integrates the incoming light. The *Bloch's law* says that the integration or persistency period is inversely proportional to the intensity of the light [10]. The brighter is the light source, the shorter is the integration period. This makes the eye have higher temporal sensitivity when the display is brighter, which is consistent with the results shown in Fig. 2.5.

<sup>8</sup>These are actually the field rates, as discussed before.

<sup>9</sup>Actually, visual sensitivity is higher to changes in the horizontal and vertical directions than in the other directions.



**Figure 2.6.** The spatial frequency response of the HVS obtained by a visual experiment. The three curves result from different stabilization settings used to remove the effect of saccadic eye movements. Filled circles were obtained under normal, unstabilized condition; Open squares, with optimal gain setting for stabilization; Open circles, with the gain changed about 5%. From [6, Fig. 6].

filter, with a peak response at about 3 - 5 cpd, and diminishes at about 30 cpd. The result of a study conducted by Kelly [6] is shown in Fig. 2.6. This result is obtained by viewing a vertical bar pattern that changes sinusoidally in the horizontal direction, i.e.,

$$\psi(x, y, t) = B(1 + m \cos 2\pi f x). \quad (2.4.2)$$

The pattern is displayed on a CRT monitor with a very high refresh rate (1000 frame/second) so that it can be considered as temporally stationary. For each given spatial frequency  $f$ , the modulation  $m$  is changed and the viewer is asked to identify the minimum modulation level,  $m_{\min}$ , at which the spatial change becomes just noticeable. The vertical axis in Fig. 2.6 represents  $m_{\min}$ . Three curves are shown, which were obtained using different stabilization settings to remove the effect of eye movements.

The eye often jumps from one fixation position to another very fast when observing a scene. This is known as *saccadic eye movement* [4]. It has been found that the saccadic eye movement has the effect of enhancing the contrast sensitivity, but reduces the frequency at which peak responses occur. For example, in Fig. 2.6, viewer's sensitivity is about 10 fold higher with normal eye movement than without (i.e., with stabilization). On the other hand, the peak response occurs at about 3.5

cpd with normal eye movement. The peak is shifted to 5 cpd with complete removal of eye movement. Therefore, saccadic eye movement enhances the smoothing effect, but reduces the edge-sharpening effect.

The band-pass character of the spatial frequency response can be attributed to the band-pass filtering operation effectively conducted by the HVS. It has been found that the impulse response of the HVS has a shape similar to the Laplacian of Gaussian (the shape of a Mexican sombrero hat), with positive weightings to nearby visual cells but negative weightings to farther apart cells. The positive portion provides smoothing, while the negative portion effectively enhances sharp edges. The negative weighting effect is caused by lateral inhibition in the visual cortex [1].

In Sec. 3.3.1, we will show how the spatial frequency response of the HVS is taken into consideration in determining the horizontal and vertical sampling rate in a video system.

### 2.4.3 Spatio-Temporal Frequency Response

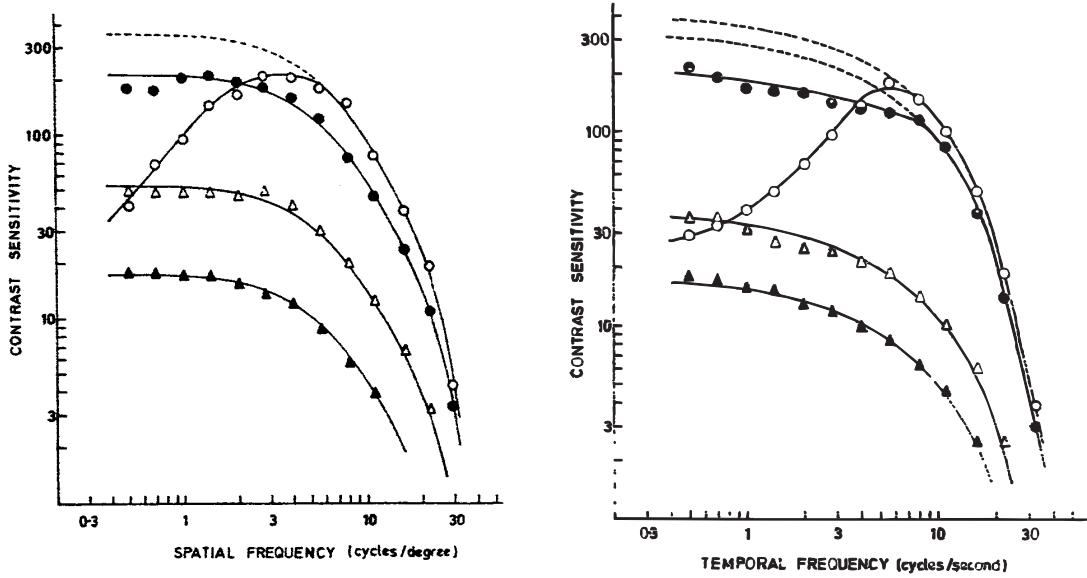
The spatial frequency response presented previously is evaluated at the zero temporal frequency, and vice versa. Here we discuss the visual response when both the spatial and temporal frequencies are non-zero. It has been found that, at higher temporal frequencies, both the peak and cut-off frequencies in the spatial frequency response shift downwards. A similar trend happens with the temporal frequency response.

Figure 2.7 shows the experimental results by Robson [12]. The test pattern in this experiment is described by

$$\psi(x, y, t) = B(1 + m \cos(2\pi f_x x) \cos(2\pi f_t t)). \quad (2.4.3)$$

For a fixed set of  $f_x$  and  $f_t$ , the modulation  $m$  is varied and the viewer is asked to identify the minimum modulation  $m_{\min}$  at which both spatial and temporal variation is just noticeable. The vertical axis in Fig. 2.7 represents  $1/m_{\min}$ . We can see that, at nearly zero temporal (resp. spatial) frequencies, the spatial (resp. temporal) frequency response has a band-pass characteristics. This is consistent with the results shown previously. But at higher temporal (resp. spatial) frequencies, the spatial (resp. temporal) response becomes more like low-pass, with the peak response decreases when the temporal (resp. spatial) frequency increases. This reveals that, when an image pattern moves very fast, the eye will not be able to differentiate the very high spatial frequency in this pattern. The eye can resolve a much higher spatial frequency when the pattern is stationary. Similarly, at higher spatial frequencies, the temporal response becomes low-pass, with the transition frequency shifting downwards at higher spatial frequencies.

One implication of this reciprocal relation between the spatial and temporal responses for the design of a video system is that one can trade-off the spatial resolution with temporal resolution, and vice versa. This property has been exploited judiciously in the interlaced scan mechanism used in analog TV systems. With in-

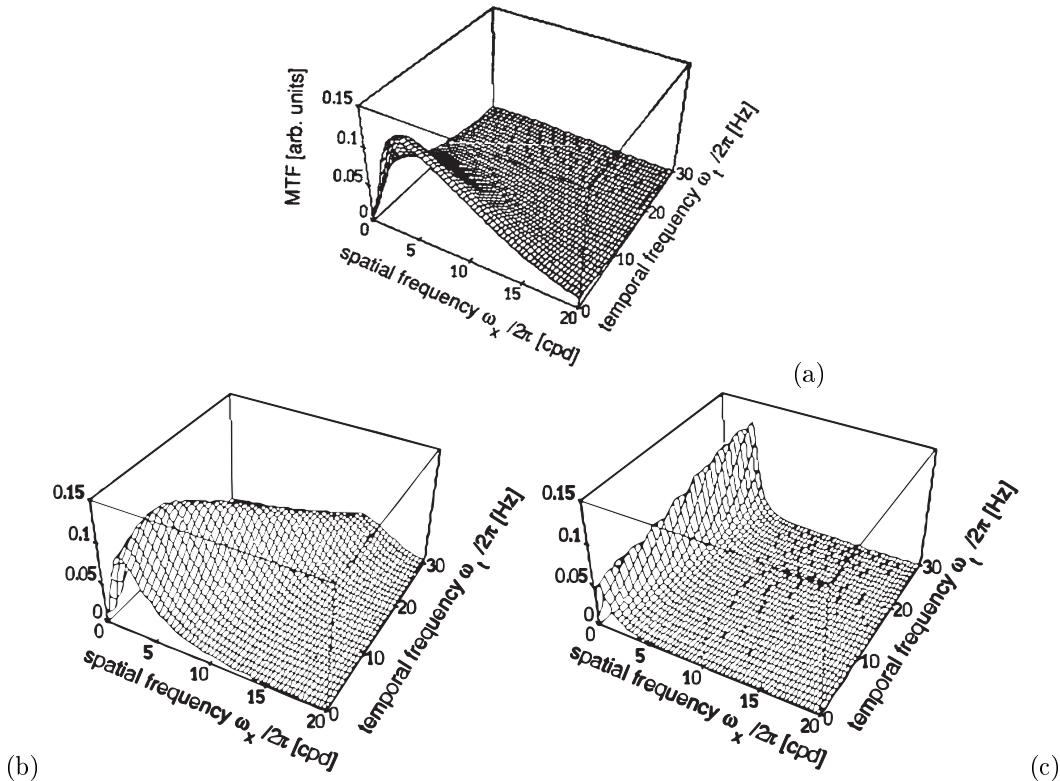


**Figure 2.7.** Spatio-temporal frequency response of the HVS. (a) Spatial frequency responses for different temporal frequencies of 1 Hz (open circles), 6 Hz (filled circles), 16 Hz (open triangles), and 22 Hz (filled triangle). (b) Temporal frequency responses for different spatial frequencies of 0.5 cpd (open circles), 4 cpd (filled circles), 16 cpd (open triangles), and 22 cpd (filled triangle). From [12].

terlaced scan, in order to render very fast changing scenes with a limited frame rate, a frame is split into two fields, each with a half number of lines in a frame. Because the HVS has a reduced spatial frequency response when the temporal frequency is high, the eye will not be able to differentiate the very high spatial frequencies in a fast changing scene, even if it is rendered at a high spatial resolution. Therefore, using a lower spatial resolution is justified. On the other hand, when the imaged scene is stationary, the lines in two separate fields combine to provide a high spatial resolution, to accommodate the higher spatial frequency resolving power of the HVS.

The reciprocity between the spatial and temporal responses also explains why we perceive *line flicker* when we watch TV within a close distance. In this case the lines are separated further apart and the angular frequency becomes low. At low angular frequencies, the human eye is more temporally acute and can therefore perceive flickers more easily.

The experiment conducted by Robson is under normal saccadic eye movement. Kelly later on conducted an experiment when the effect of saccadic motion is removed by using stabilization techniques [7]. For testing the temporal response, he used a traveling wave, instead of a flickering wave. The trends in the resulting



**Figure 2.8.** Spatio-temporal response of the HVS under smooth pursuit eye movements. (a) without smooth pursuit eye movement; (b) with eye velocity of 2 deg/s; (c) with eye velocity of 10 deg/s. From [3, Figs. 5.1 and 5.3]

spatial-temporal frequency responses are similar, but the temporal response caused by motion is about 2 fold higher than that by flickering. This means that the eye is more sensitive to temporal variations caused by motion than by flickering.

#### 2.4.4 Smooth Pursuit Eye Movements

Although the experimental results described in Sec. 2.4.1 indicate that the HVS cannot resolve temporal frequencies more than 80 Hz, in reality the human eye can see clearly a fast moving object when the eye is tracking the object, e.g., when a baseball batter tracks a fast ball. This is because, when the eye is tracking the object, the relative motion of the object becomes smaller. If the tracking were perfect, the object would have appeared still. The phenomenon that the eyes automatically move to track the observed object is known as *smooth pursuit eye movement* [11, 4].

To account for the eye tracking effect, we need to translate the spatio-temporal

frequency at the display screen to the retina coordinate when the eye is moving [3]. Assuming the eye is moving with a speed of  $v_x$  and  $v_y$ , then the image observed at the retina coordinate,  $\tilde{\psi}(x, y, t)$ , is related to the image at the display screen,  $\psi(x, y, t)$ , by

$$\tilde{\psi}(x, y, t) = \psi(x + v_x t, y + v_y t, t). \quad (2.4.4)$$

Taking CSFT on both sides yield

$$\tilde{\Psi}(f_x, f_y, f_t) = \Psi(f_x, f_y, f_t - v_x f_x - v_y f_y). \quad (2.4.5)$$

The above relation means that the observed frequencies in the retina,  $(\tilde{f}_x, \tilde{f}_y, \tilde{f}_t)$ , is related to the frequencies in the display screen,  $(f_x, f_y, f_t)$ , by

$$\tilde{f}_x = f_x, \quad \tilde{f}_y = f_y, \quad \tilde{f}_t = f_t + v_x f_x + v_y f_y. \quad (2.4.6)$$

When the temporal frequency of the displayed object is zero, the perceived temporal frequency increases with the eye motion speed as well as the spatial frequencies of the observed object. When the displayed object is moving with a constant speed, recall that the resulting temporal frequency is described by (2.3.5). Therefore, the effective temporal frequency at the retina is zero, when the eye movement matches that of the object movement. In general, by tracking the object motion, the effective temporal frequency at the retina becomes smaller.

Using the coordinate transformation of Eq. (2.4.6), Girod redrew the visual frequency response measured by Robson. Figure 2.8 shows the visual frequency response without smooth pursuit eye movement, and that under two different eye movement speeds. Clearly, the smooth-pursuit eye movement has the effect of extending the non-zero region of the visual response to a larger temporal frequency range. In fact, temporal frequencies up to 1000 Hz can be perceived [3, 13].

Smooth pursuit eye movements have important consequences for the design of video display and processing systems, as suggested by Girod [3]. In order to avoid visible blur of fast moving objects, the display system needs to be able to display temporal frequencies up to 1000 Hz. Likewise, any type of temporal filtering has to be carried out with motion compensation (i.e., filtering along the motion trajectory) to avoid visible blur of the picture content.

## 2.5 Summary

### Fourier Analysis of Multi-dimensional Signals and Systems (Secs. 2.1,2.2)

- A K-dimensional continuous (resp. discrete) signal can be represented by a continuous (resp. discrete) space Fourier transform (Eqs. (2.1.6,2.1.7) for CSFT or Eqs. (2.2.3,2.2.4) for DSFT). The inverse transform means that the signal can be decomposed into infinitely many complex sinusoidal functions with different frequencies.

- A LTI system can be completely characterized by its impulse response, or equivalently, its frequency response, the Fourier transform of the impulse response. (Theorem 2.3)
- The input and output signals of a LTI system are related by a linear convolution with the impulse response in the spatial domain, or a product with the frequency response in the transform domain, as given in Eq. (2.1.13) or (2.1.14).

### Spatial and Temporal Frequencies in Video Signals (Sec. 2.3)

- The 2D spatial frequency can be specified along any two orthogonal dimensions, usually horizontal and vertical directions.
- Temporal frequency of a moving object depends on its velocity as well as its spatial frequency. More specifically, it only depends on the projection of the velocity vector onto the spatial gradient direction at any position on the object, as given by Eq. (2.3.5) and Fig. 2.4.

### Spatial-Temporal Frequency Responses of the HVS (Sec. 2.4)

- The human eye is most sensitive to some intermediate spatial and temporal frequency components. At normal viewing conditions, the eye is most sensitive to spatial frequencies between 3-5 cpd, and temporal frequencies 5-20 Hz.
- Beyond certain cut-off frequencies, the eye cannot perceive the spatial-temporal variations, which occur at about 30 cpd in spatial frequency, and between 20-80 Hz in temporal frequency. These visual thresholds set the guide lines for designing video processing/display systems. The display system only needs to accommodate frequencies up to twice the visual thresholds.
- The eye will not perceive individual frames as long as the frame rate (field rate for interlaced display) is beyond the critical flicker frequency, which is below 80 Hz for most people.
- The visual frequency responses depend on the mean brightness of the display. In general, a brighter display can make the eye more sensitive to high frequency components.
- By tracking a moving object, the perceived temporal frequency at the retina is reduced. If the tracking is perfect, than the temporal frequency at the retina plane is zero.
- The cut-off frequencies of the HVS are driving factors for determining the spatial and temporal sampling resolutions in video capture, processing and display. This is further discussed in Sec. 3.3.1.

## 2.6 Problems

- 2.1** The impulse response of a camera can usually be modeled by

$$h(x, y, t) = \begin{cases} 1 & |x| < T_x, |y| < T_y, t \in (0, T_e), \\ 0 & \text{otherwise,} \end{cases} \quad (2.6.1)$$

where  $2T_x, 2T_y$  are the horizontal and vertical size of the camera aperture,  $T_e$  is the exposure time. Find the CSFT of  $h(x, y, t)$ . Plot the magnitude response. Comment on the effect of the parameters  $T_x, T_y, T_e$  on the frequency response of the camera.

- 2.2** Suppose the camera described in the previous problem is looking at a scene that consists of a cube moving in parallel with the camera imaging plane. The projected image on the camera plane can be described by

$$\psi(x, y, t) = \begin{cases} 1 & -B + v_x t < x < B + v_x t, -B < y < B, \\ 0 & \text{otherwise} \end{cases} \quad (2.6.2)$$

Derive what is the captured signal by the camera.

- 2.3** What are the considerations you would use to determine the frame rate and line number when designing a video capture/display system? Are the parameters chosen for NTSC color TV broadcasting appropriate?

- 2.4** Why does a computer monitor use a higher temporal refresh rate and more line numbers than does a typical TV monitor?

- 2.5** Consider a horizontal bar pattern on a TV screen that has 100 cycle/picture-height. If the picture-height is 1 meter, and the viewer sits at 3 meters from the screen, what is the equivalent angular frequency in cycle/degree? What if the viewer sits at 1 meter or 5 meters away? In either case, would the viewer be able to perceive the vertical variation properly?

- 2.6** Consider an object that has a flat homogeneous texture surface with maximum spatial frequency of  $(f_x, f_y) = (3, 4)$  cycle/meter, and is moving at a constant speed of  $(v_x, v_y) = (1, 1)$  meter/second. What is the temporal frequency of the object surface at any point? What are the results for following different speeds (in meter/second):  $(4, -3)$ ,  $(4, 0)$ ,  $(0, 1)$ ?

- 2.7** Continue from the previous problem. Suppose that the eye tracks the moving object at a speed that is equal to the object speed. What are the perceived temporal frequencies at the retina for the different values of moving speeds? What if the eye moves at a fixed speed of  $(2, 2)$  meter/second?

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## Chapter 3

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# VIDEO SAMPLING

The very beginning step of any digital video processing task is the conversion of an intrinsically continuous video signal to a digital one. The digitization process consists of two steps: sampling and quantization. This could be implemented by a digital camera, which directly digitizes the video of a continuous physical scene, or by digitizing an analog signal produced by an analog camera. We also frequently need to convert a digital video signal from one format (in terms spatial and temporal resolution) to another, *e.g.*, converting a video recorded in the PAL format to the NTSC format. In this chapter we consider the sampling problem. The sampling rate conversion problem will be discussed in the next Chapter. The quantization problem is discussed later in Chap. 8 together with other means for signal compression.

When designing a video sampling system, three questions need to be resolved: 1) what are the necessary sampling frequencies in the spatial and temporal directions, 2) given an overall sampling rate (i.e., product of the horizontal, vertical, and temporal sampling rates), how to sample in the 3D space to obtain the best representation, and 3) how can we avoid the sampling artifact known as aliasing, given that we can only use a finite sampling rate. In this chapter, we start by describing the general theory for sampling a multi-dimensional signal, which relates the frequency spectrum of a signal and its necessary sampling pattern (Secs. 3.1 and 3.2). We then focus on the sampling of video signals (Sec. 3.3). We will first describe the factors that determine the necessary sampling rates for video and trade-offs that are often made in practical systems. We then discuss 2D sampling schemes and compare the progressive and interlaced scans. We also consider how to sample a color video, and as an example, revisit the BT.601 format. Then we introduce several 3D sampling lattices and compare their efficiency. Finally, we will describe how common video cameras and displays accomplish pre-filtering and interpolation functions implicitly, and some practical limitations (Sec. 3.4).

### 3.1 Basics of the Lattice Theory

In this and next section, we extend the well-known sampling theorem for 1D and 2D signals to a general multi-dimensional space. Recall that, for 1D signals, samples are usually taken at a regular spacing. With 2D signals, samples are normally taken

on a rectangular grid. In fact, one can also take samples on a non-rectangular grid, as long as the grid has a structure that allows the specification of the grid points using integer vectors. Mathematically, this type of grid is known as a lattice. In this section, we introduce the concept of lattice in a K-D space. In the next section, we describe how to sample a continuous signal using a lattice.

The theory of sampling multi-dimensional signals on a lattice was first presented by Petersen and Middleton [7]. An excellent review of the related theory and an extension to sampling using the union of selected cosets of a sub-lattice in a lattice was given by Dubois [4]. In this chapter, we only introduce concepts and properties that are essential for us to analyze the sampling process in a multi-dimensional space. For a comprehensive treatment of the lattice theory, the readers are referred to [2]. The definitions and theorems introduced here are based primarily on the paper by Dubois [4].

**Definition 3.1:** A lattice,  $\Lambda$ , in the real K-D space,  $\mathcal{R}^K$ , is the set of all possible vectors that can be represented as integer-weighted combinations of a set of  $K$  linearly independent basis vectors,  $\mathbf{v}_k \in \mathcal{R}^K, k \in \mathcal{K} = \{1, 2, \dots, K\}$ . That is,

$$\Lambda = \{\mathbf{x} \in \mathcal{R}^K \mid \mathbf{x} = \sum_{k=1}^K n_k \mathbf{v}_k, \forall n_k \in \mathbb{Z}\}. \quad (3.1.1)$$

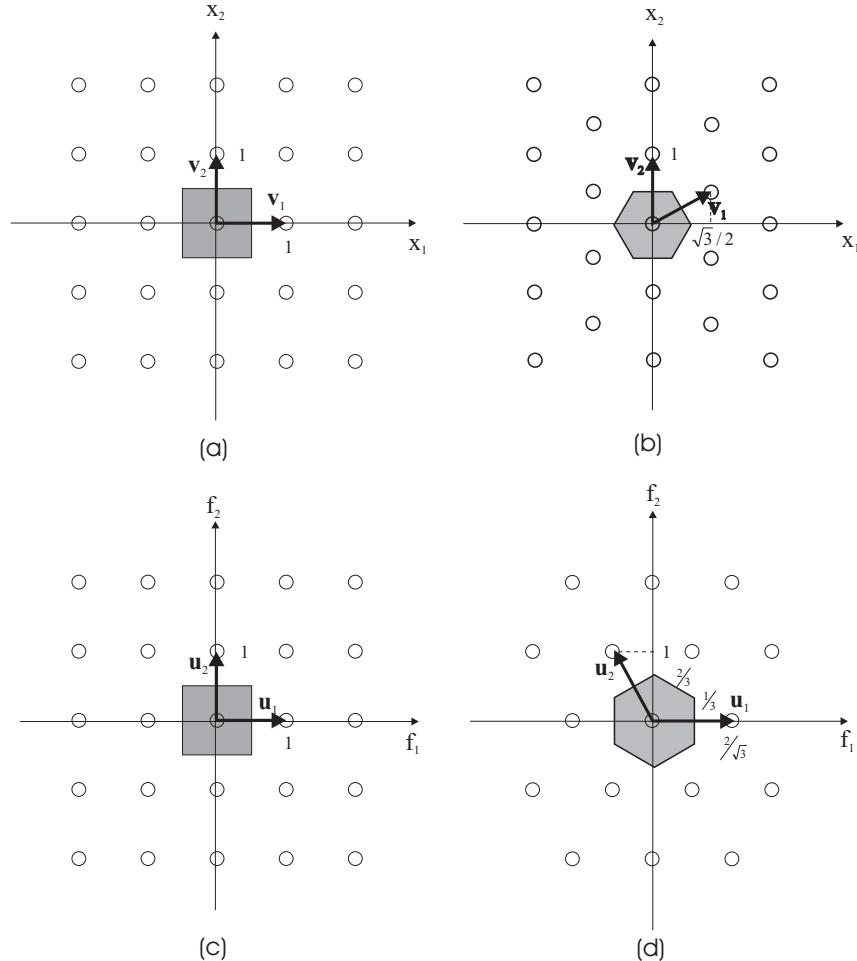
The matrix  $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_K]$  is called the generating matrix.<sup>1</sup>

**Example 3.1:** Consider two lattices in  $\mathcal{R}^2$  with the following generating matrices:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \mathbf{V}_2 = \begin{bmatrix} \sqrt{3}/2 & 0 \\ 1/2 & 1 \end{bmatrix}. \quad (3.1.2)$$

To sketch the actual lattice from a given generating matrix, we first draw the two points corresponding to the basis vectors, and then determine points corresponding to typical integer combinations of the basis vectors, for example,  $\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2$ , etc. Based on these points, we can extrapolate all the other possible points usually by visual inspection. Using this procedure, we draw the lattices determined by the two given generating matrices in Figs. 3.1(a) and 3.1(b). As expected, the first lattice is a simple rectangular grid because  $\mathbf{V}_1$  is a diagonal matrix. The second lattice is the so-called hexagonal lattice. Although a set of basis vectors or a generating matrix completely defines a lattice, the basis or generating matrix associated with a lattice is non-unique. In fact, one can easily find more than one basis or generating matrix that can generate the same lattice. For example, one can verify that for the lattice in Fig. 3.1(b), another set of basis vectors is  $\mathbf{v}_1 = [\sqrt{3}/2, 1/2]^T, \mathbf{v}_2 = [\sqrt{3}, 0]^T$ .

<sup>1</sup>It is also referred to as *sampling matrix*.

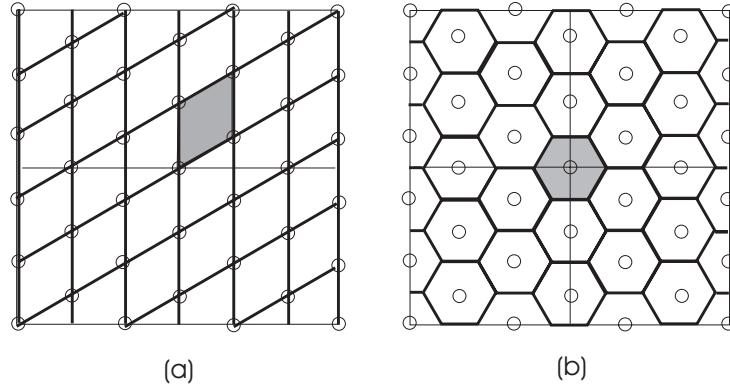


**Figure 3.1.** Example of lattices and their reciprocals: (a) a rectangular lattice; (b) a hexagonal lattice; (c) the reciprocal of the rectangular lattice; (d) the reciprocal of the hexagonal lattice. The shaded area in each lattice indicates the Voronoi cell of that lattice.

For a given generating matrix, any point in the lattice can be indexed by the integer coefficients,  $n_k, k \in \mathcal{K}$ , associated with it. Using the matrix notation, we can represent any point in the lattice by an integer vector  $\mathbf{n} = [n_1, n_2, \dots, n_K]^T \in \mathbb{Z}^K$ . The actual position of this point is  $\mathbf{x} = \mathbf{V}\mathbf{n}$ . *The fact that any point in a lattice can be indexed by an integer vector makes the lattice an elegant tool for sampling a continuous signal with a regular geometry.*

In the following, we introduce some important properties of the lattice.

**Theorem 3.1:** *Given a lattice  $\Lambda$ , one can find a unit cell  $\mathcal{U}(\Lambda)$  so that its transla-*



**Figure 3.2.** Unit cells associated with the lattice shown in Fig. 3.1(b): (a) the fundamental parallelepiped; (b) the Voronoi cell. A unit cell and its translations to all lattice points form a non-overlapping covering of the underlying continuous space.

tions to all lattice points form a non-overlapping covering of the entire space  $\mathcal{R}^K$ , i.e.

$$\bigcup_{\mathbf{x} \in \Lambda} (\mathcal{U} + \mathbf{x}) = \mathcal{R}^K \text{ and } (\mathcal{U} + \mathbf{x}) \cap (\mathcal{U} + \mathbf{y}) = \emptyset, \text{ if } \mathbf{x} \neq \mathbf{y}, \quad (3.1.3)$$

where  $\mathcal{U} + \mathbf{x} = \{\mathbf{p} + \mathbf{x} | \mathbf{p} \in \mathcal{U}\}$  denotes the translation of  $\mathcal{U}$  by  $\mathbf{x}$ , and  $\emptyset$  represents the empty set.

The above theorem tells us that the space  $\mathcal{R}^K$  can be represented as a tiling by a chosen unit cell and its translations, as illustrated in Fig. 3.1. This representation is useful when we consider the quantization of the space  $\mathcal{R}^K$ . The two partitions shown in Fig. 3.1 are two ways of uniformly quantizing the space  $\mathcal{R}^2$ .

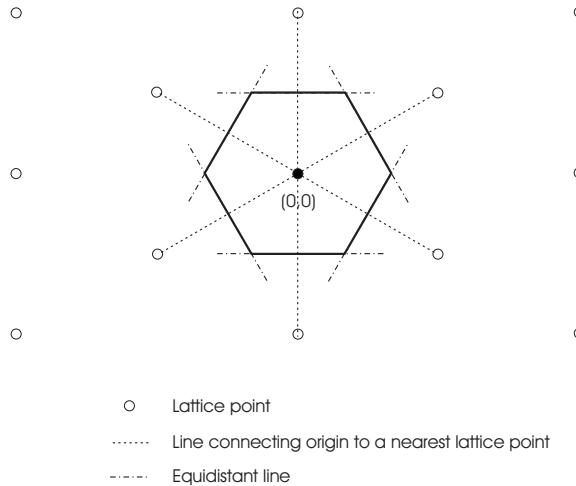
The unit cell associated with a lattice is non-unique. In fact, if  $\mathcal{U}$  is a unit cell of  $\Lambda$ , then  $\mathcal{U} + \mathbf{x}, \forall \mathbf{x} \in \Lambda$  is also a unit cell. Among several interesting unit cells, the fundamental parallelepiped and the Voronoi cell are most useful, which are introduced below.

**Definition 3.2:** The fundamental parallelepiped of a lattice with basis vectors  $\mathbf{v}_k, k \in \mathcal{K}$ , is the set defined by

$$\mathcal{P}(\Lambda) = \{\mathbf{x} \in \mathcal{R}^K | \mathbf{x} = \sum_k \alpha_k \mathbf{v}_k, \forall 0 \leq \alpha_k \leq 1\}. \quad (3.1.4)$$

In words, this is the polygon enclosed by the vectors corresponding to the basis vectors.

The fundamental parallelepiped of the lattice given in Fig. 3.1(b) is shown in Fig. 3.2(a). As shown in the figure, the fundamental parallelepiped and its translations to all the lattice points form a partition of the space  $\mathcal{R}^K$ , and therefore the



**Figure 3.3.** Determining the Voronoi cell by drawing equidistant lines.

fundamental parallelepiped is a unit cell. Obviously, there are many fundamental parallelepipeds associated with a lattice, because of the non-uniqueness of the generating basis.

**Definition 3.3:** *The Voronoi cell of a lattice is the set of points which are closer to the origin than any other points in the lattice. That is:*

$$\mathcal{V}(\Lambda) = \{\mathbf{x} \in \mathcal{R}^K | d(\mathbf{x}, \mathbf{0}) \leq d(\mathbf{x}, \mathbf{p}), \forall \mathbf{p} \in \Lambda\}. \quad (3.1.5)$$

The Voronoi cell of the lattice given in Fig. 3.1(b) is shown in Fig. 3.2(b). As with the fundamental parallelepiped, the Voronoi cell and its translations to all the lattice points also form a partition of the space  $\mathcal{R}^K$ . Therefore, the Voronoi cell is also a unit cell. We will see that the Voronoi cell is very useful for analyzing the sampling process. In Fig. 3.1, the shaded region in each lattice indicates its Voronoi cell. As expected, the Voronoi cell of a rectangular lattice is simply a rectangle. The Voronoi cell of the second lattice is a hexagon with 6 equal length sides. This is why the lattice is called the hexagonal lattice.

In the 2D case, the Voronoi cell of a lattice can be determined by first drawing a straight line between the origin and each one of the closest non-zero lattice points, and then drawing a perpendicular line that is half way between the two points. This line is the equidistant line between the origin and this lattice point. The polygon formed by all such equidistant lines surrounding the origin is then the Voronoi cell. This is illustrated in Fig. 3.3. In the 3D case, the above procedure can be extended by replacing the equidistant lines by equidistant planes.

**Volume of the Unit Cell and Sampling Density** Note that although the unit cell associated with a lattice is not unique, the volume of the unit cell is unique. This is

because the same number of unit cells is required to cover a finite subspace of  $\mathcal{R}^K$ , regardless the shape of the unit cell. From the basic algebraic theory, the volume of a parallelepiped formed by column vectors in a matrix  $\mathbf{V}$  is equal to the absolute value of the determinant of the matrix. Therefore, the volume of the fundamental parallelepiped and henceforth any unit cell is  $|\det \mathbf{V}|$ . Obviously, the smaller is the unit cell, the more lattice points exist in a given volume. Therefore, the inverse of the volume of the unit cell measures the *sampling density*, which will be denoted by

$$d(\Lambda) = 1/|\det \mathbf{V}|. \quad (3.1.6)$$

This variable describes how many lattice points exist in a unit volume in  $\mathcal{R}^K$ . For the two lattices given in Figs. 3.1(a) and 3.1(b), it is easy to verify that their sampling densities are  $d_1 = 1$  and  $d_2 = 2/\sqrt{3}$ , respectively.

**Definition 3.4:** Given a lattice with a generating matrix  $V$ , its reciprocal lattice  $\Lambda^*$  is defined as a lattice with a generating matrix

$$\mathbf{U} = (\mathbf{V}^T)^{-1} \quad \text{or} \quad \mathbf{V}^T \mathbf{U} = \mathbf{I}. \quad (3.1.7)$$

By definition, if  $\mathbf{x} = \mathbf{V}\mathbf{m} \in \Lambda, \mathbf{y} = \mathbf{U}\mathbf{n} \in \Lambda^*$ , then  $\mathbf{x}^T \mathbf{y} = \mathbf{m}^T \mathbf{V}^T \mathbf{U} \mathbf{n} = \mathbf{m}^T \mathbf{n} \in \mathbb{Z}$ . That is, the inner-product of any two points, one from each lattice, is an integer. The relation in Eq. (3.1.7) implies that the basis vectors in  $\Lambda$  and  $\Lambda^*$  are orthonormal to each other, i.e.,  $\mathbf{v}_k^T \mathbf{u}_l = \delta_{kl}, \forall k, l \in \mathcal{K}$ , where  $\delta_{kl} = 1$ , if  $k = l$ ;  $= 0$ , otherwise. Because  $\det \mathbf{V} = 1/\det \mathbf{U}$ , the sampling densities of the two lattices are inversely related by  $(\Lambda^*) = \frac{1}{(\Lambda)}$ . This means that the denser is  $\Lambda$ , the sparser is  $\Lambda^*$ , and vice versa.

**Example 3.2:** For the two lattices given in Example 3.1 and shown in Figs. 3.1(a) and 3.1(b), the generating matrices of their reciprocals are

$$\mathbf{U}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{U}_2 = \begin{bmatrix} 2/\sqrt{3} & -1/\sqrt{3} \\ 0 & 1 \end{bmatrix}. \quad (3.1.8)$$

Based on the basis vectors in  $\mathbf{U}_i$ , we can determine all the points in the reciprocal lattices, using the same procedure for determining the original lattices. The results are shown in Figs. 3.1 (c) and 3.1 (d). The Voronoi cells of the reciprocal lattices are also indicated in these two figures. We see that with the rectangular lattice, the Voronoi cell in both the original and the reciprocal lattices are rectangles, while with the hexagonal lattice, the original and reciprocal Voronoi cells are both hexagons, but with different orientations. We also see that  $\Lambda_2$  is denser than  $\Lambda_1$ , but  $\Lambda_2^*$  is sparser than  $\Lambda_1^*$ .

**Theorem 3.2:** Let  $\Lambda_1$  and  $\Lambda_2$  represent two lattices with generating matrices  $\mathbf{V}_1$  and  $\mathbf{V}_2$ , respectively. If  $\mathbf{V}_1^{-1}\mathbf{V}_2$  is a matrix of rational numbers, then the intersection

$$\Lambda_1 \cap \Lambda_2 = \{\mathbf{x} \in \mathcal{R}^K | \mathbf{x} \in \Lambda_1 \text{ and } \mathbf{x} \in \Lambda_2\}$$

is also a lattice, and it is the largest lattice that is contained in both  $\Lambda_1$  and  $\Lambda_2$ . Under the same condition, the sum

$$\Lambda_1 + \Lambda_2 = \{\mathbf{x} + \mathbf{y} | \forall \mathbf{x} \in \Lambda_1, \forall \mathbf{y} \in \Lambda_2\}$$

is also a lattice and it is the smallest lattice that contains both  $\Lambda_1$  and  $\Lambda_2$ . Furthermore,  $(\Lambda_1 + \Lambda_2)^* = \Lambda_1^* \cap \Lambda_2^*$ .

**Theorem 3.3:** If  $\Lambda_1 \subset \Lambda_2$ , then  $\Lambda_1^* \supset \Lambda_2^*$ . Also, if  $\Lambda_1 \subset \Lambda_2$ ,  $\mathcal{V}(\Lambda_1) \supset \mathcal{V}(\Lambda_2)$ .

The above results can be proven by making use of the fact that a lattice must contain points that can be represented as  $\mathbf{V}\mathbf{n}$  where  $\mathbf{n} \in \mathcal{Z}^K$  and  $\mathbf{V}$  is a non-singular matrix. The usefulness of this theorem will become clear in Sec. 4.1 when we discuss sampling rate conversion.

### Characterization of the Periodicity Using the Lattice Concept

We are all familiar with the concept of periodicity in 1D. We say a function is periodic with a period of  $T$  if  $\psi(x) = \psi(x+nT), \forall n \in \mathbb{Z}$ . One important application of the concept of lattice is for describing the periodicity of a multi-dimensional function, as defined below.

**Definition 3.5:** A function is periodic with a non-singular periodicity matrix  $\mathbf{V}$  if  $\psi(\mathbf{x}) = \psi(\mathbf{x} + \mathbf{V}\mathbf{n})$ , for any  $\mathbf{n} \in \mathcal{Z}^K$ .

Obviously, the set of all repetition centers,  $\mathbf{V}\mathbf{n}, \mathbf{n} \in \mathcal{Z}^K$ , form a lattice with a generating matrix  $\mathbf{V}$ . For this reason, when a lattice is used to describe the periodicity of a function, it is also referred to as a *periodicity lattice*. Recall that the entire space  $\mathcal{R}^K$  can be partitioned into infinitely many shifted versions of a unit cell of the lattice. Therefore, one can think of a periodic function as one that repeats a basic pattern over a chosen unit cell at all other translated unit cells. We call the Voronoi cell of the lattice as the *fundamental period* of this function. For a 1D function with a period of  $T$ , the fundamental period is the interval centered at the origin,  $(-T/2, T/2)$ . A periodic function essentially consists of its fundamental period and the shifted versions of this period over all lattice points. Obviously, one only needs to specify a periodic function over its fundamental period.

## 3.2 Sampling Over Lattices

The lattice structure provides an elegant tool for sampling a continuous signal with a uniform but not necessarily rectangular or more generally hyper-cubic grid. It

enables one to index all samples with integer vectors, which simplifies the description of the resulting sampled signal. Sampling over a lattice is a generalization of uniform sampling over a rectangular grid in 2D or a hyper-cubic grid in K-D. As shown later, with this sampling structure, all the theorems known for 1D and 2D uniform sampling still apply. In particular, a generalized Nyquist sampling theory exists, which governs the necessary density and structure of the sampling lattice for a given signal spectrum.

### 3.2.1 Sampling Process and Sampled Space Fourier Transform

**Definition 3.6:** Given a continuous signal  $\psi_c(\mathbf{x})$ ,  $\mathbf{x} \in \mathcal{R}^K$ , a sampled signal over a lattice  $\Lambda$  with a generating matrix  $\mathbf{V}$  is defined as:

$$\psi_s(\mathbf{n}) = \psi_c(\mathbf{V}\mathbf{n}), \quad \mathbf{n} \in \mathcal{Z}^K \quad (3.2.1)$$

An alternative way of defining the sampled signal is by considering it as a continuous-space signal with impulses at the sampled points, i.e.,

$$\psi_s(\mathbf{x}) = \sum_{\mathbf{n} \in \mathcal{Z}^K} \psi_s(\mathbf{n}) \delta(\mathbf{x} - \mathbf{V}\mathbf{n}), \quad \mathbf{x} \in \mathcal{R}^K, \quad (3.2.2)$$

Previously in Sec.2.2, we defined discrete space Fourier transform (DSFT). Similarly, we can define the Fourier transform of a sampled signal over a lattice [5, 4]. Here we call it *sampled space Fourier transform* (SSFT). Compared to DSFT, which is defined in terms of the abstract indices of sampled points, the definition of SSFT keeps the underlying sampling structure of a discrete signal in the picture, and enables one to make connections with actual physical dimensions.

**Definition 3.7:** The sampled space Fourier transform (SSFT) of a signal sampled over a lattice  $\Lambda$  with a generating matrix  $\mathbf{V}$  is defined as:

$$\Psi_s(\mathbf{f}) = \sum_{\mathbf{n} \in \mathcal{Z}^K} \psi_s(\mathbf{n}) \exp(-j2\pi\mathbf{f}^T \mathbf{V}\mathbf{n}) \quad (3.2.3)$$

Using the definition in Eq. (3.2.2), it can be shown that the CSFT of the sampled signal is equal to the above defined SSFT. Thus the definition of the SSFT is consistent with the CSFT. Also, the SSFT reduces to the previously defined DSFT when the lattice is a hypercube, i.e.,  $\mathbf{V}$  is a K-D identity matrix.

Notice that because  $\exp(j2\pi\mathbf{f}^T \mathbf{x}) = 1$ , if  $\mathbf{f}^T \mathbf{x} = n \in \mathcal{Z}$ , we have

$$\Psi_s(\mathbf{f} + \mathbf{U}\mathbf{m}) = \Psi_s(\mathbf{f}), \quad \text{with } \mathbf{U} = (\mathbf{V}^T)^{-1}.$$

This means that the SSFT is periodic with a periodicity matrix  $\mathbf{U}$ , and that the spectral repetition centers fall on the reciprocal of the sampling lattice,  $\Lambda^*$ . The fundamental period is the Voronoi cell of the reciprocal lattice,  $\mathcal{V}(\Lambda^*)$ . For this reason, one only needs to specify the SSFT of a sampled signal over  $\mathcal{V}(\Lambda^*)$ . This basic pattern repeats at all points in the reciprocal lattice. This is illustrated in Fig. 3.4. We will come back to this figure later.

**Theorem 3.4 (The Inverse Sampled Space Fourier Transform)** *A sampled signal over a lattice  $\Lambda$  with generating matrix  $\mathbf{V}$  can be obtained from its SSFT by*

$$\psi_s(\mathbf{n}) = \frac{1}{d(\Lambda)} \int_{\mathcal{V}(\Lambda^*)} \Psi_s(\mathbf{f}) \exp(j2\pi\mathbf{f}^T \mathbf{V}\mathbf{n}) d\mathbf{f}, \text{ for } \mathbf{n} \in \mathbb{Z}^K \quad (3.2.4)$$

The validity of the above inverse relation can be proven by substituting  $\Psi_s(\mathbf{f})$  in Eq. (3.2.4) with the forward transform in Eq. (3.2.3), and making use of the equality:

$$\int_{\mathcal{V}(\Lambda^*)} \exp(j2\pi\mathbf{x}^T \mathbf{f}) d\mathbf{f} = \begin{cases} \text{volume}\{\mathcal{V}(\Lambda^*)\} = d(\Lambda), & \text{for } \mathbf{x} = \mathbf{0}, \\ 0, & \text{otherwise.} \end{cases} \quad (3.2.5)$$

The inverse relation in Eq. (3.2.4) reveals that one can represent a sampled signal as an infinite sum of complex exponential functions with frequencies defined in the Voronoi cell of the reciprocal lattice.

All the properties of the CSFT can be carried over to the SSFT. Here we only state the convolution theorem. The proof is straight forward and is left as an exercise (Prob. 3.4).

**Definition 3.8:** *The linear convolution of two sampled signals over the same lattice  $\Lambda$  is defined as:*

$$\psi_s(\mathbf{n}) * h_s(\mathbf{n}) = \sum_{\mathbf{m} \in \mathbb{Z}^K} \psi_s(\mathbf{n} - \mathbf{m}) h_s(\mathbf{m}) = \sum_{\mathbf{x}_m \in \Lambda} \psi_c(\mathbf{x}_n - \mathbf{x}_m) h_c(\mathbf{x}_m), \quad (3.2.6)$$

where  $\psi_c(\mathbf{x})$  and  $h_c(\mathbf{x})$  are the continuous versions of  $\psi_s(\mathbf{n})$  and  $h_s(\mathbf{n})$ , respectively, and  $\mathbf{x}_n, n \in \mathbb{Z}^K$  denote the sample positions.

**Theorem 3.5 (Convolution Theorem over the Sampled Space)** *As with CSFT and DSFT, the convolution of two signals in a sampled space is equivalent to the product of their SSFT's, i.e.,*

$$\psi_s(\mathbf{n}) * h_s(\mathbf{n}) \longleftrightarrow \Psi_s(\mathbf{f}) H_s(\mathbf{f}). \quad (3.2.7)$$

**Characterization of Sampled Linear Shift-Invariant Systems** Given a linear shift-invariant system over the continuous space  $\mathcal{R}^K$  with an impulse response  $h_c(\mathbf{x})$ , if one samples the input and output signals over a lattice  $\Lambda$ , it can be shown (Prob. 3.5) that the sampled input signal  $\psi_s(\mathbf{n}) = \psi_c(\mathbf{V}\mathbf{n})$  and output signal  $\phi_s(\mathbf{n}) = \phi_c(\mathbf{V}\mathbf{n})$  are related by the above defined linear convolution in  $\Lambda$ , and the filter will be exactly the sampled impulse response  $h_s(\mathbf{n}) = h_c(\mathbf{V}\mathbf{n})$ . Therefore, the sampled system is completely characterized by the sampled impulse response  $h_s(\mathbf{n})$ .

### 3.2.2 The Generalized Nyquist Sampling Theorem

**Theorem 3.6 (The Generalized Nyquist Sampling Theorem)** *If a continuous signal  $\psi_c(\mathbf{x}), \mathbf{x} \in \mathcal{R}^K$  is sampled over a lattice  $\Lambda$  with a generating matrix  $\mathbf{V}$ ,*

then the SSFT of the sampled signal,  $\psi_s(\mathbf{n}) = \psi_c(\mathbf{V}\mathbf{n})$ ,  $\mathbf{n} \in \mathcal{Z}^K$ , is the summation of the CSFT of the original continuous signal and its translated versions at all points of the reciprocal lattice  $\Lambda^*$ , scaled by the density of the lattice. That is,

$$\Psi_s(\mathbf{f}) = d(\Lambda) \sum_{\mathbf{f}_m \in \Lambda^*} \Psi_c(\mathbf{f} - \mathbf{f}_m) = d(\Lambda) \sum_{\mathbf{m} \in \mathcal{Z}^K} \Psi_c(\mathbf{f} - \mathbf{U}\mathbf{m}). \quad (3.2.8)$$

It is possible to recover the original continuous signal from the sampled signal perfectly if and only if the non-zero region (known as the support region) of the CSFT of the original signal is limited within the Voronoi cell of the reciprocal lattice, i.e.,

$$\Psi_c(\mathbf{f}) = 0, \quad \text{for } \mathbf{f} \notin \mathcal{V}(\Lambda^*). \quad (3.2.9)$$

Furthermore, the perfect reconstruction can be accomplished by filtering the sampled signal using a reconstruction filter with the following frequency response (in terms of CSFT):

$$H_r(\mathbf{f}) = \begin{cases} \frac{1}{d(\Lambda)}, & \mathbf{f} \in \mathcal{V}(\Lambda^*), \\ 0, & \text{otherwise.} \end{cases} \quad (3.2.10)$$

One can easily recognize that the above theorem is a generalization of the 1D Nyquist sampling theorem. In that special case,  $\mathbf{V} = [\Delta]$ ,  $\mathbf{U} = [1/\Delta]$ , and  $d(\Lambda) = 1/\Delta$ , where  $\Delta$  is the sampling interval. As with the 1D and 2D signals, we refer to the repeated spectra as *alias components*. The theorem tells us that *if  $\Lambda$  is the sampling lattice in the spatial domain, then  $\Lambda^*$  encompasses centers of the alias components in the frequency domain*. If the continuous signal has a support region larger than  $\mathcal{V}(\Lambda^*)$ , than adjacent alias components would overlap. This effect is known as *aliasing*. As we have learned before, the denser is  $\Lambda$ , the sparser is  $\Lambda^*$ . Therefore using a denser sampling grid will separate the alias components in the frequency domain farther apart, and thus reducing the aliasing artifact in the sampled signal.

For a fixed sampling lattice, in order to avoid aliasing, the original continuous signal should be band-limited using the following pre-filter:

$$H_p(\mathbf{f}) = \begin{cases} 1, & \mathbf{f} \in \mathcal{V}(\Lambda^*), \\ 0, & \text{otherwise.} \end{cases} \quad (3.2.11)$$

We see that both the pre-filter and the reconstruction filter should be, in the ideal case, low-pass filters with a support region defined by the Voronoi cell of the reciprocal lattice,  $\mathcal{V}(\Lambda^*)$ .

To prove the above theorem, we can make use of the inverse CSFT. From Eq. (2.1.7),

$$\begin{aligned} \psi_s(\mathbf{n}) &= \psi_c(\mathbf{V}\mathbf{n}) = \int_{\mathcal{R}^K} \Psi_c(\mathbf{f}) \exp(j2\pi\mathbf{f}^T \mathbf{V}\mathbf{n}) d\mathbf{f} \\ &= \sum_{\mathbf{m} \in \mathcal{Z}^K} \int_{\mathcal{V}(\Lambda^*)} \Psi_c(\mathbf{f} - \mathbf{U}\mathbf{m}) \exp(j2\pi(\mathbf{f} - \mathbf{U}\mathbf{m})^T \mathbf{V}\mathbf{n}) d\mathbf{f} \end{aligned}$$

$$\begin{aligned}
&= \sum_{\mathbf{m} \in \mathcal{Z}^K} \int_{\mathcal{V}(\Lambda^*)} \Psi_c(\mathbf{f} - \mathbf{Um}) \exp(j2\pi\mathbf{f}^T \mathbf{V}\mathbf{n}) d\mathbf{f} \\
&= \int_{\mathcal{V}(\Lambda^*)} \sum_{\mathbf{m} \in \mathcal{Z}^K} \Psi_c(\mathbf{f} - \mathbf{Um}) \exp(j2\pi\mathbf{f}^T \mathbf{V}\mathbf{n}) d\mathbf{f}
\end{aligned}$$

Comparing the last equality with (3.2.4), the result in (3.2.8) follows.

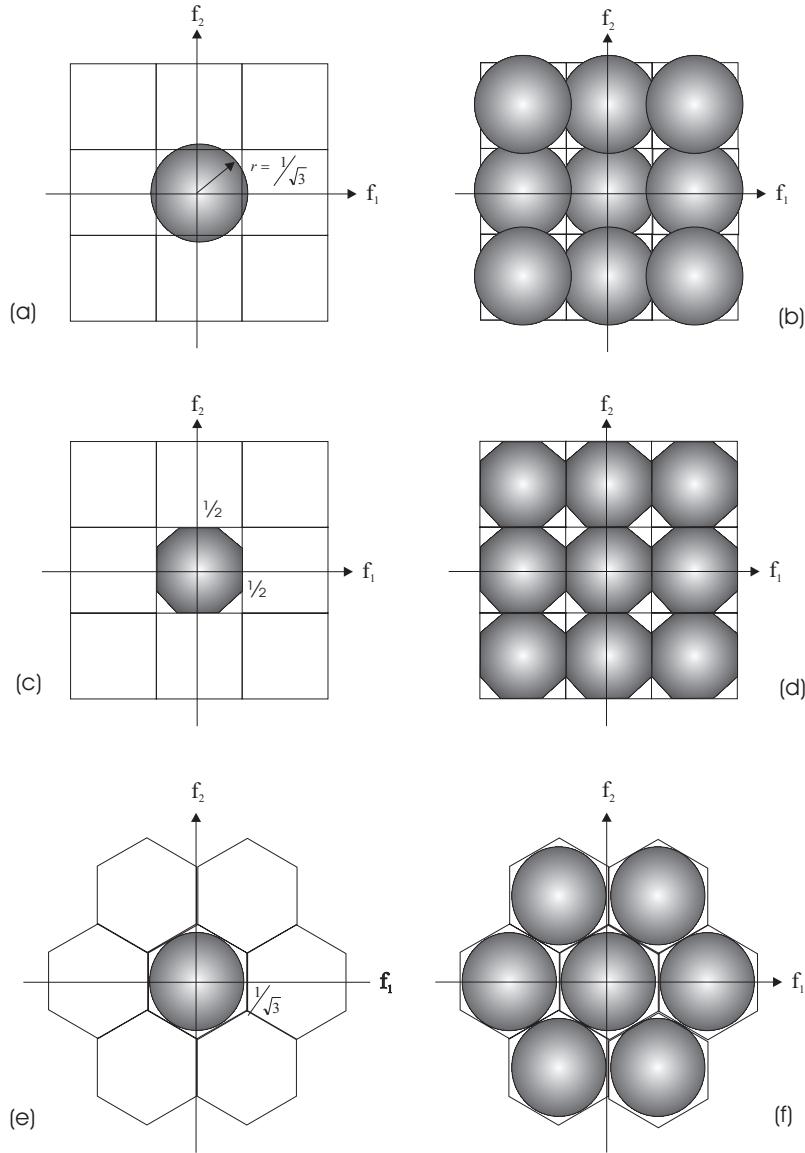
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**Example 3.3:** Consider a 2D signal with a spectrum shown in Fig. 3.4 (a), which has a circular support region with radius  $r = 1/\sqrt{3}$ . If we use the lattice presented in Fig. 3.1(a) to sample this signal, which is equivalent to using non-interlaced sampling with sampling intervals  $\Delta_x = \Delta_y = 1$ , the spectrum of the sampled signals will be as shown in Figs. 3.4(b), which is obtained by repeating the original spectra over points in the reciprocal lattice, shown previously in Fig. 3.1(c). Obviously, because the signal support region is larger than the Voronoi cell of the reciprocal lattice, there is overlap between the alias components in the sampled spectrum. To avoid the overlap, we can pre-filter the continuous signal so that its spectrum is non-zero only in the Voronoi cell of the reciprocal lattice, as illustrated in Fig. 3.4(c). If we now sample this signal with the same lattice, all the alias components will cover the entire spectrum exactly, without overlapping, as shown in Fig. 3.4(d). The reconstructed signal using the ideal reconstruction filter will be exactly the same as the pre-filtered signal, given in Fig. 3.4(c). Obviously, this signal has lost some frequency components compared to the original signal. If the application demands retention of the full bandwidth of the original signal, then a sampling lattice with a reciprocal Voronoi cell equal to or larger than the support region of the signal is needed. For example, if we use the hexagonal lattice in Fig. 3.1(b), then the signal spectrum fits entirely within the Voronoi cell of its reciprocal lattice, as illustrated in Fig. 3.4(e), and the spectrum of the sampled signal will be as shown in Fig. 3.4(f), with no aliasing. In this case, the reconstructed signal will be perfect. Rather than using the hexagonal lattice given in Fig. 3.1(b), one can still use a rectangular lattice, with a smaller sampling interval  $\Delta_x = \Delta_y = \sqrt{3}/2 < 1$ . This will lead to a higher sampling density. Recall that the sampling densities of the first and second lattices are 1 and  $2/\sqrt{3}$ , respectively, as derived previously, while the above scaled rectangular lattice has a sampling density of  $4/3$ , higher than that of the hexagonal lattice. From this example, we can see that the hexagonal lattice is more efficient than a rectangular one, because it requires a lower sampling density to obtain alias-free sampling.

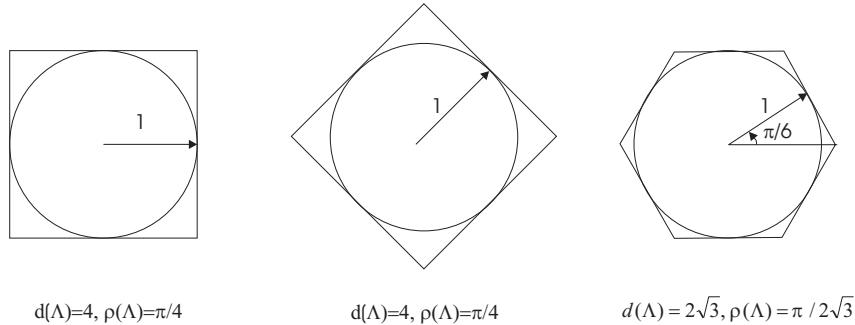
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### 3.2.3 Sampling Efficiency

As already illustrated by Example 3.3, to avoid aliasing, the sampling lattice must be designed so that the Voronoi cell of its reciprocal lattice completely covers the signal spectrum. Obviously, one can always design a very dense lattice to satisfy this requirement. This however will require a very high sampling density. To minimize the sampling density, one should design the lattice so that its reciprocal Voronoi cell covers the signal spectrum as tightly as possible. The solution will depend on the signal spectrum, which in general could have an arbitrarily shaped support region.



**Figure 3.4.** Illustration of the sampling process in the spectral domain. (a) the original signal; (b) the sampled signal using the rectangular lattice shown in Fig. 3.1(a); (c) the pre-filtered signal; (d) the sampled signal after pre-filtering; (e) the original signal spectrum overlaid on the hexagonal partition given in Fig. 3.2(b); (f) the sampled signal using the hexagonal lattice.



**Figure 3.5.** Illustration of the sphere packing capabilities of 2D rectangular, diamond, and hexagonal lattices.

Fortunately, most real-world signals are symmetric in frequency contents in different directions (after properly scaling the frequency axes) so that their spectrum support regions can be approximated well by a sphere. Therefore, to compare the sampling efficiency of different lattices, we can evaluate the sampling densities required for a signal that has a spherical support with radius one. To avoid aliasing, the Voronoi cell of the reciprocal of the sampling lattice,  $\mathcal{V}(\Lambda^*)$ , must enclose the sphere. The tighter fit is  $\mathcal{V}(\Lambda^*)$  to the sphere, the lower is the required sampling density. This leads to the definition of the *sampling efficiency* by<sup>2</sup>

$$\rho(\Lambda) = \frac{\text{volume (Unit Sphere)}}{\text{volume}(\mathcal{V}(\Lambda^*))} = \frac{\text{volume}( \text{Unit Sphere})}{d(\Lambda)}. \quad (3.2.12)$$

The closer is the above value to 1, the more efficient is the lattice.

Recall that the volume of a unit sphere is  $\pi$  in the 2D case, and  $\frac{4}{3}\pi$  in the 3D case. On the other hand, the minimum cube covering the sphere has a volume of 4 in 2D, and 8 in 3D. Therefore, the efficiency of the cubic lattice is  $\pi/4$  in 2D and  $\pi/6$  in 3D. Thus, the cubic lattice is not very efficient for sampling a signal with spherical support.

Figure 3.5 shows the efficiency in sphere covering of several 2D lattices. It can be seen that a hexagonal lattice can cover the sphere most tightly. Because of this reason, the sampling efficiency can also be defined relative to the density of the hexagonal lattice. Then the efficiency of the hexagonal lattice becomes one.

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<sup>2</sup> An alternative definition of the sampling efficiency [4] is by comparing the volume of  $\mathcal{V}(\Lambda^*)$  to that of the unit cube covering the sphere, i.e.,

$$\rho(\Lambda) = \frac{\text{volume}(\mathcal{V}(\Lambda^*))}{\text{volume (Unit Cube)}}.$$

With this definition, a smaller value represents a higher efficiency.

### 3.2.4 Implementation of Pre-filter and Reconstruction Filter

The ideal pre-filter and reconstruction filter are described in the frequency domain in the previous discussion. One can obtain the equivalent filter in the spatial domain by taking the inverse CSFT. Although the pre-filtering operation needs to be implemented entirely over the continuous domain, the reconstruction operation can be simplified because the input signal only exists over sampled points. To derive the equivalent interpolation operation in the spatial domain, let us think of the sampled signal as a continuous signal with impulses at the sampled points, as described by Eq. (3.2.2). The filtering operation can then be simplified to:

$$\begin{aligned}\psi_r(\mathbf{x}) &= \int_{\mathbf{y} \in \mathcal{R}^K} h(\mathbf{x} - \mathbf{y}) \psi_s(\mathbf{y}) d\mathbf{y} \\ &= \sum_{\mathbf{n} \in \mathcal{Z}^K} \psi_s(\mathbf{n}) \int_{\mathbf{y} \in \mathcal{R}^K} h_r(\mathbf{x} - \mathbf{y}) \delta(\mathbf{y} - \mathbf{V}\mathbf{n}) d\mathbf{y} \\ &= \sum_{\mathbf{n} \in \mathcal{Z}^K} \psi_s(\mathbf{n}) h_r(\mathbf{x} - \mathbf{V}\mathbf{n}).\end{aligned}\quad (3.2.13)$$

So the interpolated value at  $\mathbf{x}$  is a weighted sum of all sampled values with the weight for sample  $\mathbf{n}$  being  $h(\mathbf{x} - \mathbf{V}\mathbf{n})$ . This is the K-D version of the well known interpolation formula in 1D.

Note that the ideal pre-filter and interpolation filter will have infinite spatial extent because of the sharp transition band in the desired frequency response. In practice, only finite extent filters are implementable. We will discuss the filter design problem in Sec. 4.1.4.

### 3.2.5 Relation between Fourier Transforms over Continuous, Discrete and Sampled Spaces

Note that the sampled signal  $\psi_s(\mathbf{n})$  is a special case of the discrete space signal, considered in Sec. 2.2. Thus one can apply the DSFT defined in (2.2.3) to a sampled signal over an arbitrary lattice. Compared to the SSFT, the DSFT does not take into account the actual sampling geometry. By definition, we have

$$\Psi_s(\mathbf{f}) = \sum_{\mathbf{n} \in \mathcal{Z}^K} \psi_s(\mathbf{n}) \exp(-j2\pi\mathbf{f}^T \mathbf{V}\mathbf{n}), \quad (3.2.14)$$

$$\Psi_d(\mathbf{f}) = \sum_{\mathbf{n} \in \mathcal{Z}^K} \psi_s(\mathbf{n}) \exp(-j2\pi\mathbf{f}^T \mathbf{n}). \quad (3.2.15)$$

Comparing the above equations, we see that

$$\Psi_d(\mathbf{f}) = \Psi_s(\mathbf{U}\mathbf{f}), \quad \mathbf{f} \in \mathcal{I}^K, \quad \text{and} \quad \Psi_s(\mathbf{f}) = \Psi_d(\mathbf{V}^T \mathbf{f}), \quad \mathbf{f} \in \mathcal{V}(\Lambda^*). \quad (3.2.16)$$

*This tells us that the DSFT in the unit hypercube  $\mathcal{I}^K$  is the image of the SSFT over  $\mathcal{V}(\Lambda^*)$  warped to  $\mathcal{I}^K$ , or the SSFT in  $\mathcal{V}(\Lambda^*)$  is the image of the DSFT over  $\mathcal{I}^K$*

warped to  $\mathcal{V}(\Lambda^*)$ .<sup>3</sup> The discrete frequency variable  $\mathbf{f}_d$  is related to the continuous frequency variable  $\mathbf{f}_c$  by:

$$\mathbf{f}_c = \mathbf{U}\mathbf{f}_d = (\mathbf{V}^T)^{-1}\mathbf{f}_d, \text{ and } \mathbf{f}_d = \mathbf{V}^T\mathbf{f}_c = \mathbf{U}^{-1}\mathbf{f}_c. \quad (3.2.17)$$

Recall the relation between  $\Psi_s(\mathbf{f})$  and  $\Psi_c(\mathbf{f})$  in Eq. (3.2.8), we have:

$$\Psi_d(\mathbf{f}) = d(\Lambda) \sum_{\mathbf{m} \in \mathcal{Z}^K} \Psi_c(\mathbf{U}\mathbf{f} - \mathbf{Um}). \quad (3.2.18)$$

If  $\psi_c(\mathbf{x})$  is band-limited within  $\mathcal{V}(\Lambda^*)$ , then

$$\Psi_d(\mathbf{f}) = d(\Lambda)\Psi_c(\mathbf{U}\mathbf{f}), \quad \mathbf{f} \in \mathcal{I}^K. \quad (3.2.19)$$

Obviously,  $\Psi_d(\mathbf{f})$  is periodic with repetition centers at all integer points, which correspond to the points in  $\Lambda^*$  in the continuous frequency domain.

Remember in the 1D case, if we sample a continuous signal with a sampling frequency  $f_s$ , the discrete frequency is related to the continuous frequency by  $f_d = f_c/f_s$ , and the basic period  $(-1/2, 1/2)$  in the discrete frequency domain corresponds to the basic period  $(-f_s/2, f_s/2)$  in the continuous frequency domain. This is a special case described here for the K-D signal, with  $\mathbf{V} = [1/f_s]$ ,  $\mathbf{U} = [f_s]$ .

### 3.3 Sampling of Video Signals

The previous sections presented the sampling theory for general K-D signals. In the remaining sections of this chapter, we focus on 3D video signals. The two spatial dimensions and the temporal dimension are not symmetric in that they have different characteristics and that the visual sensitivities to the spatial and temporal frequencies are different. This asymmetry has led to the development of several interesting techniques for video sampling.

Although a video signal is continuously varying in space and time, cameras of today cannot capture the entire signal continuously in all three dimensions. Most movie cameras sample the video in the temporal direction, and stores a sequence of analog video frames on films. On the other hand, most TV cameras capture a video sequence by sampling it in temporal and vertical directions. The resulting signal is stored in a 1D raster scan, which is a concatenation of the color variations along successive horizontal scan-lines. To obtain a full digital video, one can sample in 2D the analog video frames resulting from a movie camera, sample in 1D along the raster scan resulting from a TV camera, or acquire discrete video frames directly using a digital camera by sampling the imaged scene in 3D. Different sampling schemes correspond to different sampling lattices. In designing a video sampling system, two questions to be answered are: i) what is the necessary sampling rates for video, and

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<sup>3</sup>Note that the spatial region corresponding to  $\mathbf{U}\mathbf{f}$  for  $\mathbf{f} \in \mathcal{I}^K$  does not correspond to  $\mathcal{V}(\Lambda^*)$  in general. But because of the periodicity of both  $\Psi_s(\mathbf{f})$  and  $\Psi_d(\mathbf{f})$ , the function  $\Psi_d(\mathbf{f})$  over  $\mathbf{f} \in \mathcal{I}^K$  comes from different parts of  $\mathcal{V}(\Lambda^*)$ . This is further considered in Problem 3.6.

ii) which sampling lattice is the most efficient under a given total sampling rate. In this section, we first describe the factors that determine the necessary sampling rates for video. We then discuss 2D sampling schemes and compare the progressive and interlaced scans. Next, we introduce several 3D sampling lattices and compare their efficiency. Finally, we will illustrate the effect of spatial and temporal aliasing by presenting a special example.

### 3.3.1 Required Sampling Rates

The very first and fundamental question one needs to answer when designing a video sampling system, either in 1D, 2D or 3D, is what should be the spatial and temporal sampling resolutions. This is governed by several factors, the frequency content of the underlying signal, visual spatial and temporal cut-off frequencies, the capture and display device characteristics, and finally the affordable processing, storage, and transmission cost. Based on the sampling theorem, the sampling rate in each dimension should be at least twice of the highest frequency along that direction. On the other hand, from Sec. 2.4, we know that human being cannot observe spatial and temporal variations beyond certain high frequencies. Given that the maximum frequency in the signal could vary significantly, the visual cut-off frequencies, which are the highest spatial and temporal frequencies that can be observed by the HVS, should be the driving factor in determining the sampling rates for video. There is no need to accommodate frequency components beyond these values. It is fair to assume that the technology can always be advanced to accommodate the human demand.

As described in Sec. 2.4, the visual sensitivity depends on the mean-brightness of the display. For TV signals, which are very bright, the visual temporal and spatial thresholds call for a frame rate of over 70 Hz, and a spatial resolution of at least 30 cpd. At a normal viewing distance of 3 times the screen height, a spatial frequency of 25 cpd translates to about 500 line/frame. To sample each line, the horizontal sampling interval should approximately match the vertical interval, so that the resulting pixels are square (i.e., having a PAR of 1). This leads to about 670 pixel/line for a display with 500 lines and an IAR of 4:3.

The above sampling rates, required by the visual cut-off frequencies, are beyond what practical technologies could offer at the time analog TV systems were developed. In order to reduce the data rate and consequently the cost for video capture, transmission, and display, interlaced scan was developed, which trades-off the vertical resolution for increased temporal resolution, for a given total data rate (product of the frame rate and line rate). For example, as explained in Sec. 1.4, in the NTSC analog TV system, 60 fields are captured per second, but each field contains only half the desired number of lines (480/2 active line/field). The total data rate is the same as that with a progressive scan using 30 frame/s and 480 line/frame. By interlacing the lines scanned in each field, it can produce the same quality as a progressive scan using 60 frame/second and 480 line/frame if the underlying scene is stationary. But for a high motion scene with special patterns (vertical

line patterns), it can lead to the notorious “interlacing artifact.” The next generation digital TV systems can operate in either the 30 frame/s and 480 line/frame mode, or the 60 field/s and 240 line/field mode. The interlaced format is retained mainly for compatibility with the analog TV system. The HDTV systems further enhance the visual impact by employing a wider screen size with an IAR of 16:9, and a sampling resolution of 60 frame/s, and 720 line/frame. Again, for compatibility purpose, an interlaced format with 60 field/s and 540 line/field can also be used (cf. Table 1.3).

For movies, because of the reduced visual sensitivity in a movie theater where the ambient brightness is kept low, a frame rate of 24 fps (progressive) is usually sufficient. As described in Sec. 2.4.1, although the original movie is captured at 24 fps, but when played back, a rotating blade is placed before the projection lens which rotates 3 times per frame, so that the effective playback rate is 72 fps. This is to suppress the flicker artifacts that may be experienced by some acute viewers. Old movies are made using a screen IAR of 4:3. For more dramatic visual impact, newer movies are usually made with a IAR of up to 2:1.

For computer displays, much higher temporal and spatial sampling rates are needed. For example, a SVGA display has a frame rate of 72 fps (progressive) and a spatial resolution of 1024x720 pixels. This is to accommodate the very close viewing distance (normally between 1 and 2 times of the picture height) and the high frequency content of the displayed material (line graphics and text).

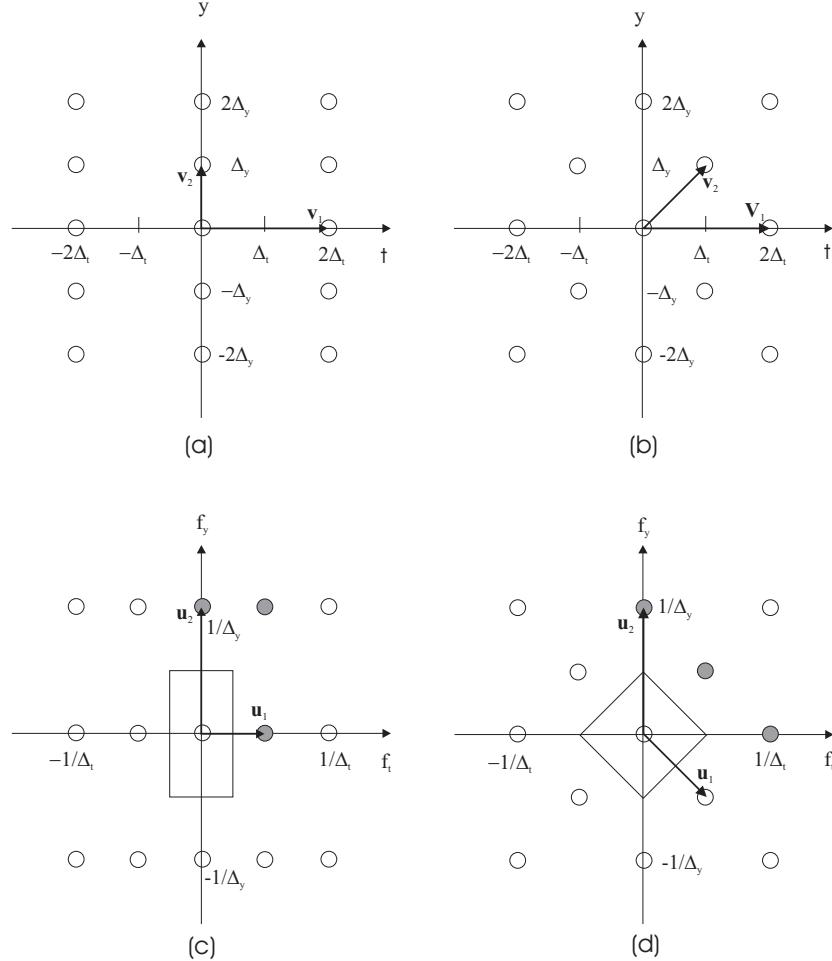
### 3.3.2 Sampling Video in 2D: Progressive vs. Interlaced Scans

We have described the mechanism for raster scan in Sec. 1.3, including both progressive and interlaced scans. It should be clear now that a video raster is actually a sampled version of a 3D video signal in the temporal and vertical directions. As already hinted, the motivation for using the interlaced scan is to trade off the vertical resolution for an enhanced temporal resolution, given the total number of lines that can be recorded within a given time. In this section, we attempt to gain more insights to the artifacts associated with these two sampling schemes by analyzing their aliasing patterns in the spectral domain.

To ease the discussion, we ignore the horizontal direction, and consider the video signal as a 2D signal in the space spanned by the temporal and vertical directions. Let  $\Delta_t$  present the field interval and  $\Delta_y$  the line interval. Then the sampling lattices employed by the progressive and interlaced scans are as shown in Fig. 3.6(a) and 3.6(b), respectively. In these figures, we also indicate the basis vectors for generating each lattice. From these basis vectors, we derive the following generating matrices for the original lattices and the reciprocal lattices:

$$\text{Progressive Scan : } \mathbf{V}_1 = \begin{bmatrix} 2\Delta_t & 0 \\ 0 & \Delta_y \end{bmatrix}, \quad \mathbf{U}_1 = \begin{bmatrix} 1/2\Delta_t & 0 \\ 0 & 1/\Delta_y \end{bmatrix}, \quad (3.3.1)$$

$$\text{Interlaced Scan : } \mathbf{V}_2 = \begin{bmatrix} 2\Delta_t & \Delta_t \\ 0 & \Delta_y \end{bmatrix}, \quad \mathbf{U}_2 = \begin{bmatrix} 1/2\Delta_t & 0 \\ -1/2\Delta_y & 1/\Delta_y \end{bmatrix}. \quad (3.3.2)$$



**Figure 3.6.** Comparison of progressive and interlaced scans: (a) sampling lattice for progressive scan; (b) sampling lattice for interlaced scan; (c) reciprocal lattice for progressive scan; (d) reciprocal lattice for interlaced scan. Filled circles in (c) and (d) indicate the nearest aliasing components.

Based on the matrices  $\mathbf{U}_1$  and  $\mathbf{U}_2$ , we draw the reciprocal lattices in Figs. 3.6 (c) and (d), in which we have highlighted the three points (filled circles) which are nearest to the origin in the first quadrant of the frequency plane. These are the centers of the alias components that are closest to the original signal spectrum, and are the primary causes for perceivable distortion.

Note that when drawing the above lattices, we have scaled the spatial and temporal dimensions so that a spatial frequency equal to the vertical sampling rate

$f_{s,y} = 1/\Delta_y$  is assigned the same length as a temporal frequency equal to the field rate  $f_{s,t} = 1/\Delta_t$ . Similarly, the spatial interval  $\Delta_y$  and the temporal interval  $\Delta_t$  are given the same length. Ideally, we would like to equate the temporal and spatial frequencies based on the visual sensitivity, by assigning the same length to the spatial and temporal cut-off frequencies. Because the spatial and temporal sampling rates are usually chosen based on the respective cut-off frequencies, equating the sampling frequencies in different directions is in general appropriate.

Comparing the original and reciprocal lattices of these two scans, we arrive at the following observations:

1. They have the same sampling density, *i.e.*,  $d_1(\Lambda) = d_2(\Lambda) = 1/2\Delta_t\Delta_y$ .
2. They have the same nearest aliases, at  $1/\Delta_y$ , along the vertical frequency axis. This means that in the absence of motion, the two sampling lattices have the same vertical resolving power. This is because although there are only half the number of lines in each field, when the imaged scene is stationary, the lines sampled in two separate field times appear as if they were sampled at the same time. When there is motion in the scene, however, the vertical resolution of the interlaced scan is lower than that of the progressive scan.
3. They have different nearest aliases along the temporal frequency axis. For the progressive scan, the first alias occurs at  $1/2\Delta_t$ , while for the interlaced scan, this occurs at  $1/\Delta_t$ . Recall that a temporal frequency component that is larger than half of the nearest temporal alias will cause a flicker artifact. We can see that with the interlaced scan, flickering is less likely to happen, when the object has a flat or slowly varying vertical pattern.
4. They have different mixed aliases. The mixed alias is defined as the nearest off-axis alias component. A frequency component that is close to the mixed alias causes inter-line flicker and line crawl. For the progressive scan, the mixed alias occurs at  $(1/2\Delta_t, 1/\Delta_y)$ , while for the interlaced scan, this occurs at  $(1/2\Delta_t, 1/2\Delta_y)$ . Because the interlaced scan has a mixed alias that is closer to the origin, the inter-line flicker and line crawl artifacts are more visible in interlaced scans. These are the notorious interlacing artifacts.
5. For a signal with isotropic spectral support, the interlaced scan is more efficient. Specifically, the maximum radius of the signal spectrum that can be represented without aliasing is equal to  $1/4\Delta_t$  with the progressive scan; while this is increased to  $1/2\sqrt{2}\Delta_t$  with the interlaced scan. In interpreting this result, it is important to remember that this is based on our way of equating the spatial and temporal frequencies.

Note that the above comparison is between an interlaced scan and a progressive scan that have the same total sampling rate. In this case, the interlaced scan has some advantages over progressive scans, as shown above. To improve the quality further, the interlaced scan can be deinterlaced, to yield a progressive scan that

has twice the total sampling rate. We will consider the deinterlacing problem in Sec. 4.2.1.

An interesting analysis of the spectrum of an interlaced frame under constant linear motion is given by Beuker and Shah [1]. Based on this analysis, techniques for deinterlacing and other rate conversion problems are considered.

### 3.3.3 Sampling a Raster Scan: BT.601 Format Revisited

As described in the beginning of this chapter, one way to obtain a full digital video is to sample a raster scan. Recall that a raster is a 1D signal consisting of successive horizontal scan-lines in successive frames (fields for an interlaced raster). Therefore, the sampling interval along the scan line directly determines the horizontal sampling interval. To determine the sampling interval, several factors need to be taken into account. First, the resulting horizontal sampling spacing should match with the vertical spacing between scan lines so that the sampling frequencies in the horizontal and vertical directions are similar. Second, the resulting samples in the 3D space should follow a desired sampling lattice. For example, if the samples on each frame/field are to form a rectangular grid, the sampling interval should be a integer divisor of the line interval. These criteria were used in designing the BT.601 digital video format described in Sec. 1.5.2. By selecting the sampling interval according to Eq. (1.5.2), the samples on successive horizontal scan-lines are vertically aligned, with the horizontal and vertical sampling spacings being approximately equal.

The above discussions assume that the underlying video raster has a single luminance component. For a color video raster, including one luminance and two chrominance components, a straight forward approach is to use the same sampling frequency for all components. This will lead to the BT.601 4:4:4 format, previously shown in Fig. 1.8(a). However, it is well known that the HVS is more sensitive to the luminance information than to the chrominance. Therefore it is wise to sample the chrominance components at resolutions lower than that for the luminance. Ideally, sampling frequencies in all three dimensions should be reduced equally. However, because a raster is a stack of horizontal scan-lines, reducing the sampling frequency for the raster only lowers the horizontal sampling rate. The vertical and temporal sampling rates are unaffected. For example, if the sampling rate for the chrominance components,  $f_{s,c}$ , is half of that for the luminance component,  $f_s$ , the temporal and vertical sampling intervals are the same for the chrominance component and the luminance component, but the horizontal sampling interval for the chrominance is twice of that for the luminance. This in fact is the color sub-sampling format used in the BT.601 4:2:2 format shown previously in Fig. 1.8(b). If we down-sample the chrominance component by a factor of 4 along the raster scan, we will arrive at the BT.601 4:1:1 format of Fig. 1.8(c). Given a 4:2:2 signal, we can further down-sample the vertical dimension by a factor of two, which will lead to the BT.601 4:2:0 format, given in Fig. 1.8(d). Note that the 4:2:0 format cannot be obtained directly from the raster signal by using uniform sampling. Conversion between BT.601 4:2:2 signals to 4:2:0 signals is considered in Chapter 4, Problems 4.6 and 4.7.

The 4:1:1 and the 4:2:0 formats have the same overall sample rates (samples/s), but the 4:2:0 format can represent a video more accurately because it captures horizontal and vertical details in the chrominance components with the same resolution, half of that for the luminance component. On the other hand, with the 4:1:1 format, the vertical detail is represented with the same resolution as the luminance component, whereas the horizontal is represented with only a quarter of that resolution.

The above discussion assumes that the three color components in the raster signal are separated. To sample a composite color video, one needs to first separate the individual color components and then perform sampling as discussed before.

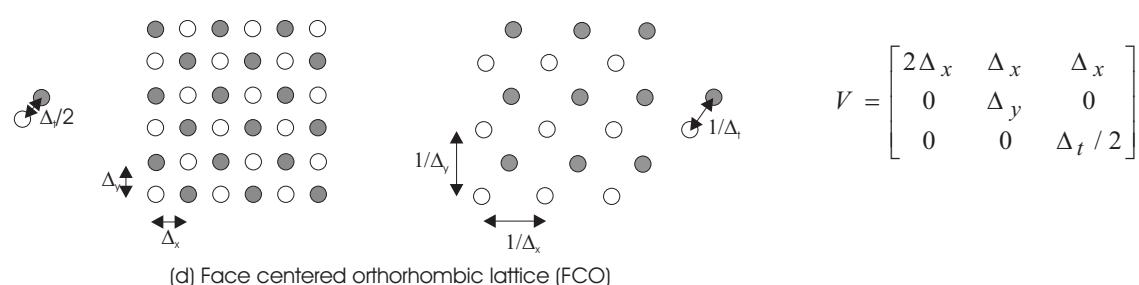
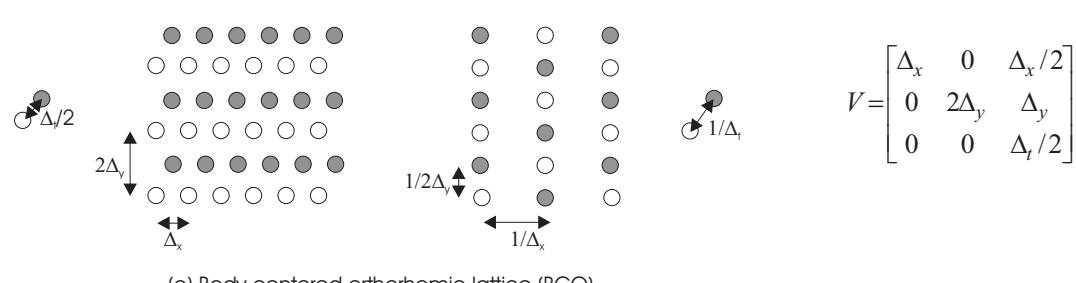
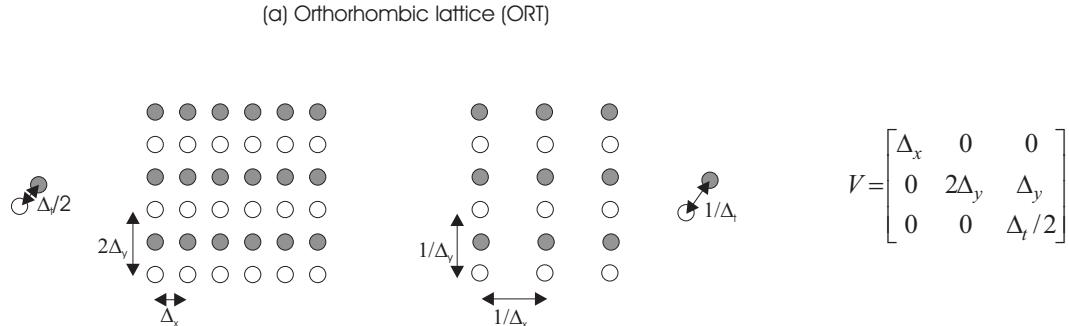
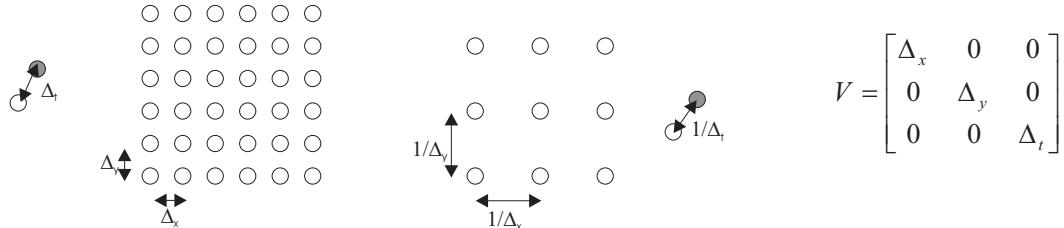
### 3.3.4 Sampling Video in 3D

In the sampling schemes described in the previous section, the horizontal samples are aligned vertically in all fields. Such a sampling scheme is by no means optimal. One can also take samples in an interlaced or more complicated pattern over the x-y plane. More generally, one can directly sample the 3D space using a desired lattice. It is difficult to make a camera that can implement a complicated 3D sampling structure. However, one can first acquire samples using a dense but simply structured lattice and then down-convert it to the desired lattice. In this section, we compare the efficiency of several lattices that have been used for video sampling. The materials presented here are from [4]. To ease the presentation, we will assume that the frequency axes,  $f_x, f_y, f_t$ , are scaled by the maximum frequencies of the signal,  $1/f_{x,\max}, 1/f_{y,\max}, 1/f_{t,\max}$ , respectively. The horizontal, vertical, and temporal axes,  $x, y, t$ , are correspondingly scaled by  $f_{x,\max}, f_{y,\max}, f_{t,\max}$ . The following discussion is in terms of the scaled unit-less variables. To evaluate the sampling efficiency of each lattice, we further assume the support region of the signal is a sphere with radius one, in the scaled frequency space.

First, let us consider the sampling of a progressively scanned raster with frame interval  $\Delta_t$  and line interval  $\Delta_y$ . If the samples are aligned vertically with a horizontal interval of  $\Delta_x$ , then the equivalent 3D sampling lattice is simply cubic or orthorhombic (denoted by ORT), as given in Fig. 3.7(a). Obviously, to avoid aliasing, we need to have  $\Delta_x = \Delta_y = \Delta_t = \frac{1}{2}$ , with a sampling density  $d(\text{ORT}) = 8$ .

Next, let us consider the sampling of an interlaced raster with a field interval of  $\Delta_t/2$ , and line interval within each field  $2\Delta_y$ . If the samples in different fields are aligned, with horizontal interval  $\Delta_x$ , the equivalent 3D sampling lattice is illustrated in Fig. 3.7(b), and is denoted by ALI. It can be shown that to avoid aliasing, we must have  $\Delta_x = \Delta_y = \frac{1}{2}, \Delta_t = \frac{1}{\sqrt{3}}$ , which leads to a sampling density of  $d(\text{ALI}) = 4\sqrt{3}$ . This is more efficient than the ORT lattice.

To improve the sampling efficiency, we can shift the samples in the even and odd lines by half, as shown in Fig. 3.7(c). This lattice is called body-centered orthorhombic lattice or BCO. It can be shown that the required sampling intervals for avoiding aliasing are  $\Delta_x = \Delta_t = \frac{1}{\sqrt{2}}, \Delta_y = \frac{1}{2\sqrt{2}}$ , with a sampling density of  $4\sqrt{2}$ . Therefore, this lattice is more efficient than the ALI.



**Figure 3.7.** Several 3D video sampling lattices and their reciprocals. In each sub-figure, left is the sampling lattice, and right is the reciprocal lattice. The matrix  $V$  indicates the generating matrix for the sampling lattice. The open and filled circles represent lattice points on two different time (left) or temporal frequency (right) planes, with a distance indicated by the arrows shown on the left and right, respectively. For example, for (b), on the left, the open circles represent the samples taken at time  $t = 0$ , and the filled circles at time  $t = \Delta_t/2$ . On the right, the open circles represent the aliasing components at temporal frequency  $f_t = 0$ , and filled circles at  $f_t = 1/\Delta_t$ . For (a), the filled circles fall directly behind the open circles and are therefore not shown.

**Table 3.1.** Alias-Free Sampling Lattices for Signals with a Unit Sphere Spectrum.

Lattice	$\Delta_x$	$\Delta_y$	$\Delta_t$	Density	Efficiency
ORT	$1/2$	$1/2$	$1/2$	8	$\pi/6 = 0.524$
ALI	$1/2$	$1/2$	$1/\sqrt{3}$	$4\sqrt{3}$	$\pi/3\sqrt{3} = 0.605$
BCO	$1/\sqrt{2}$	$1/2\sqrt{2}$	$1/\sqrt{2}$	$4\sqrt{2}$	$\pi/3\sqrt{2} = 0.740$
FCO	$1/2$	$1/2$	$1/\sqrt{2}$	$4\sqrt{2}$	$\pi/3\sqrt{2} = 0.740$

A variation of the BCO sampling lattice is shown in Fig. 3.7(d). This lattice is known as face-centered orthorhombic (FCO). With this sampling structure, each field contains all the lines, but the samples in the same field are interleaved in the even and odd lines. The sampling efficiency of this lattice is the same as BCO. But because samples in the even and odd fields are more evenly blended, it usually leads to a better visual quality.

The minimum sampling density and the associated sampling intervals for the above lattices are summarized in Table 3.1.

### 3.3.5 Spatial and Temporal Aliasing

In Sec. 2.3.2, we described the relation between the temporal frequency, spatial frequency, and velocity of an object undergoing a linear motion. In general, the temporal frequency of an object is larger if it has a higher velocity (unless it is moving in a direction orthogonal to the spatial frequency direction). Here, we illustrate the effect of temporal aliasing by again considering an object with linear motion.

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**Example 3.4:** Consider a scene with a pure vertical sinusoidal color pattern with a horizontal frequency of 1 cycle/cm (cf. Fig. 2.1(a)). Suppose the pattern is moving horizontally with a velocity  $v_x = 3$  cm/s. If we sample it using a rate of  $f_{s,x} = f_{s,y} = 3$  samples/cm,  $f_{s,t} = 3$  frame/s, what will be the apparent motion and sinusoidal frequency?

Because the signal has a spatial frequency of 1 cycle/cm and is moving at 3 cm/s, it advances 3 cycle/s or 1 cycle/frame at a temporal sampling rate of 3 frame/s. Thus the captured frames at two succeeding frames are exactly the same, as illustrated in Fig. 3.8(a). Because the spatial sampling rate is high enough ( $f_{s,x} > 2f_{x,0}$ ), there is no spatial aliasing. Therefore, the sampled signal will appear as a stationary sinusoidal with the same pattern as the continuous signal.

Now, let us explain the above phenomenon using the lattice sampling theorem. Obviously, this signal has a pair of spatial frequency components at  $(f_{x,0}, f_{y,0}) = \{(1, 0), (-1, 0)\}$ . Based on Eq. (2.3.4), the temporal frequency caused by the motion is  $f_{t,0} = -(f_{x,0}v_x + f_{y,0}v_y) = \{-3, 3\}$  (cycle/s). Therefore, the spectrum of the signal has a single pair of impulses at frequencies  $(f_{x,0}, f_{y,0}, f_{t,0}) = \{(1, 0, -3), (-1, 0, 3)\}$ , as illustrated in Fig. 3.8(b). In this figure, we only draw the  $f_x - f_t$  plane at  $f_y = 0$ , as the signal is zero in all other planes. After sampling, the signal spectrum will replicate at all multiples of the sampling

frequencies as shown in the figure. We can see that because  $f_{s,t} < 2f_{t,0}$ , a pair of alias components fall in the Voronoi region of the reciprocal lattice, which in this case is simply a rectangular region bounded by  $\pm f_{s,x}/2$  and  $\pm f_{s,t}/2$ . In fact, the original signal component fall out of this region. If an ideal low-pass filter is used to recover the continuous signal from the sampled one, then the recovered signal would be a pure sinusoid with a spatial frequency of  $(1, 0)$  and temporal frequency of 0. This corresponds to a stationary vertical sinusoid with horizontal frequency of 1 cycle/cm. This is the signal that will be perceived by the eye, which functions as a low-pass filter.

In order to retain the true motion in the sampled signal, the temporal sampling rate should be at least  $2f_{t,0} = 6$  frame/s. At this rate, each frame is the previous frame moved by half a cycle.

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The above example is an extreme case of the temporal aliasing effect: a moving object becomes stationary. More generally, a fast moving object may appear to move at a slower speed when the frame rate is not high enough. Depending on the spatial pattern of the moving object, the directions of some line patterns may also alter. This is why a moving stagecoach wheel in cowboy movies sometimes appears to turn backward and move slower.

### 3.4 Filtering Operations in Cameras and Display Devices

The previous sections presented the sampling theory for general K-D signals as well as for video signals. We have derived, for several typical video formats, the required pre-filters and reconstruction filters. In this section, we discuss how practical cameras and display devices accomplish these tasks in a crude way and how the human visual system partially accomplishes the required interpolation task.

#### 3.4.1 Camera Aperture

Consider a camera that samples a continuously varying scene at regular intervals  $\Delta_x, \Delta_y, \Delta_t$  in the horizontal, vertical, and temporal directions, respectively. This corresponds to using a simple cubic lattice. The sampling frequencies are  $f_{s,x} = 1/\Delta_x, f_{s,y} = 1/\Delta_y, f_{s,t} = 1/\Delta_t$ . The ideal pre-filter should be a low-pass filter with cut-off frequencies at half of the sampling frequencies. In the following, we discuss the actual pre-filters implemented in typical cameras.

**Temporal Aperture** A video camera typically accomplishes a certain degree of pre-filtering in the capturing process. First, the intensity values read out at any frame instant is not the sensed value at that time, rather, the average of the sensed signal over a certain time interval,  $\Delta_e$ , known as exposure time. Therefore, the camera is applying a pre-filter in the temporal domain with an impulse response of the form:

$$h_{p,t}(t) = \begin{cases} \frac{1}{\Delta_e} & t \in (0, \Delta_e), \\ 0 & \text{others.} \end{cases} \quad (3.4.1)$$

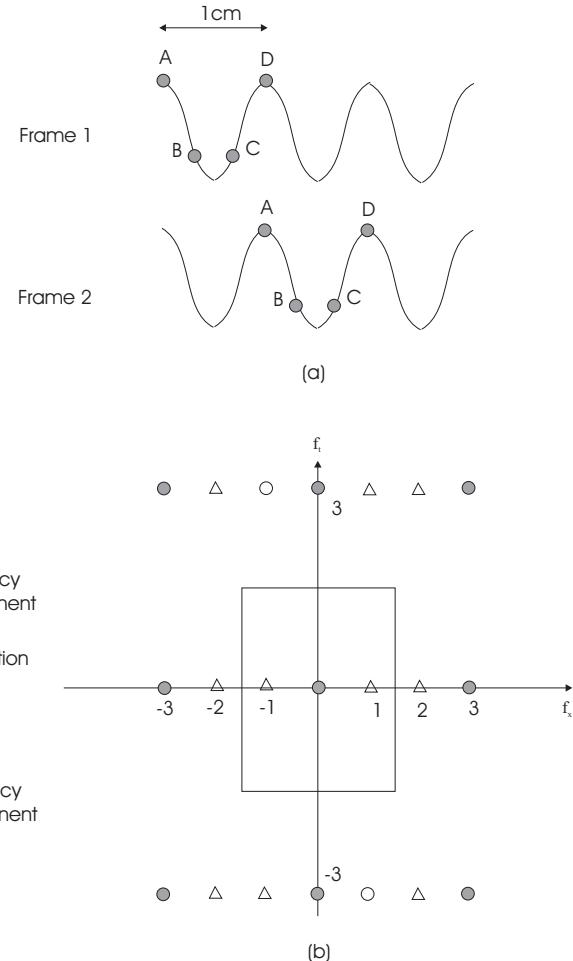


Figure 3.8. Sampling a sinusoidal pattern. See Example 3.4.

The spectrum (i.e., CSFT) of this filter is:

$$H_{p,t}(f_t) = \exp(-j\pi f_t \Delta_e) \frac{\sin(\pi f_t \Delta_e)}{\pi f_t}. \quad (3.4.2)$$

We can see that it reaches zero at  $f_t = 1/\Delta_e$ . Recall that  $1/\Delta_t$  is the temporal sampling rate and that the ideal pre-filter for this task is a low-pass filter with cut-off frequency at half of the sampling rate. By choosing  $\Delta_e \geq \Delta_t$ , the camera can suppress temporal alias component near the sampling rate. But too large a  $\Delta_e$  will make the signal blurred. In practice, the effect of blurring is sometimes more noticeable than aliasing. Therefore, the exposure time  $\Delta_e$  must be chosen to reach

a proper trade-off between aliasing and blurring effects.

**Spatial Aperture** In addition to temporal integration, the camera also performs spatial integration. The value read-out at any pixel (a position on a scan line with a tube-based camera or a sensor in a CCD camera) is not the optical signal at that point alone, rather a weighted integration of the signals in a small window surrounding it, called *aperture*. The shape of the aperture and the weighting values constitute the camera *spatial aperture function*. This aperture function serves as the spatial pre-filter, and its Fourier transform is known as the *modulation transfer function* (MTF) of the camera. With most cameras, the spatial aperture function can be approximated by a circularly symmetric Gaussian function,

$$h_{p,x,y}(x, y) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-(x^2 + y^2)/2\sigma^2). \quad (3.4.3)$$

The spectrum of this function is also a Gaussian:

$$H_{p,x,y}(f_x, f_y) = \exp(-(f_x^2 + f_y^2)/2\beta^2), \text{ with } \beta = \frac{1}{2\sigma\pi}. \quad (3.4.4)$$

The value of  $\sigma$  or  $\beta$  depends on the aperture size and shape, which are usually designed so that the frequency response is 0.5 at half of the vertical and horizontal sampling rates. Assuming  $f_{s,x} = f_{s,y} = f_{s,x,y}$ , we see that this requires  $\beta = f_{s,x,y}/2\sqrt{\ln 2}$ . For the MTF of several camera tubes, please see [6].

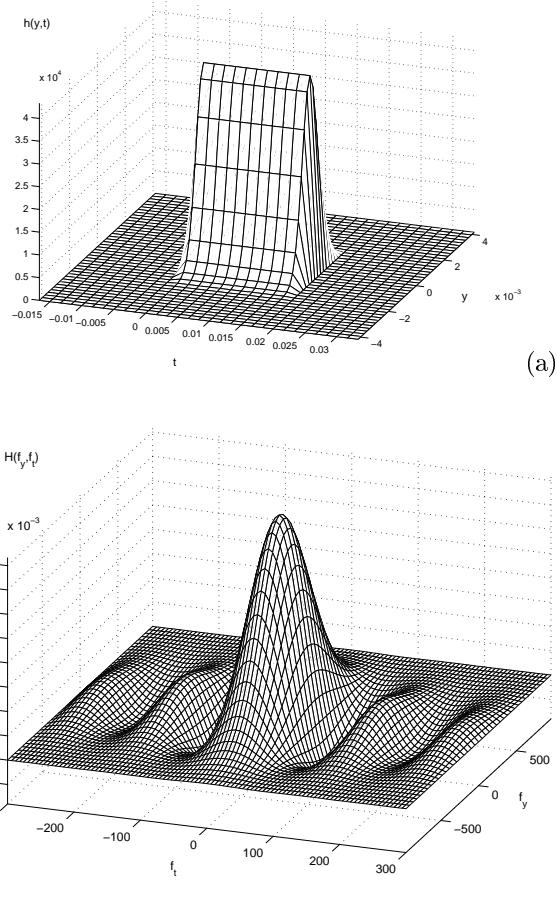
**Combined Aperture** The overall camera aperture function or pre-filter is

$$h_p(x, y, t) = h_{p,t}(t)h_{p,x,y}(x, y), \quad (3.4.5)$$

with a frequency response

$$H_p(f_x, f_y, f_t) = H_{p,t}(f_t)H_{p,x,y}(f_x, f_y). \quad (3.4.6)$$

The impulse response of a camera with  $\Delta_e = \Delta_t = 1/60$  second and  $f_{s,y} = 480$  (line/picture-height) is shown in Fig. 3.9(a). Its frequency response is given in Fig. 3.9(b). Only the  $f_y - f_t$  plane at  $f_x = 0$  is shown. Obviously, it is far from the ideal half-band low-pass filter, which should have a square pass-band defined by  $|f_y| \leq f_{s,y}/2, |f_t| \leq f_{s,t}/2$ . On one hand, it attenuates the frequency components inside the desired pass-band (the Voronoi cell), thereby reducing the signal resolution unnecessarily; on the other hand, it does not remove completely the frequency components in the desired stop-band, which will cause aliasing in the sampled signal. It has been found that the human eye is more annoyed by the loss of resolution than the aliasing artifact. This is partly because the aliasing artifact (fold-over of an alias component) only cause noticeable visual artifacts when the image contains high frequency periodic patterns that are close to the lowest aliasing frequency, which are rare in natural scene imagery. For this reason, preservation of the signal in the pass-band is more important than suppression of the signal outside the pass-band.



**Figure 3.9.** The aperture function of a typical camera. (a) The impulse response; (b) The frequency response.

In order to accomplish the pre-filtering more precisely, one can use digital filters. This would involve three steps: i) sample the signal at a rate higher than the desired sampling rate, ii) use a digital filter to suppress the frequencies outside the desired pass-band, and iii) down-convert this digital signal to the desired sampling rate. Although a filter with very sharp transition gives better results in terms of mean square error, such a filter will cause ringing at sharp edges, known as the Gibbs' effect. Furthermore, to realize a sharp transition, a very high order filter is needed, which may not be feasible in video applications.

### 3.4.2 Display Aperture

As described in Sec. 1.2, with a CRT monitor, an electronic gun emits an electronic beam across the screen line by line, striking phosphors with intensities proportional to the intensity of the video signal at corresponding locations. To display a color image, three beams are emitted by three separate guns, striking red, green, and blue phosphors with the desired intensity combination at each location. The beam thickness essentially determines the vertical filtering: a very thin beam will make the image look sharper but will also cause the perception of the scan lines if the viewer sits too close to the screen; on the other hand, a thicker beam will blur the image. Normally to minimize the loss of spatial resolution, thin beams are used so that very little vertical filtering is exercised by the display device. Temporal filtering is determined by the phosphors used. The P22 phosphors used in color television decay to less than 10 percent of peak responses in 10  $\mu\text{s}$  to 1 ms [3, 4], much shorter than a field time (16.7 ms). Thus virtually no temporal filtering is performed.

Fortunately, the human visual system (HVS) has a low-pass or band-pass characteristic, depending on the regime of the temporal and spatial frequencies in the imagery. This has been described in Sec. 2.4. Therefore, the eye performs to some degree the required interpolation task. For improved performance, one can use a digital filter to first up-convert the sampled signal to a higher resolution, which is then fed to a high resolution display system.

It has been found that the combination of the camera and display apertures in most of current television systems results in a maximum vertical resolution of only about 0.7 of the theoretical limit (*i.e.* half of the line rate). In other words, the maximum vertical frequency that can be rendered properly by a TV system with a line number of  $N_y$  is about  $0.7N_y/2$ . This factor is referred to as the *Kell factor* [4], denoted by  $K$ . For a discussion of how the parameters of an imaging system can influence the Kell factor, the reader is referred to [8]. By using digital filters for pre-filtering and interpolation, as described before, Kell factors much closer to unity may be possible. Previously, in Sec. 1.4.3, we have seen how does the Kell factor affect the bandwidth of the TV signal.

## 3.5 Summary

### Sampling of K-D Signals over Arbitrary Lattices (Sec. 3.2)

- A K-D continuous space signal can be sampled over a lattice. One can define a sampled space Fourier transform (SSFT), which satisfies similar properties as DSFT.
- A SSFT is periodic with alias centers located at points of the reciprocal of the sampling lattice.
- The original signal can be reconstructed perfectly from the sampled signal if the spectrum (*i.e.*, CSFT) of the original signal is non-zero only within the

Voronoi cell of the reciprocal lattice.

- A sampling lattice is more efficient if the Voronoi cell of its reciprocal lattice covers the support region of the signal more tightly. For example, for a signal with circular spectrum support, the rectangular sampling lattice is less efficient than a hexagonal or diamond lattice.

### Sampling of Video Signals (Sec. 3.3)

- The spatial and temporal sampling frequencies required for a video signal depend on the cut-off frequencies of the spatio-temporal frequency response of the HVS, in addition to the spectrum content of the signal.
- Interlaced scan is a way to trade-off vertical resolution for enhanced temporal resolution. Interlaced scan can be considered as using a non-rectangular sampling lattice on the vertical-temporal plane.
- More generally, one can use interleaved scan in both horizontal and vertical dimensions. These correspond to non-cubic lattices in 3D. Compared to cubic lattices, they can reduce aliasing artifacts under a given total sampling rate.

## 3.6 Problems

- 3.1** Suppose the basis vectors of two sampling lattices are:

- A)  $\mathbf{v}_1 = [\sqrt{3}, 1]^T$  and  $\mathbf{v}_2 = [0, 2]^T$ ;  
 B)  $\mathbf{v}_1 = [2\sqrt{2}, 0]^T$  and  $\mathbf{v}_2 = [\sqrt{2}, \sqrt{2}]^T$ ;

For each of the sampling lattices defined above:

- Sketch the basis vectors and the sample points. Illustrate the Voronoi unit cell. Illustrate how the entire spatial domain is tiled up by the shifted versions of the unit cell. Determine the sampling density.
- Determine the basis vectors of the reciprocal lattice. Repeat part (a) for this lattice.
- For a signal with a circular spectrum (i.e, the support region is circle), which lattice is better? Construct a spectrum (sketch its support region) that will cause aliasing by one lattice but not the other.
- The basis vectors for a given lattice are not unique. Can you find another set of basis vectors for the lattices A and B?

- 3.2** Prove Theorem 3.2.

- 3.3** Prove Theorem 3.3.

- 3.4** Prove Theorem 3.5.

**3.5** Consider a linear shift-invariant system over the continuous space  $\mathcal{R}^K$  with an impulse response  $h_c(\mathbf{x})$ . If one samples the input and output signals over a lattice  $\Lambda$ , show that the sampled input signal  $\psi_s(\mathbf{n}) = \psi_c(\mathbf{V}\mathbf{n})$  and output signal  $\phi_s(\mathbf{n}) = \phi_c(\mathbf{V}\mathbf{n})$  are related by a sampled space convolution in  $\Lambda$ , and the filter will be exactly the sampled impulse response  $h_s(\mathbf{n}) = h_c(\mathbf{V}\mathbf{n})$ .

**3.6** In Section 3.2.5, we showed that the DSFT in the unit hypercube  $\mathcal{I}^K$  is the warped image of the SSFT in a region warped from  $\mathcal{I}$  using the transformation  $\mathbf{Uf}$ . Illustrate the warping process, for the case where the sampling lattice is hexagonal. More specifically, consider a signal with a spherical support. Start with a signal sampled over the hexagonal lattice, draw both SSFT and DSFT.

**3.7** Consider a 2D signal defined by

$$\psi(x, y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2+y^2}{2}}. \quad (3.6.1)$$

(a) Determine its CSFT.

(b) Suppose we want to sample this signal using a hexagonal lattice  $\Lambda$  given by

$$\mathbf{V} = \alpha \begin{bmatrix} \sqrt{3}/2 & 0 \\ 1/2 & 1 \end{bmatrix} \quad (3.6.2)$$

Choose an appropriate scaling factor  $\alpha$  so that aliasing effect is not severe. For example, choose  $\alpha$  so that  $\Psi(f_x, f_y) = 0.1$  at the border of the Voronoi cell of  $\Lambda^*$ .

(c) Determine the sampled signal and its spectrum (SSFT).

(d) Draw the original continuous signal, the sampled signal, the continuous and sampled spectrum using Matlab. Do you see the repeated peaks in SSFT?

(e) Determine and draw the DSFT of the sampled signal, and compare it with SSFT. Are the DSFT and SSFT related by the warping relation described by Eq. (3.2.16)?

**3.8** For the 3D sampling lattices shown in Fig. 3.7, prove that, in order to avoid aliasing, the sampling intervals should be chosen as in Table 3.1 if the support region of the signal is a unit sphere.

**3.9** Consider a sinusoidal bar pattern described by

$$\psi(x, y) = \sin(4\pi(x - y)).$$

Assume the unit for  $x$  and  $y$  directions is meter (m). Suppose this pattern is moving at a speed of  $(v_x, v_y)$  m/s. Determine the necessary sampling rates in horizontal, vertical, and temporal directions, for the following velocities: (a)  $(v_x, v_y) = (1, 1)$ ; (b)  $(v_x, v_y) = (-1, 1)$ ; (c)  $(v_x, v_y) = (2, 1)$ .

- 3.10** Continue from the above example. Suppose the sinusoidal bar is moving at a speed of  $(v_x, v_y) = (3, 0)$ . If the signal is sampled at  $f_{s,x} = f_{s,y} = 6$  sample/m, and  $f_{s,t} = 6$  sample/s. What are the apparent spatial frequency, orientation, and the speed of the sampled bar pattern?

- 3.11** Consider a scene under constant linear motion so that

$$\psi(x, y, t) = \psi(x - v_x t, y - v_y t, 0) = \psi_0(x - v_x t, y - v_y t). \quad (3.6.3)$$

If this continuous imagery is sampled using an interlaced lattice to form even and odd fields, how is the DSFT of the even field at time  $t$  related to the CSFT of  $\psi_0(x, y, t)$ ? What about the DSFT of the odd field and that of the interlaced frame?

Hint: Refer to [1].

- 3.12** In Sec. 5.1, we considered a camera with a spatial aperture function that has a Gaussian shape. Show that indeed its spectrum is as given by Eq. (3.4.4). What should be the value of  $\beta$  for the frequency response to be 0.5 at  $f_x = f_{s,x}/2$ ,  $f_y = f_{s,y}/2$ .

- 3.13** For some practical cameras, their spatial aperture functions can be better modeled by a box function:

$$h_{p,x,y}(x, y) = \begin{cases} 1 & |x| < T_x/2, |y| < T_y/2, \\ 0 & \text{otherwise.} \end{cases} \quad (3.6.4)$$

Assume that the temporal impulse response is the same as that given in Eq. (3.4.1). Find the CSFT of the overall aperture function of the camera,  $h_p(x, y, t)$ . Plot the magnitude response, assuming  $T_x = \Delta_x = 1/720$  picture-width,  $T_y = \Delta_y = 1/480$  picture-height,  $\Delta_e = \Delta_t = 1/60$  s. Compare the spectrum with that of the camera with a Gaussian aperture (Fig. 3.9). Which one performs pre-filtering more effectively?

### 3.7 Bibliography

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## Chapter 4

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# VIDEO SAMPLING RATE CONVERSION

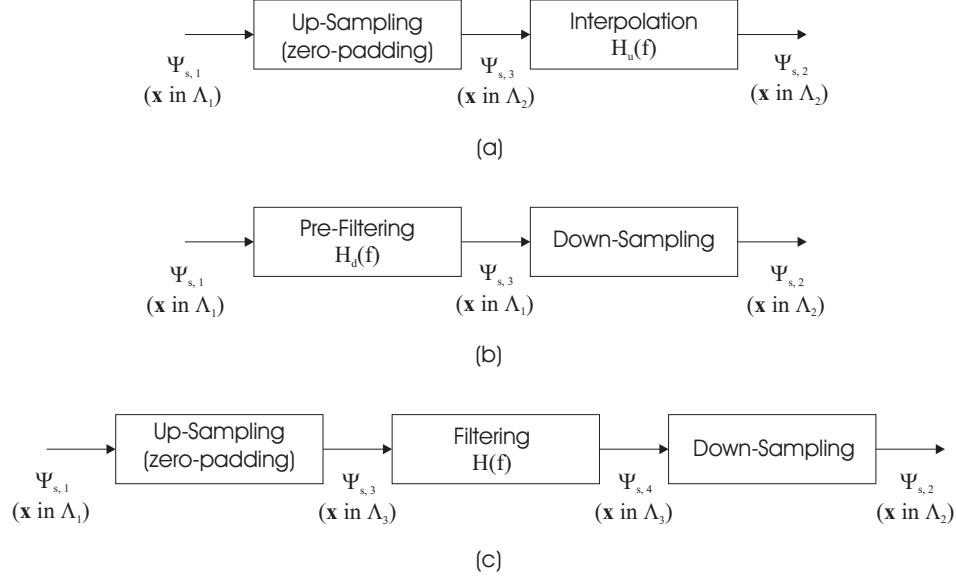
In the previous chapter, we considered the sampling problem, which is required for converting an intrinsically continuous video signal to a digital one. Once in the digital domain, we often need to convert a digital video signal from one format (in terms spatial and temporal resolution) to another, *e.g.*, converting a video recorded in the PAL format to the NTSC format, from interlaced to progressive, from a HDTV format signal to SDTV format, etc. In this chapter we consider the sampling rate conversion problem. Section 4.1 describes the general theory governing the conversion of multi-dimensional signals sampled on different lattices. Then Section 4.2 focuses on the sampling rate conversion problem of 3D video signals and, as examples, discusses some of the above video conversion problems.

### 4.1 Conversion of Signals Sampled on Different Lattices

Given a sampled signal defined over one lattice  $\Lambda_1$ , it is often desired to generate a signal defined over another lattice  $\Lambda_2$ . This constitutes the sampling rate conversion problem. The solution depends on the relation between the two lattices. For example, if  $\Lambda_1 \subset \Lambda_2$ , i.e., every point in  $\Lambda_1$  is also in  $\Lambda_2$ , then the problem is an up-conversion (or interpolation) problem. We can first fill all points in  $\Lambda_2$  but not in  $\Lambda_1$  by zeros (*i.e.*, zero-padding), and then estimate the values of these points, which can be accomplished by an interpolation filter over  $\Lambda_2$ . We will discuss the desired form of this interpolation filter later.

On the other hand, if  $\Lambda_1 \supset \Lambda_2$ , this is a down-conversion (also known as decimation) problem. We could simply take those samples from  $\Lambda_1$  that are also in  $\Lambda_2$ . However, in order to avoid aliasing in the down-sampled signal, we need to pre-filter the signal to limit its bandwidth to the Voronoi cell of  $\Lambda_2^*$ . The process for up- and down-conversion is illustrated in Figs. 4.1(a) and 4.1(b).

In general, if  $\Lambda_1$  and  $\Lambda_2$  are not contained in each other, we need to find another lattice  $\Lambda_3$  that contains both  $\Lambda_1$  and  $\Lambda_2$ . We can first up-sample the signal from  $\Lambda_1$  to  $\Lambda_3$ , and then down-sample the signal from  $\Lambda_3$  to  $\Lambda_2$ . This process is illustrated



note: change  $\Psi_{s,i}(\cdot)$  to  $\psi_{s,i}(\cdot)$ .

**Figure 4.1.** Sampling rate conversion: (a) up-conversion,  $\Lambda_1 \subset \Lambda_2$ ; (b) down-conversion,  $\Lambda_2 \subset \Lambda_1$ ; and (c) arbitrary rate conversion,  $\Lambda_3 = \Lambda_1 + \Lambda_2$ .

in Fig. 4.1(c). There the intermediate filter in  $\Lambda_3$  fulfills two purposes: first it interpolates the missing samples in  $\Lambda_1$ ; second it limits the spectrum of the signal in  $\Lambda_3$  to the Voronoi cell of  $\Lambda_2^*$ .

In the following subsections, we discuss the above scenarios separately. For notation simplicity, we will use  $\mathcal{V}_i^*$  to represent  $\mathcal{V}(\Lambda_i^*)$ , the Voronoi cell of  $\Lambda_i^*$ .

#### 4.1.1 Up-Conversion

As discussed in the introduction, if  $\Lambda_1 \subset \Lambda_2$ , then we can first transfer all the samples in  $\Lambda_1$  to  $\Lambda_2$  with the missing samples filled by zeros. This *up-sampling* process can be described by:

$$\psi_{s,3}(x) = \mathcal{U}\{\psi_{s,1}\}(x) = \begin{cases} \psi_{s,1}(x), & x \in \Lambda_1, \\ 0, & x \in \Lambda_2 \setminus \Lambda_1, \end{cases} \quad (4.1.1)$$

where  $\Lambda_2 \setminus \Lambda_1$  represents the set of points in  $\Lambda_2$  but not in  $\Lambda_1$ . This signal is now defined over the lattice  $\Lambda_2$ .

To fill the zero-padded samples, we need to apply an interpolation filter that is defined over  $\Lambda_2$ . To determine the appropriate interpolation filter, let us go back to the original continuous signal  $\psi_c(x)$  defined over the continuous space  $\mathcal{R}^K$ . If this signal is directly sampled over  $\Lambda_2$  with a generating matrix  $\mathbf{V}_2$ , then the SSFT of

the sampled signal would be

$$\Psi_{s,2}(\mathbf{f}) = d(\Lambda_2) \sum_{\mathbf{m} \in \mathcal{Z}^K} \Psi_c(\mathbf{f} - \mathbf{U}_2 \mathbf{m}), \quad (4.1.2)$$

with  $\mathbf{U}_2 = (\mathbf{V}_2^T)^{-1}$ . On the other hand, the SSFT of the signal sampled over  $\Lambda_1$  with a generating matrix  $\mathbf{V}_1$  is:

$$\Psi_{s,1}(\mathbf{f}) = d(\Lambda_1) \sum_{\mathbf{m} \in \mathcal{Z}^K} \Psi_c(\mathbf{f} - \mathbf{U}_1 \mathbf{m}), \quad (4.1.3)$$

with  $\mathbf{U}_1 = (\mathbf{V}_1^T)^{-1}$ . If  $\Lambda_1$  satisfies the alias free condition in Eq. (3.2.9), then there will be no overlapping between the alias components in  $\Lambda_1^*$ .

Because  $\Lambda_1 \subset \Lambda_2$ , according to Theorem 3.3,  $\Lambda_1^* \supset \Lambda_2^*$ , and consequently,  $\mathcal{V}_1^* \subset \mathcal{V}_2^*$ . To derive  $\Psi_{s,2}$  from  $\Psi_{s,1}$ , we just need to remove from  $\mathcal{V}_2^*$ , the portion of  $\Psi_{s,1}$  that is not in the original spectrum. Given that the original spectrum is not known exactly, the best effort is to assume that it is the same as the portion of  $\Psi_{s,1}$  in its Voronoi cell,  $\mathcal{V}_1^*$ . Therefore, the frequency response of the interpolation filter should be, in the ideal case:

$$H_u(\mathbf{f}) = \begin{cases} d(\Lambda_2)/d(\Lambda_1), & \mathbf{f} \in \mathcal{V}_1^*, \\ 0, & \mathbf{f} \in \mathcal{V}_2^* \setminus \mathcal{V}_1^*. \end{cases} \quad (4.1.4)$$

In the above equation,  $H_u(\mathbf{f})$  represents the SSFT of a sampled space filter defined over  $\Lambda_2$ .

**Example 4.1:** Consider the up-conversion from the sampling lattice  $\Lambda_1$  to  $\Lambda_2$  in Fig. 4.2. Their respective generating matrices are

$$\mathbf{V}_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{V}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (4.1.5)$$

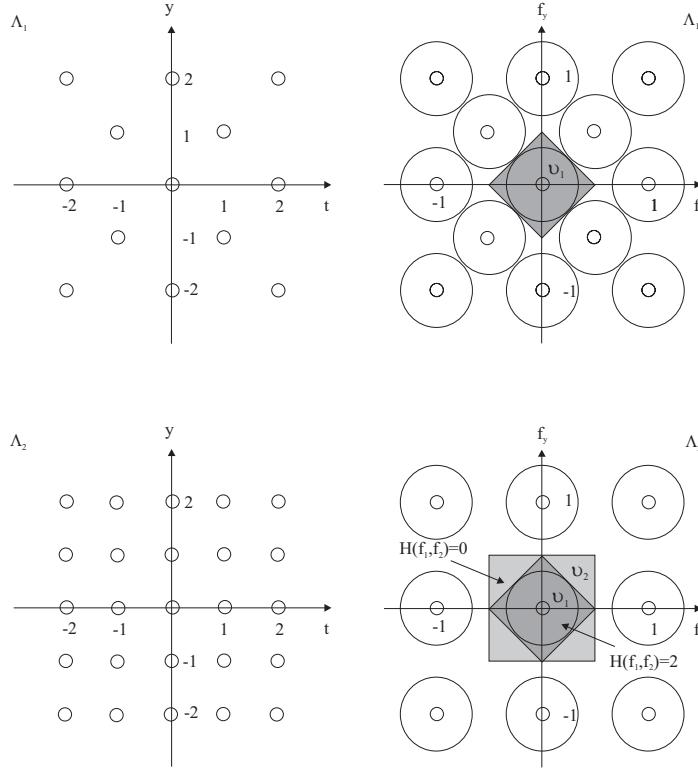
Using Eq. (3.1.7), the generating matrices for the reciprocal lattices are

$$\mathbf{U}_1 = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}; \quad \mathbf{U}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (4.1.6)$$

The reciprocal lattices and their Voronoi cells are also shown in Fig. 4.2. Assuming the spectrum of the original continuous signal has a circular support, we also draw in the figure the replicated spectra corresponding to different sampling patterns. We can see that, indeed,  $\mathcal{V}_2^* \supset \mathcal{V}_1^*$ . The ideal interpolation filter is

$$H_u(\mathbf{f}) = \begin{cases} d(\Lambda_2)/d(\Lambda_1) = 2, & \mathbf{f} \in \mathcal{V}_1^*, \\ 0, & \mathbf{f} \in \mathcal{V}_2^* \setminus \mathcal{V}_1^*. \end{cases}, \quad (4.1.7)$$

as illustrated on lattice  $\Lambda_2^*$  in Fig. 4.2. The filter serves to eliminate alias components in  $\Lambda_1^*$  that should not appear in  $\Lambda_2^*$ .



note: change  $\mathcal{V}_1, \mathcal{V}_2$  on the right to  $\mathcal{V}_1^*, \mathcal{V}_2^*$ .

**Figure 4.2.** An example of up-conversion, from lattice  $\Lambda_1$  to lattice  $\Lambda_2$ . The ideal interpolation filter is illustrated in  $\Lambda_2^*$ . See Example 4.1.

#### 4.1.2 Down-Conversion

If  $\Lambda_1 \supset \Lambda_2$ , then we can obtain  $\psi_{s,2}$  easily by retaining the samples in  $\Lambda_1$  that are also in  $\Lambda_2$ , and discarding all other samples. This down-sampling process is described by:

$$\psi_{s,2}(\mathbf{x}) = \mathcal{D}\{\psi_{s,1}\}(\mathbf{x}) = \psi_{s,1}(\mathbf{x}), \text{ for } \mathbf{x} \in \Lambda_2. \quad (4.1.8)$$

The signal so obtained would be exactly the same as the one resulting from sampling the original continuous signal over  $\Lambda_2$  if the same pre-filter is used. Let us assume that the support region of the original continuous signal or the one after pre-filter when obtaining  $\psi_{s,1}$  equal to or smaller than  $\mathcal{V}_1^*$ , so that there is no aliasing in  $\psi_{s,1}$ . Then in general, there will be aliasing in  $\psi_{s,2}$  as  $\mathcal{V}_2^*$  is smaller. To avoid aliasing, one needs to pre-filter  $\psi_{s,1}$  to remove the frequency content that is in  $\mathcal{V}_1^*$  but not in

$\mathcal{V}_2^*$ . Therefore, the appropriate pre-filter over  $\Lambda_1$  is, in terms of the SSFT over  $\Lambda_1$ :

$$H_d(\mathbf{f}) = \begin{cases} d(\Lambda_2)/d(\Lambda_1), & \mathbf{f} \in \mathcal{V}(\Lambda_2^*), \\ 0, & \mathbf{f} \in \mathcal{V}_1^* \setminus \mathcal{V}_2^*. \end{cases} \quad (4.1.9)$$

**Example 4.2:** Consider the down-conversion example given in Fig. 4.3. The associated generating matrices are:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{V}_2 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}; \quad (4.1.10)$$

$$\mathbf{U}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{U}_2 = \begin{bmatrix} 1/2 & 0 \\ -1/4 & 1/2 \end{bmatrix}. \quad (4.1.11)$$

The ideal pre-filter is

$$H_d(\mathbf{f}) = \begin{cases} 1/4, & \mathbf{f} \in \mathcal{V}(\Lambda_2^*), \\ 0, & \mathbf{f} \in \mathcal{V}_1^* \setminus \mathcal{V}_2^*, \end{cases} \quad (4.1.12)$$

which is illustrated in  $\Lambda_1^*$  in Fig. 4.3. Without pre-filtering, the original circular spectrum would have caused aliasing in  $\Lambda_2^*$ . By applying the pre-filter in  $\Lambda_1^*$ , only the portion of the spectrum that is contained in  $\mathcal{V}_2^*$  is retained, so that no aliasing will occur. Note that if there exists aliasing in  $\psi_{s,1}$ , it will be retained in  $\psi_{s,2}$ . On the other hand, if the original signal has a support region that is within  $\mathcal{V}_2^*$ , then there is no need for using the pre-filter, when obtaining the original sampled signal over  $\Lambda_1$ , and when down sampling  $\psi_{s,1}$  to obtain  $\psi_{s,2}$ .

### 4.1.3 Conversion between Arbitrary Lattices

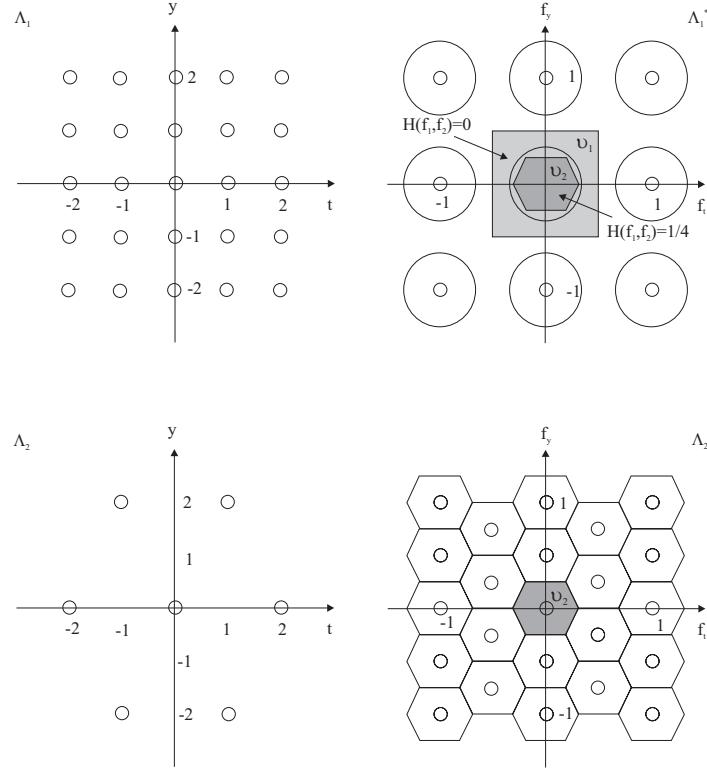
As explained in the beginning of this section, in the general case when  $\Lambda_1$  and  $\Lambda_2$  are not subsets of one another, we need to introduce another lattice  $\Lambda_3$  so that both  $\Lambda_1$  and  $\Lambda_2$  are subsets of  $\Lambda_3$ . Obviously, to minimize the sampling density in  $\Lambda_3$ , we would like to find the smallest lattice that will satisfy the above criterion. Once  $\Lambda_3$  is determined, the conversion from  $\Lambda_1$  to  $\Lambda_2$  can be accomplished by concatenating the up-conversion and down-conversion steps discussed in the previous two subsections. The interpolation filter and the pre-filter in these two steps are, respectively,

$$H_u(\mathbf{f}) = \begin{cases} d(\Lambda_3)/d(\Lambda_1), & \mathbf{f} \in \mathcal{V}_1^*, \\ 0, & \mathbf{f} \in \mathcal{V}_3^* \setminus \mathcal{V}_1^*, \end{cases} \quad (4.1.13)$$

$$H_d(\mathbf{f}) = \begin{cases} d(\Lambda_2)/d(\Lambda_3), & \mathbf{f} \in \mathcal{V}_2^*, \\ 0, & \mathbf{f} \in \mathcal{V}_3^* \setminus \mathcal{V}_2^*. \end{cases} \quad (4.1.14)$$

Obviously, these two filters, both defined over  $\Lambda_3$ , can be merged into one filter, with the following frequency response in terms of SSFT:

$$H(\mathbf{f}) = \begin{cases} d(\Lambda_2)/d(\Lambda_1), & \mathbf{f} \in (\mathcal{V}_1^* \cap \mathcal{V}_2^*), \\ 0, & \mathbf{f} \in \mathcal{V}_3^* \setminus (\mathcal{V}_1^* \cap \mathcal{V}_2^*). \end{cases} \quad (4.1.15)$$



note: change  $\mathcal{V}_1, \mathcal{V}_2$  on the right to  $\mathcal{V}_1^*, \mathcal{V}_2^*$ .

**Figure 4.3.** An example of down-conversion, from lattice  $\Lambda_1$  to  $\Lambda_2$ . The ideal pre-filter is illustrated in  $\lambda_1^*$  in the figure. See example 4.2.

The above discussion assumed that  $\Lambda_3$  can somehow be determined. Based on Theorem 3.2,  $\Lambda_3$  can be determined by the sum of  $\Lambda_1$  and  $\Lambda_2$ , i.e.,  $\Lambda_3 = \Lambda_1 + \Lambda_2$ , if  $\mathbf{V}_1^{-1}\mathbf{V}_2$  contains only rational numbers. Recall that in the 1D case, to enable conversion between two sampling rates, a necessary condition is that the ratio between the two sampling frequencies,  $f_{s,1}$  and  $f_{s,2}$ , or equivalently, the two sampling intervals,  $\Delta_1$  and  $\Delta_2$ , must be a rational number, i.e.,  $r = f_{s,2}/f_{s,1} = \Delta_1/\Delta_2 = p/q$ , where  $p$  and  $q$  are both integers. In this case, the sampling rate conversion by a factor of  $r$  can be accomplished by first up-sampling by a factor of  $p$  and then down-sampling by a factor of  $q$ . Obviously, this is a special case of the more general procedure described here. Specifically, the above two steps correspond to the up-conversion from  $\Lambda_1$  to  $\Lambda_3$ , and down-conversion from  $\Lambda_3$  to  $\Lambda_2$ , respectively. When the condition of Theorem 3.2 is not satisfied, one can first determine the union of  $\Lambda_1$  and  $\Lambda_2$ , which may not be a legitimate lattice. One then needs to extend this set to a lattice by adding additional points, usually by inspection.

**Example 4.3:** Consider the conversion between two sampling lattices  $\Lambda_1$  and  $\Lambda_2$  with generating matrices:

$$\mathbf{V}_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{V}_2 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}. \quad (4.1.16)$$

The generating matrices of their corresponding reciprocal lattices are:

$$\mathbf{U}_1 = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}, \quad \mathbf{U}_2 = \begin{bmatrix} 1/2 & 0 \\ -1/4 & 1/2 \end{bmatrix}. \quad (4.1.17)$$

First, we sketch the input and output sampling lattices  $\Lambda_i$  and reciprocal lattices  $\Lambda_i^*$ ,  $i = 1, 2$ , based on the above generating matrices. These are illustrated in Fig. 4.4. In this case,  $\Lambda_1$  and  $\Lambda_2$  are not subset of one another. Therefore, we need to determine a lattice  $\Lambda_3$  that is the smallest lattice that contains both  $\Lambda_1$  and  $\Lambda_2$ . Because  $\mathbf{V}_1^{-1}\mathbf{V}_2$  includes only rational numbers,  $\Lambda_3 = \Lambda_1 + \Lambda_2$ . In general, it is hard to determine the generating matrix for  $\Lambda_3$  analytically. However, by visual inspection, one can see that  $\Lambda_3$  in this case must be the square lattice sketched in Fig. 4.4. For this lattice, one obvious generating matrix is the identity matrix, i.e.,

$$\mathbf{V}_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (4.1.18)$$

The reciprocal lattice is also a square lattice with the generating matrix being the identity matrix, i.e.,  $\mathbf{U}_3 = (\mathbf{V}_3^T)^{-1} = \mathbf{V}_3$ .

From the sketched reciprocal lattices,  $\Lambda_i^*$ ,  $i = 1, 2, 3$ , we can determine their Voronoi cells  $\mathcal{V}_i^*$  individually, which are shown in Fig. 4.4. In this case,  $\mathcal{V}_1^* \supset \mathcal{V}_2^*$  so that  $\mathcal{V}_1^* \cap \mathcal{V}_2^* = \mathcal{V}_2^*$ . Therefore, the intermediate filter should be:

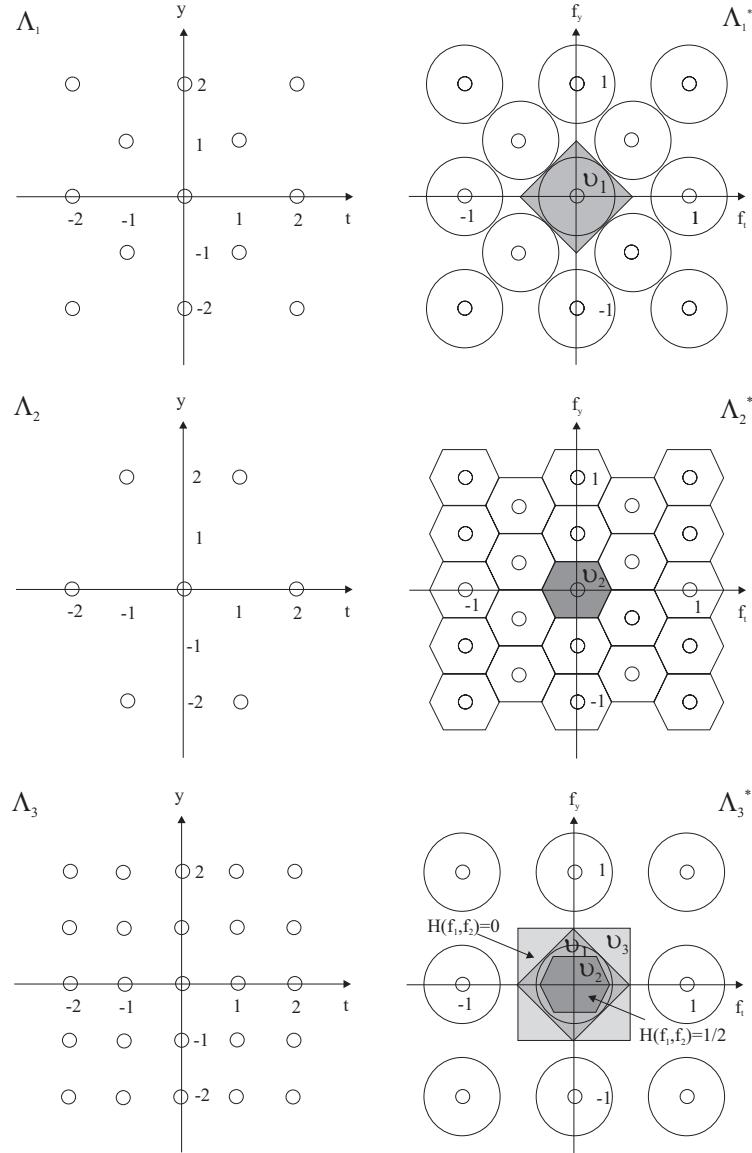
$$H(\mathbf{f}) = \begin{cases} \frac{d(\Lambda_2)}{d(\Lambda_1)} = \frac{1}{2}, & \mathbf{f} \in \mathcal{V}_2^*, \\ 0, & \mathbf{f} \in \mathcal{V}_3^* \setminus \mathcal{V}_2^*, \end{cases}$$

as illustrated on top of  $\Lambda_3^*$  in Fig. 4.4.

#### 4.1.4 Filter Implementation, Design, and Other Interpolation Approaches

In the previous discussion, we have only specified the desired filter in terms of its SSFT. The equivalent filter in the spatial domain  $h(\mathbf{x}_n)$ ,  $\mathbf{x}_n \in \Lambda_3$  can be determined by the inverse SSFT. Theoretically, to implement the filtering operation, we need to first up-sample the input signal from  $\Lambda_1$  to  $\Lambda_3$ , filter it in  $\Lambda_3$ , and then down-sample it from  $\Lambda_3$  to  $\Lambda_2$ , as shown in Fig. 4.1(c). The filtering operation in  $\Lambda_3$  is:

$$\psi_{s,4}(\mathbf{x}_n) = \psi_{s,3}(\mathbf{x}_n) * h(\mathbf{x}_n) = \sum_{\mathbf{y}_m \in \Lambda_3} \mathcal{U}\{\psi_{s,1}\}(\mathbf{y}_m)h(\mathbf{x}_n - \mathbf{y}_m), \quad \mathbf{x}_n \in \Lambda_3. \quad (4.1.19)$$



note: change  $\mathcal{V}_1, \mathcal{V}_2$  on the right to  $\mathcal{V}_1^*, \mathcal{V}_2^*$ .

**Figure 4.4.** Conversion between two non-inclusive lattices,  $\Lambda_1$  and  $\Lambda_2$ .  $\Lambda_3$  is the intermediate lattice that includes both  $\Lambda_1$  and  $\Lambda_2$ . The ideal filter is illustrated in  $\Lambda_3^*$ . See example 4.3.

In reality, the filtering output does not have to be calculated for all the samples in  $\Lambda_3$ , as only the samples in  $\Lambda_2$  are desired in the output. In addition, because the up-sampled input signals are non-zero only in  $\Lambda_1$ , the sum in the above equation involves only input samples in  $\Lambda_1$ . Overall, the output signal over  $\Lambda_2$  can be directly obtained from the input over  $\Lambda_1$  by

$$\psi_{s,2}(\mathbf{x}_n) = \mathcal{D}\{\psi_{s,4}\}(\mathbf{x}_n) = \sum_{\mathbf{y}_m \in \Lambda_1} \psi_{s,1}(\mathbf{y}_m) h(\mathbf{x}_n - \mathbf{y}_m), \mathbf{x}_n \in \Lambda_2. \quad (4.1.20)$$

This means that a sample  $\mathbf{x}_n \in \Lambda_2$  is estimated by a weighted average of the samples  $\mathbf{y}_m \in \Lambda_1$ , with weights equal to  $h(\mathbf{x}_n - \mathbf{y}_m)$ .

In previous discussions, we have simply given the desired frequency responses of the pre-filter and reconstruction filter, without considering how to design the filters. From the specified frequency response  $H(\mathbf{f})$ ,  $\mathbf{f} \in \mathcal{V}_3^*$ , in Eq. (4.1.15), we can perform the inverse SSFT to obtain the ideal filter  $h(\mathbf{x}_n)$ ,  $\mathbf{x}_n \in \Lambda_3$ . However, this filter will have an infinite extent, because the very sharp transition in  $H(\mathbf{f})$ . If we simply truncate the filter, it will cause the notorious ringing artifacts in the filtered signal. Therefore, one needs to carefully modify the desired filter response by applying an appropriate windowing function, so that the corresponding filter has a finite extent. One can also use various optimization techniques to directly design finite extent filters so that their frequency responses are as close to the desired ones as possible. Filter design techniques are beyond the scope of this book. Interested readers are referred to [4], which has a good coverage on multi-dimensional filter design. Most of the digital filter design techniques are developed assuming the frequency response is specified in terms of the discrete frequency. To solve the problem here, one needs to warp the desired frequency response in terms of the continuous frequency to the discrete frequency, as specified in Eq. (3.2.16). Based on this warped frequency response, one can apply the digital filter design technique to derive the desired filter over the discrete space. One can implement the convolution operation in the abstract discrete space, or in the sampled space directly.

As will be seen later, for video signal sampling and sampling rate conversion, simple filters that are far from the ideal frequency response are often used, in order to reduce the computational complexity. Example filters for several important conversion problems are described in Sec. 4.2.

Until now, we have derived the desired interpolation filters based on the frequency domain consideration. One can also develop interpolation methods based on sample domain requirements. For the interpolation of a continuous signal from samples, the problem is to construct a continuous surface that passes through the given sample values. For the up-conversion task, the problem is to estimate some missing samples from surrounding known samples. The general approach is to use a polynomial function with a finite order to approximate the continuous signal over each small region, and determine the polynomial coefficients based on the specified sample values in that region using a least squares fitting procedure. Once the polynomial is determined, one can either reconstruct the entire continuous surface, or derive the missing samples. To maintain the continuity of the surfaces derived

for adjacent regions, one can use spline functions which are special classes of the piecewise polynomials. The most popular one being the cubic-spline approach that has been described for 2D signal interpolation [6].

## 4.2 Sampling Rate Conversion of Video Signals

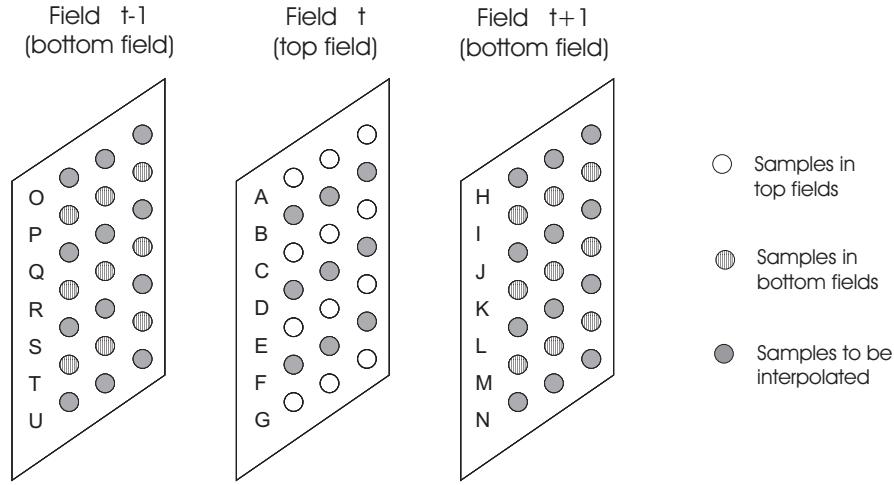
Sampling rate conversion is often required in video systems. For example, display of a PAL signal on a NTSC TV system requires the conversion of an interlaced signal with sampling rate of  $f_{s,t}=50$  field/s,  $f_{s,y}=625$  line/picture-height into an interlaced signal with  $f_{s,t}=60$  field/s,  $f_{s,y}=525$  line/picture-height. To convert a movie on film for NTSC TV broadcasting/display requires the conversion from  $f_{s,t}=24$  frame/s to  $f_{s,t}=60$  field/s. Another interesting problem is to convert an interlaced raster into a progressive one, known as *deinterlacing*, which is required to display a made-for-TV material on a computer screen which uses progressive display. Some advanced TV systems already have a built-in deinterlacing function so that interlaced material is displayed in a progressive mode for improved visual quality.

As described in Sec. 4.1, the general procedure for solving the sampling rate conversion problem is as follows: i) determine the equivalent sampling lattices of the input and output signals and an intermediate lattice that covers the samples in both input and output signals; ii) Based on the Voronoi cells of the three lattices, determine the desired filter frequency response; iii) Design a filter that approximates the desired response. In reality, because of the very high data rate of the video signal, one cannot afford to use a filter with many coefficients, especially in the temporal direction. To reduce the computational complexity, one can also decompose a spatio-temporal conversion problem into spatial conversion followed by temporal conversion, or vice versa.

In the following, we describe the solutions for deinterlacing and for conversion between PAL and NTSC in more detail. We will consider operations in the vertical and temporal directions only, as the sampling rate conversion in the horizontal direction can be done in a separate step with conventional 1D techniques. The book by Haskell, Puri and Netravali [5] has an extensive discussion of various video conversion problems and their practical solutions. Some of these problems are considered in the Problem section.

### 4.2.1 Deinterlacing

The problem of deinterlacing is to fill in the skipped lines in each field, as illustrated in Fig. 4.5. Consider the interlaced NTSC signal, with field rate  $f_{s,t}=60$  field/s, and line rate  $f_{s,y}=525$  line/picture-height. The sampling intervals are  $\Delta_t=1/60$  s,  $\Delta_y=1/525$  picture-height. The corresponding sampling lattice and its reciprocal are illustrated in Fig. 4.6 (a). The sampling lattice for the deinterlaced signal is a square lattice, shown in Fig. 4.6 (b). As before, we have scaled the vertical and temporal axes so that  $\Delta_y$  and  $\Delta_t$  are represented with the same length. Similarly,  $f_{s,y}$  and  $f_{s,t}$  are assigned the same length in the frequency domain.



**Figure 4.5.** The deinterlacing process. Fields  $t$  and  $t + 1$  form one interlaced frame.

Obviously, the generating matrices for these two lattices and their reciprocal lattices are

$$\mathbf{V}_1 = \begin{bmatrix} 2\Delta_t & \Delta_t \\ 0 & \Delta_y \end{bmatrix}, \quad \mathbf{U}_1 = \begin{bmatrix} 1/2\Delta_t & 0 \\ -1/2\Delta_y & 1/\Delta_y \end{bmatrix}, \quad (4.2.1)$$

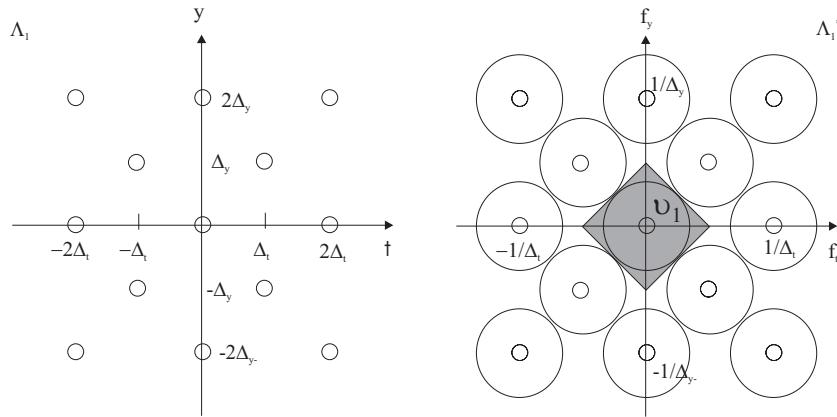
$$\mathbf{V}_2 = \begin{bmatrix} \Delta_t & 0 \\ 0 & \Delta_y \end{bmatrix}, \quad \mathbf{U}_2 = \begin{bmatrix} 1/\Delta_t & 0 \\ 0 & 1/\Delta_y \end{bmatrix}. \quad (4.2.2)$$

In this case,  $\Lambda_2 \supset \Lambda_1$ , and the problem is an up-conversion problem, and the ideal interpolation filter is:

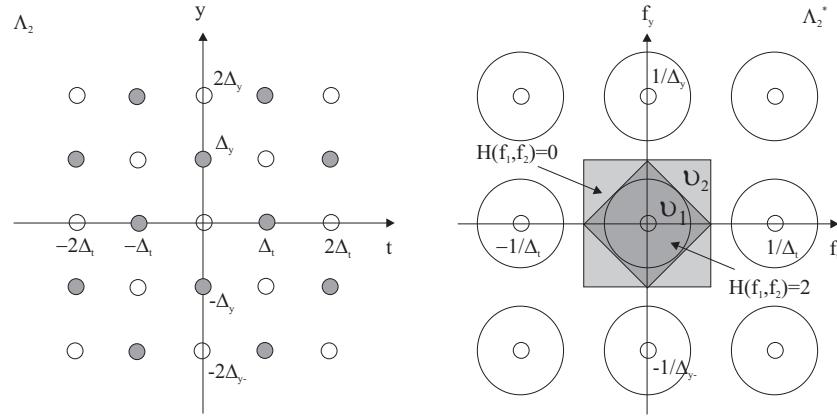
$$H(\mathbf{f}) = \begin{cases} \frac{d(\Lambda_2)}{d(\Lambda_1)} = 2, & \mathbf{f} \in \mathcal{V}_1^*, \\ 0, & \mathbf{f} \in \mathcal{V}_2^* \setminus \mathcal{V}_1^*. \end{cases} \quad (4.2.3)$$

The Voronoi cells  $\mathcal{V}_1^*$  and  $\mathcal{V}_2^*$  are illustrated in Figs. 4.6(a) and 4.6(b). The magnitude of the frequency response of this filter is shown in Fig. 4.7(a). We can see that this is a low-pass filter with a diamond-shaped pass-band. Notice that this filter is not separable (*i.e.* cannot be decomposed into a product of a temporal filter and a vertical filter), so one has to use two dimensional filter design technique. To simplify the design problem, one can first design a separable low-pass filter with a square-shaped pass-band, and then rotate the resulting filter.

In practice, many simpler filters have been proposed for deinterlacing. One option is to use vertical interpolation within the same field, which is a 1D factor-of-two up-conversion problem. The ideal vertical filter is a half-band low-pass filter [7]. However, this filter requires infinite length and is not realizable. Much shorter filters are used in practice. The simplest one is *line averaging*, which estimates a



(a) The sampling lattice and its reciprocal corresponding to interlaced scan



(b) The sampling lattice and its reciprocal after deinterlacing

**Figure 4.6.** The sampling lattices and their reciprocal lattices before and after deinterlacing. Open circles represent original samples, filled circles the samples to be interpolated.

missing line by the average of the line above and below. For example, in Fig. 4.5, for field  $t$ ,  $D = (C + E)/2$ . The equivalent filter in  $\Lambda_2$  is

$$h(y, t) = \begin{cases} 1, & (y, t) = (0, 0), \\ 1/2, & (y, t) = (\Delta_y, 0), (-\Delta_y, 0), \\ 0, & \text{others.} \end{cases} \quad (4.2.4)$$

The frequency response of this filter (i.e., the SSFT of  $h(y, t)$  over  $\Lambda_2$ ) is, using

(3.2.3),

$$H(f_y, f_t) = 1 + \frac{1}{2} (e^{j2\pi\Delta_y f_y} + e^{-j2\pi\Delta_y f_y}) = 1 + \cos(2\pi\Delta_y f_y). \quad (4.2.5)$$

This frequency response is illustrated in Fig. 4.7(b). We can see that it deviates from the desired response shown in Fig. 4.7(a) significantly. Because no temporal filtering is applied, it has an all-pass characteristics along the temporal frequency axis. Along the vertical frequency axis, it is far from the ideal half-band low-pass filter.

To improve the performance, longer vertical interpolation filters that have frequency responses closer to the ideal half-band low-pass filter can be used. For example, in Fig. 4.5, a satisfactory interpolation method for lines in field  $t$  is  $D = (A + 7C + 7E + G)/16$ . The equivalent interpolation filter over the deinterlaced grid (the vertical axis in  $\Lambda_2$ ) is

$$h = [1, 0, 7, 16, 7, 0, 1]/16. \quad (4.2.6)$$

Following the same approach as before, we can easily show that the frequency response of this filter is

$$H(f_y, f_t) = 1 + \frac{7}{8} \cos(2\pi\Delta_y f_y) - \frac{1}{8} \cos(6\pi\Delta_y f_y), \quad (4.2.7)$$

which is shown in Fig. 4.7(c).

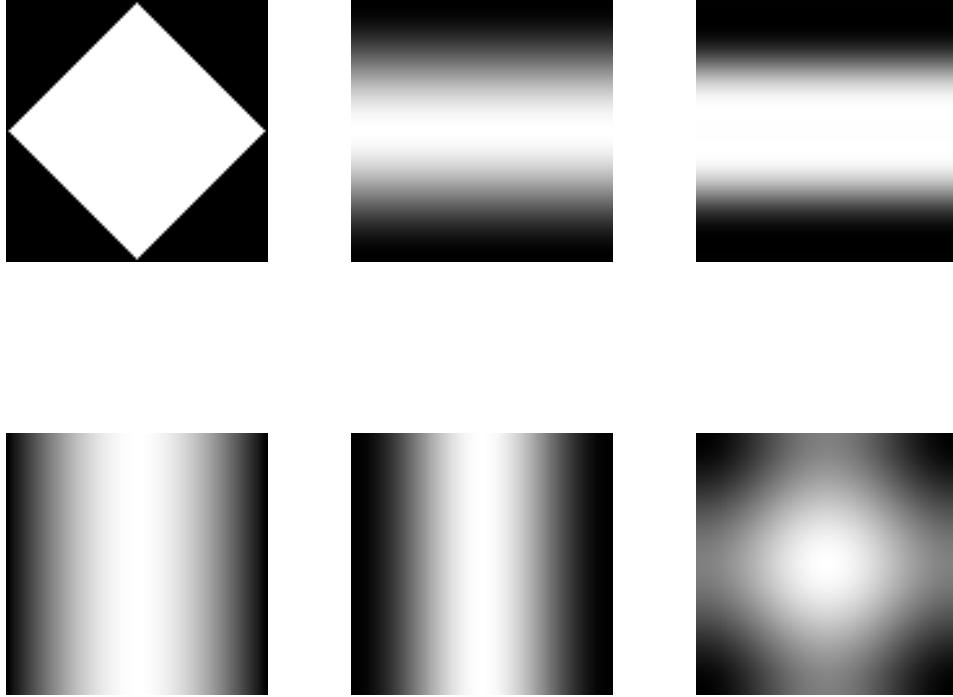
The previous two approaches use vertical interpolation only. An alternative is to use temporal interpolation. Notice that for every missing line in a field, there is a corresponding line in the other field of the same frame (cf. Fig. 4.5). A simple temporal interpolation scheme is to copy this corresponding line. That is,  $D = K, J = C$ . This method is known as *field merging*, because each deinterlaced frame is obtained by merging two fields. The equivalent filter used for deinterlacing the top field (assuming it appears before the bottom field) is:

$$h(y, t) = \begin{cases} 1, & (y, t) = (0, 0), (0, -\Delta_t), \\ 0, & \text{others.} \end{cases} \quad (4.2.8)$$

The filter for the bottom field is a time reversed version of the above filter. The frequency response of the filter in (4.2.8) is

$$H(f_y, f_t) = 1 + e^{-j2\pi\Delta_t f_t}. \quad (4.2.9)$$

Because the filter is asymmetric in time, it has a complex frequency response. In addition, the temporal interpolation direction is opposite with the two fields in a frame. These may sometimes yield visual artifacts for some special patterns. The magnitude response is illustrated in Fig. 4.7(d), which again deviates from the desired response significantly. Because filtering is applied only in the temporal direction, it is all-pass in the vertical direction.



**Figure 4.7.** Frequency responses of several deinterlacing filters. From left to right, and top to bottom: (a) The ideal filter (Eq. 4.2.3); (b) Line-averaging (Eq. 4.2.5); (c) 4-point-vertical interpolation (Eq. 4.2.7); (d) field-merging (Eq. 4.2.9); (e) field-averaging (Eq. 4.2.11); (f) Line-field averaging (Eq. 4.2.13). In each figure, the vertical axis represents the vertical frequency, and the horizontal axis the temporal frequency. The frequency range is  $|f_y| \leq f_{s,y}/2, |f_t| \leq f_{s,t}/2$ .

To improve the performance, a symmetric filter can be used, for example, averaging the corresponding lines in the previous and following fields to obtain a missing line in the current field. For example, for field  $t$ ,  $D = (K + R)/2$ . We refer to this method as *field averaging*. The equivalent interpolation filter is:

$$h(y, t) = \begin{cases} 1, & (y, t) = (0, 0), \\ 1/2, & (y, t) = (0, \Delta_t), (0, -\Delta_t), \\ 0, & \text{others,} \end{cases} \quad (4.2.10)$$

or

$$H(f_y, f_t) = 1 + \cos(2\pi\Delta_t f_t). \quad (4.2.11)$$

The frequency response is shown in Fig. 4.7(e). Although this filter has a better frequency response than the previous one, it involves 3 fields for interpolating any

field. This requires the storage of two frames, which represents a non-trivial increase in memory and delay requirement compared to the previous field merging approach.

To achieve a compromise between spatial and temporal artifacts, a better approach is to use both vertical and temporal interpolation. For example, one can interpolate a missing pixel by taking the average of the pixels above and below in the same field, and the pixels in the previous and following fields. In Fig. 4.5, this corresponds to  $D = (C + E + K + R)/4$ . This method is referred to as *line and field averaging*. The equivalent filter on the deinterlaced lattice is:

$$h(y, t) = \begin{cases} 1, & (y, t) = (0, 0), \\ 1/4, & (y, t) = (\Delta_y, 0), (-\Delta_y, 0), (0, \Delta_t), (0, -\Delta_t), \\ 0, & \text{others.} \end{cases} \quad (4.2.12)$$

The frequency response is

$$H(f_y, f_t) = 1 + \frac{1}{2} (\cos(2\pi f_y \Delta_y) + \cos(2\pi f_t \Delta_t)), \quad (4.2.13)$$

which is illustrated in Fig. 4.7(f). We can see that it is closer to the desired one than all previous filters. As with field averaging, this method requires the storage of two frames. An approach that requires only one frame storage is discussed in Problem 4.1.

When the imaged scene is stationary between adjacent fields, the missing even lines in an odd field should be exactly the same as the corresponding even lines in the previous and following even field. Therefore, temporal interpolation will yield perfect estimates. On the other hand, when there is motion in the scene, the corresponding lines in adjacent fields may not correspond to the same object location and temporal interpolation will yield unacceptable artifacts. Imagine a vertical grating pattern moving horizontally so that the grating pattern captured in two adjacent fields are shifted by half of the grating interval. Field averaging will yield serrated vertical gratings. The methods that use both spatial and temporal averaging will produce less severe artifacts, but the resulting video may still be unacceptable for certain applications. For better quality, motion compensated interpolation should be used, which is discussed in Sec. 4.2.3.

### 4.2.2 Conversion between NTSC and PAL Signals

In this section we consider the conversion from PAL to NTSC format signals, both are interlaced. The conversion from NTSC to PAL will follow the same principle, and is considered in Problem 4.4. Recall that the line and field rates of the PAL signal are  $f_{y,1}=625$  line/picture-height and  $f_{t,1}=50$  field/s, and those for the NTSC signal are  $f_{y,2}=525$  line/picture-height and  $f_{t,2}=60$  field/s. The sampling lattices for the PAL ( $\Lambda_1$ ) and NTSC ( $\Lambda_2$ ) formats are sketched in Figs. 4.8(a) and 4.8(b), respectively, where  $\Delta_t=1/60$  s,  $\Delta_y=1/525$  picture-height. Based on the points on each sampling lattice, one can easily determine its generating matrix, and consequently determine the  $\mathbf{V}$  and  $\mathbf{U}$  matrices. From the  $\mathbf{U}$  matrix, one can then determine the

reciprocal lattice and its Voronoi cell. All these are given in Fig. 4.8. *In drawing the lattices, we have scaled the temporal and vertical axes by equating  $\Delta_t$  and  $\Delta_y$  in the spatial-temporal domain, and equating  $1/\Delta_t$  and  $1/\Delta_y$  in frequency domain.*

The conversion between PAL and NTSC is difficult because the vertical and temporal sampling rates of these two lattices are not integer multiples of each other. First we derive the ideal solution using the technique described in Sec. 4.1. Towards this purpose, we need to determine a lattice  $\Lambda_3$  that contains both  $\Lambda_1$  and  $\Lambda_2$ . By observing that the minimal common divisor of  $f_{y,1} = 625$  and  $f_{y,2} = 525$  is  $f_{y,3} = 13125$ , and that for  $f_{t,1} = 50$  and  $f_{t,2} = 60$  is  $f_{t,3} = 300$ , we can see that a rectangular lattice with line rate of  $f_{y,3}$  and frame rate of  $f_{t,3}$  is the desired intermediate lattice, which is the lattice  $\Lambda_3$  in Fig. 4.8(c). From the figure, it is easy to see that the desired frequency response of the filter over  $\Lambda_3$  is

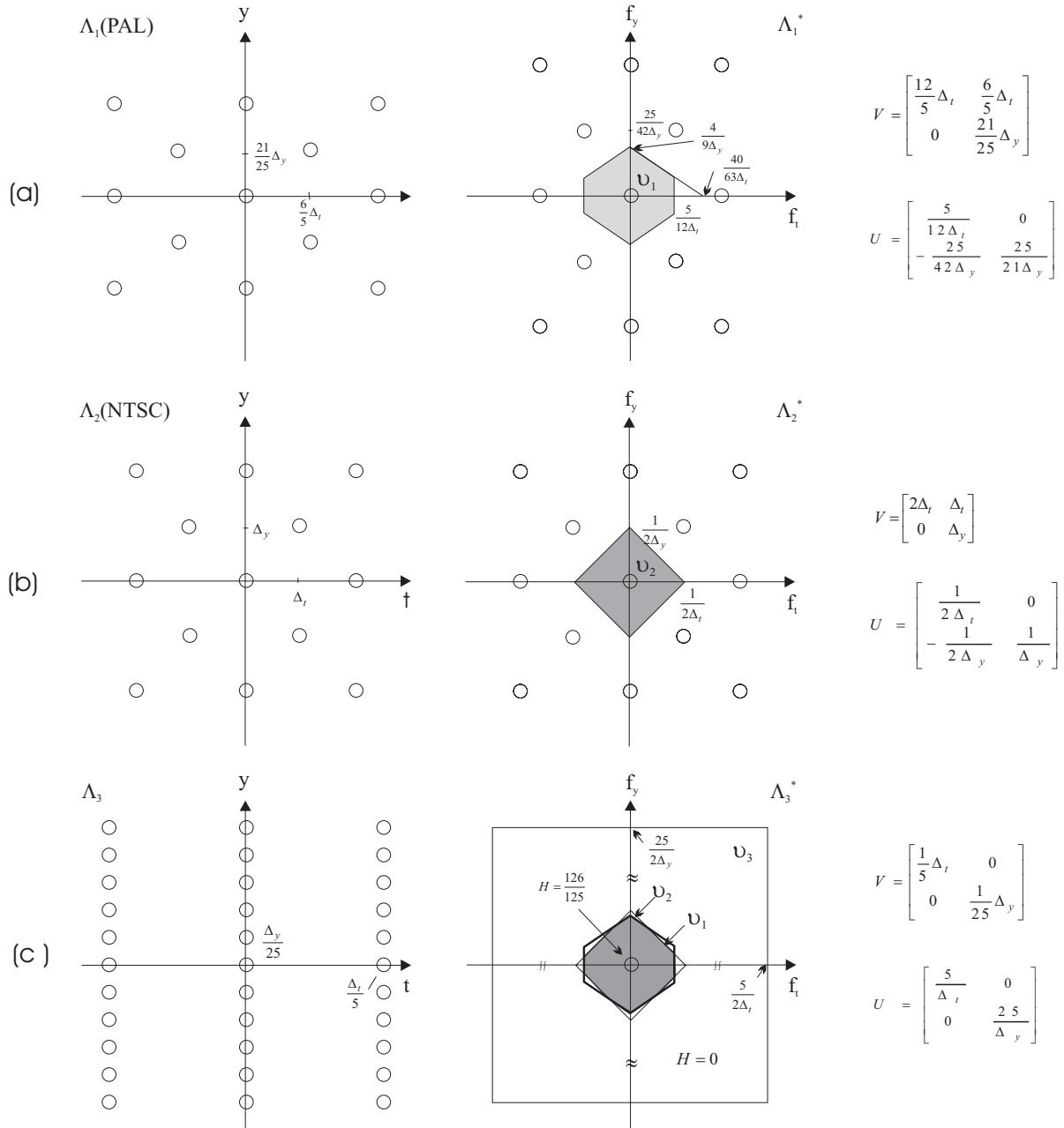
$$H(f_y, f_t) = \begin{cases} \frac{d(\Lambda_2)}{d(\Lambda_1)} = 126/125, & (f_y, f_t) \in \mathcal{V}_1^* \cap \mathcal{V}_2^*, \\ 0, & (f_y, f_t) \in \mathcal{V}_3^* \setminus (\mathcal{V}_1^* \cap \mathcal{V}_2^*). \end{cases} \quad (4.2.14)$$

This ideal conversion filter is illustrated on  $\Lambda_3^*$  in Fig. 4.8(c), and also in Fig. 4.11(a). Obviously, because of the very complicated shape of  $\mathcal{V}_1^* \cap \mathcal{V}_2^*$ , the above filter is not easy to design.

The above solution calls for directly converting a signal from  $\Lambda_1$  to  $\Lambda_3$  and then back to  $\Lambda_2$ , as shown in Fig. 4.9(a). In practice, the problem is more often solved in four steps: i) deinterlacing each field in PAL video so that each field contains 625 lines, ii) line rate down conversion from 625 to 525 lines within each deinterlaced field, iii) field rate up-conversion from 50 to 60 Hz, and finally iv) down-sample every field to  $525/2$  lines. This sequential implementation is illustrated in Fig. 4.9(b).

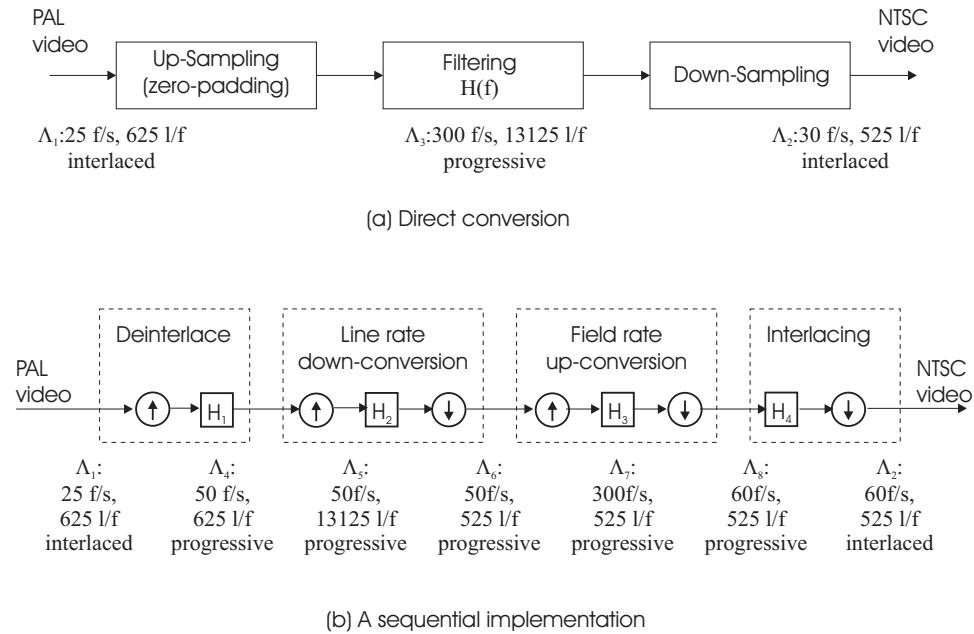
The general deinterlacing problem has been discussed in Sec. 4.2.1, and the solution for the current specific set-up is illustrated in Fig. 4.10(a). For the vertical conversion from 625 to 525 lines, ideally one should first convert the line rate to 13125 lines, filtering the signal in that high resolution, and then down-sample to 525 lines, as indicated in Fig. 4.9(b). The ideal solution is illustrated in Fig. 4.10(a-c). For the temporal conversion from 50 to 60 frames, one needs to first convert the frame rate to 300 frame/s, filtering the signal in that high resolution, and then down-sample to 50 frames, as indicated in Fig. 4.9(b). The ideal solution is illustrated in Fig. 4.10(c-e). Finally, to create an interlaced signal from the progressive frames, one can simply skip every other lines in each frame. But to avoid aliasing, a pre-filter is needed, and the solution is given in Fig. 4.10(e). The equivalent overall filter response is shown in Fig. 4.10(f). Compared to the ideal filter shown in Fig. 4.8(a), this filter has the same pass-band, but a different stop-band. The frequency response corresponding to the concatenation of  $H_2$  and  $H_3$  is shown in Fig. 4.11(b). Although the direct conversion and the sequential method leads to the same frequency response, it can be shown (see Problem 4.3) that the sequential operation can reduce the computation requirement.

Note that to convert 625 lines to 525 lines is equivalent to convert every 25 lines to 21 lines [5]. Instead of using the ideal interpolation filter in the 13125 line



Note to graphics illustrator: change all  $v_i$  to  $v_i^*$

**Figure 4.8.** The sampling lattices involved in PAL to NTSC Conversion.  $\Lambda_1$  and  $\Lambda_2$  are sampling lattices for PAL and NTSC formats, and  $\Lambda_3$  is the intermediate lattice including both  $\Lambda_1$  and  $\Lambda_2$ .



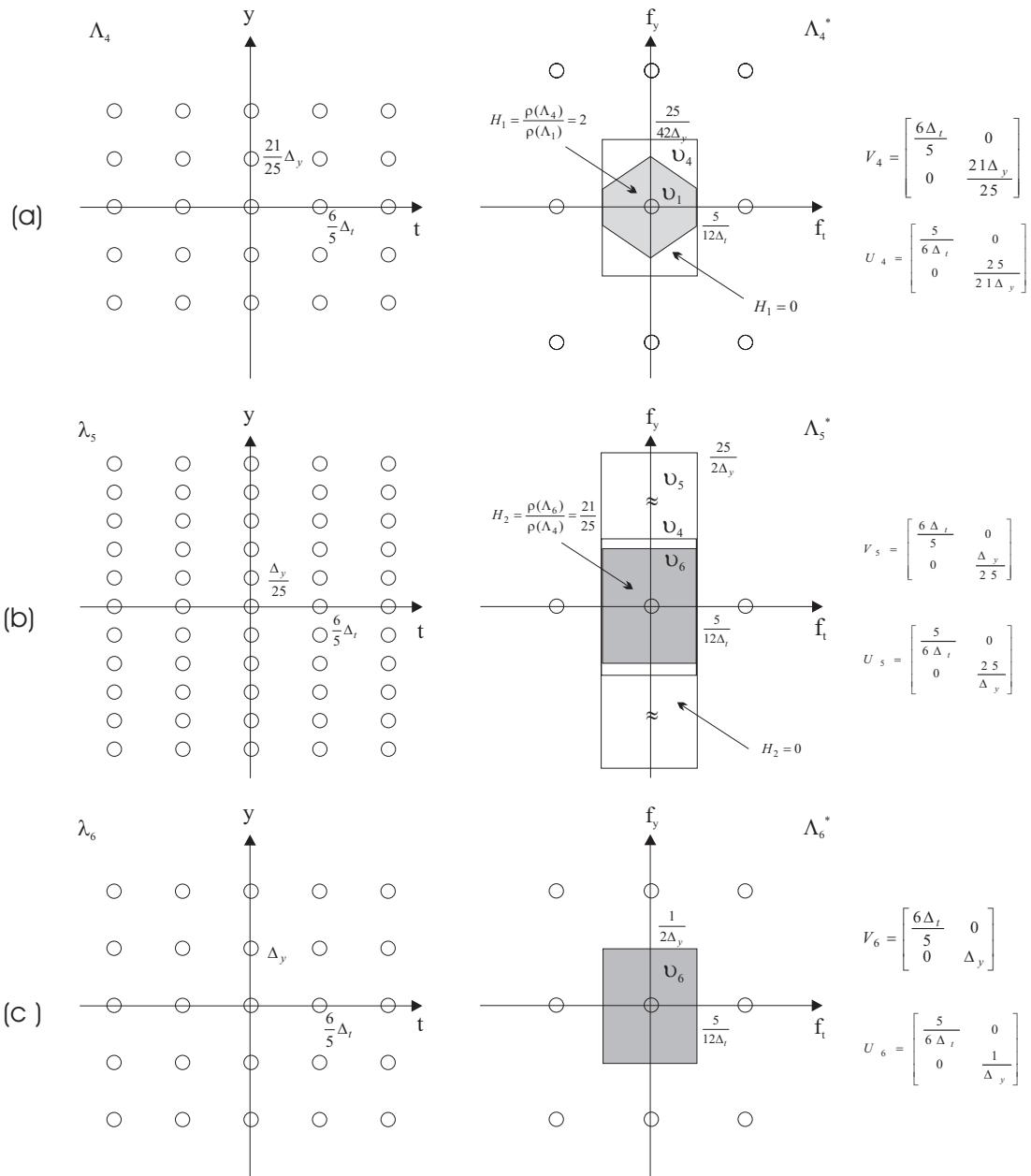
**Figure 4.9.** Different ways for converting PAL to NTSC formats.

resolution, a simple approach is to find, for each of the 21 lines to be determined, its two nearest neighbors in the given 25 lines, and use linear interpolation to determine the sample values, as shown in Fig. 4.12. To determine the interpolation coefficients associated with the two known lines, one should determine their distances to the line to be interpolated. If the distance to the left known line is  $d_l$ , and that to the right is  $d_r$ , the coefficients should be  $h_l = d_r/(d_l + d_r)$  and  $h_r = d_l/(d_l + d_r)$ , respectively. Let  $\Delta_{y,3}$  represent the spacing between every two lines in the 13125-line grid,  $\Delta_{y,1}$  and  $\Delta_{y,2}$  the spacings in the 625- and 525-line grids, respectively. Obviously  $\Delta_{y,1} = 21\Delta_{y,3}$ ,  $\Delta_{y,2} = 25\Delta_{y,3}$ . For line  $l$  in the 525-line grid, if  $k = l * \Delta_{y,2}/\Delta_{y,1} = \frac{25}{21}l$  is an integer, then it can be directly copied from line  $k$  in the 625 line-grid. This happens when  $l = 0, 21, 42, \dots$ . For the in-between lines, the indices for the left and right adjacent lines in the 625-line grid are  $k = \lfloor l * \Delta_{y,2}/\Delta_{y,1} \rfloor$  and  $k + 1$ , respectively. The corresponding distances are  $d_l = l\Delta_{y,2} - k\Delta_{y,1}$ , and  $d_r = (k + 1)\Delta_{y,1} - l\Delta_{y,2}$ . Therefore, interpolation coefficients are  $h_l = k + 1 - l * 25/21$  and  $h_r = l * 25/21 - k$ . The weight for the left known line for each interpolated line is indicated in Fig. 4.12.

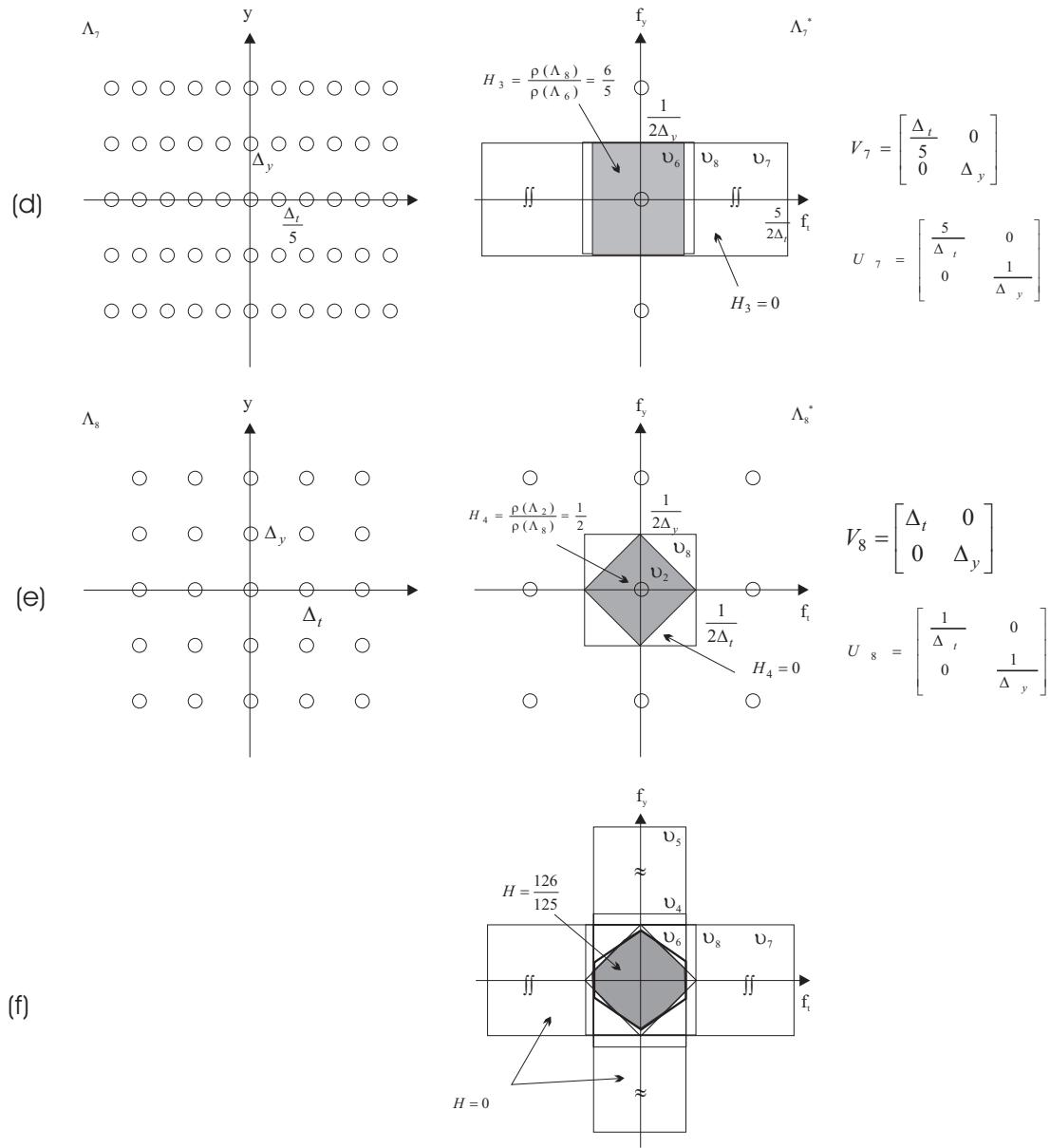
The equivalent vertical filter in  $\Delta_3$  is

$$h_v(y_n) = \begin{cases} (1 - |n|/21), & y_n = n\Delta_{y,3}, |n| = 0, 1, \dots, 20, \\ 0 & \text{otherwise.} \end{cases} \quad (4.2.15)$$

This is the linear interpolation filter for interpolating from every 625 lines to 13125 lines, or inserting 21 lines between every two lines. But at most one of the lines is

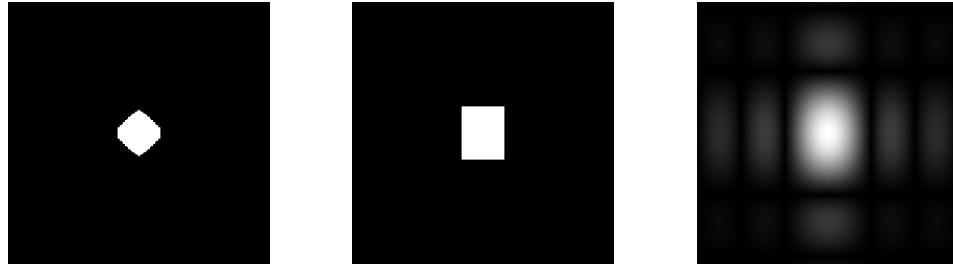


Note to graphics illustrator: change all  $v_i$  to  $v_i^*$



Note to graphics illustrator: change all  $v_i$  to  $v_i^*$

**Figure 4.10.** Ideal operations involved in the sequential implementation of PAL to NTSC conversion.



**Figure 4.11.** Frequency responses of PAL to NTSC conversion filters. from left to right: (a) The ideal filter; (b) The composite filter corresponding to concatenation of  $H_2$  and  $H_3$  in Fig. 4.9, when  $H_2$  and  $H_3$  are both ideal filters; (c) The composite filter when line-rate down-conversion ( $H_2$ ) and frame-rate up-conversion ( $H_3$ ) are implemented by Eqs. (4.2.15,4.2.16). Part (a) considers deinterlacing and interlacing filters, whereas (b) and (c) do not. In each figure, the vertical axis represents the vertical frequency, and the horizontal axis the temporal frequency. Only the frequency region defined by  $|f_y| \leq 2.5/\Delta_y, |f_t| \leq 2.5/\Delta_t$  is drawn, although the filter response is defined over  $|f_y| \leq 12.5/\Delta_y, |f_t| \leq 2.5/\Delta_t$ .

retained in the 525-line grid, using two out of 21 possible interpolation coefficients given in Eq. (4.2.15).

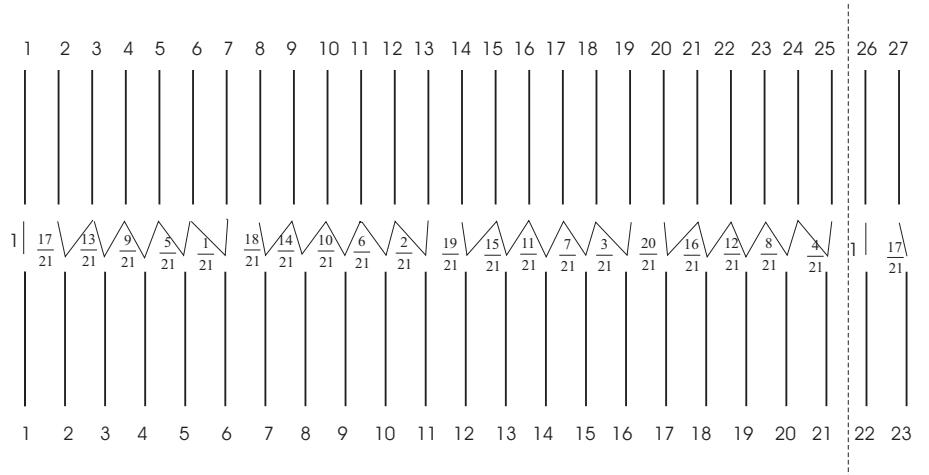
To implement the temporal conversion from 50 Hz to 60 Hz, instead of up converting the frame rate first from 50 to 300 and then down sample to 60, a much simpler approach is to convert every 5 frames to 6 frames as illustrated in Fig. 4.13 [5]. As shown there, each of the 6 output frames is interpolated from 2 of the 5 input frames, where the interpolation coefficient associated with each input frame is inversely proportional to the distance between the output frame and the input frame. Following the procedure described previously for the line rate conversion, one can determine the interpolation coefficients, which are shown in Fig. 4.13. The equivalent temporal filter over the 300-frame grid is

$$h_t(t_k) = \begin{cases} (1 - |k|/6), & t_k = k\Delta_{t,3}, |k| = 0, 1, \dots, 5, \\ 0 & \text{otherwise.} \end{cases} \quad (4.2.16)$$

The frequency response of the combined spatio-temporal filter is illustrated in Fig. 4.11(c). Compared to the desired response shown in Fig. 4.11(b), we see linear interpolation leads to blurred transitions and many ripples in the stop band.

### 4.2.3 Motion Adaptive Interpolation

From the examples given in the previous two sections, we can see that there are pros and cons with using interpolation in the spatial and temporal directions, respectively. In regions that are stationary, temporal interpolation will yield an accurate result, which will indeed increase the resolution of a given frame. On the other hand,



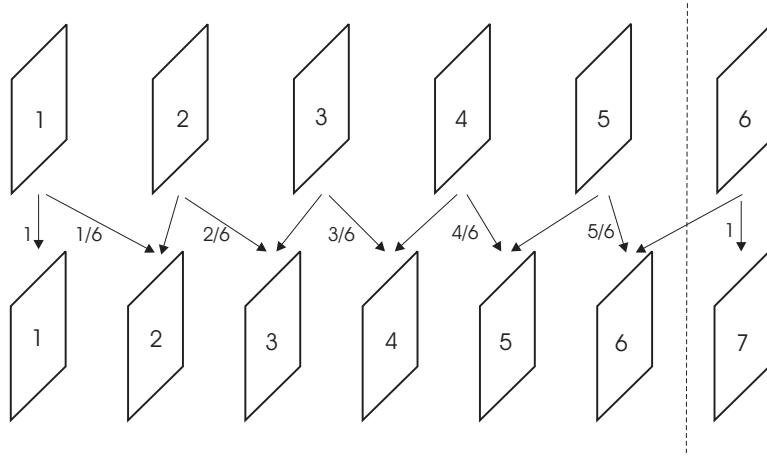
Note to graphics illustrator: each short slanted line should be replaced with an arrowed line with arrow downward.

**Figure 4.12.** Accomplishing 625 lines to 525 lines down-conversion by converting every 25 lines to 21 lines and using two nearest known lines (in the upper row) for interpolating each new line (in the lower row). For each line in the bottom row, the interpolation coefficient associated with the left neighboring line in the upper row is given in the figure. The interpolation coefficient for the right neighboring line is one minus the left coefficient. Modified from [5, Fig. 5.23].

in regions undergoing rapid temporal changes between successive fields/frames, pixels with same spatial indices in two separate fields/frames may correspond to different object regions in the scene. In this case, temporal interpolation yields erroneous results, and spatial alone interpolation may be better. To overcome such problems, motion-adaptive interpolation filters can be used. With such a filter, one switches between vertical and temporal interpolation at each pixel, depending on the output of a motion detector. If motion is detected, vertical only interpolation is used; otherwise, temporal only interpolation is used. Motion detection at each pixel can be based on the temporal variation in a small neighborhood surrounding the pixel. A key to the success of this approach is the accuracy of the motion detection process.

Rather than using hard switching between temporal and spatial interpolation, one can also use a weighted average of both, where the weights can be determined by certain measures of spatial and temporal variation in a small neighborhood surrounding the pixel being interpolated. Let  $\sigma_s^2$  and  $\sigma_t^2$  represent the spatial and temporal variation measures, respectively, and  $\hat{\psi}_s$  and  $\hat{\psi}_t$  the interpolated value using spatial and temporal interpolations, respectively. The final result will be:

$$\hat{\psi} = w_t \hat{\psi}_t + w_s \hat{\psi}_s, \text{ with } w_t = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_t^2}, \quad w_s = \frac{\sigma_t^2}{\sigma_s^2 + \sigma_t^2}. \quad (4.2.17)$$



**Figure 4.13.** Accomplishing 50 to 60 fields up-conversion by converting every 5 fields into 6 fields and using two nearest known fields for interpolating each new field. For each field in the bottom row, the interpolation coefficient associated with the left neighboring field in the upper row is given in the figure. The interpolation coefficient for the right neighboring field is one minus the left coefficient. Modified from [5, Fig. 5.23].

To further improve the performance in regions undergoing motion, motion-compensated interpolation has also been investigated. With this approach, if a pixel is deemed to fall in a motion region, its corresponding position in adjacent fields is determined and used to provide the temporal interpolation result, i.e.,  $\hat{\psi}_t$  in Eq. (4.2.17). The most challenging problem is motion estimation, i.e., finding corresponding points in adjacent fields/frames. We will discuss motion estimation at length in Chapter 6. For a more in-depth study of motion-compensated sampling rate conversion, the reader is referred to [2, 3]. Another alternative is to adapt the sampling grid based on the spatial frequency content [1].

### 4.3 Summary

#### Ideal solution for sampling rate conversion (Sec. 4.1)

- The ideal solutions for different conversion problems are illustrated in Fig. 4.1. The ideal filters are given in Eqs. (4.1.4,4.1.9,4.1.15). The up-sampling problem is a special case of the general conversion problem with  $\Lambda_3 = \Lambda_2$ ; the down-sampling problem is a special case with  $\Lambda_3 = \Lambda_1$ .
- In the general rate conversion problem, the filtering operation does not need to be implemented in  $\Lambda_3$ . To interpolate each sample in  $\Lambda_2$ , filtering only need to be applied to samples in  $\Lambda_1$ , as indicated by Eq. (4.1.20).

### Practical solutions for video sampling rate conversion (Sec. 4.2)

- Common video rate conversion problems include: deinterlacing, NTSC to/from PAL or equivalently BT.601 525/60 to/from 625/50, SDTV to/from HDTV, BT.601 4:2:2 to/from 4:2:0, film to/from NTSC/PAL.
- Very short temporal and spatial filters are usually used in practice, to reduce the storage/computation requirements.
- It is important to determine the ideal solution and use it as a guideline to evaluate practical schemes.
- Motion-adaptive operations are important for temporal direction filtering. The performance depends on the accuracy of motion detection.

## 4.4 Problems

**4.1** In Sec. 4.2.1, we discussed several practical filters for deinterlacing. We see that the line-field averaging method is closest to the ideal filter. This filter however requires storage of two frames. To minimize the frame storage, a single frame interpolation filter has been proposed [5]. It uses vertical interpolation within the same field and temporal interpolation from the opposite field in the same frame. Refer to Fig. 4.5, line  $D$  in field  $t$  is estimated by  $D = (C+E)/2 + K/4 - (I+M)/8$ . Determine the equivalent filter in the deinterlaced lattice and its frequency response. Draw the magnitude response (using Matlab) and compare it to those shown in Fig. 4.7. Does this filter has a real frequency response? If not, why?

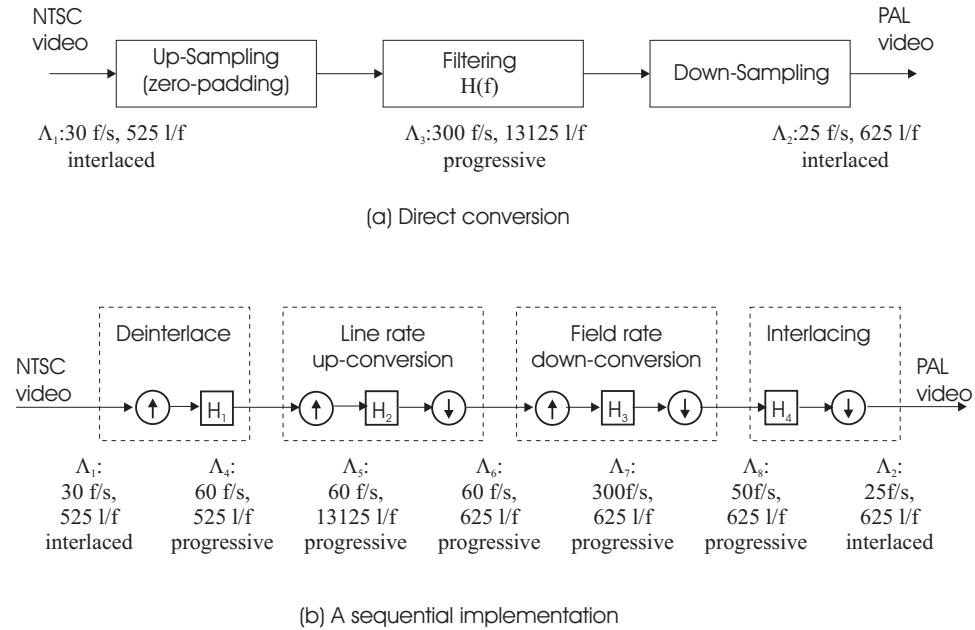
**4.2** Consider the following deinterlacing method:

$$\hat{f}(t, m) = \frac{1}{2}f(t-1, m) + \frac{9}{32}[f(t, m-1) + f(t, m+1)] - \frac{1}{32}[f(t, m-3) + f(t, m+3)]$$

Here,  $f(t, m)$  represent the image value at field  $t$  and line  $m$ . For field  $t$ , we assume that the lines  $m + 2k, k = 0, 1, \dots$  are missing.

Find the equivalent interpolation filter for the given operation, calculate its frequency response. Use Matlab to draw the frequency response and compare it to the ideal interpolation filter for deinterlacing.

**4.3** Compare the computation requirements for the direct and sequential methods for PAL to NTSC conversion, discussed in Sec. 4.2.2. Assume that the filtering over a domain with a total number of  $N$  samples takes  $\alpha N \log_2 N$  operations. Compare the total numbers of operations for the two methods. You only need to count the operations involved in filtering. The operations involved for up and down conversion are trivial and can be neglected.

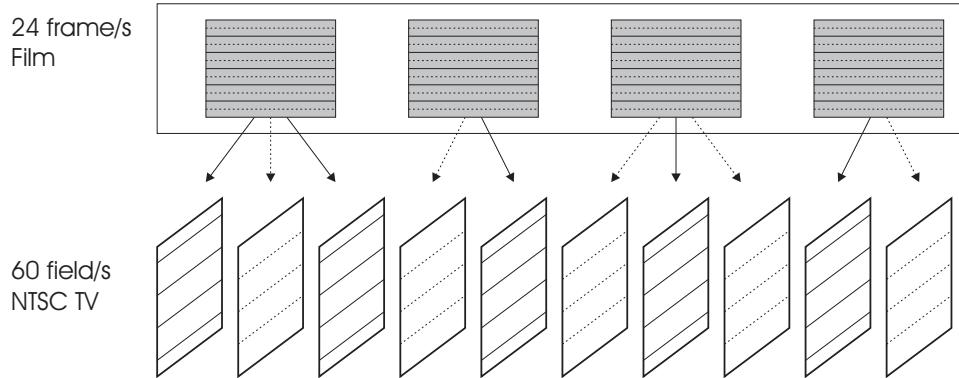


**Figure 4.14.** Different ways for converting NTSC to PAL formats.

**4.4** Consider the two methods shown in Fig. 4.14 for conversion of the NTSC signal (525 line/frame, 30 frame/second, interlaced) to PAL (625 line/frame, 25 frame/second, interlaced).

- 1) For each method, draw the sampling lattice and its reciprocal lattice at each intermediate step, and determine the desired frequency responses of the filters involved. For simplicity, do the above in the temporal–vertical plane only. When sketching the lattice and Voronoi regions in the frequency domain, assume that the temporal and spatial frequency axis have been scaled so that  $\frac{1}{\Delta t}$  cycle/second =  $\frac{1}{\Delta y}$  cycle/picture-height, for  $\Delta t = \frac{1}{60}$  sec,  $\Delta y = \frac{1}{525}$  picture-height.
- 2) The sequential implementation can be combined so that they can be accomplished by a single operation as with the direct method. Find the response of the equivalent overall filter and compare it to that of the Method 1.
- 3) Compare the total numbers of operations for the two methods, using the same assumptions as in Problem 4.3.
- 4) Based on your solutions for 2) and 3), comment on the pros and cons of the two methods.

**4.5** Consider the problem of converting a film captured at 24 frame/s, progressive to an NTSC TV format with 30 frame/s, interlaced.



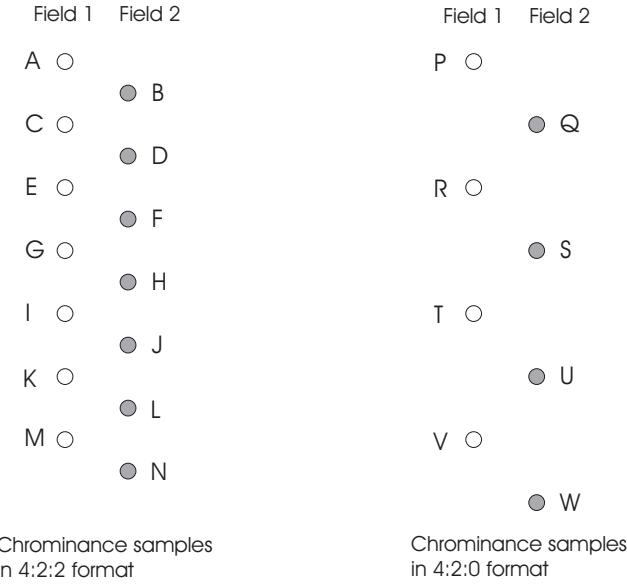
**Figure 4.15.** The 3:2 pull-down operation used for converting 24 frame/s film to 60 field/s NTSC TV. Modified from [5, Fig.5.24]

- (a) Draw the lattices corresponding to these two sampling formats. Determine the intermediate lattice for the conversion, and the ideal filter over this lattice.
  - (b) In practice, a simple 3:2 pull-down operation illustrated in Fig. 4.15 is often used. We can see that half of the frames are converted into 3 field, while the other half are split into 2 fields. For simplicity, no filtering is used. Can you find the equivalent filter in the intermediate lattice determined in (a)?
  - (c) Can you come up with other simple approaches that are better than (b)?
- 4.6** Consider the conversion of a BT.601 4:2:2 signal to BT.601 4:2:0. Remember that the Y-component is the same in these two formats, but the Cr or Cb component has half the number of the lines in 4:2:0 format than in 4:2:2. Because both formats are interlaced, this is not a simple 2:1 down-sampling problem. The conversion problem for the chrominance component is illustrated in Fig. 4.16.
- (a) Illustrate the sampling lattices for both BT.601 4:2:2 and 4:2:0 formats, and determine the intermediate lattice and the ideal interpolation filter.
  - (b) For simplicity, one can restrict filtering within the same field, i.e., samples in field 1 (resp. field 2) are down-sampled by filtering in field 1 (resp. field 2) only. In fact, this is the technique recommended in MPEG2 standard for converting 4:2:2 format to 4:2:0 [5, Sec.5.4.1]. In this case, field 1 needs to be down-sampled by a factor of two. The recommended pre-filtering operation is

$$G' = (-29A + 88E + 138G + 88I - 29M)/138.$$

For field 2, a factor of 2 interpolation needs to be applied to obtain the desired samples. The recommended interpolation operation is

$$S = (D + 7F + 7H + J)/16.$$



**Figure 4.16.** Relation between Chrominance Samples in BT.601 4:2:2 and 4:2:0 formats.  
Modified from [5, Fig.5.18]

For the above resampling scheme, determine the equivalent vertical filter for both field 1 and field 2, as well the overall vertical-temporal filter. Find their frequency responses. Compare the result with the ideal filter of (a).

**4.7** The previous problem considered the conversion from 4:2:2 to 4:2:0 format. Here we consider the conversion from 4:2:0 to 4:2:2 format.

(a) Draw the intermediate lattice and determine the ideal interpolation filter over this lattice.

(b) Determine the equivalent filter and its frequency response for the following practical scheme [5]. For field 1, a factor of 2 interpolation is needed, and the missing samples are obtained by an average of two neighboring samples, i.e., ,

$$G = (R + T)/2.$$

For field 2, a factor of 4 interpolation is needed, followed by a factor 2 down-sampling. The end-operations are

$$D = (3Q + S)/4, F = (Q + 3S)/4.$$

Determine the equivalent vertical filters for both field 1 and field 2, as well as the overall vertical-temporal filter, in the interpolated grid. Find their frequency responses. Compare with the result in (a).

## 4.5 Bibliography

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## Chapter 5

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# VIDEO MODELING

One of the tasks of digital video processing is to describe the change between consecutive images of a video sequence in terms of object motion and other effects like illumination changes and camera motion. In order to relate changes in the real world to changes in a video sequence, we need parametric models describing the real world and the image generation process. The most important models are *scene model*, *object model*, *camera model* and *illumination model*. These models describe the assumptions that we make about the real world. Depending on the selected model, we are able to describe the real world with more or less detail and precision. Using image analysis tools, we estimate the parameters of the parametric models from the video sequence of the real world. Using the parametric models and their estimated parameters, we can reconstruct a model world that is an approximation of the real world. Table 5.1 shows the terms that we use to name the parametric models, their corresponding real world entity and the entity reconstructed according to the parametric model and presented in the model world. We distinguish items of the real world from items in the model world using the prefix *real* and *model*. When the context is clear, we may omit the prefix.

In the following sections, we describe the camera model (Section 5.1), the illumination model (Section 5.2), the object model (Section 5.3), and the scene model (Section 5.4). Today, object models in most video processing applications assume objects with 2D motion. We present the relevant 2D motion models in Section 5.5.

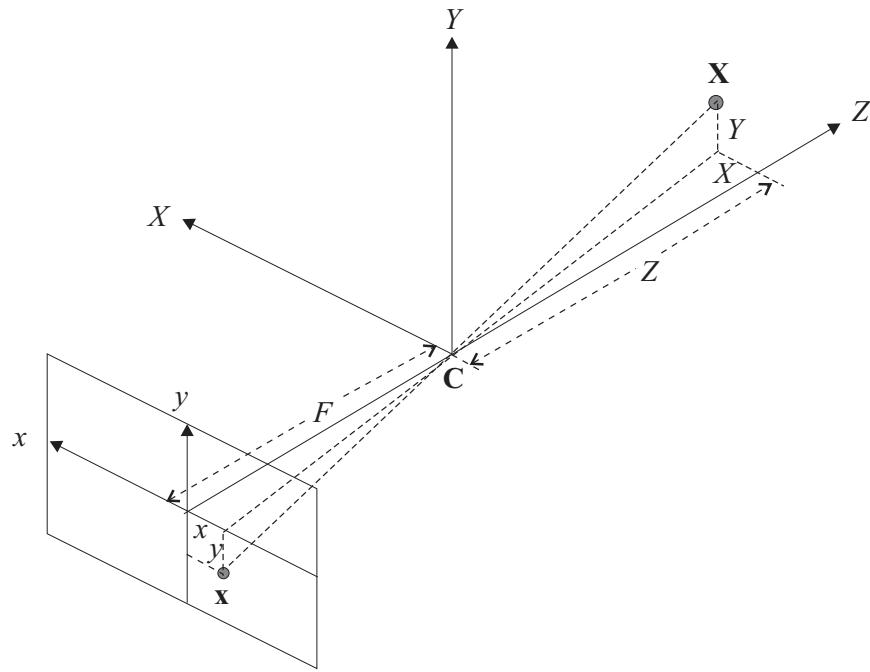
Depending on the model used, the estimation of the model parameters can be difficult. In Chapters 6 and 7, we will discuss the estimation of the model parameters for 2D and 3D motions, respectively.

### 5.1 Camera Model

The camera model describes the projection of real objects in the real scene onto the *imaging plane* of the real camera. The imaging plane is also referred to as the *camera target*. The image on the imaging plane is then converted into a digital image as described in Chapter 1.

**Table 5.1.** Naming conventions for terms of the real scene with "real {term}", for parametric models with "{term} model", and for the reconstructed models with "model {term}". When the context is clear, we omit the prefix *real* and *model*.

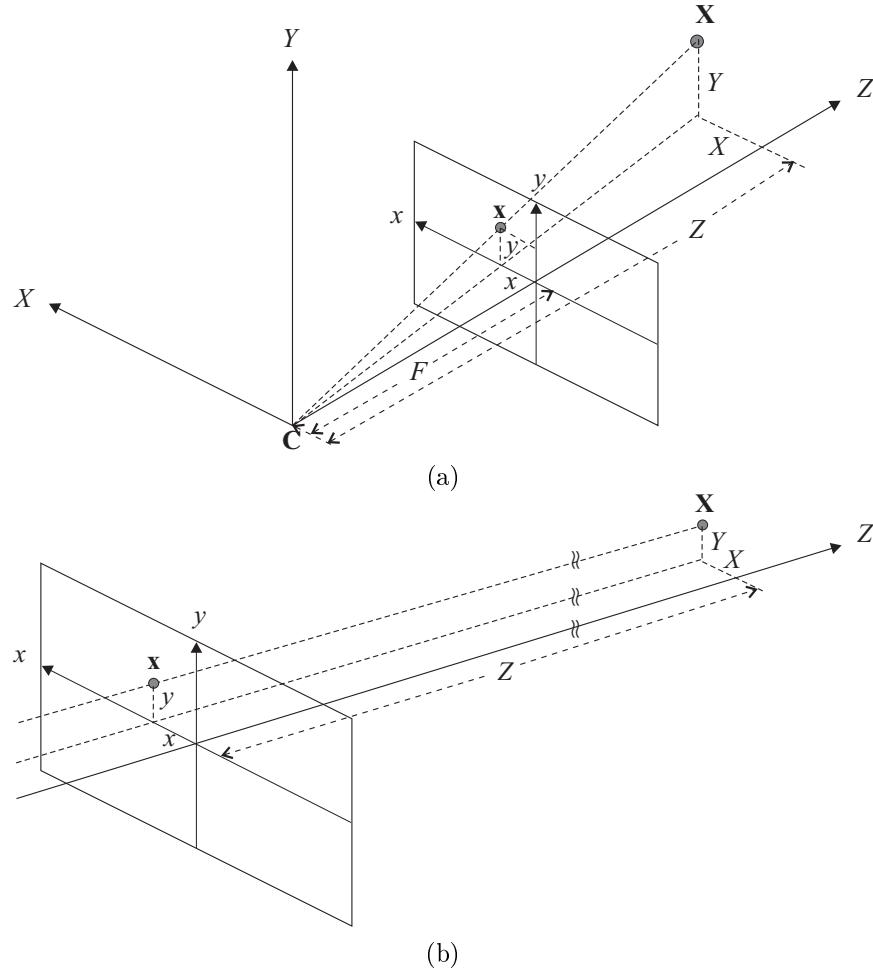
real world	parametric model	model world
real scene	scene model	model scene
real object	object model	model object
real texture	texture model	model texture
real shape	shape model	model shape
real motion	motion model	model motion
real camera	camera model	model camera
real image	image model	model image
real illumination	illumination model	model illumination



**Figure 5.1.** Perspective Projection by a Pin-Hole Camera.

### 5.1.1 Pinhole Model

A widely used approximation of the projection of real objects onto a real camera target is the *pinhole camera*, illustrated in Figure 5.1. In this figure,  $F$  represents



**Figure 5.2.** Camera model: (a) using perspective projection as in a pin-hole camera, (b) using parallel projection as an approximation of a pinhole camera. In both cases, perfect alignment of XY-plane of the world coordinate system with the image plane is assumed.

the *focal length* of the camera, and  $\mathbf{C}$  the *focal center* (also known as camera center). The projected position  $\mathbf{x}$  of a 3D point  $\mathbf{X}$  is the intersection of the line connecting  $\mathbf{X}$  and  $\mathbf{C}$  with the imaging plane. Because the imaging plane is behind the focal center, the imaged position is reversed from its true 3D position. Most camera models used in video processing place the imaging plane on the same side of the focal center as the objects (Fig. 5.2(a)). This avoids to deal with reversed positions in the imaging plane.

Without loss of generality, we assume (cf. Fig. 5.2(a)) that the origin of the

3D world coordinate is located at the focal center, its XY-plane is parallel to the imaging plane, and finally the world coordinate  $(X, Y, Z)$  follows the right-hand system, with the positive direction of the Z-axis being in the imaging direction. Furthermore, we assume the imaging plane uses the same distance unit as the 3D coordinate. From the similar triangles illustrated in Fig. 5.2(a), we have

$$\frac{x}{F} = \frac{X}{Z}, \quad \frac{y}{F} = \frac{Y}{Z}, \quad (5.1.1)$$

or

$$x = F \frac{X}{Z}, \quad y = F \frac{Y}{Z}. \quad (5.1.2)$$

The above relation is known as *perspective projection*. A notable character of perspective projection is that the image of an object is smaller if it is farther away from the camera. Mathematically, this is described by the inverse relation between the projected  $x$  and  $y$  values and the depth value  $Z$ .

When the imaged object is very far from the camera plane, the perspective projection can be approximated by the *orthographic projection*, which is also known as *parallel projection* (Fig. 5.2(b)):

$$x = X, \quad y = Y. \quad (5.1.3)$$

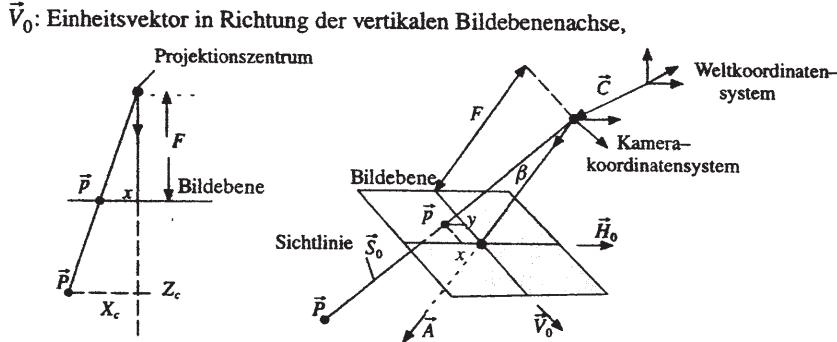
Obviously, this relation is much simpler and can greatly simplify transformation of 3D to/from 2D. In practice, the condition that the object must be far away from the camera can be relaxed. As long as the relative depth variation of the object surface is negligible compared to the distance of the object to the camera, the above approximation can be used reliably.

Note that both perspective and orthographic projections are many-to-one mappings. In fact, all the 3D points on a ray connecting the focal center  $\mathbf{C}$  and the image point  $\mathbf{x}$  would have  $\mathbf{x}$  as its projection. We name this ray *line of sight*. On the imaging plane we only see the first object point that this ray encounters. The many-to-one mapping of the projection used in camera models makes the estimation of 3D structure and motion of objects based on their 2D images a very difficult and challenging problem.

The pinhole camera model with its perspective projection is only an approximation of real cameras. It does not consider the misalignment of the camera axis and the image center, the lowpass filter effect of the finite size aperture of a real lens, the finite exposure time, and other distortions of the lens.

### 5.1.2 CAHV Model

For some applications, we like to describe the camera such that we can accommodate camera motion. Furthermore, we would like to calibrate a camera in order to compensate for geometrical differences between the pinhole camera model and the real camera. The *CAHV camera model* [16] describes perspective projection for a pinhole camera model using



**Figure 5.3.** Perspective Projection of a point  $\mathbf{P}$  in space onto point  $\mathbf{p}$  in the imaging plane using the CAHV camera model. Adapted from [8, Fig. 2.5]

**C:** Vector to the camera center,

**A:** Unit vector in the direction of the optical axis,

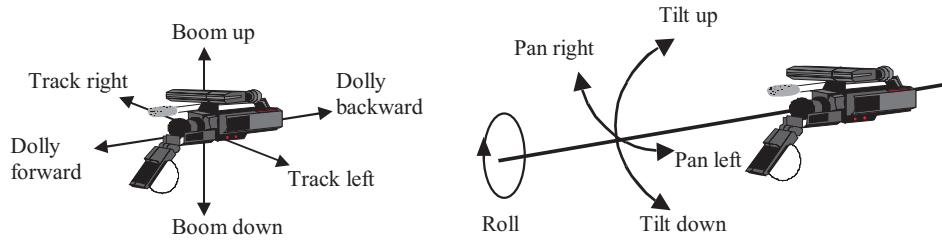
**H<sub>0</sub>:** Unit vector in the direction of the horizontal axis of the imaging plane,

**V<sub>0</sub>:** Unit vector in the direction of the vertical axis of the imaging plane.

This geometry is depicted in Fig. 5.3. Adapting the perspective projection of Eq. (5.1.2) by projecting the vector  $(\mathbf{P} - \mathbf{C})$  onto the camera axis  $\mathbf{A}$  and the imaging plane axis  $\mathbf{H}_0$  and  $\mathbf{V}_0$ , a point  $\mathbf{P}$  is projected onto image point  $\mathbf{p}$  according to

$$\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{F}{\mathbf{A}^T \cdot (\mathbf{P} - \mathbf{C})} \cdot \begin{pmatrix} \mathbf{H}_0^T \cdot (\mathbf{P} - \mathbf{C}) \\ \mathbf{V}_0^T \cdot (\mathbf{P} - \mathbf{C}) \end{pmatrix}. \quad (5.1.4)$$

In reality, the imaging plane coordinate and the world coordinate are not always in perfect alignment as assumed by the pinhole camera model according to Eq. (5.2). The CAHV camera model can characterize a practical camera system by its extrinsic parameters  $\mathbf{C}$ ,  $\mathbf{A}$  and its intrinsic parameters  $\mathbf{H}_0$ ,  $\mathbf{V}_0$ ,  $F$ . These parameters enable us to describe an imaging plane that is off the camera axis as well as some distortions introduced by the optical system. Additional intrinsic camera parameters describe radial distortions of the lens and the modulation transfer function (MTF) of the lens. The MTF can be modeled by a lowpass filter. Camera calibration algorithms are used to estimate these parameters. Interested readers are referred to [2]. Precise knowledge of camera parameters is especially useful when estimating 3D shapes and motion from a video sequence. Recently, algorithms were described that do not require a calibrated camera in order to extract 3D shape and motion from an image sequence [10].



**Figure 5.4.** Typical types of camera motion. [Courtesy of MPEG-7].

### 5.1.3 Camera Motions

Fig. 5.4 show typical camera motions. Translations of the camera along the horizontal ( $\mathbf{H}_0$ ) and vertical ( $\mathbf{V}_0$ ) axes of the imaging plane are called *track* and *boom*, respectively. Translation in the direction of the optical camera axis  $\mathbf{A}$  is called *dolly*. *Pan* refers to the turning around the vertical axis ( $\mathbf{V}_0$ ), whereas *tilt* refers to the turning around the horizontal axis ( $\mathbf{H}_0$ ). The camera rotation around the optical axis ( $\mathbf{A}$ ) is called *roll*. In addition to the above motion types, a camera can also change its focal length, which is called *zoom*. As mentioned before, the CAHV camera model is well suited to express various camera motions.

## 5.2 Illumination Model

As described in Sec. 1.1.1, there are two types of light sources: illuminating and reflecting lights. The illumination model describes how the light incident on an object influences the reflected light distribution, which is what we see. In this section, we describe several such models. Illumination models can be divided into *spectral* and *geometric* models. Spectral illumination models are used if we want to model changes of color resulting from several colored light sources, indirect illumination with objects of different color or colored reflecting surfaces. Geometric models describe the amplitude and directional distribution of incident light. The spectral properties have been discussed in Sec. 1.1.1. Geometric models are discussed in this section.

Geometric illumination models are available for ambient and for point light sources. For each type of light source, we can also distinguish between local and global illumination models. A *local illumination model* assumes that the illumination is independent of object location and other objects. *Global illumination models* also consider effects between objects like shadows and light reflected from object surfaces.

### 5.2.1 Diffuse and Specular Reflection

We can see only objects that reflect light and the perceived color depends on the range of wavelengths reflected. In general, the reflection can be decomposed into two components: the *diffuse reflection*, which has equal energy distribution in all directions, and the *specular reflection*, which is strongest in the mirror direction of the incident light. For a good treatment of both diffuse and specular reflection, see [3]. The type of surface that exhibits only diffuse reflection is known as *Lambertian surface*, more commonly described as dull and matte. Wood surfaces and cement walls belong to this category. Due to diffuse reflection, we can perceive the color of the object. Specular reflection can be observed on shiny surfaces and mirrors. Specular reflection does not show the color of the object but the color of the incident light. Therefore, we cannot really perceive the color of an object with a surface that only shows specular reflection. Except for mirrors, surfaces usually have diffuse as well as specular reflections. Only the diffuse reflection determines the color of the object surface. Looking at a human eye in daylight, we usually see the eye itself except for a white spot which is caused by specular reflection of the sunlight. From experience, we know that this spot moves on the surface if we change our position relative to the eye.

### 5.2.2 Radiance Distribution Under Different Illumination and Reflection Conditions

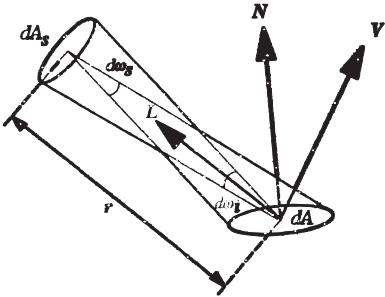
In video processing, the illumination model is mainly used to describe the temporal changes in the video sequence caused by the changing illumination of the real world. The illumination of a background may change because of an object that moves together with its cast shadow. Since the object surface reflects light, this reflecting source changes the overall illumination of the scene.

When discussing the interaction of a light source with an object surface, there are three types of energy involved. First, *incident flux* refers to the rate at which energy is emitted from the light source, measured in watts (W). Second, *incident irradiance* is the incident flux per unit surface area on the object and has a unit of W/m<sup>2</sup>. Note that the irradiance at an object point depends on the angle between the incident light and the surface normal at that point. Finally, *reflected radiance* measures the light energy reflected from an object surface.

The distribution of the reflected radiance  $C$  depends on the distribution of incident irradiance  $E$  and the object surface *reflectance function*  $r$  at this point. The most general relation can be described by

$$C(\mathbf{L}, \mathbf{V}, \mathbf{N}, \mathbf{X}, t, \lambda) = r(\mathbf{L}, \mathbf{V}, \mathbf{N}, \mathbf{X}, t, \lambda) \cdot E(\mathbf{L}, \mathbf{N}, \mathbf{X}, t, \lambda), \quad (5.2.1)$$

where  $\mathbf{X}$  is the location on the object surface,  $\mathbf{N}$  is the surface normal vector at the location  $\mathbf{X}$ ,  $\mathbf{L}$  is the illumination direction,  $\mathbf{V}$  is the viewing direction connecting  $\mathbf{X}$  to the focal point of the camera and  $\lambda$  the wavelength of light (Fig. 5.5) [12]. Obviously,  $\mathbf{L}$ ,  $\mathbf{V}$ , and  $\mathbf{N}$  are functions of  $\mathbf{X}$  and  $t$ . The reflectance function  $r$  is



**Figure 5.5.** Surface patch  $dA$  with normal vector  $N$  illuminated from direction  $\mathbf{L}$  by a point light source with an infinitesimal small area  $dA_s$ . The patch is viewed from direction  $\mathbf{V}$ . From [12].

defined as the ratio between the reflected light intensity (i.e., the flux) and the incident light intensity. This scalar function  $r$  is also known as *diffuse-reflection coefficient*, or *reflection coefficient* for short. The reflectance function depends on the wavelength of the incident light, the surface geometry and material property. When the object is moving, the reflection coefficient changes in time at the same location. Note that  $r(\mathbf{L}, \mathbf{V}, \mathbf{N}, \mathbf{X}, t, \lambda)$  is defined only for those  $\mathbf{X}$  that belong to a surface at time  $t$ .

In the following paragraphs, we introduce several simplifying assumptions in order to learn more about the reflected radiance (Eq. 5.2.1). We start with assuming opaque object surfaces and temporally invariant illumination direction as well as viewing direction. In this case, Eq. (5.2.1) simplifies to

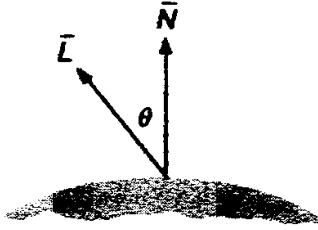
$$C(\mathbf{N}, \mathbf{X}, t, \lambda) = r(\mathbf{N}, \mathbf{X}, t, \lambda) \cdot E(\mathbf{N}, \mathbf{X}, t, \lambda). \quad (5.2.2)$$

Note that although the illumination and viewing directions ( $\mathbf{V}$  and  $\mathbf{L}$ ) are fixed, the incident irradiance is still time varying because the object is moving.

An *ambient source* radiates the same amount of energy in every direction at any point, hence it illuminates objects without casting shadows. When the incident light is such an ambient source and the object surface is diffuse reflecting, the reflected radiance intensity distribution is

$$C(\mathbf{X}, t, \lambda) = r(\mathbf{X}, t, \lambda) \cdot E_a(t, \lambda), \quad (5.2.3)$$

where  $E_a(t, \lambda)$  represents the intensity of the ambient light at time  $t$ . Since the light source is ambient,  $E_a$  does not depend on the surface location  $\mathbf{X}$  or surface normal  $\mathbf{N}$ . Since the surface is diffuse reflecting, the reflectance function  $r$  does not depend on the surface normal  $\mathbf{N}$ . This ambient light source model is a local illumination model since we cannot model global effects like shadows anymore. In rooms, illuminated white walls can often be modeled as ambient light sources. Outdoors, the sun, when covered by clouds, provides ambient illumination.



**Figure 5.6.** Diffuse reflection of a point light source is constant in all directions. The intensity depends on the angle  $\theta$  between the surface normal  $\mathbf{N}$  and the direction  $\mathbf{L}$  to the light source. (From [3, Fig. 14.1])

Now we discuss the reflected radiance due to a *point source*. We assume that the light source is far away from the scene, such that the position of an object has no influence on the incident light. With a local illumination model and a diffuse reflecting surface, the reflected radiance at any object surface point  $\mathbf{X}$  depends on the angle between the incident light direction  $\mathbf{L}$  and the surface normal  $\mathbf{N}$  at this point, denoted by  $\theta$ . Obviously,  $\mathbf{L}^T \mathbf{N} = \cos(\theta)$  holds. These quantities are illustrated in Fig. 5.6. Let  $E_p(t, \lambda)$  represents the maximum irradiance from the light source, which is the irradiance intensity when the light is perpendicular to the surface. Then the irradiance with the light at an arbitrary angle  $\theta$  is  $E_p(t, \lambda) \cos \theta$ . Starting with Eq. (5.2.2), the reflected radiance intensity at  $\mathbf{X}$  simplifies to [3]

$$C(\mathbf{X}, t, \lambda) = r(\mathbf{X}, t, \lambda) \cdot E_p(t, \lambda) \cdot \max(0, \mathbf{L}^T \mathbf{N}). \quad (5.2.4)$$

The max operator in Eq. (5.2.4) prevents negative reflected intensities from those parts of the object that do not receive light from the point light source. Typical point light sources are spot lights and the sun. When both ambient and point light sources are present, the total reflected radiance at any point is the superposition of the reflected radiance from each light source according to Eq. (5.2.3) and Eq. (5.2.4).

If a point light source is far away from the object surface, we can approximate the incident light as a parallel light. This approximation is valid for daylight and sometimes even indoors. In this case, the maximum incident light irradiance  $E_p$  is not dependent on the surface point  $\mathbf{X}$  anymore, but on the surface normal  $\mathbf{N}$ . Assuming the scene is illuminated by one stationary distant point light source and an ambient light, both invariant in time and space, simplifies the description of the incident irradiance to:

$$E(\mathbf{N}, \lambda) = E_a(\lambda) + E_p(\lambda) \cdot \max(0, \mathbf{L}^T \cdot \mathbf{N}). \quad (5.2.5)$$

The above is the shading model of Phong used in early computer graphics [11]. Assuming the object has a homogeneous surface, that is  $r(\mathbf{X}, \lambda) = r(\lambda)$ , the corre-

sponding reflected radiance becomes

$$C(\mathbf{N}, \lambda) = r(\lambda) \cdot E(\mathbf{N}, \lambda). \quad (5.2.6)$$

### 5.2.3 Changes in the Imaged Function due to Object Motion

Now, we investigate how the illumination model can be used to describe illumination effects in the image plane. Assuming white illumination and that the surface normals  $\mathbf{N}$  and therefore the object shape are known, the illumination model in Eq. (5.2.5) can be used to determine the illumination changes in an image sequence due to object motion. Three parameters have to be estimated: the ratio between ambient and point light irradiance  $E_a/E_p$  and the two angles describing the direction  $\mathbf{L}$  of the point light source.

The model given by Eq. (5.2.5) has been implemented in an object-based analysis-synthesis coder by Stauder [13]. In the 2D image plane, he assumes that the luminance  $\psi$  at pixel  $\mathbf{x}$  and time  $t$  is proportional to the reflected radiance at the 3D point  $\mathbf{X}$  corresponding to  $\mathbf{x}$ , i.e.,

$$\psi(\mathbf{x}, t) = k \cdot C(\mathbf{X}) = k \cdot r(\mathbf{X}) \cdot E(\mathbf{N}), \quad (5.2.7)$$

where  $k$  is a constant,  $\mathbf{N}$  denotes the normal direction corresponding to  $\mathbf{X}$ . Thus, the luminance intensity of a point moving from  $\mathbf{x}_k$  to  $\mathbf{x}_{k+1}$  (with corresponding surface normals  $\mathbf{N}_k$  and  $\mathbf{N}_{k+1}$ ), from time  $t$  to  $t + d_t$ , changes according to

$$\psi(\mathbf{x}_{k+1}, t + d_t) = \psi(\mathbf{x}_k, t) \cdot \frac{E(\mathbf{N}_{k+1})}{E(\mathbf{N}_k)}. \quad (5.2.8)$$

Another simple illumination model assumes that the image signal  $\psi(\mathbf{x})$  depends on the illumination  $E(\mathbf{x})$  and the reflectance function  $r(\mathbf{x})$ , both defined in the image plane. The illumination  $E(\mathbf{x})$  in general depends on both ambient and point light sources. Assuming ambient illumination, diffuse reflecting surfaces and a camera model with orthographic projection, the image signal is given by the reflection model

$$\psi(\mathbf{x}, t) = k_b \cdot E(\mathbf{x}) \cdot r(\mathbf{x}, t), \quad (5.2.9)$$

where  $k_b$  is a constant. In the image plane, the signal  $\psi(\mathbf{x})$  of a point moving from  $\mathbf{x}_k$  to  $\mathbf{x}_{k+1}$  changes according to

$$\psi(\mathbf{x}_{k+1}, t + d_t) = k_b \cdot E(\mathbf{x}_{k+1}) \cdot r(\mathbf{x}_k, t), \quad (5.2.10)$$

because  $r(x_{k+1}, t + d_t) = r(x_k, t)$  holds, i.e. the reflectance function moves with the object. Equation (5.2.10) assumes that the illumination of the scene does not change due to the motion, i.e. the object does not cast shadows in the scene. This has proven to be useful in 2D motion estimation using block matching, 3D motion estimation and segmentation of moving objects and their shadows [1, 4, 9, 14].

The simplest and yet most widely used illumination model simply assumes that  $E(\mathbf{x})$  is a constant. In this case, the luminance of a moving point does not change, and Eq. (5.2.10) simplifies to

$$\psi(\mathbf{x}_{k+1}, t + d_t) = \psi(\mathbf{x}_k, t). \quad (5.2.11)$$

Sometimes, the above is referred to as *constant intensity assumption*. It is valid for spatially and temporally invariant ambient illumination sources and diffuse reflecting surfaces. Under these conditions, there are no moving shadows and no reflections due to glossy surfaces. Unless mentioned otherwise, the algorithms discussed in this book make use of this assumption.

In the above equations, we have assumed that the incident light intensity is time-invariant. However, the reflected radiance and hence the luminance image is in general space-and-time-varying, as long as the object surface is not completely flat and stationary. The discussion above can be extended to the chrominance components of a video by assuming a colored incident light. Therefore,  $\psi$  in Eqs. (5.2.7) to (5.2.11) can represent either a luminance or a color function.

### 5.3 Object Model

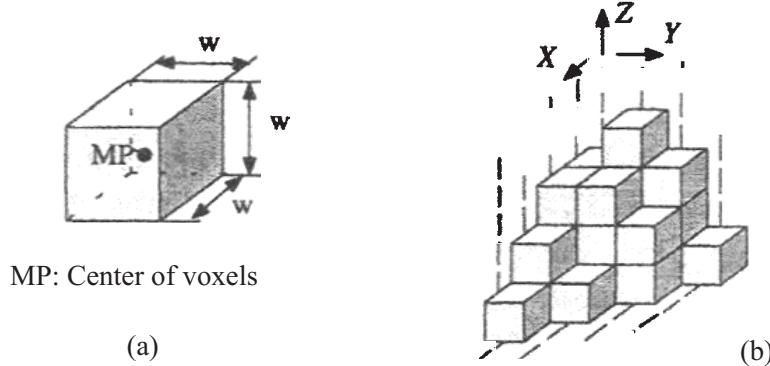
The object model describes the assumptions about real objects. Objects are entities in a scene that we consider to be separate items. The interpretation of what an object might be can change depending on the application. For traffic surveillance, we might consider each car to be an object, whereas for video coding, we might define an object according to its motion thus segmenting the car into the rotating wheels and the translationally moving body. An object is described by a shape, motion and texture model [7].

The *texture model* describes the surface properties of an object. Here, we assume that the texture of an object  $m$  is described by the color parameters  $S_m$ . These color parameters contain the luminance as well as the chrominance reflectance. Since we usually assume constant intensity according to Eq. (5.2.11), these color parameters correspond to the image signal  $\psi(\mathbf{x})$ .

While all object models described in this book use the same texture model, they employ different *motion* and *shape models* as discussed in Section 5.5 and Chapter 7. Motion and shape models are usually derived from an approximate 3D description of objects in order to simplify the estimation of the model parameters. Here we start with the most general shape and motion models. Simplification of this model will be introduced as needed (Section 5.5 and Chapter 7).

#### 5.3.1 Shape Model

The shape of a 3D object is described by the 3D space that it occupies. If we quantize the 3D space into *voxels*, small cubes of edge length  $w$ , we can determine for each voxel whether it belongs to the object or not. This is illustrated in Fig. 5.7, where  $w$  denotes the length of each side of the cube. Object shapes can be convex, concave or



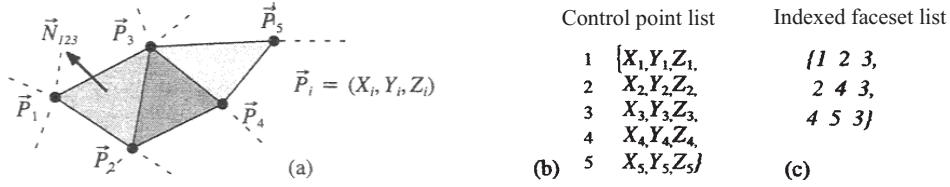
**Figure 5.7.** Voxels: (a) volume element, (b) shape representation with voxels in a 3D lattice. From [8, Fig. 2.10].

have holes. We assume that an object is topologically connected, i.e. we can draw a path from any point in the object to any other point without leaving the object. The projection of an object onto the image plane might result in topologically disconnected regions due to covered object parts or parts of the object being outside of the image. Since we are not so much interested in the interior of an object, a voxel representation is inefficient in terms of memory. Therefore, we usually describe the shape of an object by its surface. Often, a mesh of triangles commonly referred to as *wireframe* is used. The mesh of triangles is put up by vertices referred to as *control points*  $\mathbf{P}_C^{(i)}$ . In addition to these control points, we need a list that defines which control points define a triangle (Fig. 5.8). Assuming that the list of control points is numbered as in Fig. 5.8(b), we can use these numbers as indices for the control point. This simplifies the definition of surface patches or face by giving the indices that define the patch. This list of indices is called *indexed faceset* (Fig. 5.8(c)). Sometimes, faces are defined by more than 3 points.

The control points of a wireframe are located on the object surface. The number and location of control points is determined by the object shape as well as the accuracy with which the wireframe model is to describe the object shape. As long as control points cannot be moved independently of each other, the object is *rigid* and cannot change its shape. Otherwise, the object is *flexible*, i.e. it can change its shape.

### 5.3.2 Motion Model

As described in Sec. 5.3.1, an object can be rigid or flexible. The motion of a rigid object can be described in terms of a translation vector  $\mathbf{T} = (T_x, T_y, T_z)^T$  and a rotation matrix  $[\mathbf{R}]$ . Let us first describe how a point of an object moves in 3D space using the motion parameters  $\mathbf{T}$  and  $[\mathbf{R}]$ . The translation vector  $\mathbf{T}$  describes a displacement of a point from  $\mathbf{X}$  to  $\mathbf{X}'$  by  $T_x, T_y, T_z$  in the directions of the coordinate



**Figure 5.8.** Wireframe for object shape description: (a) mesh of triangles to represent the object surface, (b) representation of the mesh of triangles as a list of control points and a list of triangles that references the list of control points. From [8, Fig. 2.13].

axes  $X, Y, Z$ , respectively:

$$\mathbf{X}' = \mathbf{X} + \mathbf{T}. \quad (5.3.1)$$

If an object is translated, Eq. (5.3.1) holds for all object points.

If an object rotates around the origin of the 3D space, we describe the motion of its points with the rotation matrix  $[\mathbf{R}]$ :

$$[\mathbf{R}] = [\mathbf{R}_z] \cdot [\mathbf{R}_y] \cdot [\mathbf{R}_x]. \quad (5.3.2)$$

This rotation matrix rotates a point around the axes  $X, Y$  and  $Z$  in this order. It is computed from the rotation matrices that rotate a point just around one axis. These individual rotation matrices are

$$[\mathbf{R}_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{bmatrix}, \quad (5.3.3)$$

$$[\mathbf{R}_y] = \begin{bmatrix} \cos \theta_y & 0 & \sin \theta_y \\ 0 & 1 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y \end{bmatrix}, \quad (5.3.4)$$

and

$$[\mathbf{R}_z] = \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5.3.5)$$

where  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  are the rotation angles w.r.t each axis. Finally, we have  $[\mathbf{R}]$  as a function of the rotation angles  $\theta_x$ ,  $\theta_y$  and  $\theta_z$

$$[\mathbf{R}] = \begin{bmatrix} \cos \theta_y \cos \theta_z & \sin \theta_x \sin \theta_y \cos \theta_z - \cos \theta_x \sin \theta_z & \cos \theta_x \sin \theta_y \cos \theta_z + \sin \theta_x \sin \theta_z \\ \cos \theta_y \sin \theta_z & \sin \theta_x \sin \theta_y \sin \theta_z + \cos \theta_x \cos \theta_z & \cos \theta_x \sin \theta_y \sin \theta_z - \sin \theta_x \cos \theta_z \\ -\sin \theta_y & \sin \theta_x \cos \theta_y & \cos \theta_x \cos \theta_y \end{bmatrix}. \quad (5.3.6)$$

The rotation matrix  $[\mathbf{R}]$  is an orthonormal matrix, i.e. it satisfies

$$[\mathbf{R}]^T = [\mathbf{R}]^{-1}, \quad (5.3.7)$$

$$\det[\mathbf{R}] = \pm 1. \quad (5.3.8)$$

Now, we can express the motion of a point  $\mathbf{X}$  on the object surface from  $\mathbf{X}$  to  $\mathbf{X}'$  as

$$\mathbf{X}' = [\mathbf{R}] \cdot \mathbf{X} + \mathbf{T}. \quad (5.3.9)$$

For many motion estimation algorithms, the non-linear rotation matrix according to Eq. (5.3.2) has to be linearized with respect to the rotation angles. Assuming rotation angles are small so that the approximations  $\cos(\alpha) \approx 1$  and  $\sin(\alpha) \approx \alpha$  are valid, Eq. (5.3.6) simplifies to

$$[\mathbf{R}] \approx [\mathbf{R}'] = \begin{bmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{bmatrix}. \quad (5.3.10)$$

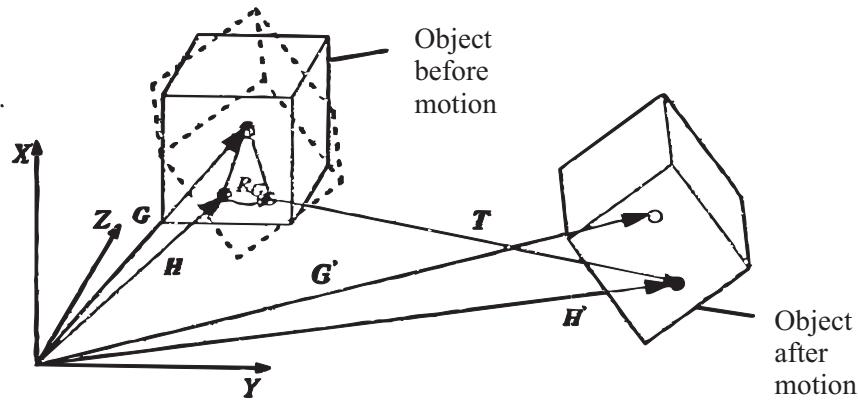
Equation (5.3.9) rotates the point  $\mathbf{X}$  on the object surface around the center of the world coordinate system. In the case that the object is far away from the center of the coordinate system and the object is just rotating around its own center, a large translation vector is required in this motion representation. This can be avoided by defining a local coordinate system for each object and by defining rotation and translation of the object with respect to its own center  $\mathbf{C} = (C_x, C_y, C_z)^T$ , which would also be the center of the local coordinate system (cf. Fig. 5.9):

$$\mathbf{X}' = [\mathbf{R}] \cdot (\mathbf{X} - \mathbf{C}) + \mathbf{C} + \mathbf{T}. \quad (5.3.11)$$

Assuming  $\mathbf{C} \neq 0$ , the rotation matrices in Eq. (5.3.9) and Eq. (5.3.11) are identical whereas the translation vectors differ for identical physical motion.

An object is rigid, if all its points move with the same 3D motion parameters. As shown above, the motion of a rigid object can be described by the parameters  $A^{(m)} = (T_x^{(m)}, T_y^{(m)}, T_z^{(m)}, \theta_x^{(m)}, \theta_y^{(m)}, \theta_z^{(m)})$ . A rigid object does not change its shape when it is moved with the parameter set  $A^{(m)}$ .

Not all real objects are rigid. One way to describe a flexible object is by decomposing it into two or more rigid *components*. Each component has its own set of motion parameters according to Eq. (5.3.11). Since each component is defined by its control points, the components are linked by triangles interconnecting control points in different components. Through these triangles, components are flexibly connected. Fig. 5.10 shows a scene with the objects “Background” and “Claire”. The model object “Claire” consists of two components “Head” and “Shoulder”. Alternatively, flexible objects can be described by superposing small local motions onto the rigid motion parameters  $A^{(m)}$ . If we have a sailing ship with a flag, we might describe the motion of the entire ship using a set of rigid parameters  $A^{(m)}$  and use local motion to model the waving of its flag in the wind. Since flexible objects change their shape due to local motion, it is not obvious whether this flexibility should be described by shape parameters or by motion parameters.



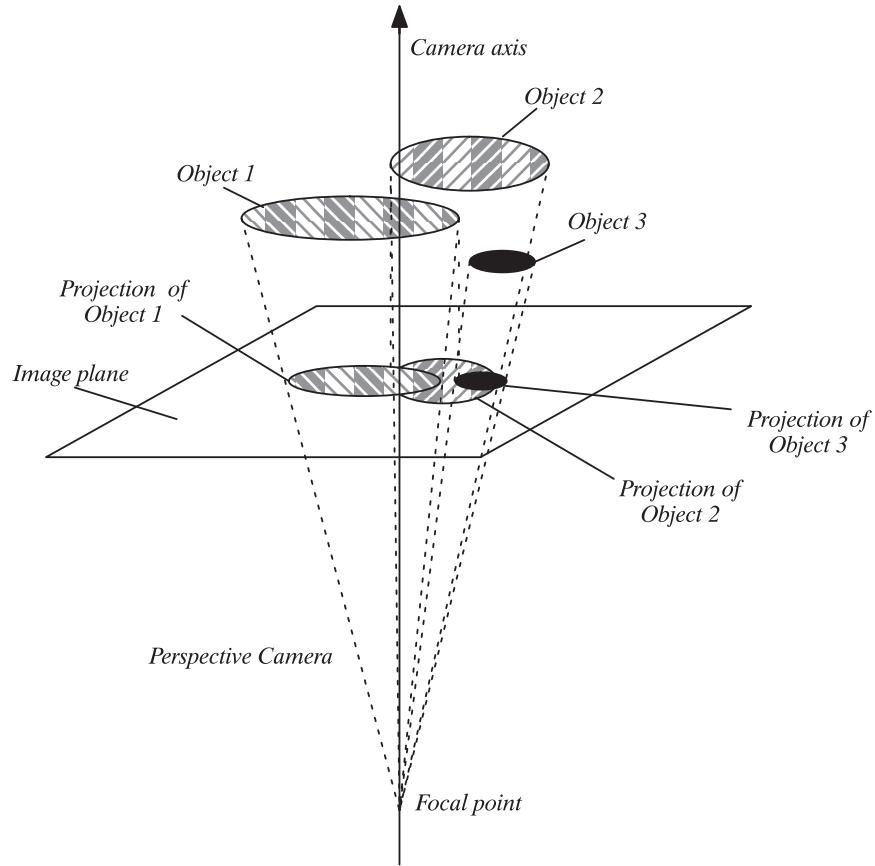
**Figure 5.9.** World coordinate system showing the motion of a point  $\mathbf{X}$ .



**Figure 5.10.** A model object consisting of two components. The shape is represented by a wireframe. The surface texture is extracted from an image.

#### 5.4 Scene Model

Having discussed how to model the illumination source, objects, and cameras, we are now ready to consider the modeling of an imaged scene. The scene model describes the world with illumination sources, objects and cameras. Depending on the models used for different components, we arrive at different scene models. Figure 5.11 shows a complex model scene that is created using a *3D scene model*. It is obtained using a camera model assuming perspective projection. This model allows for describing object motion, object occlusion as well as cast shadows. Each dashed line describes a *line of sight* according to the camera model. A point on the object surface is projected on the image plane where the line of sight intersects the image plane. The projection onto the image plane changes as the objects move. Objects 1 and 3 occlude part of Object 2. All objects occlude part of the *image*

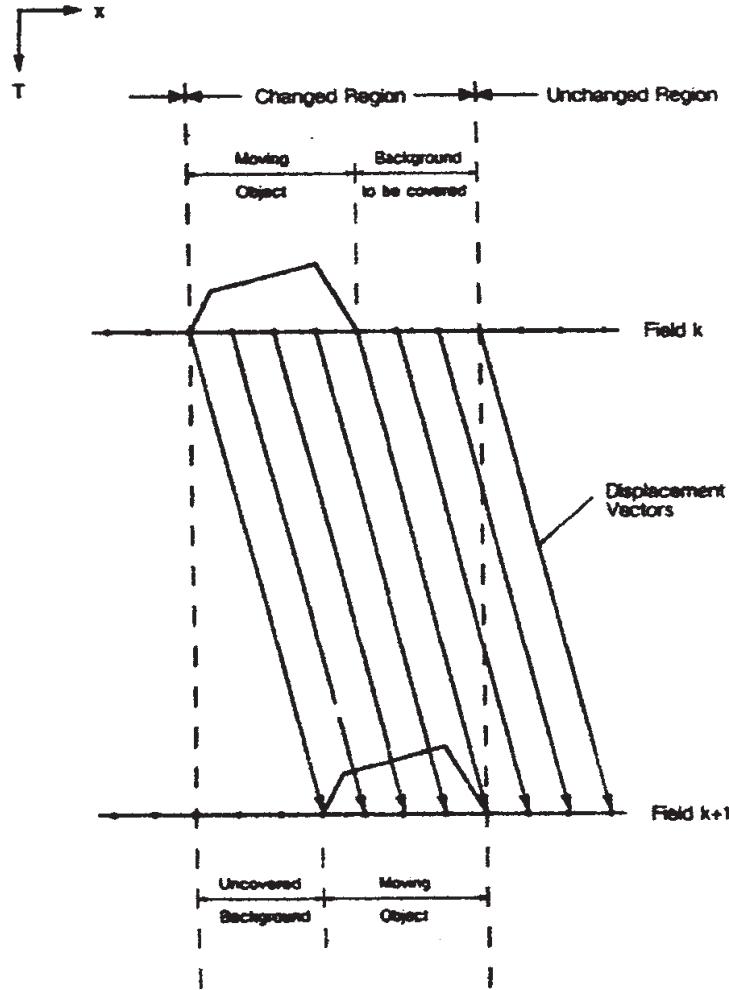


Editing instructions: Replace Object 2 by an rectangle.

**Figure 5.11.** A complex 3D scene with several objects, cameras and diffuse illumination. The diffuse illumination is not shown.

*background* which is defined as the image that we would have if there were no moving objects present in the scene. If the objects are in motion, we distinguish between four image areas: *static background*, *moving object*, *uncovered background* and *newly covered background*.

Fig. 5.12 shows two image frames with a moving object. Comparing frames  $k$  and  $k + 1$ , we can distinguish the *changed regions* and the *unchanged regions*. The unchanged regions show the static background in both images. The moving object is part of the changed regions in image  $k$  and  $k + 1$ . In frame  $k$ , the changed region is defined as the area of the moving object and the *background to be covered* in frame  $k + 1$  due to the object motion. In frame  $k + 1$ , the changed region is defined as the area of the moving object and the *uncovered background* that was not visible in

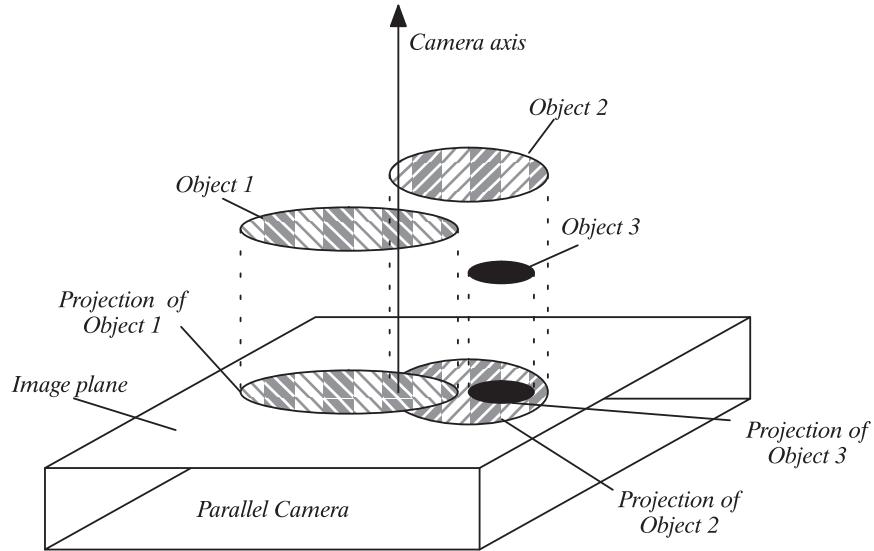


Note to the graphics illustrator: remove text 'displacement vectors' and the right 3 arrows, rename Field to frame

**Figure 5.12.** The separation of changed areas into moving objects, uncovered background and background to be covered (covered background). From[5, Fig. 5].

frame  $k$ . Due to the perspective projection of the camera model, Object 3 extends to image areas not covered by Object 2. The scene in Fig. 5.11 is illuminated by diffuse illumination only. Therefore, we do not see any shadows.

In Fig. 5.13, we see a scene with layered 2D objects. The camera model uses



**Figure 5.13.** A 2.5 D scene with several objects, a camera and ambient illumination. The ambient illumination is not shown.

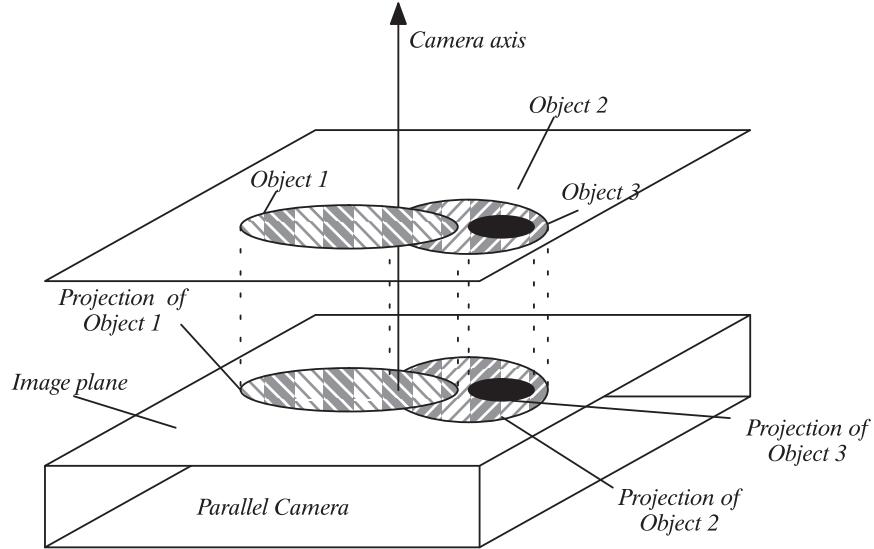
Note to graphics illustrator: Change 'Parallel' to 'Orthographic'

orthographic projection instead of perspective projection. Therefore depth has no influence on the image of this scene. We can describe how objects are layered on top of each other when viewed from the camera. This scene model does not allow describing effects due to 3D motion, since we do not explicitly use the 3rd dimension of space. This scene model is commonly referred to as a *2.5D scene model*. The MPEG-4 video coding standard supports layering of video objects and hence supports this scene model (Chapter 13).

Objects 1, 2 and 3 in Fig. 5.13 are at the same 3D location as in Fig. 5.11. Due to the use of parallel projection, the lines of sight are now parallel. Hence, Object 3 does not extend beyond Object 2 in the image plane anymore.

Fig. 5.14 shows the simplest scene model useful for image processing. It assumes all objects are flat and lay on the same image plane. 2D objects are limited to move in a 2D plane. The concept of object layering is not supported. This simple *2D scene model* is used in the video coding standards H.261, H.263, MPEG-1 and MPEG-2 (Chap. 13).

The selection of a specific scene model limits the choices for illumination, camera and object models. If we choose a 2D scene model, it makes no sense to pick an object model that explicitly describes 3D shapes. For some applications however, it may be useful to choose a 3D scene model and an object model that only allows for 2D shapes.



**Figure 5.14.** A 2D scene with several objects, a camera and ambient illumination. The ambient illumination is not shown.

Note to graphics illustrator: Change 'Parallel' to 'Orthographic'

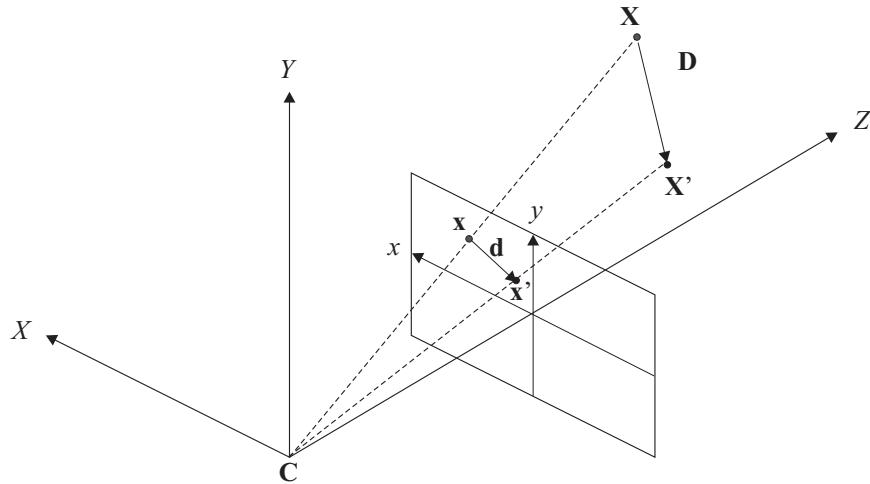
## 5.5 2D Motion Models

### 5.5.1 Definition and Notation

When the camera or the object in the scene is moving, the image of the same 3D object point will also change. Figure 5.15 illustrates the projection of a moving object onto an image plane and the relation between 3D and 2D motions. When an object point is moved from  $\mathbf{X} = [X, Y, Z]^T$  at time  $t_1$  to  $\mathbf{X}' = [X', Y', Z']^T = [X + D_X, Y + D_Y, Z + D_Z]^T$  at time  $t_2 = t_1 + d_t$ , its projected image is changed from  $\mathbf{x} = [x, y]^T$  to  $\mathbf{x}' = [x', y']^T = [x + d_x, y + d_y]^T$ . We call the 3D displacement,  $\mathbf{D}(\mathbf{X}; t_1, t_2) = \mathbf{X}' - \mathbf{X} = [D_X, D_Y, D_Z]^T$ , the *3D motion vector* at  $\mathbf{X}$ , while the 2D displacement,  $\mathbf{d}(\mathbf{x}; t_1, t_2) = \mathbf{x}' - \mathbf{x} = [d_x, d_y]^T$ , the *2D motion vector* at  $\mathbf{x}$ . Note that in general the motion vectors (MVs) are position-dependent. As a function of all image positions  $\mathbf{x}$  at time  $t_1$ ,  $\mathbf{d}(\mathbf{x}; t_1, t_2)$  represents a *2D motion field* from  $t_1$  to  $t_2$ . When the underlying  $t_1$  and  $t_2$  are clear, we simply write  $\mathbf{d}(\mathbf{x})$ .

Instead of describing the displacement between corresponding points, it is sometimes more convenient to specify, for each point  $\mathbf{x}$  at time  $t_1$ , its corresponding position at  $t_2$ ,  $\mathbf{w}(\mathbf{x}; t_1, t_2) = \mathbf{x}'$ . We call  $\mathbf{w}(\mathbf{x}; t_1, t_2)$  or  $\mathbf{w}(\mathbf{x})$  for short, the *mapping function*. Obviously, it is uniquely related to the motion field by  $\mathbf{w}(\mathbf{x}) = \mathbf{x} + \mathbf{d}(\mathbf{x})$ .

In this book, we only deal with a digital video signal that has a finite and discrete image domain, described by a truncated lattice,  $\Lambda$ . The notation  $\mathbf{x} = [x, y]^T \in \Lambda$  represents a pixel index. We further assume the time interval  $d_t = t_2 - t_1$  is

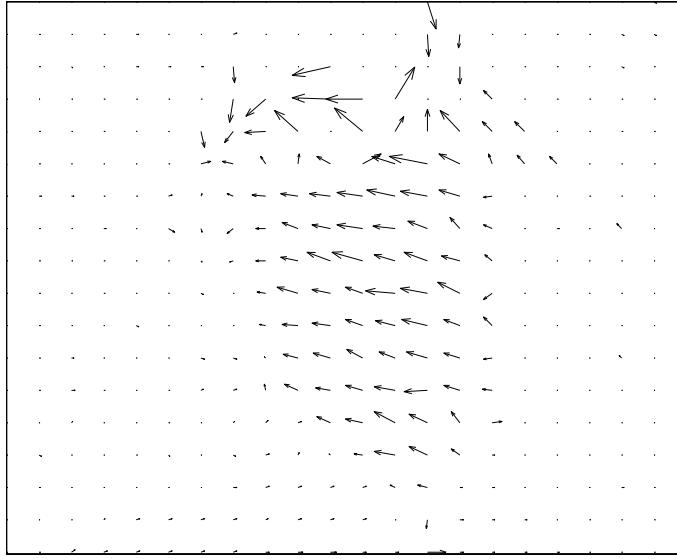


**Figure 5.15.** Projection of a moving object: relation between 3D and 2D motion vectors

either equal to the temporal sampling interval (i.e., the frame interval) or an integer multiple of this interval. The motion field for a given time interval is a finite set of 2D vectors arranged in a 2D array, in the same order as the pixels. Such a *discrete motion field* is often depicted by a vector graph, as shown in Fig. 5.16, where the direction and magnitude of each arrow represents the direction and magnitude of the MV at the pixel where the arrow originates.

Instead of using the actual displacements between a given time interval, one can also use the velocity vectors to characterize the motion. The velocity vectors are known as *flow vectors*, defined as  $\mathbf{v} = \frac{\partial \mathbf{d}}{\partial t} = \left[ \frac{\partial d_x}{\partial t}, \frac{\partial d_y}{\partial t} \right]^T$ . If the time interval  $d_t$  is sufficiently small, the motion within this time interval can be assumed to be constant, i.e.,  $\mathbf{v} = \mathbf{d}/d_t$ . When dealing with digital video, we often consider  $d_t = 1$ , so that the flow vectors are synonymous with the motion vectors. As with the definition of the motion field, one can define the *flow field* over the entire image domain, as  $\mathbf{v}(\mathbf{x}; t_1, t_2), \mathbf{x} \in \Lambda$ .

In a real video sequence, motion vectors are not always defined everywhere. As described in Sec. 5.4, there may be overlapping objects in a scene, one behind the other. The most common example (cf. Fig. 5.12) is a moving object in front of a static background. The part of the background that is exposed in a new frame is called *uncovered* region, while the part that is being covered by the front object the *covered* or *occluded* region. Motion vectors are not defined over uncovered regions. A complete definition of the 2D motion field at any given time should consist of a masking image and a motion field image. For example, in Fig. 5.12, if the motion is defined from frame  $k + 1$  to frame  $k$ , then the pixels in the uncovered region in frame  $k + 1$  should be labeled as uncovered.



**Figure 5.16.** A typical 2D motion field

### 5.5.2 2D Motion Models Corresponding to Typical Camera Motions

In this section, we describe 2D motion models corresponding to some of the camera motions depicted in Fig. 5.4. These models are derived by assuming the perspective imaging geometry shown in Fig. 5.2(a). That is, the image coordinate  $(x, y)$  are related to the world coordinate  $(X, Y, Z)$  by the perspective mapping given in Eq. (5.1.2).

To derive the 2D motion resulting from a camera motion, we first determine the new 3D coordinate of an object point with respect to the new camera coordinate, and then determine its image position by using Eq. (5.1.2).

**Camera Translation within the Image Plane (Track and Boom):** Recall that track and boom refer to the translations of the optical center along the X- and Y-axes, respectively (cf. Fig. 5.4(a)). Let  $T_x$  and  $T_y$  denote the actual translations. Then the 3D position of any point  $(X, Y, Z)$  in the old camera coordinate will change to

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ 0 \end{bmatrix}. \quad (5.5.1)$$

Using (5.1.2), the imaged position  $(x, y)$  changes to

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} FT_x/Z \\ FT_y/Z \end{bmatrix}.$$

Therefore, the displacement of a pixel  $(x, y)$  depends on the  $Z$  coordinate of its corresponding 3D point. However, when the relative variation of the depth of the imaged object is small compared to the average distance  $\bar{Z}$  between the object and the camera center, one can approximate the motion field by a constant vector, i.e.,

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix},$$

where  $t_x = FT_x/\bar{Z}$ ,  $t_y = FT_y/\bar{Z}$ . Note that a translation along the Z-axis (i.e., dolly) will lead to a substantially more complex 2D motion field, as will become clear later.

**Camera Pan and Tilt:** Pan and tilt refer to the rotation of the camera around the Y- and X-axes, respectively (cf. Fig. 5.4(b)). Let  $\theta_y$  and  $\theta_x$  denote the corresponding rotation angles. The new camera coordinate is related to the old one by

$$\mathbf{X}' = [\mathbf{R}_x][\mathbf{R}_y]\mathbf{X} \quad (5.5.2)$$

where  $[\mathbf{R}_x]$  and  $[\mathbf{R}_y]$  are as defined previously in Eqs. (5.3.3,5.3.4). When the rotation angles are small, we have

$$[\mathbf{R}_x][\mathbf{R}_y] = \begin{bmatrix} 1 & 0 & \theta_y \\ 0 & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{bmatrix}. \quad (5.5.3)$$

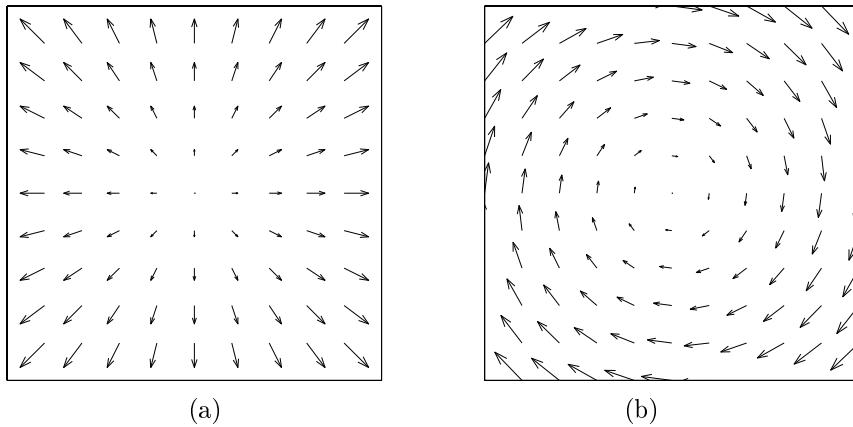
If  $Y\theta_x \ll Z$  and  $X\theta_y \ll Z$ , then  $Z' \approx Z$ . Using (5.1.2), we arrive at

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \theta_y F \\ -\theta_x F \end{bmatrix}. \quad (5.5.4)$$

or

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} \theta_y F \\ -\theta_x F \end{bmatrix}. \quad (5.5.5)$$

The above result implies that when the pan and tilt angles are small, and when the imaged object is far from the camera, so that its extension in the  $X$  and  $Y$  directions are significantly smaller than its distance from the camera, the image of the object is displaced uniformly.



**Figure 5.17.** The motion fields corresponding to (a) a camera zoom; and (b) a camera rotation.

**Camera Zoom:** Let  $F$  and  $F'$  represent the focal lengths before and after zooming. Using Eq. (5.1.2), it is easily shown that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \rho x \\ \rho y \end{bmatrix}, \quad (5.5.6)$$

where  $\rho = F'/F$  is the zoom factor. The 2D motion field is

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} (1 - \rho)x \\ (1 - \rho)y \end{bmatrix}. \quad (5.5.7)$$

A typical motion field corresponding to zoom is illustrated in Fig. 5.17(a).

**Camera Roll:** Recall that this refers to the rotation around the  $Z$ -axis (cf. Fig. 5.4(b)). Because such a rotation does not change the  $Z$ -coordinate, the net-effect after perspective projection is that the image coordinate undergoes a rotation, i.e.,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta_z & -\sin \theta_z \\ \sin \theta_z & \cos \theta_z \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \approx \begin{bmatrix} 1 & -\theta_z \\ \theta_z & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad (5.5.8)$$

where the approximation is valid when  $\theta_z$  is small. The corresponding motion field is

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} -\theta_z y \\ \theta_z x \end{bmatrix}. \quad (5.5.9)$$

Figure 5.17(b) illustrates a typical motion field due to rotation.

**Four Parameter Model:** Consider a camera that goes through translation, pan, tilt, zoom, and rotation in sequence. Using the approximations presented above, the imaged positions of the same 3D point before and after the camera motions are related by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \rho \begin{bmatrix} \cos \theta_z & -\sin \theta_z \\ \sin \theta_z & \cos \theta_z \end{bmatrix} \begin{bmatrix} x + \theta_y F + t_x \\ y - \theta_x F + t_y \end{bmatrix} \quad (5.5.10)$$

$$= \begin{bmatrix} c_1 & -c_2 \\ c_2 & c_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_3 \\ c_4 \end{bmatrix}. \quad (5.5.11)$$

The above mapping function has four parameters and is a special case of the affine mapping, which generally has six parameters. This specialized form is known as the *geometric mapping*, which can characterize any combination of scaling, rotation and translation in 2D. If the order of pan, zoom, and rotation is changed, the above form would still be valid, albeit the relations between the parameters  $c_k$  and the camera motion parameters,  $\rho, \theta_x, \theta_y, \theta_z, t_x, t_y$  would change.

### 5.5.3 2D Motion Corresponding to 3D Rigid Motion

The previous 2D motion models do not cover all possible 3D motions of the camera. Specifically, it does not consider camera translation in the z-direction nor does it consider rotation around an arbitrary axis. An arbitrary camera motion in 3D can be decomposed into a 3D rotation and a 3D translation. Similarly, an imaged object may undergo a rigid motion (rotation plus translation) while the camera is held steady. In general, both the camera and object can undergo a rigid motion, and it is hard to separate the motion of the camera and object. The overall effect however is equivalent to assuming one item (object or camera) is stationary and another item (camera or object) is undergoing a rigid motion. In this section, we consider the 2D motion induced by such an arbitrary 3D rigid motion. Without loss of generality, we will assume that the camera is stationary and the object is undergoing a rigid motion.

#### General Case

As described in Sec. 5.3, the 3D positions of any object point before and after a rigid motion are related by

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}. \quad (5.5.12)$$

Although the rotation matrix has nine entries, it is completely determined by three rotation angles (cf. Eq. (5.3.6)). Together with the 3 translation parameters, there are six parameters.

Substituting Eq. (5.1.2) into Eq. (5.5.12), after some manipulation, one can obtain the relation between the image coordinates before and after the motion:

$$\begin{aligned}x' &= F \frac{(r_1x + r_2y + r_3F)Z + T_xF}{(r_7x + r_8y + r_9F)Z + T_zF}, \\y' &= F \frac{(r_4x + r_5y + r_6F)Z + T_yF}{(r_7x + r_8y + r_9F)Z + T_zF}.\end{aligned}\quad (5.5.13)$$

Note that if we scale the translational parameters  $T_x, T_y, T_z$  and the depth  $Z$  by the same factor, the correspondence between  $(x, y)$  and  $(x', y')$  will not change. This indicates that based on the correspondences of imaged coordinates, the parameters  $T_x, T_y, T_z$ , are unique only up to a scaling factor. Alternatively, we say that only the direction but not the length of the translation vector can be determined. In general, when the object surface (i.e., the  $Z$  value corresponding to every  $(x, y)$ ) is spatially varying, the mapping function changes from point to point, and hence cannot be characterized by a small number of parameters.

### Projective Mapping

When there is no translational motion in the  $Z$ -direction, or when the imaged object has a planar surface,<sup>1</sup> it is easy to show that (Prob. 5.4) Eq. (5.5.13) can be simplified into the following 8-parameter *projective mapping*:<sup>2</sup>

$$x' = \frac{a_0 + a_1x + a_2y}{1 + c_1x + c_2y}, \quad y' = \frac{b_0 + b_1x + b_2y}{1 + c_1x + c_2y}.\quad (5.5.14)$$

There are eight free parameters in the projective mapping, which are dependent on the five motion parameters (recall that the three translation parameters are unique only up to a scale and therefore only two of them are free) and three plane parameters of the object surface.

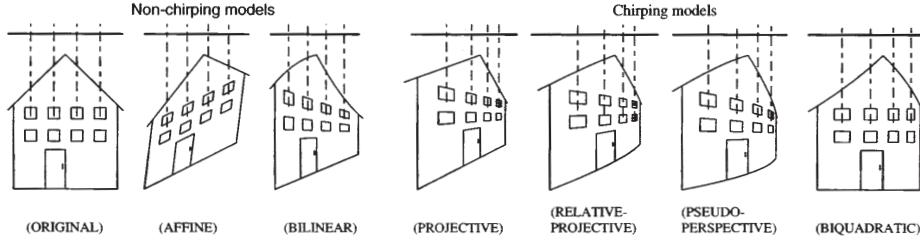
The projective mapping is an important relation in the study of motion between two video frames, or equivalently the registration of two video frames. It models *exactly* the relation between images of the same object after camera or object motion, if either of the following is true: i) the object has a planar surface (in this case, the camera or object or both can undergo arbitrary 3D rotation and translation); ii) neither the camera nor the object undergoes translation along the imaging axis (i.e., the  $Z$ -axis)<sup>3</sup> (in this case, the object geometry can be arbitrary). In practice, when the imaged scene does not have a planar surface, we can divide the imaged domain into small elements so that the 3D patch corresponding to each 2D element

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<sup>1</sup>This means that the 3D coordinates of points on the object satisfy  $aX + bY + cZ = 1$  for some constants  $a, b, c$ .

<sup>2</sup>Such a mapping is sometimes referred to as *perspective mapping*. Here to avoid the confusion with the relation between 3D and 2D coordinates in a perspective imaging system, we use the term “projective”.

<sup>3</sup>A motion that does not include translation in the imaging direction is sometimes referred as zero-parallax motion.



**Figure 5.18.** Pictorial effects of different mapping functions. From [6, Fig. 3]

is flat and make use of the above projective model over each patch. Note that the projective mapping includes as special cases pure translation and affine mapping.

When the imaging geometry can be approximated by an orthographic projection (cf. Eq. (5.1.3)), it is easy to show that a planar patch undergoing 3D rigid motion can be described by an affine function. See Prob. 5.2.

The projective mapping can characterize the two unique phenomena associated with perspective imaging [6]: i) the chirping effect, which refers to the increase in the perceived spatial frequency as the distance from the camera increases; and ii) the converging effect (also known as the keystone effect), which refers to the fact that two parallel lines appear to move closer at distance and eventually converge to a single point. These effects are illustrated in Fig. 5.18.

Another important property of projective mapping is that the concatenation of two projective mappings is another projective mapping [6]. This has a beneficial implication for estimation of a projective mapping between two images: one can estimate the parameters in several iterations; as long as each step leads to a projective mapping, the overall mapping is guaranteed to be projective.

The projective mapping acting on a square block is illustrated in Fig. 5.19(d). The projective function can map between two arbitrary quadrilaterals. Instead of using the parameters  $a_l, b_l$  and  $c_l$ , one can also use the displacement  $\mathbf{d}_k = (x'_k - x_k, y'_k - y_k)$ ,  $k = 1, 2, 3, 4$  between two quadrangles at the four corners to parameterize a projective mapping. Problem 5.9 considers how to determine the projective mapping parameters from  $\mathbf{x}_k$  and  $\mathbf{d}_k$ .

The motion field corresponding to a projective mapping is

$$d_x(x, y) = \frac{a_0 + a_1 x + a_2 y}{1 + c_1 x + c_2 y} - x, \quad d_y(x, y) = \frac{b_0 + b_1 x + b_2 y}{1 + c_1 x + c_2 y} - y. \quad (5.5.15)$$

We would like to point out that the motion field itself cannot be described by a projective function. In [6], the mapping corresponding to a motion field that is approximated by a projective mapping is called *relative projective*. Its effect is shown in Fig. 5.18. We can see that it can approximate a projective mapping relatively well.

### 5.5.4 Approximations of Projective Mapping

In practice, in order to circumvent the problem caused by the rational form, the projective mapping is often approximated by polynomial mapping. In this section, we discuss some of the common approximations.

**Affine Motion:** The *affine motion* has the form

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} a_0 + a_1x + a_2y \\ b_0 + b_1x + b_2y \end{bmatrix}. \quad (5.5.16)$$

It has six parameters,  $a_k, b_k, k = 0, 1, 2$ . As we already show, the projected 2D motion of most camera motions can be described by an affine function. Furthermore, a planar patch undergoing arbitrary 3D rigid motion can be described by an affine function under orthographic projection. The affine model, however, cannot capture either the chirping or converging effect of the projective mapping, as illustrated in Fig. 5.18.

The affine motion can be visualized as the deformation of a triangle into another, by moving the corners of the triangle. The affine parameters are completely determined by the MVs of the three corners. Therefore, an affine motion can also be parameterized by these MVs. In the example shown in Fig. 5.19(b), one can only specify the displacements at three corners, and the displacement at the fourth corner is determined by the MVs at the other three corners.

**Bilinear Motion:** The *bilinear motion* has the form of

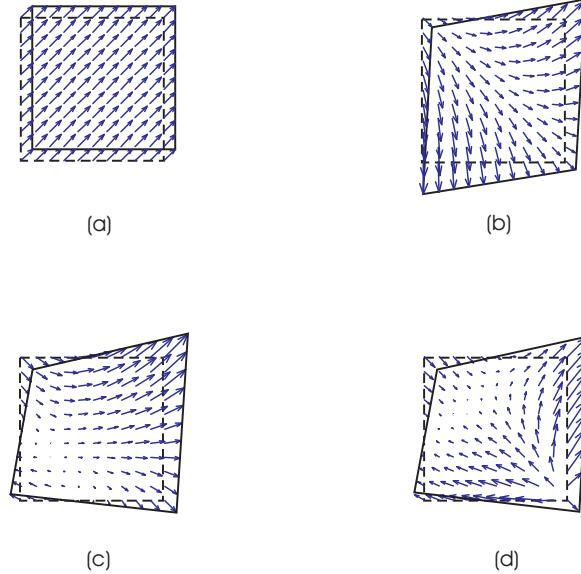
$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} a_0 + a_1x + a_2y + a_3xy \\ b_0 + b_1x + b_2y + b_3xy \end{bmatrix}. \quad (5.5.17)$$

It has eight parameters,  $a_k, b_k, k = 0, 1, 2, 3$ . From Fig. 5.18, we see that the bilinear mapping can capture the converging effect of the projective mapping, but not the chirping effect.

The bilinear motion can be visualized as the warping of a square into a quadrangle, as illustrated in Fig. 5.19(c). The eight parameters are completely determined by the MVs of the four corners in the original quadrilateral. Note that, unlike the projective mapping, the bilinear function cannot map between two arbitrary quadrangles, even though such a mapping has only 8 degrees of freedom. This is because the bilinear function maps a straight line to a curved line, unless the original line is horizontal or vertical. In general, the bilinear function maps a quadrilateral to a curvilinear quadrilateral [15].

**Other Polynomial Models:** In general, any motion function can be approximated by a polynomial mapping of the form

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \sum_{\substack{0 \leq i, j \leq N_1 \\ i + j \leq N_2}} \begin{bmatrix} a_{i,j} \\ b_{i,j} \end{bmatrix} x^i y^j. \quad (5.5.18)$$



**Figure 5.19.** Illustration of basic motion models: (a) translational; (b) affine; (c) bilinear; (d) projective.

The translation, affine, and bilinear motions are special cases of this model with  $N_1 = N_2 = 0$ , for translation,  $N_1 = N_2 = 1$  for affine, and  $N_1 = 1, N_2 = 2$  for bilinear. The case corresponding to  $N_1 = 2, N_2 = 2$  is

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy \\ b_0 + b_1x + b_2y + b_3x^2 + b_4y^2 + b_5xy \end{bmatrix}. \quad (5.5.19)$$

This is known as *biquadratic mapping*, which has 12 parameters. As shown in Fig. 5.18, the biquadratic mapping can produce the chirping effect but not the converging effect.

In [6], Mann and Picard compared the pros-and-cons of various polynomial approximations for the projective mapping. They have found that the affine function, although having fewer parameters, is more susceptible to noise because it lacks the correct degrees of freedom needed to properly track the actual image motion. On the other hand, the bilinear function, although possessing the correct number of parameters, fails to capture the chirping effect. A better eight-parameter model is the following *pseudo-perspective mapping*:

$$\begin{bmatrix} d_x(x, y) \\ d_y(x, y) \end{bmatrix} = \begin{bmatrix} a_0 + a_1x + a_2y + a_3xy + b_3x^2 \\ b_0 + b_1x + b_2y + b_3xy + a_3y^2 \end{bmatrix}. \quad (5.5.20)$$

From Fig. 5.18, we see that this mapping can produce both the chirping and converging effect, and is the best approximation of the projective mapping using low-order polynomials.

## 5.6 Summary

In this chapter, we discussed parametric models that allow us to describe how an image sequence is generated. Depending on the application, we can use models that model the real world with an appropriate amount of detail.

### Camera Model

The camera model describes the projection of the 3D world on to the image plane by means of the real camera. Depending on our application, we can use camera models of different complexity. If the objects in the 3D world are far away from the camera we might use a simple camera model with orthographic projection. A pinhole camera model uses a perspective projection and enables us to describe the change of object size in an image sequence as an object changes its distance from the camera.

### Illumination Model

In order to see an object, we need to illuminate the scene we are looking at. Describing illumination and the reflection of light on object surfaces requires usually complex models. However, we assume the most simple illumination model for many video processing applications: ambient light and diffuse reflecting surfaces. Under this model, the surface reflectance of an object does not change as the object moves. This model is sometimes referred to as *constant intensity assumption*. The motion estimation algorithms presented in Chaps. 6 and 7 make use of this assumption extensively. Note that, with this model, we cannot describe shadows or glossy reflections.

### Object Model

Objects are described by shape, motion and texture. The 3D shape of an object can be described by its volume using cubes of unit length named voxels or by its surface using a wireframe. We express 3D object motion by means of a 3D translation vector and a 3x3 rotation matrix that is computed from the rotation angles around the three coordinate axes. Simple models can be derived by simplifying the 3D object shape and motion models.

### Scene Model

The scene model describes how the moving objects and the camera of a 3D scene are positioned with respect to each other. In video coding, we often use a 2D scene model that assumes 2D objects moving parallel to the imaging plane of the camera. The slightly more complex 2.5D scene model allows us to handle object occlusion. Finally, the 3D scene model can describe the real world realistically.

## 2D Motion Model

Object or camera motions in 3D lead to 2D motions. The 2D motion model depends not only on the 3D motion model, but also the illumination model and the camera model. The most important 2D motion model is the projective mapping, which is valid over small 2D regions, when the object/camera undergoes rigid motion, the camera can be modeled by perspective projection, and the constant intensity assumptions applies. In practice, this mapping is often approximately by affine or bilinear mappings.

## 5.7 Problems

- 5.1** Assume a pinhole camera with focal length  $F = 9\text{mm}$ , a target of  $1'' \times 1.33''$  and an image resolution of  $352 \times 288$  pels and an object point with a distance of  $Z = 2\text{ m}$  from the camera center. Determine the projection of this point into the image as a function of its  $(X, Y)$  position in 3D space. How much does the point have to move in the direction of the  $Z$ -axis in order to move its image point by 1 pel? What is the answer if we assume a camera model with orthographic projection?
- 5.2** Show that, under orthographic projection, the projected 2D motion of a planar patch undergoing translation, rotation and scaling (because of camera zoom) can be described by an affine function.
- 5.3** Show that, under perspective projection, the projected 2D motion of a planar patch undergoing rigid motion can be described by a projective mapping.
- 5.4** Show that Eq. (5.5.13) can be simplified into the *projective mapping* giving in Eq. 5.5.14, when the imaged object has a planar surface. That is, 3D coordinates of points on the object satisfy  $aX+bY+cZ = 1$  for some constants  $a, b, c$ .
- 5.5** Consider a triangle whose original corner positions are at  $\mathbf{x}_k, k = 1, 2, 3$ . Suppose each corner is moved by  $\mathbf{d}_k$ . Determine the affine parameters that can realize such a mapping.
- 5.6** Suppose that you are given the motion vectors  $\mathbf{d}_k$  at  $K > 3$  points  $\mathbf{x}_k$  in an image, and that you want to use the affine mapping to approximate the overall motion. How do you determine the affine parameters?  
Hint: use a least squares fitting method.
- 5.7** Repeat 5.5 for the bilinear mapping between two quadrangles. Assume that displacements at  $K=4$  points are available.
- 5.8** Repeat 5.6 for the bilinear mapping, assuming  $K > 4$ .

- 5.9** Repeat 5.5 for the projective mapping between two quadrangles. Assume that displacements at  $K=4$  points are available.

Hint: you can still set-up a linear equation in terms of the parameters  $a_k, b_k, c_k$ . The constants in the left side of the equation will contain both  $\mathbf{x}_k$  and  $\mathbf{x}'_k = \mathbf{x}_k + \mathbf{d}_k$ .

- 5.10** Repeat 5.6 for the projective mapping, assuming  $K > 4$ .

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## Chapter 6

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# TWO DIMENSIONAL MOTION ESTIMATION

Motion estimation is an important part of any video processing system. In this chapter, we are only concerned with the estimation of 2D motion. In Chapter 7, we will discuss estimation of actual 3D motion. As will be seen, 2D motion estimation is often a preprocessing step required for 3D structure and motion estimation. Also, 2D motion estimation itself has a wide range of applications, including video compression, video sampling rate conversion, video filtering, etc. Depending on the intended applications for the resulting 2D motion vectors, motion estimation methods could be very different. For example, for computer vision applications, where the 2D motion vectors are to be used to deduce 3D structure and motion parameters, a sparse set of 2D motion vectors at critical feature points may be sufficient. The motion vectors must be physically correct for them to be useful. On the other hand, for video compression applications, the estimated motion vectors are used to produce a motion-compensated prediction of a frame to be coded from a previously coded reference frame. The ultimate goal is to minimize the total bits used for coding the motion vectors and the prediction errors. There is a trade-off that one can play between the accuracy of the estimated motion, and the number of bits used to specify the motion. Sometimes, even when the estimated motion is not an accurate representation of the true physical motion, it can still lead to good temporal prediction and in that regard is considered a good estimate. In this chapter, we focus on the type of motion estimation algorithms targeted for motion compensated processing (prediction, filtering, interpolation, etc.). For additional readings on this topic, the reader is referred to the review papers by Musmann *et al.* [28] and by Stiller and Konrad [38]. For a good treatment of motion estimation methods for computer vision applications, please see the article by Aggarwal and Nandhahumar [1].

All the motion estimation algorithms are based on temporal changes in image intensities (more generally color). In fact, the observed 2D motions based on intensity changes may not be the same as the actual 2D motions. To be more precise, the velocity of observed or apparent 2D motion vectors are referred to as *optical*

*flow.* Optical flow can be caused not only by object motions, but also camera movements or illumination condition changes. In this chapter, we start by defining optical flow. We then derive the optical flow equation, which imposes a constraint between image gradients and flow vectors. This is a fundamental equality that many motion estimation algorithms are based on. Next we present the general methodologies for 2D motion estimation. As will be seen, motion estimation problem is usually converted to an optimization problem and involves three key components: parameterization of the motion field, formulation of the optimization criterion, and finally searching of the optimal parameters. Finally, we present motion estimation algorithms developed based on different parameterizations of the motion field and different estimation criteria. Unless specified otherwise, the word “motion” refers to 2D motion, in this chapter.

## 6.1 Optical Flow

### 6.1.1 2D Motion vs. Optical Flow

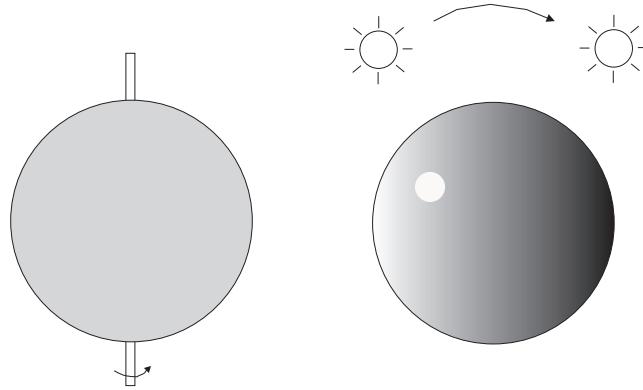
The human eye perceives motion by identifying corresponding points at different times. The correspondence is usually determined by assuming that the color or brightness of a point does not change after the motion. It is interesting to note that the observed 2D motion can be different from the actual projected 2D motion under certain circumstances. Figure 6.1 illustrates two special cases. In the first example, a sphere with a uniform flat surface is rotating under a constant ambient light. Because every point on the sphere reflects the same color, the eye cannot observe any change in the color pattern of the imaged sphere and thus considers the sphere as being stationary. In the second example, the sphere is stationary, but is illuminated by a point light source that is rotating around the sphere. The motion of the light source causes the movement of the reflecting light spot on the sphere, which in turn can make the eye believe the sphere is rotating. The observed or apparent 2D motion is referred to as *optical flow* in computer vision literature. The above examples reveal that the optical flow may not be the same as the true 2D motion. When only image color information is available, the best one can hope to estimate accurately is the optical flow. However, in the remaining part of this chapter, we will use the term 2D motion or simply motion to describe optical flow. The readers should bear in mind that sometimes it may be different from the true 2D motion.

### 6.1.2 Optical Flow Equation and Ambiguity in Motion Estimation

Consider a video sequence whose luminance variation is represented by  $\psi(x, y, t)$ .<sup>1</sup> Suppose an imaged point  $(x, y)$  at time  $t$  is moved to  $(x + d_x, y + d_y)$  at time  $t + d_t$ . Under the *constant intensity assumption* introduced in Sec. 5.2.3 (Eq. (5.2.11)), the

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<sup>1</sup>In this book, we only consider motion estimation based on the luminance intensity information, although the same methodology can be applied to the full color information.



**Figure 6.1.** The optical flow is not always the same as the true motion field. In (a), a sphere is rotating under a constant ambient illumination, but the observed image does not change. In (b), a point light source is rotating around a stationary sphere, causing the highlight point on the sphere to rotate. Adapted from [17, Fig.12-2].

images of the same object point at different times have the same luminance value. Therefore,

$$\psi(x + d_x, y + d_y, t + d_t) = \psi(x, y, t). \quad (6.1.1)$$

Using Taylor's expansion, when  $d_x, d_y, d_t$  are small, we have

$$\psi(x + d_x, y + d_y, t + d_t) = \psi(x, y, t) + \frac{\partial\psi}{\partial x}d_x + \frac{\partial\psi}{\partial y}d_y + \frac{\partial\psi}{\partial t}d_t. \quad (6.1.2)$$

Combining Eqs. (6.1.1) and (6.1.2) yields

$$\frac{\partial\psi}{\partial x}d_x + \frac{\partial\psi}{\partial y}d_y + \frac{\partial\psi}{\partial t}d_t = 0. \quad (6.1.3)$$

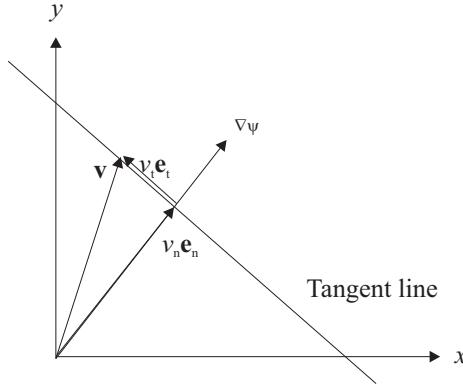
The above equation is written in terms of the motion vector  $(d_x, d_y)$ . Dividing both sides by  $d_t$  yields

$$\frac{\partial\psi}{\partial x}v_x + \frac{\partial\psi}{\partial y}v_y + \frac{\partial\psi}{\partial t} = 0 \quad \text{or} \quad \nabla\psi^T \mathbf{v} + \frac{\partial\psi}{\partial t} = 0. \quad (6.1.4)$$

where  $(v_x, v_y)$  represents the velocity vector,  $\nabla\psi = \left[ \frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y} \right]^T$  is the spatial gradient vector of  $\psi(x, y, t)$ . In arriving at the above equation, we have assumed that  $d_t$  is small, so that  $v_x = d_x/d_t, v_y = d_y/d_t$ . The above equation is commonly known as the *optical flow equation*.<sup>2</sup> The conditions for this relation to hold is the same as that for the constant intensity assumption, and have been discussed previously in Sec. 5.2.3.

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<sup>2</sup> Another way to derive the optical flow equation is by representing the constant intensity assumption as  $\frac{d\psi(x,y,t)}{dt} = 0$ . Expanding  $\frac{d\psi(x,y,t)}{dt}$  in terms of the partials will lead to the same equation.



**Figure 6.2.** Decomposition of motion  $\mathbf{v}$  into normal ( $v_n \mathbf{e}_n$ ) and tangent  $v_t \mathbf{e}_t$  components. Given  $\nabla\psi$  and  $\frac{\partial\psi}{\partial t}$ , any MV on the tangent line satisfies the optical flow equation.

As shown in Fig. 6.2, the flow vector  $\mathbf{v}$  at any point  $\mathbf{x}$  can be decomposed into two orthogonal components as

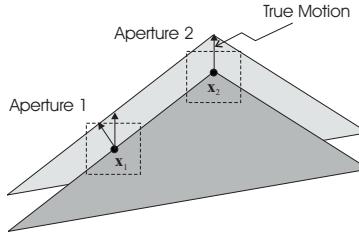
$$\mathbf{v} = v_n \mathbf{e}_n + v_t \mathbf{e}_t, \quad (6.1.5)$$

where  $\mathbf{e}_n$  is the direction vector of the image gradient  $\nabla\psi$ , to be called the normal direction, and  $\mathbf{e}_t$  is orthogonal to  $\mathbf{e}_n$ , to be called the tangent direction. The optical flow equation in Eq. (6.1.4) can be written as

$$v_n \|\nabla\psi\| + \frac{\partial\psi}{\partial t} = 0, \quad (6.1.6)$$

where  $\|\nabla\psi\|$  is the magnitude of the gradient vector. Three consequences from Eq. (6.1.4) or (6.1.6) are:

1. At any pixel  $\mathbf{x}$ , one cannot determine the motion vector  $\mathbf{v}$  based on  $\nabla\psi$  and  $\frac{\partial\psi}{\partial t}$  alone. There is only one equation for two unknowns ( $v_x$  and  $v_y$ , or  $v_n$  and  $v_t$ ). In fact, the underdetermined component is  $v_t$ . To solve both unknowns, one needs to impose additional constraints. The most common constraint is that the flow vectors should vary smoothly spatially, so that one can make use of the intensity variation over a small neighborhood surrounding  $\mathbf{x}$  to estimate the motion at  $\mathbf{x}$ .
2. Given  $\nabla\psi$  and  $\frac{\partial\psi}{\partial t}$ , the projection of the motion vector along the normal direction is fixed, with  $v_n = -\frac{\partial\psi}{\partial t} / \|\nabla\psi\|$ , whereas the projection onto the tangent direction,  $v_t$ , is undetermined. Any value of  $v_t$  would satisfy the optical flow equation. In Fig. 6.2, this means that any point on the tangent line will satisfy the optical flow equation. This ambiguity in estimating the motion vector is known as the *aperture problem*. The word “aperture” here refers to



**Figure 6.3.** The aperture problem in motion estimation: To estimate the motion at  $\mathbf{x}_1$  using aperture 1, it is impossible to determine whether the motion is upward or perpendicular to the edge, because there is only one spatial gradient direction in this aperture. On the other hand, the motion at  $\mathbf{x}_2$  can be determined accurately, because the image has gradient in two different directions in aperture 2. Adapted from [39, Fig. 5.7].

the small window over which to apply the constant intensity assumption. The motion can be estimated uniquely only if the aperture contains at least two different gradient directions, as illustrated in Fig. 6.3.

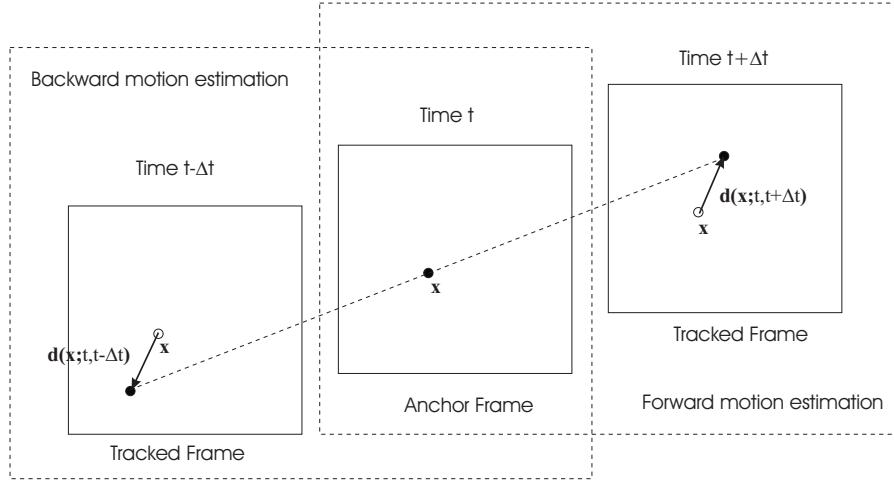
3. In regions with constant brightness so that  $\|\nabla\psi\| = 0$ , the flow vector is indeterminate. This is because there is no perceived brightness changes when the underlying surface has a flat pattern. The estimation of motion is reliable only in regions with brightness variation, i.e., regions with edges or non-flat textures.

The above observations are consistent with the relation between spatial and temporal frequencies discussed in Sec. 2.3.2. There, we have shown that the temporal frequency of a moving object is zero if the spatial frequency is zero, or if the motion direction is orthogonal to the spatial frequency. When the temporal frequency is zero, no changes can be observed in image patterns, and consequently, motion is indeterminate.

As will be seen in the following sections, the optical flow equation or, equivalently, the constant intensity assumption plays a key role in all motion estimation algorithms.

## 6.2 General Methodologies

In this chapter, we consider the estimation of motion between two given frames,  $\psi(x, y, t_1)$  and  $\psi(x, y, t_2)$ . Recall from Sec. 5.5.1, the MV at  $\mathbf{x}$  between time  $t_1$  and  $t_2$  is defined as the displacement of this point from  $t_1$  to  $t_2$ . We will call the frame at time  $t_1$  the *anchor frame*, and the frame at  $t_2$  the *tracked frame*. Depending on the intended application, the anchor frame can be either before or after the tracked frame in time. As illustrated in Fig. 6.4, the problem is referred to as *forward motion estimation*, when  $t_1 < t_2$ , and as *backward motion estimation*, when  $t_1 > t_2$ . For notation convenience, from now on, we use  $\psi_1(\mathbf{x})$  and  $\psi_2(\mathbf{x})$  to denote the



**Figure 6.4.** Forward and backward motion estimation. Adapted from [39, Fig. 5.5].

anchor and tracked frames, respectively. In general, we can represent the motion field as  $\mathbf{d}(\mathbf{x}; \mathbf{a})$ , where  $\mathbf{a} = [a_1, a_2, \dots, a_L]^T$  is a vector containing all the motion parameters. Similarly, the mapping function can be denoted by  $\mathbf{w}(\mathbf{x}; \mathbf{a}) = \mathbf{x} + \mathbf{d}(\mathbf{x}; \mathbf{a})$ . The motion estimation problem is to estimate the motion parameter vector  $\mathbf{a}$ . Methods that have been developed can be categorized into two groups: feature-based and intensity-based. In the *feature-based* approach, correspondences between pairs of selected feature points in two video frames are first established. The motion model parameters are then obtained by a least squares fitting of the established correspondences into the chosen motion model. This approach is only applicable to parametric motion models and can be quite effective in, say, determining global motions. The *intensity-based* approach applies the constant intensity assumption or the optical flow equation at every pixel and requires the estimated motion to satisfy this constraint as closely as possible. This approach is more appropriate when the underlying motion cannot be characterized by a simple model, and that an estimate of a pixel-wise or block-wise motion field is desired.

In this chapter, we only consider intensity-based approaches, which are more widely used in applications requiring motion compensated prediction and filtering. In general, the intensity-based motion estimation problem can be converted into an optimization problem, and three key questions need to be answered: i) how to parameterize the underlying motion field? ii) what criterion to use to estimate the parameters? and iii) how to search for the optimal parameters? In this section, we first describe several ways to represent a motion field. Then we introduce different types of estimation criteria. Finally, we present search strategies commonly used for motion estimation. Specific motion estimation schemes using different motion representations and estimation criteria will be introduced in subsequent sections.

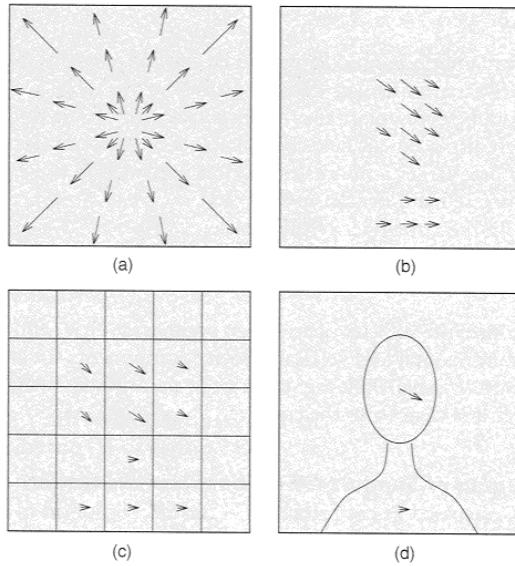
### 6.2.1 Motion Representation

A key problem in motion estimation is how to parameterize the motion field. As shown in Sec. 5.5, the 2D motion field resulting from a camera or object motion can usually be described by a few parameters. However, usually, there are multiple objects in the imaged scene that move differently. Therefore, a global parameterized model is usually not adequate. The most direct and unconstrained approach is to specify the motion vector at every pixel. This is the so-called *pixel-based representation*. Such a representation is universally applicable, but it requires the estimation of a large number of unknowns (twice the number of pixels!) and the solution can often be physically incorrect unless a proper physical constraint is imposed during the estimation step. On the other hand, if only the camera is moving or the imaged scene contains a single moving object with a planar surface, one could use a *global motion representation* to characterize the entire motion field. In general, for scenes containing multiple moving objects, it is more appropriate to divide an image frame into multiple regions so that the motion within each region can be characterized well by a parameterized model. This is known as *region-based motion representation*,<sup>3</sup> which consists of a region segmentation map and several sets of motion parameters, one for each region. The difficulty with such an approach is that one does not know in advance which pixels have similar motions. Therefore, segmentation and estimation have to be accomplished iteratively, which requires intensive amount of computations that may not be feasible in practice.

One way to reduce the complexity associated with region-based motion representation is by using a fixed partition of the image domain into many small blocks. As long as each block is small enough, the motion variation within each block can be characterized well by a simple model and the motion parameters for each block can be estimated independently. This brings us to the popular *block-based representation*. The simplest version models the motion in each block by a constant translation, so that the estimation problem becomes that of finding one MV for each block. This method provides a good compromise between accuracy and complexity, and has found great success in practical video coding systems. One main problem with the block-based approach is that it does not impose any constraint on the motion transition between adjacent blocks. The resulting motion is often discontinuous across block boundaries, even when the true motion field is changing smoothly from block to block. One approach to overcome this problem is by using a *mesh-based representation*, by which the underlying image frame is partitioned into non-overlapping polygonal elements. The motion field over the entire frame is described by the MVs at the nodes (corners of polygonal elements) only, and the MVs at the interior points of an element are interpolated from the nodal MVs. This representation induces a motion field that is continuous everywhere. It is more appropriate than the block-based representation over interior regions of an object,

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<sup>3</sup>This is sometimes called *object-based motion representation* [27]. Here we use the word “region-based” to acknowledge the fact that we are only considering 2D motions, and that a region with a coherent 2D motion may not always correspond to a physical object.



**Figure 6.5.** Different motion representations: (a) global, (b) pixel-based, (c) block-based, and (d) region-based. From [38, Fig. 3].

which usually undergoes a continuous motion, but it fails to capture motion discontinuities at object boundaries. Adaptive schemes that allow discontinuities when necessary is needed for more accurate motion estimation. Figure 6.5 illustrates the effect of several motion representations described above for a head-and-shoulder scene. In the next few sections, we will introduce motion estimation methods using different motion representations.

### 6.2.2 Motion Estimation Criteria

For a chosen motion model, the problem is how to estimate the model parameters. In this section, we describe several different criteria for estimating motion parameters.

#### Criterion based on Displaced Frame Difference

The most popular criterion for motion estimation is to minimize the sum of the errors between the luminance values of every pair of corresponding points between the anchor frame  $\psi_1$  and the tracked frame  $\psi_2$ . Recall that  $\mathbf{x}$  in  $\psi_1$  is moved to  $\mathbf{w}(\mathbf{x}; \mathbf{a})$  in  $\psi_2$ . Therefore, the objective function can be written as,

$$E_{\text{DFD}}(\mathbf{a}) = \sum_{\mathbf{x} \in \Lambda} |\psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})|^p, \quad (6.2.1)$$

where  $\Lambda$  is the domain of all pixels in  $\psi_1$ , and  $p$  is a positive number. When  $p = 1$ , the above error is called mean absolute difference (MAD), and when  $p = 2$ , the mean squared error (MSE). The error image,  $e(\mathbf{x}; \mathbf{a}) = \psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})$ , is usually called *displaced frame difference (DFD) image*, and the above measure the *DFD error*.

The necessary condition for minimizing  $E_{\text{DFD}}$  is that its gradient vanishes. In the case of  $p = 2$ , this gradient is

$$\frac{\partial E_{\text{DFD}}}{\partial \mathbf{a}} = 2 \sum_{\mathbf{x} \in \Lambda} (\psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})) \frac{\partial \mathbf{d}(\mathbf{x})}{\partial \mathbf{a}} \nabla \psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) \quad (6.2.2)$$

where

$$\frac{\partial \mathbf{d}}{\partial \mathbf{a}} = \left[ \begin{array}{cccc} \frac{\partial d_x}{\partial a_1} & \frac{\partial d_x}{\partial a_2} & \dots & \frac{\partial d_x}{\partial a_L} \\ \frac{\partial d_y}{\partial a_1} & \frac{\partial d_y}{\partial a_2} & \dots & \frac{\partial d_y}{\partial a_L} \end{array} \right]^T.$$

### Criterion based on Optical Flow Equation

Instead of minimizing the DFD error, another approach is to solve the system of equations established based on the optical flow constraint given in Eq. (6.1.3). Let  $\psi_1(x, y) = \psi(x, y, t)$ ,  $\psi_2(x, y) = \psi(x, y, t + d_t)$ . If  $d_t$  is small, we can assume  $\frac{\partial \psi}{\partial t} dt = \psi_2(\mathbf{x}) - \psi_1(\mathbf{x})$ . Then, Eq. (6.1.3) can be written as

$$\frac{\partial \psi_1}{\partial x} d_x + \frac{\partial \psi_1}{\partial y} d_y + (\psi_2 - \psi_1) = 0 \quad \text{or} \quad \nabla \psi_1^T \mathbf{d} + (\psi_2 - \psi_1) = 0. \quad (6.2.3)$$

This discrete version of the optical flow equation is more often used for motion estimation in digital videos. Solving the above equations for all  $\mathbf{x}$  can be turned into a minimization problem with the following objective function:

$$E_{\text{flow}}(\mathbf{a}) = \sum_{\mathbf{x} \in \Lambda} |\nabla \psi_1(\mathbf{x})^T \mathbf{d}(\mathbf{x}; \mathbf{a}) + \psi_2(\mathbf{x}) - \psi_1(\mathbf{x})|^p. \quad (6.2.4)$$

The gradient of  $E_{\text{flow}}$  is, when  $p = 2$ ,

$$\frac{\partial E_{\text{flow}}}{\partial \mathbf{a}} = 2 \sum_{\mathbf{x} \in \Lambda} (\nabla \psi_1(\mathbf{x})^T \mathbf{d}(\mathbf{x}; \mathbf{a}) + \psi_2(\mathbf{x}) - \psi_1(\mathbf{x})) \frac{\partial \mathbf{d}(\mathbf{x})}{\partial \mathbf{a}} \nabla \psi_1(\mathbf{x}). \quad (6.2.5)$$

If the motion field is constant over a small region  $\Lambda'$ , i.e.,  $\mathbf{d}(\mathbf{x}; \mathbf{a}) = \mathbf{d}_0, \mathbf{x} \in \Lambda'$ , then Eq. (6.2.5) becomes

$$\frac{\partial E_{\text{flow}}}{\partial \mathbf{d}_0} = \sum_{\mathbf{x} \in \Lambda'} (\nabla \psi_1(\mathbf{x})^T \mathbf{d}_0 + \psi_2(\mathbf{x}) - \psi_1(\mathbf{x})) \nabla \psi_1(\mathbf{x}). \quad (6.2.6)$$

Setting the above gradient to zero yields the least squares solution for  $\mathbf{d}_0$ :

$$\mathbf{d}_0 = \left( \sum_{\mathbf{x} \in \Lambda'} \nabla \psi_1(\mathbf{x}) \nabla \psi_1(\mathbf{x})^T \right)^{-1} \left( \sum_{\mathbf{x} \in \Lambda'} (\psi_1(\mathbf{x}) - \psi_2(\mathbf{x})) \nabla \psi_1(\mathbf{x}) \right). \quad (6.2.7)$$

When the motion is not a constant, but can be related to the model parameters linearly, one can still derive a similar least-squares solution. See Prob. 6.6 in the Problem section.

An advantage of the above method is that the minimizing function is a quadratic function of the MVs, when  $p = 2$ . If the motion parameters are linearly related to the MVs, then the function has a unique minimum and can be solved easily. This is not true with the DFD error given in Eq. (6.2.1). However, the optical flow equation is valid only when the motion is small, or when an initial motion estimate  $\tilde{\mathbf{d}}(\mathbf{x})$  that is close to the true motion can be found and one can pre-update  $\psi_2(\mathbf{x})$  to  $\psi_2(\mathbf{x} + \tilde{\mathbf{d}}(\mathbf{x}))$ . When this is not the case, it is better to use the DFD error criterion, and find the minimal solution using the gradient descent or exhaustive search method.

### Regularization

Minimizing the DFD error or solving the optical flow equation does not always give physically meaningful motion estimate. This is partially because the constant intensity assumption is not always correct. The imaged intensity of the same object point may vary after an object motion because of the various reflectance and shadowing effects. Secondly, in a region with flat texture, many different motion estimates can satisfy the constant intensity assumption or the optical flow equation. Finally, if the motion parameters are the MVs at every pixel, the optical flow equation does not constrain the motion vector completely. These factors make the problem of motion estimation a ill-posed problem.

To obtain a physically meaningful solution, one needs to impose additional constraints to regularize the problem. One common regularization approach is to add a penalty term to the error function in (6.2.1) or (6.2.4), which should enforce the resulting motion estimate to bear the characteristics of common motion fields. One well-known property of a typical motion field is that it usually varies smoothly from pixel to pixel, except at object boundaries. To enforce the smoothness, one can use a penalty term that measures the differences between the MVs of adjacent pixels, i.e.,

$$E_s(\mathbf{a}) = \sum_{\mathbf{x} \in \Lambda} \sum_{\mathbf{y} \in \mathcal{N}_{\mathbf{x}}} \|\mathbf{d}(\mathbf{x}; \mathbf{a}) - \mathbf{d}(\mathbf{y}; \mathbf{a})\|^2, \quad (6.2.8)$$

where  $\|\cdot\|$  represents the 2-norm,  $\mathcal{N}_{\mathbf{x}}$  represents the set of pixels adjacent to  $\mathbf{x}$ . Either the 4-connectivity or 8-connectivity neighborhood can be used.

The overall minimization criterion can be written as

$$E = E_{\text{DFD}} + w_s E_s. \quad (6.2.9)$$

The weighting coefficient  $w_s$  should be chosen based on the importance of motion smoothness relative to the prediction error. To avoid over-blurring, one should reduce the weighting at object boundaries. This, however, requires accurate detection of object boundaries, which is not a trivial task.

### Bayesian Criterion

The Bayesian estimator is based on a probabilistic formulation of the motion estimation problem, pioneered by Konrad and Dubois [22, 38]. Under this formulation, given an anchor frame  $\psi_1$ , the image function at the tracked frame  $\psi_2$  is considered a realization of a random field  $\Psi$ , and the motion field  $\mathbf{d}$  is a realization of another random field  $\mathcal{D}$ . The *a posterior* probability distribution of the motion field  $\mathcal{D}$  given a realization of  $\Psi$  and  $\psi_1$  can be written, using the Bayes rule

$$P(\mathcal{D} = \mathbf{d} | \Psi = \psi_2; \psi_1) = \frac{P(\Psi = \psi_2 | \mathcal{D} = \mathbf{d}; \psi_1) P(\mathcal{D} = \mathbf{d}; \psi_1)}{P(\Psi = \psi_2; \psi_1)}. \quad (6.2.10)$$

In the above notation, the semicolon indicates that subsequent variables are deterministic parameters. An estimator based on the Bayesian criterion attempts to maximize the *a posterior* probability. But for given  $\psi_1$  and  $\psi_2$ , maximizing the above probability is equivalent to maximizing the numerator only. Therefore, the maximum a posterior (MAP) estimate of  $\mathbf{d}$  is

$$\mathbf{d}_{\text{MAP}} = \operatorname{argmax}_{\mathbf{d}} \{P(\Psi = \psi_2 | \mathcal{D} = \mathbf{d}; \psi_1) P(\mathcal{D} = \mathbf{d}; \psi_1)\}. \quad (6.2.11)$$

The first probability denotes the likelihood of an image frame given the motion field and the anchor frame. Let  $\mathcal{E}$  represent the random field corresponding to the DFD image  $e(\mathbf{x}) = \psi_2(\mathbf{x} + \mathbf{d}) - \psi_1(\mathbf{x})$  for given  $\mathbf{d}$  and  $\psi_1$ , then

$$P(\Psi = \psi_2 | \mathcal{D} = \mathbf{d}; \psi_1) = P(\mathcal{E} = e),$$

and the above equation becomes

$$\begin{aligned} \mathbf{d}_{\text{MAP}} &= \operatorname{argmax}_{\mathbf{d}} \{P(\mathcal{E} = e) P(\mathcal{D} = \mathbf{d}; \psi_1)\} \\ &= \operatorname{argmin}_{\mathbf{d}} \{-\log P(\mathcal{E} = e) - \log P(\mathcal{D} = \mathbf{d}; \psi_1)\}. \end{aligned} \quad (6.2.12)$$

From the source coding theory (Sec. 8.3.1), the minimum coding length for a source  $\mathcal{X}$  is its entropy,  $-\log P(\mathcal{X} = x)$ . We see that the MAP estimate is equivalent to minimizing the sum of the coding length for the DFD image  $e$  and that for the motion field  $\mathbf{d}$ . As will be shown in Sec. 9.3.1, this is precisely what a video coder using motion-compensated prediction needs to code. Therefore, the MAP estimate for  $\mathbf{d}$  is equivalent to a *minimum description length* (MDL) estimate [34]. Because the purpose of motion estimation in video coding is to minimize the bit rate, the MAP criterion is a better choice than minimizing the prediction error.

The most common model for the DFD image is a zero-mean independently identically distributed (i.i.d.) Gaussian field, with distribution

$$P(\mathcal{E} = e) = (2\pi\sigma^2)^{-|\Lambda|/2} \exp \frac{-\sum_{\mathbf{x} \in \Lambda} e^2(\mathbf{x})}{2\sigma^2}, \quad (6.2.13)$$

where  $|\Lambda|$  denotes the size of  $\Lambda$  (i.e., the number of pixels in  $\Lambda$ ). With this model, minimizing the first term in Eq. (6.2.12) is equivalent to minimizing the previously defined DFD error (when  $p = 2$ ).

For the motion field  $\mathcal{D}$ , a common model is a Gibbs/Markov random field [11]. Such a model is defined by a neighborhood structure called *clique*. Let  $\mathcal{C}$  represent the set of cliques, the model assumes

$$P(\mathcal{D} = \mathbf{d}) = \frac{1}{Z} \exp\left(-\sum_{c \in \mathcal{C}} V_c(\mathbf{d})\right), \quad (6.2.14)$$

where  $Z$  is a normalization factor. The function  $V_c(\mathbf{d})$  is called the potential function, which is usually defined to measure the difference between pixels in the same clique:

$$V_c(\mathbf{d}) = \sum_{(\mathbf{x}, \mathbf{y}) \in c} |\mathbf{d}(\mathbf{x}) - \mathbf{d}(\mathbf{y})|^2. \quad (6.2.15)$$

Under this model, minimizing the second term in Eq. (6.2.12) is equivalent to minimizing the smoothing function in Eq. (6.2.8). Therefore, the MAP estimate is equivalent to the DFD-based estimator with an appropriate smoothness constraint.

### 6.2.3 Minimization Methods

The error functions presented in Sec. 6.2.2 can be minimized using various optimization methods. Here we only consider exhaustive search and gradient-based search methods. Usually, for the exhaustive search, the MAD is used for reasons of computational simplicity, whereas for the gradient-based search, the MSE is used for its mathematical tractability.

Obviously, the advantage of the exhaustive search method is that it guarantees reaching the global minimum. However, such search is feasible only if the number of unknown parameters is small, and each parameter takes only a finite set of discrete values. To reduce the search time, various fast algorithms can be developed, which achieve sub-optimal solutions.

The most common gradient descent methods include the *steepest gradient descent* and the *Newton-Ralphson method*. A brief review of these methods is provided in Appendix B. A gradient-based method can handle unknown parameters in a high dimensional continuous space. However, it can only guarantee the convergence to a local minimum. The error functions introduced in the previous section in general are not convex and can have many local minima that are far from the global minimum. Therefore, it is important to obtain a good initial solution through the use of *a prior* knowledge, or by adding a penalty term to make the error function convex.

With the gradient-based method, one must calculate the spatio-temporal gradients of the underlying signal. Appendix A reviews methods for computing first and second order gradients from digital sampled images. Note that the methods used for calculating the gradient functions can have profound impact on the accuracy and robustness of the associated motion estimation methods, as have been shown by Barron *et al.* [4]. Using a Gaussian pre-filter followed by a central difference generally leads to significantly better results than the simple two point difference approximation.

One important search strategy is to use a multi-resolution representation of the motion field and conduct the search in a hierarchical manner. The basic idea is to first search the motion parameters in a coarse resolution, propagate this solution into a finer resolution, and then refine the solution in the finer resolution. It can combat both the slowness of exhaustive search methods and the non-optimality of gradient-based methods. We will present the multi-resolution method in more detail in Sec. 6.9.

### 6.3 Pixel-Based Motion Estimation

In pixel-based motion estimation, one tries to estimate a motion vector for every pixel. Obviously, this problem is ill-defined. If one uses the constant intensity assumption, for every pixel in the anchor frame, there are many pixels in the tracked frame that have exactly the same image intensity. If one uses the optical flow equation instead, the problem is again indeterminate, because there is only one equation for two unknowns. To circumvent this problem, there are in general four approaches. First, one can use the regularization technique to enforce some smoothness constraints on the motion field, so that the motion vector of a new pixel is constrained by those found for surrounding pixels. Second, one can assume the motion vectors in a neighborhood surrounding each pixel are the same, and apply the constant intensity assumption or the optical flow equation over the entire neighborhood. The third approach is to make use of additional invariance constraints. In addition to intensity invariance, which leads to the optical flow equation, one can assume that the intensity gradient is invariant under motion, as proposed in [29, 26, 15]. Finally, one can also make use of the relation between the phase functions of the frame before and after motion [9]. In [4], Barron, *et al.* evaluated various methods for optical flow computation, by testing these algorithms on both synthetic and real world imageries. In this section, we will describe the first two approaches only. We will also introduce the pel-recursive type of algorithms which are developed for video compression applications.

#### 6.3.1 Regularization Using Motion Smoothness Constraint

Horn and Schunck [16] proposed to estimate the motion vectors by minimizing the following objective function, which is a combination of the flow-based criterion and a motion smoothness criterion:

$$E(\mathbf{v}(\mathbf{x})) = \sum_{\mathbf{x} \in \Lambda} \left( \frac{\partial \psi}{\partial x} v_x + \frac{\partial \psi}{\partial y} v_y + \frac{\partial \psi}{\partial t} \right)^2 + w_s (\|\nabla v_x\|^2 + \|\nabla v_y\|^2). \quad (6.3.1)$$

In their original algorithm, the spatial gradient of  $v_x$  and  $v_y$  are approximated by  $\nabla v_x = [v_x(x, y) - v_x(x-1, y), v_x(x, y) - v_x(x, y-1)]^T$ ,  $\nabla v_y = [v_y(x, y) - v_y(x-1, y), v_y(x, y) - v_y(x, y-1)]^T$ . The minimization of the above error function is accomplished by a gradient-based method known as Gauss-Siedel method.

Nagle and Enkelmann conducted a comprehensive evaluation of the effect of smoothness constraints on motion estimation [30]. In order to avoid over-smoothing of the motion field, Nagel suggested an *oriented-smoothness* constraint in which smoothness is imposed along the object boundaries, but not across the boundaries [29]. This has resulted in significant improvement in motion estimation accuracy [4].

### 6.3.2 Using a Multipoint Neighborhood

In this approach, when estimating the motion vector at a pixel  $\mathbf{x}_n$ , we assume that the motion vectors of all the pixels in a neighborhood  $\mathcal{B}(\mathbf{x}_n)$  surrounding it are the same, being  $\mathbf{d}_n$ . To determine  $\mathbf{d}_n$ , one can either minimize the prediction error over  $\mathcal{B}(\mathbf{x}_n)$ , or solve the optical flow equation using a least squares method. Here we present the first approach. To estimate the motion vector  $\mathbf{d}_n$  for  $\mathbf{x}_n$ , we minimize the DFD error over  $\mathcal{B}(\mathbf{x}_n)$ :

$$E_n(\mathbf{d}_n) = \frac{1}{2} \sum_{\mathbf{x} \in \mathcal{B}(\mathbf{x}_n)} w(\mathbf{x}) (\psi_2(\mathbf{x} + \mathbf{d}_n) - \psi_1(\mathbf{x}))^2, \quad (6.3.2)$$

where  $w(\mathbf{x})$  are the weights assigned to pixel  $\mathbf{x}$ . Usually, the weight decreases as the distance from  $\mathbf{x}$  to  $\mathbf{x}_n$  increases.

The gradient with respect to  $\mathbf{d}_n$  is

$$\mathbf{g}_n = \frac{\partial E_n}{\partial \mathbf{d}_n} = \sum_{\mathbf{x} \in \mathcal{B}(\mathbf{x}_n)} w(\mathbf{x}) e(\mathbf{x}, \mathbf{d}_n) \left. \frac{\partial \psi_2}{\partial \mathbf{x}} \right|_{\mathbf{x}+\mathbf{d}_n}, \quad (6.3.3)$$

where  $e(\mathbf{x}, \mathbf{d}_n) = \psi_2(\mathbf{x} + \mathbf{d}_n) - \psi_1(\mathbf{x})$  is the DFD at  $\mathbf{x}$  with the estimate  $\mathbf{d}_n$ . Let  $\mathbf{d}_n^l$  represent the estimate at the  $l$ -th iteration, the first order gradient descent method would yield the following update algorithm

$$\mathbf{d}_n^{l+1} = \mathbf{d}_n^l - \alpha \mathbf{g}_n(\mathbf{d}_n^l). \quad (6.3.4)$$

From Eq. (6.3.3), the update at each iteration depends on the sum of the image gradients at various pixels scaled by the weighted DFD values at those pixels.

One can also derive an iterative algorithm using the Newton-Ralphson method. From Eq. (6.3.3), the Hessian matrix is

$$\begin{aligned} \mathbf{H}_n &= \frac{\partial^2 E_n}{\partial \mathbf{d}_n^2} = \sum_{\mathbf{x} \in \mathcal{B}(\mathbf{x}_n)} w(\mathbf{x}) \left. \frac{\partial \psi_2}{\partial \mathbf{x}} \frac{\partial \psi_2}{\partial \mathbf{x}}^T \right|_{\mathbf{x}+\mathbf{d}_n} + w(\mathbf{x}) e(\mathbf{x}, \mathbf{d}_n) \left. \frac{\partial^2 \psi_2}{\partial \mathbf{x}^2} \right|_{\mathbf{x}+\mathbf{d}_n} \\ &\approx \sum_{\mathbf{x} \in \mathcal{B}(\mathbf{x}_n)} w(\mathbf{x}) \left. \frac{\partial \psi_2}{\partial \mathbf{x}} \frac{\partial \psi_2}{\partial \mathbf{x}}^T \right|_{\mathbf{x}+\mathbf{d}_n}. \end{aligned}$$

The Newton-Ralphson update algorithm is then (See Appendix B):

$$\mathbf{d}_n^{l+1} = \mathbf{d}_n^l - \alpha \mathbf{H}(\mathbf{d}_n^l)^{-1} \mathbf{g}_n(\mathbf{d}_n^l). \quad (6.3.5)$$

This algorithm converges faster than the first order gradient descent method, but it requires more computation in each iteration.

Instead of using gradient-based update algorithms, one can also use exhaustive search to find the  $\mathbf{d}_n$  that yields the minimal error within a defined search range. This will lead to the exhaustive block matching algorithm (EBMA) to be presented in Sec. 6.4.1. The difference from the EBMA is that the neighborhood used here is a sliding window and a MV is determined for each pixel by minimizing the error in its neighborhood. The neighborhood in general does not have to be a rectangular block.

### 6.3.3 Pel-Recursive Methods

In a video coder using motion compensated prediction, one needs to specify both the MVs and the DFD image. With a pixel-based motion representation, one would need to specify a MV for each pixel, which is very costly. In pel-recursive motion estimation methods, which are developed for video coding applications, the MVs are obtained recursively. Specifically, the MV at a current pixel is updated from those of its neighboring pixels that are coded before. The decoder can derive the same MV following the same update rule, so that the MVs do not need to be coded. A variety of such algorithms have been developed, where the update rules all follow some types of gradient-descent methods [31].

Although pel-recursive methods are very simple, their motion estimation accuracy is quite poor. As a result, the prediction error is still large and requires a significant number of bits to code. These algorithms have been used in earlier generations of video codecs because of their simplicity. Today's codecs use more sophisticated motion estimation algorithms, which can provide a better trade-off between the bits used for specifying MVs and the DFD image. The most popular one is the block matching algorithm to be discussed in the next section.

## 6.4 Block Matching Algorithm

As already seen, a problem with pixel-based motion estimation is that one must impose some smoothness constraints to regularize the problem. One way of imposing smoothness constraints on the estimated motion field is to divide the image domain into non-overlapping small regions, called blocks, and assume that the motion within each block can be characterized by a simple parametric model, e.g., constant, affine, or bilinear. If the block is sufficiently small, then this model can be quite accurate. In this section, we describe motion estimation algorithms developed using the block-based motion representation. We will use  $\mathcal{B}_m$  to represent the  $m$ -th image block,  $M$  the number of blocks, and  $\mathcal{M} = \{1, 2, \dots, M\}$ . The partition into blocks should satisfy

$$\bigcup_{m \in \mathcal{M}} \mathcal{B}_m = \Lambda, \quad \text{and} \quad \mathcal{B}_m \cap \mathcal{B}_n = \emptyset, \quad m \neq n.$$

Theoretically, a block can have any polygonal shape. But in practice, the square shape is used almost exclusively. The triangular shape has also been used, which is more appropriate when the motion in each block is described by an affine model.

In the simplest case, the motion in each block is assumed to be constant, i.e., the entire block undergoes a translation. This is called the *block-wise translational model*. In this section, we consider this simple case only, where the motion estimation problem is to find a single MV for each block. This type of algorithm is collectively referred as block matching algorithm (BMA). In the next section, we will consider the more general case where the motion in each block is characterized by a more complex model.

#### 6.4.1 The Exhaustive Search Block Matching Algorithm (EBMA)

Given an image block in the anchor frame  $\mathcal{B}_m$ , the motion estimation problem at hand is to determine a matching block  $\mathcal{B}'_m$  in the tracked frame so that the error between these two blocks is minimized. The displacement vector  $\mathbf{d}_m$  between the spatial positions of these two blocks (the center or a selected corner) is the MV of this block. Under the block-wise translational model,  $\mathbf{w}(\mathbf{x}; \mathbf{a}) = \mathbf{x} + \mathbf{d}_m, \mathbf{x} \in \mathcal{B}_m$ , the error in Eq. (6.2.1) can be written as:

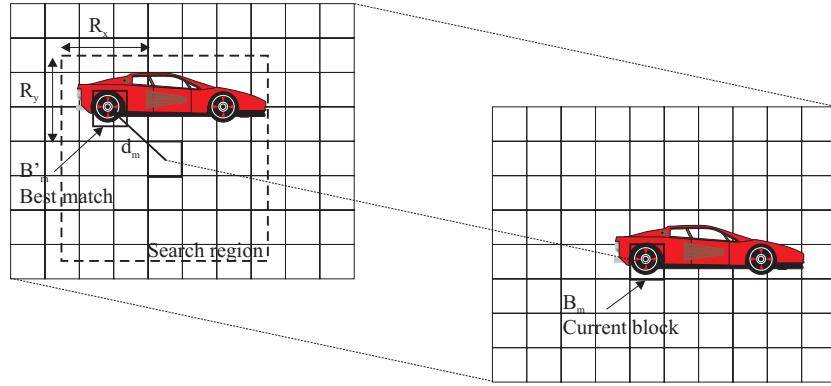
$$E(\mathbf{d}_m, \forall m \in \mathcal{M}) = \sum_{m \in \mathcal{M}} \sum_{\mathbf{x} \in \mathcal{B}_m} |\psi_2(\mathbf{x} + \mathbf{d}_m) - \psi_1(\mathbf{x})|^p. \quad (6.4.1)$$

Because the estimated MV for a block only affects the prediction error in this block, one can estimate the MV for each block individually, by minimizing the prediction error accumulated over this block only, which is:

$$E_m(\mathbf{d}_m) = \sum_{\mathbf{x} \in \mathcal{B}_m} |\psi_2(\mathbf{x} + \mathbf{d}_m) - \psi_1(\mathbf{x})|^p, \quad (6.4.2)$$

One way to determine the  $\mathbf{d}_m$  that minimizes the above error is by using exhaustive search and this method is called exhaustive block matching algorithm (EBMA). As illustrated in Fig. 6.6, the EBMA determines the optimal  $\mathbf{d}_m$  for a given block  $\mathcal{B}_m$  in the anchor frame by comparing it with all candidate blocks  $\mathcal{B}'_m$  in the tracked frame within a predefined search region and finding the one with the minimum error. The displacement between the two blocks is the estimated motion vector.

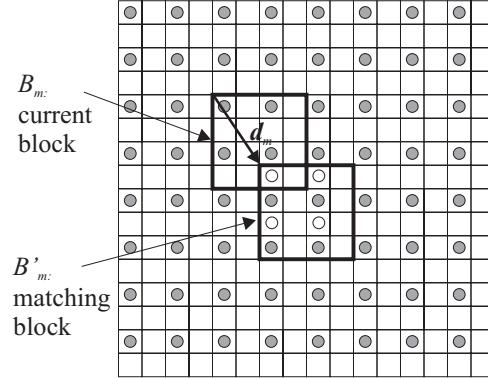
To reduce the computational load, the MAD error ( $p = 1$ ) is often used. The search region is usually symmetric with respect to the current block, up to  $R_x$  pixels to the left and right, and up to  $R_y$  pixels above and below, as illustrated in Fig. 6.6. If it is known that the dynamic range of the motion is the same in horizontal and vertical directions, then  $R_x = R_y = R$ . The estimation accuracy is determined by the search stepsize, which is the distance between two nearby candidate blocks in the horizontal or vertical direction. Normally, the same stepsize is used along the two directions. In the simplest case, the stepsize is one pixel, which is known as *integer-pel accuracy search*.



**Figure 6.6.** The search procedure of the exhaustive block matching algorithm.

Let the block size be  $N \times N$  pixels, and the search range be  $\pm R$  pixels in both horizontal and vertical directions (cf. Fig. 6.6). With a stepsize of one pixel, the total number of candidate matching blocks for each block in the anchor frame is  $(2R+1)^2$ . Let an operation be defined as consisting of one subtraction, one absolute value computation, and one addition. The number of operations for calculating the MAD for each candidate estimate is  $N^2$ . The number of operations for estimating the MV for one block is then  $(2R+1)^2N^2$ . For an image of size  $M \times M$ , there are  $(M/N)^2$  blocks (assuming  $M$  is a multiple of  $N$ ). The total number of operations for a complete frame is then  $M^2(2R+1)^2$ . It is interesting to note that the overall computational load is independent of the block size  $N$ .

As an example, consider  $M = 512, N = 16, R = 16$ , the total operation count per frame is  $2.85 \times 10^8$ . For a video sequence with a frame rate of 30 fps, the operation required per second is  $8.55 \times 10^9$ , an astronomical number! This example shows that EBMA requires intense computation, which poses a challenge to applications requiring software-only implementation. Because of this problem, various fast algorithms have been developed, which trade off the estimation accuracy for reduced computations. Some fast algorithms are presented in Sec. 6.4.3. One advantage of EBMA is that it can be implemented in hardware using simple and modular design, and speed-up can be achieved by using multiple modules in parallel. There have been many research efforts on efficient realization of the EBMA using VLSI/ASIC chips, which sometimes involve slight modification of the algorithm to trade off the accuracy for reduced computation or memory space or memory access. For a good review of VLSI architecture for implementing EBMA and other fast algorithms for block matching, see [21, 32, 14].



**Figure 6.7.** Half-pel accuracy block matching. Filled circles are samples existing in the original tracked frame, open circles are samples to be interpolated for calculating the matching error, for a candidate MV  $\mathbf{d}_m = (-1, -1.5)$ . Instead of calculating these samples on-demand for each candidate MV, a better approach is to pre-interpolate the entire tracked frame.

#### 6.4.2 Fractional Accuracy Search

As already hinted, the stepsize for searching the corresponding block in the BMA algorithm does not have to be an integer. For more accurate motion representation, fractional-pel accuracy is needed. A problem with using fractional stepsizes is that there may not be corresponding sample points in the tracked frame for certain sample points in the anchor frame. These samples need to be interpolated from the available sample points. Bilinear interpolation is commonly used for this purpose. In general, to realize a stepsize of a  $1/K$  pixel, the tracked frame needs to be interpolated by a factor of  $K$  first. An example of  $K = 2$  is shown in Fig. 6.7, which is known as *half-pel accuracy search*. It has been shown that half-pel accuracy search can provide a significant improvement in the estimation accuracy over the integer-pel accuracy search, especially for low-resolution videos.

A question that naturally arises is what is the appropriate search stepsize for motion estimation. Obviously, it depends on the intended application of estimated motion vectors. For video coding, where the estimated motion is used to predict a current frame (the anchor frame) from a previously coded frame (the tracked frame), it is the prediction error (i.e., the DFD error) that should be minimized. A statistical analysis of the relation between the prediction error and search precision has been considered by Girod [12] and will be presented in Sec. 9.3.5.

Obviously, with a fractional-pel stepsize, the complexity of the EBMA is further increased. For example, with half-pel search, the number of search points is quadrupled over that using integer-pel accuracy. The overall complexity is more than quadrupled, considering the extra computation required for interpolating the tracked frame.

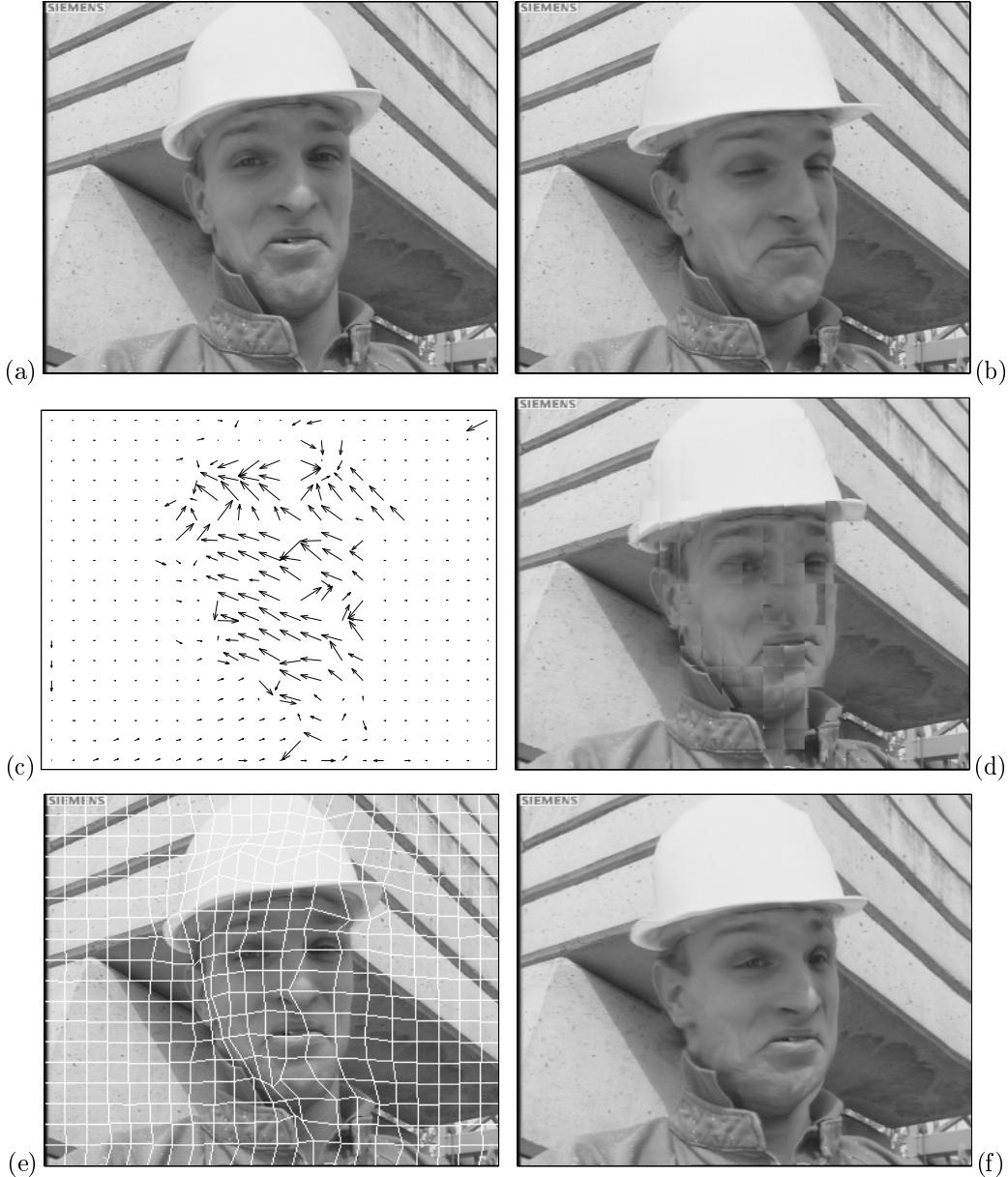
**Example 6.1:** Figure 6.8(c) shows the estimated motion field by a half-pel EBMA algorithm for two given frames in Figure 6.8(a-b). Figure 6.8(d) shows the predicted anchor frame based on the estimated motion. This is obtained by replacing each block in the anchor frame by its best matching block in the tracked frame. The image size is  $352 \times 288$  and the block size is  $16 \times 16$ . We can see that a majority of blocks are predicted accurately, however, there are blocks that are not well-predicted. Some of these blocks undergo non-translational motions, such as those blocks covering the eyes and the mouth. Other blocks contain both the foreground object and the background and only the foreground object is moving. They are also blocks where the image intensity change is due to the change in the reflection patterns when the head turns. The motion variation over these blocks cannot be approximated well by a constant MV, and the EBMA algorithm simply identifies a block in the tracked frame that has the smallest absolute error from a given block in the anchor frame. Furthermore, the predicted image is discontinuous along certain block boundaries, which is the notorious blocking artifact common with the EBMA algorithm. These artifacts are due to the inherent limitation of the block-wise translational motion model, and the fact that the MV for a block is determined independent of the MVs of its adjacent blocks.

The accuracy between a predicted image and the original one is usually measured by PSNR defined previously in Eq. (1.5.6). The PSNR of the predicted image by the half-pel EBMA is 29.72 dB. With the integer-pel EBMA, the resulting predicted image is visually very similar, although the PSNR is slightly lower.

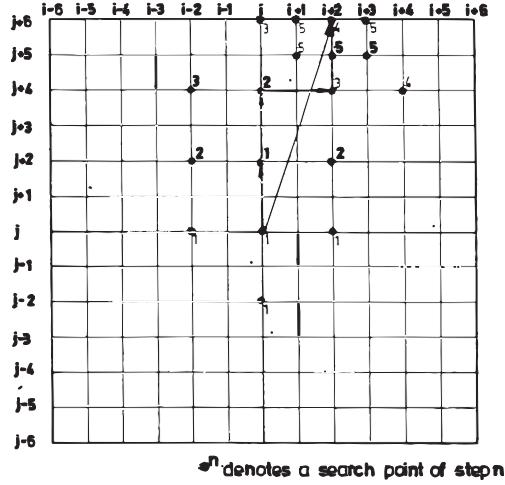
#### 6.4.3 Fast Algorithms

As shown above, EBMA requires a very large amount of computation. To speed up the search, various fast algorithms for block matching have been developed. The key to reduce the computation is by reducing the number of search candidates. As described before, for a search range of  $\pm R$  and a stepsize of 1 pixel, the total number of candidates is  $(2R + 1)^2$  with EBMA. Various fast algorithms differ in the way to skip those candidates that are unlikely to have small errors.

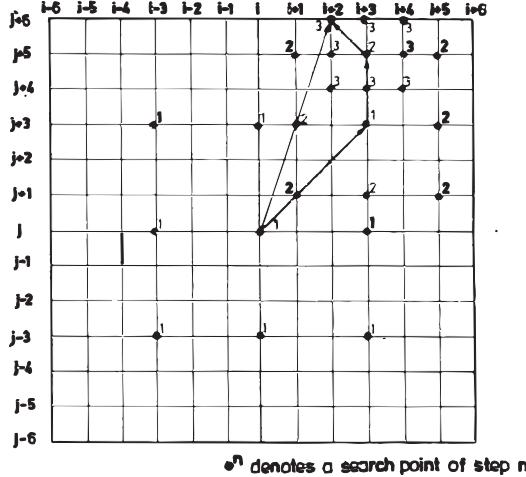
**2D-Log Search Method** One popular fast search algorithm is the *2D-log search* [19], illustrated in Fig. 6.9. It starts from the position corresponding to zero-displacement. Each step tests five search points in a diamond arrangement. In the next step, the diamond search is repeated with the center moved to the best matching point resulting from the previous step. The search stepsize (the radius of the diamond) is reduced if the best matching point is the center point, or on the border of the maximum search range. Otherwise, the stepsize remains the same. The final step is reached when the stepsize is reduced to 1 pel, and nine search points are examined at this last step. The initial stepsize is usually set to half of the maximum search range. With this method, one cannot pre-determine the number of steps and the total number of search points, as it depends on the actual MV. But the best case (requiring the minimum number of search points) and the



**Figure 6.8.** Example motion estimation results: (a) the tracked frame; (b) the anchor frame; (c-d) motion field and predicted image for the anchor frame ( $\text{PSNR}=29.86 \text{ dB}$ ) obtained by half-pel accuracy EBMA ; (e-f) motion field (represented by the deformed mesh overlaid on the tracked frame) and predicted image ( $\text{PSNR}=29.72 \text{ dB}$ ) obtained by the mesh-based motion estimation scheme in [43].



**Figure 6.9.** The 2D-logarithmic search method. The search points in a tracked frame are shown with respect to a block center at  $(i,j)$  in the anchor frame. In this example, the best matching MVs in steps 1 to 5 are  $(0,2)$ ,  $(0,4)$ ,  $(2,4)$ ,  $(2,6)$ , and  $(2,6)$ . The final MV is  $(2,6)$ . From [28, Fig. 11]



**Figure 6.10.** The three-step search method. In this example, the best matching MVs in steps 1 to 3 are  $(3,3)$ ,  $(3,5)$ , and  $(2,6)$ . The final MV is  $(2,6)$ . From [28, Fig. 12]

worst case (requiring the maximum number) can be analyzed.

**Three-Step Search Method** Another popular fast algorithm is the *three-step search* algorithm [20]. As illustrated in Fig. 6.10, the search starts with a step-

**Table 6.1.** Comparison of Fast Search Algorithms for a Search Range of  $R = 7$ . From [14, Table 1]

Search Algorithm	Number of Search Points		Number of Search Steps	
	Minimum	Maximum	Minimum	Maximum
EBMA	225	225	1	1
2D-log [19]	13	26	2	8
three-step [20]	25	25	3	3

size equal to or slightly larger than half of the maximum search range. In each step, nine search points are compared. They consist of the central point of the search square, and eight search points located on the search area boundaries. The stepsize is reduced by half after each step, and the search ends with stepsize of 1 pel. At each new step, the search center is moved to the best matching point resulting from the previous step. Let  $R_0$  represent the initial search stepsize, there are at most  $L = \lceil \log_2 R_0 + 1 \rceil$  search steps, where  $[x]$  represents the lower integer of  $x$ . If  $R_0 = R/2$ , then  $L = \lceil \log_2 R \rceil$ . At each search step, eight points are searched, except in the very beginning, when nine points need to be examined. Therefore, the total number of search points is  $8L + 1$ . For example, for a search range of  $R = 32$ , with EBMA, the total number of search points is 4225, whereas with the three-step method, the number is reduced to 41, a saving factor of more than 100. Unlike the 2D-log search method, the three-step method has a fixed, predictable number of search steps and search points. In addition, it has a more regular structure. These features make the three-step method more amenable to VLSI implementation than the 2D-log method and some other fast algorithms.

**Comparison of Different Fast Algorithms** Table 6.1 compares the minimum and maximum numbers of search points and the number of search steps required by several different search algorithms. As can be seen, some algorithms have more regular structure and hence fixed number of computations, while others have very different best case and worst case numbers. For VLSI implementation, structure regularity is more important, whereas for software implementation, the average case complexity (which is more close to the best case in general) is more important. For an analysis on the implementation complexity and cost using VLSI circuits for these algorithms, see the article by Hang *et al.* [14].

The above discussions assume that the search accuracy is integer-pel. To achieve half-pel accuracy, one can add a final step in any fast algorithm, which searches with a half-pel stepsize in a  $\pm 1$  pel neighborhood of the best matching point found from the integer-pel search.

#### 6.4.4 Imposing Motion Smoothness Constraints

From Figures 6.8(c), we see that the motion field obtained using EBMA is quite chaotic. This is because no constraints are imposed on the spatial variation of the block MVs. Several approaches have been developed to make the estimated motion field smoother so that it is closer to the physical motion field. One effective approach is to use a hierarchical approach, which estimates the MVs in a coarser spatial resolution first, and then continuously refine the MVs in successive finer resolutions. The propagation of the MVs from a coarser resolution to a finer one is accomplished by spatial interpolation, which induces a certain degree of spatial continuity on the resulting motion field. This technique will be treated in more detail in Sec. 6.9. Another approach is to explicitly impose a smoothness constraint by adding a smoothing term in the error criterion in Eq. (6.4.2), which measures the variation of the MVs of adjacent blocks. The resulting overall error function will be similar to that in Eq. (6.2.9), except that the motion vectors are defined over blocks and the prediction error is summed over a block. The challenge is to determine a proper weighting between the prediction error term and the smoothing term so that the resulting motion field is not over-smoothed. Ideally, the weighting should be adaptive: it should not be applied near object boundaries. A more difficult task is to identify object boundaries where motion discontinuity should be allowed.

#### 6.4.5 Phase Correlation Method\*

Instead of minimizing the DFD, another motion estimation method is by identifying peaks in the phase-correlation function. Assume that two image frames are related by a pure translation, so that

$$\psi_1(\mathbf{x}) = \psi_2(\mathbf{x} + \mathbf{d}). \quad (6.4.3)$$

Taking the Fourier transform on both sides and using the Fourier shift theorem, we get

$$\Psi_1(\mathbf{f}) = \Psi_2(\mathbf{f}) \cdot e^{j2\pi\mathbf{d}^T \mathbf{f}}. \quad (6.4.4)$$

The normalized cross power spectrum between  $\psi_1(\mathbf{x})$  and  $\psi_2(\mathbf{x})$  is

$$\tilde{\Psi}(\mathbf{f}) = \frac{\Psi_1(\mathbf{f}) \cdot \Psi_2^*(\mathbf{f})}{|\Psi_1(\mathbf{f}) \cdot \Psi_2^*(\mathbf{f})|} = e^{j2\pi\mathbf{d}^T \mathbf{f}}, \quad (6.4.5)$$

where the superscript \* indicates complex conjugation. Taking the inverse Fourier transform results in the *phase correlation function* (PCF):<sup>4</sup>

$$\text{PCF}(\mathbf{x}) = \mathcal{F}^{-1}\{\tilde{\Psi}(\mathbf{f})\} = \delta(\mathbf{x} + \mathbf{d}). \quad (6.4.6)$$

---

<sup>4</sup>The name comes from the fact that it is the cross correlation between the phase portions of the functions  $\psi_1(\mathbf{x})$  and  $\psi_2(\mathbf{x})$ , respectively.

We see that *the PCF between two images that are translation of each other is an impulse function, with the impulse located at a position exactly equal to the translation between the two images*. By identifying the peak location of the PCF, one can estimate the translation between two images. This approach was first used by Kuglin [23] for image alignment.

The above derivation assumes that both images are continuous in space and have infinite size. On the other hand, the actual image signal is discrete and finite. In practice, we apply DSFT over the available image domain, which is equivalent to the periodic extension of the CSFT over an infinite image that is zero outside the given image domain (see Secs. 2.1 and 2.2). In order to suppress the aliasing effect caused by sampling, a frequency-domain weighting function  $W(\mathbf{f})$  is usually applied when computing (6.4.5). In [13], a Kaiser window with  $\alpha = 0.2$  is used as the weighting function. To reduce the effect of boundary samples, a space domain weighting function  $w(\mathbf{x})$  can also be applied to  $\psi_1(\mathbf{x}), \psi_2(\mathbf{x})$  before computing DSFT.

Phase correlation is used extensively in image registration, where entire images have to be aligned [33][10]. For motion estimation, the underlying two frames are usually not related by a global translation. To handle this situation, the phase correlation method is more often applied at the block level. For motion estimation over non-overlapping blocks of size  $N \times N$ , both frames are usually divided into overlapping range blocks of size  $L \times L$ . For a search range of  $\pm R$ , the range block size should be  $L \geq N + 2R$ . To determine the MV of a block in  $\psi_1(\mathbf{x})$ , a size  $L \times L$  discrete Fourier transform (DFT) is applied to both this block and its corresponding block in  $\psi_2(\mathbf{x})$ . Then the PCF is computed using inverse DFT of the same size, and the peak location is identified. To enable the use of fast Fourier transform (FFT) algorithms,  $L$  is usually chosen to be a power of 2. For example, if  $N = 16$ ,  $R = 16$ ,  $L = 64$  would be appropriate.

The above method assumes that there is a global translation between the two corresponding range blocks. This assumption does not hold for general video sequences. When there are several patches in the range block in  $\psi_1(\mathbf{x})$  that undergo different motions, we will observe several peaks in the PCF. Each peak corresponds to the motion of one patch. The location of the peak indicates the MV of the patch, whereas the amplitude of the peak is proportional to the size of the patch [40]. In this sense, the PCF reveals similar information as the MV histogram over a block. To estimate the dominant MV of the block, we first extract local maxima of the PCF. We then examine the DFD at the corresponding MV's. The MV yielding the minimum DFD will be considered the block MV. Since only a small number of candidate MV's are examined, significant savings in computational complexity may be achieved compared to the full-search method.

This approach can be extended to motion estimation with fractional pel accuracy. In [13], the integer-pel candidate motion vectors are augmented by varying the length of the candidate motion vectors by up to  $\pm 1$  pel. In [37] and [40], alternative methods are suggested.

An advantage of the phase correlation method for motion estimation is its insensitivity to illumination changes (see Sec. 5.2). This is because changes in the

mean value of an image or multiplication of an image with a constant do not affect the phase information. This is not true for the DFD-based methods.

#### 6.4.6 Binary Feature Matching\*

In this scheme, known as Hierarchical Feature Matching Motion Estimation Scheme (HFM-ME) [25], a Sign Truncated Feature (STF) is defined and used for block template matching, as opposed to the pixel intensity values used in conventional block matching methods. Using the STF definition, a data block is represented by a mean and a binary bit pattern. The block matching motion estimation is divided to mean matching and binary phase matching. This technique enables a significant reduction in computational complexity compared with EBMA because binary phase matching only involves Boolean logic operation. The use of STF also significantly reduces the data transfer time between the frame buffer and motion estimator. Tests have shown that HFM-ME can achieve similar prediction accuracy as EBMA under the same search ranges, but can be implemented about 64 times faster. When the search range is doubled for HFM-ME, it can achieve significantly more accurate prediction than EBMA, still with nontrivial time savings [25].

The STF vector of a block of size  $2^N \times 2^N$  consists of two parts. The first part is the multiresolution mean vectors, and the second part is the sign-truncated binary vectors. The mean vectors are determined recursively as follows

$$\begin{aligned} \text{Mean}^n(i, j) &= \left[ \frac{1}{4} \sum_{p=0}^1 \sum_{q=0}^1 \text{Mean}^{n+1}(2i+p, 2j+q) \right], \quad 0 \leq i, j \leq 2^n - 1, 0 \leq n \leq N-1 \\ \text{Mean}^N(i, j) &= \psi(i, j), \quad 0 \leq i, j \leq 2^N - 1, \end{aligned} \quad (6.4.7)$$

where  $\{\psi(i, j); 0 \leq i, j \leq 2^N - 1\}$  are the pixel intensity values of the original block.

The sign-truncated vectors are obtained by

$$\text{ST\_pattern}^n(i, j) = \begin{cases} 0 & \text{if } \text{Mean}^n(i, j) \geq \text{Mean}^{n-1}(\lfloor \frac{i}{2} \rfloor, \lfloor \frac{j}{2} \rfloor), \\ 1 & \text{otherwise.} \end{cases} \quad (6.4.8)$$

The STF vectors, decomposed to the  $n$ -th level for a  $2^N \times 2^N$  block can then be represented as

$$\text{STFV}_n^N = \{\text{ST\_pattern}^N, \text{ST\_pattern}^{N-1}, \dots, \text{ST\_pattern}^{N-n+1}, \text{mean}^{N-n}\}. \quad (6.4.9)$$

When  $n=N$ , a block is fully decomposed with the following STF vector

$$\text{STFV}_N^N = \{\text{ST\_pattern}^N, \text{ST\_pattern}^{N-1}, \dots, \text{ST\_pattern}^1, \text{mean}^0\}. \quad (6.4.10)$$

All the intermediate mean vectors are only used to generate ST-patterns and can be discarded. Therefore, the final STF representation consists of a multiresolution binary sequence with  $\frac{4}{3}(4^N - 1)$  bits and a one-byte mean. This represents a much reduced data set compared to the original  $4^N$  bytes of pixel values. Also, this feature set allows binary Boolean operation for the block matching purpose.

As an example, let us consider how to form the STF vector for a 4x4 block with 2 layers. First, the mean pyramid is formed as

$$\begin{bmatrix} 158 & 80 & 59 & 74 \\ 80 & 69 & 59 & 74 \\ 87 & 86 & 65 & 62 \\ 116 & 100 & 72 & 58 \end{bmatrix} \Rightarrow \begin{bmatrix} 97 & 67 \\ 97 & 64 \end{bmatrix} \Rightarrow 81$$

The STF vectors is then obtained as:

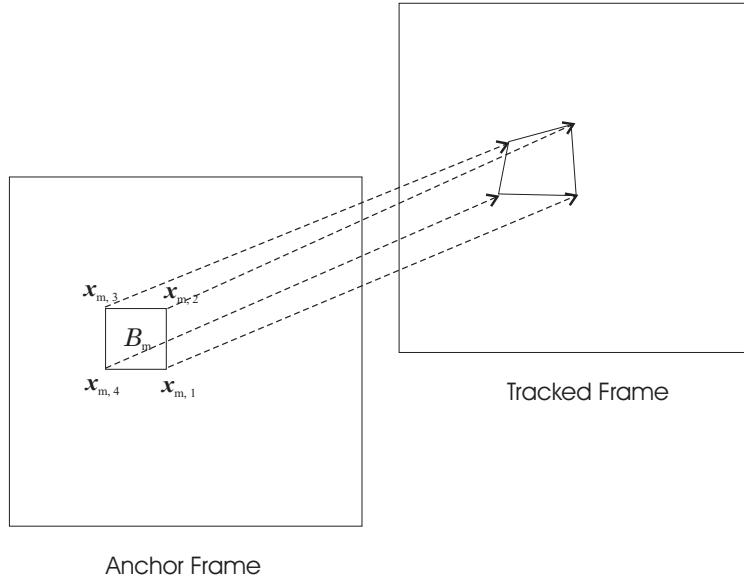
$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad 81$$

The STF vector decomposed to one layer for the above example is  $\{0110\ 1110\ 0010\ 0001, (97, 67, 97, 64)\}$ . The completely-decomposed STF vector is  $\{0101, 0110\ 1110\ 0010\ 0001, 81\}$ . It consists of a 20-bit binary pattern, which includes a 2x2 second layer sign-pattern and a 4x4 first layer sign-pattern, and a mean value. In practical implementations, either completely-decomposed or mixed-layer STF vectors can be used.

Comparison of two STF vectors is accomplished by two parallel decision procedures: i) calculating the absolute error bewteen the mean values, and ii) determining the Hamming distance between the binary patterns. The later can be accomplished extremely fast by using an XOR Boolean operator. Therefore, the main computational load of the HFM-ME lies in the computation of the mean pyramid for the current and all candidate matching blocks. This computation can however be done in advance, only once for every possible block. For a detailed analysis of the computation complexity and a fast algorithm using logarithmic search, see [25].

## 6.5 Deformable Block Matching Algorithms (DBMA)\*

In the block matching algorithms introduced previously, each block is assumed to undergo a pure translation. This model is not appropriate for blocks undergoing rotation, zooming, etc. In general, a more sophisticated model, such as the affine, bilinear, or projective mapping, can be used to describe the motion of each block. Obviously, it will still cover the translational model as a special case. With such models, a block in the anchor frame is in general mapped to a non-square quadrangle, as shown in Fig. 6.11. Therefore, we refer to the class of block-based motion estimation methods using higher order models as *deformable block matching algorithms* (DBMA) [24]. It is also known as *generalized block matching algorithm* [36]. In the following, we first discuss how to interpolate the MV at any point in a block using only the MVs at the block corners (called nodes), and then we present an algorithm for estimating nodal MVs.



**Figure 6.11.** The deformable block matching algorithm finds the best matching quadrangle in the tracked frame for each block in the anchor frame. The allowed block deformation depends on the motion model used for the block. Adapted from [39, Fig. 6.9].

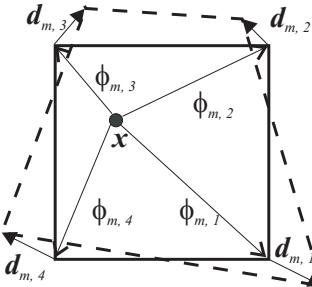
### 6.5.1 Node-Based Motion Representation

In Sec. 5.5, we described several 2D motion models corresponding to different 3D motions. All these models can be used to characterize the motion within a block. In Sec. 5.5.4, we showed how the most general model, the projective mapping, can be approximated by a polynomial mapping of different orders. In this section, we introduce a node-based block motion model [24], which can characterize the same type of motions as the polynomial model, but is easier to interpret and specify.

In this model, we assume that a selected number of control nodes in a block can move freely and that the displacement of any interior point can be interpolated from nodal displacements. Let the number of control nodes be denoted by  $K$ , and the MVs of the control nodes in  $\mathcal{B}_m$  by  $\mathbf{d}_{m,k}$ , the motion function over the block is described by

$$\mathbf{d}_m(\mathbf{x}) = \sum_{k=1}^K \phi_{m,k}(\mathbf{x}) \mathbf{d}_{m,k}, \quad \mathbf{x} \in \mathcal{B}_m. \quad (6.5.1)$$

The above equation expresses the displacement at any point in a block as an interpolation of nodal displacements, as shown in Fig. 6.12. The interpolation kernel  $\phi_{m,k}(\mathbf{x})$  depends on the desired contribution of the  $k$ -th control point in  $\mathcal{B}_m$  to  $\mathbf{x}$ . One way to design the interpolation kernels is to use the shape functions associated with the corresponding nodal structure. We will discuss more about the design of



**Figure 6.12.** Interpolation of motion in a block from nodal MVs.

shape functions in Sec. 6.6.1.

The translational, affine, and bilinear models introduced previously are special cases of the node-based model with one, three, and four nodes, respectively. A model with more nodes can characterize more complex deformation. The interpolation kernel in the one-node case (at the block center or a chosen corner) is a pulse function, corresponding to nearest neighbor interpolation. The interpolation functions in the three-node (any three corners in a block) and four-node (the four corners) cases are affine and bilinear functions, respectively. Usually, to use an affine model with a rectangular block, the block is first divided into two triangles, and then each triangle is modeled by the three-node model.

Compared to the polynomial-based representation introduced previously, the node-based representation is easier to visualize. Can you picture in your head the deformed block given 8 coefficients of a bilinear function? But you certainly can, given locations of the four corner points in the block! Furthermore, the nodal MVs can be estimated more easily and specified with a lower precision than for the polynomial coefficients. First, it is easy to determine appropriate search ranges and search stepsizes for the nodal MVs than the polynomial coefficients, based on the *a priori* knowledge about the dynamic range of the underlying motion and the desired estimation accuracy. Secondly, all the motion parameters in the node-based representation are equally important, while those in the polynomial representation cannot be treated equally. For example, the estimation of the high order coefficients is much harder than the constant terms. Finally, specification of the polynomial coefficients requires a high degree of precision: a small change in a high order coefficient can generate a very different motion field. On the other hand, to specify nodal MVs, integer or half-pel accuracy is usually sufficient. These advantages are important for video coding applications.

### 6.5.2 Motion Estimation Using Node-Based Model

Because the estimation of nodal movements are independent from block to block, we skip the subscript  $m$  which indicates which block is being considered. The

following derivation applies to any block  $\mathcal{B}$ . With the node-based motion model, the motion parameters for any block are the nodal MVs, i.e.,  $\mathbf{a} = [\mathbf{d}_k, k \in \mathcal{K}]$ , where  $\mathcal{K} = \{1, 2, \dots, K\}$ . They can be estimated by minimizing the prediction error over this block, i.e.,

$$E(\mathbf{a}) = \sum_{\mathbf{x} \in \mathcal{B}} |\psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})|^p, \quad (6.5.2)$$

where

$$\mathbf{w}(\mathbf{x}; \mathbf{a}) = \mathbf{x} + \sum_{k \in \mathcal{K}} \phi_k(\mathbf{x}) \mathbf{d}_k. \quad (6.5.3)$$

As with the BMA, there are many ways to minimize the error in Eq. (6.5.2), including exhaustive search and a variety of gradient-based search method. The computational load required of the exhaustive search, however, can be unacceptable in practice, because of the high dimension of the search space. Gradient-based search algorithms are more feasible in this case. In the following, we derive a Newton-Ralphson search algorithm, following the approach in [24].

Define  $\mathbf{a} = [\mathbf{a}_x^T, \mathbf{a}_y^T]^T$  with  $\mathbf{a}_x = [d_{x,1}, d_{x,2}, \dots, d_{x,K}]^T$ ,  $\mathbf{a}_y = [d_{y,1}, d_{y,2}, \dots, d_{y,K}]^T$ . One can show that

$$\frac{\partial E}{\partial \mathbf{a}}(\mathbf{a}) = \left[ \frac{\partial E}{\partial \mathbf{a}_x}(\mathbf{a}), \frac{\partial E}{\partial \mathbf{a}_y}(\mathbf{a}) \right]^T,$$

with

$$\begin{aligned} \frac{\partial E}{\partial \mathbf{a}_x}(\mathbf{a}) &= 2 \sum_{\mathbf{x} \in \mathcal{B}} e(\mathbf{x}; \mathbf{a}) \frac{\partial \psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a}))}{\partial x} \phi(\mathbf{x}), \\ \frac{\partial E}{\partial \mathbf{a}_y}(\mathbf{a}) &= 2 \sum_{\mathbf{x} \in \mathcal{B}} e(\mathbf{x}; \mathbf{a}) \frac{\partial \psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a}))}{\partial y} \phi(\mathbf{x}). \end{aligned}$$

In the above equations,  $e(\mathbf{x}; \mathbf{a}) = \psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})$  and  $\phi(\mathbf{x}) = [\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \dots, \phi_K(\mathbf{x})]^T$ . By dropping the terms involving second order gradients, the Hessian matrix can be approximated as

$$[\mathbf{H}(\mathbf{a})] = \begin{bmatrix} \mathbf{H}_{xx}(\mathbf{a}) & \mathbf{H}_{xy}(\mathbf{a}) \\ \mathbf{H}_{xy}(\mathbf{a}) & \mathbf{H}_{yy}(\mathbf{a}) \end{bmatrix},$$

with

$$\begin{aligned} \mathbf{H}_{xx}(\mathbf{a}) &= 2 \sum_{\mathbf{x} \in \mathcal{B}} \left( \frac{\partial \psi_2}{\partial x} \right)^2 \Big|_{\mathbf{w}(\mathbf{x}; \mathbf{a})} \phi(\mathbf{x}) \phi(\mathbf{x})^T, \\ \mathbf{H}_{yy}(\mathbf{a}) &= 2 \sum_{\mathbf{x} \in \mathcal{B}} \left( \frac{\partial \psi_2}{\partial y} \right)^2 \Big|_{\mathbf{w}(\mathbf{x}; \mathbf{a})} \phi(\mathbf{x}) \phi(\mathbf{x})^T. \\ \mathbf{H}_{xy}(\mathbf{a}) &= 2 \sum_{\mathbf{x} \in \mathcal{B}} \frac{\partial \psi_2}{\partial x} \frac{\partial \psi_2}{\partial y} \Big|_{\mathbf{w}(\mathbf{x}; \mathbf{a})} \phi(\mathbf{x}) \phi(\mathbf{x})^T. \end{aligned}$$

The Newton-Ralphson update algorithm is:

$$\mathbf{a}^{l+1} = \mathbf{a}^l - \alpha [\mathbf{H}(\mathbf{a}^l)]^{-1} \frac{\partial E}{\partial \mathbf{a}}(\mathbf{a}^l). \quad (6.5.4)$$

The update at each iteration thus requires the inversion of the  $2K \times 2K$  symmetric matrix  $[\mathbf{H}]$ .

To reduce numerical computations, we can update the displacements in  $x$  and  $y$  directions separately. Similar derivation will yield:

$$\mathbf{a}_x^{l+1} = \mathbf{a}_x^l - \alpha [\mathbf{H}_{xx}(\mathbf{a}^l)]^{-1} \frac{\partial E}{\partial \mathbf{a}_x}(\mathbf{a}^l), \quad (6.5.5)$$

$$\mathbf{a}_y^{l+1} = \mathbf{a}_y^l - \alpha [\mathbf{H}_{yy}(\mathbf{a}^l)]^{-1} \frac{\partial E}{\partial \mathbf{a}_y}(\mathbf{a}^l). \quad (6.5.6)$$

In this case we only need to invert two  $K \times K$  matrices in each update. For the four-node case,  $[\mathbf{H}]$  is an  $8 \times 8$  matrix, while  $[\mathbf{H}_{xx}]$ , and  $[\mathbf{H}_{yy}]$  are  $4 \times 4$  matrices.

As with all gradient-based iterative processes, the above update algorithm may reach a bad local minimum that is far from the global minimum, if the initial solution is not chosen properly. A good initial solution can often be provided by the EBMA. For example, consider the four-node model with four nodes at the corners of each block. One can use the average of the motion vectors of the four blocks attached to each node as the initial estimate of the MV for that node. This initial estimate can then be successively updated by using Eq. (6.5.4).

Note that the above algorithm can also be applied to polynomial-based motion representation. In that case,  $\mathbf{a}_x$  and  $\mathbf{a}_y$  would represent the polynomial coefficients associated with horizontal and vertical displacements, respectively, and  $\phi_k(\cdot)$  would correspond to the elementary polynomial basis functions. However, it is difficult to set the search range for  $\mathbf{a}_x$  and  $\mathbf{a}_y$  and check the feasibility of the resulting motion field.

## 6.6 Mesh-Based Motion Estimation\*

With the block-based model used either in BMA or DBMA, motion parameters in individual blocks are independently specified. Unless motion parameters of adjacent blocks are constrained to vary smoothly, the estimated motion field is often discontinuous and sometimes chaotic, as sketched in Fig. 6.13(a). One way to overcome this problem is by using mesh-based motion estimation. As illustrated in Figs. 6.13(b), the anchor frame is covered by a mesh, and the motion estimation problem is to find the motion of each node so that the image pattern within each element in the anchor frame matches well with that in the corresponding deformed element in the tracked frame. The motion within each element is interpolated from nodal MVs. *As long as the nodes in the tracked frame still form a feasible mesh, the mesh-based motion representation is guaranteed to be continuous and thus be free from the blocking artifacts associated with block-based representation.* Another

benefit of the mesh-based representation is that it enables continuous tracking of the same set of nodes over consecutive frames, which is desirable in applications requiring object tracking. As shown in Fig. 6.13(c), one can generate a mesh for an initial frame, and then estimate the nodal motions between every two frames. At each new frame (the anchor frame), the tracked mesh generated in the previous step is used, so that the same set of nodes are tracked over all frames. This is not possible with the block-based representation, because it requires that each new anchor frame be reset to a partition consisting of regular blocks.

Note that the inherent continuity with the mesh-based representation is not always desired. The type of motion that can be captured by this representation can be visualized as the deformation of a rubber-sheet, which is continuous everywhere. In real-world video sequences, there are often motion discontinuities at object boundaries. A more accurate representation would be to use separate meshes for different objects. As with the block-based representation, the accuracy of the mesh-based representation depends on the number of nodes. A very complex motion field can be reproduced as long as a sufficient number of nodes are used. To minimize the number of nodes required, the mesh should be adapted to the imaged scene, so that the actual motion within each element is smooth (i.e., can be interpolated accurately from the nodal motions). If a regular, non-adaptive mesh is used, a larger number of nodes are usually needed to approximate the motion field accurately.

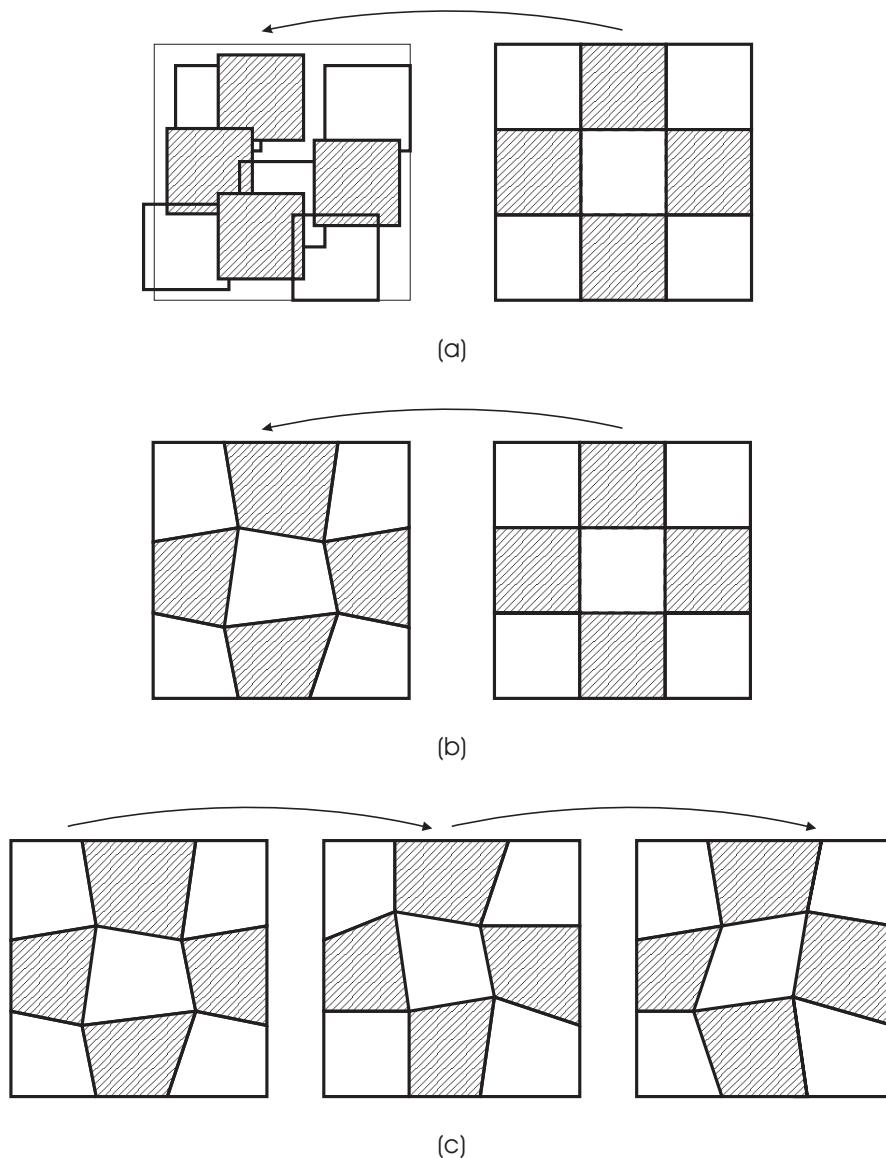
In the following, we first describe how to specify a motion field using a mesh-based representation. We then present algorithms for estimating nodal motions in a mesh.

### 6.6.1 Mesh-Based Motion Representation

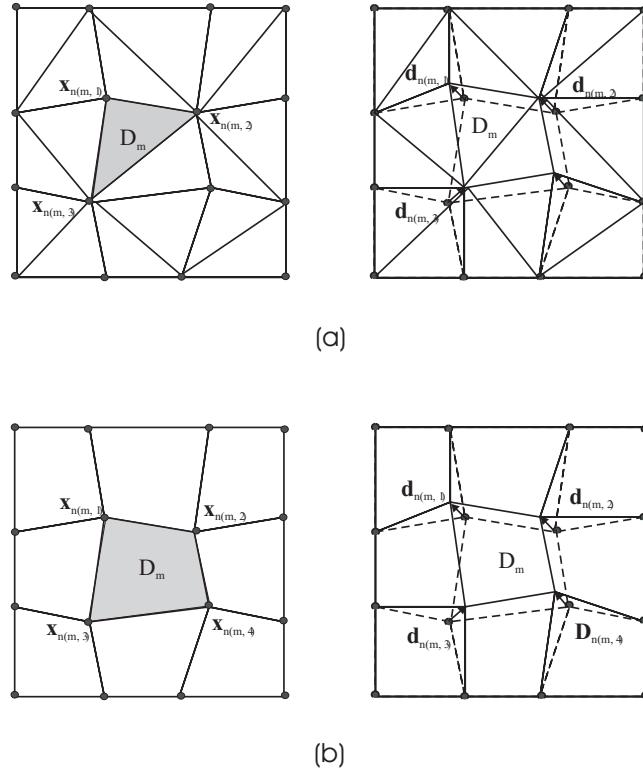
With the mesh-based motion representation, the underlying image domain in the anchor frame is partitioned into non-overlapping polygonal elements. Each element is defined by a few nodes and links between the nodes, as shown in Fig. 6.14. Such a mesh is also known as a *control grid*. In the mesh-based representation, the motion field over the entire frame is described by MVs at the nodes only. The MVs at the interior points of an element are interpolated from the MVs at the nodes of this element. The nodal MVs are constrained so that the nodes in the tracked frame still form a feasible mesh, with no flip-over elements.

Let the number of elements and nodes be denoted by  $M$  and  $N$  respectively, and the number of nodes defining each element by  $K$ . For convenience, we define the following index sets:  $\mathcal{M} = \{1, 2, \dots, M\}$ ,  $\mathcal{N} = \{1, 2, \dots, N\}$ ,  $\mathcal{K} = \{1, 2, \dots, K\}$ . Let the  $m$ -th element and  $n$ -th node in frame  $t$  ( $t=1$  for the anchor frame and  $t=2$  for the tracked frame) be denoted by  $\mathcal{B}_{t,m}$ ,  $m \in \mathcal{M}$  and  $\mathbf{x}_{t,n}$ ,  $n \in \mathcal{N}$ , and the MV of the  $n$ -th node by  $\mathbf{d}_n = \mathbf{x}_{2,n} - \mathbf{x}_{1,n}$ . The motion field in element  $\mathcal{B}_{1,m}$  is related to the nodal MVs  $\mathbf{d}_n$  by:

$$\mathbf{d}_m(\mathbf{x}) = \sum_{k \in \mathcal{K}} \phi_{m,k}(\mathbf{x}) \mathbf{d}_{n(m,k)}, \quad \mathbf{x} \in \mathcal{B}_{1,m}, \quad (6.6.1)$$



**Figure 6.13.** Comparison of block-based and mesh-based motion representations: (a) Block-based motion estimation between two frames, using a translational model within each block in the anchor frame; (b) Mesh-based motion estimation between two frames, using a regular mesh at the anchor frame; (d) Mesh-based motion tracking, using the tracked mesh for each new anchor frame.



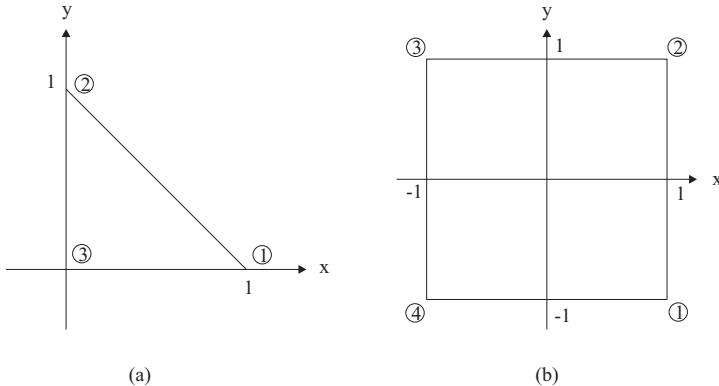
**Figure 6.14.** Illustration of mesh-based motion representation: (a) using a triangular mesh, with 3 nodes attached to each element, (b) using a quadrilateral mesh, with 4 nodes attached to each element. In the shown example, the two meshes have the same number of nodes, but the triangular mesh has twice the number of elements. The left column shows the initial mesh over the anchor frame, the right column the deformed mesh in the tracked frame.

where  $n(m, k)$  specifies the global index of the  $k$ -th node in the  $m$ -th element (cf. Fig. 6.14). The function  $\phi_{m,k}(\mathbf{x})$  is the *interpolation kernel* associated with node  $k$  in element  $m$ . It depends on the desired contribution of the  $k$ -th node in  $\mathcal{B}_{1,m}$  to the MV at  $\mathbf{x}$ . This interpolation mechanism has been shown previously in Fig. 6.12. To guarantee continuity across element boundary, the interpolation kernels should satisfy:

$$0 \leq \phi_{m,k}(\mathbf{x}) \leq 1, \sum_k \phi_{m,k}(\mathbf{x}) = 1, \forall \mathbf{x} \in \mathcal{B}_m,$$

and

$$\phi_{m,k}(\mathbf{x}_l) = \delta_{k,l} = \begin{cases} 1 & k = l, \\ 0 & k \neq l. \end{cases}$$



**Figure 6.15.** (a) A standard triangular element; (b) A standard quadrilateral element (a square).

In the finite element method (FEM) analysis, these functions are called *shape functions* [45]. If all the elements have the same shape, then all the shape functions are equal, i.e.,  $\phi_{m,k}(\mathbf{x}) = \phi_k(\mathbf{x})$ .

Standard triangular and quadrilateral elements are shown in Fig. 6.15. The shape functions for the standard triangular element are:

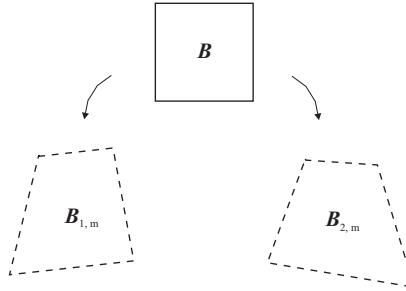
$$\phi_1^t(x, y) = x, \quad \phi_2^t(x, y) = y, \quad \phi_3^t(x, y) = 1 - x - y. \quad (6.6.2)$$

The shape functions for the standard quadrilateral element are:

$$\begin{aligned} \phi_1^q(x, y) &= (1 + x)(1 - y)/4, & \phi_2^q(x, y) &= (1 + x)(1 + y)/4, \\ \phi_3^q(x, y) &= (1 - x)(1 + y)/4, & \phi_4^q(x, y) &= (1 - x)(1 - y)/4. \end{aligned} \quad (6.6.3)$$

We see that the shape functions for these two cases are affine and bilinear functions, respectively. The readers are referred to [41] for the shape functions for arbitrary triangular elements. The coefficients of these functions depend on the node positions.

Note that the representation of the motion within each element in Eq. (6.6.1) is the same as the node-based motion representation introduced in Eq. (6.5.1), except that the nodes and elements are denoted using global indices. This is necessary because the nodal MVs are not independent from element to element. It is important not to confuse the mesh-based model with the node-based model introduced in the previous section. There, although several adjacent blocks may share the same node, the nodal MVs are determined independently in each block. Going back to Fig. 6.14(b), in the mesh-based model, node  $n$  is assigned a single MV, which will affect the interpolated motion functions in four quadrilateral elements attached to this node. In the node-based model, node  $n$  can have four different MVs, depending on in which block it is considered.



**Figure 6.16.** Mapping from a master element  $\tilde{\mathcal{B}}$  to two corresponding elements in the anchor and tracked frames  $\mathcal{B}_{1,m}$  and  $\mathcal{B}_{2,m}$ .

### 6.6.2 Motion Estimation Using Mesh-Based Model

With the mesh-based motion representation, there are in general two sets of problems to be solved: 1) Given a mesh or equivalently nodes in the current frame, how to determine the nodal positions in the tracked frame. This is essentially a motion estimation problem. 2) How to set up the mesh in the anchor frame, so that the mesh conforms to the object boundaries. Note that a mesh in which each element corresponds to a smooth surface patch in a single object can lead to more accurate motion estimation, than an arbitrarily configured mesh (e.g., a regular mesh). An object adaptive mesh would also be more appropriate for motion tracking over a sequence of frames. In this book, we only consider the first problem. For solutions to the mesh generation problem, see, e.g., [42, 3].

With the mesh-based motion representation described by Eq. (6.6.1), the motion parameters include the nodal MVs, i.e.,  $\mathbf{a} = \{\mathbf{d}_n, n \in \mathcal{N}\}$ . To estimate them, we can again use an error minimization approach. Under the mesh-based motion model, the DFD error in Eq. (6.2.1) becomes

$$E(\mathbf{d}_n, n \in \mathcal{N}) = \sum_{m \in \mathcal{M}} \sum_{\mathbf{x} \in \mathcal{B}_{1,m}} |\psi_2(\mathbf{w}_m(\mathbf{x})) - \psi_1(\mathbf{x})|^p, \quad (6.6.4)$$

where, following Eq. (6.6.1),

$$\mathbf{w}_m(\mathbf{x}) = \mathbf{x} + \sum_{k \in \mathcal{K}} \phi_{m,k}(\mathbf{x}) \mathbf{d}_{n(m,k)}, \quad \mathbf{x} \in \mathcal{B}_{1,m}.$$

In general, the error function in Eq. (6.6.4) is difficult to calculate because of the irregular shape of  $\mathcal{B}_{1,m}$ . To simplify the calculation, we can think of  $\mathcal{B}_{t,m}, t = 1, 2$ , as being deformed from a master element with a regular shape. In general, the master element for different elements could differ. Here, we only consider the case when all the elements have the same topology that can be mapped from the same master element, denoted by  $\tilde{\mathcal{B}}$ . Fig. 6.16 illustrates such a mapping.

Let  $\tilde{\phi}_k(\mathbf{u})$  represent the shape function associated with the  $k$ -th node in  $\tilde{\mathcal{B}}$ , the mapping functions from  $\tilde{\mathcal{B}}$  to  $\mathcal{B}_{t,m}$  can be represented as

$$\tilde{\mathbf{w}}_{t,m}(\mathbf{u}) = \sum_{k \in \mathcal{K}} \tilde{\phi}_k(\mathbf{u}) \mathbf{x}_{t,n(m,k)}, \quad \mathbf{u} \in \tilde{\mathcal{B}}, \quad t = 1, 2, \quad (6.6.5)$$

Then the error in Eq. (6.6.4) can be calculated over the master element as

$$E(\mathbf{d}_n, n \in \mathcal{N}) = \sum_{m \in \mathcal{M}} \sum_{\mathbf{u} \in \tilde{\mathcal{B}}} |\tilde{e}_m(\mathbf{u})|^p J_m(\mathbf{u}), \quad (6.6.6)$$

where

$$\tilde{e}_m(\mathbf{u}) = \psi_2(\tilde{\mathbf{w}}_{2,m}(\mathbf{u})) - \psi_1(\tilde{\mathbf{w}}_{1,m}(\mathbf{u})) \quad (6.6.7)$$

represents the error between the two image frames at points that are both mapped from  $\mathbf{u}$  in the master element (cf. Fig 6.16). The function  $J_m(\mathbf{u}) = \left| \det \left( \frac{\partial \mathbf{w}_{1,m}(\mathbf{u})}{\partial \mathbf{u}} \right) \right|$  denotes the Jacobian of the transformation:<sup>5</sup>

$$\tilde{\mathbf{w}}_{1,m}(\mathbf{u}). \quad (6.6.8)$$

For motion tracking over a set of frames, because the mesh used in each new anchor frame is the tracked mesh resulting from a previous step, the shape of  $\mathcal{B}_{1,m}$  is in general irregular (cf. Fig. 6.13(c)). Consequently the mapping function  $\tilde{\mathbf{w}}_1(\mathbf{u})$  and the Jacobian  $J_m(\mathbf{u})$  depend on the nodal positions in  $\mathcal{B}_{1,m}$ . On the other hand, for motion estimation between two frames, to reduce the complexity, one can use a regular mesh for the anchor frame so that each element is itself equivalent to the master element (cf. Fig. 6.13(b)). In this case, the mapping function in the anchor frame is trivial, i.e.,  $\mathbf{w}_{1,m}(\mathbf{u}) = \mathbf{u}$ , and  $J_m(\mathbf{u}) = 1$ .

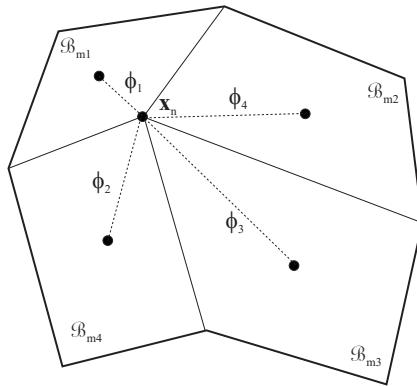
The gradient of the error function in Eq. (6.6.6) is, when  $p = 2$ ,

$$\frac{\partial E_p}{\partial \mathbf{d}_n} = 2 \sum_{m \in \mathcal{M}_n} \sum_{\mathbf{u} \in \tilde{\mathcal{B}}} \tilde{e}_m(\mathbf{u}) \tilde{\phi}_{k(m,n)}(\mathbf{u}) \frac{\partial \psi_2(\mathbf{x})}{\partial \mathbf{x}} \Big|_{\tilde{\mathbf{w}}_{2,m}(\mathbf{u})} J_m(\mathbf{u}), \quad (6.6.9)$$

where  $\mathcal{M}_n$  includes the indices of the elements that are attached to node  $n$ , and  $k(m, n)$  specifies the local index of node  $n$  in the  $m$ -th adjacent element. Figure 6.17 illustrates the neighboring elements and shape functions attached to node  $n$  in the quadrilateral mesh case.

It can be seen that the gradient with respect to one node only depends on the errors in the several elements attached to it. Ideally, in each iteration of a gradient-based search algorithm, to calculate the above gradient function associated with any node, one should assume the other nodes are fixed at the positions obtained in the previous iteration. All the nodes should be updated at once at the end of

<sup>5</sup>Strictly speaking, the use of Jacobian is correct only when the error is defined as an integral over  $\tilde{\mathcal{B}}$ . Here we assume the sampling over  $\tilde{\mathcal{B}}$  is sufficiently dense when using the sum to approximate the integration.

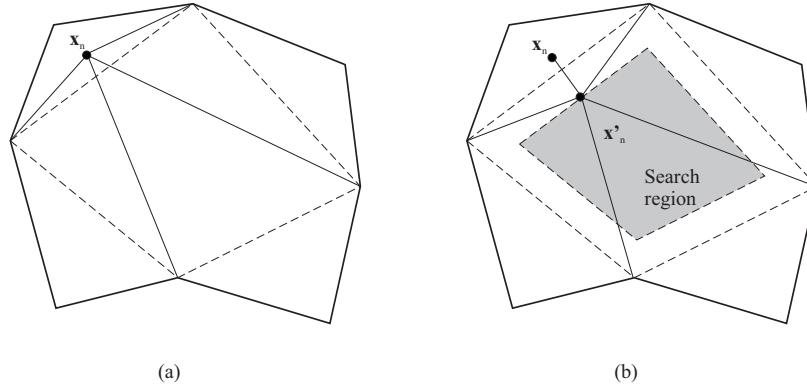


**Figure 6.17.** Neighborhood structure in a quadrilateral mesh: For a given node  $n$ , there are four elements attached to it, each with one shape function connected to this node.

the iteration, before going to the next iteration. But in reality, to speed up the process, one can update one node at a time, while fixing its surrounding nodes. Of course, this sub-optimal approach could lead to divergence or convergence to a local minimum that is worse than the one obtained by updating all the nodes simultaneously. Instead of updating the nodes in the usual raster order, to improve the accuracy and convergence rate, one can order the nodes so that the nodes whose motion vectors can be estimated more accurately be updated first. Because of the uncertainty in motion estimation in smooth regions, it may be better to update the nodes with large edge magnitude and small motion compensation error first. This is known as *highest confidence first* [7] and this approach was taken in [2]. Another possibility is to divide all the nodes into several groups so that the nodes in the same group do not share same elements, and therefore are independent in their impact on the error function. Sequential update of the nodes in the same group is then equivalent to simultaneous updates of these nodes. This is the approach adopted in [42]. Either the first order gradient descent method or the second order Newton-Ralphson type of update algorithm could be used. The second order method will converge much faster, but it is also more liable to converge to bad local minima.

The newly updated nodal positions based on the gradient function can lead to overly deformed elements (including flip-over and obtuse elements). In order to prevent such things from happening, one should limit the search range where the updated nodal position can fall into. If the updated position goes beyond this region, then it should be projected back to the nearest point in the defined search range. Figure 6.18 shows the legitimate search range for the case of a quadrilateral mesh.

The above discussion applies not only to gradient-based update algorithms, but also exhaustive search algorithms. In this case, one can update one node at a time, by searching for the nodal position that will minimize the prediction errors



**Figure 6.18.** The search range for node  $n$  given the positions of other nodes: the diamond region (dash line) is the theoretical limit, the inner diamond region (shaded) is used in practice. When  $\mathbf{x}_n$  falls outside the diamond region (a), at least one element attached to it becomes obtuse. By projecting  $\mathbf{x}_n$  onto the inner diamond (b), all four elements would not be overly deformed.

in elements attached to it in the search range illustrated in Fig. 6.18. For each candidate position, one calculates the error accumulated over the elements attached to this node, i.e., replacing  $n \in \mathcal{M}$  by  $n \in \mathcal{M}_n$  in Eq. (6.6.4). The optimal position is the one with the minimal error. Here again, the search order is very important.

**Example 6.2:** Figure 6.8 shows the motion estimation result obtained by an exhaustive search approach for backward motion estimation using a rectangular mesh at each new frame [43]. Figure 6.8(e) is the deformed mesh overlaid on top of the tracked frame, and Figure 6.8(f) is the predicted image for the anchor frame. Note that each deformed quadrangle in Fig. 6.8(e) corresponds to a square block in the anchor frame. Thus a narrow quadrangle in the right side of the face indicates it is expanded in the anchor frame. We can see that the mesh is deformed smoothly, which corresponds to a smooth motion field. The predicted image does not suffer from the blocking artifact associated with the EBMA (Fig. 6.8(d) vs. Fig. 6.8(f)) and appears to be a more successful prediction of the original. A careful comparison between the predicted image (Fig. 6.8(f)) and the actual image (Fig. 6.8(b)) however will reveal that the eye closing and mouth movement are not accurately reproduced, and there are some artificial warping artifacts near the jaw and neck. In fact, the PSNR of the predicted image is lower than that obtained by the EBMA.

Until now, we have assumed that a single mesh is generated (or propagated from the previous frame in the forward tracking case) for the entire current frame, and every node in this mesh is tracked to one and only one node in the tracked frame, so that the nodes in the tracked frame still form a mesh that covers the entire frame. In order to handle newly appearing or disappearing objects in a scene, one should

allow the deletion of nodes corresponding to disappeared objects, and creation of new nodes in newly appearing objects. For a solution to such problem, see [3].

## 6.7 Global Motion Estimation

In Sec. 5.5, we showed that, depending on the camera and object motion and the object surface geometry, the mapping function between the two images of the same imaged object can be described by a translation, a geometric transformation, an affine mapping, or a projective mapping. Such a model can be applied to the entire frame if the entire motion field is caused by a camera motion, or if the imaged scene consists of a single object that is undergoing a rigid 3D motion.<sup>6</sup>

In practice, one can hardly find a video sequence that contains a single object. There are usually at least two objects: a stationary background and a moving foreground. More often, there are more than one foreground objects. Fortunately, when the foreground object motion is small compared to the camera motion, and the camera does not move in the Z-direction, the motion field can be approximated well by a global model. This will be the case, for example, when the camera pans over a scene or zooms into a particular subject in a relatively fast speed. Such camera motions are quite common in sports video and movies. Even when the actual 2D motion field cannot be represented by a single global motion accurately, as long as the effect of the camera motion dominates over other motions (motion of individual small objects), determination of this dominant global motion is still very useful. In this section, we discuss the estimation of global motions.

There are in general two approaches for estimating the global motion. One is to estimate the global motion parameters directly by minimizing the prediction errors under a given set of motion parameters. The other is to first determine pixel-wise or block-wise motion vectors, using the techniques described previously, and then using a regression method to find the global motion model that best fits the estimated motion field. The latter method can also be applied to motion vectors at selected feature points, such as points with strong edges.

### 6.7.1 Robust Estimator

A difficulty in estimating the global motion is that a pixel may not experience only the global motion. Usually, the motion at any pixel can be decomposed into a global motion (caused by camera movement) and a local motion because of the movement of the underlying object. Therefore, the prediction error obtained by using the global motion model alone may not be small, even if the correct global motion parameters are available. In other instances, not all the pixels in the same frame experience the global motion and ideally one should not apply the same motion

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<sup>6</sup>Recall that in the case that the camera or the object moves in the Z-direction, the motion field can be represented by a projective mapping only if the object surface is planar. When the object surfaces is spatially varying, the mapping function at any point also depends on the surface depth of that point and cannot be represented by a global model.

model to the entire frame. These problems can be overcome by a *robust estimation* method [35], if the global motion is dominant over other local motions, in the sense that the pixels that experience the same global motion and only the global motion occupy a significantly larger portion of the underlying image domain than those pixels that do not.

The basic idea in robust estimation is to consider the pixels that are governed by the global motion as *inliers*, and the remaining pixels as *outliers*. Initially, one assumes that all the pixels undergo the same global motion, and estimate the motion parameters by minimizing the prediction or fitting error over all the pixels. This will yield an initial set of motion parameters. With this initial solution, one can then calculate the prediction or fitting error over each pixel. The pixels where the errors exceed a certain threshold will be classified as outliers and be eliminated from the next iteration. The above process is then repeated to the remaining inlier pixels. This process iterates until no outlier pixels exist. This approach is called *Hard Threshold Robust Estimator*.

Rather than simply labeling a pixel as either inlier or outlier at the end of each iteration, one can also assign a different weight to each pixel, with a large weight for a pixel with small error, and vice versa. At the next minimization or fitting iteration, a weighted error measure is used, so that the pixels with larger errors in the previous iteration will have less impact than those with smaller errors. This approach is known as *Soft Threshold Robust Estimator*.

### 6.7.2 Direct Estimation

In either the hard or soft threshold robust estimator, each iteration involves the minimization of an error function. Here we derive the form of the function when the model parameters are directly obtained by minimizing the prediction error. We only consider the soft-threshold case, as the hard-threshold case can be considered as a special case where the weights are either one or zero. Let the mapping function from the anchor frame to the tracked frame be denoted by  $\mathbf{w}(\mathbf{x}; \mathbf{a})$ , where  $\mathbf{a}$  is the vector that contains all the global motion parameters. The prediction error can be written as, following Eq. (6.2.1):

$$E_{DFD} = \sum_{\mathbf{x} \in \Lambda} w(\mathbf{x}) |\psi_2(\mathbf{w}(\mathbf{x}; \mathbf{a})) - \psi_1(\mathbf{x})|^p \quad (6.7.1)$$

where  $w(\mathbf{x})$  are the weighting coefficients for pixel  $\mathbf{x}$ . Within each iteration of the robust estimation process, the parameter vector  $\mathbf{a}$  is estimated by minimizing the above error, using either gradient-based method or exhaustive search. The weighting factor at  $\mathbf{x}$ ,  $w(\mathbf{x})$ , in a new iteration will be adjusted based on the DFD at  $\mathbf{x}$  calculated based on the motion parameters estimated in the previous iteration.

### 6.7.3 Indirect Estimation

In this case, we assume that the motion vectors  $\mathbf{d}(\mathbf{x})$  have been estimated at a set of sufficiently dense points  $\mathbf{x} \in \Lambda' \subset \Lambda$ , where  $\Lambda$  represent the set of all pixels.

This can be accomplished, for example, using either the block-based or mesh-based approaches described before. One can also choose to estimate the motion vectors at only selected feature points, where the estimation accuracy is high. The task here is to determine  $\mathbf{a}$  so that the model  $\mathbf{d}(\mathbf{x}; \mathbf{a})$  can approximate the pre-estimated motion vectors  $\mathbf{d}(\mathbf{x}), \mathbf{x} \in \Lambda'$  well. This can be accomplished by minimizing the following fitting error:

$$E_{\text{fitting}} = \sum_{\mathbf{x} \in \Lambda'} w(\mathbf{x}) |\mathbf{d}(\mathbf{x}; \mathbf{a}) - \mathbf{d}(\mathbf{x})|^p \quad (6.7.2)$$

As shown in Sec. 5.5.4, a global motion can usually be described or approximated by a polynomial function. In this case,  $\mathbf{a}$  consists of the polynomial coefficients and  $\mathbf{d}(\mathbf{x}; \mathbf{a})$  is a linear function of  $\mathbf{a}$ , i.e.,  $\mathbf{d}(\mathbf{x}; \mathbf{a}) = [\mathbf{A}(\mathbf{x})]\mathbf{a}$ . If we choose  $p = 2$ , then the above minimization problem becomes a weighted least squares problem. By setting  $\frac{\partial E_{\text{fitting}}}{\partial \mathbf{a}} = 0$ , we obtain the following solution

$$\mathbf{a} = \left( \sum_{\mathbf{x} \in \Lambda'} w(\mathbf{x}) [\mathbf{A}(\mathbf{x})]^T [\mathbf{A}(\mathbf{x})] \right)^{-1} \left( \sum_{\mathbf{x} \in \Lambda'} w(\mathbf{x}) [\mathbf{A}(\mathbf{x})]^T \mathbf{d}(\mathbf{x}) \right). \quad (6.7.3)$$

As an example, consider the affine motion model given in Eq. (5.5.16). The motion parameter vector is  $\mathbf{a} = [a_0, a_1, a_2, b_0, b_1, b_2]^T$ , and the matrix  $[\mathbf{A}(\mathbf{x})]$  is

$$[\mathbf{A}(\mathbf{x})] = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix}.$$

In fact, the parameters for the  $x$  and  $y$  dimensions are not coupled and can be estimated separately, which will reduce the matrix sizes involved. For example, to estimate the  $x$  dimensional parameter  $\mathbf{a}_x = [a_0, a_1, a_2]$ , the associated matrix is  $[\mathbf{A}_x(\mathbf{x})] = [1, x, y]$ , and the solution is:

$$\mathbf{a}_x = \left( \sum_{\mathbf{x} \in \Lambda'} w(\mathbf{x}) [\mathbf{A}_x(\mathbf{x})]^T [\mathbf{A}_x(\mathbf{x})] \right)^{-1} \left( \sum_{\mathbf{x} \in \Lambda'} w(\mathbf{x}) [\mathbf{A}_x(\mathbf{x})]^T d_x(\mathbf{x}) \right). \quad (6.7.4)$$

## 6.8 Region-Based Motion Estimation\*

As already pointed out in the previous section, there are usually multiple types of motions in the imaged scene, which correspond to motions associated with different objects. By region-based motion estimation, we mean to segment the underlying image frame into multiple regions and estimate the motion parameters of each region. The segmentation should be such that a single parametric motion model can represent well the motion in each region. Obviously, region segmentation is dependent on the motion model used for characterizing each region. The simplest approach is to require each region undergo the same translational motion. This requirement however can result in too many small regions, because the 2-D motion

in a region corresponding to a physical object can rarely be modeled by a simple translation. Such a region would have to be split into many small sub-regions for each sub-region to have a translational motion. For a more efficient motion representation, an affine or bilinear or perspective motion model should be used.

In general, there are three approaches for accomplishing region-based motion estimation. With the first approach, one first segments the image frame into different regions based on texture homogeneity, the edge information, and sometimes the motion boundary obtained by analyzing the difference image between two frames, and then estimates the motion in each region. The latter can be accomplished by applying the global motion estimation method described in Sec. 6.7 to each region. We call such a method *region-first*. With the second approach, one first estimates the motion field over the entire frame, and then segments the resulting motion field so that the motion in each region can be modeled by a single parametric model. We call this method *motion-first*. The resulting region can be further refined subject to some spatial connectivity constraints. The first step can be accomplished using the various motion estimation methods described previously, including the pixel-, block-, and mesh-based methods. The second problem involves motion-based segmentation, which is discussed further in Sec. 6.8.1. Yet the third approach is to jointly estimate region partition and motion in each region. In general, this is accomplished by an iterative process, which performs region segmentation and motion estimation alternatively. This approach is described in Sec. 6.8.2.

### 6.8.1 Motion-Based Region Segmentation

As described already, motion-based segmentation refers to the partitioning of a motion field into multiple regions so that the motion within each region can be described by a single set of motion parameters. Here we present two approaches for accomplishing this task. The first approach uses a clustering technique to identify similar motion vectors. The second approach uses a layered approach to estimate the underlying region and associated motions sequentially, starting from the region with the most dominant motion.

**Clustering** Consider the case when the motion model for each region is a pure translation. Then the segmentation task is to group all spatially connected pixels with similar motion vectors into one region. This can be easily accomplished by an automated clustering method, such as the K-means or the ISODATA method [8]. This is an iterative process: Starting from an initial partitioning, the mean motion vector, known as the centroid, of each region is calculated. Then each pixel is reclassified into the region whose centroid is closest to the motion vector of this pixel. This results in a new partition and the above two steps can be repeated until the partition does not change any more. In this process, the spatial connectivity is not considered. Therefore, the resulting regions may contain pixels that are not spatially connected. Postprocessing steps may be applied at the end of the iterations to improve the spatial connectivity of the resulting regions. For example, a single region may be split into several sub-regions so that each region is a spatially

connected subset, isolated pixels may be merged into its surrounding region, and finally region boundaries can be smoothed using morphological operators.

When the motion model for each region is not a simple translation, motion-based clustering is not as straight forward. This is because one cannot use the similarity between motion vectors as the criterion for performing clustering. One way is to find a set of motion parameters for each pixel by fitting the motion vectors in its neighborhood into a specified model. Then one can employ the clustering method described previously, by replacing the raw motion vector with the motion parameter vector. If the original motion field is given in the block-based representation using a higher order model, then one can cluster blocks with similar motion parameters into the same region. Similarly, with the mesh-based motion representation, one can derive a set of motion parameters for each element based on its nodal displacements, and then clustering the elements with similar parameters into the same region. This is the parallel approach described in [44].

**Layered Approach** Very often, the motion field in a scene can be decomposed into layers, with the first layer representing the most dominant motion, the second layer the less dominant one, and so on. Here, the dominance of a motion is determined by the area of the region undergoing the corresponding motion. The most dominant motion is often a reflection of the camera motion, which affects the entire imaged domain. For example, in a video clip of a tennis play, the background will be the first layer, which usually undergoes a coherent global camera motion; the player the second layer (which usually contains several sub-object level motions corresponding to the movements of different parts of the body), the racket the third, and the ball the fourth layer. To extract the motion parameters in different layers, one can use the robust estimator method described in Sec. 6.7.1 recursively. First, we try to model the motion field of the entire frame by a single set of parameters, and continuously eliminate outlier pixels from the remaining inlier group, until all the pixels within the inlier group can be modeled well. This will yield the first dominant region (corresponding to the inlier region) and its associated motion. The same approach can then be applied to the remaining pixels (the outlier region) to identify the next dominant region and its motion. This process continues until no more outlier pixels exist. As before, postprocessing may be invoked at the end of the iterations to improve the spatial connectivity of the resulting regions. This is the sequential approach described in [44].

For the above scheme to work well, the inlier region must be larger than the outlier region at any iteration. This means that the largest region must be greater than the combined area of all other regions, the second largest region must be greater than the combined area of the remaining regions, and so on. This condition is satisfied in most video scenes, which often contain a stationary background that covers a large portion of the underlying image, and different moving objects with varying sizes.

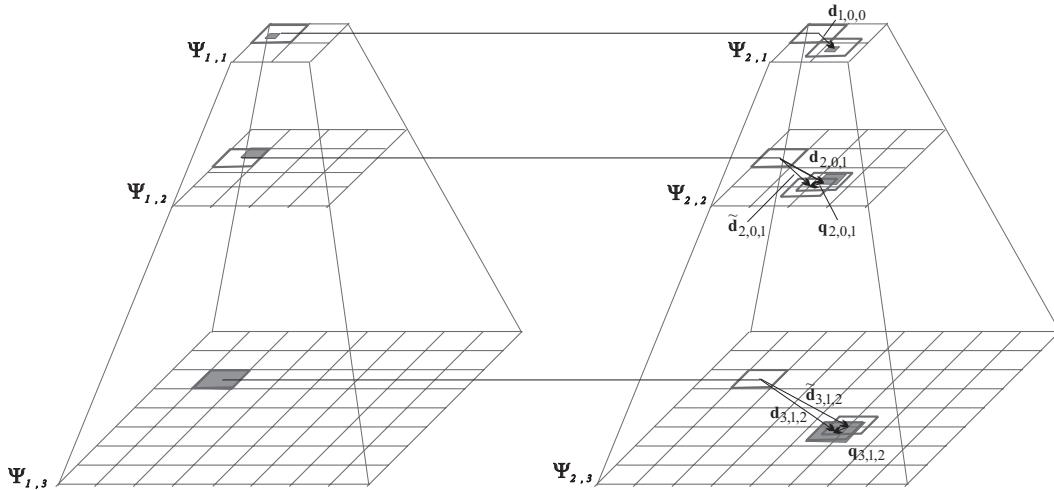
### 6.8.2 Joint Region Segmentation and Motion Estimation

Theoretically, one can formulate the joint estimation of the region segmentation map and the motion parameters of each region as an optimization problem. The function to be minimized could be a combination of the motion compensated prediction error and a region smoothness measure. The solution of the optimization problem however is difficult because of the very high dimension of the parameter space and the complicated interdependence between these parameters. In practice, a sub-optimal approach is often taken, which alternates between the estimation of the segmentation and motion parameters. Based on an initial segmentation, the motion of each region is first estimated. In the next iteration, the segmentation is refined, e.g., by eliminating outlier pixels in each region where the prediction errors are large, and by merging pixels sharing similar motion models. The motion parameters for each refined region are then be re-estimated. This process continues until no more changes in the segmentation map occur.

An alternative approach is to estimate the regions and their associated motions in a layered manner, similar to the layered approach described previously. There, we assume that the motion vector at every point is already known, and the identification of the region with the most dominant motion (*i.e.* the inlier) is accomplished by examining the fitting error induced by representing individual MVs using a set of motion parameters. This is essentially the indirect robust estimator presented in Sec. 6.7.3. In the joint region segmentation and motion estimation approach, to extract the next dominant region and its associated motion from the remaining pixels, we can use the direct robust estimator. That is, we directly estimate the motion parameters, by minimizing the prediction errors at these pixels. Once the parameters are determined, we determine whether a pixel belongs to the inlier by examining the prediction error at this pixel. We then re-estimate the motion parameters by minimizing the prediction errors at the inlier pixels only. This approach has been taken by Hsu [18].

## 6.9 Multi-Resolution Motion Estimation

As can be seen from previous sections, various motion estimation approaches can be reduced to solving an error minimization problem. Two major difficulties associated with obtaining the correct solution are: i) the minimization function usually has many local minima and it is not easy to reach the global minimum, unless it is close to the chosen initial solution; and ii) the amount of computation involved in the minimization process is very high. Both problems can be combated by the multi-resolution approach, which searches the solution for an optimization problem in successively finer resolutions. By first searching the solution in a coarse resolution, one can usually obtain a solution that is close to the true motion. In addition, by limiting the search in each finer resolution to a small neighborhood of the solution obtained in the previous resolution, the total number of searches can be reduced, compared to that required by directly searching in the finest resolution over a large



**Figure 6.19.** Illustration of the Hierarchical Block Matching Algorithm.

range.

In this section, we first describe the multi-resolution approach for motion estimation in a general setting, which is applicable to any motion models. We then focus on the block-translation model, and describe a hierarchical block matching algorithm.

### 6.9.1 General Formulation

As illustrated in Fig. 6.19, pyramid representations of the two raw image frames are first derived, in which each level is a reduced resolution representation of the lower level, obtained by spatial low-pass filtering and sub-sampling. The bottom level is the original image. Then the motion field between corresponding levels of the two pyramids is estimated, starting from the top coarsest level, and progressing to the next finer level repeatedly. At each new finer resolution level, the motion field obtained at the previous coarser level is interpolated to form the initial solution for the motion at the current level. The most common pyramid structure is the one in which the resolution is reduced by half both horizontally and vertically between successive levels. Usually, a simple  $2 \times 2$  averaging filter is used for low-pass filtering. For better performance, a Gaussian filter can be employed.

Assume that the number of levels is  $L$ , with the  $L$ -th level being the original image. Let the  $l$ -th level images of the anchor and tracked frames be represented by  $\psi_{t,l}(\mathbf{x})$ ,  $\mathbf{x} \in \Lambda_l$ ,  $t = 1, 2$ , where  $\Lambda_l$  is the set of pixels at level  $l$ . Denote the total motion field obtained from levels 1 to  $l - 1$  by  $\mathbf{d}_{l-1}(\mathbf{x})$ . At the  $l$ -th level, we first interpolate  $\mathbf{d}_{l-1}(\mathbf{x})$  to the resolution of level  $l$ , to produce an initial motion estimate  $\tilde{\mathbf{d}}_l(\mathbf{x}) = \mathcal{U}(\mathbf{d}_{l-1}(\mathbf{x}))$ , where  $\mathcal{U}$  represents the interpolation operator. We

then determine the update  $\mathbf{q}_l(\mathbf{x})$  at this level so that the error

$$\sum_{\mathbf{x} \in \Lambda_l} \left| \psi_{2,l}(\mathbf{x} + \tilde{\mathbf{d}}_l(\mathbf{x}) + \mathbf{q}_l(\mathbf{x})) - \psi_{1,l}(\mathbf{x}) \right|^p$$

is minimized. The new motion field obtained after this step is  $\mathbf{d}_l(\mathbf{x}) = \mathbf{q}_l(\mathbf{x}) + \tilde{\mathbf{d}}_l(\mathbf{x})$ . Upon completion of successive refinements, the total motion at the finest resolution is

$$\mathbf{d}(\mathbf{x}) = \mathbf{q}_L(\mathbf{x}) + \mathcal{U}(\mathbf{q}_{L-1}(\mathbf{x}) + \mathcal{U}(\mathbf{q}_{L-2}(\mathbf{x}) + \cdots + \mathcal{U}(\mathbf{q}_1(\mathbf{x}) + \mathbf{d}_0(\mathbf{x})) \cdots)).$$

The initial conditions for the above procedure is  $\mathbf{d}_0(\mathbf{x}) = \mathbf{0}$ . One can either directly specify the total motion  $\mathbf{d}(\mathbf{x})$ , or the motion updates at all levels,  $\mathbf{q}_l(\mathbf{x}), l = 1, 2, \dots, L$ . The latter represents the motion in a layered structure, which is desired in applications requiring progressive retrieval of the motion field.

The benefits of the multi-resolution approach are two folds. First, the minimization problem at a coarser resolution is less ill-posed than at a finer resolution, and therefore, the solution obtained at a coarser level is more likely to be close to the true solution at that resolution. The interpolation of this solution to the next resolution level provides a good initial solution that is close to the true solution at that level. By repeating this step successively from the coarsest to the finest resolution level, the solution obtained at the finest resolution is more likely to be close to the true solution (the global minimum). Second, the estimation at each resolution level can be confined to a significantly smaller search range than the true motion range at the finest resolution, so that the total number of searches to be conducted is smaller than the number of searches required in the finest resolution directly. The actual number of searches will depend on the search ranges set at different levels.

The use of multi-resolution representation for image processing was first introduced by Burt and Adelson [6]. Application to motion estimation depends on the motion model used. In the above presentation, we have assumed that motion vectors at all pixels are to be estimated. The algorithm can be easily adapted to estimate block-based, mesh-based, global or object level motion parameters. Because the block-wise translational motion model is the most popular for practical applications, we consider this special case in more detail in the following.

### 6.9.2 Hierarchical Block Matching Algorithm (HBMA)

As indicated earlier in Sec. 6.4.1, using an exhaustive search scheme to derive block MVs requires an extremely large number of computations. In addition, the estimated block MVs often lead to a chaotic motion field. In this section, we introduce a hierarchical block matching algorithm (HBMA), which is a special case of the multi-resolution approach presented before. Here, the anchor and tracked frames are each represented by a pyramid, and the EBMA or one of its fast variants is employed to estimate MVs of blocks in each level of the pyramid. Fig. 6.19 illustrates the process when the spatial resolution is reduced by half both horizontally

and vertically at each increasing level of the pyramid. Here, we assume that the same block size is used at different levels, so that the number of blocks is reduced by half in each dimension as well. Let the MV for block  $(m, n)$  at level  $l$  be denoted by  $\mathbf{d}_{l,m,n}$ . Starting from level 1, we first find the MVs for all blocks in this level,  $\mathbf{d}_{1,m,n}$ . At each new level  $l > 1$ , for each block, its initial MV  $\tilde{\mathbf{d}}_{l,m,n}$  is interpolated from a corresponding block in level  $l - 1$  by

$$\tilde{\mathbf{d}}_{l,m,n} = \mathcal{U}(\mathbf{d}_{l-1,\lfloor m/2 \rfloor, \lfloor n/2 \rfloor}) = 2\mathbf{d}_{l-1,\lfloor m/2 \rfloor, \lfloor n/2 \rfloor}. \quad (6.9.1)$$

Then a correction vector  $\mathbf{q}_{l,m,n}$  is searched, yielding the final estimated MV

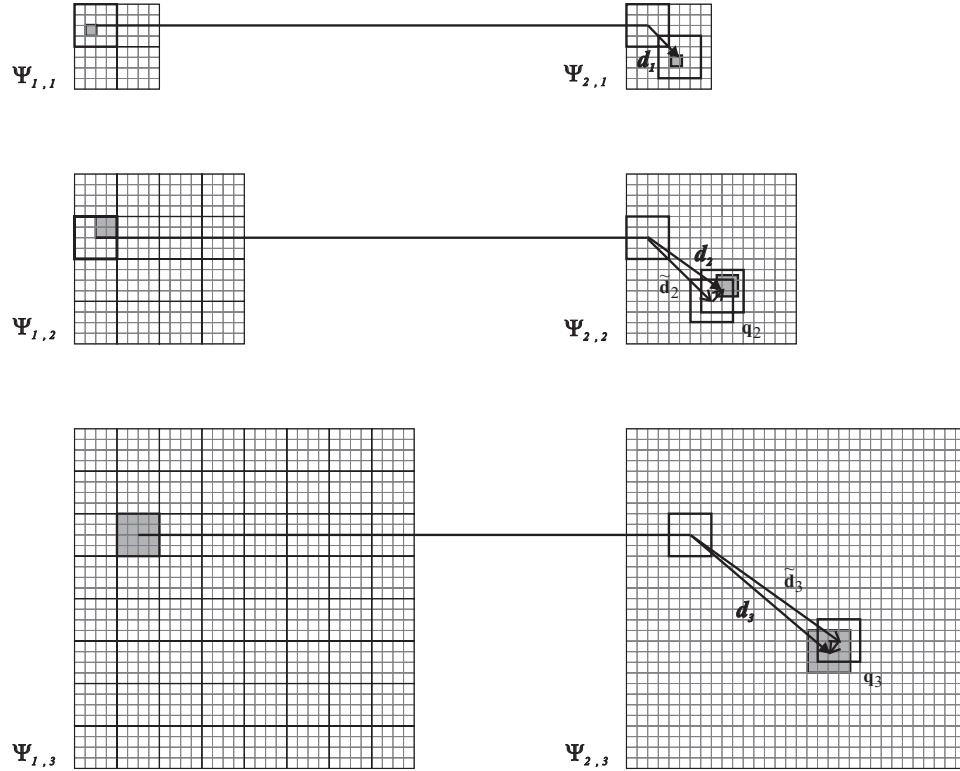
$$\mathbf{d}_{l,m,n} = \tilde{\mathbf{d}}_{l,m,n} + \mathbf{q}_{l,m,n}. \quad (6.9.2)$$

**Example 6.3:** In Fig. 6.20, we show two video frames, of size  $32 \times 32$ , in which a gray block in the anchor frame moved by a displacement of  $(13, 11)$ . We show how to use a three level HBMA to estimate the block motion field. The block size used at each level is  $4 \times 4$ , and the search stepsize is 1 pixel. Starting from level 1, for block  $(0,0)$ , the MV is found to be  $\mathbf{d}_{1,0,0} = \mathbf{d}_1 = (3, 3)$ . When going to level 2, for block  $(0,1)$ , it is initially assigned the MV  $\tilde{\mathbf{d}}_{2,0,1} = \mathcal{U}(\mathbf{d}_{1,0,0}) = 2\mathbf{d}_1 = (6, 6)$ . Starting with this initial MV, the correction vector is found to be  $\mathbf{q}_2 = (1, -1)$ , leading to the final estimated MV  $\mathbf{d}_{2,0,1} = \mathbf{d}_2 = (7, 5)$ . Finally at level 3, block  $(1,2)$  is initially assigned a MV of  $\tilde{\mathbf{d}}_{3,1,2} = \mathcal{U}(\mathbf{d}_{2,0,1}) = 2\mathbf{d}_2 = (14, 10)$ . With a correction vector of  $\mathbf{q}_3 = (-1, 1)$ , the final estimated MV is  $\mathbf{d}_{3,1,2} = \mathbf{d}_3 = (13, 11)$ .

Note that using a block width  $N$  at level  $l$  corresponds to a block width of  $2^{L-l}N$  at the full resolution. The same scaling applies to the search range and stepsize. Therefore, by using the same block size, search range and stepsize at different levels, we actually use a larger block size, a larger search range, as well as a larger stepsize in the beginning of the search, and then gradually reduce (by half) these in later steps.

The number of operations involved in the HBMA depends on the search range at each level. If the desired search range is  $R$  in the finest resolution, then with a  $L$ -level pyramid, one can set the search range to be  $R/2^{L-1}$  at the first level. For the remaining levels, because the initial MV interpolated from the previous level is usually quite close to the true MV, the search range for the correction vector do not need to be very large. However, for simplicity, we assume every level uses a search range of  $R/2^{L-1}$ . If the image size is  $M \times M$  and block size is  $N \times N$  at every level, the number of blocks at the  $l$ -th level is  $(M/2^{L-l}N)^2$ , and the number of searches is  $(M/2^{L-l}N)^2 * (2R/2^{L-1} + 1)^2$ . Because the number of operations required for each search is  $N^2$ , the total number of operations is

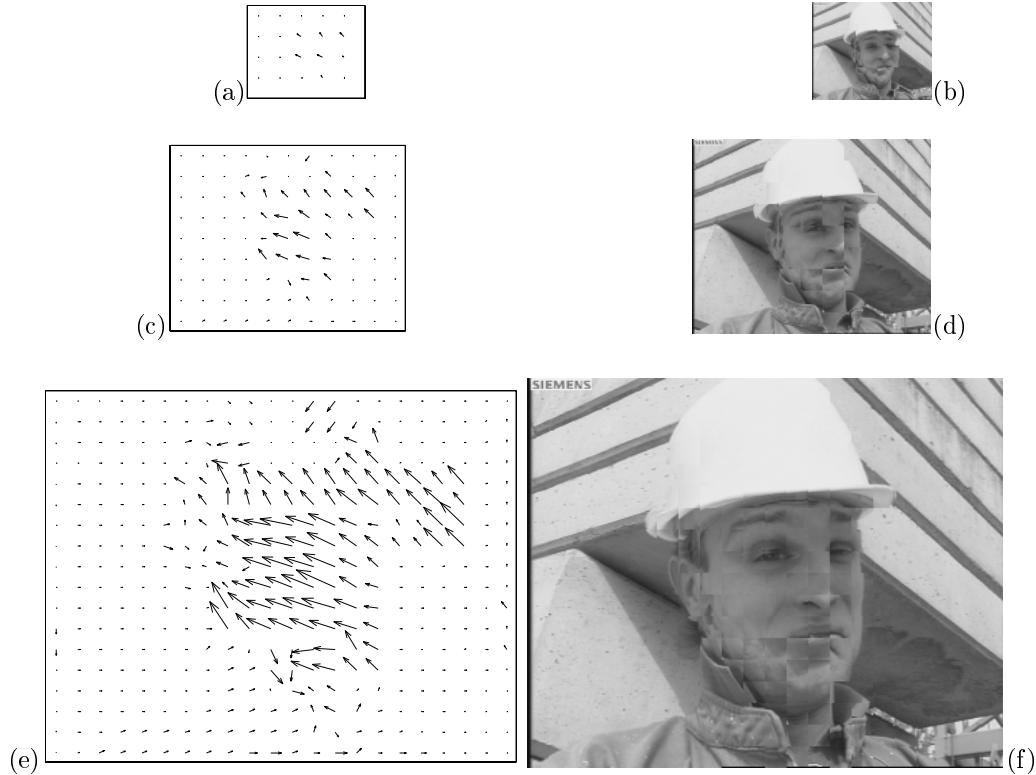
$$\begin{aligned} \sum_{l=1}^L (M/2^{L-l})^2 (2R/2^{L-1} + 1)^2 &= \frac{4}{3}(1 - 4^{-(L-1)})M^2(2R/2^{L-1} + 1)^2 \\ &\approx \frac{1}{3}4^{-(L-2)}4M^2R^2. \end{aligned}$$



**Figure 6.20.** An example of using 3-level HBMA for block motion estimation. See Example 6.3.

Recall that the operation count for EBMA is  $M^2(2R+1)^2 \approx 4M^2R^2$  (cf. Sec. 6.4.1). Therefore, the hierarchical scheme using the above parameter selection will reduce the computation by a factor of  $3 \cdot 4^{L-2}$ . Typically, the number of levels  $L$  is 2 or 3.

**Example 6.4:** Figure 6.21 shows the estimation results obtained with the HBMA approach, for the same pair of video frames given in Figure 6.8. For this example, a three-level pyramid is used. The search range in each level is set to 4, so that the equivalent search range in the original resolution is  $R = 16$ . Integer-pel accuracy search is used in all the levels. The final integer-accuracy solution is further refined to the half-pel accuracy by using a half-pel stepsize search in a search range of one pixel. Comparing the result in the final level with the ones shown in Figs. 6.8(c) and 6.8(d), we can see that the multi-resolution approach indeed yields a smoother motion field than EBMA. Visual observation also reveals that this motion field represents more truthfully the motion between the two image frames in Figs. 6.8(a) and 6.8(b). This is true in spite of the fact that the EBMA yields a higher PSNR. In terms of computational complexity, the half-pel accuracy EBMA algorithm used for



**Figure 6.21.** Example motion estimation results by HBMA for the two images shown in Fig. 6.8: (a-b) the motion field and predicted image at level 1; (c-d) the motion field and predicted image at level 2; (e-f) the motion field and predicted image at the final level ( $\text{PSNR}=29.32$ ); A three-level HBMA algorithm is used. The block size is  $16 \times 16$  at all levels. The search range is 4 at all levels with integer-pel accuracy. The result in the final level is further refined by a half-pel accuracy search in the range of  $\pm 1$ .

*Fig. 6.8(c-d) requires  $4.3E+8$  operations, while the three level algorithm here uses only  $1.1E+7$  operations, if we neglect the final refinement step using half-pel search.*

There are many variants to implementation of HBMA. Bierling is the first who applied this idea to block-based motion model [5]. A special case of hierarchical BMA is known as variable size or quad-tree BMA, which starts with a larger block size, and then repeatedly divides a block into four if the matching error for this block is still larger than a threshold. In this case, all the processings are done on the original image resolution.

## 6.10 Summary

### Relation between Image Intensity and Motion

Almost all motion estimation algorithms are based on the constant intensity assumption (Eq. (6.1.1) or Eq. (5.2.11)), or the optical flow equation (Eq. (6.1.3)) derived based on this assumption. This enables us to estimate motion by identifying pixels with similar intensity, subject to some motion models. Note that this assumption is valid only when the illumination source is ambient and temporally invariant, and that the object surface is diffusely reflecting (Sec. 5.2).

When the motion direction is orthogonal to image intensity gradient, or if the image gradient is zero, motion does not induce changes in image intensity. This is the inherent limit of intensity based motion estimation methods.

### Key Components in Motion Estimation

**Motion representation:** This depends on the partition used to divide a frame (pixel-based, block-based, mesh-based, region-based, global), the motion model used for each region of the partition (block, mesh-element, object-region, or entire frame), and the constraint between motions in adjacent regions.

**Motion estimation criterion:** We presented three criteria for estimating the motion parameters over each region: i) minimizing DFD (when the motion is small, this is equivalent to the method based on the optical-flow equation); ii) making the resulting motion field as smooth as possible across regions, while minimizing DFD; and iii) maximizing the *a posteriori* probability of the motion field given the observed frames. We showed that iii) essentially requires i) and ii). Instead of minimizing DFD, one can also detect peaks in the PCF, when the motion in a region is a pure translation.

**Optimization methods:** For chosen representation and criterion, the motion estimation problem is usually converted to an optimization (minimization or maximization) problem, which can be accomplished by using exhaustive search or gradient-based search. To speed up the search and avoid being trapped in local minima, a multi-resolution procedure can be used.

### Application of Motion Estimation in Video Coding

Motion estimation is a key element in any video coding system. As will be discussed in Sec. 9.3.1, an effective video coding method is to use block-wise temporal prediction, by which a block in a frame to be coded is predicted from its corresponding block in a previously coded frame, then the prediction error is coded. To minimize the bit rate for coding the prediction error, the appropriate criterion for motion estimation is to minimize the prediction error. The fact that the estimated motion field does not necessarily resemble the actual motion field is not problematic in such applications. Therefore, block matching algorithms described in Sec. 6.4 (EBMA and its fast variant including HBMA) offer simple and effective solutions. Instead of

using the MV estimated for each block directly for the prediction of that block, one can use a weighted average of the predicted values based on the MV's estimated for its neighboring blocks. This is known as overlapping block motion compensation, which will be discussed in Sec. 9.3.2.

Note that in the above video coding method, the motion vectors also need to be coded, in addition to the prediction error. Therefore, minimizing the prediction error alone is not the best criterion to use. Since a smoother motion field requires fewer bits to code, imposing smoothness in the estimated motion field, if done properly, can help improve the overall coding efficiency. More advanced motion estimation algorithms therefore operate by minimizing the total bit rate used for coding the MVs and the prediction errors. This subject is discussed further in Sec. 9.3.3.

To overcome the blocking artifacts produced by block-based motion estimation methods, high-order block-based (DBMA), mesh-based or a combination of block-based, mesh-based, and/or DBMA can be applied. However, these more complicated schemes usually do not lead to significant gain in the coding efficiency.

In more advanced video coding schemes (Chap 10), global motion estimation is usually applied to the entire frame, prior to block-based motion estimation, to compensate the effect of camera motion. Moreover, an entire frame is usually segmented into several regions or objects, and the motion parameters for each region or object are estimated using the global motion estimation method discussed here.

## 6.11 Problems

- 6.1** Describe the pros and cons of different motion representation methods (pixel-based, block-based, mesh-based, global).
- 6.2** Describe the pros and cons of the exhaustive search and gradient descent methods. Also compare between first order and second order gradient descent methods.
- 6.3** What are the main advantages of the multi-resolution estimation method, compared to an approach using a single resolution? Are there any disadvantages?
- 6.4** In Sec. 6.3.2, we derived the multipoint neighborhood method using the gradient descent method. Can you find a closed form solution using the optical flow equation? Under what condition, will your solution be valid?
- 6.5** In Sec. 6.4.1, we described an exhaustive search algorithm for determine block MVs in the block-based motion representation. Can you find a closed form solution using the optical flow equation? Under what condition, will your solution be valid?
- 6.6** In Eq. (6.2.7), we showed that, if the motion field is a constant, one can use the optical flow equation to set up a least squares problem, and obtain a closed-

form solution. Suppose that the motion field is not a constant, but can be modeled by a polynomial mapping. Can you find a closed-form solution for the polynomial coefficients?

Hint: any polynomial mapping function can be represented as  $\mathbf{d}(\mathbf{x}; \mathbf{a}) = [\mathbf{A}(\mathbf{x})]\mathbf{a}$ , where  $\mathbf{a}$  contains all the polynomial coefficients.

- 6.7** In Sec. 6.4.5, we said that when there are several patches in a range block in  $\phi_1(\mathbf{x})$  that undergo different motions, there will be several peaks in the PCF. Each peak corresponds to the motion of one patch. The location of the peak indicates the MV of the patch, whereas the amplitude of the peak is proportional to the size of the patch. Can you prove this statement at least qualitatively? You can simplify your derivation by considering the 1-D case only.
- 6.8** With EBMA, does the computational requirement depends on the block size?
- 6.9** In Sec. 6.9.2, we derived the number of operations required by HBMA, if the search range at every level is  $R/2^{L-1}$ . What would be the number if one uses a search range of  $\pm 1$  pel in every level, except at the first level, where the search range is set to  $R/2^{L-1}$ ? Is this parameter set-up appropriate?
- 6.10** Consider a CCIR601 format video, with Y-component frame size of  $720 \times 480$ . Compare the required computation by an EBMA algorithm (integer-pel) with block size  $16 \times 16$  and that by a two-level HBMA algorithm. Assume the maximum motion range is  $\pm 32$ . You can compare the computation by the operation number with each operation including one subtraction, one absolute value computation, and one addition. You can make your own assumption about the search range at different levels with HBMA. For simplicity, ignore the computations required for generating the pyramid and assume only integer-pel search.
- 6.11** Repeat the above for a three-level HBMA algorithm.
- 6.12** Write a C or *Matlab* code for implementing EBMA with integer-pel accuracy. Use a block size of  $16 \times 16$ . The program should allow the user to choose the search range, so that you can compare the results obtained with different search ranges. Note that the proper search range depends on the extent of the motion in your test images. Apply the program to two adjacent frames of a video sequence. Your program should produce and plot the estimated motion field, the predicted frame, the prediction error image. It should also calculate the PSNR of the predicted frame compared to the original tracked frame. With *Matlab*, you can plot the motion field using the function *quiver*.
- 6.13** Repeat the above exercise for EBMA with half-pel accuracy. Compare the PSNR of the predicted image obtained using integer-pel and that using half-pel accuracy estimation. Which method gives you more accurate prediction?

Which requires more computation time?

- 6.14** You can obtain a dense (i.e., pixel-based) motion field from a block-based one by spatial interpolation. Write a C or *Matlab* code that can interpolate the motion field resulting from Prob. 6.12 by assuming the MV for each block is actually the MV of the block center. Use bilinear interpolation. Using the interpolated pel-wise motion field, you can again produce the predicted image and the prediction error image. Compare the motion field, predicted image, and prediction error image obtained in Probs. 6.12 and 6.13 with those obtained here. Which method gives you more accurate prediction? Which requires more computation time?
- 6.15** Implement the HBMA method in C or *Matlab*. You can choose to use either two or three levels of resolution. You can use integer-pel search at all levels, but refine your final result by half-pel accuracy search within a  $\pm 1$  neighborhood. Use a block size of  $16 \times 16$  at all levels. The search range should be chosen so that the equivalent search range in the original resolution is  $\pm 32$ . Compare the results with that obtained in Probs. 6.12 and 6.13, in terms of both accuracy and computation time.
- 6.16** In Sec. 6.7, we say that the fitting error in Eq. (6.7.2) is minimized with the solution given in Eq. (6.7.3). Prove this result yourself.
- 6.17** Assume the motion between two frames can be modeled by a global affine model. We want to estimate the affine parameters directly based on the DFD criterion. Set up the optimization problem, and derive an iterative algorithm for solving the optimization problem. You can use either the first order gradient descent or the Newton-Ralphson method. Write a C or *Matlab* code for implementing your algorithm. Apply to two video frames that are undergoing predominantly camera motion (e.g., , the *flower garden* sequence). Compare the resulting motion field and predicted frame with that obtained with EBMA.
- 6.18** Repeat Prob. 6.17, but uses an indirect method to derive the affine parameters from given block motion vectors. Derive the regression equation and the closed-form solution. Write a C or *Matlab* code for implementing your algorithm. You can use your previous code for integer-pel EBMA to generate block MVs. Compare the result obtained with the direct method (Prob. 6.17).

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## Chapter 7

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# THREE DIMENSIONAL MOTION ESTIMATION

3D motion estimation allows for describing the motion of an object in 3D space. The motion is estimated from images of the moving objects. It is used in computer vision applications for object tracking as in vehicle guidance and robotics as well as in object-based video coding for object tracking and motion compensation. In some applications, a moving camera moves through a static scene, in others, a static camera observes moving objects, and for some applications such as surveillance from an airplane, the camera and objects move.

Similar to 2D motion estimation, 3D motion estimation is an ill-posed problem. In order to ensure useful solutions, we make simplifying assumptions like rigid objects, usually of known shape. The task is to estimate the six motion parameters of an object given in Eq. (5.3.9). Most algorithms assume that the object shape is planar or parabolic, if the shape of an object is not known.

Both orthographic projection and perspective projections can be used as camera models for 3D motion estimation. Naturally, the use of orthographic projection simplifies the estimation process. However, orthographic projection does not allow us to observe depth and causes estimation errors if the object moves along the optical axes of the camera or rotates around an axis other than the optical axis.

Another classification of 3D motion estimation algorithms distinguishes between direct methods that use the image signal as their input and indirect methods that rely on feature correspondences that have been established with a separate algorithm like block matching or feature matching. In section 7.1, we discuss algorithms that estimate motion from feature correspondences. The precision of the estimated motion parameters depends to a large extent on the precision of the feature correspondences. Given these correspondences, the algorithms can easily estimate large motion. Since motion is not estimated directly from the image signal but from features derived from the image signal, we call these algorithms *indirect motion estimation* methods. In section 7.2, we present *direct motion estimation methods*. They have the advantage of not relying on feature correspondences. However, these algorithms can only estimate small motion. Therefore, they are used in an iterative

estimation loop.

## 7.1 Feature-based Motion Estimation

Feature-based motion estimation assumes that identical features on an object have been identified in two images. In order to allow for reliable feature detection, these features usually correspond to physical landmarks on the object for which the motion has to be estimated. Features can be described as points, lines or corners (connection between lines) in the image plane. In the image plane, displacement vectors may be used to establish the correspondence between feature points. Feature correspondences may be identified using simple block matching. More reliable results can be achieved if we limit the matching to image areas with high spatial gradients. Therefore, we first extract edges in the two images and then try to match the edges. In this section, we will limit our discussion to algorithms that rely on feature correspondences. For a discussion of line and corner correspondences see [2] or the overview [11].

In the following sections, we will discuss four algorithms for feature-based motion estimation. Every algorithm assumes rigid 3D objects. The first (Section 7.1.1) assumes a camera model with orthographic projection and known object shape. The second algorithm assumes a camera model with perspective projection and known shape (Section 7.1.2). The algorithm in Section 7.1.3 is suitable for objects that can be approximated by planar surfaces. Finally, we present an algorithm that is based on perspective projection and does not require any knowledge about the object shape in Section 7.1.4.

### 7.1.1 Objects of Known Shape under Orthographic Projection

In this section, we assume that we know the shape of the moving object. Therefore, we know for each point  $\mathbf{x} = [x, y]^T$  in image  $k$  the 3D coordinates  $\mathbf{X} = [X, Y, Z]^T$  of the object surface point associated with  $\mathbf{x}$ . We use a camera model with orthographic projection according to Fig. 5.2(b) and Eq. (5.1.3). With this camera model and Eq. (5.3.9) for 3D motion, the mapping of a point  $\mathbf{X}$  onto the image plane before and after motion becomes

$$\mathbf{x}' = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} r_1 x + r_2 y + (r_3 Z + T_x) \\ r_4 x + r_5 y + (r_6 Z + T_y) \end{pmatrix} \quad (7.1.1)$$

with the rotation matrix

$$[R] = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \quad (7.1.2)$$

according to Eq. (5.3.6) and the translation vector  $\mathbf{T} = [T_x, T_y, T_z]^T$ . This is an affine mapping of point  $\mathbf{x}$  in image  $k$  onto image point  $\mathbf{x}'$  in image  $k+1$ . Linearizing

the elements  $r_i$  of the rotation matrix according to Eq. (5.3.6) to Eq. (5.3.10), we have an affine relation between  $\mathbf{x}$  and  $\mathbf{x}'$ :

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x - \theta_z y + (\theta_y Z + T_x) \\ \theta_z x + y - (\theta_x Z + T_y) \end{pmatrix}. \quad (7.1.3)$$

Given at least 3 point correspondences, the solution for the five unknown motion parameters  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$  and  $T_x$ ,  $T_y$  can be estimated using the least squares method. In order to increase the reliability of the estimated motion parameters, it is recommended to use at least 6 correspondences.

Eq. (7.1.3) uses the linearized rotation matrix. Therefore, the estimated motion parameters can only be an approximation of the real motion parameters. Therefore, the algorithm has to be used in an iterative fashion in order to estimate large motion parameters. With each iteration, the estimated motion parameters get smaller. At the same time the error due to the use of the linearized rotation matrix decreases. The iteration should stop as soon as the motion parameter updates approach 0. We present a more detailed example of iterative motion estimation in Section 7.3.

In Eq. (7.1.3), the motion parameters  $\theta_x$  and  $\theta_y$  are multiplied by the distance  $Z$  between the object surface point  $\mathbf{X}$ . Hence,  $\theta_x$  and  $\theta_y$  can only be estimated accurately if the object shape and hence  $Z$  is known precisely. Since the object shape or its depth is not always known, several proposals for estimating the object shape in conjunction with this motion estimation method have been proposed. In [1], a two step approach for each iteration is proposed that estimates the motion parameters and then updates the depth estimates of the object. In [4] a further improvement is suggested by updating the depth estimates  $Z$  according to the gradient of an error function. If reasonable depth estimates are not available, (7.1.3) can be used for objects that move translationally mainly parallel to the image plane and mainly rotate around the camera axis. That is, we assume  $\theta_x = \theta_y = 0$  and estimate  $\theta_z$ ,  $T_x$  and  $T_y$  only.

### 7.1.2 Objects of Known Shape under Perspective Projection

In order to estimate arbitrary motion of 3D objects with arbitrary but known shape, we assume that a known point  $\mathbf{X}$  moves to the unknown location  $\mathbf{X}'$ . Projecting  $\mathbf{X}'$  into the image plane using a camera model with perspective projection in Eq. (5.1.2) gives

$$\mathbf{x}' = \begin{bmatrix} x' \\ y' \end{bmatrix} = F \cdot \frac{1}{Z'} \begin{bmatrix} X' \\ Y' \end{bmatrix}. \quad (7.1.4)$$

If we now replace  $\mathbf{X}'$  with  $\mathbf{X}$  using the 3D motion equation (5.3.9), we get

$$\mathbf{x}' = F \cdot \begin{bmatrix} \frac{r_1 X + r_2 Y + r_3 Z + T_x}{r_7 X + r_8 Y + r_9 Z + T_z} \\ \frac{r_4 X + r_5 Y + r_6 Z + T_y}{r_7 X + r_8 Y + r_9 Z + T_z} \\ \frac{r_2 X + r_4 Y + r_6 Z + T_z}{r_7 X + r_8 Y + r_9 Z + T_z} \end{bmatrix}. \quad (7.1.5)$$

The problem is to solve for the motion parameters  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$ ,  $T_x$ ,  $T_y$ ,  $T_z$  assuming  $\mathbf{x}$ ,  $\mathbf{x}'$  and  $\mathbf{X}$  are known. In order to solve this equation using a linear equation

system, we assume small rotation angles and use the linearized rotation matrix according to Eq. (5.3.10). This yields

$$\mathbf{x}' = F \cdot \begin{bmatrix} \frac{X - \theta_z Y + \theta_y Z + T_x}{-\theta_y X + \theta_x Y + Z + T_z} \\ \frac{-\theta_y X + \theta_x Y + Z + T_z}{\theta_z X + Y - \theta_x Z + T_y} \\ \frac{\theta_z X + Y - \theta_x Z + T_y}{-\theta_y X + \theta_x Y + Z + T_z} \end{bmatrix}. \quad (7.1.6)$$

After further computation, we get

$$\begin{bmatrix} x' \cdot \frac{Z}{F} - X \\ y' \cdot \frac{Z}{F} - Y \end{bmatrix} = \begin{bmatrix} -Y \frac{x'}{F} & Z + X \frac{x'}{F} & -Y & 1 & 0 & -\frac{x'}{F} \\ -Z - Y \frac{y'}{F} & X \frac{y'}{F} & X & 0 & 1 & -\frac{y'}{F} \end{bmatrix} \cdot \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ T_x \\ T_y \\ T_z \end{bmatrix} \quad (7.1.7)$$

where the points  $\mathbf{x}'$  and  $\mathbf{X}$  are known and the six motion parameters are unknown. As before, each point correspondence provides two equations. In order to allow for inaccuracies in the feature point estimation, we should solve Eq. (7.1.7) for more than three points using a least squares algorithm. Due to the use of the linearized rotation matrix, we need to estimate the motion parameters iteratively.

### 7.1.3 Planar Objects

Estimation of motion parameters of a plane moving arbitrarily in space is an important problem. Often, we can approximate an object surface with one or more planes. For example, if we approximate an object surface using a wireframe, each triangle on the wireframe represents a planar patch. The algorithm here does not assume any knowledge of the orientation of the plane in space. Therefore, we will estimate 8 parameters that determine the plane orientation and motion.

Assuming the camera model with perspective projection according to Eq. (5.1.2), rigid objects moving according to Eq. (5.3.9) and the object plane

$$aX + bY + cZ = 1, \quad (7.1.8)$$

we can describe the mapping of point  $\mathbf{x}$  from image k to image k+1 as a projective mapping given in Eq. (5.5.14) [17][5]

$$[x', y']^T = \left[ \frac{a_0 + a_1 x + a_2 y}{1 + c_1 x + c_2 y}, \frac{b_0 + b_1 x + b_2 y}{1 + c_1 x + c_2 y} \right]^T \quad (7.1.9)$$

with the eight unknown motion and structure parameters  $a_i, b_i, c_i$ . Sometimes, these parameters are referred to as *Pure Parameters* [18].

We can solve for the pure parameters by having at least four point correspondences. If we limit ourselves to four point correspondences, no three of the four points are allowed to be co-linear [6]. In order to increase the reliability of the

results, we suggest to use more than 8 correspondences and use a least squares method to solve for the pure parameters.

In [20], Tsai and Huang estimate the 3D motion parameters and the orientation of the object plane from the 8 pure parameters using singular value decomposition (SVD). This is only useful if the pure parameters can be estimated reliably. Please note that this algorithm for estimating the pure parameters cannot easily be used for object tracking. After estimation of the parameters from two images, the object orientation is fixed. Therefore, there are only 6 parameters to be estimated for the next image. If we estimate again 8 pure parameters as done in [5], we allow the image plane to have two different orientations for one image frame. Normally, this algorithm is used for backward motion estimation only, i.e., for any small region in frame  $k+1$ , we assume that it corresponds to a planar patch in 3-D but with unknown plane orientation, and estimate its motion from frame  $k+1$  to frame  $k$  together with the plane orientation. This does not allow us to continuously track the same 3-D patch over multiple frames.

#### 7.1.4 Objects of Unknown Shape Using the Epipolar Line

In this section, we describe a motion estimation algorithm that allows to estimate 3D motion and shape without any knowledge of the object shape [8]. Without loss of generality, we set the camera focal length  $F$  to unity. We assume a rigid object and perspective projection. Starting with Eq. (7.1.5) and replacing  $\mathbf{X}$  with its projection  $\mathbf{x}$  into the image plane and  $F = 1$ , we get

$$\mathbf{x}' = \begin{pmatrix} \frac{r_1x+r_2y+r_3+T_x/Z}{r_7x+r_8y+r_9+T_z/Z} \\ \frac{r_4x+r_5y+r_6+T_y/Z}{r_7x+r_8y+r_9+T_z/Z} \\ \frac{r_7x+r_8y+r_9+T_z/Z}{r_7x+r_8y+r_9+T_z/Z} \end{pmatrix}. \quad (7.1.10)$$

Note that the above equation is the same as that presented in Eq. (5.5.13) with  $F = 1$ . This equation does not change if we multiply  $\mathbf{T}$  and  $Z$  by a constant value. Therefore, we will only be able to determine the direction of  $\mathbf{T}$  but not its absolute length. Its length depends on the object size and the distance of the object from the camera.

In section 7.1.2, we solved this non-linear equation using iterative methods and linearizations, assuming the object shape is known. Using an intermediate matrix allows for estimating the motion without knowing the shape [8] [19]. This intermediate matrix is known as *E-matrix* or *Essential matrix* with its nine *essential parameters*. Eliminating  $Z$  in Eq. (7.1.10) we can determine the relationship between  $\mathbf{x}$  and  $\mathbf{x}'$  as

$$[x', y', 1] \cdot [\mathbf{E}] \cdot [x, y, 1]^T = 0, \quad (7.1.11)$$

with

$$[\mathbf{E}] = \begin{bmatrix} e_1 & e_2 & e_3 \\ e_4 & e_5 & e_6 \\ e_7 & e_8 & e_9 \end{bmatrix} = [\mathbf{T}][\mathbf{R}]. \quad (7.1.12)$$

The matrix  $[\mathbf{T}]$  is defined as

$$[\mathbf{T}] = \begin{bmatrix} 0 & T_z & -T_y \\ -T_z & 0 & T_x \\ T_y & -T_x & 0 \end{bmatrix} \quad (7.1.13)$$

and the rotation matrix  $[\mathbf{R}]$  according to Eq. (5.3.6). Multiplying Eq. (7.1.11) with  $Z \cdot Z'$ , we get

$$[X', Y', Z'] \cdot [\mathbf{E}] \cdot [X, Y, Z]^T = 0. \quad (7.1.14)$$

Equation (7.1.14) can only be used for motion estimation for  $\mathbf{T} \neq 0$ , i.e. if the object rotates only, we cannot estimate its motion. Therefore we assume that the object of interest also moves translationally.

### Epipolar Line

Equation (7.1.11) defines a linear dependency between the corresponding image points  $\mathbf{x}$  and  $\mathbf{x}'$ . Consequently, the possible positions of  $\mathbf{x}'$  of a point  $\mathbf{x}$  after motion lie on a straight line. This line is called *epipolar line* and is defined by the motion parameters according to

$$a(\mathbf{x}, [\mathbf{E}])x' + b(\mathbf{x}, [\mathbf{E}])y' + c(\mathbf{x}, [\mathbf{E}]) = 0 \quad (7.1.15)$$

with

$$\begin{aligned} a(\mathbf{x}, [\mathbf{E}]) &= e_1x + e_2y + e_3, \\ b(\mathbf{x}, [\mathbf{E}]) &= e_4x + e_5y + e_6, \\ c(\mathbf{x}, [\mathbf{E}]) &= e_7x + e_8y + e_9. \end{aligned} \quad (7.1.16)$$

Figure 7.1 shows the epipolar line in image 2 for a point  $\mathbf{x}$  in image 1.

Motion estimation is carried out in two steps. First the E-matrix is estimated, then the E-matrix is decomposed into the rotation matrix and the translation vector.

### E-matrix Estimation

It is obvious that the E-matrix can only differ from 0 for  $\mathbf{T} \neq 0$ . With Eq. (7.1.11), we define one equation for each point correspondence. Since Eq. (7.1.11) is a homogeneous equation we can set an arbitrary parameter of the matrix to 1 like

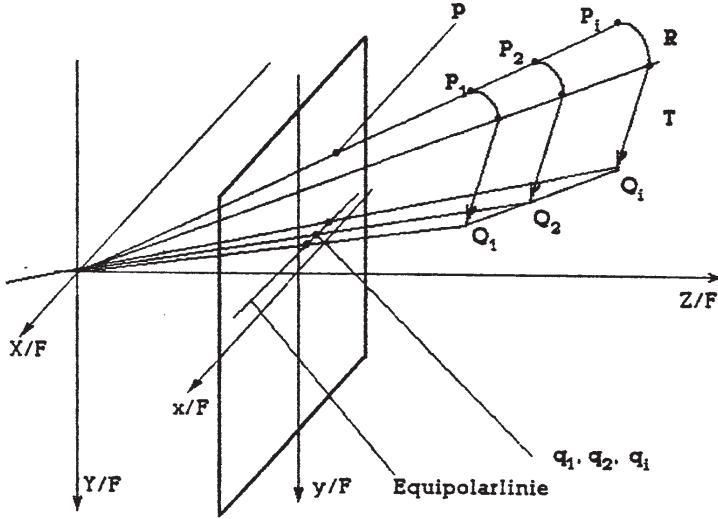
$$e_9 = 1. \quad (7.1.17)$$

With this constraint, we only need a minimum of 8 equations in order to estimate the E-matrix. For each measured point correspondence between  $\mathbf{x}^{(j)}$  and  $\mathbf{x}'^{(j)}$ , we can setup the following equation according to Eq. (7.1.11)

$$\mathbf{a}_j^T \mathbf{e}' = r_j \quad (7.1.18)$$

with  $\mathbf{a}_j = [x'_j x_j, x'_j y_j, x'_j, y'_j x_j, y'_j y_j, y'_j, x_j, y_j, 1]^T$  and  $\mathbf{e}' = [e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, 1]^T$ . All the point correspondences will lead to the following system of equations

$$[\mathbf{A}]\mathbf{e}' = \mathbf{r} \quad (7.1.19)$$



**Figure 7.1.** The epipolar line for a point  $x$  as defined by object motion

with

$$[\mathbf{A}] = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_J]^T. \quad (7.1.20)$$

We can then solve the over-determined system in Eq. (7.1.19) using a least squares method minimizing  $\|\mathbf{r}\|^2$ .

It can be shown that setting one of the components of the E-matrix to a constant value like in Eq. (7.1.17) may not result in the minimal achievable residual when solving for the E-matrix. An alternative approach is to require the vector  $\mathbf{e} = [e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9]^T$  to have unit norm, i.e.,

$$\|\mathbf{e}\| = 1. \quad (7.1.21)$$

Then, we can solve for the E-matrix using a constrained minimization problem

$$\|[\mathbf{A}]\mathbf{e}\| \xrightarrow{\mathbf{e}} \min \quad \text{subject to } \|\mathbf{e}\| = 1. \quad (7.1.22)$$

This will result in the minimal residual and a better solution for the E-matrix. The solution is the unit eigenvector of  $\mathbf{A}^T \mathbf{A}$  associated with the smallest eigenvalue. In order to avoid numerical instabilities, eigenvalues and eigenvectors of  $\mathbf{A}$  can be computed using Singular Value Decomposition (SVD) according to

$$\mathbf{A} = \mathbf{U} \Sigma \mathbf{V}^T. \quad (7.1.23)$$

The matrix  $\Sigma$  contains only the positive square roots of the non-negative eigenvalues of  $\mathbf{A}^T \mathbf{A}$  [14].

Following Problem 6 in Section 7.5, we see that the influence of an error in a measured point correspondence  $(\mathbf{x}, \mathbf{x}')$  depends on the actual image coordinates  $\mathbf{x}$ . Errors in correspondences close to the image center do not influence the solution as much as those at the image boundaries.

### Estimation of Rotation and Translation

Since the E-matrix is defined to describe the 3D motion of a rigid object, it has 5 degrees of freedom considering that we can only recover the orientation of the translation vector. However, during the estimation of the E-matrix we allow for 8 degrees of freedom. Therefore, the extraction of rotation and translation from the E-matrix requires to solve an optimization problem.

Following [21], we first estimate the translation vector. With  $[\mathbf{T}]$  defined in Eq. (7.1.13), we have

$$[\mathbf{T}]^T = -[\mathbf{T}] \quad \text{and} \quad [\mathbf{T}]\mathbf{T} = \mathbf{0}. \quad (7.1.24)$$

Multiply Eq. (7.1.12) by  $\mathbf{T}$ , we get

$$[\mathbf{E}]^T \cdot \mathbf{T} = [\mathbf{R}]^T [\mathbf{T}]^T \cdot \mathbf{T} = -[\mathbf{R}]^T [\mathbf{T}] \cdot \mathbf{T} = 0. \quad (7.1.25)$$

Hence, all row vectors of  $[\mathbf{E}]$  are linearly dependent ( $\det([\mathbf{E}]) = 0$ ) and  $\mathbf{T}$  is orthogonal to the plane defined by the column vectors of  $[\mathbf{E}]$ . If the E-matrix is estimated according to Eq. (7.1.22) for noisy point correspondences, we cannot assume Eq. (7.1.25) to hold exactly. Therefore, we estimate the translation vector with

$$\|[\mathbf{E}]^T \mathbf{T}\|_{\mathbf{T}} \rightarrow \min \quad \text{subject to } \|\mathbf{T}\| = 1, \quad (7.1.26)$$

preferably using SVD. Similar to the solution for  $\mathbf{e}$  in Eq. (7.1.22),  $\mathbf{T}$  is the unit eigenvector of  $[\mathbf{E}][\mathbf{E}]^T$  with minimal eigenvalue. From Eq. (7.1.25) it is clear that the sign of  $\mathbf{T}$  cannot be determined. Similarly, the solution for  $[\mathbf{E}]$  is not unique. Given an estimated E-matrix  $[\hat{\mathbf{E}}]$ , we know that  $-[\hat{\mathbf{E}}]$  is also a valid solution. In [22], it is shown that the correct sign for  $\mathbf{T}$  can be determined by selecting  $\mathbf{T}$  such that the following condition holds for all  $\mathbf{x}$ :

$$\sum_{\mathbf{x}} [\mathbf{T}[x', y', 1]]^T \cdot [\mathbf{E}] \cdot [x, y, 1]^T > 0. \quad (7.1.27)$$

This equation assures that  $\mathbf{X}$  and  $\mathbf{X}'$  are located in front of the camera before and after motion.

In a second step, we estimate the rotation matrix  $[\mathbf{R}]$ . Equation (7.1.12) can also be written as

$$[\mathbf{E}]^T = [\mathbf{R}]^T [\mathbf{T}]^T = -[\mathbf{R}]^T [\mathbf{T}]. \quad (7.1.28)$$

Therefore,  $[\mathbf{R}]$  can be found by the following optimization problem

$$\|[\mathbf{R}]^T [-\mathbf{T}] - \mathbf{E}^T\|_{[\mathbf{R}]} \rightarrow \min \quad \text{subject to } [\mathbf{R}]^T = [\mathbf{R}]^{-1} \text{ and } \det([\mathbf{R}]) = 1. \quad (7.1.29)$$

The constraints are valid for any rotation matrix  $[\mathbf{R}]$ . Equation (7.1.29) is a special case of the general problem

$$\|[\mathbf{F}][\mathbf{C}] - [\mathbf{D}]\|_{[\mathbf{F}]} \rightarrow \min \quad (7.1.30)$$

where  $[\mathbf{F}]$  must satisfy the properties of a rotation matrix, and  $[\mathbf{C}]$  and  $[\mathbf{D}]$  are arbitrary  $3 \times 3$  matrices. The solution is computed using the  $4 \times 4$  matrix  $[\mathbf{B}]$  [21]:

$$[\mathbf{B}] = \sum_{i=1}^3 [\mathbf{B}_i]^T [\mathbf{B}_i] \quad (7.1.31)$$

with

$$[\mathbf{B}_i] = \begin{bmatrix} 0 & c_{1i} - d_{1i} & c_{2i} - d_{2i} & c_{3i} - d_{3i} \\ d_{1i} - c_{1i} & 0 & -(c_{3i} + d_{3i}) & c_{2i} + d_{2i} \\ d_{2i} - c_{3i} & c_{3i} + d_{3i} & 0 & -(c_{1i} + d_{1i}) \\ d_{3i} - c_{2i} & -(c_{2i} + d_{2i}) & c_{1i} + d_{1i} & 0 \end{bmatrix} \quad (7.1.32)$$

where  $c_{ji}, d_{ji}$  are elements of  $[\mathbf{C}]$  and  $[\mathbf{D}]$ . First, we determine the unit eigenvector  $\mathbf{g} = [g_1, g_2, g_3, g_4]^T$  of  $[\mathbf{B}]$  with the smallest eigenvalue, preferable using SVD. Then the matrix  $[\mathbf{F}]$  can be obtained by

$$[\mathbf{F}] = \begin{bmatrix} g_1^2 + g_2^2 + g_3^2 + g_4^2 & 2(g_2g_3 - g_1g_4) & 2(g_2g_4 + g_1g_3) \\ 2(g_2g_3 + g_1g_4) & g_1^2 - g_2^2 + g_3^2 - g_4^2 & 2(g_3g_4 - g_1g_2) \\ 2(g_2g_4 - g_1g_3) & 2(g_3g_4 + g_1g_2) & g_1^2 - g_2^2 - g_3^2 + g_4^2 \end{bmatrix}. \quad (7.1.33)$$

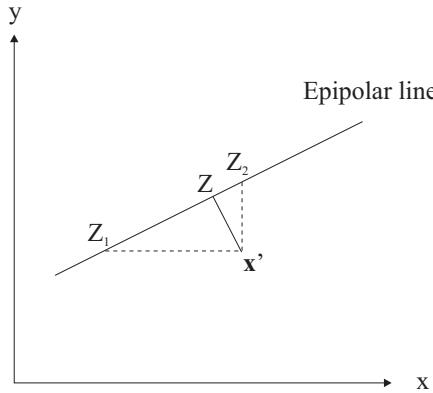
Setting  $[\mathbf{C}] = -[\mathbf{T}]$ ,  $[\mathbf{D}] = [\mathbf{E}]^T$ , then the rotation matrix is related to the above solution by  $[\mathbf{R}] = [\mathbf{F}]^T$ .

### Estimation accuracy

In this section we want to find out how well the estimated parameters represent the feature point correspondences. A suitable measure for the accuracy of the derived motion parameters is to compute the epipolar line for each feature point  $\mathbf{x}$  according to Eq. (7.1.15) using the estimated rotation and translation parameters. Then we measure the distance  $d^2$  between  $\mathbf{x}'$  and its epipolar line:

$$d^2 = \frac{(\mathbf{x}'[\mathbf{E}]\mathbf{x})^2}{a(\mathbf{x}, [\mathbf{E}])^2 + b(\mathbf{x}, [\mathbf{E}])^2} \quad (7.1.34)$$

The average distance  $\bar{d}$  indicates how good the E-matrix approximates the given correspondences. We have two possibilities of computing  $\bar{d}$ . We can use the estimated E-matrix according to Eq. (7.1.22) or we can use an E-matrix  $[\hat{\mathbf{E}}] = [\mathbf{T}][\mathbf{R}]$  that we compute from the estimated translation and rotation parameters according to Eq.(7.1.26) and (7.1.29), which are in turn calculated from the estimated



**Figure 7.2.** The projection of a point  $\mathbf{x}'$  onto the epipolar line at  $\mathbf{z}$  determines the location of the corresponding object point in the 3-D space.  $z_1$  and  $z_2$  are computed using Eq. (7.1.35) and (7.1.36).

E-matrix. It is important to note that  $\bar{d}$  computed using the second option is often an order of magnitude higher than  $\bar{d}$  computed using the first option. This indicates that motion estimation using the E-matrix is very sensitive to noise in the correspondences. Advanced algorithms use (7.1.22) as a starting point for an iterative algorithm that estimates the E-matrix subject to the constraints that  $\mathbf{E}$  is defined by  $\mathbf{R}$  and  $\mathbf{T}$  through Eq. (7.1.12).

### Estimation of Object Shape

Knowing the corresponding points  $\mathbf{x}$  and  $\mathbf{x}'$  as well as the motion  $\mathbf{R}$  and  $\mathbf{T}$ , we can compute  $Z$  for a point according to Eq. (7.1.10) in two ways:

$$Z = \frac{T_y - y'T_z}{(xr_7 + yr_7 + r_9)y' - (xr_4 + yr_5 + r_6)}, \quad (7.1.35)$$

or

$$Z = \frac{T_x - x'T_z}{(xr_7 + yr_8 + r_9)x' - (xr_1 + yr_2 + r_3)}. \quad (7.1.36)$$

If  $\mathbf{x}'$  is not located on the epipolar line, the above two equations give different estimates for  $Z$ . Figure 7.2 shows how (7.1.35) and (7.1.36) project  $\mathbf{x}'$  onto the epipolar line. A better approach to determine  $Z$  is to choose  $Z$  at the orthogonal projection of  $\mathbf{x}'$  onto the epipolar line. With  $Z$  known,  $X$  and  $Y$  can be determined from  $\mathbf{x}$  using inverse perspective projection.

## 7.2 Direct Motion Estimation

In the previous section, we assumed having a small number of accurate feature point correspondences for features that are visible in two images. Since these accurate

feature point correspondences are not always available, we now introduce motion estimation algorithms that use the image signal and its gradients directly. Since the image signal tends to be noisy we expect to use a rather large number of image points for determining 3D motion. We call these points *observation points*. We do not know the position of an observation point in a previous or following image. If the assumptions that we make with respect to object shape, illumination and camera model hold, these algorithms can measure large motion. An obvious consequence of the assumptions is that the projections of an object in two consecutive images have to overlap. Therefore, the motion estimation range of direct motion estimation algorithms may be smaller than that of indirect motion estimation algorithms.

We will first start with deriving the basic relationship between images and motion (Section 7.2.1). Then, we will present an algorithm similar to Section 7.1.2 that allows to estimate the 3D motion of a 3D object with known shape without feature points (Section 7.2.2). Furthermore we will apply this knowledge to extend the algorithm of Section 7.1.3 to work without feature points (Section 7.2.3). In Section 7.2.4, we present several methods with which the robustness of these estimation algorithms with respect to the estimation accuracy can be increased.

### 7.2.1 Image Signal Models and Motion

In order to derive a direct motion estimation algorithm, it is assumed that differences between two consecutive luminance images  $\psi_k(\mathbf{x})$  and  $\psi_{k+1}(\mathbf{x})$  are due to object motion only. This implies that objects have diffuse reflecting surfaces and that the scene is illuminated using ambient light sources (see Section 5.2). In [16] and [4] illumination effects are considered as well. We now relate the frame difference between the two video frames  $\psi_k(\mathbf{x})$  and  $\psi_{k+1}(\mathbf{x})$  to motion.

First, we need an analytical description of the image signal. Using the first order Taylor expansion of  $\psi(\mathbf{x} + \Delta\mathbf{x})$  at  $\mathbf{x}$ , we obtain

$$\psi(\mathbf{x} + \Delta\mathbf{x}) \approx \psi(\mathbf{x}) + \frac{\partial\psi}{\partial x}\Delta x + \frac{\partial\psi}{\partial y}\Delta y. \quad (7.2.1)$$

Obviously, this linear signal model is only a crude approximation of a real image. Furthermore, the approximation is only valid for small  $\Delta\mathbf{x}$ . In Appendix A, we describe several methods for computing the image gradient  $\mathbf{g}(\mathbf{x}) = [\frac{\partial\psi(\mathbf{x})}{\partial x}, \frac{\partial\psi(\mathbf{x})}{\partial y}]^T$  at a location  $\mathbf{x}$  in an image.

Let us consider an observation point on an object surface that is located at  $\mathbf{X}$  at time instant  $k$  and at  $\mathbf{X}'$  at time instant  $k + 1$ . With  $\mathbf{X}$  projected onto  $\mathbf{x}$  in image  $\psi_k$  and the same point  $\mathbf{X}'$  projected onto  $\mathbf{x}'$  in image  $\psi_{k+1}$  after motion, the luminance difference between images  $k$  and  $k + 1$  at position  $\mathbf{x}$  is

$$\begin{aligned} \Delta\psi_k(\mathbf{x}) &= \psi_{k+1}(\mathbf{x}) - \psi_k(\mathbf{x}) \\ &= \psi_{k+1}(\mathbf{x}) - \psi_{k+1}(\mathbf{x}'). \end{aligned} \quad (7.2.2)$$

Eq. (7.2.2) holds because we assume that all changes are due to object motion according to the constant intensity assumption  $\psi_{k+1}(\mathbf{x}') = \psi_k(\mathbf{x})$  (Eq. 5.2.11).

With the linear signal model of Eq. (7.2.1) and  $\Delta\mathbf{x} = \mathbf{x}' - \mathbf{x}$ , we can rewrite Eq. (7.2.2) as

$$\Delta\psi_k(\mathbf{x}) = -\mathbf{g}_{k+1}(\mathbf{x}) \cdot (\mathbf{x}' - \mathbf{x}). \quad (7.2.3)$$

Due to the linear signal model,  $\mathbf{g}_{k+1}(\mathbf{x}) = \mathbf{g}_k(\mathbf{x})$  holds. Therefore, Eq. (7.2.4) can be also represented as

$$\Delta\psi_k(\mathbf{x}) = -\mathbf{g}_k(\mathbf{x}) \cdot (\mathbf{x}' - \mathbf{x}). \quad (7.2.4)$$

Note that the above equation is equivalent to the discrete version of the optical flow equation given in Eq. (6.2.3).

Equation (7.2.4) gives us a relationship between an observation point  $\mathbf{x}$  in image  $k$  and its displaced position  $\mathbf{x}'$  in image  $k + 1$ . Fig. 7.3 illustrates this relationship between image gradient, frame difference and 2D motion. For Fig. 7.3(a), Eq. (7.2.4) holds exactly because the image signals  $\psi_k$  and  $\psi_{k+1}$  are linear. In this example, the luminance difference is  $\Delta\psi_k(\mathbf{x})$  is negative, and the image gradient  $\mathbf{g}_k(\mathbf{x})$  is positive resulting in a positive displacement  $(\mathbf{x}' - \mathbf{x}) > 0$ . The situation in Fig. 7.3(b) is similar. However, the image signal is not linear anymore. In this example, the measured frame difference  $\Delta\psi_k(\mathbf{x})$  is smaller than the linear signal model would suggest. Hence, we will estimate a displacement with the correct sign but a too small amplitude. Due to this effect, we will have to estimate motion iteratively.

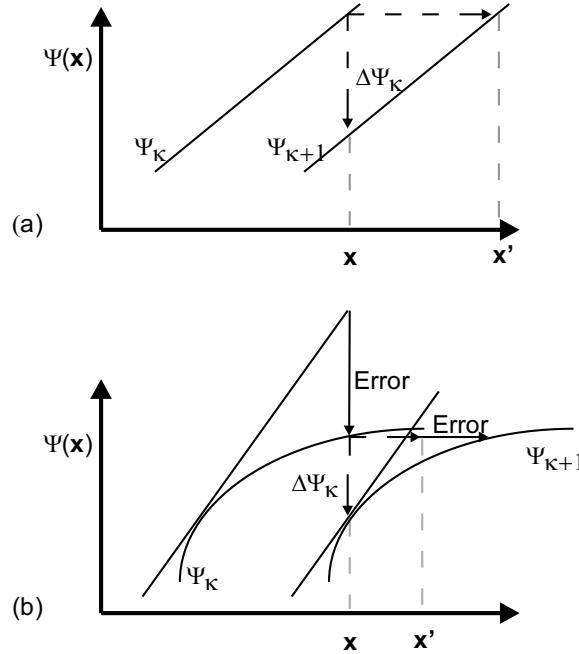
Looking at Fig. 7.3(b), we can imagine that we would measure a luminance difference with an unpredictable sign if the image signal contains very high frequencies. In this case, the displacement  $(\mathbf{x}' - \mathbf{x})$  may be larger than the period of the image signal around  $\mathbf{x}$ . This may result in estimating displacements with an incorrect sign. Therefore, we recommend to use a lowpass filtered version of the image for the first couple of iterations.

The motion estimation algorithms described in the following two sections are based on an image signal model in order to derive a relationship between frame differences and 3D motion. They relate the motion of a point  $\mathbf{x}$  to the motion in 3D space using the camera model, the motion equation and object shape. Since Eq. (7.2.4) is based on a linear approximation of the image signal, the estimation algorithm will be used within an iterative estimation process until the correct motion is estimated. In order to speed up the convergence of the algorithms, a higher order approximation of the image signal is desirable. According to [3], we can approximate the image signal by a quadratic image signal model using a Taylor series expansion of second order without explicitly computing the second derivatives. We compute the gradient  $\bar{\mathbf{g}}(\mathbf{x})$  of the second order image signal model by averaging the linear image gradients at position  $\mathbf{x}$  in the images  $\psi_k$  and  $\psi_{k+1}$ :

$$\bar{\mathbf{g}}(\mathbf{x}) = \frac{1}{2} (\mathbf{g}_k(\mathbf{x}) + \mathbf{g}_{k+1}(\mathbf{x})) \quad (7.2.5)$$

resulting in this more suitable equation

$$\Delta\psi_k(\mathbf{x}) = -\bar{\mathbf{g}}(\mathbf{x}) \cdot (\mathbf{x}' - \mathbf{x}). \quad (7.2.6)$$



**Figure 7.3.** Two examples showing how Eq. (7.2.4) relates the image signal to 2D motion of an observation point. a) No approximation error because the linear signal model holds. b) Since the linear signal model cannot approximate the image signal very well, we measure a luminance difference that is smaller than required. As a result, we estimate a displacement that is too small.

Since this quadratic image signal model enables a better image signal approximation, yields better motion estimates and is simple to compute, we recommend this model whenever appropriate.

### 7.2.2 Objects of Known Shape

Here we extend the algorithm of Section 7.1.2 to work without feature points. During motion estimation, we use a large set of observation points to represent the object. Since we assume that we know the object shape, the 3D positions of these points are known. Replacing image coordinate  $\mathbf{x}$  in Eq. (7.2.6) with world coordinates  $\mathbf{X}$  using the pinhole camera model according to Eq. (5.1.2), we get

$$\Delta\psi_k(\mathbf{x}) = F \cdot \bar{g}_x \left( \frac{X'}{Z'} - \frac{X}{Z} \right) + F \cdot \bar{g}_y \left( \frac{Y'}{Z'} - \frac{Y}{Z} \right). \quad (7.2.7)$$

where  $\bar{g}_x$  and  $\bar{g}_y$  are the  $x$  and  $y$  components of  $\bar{g}(\mathbf{x})$ . Assuming small rotation angles and object rotation around its center  $\mathbf{C} = [\mathbf{C}_x, \mathbf{C}_y, \mathbf{C}_z]^T$ , Eq. (5.3.10) and

Eq. (5.3.11) result in

$$\mathbf{X}' = [\mathbf{R}'] \cdot (\mathbf{X} - \mathbf{C}) + \mathbf{C} + \mathbf{T}. \quad (7.2.8)$$

Substituting (7.2.8) for  $\mathbf{X}'$  in (7.2.7) results in

$$\begin{aligned} \Delta\psi_k(\mathbf{x}) = & F \cdot \bar{g}_x / Z \cdot T_x \\ & + F \cdot \bar{g}_y / Z \cdot T_y \\ & - [(X\bar{g}_x + Y\bar{g}_y)F/Z^2 + \Delta\psi_k(\mathbf{x})/Z] \cdot T_z \\ & - [(X\bar{g}_x(Y - C_y) + Y\bar{g}_y(Y - C_y) + Z\bar{g}_y(Z - C_z))F/Z^2 + \\ & \quad \Delta\psi(\mathbf{x})/Z(Y - C_y)] \cdot \theta_x \\ & + [(Y\bar{g}_y(X - C_x) + X\bar{g}_1(X - C_x) + Z\bar{g}_x(Z - C_z))F/Z^2 + \\ & \quad \Delta\psi(\mathbf{x})/Z(X - C_x)] \cdot \theta_y \\ & - [\bar{g}_x(Y - C_y) - \bar{g}_y(X - C_x)]F/Z \cdot \theta_z \end{aligned} \quad (7.2.9)$$

where  $\mathbf{T} = (T_x, T_y, T_z)^T$  and  $\mathbf{R} = (\theta_x, \theta_y, \theta_z)^T$  are the six unknown motion parameters,  $\mathbf{X} = (X, Y, Z)^T$  is the point on the object surface,  $\bar{\mathbf{g}} = [\bar{g}_x, \bar{g}_y]^T$  is the image gradient according to Eq. (7.2.5) and  $F$  is the camera focal length, and  $\mathbf{x}$  is the location of the observation point in frame  $k$  prior to motion.

In order to get reliable estimates for the six motion parameters, Eq. (7.2.9) has to be established for many observation points on the surface of the moving object resulting in an over-determined system of linear equations

$$[\mathbf{A}] \cdot \phi - \mathbf{b} = \mathbf{r} \quad (7.2.10)$$

with the residual  $\mathbf{r} = [r_1, \dots, r_J]^T$ ,  $\phi = [T_x, T_y, T_z, \theta_x, \theta_y, \theta_z]^T$ ,  $\mathbf{b} = [\Delta\psi_k(\mathbf{x}^{(1)}), \dots, \Delta\psi_k(\mathbf{x}^{(J)})]^T$ ,  $[\mathbf{A}] = (\mathbf{a}_1, \dots, \mathbf{a}_J)^T$  and  $\mathbf{a}_j^T$  is defined according to Eq. (7.2.9) for the  $j$ -th observation point. This system of linear equations can be solved by the following optimization problem

$$\|[\mathbf{A}]\phi - \mathbf{b}\|^2 = \|\mathbf{r}\|^2 \xrightarrow{\phi} \min, \quad (7.2.11)$$

which has the effect of minimizing the motion compensated frame difference measured at the observation points  $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(J)}$  of the object. The motion parameters are given by

$$\hat{\phi} = ([\mathbf{A}]^T [\mathbf{A}])^{-1} [\mathbf{A}]^T \mathbf{b}. \quad (7.2.12)$$

In order to avoid the inversion of large matrices, we do not compute  $\mathbf{A}$  but immediately the  $6 \times 6$  matrix  $\mathbf{A}^T \mathbf{A}$  and the vector  $[\mathbf{A}]^T \mathbf{b}$ .

Due to the linearizations in Eq. (7.2.8) and Eq. (7.2.6), motion parameters have to be estimated iteratively. After every iteration, the model object and its observation points are moved according to the non-linear 3D motion equation (5.3.11) using the estimated motion parameters  $\hat{\phi}$ . Then, a new set of motion equations is established, giving new motion parameter updates. The motion parameter updates decrease during the iterations assuming that the motion estimation algorithm converges. Since the motion parameter updates approach zero during the iterations,

the error due to the linearization of the rotation matrix also approaches 0. The iteration process terminates if the decrease of the residual  $\|\mathbf{A}\phi - \mathbf{b}\|^2$  becomes negligible.

This algorithm estimates object motion from frame  $k$  to  $k + 1$ . Due to this forward motion estimation, we can track an object as it is moving forward in a video scene.

### 7.2.3 Planar Objects

Planar objects are often used as an approximation for real objects (Sec. 5.5.3). Since these approximations are usually not used for tracking but for motion compensation in video coding, we now extend the algorithm presented in Section 7.1.3 to a direct estimation method [5] that estimates motion backward. Starting with Eq. (7.1.9) defining the mapping  $\mathbf{A}(x, y)$  of a point  $\mathbf{x}$  in image  $k + 1$  to  $\mathbf{x}'$  in image  $k$  with

$$\begin{aligned} \mathbf{x}' &= (x', y')^T \\ &= [A_x(\mathbf{x}), A_y(\mathbf{x})]^T \\ &= \left[ \frac{a_1 x + a_2 y + a_3}{a_7 x + a_8 y + 1}, \frac{a_4 x + a_5 y + a_6}{a_7 x + a_8 y + 1} \right]^T \\ &= \mathbf{A}(\mathbf{x}) \end{aligned} \quad (7.2.13)$$

and assuming again that all changes in the image sequence are due to object motion, we get

$$\psi_{k+1}(\mathbf{x}) = \psi_k(\mathbf{x}') = \psi_k(\mathbf{A}(\mathbf{x})). \quad (7.2.14)$$

The frame difference similar to Eq. (7.2.2) becomes

$$\begin{aligned} \Delta\psi_k(\mathbf{x}) &= \psi_{k+1}(\mathbf{x}) - \psi_k(\mathbf{x}) \\ &= \psi_k(\mathbf{A}(\mathbf{x})) - \psi_k(\mathbf{x}). \end{aligned} \quad (7.2.15)$$

With a Taylor series expansion we express the luminance function at an observation point  $\mathbf{x}$  with respect to the motion parameters  $a_i$ :

$$\begin{aligned} \psi_k(\mathbf{A}(\mathbf{x})) &= \\ \psi_k(\mathbf{x}_k) &+ \sum_{i=1}^8 \frac{\partial\psi_k(\mathbf{x})}{\partial a_i} \Delta a_i + \frac{1}{2} \sum_{i=1}^8 \sum_{j=1}^8 \frac{\partial\psi_k(\mathbf{x})}{\partial a_i} \frac{\partial\psi_k(\mathbf{x})}{\partial a_j} \Delta a_i \Delta a_j + R_2(\mathbf{x}) \end{aligned} \quad (7.2.16)$$

Using this in Eq. (7.2.15), results in

$$\begin{aligned} \Delta\psi_k(\mathbf{x}) &= \\ \sum_{i=1}^8 \frac{\partial\psi_k(\mathbf{x})}{\partial a_i} \Delta a_i &+ \frac{1}{2} \sum_{i=1}^8 \sum_{j=1}^8 \frac{\partial\psi_k(\mathbf{x})}{\partial a_i} \frac{\partial\psi_k(\mathbf{x})}{\partial a_j} \Delta a_i \Delta a_j + R_2(\mathbf{x}). \end{aligned} \quad (7.2.17)$$

with  $\Delta\mathbf{a} = \mathbf{a} - \mathbf{e}$ ,  $\mathbf{e} = (1, 0, 0, 0, 1, 0, 0, 0)^T$ ,  $\mathbf{a} = (a_1, \dots, a_8)^T$ ,  $\frac{\partial\psi_k(\mathbf{x})}{\partial a_i} = \frac{\partial\psi_k(\mathbf{x})}{\partial a_i} \frac{\partial\psi_k(\mathbf{x})}{\partial \mathbf{x}}$  and  $R_2(\mathbf{x})$  denoting the higher order terms of the Taylor series expansion. If

the quadratic image signal model is valid, this residual  $R_2(\mathbf{x})$  is zero simplifying Eq. (7.2.17) to

$$\begin{aligned}\Delta\psi_k(\mathbf{x}) &= \bar{g}_x x \Delta a_1 + \bar{g}_x y \Delta a_2 + \bar{g}_x \Delta a_3 + \bar{g}_y x \Delta a_4 + \bar{g}_y y \Delta a_5 \\ &\quad + \bar{g}_y \Delta a_6 - x(\bar{g}_x x + \bar{g}_y y) \Delta a_7 - y(\bar{g}_x x + \bar{g}_y y) \Delta a_8 \\ &= \mathbf{h}^T \Delta \mathbf{a}.\end{aligned}\quad (7.2.18)$$

with  $\frac{\partial\psi_k(\mathbf{x})}{\partial a_i} = \frac{\partial\psi_k(\mathbf{x})}{\partial x} \frac{\partial A_x(\mathbf{x})}{\partial a_i} + \frac{\partial\psi_k(\mathbf{x})}{\partial y} \frac{\partial A_y(\mathbf{x})}{\partial a_i}$  and  $\bar{\mathbf{g}}(\mathbf{x}) = [\bar{g}_x, \bar{g}_y]^T$  according to Eq. (7.2.6).

Eq. (7.2.18) defines an equation for one observation point of the object with the 8 unknown motion parameters. In order to estimate the motion, we set up Eq. (7.2.18) for  $J$  observation points  $\mathbf{x}$  and solve the following system of  $J$  linear equations

$$[\mathbf{H}] \Delta \mathbf{a} = \Delta \psi. \quad (7.2.19)$$

The row vectors of matrix  $[\mathbf{H}]$  are the  $\mathbf{h}^T$  determined according to Eq. (7.2.18),  $\Delta\psi$  is a vector with the frame differences between the two images according to Eq. (7.2.18). Solving this system of linear equations with the least squares method, we get, similar to (7.2.12),

$$\Delta \mathbf{a} = ([\mathbf{H}]^T [\mathbf{H}])^{-1} [\mathbf{H}]^T \Delta \psi. \quad (7.2.20)$$

Due to the approximations used in the image signal model, this estimation process has to be carried out iteratively. After each iteration, we use the estimated motion parameters to motion compensate frame  $k+1$  using

$$\hat{\Psi}_k(\mathbf{x}) = \Psi_{k+1}(\mathbf{A}'(\mathbf{x})). \quad (7.2.21)$$

The displaced frame difference (DFD) becomes

$$DFD(\mathbf{x}) = \psi_{k+1}(\mathbf{A}'(\mathbf{x})) - \psi_k(\mathbf{x}) \quad (7.2.22)$$

where the inverse mapping  $\mathbf{A}'(\mathbf{x})$  is defined as  $\mathbf{A}'(\mathbf{A}(\mathbf{x})) = \mathbf{x}$ . During the iteration, Eq. (7.2.22) replaces Eq. (7.2.15). Furthermore, the image gradients in Eq. (7.2.18) have to be computed using the motion compensated image and image  $k$ . If we are interested in the overall motion parameter, the estimated parameters during each iteration have to be concatenated appropriately (see Problem 9).

### 7.2.4 Robust Estimation

In order to enable motion estimation that is not easily affected by image noise or violation of our model assumptions, we have to take care in the selection and use of observation points. Two types of errors affect motion estimation: *measurement errors* due to image noise occurred when we sample the image signal, and *modeling errors* occurred when the model assumptions of our estimation algorithm are not

valid. In this section, we deal with modeling errors and how to minimize their effect on the estimated motion parameters.

The algorithms in the previous two sections are based on a quadratic image signal model. Since this is only valid for a small area around the location of an observation point at most, the selection of appropriate observation points is an important factor in developing a robust direct motion estimator. Here we present concepts that are valid for any direct motion estimation algorithm.

Considering image noise, we can derive a basic criterion for selecting observation points. Assuming the camera noise in two consecutive frames are i.i.d and the noise variance is  $\sigma_n^2$ , the noise of the image difference signal will be

$$\sigma_{\Delta\psi_k}^2 = 2 \cdot \sigma_n^2. \quad (7.2.23)$$

Since we use the frame difference for computing motion, we have to take care that we consider only observation points where the influence of the camera noise is small, i.e. we prefer observation points where the frame difference exceeds a certain minimum.

According to Eq. (7.2.4), we represent the local displacement  $\mathbf{x}' - \mathbf{x}$  as a function of the noise-free luminance signal. In a noisy environment, the gradient  $\mathbf{g}(\mathbf{x})$  should have a large absolute value in order to limit the influence of camera noise. Hence, we select as observation points only points with a gradient larger than a threshold  $T_G$ :

$$\begin{aligned} |g_x(\mathbf{x})| &> T_G \\ |g_y(\mathbf{x})| &> T_G \end{aligned} \quad (7.2.24)$$

Relatively large gradients allow also for a precise estimation of the motion parameters, because a slight mismatch would immediately result in a high displaced frame difference or  $\Delta\psi(\mathbf{x})$ . If we choose an observation point that has a zero-gradient or a very small gradient, the point could be moved by several pels without causing any significant displaced frame difference. Figure 7.4 shows an image of the test sequence *Claire* and its observation points. We selected all points  $\mathbf{x}$  with  $|g_x(\mathbf{x})| > 32$  and  $|g_y(\mathbf{x})| > 32$  as observation points. With this initial selection of observation points, we look now into methods of improving the performance of the motion estimation algorithms.

Equations (7.2.10) and (7.2.19) are solved such that the variances of the residual errors are minimized. However, this approach is sensitive to modeling errors [10]. Modeling errors occur because Eq. (7.2.10) and Eq. (7.2.19) are based on several model assumptions and approximations that tend to be valid for the majority of observation points but not all. Observation points that violate these assumptions are named *outliers* [15]. When using a least squares method for solving Eq. (7.2.10) and Eq. (7.2.19), outliers have a significant influence on the solution. Therefore, we have to take measures that limit the influence of these outliers on the estimation process. Sometimes, the following assumptions are not valid:

1. rigid real object,



**Figure 7.4.** (a) Image of the test sequence *Claire*. (b) Observation points for this image

2. quadratic image signal model,
3. small deviations of model object shape from real object shape,
4. diffuse illumination and diffuse reflecting surfaces.

In the following, we discuss the influence of each of the above assumptions on motion estimation accuracy.

**Assumption (1):** In case that parts of the real object are non-rigid, i.e. the object is flexible, we have image areas that cannot be described by motion parameters and the shape and color parameters of the object. These image areas are named *model failures* and can be detected due to their potentially high frame difference  $\Delta\psi(\mathbf{x})$ . Observation points with a high frame difference can be classified as outliers. During an iteration, we will only consider observation points  $\mathbf{x}$  for which

$$\Delta\psi(\mathbf{x}) < \sigma_{\Delta\psi} \cdot T_{ST} \quad (7.2.25)$$

with

$$\sigma_{\Delta\psi}^2 = \frac{1}{J} \sum_{j=0}^J (\Delta\psi(\mathbf{x}^{(j)}))^2 \quad (7.2.26)$$

holds. The threshold  $T_{ST}$  is used to remove the outliers from consideration. The variance  $\sigma_{\Delta\psi}^2$  should be recalculated after each iteration.

**Assumption (2):** According to (7.2.6), motion estimation is based on a quadratic image signal model. This is at most valid for a small area around an observation point  $\mathbf{x}$ . It allows only for estimating small displacements  $(\mathbf{x}' - \mathbf{x})$  in one iteration step [12]. Given an image gradient  $\mathbf{g}(\mathbf{x})$  and a maximum displacement  $d_{max}$  that we want to allow for estimation, we can limit the allowable frame difference  $\Delta\psi(\mathbf{x})$  for each observation point and if necessary exclude an observation point for a given iteration. We assume that this point does not conform to the image signal model.

**Assumption (3):** The shape difference between the model object and the real object can be modeled by means of a spatial uncertainty of an observation point along the line of sight. This can be considered using a Kalman filter during motion estimation [9].

**Assumption (4):** Although influence of non-diffuse illumination and reflections disturb image analysis, they are usually not modeled separately. To some extent, these image areas are automatically detected when detecting model failures (see assumption 1)

Summarizing these observations, we conclude that we should select observation points with large absolute image gradients according to Eq. (7.2.24). Only observation points that pass this initial test will be considered during motion estimation. Eq. (7.2.25) provides a selection criterion for the observation points that we will use for a given iteration step. In [13], further details on robust direct motion estimation and maximum-likelihood estimation are discussed. In the following, we will briefly focus on an efficient estimator described in [13] and [7].

Instead of using the binary selection criteria for observation points according to Eq. (7.2.25), we can use continuous cost functions to control the influence of an observation point on the parameter estimation. We use the residual  $r_j$  according to Eq. (7.2.10) as measure for the influence of an observation point [14][23]. A critical point for selecting an appropriate measure is the treatment of outliers. Ideally, we would know the probability density function of the outliers. Furthermore, we want outliers to have no influence on the estimated motion parameters. However, the probability density function of outliers is often unknown. Therefore heuristic solutions using the cost function  $w = (1 - r^2/b^2)^2$  as suggested by Tukey were found [7][23]:

$$r'_j = \begin{cases} r_j \cdot \left(1 - \frac{r_j^2}{b^2}\right) & \text{if } |r_j| < b \\ 0 & \text{otherwise.} \end{cases} \quad (7.2.27)$$

Instead of  $\sum_j |r_j|^2 \rightarrow \min$  we now minimize

$$\sum_j |r_j|^2 |w_j|^2 \rightarrow \min. \quad (7.2.28)$$

The cost  $w = (1 - r^2/b^2)^2$  increases to 1 when  $|r|$  decreases. Observation points with  $|r_j| \geq b$  are excluded from the next iteration. The constant  $b$  is the threshold for detecting outliers. In order to adapt the outlier detection to the image difference signal  $\Delta\psi$  we select  $b$  proportional to  $\sigma_{\Delta\psi}$  in Eq. (7.2.26). During motion estimation, we multiply the equation of each observation point  $\mathbf{x}^{(j)}$  with its individual cost  $w_j = (1 - r_j^2/b^2)^2$  therefore adapting its influence on the estimated motion parameters according to Eq. (7.2.11) and Eq. (7.2.20).

### 7.3 Iterative Motion Estimation

Direct motion estimation algorithms can only estimate small motion, mainly due to the limited range for which the assumptions of the image signal model hold. In order to estimate larger motion, we have to use an iterative algorithm.

Let us assume that we estimate the motion parameters starting with a set of observation points and equations as described in one of the previous sections. During this first iteration we solve for the motion parameters  $\mathbf{R}_i$  and  $\mathbf{T}_i$  with  $i = 1$  giving the residual  $r_i$ . We use these motion parameters to motion compensate the object and its observation points. If appropriate, we use the non-linear motion equations for motion compensation even if the motion parameters are solved by using a linearized rotation matrix. We now set up a new set of equations following the same procedure as for iteration 1 resulting in the motion parameters  $\mathbf{R}_i$  and  $\mathbf{T}_i$  with  $i = 2$ . We continue these iterations as long as the new motion parameters decrease the residual, i.e.  $(r_j^2 - r_{j+1}^2) > T$  with threshold  $T$  chosen appropriately. The final estimated motion parameters from image  $k$  to image  $k + 1$  can be computed by concatenating the motion parameters of each iteration.

To show the principle of motion parameter concatenation, we compute the motion parameter concatenation for the algorithm presented in Section 7.2.2. Assuming motion according to Eq. (5.3.9), the estimated rotation matrix  $[\hat{\mathbf{R}}]$  becomes

$$[\hat{\mathbf{R}}] = \prod_{i=1}^I [\mathbf{R}_{I+1-i}] \quad (7.3.1)$$

with the rotation matrix  $[\mathbf{R}_i]$  estimated in iteration  $i$ . The computation of the translation vector  $\hat{\mathbf{T}}$  has to consider the center of rotation. Assuming that we compute rotation around  $\mathbf{C}_i$  during iteration  $i$ , we get

$$\hat{\mathbf{T}} = \mathbf{T}_I + \mathbf{C}_I + [\mathbf{R}_I](\mathbf{T}_{I-1} + \mathbf{C}_{I-1} - \mathbf{C}_I + [\mathbf{R}_{I-1}](\mathbf{T}_{I-2} + \mathbf{C}_{I-2} - \mathbf{C}_{I-1} + \dots)) \quad (7.3.2)$$

assuming rotation around the coordinate center, i.e. the object moves according to

$$\mathbf{X}' = [\hat{\mathbf{R}}]\mathbf{X} + \hat{\mathbf{T}}. \quad (7.3.3)$$

Compared to indirect motion estimation, direct motion estimation usually requires iterative algorithms that tend to be more compute intensive. However, direct motion estimation has the advantage of considering the image signal during estimation.

### 7.4 Summary

In this chapter, we discussed several methods of estimating 3D motion. We presented two estimation methods, indirect motion estimation and direct motion estimation.

Indirect motion estimation algorithms (Sec. 7.1) depend on feature points on the moving object. The feature points have to be reliably marked on two images. Then

we can estimate 3D motion from these correspondences. The algorithms allow for estimating large motion. However, the estimation algorithms tend to be sensitive to errors in the feature point correspondences. Different algorithms can be used depending on whether we want to assume orthographic (Sec. 7.1.1) or perspective (Sec. 7.1.2, 7.1.4) projection and whether we know the object shape. Using an orthographic camera model is acceptable if the object does not move significantly in the direction of the camera axis and object rotation is mainly within the camera plane. Otherwise we use an algorithm with a perspective camera model, which requires more computations than an orthographic model. Estimating the essential matrix from feature point correspondences (Sec. 7.1.4) enables us to determine the motion of the object as well as the 3D location of these feature points. In order to gain robustness against noise, we suggest to use the epipolar line constraint and minimize the distance of the feature points from this line (Eq. 7.1.34) when determining the motion and shape parameters from the essential matrix.

Direct motion estimation algorithms (Sec. 7.2) approximate the image signal using a Taylor series expansion (Section 7.2.1) and derive pel-based motion (optical flow) from the frame difference signal. This pel-based motion is used directly to compute the 3D motion parameters. The algorithms are used if we have small object motion from one frame to another. The motion estimation range can be extended using an iterative approach (Sec. 7.3). Usually, we are required to make some assumption about the 3D object shape. The algorithm described in Sec. 7.2.3 assumes a planar object shape. Key to the successful use of direct motion estimation algorithms is the careful selection of those observation points where we measure the frame difference signal. In Sec. 7.2.4 we describe robust methods for selecting these points during motion estimation.

The selection of the estimation method depends to a large extend on what measurements are available. Indirect motion estimation is the method of choice if we have only a few highly accurate feature point locations. Direct motion estimation should be used if we can reliably estimate motion from the image signal using optical flow. As always, a combination of these approaches might results in the best results for some applications.

## 7.5 Problems

1. What are the differences between a direct and an indirect motion estimator.  
What are the potential advantages and disadvantages?
2. Derive Eq. (7.1.9). Start with the general 3D motion equation  $\mathbf{X}' = [\mathbf{R}]\mathbf{X} + \mathbf{T}$  and the plane equation in Eq. (7.1.8).
3. Show that Eq. (7.1.10) can be written as Eq. (7.1.11).
4. Show that the solution to the optimization problem in Eq. (7.1.22) is the unit eigenvector of  $\mathbf{A}^T \mathbf{A}$  associated with the smallest eigenvalue.

5. Consider an image point  $\mathbf{x}$  that is the projection of  $\mathbf{X}$ . This point moves to  $\mathbf{x}'$  and  $\mathbf{X}'$ , respectively. Derive the epipolar line for this point  $\mathbf{x}$  for the motion parameters  $\mathbf{T}$  and  $[\mathbf{R}]$ . For an image point  $\mathbf{x}$  and its epipolar line, compute and draw a curve that shows how the distance of a point  $\mathbf{X}$  from the camera center determines its location on the epipolar line?
6. When estimating the E-matrix, the influence of a measurement error  $\delta$  in the point correspondence depends on the absolute coordinates of these correspondences. Show how the residual of a point correspondence depends on the coordinates of the corresponding points. In Eq. (7.1.11) and Eq. (7.1.19) replace  $\mathbf{x}'$  by  $\mathbf{x}' + \delta$ . What would be the weighting factors for each equation in (7.1.22) such that the image coordinates of a correspondence do not influence the estimation of the E-matrix?
7. Present a polynomial representation of the luminance signal  $\psi(\mathbf{x})$  around  $\mathbf{x}$  using a linear and a quadratic image signal model.
8. Derive Eq. (7.2.6) and Eq. (7.2.5) using a second order Taylor series expansion for the image signal.
9. In order to implement the direct motion estimation algorithm for planar objects (Section 7.2.3), the inverse mapping of  $\mathbf{A}$  has to be known. Show that the mapping  $\mathbf{A}$  fulfills the four axioms of a group, namely closure, existence of an inverse, existence of an identity and associativity. Compute the inverse of a mapping  $\mathbf{A}$  and the sum of the motion parameter estimates after two iterations.
10. Discuss measures to increase the robustness of a direct motion estimation algorithm.

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## Chapter 8

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# FOUNDATIONS FOR VIDEO CODING

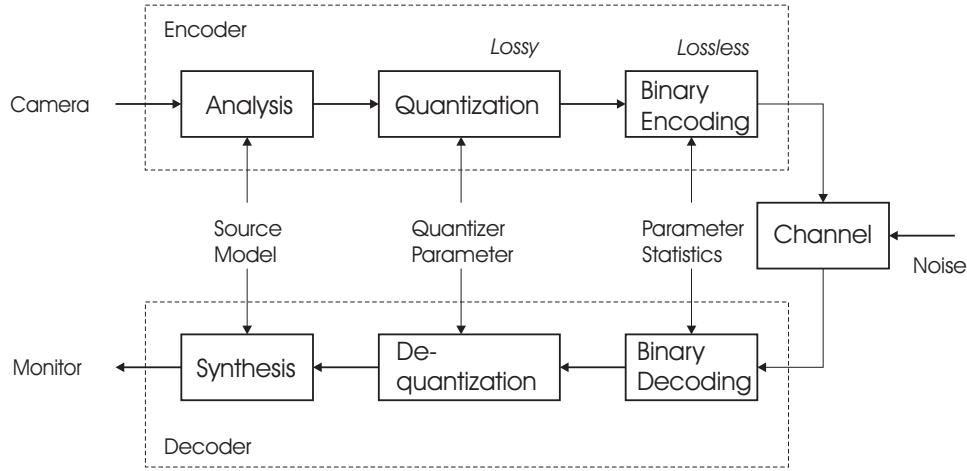
Video coding is an important application of digital video processing. The goal of video coding is to reduce the data rate of a video sequence so that it is feasible to transmit the video in real-time over a given communication channel. The channel bandwidth varies depending on the application and the transmission media. For video phone applications using regular telephone lines, 20 Kbit/s may be all that is available for video coding. For broadcasting of standard definition television signals over satellite, a data rate of 6 Mbit/s may be available. In addition to communications applications, video coding is also necessary for storage and retrieval applications, where different storage media have different capacity and access rates, thus demanding varying amount of compression. Due to this wide range of data rates, different algorithms have been developed. One class of algorithms allows efficient coding of arbitrary video signals without trying to analyze the video content. A second class of algorithms identifies regions and objects in a video sequence and codes those. We call the former a *waveform-based* video coder and the latter a *content-dependent* video coder.

In this chapter, we first give an overview of different waveform-based and content-dependent video coders in Sec. 8.1. Then, following a review of basic probability and information theory concepts in Sec. 8.2, information theory used to optimize lossless coding and rate distortion theory used to optimize lossy coding are introduced in Sec. 8.3. This will be followed by description of lossless coding (Sec. 8.4) and the most basic lossy coding techniques including scalar and vector quantization (Secs. 8.5 and 8.6). Chapters 9 and 10 will discuss techniques for waveform-based video coding and content-dependent video coding, respectively.

### 8.1 Overview of Coding Systems

#### 8.1.1 General Framework

The components of a video coding algorithm are determined to a large extent by the *source model* that is adopted for modeling the video sequences. The video



**Figure 8.1.** Overview of a video coding system.

coder seeks to describe the contents of a video sequence by means of its source model. The source model may make assumptions about the spatial and temporal correlation between pixels of a sequence. It might also consider the shape and motion of objects or illumination effects. In Fig. 8.1, we show the basic components in a video coding system. In the encoder, the digitized video sequence is first described using the parameters of the source model. If we use a source model of statistically independent pixels, then the parameters of this source model would be the luminance and chrominance amplitudes of each pixel. On the other hand, if we use a model that describes a scene as several objects, the parameters would be the shape, texture, and motion of individual objects. In the next step, the parameters of the source model are quantized into a finite set of symbols. The quantization parameters depend on the desired trade-off between the bit rate and distortion. The quantized parameters are finally mapped into binary codewords using lossless coding techniques, which further exploit the statistics of the quantized parameters. The resulting bit stream is transmitted over the communication channel. The decoder retrieves the quantized parameters of the source model by reversing the binary encoding and quantization processes of the encoder. Then, the image synthesis algorithm at the decoder computes the decoded video frame using the quantized parameters of the source model.

### 8.1.2 Categorization of Video Coding Schemes

In this subsection, we provide an overview of several popular video coding algorithms and put them into context by looking at their underlying source models. The source model of a coding algorithm can be determined by looking at the coded parameter set and at the image synthesis algorithm that composes the decoded image based

**Table 8.1.** Comparison of different source models, their parameter sets and coding techniques

Source Model	Encoded Parameters	Coding Technique
Statistically independent pels	Color of each pel	PCM
Statistically dependent pels	Color of each block	Transform coding, Predictive Coding, & Vector quantization
Translatoryically moving blocks	Color and motion vector of each block	Motion compensated hybrid coding
Moving unknown objects	Shape, motion, and color of each object	Analysis-synthesis coding
Moving known object	Shape, motion, and color of each known object	Knowledge-based coding
Moving known object with known behavior	Shape, color, and behavior of each object	Semantic coding

on the decoded parameters.

### Waveform-Based Coding

The techniques in this category all try to represent the color values of individual pixels as accurately as possible, without considering the fact that a group of pixels may represent a physical object.

Assuming statistically independent pixels results in the simplest source model (Table 8.1). The related coding technique is named *Pulse Coded Modulation* (PCM). A PCM representation of the image signal is usually not used for video coding due to its inefficiency compared to other source models.

In most images, we find that the colors of neighboring pixels are highly correlated. For bit rate reduction, this property is best exploited using a *transform* like Karhunen-Loeve transform (KLT), Discrete Cosine transform (DCT) or wavelet transforms (Secs. 9.1 and 11.3). The transform serves to decorrelate the original sample values and concentrate the energy of the original signal into a few coefficients. The parameters to be quantized and coded are the coefficients of the transform. Another way to exploit the correlation among adjacent samples is predictive coding, by which the sample value to be coded is first predicted from previously coded samples. The prediction error, which has less correlation and lower energy than the original signal, is then quantized and coded. Both transform and predictive coding can be considered as a special case of vector quantization, which quantizes a block of samples (a vector) at a time. Essentially, it finds the typical block patterns

occurring in the signal and approximates any block by one of the typical patterns.

Today's video coding standards, H.261, H.263, MPEG-1, MPEG-2, and MPEG-4 (Chap. 13), are based on *motion-compensated hybrid coding*, which combines predictive coding with transform coding (Sec. 9.2.4). This coding technique subdivides each image into fixed sized blocks. Each block of a frame  $k$  is synthesized using a block of the same size at a displaced position of the previous frame  $k-1$ . This is done for all blocks of frame  $k$ . The resulting image is called the predicted image. The encoder transmits the 2-D motion vectors for all blocks to the decoder such that the decoder can compute the same predicted image. The encoder subtracts this predicted image from the original image, resulting in the prediction error image. If the synthesis of a block using the predicted image is not sufficiently accurate, i.e., the prediction error of a block exceeds a threshold, then the encoder uses a transform coder for transmitting the prediction error of this block to the decoder. The decoder adds the prediction error to the predicted image and thus synthesizes the decoded image. Therefore, motion compensated hybrid coding is based on the source model of translatorically moving blocks. In addition to the color information encoded as transform coefficients of the prediction error, motion vectors have to be transmitted. It is worth noting that such a coder can *switch* to the simpler source model of statistically dependent pixels. This is done whenever the coding of a block is achieved more efficiently without referring to the previous frame.

### Content-Dependent Coding

The above block-based hybrid coding techniques essentially approximate the shape of objects in the scene with square blocks of fixed size. Therefore, this coder generates high prediction errors in blocks on object boundaries. These boundary blocks contain two objects with different motions, which cannot be accounted for by a single motion vector. Content-dependent coders recognize such problems, and try to segment a video frame into regions corresponding to different objects and code these objects separately. For each object, shape information has to be transmitted in addition to motion and texture information.

In *object-based analysis-synthesis coding* (OBASC) (Sec. 10.5), each moving object of the video scene is described by a model object. In order to describe the shape of an object, OBASC employs a segmentation algorithm. Further, motion and texture parameters of each object are estimated. In the simplest case, a 2-D silhouette describes the shape of an object, a motion vector field describes its motion, and the color waveform represents its texture. Other approaches describe an object with a 3-D wireframe. An object in frame  $k$  is described by the shape and color of an object in frame  $k-1$  and an update of shape and motion parameters. The decoder synthesizes the object using the current motion and shape parameters as well as the color parameters of the preceding frame. Only for those image regions where the image synthesis fails, color information is transmitted.

In the case that the object type in a video sequence belongs to some known object classes, *knowledge-based coding* can be employed, which uses a wireframe specially

designed to describe the recognized object type (Sec. 10.6). Several approaches have been developed to code human heads using predefined wireframes. Using a predefined wireframe increases coding efficiency because it is adapted to the shape of the object. Sometimes, this technique is also referred to as *model-based coding*.

When possible object types as well as their behaviors are known, semantic coding may be used. For example, for a human face object, the behavior refers to the temporal trajectory of a set of facial feature points associated with a particular facial expression. The possible behaviors of a human face include typical facial expressions such as happiness, sadness, anger, etc. In this case, the parameters for describing the behavior of an object are estimated and transmitted to the decoder. This coding technique has the potential for achieving very high coding efficiency, because the number of possible behaviors for an object like a face is very small, and the number of bits required to specify the behavior is much smaller than that for describing the actual action with conventional motion and color parameters.

## 8.2 Basic Notions in Probability and Information Theory

In video coding or coding of any signal, we treat a given signal as the realization of a random process. In source coding theory, a random process is referred to as a source. The efficiency of a coding technique depends on how fully the source statistics are exploited. Before moving into the theory and techniques for source coding, we first review how to characterize a random process using probability distributions, and introduce the concepts of entropy and mutual information from information theory.

### 8.2.1 Characterization of Stationary Sources

We only consider sources defined over a discrete index set so that a source is a random sequence. We use  $\mathcal{F} = \{\mathcal{F}_n\}$  to represent a source, in which  $\mathcal{F}_n$  represent the random variable (RV) corresponding to the  $n$ -th sample. The actual value that  $\mathcal{F}_n$  takes in a given realization is denoted by  $f_n$ . Note that for a discrete signal over a multi-dimensional space, we can either order it into a 1-D sequence, so that  $n$  refers to the 1-D index of a sample, or we can assume  $n$  is a multi-dimensional index. Also,  $\mathcal{F}_n$  can be a scalar or a vector. In the latter case, it is also called a random vector. If  $\mathcal{F}_n$  can only take symbols from a finite alphabet  $\mathcal{A} = \{a_1, a_2, \dots, a_L\}$ , then we call  $\mathcal{F}_n$  a discrete RV and  $\mathcal{F}$  a *discrete source*. If  $\mathcal{F}_n$  takes value over a continuous supporting range or the union of several continuous ranges, denoted by  $\mathcal{B}$ , then  $\mathcal{F}_n$  is called a continuous RV and  $\mathcal{F}$  a *continuous source*.<sup>1</sup>

As an example, consider a digital color video. In this case, the index  $n$  represents a particular combination of pixel location and frame number, and  $\mathcal{F}_n$  is a 3-D random vector, representing the three color values taken by the  $n$ -th pixel. Because each color value is quantized into 256 levels, a digital video is a discrete source with

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<sup>1</sup>Note that in this chapter, the word “discrete” and “continuous” refer to the sample amplitude, whereas in earlier chapters, they refer to the space over which the signal is defined. A digital signal is a discrete space signal in which each sample takes only discrete values.

an alphabet size of 256<sup>3</sup>. Any given video sequence is a particular realization of this discrete random process. An analog video, on the other hand, is a continuous space continuous amplitude random process. The sampling process converts it to a discrete space continuous amplitude process. Only after quantization, it becomes a discrete space discrete amplitude process (i.e., a discrete source). In the real world, a discrete source is often obtained from a continuous space continuous amplitude source by sampling and quantization.

In this book, we only consider *stationary* processes, in which the probability distribution of  $\mathcal{F}_n$  does not depend on the index  $n$ , and the joint distribution of a group of  $N$  samples is invariant with respect to a common shift in the index. In this case, we use  $p_{\mathcal{F}_n}(f)$  to represent the *probability mass function* (pmf), in the case of a discrete source, or the *probability density function* (pdf), in the case of a continuous source, of any sample  $\mathcal{F}_n$ . In addition, we use  $p_{\mathcal{F}_{n+1}, \mathcal{F}_{n+2}, \dots, \mathcal{F}_{n+N}}(f_1, f_2, \dots, f_N)$  to represent the joint pmf or pdf of any  $N$  successive samples in  $\mathcal{F}$ . We also use  $p_{\mathcal{F}_n | \mathcal{F}_{n-1}, \mathcal{F}_{n-2}, \dots, \mathcal{F}_{n-M}}(f_{M+1} | f_M, f_{M-1}, \dots, f_1)$  to represent the conditional pmf or pdf of any sample  $\mathcal{F}_n$  given its previous  $M$  samples. When it is clear from the underlying context, we simply use  $p(f)$ ,  $p(f_1, f_2, \dots, f_N)$ , and  $p(f_{M+1} | f_M, f_{M-1}, \dots, f_1)$  to denote the above functions.

An important class of stationary sources is known as *independent and identically distributed* (i.i.d.) sources, which satisfy  $p(f_1, f_2, \dots, f_N) = p(f_1)p(f_2) \cdots p(f_N)$ , and  $p(f_{M+1} | f_M, f_{M-1}, \dots, f_1) = p(f_{M+1})$ . An i.i.d. source is also referred to as *memoryless*. Another important class is the *Markov process*, in which a sample only depends on its immediate predecessor, i.e.,  $p(f_{M+1} | f_M, f_{M-1}, \dots, f_1) = p(f_{M+1} | f_M)$ . More generally, an  $M$ -th order *Markov process* is the one in which a sample only depends on its previous  $M$  samples. A process in which any  $N$  samples follow an  $N$ -th order Gaussian distribution is called a *Gaussian process*. A Gaussian process is Markov if the covariance between two samples  $\mathcal{F}_n$  and  $\mathcal{F}_m$  has the form  $C(\mathcal{F}_n, \mathcal{F}_m) = \sigma^2 \rho^{-(n-m)}$ . Such a process is called a *Gauss-Markov process*, or a *Gauss-Markov field (GMF)* when the underlying process is two dimensional. In image and video processing, a real image or video frame is often modeled as a GMF.

### 8.2.2 Entropy and Mutual Information for Discrete Sources

In this section, we introduce two very important notions from information theory: entropy and mutual information. As will be seen later, they provide bounds on the minimal bit rates achievable using lossless and lossy coding, respectively. We first define entropy and mutual information for discrete RVs, and then apply these definitions to samples in a discrete source.

**Definition 8.1 (Entropy of a Discrete RV)** *The entropy of a discrete RV  $\mathcal{F}$  with an alphabet  $\mathcal{A}$  and a pmf  $p_{\mathcal{F}}(f)$  is defined by*

$$H(\mathcal{F}) = - \sum_{f \in \mathcal{A}} p_{\mathcal{F}}(f) \log_2 p_{\mathcal{F}}(f). \quad (8.2.1)$$

In general, the log can be taken to any base. Here we only consider the base-2 case, so that entropy is expressed in bits. Also, it is assumed that  $0 \log 0 = 0$ , so that adding a symbol with zero probability to the alphabet does not change the entropy.

The entropy of a discrete RV is always non-negative, because  $0 \leq p(f) \leq 1$  for any pmf. Among all RVs with an alphabet size  $L$ , the uniform distribution,  $p(f) = 1/L, f \in \mathcal{A}$ , achieves the maximum entropy,  $H_{\max} = \log_2 L$ .

Entropy is a measure of the uncertainty about a RV  $\mathcal{F}$ . It depends on the pmf of  $\mathcal{F}$ , not the actual alphabet used by  $\mathcal{F}$ . When  $\mathcal{F}$  can take on any value in  $\mathcal{A}$  with equal probability, it is most uncertain, hence it has maximum entropy. On the other hand, if  $\mathcal{F}$  takes a particular symbol in  $\mathcal{A}$  with probability 1, it has no uncertainty, and hence the entropy is zero. The uncertainty about a RV can in turn be considered as the information that may be conveyed by a RV: when one is shown the actual value taken by  $\mathcal{F}$ , one is told a lot more in the first case than in the second.

**Definition 8.2 (Joint Entropy of Two Discrete RVs)** *Let  $\mathcal{F}$  and  $\mathcal{G}$  represent two discrete RVs, with a joint pmf  $p_{\mathcal{F},\mathcal{G}}(f,g), f \in \mathcal{A}_f, g \in \mathcal{A}_g$ . Their joint entropy is defined as*

$$H(\mathcal{F}, \mathcal{G}) = - \sum_{f \in \mathcal{A}_f} \sum_{g \in \mathcal{A}_g} p_{\mathcal{F},\mathcal{G}}(f,g) \log_2 p_{\mathcal{F},\mathcal{G}}(f,g). \quad (8.2.2)$$

**Definition 8.3 (Conditional Entropy between Two Discrete RVs)** *Let  $\mathcal{F}$  and  $\mathcal{G}$  represent two discrete RVs, with marginal pmf  $p_{\mathcal{G}}(g)$  and conditional pmf  $p_{\mathcal{F}|\mathcal{G}}(f|g), f \in \mathcal{A}_f, g \in \mathcal{A}_g$ . The conditional entropy of  $\mathcal{F}$  given  $\mathcal{G}$  is defined as*

$$\begin{aligned} H(\mathcal{F}|\mathcal{G}) &= \sum_{g \in \mathcal{A}_g} p_{\mathcal{G}}(g) H(\mathcal{F}|g) \\ &= - \sum_{g \in \mathcal{A}_g} p_{\mathcal{G}}(g) \sum_{f \in \mathcal{A}_f} p_{\mathcal{F}|\mathcal{G}}(f|g) \log_2 p_{\mathcal{F}|\mathcal{G}}(f|g). \end{aligned} \quad (8.2.3)$$

**Definition 8.4 (Mutual Information between Two Discrete RVs)** *Let  $\mathcal{F}$  and  $\mathcal{G}$  represent two discrete RVs, with a joint pmf  $p_{\mathcal{F},\mathcal{G}}(f,g), f \in \mathcal{A}_f, g \in \mathcal{A}_g$  and marginal distributions  $p_{\mathcal{F}}(f)$  and  $p_{\mathcal{G}}(g)$ . The mutual information between  $\mathcal{F}$  and  $\mathcal{G}$  is defined as*

$$I(\mathcal{F}; \mathcal{G}) = \sum_{f \in \mathcal{A}_f} \sum_{g \in \mathcal{A}_g} p_{\mathcal{F},\mathcal{G}}(f,g) \log_2 \frac{p_{\mathcal{F},\mathcal{G}}(f,g)}{p_{\mathcal{F}}(f)p_{\mathcal{G}}(g)}. \quad (8.2.4)$$

The mutual information between  $\mathcal{F}$  and  $\mathcal{G}$  specifies the amount of information provided by  $\mathcal{G}$  about  $\mathcal{F}$ . In other words, it measures the reduction in the number of bits required to specify  $\mathcal{F}$ , given the realization of  $\mathcal{G}$ . The mutual information is always non-negative. It is easy to show that

$$I(\mathcal{F}; \mathcal{G}) = I(\mathcal{G}; \mathcal{F}) \geq 0. \quad (8.2.5)$$

**Relations between Entropy, Conditional Entropy, and Mutual Information** We summarize some important relations between entropy, conditional entropy and mutual information below. These relations can be proved quite easily (Prob. 8.2).

$$H(\mathcal{F}, \mathcal{G}) = H(\mathcal{G}) + H(\mathcal{F}|\mathcal{G}) = H(\mathcal{F}) + H(\mathcal{G}|\mathcal{F}) \quad (8.2.6)$$

$$I(\mathcal{F}; \mathcal{G}) = H(\mathcal{F}) - H(\mathcal{F}|\mathcal{G}) = H(\mathcal{G}) - H(\mathcal{G}|\mathcal{F}) \quad (8.2.7)$$

$$I(\mathcal{F}; \mathcal{G}) = H(\mathcal{F}) + H(\mathcal{G}) - H(\mathcal{F}, \mathcal{G}) \quad (8.2.8)$$

$$I(\mathcal{F}; \mathcal{G}) \leq H(\mathcal{F}), \quad I(\mathcal{F}; \mathcal{G}) \leq H(\mathcal{G}) \quad (8.2.9)$$

$$H(\mathcal{F}) \geq H(\mathcal{F}|\mathcal{G}), \quad H(\mathcal{G}) \geq H(\mathcal{G}|\mathcal{F}) \quad (8.2.10)$$

$$H(\mathcal{F}, \mathcal{G}) \leq H(\mathcal{F}) + H(\mathcal{G}) \quad (8.2.11)$$

$$I(\mathcal{F}; \mathcal{F}) = H(\mathcal{F}) = H(\mathcal{F}, \mathcal{F}) \quad (8.2.12)$$

Equation (8.2.10) says that conditioning reduces the entropy (i.e., uncertainty) of a RV. From Eq. (8.2.7), we can see that  $I(\mathcal{F}; \mathcal{G})$  describes the reduction in the uncertainty (or information) of  $\mathcal{F}$  due to the knowledge of  $\mathcal{G}$ .

The above definitions are given for a RV or between two RVs. They can be similarly defined for random vectors. By applying these definitions to random vectors consisting of successive samples in a discrete stationary source, we arrive at the following definitions.

**Definition 8.5 (Entropy of a Discrete Source)** *The  $N$ -th order entropy of a discrete stationary source  $\mathcal{F}$  with  $N$ -th order joint pmf  $p(f_1, f_2, \dots, f_N)$  is the joint entropy among  $N$  successive samples  $\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_N$  of  $\mathcal{F}$ :*

$$\begin{aligned} H_N(\mathcal{F}) &= H(\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_N) \\ &= - \sum_{[f_1, f_2, \dots, f_N] \in \mathcal{A}^N} p(f_1, f_2, \dots, f_N) \log_2 p(f_1, f_2, \dots, f_N), \end{aligned} \quad (8.2.13)$$

where  $\mathcal{A}^N$  represents the  $N$ -fold Cartesian product of  $\mathcal{A}$ .

**Definition 8.6 (Conditional Entropy of a Discrete Source)** *The  $M$ -th order conditional entropy of a discrete stationary source  $\mathcal{F}$  with  $M$ -th order joint pmf  $p(f_1, f_2, \dots, f_M)$  and  $M$ -th order conditional pmf  $p(f_{M+1}|f_M, f_{M-1}, \dots, f_1)$  is the conditional entropy of a sample  $\mathcal{F}_{M+1}$  given its previous  $M$  samples  $\mathcal{F}_M, \mathcal{F}_{M-1}, \dots, \mathcal{F}_1$ :*

$$\begin{aligned} H_{c,M}(\mathcal{F}) &= H(\mathcal{F}_{M+1}|\mathcal{F}_M, \mathcal{F}_{M-1}, \dots, \mathcal{F}_1) \\ &= - \sum_{[f_1, f_2, \dots, f_M] \in \mathcal{A}^M} p(f_1, f_2, \dots, f_M) H(\mathcal{F}_{M+1}|f_M, f_{M-1}, \dots, f_1) \end{aligned} \quad (8.2.14)$$

where

$$\begin{aligned} H(\mathcal{F}_{M+1}|f_M, f_{M-1}, \dots, f_1) &= \\ &- \sum_{f_{M+1} \in \mathcal{A}} p(f_{M+1}|f_M, f_{M-1}, \dots, f_1) \log_2 p(f_{M+1}|f_M, f_{M-1}, \dots, f_1). \end{aligned} \quad (8.2.15)$$

**Definition 8.7 (Entropy Rate of a Discrete Source)** *It can be shown that  $\frac{1}{N}H_N(\mathcal{F})$  and  $H_{c,N}(\mathcal{F})$  are both non-increasing function of  $N$ . Furthermore,  $\lim_{N \rightarrow \infty} \frac{1}{N}H_N(\mathcal{F})$  and  $\lim_{N \rightarrow \infty} H_{c,N}(\mathcal{F})$  both exist and are equal. This limit is defined as the entropy rate of the source:*

$$\bar{H}(\mathcal{F}) = \lim_{N \rightarrow \infty} \frac{1}{N}H_N(\mathcal{F}) = \lim_{N \rightarrow \infty} H_{c,N}(\mathcal{F}). \quad (8.2.16)$$

**Relation between joint entropy, conditional entropy and entropy rate** It can be shown (Prob. 8.3) that, for any finite  $N$  and  $M$ ,

$$H_N(\mathcal{F}) = \sum_{m=1}^{N-1} H_{c,m}(\mathcal{F}) + H_1(\mathcal{F}) \quad (8.2.17)$$

$$\bar{H}(\mathcal{F}) \leq H_{c,N-1}(\mathcal{F}) \leq \frac{1}{N}H_N(\mathcal{F}) \leq H_1(\mathcal{F}), \quad (8.2.18)$$

If  $\mathcal{F}$  is i.i.d., then

$$\bar{H}(\mathcal{F}) = \frac{1}{N}H_N(\mathcal{F}) = H_{c,M}(\mathcal{F}) = H_1(\mathcal{F}). \quad (8.2.19)$$

As will be shown in Sec. 8.3.1, the entropy rate is the lower bound on the minimal bit rate required to code a discrete source losslessly. The bound can be achieved only when an infinite number of samples are coded together. On the other hand, the first order entropy and conditional entropy provide lower bounds on the achievable bit rate when one sample is coded independently, or conditioned on the previous samples.

**Definition 8.8 (Mutual Information between Discrete Sources)** *Let  $\mathcal{F}$  and  $\mathcal{G}$  represent two discrete stationary sources, with  $N$ -th order pmf  $p(f_1, f_2, \dots, f_N)$  and  $p(g_1, g_2, \dots, g_N)$ , respectively. Further let  $p(f_1, f_2, \dots, f_N, g_1, g_2, \dots, g_N)$  represent the joint pmf of  $N$  samples each from  $\mathcal{F}$  and  $\mathcal{G}$ . The  $N$ -th order mutual information between  $\mathcal{F}$  and  $\mathcal{G}$  is defined as*

$$I_N(\mathcal{F}; \mathcal{G}) = \sum_{[f_1, f_2, \dots, f_N] \in \mathcal{A}_f^N} \sum_{[g_1, g_2, \dots, g_N] \in \mathcal{A}_g^N} p(f_1, f_2, \dots, f_N, g_1, g_2, \dots, g_N) \cdot \log_2 \frac{p(f_1, f_2, \dots, f_N, g_1, g_2, \dots, g_N)}{p(f_1, f_2, \dots, f_N)p(g_1, g_2, \dots, g_N)} \quad (8.2.20)$$

In lossy source coding, a source  $\mathcal{F} = \{\mathcal{F}_n\}$  is represented by a quantized version  $\mathcal{G} = \{\mathcal{G}_n\}$ . The first order mutual information  $I_1(\mathcal{F}; \mathcal{G})$  measures the amount of information provided by a quantized sample  $\mathcal{G}_n$  about the original sample  $\mathcal{F}_n$ . On the other hand,  $I_N(\mathcal{F}; \mathcal{G})$  measures the information provided by a block of  $N$  quantized samples about original  $N$  samples. As will be shown in Sec. 8.3.2,  $\lim_{N \rightarrow \infty} \min \frac{1}{N}I_N(\mathcal{F}; \mathcal{G})$  provides a lower bound on the minimal bit rate required for a desired distortion  $D$  between  $\mathcal{F}$  and  $\mathcal{G}$ , where the minimum is taken over all possible  $\mathcal{G}$ 's that have distortion  $D$  from  $\mathcal{F}$ .

### 8.2.3 Entropy and Mutual Information for Continuous Sources

For a continuous source, the pmf for any possible value in its support region is zero. Direct application of the previously defined entropy would yield an infinite entropy.<sup>2</sup> Instead, we use the pdf of the source to define differential entropy.

**Definition 8.9 (Differential Entropy of a Continuous RV)** *The differential entropy of a continuous RV  $\mathcal{F}$  with a support range  $\mathcal{B}$  and a pdf  $p_{\mathcal{F}}(f)$  is defined by*

$$h(\mathcal{F}) = - \int_{f \in \mathcal{B}} p_{\mathcal{F}}(f) \log_2 p_{\mathcal{F}}(f) df. \quad (8.2.21)$$

As with entropy, differential entropy measures the uncertainty or information content of a continuous RV. Unlike entropy, differential entropy can take on negative values and in fact can vary in the range of  $(-\infty, \infty)$ , depending on the pdf.

All other definitions (including joint entropy, conditional entropy, mutual information) given in Sec. 8.2.2 for discrete sources can be similarly given for continuous sources, by replacing any involved pmf with a corresponding pdf. Similarly, the relations between entropy, conditional entropy, and mutual information are still valid, with entropy replaced by differential entropy.

#### Gaussian Sources

A very important continuous source is the Gaussian source, in which each sample follows a Gaussian distribution and each group of  $N$  samples follows a joint Gaussian distribution. Here, we give the differential entropy for a Gaussian RV and a Gaussian random vector.

A Gaussian RV with mean  $\eta$  and variance  $\sigma^2$  has a pdf

$$p(f) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(f-\eta)^2/2\sigma^2}, f \in (-\infty, \infty). \quad (8.2.22)$$

It can be shown (Problem 8.4) that its differential entropy is

$$h_{\text{Gaussian}} = \frac{1}{2} \log_2(2\pi e \sigma^2). \quad (8.2.23)$$

An  $N$ -dimensional Gaussian random vector  $\mathcal{F}$  with mean  $\boldsymbol{\eta}$  and covariance matrix  $[\mathbf{C}]$  has a joint pdf

$$p(\mathbf{f}) = \frac{1}{(\sqrt{2\pi})^N |\det[\mathbf{C}]|^{1/2}} \exp\left(-\frac{1}{2} (\mathbf{f} - \boldsymbol{\eta})^T [\mathbf{C}]^{-1} (\mathbf{f} - \boldsymbol{\eta})\right). \quad (8.2.24)$$

It can be shown that the differential entropy of this random vector (or joint differential entropy among the  $N$  components) is

$$h_{N\text{-DGaussian}} = \frac{1}{2} \log_2 ((2\pi e)^N |\det[\mathbf{C}]|) = \frac{1}{2} \log_2 \left( (2\pi e)^N \prod_n \lambda_n \right), \quad (8.2.25)$$

---

<sup>2</sup>The previously defined entropy based on the pmf is also known as absolute entropy.

where  $\lambda_n, n = 1, 2, \dots, N$  are the eigenvalues of  $[\mathbf{C}]$ .

*An important property of the Gaussian distribution is that the Gaussian RV has the largest entropy among all continuous RVs with the same variance. Similarly, a Gaussian random vector has the largest entropy among all random vectors with the same covariance matrix. This makes the Gaussian source most difficult to code, in that it requires a higher bit rate to represent than any other sources, for a given distortion criterion.*

### 8.3 Information theory for Source Coding

The theoretical foundation for source coding is established by several important results of information theory.<sup>3</sup> These results, due to Shannon, the father of information theory, establish bounds on the minimal bit rate required to realize lossless and lossy coding. A rigorous and comprehensive treatment of this subject is beyond the scope of this book. The intent of this section is to help the reader to understand the theoretical limits of source coding and how they may be used to guide the design of practical coding techniques. The reader is referred to the books by Cover and Thomas [3] and by Berger [1] for more in-depth exposition.

#### 8.3.1 Bound for Lossless Coding

##### Scalar Lossless Coding

For a particular realization  $\{f_n\}$  of a discrete source  $\mathcal{F}$  with an alphabet  $\mathcal{A} = \{a_1, a_2, \dots, a_L\}$ , scalar lossless coding refers to assigning a binary codeword  $c_n$  for each sample  $f_n$ . This requires that one predesign a codebook  $\mathcal{C} = \{c(a_1), c(a_2), \dots, c(a_L)\}$ , where  $c(a_i)$  is the codeword for symbol  $a_i$ . Then the codeword for  $f_n$  will be  $c_n = c(f_n)$ . For a codebook to be useful, a coded sequence must be *uniquely decodable*, i.e., a sequence of codewords corresponds to one and only one possible sequence of source symbols. Note that this is a stronger requirement than the one that the mapping between the alphabet and the codebook be one-to-one. Let  $l(a_i)$  denote the length (i.e., the number of bits) of  $c(a_i)$ , then the *bit rate*, defined as the average number of bits per sample,<sup>4</sup> will be equal to the average length per codeword, i.e.,

$$R = \sum_{a_i \in \mathcal{A}} p(a_i)l(a_i). \quad (8.3.1)$$

---

<sup>3</sup>Source coding refers to the process of converting samples from a source into a stream of binary bits, whereas channel coding refers to further manipulating the source bits to add protection against transmission errors.

<sup>4</sup>Elsewhere in this book, we also define bit rate as the number of bits per second (bps). For a source outputting  $f_s$  sample/s and representing each sample with  $R$  bits, the bit rate in terms of bps is  $f_s R$ .

**Theorem 8.1 (Bound for Scalar Lossless Coding)** *The minimum bit rate  $\bar{R}_1(\mathcal{F})$  required to represent a discrete stationary source  $\mathcal{F}$  by assigning one codeword to each sample satisfies*

$$H_1(\mathcal{F}) \leq \bar{R}_1(\mathcal{F}) \leq H_1(\mathcal{F}) + 1. \quad (8.3.2)$$

*The lower bound can be achieved when the pmf of the source is a function of power 2. That is, there exist a set of integers  $\{m_1, m_2, \dots, m_L\}$  such that  $p(a_i) = 2^{-m_i}$ . In this case,  $l(a_i) = -\log_2 p(a_i) = m_i$ .*

The above theorem tells us that the first-order entropy of a discrete source  $H_1(\mathcal{F})$  determines the range of the minimum bit rate for scalar coding the source. This is not surprising, as  $H_1(\mathcal{F})$  measures the information content (in bit/sample) carried by each new sample in a source.

### Vector Lossless Coding

The above theorem tells us that, depending on the source, the minimal achievable rate can be off from the first order entropy by as much as 1 bit/sample. To further reduce the bit rate, one can treat each successive  $N$  samples in a discrete sequence as a vector-sample, and assign one codeword for each possible vector-symbol in  $\mathcal{A}^N$ . The first order entropy of the vector source would be the  $N$ -th order entropy of the original source. If  $\bar{R}^N$  represent the minimal number of bits per vector, then applying Theorem 8.1 to the above vector source yields

$$H_N(\mathcal{F}) \leq \bar{R}^N(\mathcal{F}) \leq H_N(\mathcal{F}) + 1 \quad (8.3.3)$$

Let  $\bar{R}_N = \bar{R}^N/N$  represent the minimal number of bits per sample, we arrive at the following theorem.

**Theorem 8.2 (Bound for Vector Lossless Coding)** *The minimum bit rate  $\bar{R}_N(\mathcal{F})$  required to represent a discrete stationary source  $\mathcal{F}$  by assigning one codeword to each group of  $N$  successive samples satisfies*

$$H_N(\mathcal{F})/N \leq \bar{R}_N(\mathcal{F}) \leq H_N(\mathcal{F})/N + 1/N. \quad (8.3.4)$$

*In the limit when  $N \rightarrow \infty$ , from Eq. (8.2.16), we have*

$$\lim_{N \rightarrow \infty} \bar{R}_N(\mathcal{F}) = \bar{H}(\mathcal{F}). \quad (8.3.5)$$

The above theorem tells us that the bit rate can be made arbitrarily close to the source entropy rate by coding many samples together.

### Conditional Lossless Coding

We have seen that one can improve upon the efficiency of scalar coding by vector coding. An alternative is conditional coding, also known as *context-based coding*. With  $M$ -th order conditional coding, the codeword for a current sample depends on

the pattern formed by the previous  $M$  samples. Such a pattern is called the *context*. Specifically, a separate codebook is designed for each possible context, based on the conditional distribution of the output sample given this context. If the source alphabet size is  $L$ , then the maximum number of contexts, and consequently the maximum number of codebooks, is  $L^M$ . Applying Theorem 8.1 to the conditional distribution under context  $m$ , the minimal bit rate under this context satisfies

$$H_{c,M}^m(\mathcal{F}) \leq \bar{R}_{c,M}^m(\mathcal{F}) \leq H_{c,M}^m(\mathcal{F}) + 1, \quad (8.3.6)$$

where  $H_{c,M}^m(\mathcal{F})$  represents the entropy of the conditional distribution under context  $m$ . Let  $p_m$  represent the probability of context  $m$ , then the average minimal bit rate with a conditioning order  $M$  is

$$\bar{R}_{c,M}(\mathcal{F}) = \sum_m p_m \bar{R}_M^m(\mathcal{F}). \quad (8.3.7)$$

Substituting Eq. (8.3.6) to the above yields the following result.

**Theorem 8.3 (Bound for Conditional Lossless Coding)** *The minimum bit rate  $\bar{R}_{c,M}(\mathcal{F})$  required to represent a discrete stationary source  $\mathcal{F}$  by using  $M$ -th order conditional coding satisfies*

$$H_{c,M}(\mathcal{F}) \leq \bar{R}_{c,M}(\mathcal{F}) \leq H_{c,M}(\mathcal{F}) + 1. \quad (8.3.8)$$

*In the limit when  $M \rightarrow \infty$ , we have*

$$\bar{H}(\mathcal{F}) \leq \lim_{M \rightarrow \infty} \bar{R}_{c,M}(\mathcal{F}) \leq \bar{H}(\mathcal{F}) + 1. \quad (8.3.9)$$

Compared to Eq. (8.3.2), conditional coding can achieve a lower bit rate than scalar coding, because  $\bar{H}(\mathcal{F}) < H_1(\mathcal{F})$ , unless the source is i.i.d. However, because one still codes one sample at a time, the upper bound is still off from the entropy rate by 1 bit, even when the conditioning order  $M$  goes to infinity. Comparing  $N$ -th order vector lossless coding with  $(N-1)$ -th order conditional coding, the former always has the same or larger lower bound according to Eq. (8.2.18). On the other hand, vector coding will have a smaller upper bound if  $\frac{1}{N}H_N(\mathcal{F}) - H_{c,N-1}(\mathcal{F}) < \frac{N-1}{N}$ . Therefore, which method is more efficient depends on the actual source statistics.

### 8.3.2 Bound for Lossy Coding\*

When a source is continuous, it is impossible to represent it exactly with a finite number of bits. Recall that the absolute entropy rate of a continuous source is infinite. Therefore, applying the bound on lossless coding (Eq. (8.3.5)) to a continuous source leads to a minimal bit rate that is infinite. To describe a continuous source with a finite number of bits, one must in some way quantize the source so that each sample or a group of samples can only take symbols from a finite alphabet. Such quantization will inevitably induce distortion between the original source and

the reconstructed source based on the quantized representation. One can directly quantize the source samples, either one at a time (scalar quantization) or a group of samples together (vector quantization), or transform the samples into some other parameters and quantize the resulting parameters. The quantized symbols can then be encoded losslessly into binary codewords. At the decoder, the codewords are first mapped back to the quantized symbols, from which each sample is reconstructed.

In general, a lossy coding process can be considered as mapping every input vector  $\mathbf{f} = \{f_1, f_2, \dots, f_N\}$  consisting of  $N$  samples from the original source  $\mathcal{F}$  into a quantized vector  $\mathbf{g} = Q(\mathbf{f})$ . The vector  $\mathbf{g}$  must belong to a pre-designed *reconstruction codebook* of a finite size  $L$ ,  $\mathcal{C} = \{\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_L\}$ . We will assume that each component of  $\mathbf{f}$  and  $\mathbf{g}$  can only take values from the same support range  $\mathcal{B}$ . Using fixed length coding, each quantized vector is represented by  $\log_2(L)$  bits, so that the bit rate (bit/sample) of the coded sequence is

$$R_N = \frac{1}{N} \log_2(L). \quad (8.3.10)$$

Given a designed reconstruction codebook, the bit rate can be further reduced from Eq. (8.3.10) by applying variable length coding to the indices of codewords. However, in the remaining discussions, we assume the bit rate is determined according to Eq. (8.3.10).

To meet a given distortion criterion between  $\mathcal{F}$  and  $\mathcal{G}$ , the required codebook size  $L$  depends on the coding scheme, i.e., how is the codebook designed. Intuitively, the bit rate can be reduced by increasing the vector length  $N$ , especially when the source is not i.i.d. The rate distortion theory defines the lower bound on the bit rate required to achieve a given distortion, when  $N$  approaches infinity.

Note that lossy coding can be applied to both continuous and discrete sources. Recall that for a discrete source, the minimal bit rate required for lossless (zero distortion) coding is bounded by the source entropy rate. To reach an even lower rate, one must quantize the samples to a smaller alphabet, thereby incurring a certain degree of distortion. The following discussions apply to both continuous and discrete sources.

### Distortion Measure

Until now, we have used the word “distortion” without formally defining it. To measure the distortion between an original source  $\mathcal{F}$  and the reconstructed source  $\mathcal{G}$ , we first define a distortion measure  $d_1(f, g)$  between two scalars  $f$  and  $g$ . When  $f$  and  $g$  are both real values, the most common distortion measures are the squared error,  $d_1(f, g) = (f - g)^2$ , and absolute error,  $d_1(f, g) = |f - g|$ . Then the average distortion per sample between an original vector  $\mathbf{f}$  and the reconstructed one  $\mathbf{g}$  is

$$d_N(\mathbf{f}, \mathbf{g}) = \frac{1}{N} \sum_n d_1(f_n, g_n). \quad (8.3.11)$$

The above distortion is defined for a given vector and its reconstruction vector. To evaluate the average performance over all possible input vectors, we define the

following expected distortion between  $\mathcal{F}$  and  $\mathcal{G}$  when the vector length is  $N$ :

$$E\{d_N(\mathcal{F}, \mathcal{G})\} = \int_{\mathbf{f} \in \mathcal{B}^N} \int_{\mathbf{g} \in \mathcal{B}^N} p_N(\mathbf{f}) q_N(\mathbf{g}|\mathbf{f}) d_N(\mathbf{f}, \mathbf{g}) d\mathbf{f} d\mathbf{g}, \quad (8.3.12)$$

where  $p_N(\mathbf{f})$  represents the joint pdf of  $N$  successive samples in  $\mathcal{F}$ ,  $q_N(\mathbf{g}|\mathbf{f})$  represents the conditional probability that  $\mathbf{f}$  is reconstructed as  $\mathbf{g}$ ,  $\mathcal{B}^N$  represents the  $N$  fold Cartesian product of  $\mathcal{B}$ . Note that  $q_N(\mathbf{g}|\mathbf{f})$  essentially characterizes the coding scheme, although in practice, a deterministic mapping between  $\mathbf{f}$  and  $\mathbf{g}$  is used. When  $d_1(f, g)$  is the squared error, the above measure is known as *mean square error* (MSE), which is widely used, primarily because of its mathematical tractability.

In the above, we have assumed the source is continuous, and the functions  $p_N(\cdot), q_N(\cdot)$  represent pdf's. When the source is discrete, the integration over  $\mathcal{B}^N$  should be placed by a sum over  $\mathcal{A}^N$ , and  $p_N(\cdot), q_N(\cdot)$  would represent corresponding pmf's.

### Rate-Distortion Bound

The performance of a lossy coder is measured by its *operational rate-distortion (RD) function*, denoted by  $R(D)$ , which describes the bit rate  $R$  required to achieve a given distortion  $D$ , for given source characteristics. The rate-distortion theorem establishes the bound on the minimal rate required among all possible coders, when the vector length  $N$  reaches infinity. We call the function relating the minimal rate and the distortion the *rate-distortion (RD) bound*, denoted by  $\bar{R}(D)$ . Let  $R_N(D; q_N(\mathbf{g}|\mathbf{f}))$  represent the bit rate required to achieve distortion  $D$  by a coding scheme described by  $q_N(\mathbf{g}|\mathbf{f})$ . The RD bound is defined as

$$\bar{R}(D) = \lim_{N \rightarrow \infty} \min_{q_N(\mathbf{g}|\mathbf{f}) \in Q_{D,N}} R_N(D; q_N(\mathbf{g}|\mathbf{f})) \quad (8.3.13)$$

where

$$Q_{D,N} = \{q_N(\mathbf{g}|\mathbf{f}) : E\{d_N(\mathcal{F}, \mathcal{G})\} \leq D\} \quad (8.3.14)$$

is the set of conditional probabilities (i.e., coding schemes) that satisfy the specified distortion constraint.

Instead of representing  $R$  as a function of  $D$ , one can also represent  $D$  in terms of  $R$ , which will lead to the *operational distortion-rate function*  $D(R)$  or *distortion-rate bound*  $\bar{D}(R)$ .

**Theorem 8.4 (Bound on Lossy Coding)** *The RD bound for coding a stationary source  $\mathcal{F}$  at a distortion  $D$  is given by<sup>5</sup>*

$$\bar{R}(D) = \lim_{N \rightarrow \infty} \min_{q_N(\mathbf{g}|\mathbf{f}) \in Q_{D,N}} \frac{1}{N} I_N(\mathcal{F}; \mathcal{G}). \quad (8.3.15)$$

<sup>5</sup>In information theory, the RD function of a source  $\mathcal{F}$  is defined as the right side of Eq. (8.3.15). The Shannon source coding theorem states that there exists a coding scheme by which the bit rate required to achieve a distortion  $D$  approaches  $\bar{R}(D)$ .

When the source is i.i.d., the RD bound becomes

$$\bar{R}(D) = \min_{q_1(\mathbf{g}|\mathbf{f}) \in Q_{D,1}} I_1(\mathcal{F}; \mathcal{G}). \quad (8.3.16)$$

Recall that  $I_N(\mathcal{F}; \mathcal{G})$  describes the amount of information provided by  $N$  samples of  $\mathcal{G}$  about corresponding  $N$  samples of  $\mathcal{F}$ . Therefore, it is not surprising that  $I_N(\mathcal{F}; \mathcal{G})$  determines the number of bits required to specify  $\mathcal{F}$  with distortion  $D$  when  $N \rightarrow \infty$ .

An important result from information theory is that, even when the source is i.i.d., one can reduce the bit rate  $R$  by coding many samples together. In fact, the RD bound is achievable only when the vector length  $N$  reaches infinity.

In general, the above bound is difficult to calculate for an arbitrary source. A more useful bound is described by the following theorem.

**Theorem 8.5:** Under the MSE distortion criterion, the RD bound for any stationary source  $\mathcal{F}$  satisfies

$$\bar{R}_L(D) \leq \bar{R}(D) \leq \bar{R}_G(D), \quad (8.3.17)$$

where  $\bar{R}_G(D)$  is the RD bound for a Gaussian source with the same variance as  $\mathcal{F}$ , and  $\bar{R}_L(D)$  is known as the Shannon lower bound, with

$$\bar{R}_L(D) = \bar{h}(\mathcal{F}) - \frac{1}{2} \log_2 2\pi e D = \frac{1}{2} \log_2 \frac{Q(\mathcal{F})}{D}, \quad (8.3.18)$$

where  $\bar{h}(\mathcal{F})$  is the entropy rate of  $\mathcal{F}$ , and  $Q(\mathcal{F}) = \frac{1}{2\pi e} 2^{2\bar{h}(\mathcal{F})}$  is called the entropy power of  $\mathcal{F}$ . For a Gaussian source,  $\bar{R}_L(D) = \bar{R}_G(D)$ .

The above theorem tells us that among all sources with the same variance, the Gaussian source requires the highest bit rate, to satisfy the same distortion criterion. In fact, for any source, the optimal coding scheme that achieves the RD bound is the one by which the quantization error sequence is an i.i.d. Gaussian source with variance  $D$ , with a differential entropy  $\frac{1}{2} \log_2 2\pi e D$ . Therefore, the lower bound given in Eq. (8.3.18) is equal to the difference between the differential entropy of the original source and that of the quantization error.

### 8.3.3 Rate-Distortion Bounds for Gaussian Sources\*

For most sources, a closed-form solution for the RD bound is difficult to derive. One exception is the various types of Gaussian sources, for which closed-form solutions have been found, when the distortion criterion is MSE. We summarize these results below.

**i.i.d. Gaussian source:** The RD bound for an i.i.d. Gaussian source with variance  $\sigma^2$  is

$$\bar{R}(D) = \begin{cases} \frac{1}{2} \log_2 \frac{\sigma^2}{D}, & 0 \leq D \leq \sigma^2, \\ 0, & D > \sigma^2. \end{cases} \quad (8.3.19)$$

Alternatively, one can write

$$\bar{D}(R) = \sigma^2 2^{-2R}. \quad (8.3.20)$$

**i.i.d. Vector Gaussian Source with independent components:** Consider an i.i.d. vector Gaussian source, in which each vector sample consists of  $N$  independent Gaussian RVs with variances  $\sigma_n^2, n = 1, 2, \dots, N$ . The RD bound for this source is given by

$$\begin{cases} R(\alpha) = \frac{1}{N} \sum_n \max\{0, \frac{1}{2} \log_2 \frac{\sigma_n^2}{\alpha}\}, \\ D(\alpha) = \frac{1}{N} \sum_n \min\{\alpha, \sigma_n^2\}, \end{cases} \quad (8.3.21)$$

where  $\alpha$  is an intermediate parameter. The RD bound for the feasible distortion region can be obtained by letting  $\alpha$  traversing through the range  $(0, \max\{\sigma_n^2\})$ .

In the case when  $D$  is sufficiently small, so that  $D \leq \min\{\sigma_n^2\}$ , the above reduces to

$$\bar{R}(D) = \frac{1}{2} \log_2 \frac{(\prod_n \sigma_n^2)^{1/N}}{D}, \quad (8.3.22)$$

or

$$\bar{D}(R) = (\prod_n \sigma_n^2)^{1/N} 2^{-2R}. \quad (8.3.23)$$

The above RD bound is achieved by coding each component in a vector sample independently,<sup>6</sup> with  $R_n = \max\{0, \frac{1}{2} \log_2 \frac{\sigma_n^2}{\alpha}\}$  bits used for component  $n$ . For a component with  $\sigma_n^2 \leq \alpha$ , it can be always represented with zero, thus  $R_n = 0$ . For the remaining components, the bits are allocated so that they all have equal distortion  $D_n = \alpha$ . This result is obtained by optimally allocating the total bit rate  $NR$  among the  $N$  components so that the average distortion  $D$  is minimized. Essentially, it requires that the bit rates  $R_n$  being chosen such that the slopes of the distortion-rate functions for different components,  $\frac{\partial D_n(R_n)}{\partial R_n}$  are equal at these rates.

**i.i.d. Vector Gaussian Source with Correlated Components:** Let  $[\mathbf{C}]$  represent the covariance matrix of the  $N$  components of each vector sample, and let  $\lambda_n, n = 1, 2, \dots, N$  represent the eigenvalues of  $[\mathbf{C}]$ . The RD bound for this source is given by

$$\begin{cases} R(\alpha) = \frac{1}{N} \sum_n \max\{0, \frac{1}{2} \log_2 \frac{\lambda_n}{\alpha}\}, \\ D(\alpha) = \frac{1}{N} \sum_n \min\{\alpha, \lambda_n\}, \end{cases} \quad (8.3.24)$$

where  $\alpha \in (0, \max\{\lambda_n\})$  is an intermediate parameter.

In the case when  $D$  is sufficiently small, so that  $D \leq \min\{\lambda_n\}$ , the above reduces to

$$\bar{R}(D) = \frac{1}{2} \log_2 \frac{(\prod_n \lambda_n)^{1/N}}{D} = \frac{1}{2} \log_2 \frac{|\det[\mathbf{C}]|^{1/N}}{D} \quad (8.3.25)$$

or

$$\bar{D}(R) = (\prod_n \lambda_n)^{1/N} 2^{-2R} = |\det[\mathbf{C}]|^{1/N} 2^{-2R}. \quad (8.3.26)$$

---

<sup>6</sup>But the  $n$ -th components from successive samples must be coded together to achieve the RD bound.

The above result is obtained by transforming the original vector using a transform matrix consisting of the eigenvectors of  $[\mathbf{C}]$ , and then applying the RD bound in Eq. (8.3.21) to the transformed vector, which now has independent components.

**General Gaussian Source:** An arbitrary stationary Gaussian source with mean  $\eta$  can be described by its auto-covariance function  $C(k) = E\{(\mathcal{F}_n - \eta)(\mathcal{F}_{n+k} - \eta)\}, k = 0, \pm 1, \dots$  or its power spectrum  $S(e^{j\omega})$ .<sup>7</sup> The RD bound for such a source is the limit of the RD bound given in Eq. (8.3.24) when  $N \rightarrow \infty$ . The result is

$$\begin{cases} R(\alpha) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \max \left\{ 0, \frac{1}{2} \log_2 \frac{S(e^{j\omega})}{\alpha} \right\} d\omega, \\ D(\alpha) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \min \left\{ \alpha, S(e^{j\omega}) \right\} d\omega. \end{cases} \quad (8.3.27)$$

When  $D$  is sufficiently small so that the region of  $\omega$  where  $D > S(e^{j\omega})$  can be neglected, we have

$$\bar{R}(D) = \frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2 \frac{S(e^{j\omega})}{D} d\omega. \quad (8.3.28)$$

Equation (8.3.27) tells us that to represent a source with a spectrum  $S(e^{j\omega})$  at distortion  $\alpha$ , the frequency range over which  $S(e^{j\omega}) < \alpha$  does not need to be coded, whereas the remaining range should be coded so that the error signal has an power equal to  $\alpha$  over this range. This bit allocation procedure is commonly known as *reverse water filling*.

In the special case of a first order Gauss Markov process with  $C(k) = \sigma^2 |\rho|^k$ , we have

$$R(D) = \frac{1}{2} \log_2 \frac{(1 - \rho^2)\sigma^2}{D}, \quad (8.3.29)$$

or

$$D(R) = (1 - \rho^2)\sigma^2 2^{-2R}. \quad (8.3.30)$$

## 8.4 Binary Encoding

Recall that binary encoding is the process of representing each possible symbol from a finite alphabet source by a sequence of binary bits, called codeword. The codewords for all possible symbols form a codebook. A symbol may correspond to one or several original or quantized pixel values or model parameters.

For a binary code to be useful, it should satisfy the following properties: i) it should be uniquely decodable, which means that there is a one-to-one mapping between the codeword and the symbol; and ii) the code should be *instantaneously decodable*, which means that one can immediately decode a group of bits if they match a codeword, without examining subsequent bits in the coded sequence (cf. Fig. 8.2). This second property requires that no prefix of any codeword is another valid codeword. Such type of code is called *prefix code*. Although the instantaneous decodability is a stronger requirement than the uniqueness and enables fast

---

<sup>7</sup>  $S(e^{j\omega})$  is the discrete time Fourier transform of  $C(k)$ .

Codebook 1 (a prefix code)		Codebook 2 (not a prefix code)	
Symbol	Codeword	Symbol	Codeword
a1	0	a1	0
a2	10	a2	01
a3	110	a3	100
a4	111	a4	011

Bitstream: .....0 0 1 1 0 1 0 1 1 0 1 0 0

Decoded string based on codebook 1: 0|0|1 1 0|1 0|1 1 0|1 0|0-->a1 a1 a3 a2 a3 a2 a1  
(can decode instantaneously)

Decoded string based on codebook 2: 0|0 1 1|0 1|0 1 1|0|1 0 0-->a1 a4 a2 a4 a1 a3  
(need to look ahead to decode)

**Figure 8.2.** With the codebook on the left (a prefix code), the given bit stream can be instantaneously decoded upon the detection of a complete codeword. With the codebook on the right (a non-prefix code), one does not know whether the first ‘0’ in the bit stream corresponds to the codeword ‘0’, or it corresponds to the first bit in the second or fourth codeword. One needs to examine additional bits to make the decision. Because the second bit is still ‘0’, and yet there is no codeword that starts with two ‘0’s, the first ‘0’ must correspond to the codeword ‘0’.

decoding, it does not limit the coding efficiency. It can be shown that a prefix code can produce the minimal bit rate among all uniquely decodable codes, for the same source [4]. All practical coding methods produce prefix codes.

Obviously, the simplest binary code is the fixed-length binary representation of all possible symbols. If the number of symbols is  $L$ , then the bit rate will be  $\lceil \log_2 L \rceil$  bit/symbol. From Sec. 8.3.1, we know that the lowest possible bit rate by any codebook is the entropy rate of the source. Unless the source has a uniform distribution, the fixed length coding scheme will be very inefficient in that the bit rate is much higher than the source entropy. To reduce the bit rate, *variable length coding* (VLC) is needed, which codes a symbol with a high probability using a shorter codeword, so that the average bit rate is low. Because the bit rate of an appropriately designed variable length coder can approach the entropy of the source, VLC is also referred to as *entropy coding*.

There are three popular VLC methods. Huffman coding converts a fixed number of symbols into a variable length codeword; the LZW method converts a variable number of symbols into a fixed length codeword; and finally, arithmetic coding converts a variable number of symbols into a variable length codeword. Huffman

and arithmetic methods are probability-model-based and both can reach the entropy bound asymptotically. The arithmetic coding method can more easily achieve the asymptotic performance and can easily adapt to variations in signal statistics. But it is also more complex than Huffman coding. The LZW method [12, 10] does not require the knowledge of the signal statistics and thus is universally applicable. But it is less efficient than the other two methods. Huffman and arithmetic coding methods have been employed in various video coding standards. We describe these two methods in the following two subsections.

### 8.4.1 Huffman Coding

Huffman coding is the most popular approach for lossless coding of a discrete source with alphabet  $\mathcal{A} = \{a_1, a_2, \dots, a_L\}$  and pmf  $p(a_l)$ . It designs a codebook for all possible symbols, so that a symbol appearing more frequently is assigned a shorter codeword. The basic procedure for codebook design using Huffman coding is as follows:

**Step i:** Arrange the symbol probabilities  $p(a_l), l = 1, 2, \dots, L$  in a decreasing order and consider them as leaf nodes of a tree.

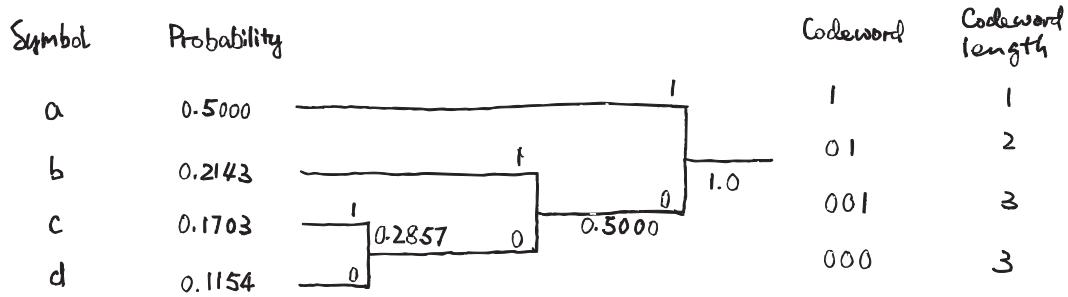
**Step ii:** While there is more than one node:

- (a) Find the two nodes with the smallest probability and arbitrarily assign 1 and 0 to these two nodes.
- (b) merge the two nodes to form a new node whose probability is the sum of the two merged nodes. Go back to Step i.

**Example 8.1 (Scalar Huffman Coding)** An example of Huffman coding is shown in Fig. 8.3, in which the source consists of 4 symbols. The symbols and their probabilities are shown on the left two columns. The resulting codewords and the codeword lengths for these symbols are shown on the right two columns. Also given are the average bit rate  $R$  and the first-order entropy of the source  $H_1$ . We can see that indeed,  $H_1 < R < H_1 + 1$ .

A notorious disadvantage of Huffman Coding, when applied to individual samples, is that at least one bit has to be used for each sample. To further reduce the bit rate, one can treat each group of  $N$  samples as one entity and give each group a codeword. This leads to Vector Huffman Coding, a special case of vector lossless coding. This is shown in the next example.

**Example 8.2 (Vector Huffman Coding)** Consider a source with the same alphabet and the pmf as in Example 8.1. Suppose we further know that the conditional distribution of a sample  $\mathcal{F}_n$  given its previous sample  $\mathcal{F}_{n-1}$  is described by the fol-



$$\text{Bit Rate } R = 1.7857 \quad \text{Entropy } H_1 = 1.7707$$

Fig. 7.2

Figure 8.3. Example of Scalar Huffman Coding. See Example 8.1

lowing matrix

$$[\mathbf{Q}] = \begin{bmatrix} 0.6250 & 0.3750 & 0.3750 & 0.3750 \\ 0.1875 & 0.3125 & 0.1875 & 0.1875 \\ 0.1250 & 0.1875 & 0.3125 & 0.1250 \\ 0.0625 & 0.1250 & 0.1250 & 0.3125 \end{bmatrix} \quad (8.4.1)$$

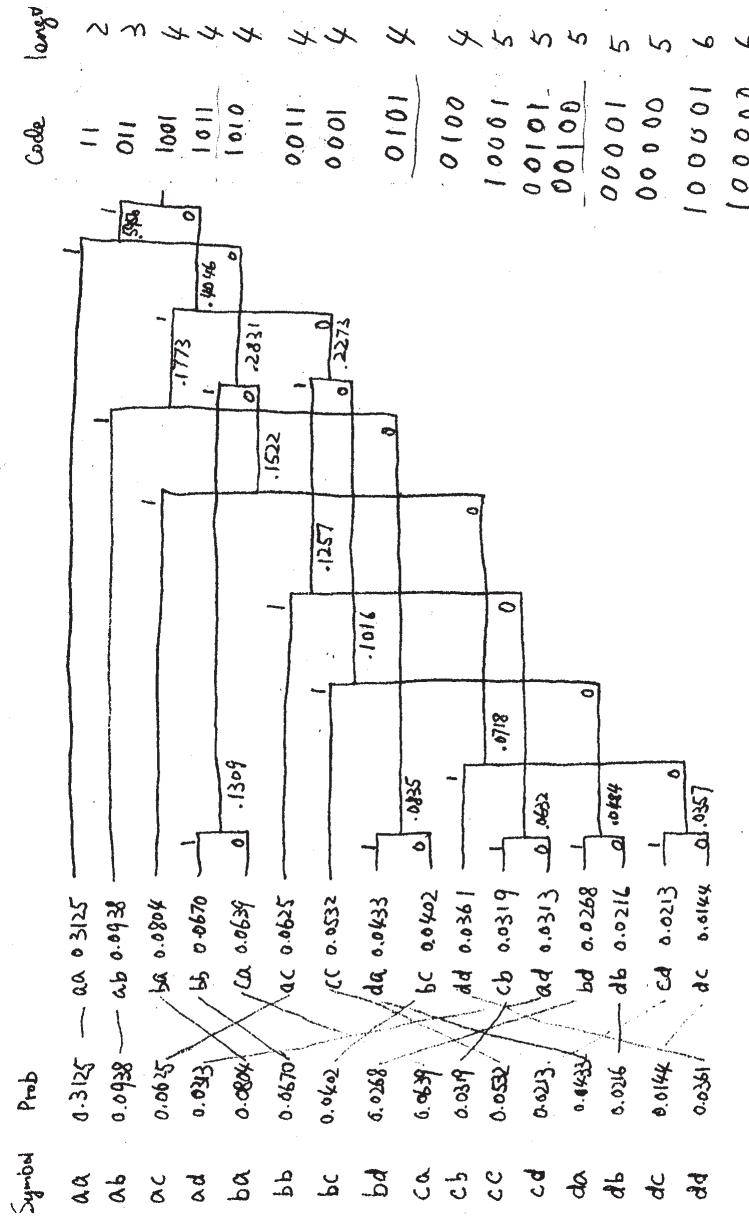
where the element in row  $i$  and column  $j$  specifies the conditional probability  $q(i|j)$ , which is the probability that  $\mathcal{F}_n$  takes the  $i$ -th symbol, given that  $\mathcal{F}_{n-1}$  equals the  $j$ -th symbol. We can determine the joint pmf of every two samples by

$$p(f_{n-1}, f_n) = p(f_{n-1})q(f_n|f_{n-1}). \quad (8.4.2)$$

Applying the above to all possible combination of two symbols, we obtain the probabilities of all possible 2-D vector symbols, which are given in the first column of Fig. 8.4. Then we can apply Huffman coding to this new source and the resulting codebook is also shown in Fig. 8.4. In this case, the bit rate per vector sample is  $R^2 = 3.5003$ , and bit rate per sample is  $R_2 = R^2/2 = 1.75015$ . On the other hand,  $H_2/2 = 1.7314$ . Indeed, we have  $\frac{1}{2}H_2 < R_2 < \frac{1}{2}H_2 + 1/2$ . Compared to the result in Example 8.1, we see that the bit rate is reduced by using vector coding.

Instead of coding two samples together, we can also use conditional Huffman coding, which uses a different codebook depending on the symbols taken by previous samples. This is shown in the next example.

**Example 8.3 (Conditional Huffman Coding)** Continue with the previous example, but we now design a separate codebook for each possible context consisting



$$R^2 = 3.5003, \quad H_2 = 3.4629$$

Figure 8.4. Example of Vector Huffman Coding. See Example 8.2

Symbol	Prob		Code word	Length
a/b	0.3750		1	1
b/b	0.3125		01	2
c/b	0.1875		001	3
d/b	0.1250		000	3

$$R_{c,\text{b}'} = 1.9375 \quad , \quad H_{c,\text{b}'} = 1.8829$$

**Figure 8.5.** Example of Conditional Huffman Coding, when the conditioning context is "b". See Example 8.3

of one previous sample. In this case, there are only 4 possible contexts. Figure 8.5 shows the codebook design for the context “b” (i.e., , the previous symbol being “b”), based on the conditional probabilities given in the second column of the matrix given in Eq. (8.4.1). Following the same procedure, one can design the codebook for the other three possible contexts. The resulting bit rates for the four contexts “a”, “b”, “c”, “d” are 1.5625, 1.9375, 1.9375, 1.9375, and the average bit rate over all possible contexts is  $R_{c,1} = 1.7500$ . On the other hand, the conditional entropy for the four contexts are 1.5016, 1.8829, 1.8829, and 1.8829, respectively, with an average conditional entropy being  $H_{c,1} = 1.6922$ . As expected, we have  $H_{c,1} < R_{c,1} < H_{c,1} + 1$ . In this particular example,  $R_{c,1} < R_2$ , although this may not be the case in general.

If the source is a Markov-1 process, then the entropy rate of the source is equal to its first order conditional entropy. Hence  $H_{c,1}$  is the lowest possible bit rate achievable by vector coding even when the vector length is infinite.

## 8.4.2 Arithmetic Coding

A disadvantage of Huffman coding is that it cannot reach the entropy bound closely unless many samples are coded together. This is because each sample (or group of sample) uses at least 1 bit. Therefore, even for a very small alphabet, the bit rate cannot be lower than 1 bit/sample, in the case of scalar coding, or 1 bit/vector-sample, in the case of vector coding. One way to avoid such problems is by converting a variable number of samples into a variable length codeword. This method is known as *arithmetic coding*. It can approach the entropy bound more

closely than Huffman coding for common signals.

The idea in arithmetic coding is to represent a sequence of symbols by an interval in the line segment from 0 to 1, with length equal to the probability of the sequence. Because the probabilities of all possible sequences sum to 1, the intervals corresponding to all possible sequences will fill up the entire line segment. The coded bits for a sequence is essentially the binary representation of any point in the interval corresponding to the sequence. Instead of waiting for the entire sequence to appear before deciding on the interval and its binary representation, one starts from an initial interval determined based on the first symbol, and then recursively divides the previous interval after each new symbol appears. To specify an interval, the lower and upper boundaries of the interval are represented in binary. Whenever the most significant bit (MSB) of the lower boundary is the same as that of the upper boundary, this bit is shifted out. At the end of the source sequence, all the bits that have been collected would be the binary representation of an intermediate point in the interval corresponding to the sequence. The more likely is a sequence, the longer will be the interval, and the fewer bits will be needed to specify the interval.

Let  $a_l, l = 1, 2, \dots, L$  represent the  $L$  possible symbols of the source,  $p_l$  the probability of symbol  $a_l$ , and finally  $q_l = \sum_{k=1}^l p_k$  the cumulative probability up to the  $l$ -th symbol. Let  $d_n, l_n, u_n$  represent the length, lower boundary and upper boundary of the interval at step  $n$ , respectively, with  $l_0 = 0, u_0 = 1, d_0 = 1$ . Upon receiving the  $n$ -th symbol of the source sequence, if the symbol is  $a_l$ , the lower and upper boundaries are computed using

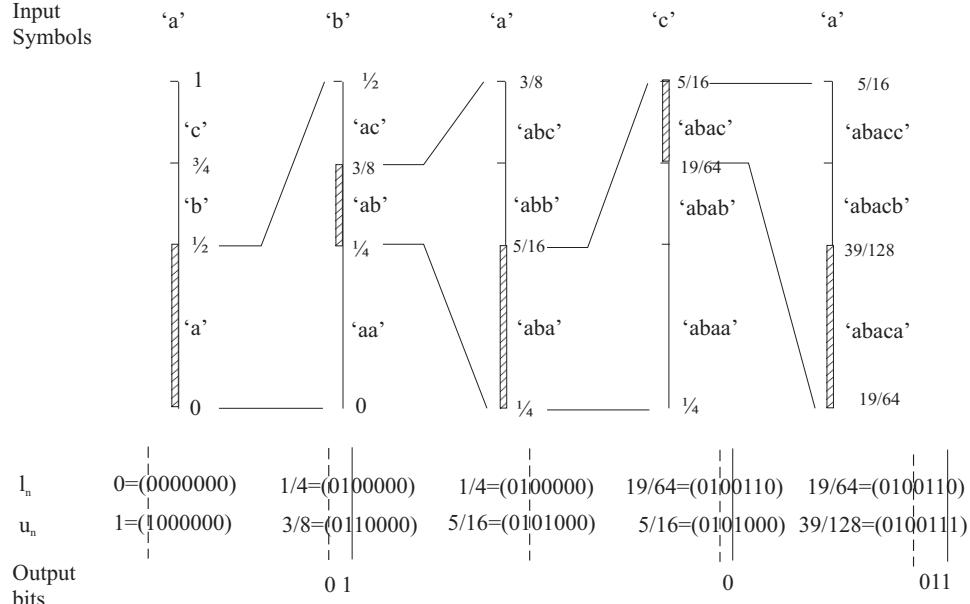
$$d_n = d_{n-1} * p_l; \quad l_n = l_{n-1} + d_{n-1} * q_{l-1}; \quad u_n = l_n + d_n. \quad (8.4.3)$$

Let the binary representation of  $l_n$  and  $u_n$  be  $(b_1^l, b_2^l, \dots, b_K^l)$  and  $(b_1^u, b_2^u, \dots, b_K^u)$ , respectively.<sup>8</sup> If the first few  $b_k^l$  and  $b_k^u$  are the same, then they are shifted out. Otherwise, all the bits are retained. The algorithm then proceeds by examining the next symbol.

**Example 8.4:** Figure 8.6(a) shows an example of arithmetic encoding. The source alphabet contains 3 symbols, ‘a’, ‘b’, and ‘c’, with probability distribution  $p(a) = 1/2, p(b) = 1/4, p(c) = 1/4$ . The intervals corresponding to these symbols are shown in the first column in Fig. 8.6(a). The first source symbol is ‘a’, which corresponds to the first interval in the first column. The lower and upper boundaries are  $l_1 = 0 = (0000\dots)$  and  $u_1 = 1/2 = (1000\dots)$ . Because the MSB’s of  $l_1$  and  $u_1$  are different, no bits are shifted out. The next source symbol is ‘b’, and therefore, we take the second interval in column 2. The boundaries of this interval are, using Eq. (8.4.3),  $l_2 = 1/4 = (01000\dots)$  and  $u_2 = 3/8 = (011000\dots)$ . The first two bits are common, and therefore, we can shift out ‘01’ in the coded bit stream. Following this procedure, we can determine the coded bits for all the following source symbols.

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<sup>8</sup>The binary representation of a number  $f$  between 0 and 1 is given by  $(b_1, b_2, \dots, b_K)$  if  $f = \sum_{k=1}^K b_k 2^{-k}$ .



	Received bits	Interval	Decoded symbol
0		$[0,1/2)$	'a'
01		$[1/4,1/2)$	--
010		$[1/4,3/8)$	'b'
0100		$[1/4,5/16)$	'a'
01001		$[9/32,5/16)$	--
010011		$[19/64,5/16)$	'c'
...	...	...	...

(b)

**Figure 8.6.** Example of arithmetic coding. (a) Encoding a source sequence 'abaca...'. The shaded interval in the  $n$ -th column indicates the interval corresponding to the sequence up to the  $n$ -th symbol. (b) Decoding a bit stream '010011...'.

To decode a received bit stream, one determines the lower and upper boundaries of an interval corresponding to the received bits. If the interval falls in a range that can only be associated with a particular source symbol, that symbol is decoded. If not, additional bits are examined.

**Example 8.5:** *The decoding process for the previous example is shown in Fig. 8.6(b). The first received bit is '0', and the possible lower and upper boundaries both with '0' as the MSB are  $l = (0000\dots) = 0$  and  $u = (0111\dots) = (1000\dots) = 1/2$ . From the first column in Fig. 8.6(a), this interval corresponds to 'a', therefore, one can output 'a' as a decoded symbol. With the next received bit '1', the possible lower and upper boundaries both with '01' as the first two bits are  $l = (0100\dots) = 1/4$  and  $u = (0111\dots) = (1000\dots) = 1/2$ . From the second column in Fig. 8.6(a), the range  $(1/4, 1/2)$  can come from either 'ab' or 'ac'. Therefore, one cannot decode a symbol at this step. With the third bit '0', the lower and upper boundaries are  $l = (010000\dots) = 1/4$  and  $u = (010111\dots) = (011000\dots) = 3/8$ . From the second column in Fig. 8.6(a), this interval corresponds to 'ab'. Therefore, one can output 'b' in this step. Continuing this process, one can decode all the symbols for the given bit stream.*

The above description of arithmetic coding is on a conceptual level only, which assumes infinite precision in calculation of the interval boundaries. As more source symbols are processed, the interval corresponding to the input sequence becomes increasingly small. To process a reasonably long sequence, one would need a computer with an extremely high precision to specify the boundary values. Fortunately, it is not necessary to record the absolute boundary values for the encoding and decoding process to work. A previously determined interval can be re-scaled to length 1 once a common bit of the lower and upper boundaries is shifted out. Such type of implementation is known as *finite precision arithmetic coding*. Instead of using fractional arithmetic for the lower and upper bound calculation, integer implementation, which is simpler, has also been developed. A full description of practical encoding and decoding algorithms is beyond the scope of this book. The reader is referred to the excellent tutorial by Witten et al. [11]. A good coverage of both Huffman and arithmetic coding can also be found in the book by Sayood [9].

The bit rate of arithmetic coding is bounded by [9]

$$H_N(\mathcal{F})/N \leq R \leq H_N(\mathcal{F})/N + 2/N, \quad (8.4.4)$$

where  $N$  is the number of symbols in the sequence being coded,  $H_N(\mathcal{F})$  is the  $N$ -th order entropy of the sequence. Recall that if we code the entire length  $N$  sequence using vector Huffman coding, the bit rate will be bounded as in Eq. (8.3.4). Therefore, both methods can approach the entropy bound closely when  $N$  is sufficiently large. However, with Huffman coding, one must design and store the codebook for all possible sequences of length  $N$ , which has complexity that grows exponentially with  $N$ . This limits the sequence length that can be used in practice. With arithmetic coding, no codebook needs to be designed in advance for each possible

source sequence. Rather, bits are sequentially obtained whenever the lower and upper boundaries of the identified interval have common MSB. The coded sequence length can be as long as the length of the source. Therefore, in practice, arithmetic coding can achieve the entropy bound closely.

Another advantage of arithmetic coding is that one can adapt to changes in the source statistics simply by updating the symbol probability table. One can also easily realize conditional coding, by using different probability tables for different conditioning states. With Huffman coding, one would have to redesign the codebook based on an updated probability table, or have multiple codebooks for different conditioning states.

Because of the higher coding efficiency and ease of adaptation, arithmetic coding is a better alternative than Huffman coding, as long as the computation involved is acceptable.

## 8.5 Scalar Quantization

### 8.5.1 Fundamentals

The most basic lossy coding method is scalar quantization (SQ), which quantizes each sample in a source signal to one of the reconstruction values in a pre-designed reconstruction codebook. In general, the original source can be either continuous or discrete. Here we only consider the first case, where the original sample can take on any value in a continuous support range  $\mathcal{B}$  over the real line. This support range is divided into a number of regions,  $\mathcal{B}_l, l = 1, 2, \dots, L$ , so that values in the same region  $\mathcal{B}_l$  are mapped into the same reconstruction value  $g_l$ . When the source is discrete, but all possible symbols can be ordered along the real line, we can treat each sample as the output from a fictitious continuous source, which only produces certain discrete values over a continuous range. With this approach, the theory and techniques discussed below for continuous sources can be applied to discrete sources as well.

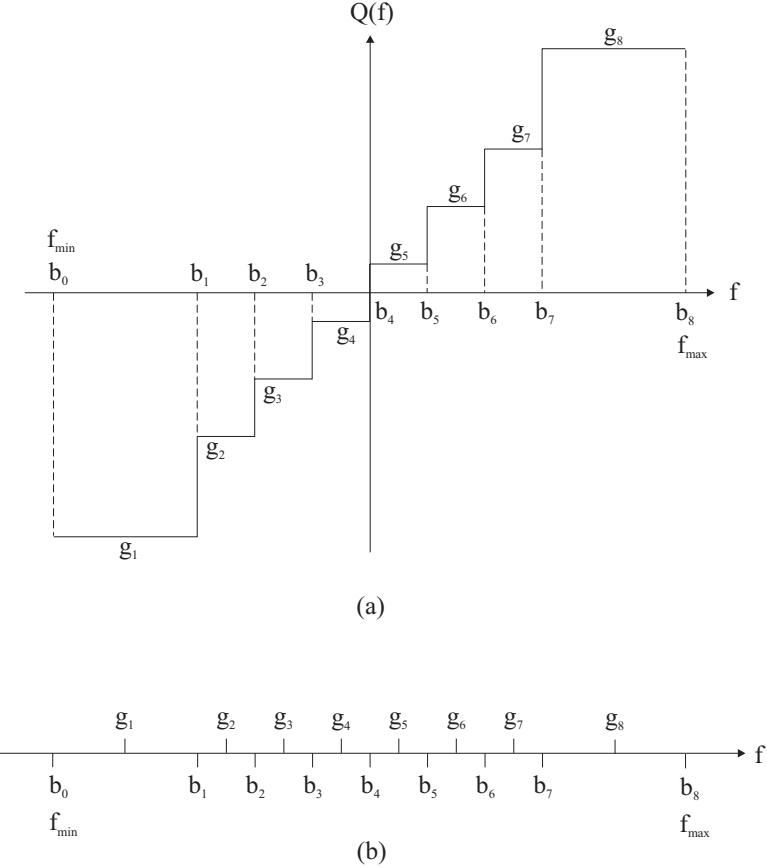
A quantizer is described by the *number of reconstruction values*,  $L$ , the *boundary values*,  $b_l, l = 0, 1, \dots, L$ , and *reconstruction values*,  $g_l, l = 1, 2, \dots, L$ . The boundary values can alternatively be described by the *partition regions*  $\mathcal{B}_l = [b_{l-1}, b_l)$ .<sup>9</sup> Letting  $\mathcal{L} = \{1, 2, \dots, L\}$ , the quantization function is described by:

$$Q(f) = g_l \quad \text{if } f \in \mathcal{B}_l, \quad l \in \mathcal{L}. \quad (8.5.1)$$

This function is illustrated in Fig. 8.7. In a digital computer, the reconstruction value  $g_l$  is simply specified by the integer index  $l$ , which, in a fixed length binary representation, requires  $R = \lceil \log_2 L \rceil$  bits. Here, the symbol  $\lceil x \rceil$  denotes the upper integer of  $x$ , i.e., the smallest integer that is equal to or greater than  $x$ .

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<sup>9</sup>The use of a closed left boundary is arbitrary. One can also use  $\mathcal{B}_l = (b_{l-1}, b_l]$ .



**Figure 8.7.** Illustration of a scalar quantizer: (a) representation as a function; (b) representation as line partition.

### Distortion of a Quantizer

Previously in Sec. 8.3.2, we have defined a distortion measure between the original and the quantized sources, when  $N$  samples are quantized together. In the case of a scalar quantizer,  $N = 1$ , the distortion measure given in Eq. (8.3.12) becomes

$$D_q = E\{d_1(\mathcal{F}, Q(\mathcal{F}))\} = \int_{f \in \mathcal{B}} d_1(f, Q(f))p(f) df \quad (8.5.2)$$

$$= \sum_{l \in \mathcal{L}} P(\mathcal{B}_l) D_{q,l} \quad (8.5.3)$$

with

$$D_{q,l} = \int_{f \in \mathcal{B}_l} d_1(f, g_l) p(f|f \in \mathcal{B}_l) df. \quad (8.5.4)$$

In the above equations,  $\mathcal{F}$  represents the RV corresponding to any sample in a source  $\mathcal{F}$ ,  $p(f), f \in \mathcal{B}$ , the pdf of  $\mathcal{F}$ ,  $P(\mathcal{B}_l) = \int_{\mathcal{B}_l} p(f) df$  the probability that  $f \in \mathcal{B}_l$ , and  $p(f|f \in \mathcal{B}_l) = p(f)/P(\mathcal{B}_l)$  the conditional pdf of  $\mathcal{F}$  given  $\mathcal{F} \in \mathcal{B}_l$ . Obviously,  $D_{q,l}$  denotes the mean distortion incurred in region  $\mathcal{B}_l$ .

When  $d_1(f, g) = (f - g)^2$ , the above distortion reduces to the MSE, which is equal to the variance of the quantization error  $\mathcal{F} - Q(\mathcal{F})$ , denoted by  $\sigma_q^2$ . Specifically,

$$\sigma_q^2 = E\{|\mathcal{F} - Q(\mathcal{F})|^2\} = \sum_{l \in \mathcal{L}} P(\mathcal{B}_l) \int_{b_{l-1}}^{b_l} (f - g_l)^2 p(f|\mathcal{B}_l) df. \quad (8.5.5)$$

### 8.5.2 Uniform Quantization

The simplest scalar quantizer is a uniform quantizer, which has equal distances between adjacent boundary values and between adjacent reconstruction values:

$$b_l - b_{l-1} = g_l - g_{l-1} = q \quad (8.5.6)$$

where  $q$  is called the *quantization stepsize*. Such a quantizer is only applicable to a signal with a finite dynamic range  $B$ . Let  $f_{\min}$  and  $f_{\max}$  represent the minimal and maximal values, then  $B = f_{\max} - f_{\min}$ . The quantizer is specified by the number of quantization levels  $L$ , or the bit rate  $R$ , or the quantization stepsize  $q$ . These parameters are related by  $q = B/L = B2^{-R}$ . The boundary and reconstruction values are given by

$$b_l = l * q + f_{\min}, \quad g_l = (l - 1) * q + q/2 + f_{\min}. \quad (8.5.7)$$

The quantization function can be described in a closed form:

$$Q(f) = \lfloor \frac{f - f_{\min}}{q} \rfloor * q + \frac{q}{2} + f_{\min}. \quad (8.5.8)$$

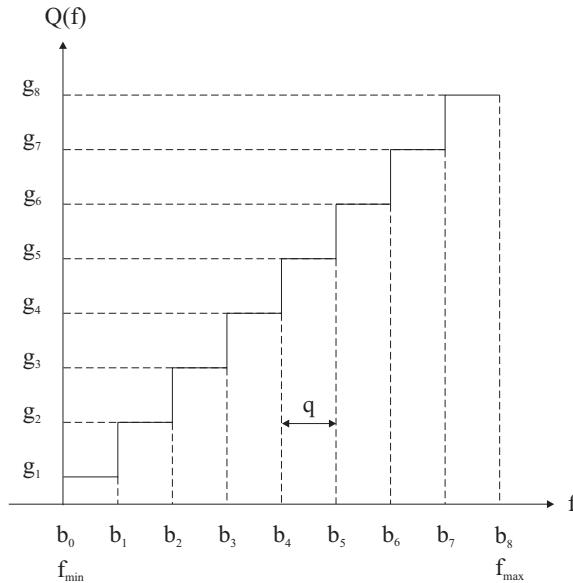
In the above,  $\lfloor x \rfloor$  represents the floor function of  $x$ , i.e., the largest integer that is equal or smaller than  $x$ . See Fig. 8.8.

If the source is uniformly distributed, i.e.,

$$p(f) = \begin{cases} 1/B & f \in (f_{\min}, f_{\max}) \\ 0 & \text{otherwise} \end{cases} \quad (8.5.9)$$

then substituting Eqs. (8.5.7) and (8.5.9) into Eq. (8.5.5) yields

$$\sigma_q^2 = \frac{q^2}{12} = \sigma_f^2 2^{-2R} \quad (8.5.10)$$



**Figure 8.8.** A uniform quantizer.

where  $\sigma_f^2 = B^2/12$  is the variance of the original signal  $\mathcal{F}$ . The signal to noise ratio (SNR) of the quantizer is

$$\text{SNR} = 10 \log_{10} \frac{\sigma_f^2}{\sigma_q^2} = (20 \log_{10} 2) R = 6.02R \text{ (dB)} \quad (8.5.11)$$

Therefore, *every additional bit in a uniform quantizer leads to a 6.02 dB gain in SNR for a uniform source.* This is a well known result in the quantization theory.

### 8.5.3 Optimal Scalar Quantizer

The uniform quantizer introduced thus far are simple to implement, but may not be most efficient in representing a source. In this section, we examine how to design an optimal scalar quantizer, so that the distortion is minimized for a fixed bit rate, given the source pdf  $p(f)$ .

#### Minimal Mean Square Error (MMSE) Quantizer

In such a quantizer,  $b_l, g_l$  are chosen such that the MSE  $\sigma_q^2$  is minimized for a given  $L$ . From the calculus of variations, the necessary conditions are  $\frac{\partial \sigma_q^2}{\partial b_l} = 0$  and

$$\frac{\partial \sigma_q^2}{\partial g_l} = 0, \forall l. \text{ From (8.5.5),}$$

$$\begin{aligned}\frac{\partial \sigma_q^2}{\partial b_l} &= \frac{\partial}{\partial b_l} \left( \int_{b_{l-1}}^{b_l} (f - g_l)^2 p(f) df + \int_{b_l}^{b_{l+1}} (f - g_{l+1})^2 p(f) df \right) \\ &= (b_l - g_l)^2 p(b_l) - (b_l - g_{l+1})^2 p(b_l) = 0, \\ \frac{\partial \sigma_q^2}{\partial g_l} &= \frac{\partial}{\partial g_l} \int_{b_{l-1}}^{b_l} (f - g_l)^2 p(f) df = - \int_{b_{l-1}}^{b_l} 2(f - g_l)p(f) df = 0.\end{aligned}$$

Simplification of the preceding equations yields:

$$b_l = \frac{g_l + g_{l+1}}{2}, \quad (8.5.12)$$

$$g_l = E\{\mathcal{F} | \mathcal{F} \in \mathcal{B}_l\} = \int_{\mathcal{B}_l} f p(f | f \in \mathcal{B}_l) df. \quad (8.5.13)$$

The conditional mean  $E\{\mathcal{F} | \mathcal{F} \in \mathcal{B}_l\}$  is referred to as the *centroid* of  $\mathcal{B}_l$ .

The results in Eqs. (8.5.12) and (8.5.13) state that *the optimum boundary values lie halfway between the optimal reconstruction values, which, in turn, lie at the centroids of the regions between the boundary values.* Note that requiring  $b_l = (g_l + g_{l+1})/2$  is equivalent to quantizing any  $f \in (g_l, g_{l+1})$  to  $g_l$  if  $f$  is closer to  $g_l$  than  $g_{l+1}$ . A more general statement is that any value  $f$  is quantized to the reconstruction value that is closest to  $f$ . Therefore, Eq. (8.5.12) can be equivalently written as

$$\mathcal{B}_l = \{f : d_1(f, g_l) \leq d_1(f, g'_l), \forall l' \neq l\}. \quad (8.5.14)$$

The condition given by Eq. (8.5.12) or Eq. (8.5.14) is called the *nearest neighbor condition*. On the other hand, equation (8.5.13) is known as the *centroid condition*.

It can be shown that the nearest neighbor condition given in Eq. (8.5.14) is still valid with a distortion measure rather than the MSE. The centroid condition in Eq. (8.5.13) however has to be changed to

$$g_l = \operatorname{argmin}_g \{E\{d_1(\mathcal{F}, g) | \mathcal{F} \in \mathcal{B}_l\}\}, \quad (8.5.15)$$

where  $\operatorname{argmin}_g \{E(g)\}$  represents the argument  $g$  that minimizes the function  $E(g)$ . The  $g_l$  defined in Eq. (8.5.15) is called the *generalized centroid* of  $\mathcal{B}_l$ , and its solution depends on the definition of  $d_1(f, g)$ . For example, if  $d_1(f, g) = |f - g|$ , then  $g_l$  is the median value in  $\mathcal{B}_l$ . See Prob. 8.12.

An important property of a MMSE quantizer is that it equalizes the quantization error in different partition regions, i.e.,

$$P(\mathcal{B}_l)D_{g,l} = D_q, \forall l \in \mathcal{L} \quad (8.5.16)$$

For other important statistical properties of an optimal quantizer, see [5, 4]. Also, see Prob. 8.8.

### MMSE Quantizer for a Uniform Source

Substituting the uniform distribution in Eq. (8.5.9) into Eqs. (8.5.12) and (8.5.13) will yield the solution given in Eq. (8.5.7). Thus, as expected, *the MMSE quantizer for a uniform source is uniform.*

### MMSE Quantizer for a Non-Uniform Source

For an arbitrary pdf  $p(f)$ , the conditions in Eqs. (8.5.12) and (8.5.13) do not always have a closed-form solution. Numerical procedures need to be used to determine the optimal set of  $b_l, g_l$ . Table 4.3 in [5] gives the optimal  $\tilde{b}_l$  and  $\tilde{g}_l$  for uniform, Gaussian, Laplacian, and Gamma sources with unit variance and zero mean. It can be shown (Prob. 8.10) that the optimal  $b_l$  and  $g_l$  for a source with arbitrary mean  $\mu_f$  and variance  $\sigma_f^2$  can be obtained by:

$$b_l = \sigma_f \tilde{b}_l + \mu_f, \quad g_l = \sigma_f \tilde{g}_l + \mu_f. \quad (8.5.17)$$

The MMSE quantizer for a non-uniform source is in general non-uniform, i.e., the partition regions are not equal in length. In this case, the quantized value cannot be determined by a closed-form formula as with a uniform quantizer. In general, one must compare the given value  $f$  with all boundary values, until the region for which  $b_{l-1} \leq f < b_l$  is found. Alternatively, one can find  $g_l$  that is closest to  $f$ . A sequential search would require up to  $L$  comparisons. Using a binary tree search procedure, the number of comparisons can be reduced to  $\lceil \log_2 L \rceil$ .

### Asymptotic Performance of MMSE Quantizer

For a source with an arbitrary pdf, it is difficult to derive a closed-form solution for the quantizer parameters  $b_l, g_l$  and consequently the quantization error  $\sigma_q^2$ , using the nearest neighbor condition and the centroid condition. However, when the bit rate  $R$  and consequently the quantization level  $L = 2^R$  is very high, a closed form solution for the MMSE quantizer in terms of the pdf can be derived [5, 4]. The associated quantization error can be expressed as

$$\sigma_q^2 = \epsilon^2 \sigma_f^2 2^{-2R} \quad (8.5.18)$$

with

$$\epsilon^2 = \frac{1}{12} \left( \int_{-\infty}^{\infty} \tilde{p}(f)^{1/3} df \right)^3, \quad (8.5.19)$$

where  $\tilde{p}(f) = \sigma_f p(\sigma_f f)$  denotes the pdf of a normalized source with unit variance. Comparing the above with Eq. (8.5.10), we see that  $\epsilon^2 = 1$  for a uniform source. For a non-uniform source,  $\epsilon^2 > 1$  in general. In particular, for a Gaussian source,  $\epsilon^2 = 2.71$ . Recall that the distortion-rate bound for an i.i.d Gaussian source is given in Eq. (8.3.20). Therefore, using the best scalar quantizer is still off from the bound by  $\frac{1}{2} \log_2 \epsilon^2 = 0.7191$  bit/sample. For the values of  $\epsilon^2$  for several other sources, see [5,

Table 4.8]. To be closer to the bound, multiple samples must be quantized together. The factor  $\epsilon^2$  can be reduced by applying variable length coding to the quantized indices. This is discussed further in Sec. 8.6.4.

### Lloyd Algorithm for MMSE Quantizer Design Based on Training Data

When the distribution of the signal to be quantized is unknown, the quantizer can be designed based on a training set containing representative samples to be quantized. A popular method for designing a quantizer based on the training data is the Lloyd algorithm [7, 4]. As shown in Fig. 8.9, the algorithm updates the reconstruction and boundary values iteratively. At each iteration, one first recalculates the reconstruction values based on the centroid condition, and then partition all the training samples based on the nearest neighbor condition. The statistical expectation required in the calculation of the generalized centroid and distortion are approximated by sample averages. Under the MSE criterion, the reconstruction value for a partition is simply the mean value of training samples belonging to that partition, whereas the boundary values are simply the average of two adjacent reconstruction values.

## 8.6 Vector Quantization

### 8.6.1 Fundamentals

Rather than quantizing one sample at a time, one can quantize a group of  $N$  samples together. Each sample group is known as a vector, and the process of quantizing a vector at a time is called vector quantization (VQ). In the image or video case, a vector usually corresponds to a square block of pixels. The motivation for using vector quantization (VQ) primarily stems from the observation that in a typical image or video frame, samples in a block are correlated so that there are certain block patterns that are more likely than others. The task of VQ is essentially to find the  $L$  most popular patterns, and replace any given block by one of the representative patterns that resembles the original pattern closest.

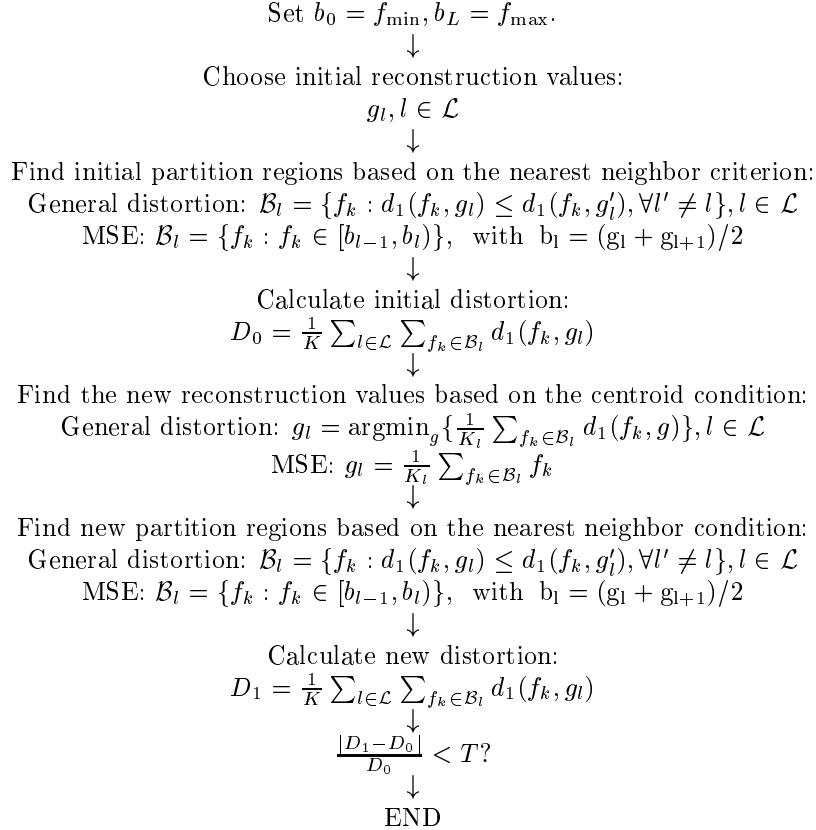
The above discussion assumes that a vector corresponds to a group of sample values. In a source in which each sample is described by multiple values, each sample can itself be quantized using a vector quantizer. For example, each sample in a color image contains three components. To display a 24 bit color image on a computer monitor using an 8 bit graphics card, one has to find 256 colors that best represent all possible  $2^{24} = 16$  million colors.

Consider each possible  $N$ -D vector  $\mathbf{f}$  as a point in an  $N$ -D space, the VQ problem is to partition the  $N$ -D space into  $L$  regions  $\mathcal{B}_l$ , and representing all the points in region  $\mathcal{B}_l$  by a representative point  $\mathbf{g}_l$ . This process is illustrated in Fig. 8.10 for the case of  $N = 2$ . We call  $\mathcal{B}_l$  the partition regions and  $\mathbf{g}_l$  the reconstruction vectors or *codewords*. The set containing all codewords  $\{\mathbf{g}_l, l \in \mathcal{L}\}$  is called the *codebook*.<sup>10</sup>

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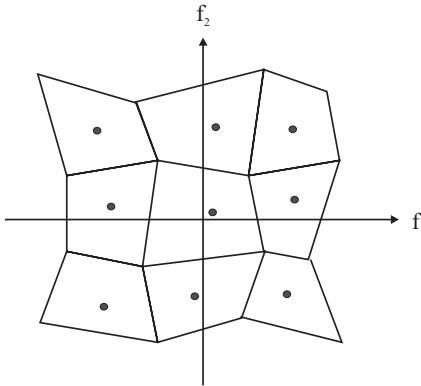
<sup>10</sup>Note that each reconstruction codeword needs to be mapped into a binary codeword, to

Given samples  $f_k, k = 1, 2, \dots, K$ , in a training set and the quantization level  $L$ .



note to graphics illustrator: 1) put a box around the text between each pair of arrows. 2) add a loop back arrowed line from  $\frac{|D_1 - D_0|}{D_0} < T?$  to "find the new reconstruction values ...".

**Figure 8.9.** Lloyd algorithm for designing an optimal scalar quantizer from training data.



Note to graphical illustrator: change solid circle to open circle. Be consistent with Figure 8.12.

**Figure 8.10.** Illustration of vector quantization with vector dimension  $N = 2$ . The solid circle inside each partition region represents the reconstruction codeword of that region.

The quantization function can be described as

$$Q(\mathbf{f}) = \mathbf{g}_l, \text{ if } \mathbf{f} \in \mathcal{B}_l. \quad (8.6.1)$$

If the reconstruction codewords are converted into binary bits using fixed length binary coding, then each group of  $N$  samples requires  $\lceil \log_2 L \rceil$  bits, and the bit rate (bit/sample) is

$$R = \frac{1}{N} \lceil \log_2 L \rceil. \quad (8.6.2)$$

Comparing Eq. (8.6.1) with Eq. (8.5.1) and Fig. 8.10 with Fig. 8.7(b), it can be seen that the scalar quantizer is just a special case of the vector quantizer, where  $\mathbf{f}$  is a scalar and  $\mathcal{B}_l$  is a line segment. In general, the partition region  $\mathcal{B}_l$  cannot be described simply by a few decision values, as in the scalar case.

To evaluate the quantization error introduced by a vector quantizer, let  $\mathcal{F}$  and  $Q(\mathcal{F})$  represent the original and the quantized vector, and let  $p_N(\mathbf{f})$  represent the joint pdf of the components in  $\mathcal{F}$ . Using the notation in Sec. 8.3.2, the conditional probability  $q_N(\mathbf{g}|\mathbf{f})$  for the vector quantizer described by Eq. (8.6.1) is

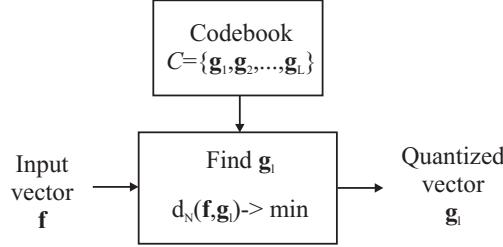
$$q_N(\mathbf{g}|\mathbf{f}) = \begin{cases} 1, & \text{if } \mathbf{f} \in \mathcal{B}_l, \mathbf{g} = \mathbf{g}_l, \\ 0, & \text{otherwise.} \end{cases} \quad (8.6.3)$$

The distortion criterion in Eq. (8.3.12) reduces to, in this case,

$$D_q = E\{d_N(\mathcal{F}, Q(\mathcal{F}))\} = \int_{\mathcal{B}} p_N(\mathbf{f}) d_N(\mathbf{f}, Q(\mathbf{f})) d\mathbf{f} \quad (8.6.4)$$

---

represent the quantized signal using binary bits. The words “codeword” and “codebook” in this section refers to the reconstruction codeword and codebook, whereas those in Secs. 8.3.1 and 8.4 refer to binary codeword and codebook.



**Figure 8.11.** Operation in a nearest neighbor quantizer.

$$= \sum_{l=1}^L P(\mathcal{B}_l) D_{q,l} \quad (8.6.5)$$

with

$$D_{q,l} = E\{d_N(\mathcal{F}, Q(\mathcal{F})) | \mathcal{F} \in \mathcal{B}_l\} = \int_{\mathbf{f} \in \mathcal{B}_l} p_N(\mathbf{f} | \mathbf{f} \in \mathcal{B}_l) d_N(\mathbf{f}, \mathbf{g}_l) d\mathbf{f}. \quad (8.6.6)$$

### Nearest Neighbor Quantizer and Complexity of VQ

In general, a vector quantizer is specified by both the codewords  $\mathbf{g}_l$  and the partition regions  $\mathcal{B}_l$ . With a *nearest neighbor quantizer*, for any input vector  $\mathbf{f}$ , the quantized vector is determined by comparing  $\mathbf{f}$  with all codewords and finding the one that is closest to it under the distance measure  $d_N(\mathbf{f}, \mathbf{g}_l)$ . In other words, the partition regions are implicitly determined by the codewords through

$$\mathcal{B}_l = \{\mathbf{f} \in \mathcal{R}^N : d_N(\mathbf{f}, \mathbf{g}_l) \leq d_N(\mathbf{f}, \mathbf{g}'_l), \forall l' \neq l\}. \quad (8.6.7)$$

As will be shown in Sec. 8.6.3, this is a necessary condition for minimizing the mean quantization error. Essentially, all quantizers used in practice belong to this category. Figure 8.11 illustrates the operation of a nearest neighbor quantizer.

With a nearest neighbor quantizer, for each input vector, one needs to evaluate the distortion between this vector and all  $L$  reconstruction codewords. With the squared error measure,

$$d_N(\mathbf{f}, \mathbf{g}) = \frac{1}{N} \sum_{n=1}^N (f_n - g_n)^2, \quad (8.6.8)$$

each evaluation requires  $N$  operations (an operation consists of a subtraction, a multiplication and an addition). Hence, the total number of operations required to quantize one vector is  $NL$ . From Eq. (8.6.2), we have  $L = 2^{NR}$  and hence the number of operations is  $N2^{NR}$ . In terms of the storage requirement, each codeword requires  $N$  basic memory unit and all  $L$  codewords require  $NL = N2^{NR}$  memory units. *Thus, both the complexity and storage requirements of VQ increase*

*exponentially with the vector dimension N.* Recall that Theorem 8.4 tells us that the RD bound for a source is usually achieved only when  $N$  reaches infinity. In general, the larger is  $N$ , the more efficient is the coder. However, the exponential growth of the complexity with  $N$  limits the vector dimension that can be used in practice. Consider the quantization of video frames, by which each block of  $N$  pixels are quantized using a size  $L$  codebook. Assuming the frame rate is  $f_t$  and the frame size is  $N_x \times N_y$ , then the number of operations per second will be  $(f_t N_x N_y / N) \cdot N 2^{NR} = f_t N_x N_y 2^{NR}$ . For a CCIR 601 video Y-component,  $f_t = 30, N_x = 720, N_y = 480$ , a vector size of  $4 \times 4$  and bit rate of 1 bit/sample lead to an operation count of  $6.8E + 11$  per second! For image and video coding,  $N$  is usually limited to  $4 \times 4$  or below.

To reduce the complexity, similar to binary search in the SQ case, one does not have to do exhaustive search if each partition region is a polytope. In this case, different partition regions are defined by intersecting hyperplanes. One can compare a given vector with one hyperplane at a time, each time eliminating codewords on one side of the plane. This way, the number of comparisons can be reduced to a number significantly lower than  $L$ , but may not be as low as  $\log_2 L$ , unless the codebook has a special tree-structure. To further reduce the complexity, various fast VQ schemes have been developed, which impose certain constraints on the codebook structure, so that the number of comparisons can be reduced. The book by Gersho and Gray [4] provides an extensive coverage of various VQ techniques.

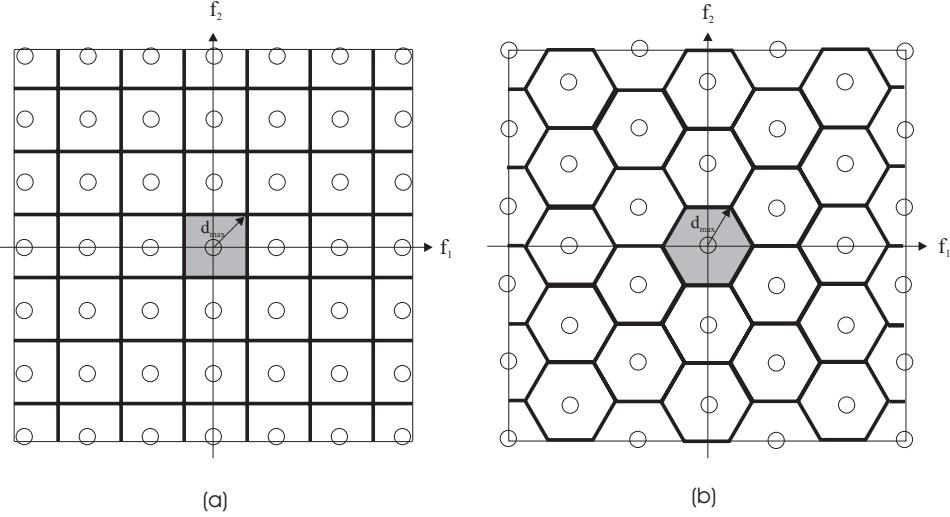
### 8.6.2 Lattice Vector Quantizer

As we have learned in Sec. 8.5.2, the simplest scalar quantizer is a uniform quantizer, in which a certain range of the real line is partitioned into equal-length quantization bins. An analog of the uniform quantizer in the VQ case is a lattice quantizer, in which all the partition regions have the same shape and size, and in fact are all shifted versions of a basic region, which is the Voronoi region of the lattice. Figure 8.12 shows two lattice quantizers, using rectangular and hexagonal lattices, respectively. Previously in Chap. 3, we have introduced the concept of lattice for uniform sampling over a multi-dimensional space. Here we see that lattices can also be used to define uniform quantizers over a multi-dimensional space.

Recall that a lattice  $\Lambda$  in an  $N$ -D real space  $\mathcal{R}^N$  is defined by a set of basis vectors  $\mathbf{v}_n, n = 1, 2, \dots, N$  or equivalently a generating matrix  $[\mathbf{V}] = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_N]$ . The points in a lattice are those vectors that can be represented as

$$\mathbf{g}_l = \sum_{n=1}^N m_{l,n} \mathbf{v}_n = [\mathbf{V}] \mathbf{m}_l, \quad (8.6.9)$$

where  $m_{l,n} \in \mathbb{Z}$  are integers or  $\mathbf{m}_l \in \mathbb{Z}^N$  is an  $N$ -D integer vector. Usually a codeword  $\mathbf{g}_l$  is indexed by its corresponding  $\mathbf{m}_l$ . The Voronoi region  $\mathcal{V}$  of  $\Lambda$  is the region in which all points are closer to the origin than any other non-zero lattice points. With a lattice quantizer, the codebook consists of all points in a lattice,



**Figure 8.12.** Lattice quantizers using (a) rectangular and (b) hexagonal lattices. The shaded regions indicate the Voronoi regions.  $d_{\max}$  is the maximal distance between an input vector and its quantized version.

or a coset of a lattice, which is a shifted version of a lattice. Such quantizers have infinite number of codewords. In reality, a truncated version of a lattice or its coset is more often used, which only take sample points that are within a certain support range in  $\mathcal{R}^N$ .

For a uniformly distributed source over a certain finite region in  $\mathcal{R}^N$ , if we assume all the partition regions fit in the support region perfectly,<sup>11</sup> then all the partition regions are equally likely, and the distortion in each region is the same as that over the Voronoi region. Recall that the volume of the Voronoi region  $\mathcal{V}$  is equal to  $|\det[\mathbf{V}]|$ . Thus, the conditional pdf  $p(\mathbf{f}|\mathbf{f} \in \mathcal{V}) = 1/|\det[\mathbf{V}]|$ . The mean quantization error is therefore

$$D_q = D_{q,l} = \frac{1}{|\det[\mathbf{V}]|} \int_{\mathcal{V}} d_N(\mathbf{f}, \mathbf{0}) d\mathbf{f}. \quad (8.6.10)$$

With the MSE criterion, we have

$$D_q = \frac{1}{|\det[\mathbf{V}]|} \int_{\mathcal{V}} \|\mathbf{f}\|^2 d\mathbf{f}. \quad (8.6.11)$$

The values of  $D_q$  for several high dimensional lattices have been tabulated by Conway and Sloane [2].

<sup>11</sup>This is in general not true unless a hypercubic lattice is used. But when  $L$  is large, the number of partition regions on the boundary of the support region is small and the impact due to those regions can be ignored.

It is worth noting that even for an i.i.d. source, VQ can improve the coding efficiency over SQ. This is because VQ allows more flexibility in packing a space using partition regions. Consider the two examples given in Fig. 8.12. In either case, the maximum distortion in representing a point in any region by its center (the codeword) is  $d_{\max} = 1$ , so that the two quantizers will yield the same maximum distortion. But to fill the same 2-D space, fewer number of hexagons are needed, because the Voronoi region for the hexagonal lattice is larger than that for the rectangular lattice. Specifically, the areas of the Voronoi regions for the rectangular and hexagonal lattices in this example are  $2d_{\max}^2$  and  $2.59d_{\max}^2$ , respectively. In the entire support region considered in the figure, the rectangular lattice has more than 36 partition regions, whereas the hexagonal lattice has 30 regions.<sup>12</sup> Now, the rectangular lattice quantizer can also be realized by applying a uniform scalar quantizer in each dimension. This means that even when two samples are independent and both are uniformly distributed, quantizing them together as a vector can reduce the bit rate, with a well-designed vector quantizer.

**Implementation of a lattice quantizer** As described before, for an arbitrary nearest neighbor quantizer, the quantization of a given vector involves an exhaustive search. This is not needed with a lattice quantizer. In this case, one can first determine a real index vector of the quantized vector by

$$\mathbf{m} = [\mathbf{V}]^{-1}\mathbf{f}. \quad (8.6.12)$$

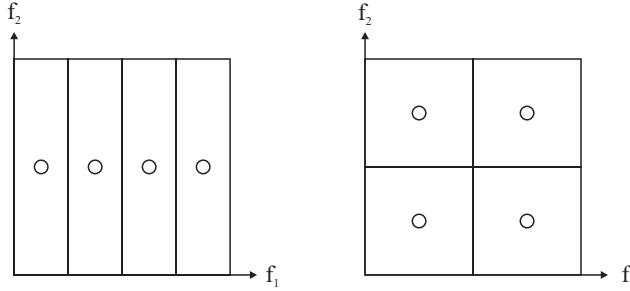
Then one can evaluate the distortions associated with all the integer index vectors neighboring to  $\mathbf{m}$ , by taking the lower and upper integer of each component of  $\mathbf{m}$ , and see which one gives the minimal distortion. For a vector dimension  $N$ , at most  $2N$  candidate vectors need to be evaluated. To further reduce the complexity, one can also simply round  $\mathbf{m}$  to the nearest integer vector. The reader is referred to [2] for a more detailed discussion on lattice encoding algorithms and performances.

### 8.6.3 Optimal Vector Quantizer

Although lattice quantizers are simple to implement, they may not be the most efficient when the source is not uniformly distributed. For a given vector dimension  $N$  and the source pdf  $p_N(\mathbf{f})$ , it is desirable to design an *optimal* quantizer that minimizes a given distortion criterion for a fixed bit rate  $R$  or equivalently the codebook size  $L$ . Ideally, one needs to determine the codewords  $\mathbf{g}_l$  and the partition regions  $\mathcal{B}_l$  simultaneously to minimize the average distortion  $D_q$ . The optimal solution is however difficult to find. Instead, one can proceed in two stages. First, for given codewords  $\mathbf{g}_l, l \in \mathcal{L}$ , the partition regions  $\mathcal{B}_l, l \in \mathcal{L}$ , should be such that the average distortion over all regions,  $D_q$ , is minimized. Second, for a given partition region  $\mathcal{B}_l$ , the codeword  $\mathbf{g}_l$  should be chosen to minimize the distortion within  $\mathcal{B}_l$ ,  $D_{q,l}$ .

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<sup>12</sup>Recall that from Chap. 3, we know that the hexagonal lattice is more efficient in covering a space.



**Figure 8.13.** The two vector quantizers both satisfy the nearest neighbor condition and the centroid condition. But the quantizer on the right has a lower distortion.

For the first problem, from Eq. (8.6.4), it is easy to see that  $D_q$  will be minimized if  $Q(\mathbf{f})$  is chosen so that  $d_N(\mathbf{f}, Q(\mathbf{f}))$  is minimized for any  $\mathbf{f}$ . That is,

$$Q(\mathbf{f}) = \mathbf{g}_l = \operatorname{argmin}_{\mathbf{g}'_l, l' \in \mathcal{L}} \{d_N(\mathbf{f}, \mathbf{g}'_l)\}. \quad (8.6.13)$$

In other words,  $\mathbf{f}$  should be quantized to the codeword nearest to it. The above assignment is equivalent to set the partition region  $\mathcal{B}_l$  according to

$$\mathcal{B}_l = \{\mathbf{f} : d_N(\mathbf{f}, \mathbf{g}_l) \leq d_N(\mathbf{f}, \mathbf{g}'_l), \forall l' \neq l\}. \quad (8.6.14)$$

We call either Eq. (8.6.13) or Eq. (8.6.14) the *nearest neighbor condition*.

Now, consider the second problem. Obviously, we should choose  $\mathbf{g}_l$  such that  $D_{q,l}$  in Eq. (8.6.6) is minimized, i.e.,

$$\mathbf{g}_l = \operatorname{argmin}_{\mathbf{g}} E\{d_N(\mathcal{F}, \mathbf{g}) | \mathcal{F} \in \mathcal{B}_l\}. \quad (8.6.15)$$

We call the above  $\mathbf{g}_l$  the *generalized centroid* of  $\mathcal{B}_l$ , and the above equation the *centroid condition*.

When the distortion measure is MSE,

$$D_{q,l}(\mathbf{g}) = \frac{1}{N} E\{\|\mathcal{F} - \mathbf{g}\|^2 | \mathcal{F} \in \mathcal{B}_l\} = \frac{1}{N} \int_{\mathcal{B}_l} \|\mathbf{f} - \mathbf{g}\|^2 p(\mathbf{f} | \mathbf{f} \in \mathcal{B}_l) d\mathbf{f} \quad (8.6.16)$$

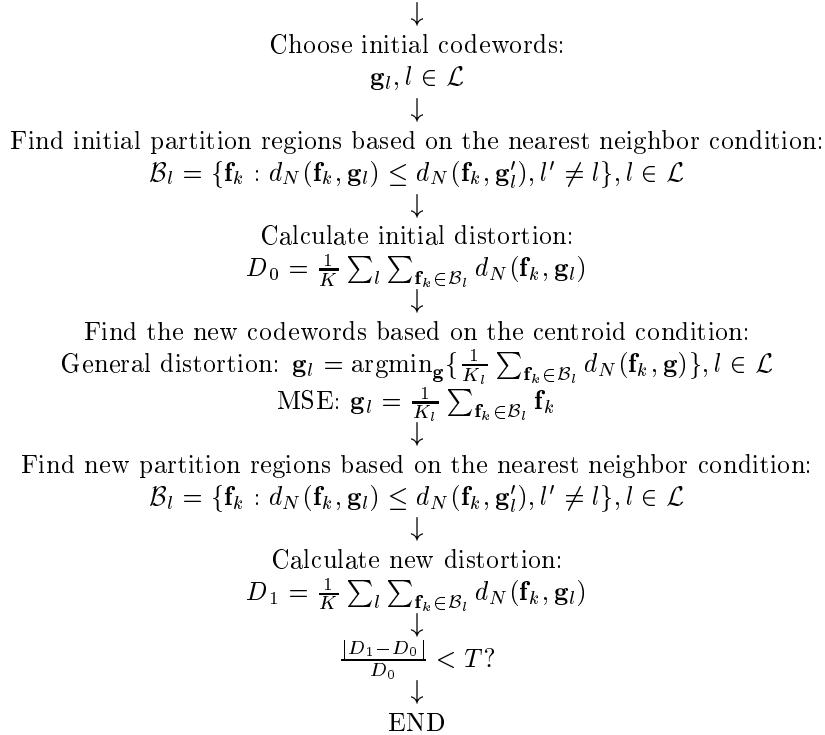
The  $\mathbf{g}_l$  that minimizes  $D_{q,l}$  must satisfy  $\frac{\partial D_{q,l}}{\partial \mathbf{g}} = 0$ . This yields,

$$\mathbf{g}_l = \int_{\mathcal{B}_l} p(\mathbf{f} | \mathbf{f} \in \mathcal{B}_l) \mathbf{f} d\mathbf{f} = E\{\mathcal{F} | \mathcal{F} \in \mathcal{B}_l\} \quad (8.6.17)$$

That is,  $\mathbf{g}_l$  is the conditional mean or centroid of  $\mathcal{B}_l$ .

Note that the nearest neighbor condition and the centroid condition are necessary but not sufficient conditions for minimizing  $D_q$ . A quantizer satisfying both conditions may not achieve the global minimum of  $D_q$ . Figure 8.13 shows two 2-D vector quantizers for a uniformly distributed signal. It is easy to verify that both quantizers satisfy above conditions. But clearly, the quantizer on the right will have a lower MSE. Therefore, the quantizer on the left is only locally optimal.

Given sample vectors  $\mathbf{f}_k, k = 1, 2, \dots, K$ , in a training set and quantization level  $L$ .



note to graphics illustrator: add a loop back arrowed line from  $\frac{|D_1 - D_0|}{D_0} < T$ ? to "find the new codewords ...".

**Figure 8.14.** Generalized Lloyd algorithm for designing an optimal vector quantizer from training data.

### Generalized Lloyd Algorithm for Designing Optimal Vector Quantizer

For an arbitrary source, it is difficult to derive closed-form solutions for  $\mathbf{g}_l, \mathcal{B}_l, l \in \mathcal{L}$ . A locally optimal solution can be obtained by the *generalized Lloyd algorithm*.<sup>13</sup> Like the Lloyd algorithm for SQ, the Generalized Lloyd algorithm for VQ iteratively determines the optimal  $\mathbf{g}_l$  and  $\mathcal{B}_l$  based on some training data. Figure 8.14 shows the flow-chart of this algorithm. Comparing this figure to Fig. 8.9, one can easily see the analogy between these two algorithms.

**Example 8.6 (Vector Quantizer Design)** *Figure 8.15 shows an example of quantizer design using the Generalized Lloyd algorithm under the MSE criterion. Within each iteration, we first recalculate the centroid vector (indicated by a circle) for each previously determined partition region, by averaging all the samples in that region, and then we repartition (indicated by ovals) all the samples based on their distances to the newly updated centroid vectors. Results for the first 3 iterations are shown. Because the partition after the 3rd iteration remains the same as that after the 2nd iteration, the 4th iteration will not change the codewords and consequently the partition regions. Therefore, the result given after the 3rd iteration is the final solution.*

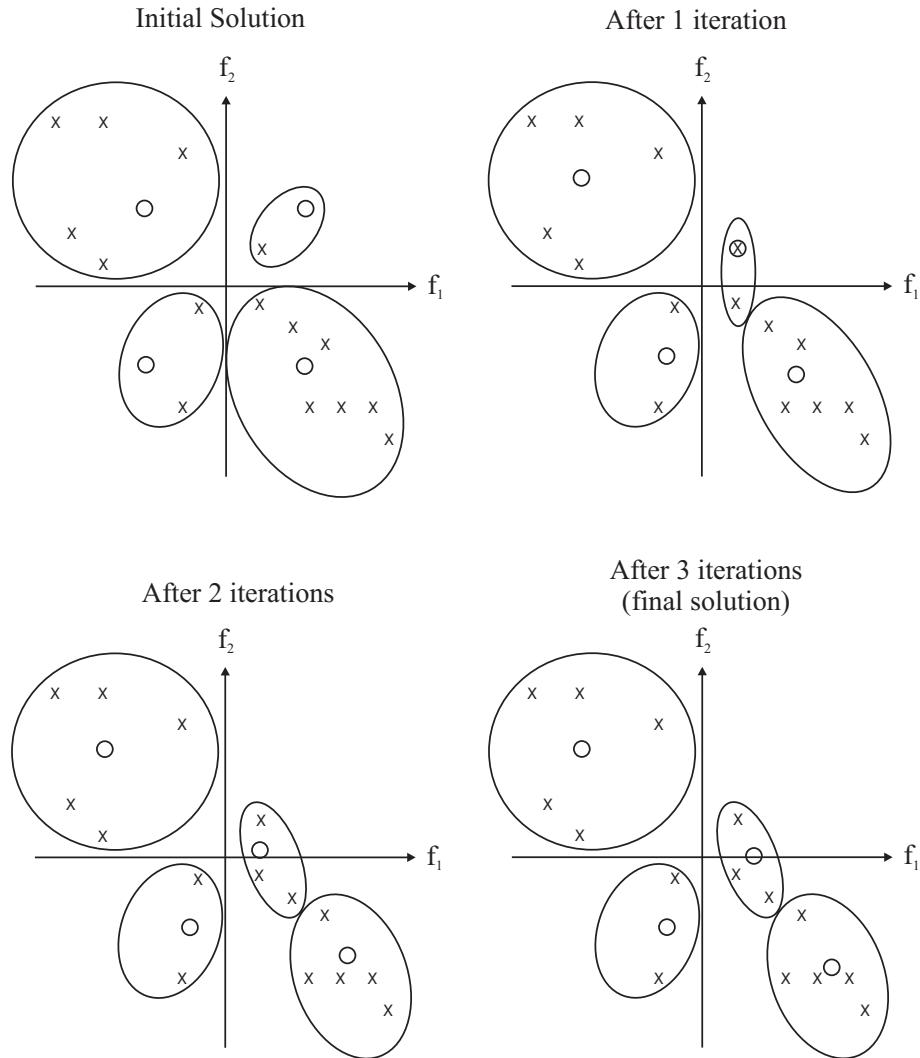
The resulting quantizer by the generalized Lloyd algorithm depends on the initial codewords. When they are not chosen properly, the algorithm may converge to a bad local minimum that is far from the global one. One simple and yet quite effective way to chose the initial codewords is by using the codewords that would have resulted from applying uniform scalar quantization in each dimension. For an extensive coverage of VQ design algorithms, including the selection of initial codewords, see [4].

#### 8.6.4 Entropy-Constrained Optimal Quantizer Design

Until now, we have assumed that each codeword in a size  $L$  codebook takes  $\lceil \log_2 L \rceil$  bits to represent. Because the reconstruction codewords usually appear with different probabilities, we can further reduce the bit rate by using Huffman coding or other variable length lossless coding methods. In the methods described for optimal quantizer design in Secs. 8.5.3 and 8.6.3, we have tried to minimize the distortion for a given bit rate  $R$  or codebook size  $L$ , by assuming  $R$  is related to  $L$  by Eq. (8.6.2). The resulting solutions in terms of  $\mathbf{g}_l, \mathcal{B}_l$  are not optimal any more, if the quantized vectors are coded using variable length coding. With entropy-constrained optimal quantizer design, we minimize the distortion subject to a rate constraint where the rate is assumed to be equal to the entropy of the quantized source. If we code one codeword at a time, then the bit rate is bounded by

$$\bar{R}_N = -\frac{1}{N} \sum_{l \in \mathcal{L}} P(\mathcal{B}_l) \log_2 P(\mathcal{B}_l), \quad (8.6.18)$$

<sup>13</sup>This algorithm is also known as the *LBG algorithm* following the authors who first generalized the Lloyd algorithm for vector quantization [6]. The same algorithm has also been used in pattern recognition for automatic clustering of data points into a few classes. There, it is known as the K-means algorithm [8].



**Figure 8.15.** An example of vector quantizer design using the generalized Lloyd algorithm. See Example 8.6.

because the probability for codeword  $\mathbf{g}_l$  is  $P(\mathcal{B}_l)$ . Given the desired bit rate  $R$  and vector dimension  $N$ , the optimization problem is to minimize  $D_q$  subject to

$$-\sum_{l \in \mathcal{L}} P(\mathcal{B}_l) \log_2 P(\mathcal{B}_l) \leq RN. \quad (8.6.19)$$

Using the Lagrange multiplier method, the above constrained optimization problem can be converted to an unconstrained one:

$$\text{minimize } J(\lambda) = D_q + \lambda \left( -\sum_{l \in \mathcal{L}} P(\mathcal{B}_l) \log_2 P(\mathcal{B}_l) - NR \right) \quad (8.6.20)$$

where  $\lambda$  has to be chosen such that the constraint in Eq. (8.6.19) is satisfied. For each fixed  $\lambda$ , the solution for  $\mathbf{g}_l$  and  $\mathcal{B}_l$  can be obtained by using the generalized Lloyd algorithm, with the distortion criterion changed from  $D_q$  to Eq. (8.6.20). Then one can calculate the bit rate required by this solution using Eq. (8.6.18). Essentially, different  $\lambda$  values yield solutions that are optimal for different bit rates.

Under the entropy constraint, the optimal quantizers for a non-uniform source tend to approach uniform when  $L$  is large. This way, the maximum distortion in each partition region is similar. However, because the codewords that correspond to regions with higher pdf will be more likely than others, they will be coded with shorter codewords.

Note that the above discussion is equally applicable to scalar quantization. Recall that the distortion-rate function of any non-entropy-constrained MMSE quantizer can be written in the form of Eq. (8.5.18) in the high rate case. The net effect of applying entropy coding is to reduce the factor  $\epsilon^2$ . For the Gaussian source, using an entropy-constrained MMSE quantizer and scalar lossless coding can reduce  $\epsilon^2$  from 2.71 to 1.42, which corresponds to a saving of 0.467 bit/sample for the same distortion. Compared to the RD bound for an i.i.d. Gaussian source, this represents a gap of only 0.255 bit/sample from the bound [5, Sec. 4.6.2].

## 8.7 Summary

### General Framework of Coding System (Sec. 8.1)

- A coding system usually consists of three components (Fig. 8.1): analysis based on the source model, which converts the signal samples into model parameters, quantization of the parameters (for lossy coding systems), and binary encoding of quantized parameters.
- The main difference between different coding systems lies in the source model used. Video coding methods can be classified into two groups, those using source models that directly characterize the pixel values (waveform-based), and those using models that describe the underlying object structures (content-dependent).

### Bounds for Lossless and Lossy Coding (Sec. 8.3)

- The minimal rate required for lossless coding for a discrete source is determined by the source entropy rate (Th. 8.2).
- The minimal rate required for coding at a given distortion (the RD bound) is determined by the minimal mutual information between the original source and the quantized one (Th. 8.4).
- The above bounds are achievable only if infinite number of samples are coded together.
- Among all the sources with the same variance, the Gaussian source requires the highest bit rate.

### Binary Encoding (Sec. 8.4)

- Binary coding can be applied to a single sample at a time, a group of samples, with or without conditioning based on previous samples.
- Huffman and arithmetic coding are the two most popular methods for binary encoding. Huffman coders are simpler to design and operate, but cannot reach the lossless coding bound with reasonable complexity. It is also hard to adapt a Huffman coder to variations in signal statistics. Arithmetic coders can more easily approach the entropy bound and are more effective for non-stationary signals, but they are also more complex to implement.

### Scalar Quantization (Sec. 8.5)

- The necessary conditions for MSE optimality are the centroid condition, Eq. (8.5.13) or Eq. (8.5.15), and the nearest neighbor condition, Eq. (8.5.12) or (8.5.14). A locally optimal quantizer can be designed using the Lloyd algorithm (Fig. 8.9).
- With an MSE optimized quantizer, the operational RD function of a source has the general form of Eq. (8.5.18) when the rate is sufficiently high. The factor  $\epsilon^2$  depends on the pdf of the source.
- The MSE optimal quantizer above assume the quantized indices are coded using fixed length coding. With variable length coding and when the rate is sufficiently high, uniform quantization is nearly optimal. For image and video coding, uniform quantization followed by entropy coding is preferred over non-uniform quantization.

### Vector Quantization (Sec. 8.6)

- VQ can exploit the correlation between samples and its efficiency increases as the vector dimension increases. It has the potential of reaching the RD bound when the vector dimension reaches infinity.

- The computational complexity and storage requirement of a general unconstrained VQ encoder grows exponentially with the vector dimension. Practical image and video coders use vector dimensions of  $4 \times 4$  or lower.
- The necessary conditions for MSE optimality are generalized version of the centroid condition, Eq. (8.6.17) or Eq. (8.6.15), and the nearest neighbor condition, Eq. (8.6.14). A locally optimal vector quantizer can be designed using the generalized Lloyd algorithm (Fig. 8.14).

## 8.8 Problems

**8.1** Consider a discrete source with an alphabet  $\mathcal{A} = \{a_1, a_2, \dots, a_L\}$ . Compute the entropy of the source for the following two cases:

- the source is uniformly distributed, with  $p(a_l) = 1/L, \forall l \in \mathcal{L}$ .
- For a particular  $k \in \mathcal{L}$ ,  $p(a_k) = 1$  and  $p(a_l) = 0, l \neq k$ .

**8.2** Prove the relations given in Eqs. (8.2.6) to (8.2.12).

**8.3** Prove the relations given in Eqs. (8.2.17) to (8.2.19).

**8.4** Prove that the differential entropy of a Gaussian RV with mean  $\eta$  and variance  $\sigma^2$  is as given in Eq. (8.2.23).

**8.5** Show that  $N$ -th order vector coding is worth than  $(N - 1)$ -th order conditional coding, if

$$\frac{1}{N}H_N(\mathcal{F}) - H_{c,N01}(\mathcal{F}) > \frac{N-1}{N}.$$

**8.6** A Markov source with three symbols  $\mathcal{A} = \{a_1, a_2, a_3\}$  has the following probability distributions:

$$p(a_1) = 1/2, p(a_2) = 1/3, p(a_3) = 1/6,$$

$$P\{a_i/a_j\} = \begin{cases} \frac{1}{2} & \text{if } i = j; \\ \frac{1}{4} & \text{otherwise.} \end{cases}$$

- Calculate the 1st order entropy, 2nd order entropy, 1st order conditional entropy, and the entropy rate of this source;
- Design the 1st order, 2nd order, and 1st order conditional Huffman codes for this source. Calculate the resulting bit rate for each case. Compare it to the corresponding lower and upper bounds defined by the entropy.
- What is the minimal bit rate achievable with this source? How could you achieve the minimal rate?

- 8.7** Encode and decode the following sequence using arithmetic coding. Using the occurrence frequency of each symbol in the sequence as the estimate of the probability of the symbol.

Source sequence:  $a\ c\ b\ a\ a\ b\ a\ c\ a\ c\ b\ a$

What is bit rate of the coded sequence? Compare the result to scalar Huffman coding.

- 8.8** Show that for a MMSE quantizer, the original RV  $\mathcal{F}$ , the quantized RV  $\mathcal{G}$  and the quantization error  $\mathcal{Q} = \mathcal{F} - \mathcal{G}$  satisfy the following statistical relations:

- (a) The quantized value is an unbiased estimate of the original value:  $E\{\mathcal{G}\} = E\{\mathcal{F}\}$
- (b) The quantized value is orthogonal to the quantization error:  $E\{\mathcal{G}\mathcal{Q}\} = 0$
- (c) The quantization process reduces the signal variance:  $\sigma_{\mathcal{G}}^2 = \sigma_{\mathcal{F}}^2 - \sigma_{\mathcal{Q}}^2$ .

- 8.9** Show that the MMSE quantizer for a uniform source is given by Eq. (8.5.7).

- 8.10** Prove that for a source with arbitrary mean  $\eta$  and variance  $\sigma^2$ , the optimal reconstruction and boundary values can be obtained from those derived for a source with zero mean and unit variance according to Eq. (8.5.17).

- 8.11** Consider a RV  $\mathcal{F}$  with pdf  $p(f) = \frac{\lambda}{2}e^{-\lambda|f|}$ . A three level quantizer is defined as

$$Q(f) = \begin{cases} b & f > a \\ 0 & -a \leq f \leq a \\ -b & f < -a \end{cases}$$

(a) Find  $b$  for a given  $a$  such that the centroid condition is satisfied when the distortion measure is the MSE.

(b) Find  $a$  for a given  $b$  such that the nearest neighbor condition is met.

(c) Find an optimal set of  $a, b$  in terms of  $\lambda$  such that both conditions are satisfied. Derive the final MSE.

- 8.12** The optimality conditions given in Eqs. (8.5.12) and (8.5.13) are derived by minimize the MSE. Here we consider other distortion measures.

(a) Derive a similar set of equations if the distortion criterion is the mean absolute error (MAE), i.e.,  $D_q = E\{|\mathcal{F} - Q(\mathcal{F})|\}$ . Show that the generalized centroid  $g_l$  for the decision region  $\mathcal{B}_l = [b_{l-1}, b_l)$  is

$$\int_{b_{l-1}}^{g_l} p(f|f \in \mathcal{B}_l) df = \int_{g_l}^{b_l} p(f|f \in \mathcal{B}_l) df.$$

That is,  $g_l$  equally splits  $\mathcal{B}_l$ . Such a  $g_l$  is also called the median of the region  $\mathcal{B}_l$ . What is the form of the nearest neighbor condition under this distortion measure?

(b) For a source with Laplacian distribution,  $p(f) = \frac{1}{2}e^{-|f|}$ , determine the generalized centroid for the region  $\mathcal{B}_l$  under the MAE criterion. Compare this centroid to that determined based on the MSE criterion  $D_q = E\{|F - Q(F)|^2\}$ .

- 8.13** Consider a quantizer for a discrete vector source  $S = \{\mathbf{f}_k, k = 1, 2, \dots, K\}$ , where  $\mathbf{f}_k$  is an  $N$ -D vector. Suppose the distortion measure between two vectors is the Hamming distance defined as

$$d(\mathbf{f}, \mathbf{g}) = \begin{cases} 0 & \text{if } \mathbf{f} = \mathbf{g}; \\ 1 & \text{otherwise.} \end{cases}$$

Assume these vectors are to be quantized into  $L < K$  codewords.

- (a) For a given partition region of the source,  $\mathcal{B}_l = \{\mathbf{f}_k, k = 1, 2, \dots, K_l\}$ , determine a new codeword for this region based on the centroid rule.
- (b) For a given set of codewords  $\mathcal{C} = \{\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_L\}$ , determine a new partition of the source based on the nearest neighbor rule. Is the partition unique?
- (c) Starting from an arbitrary initial partition of the source, how many iterations are needed to arrive at a locally optimal codebook ?

Hint: Consider two separate cases: 1) when all the training vectors are different from each other; 2) when some of the training vectors are the same.

- 8.14** A 2-D vector quantizer has two codewords:  $\mathbf{g}_1 = [1/2, 1/2]; \mathbf{g}_2 = [-1/2, -1/2]$ . Suppose the input vectors  $\mathbf{f} = [f_1, f_2]$  are uniformly distributed in the square defined by  $-1 < f_1 < 1$  and  $-1 < f_2 < 1$ . Illustrate the partition regions associated with the two codewords and determine the MSE of this quantizer (you could just write down the integral formula).

- 8.15** A 2-D lattice quantizer over a finite region is composed of the codewords  $(g_1, g_2)$  defined by  $g_1 = m$  and  $g_2 = m + 2n$ , where  $m, n$  are integers such that  $(g_1, g_2)$  fall in the specified region.

- (a) Sketch the codewords and partition regions near the origin.
- (b) Design an algorithm that will identify the nearest codeword for any given input point in the plane.
- (c) Find the nearest codeword for the point  $(123.4, 456.7)$ .
- (d) Determine the MSE of this quantizer if the input is uniformly distributed in the space spanned by all the partition regions.

- 8.16** Write a C or Matlab code to implement the generalized Lloyd algorithm based on training data generated from a 2-D i.i.d. Gaussian source. Assume the two components in each sample are independent and both have zero mean and unit variance. You can choose the number of samples used and the codebook size. Indicate the training samples and the codewords after each iteration using crosses and circles in a 2-D plot.

**8.17** Repeat the above problem, but assume the two vector components are correlated with a correlation coefficient  $\rho$ . Compare the results obtained with different values of  $\rho$ .

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## Chapter 9

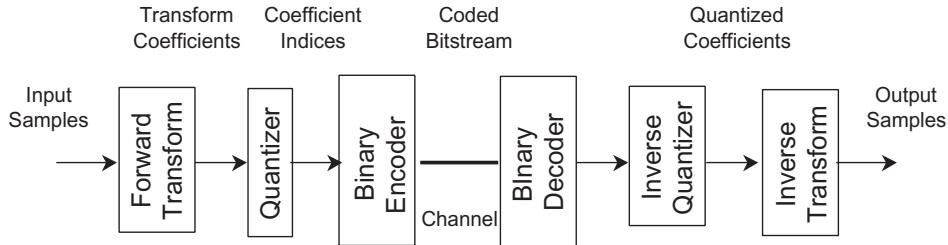
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# WAVEFORM-BASED VIDEO CODING

In Chap. 8, we introduced the fundamental techniques for video coding that are common in both waveform-based and content-dependent coders. In this chapter, we focus on techniques for waveform-based video coding. First we describe block-based transform coding techniques (Sec. 9.1), by which each block of pixels are converted into a set of uncorrelated coefficients, which are then quantized and coded. We will introduce the important Karhunen-Loeve transform (KLT) and its approximation, the discrete cosine transform (DCT), which is used in most modern image coding standards. A coding method using another important transform, wavelet transform, is introduced in Sec. 11.3, when we describe scalable coding. In Sec. 9.2, we introduce coding techniques using spatial and temporal prediction. These techniques reduce the data rate of a video sequence by exploiting spatial correlation between neighboring pixels as well as temporal correlation between consecutive frames of an video sequence. The spatial correlation is due to the fact that color values of adjacent pixels in the same video frame usually change smoothly. The temporal correlation refers to the fact that consecutive frames of a sequence usually show the same physical scene, occupied by the same objects that may have moved. We conclude this chapter by presenting a coding scheme that combines transform and predictive coding (Sec. 9.3), which is a popular and effective technique and has been adopted by all international video coding standards.

### 9.1 Block-Based Transform Coding

Transform coding has proven to be specially effective for compression of still images and video frames. Ideally, a transform should be applied to an entire image or video frame, to fully exploit the spatial correlation among pixels. But to reduce computational complexity, block-based transform coding is more often used in practice, which divides an image into non-overlapping blocks, and applies the transform over each block. In this chapter, we only describe block-based transform coding, with focus on the type of algorithm used in standard image and video coders. For a more



**Figure 9.1.** Block Diagrams for the Encoder and Decoder of a Block Transform Coding System.

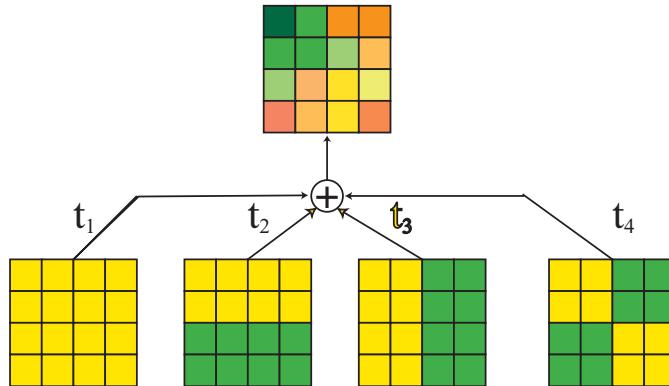
comprehensive treatment of transform coding, the reader is referred to the book by Clark [6].

We start with an overview of transform coding, describing how and why it works on a conceptual level (Sec. 9.1.1). We then formally define 1D and 2D unitary transforms (Secs. 9.1.2 and 9.1.3), and as an example, the DCT (Sec. 9.1.4). Section 9.1.5 discusses how to optimally allocate bits among the transform coefficients, so as to minimize the mean square error. This section also derives the gain of transform coding over scalar quantization. These results are obtained by assuming an arbitrary unitary transform. The next section (Sec. 9.1.6) derives the necessary conditions for a transform to achieve the highest coding gain, among all unitary transforms, and shows that the KLT is an optimal transform in this sense. The KLT depends on the statistics of the signals and is difficult to compute. Fortunately, the DCT is a good approximation of the KLT for most image signals. For this reason, DCT-based coding has been adopted by all image and video coding standards. Section 9.1.7 presents the DCT-based image coding algorithm, as used by the JPEG standard. The modification of this algorithm for video coding will be described in Sec. 9.3.1 and in Chap. 13. We conclude this section by briefly describing vector transforms in Sec. 9.1.8.

### 9.1.1 Overview

One of the most popular waveform-based coding schemes for images is transform coding. In block-based transform coding, one divides an image into non-overlapping blocks. Each block is transformed into a set of coefficients. These coefficients are quantized separately using scalar quantizers. The quantized coefficient indices are finally converted into binary bits using variable length coding. In the decoder, the image block is recovered from the quantized coefficients through an inverse transform. Figure 9.1 shows the encoder and decoder operations of a typical transform coder.

**Transform as Projection onto Selected Basis Functions** One can think of the transform process as representing an image block as a linear combination of a set of



**Figure 9.2.** A 2-D transform is a projection of an image onto a set of basic block patterns, each known as a transform basis.

basic patterns (known as transform basis functions), as illustrated in Fig. 9.2. The contribution from each basic pattern is the transform coefficient corresponding to that transform basis function. The process of deriving the transform coefficients for a given image block is the forward transform, whereas reconstruction of the image block from the transform coefficients the inverse transform.

**Transform Design Criterion** Obviously, the performance of a transform coder depends on the basis functions used. A good transform should i) decorrelate the signal to be quantized so that scalar quantization over individual values can be effectively used without losing too much coding efficiency in comparison with vector quantization; and ii) compact the energy of the original pixel block into as few coefficients as possible. The latter property allows one to represent the original block by a few coefficients with large magnitudes. As will be shown in Sec. 9.1.6, the best transform under these criteria is the KLT. But, because the KLT depends on the second order statistics of the signal, and it is difficult to calculate, in practice, one uses a fixed transform to approximate the KLT. The transform that best approximates the KLT for common image signals is the Discrete Cosine Transform (DCT). Therefore, in almost all transform-based image coders, the DCT is used.

**Transform Coding vs. Vector Quantization** Because adjacent pixels in an image are usually correlated, independent representation of each pixel value is not efficient. One way to exploit the correlation among adjacent pixels is to quantize a block of pixels together using vector quantization, which replaces each image block by one of the typical block patterns that is closest to the original block. The larger is the block size, the correlation among pixels can be exploited more completely, and a higher compression gain can be achieved. Unfortunately, as we have learned from Sec. 8.6, the complexity in searching for the best matching pattern also grows exponentially with the block size. Transform coding is one way to implement a

constrained vector quantizer without using an exhaustive search. Specifically, the codewords in this quantizer are those representable by linear combinations of basis vectors with quantized coefficients. Because each coefficient depends on the entire image block, scalar quantization of the transform coefficients essentially accomplishes vector quantization of the entire block, but at a reduced complexity.

### 9.1.2 1-D Unitary Transform

#### General 1-D Linear Transform

As described already, the transform process can be thought of as decomposing an image block into a set of basic block patterns. This is in fact a 2-D linear transform. Before formally defining the 2-D transform, we first define 1-D transform. In this case, a 1-D signal is divided into vectors of length  $N$ , and each vector is represented as a linear combination of a set of basis vectors. Each transform coefficient represents the contribution from one basis vector. Mathematically, these coefficients are obtained by a matrix operation. Let  $\mathbf{s} = [s_1, s_2, \dots, s_N]^T$  represent the vector consisting of original samples,  $\mathbf{t} = [t_1, t_2, \dots, t_N]^T$  the vector containing all the transformed coefficients. Further, let  $\mathbf{u}_k = [u_{k;1}, u_{k;2}, \dots, u_{k;N}]^T$  denote the vector corresponding to the  $k$ -th transform basis and  $[\mathbf{U}] = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N]$  the matrix containing all basis vectors. Then the inverse transform represents the sample vector  $\mathbf{s}$  as

$$\text{Inverse Transform: } \mathbf{s} = \sum_{k \in \mathcal{N}} t_k \mathbf{u}_k = [\mathbf{U}] \mathbf{t} \quad (9.1.1)$$

where  $\mathcal{N} = \{1, 2, \dots, N\}$ . In order to be able to represent any  $N$ -D vector with  $N$  basis vectors, the basis vectors must be linearly independent. In this case, the matrix  $[\mathbf{U}]$  must be invertible and the transform coefficients can be determined by

$$\text{Forward Transform: } \mathbf{t} = [\mathbf{U}]^{-1} \mathbf{s} = [\mathbf{V}] \mathbf{s} \quad (9.1.2)$$

For generality, we assume that the basis vectors  $\mathbf{u}_k$  may contain complex components, so that  $\mathbf{u}_k$  belongs to the  $N$ -D complex space  $\mathcal{C}^N$ . In this case, even if  $\mathbf{s}$  is a real vector,  $\mathbf{t}$  is in general a complex vector. In the remaining discussion we assume both  $\mathbf{s}$  and  $\mathbf{t}$  belong to  $\mathcal{C}^N$ .

#### 1-D Unitary Transform

The above discussion does not impose any constraint on the transform basis vectors. A special class of linear transform is the unitary transform, in which the basis vectors are orthonormal to each other.

Before formally defining the unitary transform, we first review the definition of inner product and norm in a complex space  $\mathcal{C}^N$ . The *inner product* of two vectors  $\mathbf{s}_1, \mathbf{s}_2$ , both in  $\mathcal{C}^N$ , are defined as

$$\langle \mathbf{s}_1, \mathbf{s}_2 \rangle = \mathbf{s}_1^H \mathbf{s}_2 = \sum_{n \in \mathcal{N}} s_{1;n}^* s_{2;n}, \quad (9.1.3)$$

where the superscript  $H$  denotes vector transpose followed by element conjugation. Two vectors are said to be *orthogonal* to each other if  $\langle \mathbf{s}_1, \mathbf{s}_2 \rangle = 0$ . The *norm*  $\|\mathbf{s}\|$  of  $\mathbf{s} \in \mathcal{C}^N$  is defined as

$$\|\mathbf{s}\|^2 = \langle \mathbf{s}, \mathbf{s} \rangle = \sum_{n \in \mathcal{N}} |s_n|^2. \quad (9.1.4)$$

The norm  $\|\mathbf{s}\|$  represents the length or magnitude of  $\mathbf{s}$ , and the norm square  $\|\mathbf{s}\|^2$  the *energy* of  $\mathbf{s}$ . A vector with a unit norm is said to be a *normalized* vector. If two vectors are orthogonal to each other, and each is normalized, they are said to be *orthonormal*.

If the basis vectors in the linear transform defined in Eq. (9.1.2) are orthonormal with each other, i.e.,

$$\langle \mathbf{u}_k, \mathbf{u}_l \rangle = \sum_{n \in \mathcal{N}} u_{k;n}^* u_{l;n} = \delta_{k,l} = \begin{cases} 1 & \text{if } k = l, \\ 0 & \text{if } k \neq l, \end{cases} \quad (9.1.5)$$

then the matrix  $[\mathbf{U}]$  satisfies

$$[\mathbf{U}]^H [\mathbf{U}] = [\mathbf{U}] [\mathbf{U}]^H = [\mathbf{I}]_N \quad (9.1.6)$$

where  $[\mathbf{I}]_N$  represents an  $N \times N$  identity matrix. A matrix satisfying the above condition is called *unitary*. A linear transform where the transform matrix  $[\mathbf{U}]$  is unitary is called a *unitary transform*. In this case,  $[\mathbf{U}]^{-1} = [\mathbf{U}]^H$  and the transform coefficients can be found by  $\mathbf{t} = [\mathbf{U}]^H \mathbf{s}$  or  $t_k = \langle \mathbf{u}_k, \mathbf{s} \rangle = \mathbf{u}_k^H \mathbf{s}$ . That is, the  $k$ -th transform coefficient  $t_k$  can be found simply by the inner product of  $\mathbf{s}$  with  $\mathbf{u}_k$ .

To summarize, a unitary transform pair is specified by

$$\begin{aligned} \text{Forward transform: } t_k &= \langle \mathbf{u}_k, \mathbf{s} \rangle \quad \text{or} \quad \mathbf{t} = [\mathbf{U}]^H \mathbf{s} = [\mathbf{V}] \mathbf{s} \\ \text{Inverse transform: } \mathbf{s} &= \sum_{k \in \mathcal{N}} t_k \mathbf{u}_k = [\mathbf{U}] \mathbf{t} = [\mathbf{V}]^H \mathbf{t}. \end{aligned} \quad (9.1.7)$$

Usually, we refer to a transform by its forward transform matrix  $[\mathbf{V}] = [\mathbf{U}]^H$ . The basis vectors of the transform are the conjugate of the row vectors in  $[\mathbf{V}]$ .

**Example 9.1 (1-D DFT)** Recall that the  $N$ -point DFT is specified by

$$\begin{aligned} \text{Forward transform: } t_k &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n e^{-j2\pi k n / N}, \quad k = 0, 1, \dots, N-1, \\ \text{Inverse transform: } s_n &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} t_k e^{j2\pi k n / N}, \quad n = 0, 1, \dots, N-1. \end{aligned} \quad (9.1.8)$$

Using the notation described here, we see that it is a unitary transform with basis vectors defined by:

$$u_{k;n} = \frac{1}{\sqrt{N}} e^{j2\pi \frac{k n}{N}}, \quad n = 0, 1, \dots, N-1, \quad k = 0, 1, \dots, N-1, \quad (9.1.9)$$

or

$$\mathbf{u}_k = \frac{1}{\sqrt{N}} \left[ \begin{array}{cccc} 1 & e^{j2\pi \frac{k}{N}} & \dots & e^{j2\pi \frac{k(N-1)}{N}} \end{array} \right]^T, \quad k = 0, 1, \dots, N-1. \quad (9.1.10)$$

Note that in this example, we follow the convention of using an index range of 0 to  $N-1$  for  $k$  and  $n$ , instead of 1 to  $N$ .

### Properties of Unitary Transforms

Until now, we have assumed that  $\mathbf{s}$  is a given sample vector, and  $\mathbf{t}$  is its transform. In reality,  $\mathbf{s}$  is a realization of a random vector  $\mathcal{S} = [\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_N]^T$ , which corresponds to any  $N$  samples extracted from a source. Similarly,  $\mathbf{t}$  is a realization of a random vector  $\mathcal{T} = [\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_N]^T = [\mathbf{V}] \mathcal{S}$ . In transform coding, we are interested in not only the relations between  $\mathbf{s}$  and  $\mathbf{t}$ , but also relations between the statistics of  $\mathcal{S}$  and  $\mathcal{T}$ . With an unitary transform, the following relations exist.

1. The mean vectors  $\boldsymbol{\eta}_s = E\{\mathcal{S}\}$  and  $\boldsymbol{\eta}_t = E\{\mathcal{T}\}$  are related by

$$\boldsymbol{\eta}_t = [\mathbf{V}] \boldsymbol{\eta}_s, \quad \boldsymbol{\eta}_s = [\mathbf{V}]^H \boldsymbol{\eta}_t. \quad (9.1.11)$$

The covariance matrices  $[\mathbf{C}]_s = E\{(\mathcal{S} - \boldsymbol{\eta}_s)(\mathcal{S} - \boldsymbol{\eta}_s)^H\}$  and  $[\mathbf{C}]_t = E\{(\mathcal{T} - \boldsymbol{\eta}_t)(\mathcal{T} - \boldsymbol{\eta}_t)^H\}$  are related by:

$$[\mathbf{C}]_t = [\mathbf{V}] [\mathbf{C}]_s [\mathbf{V}]^H, \quad [\mathbf{C}]_s = [\mathbf{V}]^H [\mathbf{C}]_t [\mathbf{V}]. \quad (9.1.12)$$

2. The total energy of the transformed vector equals that of the sample vector. This is true both for a given realization and the ensemble average, i.e.,

$$\sum_{n \in \mathcal{N}} s_n^2 = \sum_{k \in \mathcal{N}} t_k^2, \quad (9.1.13)$$

$$\sum_{n \in \mathcal{N}} \sigma_{s,n}^2 = \sum_{k \in \mathcal{N}} \sigma_{t,k}^2. \quad (9.1.14)$$

where  $\sigma_{s,n}^2 = E\{(\mathcal{S}_n - \eta_{s,n})^2\}$  and  $\sigma_{t,k}^2 = E\{(\mathcal{T}_k - \eta_{t,k})^2\}$  are the variances of  $\mathcal{S}_n$  and  $\mathcal{T}_k$ , respectively.

This property is equivalent to the Parseval's theorem for the Fourier transform.

3. Suppose we use only the first  $K \leq N$  coefficients to approximate  $\mathbf{s}$ , with the approximated vector being  $\hat{\mathbf{s}}_K = \sum_{k=1}^K t_k \mathbf{u}_k$ , the approximation error signal is  $\mathbf{e} = \mathbf{s} - \hat{\mathbf{s}}_K = \sum_{k=K+1}^N t_k \mathbf{u}_k$ . The approximation error energy for a given  $\mathbf{s}$  is

$$\|\mathbf{e}\|^2 = \sum_{n \in \mathcal{N}} e_n^2 = \sum_{k=K+1}^N t_k^2. \quad (9.1.15)$$

The variance of  $\mathcal{E} = \mathcal{S} - \hat{\mathcal{S}}$  over the ensemble of  $\mathcal{S}$  is

$$E\{\|\mathcal{E}\|^2\} = \sum_{n \in \mathcal{N}} \sigma_{e,n}^2 = \sum_{k=K+1}^N \sigma_{t,k}^2. \quad (9.1.16)$$

Because the sum of all coefficient squares or variances is a constant (Eq. (9.1.13) or (9.1.14)), the approximation error for a particular signal vector is minimized if one chooses  $K$  coefficients that have the largest values to approximate the original signal. Similarly, the mean approximation error is minimized by choosing  $K$  coefficients with the largest variances.

The proof of the above relations is quite straight-forward, and is left as an exercise (Prob. 9.1).

### 9.1.3 2-D Unitary Transform

The above discussion has assumed that the input samples are from a 1-D signal. For a 2-D image block, the above descriptions still apply if we think of  $\mathbf{s}$  as a vector consisting of the samples from an image block arranged in a 1-D order, and each  $\mathbf{u}_k$  as a vector representation of a basis image, using the same 1-D ordering. However, one can also extend the definition of the 1-D transform to 2-D. In this case, an  $M \times N$  image block  $[\mathbf{S}] = [S_{m,n}]$  is represented as a linear combination of  $M \times N$  basis images  $[\mathbf{U}]_{k,l} = [U_{k,l;m,n}]$  as (cf. Fig. 9.2)

$$[\mathbf{S}] = \sum_{k \in \mathcal{M}} \sum_{l \in \mathcal{N}} T_{k,l} [\mathbf{U}]_{k,l}, \quad (9.1.17)$$

where  $\mathcal{M} = \{1, 2, \dots, M\}$ ,  $\mathcal{N} = \{1, 2, \dots, N\}$ .

Let  $\mathcal{C}^{M \times N}$  represent the  $M \times N$  dimensional complex space. Let  $[\mathbf{S}]_1 = [S_{1;m,n}]$ ,  $[\mathbf{S}]_2 = [S_{2;m,n}]$  be two matrices in this space. Then the *inner product* of the two are defined by:

$$\langle [\mathbf{S}]_1, [\mathbf{S}]_2 \rangle = \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} S_{1;m,n}^* S_{2;m,n}. \quad (9.1.18)$$

The *norm* of a matrix  $[\mathbf{S}] = [S_{m,n}]$  is defined as:

$$\|[\mathbf{S}]\|^2 = \langle [\mathbf{S}], [\mathbf{S}] \rangle = \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} |S_{m,n}|^2. \quad (9.1.19)$$

With above definitions, a set of basis images  $[\mathbf{U}]_{k,l}$ ,  $k \in \mathcal{M}$ ,  $l \in \mathcal{N}$ , are said to be orthonormal with each other, if

$$\langle [\mathbf{U}]_{k,l}, [\mathbf{U}]_{i,j} \rangle = \delta_{k,i} \delta_{l,j} = \begin{cases} 1, & \text{if } k = i, l = j; \\ 0 & \text{otherwise.} \end{cases} \quad (9.1.20)$$

With an orthonormal set of basis images, one can easily find the transform coefficient by  $T_{k,l} = \langle [\mathbf{U}]_{k,l}, [\mathbf{S}] \rangle$ . The forward and inverse transform relations for an  $M \times N$  2-D unitary transforms are

$$\begin{aligned} \text{Forward transform: } T_{k,l} &= \langle [\mathbf{U}]_{k,l}, [\mathbf{S}] \rangle, \\ \text{Inverse Transform: } [\mathbf{S}] &= \sum_{k \in \mathcal{M}} \sum_{l \in \mathcal{N}} T_{k,l} [\mathbf{U}]_{k,l}. \end{aligned} \quad (9.1.21)$$

**Example 9.2 (2-D DFT)** The 2-D DFT of length  $M \times N$  is defined as:

$$\begin{aligned} \text{Forward transform: } T_{k,l} &= \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} S_{m,n} e^{-j2\pi(\frac{km}{M} + \frac{ln}{N})}, \\ &k = 0, 1, \dots, M-1, l = 0, 1, \dots, N-1; \\ \text{Inverse transform: } S_{m,n} &= \frac{1}{\sqrt{MN}} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} T_{k,l} e^{j2\pi(\frac{km}{M} + \frac{ln}{N})}, \\ &m = 0, 1, \dots, M-1, n = 0, 1, \dots, N-1. \end{aligned} \quad (9.1.22)$$

One can see that the above transform is a 2-D unitary transform with basis images defined by:

$$[\mathbf{U}]_{k,l;m,n} = \frac{1}{\sqrt{MN}} e^{j2\pi(\frac{km}{M} + \frac{ln}{N})}, m, k = 0, 1, \dots, M-1, n, l = 0, 1, \dots, N-1. \quad (9.1.23)$$

### Separable Transforms

One particularly important class of 2-D transforms are those in which each basis image is the outer-product of two basis vectors each from a 1-D transform. Virtually all the 2-D transforms in use belong to this type. Specifically, let  $\mathbf{h}_k, k \in \mathcal{M}$  and  $\mathbf{g}_l, l \in \mathcal{N}$  represent basis vectors in  $\mathcal{C}^M$  and  $\mathcal{C}^N$  respectively, then one can form basis images in  $\mathcal{C}^{M \times N}$  by:

$$[\mathbf{U}]_{k,l} = \mathbf{h}_k \mathbf{g}_l^T \quad \text{or} \quad [\mathbf{U}]_{k,l;m,n} = h_{k;m} g_{l;n}, k \in \mathcal{M}, l \in \mathcal{N}. \quad (9.1.24)$$

It can be shown that  $[\mathbf{U}]_{k,l}$  will form an orthonormal basis set in  $\mathcal{C}^{M \times N}$  as long as  $\mathbf{h}_k$  and  $\mathbf{g}_l$  form orthonormal basis sets in  $\mathcal{C}^M$  and  $\mathcal{C}^N$ , respectively.

**Example 9.3 (2-D DFT as separable transforms)** Recall that the basis images for  $M \times N$  point 2-D DFT are:

$$U_{k,l;m,n} = \frac{1}{\sqrt{MN}} e^{j2\pi(\frac{km}{M} + \frac{ln}{N})},$$

whereas the basis vectors for  $M$ -point DFT and  $N$ -point DFT are respectively

$$h_{k;m} = \frac{1}{\sqrt{M}} e^{j2\pi \frac{km}{M}}, g_{l;n} = \frac{1}{\sqrt{N}} e^{j2\pi \frac{ln}{N}}.$$

Obviously,  $U_{k,l;m,n} = h_{k;m} g_{l;n}$ . Therefore, the  $M \times N$ -point 2-D DFT is a separable transform constructed from an  $M$ -point 1-D DFT and an  $N$ -point 1-D DFT.

When the transform is separable, one can first accomplish 1-D transform for each row using basis matrix  $[\mathbf{G}] = [\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_N]$ , and then perform 1-D transform for each column of the intermediate image using basis matrix  $[\mathbf{H}] = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_M]$ . This operation can be represented as

$$\begin{aligned} \text{Forward Transform: } & [\mathbf{T}] = [\mathbf{H}]^H [\mathbf{S}] [\mathbf{G}], \\ \text{Inverse Transform: } & [\mathbf{S}] = [\mathbf{H}] [\mathbf{S}] [\mathbf{G}]^H. \end{aligned} \quad (9.1.25)$$

When a 2-D transform is separable, its equivalent 1-D transform representation will have a transform matrix that is the Kronecker product of the matrix  $[\mathbf{G}]$  and  $[\mathbf{H}]$ . For more detailed discussion on this topic and additional properties of separable transforms, see [18, Sec. 5.2].

**Computational savings of separable transforms** An  $M \times N$  transform in general takes  $M \times N$  calculations for determining each coefficient. To determine all  $M \times N$  coefficients, the total number of calculations is  $M^2N^2$ , which becomes  $N^4$  when  $M = N$ . On the other hand, when the transform is separable, one first calculates  $N$ -point 1-D transform along each of  $M$  rows, each requiring  $N^2$  calculations. One then calculates  $M$ -pt 1-D transform along each of  $N$  columns, each requiring  $M^2$  calculations. The total number of calculations is  $MN^2 + NM^2$ , which is  $2N^3$  if  $M = N$ . When  $N$  is large, the reduction from  $N^4$  to  $2N^3$  is significant.

If there exists a fast algorithm for calculating the 1-D transform, then further savings are obtainable. For example, if an  $N$ -point 1-D transform takes  $N \log_2 N$  operations, then an  $N \times N$  2-D separable transform can be done in  $2N^2 \log N$  operations.

#### 9.1.4 Signal Independent Transforms and the DCT

Besides the DFT, there are many other transforms that have been developed for signal analysis, including Discrete Cosine Transform (DCT), Discrete Sine Transform (DST), Hadamard Transform, Walsh Transform, Haar Transform, Slant Transforms. See [12] or [18] for definitions of the basis vectors of these transforms. All these transforms are originally defined in the 1-D form, and can be used to construct 2-D separable transforms. The DFT is most widely used for performing frequency domain analysis for discrete signals. On the other hand, the DCT has been found to be more useful for source coding, especially image coding. For this reason, we introduce the DCT in this section and show examples of representing images using DCT.

The basis vectors of the 1-D  $N$ -point DCT are defined by:

$$u_{k;n} = \alpha(k) \cos \frac{(2n+1)k\pi}{2N}, n = 0, 1, \dots, N-1 \quad (9.1.26)$$

with

$$\alpha(k) = \begin{cases} \sqrt{\frac{1}{N}} & k = 0 \\ \sqrt{\frac{2}{N}} & k = 1, 2, \dots, N-1. \end{cases} \quad (9.1.27)$$

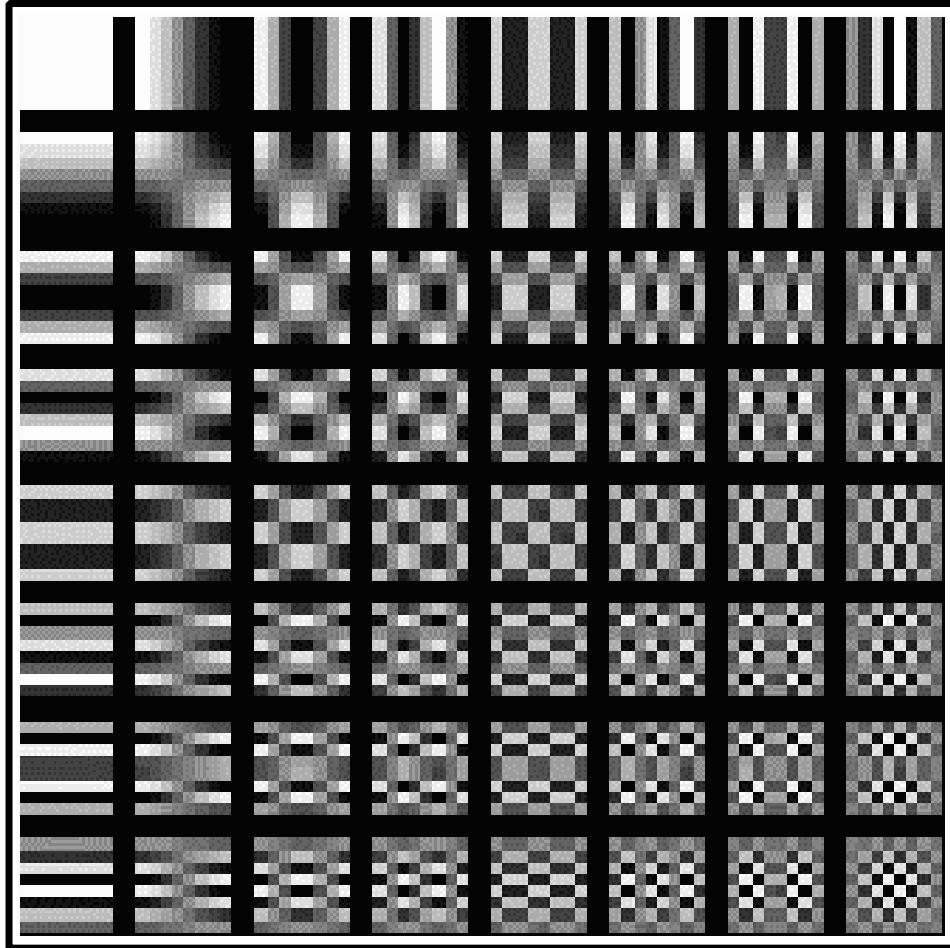
$$(9.1.28)$$

The forward and inverse transforms are described by:

$$t_k = \sum_{n=0}^{N-1} u_{k;n}^* s_n = \alpha(k) \sum_{n=0}^{N-1} s_n \cos \frac{(2n+1)k\pi}{2N}, \quad (9.1.29)$$

$$s_n = \sum_{k=0}^{N-1} u_{k;n} t_k = \sum_{k=0}^{N-1} \alpha(k) t_k \cos \frac{(2n+1)k\pi}{2N} \quad (9.1.30)$$

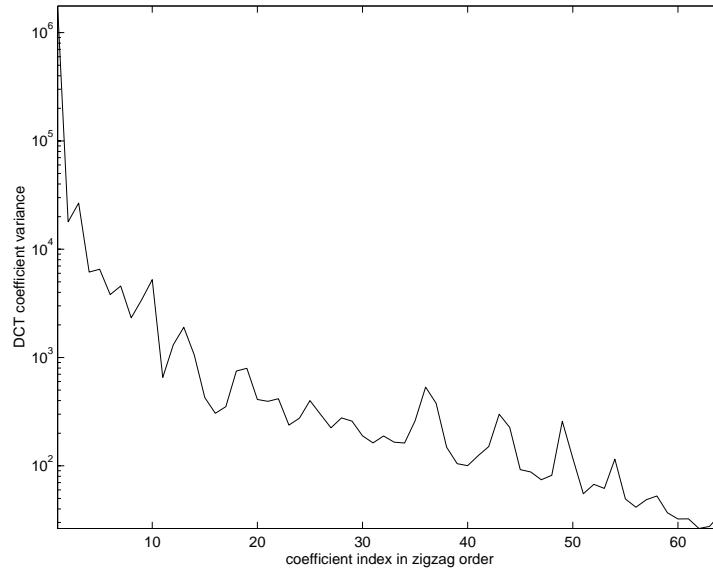
As with the DFT, we follow the convention of using an index range of 0 to  $N - 1$  for  $k$  and  $n$ .



**Figure 9.3.** Basis images of  $8 \times 8$  DCT transform.

Notice that the DCT basis vectors are real vectors, varying in a sinusoidal pattern with increasing frequency. Each DCT coefficient specifies the contribution of a sinusoidal pattern at a particular frequency to the actual signal. The lowest coefficient, known as the *DC coefficient*, represents the average value of the signal. The other coefficients, known as *AC coefficients*, are associated with increasingly higher frequencies.

The 2-D  $M \times N$  point DCT is constructed from the 1-D  $M$ -point DCT basis and  $N$ -point DCT basis. That is, each  $M \times N$  basis image is the outer-product of an  $M$ -point DCT basis vector with an  $N$ -point DCT basis vector. The basis images corresponding to 8x8 DCT are shown in Fig. 9.3. To obtain 2-D DCT of an image



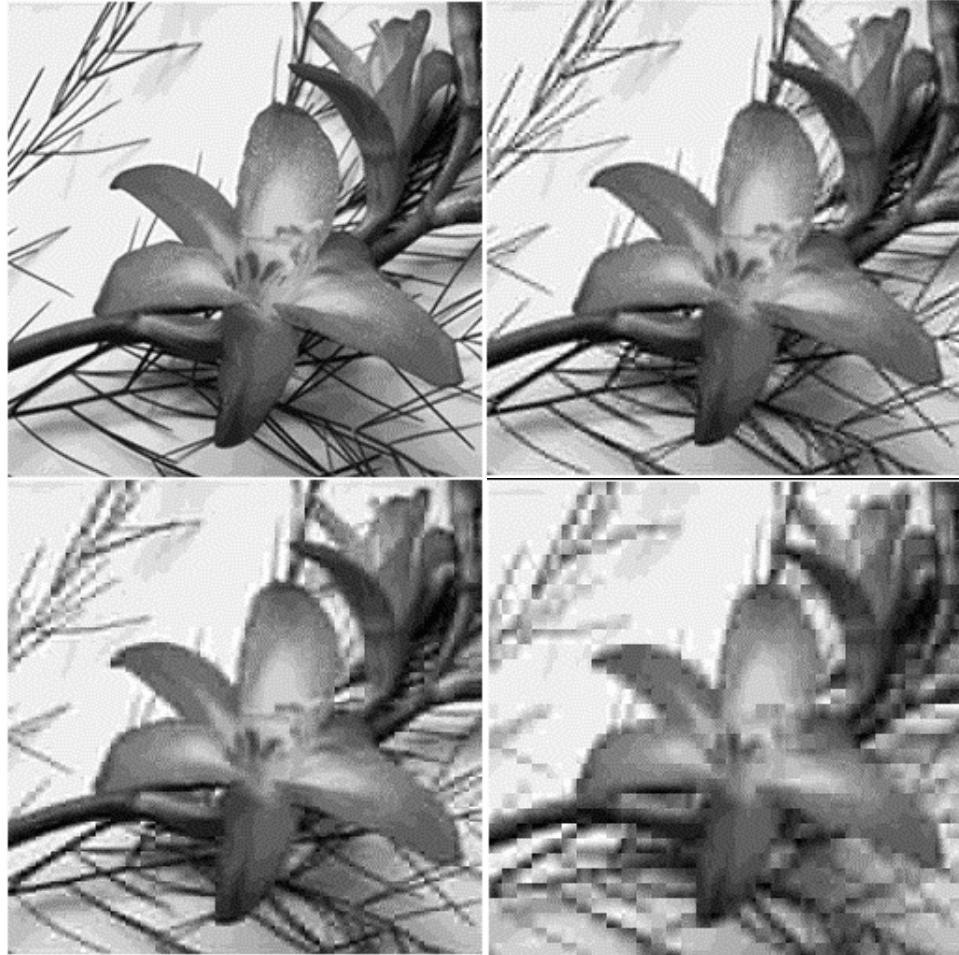
**Figure 9.4.** Energy distribution of the  $8 \times 8$  DCT coefficients of the test image “flower”.

block, one can first apply the above 1-D DCT to each row of the image block, and then apply the 1-D DCT to each column of the row transformed block. There exist several fast algorithms for computing  $N$ -point 1-D DCT with  $N \log_2 N$  operations [4, 21].

The reason that DCT is well suited for image coding is that an image block can often be represented with a few low frequency DCT coefficients. This is because the intensity values in an image are usually varying smoothly and very high frequency components only exist near edges. Figure 9.4 shows the energy (i.e., the variance) distribution of the  $8 \times 8$  DCT coefficients for a test image. The coefficients are ordered using the zigzag scan (cf. Fig. 9.8), so that the coefficients with low indices in both horizontal and vertical directions are put in the front. We can see that the energy of the DCT coefficients drops quickly as the frequency index increases. Figure 9.5 shows the approximation of the same image using different number of DCT coefficients. We can see that with only 16 out of 64 coefficients, we can already represent the original block quite well.

### 9.1.5 Bit Allocation and Transform Coding Gain

In this section, we consider the rate-distortion performance of a transform coder. Although we are mostly interested in 2-D transforms, we will use the 1-D transform representation in our analysis. Since any 2-D signal can be mapped to 1-D and then transformed, this choice does not limit the applicability of the analysis result.



**Figure 9.5.** Reconstructed images with different numbers of DCT coefficients. The top left is the original image with 64 coefficients per block, the top right with 16 coefficients, the bottom left with 8 coefficients, and the bottom right with 4 coefficients. The DCT coefficients are arranged in the zigzag order.

We will assume the transform is an arbitrary 1-D unitary transform. We further assume that the transform coefficients are quantized separately, each using a scalar quantizer optimized based on the pdf of that coefficient, and that the quantized coefficients are converted into bits using fixed length coding. Our goal is to derive the relation between the average distortion per sample in the vector  $s$  and average bit rate per sample. We will also compare the performance achievable by transform coding with that of a PCM system in which a scalar quantizer is applied to each

original sample.

### Relation between distortions in the samples and coefficients

First we must relate the quantization errors in the reconstructed samples with those in the transform coefficients. We will use the MSE as the distortion criterion exclusively in this section. As before, we use  $\mathcal{S}_n$  and  $\mathcal{T}_k$  to denote the RV's corresponding to  $s_n$  and  $t_k$ , respectively, and  $\mathbf{S}$  and  $\mathbf{T}$  the random vector corresponding to  $\mathbf{s}$  and  $\mathbf{t}$ . In addition, we use  $\hat{s}_n$ ,  $\hat{\mathbf{s}}$ ,  $\hat{\mathcal{S}}_n$ ,  $\hat{\mathbf{S}}$  to represent the quantized versions of  $s_n$ ,  $\mathbf{s}$ ,  $\mathcal{S}_n$ ,  $\mathbf{S}$ , and  $\hat{t}_n$ ,  $\hat{\mathbf{t}}$ ,  $\hat{\mathcal{T}}_n$ ,  $\hat{\mathbf{T}}$  the quantized versions of  $t_n$ ,  $\mathbf{t}$ ,  $\mathcal{T}_n$ ,  $\mathbf{T}$ . The MSE per sample between the original samples and the reconstructed ones is

$$D_s = \frac{1}{N} E\{\|\mathbf{S} - \hat{\mathbf{S}}\|^2\} = \frac{1}{N} \sum_{n \in \mathcal{N}} D_{s,n} \quad (9.1.31)$$

with

$$D_{s,n} = E\{(\mathcal{S}_n - \hat{\mathcal{S}}_n)^2\}. \quad (9.1.32)$$

Similarly, the MSE per coefficient between the original coefficients and the quantized ones is

$$D_t = \frac{1}{N} E\{\|\mathbf{T} - \hat{\mathbf{T}}\|^2\} = \frac{1}{N} \sum_{k \in \mathcal{N}} D_{t,k} \quad (9.1.33)$$

with

$$D_{t,k} = E\{(\mathcal{T}_k - \hat{\mathcal{T}}_k)^2\}. \quad (9.1.34)$$

Using the relation

$$\mathbf{S} = [\mathbf{V}]^H \mathbf{T} \text{ and } \hat{\mathbf{S}} = [\mathbf{V}]^H \hat{\mathbf{T}}, \quad (9.1.35)$$

we have

$$\begin{aligned} D_s &= \frac{1}{N} E\{\|\mathbf{S} - \hat{\mathbf{S}}\|^2\} = \frac{1}{N} E\{\|[\mathbf{V}]^H (\mathbf{T} - \hat{\mathbf{T}})\|^2\} \\ &= \frac{1}{N} E\{(\mathbf{T} - \hat{\mathbf{T}})^H [\mathbf{V}] [\mathbf{V}]^H (\mathbf{T} - \hat{\mathbf{T}})\} = \frac{1}{N} E\{\|\mathbf{T} - \hat{\mathbf{T}}\|^2\} = D_t. \end{aligned}$$

The fourth equality is due to the unitary property of  $[\mathbf{V}]$ . The above result tells us that the average quantization error in the sample domain is equal to the average quantization error in the transform domain.

Let  $\sigma_{t,k}^2$  represent the variance of coefficient  $t_k$  and let  $R_k$  and  $D_{t,k}$  denote the bit rate and distortion for  $t_k$ . From Eq. (8.5.18), when  $R_k$  is sufficiently high, we have

$$D_{t,k}(R_k) = \epsilon_{t,k}^2 \sigma_{t,k}^2 2^{-2R_k} \quad (9.1.36)$$

where  $\epsilon_{t,k}^2$  depends on the pdf of  $\mathcal{T}_k$ . Therefore, the average distortion using transform coding is

$$D_{TC} = D_s = D_t = \frac{1}{N} \sum_{k \in \mathcal{N}} \epsilon_{t,k}^2 \sigma_{t,k}^2 2^{-2R_k}. \quad (9.1.37)$$

### Optimal bit allocation

Given the desired average bit rate  $R$ , the question is how should we allocate the total bits  $RN$  among the  $N$  coefficients so that the error in Eq. (9.1.37) is minimized. Using the Lagrange multiplier method, this constrained minimization problem can be converted to minimization of

$$J(R_k, \forall k \in \mathcal{N}) = \sum_{k \in \mathcal{N}} \epsilon_{t,k}^2 \sigma_{t,k}^2 2^{-2R_k} + \lambda \left( \sum_{k \in \mathcal{N}} R_k - RN \right) \quad (9.1.38)$$

where  $\lambda$  must be chosen to satisfy

$$\sum_{k \in \mathcal{N}} R_k = RN \quad (9.1.39)$$

Let  $\frac{\partial J}{\partial R_k} = 0$ , we obtain

$$\frac{\partial D_{t,k}}{\partial R_k} = -2 \ln 2 D_{t,k} = -(2 \ln 2) \epsilon_{t,k}^2 \sigma_{t,k}^2 2^{-2R_k} = -\lambda, \forall k \in \mathcal{N} \quad (9.1.40)$$

To determine  $\lambda$ , we take the products of the above equations for all  $k$ . This yields

$$\lambda^N = (2 \ln 2)^N \left( \prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2 \right) 2^{-2 \sum_k R_k} = (2 \ln 2)^N \left( \prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2 \right) 2^{-2NR} \quad (9.1.41)$$

or

$$\lambda = (2 \ln 2) \left( \prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2 \right)^{1/N} 2^{-2R}. \quad (9.1.42)$$

Substituting the above to Eq. (9.1.40) gives

$$R_k = R + \frac{1}{2} \log_2 \frac{\epsilon_{t,k}^2 \sigma_{t,k}^2}{\left( \prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2 \right)^{1/N}}. \quad (9.1.43)$$

With the above bit allocation, the distortions incurred on all the coefficients are equal, i.e.,

$$D_{TC} = D_t = D_{t,k} = \left( \prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2 \right)^{1/N} 2^{-2R}. \quad (9.1.44)$$

The solution given in Eq. (9.1.43) implies that a coefficient with a larger variance should be given more bits, whereas a coefficient with a small variance should be given fewer bits. The optimal allocation is such that all the coefficients have the same quantization error.

Note that for a coefficient with sufficiently small variance, the rate determined according to Eq. (9.1.43) could be negative. In practice, the bit rate for this coefficient has to be reset to zero, corresponding to not coding the coefficient at all. The bits allocated to other coefficients would have to be reduced to satisfy the original average bit rate specification. For bit allocation algorithms that explicitly constrain  $R_k$  to be non-negative or non-negative integers, see [33, 30].

### Transform Coding Gain over PCM

Assume that the original samples come from a stationary source so that each sample has the same variance  $\sigma_s^2$ . If we apply optimal scalar quantization to individual samples separately, then the distortion is related to the rate by

$$D_{PCM} = D_{s,n} = \epsilon_s^2 \sigma_s^2 2^{-2R} \quad (9.1.45)$$

where  $\epsilon_s^2$  depends on the pdf of the source sample.

A measure of the performance of transform coding is the *transform coding gain* over PCM, which is defined as

$$G_{TC} = \frac{D_{PCM}}{D_{TC}}. \quad (9.1.46)$$

Substituting Eqs. (9.1.45) and (9.1.44) into above, we have

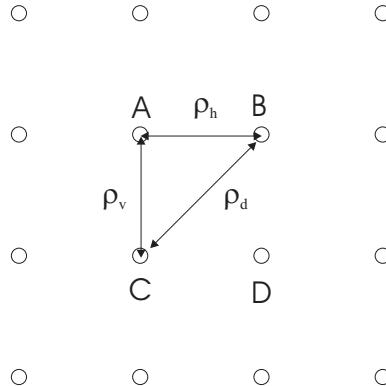
$$G_{TC} = \frac{\epsilon_s^2 \sigma_s^2}{\left(\prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2\right)^{1/N}} = \frac{\epsilon_s^2}{\left(\prod_k \epsilon_{t,k}^2\right)^{1/N}} \frac{\frac{1}{N} \sum \sigma_{t,k}^2}{\left(\prod_k \sigma_{t,k}^2\right)^{1/N}}. \quad (9.1.47)$$

The second equality results from the energy preservation property of the unitary transform given in Eq. (9.1.14) and the fact that  $\sigma_{s,n}^2 = \sigma_s^2$ . Therefore, *the transform coding gain is proportional to the ratio of the arithmetic mean of transform coefficient variances with the geometric mean of these variances*. It is a well known result that for any arbitrary set of values, the arithmetic mean is equal to or larger than the geometric mean, being equal only when all the values are the same. In general, we also have  $\epsilon_s^2 \geq \left(\prod_k \epsilon_{t,k}^2\right)^{1/N}$ . Therefore,  $G_{TC} \geq 1$  or  $D_{s,TC} \leq D_{s,PCM}$  under the same average bit rate. The more unevenly valued are the coefficient variances, the smaller is their geometric mean, and the higher will be the coding gain.

When the source is Gaussian, each sample is Gaussian. Because each transform coefficient is a linear combination of the samples, it follows a Gaussian distribution as well. In this case,  $\epsilon_s^2 = \epsilon_{t,k}^2$ , and hence,

$$G_{TC,Gaussian} = \frac{\sigma_s^2}{\left(\prod_k \sigma_{t,k}^2\right)^{1/N}} = \frac{\frac{1}{N} \sum \sigma_{t,k}^2}{\left(\prod_k \sigma_{t,k}^2\right)^{1/N}}. \quad (9.1.48)$$

**Example 9.4:** Consider applying transform coding to each  $2 \times 2$  block of an image, using a  $2 \times 2$  DCT. Assume the image is a stationary Gaussian process with variance  $\sigma_s^2$  and that the correlation coefficients between horizontally, vertically, and diagonally adjacent pixels are  $\rho_h$ ,  $\rho_v$ , and  $\rho_d$ , respectively. Figure 9.6 illustrates the correlation between the pixels in a  $2 \times 2$  block. Determine the optimal bit allocation for a given average bit rate of  $R$ , and the corresponding distortion. Compare this coder with a PCM coder that directly quantizes each sample



**Figure 9.6.** The image source considered in Examples 9.4,9.5,9.6.

using optimal scalar quantizer. Give the numerical results for the special case of  $\sigma_s^2 = 1$ ,  $\rho_h = \rho_v = \rho = 0.95$ ,  $\rho_d = \rho^2 = 0.9025$ , and  $R = 2$ .

To solve the problem, we arrange the  $2 \times 2$  pixel array into a 4-D vector, and think of the  $2 \times 2$  2-D DCT as a 4-point 1-D DCT. Using the 1-D ordering  $\mathbf{s} = [A, B, C, D]^T$ , the covariance matrix of  $\mathbf{s}$  is, assuming  $\mathbf{s}$  has zero mean,

$$\begin{aligned} [\mathbf{C}]_s &= E \left\{ \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \begin{bmatrix} A & B & C & D \end{bmatrix} \right\} = \begin{bmatrix} C_{AA} & C_{AB} & C_{AC} & C_{AD} \\ C_{BA} & C_{BB} & C_{BC} & C_{BD} \\ C_{CA} & C_{CB} & C_{CC} & C_{CD} \\ C_{DA} & C_{DB} & C_{DC} & C_{DD} \end{bmatrix} \\ &= \sigma_s^2 \begin{bmatrix} 1 & \rho_h & \rho_v & \rho_d \\ \rho_h & 1 & \rho_d & \rho_v \\ \rho_v & \rho_d & 1 & \rho_h \\ \rho_d & \rho_v & \rho_h & 1 \end{bmatrix}. \end{aligned} \quad (9.1.49)$$

The basis images of  $2 \times 2$  DCT are:

$$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \quad \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \quad (9.1.50)$$

Rearranging each basis image into a vector, the equivalent 1-D inverse transform matrix is

$$[\mathbf{U}] = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}. \quad (9.1.51)$$

The covariance matrix of the transformed vector  $[\mathbf{C}]_t$  can be obtained using the property in Eq. (9.1.12), and the variances of the transformed coefficients are the

diagonal element in  $[\mathbf{C}]_t$ . In the special case of  $\rho_h = \rho_v = \rho = 0.95$ ,  $\rho_d = \rho^2 = 0.9025$ , the coefficient variances are

$$\sigma_{t,k}^2 = \{(1 + \rho)^2, (1 - \rho^2), (1 - \rho^2), (1 - \rho)^2\} \sigma_s^2 \quad (9.1.52)$$

$$= \{3.8025, 0.0975, 0.0975, 0.0025\}. \quad (9.1.53)$$

The geometric mean of the coefficient variances is

$$\sigma_t^2 = \left( \prod_k \sigma_{t,k}^2 \right)^{1/4} = (1 - \rho^2) \sigma_s^2 = 0.0975. \quad (9.1.54)$$

If the desired average bit rate is  $R = 2$  bit/pixel (bpp), the optimal bit allocation among the 4 coefficients are, using Eq. (9.1.43):

$$R_k = \{4.64, 2, 2, -0.64\}. \quad (9.1.55)$$

The average distortion per pixel is, using Eq. (9.1.44) and  $\epsilon_{t,k}^2 = \epsilon_{\text{Gaussian}}^2$ ,

$$D_{\text{TC}} = \epsilon_{\text{Gaussian}}^2 (1 - \rho^2) \sigma_s^2 2^{-2R} = 0.0061 \epsilon_{\text{Gaussian}}^2 \quad (9.1.56)$$

If one directly quantizes each pixel, then the average distortion (assuming optimal quantizer) is, using Eq. (9.1.45)

$$D_{\text{PCM}} = \epsilon_{\text{Gaussian}}^2 \sigma_s^2 2^{-2R} = 0.0625 \epsilon_{\text{Gaussian}}^2 \quad (9.1.57)$$

The transform coding gain is, using Eq. (9.1.48)

$$G_{\text{TC}} = \frac{\sigma_s^2}{\sigma_t^2} = \frac{1}{1 - \rho^2} = 10.25. \quad (9.1.58)$$

Notice that the optimal bit allocation in Eq. (9.1.55) assumes that one can code a variable using a negative number of bits. In practice, one must set  $R_4 = 0$ , and reallocate the total number of bits among the remaining three variables. A simple solution is to set  $R_1 = 4$ , so that the modified bit allocation is

$$R_k = \{4, 2, 2, 0\}. \quad (9.1.59)$$

The average distortion in this case is

$$D_t = \frac{1}{4} \sum_k \epsilon_{\text{Gaussian}}^2 \sigma_{t,k}^2 2^{-2R_k} = 0.0074 \epsilon_{\text{Gaussian}}^2. \quad (9.1.60)$$

Although this distortion is higher than that indicated by Eq. (9.1.56), it is still much lower than that achievable by PCM.

### 9.1.6 Optimal Transform Design and the KLT

As shown above, with optimal bit allocation and optimal scalar quantization, the transform coding yields a lower average distortion than the PCM method. This is true with any unitary transform. It is natural then to question whether there exists an optimal transform that will minimize the distortion  $D_s$ , or equivalently maximize the gain  $G_{TC}$ . Because  $\epsilon_s^2 \sigma_s^2$  is fixed for a given source, the transform should be such that it yields the minimal geometric mean  $\left(\prod_k \epsilon_{t,k}^2 \sigma_{t,k}^2\right)^{1/N}$ , among all  $N \times N$  unitary transforms. Fortunately, when the source is Gaussian, the answer is a definite “yes”, and such an optimal transform is the KLT.

#### Construction of the KLT Bases

With the KLT, the basis vectors are designed based on the covariance matrix of the original signal  $[\mathbf{C}]_s$ . Let  $\lambda_k$  and  $\phi_k$  represent the  $k$ -th eigenvalue and normalized eigenvector of  $[\mathbf{C}]_s$ . Then they satisfy:

$$[\mathbf{C}]_s \phi_k = \lambda_k \phi_k, \quad \text{with } \langle \phi_k, \phi_l \rangle = \delta_{k,l}.$$

The KLT for a given covariance matrix  $[\mathbf{C}]_s$  is the one which uses  $\phi_k$  as the basis vectors. The corresponding inverse transform matrix is:

$$[\Phi] = [\phi_0, \phi_1, \dots, \phi_{N-1}]. \quad (9.1.61)$$

Replacing  $[\mathbf{V}]$  in Eq. (9.1.12) by  $[\Phi]^H$ , we get

$$[\mathbf{C}]_t = [\Phi]^H [\mathbf{C}]_s [\Phi] = \begin{bmatrix} \phi_1^H \\ \phi_2^H \\ \vdots \\ \phi_N^H \end{bmatrix} [\mathbf{C}]_s [\phi_1, \phi_2, \dots, \phi_N] \quad (9.1.62)$$

$$= \begin{bmatrix} \phi_1^H \\ \phi_2^H \\ \vdots \\ \phi_N^H \end{bmatrix} [\lambda_1 \phi_1, \lambda_2 \phi_2, \dots, \lambda_N \phi_N] = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_N \end{bmatrix} \quad (9.1.63)$$

That is, the KLT diagonalizes  $[\mathbf{C}]_s$ . Further, the variance of the  $k$ -th transformed coefficient is  $\sigma_k^2 = C_t(k, k) = \lambda_k$ .

To prove that the KLT will minimize the coding distortion, we make use of the inequality

$$\det[\mathbf{C}] \leq \prod_{k \in \mathcal{N}} \sigma_{t,k}^2 \quad (9.1.64)$$

which applies to any covariance matrix  $[\mathbf{C}]$  with diagonal entries  $\sigma_{t,k}^2$ . On the other hand, from Eq. (9.1.12), we have

$$\det[\mathbf{C}]_t = \det[\mathbf{C}]_s \quad (9.1.65)$$

for all unitary transforms. Therefore, for any unitary transform, we have

$$\prod_{k \in \mathcal{N}} \sigma_{t,k}^2 \geq \det[\mathbf{C}]_s. \quad (9.1.66)$$

But with KLT, we have

$$\prod_{k \in \mathcal{N}} \sigma_{t,k}^2 = \det[\mathbf{C}]_t = \det[\mathbf{C}]_s. \quad (9.1.67)$$

Thus, among all unitary transforms, the KLT yields the minimal geometric mean of the transform coefficient variances. Consequently, it maximizes the transform coding gain, if the source is Gaussian. Even for a non-Gaussian source, the effect of the  $\epsilon^2$  factors can usually be ignored, and the KLT can be considered the optimal transform.

Another property of the KLT is that it yields the minimal approximation error with  $K < N$  coefficients among all unitary transforms, where the error is defined as in Eq. (9.1.16). Alternatively, we say that the KLT has the highest energy compaction capability among all transforms. This is because the approximation error is directly related to the geometric mean of the coefficient variances. The lower is the geometric mean, the more unequally distributed are these variances, and consequently, the more energy can be packed into a fixed number of coefficients.

With the KLT and the Gaussian source, we have

$$D_t = \epsilon_{\text{Gaussian}}^2 (\det[\mathbf{C}]_s)^{1/N} 2^{-2R}. \quad (9.1.68)$$

Recall that the RD bound for a correlated Gaussian vector source is given by Eq. (8.3.26). Therefore, the operational RD function achievable by transform coding exceeds the bound by the constant factor  $\epsilon_{\text{Gaussian}}^2 = 2.71$ .

The coding gain using the KLT is, from Eqs. (9.1.47) and (9.1.65),

$$G_{\text{TC,KLT}} = \frac{\epsilon_s^2}{(\prod_k \epsilon_k^2)^{1/N}} \frac{\sigma_s^2}{(\prod_k \lambda_k)^{1/N}} = \frac{\epsilon_s^2}{(\prod_k \epsilon_k^2)^{1/N}} \frac{\sigma_s^2}{(\det[\mathbf{C}]_s)^{1/N}} \quad (9.1.69)$$

**Example 9.5:** Consider again the image source in Example 9.4. Instead of using DCT, we would like to use the KLT. The covariance matrix  $[\mathbf{C}]_s$  of the 4-D vector consisting of any  $2 \times 2$  pixels is given by Eq. (9.1.49). To determine the transform matrix for the KLT, we need to determine the eigenvalues and eigenvectors of  $[\mathbf{C}]_s$ . The eigenvalues are determined by solving

$$\det([\mathbf{C}]_s - \lambda[\mathbf{I}]) = 0 \quad (9.1.70)$$

For each eigenvalue  $\lambda_k$ , we can determine the corresponding eigenvector  $\phi_k$  by solving

$$([\mathbf{C}]_s - \lambda[\mathbf{I}])\phi_k = \mathbf{0} \quad (9.1.71)$$

with constraint  $\|\phi_k\|^2 = 1$ . For the special case of  $\rho_h = \rho_d = \rho$ ,  $\rho_d = \rho^2$ , the resulting eigenvalues are

$$\lambda_k = \{(1 + \rho)^2, (1 - \rho^2), (1 - \rho^2), (1 - \rho)^2\} \sigma_s^2.$$

Comparing the above result to Eq. (9.1.52), we see that the eigenvalues in this case are the same as the variances obtained by DCT in Example 9.4. Consequently, the KLT matrix, consisting of the eigenvectors, is the same as the DCT transform matrix, given in Eq. (9.1.51).<sup>1</sup> It can be easily verified that  $\det[\mathbf{C}]_s = \prod_k \lambda_k$ .

For this particular example, the DCT is equal to the KLT, for any value of  $\rho$ . It has been shown that, for an arbitrary first order Markov process, its KLT basis functions resemble closely those of the DCT [1, 6]. In general, the DCT is not as efficient as the KLT.

Although the KLT is optimal in terms of its capability in maximizing the coding gain or equivalently energy compaction and in decorrelating the signal, it is computable only for a stationary source with a known covariance matrix. In reality, the source may be temporally and/or spatially varying, and one needs to constantly update the covariance matrix based on previous signal samples and recalculate the eigenvectors, which is computationally very demanding. Furthermore, there exist no fast algorithms for the KLT derived from an arbitrary covariance matrix. Therefore, for practical applications, it is desirable to employ transforms that are signal-independent. It has been shown that the DCT is very close to the KLT for the covariance matrices of common image signals. Therefore, the DCT has been used in place of the KLT for image coding.

### 9.1.7 DCT-Based Image Coder and the JPEG Standard

The previous sections have introduced the basic principles for transform coding. As shown in Fig. 9.1, a transform coder consists of three components: transform, quantization, and binary encoding. In the analysis until now, we have assumed that each coefficient is quantized using a pdf-optimized scalar quantizer, and the quantization indices are coded using a fixed length coder. But in practice, the combination of uniform quantization and variable length coding has been found to be more effective. Here we describe a typical block transform coder, which is the basis for both the image coding standard known as JPEG [15]<sup>2</sup> and all the video

<sup>1</sup>The solutions for eigenvectors corresponding to  $\lambda_2$  and  $\lambda_3$  are not unique. Another set of solutions, e.g. is

$$[\Phi] = [\phi_1, \phi_2, \phi_3, \phi_4] = \begin{bmatrix} 1/2 & 1/\sqrt{2} & 0 & 1/2 \\ 1/2 & 0 & -1/\sqrt{2} & -1/2 \\ 1/2 & 0 & 1/\sqrt{2} & -1/2 \\ 1/2 & -1/\sqrt{2} & 0 & 1/2 \end{bmatrix}.$$

<sup>2</sup>The JPEG standard refers to the international standard for still image compression recommended by the Joint Photographic Expert Group (JPEG) of the International Standards Organization (ISO).

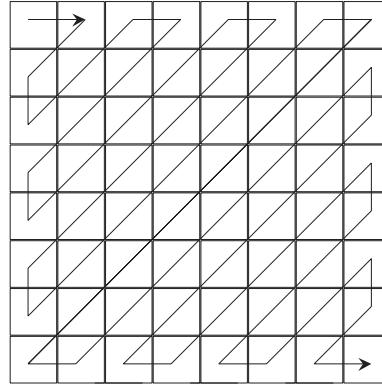
16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

**Figure 9.7.** The default normalization matrix recommended by the JPEG standards [15].

coding standards to be introduced in Chap. 13. In the JPEG standard, this method is applied to each image block directly, whereas in the video coding standards, it is applied to the error between an original image block in a current video frame and the predicted block based on a previous video frame. Occasionally, it is also applied to the original image block directly, when the prediction is not accurate or when it is desirable to reset the prediction loop.

Given an image, it is first divided into 8x8 non-overlapping blocks. An 8x8-point DCT is then applied to each block. The coefficients are then quantized using uniform quantizers, each with a different stepsize. The stepsizes for different coefficients are specified in a normalization matrix. The matrix is designed based on the visual sensitivity to different frequency components. Typically, low frequency coefficients are assigned smaller stepsizes than high frequency coefficients because the human eye is more sensitive to changes in low frequency components, as described in Chap. 2. The particular matrix used can be specified in the beginning of the compressed bit stream as side information. Alternatively, one can use a default matrix. The matrix recommended by the JPEG standard is shown in Fig. 9.7. Instead of using the normalization matrix as is, one can scale the matrix to increase or decrease the stepsizes so as to achieve the desired bit rate. The scale factor is called the *quality factor* in JPEG coders, and the *quantization parameter* (QP) in MPEG and H-series video coders.

For the binary encoding of the quantized DCT coefficients, the DCT coefficients are arranged into a 1D array following the zigzag order illustrated in Fig. 9.8. This scan order puts the low-frequency coefficients in front of the high-frequency coefficients. Because many quantized coefficients are zeros in a typical image block, it is not efficient to specify the coefficient values individually. Rather, a run-length representation is used, which starts with the DC value, followed by a series of symbols. Each symbol consists of two numbers: the number of zeros (known as *run-length*) from the last non-zero value and the following non-zero value. A special



**Figure 9.8.** The zigzag ordering of the DCT coefficients.

DCT coefficients: [5 0 0 2 3 0 0 4 0 0 0 0 0 0 1 0 0 0 0 0 ....0]

Coding symbols: (0,5), (2,2), (0,3), (2,4), (6,1), EOB

**Figure 9.9.** Converting the 64 quantized DCT coefficients resulting from the zigzag scan into symbols for binary encoding.

symbol, ‘EOB’, is placed after the last non-zero value in a block, to signal the end of the block. Figure 9.9 shows an example of how to convert the DCT coefficients of an  $8 \times 8$  block into such symbols. Ideally, the DC value and the symbols would be coded independently using two separate VLC tables. In order to avoid the use of very large VLC tables, the dynamic range of the coefficient value is partitioned into several segments. The magnitude of a non-zero coefficient is specified by the segment number and the relative value in that segment. For the DC coefficient, the segment number is Huffman coded based on the frequencies of different segments, while the relative magnitude is coded using a fixed length codeword. For the AC coefficients, each symbol is further partitioned into two parts, the part consisting of the zero run-length and the segment number of the non-zero value is Huffman coded, while the relative magnitude of the non-zero value is coded using a fixed length code. The VLC tables are usually pre-designed based on some training data. For improved coding efficiency, arithmetic coding can be used instead of Huffman coding.

When the above method is applied to original image blocks directly, the DC values of adjacent blocks are usually similar. To further improve the coding efficiency, the DC coefficient (i.e., the mean value) of a block may be predicted from the DC value of a previously coded block, and then the DC prediction error is quantized and coded using the method described above. This is the method used in the JPEG standard. Therefore, the stepsize in the top-left entry of the table given in Fig. 9.7



**Figure 9.10.** Coded versions of the image “Lena” by the JPEG algorithm at different bit rates. The top left is the original image, at 8 bpp, the top right is coded at 0.59 bpp, the bottom left at 0.37 bpp, and the bottom right at 0.22 bpp.

is actually for the DC prediction error. That is why it is higher than the stepsizes for nearby AC coefficients. For a color image, each color component can be coded separately using the above method. Figure 9.10 shows several JPEG-coded images, obtained by choosing different quality factors. These images are obtained with default normalization matrix and Huffman tables recommended by the standard.

### 9.1.8 Vector Transform Coding

Transform coding discussed thus far assume that transform coefficients are subject to scalar quantization (SQ). Although vector quantization (VQ) can be applied to the transform coefficients, the gain over SQ is not significant, as transform coefficients are already decorrelated. A method that further improves transform coding gain is by using the so-called vector transform [22]. Such type of transform converts a group of vector samples into a set of vector coefficients, each is then vector quantized. To maximize the coding efficiency, a vector transform should introduce correlation within variables in a vector coefficient, but decorrelate successive vector coefficients. For more information on vector transform and their application in image and video coding, the reader is referred to the review article by Li and Zhang [23].

## 9.2 Predictive coding

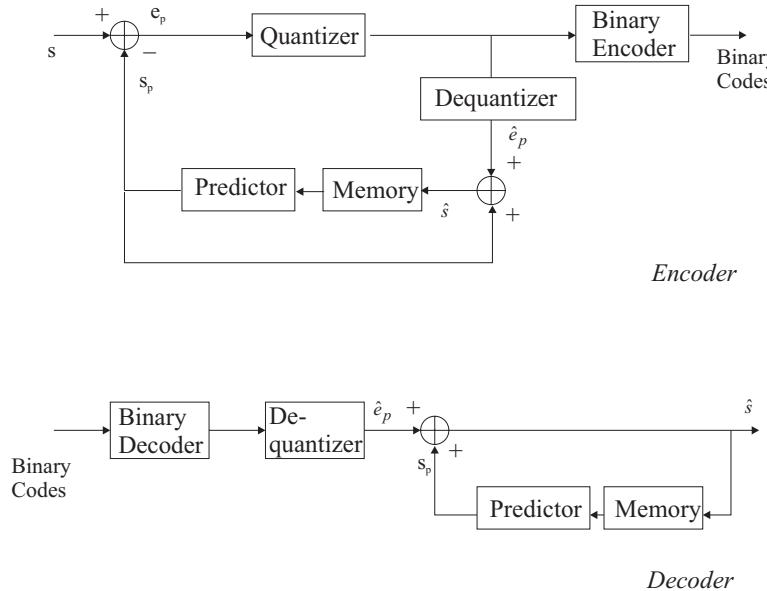
In addition to transform coding, predictive coding is another important technique for image and video coding. In fact, temporal predictive coding using motion compensated prediction is the key to the success of modern video coding standards. We start this section with an overview of predictive coding (Sec. 9.2.1). We then describe how to design the predictor to minimize the prediction error and derive the gain of predictive coding over scalar quantization (Sec. 9.2.2). The next two sections illustrate how to apply spatial and temporal prediction in image and video coding (Secs. 9.2.3 and 9.2.4).

### 9.2.1 Overview

In predictive coding, a pixel is not coded directly, rather it is predicted from those of adjacent pixels in the same frame or in a previous frame. This is motivated by the fact that adjacent pixels usually have similar color values, thus it is wasteful of bits to specify the current value independent of the past. Figure 9.11 shows the block diagrams of the encoder and decoder of a generic *lossy predictive coding* system. In the encoder, an input sample is first predicted from some previously reconstructed samples stored in the memory, then the prediction error is quantized and then coded using a variable length coder. The reconstructed value at the decoder is the predicted value plus the quantized error. To guarantee that the encoder and the decoder use exactly the same prediction value, the encoder must repeat the same process as the decoder to reproduce reconstructed samples. This is called *closed-loop prediction*. This kind of coder is generally referred to as *differential pulse coded modulation* (DPCM).<sup>3</sup>

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<sup>3</sup>Traditionally, the prediction error is uniformly quantized and then transmitted using amplitude modulation known as pulse coded modulation. With the advent of digital coding and transmission, the error can be quantized non-uniformly, various variable length coding methods can be used, and finally different digital modulation techniques can be used to carry binary bits. So the name DPCM is not an accurate description of this coding scheme any more.



**Figure 9.11.** Encoder and Decoder Block Diagrams of a Lossy Predictive Coding System

When the prediction error value is coded directly without quantization, the above system reduces to *lossless predictive coding*, which is useful for applications that require storage/transmission of original signals without any distortion.

**Error Analysis of a Lossy Predictive Coder** Refer to Fig. 9.11. Let  $s$  and  $s_p$  represent the original and predicted sample value,  $e_p = s - s_p$  and  $\hat{e}_p$  the original and quantized prediction error. Let  $e_q$  represent the quantization error for  $e$ , then  $e_p = \hat{e}_p + e_q$ . The reconstructed value for  $s$  is

$$\hat{s} = s_p + \hat{e}_p = s_p + e_p - e_q = s - e_q. \quad (9.2.1)$$

Therefore, the error between the original and the reconstructed sample value is  $e_s = s - \hat{s} = e_q$ , exactly the same as the quantization error for the prediction error. Thus, the distortion in a lossy predictive coder is completely dependent on the quantizer for the prediction error, for a fixed predictor.

For the design of the error quantizer, various quantization techniques discussed in Chap. 8 can be employed. Because the error usually has a non-uniform distribution, a non-uniform quantizer optimized for the distribution of the error signal (typically modeled by a zero mean Laplacian distribution) is usually used. For improved coding efficiency, one can also use a vector quantizer or transform coder for the error signal.

A key to the efficiency of a predictive coder is the predictor used. From Sec. 8.5, the distortion introduced by a quantizer at a given bit rate is proportional to the

variance of the signal (cf. Eq. (8.5.18)). To maximize the coding efficiency, the predictor should minimize the prediction error variance. How to design a predictor to achieve this objective is the topic of the next subsection.

### 9.2.2 Optimal Predictor Design and Predictive Coding Gain

In general, one can use various kinds of predictors, linear or non-linear. But in practice, linear predictors are used almost exclusively for the ease of implementation. Let  $s_0$  represent the current pixel, and  $s_k, k = 1, 2, \dots, K$  the previous pixels that are used to predict  $s_0$ . In a linear predictor, the predicted value for  $s_0$  is described by

$$s_p = \sum_{k=1}^K a_k s_k \quad (9.2.2)$$

where  $a_k$  are called *prediction coefficients*. The number of samples used for prediction,  $K$ , is usually called the *predictor order*. The key problem in designing a predictive coder is how to determine these coefficients. As described before, to maximize the coding efficiency, one should minimize the variance of the prediction error, which is equivalent to the MSE of the predictor. Letting  $\mathcal{S}_k$  represent the RV corresponding to  $s_k$ , and  $\mathcal{S}_p$  the RV corresponding to  $s_p$ , this error is defined as:

$$\sigma_p^2 = E\{|\mathcal{S}_0 - \mathcal{S}_p|^2\} = E\{|\mathcal{S}_0 - \sum_{k=1}^K a_k \mathcal{S}_k|^2\}. \quad (9.2.3)$$

The optimal prediction coefficients should minimize the above error. This can be accomplished by letting  $\frac{\partial \sigma_p^2}{\partial a_l} = 0$ , which yields

$$E\{(\mathcal{S}_0 - \sum_{k=1}^K a_k \mathcal{S}_k) \mathcal{S}_l\} = 0, \quad l = 1, 2, \dots, K. \quad (9.2.4)$$

Note that the above equation is precisely the *orthogonality principle* for linear minimal mean square error (LMMSE) estimator. It says that the prediction error must be orthogonal to each past sample used for prediction. Let  $R(k, l) = E\{\mathcal{S}_k \mathcal{S}_l\}$  represent the correlation between  $\mathcal{S}_k$  and  $\mathcal{S}_l$ . From Eq. (9.2.4), one can easily derive the following set of linear equations

$$\sum_{k=1}^K a_k R(k, l) = R(0, l), \quad l = 1, 2, \dots, K, \quad (9.2.5)$$

or in matrix form

$$\begin{bmatrix} R(1, 1) & R(2, 1) & \cdots & R(K, 1) \\ R(1, 2) & R(2, 2) & \cdots & R(K, 2) \\ \vdots & \vdots & \ddots & \vdots \\ R(1, K) & R(2, K) & \cdots & R(K, K) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix} = \begin{bmatrix} R(0, 1) \\ R(0, 2) \\ \vdots \\ R(0, K) \end{bmatrix} \quad (9.2.6)$$

or

$$[\mathbf{R}]\mathbf{a} = \mathbf{r}. \quad (9.2.7)$$

The above equation for solving the LMMSE predictor is usually called the Yuler-Walker equation.

From (9.2.7), the optimal prediction coefficients are:

$$\mathbf{a} = [\mathbf{R}]^{-1}\mathbf{r}. \quad (9.2.8)$$

The MSE of this predictor is:

$$\sigma_p^2 = E\{(\mathcal{S}_0 - \mathcal{S}_p)\mathcal{S}_0\} = R(0, 0) - \sum_{k=0}^K a_k R(k, 0) \quad (9.2.9)$$

$$= R(0, 0) - \mathbf{r}^T \mathbf{a} = R(0, 0) - \mathbf{r}^T \mathbf{R}^{-1} \mathbf{r}. \quad (9.2.10)$$

The first equality results from (9.2.4), the orthogonality principle.

For a stationary source, the autocorrelation of a pixel is a constant, independent of its spatial location, i.e.,  $R(l, l) = R(0, 0), l = 1, 2, \dots, K$ . Furthermore, the correlation between two pixels are symmetrical, i.e.,  $R(k, l) = R(l, k)$ . The matrix  $\mathbf{R}$  is in fact the autocorrelation matrix of the prediction vector  $\mathcal{S}_p = [\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_K]^T$ , and  $\mathbf{r} = E\{\mathcal{S}_0 \mathcal{S}\}$  is the correlation between the current pixel  $\mathcal{S}_0$  with  $\mathcal{S}_p$ . When the source is a 1-D stationary source,  $\mathcal{S}_0 = \mathcal{S}(n)$  is an arbitrary sample of the source, and  $\mathcal{S}_k = \mathcal{S}(n - k)$  are the  $k$ -th previous sample from  $\mathcal{S}(n)$ , we have  $R(k, l) = E\{\mathcal{S}(n - k)\mathcal{S}(n - l)\} = R_s(k - l)$ , the correlation between two samples separated by  $k - l$ . In this case, the matrix  $[\mathbf{R}]$  is Toeplitz in that all the entries on the same diagonal line are identical, and the corresponding upper and lower triangle lines are conjugate of each other. Such a matrix can be inverted efficiently using the *Levinson-Durbin Algorithm* [29]. When the source is 2-D, the matrix  $[\mathbf{R}]$  in general does not have such nice structure. But only a limited number of past samples is typically used for prediction, as the correlation between two pixels decreases rapidly as their distance increases. Usually, at most four pixels, on the left, top, top left, and top right of the current one, are used for prediction. In this case, direct solution of the above equation is a simple task.

The above solution for the optimal predictor assumes that the predicted value  $s_p$  is obtained from original past samples  $s_k$ . In a lossy predictive coder,  $s_k$  have to be replaced by reconstructed past samples  $\hat{s}_k$ . Ideally, the error in Eq. (9.2.3) should be replaced by

$$\sigma_p^2 = E\{|\mathcal{S}_0 - \sum_k a_k \hat{\mathcal{S}}_k|^2\} \quad (9.2.11)$$

But because  $\hat{s}_k$  is dependent on the predictor and the quantizer in a complicated relation, finding a solution that minimizes the above error is very difficult. In practice, one simply assumes that the quantization error is negligible and design the predictor by minimizing the error in Eq. (9.2.3).

### Predictive Coding Gain

As with transform coding, we can define the gain of predictive coding over PCM as the ratio of the distortions incurred by the two types of coders under the same bit rate. Recall that with predictive coding, it is the prediction error that is quantized and coded, and that the distortion on the original signal is equal to the quantization error for prediction error. Assuming optimal scalar quantization for the prediction error, the distortion is related to the rate by

$$D_{\text{DPCM}} = \epsilon_p^2 \sigma_p^2 2^{-2R} \quad (9.2.12)$$

where  $\epsilon_p^2$  depends on the pdf of the prediction error,  $\sigma_p^2$  is variance of the prediction error, which is equal to the MSE of the predictor. On the other hand, the distortion-rate function of PCM is given by Eq. (9.1.45). Therefore, the *predictive coding gain* is

$$G_{\text{DPCM}} = \frac{D_{\text{PCM}}}{D_{\text{DPCM}}} = \frac{\epsilon_s^2 \sigma_s^2}{\epsilon_p^2 \sigma_p^2} \quad (9.2.13)$$

In general, the prediction error  $\sigma_p^2$  reduces as the predictor order  $K$  increases.

With the LMMSE predictor given by Eq. (9.2.8), it can be shown [19] that the minimal prediction error when the predictor order goes to infinity is related to the power spectrum of the signal  $S(e^{j\omega})$  by

$$\sigma_{p,\min}^2 = \lim_{K \rightarrow \infty} \sigma_p^2 = \exp \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \log_e S(e^{j\omega}) d\omega \right). \quad (9.2.14)$$

The ratio  $\gamma_s = \frac{\sigma_{p,\min}^2}{\sigma_s^2}$  is called the *spectral flatness measure* of a signal. The predictive coding gain is thus inversely proportional to the spectral flatness. This is because a signal with narrowly concentrated spectrum is more amenable to prediction. A signal with flat spectrum (i.e., white noise) is unpredictable.

It can be shown that the integral in Eq. (9.2.14) can be written as [19]

$$\sigma_{p,\min}^2 = \lim_{K \rightarrow \infty} \left( \prod_k \lambda_k \right)^{1/N}, \quad (9.2.15)$$

where  $\lambda_k$  is the  $k$ -th eigenvalue of the  $N$ -th order covariance matrix of the signal.<sup>4</sup> On the other hand, the signal variance equals

$$\sigma_s^2 = \lim_{K \rightarrow \infty} \frac{1}{K} \sum_k \lambda_k \quad (9.2.16)$$

Therefore, the predictive coding gain when  $K \rightarrow \infty$  can be expressed as

$$\lim_{K \rightarrow \infty} G_{\text{DPCM}} = \frac{\epsilon_s^2 \lim_{K \rightarrow \infty} \frac{1}{K} \sum_k \lambda_k}{\epsilon_p^2 \lim_{K \rightarrow \infty} (\prod_k \lambda_k)^{1/N}}, \quad (9.2.17)$$

---

<sup>4</sup>It can be shown that  $\lambda_k = S(e^{j\omega_k})$ , where  $\omega_k$  are in general non-equidistant samples over the interval  $(-\pi, \pi)$ .

which is equal to the transform coding gain achievable with the KLT (cf. Eq. (9.1.47)), when the transform length  $N$  goes to infinity. Therefore, the two methods are equivalent when the transform length and the predictor order are both infinite. However, for any finite  $N$ , length  $N$  transform coding is less efficient than order  $N$  predictive coding [19]. This is because a length  $N$  transform only involves  $N$  samples at a time, whereas a predictive coder of any order has an effective memory that is infinite, because of the feedback of the quantization error in the closed-loop prediction.

### 9.2.3 Spatial Domain Linear Prediction

Let  $\psi(m, n)$  represent a video frame. In spatial domain linear prediction, a current pixel  $\psi(m, n)$  is predicted from its past neighboring pixels,  $\psi(k, l), (k, l) \in \mathcal{B}_{(m, n)}$  where  $\mathcal{B}_{(m, n)}$  represents a causal neighborhood of pixel  $(m, n)$ . The optimal predictor can be determined based on Eq. (9.2.6) if the correlations between  $\psi(m, n)$  and each of the pixels in  $\mathcal{B}_{(m, n)}$  are known.

**Example 9.6:** Consider the image source shown in Fig. 9.6 again. This time, we use a predictive coder, which predicts the pixel  $s_0 = \psi(m, n)$  (pixel D) from  $s_1 = \psi(m, n - 1)$  (pixel C),  $s_2 = \psi(m - 1, n)$  (pixel B), and  $s_3 = \psi(m - 1, n - 1)$  (pixel A), as shown in Fig. 9.6. Equation (9.2.6) is reduced to, in this case:

$$\begin{bmatrix} R(C, C) & R(C, B) & R(C, A) \\ R(B, C) & R(B, B) & R(B, A) \\ R(A, C) & R(A, B) & R(A, A) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} R(D, C) \\ R(D, B) \\ R(D, A) \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & \rho_d & \rho_v \\ \rho_d & 1 & \rho_h \\ \rho_v & \rho_h & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \rho_h \\ \rho_v \\ \rho_d \end{bmatrix}.$$

In the special case of  $\rho_h = \rho_v = \rho$ ,  $\rho_d = \rho^2$ , the optimal predictor is

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \rho \\ \rho \\ -\rho^2 \end{bmatrix}.$$

The MSE of the above predictor is, using Eq. (9.2.10),

$$\sigma_p^2 = R(0, 0) - \begin{bmatrix} R(0, 1) & R(0, 2) & R(0, 3) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = (1 - \rho^2)^2 \sigma_s^2.$$

With  $R = 2$  bit optimal scalar quantization, the quantization error is

$$D = \epsilon_{\text{Gaussian}}^2 \sigma_p^2 2^{-2R} = 0.000594 \epsilon_{\text{Gaussian}}^2. \quad (9.2.18)$$

The coding gain over PCM is

$$G_{DPCM} = \frac{\sigma_s^2}{\sigma_p^2} = \frac{1}{(1 - \rho^2)^2} = 105.19. \quad (9.2.19)$$

Comparing the above result with those in Examples 9.4 and 9.5, we see that, among the three methods which all involve a  $2 \times 2$  pixel block when coding, the predictive coding method is the most efficient. This is consistent with the theoretical result, that predictive coding using an  $N$ -th order predictor is better than transform coding with length  $N$  transform. Note that the theoretical analysis as well as the examples all assume optimal scalar quantization, which may not be realizable in practice. A major problem with predictive coding is that a transmission error in the coded bit stream that affects a single sample can lead to mismatch between the encoder and decoder, so that all the following reconstructed samples may be in error. This is known as the error propagation effect. The effect of transmission errors on coded bit streams and mechanisms to prevent or suppress such effect will be discussed later (Chap. 14).

Note that spatial prediction can be applied not only to original pixel values, but also transform coefficients. For example, in a block transform coder, the DC coefficients (i.e., the mean values) of adjacent blocks are often similar, and one may predict the DC value of a current block from those of the block above and to the left. In the JPEG image coding standard discussed in Sec. 9.1.7, the left block DC value is used for prediction. Usually, the correlation between AC coefficients of adjacent blocks are not strong enough to warrant the use of prediction.

In coders using wavelet transforms (Sec. 11.3), the coefficients in the same spatial position but different scales are often correlated. Therefore, prediction among these coefficients can also be applied. There, instead of predicting the actual coefficient value, a non-linear predictor is often used, which predicts locations of non-zero coefficients in a fine scale from the locations of non-zero coefficients in a coarser scale.

### 9.2.4 Motion Compensated Temporal Prediction

#### Uni-Directional Temporal Prediction

In addition to applying prediction within the same frame, one can predict a pixel value in a current frame from its corresponding pixel in a previous frame. Let  $\psi(\mathbf{x}, t)$  represent the pixel value in frame  $t$  at pixel  $\mathbf{x}$ , and let  $t_-$  denote the previous frame time. Then the prediction process is described by

$$\psi_p(\mathbf{x}, t) = \psi(\mathbf{x}, t_-). \quad (9.2.20)$$

This is known as linear temporal prediction. Such type of prediction is effective only if the underlying scene is stationary. In a real-world video, the objects in the scene as well as the camera are usually moving, so that the pixels with the same

spatial location in two adjacent frames can have very different values. In this case, *motion-compensated prediction* (MCP) is more appropriate, which uses

$$\psi_p(\mathbf{x}, t) = \psi(\mathbf{x} + \mathbf{d}(\mathbf{x}), t_-), \quad (9.2.21)$$

where  $\mathbf{d}(\mathbf{x})$  represent the motion vector (MV) of pixel  $\mathbf{x}$  from time  $t$  to  $t_-$ . Using the terminology introduced in Chap. 6, frame  $t$  is the anchor frame, and frame  $t_-$  is the tracked frame. In video coding literature, frame  $t_-$  is usually referred to as the *reference frame*, and frame  $t$  the *coded frame* or *current frame*, and  $\psi_p(\mathbf{x}, t)$  is called the *predicted frame*. The reference frame must be coded and reconstructed before the coded frame.

In a coder using MCP, both the MV and the prediction error need to be specified. Obviously, if one specifies a MV at every pixel plus a prediction error value, one may have to use more bits than specifying the original pixel value for each pixel. In a real coder, more compact motion representation is used to reduce the bit rate for coding the motion. One popular video coding method is to use the block-based motion representation, and code only one MV per block. We will discuss block-based video coders in Sec. 9.3.

The temporal predictors introduced thus far only use one pixel in a previous frame. This restriction results mainly from a practical implementation constraint, where storage of more than one previous frame is often too costly. Theoretically, however, using pixels from more than one previous frame can improve prediction accuracy. In this case, one can still use the optimal predictor design solution given in Eq. (9.2.6) to determine the predictor coefficients associated with different previous pixels.

### Bi-Directional Temporal Prediction

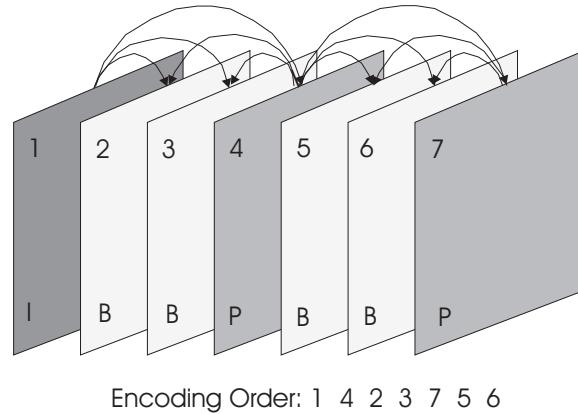
In fact, the predictor does not have to rely on pixels in previous frames only. In *bi-directional temporal prediction*, a pixel in a current frame is predicted from a pixel in a previous frame  $t_-$  as well as a pixel in a following frame  $t_+$ . The predicted value at frame  $t$  is described by

$$\psi_p(\mathbf{x}, t) = a_- \psi(\mathbf{x} + \mathbf{d}^-(\mathbf{x}), t_-) + a_+ \psi(\mathbf{x} + \mathbf{d}^+(\mathbf{x}), t_+) \quad (9.2.22)$$

where  $\mathbf{d}^-(\mathbf{x})$  and  $\mathbf{d}^+(\mathbf{x})$  represent the MV at  $\mathbf{x}$  from  $t$  to  $t_-$  and that from  $t$  to  $t_+$ . Typically, we call the prediction of the current frame from a previous ( $t_- < t$ ) reference frame *forward motion compensation*, and that from a future ( $t_- > t$ ) reference frame *backward motion compensation*.<sup>5</sup> Ideally, coefficients  $a_-$  and  $a_+$  should be determined using the optimal predictor solution given in Eq. (9.2.6), which requires the knowledge of the correlation between corresponding pixels in the three frames involved. In practice, they are typically chosen heuristically. For the predicted value to have the same mean value as the original value, they are chosen so that  $a_- + a_+ = 1$ .

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<sup>5</sup>This should not be confused with the backward and forward motion estimation defined previously in Sec. 6.2.



**Figure 9.12.** Video coding using both uni-directional and bi-directional temporal prediction. The arrows indicate the reference frames used for predicting a coded frame. Frames labeled I, P, and B are coded without prediction, with uni-directional prediction, and with bi-directional prediction, respectively.

The use of bi-directional temporal prediction can be very beneficial when there are regions in the coded frame that are uncovered by object motion. Such a region does not have a corresponding region in the previous frame, but only in the following frame. For example, in Fig. 5.12, the uncovered background in frame  $k+1$  does not have a correspondence in frame  $k$ , but it will have a correspondence in frame  $k+2$ . The prediction coefficients  $a_-$  and  $a_+$  are usually adapted based on the prediction accuracy. In the example considered above, one may choose  $a_- = 0, a_+ = 1$ . When correspondences can be found in both frame  $t_-$  and frame  $t_+$ , a good choice is  $a_- = a_+ = 0.5$ . As with uni-directional prediction, bi-directional prediction can be performed at a block level, so that only two MVs need to be specified for each block.

Note that the use of bi-directional prediction necessitates the coding of frames in an order that is different from the original temporal order. For example, a subset of frames may be coded first, using uni-directional prediction from the past coded frames only, and then the remaining frames can be coded using bi-directional prediction. This type of out-of-sequence coding is illustrated in Fig. 9.12. Although bi-directional prediction can improve the prediction accuracy and consequently the coding efficiency, it incurs encoding delay and is typically not used in real-time applications such as video phone or video conferencing. For example, the H.261/H.263 standards intended for interactive communications use only uni-directional prediction and a restricted bi-directional prediction (known as the PB-mode). On the other hand, the MPEG standard series, targeted mainly for video distribution, employ both uni- and bi-directional prediction.

### 9.3 Video Coding Using Temporal Prediction and Transform

A popular and effective video coding method is the one that uses block-based temporal prediction and transform coding. This method is essentially the core of all the international video coding standards. In this section, we first provide an overview of this coding method (Sec. 9.3.1). This will be followed by its extension that uses more sophisticated motion-compensated prediction (Sec. 9.3.2) and loop filtering (Sec. 9.3.5). As an optional advanced topic, we analyze the influence of motion estimation accuracy and loop filter on the coding efficiency (Sec. 9.3.5). Finally, we consider the issue of how to choose various coding parameters in this coding method to maximize the coding efficiency under the rate constraint imposed by the underlying application (Secs. 9.3.3 and 9.3.4).

#### 9.3.1 Block-Based Hybrid Video Coding

In this coder, each video frame is divided into blocks of a fixed size and each block is more or less processed independently, hence the name “block-based”. The word “hybrid” means that each block is coded using a combination of motion-compensated temporal prediction and transform coding. Figure 9.13 shows the key steps in this coding paradigm. First, a block is predicted from a previously coded reference frame using block-based motion estimation. The motion vector (MV) specifies the displacement between the current block and the best matching block. The predicted block is obtained from the previous frame based on the estimated MV using motion compensation.<sup>6</sup> Then, the prediction error block is coded, by transforming it using the DCT, quantizing the DCT coefficients, and converting them into binary codewords using variable length coding (VLC). The actual coding method follows that described in Sec. 9.1.7.<sup>7</sup> As with the JPEG standard, the quantization of the DCT coefficients is controlled by a *quantization parameter* (QP)<sup>8</sup>, which scales a pre-defined quantization matrix. A larger QP corresponds to large quantization stepsizes. Note that the encoder must emulate the decoder operation to deduce the same reconstructed frame as the decoder, so that there will be no mismatch between the reference frames used for prediction.

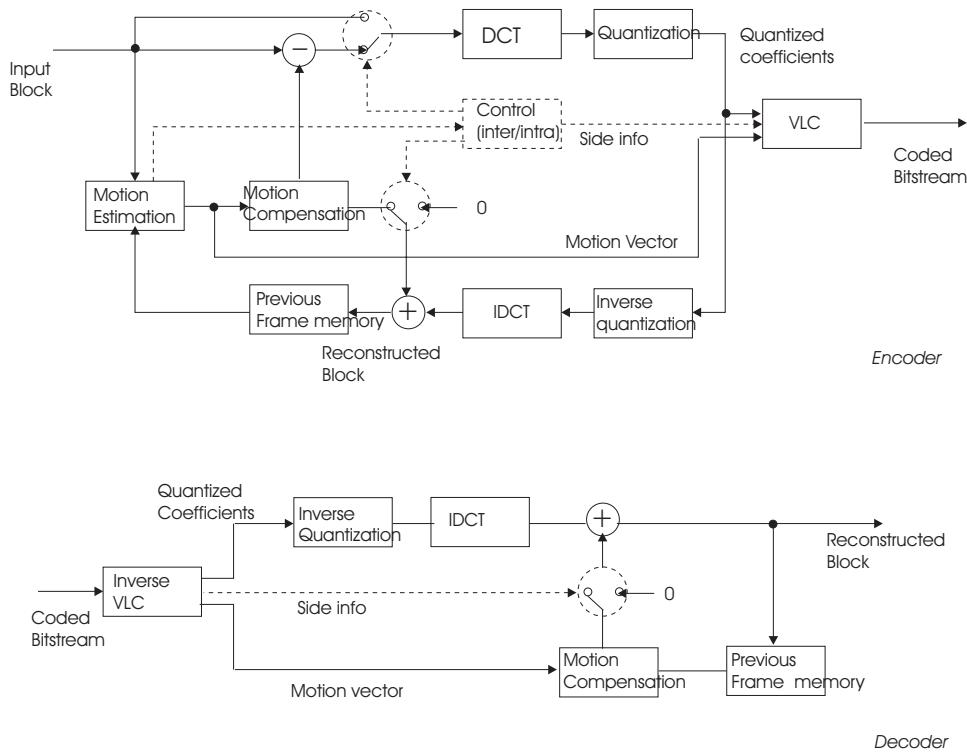
The above discussion assumes that temporal prediction is successful, in that the prediction error block requires fewer bits to code than the original image block. This method of coding is called *P-mode*. When this is not the case, the original block will be coded directly using transform coding. This is known as *intra-mode*. Instead of using a single reference frame for prediction, bi-directional prediction can be used, which finds two best matching blocks, one in a previous frame and

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<sup>6</sup>The phrase motion compensation here and subsequently in this book refers to motion compensated prediction.

<sup>7</sup>Ideally, if the prediction is very accurate, then the error pattern would be close to white noise and can be effectively coded using scalar quantization. In reality, however, the use of block-based motion model often leads to structured error patterns. Transform coding is used to exploit the correlation between error pixels.

<sup>8</sup>which is called quality factor in the JPEG standard.



**Figure 9.13.** Encoding and decoding process for a block in a typical block-based hybrid coding system.

another in a following frame, and uses a weighted average of the two matches as the prediction for the current block. In this case, two MVs are associated with each block. This is known as the *B-mode*. Both P- and B-modes are generally referred to as *inter-mode*. The mode information, the MVs, as well as other side information regarding picture format, block location, etc. are also coded using VLC.

In practice, the block size for motion estimation may not be the same as that for transform coding. Typically, motion estimation is done on a larger block known as macroblock (MB), which is sub-divided into several blocks for which the DCT is evaluated. For example, in most video coding standards, the MB size is 16x16 pels and the block size is 8x8 pels. If the color subsampling format is 4:2:0, then each MB consists of 4 Y blocks, 1 Cb block, and 1 Cr block. The coding mode (i.e., intra- or inter-mode) is decided at the MB level. Because MVs and DC coefficients of adjacent MBs or blocks are usually similar, they are typically predictively coded, using the MV and DC coefficient of the previous MB or block as the predicted values. In all the video coding standards described in Chap. 13, a number of MBs form a Group of Blocks (GOB) or a Slice, and several GOBs or Slices form a picture.

Size and shape of GOBs and Slices differ among the various video coding standards and picture sizes, and can often be tailored to the application's needs. Prediction of MVs and DC coefficients are usually restricted within the same GOB or Slice. That is, the predicted MV or DC value for the first MB or block in a GOB is set to some default value. This is to suppress the extent of error propagation when the compressed bit stream is corrupted by transmission/storage errors.

A frame may be coded entirely in the intra-mode, and such a frame is called an *intra-frame* or I-frame. The first frame of a sequence is always coded as an *intra-frame*. In applications employing high bitrate or with relaxed real-time constraints, intra-frames are also used periodically to stop potential error propagation, and to enable random-access. Low latency applications such as video conferencing cannot use such periodic refresh because the number of bits from an intra-frame is typically much greater than for a predicted frame, and thus will cause a sudden surge in the transmitted data. A *P-frame* uses only a past frame for prediction, and depending on the prediction accuracy, a MB can be coded in either intra- or P-mode, which is usually determined based on the prediction error. Finally, a *B-frame* uses bi-directional prediction, and a MB in a B-frame can be coded in intra-, P- or B-mode. A B-frame can only be coded after the surrounding I- or P-frames are coded. Both P- and B-frames are generally referred as *inter-frames*.

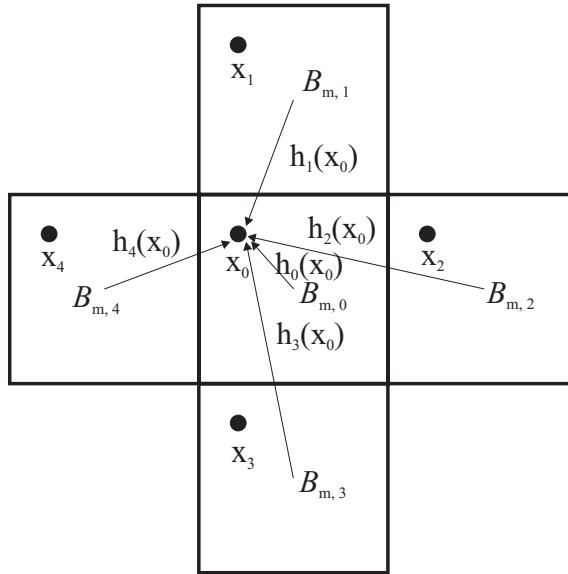
In MPEG-1 and MPEG-2 standards, the frames are partitioned into *group of pictures* (GOP's), with each GOP starting with an I-frame, followed by interleaving P- and B-frames. A typical GOP is illustrated in Fig. 9.12. This enables random access: one can access any GOP without decoding previous GOP's. The GOP structure also enables fast forwarding and rewinding: fast forwarding can be accomplished by decoding only I- or I- and P-frames. Fast rewinding can be realized by decoding only I-frames, in a backward order.

### 9.3.2 Overlapping Block Motion Compensation

In this and the next subsection, for notational simplicity, we do not specify the time index in the video function. Rather, we use  $\psi(\mathbf{x})$ ,  $\psi_r(\mathbf{x})$  and  $\psi_p(\mathbf{x})$  to represent the coded frame, the reference frame and the predicted frame, respectively. In block-based motion compensation, every pixel in a block is predicted by a single pixel in the reference frame using

$$\psi_p(\mathbf{x}) = \psi_r(\mathbf{x} + \mathbf{d}_m), \quad \mathbf{x} \in \mathcal{B}_m, \quad (9.3.1)$$

where  $\mathbf{d}_m$  represent the MV of the block  $\mathcal{B}_m$ . When  $\mathbf{d}_m$  is non-integer, an interpolation is needed to obtain  $\psi_r(\mathbf{x} + \mathbf{d}_m)$ . As described before, when the estimated MV is inaccurate, or when the block-wise translational model is not appropriate, the predicted image will suffer from the notorious blocking artifact. To overcome this problem, two approaches have been taken. One is to combine the motion estimates for neighboring blocks to produce the predicted value for a given pixel, and another is to apply filtering to the predicted image. We discuss the first approach in this section, and the filtering approach in the next section.



**Figure 9.14.** An example of overlapping block motion compensation using four neighboring blocks.

With overlapping block motion compensation (OBMC) [25, 26], a pixel is predicted based not only on the estimated MV for the block it belongs to, but also the MVs of its neighboring blocks, as illustrated in Fig. 9.14. Let  $K$  be the total number of neighboring blocks considered, and let  $\mathcal{B}_{m,k}$  and  $\mathbf{d}_{m,k}$ ,  $k \in \mathcal{K} = \{0, 1, 2, \dots, K\}$ , denote the  $k$ -th neighboring block and its MV, with  $\mathcal{B}_{m,0} = \mathcal{B}_m$ ,  $\mathbf{d}_{m,0} = \mathbf{d}_m$ . The predicted value is determined by

$$\psi_p(\mathbf{x}) = \sum_{k \in \mathcal{K}} h_k(\mathbf{x}) \psi_r(\mathbf{x} + \mathbf{d}_{m,k}), \quad \mathbf{x} \in \mathcal{B}_m. \quad (9.3.2)$$

The coefficient  $h_k(\mathbf{x})$  can be considered as the weight assigned to the estimated value based on  $\mathbf{d}_{m,k}$ . Intuitively, for a given  $\mathbf{x}$ ,  $h_k(\mathbf{x})$  should be inversely proportional to the distance between  $\mathbf{x}$  and the center position of  $\mathcal{B}_{m,k}$ . For example, with the neighborhood sketched in Fig. 9.14, for  $\mathbf{x}_0$ , the weight for  $\mathbf{d}_{m,1}$  and  $\mathbf{d}_{m,4}$  should be larger than that for  $\mathbf{d}_{m,2}$  and  $\mathbf{d}_{m,3}$ .

From an estimation-theoretic perspective, the predictor in (9.3.2) can be interpreted as a linear estimator of  $\psi(\mathbf{x})$  given the observations  $\psi_r(\mathbf{x} + \mathbf{d}_{m,k})$ . For a given  $\mathbf{x}$ , the optimal estimator  $\mathbf{h}(\mathbf{x}) = [h_k(\mathbf{x}), k \in \mathcal{K}]^T$  can be determined by minimizing the mean square prediction error

$$E\{|\psi(\mathbf{x}) - \sum_{k \in \mathcal{K}} h_k(\mathbf{x}) \psi_r(\mathbf{x} + \mathbf{d}_{m,k})|^2\}. \quad (9.3.3)$$

To preserve the mean value of the estimated image, the weights must be constrained so that

$$\sum_{k \in \mathcal{K}} h_k(\mathbf{x}) = 1.$$

Incorporating the above constraint in the optimization by using the Lagrange multiplier method gives [26]

$$\mathbf{h}(\mathbf{x}) = \mathbf{R}^{-1}(\mathbf{x}) \left( \mathbf{r}(\mathbf{x}) - \frac{\mathbf{i}^T[\mathbf{R}]^{-1}(\mathbf{x})\mathbf{r}(\mathbf{x}) - 1}{\mathbf{i}^T[\mathbf{R}]^{-1}(\mathbf{x})\mathbf{i}} \mathbf{i} \right), \quad (9.3.4)$$

where  $\mathbf{i}$  is a column vector of dimension  $K$  with all elements equal to one,  $\mathbf{R}$  and  $\mathbf{r}$  are the auto-correlation and cross-correlation matrices, respectively, with elements

$$\begin{aligned} R_{k,l}(\mathbf{x}) &= E\{\psi_r(\mathbf{x} + \mathbf{d}_{m,k})\psi_r(\mathbf{x} + \mathbf{d}_{m,l})\}, k, l \in \mathcal{K} \\ r_k(\mathbf{x}) &= E\{\psi(\mathbf{x})\psi_r(\mathbf{x} + \mathbf{d}_{m,k})\}, k \in \mathcal{K}. \end{aligned}$$

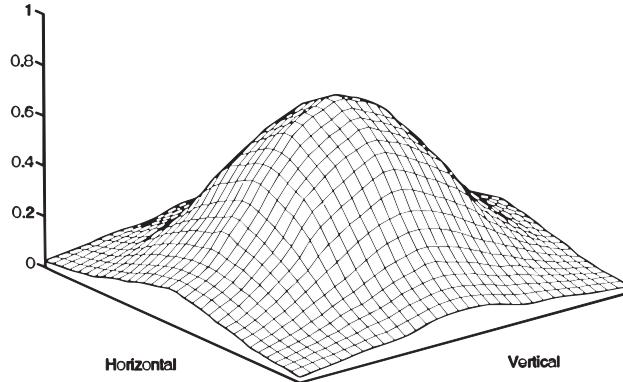
The above solution can be used to derive the weights for different neighboring block MVs for every pixel  $\mathbf{x}$  in a block. The expectation in the above equations is with respect to uncertainties in both image intensity as well as motion. The above statistics can be estimated from real video data. As indicated by the notation, they are in general spatially variant. However, assuming  $\psi(\mathbf{x})$  is stationary, the above statistics are only dependent on the relative position of the pixel in a block, not the absolute position with respect to the entire image. If the correlation between pixels is symmetric, then the solution  $h_k(\mathbf{x})$  will also be symmetric, so that only the solution for pixels in the upper left quarter of the block need to be determined.

In general, OBMC can be applied to block MVs estimated by any method, including the simple block matching method (BMA). However, the prediction error can be further reduced if motion estimation is accomplished by considering OBMC. Ideally, one should minimize the following modified MCP error:

$$\sum_{\mathbf{x} \in \mathcal{B}_m} |\psi(\mathbf{x}) - \sum_{k \in \mathcal{K}} h_k(\mathbf{x})\psi_r(\mathbf{x} + \mathbf{d}_{m,k})|^p \quad (9.3.5)$$

Because the error for a block depends on the MVs of its adjacent blocks, one cannot find one block MV at a time, as in the BMA case. One approach is to iteratively perform motion estimation and OBMC [26]. First, the conventional BMA is used to generate an initial set of MVs, where prediction error is calculated assuming (9.3.1) is used for MCP. Then OBMC is applied using these motion estimates. In the next iteration, for each block, the MV is estimated by minimizing the error given in Eq. (9.3.5), where the MVs for the neighboring blocks are assumed to be the same as those found in the previous iteration.

Instead of using iterations, a simpler approach is to apply non-equal weighting to the pixels in the current block as well as neighboring blocks, when calculating the prediction error using the BMA. To determine the proper weighting function, notice that the MV  $\mathbf{d}_m$  of block  $\mathcal{B}_m$  affects not only this block, but also its neighboring



**Figure 9.15.** The optimal window function for OBMC determined in [26, Fig. 4].

blocks  $\mathcal{B}_{m,k}$  when performing OBMC. Because the contribution of  $\mathbf{d}_m$  to pixel  $\mathbf{x}$  in  $\mathcal{B}_{m,k}$  is weighted by  $h_k(\mathbf{x})$ , we can minimize the following error

$$\sum_{k \in \mathcal{K}} \sum_{\mathbf{x} \in \mathcal{B}_{m,k}} |\psi(\mathbf{x}) - \psi_r(\mathbf{x} + \mathbf{d}_m)|^p h_k(\mathbf{x}) \quad (9.3.6)$$

Let  $\bar{\mathcal{B}}_m = \bigcup_{k \in \mathcal{K}} \mathcal{B}_{m,k}$  be the super block including  $\mathcal{B}_m$  and its neighboring blocks, the above error can be rewritten as

$$\sum_{\mathbf{x} \in \bar{\mathcal{B}}_m} |\psi(\mathbf{x}) - \psi_r(\mathbf{x} + \mathbf{d}_m)|^p \bar{h}(\mathbf{x}) \quad (9.3.7)$$

where the window function  $\bar{h}(\mathbf{x})$  is related to  $h_k(\mathbf{x})$ . For example, in Fig. 9.14,  $\bar{h}(\mathbf{x}_0) = h_0(\mathbf{x}_0), \bar{h}(\mathbf{x}_1) = h_3(\mathbf{x}_0), \bar{h}(\mathbf{x}_2) = h_4(\mathbf{x}_0), \bar{h}(\mathbf{x}_3) = h_1(\mathbf{x}_0), \bar{h}(\mathbf{x}_4) = h_2(\mathbf{x}_0)$ . The above error is a weighted MCP error over the super block  $\bar{\mathcal{B}}_m$ , and the window function  $\bar{h}(\mathbf{x})$  is a rearrangement of the original weighting coefficients  $h_k(\mathbf{x})$ . The optimal window function determined by Sullivan and Orchard is shown in Fig. 9.15.

Experimental results have shown that using OBMC can improve the prediction image quality by up to 1 dB, when OBMC is combined with standard BMA. When iterative motion estimation is applied, up to 2 dB can be obtained [26]. Due to its significant improvement in prediction accuracy, OBMC has been incorporated in the ITU-T H.263 standard for video coding, as an advanced option. The standard uses the four neighbor structure shown in Fig. 9.14, but only two of the neighbors have non-zero weights for each given pixel. The weighting coefficients  $h_k(\mathbf{x})$  used are given in Fig. 9.16, which are chosen to facilitate fast computation [17]. The equivalent window function  $\bar{h}(\mathbf{x})$  is shown in Fig. 9.17.

4 5 5 5 5 5 5 4	2 2 2 2 2 2 2 2	2 1 1 1 1 1 1 2
5 5 5 5 5 5 5 5	1 1 2 2 2 2 1 1	2 2 1 1 1 1 2 2
5 5 6 6 6 6 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
5 5 6 6 6 6 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
5 5 6 6 6 6 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
5 5 6 6 6 6 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
5 5 5 5 5 5 5 5	1 1 2 2 2 2 1 1	2 1 1 1 1 1 1 2
4 5 5 5 5 5 5 4	2 2 2 2 2 2 2 2	

(a)

(b)

(c)

**Figure 9.16.** (a) The weighting coefficients for OBMC, specified in the H.263 video coding standard [17]. (a) for prediction with motion vector of current block; (b) for prediction with motion vectors of blocks on top or bottom of current block; (c) for prediction with motion vectors of the blocks to the left or right of current block. The numbers given are 8 times of the actual weights. For example, to predict the pixel on the top left corner of the block, the weights associated with the MVs of the current MB, the top MB, and the left MB are  $4/8$ ,  $2/8$ , and  $2/8$ , respectively. For the pixel on the first row and the second column, the weights are  $5/8$ ,  $2/8$ ,  $1/8$ , respectively. (b) The equivalent

1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	2 1 1 1 1 1
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 2 2 2 2 2 1	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
2 2 2 2 2 2 2 2	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 1 2 2 4 5 5	2 2 2 2 2 2 2 2	2 1 1 1 1 1 1 1
1 1 2 2 5 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 2 2 5 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 2 2 5 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 2 2 5 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 2 2 5 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 2 2 5 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 1 2 4 5 5 5	2 2 2 2 2 2 2 2	2 1 1 1 1 1 1 1
1 1 1 2 4 5 5 5	1 1 2 2 2 2 1 1	2 2 1 1 1 1 2 2
1 1 1 2 4 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2
1 1 1 2 4 5 5 5	1 1 1 1 1 1 1 1	2 2 1 1 1 1 2 2

**Figure 9.17.** The window function corresponding to the weighting coefficients given in Fig. 9.16.

### 9.3.3 Coding Parameter Selection

In the hybrid coder described in Sec. 9.3.1, there are multiple choices that the encoder has to make, including the mode to use for each MB in an inter-coded

frame, the QP for each MB, the motion estimation methods and parameters (e.g., with or without overlapping, block size, search range), etc. Each combination of these coding parameters will lead to a different trade-off between the overall rate and distortion of the coded video. In this subsection, we address the issue of how to select these parameters.

In the earlier development of hybrid coders, such decisions are usually driven by heuristics. For example, in inter-coded pictures, the decision to code an MB in intra-mode or inter-mode can be based on the variance of the MB itself,  $\sigma_{\text{intra}}^2$ , and the variance of the motion compensation error,  $\sigma_{\text{inter}}^2$ .<sup>9</sup> If  $\sigma_{\text{intra}}^2 \leq \sigma_{\text{inter}}^2 + c$ , then the intra-mode is chosen over the inter-mode. This heuristic is motivated by the observation that the number of bits required to code a block at a given distortion is proportional to the variance of the block. The positive constant  $c$  is added to account for the additional bits required to code the motion vector in the inter-mode.

For improved performance, the choice between different parameters can be determined by a rate-distortion (R-D) optimization approach. Here, the bits required by using different parameters and the resulting distortions (say, the MSE) are determined, by actually coding the source using these parameter settings. The setting that yields the best trade-off between the rate and distortion is then chosen. Typically, there is a constraint on the average rate of the coded sequence, and the problem is to find the parameter setting that has the minimal distortion while meeting the rate constraint. Such a constrained optimization problem can be solved by either Lagrangian multiplier method [7] or the dynamic programming approach [2].

Let us consider an example of determining the coding modes for all MB's in a frame, assuming all other options are fixed and that the desired number of bits for the entire frame is  $R_d$ . Denote the distortion for the  $n$ -th MB using mode  $m_n$  by  $D_n(m_n)$  and the required number of bits by  $R_n(m_k, \forall k)$ . The reason that  $R_n$  depends on the coding modes for other MB's is because the motion vectors and DC coefficients are predictively coded from neighboring blocks. The problem is to

$$\begin{aligned} & \text{minimize} && \sum_n D_n(m_n), \\ & \text{subject to} && \sum_n R_n(m_k, \forall k) \leq R_d. \end{aligned} \quad (9.3.8)$$

Using the Lagrangian multiplier method, the above problem can be converted to

$$\text{minimize } J(m_n, \forall n) = \sum_n D_n(m_n) + \lambda \sum_n R_n(m_k, \forall k) \quad (9.3.9)$$

where  $\lambda$  must be chosen to satisfy the rate constraint.

Strictly speaking, the optimal coding modes for different MB's are interdependent. For the ease of understanding the basic concept, let us ignore the dependence of the rate  $R_n$  on the coding modes of other MB's, i.e., we assume  $R_n(m_k, \forall k) = R_n(m_n)$ . Then the coding mode for each MB can be determined one by one, by minimizing

$$J_n(m_n) = D_n(m_n) + \lambda R_n(m_n). \quad (9.3.10)$$

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<sup>9</sup>In practice, to reduce computation, the variance is often replaced by the sum of absolute differences between the value (original or prediction error) of each pixel and the block mean.

Given that there are only a small number of possible modes to choose from, the optimal mode for each block can be found by an exhaustive search.

Note that if  $m_n$  and consequently  $R_n$  is a continuous variable, then minimizing  $J_n$  is equivalent to setting  $\frac{\partial J_n}{\partial R_n} = 0$ , which leads to  $\frac{\partial D_n}{\partial R_n} = -\lambda$ . *This implies that the optimal mode in each MB is such that the resulting R-D slope  $\frac{\partial D_n}{\partial R_n}$  is the same among different MB's.*<sup>10</sup> In practice, there are only a finite number of modes to choose from. Each possible mode corresponds to an operating point on a piecewise linear R-D curve, so that each mode is associated with a range of R-D slopes. The optimal modes for different MB's found by minimizing  $J_n(m_n)$  above for a given  $\lambda$  will have similar ranges in their R-D slopes.

The result that different MB's should operate at the same R-D slope is a special case of a variety of R-D optimization problems involving multiple independent coding parameters. Recall that in the bit allocation problem in transform coding (Sec. 9.1.5), the optimal solution is such that the R-D slopes for different coefficients are the same (Eq. 9.1.40). In that case, the R-D slope is proportional to the distortion, thereby the optimal bit allocation yields identical distortion among different coefficients.

A difficult problem in the above approach is how to determine  $\lambda$  for a given desired rate. For an arbitrarily chosen  $\lambda$ , the above method will yield a solution that is optimal at a particular rate, which may or may not be close to the desired rate. A bisection method is commonly employed to search for the  $\lambda$  that will yield the desired rate. A closed-form relation between  $\lambda$  and the QP is derived in [35], which assumes the QP is chosen based on the target bit rate (See Sec. 9.3.4).

Rate-distortion optimized selection of coding modes was first considered by Wiegand, *et al.* [37]. There, to handle the interdependence among the coding modes of different MB's in the same frame and over adjacent frames, a dynamic programming scheme is employed to find the optimal coding modes for a set of MB's simultaneously. The paper by Sullivan and Wiegand [35] includes a good discussion on how to use a similar approach to determine a variety of coding options in the H.263 coder. It is worth noting that the R-D based approach leads to rather marginal gain over heuristic approaches within the H.263 framework (about 10% savings in the bit rate or 0.5 dB in PSNR) [35]. In practice, such gains may not be justified considering the quite significant increase in complexity. Thus, the R-D optimized approach mainly serves as a benchmark for evaluating the performance of a heuristic, yet practical approach.

The computationally most demanding step in the R-D based parameter selection approach is to collect the R-D data associated with different parameter settings (coding mode and QP and possibly different motion estimation methods). This would require actually coding all the MB's using all different parameters. To reduce

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<sup>10</sup>By definition,  $\frac{\partial D_n}{\partial R_n} = \lim_{\Delta R \rightarrow 0} \frac{\Delta D}{\Delta R}$ , i.e., the R-D slope measures the reduction in the distortion by each extra bit. The equal slope condition is equivalent to requiring all blocks operate in such a way that an extra bit in each block will reduce the error by the same amount. Otherwise, a block operating at a higher (resp. lower) slope should use more (resp. fewer) bits.

the computation, some R-D models have been proposed, which relate the rate and distortion, on the one hand, and the QP and coding modes, on the other hand [5, 24]. Once the R-D data are obtained, one can use either Lagrangian multiplier method or dynamic programming approach to find the optimal allocation. The Lagrangian multiplier method is simpler, but has sub-optimal performance, because it ignores interdependence between rates of adjacent MBs in the same frame or between neighboring frames.

In addition to coding parameter selection, R-D based methods can be applied in a variety of problems in image and video coding. One important area is motion estimation for video coding. Whereas traditional approaches to motion estimation focus on minimizing motion compensated prediction error only (cf. Chap. 6), R-D optimized approaches also consider the rate needed to code the resulting MV's. For example, the zero MV may be preferred over a non-zero MV, if it only leads to slightly higher prediction error, considering the extra bits required to code the non-zero MV. Furthermore, because MV's are coded in a predictive manner, smoother motion fields are preferred. Work on R-D optimized motion estimation can be found in [34, 11, 20, 32, 3].

For a more complete coverage on how to use the R-D theory to optimize image and video coding, the reader is referred to the review paper by Ortega and Ramchandran [28], which also contains a good exposition of Lagrangian and dynamic programming methods as optimization tools. More extensive coverage on this topic can be found in the book by Schuster and Katsaggelos [31].

### 9.3.4 Rate Control

One important issue in video coding is rate control, which refers to how to code a video so that the resulting bitstream satisfies a target bit rate. Within the framework of hybrid coding, this reduces to how to choose the coding parameters (frame rate, inter vs. intra, QP, etc.) to meet the rate constraint. We have already alluded to this problem in the previous subsection, where we described an R-D optimized approach for selecting the coding mode for each MB subject to a rate constraint.

For pleasant visual perception, it is better to represent a video so that it has a constant (or smoothly changing) quality. Because the scene activity (in terms of motion as well as texture complexity) changes in time, the required bits will change from frame to frame. Even if we accept changes in video quality, the bit rate cannot be made strictly constant, due to the use of VLC and possibly different coding modes on the frame level. Therefore, the best that one can hope to achieve are constant average rates over short intervals of time. The variability of the bit rate within each interval has to be handled by a smoothing buffer following the encoder. The output bits from the encoder first enter the smoothing buffer at a variable rate, and are then moved out of the buffer at a constant rate. The maximum time interval and the allowed variability within this interval depend on the delay requirement of the underlying application and the affordable buffer size.<sup>11</sup> A larger buffer will enable

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<sup>11</sup>Given the continuously diminishing cost of memory, it is reasonable to assume that the buffer

better video quality, but will also introduce longer delay.<sup>12</sup>

Rate control is typically accomplished in three steps:

1. Update the target average bit rate (in terms of bits/s) for each short time interval, also referred to as *rate update interval*<sup>13</sup>;
2. Determine the coding mode (e.g., I, P or B-frame) and the target bit budget for each frame to be coded in this interval, which is usually based on the target average rate for this interval and the current buffer fullness; and
3. Determine the coding mode and QP for each MB in a frame to meet the target rate for this frame.

Step 1 depends on the underlying applications. For video transport over a constant bit rate (CBR) network (such as a modem line or an ISDN channel), the target average rate should be constant. For networks that can accommodate variable bit rate (VBR), such as an ATM network, the coder should try to regulate the rate to maintain a constant video quality, and yet satisfying the average and peak rate requirement [14]. Finally, for networks where channel capacity and error characteristics vary in time, such as the best effort Internet and wireless channels, one needs to update the target average rate based on the channel conditions. The update interval depends on how fast the network conditions change. In all these applications, the target average bit rate per update interval should depend on not only the available bandwidth (which would need to be estimated in time-varying channels), but also the end-to-end delay requirement, and the current encoder buffer size (i.e., how many bits remain to be sent). *Essentially, the target rate should be set so that the bits from the current update interval as well as remaining bits in the buffer can reach the destination within the maximally allowed delay time.* In this sense, the rate constraint is determined by the delay constraint [27]. In lossy networks, one also needs to take into account of the fact that some previously sent data may have to be retransmitted in case they are damaged during transmission [13]. In [27], Ortega provides a good categorization of rate control problems and an extensive coverage on how to determine the target rate based on the underlying network characteristics and delay requirements. For rate estimation approaches designed for video streaming over Internet applications, see discussion in Sec. 15.3.1.

Step 2 is typically accomplished by choosing an appropriate frame rate based on the total rate budgeted for an update interval and assigning the same target rate for all the frames in the interval. Frames can be skipped when the buffer occupancy is in danger of overflow. This could happen when the bits used for

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can be as large as the end-to-end delay allows, and therefore, this physical limitation of the buffer size can be ignored [27].

<sup>12</sup>The encoder buffer is only one factor that contributes to the end-to-end delay of a video transmission. For a description of different factors contributing to the end-to-end delay, see Chap. 14.

<sup>13</sup>In practice, this may be done in a sliding window fashion. That is, at each new frame time, the target bit rate for a short period of time following this frame is determined.

the previous frames exceed the specified targets by too much, or the remaining bit budget for this interval is insufficient to code the video at the specified frame rate. This is the approach typically employed for interactive applications, where all the frames, except the first frame, are coded in the inter-mode, so as to maintain a fairly constant delay between frames. For broadcast or streaming of MPEG-2 coded video, an update interval usually corresponds to a GOP, which contains one I-frame followed by interleaving P- and B-frames. Typically, rate allocation within a GOP assumes some constant ratios between the bits used for I-, P-, and B-frames [16]. More sophisticated, R-D based approaches would try to allocate the total bits among frames based on the scene content, so that the average distortion is minimized. The look-ahead involved however will incur significant computations and additional coding delay.

Rate control within a frame (i.e., step 3) can be accomplished by adjusting the coding mode and QP for each MB, which can be done either based on some heuristic rules or using an R-D optimized approach, as described already in Sec. 9.3.3.

The above discussions focus on video transmission applications. For storage applications such as in DVD movies, the total bits used for a video are limited by the capacity of the storage medium, however the bit rate may change from time to time to accommodate changes in scene activity.

The rate control solutions discussed thus far only consider waveform-based coders. For object-based coders such as MPEG-4, rate control must consider rate allocation among multiple objects, and within each object, allocation among shape, motion and texture. The frame rates of different objects can also vary based on the relative importance of each object. These problems are addressed by Vetro, *et al.* [36].

Instead of producing a bit stream with a fixed target rate, another approach is to produce a scalable bitstream, from which the receiver can choose to extract only partial bits based on the available bandwidth. This approach is more appropriate when the same video is to be accessed by multiple users with different link capacities. Scalable coding is discussed in Chap. 11. The distribution of a scalable stream to users with different bandwidth capacities is considered in Chap. 15.

In video coding standards, rate control is not defined. Instead, the encoder has the flexibility to optimize rate control as long as the coded bitstream follows the standardized syntax. Rate control schemes used in the test models of different video coding standards will be described in Chap. 13.

A problem that is related to rate control is *rate shaping*. This refers to the function of an interface (called transcoder or filter) between the compression layer and the network transport layer or between two network segments, with which a pre-compressed video stream is converted so that the rate of the resulting stream matches the available network bandwidth. This subject is discussed in Sec. 15.3.1.

### 9.3.5 Loop Filtering\*

The sources for motion compensated prediction error can be classified into three groups: i) the motion estimation error, ii) noise in the reference image, which can be due to the accumulation of errors caused by motion estimation in prior frames and quantization errors, and iii) changes in the coded frame that are not due to motion, such as occlusion and illumination variation. In order to suppress primarily the second source of noise, *loop filtering* can be applied, which is a low-pass filter applied to the predicted image. Note that with non-integer motion estimates, an interpolation filter is used implicitly when one performs motion-compensated prediction. Similarly, loop filtering is accomplished implicitly with OBMC. However, as can be seen below, a filter can be specifically designed to reduce the effect of noise in the video signal. Motion compensation with filtering can be generally described by

$$\psi_p(x, y) = g(x, y) * \psi_r(x + d_x(x, y), y + d_y(x, y)), \quad (9.3.11)$$

where  $\psi_p(x, y)$  represents the predicted frame,  $\psi_r(x, y)$  the reference frame,  $g(x, y)$  the impulse response of the loop filter, and  $*$  denotes 2-D linear convolution. In the following, we derive the optimal filter following the approach of Girod [10, 8, 9].

To simplify the design, the following assumptions are made: i) the reference video frame is a stationary random field; ii) the errors in estimated motion vectors at different spatial positions are independent of each other; and iii) the noise term is independent of the reference frame. With these assumptions, the power spectral density (PSD) function of the error after motion compensated prediction can be shown to be [8]

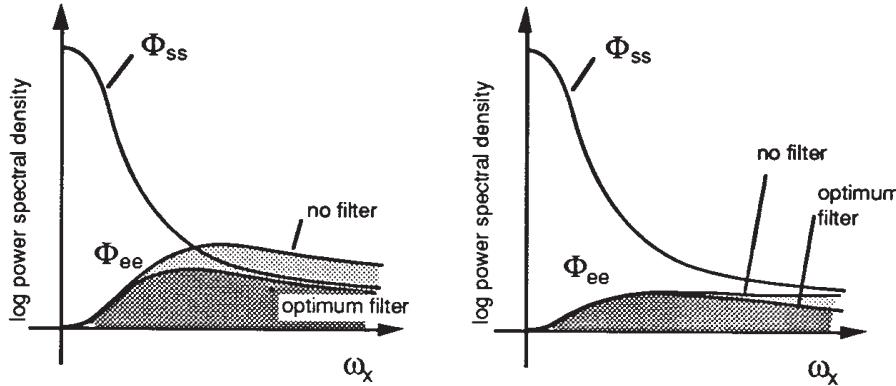
$$\begin{aligned} \Phi_{e,e}(f_x, f_y) &= \Phi_{s,s}(f_x, f_y) (1 + |G(f_x, f_y)|^2 - 2\Re\{G(f_x, f_y)P(f_x, f_y)\}) \\ &\quad + \Phi_{n,n}(f_x, f_y)|G(f_x, f_y)|^2. \end{aligned} \quad (9.3.12)$$

In the above relation,  $\Phi_{s,s}(f_x, f_y)$  and  $\Phi_{n,n}(f_x, f_y)$  represent the PSD functions of the reference frame and the noise signal, respectively,  $G(f_x, f_y)$  is the Fourier transform of  $g(x, y)$ ,  $P(f_x, f_y)$  is the Fourier transform of the probability density function (pdf) of the motion estimation error, and  $\Re$  denotes the real part of a complex variable. In this analysis, the signals are assumed to be discrete and the Fourier transforms are DSFT.

By differentiating (9.3.12) with respect to  $G(f_x, f_y)$ , we arrive at the optimal filter that minimizes the mean square prediction error:

$$G(f_x, f_y) = P^*(f_x, f_y) \frac{\Phi_{s,s}(f_x, f_y)}{\Phi_{s,s}(f_x, f_y) + \Phi_{n,n}(f_x, f_y)}, \quad (9.3.13)$$

where “ $*$ ” denotes complex conjugation. Note that, without the multiplication term  $P^*(f_x, f_y)$ , the above filter would have been the conventional Wiener filter with respect to the noise term. The factor  $P^*(f_x, f_y)$  takes into account the inaccuracy of motion estimation.



**Figure 9.18.** Power spectral density of the motion compensated prediction error for (a) moderately accurate motion estimation, and (b) very accurate motion estimation. From [10, Fig. 5.14]

In general  $S_{\psi,\psi}(f_x, f_y)$  has a low-pass characteristics. Therefore, the optimal filter  $G(f_x, f_y)$  is also low-pass in general. A careful examination of (9.3.12) will show that  $S_{e,e}(f_x, f_y)$  tends to be small at low spatial frequencies, and large at high frequencies. For example, if  $G(f_x, f_y) = 0$  for a certain high frequency range, the prediction error would have the same energy as the reference signal for this frequency range. This analysis shows that motion compensation works well for low-frequency components of the signal, but not for high frequency components, even with the optimal filter applied. Figure 9.18 shows  $\Phi_{e,e}(f_x, f_y)$  for two levels of motion estimation accuracy, with and without optimum filter applied. In both cases, it is assumed that  $\Phi_{n,n}(f_x, f_y) = 0$ .

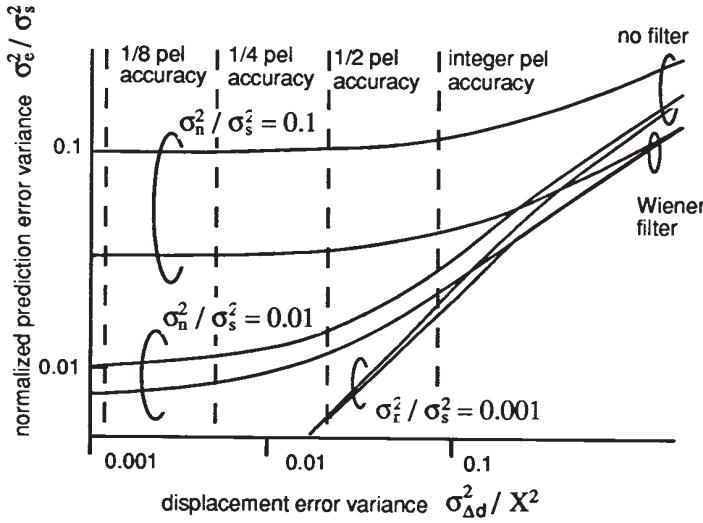
Loop filtering has been found to provide significant gain in prediction accuracy. In fact, the use of a loop filter is incorporated in the ITU-T H.263 video coding standard.

#### Relation Among Prediction Error, Motion Estimation Accuracy and Loop-Filtering\*

We now analyze the relation among the prediction error, motion estimation error, and noise power, with or without the application of optimum filtering, following the approach of Girod [8, 9, 10]. From the PSD of the prediction error in (9.3.12), one can determine the variance of the prediction error (i.e., the MSE of prediction) by

$$\sigma_e^2 = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \Phi_{e,e}(f_x, f_y) df_x df_y. \quad (9.3.14)$$

Assuming that the signal spectrum is isotropic with variance  $\sigma_s^2$ , the noise has a flat spectrum with variance  $\sigma_n^2$ , and the motion estimation error follows a Gaussian



**Figure 9.19.** Influence of motion compensation accuracy on prediction error variance for noisy signals. From [10, Fig. 5.15]

distribution with variance  $\sigma_{\Delta d}^2$ , Girod calculated the prediction error variances for various motion estimation accuracy and signal-to-noise ratio. Figure 9.19 shows the results of his analysis. The most interesting observation is that *there is a critical point for motion estimation accuracy, beyond which the possibility of further improving prediction is small*. This critical point increases with the noise level in the signal. That is, when the signal is very noisy, there is no point in trying to estimate the motion very accurately. Even with very accurate motion estimation, the predicted frame is still error prone due to the presence of noise. Also revealed by this figure is that Wiener filtering can be very helpful when the noise level is high, but not very effective in the low noise case.

In Fig. 9.19, the motion estimation error variance has been related to the search accuracy in terms of pixels. This is done by assuming the motion estimation error is due entirely to the rounding to a given fractional accuracy. The vertical dashed lines indicate the minimum displacement error variances that can be achieved with the indicated motion compensation accuracy. We can see that, in the low-noise case, no more than 1/8-pel accuracy is needed, whereas for the high noise case, 1/2-pel accuracy is quite sufficient. The above analysis results have been validated by actual experimental results [9]. It has been observed that for high quality broadcast video signals, motion estimation with 1/4-pel accuracy is sufficient for a practical coder, whereas for videophone type signals, 1/2-pel accuracy seems to be a desirable limit. In both cases, the gain over integer pel accuracy is significant.

## 9.4 Summary

### Transform Coding (Sec. 9.1)

- Transform is a way to represent a group of samples (e.g. an image block) as a linear combination of some basic patterns. The purpose of transform for coding is to decorrelate original samples and compact the energy into a few coefficients so that scalar quantization can be effectively applied. The optimal transform under these criteria is the KLT.
- To achieve maximum gain from transform coding, one must appropriately distribute bits among the coefficients. The optimal allocation (Eq. (9.1.43)) equalizes the distortion incurred by different coefficients (Eq. (9.1.44)). The final distortion depends on the geometric mean of the coefficient variances.
- The KLT depends on the second order statistics of the signal and thus is image dependent and difficult to compute. DCT is a good approximation of the KLT for most image signals and it can be computed using fast algorithms.
- DCT based coding method (Sec. 9.1.7) is an effective method for image coding and has been employed in international standards for image and video coding.
- Optimization of quantization and binary encoding process is very important in a transform coder. With similar quantization and binary encoding schemes, DCT and wavelet transforms lead to similar coding efficiency.

### Predictive Coding (Sec. 9.2)

- The purpose of prediction is also to reduce the correlation between samples to be coded, so that scalar quantization can be applied effectively. The predictor should be designed to minimize the prediction error. The LMMSE predictor can be determined by solving the Yuler-Walker equation (Eq. (9.2.6)).
- To avoid the mismatch between the reference sample used for prediction in the encoder and those in the decoder, closed-loop prediction is necessary, in which the encoder must repeat the same operation of the decoder.
- Theoretically, predictive coding outperforms transform coding when the predictor order is the same as the transform length. However, the bit stream resulting from predictive coding is very sensitive to transmission errors.
- For video coding, prediction can be performed in both spatial and temporal domain. In the temporal direction, motion compensation is necessary to account for the effect of object motion. Only very low order predictors (usually using only one previous frame) are used, to maintain a reasonable complexity, and to reduce the effect of transmission error propagation.

**Block-Based Hybrid Video Coding (Sec. 9.3)**

- The block-based hybrid coder (Sec. 9.3.1) combines motion compensated prediction and transform coding effectively. Because of its relatively low complexity and good coding efficiency, it has been adopted in various international standards for video coding (Chap. 13).
- Motion compensation accuracy can be improved by applying loop-filtering (Sec. 9.3.5), which suppresses the propagation of quantization errors in previous frames. The achievable motion compensation accuracy depends not only on the motion estimation accuracy but also the loop-filter used and the noise level in the signal (Sec. 9.3.5).
- Within the framework of hybrid coders, the performance can be improved by appropriately performing motion estimation and compensation (e.g. with or without overlapping, fixed vs. variable block size) and selecting modes of operations (intra vs. inter, prediction mode, etc.) The R-D optimized approach chooses these coding options so that the distortion is minimized under a given rate constraint. See Sec. 9.3.3.
- Rate control (Sec. 9.3.4) is an important issue in video coding, especially for real-time applications, where excessive rates will lead to deterioration (as opposed to improvement) in quality due to transmission delay and loss. Rate control in hybrid coders are typically accomplished by adjusting frame rates and quantization parameters, using either heuristic or R-D optimized approaches.
- Coding mode selection and rate control can affect the performance of a coder significantly. Note that all the video coding standards using the hybrid coding framework only define the bit stream syntax and consequently the decoder operation. This gives the encoder flexibility in optimizing its operation, including motion estimation, coding mode selection and rate control. These are typically the factors that differentiate different video coding systems that all conform to the same standard. Beyond coding efficiency considerations, another factor that can set apart different systems is how does the encoder makes the bit stream more resilient to transmission errors by judiciously selecting the coding parameters within the standard framework, and how does the decoder react to transmission errors. These issues are discussed in Chap. 14.

**9.5 Problems**

- 9.1** Prove the relations in Eqs. (9.1.11) to (9.1.16).
- 9.2** Confirm that the 1-D DCT basis functions form an orthonormal basis.
- 9.3** Prove that a 2-D basis formed by the outer product of two orthonormal 1-D bases is also orthonormal.

- 9.4** Repeat Example 9.4 for the special case of  $\rho_h = \rho_v = \rho_d = \rho$ .
- 9.5** Repeat Example 9.5 for the special case of  $\rho_h = \rho_v = \rho_d = \rho$ . Compare the KLT transform basis with the DCT.
- 9.6** Repeat Example 9.6 for the special case of  $\rho_h = \rho_v = \rho_d = \rho$ .
- 9.7** Consider a predictive image coding system in which a pixel is predicted from two pixels: one above and one to the left. Assume the image statistics is the same as that considered in Example 9.4. Derive the optimal predictor and the predictive coding gain. Compare the result with that in Example 9.6. Which predictor is more efficient?
- 9.8** Write a C or Matlab code that examines the effect of approximating an image with a partial set of DCT coefficients. Use  $8 \times 8$  DCT, reconstruct the image with  $K < 64$  coefficients, for  $K = 4, 8, 16, 32$ . How many coefficients are necessary to provide a satisfactory reconstruction?
- 9.9** Write a C or Matlab code that examines the effect of quantization in the DCT domain. Use  $8 \times 8$  DCT, and chose the quantization stepsizes to be multiples of the normalization matrix given in Fig. 9.7. Try scale factors of 0.5, 1, 2, 4, 8, 16. What is the largest scale factor that still provides a satisfactory image?
- 9.10** Two simple transform coding methods are: i) only retain first  $K$  coefficients (called *zonal coding*), and ii) only retain the coefficients that exceed a certain threshold (called *threshold coding*). Discuss the pros and cons of these two methods.
- 9.11** Consider the following three video coding methods: i) directly code a frame; ii) code the difference between every two frames; iii) perform block-based motion estimation between every two frames and code the motion compensation error image. Recall that the bit rate required to achieve a given distortion is proportional to the variance of the signal to be coded. Take two adjacent video frames from a sequence, calculate and compare the variances of i) a frame directly; ii) the direct difference between the two frames; and iii) the motion compensation error image. Based on the result you get, can you determine which of the three coding methods would be more efficient ?
- 9.12** Variance is only a crude indicator of the bit rate required. If we apply DCT coding in each of the three methods in Prob. 9.11, a more accurate measure of the bit rate is the number of DCT coefficients that are non-zero after quantization with a chosen quantization parameter. Write a C or Matlab code to perform this experiment.
- 9.13** Show that the optimal linear estimator solution for OBMC is as given in Eq. (9.3.4).

- 9.14** Show that the window function corresponding to the weighting coefficients given in Fig. 9.16 is as given in Fig. 9.17. Draw it in the style of Fig. 9.15 and comment on the similarity and difference between the two window functions.
- 9.15** Write a C- or matlab program that implements a basic form of the block-based hybrid coder. For simplicity, only consider the intra-mode and the P-mode. Except the first frame, which will be coded using the intra-mode entirely, each MB in following frames will be coded using either the P-mode or intra-mode, depending on whether the original MB or the motion compensation error has the smaller variance. For motion estimation, you can choose to implement either the EBMA or some fast algorithms. For the lossless coding part, you can either use the Huffman tables given a standard, e.g., [17], or estimating the required bit rate by calculating the entropy of the symbols to be coded. Apply the program to several test sequences with different motion levels using different parameters: QP, frame rate, intra-fresh rate, etc. Evaluate the trade-off between rate and distortion and complexity (execution time). Play the decompressed video to observe the coding artifacts with different QP's.

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# CONTENT DEPENDENT VIDEO CODING

In Section (8.1), we gave an overview on different video coding systems. One way of distinguishing different coding systems is to compare their source models. Video coding based on source models that do not adapt to the shape of moving objects in a video sequence were described in Chapter 9. In this chapter, we focus on video coding algorithms that consider the shape of the objects in the video sequence. In order to efficiently transmit arbitrarily shaped video objects, we have to code the object shape as well as the texture for these arbitrarily-shaped objects. In Section 10.1 we describe several methods for coding the 2D shape of an object. Section 10.2 presents algorithms to efficiently code the texture of these segments. Two algorithms coding shape and texture jointly are briefly described in Section 10.3. In the following sections 10.4 and 10.5, we describe how these techniques can be used in region-based and object-based coding, respectively. Object-based coding may be based on 2D and 3D object models. Using 3D object models, object-based coding can be extended to knowledge-based and semantic coding as described in Sections 10.6 and 10.7. In Section 10.8 we present a concept that allows to integrate conventional block-based hybrid coders with object-based coding, knowledge-based coding and semantic coding.

### 10.1 2D Shape Coding

The 2D shape of an object, say the  $k$ -th object, is defined by means of an *alpha*-map  $M_k$ :

$$M_k = \{m_k(x, y) | 0 \leq x \leq X, 0 \leq y \leq Y\}, \quad 0 \leq m_k \leq 255. \quad (10.1.1)$$

The shape  $M_k$  defines for each pel  $\mathbf{x} = (x, y)$  whether it belongs to the object ( $m_k(\mathbf{x}) > 0$ ) or not ( $m_k(\mathbf{x}) = 0$ ). For an opaque object, the corresponding *alpha*-values are 255, for transparent objects they range from 0 to 255. Usually, the *alpha*-map has the same spatial and temporal resolution as the luminance signal of the video sequence. In video editing applications, the *alpha*-map is used to describe object shape and object transparency. Let us assume that we have a background

image  $\psi_b(\mathbf{x})$ , the object represented by image  $\psi_o(\mathbf{x})$  and the *alpha*-map  $m_o(\mathbf{x})$ . Overlaying the object on the background is done according to

$$\psi(\mathbf{x}) = \left(1 - \frac{m_o(\mathbf{x})}{255}\right) \cdot \psi_b(\mathbf{x}) + \frac{m_o(\mathbf{x})}{255} \cdot \psi_o(\mathbf{x}) \quad (10.1.2)$$

The amplitude of the *alpha*-map determines how visible the object becomes. In this chapter, we will only deal with binary object shape, i.e.  $m_k(\mathbf{x}) \in \{0, 255\}$

There are two classes of binary shape coders. A *bitmap*-based coder (Section 10.1.1) encodes for each pel whether it belongs to the object or not. A *contour*-based coder (Section 10.1.2) encodes the outline of the object. In order to retrieve the bitmap of the object shape, the contour is filled with the object label. In the case that there is also texture transmitted with the shape information, an *implicit* shape coder can be used where the shape information can be derived from the texture (see Section 10.3). Similar to texture coding, binary shapes can be coded in a lossless or lossy fashion. Therefore, it is important to define a measure for the quality of lossily encoded shapes (Section 10.1.3).

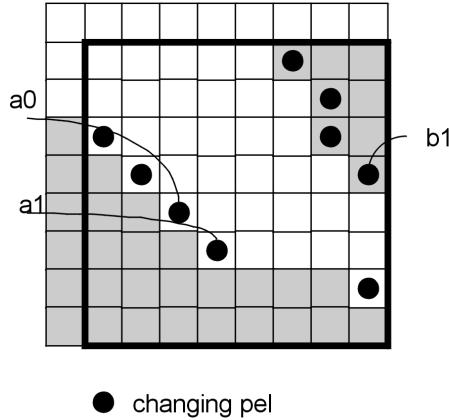
### 10.1.1 Bitmap Coding

A bitmap-based shape coder specifies a binary *alpha*-map. In its simplest form, we scan the *alpha*-map and transmit for each pel a 0 or 1 depending on whether it belongs to the object. Such an algorithm is inefficient because it does not consider statistical dependencies between neighboring pels. In this section, we describe three ways of improving coding efficiency: One class scans the bitmap in scan line order and specifies the run-lengths of black and white pels, a second class of algorithms codes each pel of the bitmap based on the colors of the neighboring pels, and the third class defines the shape as a tree of basic shapes arranged such that they completely fill the object shape.

#### Scan Line Coding

In this method, we scan the image line by line and transmit the run of black and white pels using a variable length coder thus exploiting one-dimensional correlation between pels. This method is used in the FAX standard G3 [36]. Next we discuss an extension of this coder that also considers the correlation between neighboring scan lines.

The *modified READ code* (Relative Element Address Designate) as used in the FAX standards G4 [20] and JBIG [21] codes one scan line with respect to the previous scan line thus exploiting 2D correlation. The algorithm scans each line of the document and encodes the location of *changing pels* where the scan line changes its color (Fig. 10.1). In this line-by-line scheme the position of each changing pel on the current line is coded with respect to either the position of a corresponding changing pel in the reference line, which lies immediately above the present line, or with respect to the preceding changing pel in the current line [36]. Figure 10.1 is used to explain one implementation of a modified READ coder. We assume that



**Figure 10.1.** Changing pels define the object boundary for a modified READ coder.

the top 5 rows of the block have already been coded, hence the coder knows the positions of the pels  $a_0$  and  $b_1$  in the current block. The unknown point  $a_1$  on the object boundary is encoded with reference to the two pels  $a_0$  and  $b_1$ . Pel  $a_0$  is the last boundary pel encoded prior to  $a_1$ . Pel  $b_1$  is the first changing pel on the line above  $a_0$  to the right of  $a_0$  and with the opposite color of  $a_0$ , if such point exists. If not, then  $b_1$  is the leftmost changing pel on the same line as  $a_0$ . In order to encode the distance between  $a_1$  and  $a_0$ , one of the three modes, Vertical, Horizontal or Vertical Pass, is selected. Assuming that all pels are numbered in raster-scan order starting with 0 in the top-left corner of the block, e.g. in Fig. 10.1  $\text{num}(a_0) = 34$ , and columns are numbered from left to right, e.g.  $c(a_0) = 2$ , a mode is selected according to

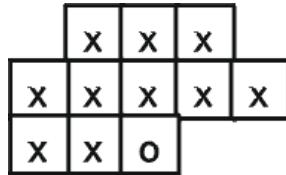
$$\text{mode} = \begin{cases} \text{Vertical} & \text{if } |c(a_1) - c(b_1)| \leq T \\ \text{Horizontal} & \text{else if } \text{num}(a_1) - \text{num}(a_0) < W \\ \text{Vertical Pass} & \text{else} \end{cases} \quad (10.1.3)$$

where  $T$  is a threshold and  $W$  the width of the block to be coded. The value of  $T$  depends on the length of the scan lines and can be as low as  $T = 5$ . In the example of Fig. 10.1,  $a_1$  will be coded in the vertical mode if  $T = 5$ .

In Vertical Mode, the distance  $c(a_1) - c(b_1)$  is encoded using one of eight VLC tables that is selected according to the object boundary direction as defined by a template positioned above pel  $b_1$  (Fig. 10.1).

In Horizontal Mode, the position of  $a_1$  is encoded as its distance to  $a_0$ . Just due to the fact that the horizontal and not the vertical mode is selected, the decoder can sometimes deduct the minimum distance between  $a_1$  and  $a_0$ . In this case, only the difference with respect to this minimum distance is encoded.

Vertical Pass Mode is selected according to Eq. 10.1.3 when there is at least one row of points following  $a_0$  without a boundary pel. In this case, we send one



**Figure 10.2.** Template for defining the context of the pel (o) to be coded.

codeword for each line without an object boundary. One last codeword codes the remaining distance to the next point on the object boundary.

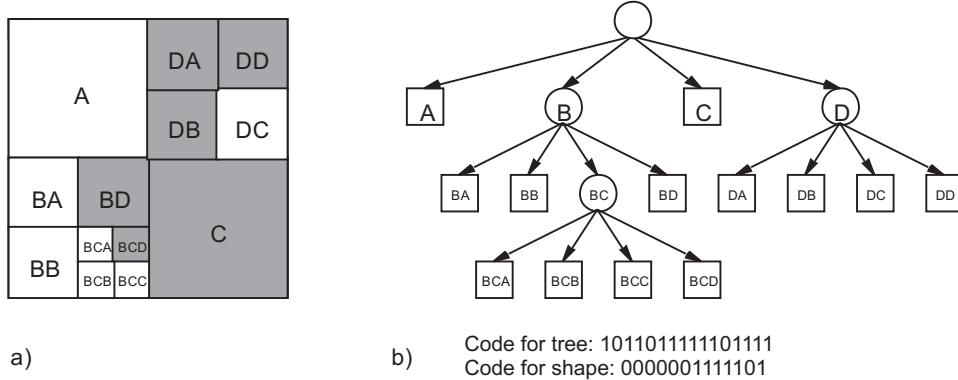
### Pel-wise Coding

This coder exploits the spatial redundancy of the binary shape information to be coded. Pels are coded in scan-line order and row by row. The color of each pel is coded using conditional entropy coding (cf. Sec. 8.3.1). A template of  $n$  pels is used to define the causal context for predicting the color of the current pel. Figure 10.2 shows a template that uses 10 pels to define the context for coding the current pel. The template extends up to 2 pels to the left, to the right and to the top of the pel to be coded [5]. This context, i.e. the colors of the pels in the template, determines the code that we use for encoding the current pel. Because each pel can only have one of two colors, there are a total of  $2^{10} = 1024$  different contexts. For a given context, since each pel can only choose from two possible colors, using a Huffman code will not lead to an increase of the coding efficiency due to the minimum codeword length of 1 bit. However, we can code the color of the pel with less than one bit by using an arithmetic coder (Sec. 8.4.2). For each context the arithmetic encoder uses a different probability table. For the template of Fig. 10.2, we would require 1024 probability tables. Given that we have only to store the probability of one symbol (say, black) and can compute the probability of the other symbol (say, white), we need only to store 1024 probabilities.

In the shape coding method of the MPEG-4 standard, the image is partitioned into square blocks. Three different types of blocks are distinguished: Transparent and opaque blocks as well as boundary blocks on the object boundary containing transparent as well as opaque pels. The boundary blocks are coded using the above method, and this coding method is called *context-based arithmetic coding* [26].

### Quad-tree Coding

A quad-tree describes the shape of an object by placing non-overlapping squares of different sizes inside the shape such that the shape gets described as accurate as possible. Typically, the size of the squares are powers of 2. We define a maximum size and a minimum size for the squares. We can describe shapes exactly if the minimum square size is 1 pel. In a first step, we place squares of maximum size  $M \times M$  pels next to each other on the image. Then, we decide based on a homogeneity



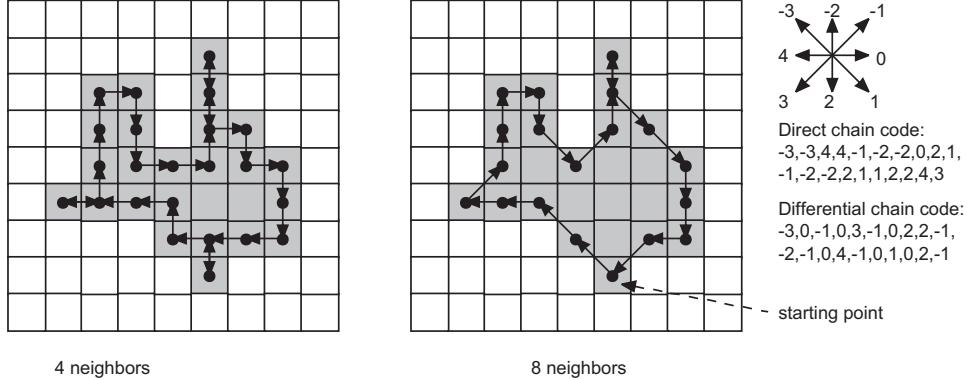
**Figure 10.3.** (a) An image is divided into a quadtree that describes the object shape. (b) Related quadtree with code that describes the tree and the label of each square such that the shape can be recovered.

criterion whether we need to subdivide a square. If yes, then we replace the square with four squares of size  $M/2 \times M/2$ . We repeat this process recursively (Fig. 10.3). This process creates an exact or an approximate shape approximation depending on the homogeneity criterion and the minimum square size. The homogeneity criterion defines the number of pels  $n$  that are allowed to be misrepresented in any given square. This number can either be fixed or vary as a function of the size of the square that is currently considered.

The quad-tree can be encoded with binary symbols. We walk through the tree in a depth-first fashion. In the example of Fig. 10.3, a ‘1’ indicates that the associated square is not further divided, a ‘0’ indicates that the square is subdivided and that the next four symbols indicate the states of the children of this square. Again, this description is repeated recursively. After the tree is described, we have to indicate for each square whether it belongs to the object. This is done by sending a binary symbol for each end node in the tree. In Fig. 10.3, we use ‘1’ for squares that belong to the object. Hence, this shape code requires as many symbols as we have ‘1’ in the quad-tree code. Using multilevel symbols for the shape code allows us to describe several objects within an image with just one quad-tree. See Prob. 10.6.

### 10.1.2 Contour Coding

Extensive work has been published on contour-based shape representation and coding. Different applications nurtured this research: For lossless and lossy encoding of object boundaries, chain coders [15, 12], polygon and higher order approximations [18, 49, 39, 16] were developed. For recognition purposes, shape representations like Fourier descriptors were developed to allow translation, rotation and scale invariant shape representations [56].

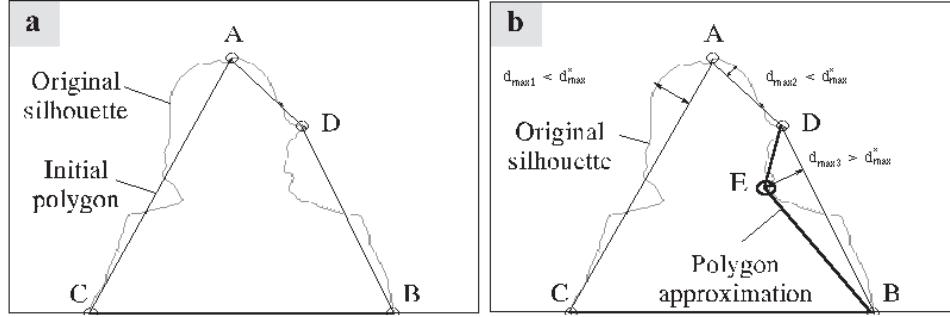


**Figure 10.4.** Chain code for pels with 4 and 8 neighbors. We give examples for a direct chain code with the symbols  $c_i$  and for a differential chain code with the symbols  $d_i$  of an 8-connected chain. The first symbol of the two codes are identical and define the starting direction. The following symbols of the differential chain code are created by aligning direction ‘0’ of the direction star with the direction of the last coded symbol.

### Chain Coding

A chain code follows the contour of an object. After coding the coordinates of a starting point on the object boundary, the chain code encodes the direction in which the next boundary pel is located (Fig. 10.4). Chain codes can code the direction of the contour (direct chain code) or the change in direction (differential chain code) [10]. The differential chain codes are more efficient because it employs the statistical dependency between successive links. Algorithms differ by whether they consider a pel having 4 or 8 neighbors for rectangular grids or six neighbors for hexagonal grids. In Fig. 10.4 we define that the boundary pels are part of the object. However, this leads to the ‘dual shape’ problem, e.g. the white pels in Fig. 10.4 have their own set of boundary pels. This boundary pel definition causes coding of redundant information if we have two objects that touch each other. Several algorithms define the object boundary between pels thus avoiding this problem [47]. Chain codes are lossless shape coders, i.e. they describe a given contour exactly. However, we can preprocess, e.g. smooth or quantize, a contour prior to coding with a chain code. Sometimes, this preprocessing is not considered separately from the chain coder. In that case, chain coding may be considered as a lossy coder.

Freeman [15] originally proposed the use of chain coding for boundary quantization and encoding, which has attracted considerable attention over the last thirty years [45, 29, 37, 25]. The curve is quantized using the grid intersection scheme [15] and the quantized curve is represented using a string of increments. Since the planar curve is assumed to be continuous, the increments between grid points are limited to the 8 grid neighbors, and hence an increment can be represented by 3 bits. For lossless encoding of boundary shapes, an average rate between 1.2 bits/boundary



**Figure 10.5.** Polygon approximation of a contour. The quality measure  $d_{max}^*$  is shown. A point  $E$  is added during the refinement of the contour description at the location with the largest  $d_{max}^*$ .

pel and 1.4 bits/boundary pel is required [12].

There have been many extensions to this basic scheme such as the generalized chain codes [45], where the coding efficiency has been improved by using links of different length and different angular resolution. In [25] a scheme is presented which utilizes patterns in a chain code string to increase the coding efficiency. There has also been interest in the theoretical performance of chain codes. In [29] the performance of different quantization schemes is compared, whereas in [37] the rate distortion characteristics of certain chain codes with preprocessing are studied.

Some chain codes also include simplifications of the contour in order to increase coding efficiency [38]. This is similar to filtering the object shape with morphological filters and then coding with a lossless chain code. Simplifications correspond to amplitude quantization in image coding. Whereas the chain code is limited to a lossless shape representation, the following shape coding algorithms allow for a lossy shape representation.

### Polygon Approximation

A polygon-based shape representation was developed for object-based analysis-synthesis coding (OBASC) [18, 19]. The object shape is approximated by a continuous chain of line segments (Fig. 10.5). Intuitively we can assume that a polygon representation is very efficient for describing geometric objects with straight edges using just a few line segments. Curved boundaries will require many line segments in order to provide a natural looking boundary.

Since a polygon representation allows for lossy shape coding, we need to define a quality measure by which we can evaluate the quality of the coded shape. The maximum of the Euclidean distances between the original and the approximated contour positions,  $d_{max}$ , is commonly used.

The efficiency of a polygon approximation depends not only on the object shape but also on the location of the starting point for the contour description. Instead of

computing the representation for all starting points and choosing the most efficient one, the polygon approximation is computed by using those two contour points with the maximum distance between them as the starting point. Then, additional points are added to the polygon where the approximation error between the polygon and the boundary are maximum. In Fig. 10.5a, the initial approximation  $\overline{AB}$  is extended by point  $C$  and then  $D$ . This is repeated until the shape approximation error is less than a threshold  $d_{max}^*$ . Figure 10.5b shows that the point  $E$  is added to the approximation where the largest error  $d_{max}$  is measured. Advanced algorithms select vertices that minimize the approximation error given a bit budget by optimally positioning the vertices on the contour or within a thin stripe along the contour[26][28].

The encoder has to transmit the vertex coordinates of the line segments to the decoder. After encoding the absolute position of one vertex, coordinates of the following vertices are differentially encoded using the image size as an upper bound for the vertex coordinates.

The polygon representation requires many vertices for lossless shape coding. Chain codes usually outperform polygon representation for lossless coding whereas polygon representations are more efficient for lossy coding.

### Spline Approximation

Spline functions are defined by selected control points. We can use spline functions to compute continuous curves from discrete points. We represent the continuous boundary coordinates as a function of the contour length  $t$  as  $\mathbf{c}(t) = [x(t), y(t)]$ . This contour can be approximated by  $n$  node points  $\mathbf{p}_n = [x_n, y_n]$  according to

$$\mathbf{c}(t) = \sum_{n=0}^{N-1} \mathbf{p}_n \cdot B_{n,q}(t) \quad (10.1.4)$$

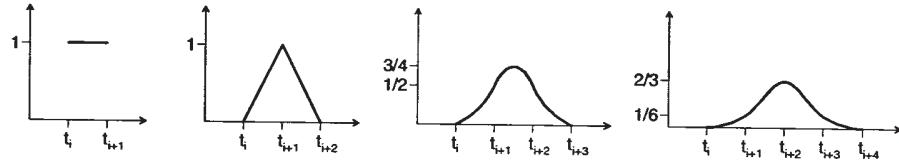
The spline function  $B_{n,q}$  is called *B-spline* where B stands for Basis.  $B_{n,q}$  depends on the degree  $q$  of the function and the position  $t_n$  of the nodes on the contour. It is determined by the following recursion:

$$B_{n,q}(t) = \frac{(t - t_n) \cdot B_{n,q-1}(t)}{t_{n+q-1} - t_n} + \frac{(t_{n+q} - t) \cdot B_{n+1,q-1}(t)}{t_{n+q} - t_{n+1}}; q > 0 \quad (10.1.5)$$

and

$$B_{n,0}(t) = \begin{cases} 1 & t_n \leq t < t_{n+1} \\ 0 & \text{else} \end{cases} \quad (10.1.6)$$

Fig. 10.6 shows the shape of different spline functions for equally spaced nodes. Nodes can also be placed at arbitrary intervals on the contour. Nodes are allowed to coincide. Properties of the spline functions of order  $q$  are that its first  $q-1$  derivatives are continuous along the contour. As can be seen, Eq. (10.1.6) is simply a hold function (Fig. 10.6a). A spline function of order  $q$  is created by convoluting the spline of order  $q-1$  with a spline of order 0. Since the extent of the spline



**Figure 10.6.** Splines of order  $q = 0, 1$  (linear),  $2$  (quadratic),  $3$  (cubic) with equidistant nodes.

function is limited, the position of a point  $\mathbf{c}(t)$  is influenced by  $q + 1$  points with  $q$  the degree of the spline. Quadratic and cubic splines are used frequently. For shape coding, we have to approximate the discrete contour  $\mathbf{c}(t_i)$  with  $I$  contour points:

$$\mathbf{c}(t_i) = \sum_{n=0}^{N-1} \mathbf{p}_n \cdot B_{n,q}(t_i) \text{ for } 0 \leq i < I \quad (10.1.7)$$

We can compute the node points  $\mathbf{p}_n$  if we choose the number of nodes  $N$  smaller than the number of contour points  $I$ . In this case, we solve this system using a least squares method.

In [22] B-spline curves are used to approximate a boundary. An optimization procedure is formulated for finding the optimal locations of the control points by minimizing the mean squared error between the boundary and the approximation. This is an appropriate objective when the smoothing of the boundary is the main problem. When, however, the resulting control points need to be encoded, the tradeoff between the encoding cost and the resulting distortion needs to be considered. By selecting the mean squared error as the distortion measure and allowing for the location of the control points to be anywhere on the plane, the resulting optimization problem is continuous and convex and can be solved easily. In order to encode the positions of the resulting control points efficiently, however, one needs to quantize them, and therefore the optimality of the solution is lost. It is well known that the optimal solution to a discrete optimization problem (quantized locations) does not have to be close to the solution of the corresponding continuous problem. In [26] a shape coder using splines is presented. Coding is optimal in the operational rate distortion sense, i.e. the control points are placed such that the required bit rate is minimum for a given distortion.

### Fourier Descriptors

Fourier descriptors were developed for applications in recognition, where shape is an important key. Fourier descriptors allow a translation, rotation and scale invariant representation of closed contours [55]. There are different methods to define the Fourier descriptors of a closed contour. Using a simple descriptor, we sample the 2D coordinates  $(x_n, y_n)$  of the contour with  $N$  pels clockwise in the image-plane.

We represent the coordinates as a complex number  $z_n = x_n + j \cdot y_n$  and compute the Fourier series as

$$Z(m) = \sum_{n=0}^{N-1} z(n) \cdot e^{-j \frac{2\pi n m}{N}} \quad (10.1.8)$$

Smooth contours will have their energy concentrated in the lower frequencies of the Fourier series. Setting higher frequency coefficients to 0 smoothes a contour.

It can be shown from Eq. (10.1.8) that a translation of the contour will only influence coefficient  $Z(0)$ , the DC value. Scaling of the contour will multiply all coefficients with the scale factor. Rotation of the contour will shift the phase of the Fourier coefficients but it will not effect their amplitude.

Another way of computing Fourier descriptors is to transform the list of 2D coordinates  $(x_n, y_n)$  into an ordered list  $(n, (y_{n+1} - y_n)/(x_{n+1} - x_n))$  with  $(x_0, y_0) = (x_N, y_N)$  and  $0 \leq n \leq N$  being the contour points. The term  $(y_{n+1} - y_n)/(x_{n+1} - x_n)$  describes the change of direction of the contour. The Fourier series of this descriptor compacts more energy into the lower coefficients. This is partially due to the fact that the DC value describes a circle, which has constant change of direction, instead of a straight line as in Eq. (10.1.8).

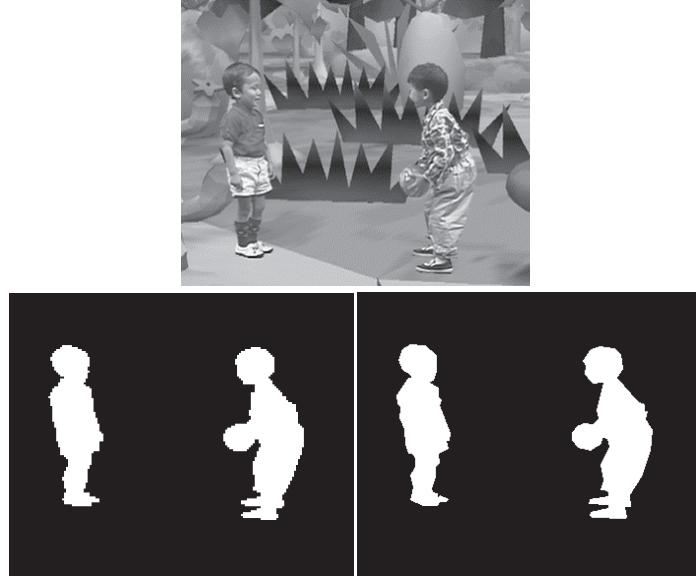
In order to preserve the main characteristics of a shape, only the large Fourier coefficients have to be maintained. Fourier descriptors are not very efficient in reconstructing polygon-like shapes with only a few coefficients. This is one of the reasons that they never became very competitive in terms of coding efficiency.

### 10.1.3 Evaluation Criteria for Shape Coding Efficiency

Lossy shape coding can create two different types of distortions:

- The contour of an object might get changed. Sharp corners get smoothed, parts of the contour get displaced by a small number of pels.
- The topology of an object gets changed, i.e. the object might get additional holes, holes are closed or parts of the object get disconnected from the original shape.

There are two commonly used quality measures for objectively assessing the quality of coded shape parameters. The peak deviation  $d_{max}^*$  is the maximum of the Euclidean distances between each coded contour point and the closest point on the original contour. This measure allows for an easy interpretation of the shape quality due to changes of the object contours. However, if lossy shape coding results in changing the topology of an object, the peak deviation  $d_{max}^*$  is not a useful measure. A second measure  $d_n$  depends of the number of pels that belong to the object and its coded representation. The metric  $d_n$  is the number of erroneously represented pels of the coded shape divided by the total number of pels belonging to the original shape. Since different objects can have very different ratios of contour pels to interior pels, a given value for  $d_n$  only allows to compare different approximations of the same object. The quality measure  $d_n$  by itself does not provide sufficient



**Figure 10.7.** Lossy encoding using a bitmap-based (left) and a contour-based (right) shape coder. The shapes describe the two children in the top picture.

information about the subjective shape quality since the subjective meaning of a given  $d_n$  depends not only on the object size but also on the contour length.

Subjective evaluation of several sequences indicated that a shape representation with an approximation error of  $d_{max}^* > 3$  pels is not useful at all for video, independent of its resolution. It was found that allowing a peak distance of  $d_{max}^* = 1.4$  pel at CIF resolution ( $352 \times 288$  pels) is sufficient to allow proper representations of objects in low bit rate video coding applications.

Subjective evaluations also showed that the above two objective measures truthfully reflect subjective quality when comparing different bitmap-based shape coders or when comparing different contour-based shape coders. For lossy shape coding, the bitmap-based shape coders create blocky object shapes whereas contour-based shape coders create an object shape showing curved distortions especially at object corners or polygon edges. Since the two classes of shape coders give different distortions (Fig.10.7), a comparison between algorithms belonging to different classes has to be done subjectively.

## 10.2 Texture Coding for Arbitrarily Shaped Regions

There are two classes of algorithms for coding the texture of arbitrarily shaped regions:

- We *extrapolate* the texture of the region to fill a rectangle. Then we code this

rectangle using an algorithm for coding the texture of rectangular regions. We give examples in Section 10.2.1. These algorithms are usually fast to compute and will create as many coefficients as there are pels in the rectangle. Therefore they have a disadvantage compared to the next class of algorithms.

- We code the pels of the region *directly* using a transform that is adapted to the shape of the region. Therefore, we compute as many coefficients as there are pels in the region. Examples are given in Section 10.2.2.

### 10.2.1 Texture Extrapolation

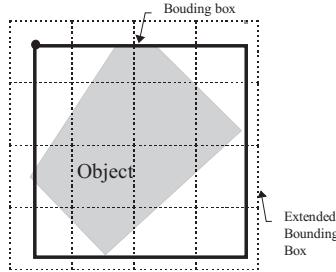
As mentioned above, texture extrapolation or *padding* algorithms use a bounding box around the region that defines the area of texture to be coded. We call pels that belong to the region opaque and pels outside of the region but inside the bounding box transparent, since they will not be visible at the decoder. The texture of the bounding box will be coded using any algorithm suitable for coding textures of rectangular regions like DCT or wavelet transforms. Fig. 10.8 shows the bounding box for a region. This bounding box gets extended such that its width and height are suitable for the texture coding algorithm. If we use a DCT we might extend the rectangle such that its width and height are multiples of 8 pels.

Since the decoder will neglect the texture of all transparent pels, we define the transparent pels inside the bounding box such that they do not unnecessarily increase the data rate. Ideally, we extrapolate the texture of the region such that the signal is extended to the transparent pels without creating high frequency components. This usually excludes setting the transparent pels to 0 or 255. A first simple approach is to set the texture to the average texture value in the region or the neighboring opaque pels. In addition, low pass extrapolation filters can be applied to these transparent pels. Each transparent pel is set to the average of its four neighbors. This process starts at one corner of the bounding box and proceeds in scan line order [27].

### 10.2.2 Direct Texture Coding

In order to limit the number of coefficients to be transmitted to the number of pels that need to be coded, several algorithms were developed that compute a transform on an arbitrary image region. Starting with the DCT transform of the bounding box of an image region, Gilge computes a transform that is orthogonal on the image segment [17]. The resulting transform computes as many coefficients as there are pels in the image segment. Furthermore, it exploits the spatial correlation between neighboring pels. For each new shape of an segment, a new transform has to be computed, which makes this approach very compute intensive.

The shape-adaptive DCT (SA-DCT) is much faster to compute using 1D DCT transforms. First, we shift all pels of an image segment vertically to the block boundary (Fig. 10.9b) [50]. Then a 1D-DCT according to the length of each column is applied. In a second step, the computed DCT coefficients are shifted horizontally



**Figure 10.8.** The circumscribing rectangle of the object or region of interest is extended such that the side lengths of the extended bounding box are multiples of 8 for DCT coding.

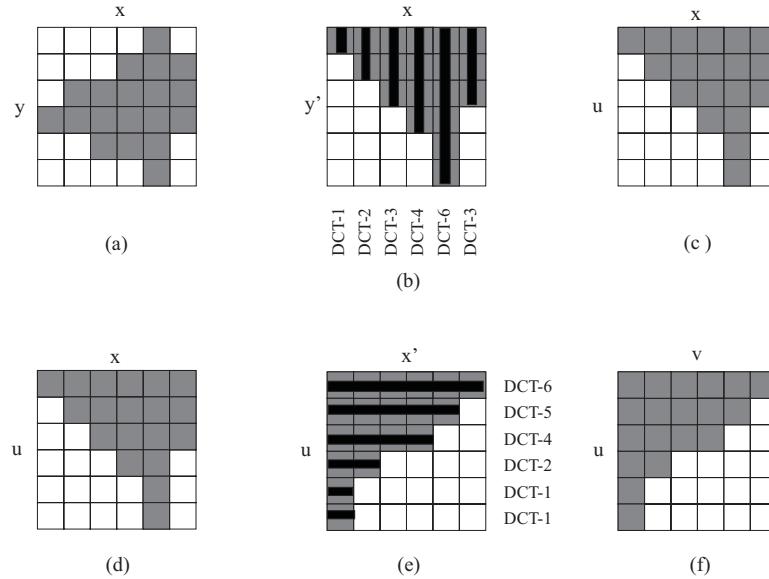
to the left block boundary (Fig. 10.9e). Again a 1D-DCT is applied with a length according to the number of DCT coefficients in the row. Finally, the coefficients are quantized and encoded in a way identical to the coefficients of the regular 2D-DCT. SA-DCT is not orthonormal due to the different lengths of the DCTs. Furthermore, the shifting of pels does not allow to fully exploit the spatial correlation between neighboring pels. Therefore, SA-DCT is not as efficient as the Gilge transform. Compared to padding, this more complex algorithm gains on average between 1dB and 3dB in PSNR measured over an image segment for the same bit rate.

Wavelet coders can also be adapted to arbitrarily shaped regions [32]. In order to compute the wavelet coefficients, the image signal at the boundary of a segment needs to be extended using periodic and symmetric extensions. Since no pels are shifted, the spatial correlation between neighboring pels is maintained.

### 10.3 Joint Shape and Texture Coding

This shape coding technique was inspired from the blue screen technique used in film and TV studios. The object to be coded is placed on a static one-colored background. The color of the background has to be outside of the color space occupied by the texture of the object. Usually, highly saturated colors (e.g. pure blue) fulfill this requirement. Since the shape information is coded as part of the texture information, we sometimes call this coding method *implicit shape coding*. We present two coding algorithms that use this feature.

Today, GIF89a [9][4] is used in Web applications to allow describing arbitrarily-shaped image objects. An image with a maximum of 256 colors is encoded based on a lossless compression scheme called LZW (after the authors Lempel, Ziv, Welch). The particular implementation is called LZ78. In the header of a GIF file, it is possible to declare one of the 256 colors as the transparent colors. All pels with



**Figure 10.9.** Shape adaptive DCT requires transforms of length  $n$ : (a) original image segment, (b) pels shifted vertically, (c) location of DCT-coefficients after vertical 1D DCT, (d) location of DCT-coefficients prior to horizontal 1D DCT, (e) DCT coefficients shifted horizontally, (f) location of DCT coefficients after horizontal DCT.

that color will not be displayed, hence they appear to be transparent.

This concept of defining a transparent color was also introduced into video coding. The video signal is encoded using a frame-based coder. To the decoder, the chroma-key (background color) is transmitted. The decoder decodes the images. Pels with a color similar to the chroma-key are considered to be transparent. Otherwise, pels belong to the object [6]. Since the shape information is typically carried by the subsampled chroma signal, this technique is not suited for lossless shape coding. Because the shape information is embedded in the texture, the shape coding is lossy as long as there is quantization of the texture. An important advantage of this method is its low computational and algorithmic complexity. As far as coding efficiency is concerned, this implicit shape coder requires a higher data rate than an explicit shape coder plus a texture coder like SA-DCT or low pass extrapolation with DCT.

## 10.4 Region-Based Video Coding

Most image and video coders are optimized for coding efficiency. Image and video quality is measured by PSNR. It is known that the simple measure of PSNR does not capture well the properties of the human visual system. This becomes very apparent

at low bit rates when blocking artifacts distort images without resulting in a low PSNR. Region-based image and video coding, also known as *Second generation image and video coding* [30], tries to pay special consideration to the human visual system. Properties of the human visual system lead to the following basic claims that build the foundation for the choices made in designing these algorithms [54]:

- The edge and contour information are very important to the human visual system and are responsible for our perception.
- The texture information has relative importance and influences our perception when taken together with contour information.

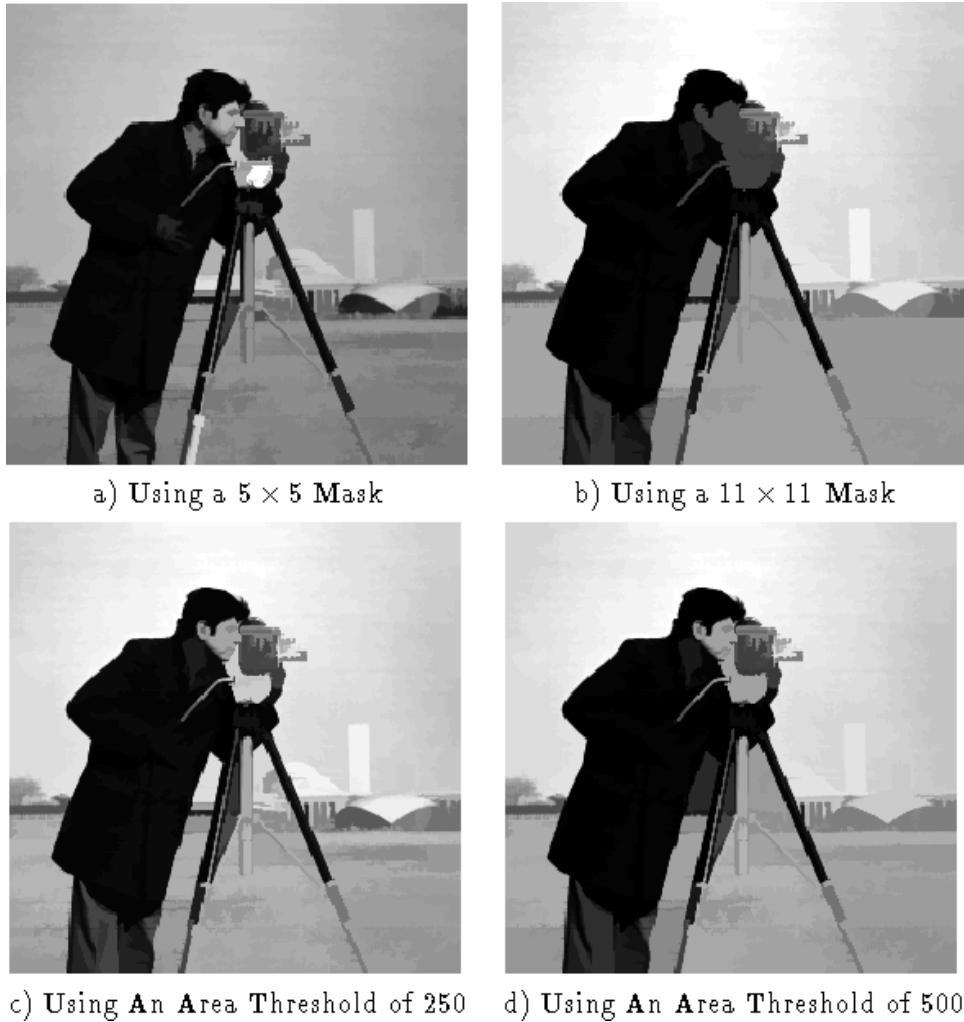
Based on these assumptions, region-based video coding puts more emphasis on coding contours than on coding textures. Motion, static objects or moving objects are not considered when determining important contours.

A region-based video coder segments each image into regions of similar textures. Since the contours are considered to be very important, the coder transmits the contours of the regions with high precision. The texture of the regions is approximated using the mean value of the texture of the original region. Fig. 10.10 shows an image segmented into regions. Depending on how stringent the similarity criterion is defined and what is the minimum region size, different segmentations result. Compared to DCT-based image coders at low data rates, this coder does not show any blocking artifacts. However, the flat texture representation creates a new kind of distortion that some people consider more disturbing. At high data rates, a DCT-based coder clearly outperforms a region-based coder because the region-based coder requires many contours to be transmitted.

This concept can be extended to video coding [48]. In order to reduce the data rate required for shape coding, regions are tracked from one image to the next. Regions with similar or identical motion get clustered. For the new image, the encoder transmits the motion of regions in the current image, their change in shape, and newly appearing regions. Changes of the texture values are also transmitted.

## 10.5 Object-Based Video Coding

Object-based analysis-synthesis coding (OBASC) [35] defines objects of uniform motion according to a 2D or 3D motion model. In contrast to region-based coding, OBASC does not segment objects further into regions of homogenous texture thus saving on transmitting shape parameters. OBASC subdivides each image of a sequence into uniformly moving objects and describes each object  $m$  by three sets of parameters  $A(m)$ ,  $M(m)$  and  $S(m)$ , defining its motion, shape, and color, respectively. Motion parameters define position and motion of the object. Color parameters denote the luminance as well as the chrominance reflectance on the surface of the object. In computer graphics, they are sometimes called texture. Fig. 10.11 explains the concept and structure of OBASC. Instead of a frame memory used in block based hybrid coding, OBASC requires a memory to store the coded



**Figure 10.10.** The image ‘camera man’ is segmented into several regions. Each region is represented by its contour and the mean value of the original texture. Different segmentations requiring different amounts of data to code are achieved by varying parameters of the segmentation algorithm.

and transmitted object parameters  $A'(m)$ ,  $M'(m)$  and  $S'(m)$ . The parameter memories in the encoder and decoder contain the same information. Evaluating these parameter sets, image synthesis computes a model image  $s'_k$  which is displayed at the decoder. The parameter sets of the memory and the current image  $s_{k+1}$  are the input to image analysis.

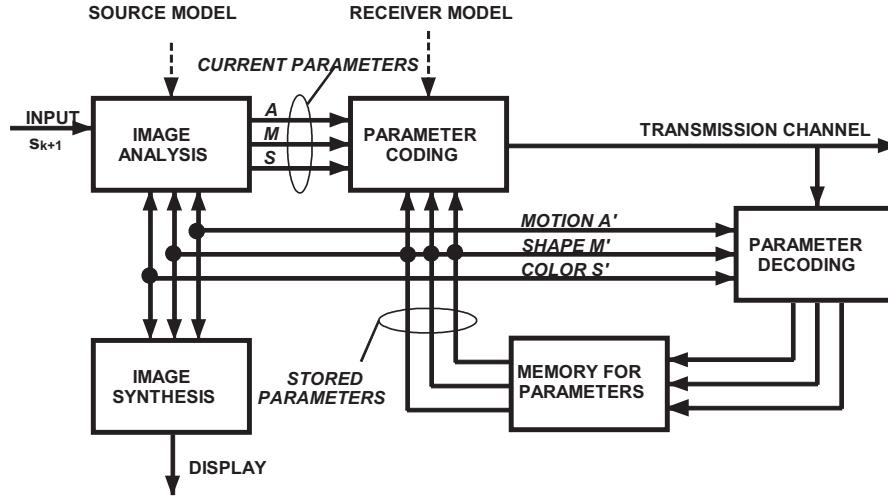
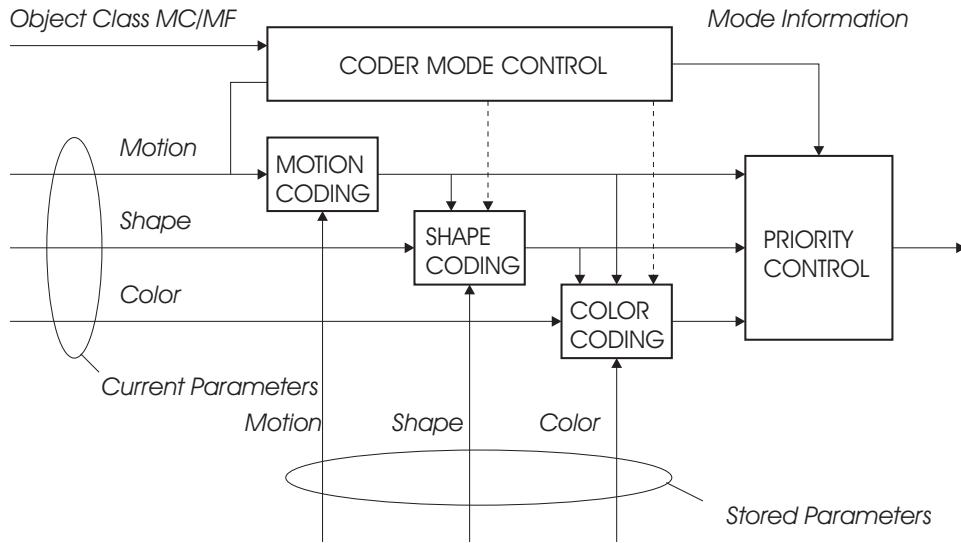


Figure 10.11. Block diagram of an object-based analysis synthesis coder.

The task of image analysis is to analyze the current image  $s_{k+1}$  to be coded and to estimate the parameter sets  $A_{k+1}(m)$ ,  $M_{k+1}(m)$  and  $S_{k+1}(m)$  of each object  $m$ . In the current image, moving and static objects are detected first. For moving objects, new motion and shape parameters are estimated in order to reuse most of the already transmitted color parameters  $S'_k(m)$ . Objects for which the correct motion and shape parameters can be estimated are denoted as MC-objects (model compliance). In the final step of image analysis, image areas which cannot be described by MC-objects using the transmitted color parameters and the new motion and shape parameters  $A_{k+1}(m)$ ,  $M_{k+1}(m)$ , respectively, are detected. These areas of model failures (MF) are defined by 2D-shape and color parameters only and are referred to as MF-objects. The detection of MF-objects exploits that small position and shape errors of the model objects - referred to as *geometrical distortions* - do not disturb subjective image quality. This assumption is valid for OBASC because the motion-compensated prediction image looks like a real image due to the object-based image description. However, this image might be semantically incorrect. Thus, MF-objects are reduced to those image regions with significant differences between the motion- and shape-compensated prediction image and the current image  $s_{k+1}$ . They tend to be small in size. This allows to code color parameters of MF-objects with high quality, thus avoiding subjectively annoying quantization errors. Since the transmission of color parameters is expensive in terms of data rate, the total area of MF-objects should not be bigger than 4% of the image area assuming 64 kbit/s, CIF and 10Hz.



**Figure 10.12.** Parameter coding for OBASC

Depending on the object class MC/MF, the parameter sets of each object are coded by parameter coding using predictive coding techniques (Fig. 10.12). Motion and shape parameters are coded, transmitted and decoded for MC-objects, as well as shape and color parameters for MF-objects. Since the coding of color parameters is most expensive in terms of bit rate, parameter coding and image analysis have to be designed jointly. Let  $R_A$ ,  $R_M$ , and  $R_S$  represent the rates for motion, shape and texture, respectively. By minimization of the total bit rate

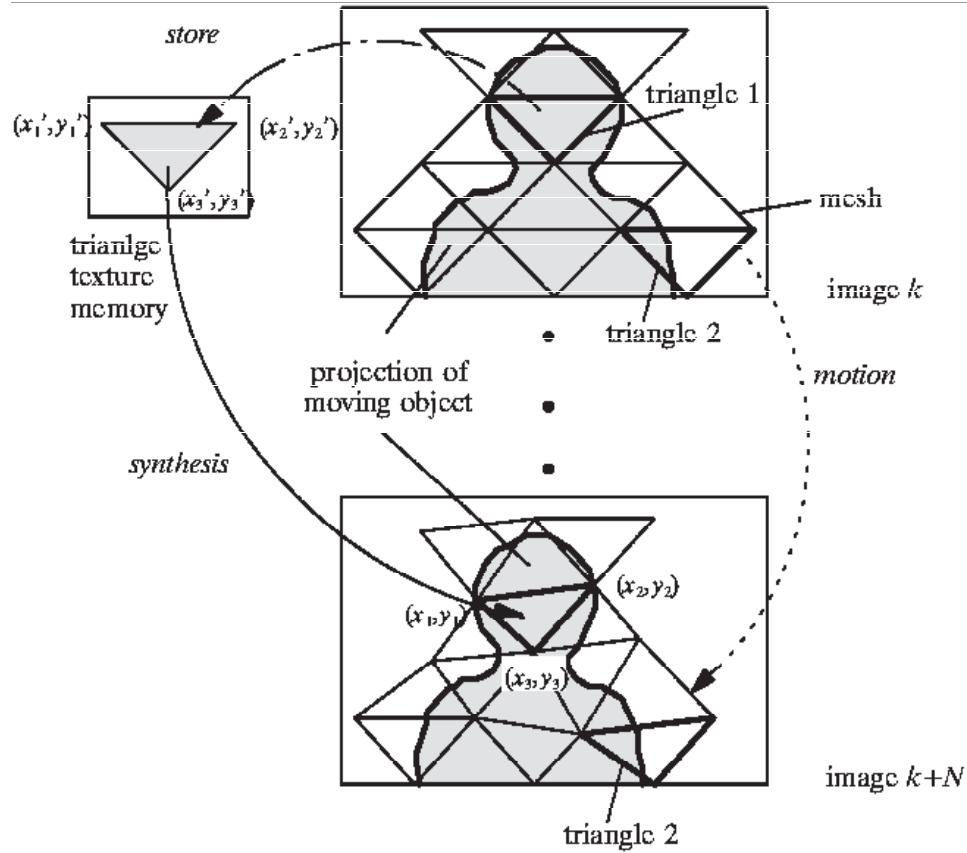
$$R = R_A + R_M + R_S \quad (10.5.1)$$

a higher coding gain than in block-based hybrid coding techniques can be achieved.

In OBASC, the suitability of source models can be judged by comparing the bit rates required for coding the same image sequence at the same image quality. Image quality is influenced mainly by the algorithm for detecting model failures and by the bit rate available for coding the color parameters of model failures. We describe below several source models that have been developed in the context of OBASC.

### 10.5.1 Source Model F2D

The source model of *flexible 2D objects (F2D)* assumes that the motion of a real object can be described by a smooth displacement vector field. This displacement vector field moves the projection of the real object into the image plane to its new position. This displacement vector field defines a vector for each pel that belongs to the projections of the object into the image plane. The shape of the object is

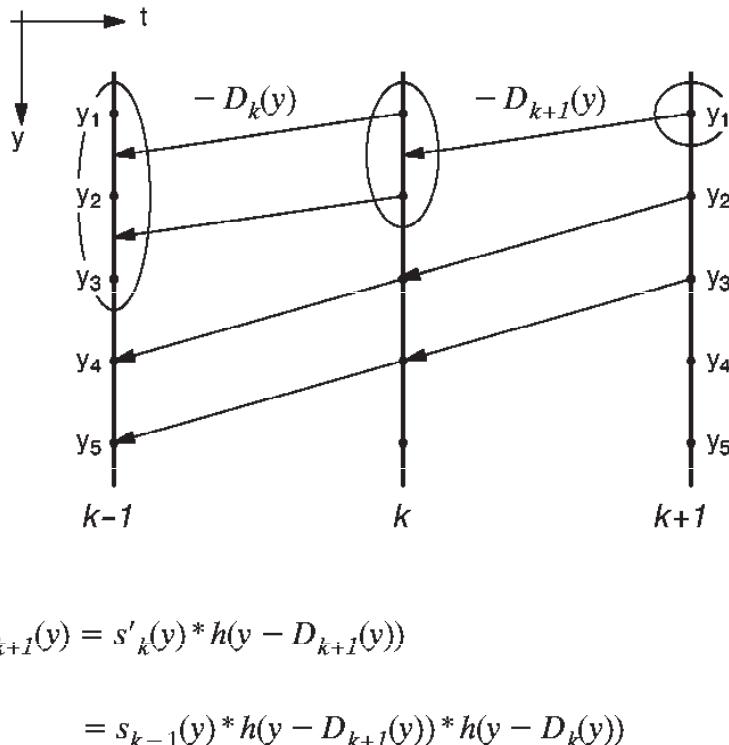


**Figure 10.13.** Image synthesis for MC objects using a triangle-based mesh as texture memory

defined by the 2D silhouette that outlines the projection of the object into the image plane.

In order to efficiently describe the displacement vector field of the object, the field is subsampled and only one vector for every  $16 \times 16$  pels is transmitted. The decoder reconstructs the displacement vector field by bilinear interpolation resulting in a smooth displacement vector field that allows to describe 2D object motion as well as some flexible deformations of an object. As an example, the source model F2D is very well suited to model a sheet of rubber moving on a flat surface. Additionally, stretching of the rubber can be described. The shape of a moving object is described by a polygon approximation.

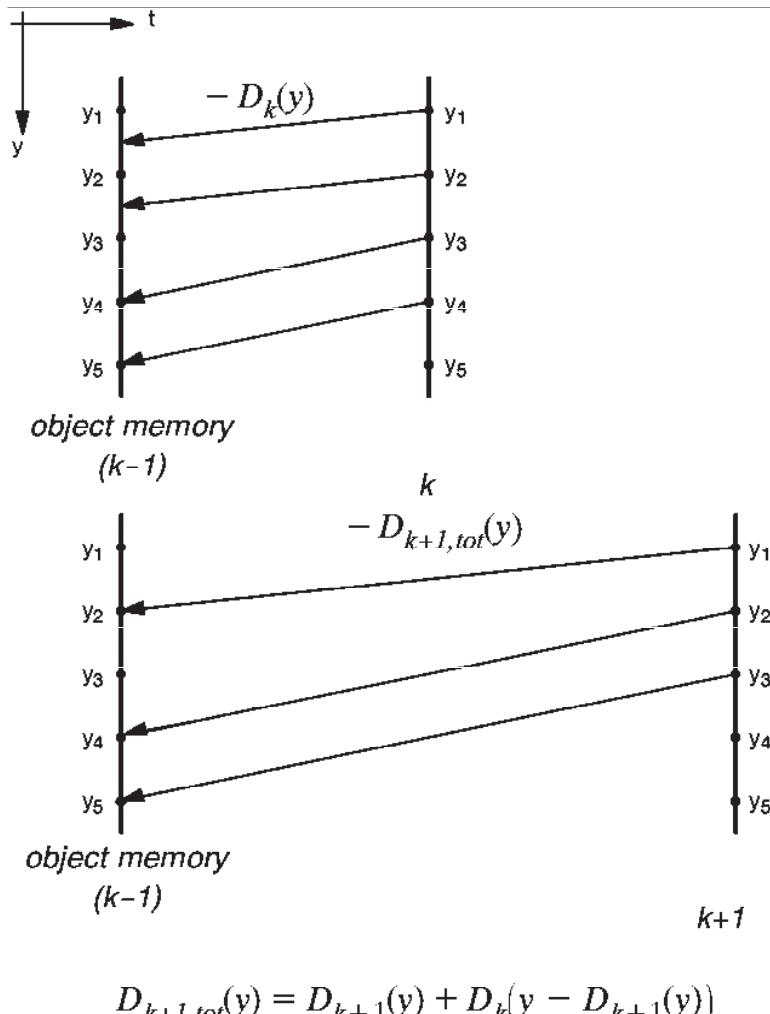
Whereas block-based hybrid coding predicts the current image  $k+1$  using motion compensated prediction from the texture of frame  $k$ , OBASC stores the texture of



**Figure 10.14.** Block-based hybrid coders perform image synthesis by filter concatenation for motion compensation

objects in a texture memory (Fig. 10.13). Using this texture memory improves the performance of motion compensation. Fig. 10.14 shows motion compensation by filter concatenation as done in motion-compensated hybrid coding. The motion compensated prediction of image  $k$  is computed by copying a pel from image  $k-1$  in case of an integer pel motion vector and by filtering with  $h(n)$  in case of a motion vector with fractional pel accuracy. The lowpass filter  $h(n)$  is typically implemented as bi-linear interpolation. If we now use frame  $k$  to predict frame  $k+1$  we see that pel  $y_1$  in frame  $k+1$  in Fig. 10.14 is computed by filtering previously filtered pels of frame  $k$ . Therefore, motion compensated prediction using filter concatenation creates images that lose sharpness as we predict later frames.

This effect of filter concatenation can be avoided using a texture memory (Fig. 10.15). Assuming again that this texture memory is initialized for frame  $k$  with the texture of frame  $k$ , we use the displacement vector field  $D_k(y)$  from frame  $k-1$  to  $k$  in order to compute the motion compensated prediction image  $k$ . This requires one filter operation for each pel with a fractional pel motion vector. Similarly, we can compute the motion compensated prediction image  $k+1$  by directly accessing



**Figure 10.15.** Image synthesis by concatenation of displacement vector fields (one-dimensional case) using texture memory.

the texture memory using the concatenated displacement vector field  $D_{k+1,tot}(y)$ . Again, we need only one filter operation for each pel with a fractional pel motion vector (Fig. 10.15).

### 10.5.2 Source Models R3D and F3D

The use of 3D scene models with 3D object models holds the promise that we can describe and model real world objects more efficiently using moving *rigid 3D model objects* ( $R_{3D}$ ) (Sec. 5.3). We describe the shape of our model objects using a wireframe. The texture of the object is described using images or texture maps. Objects may move in 3D using 3D motion according to Eq. (5.3.9). We allow an object to consist of several connected rigid components in order to enable the modeling of joints [33]. After motion and shape estimation and compensation, areas of model failure are detected. The difference image between prediction image and the original image (Fig. 10.16a) is binarized with a noise adaptive threshold (Fig. 10.16b). The resulting mask identifies areas where there are differences between the images. Some of these differences are due to a small error in motion compensation (Fig. 10.16c). We consider these errors as geometric distortions that are not of subjective importance. Of importance are areas of model failure where the content of the image changes. These areas exceed a minimum size and have shapes that are not line structures. Figure 10.16d shows MF-objects due to eye-blinking, mouth motion and specular reflections on the ear ring. We code the shape and texture only for the areas of model failure or  $MF_{R3D}$  objects.

Since the surface of a model object is described as a wireframe, it is simple to enable flexible deformations of the object. The source model of moving *flexible 3D objects* ( $F_{3D}$ ) allows control points of the wireframe to move tangential to the object surface. We describe this deformation using 2D shift vectors. These shift vectors enable us to compensate for local deformations of the real object. Describing flexible deformations for the entire model object may require a high data rate. Therefore, we limit the use of the shift vectors to the areas of  $MF_{R3D}$ -objects.

Using the source model  $F_{3D}$  requires in a first step to estimate the  $MC_{R3D}$  objects and the  $MF_{R3D}$ -objects as described in the previous paragraph. Input to the image analysis for estimating the shift vectors are the  $MC_{R3D}$  objects, the  $MF_{R3D}$ -objects, and the real image  $k+1$  (Fig. 10.17). Shift vectors are estimated for those vertices that are projected into the area of the current  $MF_{R3D}$ -objects. After compensating for the estimated shift vectors, we verify this step of image analysis by detecting model failures again, using the same algorithm as for the source model  $R_{3D}$ . Fig. 10.17 shows the estimated shift vectors and the  $MF_{F3D}$ -objects. These  $MF_{F3D}$ -objects are usually smaller than the  $MF_{R3D}$ -objects because the flexible shift vectors compensate for some flexible deformations. Since it costs less bits to code the shift vectors of the  $MC_{F3D}$  object as well as the texture and shape of the  $MF_{F3D}$ -objects instead of the texture and shape of the larger  $MF_{R3D}$ -objects, the use of  $F_{3D}$  results in an increased coding efficiency. This is possible because data rate for the shift vectors is only spent if they reduce the size of the MF-objects. When coding simple video telephony sequences, the use of  $F_{3D}$  instead of  $R_{3D}$  reduces the data rate from 64 kbit/s to 56 kbit/s [41, 42].

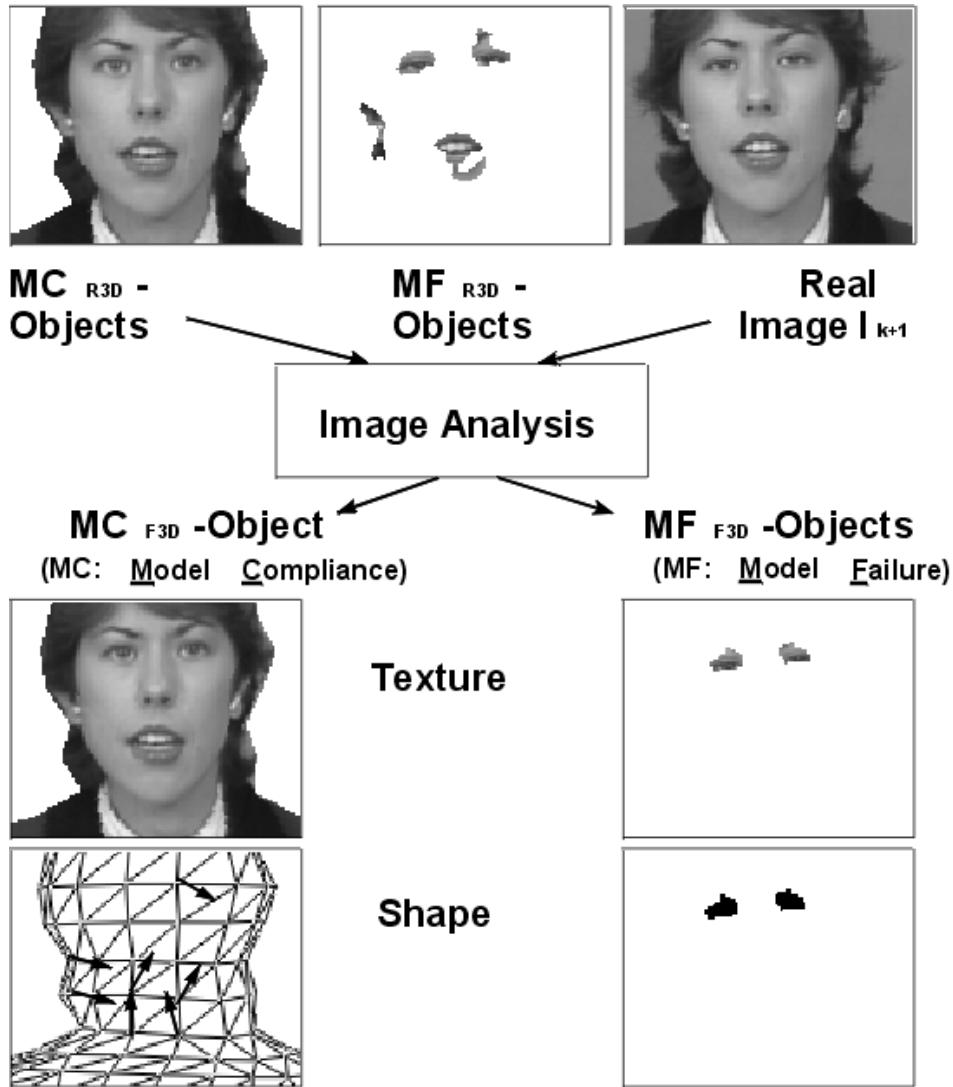
OBASC codes the color parameters at a rate of 1 bit/pel which is much higher than the 0.1 bit/pel to 0.3 bit/pel that a block-based hybrid coder typically uses.



**Figure 10.16.** Detection of model failures: (a) scaled difference image between real image  $s_{k+1}$  and model image after motion and shape compensation  $s_{k+1}^*$ ; (b) synthesis error mask; (c) geometrical distortions and perceptually irrelevant regions; (d) mask  $MF_{R3D}$  indicating regions with model failures of the source model R3D.

Therefore, OBASC shows less quantization artifacts and the images tend to look sharper than those of a hybrid coder. However, an OBASC relies heavily on image analysis. Moving objects need to be segmented correctly, and their motion needs to be estimated fairly precisely. If the image analysis fails, OBASC cannot use the shape parameters, which it has to transmit in addition to the motion and texture parameters, to increase coding efficiency. To the contrary, it might require a data rate higher than that of a hybrid coder that transmits only motion and texture parameters. Due to the lack of robustness of image analysis, OBASC is still an ongoing area of research.

Depending on the applied source model, an OBASC spends up to 80% of its data rate for coding model failures. OBASC does not require or exploit any scene knowledge. Therefore, the algorithm for the detection of model failures is not



**Figure 10.17.** Flexible shift vectors are estimated in the areas of  $MF_{R3D}$ -objects. This usually results in a reduction of the  $MF$ -object size.

controlled by scene contents. In order to incorporate knowledge about the scene, we need to extend an OBASC to a knowledge-based analysis-synthesis coder (KBASC) which evaluates scene knowledge for efficient coding.

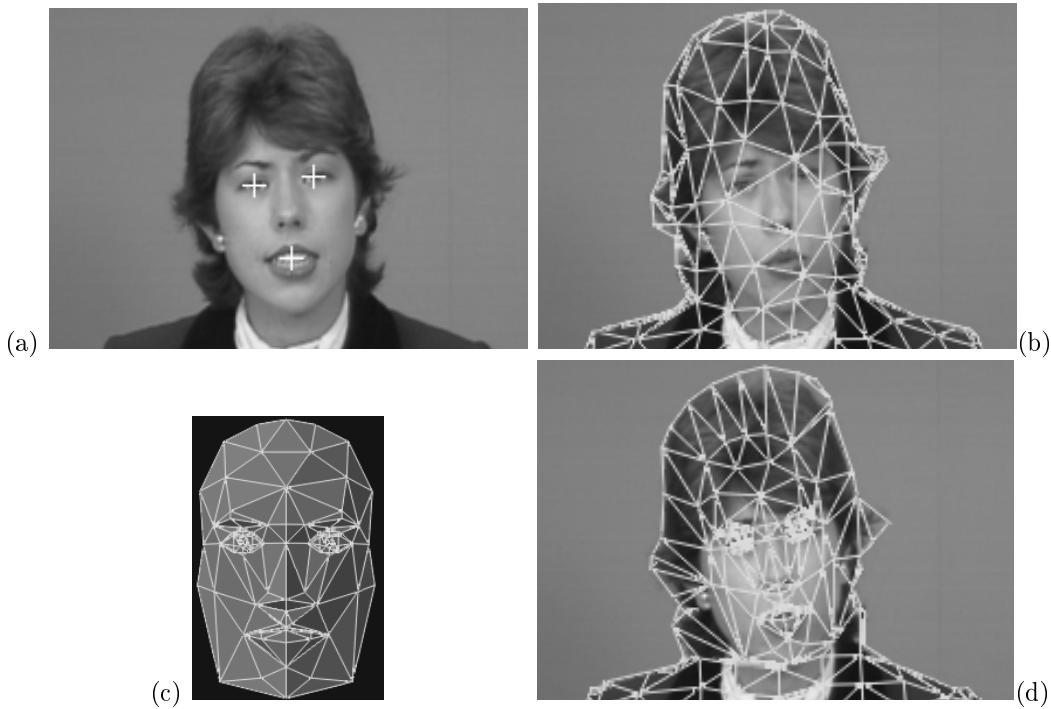
## 10.6 Knowledge-Based Video Coding

A knowledge-based analysis-synthesis coder (KBASC) is based on an OBASC. In addition, the coder tries to recognize objects like faces in a video scene. As soon as the coder recognizes an object, it switches the coding mode for the object from the generic object-based mode to a knowledge-based mode. For head and shoulder scenes, the goal is to achieve a better modelling of the human face and to use the knowledge of the face location to control the coder. In order to enable an automatic switching from an OBASC to a KBASC, an algorithm for recognizing facial features and the adaptation of a face model to the image sequences is proposed [24]. First, we need to identify eyes and mouth of the person that we want to code using KBASC (Fig. 10.18a). In a second step , we need to adapt a face model into the wireframe that currently represents the person (Fig. 10.18b). As an example, we incorporate the face model *Candide* (Fig. 10.18c)[44] into the object model. We can adapt the face model to the face of a person in an image by scaling the face model horizontally to match the distance between the two eyes and vertically to match the distance between mouth and eyes. Finally, we need to stitch the face model into the the wireframe model that we use to describe the object when coding with the OBASC layer (Fig. 10.18d).

As soon as KBASC has detected a face, its coding efficiency increases compared to OBASC. One reason is that the use of a face model instead of a generic model improves the shape representation and allows for better motion compensation. Knowing the location of the face also allows to define the joint between head and shoulders more precisely, which again results in improved motion estimation and compensation. Finally, we can use the knowledge about the face location to compute model failures in a scene-dependent fashion. While we continue to use a sensitive detection algorithm in the face area we can attribute most prediction errors outside of the face area to geometric distortions as well as illumination errors that we choose to neglect. In an experiment with the test sequence *Claire* at a spatial resolution of 352x288 pels and frame rate of 10Hz, the average area of model failures was 3% for the source model F3D. The area decreased to 2.7% when we adapted the face model into the model object. Finally, exploiting the knowledge of the face location for the detection of model failures reduced their area to 2.5% without loss of subjective image quality. The corresponding data rates are 57 kbit/s for F3D of the OBASC coder and 47 kbit/s for KBASC [24].

## 10.7 Semantic Video Coding

Previously described video coding algorithms describe each frame of a video sequence using a combination of the current image signal and a predicted image signal that is derived using temporal prediction. Semantic video coding describes a video sequence using model objects with behavior that represent real objects with their behavior. We expect semantic coding to be far more efficient than the other video coders because motion and deformation of most objects is very limited when



**Figure 10.18.** First, the positions of eye and mouth are identified (a). Using the object model of the person as a reference (b), we scale the face model (c) to the picture. Finally, we cut an opening into face area of the wireframe of the person (b) and stitch the face model into the wireframe (d).

compared to the variations possible in an array of pels required to display the object. The number of possible variations in the video representation determines the maximum entropy of the video sequence and hence the required bit rate. If we consider KBASC with a face model, changing the facial expression from neutral to joy would require a KBASC to transmit the mouth area as a MF-object. A semantic coder with an appropriate face model could just transmit the command ‘smile’ and the decoder would know how to deform the face model to make it smile.

For face models, action units describing facial expressions have been proposed [7, 2, 14, 31]. Action units (AU) are derived from the Facial Action Coding System (FACS) developed by psychologists [11]. The system was developed to “distinguish all possible visually distinguishable facial movements”. The unit of measurement being used here is action not muscle units. This is because FACS combines more than one muscle in their unitization of appearance changes. The term action unit is also used because FACS can separate more than one action from what most anatomists describe as one muscle. For example, the frontalis muscle, which raises



**Figure 10.19.** Examples of different action units from [3, Figure 2]: Examples of the six actions used in this study. AU 1: Inner brow raiser. 2: Outer brow raiser. 4: Brow lower. 5: Upper lid raiser (widening the eyes). 6: Cheek raiser. 7: Lid tightener (partial squint).

the brow, was separated into two action units, depending on whether the inner or outer portion of this muscle lifts the inner or outer portion of the eyebrow. Several action units can be exercised at the same time in order to create any facial expression. There are 46 AUs which account for changes in facial expression, and 12 AUs which describe changes in gaze direction and head orientation in coarser terms. An example of AUs is shown in Fig. 10.19.

In order to use this system for face animation, amplitudes are assigned to action units. Temporal behavior of facial expressions can be modeled according to the onset, apex and offset of AU measured on humans.

While the FACS system is based on the human physiology, other proposals define feature points on a face that are animated by means of facial animation parameters (FAP). The 68 FAPs used in MPEG-4 are based on the study of minimal perceptible actions and are closely related to muscle actions [23][53]. FAPs represent a complete set of basic facial actions including head motion, tongue, eye, and mouth control. They allow representation of natural facial expressions. Fig. 10.20 shows the points that can be used to define the head shape and the feature points that are animated.

Both the FAP system and FACS animate faces with less than 100 parameters. It has been found that FAPs or AUs can be used to animate a face with less than 2 kbit/s [52]. However, it is not yet clear how to create a face model that allows realistic animation of faces. Shape, motion, and illumination estimation are still problems waiting for robust solutions. Furthermore, extraction of semantic parameters like FAPs or AUs from a video sequence is a hard challenge. Semantic coding requires very reliable parameter estimation since an error in the estimation changes the semantic meaning of the face representation at the decoder. A mix-up between a joyful and a sad facial expression can prevent effective communications. A car might turn left instead of driving straight ahead.

Since a semantic coder provides the promise of very high coding efficiency, it will be very sensitive to errors on the communications channel. A bit error will not result in a lower picture quality but in a change of semantic content.

## 10.8 Layered Coding

Depending on the scene content, it may not be efficient to describe each moving object with a separate object model in an object-based coder. Similarly, it is not always efficient to code an image of a sequence with a reference to previous images. Hence, we need to code a video sequence with the appropriate set of source models. In [34], a layered coder is proposed (Fig. 10.21). The layered coder allows the application of different source models and different levels of scene understanding for coding of an image sequence such that the best performance is guaranteed. Whereas Layer I only transmits the color parameters of statistically dependent pels, Layer II allows the additional transmission of motion parameters for blocks of fixed size and position. Hence Layer I corresponds to an image coder or the I-frame coder of an hybrid coder. Layer II corresponds to the coding of P-frames in a hybrid coder. Layer III, the analysis-synthesis coder, allows the transmission of shape parameters in order to describe arbitrarily shaped objects. Here, Layer III is represented by an OBASC based on the source model of F3D. Using a 3D source model allows for a seamless integration with KBASC. In layer IV, knowledge of the scene contents is extracted from the video sequence and transmitted, in order to apply scene dependent object models like the face model Candide. Layer V transmits high-level symbols for describing complex behavior of the objects.

Whereas different layers of this layered coder have been investigated by many groups, the selection of the appropriate layer for encoding has been thoroughly investigated only for layers I and II, since these layers correspond to coding blocks in intra mode or predictive mode in a block-based hybrid coder. The switching between the layers II and III has only recently been addressed. In [8], a selection depending on the residual coding error and bitrate is proposed. In [40], the size of a moving object decides whether the object is coded in the block-based or in the object-based layer. Switching between the OBASC of layer III and the KBASC of layer IV is addressed in [24]. A semantic coder suitable for integration into a layered coding system has been investigated in [13]. This coder saves 35% of the data rate for coding head and shoulder video sequences when compared to the H.263 video coder (Layer I and II).

## 10.9 Summary

In this chapter, we discussed content-based video coding, which describes individual video objects and not just entire frames. In order to describe an object we use an *alpha*-map that defines for each pel of a frame whether it belongs to the object or not. In addition, the *alpha*-map allows to describe arbitrary transparency for a video object. In order to efficiently transmit the shape of an video object, we looked at 2D shape coding algorithms. Bitmap-based shape coders describe for each pel whether it belongs to the object using a context-based arithmetic coder, contour-based coders describe the outline of an object using polygons, splines or a chain code. The human visual system is relatively sensitive to shape distortions.

Therefore, we tend to code shapes with high quality.

As far as texture coding is concerned, we now need to be able to code the texture of an arbitrarily shaped region. One possibility is to low-pass extrapolate the texture such that it fills a square block that can be coded using standard transform coders like DCT. Alternatively, we can use the more efficient shape adaptive DCT or wavelet transform that creates only as many coefficients as there are pels to be coded.

Object-based analysis-synthesis coding (OBASC) integrates arbitrarily shaped video objects into the framework of video coding. Since an object is now described as a unit, the motion-compensated prediction image looks like a natural image. It does not suffer from the block distortion of the block-based motion-compensated prediction images. We use the concept of model failures to determine those parts of an image that cannot be modeled with sufficient subjective quality. OBASC has been implemented using 2D and 3D source models. 3D source models were extended to knowledge-based coding where the coder identifies predefined models like faces to describe the video scene more efficiently. Semantic coding aims at describing video using high-level descriptions. It is mainly developed for faces where parameter sets like AU or FAP are used to animate a face.

Finally, we looked at a layered coder that integrates different coding modes into a framework. The layered coder switches from one coder to another based on the coding efficiency ensuring that we always code a video scene with the most efficient source model.

Efficient content-dependent video coding depends on an image analysis that describes the image content accurately. Image analysis includes the problems of segmenting the objects or regions and estimating their motion. Segmentation involves partition of an initial video frame into regions of coherent texture or motion, and the tracking of these regions over time. In Sec. 6.8, we briefly considered the problem of segmenting a frame into regions of coherent motion. There we have seen that motion estimation and region segmentation are inter-related and should be jointly optimized. A more substantial discussion of segmentation and tracking algorithms is beyond the scope of this book. Interested readers can explore this topic further starting with these references [43][1][46][51].

## 10.10 Problems

**10.1** What is an *alpha*-map and how is it used in video?

**10.2** What are the 2 main shape coding approaches. Discuss their advantages and disadvantages. How do they differ when a shape is coded lossily?

**10.3** What objective measures for shape quality do you know?

**10.4** Imagine a diagonal line of one pel width. How does the representation of this line change if we define an object using pels with 4 or 8 neighbors? What is the advantage of an hexagonal grid for pels?

- 10.5** For the example given in Fig. 10.4, what are the direct chain code and the differential chain code with 4-neighbors?
- 10.6** For the shape given in Fig. 10.22, derive the quad-tree representation.
- 10.7** For the Fourier descriptor representation of a contour given in Eq. (10.1.8), prove that i) a translation of the contour will only influence coefficient  $Z(0)$ , the DC value; ii) a scaling of the contour will multiply all coefficients with the scale factor; and iii) a rotation of the contour will shift the phase of the Fourier coefficients but it will not affect their amplitude.
- 10.8** Explain algorithms for joint shape and texture coding. What method is commonly used on the Internet?
- 10.9** How can you extend a pel-wise shape coder to work efficiently not only in an image coder but also in a video coder?
- 10.10** What approaches to texture coding of arbitrarily shaped objects do you know? Compare their efficiency.
- 10.11** What are the two main assumptions that guided the design of region-based coders.
- 10.12** How do object-based analysis synthesis coding and region-based coding differ? Which coder will do better for natural video sequences and which for cartoons? Explain.
- 10.13** What are the most commonly used source models for OBASC?
- 10.14** Model failure detection ignores geometrical distortions. What are they and will they accumulate over time?
- 10.15** Explain image synthesis by filter concatenation and image synthesis by displacement vector field concatenation. Which method is more efficient?
- 10.16** Name at least 2 reasons why a knowledge-based coder can code a scene more efficiently than an OBASC?
- 10.17** Why does semantic coding hold the promise to be a very efficient coder. Why do we not have semantic coders today?
- 10.18** What are the differences between action units and facial animation parameters?
- 10.19** Explain the layers of a layered coder. Which layers are commonly used today?

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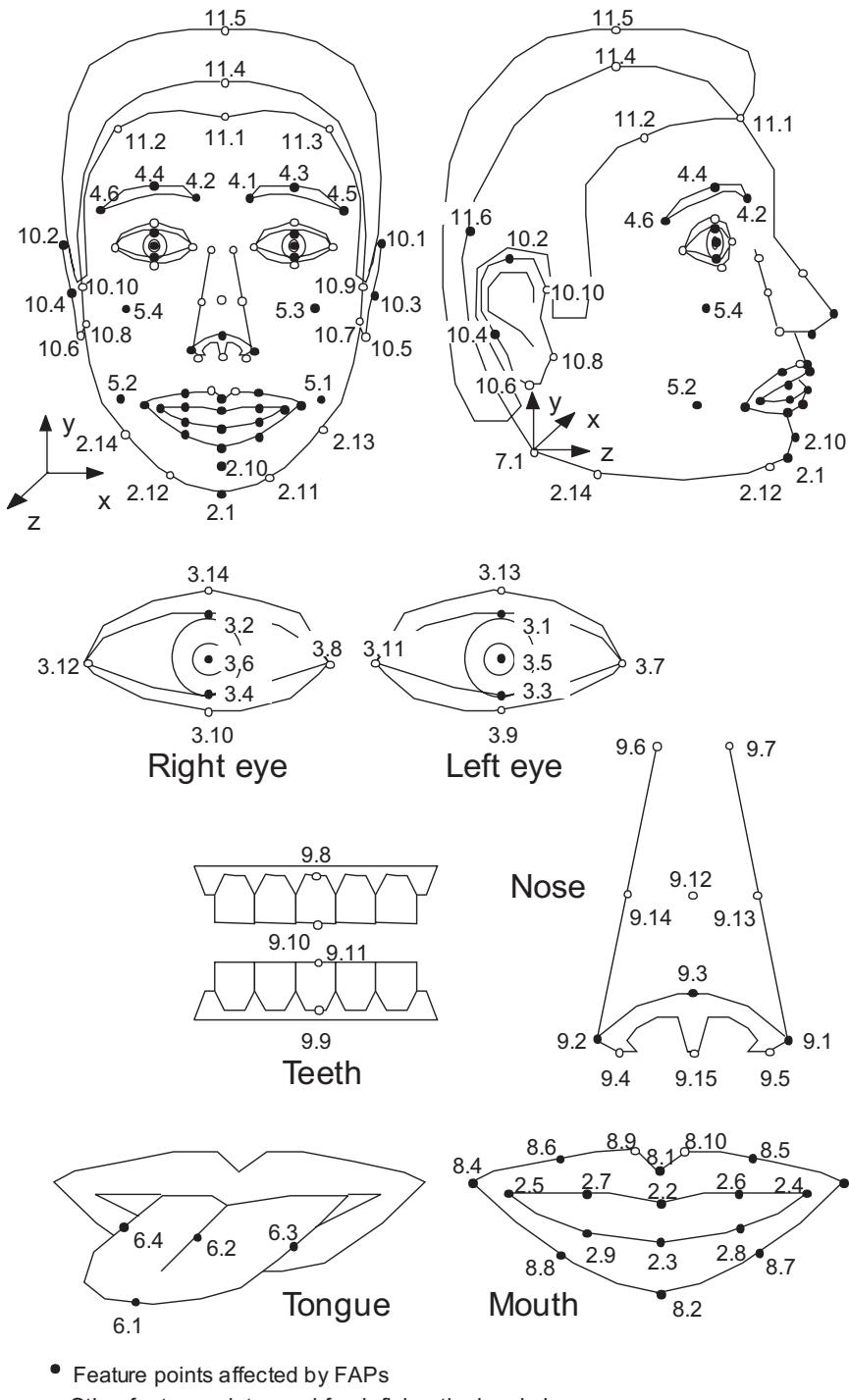
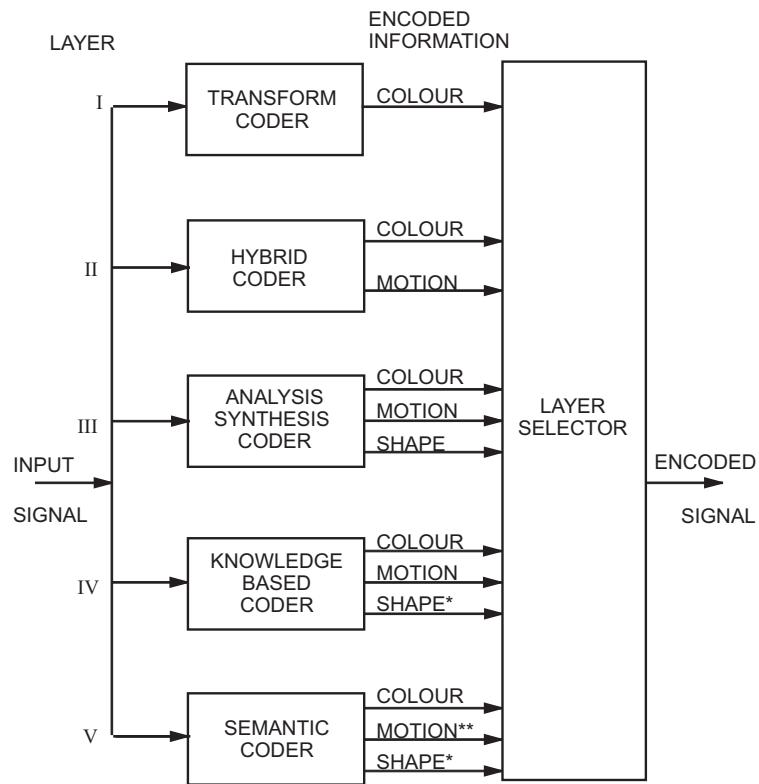


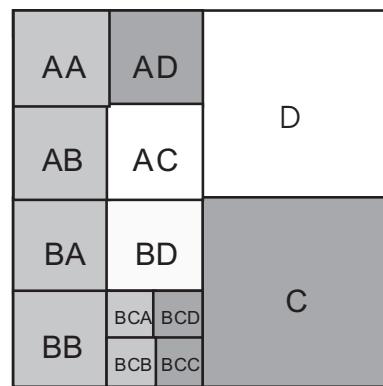
Figure 10.20. Feature points defined for a generic face.



\* including a predefined wireframe

\*\* including action units

**Figure 10.21.** Block diagram of a layered coder (from [34]).



**Figure 10.22.** An image with three different objects represented by different shades.

## Chapter 11

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# SCALABLE VIDEO CODING

The coding methods introduced in the previous chapters, including both waveform-based and content-dependent methods, aim to optimize the coding efficiency for a fixed bit rate. This presents a difficulty when multiple users try to access the same video through different communication links. For example, a MPEG-1 video coded at 1.5 Mbps can be downloaded in real-time for play back in a terminal that is connected to the server via a high-speed link (e.g., an ADSL modem). But a user with only a modem connection of 56 Kbps will not be able to receive sufficient number of bits in time for playback in real-time. *Scalability refers to the capability of recovering physically meaningful image or video information from decoding only partial compressed bitstreams.* In the previous example, if the video stream is scalable, then the user with high bandwidth connection can download the entire bitstream to view the full quality video, while the user with 56 Kbps connection will only download a subset of the stream, and see a low quality presentation.

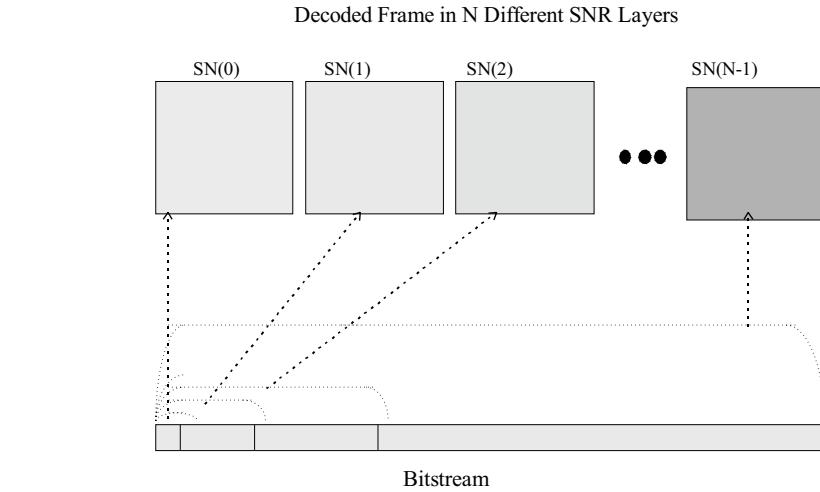
The above example only illustrates one aspect of scalability: the bandwidth scalability. A scalable stream can also offer adaptivity to varying channel error characteristics and computing powers at the receiving terminal. For wireless communications, scalability allows adjusting the source rate and applying unequal error protection in response to channel error conditions. For Internet transmission, scalability enables variable bit rate transmission, selective bits discarding, and adjusting source rate corresponding to different modem rates, changing channel bandwidth, and diverse device capabilities. As we move to the convergence of wireless, internet and multimedia, scalability becomes increasingly important for compelling rich media access to anywhere, by anyone, at anytime, with any device, and in any form.

Scalable coders can have *coarse granularity* (two or three layers, also called *layered* coders), or *fine granularity*. In the extreme case of fine granularity, the bit stream can be truncated at any point. The more bits are retained, the better will be the reconstructed image quality. We call such type of bitstream *embedded*. Embedded coders enable precise bitrate control, which is a desirable feature in many applications. For example, network filters (described later in Section 15.4.1) can select the number of bits to transmit from an embedded bit-stream to match the available bandwidth.

Scalable coding is typically accomplished by providing multiple versions of a

**Figure 11.1.** Decoded images from a scalable bitstream obtained using the MZTE method (Sec. 11.3.1). The two images in (a) are of size  $128 \times 128$ , reconstructed by decoding 80 and 144 Kbits, respectively. The two images in (b) are of size  $256 \times 256$  at a bit rate of 192 and 320 Kbits, respectively. The final image in (c) has a size  $512 \times 512$  and a bit rate of 750 Kbits. From [20, Fig. 13].

video either in terms of amplitude resolutions (called *quality scalability* or *SNR scalability*), spatial resolutions (*spatial scalability*), temporal resolutions (*temporal scalability*), frequency resolutions (*frequency scalability* or more often known as *data partition*), or combinations of these options. Figure 11.1 shows a set of images decoded from a scalable bit stream, which are obtained by combining quality scalability with spatial scalability.



Note: change "decoded Frame" to "Decoded Frames"

**Figure 11.2.**  $N$  layers of quality scalability. From [20, Fig. 2].

Scalable contents can be accessed either at frame level or object level. The latter is referred as *object-based scalability*, as defined in the MPEG4 standard [5]. In this section, we first introduce the four basic schemes for achieving scalability, including quality, spatial, temporal, and frequency scalability (Sec. 11.1). We then describe how to realize scalability at the object level (Sec. 11.2). Although similar concepts can be applied to different types of coders, we will focus our discussion on how to modify block-based hybrid coders (Sec. 9.3.1) to realize different scalability modes. Finally, we will describe wavelet-based coding methods, which, by the nature of wavelet transforms, naturally lead to fine granularity scalability.

Note that an alternative way to handle varying channel environments/receiver capabilities is by *simulcast*, which simply codes the same video several times, each with a different quality/resolution setting. Although simple, it is very inefficient, as a higher quality/resolution essentially repeats the information that is already contained in a lower quality/resolution stream, plus some additional information. On the other hand, to provide the scalability functionality, a coder has to sacrifice a certain amount of coding efficiency, compared to the state-of-the-art non-scalable coders. *The design goal in scalable coding is to minimize the reduction in coding efficiency while realizing the requirement for scalability.*

## 11.1 Basic Modes of Scalability

### 11.1.1 Quality Scalability

Quality scalability is defined as representing a video sequence with different accuracies in the color patterns. This is typically obtained by quantizing the color values (in the original or a transformed domain) with increasingly finer quantization step-sizes. Because different quantization accuracies lead to different PSNRs between the quantized video and the original one, this type of scalability is more commonly referred as SNR scalability.

Figure 11.2 shows a bitstream with  $N$  layers of quality scalability. Decoding the first layer (also called base layer) provides a low quality version of the reconstructed image. Further decoding the remaining layers (also called enhancement layers) results in a quality increase of the reconstructed image up to the highest quality. The first layer is obtained by applying a coarse quantizer to the original image or in a transform (e.g., DCT) domain. The second layer contains the quantized difference between the original image and that reconstructed from the first layer, using a quantizer that is finer than that used to produce the first layer. Similarly, each of the subsequent layers contains quantized difference between the original image and that reconstructed from previous layers, using an increasingly finer quantizer.

An encoder with two-level quality scalability is depicted in Fig. 11.3(a). For the base level, the encoder operates in the same manner as that of a typical block-based hybrid coder. For the enhanced level, the operations are performed as follows:

1. The raw video frame (or the motion compensation error frame) is DCT-transformed and quantized at the base level;
2. The base-level DCT coefficients are reconstructed by inverse quantization;
3. Subtract the base-level DCT coefficients from the original DCT coefficient;
4. The residual is quantized by a quantization parameter that is smaller than that of the base level;
5. The quantized bits are coded by VLC.

Since the enhanced level uses smaller quantization parameter, it achieves better quality than the base level.

The decoder operation is depicted in Fig. 11.3(b). For the base level, the decoder operates exactly as the non-scalable video decoder. For the enhanced level, both levels must be received, decoded by variable length decoding (VLD), and inversely quantized. Then the base-level DCT coefficient values are added to the enhanced-level DCT coefficient refinements. After this stage, the summed DCT coefficients are inversely DCT-transformed, resulting in enhanced-level decoded video.

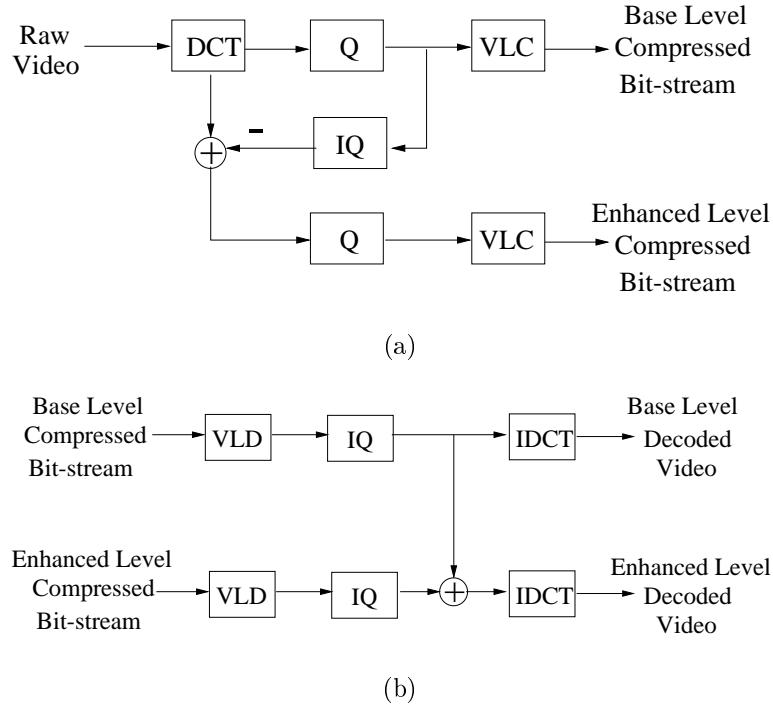
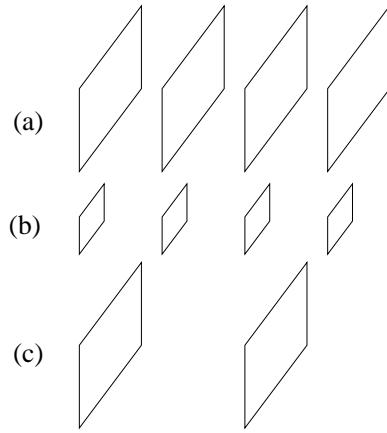


Figure 11.3. A two level quality-scalable codec. (a) encoder; (b) decoder.

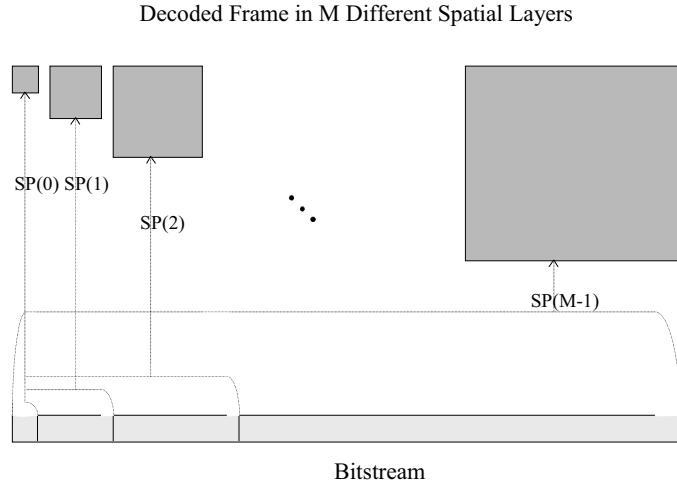
### 11.1.2 Spatial Scalability

Spatial scalability is defined as representing the same video in different spatial resolutions or sizes (see Fig. 11.4(a) and 11.4(b)). Figure 11.5 illustrates a bitstream with  $M$  layers of spatial scalability. By decoding the first layer, the user can display a preview version of the decoded image at a lower resolution. Decoding the second layer results in a larger reconstructed image. Furthermore, by progressively decoding the additional layers, the viewer can increase the spatial resolution of the image up to the full resolution of the original image.

To produce such a layered bit stream, a multi-resolution decomposition of the original image is first obtained. The lowest resolution image is coded directly to produce the first layer (i.e., the base layer). To produce the second layer, the decoded image from the first layer is first interpolated to the second lowest resolution and the difference between the original image at that resolution and the interpolated one is coded. The bitstream for each of the following resolutions is produced in the same way: first form an estimated image at that resolution, based on the previous layers, then code the difference between the estimated and the original image at that resolution.



**Figure 11.4.** Spatial/temporal scaling of a video stream: (a) original video frames; (b) frames scaled to 1/4 original size; (c) temporally scaled frames.



Note to graphics illustrator: the bottom part should be similar to that in Fig. 11.2.

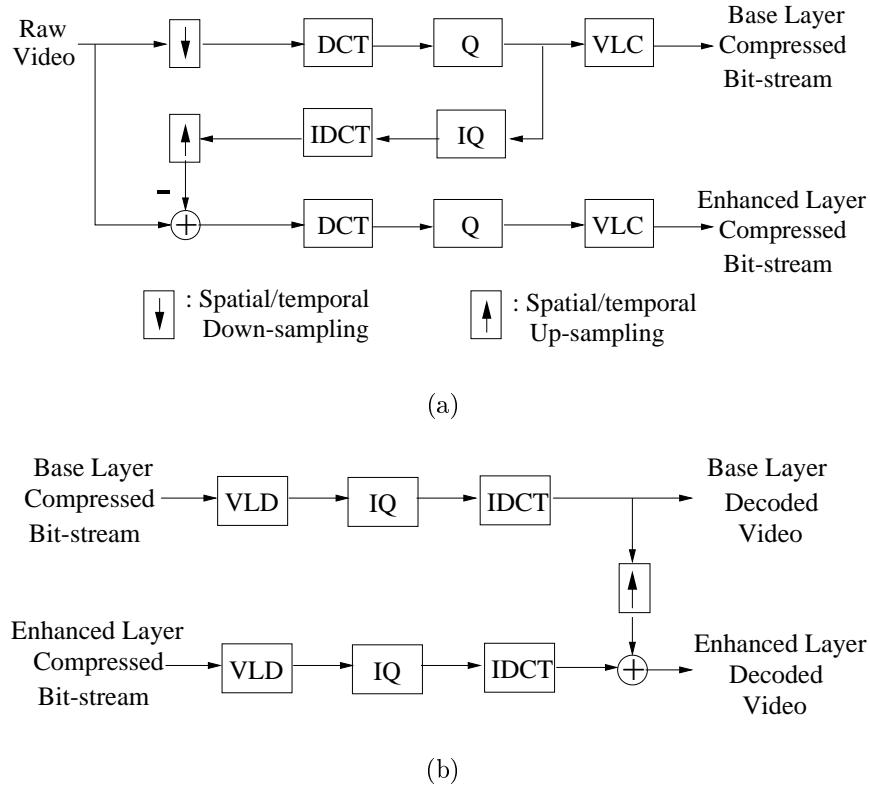
Note: change "decoded Frame" to "Decoded Frames"

Note: Change the squares that enclose up or down arrows to circles.

**Figure 11.5.**  $M$  layers of spatial scalability. From [20, Fig. 1].

Figure 11.6(a) shows a block diagram of a two-layer spatially scalable encoder. For the base layer, the raw video is first spatially down-sampled,<sup>1</sup> then DCT-

<sup>1</sup>For example, spatially down-sampling with ratio 4:1 can be accomplished by replacing every  $2 \times 2$  pixels by their average value. More sophisticated pre-filters can be used to reduce the aliasing



**Figure 11.6.** A two level spatially/temporally scalable codec. (a) encoder; (b) decoder.

transformed, quantized and VLC-coded. For the enhanced layer, the operations are performed as follows:

1. The raw video is spatially down-sampled, DCT-transformed and quantized at the base layer;
2. The base-layer image is reconstructed by inverse quantization and inverse DCT;
3. The base-layer image is spatially up-sampled;<sup>2</sup>
4. Subtract the up-sampled base-layer image from the original image;

effect in the down-sampled image, at the expense of increased complexity.

<sup>2</sup>For example, a simple way to spatially up-sampling with ratio 1:4 is to replicate each pixel four times.

5. The residual is DCT-transformed, and quantized by a quantization parameter, which is smaller than that of the base layer;
6. The quantized bits are coded by VLC.

Since the enhanced layer uses smaller quantization parameter, it achieves finer quality than the base layer.

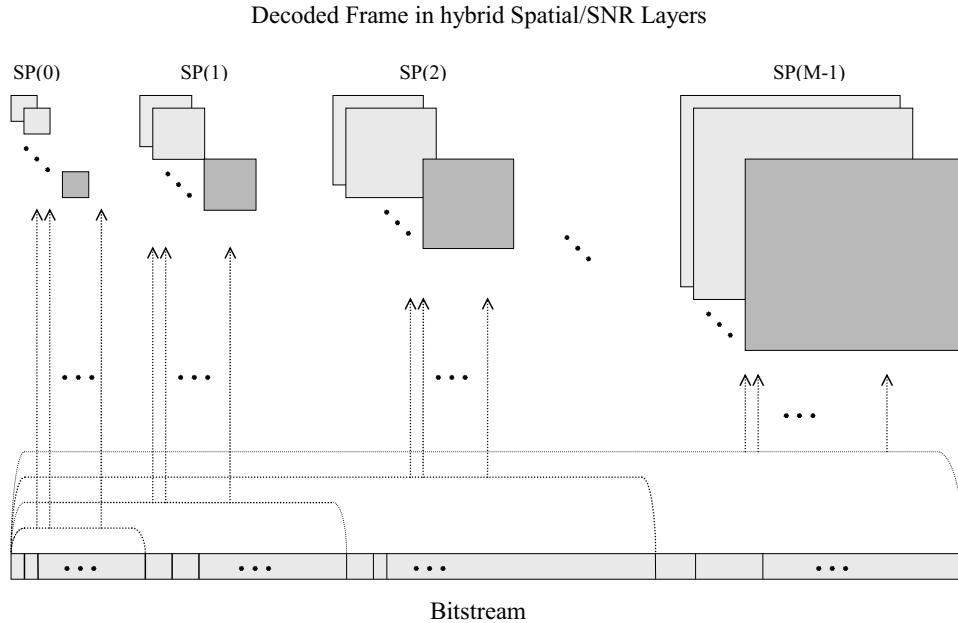
A spatially scalable decoder with two-layer scalability is depicted in Fig. 11.6(b). For the base layer, the decoder operates exactly as the non-scalable video decoder. For the enhanced layer, both layers must be received, decoded by VLD, inversely quantized and inversely DCT-transformed. Then the base-layer image is spatially up-sampled. The up-sampled base-layer image is combined with the enhanced-layer refinements to form enhanced-layer decoded video.

### 11.1.3 Temporal Scalability

Temporal scalability is defined as representing the same video in different temporal resolutions or frame rates (see Fig. 11.4(a) and 11.4(c)). Temporal scalability enables different frame rates for different layers of the contents. Typically, temporally scalable video is encoded in such an efficient way: making use of temporally up-sampled pictures from a lower layer as a prediction in a higher layer. The block diagram of temporally scalable codec is the same as that of spatially scalable codec (see Fig. 11.6). The only difference is that the spatially scalable codec uses spatial down-sampling and spatial up-sampling while the temporally scalable codec uses temporal down-sampling and temporal up-sampling. The simplest way to perform temporal down-sampling is by frame skipping. For example, temporal down-sampling with ratio 2:1 can be achieved by discarding one frame from every two frames. Temporal up-sampling can be accomplished by frame copying. For example, temporal up-sampling with ratio 1:2 can be realized by making a copy for each frame and transmit the two frames to the next stage. In this case, the base layer simply includes all the even frames, and the enhancement layer all the odd frames. For motion compensation, a base layer frame will be predicted only from previous base layer frames, whereas an enhancement layer frame can be predicted from both base layer frames and enhancement frames.

### 11.1.4 Frequency Scalability

Another way to represent a video frame in multiple layers is by including different frequency components in each layer, with the base layer containing low frequency components, and other layers containing increasingly higher frequency components. This way, the base layer will provide a blurred version of the image, and the addition of enhancement layers will yield increasingly sharper images. Such decomposition can be accomplished via whole-frame transforms, such as subband-decomposition or wavelet transforms, or via block-based transforms such as block DCT. One way to implement this idea in the block-based hybrid coders is by including the mode information, motion information as well as first few DCT coefficients of each macroblock



**Figure 11.7.** NxM layers of combined spatial/quality scalability. From [20, Fig. 3].

in the base layer, and including the remaining DCT coefficients in the enhancement layer. In the MPEG-2 standard, this is known as *data partitioning*. We will discuss wavelet-based scalable coders in Sec. 11.3.

### 11.1.5 Combination of Basic Schemes

Quality, spatial, temporal and frequency scalability are basic scalable mechanisms. They can be combined to reach finer granularity. Figure 11.7 shows a case of combined spatial/quality scalability. In this example, the bitstream consists of  $M$  spatial layers and each spatial layer includes  $N$  levels of quality scalability. In this case, both spatial resolution and quality of the reconstructed image can be improved by progressively transmitting and decoding the bitstream. The order is to improve image quality at a given spatial resolution until the best quality is achieved at that spatial resolution and then to increase the spatial resolution to a higher level and improve the quality again.

### 11.1.6 Fine Granularity Scalability

The scalability methods described above produces a bitstream that consists of several layers, a base layer followed by one or several enhancement layers. This type of coder only provides *coarse granularity*, in that the quality improvements are ob-

tained with rate increases in large discrete steps. If only a partial set of bits in a particular layer is received, the reproduced video quality is determined to a large extent by the received bits in the previous layers, not benefiting from the bits received from this layer. *Fine granularity scalability* (FGS) refers to a coding method by which the rate as well as quality increment at a much smaller step. In the limiting case when a bitstream can provide continuously improving video quality with every additional bit, the underlying coding method is called *embedded coding*. Obviously, FGS and embedded coding can adapt to bandwidth variations in real networks more effectively.

In practice, the requirement that the bitstream be completely embedded or have fine granularity is often relaxed. A base layer may be first produced to provide a low but guaranteed level of quality, and then an enhancement layer may be generated to provide improvement in fine granularity. This, e.g., is the method employed in the FGS mode of MPEG-4 [14, 11]. In this method, the conventional block-based hybrid coding method is employed to produce a base layer stream at a given frame rate, using a relatively large QP. Then, for every coded frame (intra- or inter-coded), the differences between the original DCT coefficients<sup>3</sup> and the quantized ones in the base layer (to be called *refinement coefficients*) are coded into a fine granularity stream. This is accomplished by quantizing the refinement coefficients using a very small QP<sup>4</sup> and then representing the quantized indices through successive bit plane encoding.

Specifically, the absolute values of quantized refinement coefficients in each block are specified in binary representations. Starting from the highest bit plane that contains non-zeros, each bit plane is successively coded using run-length coding block by block. The runlengths can be coded using either Huffman coding or arithmetic coding. Typically, different codebooks are used for different bit planes because the runlength distributions are different across planes. When only a partial set of the enhancement layer stream is decoded, depending on how many bit planes were included in the retained bits, the reconstructed video will have a quality in between that obtainable from the base layer only to that obtainable with the QP used on the refinement DCT coefficients. The granularity of the stream is at the bit plane level: every additional complete bit plane will yield an improvement in the quantization accuracy of the DCT coefficients by a factor of two.<sup>5</sup>

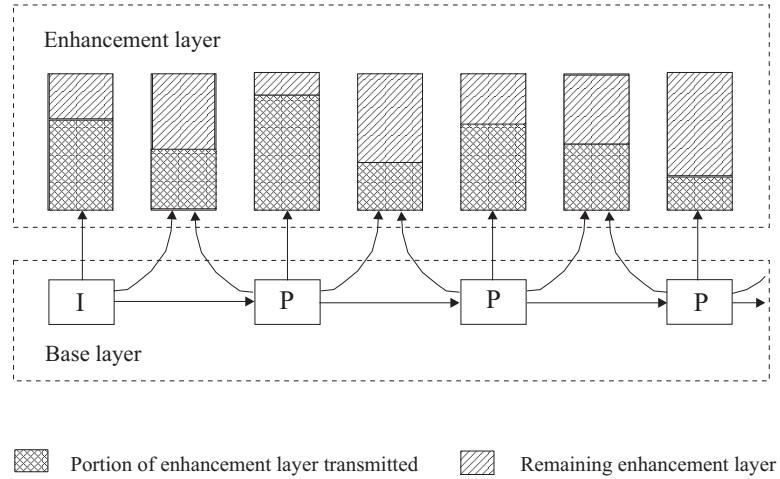
The above method only offers scalability in quality at a fixed spatial-temporal resolution. To add temporal scalability, the base layer may include a sub-sampled set of frames coded with a higher QP. For the enhancement layer, the remaining frames are bidirectionally predicted from the base layer frames. The DCT coeffi-

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<sup>3</sup>For each block, the transform is applied to the original block or the motion compensation error block, depending on whether the block is coded in the intra- or inter mode in the base layer.

<sup>4</sup>No quantization is performed if one wishes to obtain lossless representation at the highest rate.

<sup>5</sup>In fact, the granularity can be up to the codeword level for the run-length representation, as every additional complete codeword received can improve the reconstruction accuracy of some coefficients. Although, depending on the available bandwidth, it is more practical to truncate a FGS stream at the bitplane level.



**Figure 11.8.** Illustration of how to transmit portions of the enhancement layer stream for supporting joint quality-temporal scalability. Adapted from [22, Fig. 1].

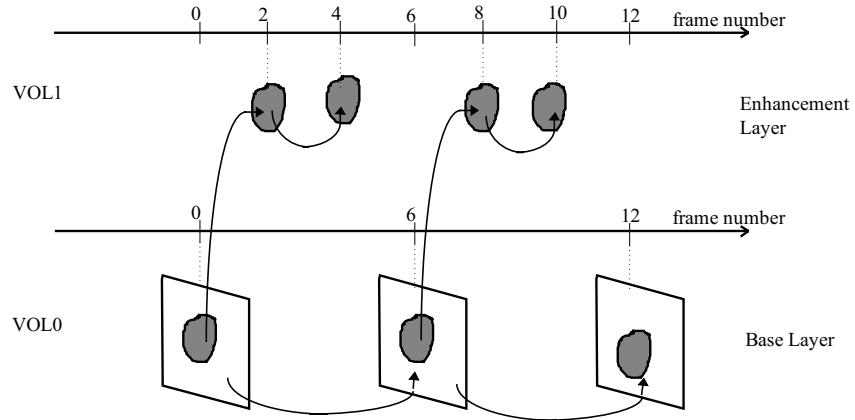
clients in the prediction error images are then quantized and coded using the bit plane encoding method, in addition to the refinement coefficients for the base layer frames [22]. This scheme is illustrated in Fig. 11.8.

A limitation with the above FGS scheme is that the base layer has to be delivered completely and without error. This may or may not be possible in practical networks. Another problem is that, when the base layer bit rate is kept low (so as to increase the scalable range of the bitstream), the coding efficiency of the FGS method will be significantly reduced compared to a non-scalable coder. This is because the reconstructed base layer frames are used as reference frames for motion compensated prediction. Although such a choice eliminates the problem of temporal error propagation in the enhancement layer, the high quantization error in base layer frames reduces the prediction gain. One approach to improve the coding efficiency of the FGS method is by periodically using an intermediate enhancement layer (that reconstructed from some but not all bit planes) as reference for motion compensated prediction [24].

It is difficult to use block-based transform to realize fully embedded coding. We will show in Sec. 11.3 how to achieve this goal with wavelet transforms.

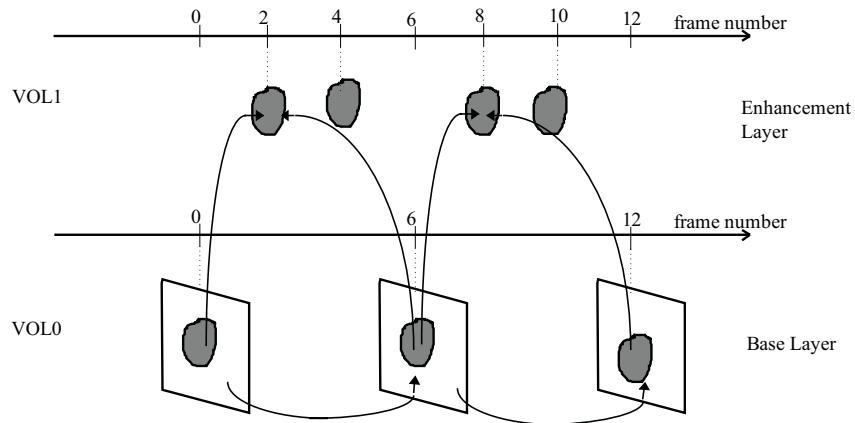
## 11.2 Object-Based Scalability

The various scalable coding methods introduced in the previous section perform the same operation over the entire video frame. In object-based temporal scalability (OTS), the frame rate of a selected object is enhanced such that it has a smoother motion than the remaining area. In other words, the frame rate of the selected



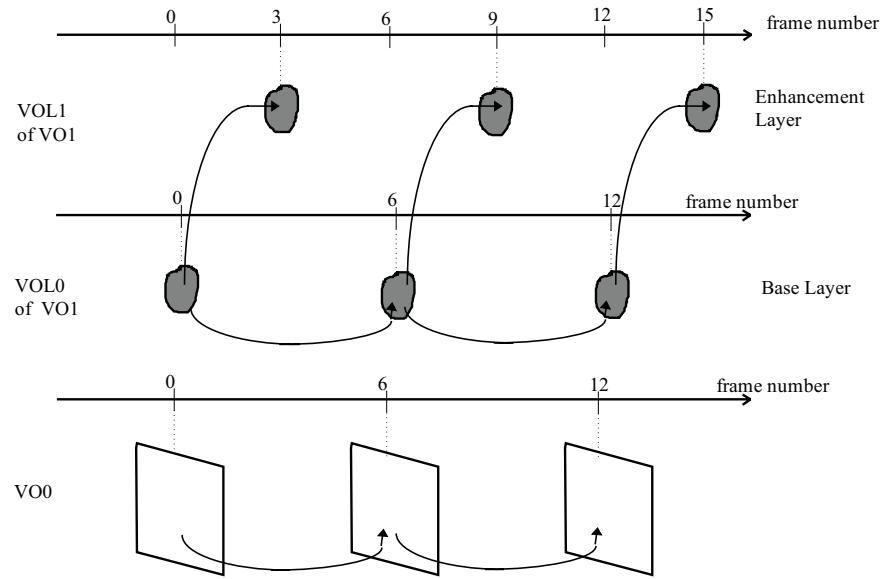
**Figure 11.9.** Enhancement structure of Type 1 with P-VOPs (Courtesy of MPEG4)

object is higher than that of the remaining area. We will mainly introduce object-based temporal scalability (OTS) based on MPEG4 implementations.



**Figure 11.10.** Enhancement structure of Type 1 with B-VOPs (Courtesy of MPEG4)

There are two types of enhancement structures in OTS. Figure 11.9 (courtesy of MPEG4) shows an example of Type 1 where VOL0 (VideoObjectLayer 0) is an entire frame with both an object and a background, while VOL1 represents the particular object in VOL0. VOL0 is coded with a low frame rate and VOL1 is



**Figure 11.11.** Enhancement structure of Type 2 (Courtesy of MPEG4)

coded to achieve a higher frame rate than VOL0. In this example, frames 2 and 4 are coded by predicting from frame 0 in VOL0, followed by overlapping the object of the enhancement layer onto the combined frame. Figure 11.10 shows another example of Type 1, which uses bidirectional predictions forming B-VOPs in the enhancement layer. In this case, frames 2 and 4 in VOL1 are predicted from frames 0 and 6 in VOL0. In both cases, two additional shape data, a forward shape and a backward shape, are encoded to perform the background composition.

Figure 11.11 shows an example of Type 2 where VO0 (VideoObject 0) is the sequence of an entire frame which only contains a background and it has no scalability layer. VO1 is the sequence of a particular object and it has two scalability layers, VOL0 and VOL1. VOL1 represents the same object as VOL0 and it is coded to achieve a higher frame rate than VOL0. In this example, VOL0 is regarded as a base layer and VOL1 is regarded as an enhancement layer of the OTS. Note that the VO0 may not have the same frame rate as other VOs.

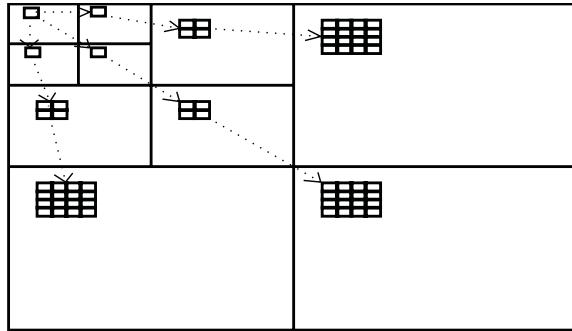
### 11.3 Wavelet Transform Based Coding

The discrete wavelet transform (DWT) has recently emerged as a powerful tool in image and video compression due to its flexibility in representing non-stationary

image signals and its ability in adapting to human visual characteristics [16, 15]. The wavelet representation provides a multiresolution/multifrequency expression of a signal with localization in both time and frequency. This property is very desirable in image and video coding applications. First, real-world image and video signals are non-stationary in nature. A wavelet transform decomposes a non-stationary signal into a set of multiscaled subbands where each component becomes relatively more stationary and hence easier to code. Also, coding schemes and parameters can be adapted to the statistical properties of each subband, and hence coding each stationary component is more efficient than coding the whole non-stationary signal. In addition, the wavelet representation matches to the spatially tuned frequency-modulated properties experienced in early human vision as indicated by the research results in psychophysics and physiology. Finally, the multiresolution/multifrequency decomposition offered by the wavelet transform lends itself easily to a scalable bit stream.

Like DCT-based approach, wavelet transform based coding for images consists of three steps: (1) wavelet transform; (2) quantization; and (3) entropy coding. Wavelet transform converts an image signal into a set of coefficients that correspond to scaled space representation at multiple resolutions and frequency segments. Wavelet coefficients are typically organized into a hierarchical data structure so bit allocation and data compaction can be employed more efficiently. Quantization allows data rate to be reduced at the expense of some distortions. Finally, entropy coding encodes the quantized coefficients into a set of compact binary bitstreams. There are many variations of wavelet transform coding. Wavelet transform is typically performed globally, but block-wise wavelet transform has also been developed to boost the implementation efficiency [9]. Vector wavelet transform and arbitrary-shape wavelet transform have also been recently developed [12, 10]. Quantization could be of several types: scalar, vector, or trellis-coded (TCQ). Entropy coding could be Huffman or arithmetic, using either fixed or adaptive code, implemented in terms of bitplanes or entire samples. When applying wavelets to video coding, some type of mechanisms to reduce temporal redundancy is needed. Motion compensation in both spatial domain and wavelet domain, 3-D wavelets, and motion-compensated 3-D wavelet video coding algorithms have been proposed [26, 21].

For image coding, wavelet has been proven to outperform DCT-based coding techniques by a wide margin in terms of compression efficiency and enhanced features such as scalability. That is the reason why both MPEG-4 and JPEG2000 have selected wavelet-based schemes as the basis for coding still texture and images [9, 20]. However, wavelet-based video coding has yet to show significant compression benefits beyond DCT-based schemes, although advanced scalability has been promised by wavelet approaches. A comparison of DCT and wavelet coding for images and video is included in [25]. Most existing video coding standards, ITU-H.261, H.263, MPEG1, MPEG2, have adopted motion-compensated DCT framework [7, 8, 3, 4]. MPEG4 [5] also uses DCT-based scheme for coding of natural video, although wavelet-based coding is used for still images and graphics. There has been very active research in wavelet-based video coding, aiming at both much



**Figure 11.12.** The parent-child relationship of wavelet coefficients. From [20, Fig. 4].

improved compression ratio and scalabilities.

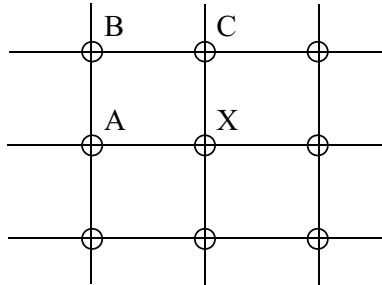
We begin this section by describing the most popular wavelet image coding scheme, namely zero-tree based scheme. It is followed by an example motion-compensated video codec to illustrate wavelet applications to video coding. Since basic mathematical formulation of wavelet transform has been extensively covered in literature (see e.g., [23, 1]), this section focuses on the compression aspect of wavelet-based coding approach.

### 11.3.1 Wavelet Coding of Still Images

#### The EZW Method

Zero-tree based concept was originally developed by Shapiro, known as the Embedded Zero-Tree Wavelet Coding (EZW) [19]. Besides superior compression performance, the advantages of zero-tree wavelet coding include simplicity, embedded bit-stream structure, scalability, and precise bit rate control. Zero-tree wavelet coding is based on three key ideas: (1) wavelet transform for decorrelation; (2) exploiting the self-similarity inherent in the wavelet transform to predict the location of significant information across scales; and (3) universal lossless data compression using adaptive arithmetic coding.

A discrete wavelet transform (DWT) decomposes the input image into a set of subbands of varying resolutions. The coarsest subband, known as the DC band, is



**Figure 11.13.** Adaptive predictive coding of the DC coefficients. From [20, Fig. 6].

a low-pass approximation of the original image, and the other subbands are finer-scale refinements. In the hierarchical subband system such as that of the wavelet transform, with the exception of the lowest frequency subband, every coefficient at a given scale can be related to a set of coefficients of similar orientation at the next finer scale. The coefficient at the coarse scale is called the parent, and all coefficients at the same spatial location, and of similar orientation at the next finer scale are called children.

As an example, Figure 11.12 shows a wavelet tree resulted from a 3-level wavelet decomposition. For the lowest frequency subband, LL3 in the example, the parent-child relationship is defined such that each parent node has three children, one in each subband at the same scale and spatial location but different orientation. For the other subbands, each parent node has four children in the next finer scale of the same orientation.

The wavelet coefficients of the DC band are encoded independently from the other bands. As shown in Figure 11.13, the current coefficient  $X$  is adaptively predicted from three other quantized coefficients in its neighborhood, i.e.  $A$ ,  $B$ , and  $C$ , and the predicted value is subtracted from the current coefficient. The predicted value is obtained as follows:

$$\hat{X} = \begin{cases} C & \text{if } |A - B| < |B - C|, \\ A & \text{otherwise.} \end{cases} \quad (11.3.1)$$

If any of the neighbors,  $A$ ,  $B$ , or  $C$ , is not in the image, its value is set to zero for the purpose of the prediction.

In the bitstream, the quantization step size is first encoded, then the magnitude of the minimum value of the differential quantization indices, called “band\_offset”, and the maximum value of the differential quantization indices, called “band\_max\_value”, are encoded into bitstream. The parameter “band\_offset” is a negative integer or zero and the parameter “band\_max\_value” is a positive integer. Therefore, only the magnitudes of these parameters are coded into the bitstream. The differential quantization indices are coded using the arithmetic coder in a raster scan order, starting from the upper left index and ending at the lower right one. The model is

updated with each encoding of a bit of the predicted quantization index to adapt to the statistics of the DC band.

EZW scans wavelet coefficients subband by subband. Parents are scanned before any of their children. Each coefficient is compared against the current threshold  $T$ . A coefficient is significant if its amplitude is greater than  $T$ . Such a coefficient is then encoded using one of the symbols, *negative significant* (NS) or *positive significant* (PS). The *zero-tree root* (ZTR) symbol is used to signify a coefficient below  $T$  with all its children also below  $T$ . The *isolated zero* (IZ) symbol signifies a coefficient below  $T$  but with at least one child not below  $T$ . For significant coefficients, EZW further encodes coefficient values using a successive approximation quantization scheme. Coding is done by bit-planes, leading to an embedded stream.

### The ZTE and MZTE Methods

EZW has been significantly improved over the years. Examples include SPIHT [18], predictive EZW [13], Zero-Tree Entropy Coding (ZTE) [17], and the more general case Multi-scale ZTE [20]. Shape adaptive wavelet coding using zero-trees has also been proposed for coding of objects with arbitrary shapes [10].

The Zero-Tree Entropy Coding (ZTE) [17] is based on, but differs significantly from, EZW coding. Similar to EZW, ZTE coding exploits the self-similarity inherent in the wavelet transform of images to predict the location of information across wavelet scales. Although ZTE does not produce a fully embedded bitstream as EZW, it gains flexibility and other advantages over EZW coding, including substantial improvement in coding efficiency, simplicity, and spatial scalability. ZTE coding is performed by assigning a zero-tree symbol to a coefficient and then coding the coefficient value with its symbol in one of the two different scanning orders. The four zero-tree symbols used in ZTE are *zero-tree root* (ZTR), *valued zero-tree root* (VZTR), *value* (VAL), and *isolated zero* (IZ). The zero-tree symbols and quantized coefficients are then losslessly encoded using an adaptive arithmetic coder with a given symbol alphabet. The arithmetic encoder adaptively tracks the statistics of the zero-tree symbols and encoded values using three models: 1) *type* to encode the zero-tree symbols, 2) *magnitude* to encode the values in a bit-plane fashion, and (3) *sign* to encode the sign of the value. For each coefficient its zero-tree symbol is encoded first and if necessary, then its value is encoded. The value is encoded in two steps. First, its absolute value is encoded in a bit-plane fashion using the appropriate probability model and then the sign is encoded using a binary probability model.

The Multi-scale Zero-Tree Entropy (MZTE) [20] coding technique is based on ZTE coding but utilizes a new framework to improve and extend ZTE coding to a fully scalable yet very efficient coding technique. At the first scalability layer, the zero-tree symbols are generated in the same way as in ZTE coding and coded with the non-zero wavelet coefficients of that scalability layer. For the next scalability layer, the zero-tree map is updated along with the corresponding value refinements. In each scalability layer, a new zero-tree symbol is coded for a coefficient only if

it was coded as ZTR or IZ in the previous scalability layer. If the coefficient is coded as VZTR or VAL in the previous layer, only its refinement value is coded in the current layer. An additional probability model, residual, is used for encoding the refinements of the coefficients that are coded with a VAL or VZTR symbol in any previous scalability layers. Furthermore, to utilize the highly correlated zero-tree symbols between scalability layers, context modeling, based on the zero-tree symbol of the coefficient in the previous scalability layer in MZTE, is used to better estimate the distribution of zero-tree symbols.

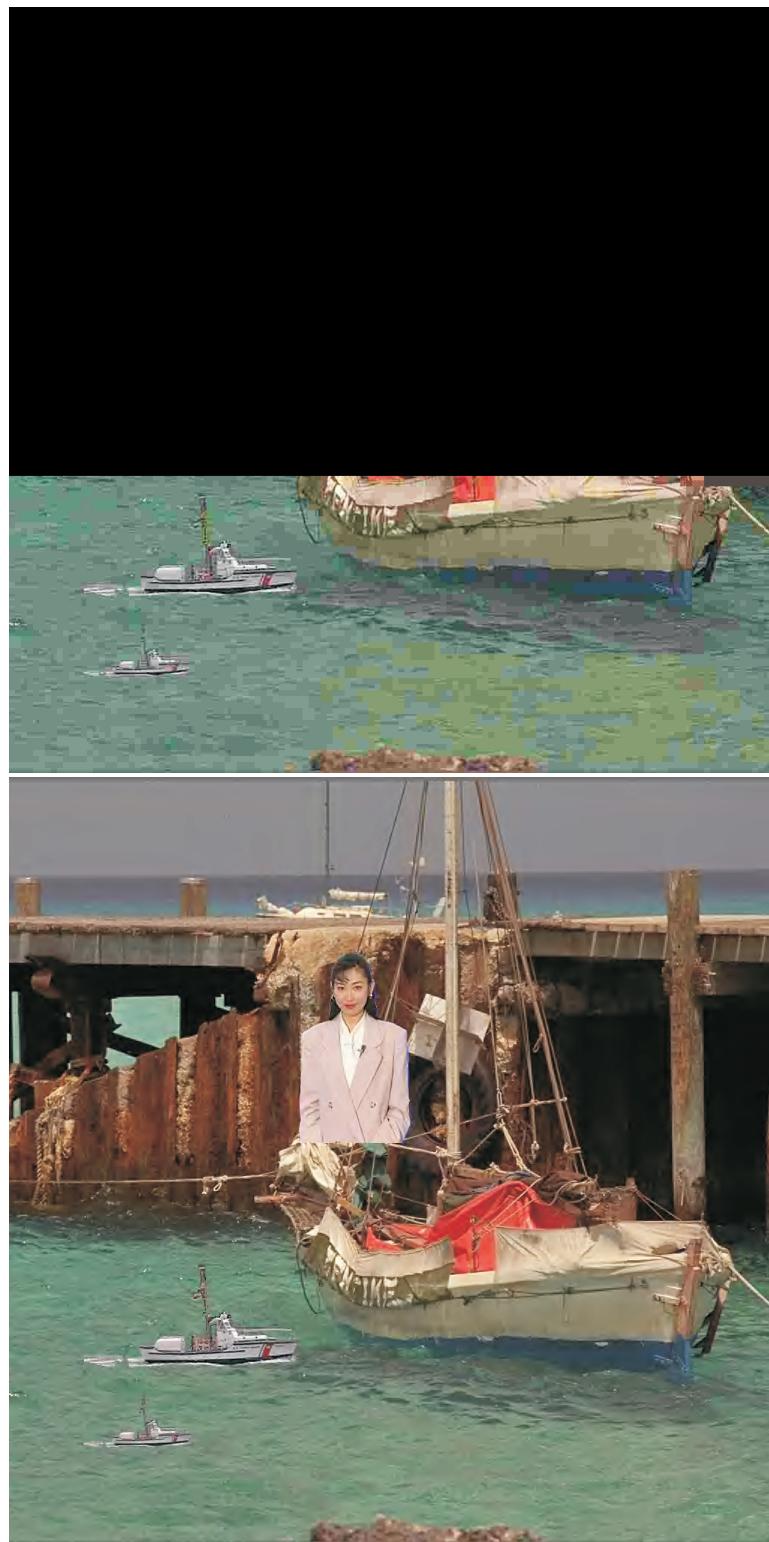
The images in Figures 11.14(a) and 11.14(b) are obtained by JPEG and MZTE compression schemes, respectively, at the same compression ratio of 45:1. The results show that the MZTE scheme generates much better image quality with good preservation of fine texture regions and absence of the blocking effect, compared with JPEG. The study in [25] showed that a wavelet-based coder can typically outperform a DCT-based coder by about 1 dB in PSNR when the bitrate is the same.

With the MZTE method, one can easily obtain spatial and quality scalabilities. By decoding increasingly more bits from the bitstream of a MZTE compressed image, one can obtain a bigger and sharper renditions of the original image. An example set of decoded images at different bit rates has previously been shown in Fig. 11.1.

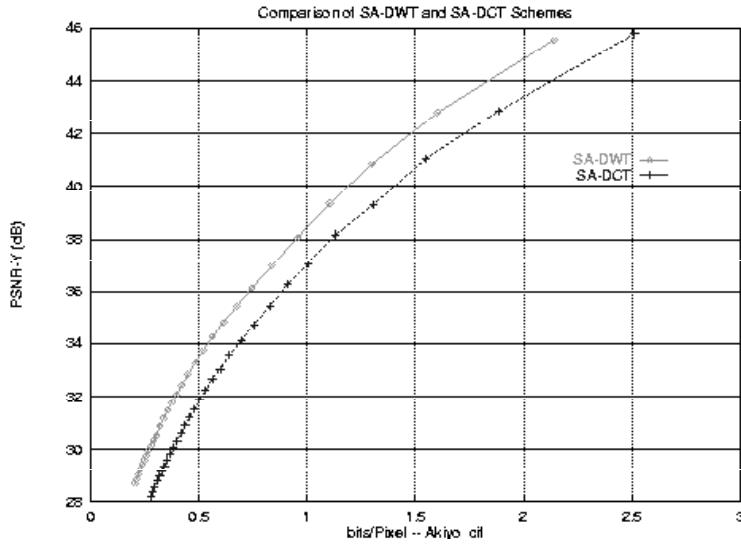
### Shape Adaptive MZTE

MZTE has also been extended to objects of arbitrary shapes [10]. In the shape adaptive ZTE (SA-ZTE) method, the object shape mask is decomposed into a pyramid of subbands so that we know which wavelet tree nodes have valid wavelet coefficients and which ones have don't-care values. We have to pay attention to the way of coding the multi-resolution arbitrarily shaped objects with these don't-care values (corresponding to the out-of-boundary pixels or out-nodes). We discuss how to extend the conventional zerotree coding method to the shape adaptive case.

As discussed in [10], the SA-ZTE decomposes the arbitrarily shaped objects in the image domain to a hierarchical structure with a set of subbands of varying resolutions. Each subband has a corresponding shape mask associated with it to specify the locations of the valid coefficients in that subband. There are three types of nodes in a tree: zeros, non-zeros, and out-nodes (with don't-care values). The task is to extend the zero-tree coding method to the case with out-nodes. A simple way is to set those don't-care values to zeros and then apply the zero-tree coding method. However, this requires bits to code the out-nodes such as a don't-care tree (the parent and all of its children have don't-care values). This is a waste of bits because out-nodes do not need to be coded as the shape mask already indicates their status. Therefore, we should treat out-nodes differently from zeros. Although we don't want to use bits to code an out-node, we have to decide what to do with its children nodes. One way is not to code any information about the status of the children nodes of the don't-care node. This way, we always assume that it has four



**Figure 11.14.** Coded images by (a) JPEG baseline method (PSNRs for Y, U, and V components are 28.36, 34.74, 34.98 dB, respectively); (b) the MZTE method (PSNRs for Y, U, and V components are 30.98, 41.68, 40.14 dB, respectively). Both at a compression ratio of 45:1. From [20, Figs. 11 and 12].



**Figure 11.15.** Comparison of SA-DWT (SA-ZTE in this case) coding with SA-DCT coding.

children to be examined further. When the decoder scans to this node, it will be informed by the shape information that this node is a don't-care node and it will continue to scan its four children nodes. By doing so, all the don't-care nodes in a tree structure need not be coded. This approach performs well when there are only sparse valid nodes in a tree structure. One disadvantage of this approach is that, even if a don't-care node has four zero-tree root children, it still needs to code four zero-tree root symbols instead of one zero-tree root symbol if the don't-care value is treated as a zero. Another way is to selectively treat an out-node as a zero. This is equivalent to creating another symbol for coding some don't-care values. Through extensive experiments, it has been found that the method of not coding out-nodes overall performs better. Detailed description of the coding algorithm can be found in [10].

Extensive experiments have been conducted on the SA-DWT coding technique. The results have been compared with that of SA-DCT coding. The object shape is coded using the MPGE-4 shape coding tool. The test results are presented in the form of PSNR-bitrate curves and the shape bits are excluded from the bitrate since they are independent of the texture coding scheme. Only the texture bitrate are used for comparison. The bitrate (in bits per pixel) is calculated based on the number of pixels in an object with the reconstructed shape and the PSNR value is also calculated over the pixels in the reconstructed shape. Figure 11.15 presents the PSNR-bitrate curves. Clearly, SA-ZTE coding achieves a better coding efficiency than SA-DCT coding with 1.5 dB to 2 dB improvement in PSNR. Figures 11.16(a)



**Figure 11.16.** Reconstructed Object using SA-DCT and SA-ZTE: (a) SA-DCT (1.0042bpp, PSNR-Y=37.09dB; PSNR-U=42.14dB; PSNR-V=42.36dB); (b) SA-ZTE (0.9538bpp, PSNR-Y=38.06dB; PSNR-U=43.43dB; PSNR-V=43.25dB)

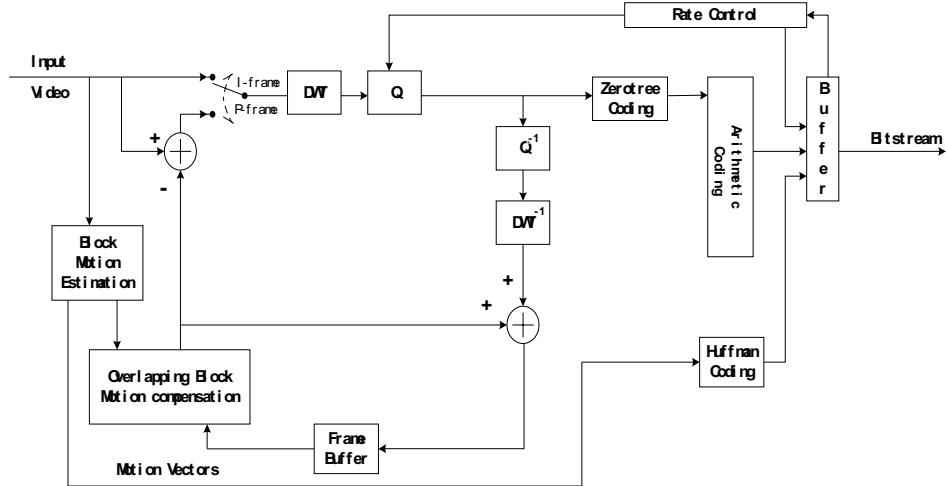
and 11.16(b) show the reconstructed objects from SA-DCT coding and SA-ZTE coding, respectively.

### 11.3.2 Wavelet Coding of Video

Wavelet coding techniques of video can be classified into three categories: (1) spatial-domain motion compensation followed by 2-D wavelet transform; (2) wavelet transform followed by frequency-domain motion compensation; (3) 3-D wavelet transforms with or without motion estimation. Different motion estimation algorithms, quantization schemes, and entropy coding methods can be applied to each of the three categories.

We use an example video codec to illustrate the wavelet-based video coding process [17]. The codec consists of five main components: (1) three-stage motion estimation and compensation; (2) adaptive wavelet transform; (3) multi-stage quantization; (4) zero-tree entropy coding; and (5) intelligent bit rate control. The block diagram of the codec is shown in Figure 11.17. First, a three-stage motion estimation and compensation process is performed including global motion estimation (GME), variable block size motion estimation (VBS-ME), and overlapped block motion compensation (OBMC). The motion-compensated residual frames are then decomposed with a wavelet filter into a set of multi-resolution pyramidal subbands, which is followed by a multi-stage quantizer with different step sizes and dead-zones. The quantized coefficients are finally entropy-coded using ZTE-like encoder but optimized for residual video wavelet coefficients.

GME is used to estimate effects of camera motion and to separate the image sequence into different layers. Each layer has its own global and local motion



**Figure 11.17.** Block diagram of a wavelet-based video codec. From [17, Fig. 1].

estimation. The global motion estimation is based on a 2D affine motion model, and is applied prior to the local motion estimation.

VBS-ME is applied to the successive frames after GME. A quad-tree structure is used to split the frames into variable size blocks ranging from  $8 \times 8$  pixels to  $256 \times 256$  pixels. Conventional block-matching is then used to find the motion vectors for each individual blocks. OBMC (cf. Sec. 9.3.2) uses a weighted average of neighboring blocks displaced by their respective motion vectors as the prediction for the current block. This results in a smoother version of the motion-compensated residual frame. Due to the variable block size used in VBS-ME, OBMC uses the smallest block size (in this case,  $8 \times 8$ ) and the associated window function as the overlapping unit for prediction.

The wavelet transform is applied to the motion-compensated residual frames. Since the energy distribution across different subbands tend to be uniform for residual error frames, the normal criterion for the choice of wavelet filters, compact support, becomes less important. A more important criterion here is the time localization of a wavelet filter. A wavelet filter with good time localization tends to localize edges which are more often encountered in residual frames than original images. This suggests that short-tap filters (with better time localization but less compact support) performs better than long-tap filters (with better compact support but bad time resolution). Indeed, the Haar transform has been found to perform reasonably well.

The DWT is followed by scalar quantization of the wavelet transform coefficients, which is then followed by ZTE. The bit allocation and rate control in the existing coder can be divided into three stages: (a) temporal domain bit allocation with

variable frame rates; (b) inter-frame bit allocation to give optimal rate assignment among frames; and (c) intra-frame bit allocation to produce optimal bit allocation within a frame.

The above method has been compared to the MPEG-4's DCT-based coder [6]. The two methods produced similar performance in terms of PSNR [25]. It has been found that optimization of quantization, entropy coding, and the complex interplay among elements in the coding system is more important than optimizing the transform itself.

## 11.4 Summary

- Scalability is a very useful feature since it allows flexibility and adaptivity to (1) changing network bandwidth; (2) varying channel conditions; and (3) different computing capabilities. Scalable coders are desirable for transport over networks that have time varying bandwidth or error characteristics, or to receivers that have varying access rates or processing capabilities. Scalability is also desirable for video streaming applications, where the same video file may be accessed by users with different communication links and computing powers. Video streaming is discussed further in Chap. 15.
- Scalable coders can have either coarse granularity (two or three layers, also called layered coders) or fine granularity. In the extreme case of fine granularity, the bit stream is fully embedded and can be truncated at any point.
- Quality, spatial, temporal, and frequency scalability are basic scalable mechanisms. These schemes can be combined to achieve finer granularity.
- One approach to obtain embedded streams at fixed spatial-temporal resolutions is by successive bit plane encoding, as used both in fine granularity scalability mode of the MPEG-4 standard, and in most wavelet-based coders.
- Wavelet-based coders can be easily made to produce an embedded stream, because the wavelet transform provides a multi-scale representation of the signal.

## 11.5 Problems

- 11.1 What are the differences between layered and embedded coders? Give example coders in each category.
- 11.2 Describe the four basic scalability modes: quality, spatial, temporal, frequency.
- 11.3 How does the FGS method described in Sec. 11.1.6 work? What is the difference between FGS and quality scalability?

- 11.4** Consider i.i.d. Gaussian processes. Suppose that one can design a one layer coder that reaches the distortion-rate bound, i.e., , the distortion and rate is related by  $D = \sigma_x^2 2^{-2R}$ , where  $\sigma_x^2$  is the source signal variance. Show that a quality scalable coder with two quantization layers will reach the same D-R bound, as the one-layer coder. Extend the proof to an arbitrary number of layers. This means that for Gaussian processes, SNR scalability can achieve the layered functionality without sacrificing coding efficiency. This result is known as the successive refinement property of the Gaussian process. To explore further on this topic, see [2].
- 11.5** Starting from the program you created for the block-based hybrid video coder for Prob. 9.15, generate a code for implementing two-layer quality scalability. Compare the coding efficiency of the original code and the layered coder for a test sequence.
- 11.6** Starting from the program you created for the block-based hybrid video coder for Prob. 9.15, generate a code for implementing two-layer spatial scalability. Compare the coding efficiency of the original code and the layered coder for a test sequence.
- 11.7** What are the main differences between a block-wise transform such as the DCT and the wavelet transform? What are the pros and cons of each?
- 11.8** How would you describe the EZW method to a friend who wants to know how does it work?
- 11.9** Why is the wavelet transform more suitable for producing embedded streams? How would you categorize the type of scalability that is offered by, say, the EZW method? Does it offer quality, spatial, and/or frequency scalability?
- 11.10** Implement the 2-stage wavelet transform for an image using the Haar wavelet filter. Examine the resulting image quality when coefficients in a chosen sub-band are set to zero. Based on these images, comment on what type of features does a particular band of wavelet coefficients reveal.

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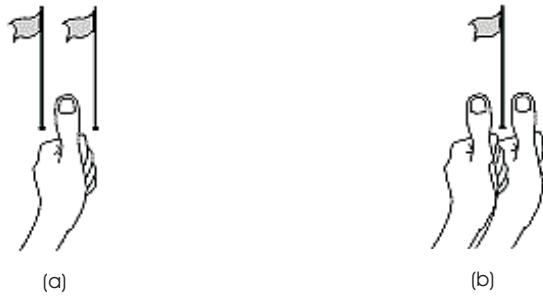
# STEREO AND MULTIVIEW SEQUENCE PROCESSING

Acquisition and display of stereoscopic and more generally multiview sequences have many applications. In industries, stereo display is used for object handling and inspection by robots in environment under hazard conditions. For medical applications, stereoscopic probes are providing more useful information than a monoscopic one. For scientific exploration and visualization, stereoscopic or multiview display enables the human being to experience 3-D environment over a flat 2-D screen. Such applications have long existed, where stereo sequences have traditionally been acquired, stored, and displayed in an analog format. A recent boost for 3-D vision however comes from the advancement of digital television technology. As digital TV including HDTV becomes a reality, digital 3DTV is foreseen to be the next quantum leap for the television industry, promising to offer 3-D perception, in addition to sharper images. Another more recent application for stereo/3-D display lies in virtual reality and human-computer interface, where 3-D rendering and display gives viewers an illusion of physically interacting with people or objects in a remote (or non-existent) site.

In this chapter, we start with a brief discussion of the mechanism for perceiving depth in the human visual system (Sec. 12.1). We then move on to the technical aspects of stereo imaging and show how the 3-D position of an imaged point is related to its projections in a stereo image pair (Sec. 12.2). Next we discuss the technically most challenging problem in stereo sequence processing: disparity and consequently depth from stereo (Sec. 12.3). Finally, we present approaches for coding of stereoscopic and multiview video (Sec. 12.5). These topics are chosen because they belong to signal processing aspects of stereo and multiview systems. For coverage of stereo and multiview capture and display systems, see e.g., [16].

### 12.1 Depth Perception Through Stereopsis

Stereoscopic imaging and display is motivated by how humans perceive depth: through two separate eyes. Although there are several monocular cues that the



**Figure 12.1.** An experiment for understanding stereopsis: (a) When your eyes converge on the thumb, the background appears as a double image; (b) When your eyes converge on the background, you start to see two thumbs in the foreground. From [17].

human brain uses to differentiate objects at different depths, e.g., through relative size and motion parallax, the most important and effective mechanism is through a binocular cue known as stereopsis. To understand how and why stereoscopic systems work, it is essential to understand this phenomenon. In this section, we first describe how stereopsis works. We then present the visual sensitivity in depth perception, which is important for designing 3-D video systems.

### 12.1.1 Binocular Cues — Stereopsis

To explain stereopsis, we take the example from [17]. Try this experiment: hold one finger in front of your face. When you look at the finger, your eyes are *converging* on the finger. This is accomplished by the muscle movement of your eyes so that the images of the finger fall on the fovea of each eye, i.e., the central portion of each retina. If you continue to converge your eyes on your finger, while paying attention to the background, you'll notice the background appears to be double. On the other hand, if you try to focus on the background, your finger will now appear double. This phenomenon is illustrated in Figure 12.1.

If we could take the images that are on your left and right retinæ and superimpose them, we would see two almost overlapping images. *Retinal disparity* refers to the horizontal distance between the corresponding left and right image points of the superimposed retinal images. The points for which the disparity is zero are where the eyes are converged. The mind's ability to combine two different perspective views into one image is called *fusion*, and the resultant sense of depth is called *stereopsis*.

### 12.1.2 Visual Sensitivity Thresholds for Depth Perception

Having learned how human beings perceive depth information, the next question is how sensitive is the human visual system to changes in depth, both in space and

time. In Chapter 2.4, we discussed visual thresholds in detecting spatial (in the same 2-D plane) and temporal changes in monoscopic images. We learned that the contrast sensitivity as a function of spatial frequency exhibits a band-pass characteristics at low temporal frequencies (maximum sensitivity at approximately 4 cpd) and a low-pass characteristics at high temporal frequencies. Similarly, the visual sensitivity with respect to temporal changes is band-pass at low spatial frequencies (maximum sensitivity at approximately 8–15 Hz), and is low-pass at high spatial frequencies. These findings have guided the design of video display systems. In order to portray as much as the eye can see, a display should be able to produce a spatial frequency of at least 64 cpd (the resolution in terms of pixels depends on the picture height and width and the viewing distance) and a temporal frequency of 70 Hz.

To design a 3-D video capture and display system, an important issue is the visual threshold in depth perception. Because the depth is perceived through retinal disparities, visual sensitivity to depth can be evaluated in terms of the variation in disparity. Results quoted below are from Pastoor [11].

**Sensitivity to depth variation in space and time** It has been found that the visual sensitivity for spatial and temporal changes of the disparity lies significantly below the sensitivity for image contrast variations. Thus, a display system that meets the spatial and temporal bandwidth requirement of the contrast sensitivity function would also enable satisfactory perception of depth variation, both spatially and temporally. It has been found that the visual sensitivity in terms of the spatial frequency of the disparity field has a band-pass characteristic with a bandwidth of 3–4 cpd. The maximum sensitivity lies in the range of 0.2–0.5 cpd. The upper limit for temporal changes in disparity which are still perceptible as movements in depth lies in the region of 4–6 Hz.

**Sensitivity to disparity quantization** It has been found that the threshold of stereoscopic acuity, that is, the ability to resolve stimuli in depth, is about 2 seconds of arc.<sup>1</sup> Such fine differences in depth can be perceived if the horizontal image resolution for both left and right images are at least 24 cpd. With digital display, the projected disparity and consequently the apparent depth is quantized. When the quantization is too coarse, the objects in the scene may appear to belong to layers of flat scenery at different depths and object motion in depth may also appear jerky.

**Sensitivity to view angle quantization** In natural vision, when a viewer moves, his/her viewpoint moves continuously. With 3-D display systems offering multiple viewpoints, only a limited number of viewpoints can be realized, which can be thought of as quantizing the direction of the viewing angle. Such quantization has the effect that, when the observer changes his/her position in front of the display, sudden shifts take place between perceived views. It has been demonstrated that view angle shifts must lie within a threshold of approximately 1 min of arc for the subjective quality to be rated “good” on the CCIR quality assessment scale.

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<sup>1</sup> 1 second of arc is  $1/3600$  of a degree.

**Asymmetric spatial resolution requirement** Various experiments have shown that in stereo display, the resolution of one image can be considerably reduced (up to half) without causing deterioration in the subjective impression of sharpness, when the other image is displayed with sufficient resolution as dictated by the visual spatial-temporal thresholds, at least for a short period. It is not known to what degree such a display leads to visual strain when viewed over a long period of time. This asymmetrical property has been explored in stereo sequence compression (see Sec. 12.5.3). By reducing the sampling resolution of one view by half in both horizontal and vertical directions, a compression factor of four can be achieved immediately.

**Asynchronous fusion of binocular views** Within certain limits of disparity values, the visual system is capable of fusing the two images of a stereo pair presented asynchronously as long as the delay between the presentations does not exceed 50 ms [14]. This fact has been taken advantage in field sequential display systems. The delay between the left and right views can lead to distortions of depth when the objects being displayed move. It has been found that a delay of 160 ms can create visible depth distortion.

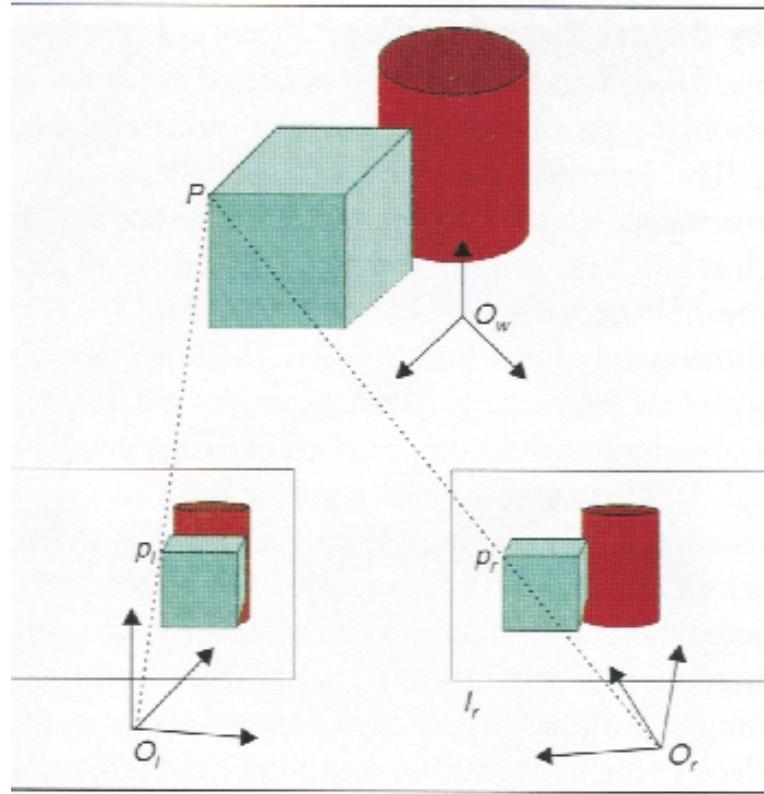
## 12.2 Stereo Imaging Principle

Stereoscopic imaging and display systems are designed to emulate human stereo perception: capturing a 3-D scene using two cameras located in slightly shifted positions, and then presenting the two images separately to each eye. In this section, we describe the principle of stereoscopic imaging and show how to deduce the depth of a 3-D point from its image positions in two different imaging planes. We will see that, in fact, to capture the depth information, the two cameras do not have to be parallel as our human eyes, although such an arrangement leads to mathematically simple relationships between depth and image correspondence. We will start by considering a system with two cameras in arbitrary positions. We will then focus on two special cases: one with two cameras sharing the same imaging plane, another with two cameras oriented towards each other.

### 12.2.1 Arbitrary Camera

We start by considering two arbitrarily positioned cameras. Consider an arbitrary camera configuration shown in Fig. 12.2. Let  $[\mathbf{R}]_l$  and  $\mathbf{t}_l$  (resp.  $[\mathbf{R}]_r$  and  $\mathbf{t}_r$ ) denote the rotation matrix and the translation vector required to align the left (resp. right) camera coordinate ( $\mathbf{C}_l$  and  $\mathbf{C}_r$ ) with a chosen world coordinate ( $\mathbf{C}_w$ ). Then the coordinates of the left and right cameras,  $\mathbf{X}_l = [X_l, Y_l, Z_l]^T$  and  $\mathbf{X}_r = [X_r, Y_r, Z_r]^T$ , are related to the world coordinate  $\mathbf{X} = [X, Y, Z]^T$  by

$$\mathbf{X}_l = [\mathbf{R}]_l \mathbf{X} + \mathbf{t}_l; \quad \mathbf{X}_r = [\mathbf{R}]_r \mathbf{X} + \mathbf{t}_r. \quad (12.2.1)$$



note to graphics illustrator: change  $P, P_l, P_r$  to  $\mathbf{X}, \mathbf{x}_l, \mathbf{x}_r$ . Change  $O_w, O_l, O_r$  to  $\mathbf{C}_w, \mathbf{C}_l, \mathbf{C}_r$ .

**Figure 12.2.** A stereo imaging system. From [18, Fig. 2].

The matrices  $[\mathbf{R}]_l$  and  $[\mathbf{R}]_r$  are orthonormal by definition. Eliminating  $\mathbf{X}$  from (12.2.1) yields

$$\mathbf{X}_r = [\mathbf{R}]_r [\mathbf{R}]_l^T (\mathbf{X}_l - \mathbf{t}_l) + \mathbf{t}_r = [\mathbf{R}]_{rl} \mathbf{X}_l + \mathbf{t}_{rl}, \quad (12.2.2)$$

with

$$[\mathbf{R}]_{rl} = [\mathbf{R}]_r [\mathbf{R}]_l^T, \mathbf{t}_{rl} = \mathbf{t}_r - [\mathbf{R}]_r [\mathbf{R}]_l^T \mathbf{t}_l. \quad (12.2.3)$$

The above equation describes the relation between the left and right camera coordinates.

Recall that when the world coordinate is coincident with the camera coordinate, the world coordinate of a 3-D point  $\mathbf{X} = [X, Y, Z]^T$  is related to the image coordinate  $\mathbf{x} = [x, y]^T$  by Eq. (5.1.2). Applying this relation to  $\mathbf{X}_l$  and  $\mathbf{X}_r$  respectively yields

the left and right image coordinates,  $\mathbf{x}_l = [x_l, y_l]^T$  and  $\mathbf{x}_r = [x_r, y_r]^T$ , of any world point  $\mathbf{X}$ :

$$\begin{aligned} x_l &= F_l \frac{X_l}{Z_l}, & y_l &= F_l \frac{Y_l}{Z_l}; \\ x_r &= F_r \frac{X_r}{Z_r}, & y_r &= F_r \frac{Y_r}{Z_r}. \end{aligned} \quad (12.2.4)$$

Substituting the above relation into (12.2.2) yields

$$\frac{Z_r}{F_r} \begin{bmatrix} x_r \\ y_r \\ F_r \end{bmatrix} = \frac{Z_l}{F_l} [\mathbf{R}]_{rl} \begin{bmatrix} x_l \\ y_l \\ F_l \end{bmatrix} + \mathbf{t}_{rl}. \quad (12.2.5)$$

Using the above relation, one can determine  $Z_r$  and  $Z_l$  of any 3-D point if its image positions  $\mathbf{x}_l$  and  $\mathbf{x}_r$  are known. Although there are three equations, only two are independent, from which a unique solution is obtainable. Once  $Z_l$  and  $Z_r$  are known, one can then determine  $X_l, X_r, Y_l, Y_r$  using (12.2.4), and finally derive the reference coordinate  $(X, Y, Z)$  using (12.2.1). This is the underlying principle for determining the structure of a 3-D object (i.e., the 3-D positions of its various points) from stereo images. This procedure is commonly known as *stereo triangulation*.

The reconstructed 3-D point  $(X, Y, Z)$  is in fact the intersection of the projection lines from the left and right image points. If the two image points are not exactly the projections of a 3-D point, their projection lines would not intersect. This will make Eq. (12.2.5) inconsistent. The least squares solution will be the mid-point in the shortest line that connects the two projection lines [22].

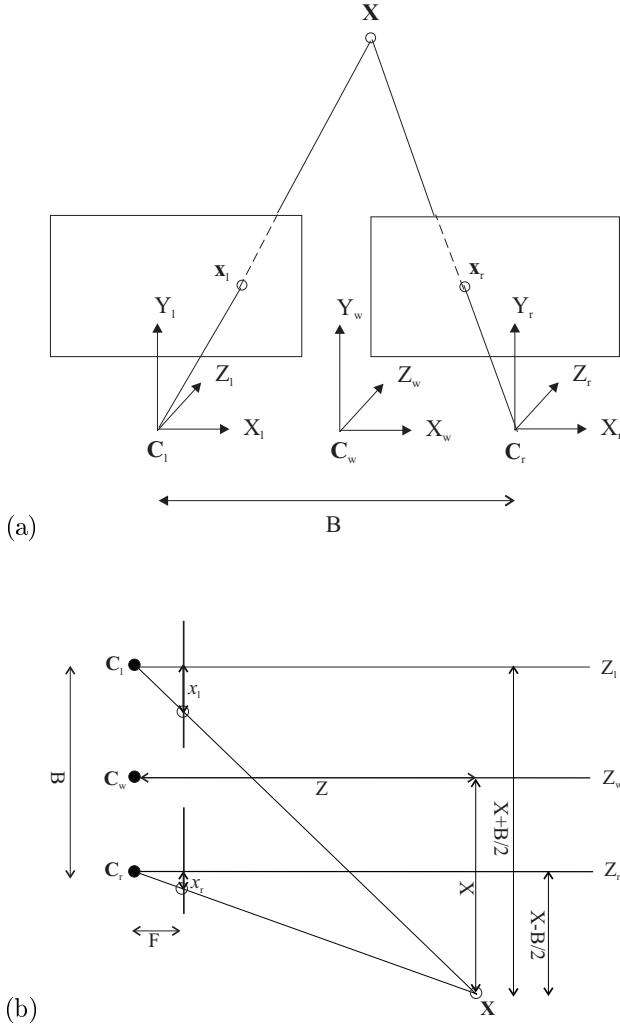
The displacement vector between the left and right image coordinates of the same 3-D position is known as the *disparity vector* (DV) or simply *disparity*.<sup>2</sup> The disparity vector can be defined over either the left or right image coordinate. For example, if we use the right image position as the reference, the disparity is defined as the displacement of the left coordinate with respect to a given right coordinate, i.e.,  $\mathbf{d}(\mathbf{x}_r) = \mathbf{x}_l(\mathbf{x}_r) - \mathbf{x}_r$ .<sup>3</sup> The main difficulty in 3-D structure estimation from stereo images lies in determining the disparity vectors, or equivalently, establishing the correspondence between the left and right image points.

### 12.2.2 Parallel Camera

The most popular configuration for stereo imaging is to use two cameras with parallel imaging planes that are located on the same  $X$ - $Y$  plane of the world coordinate. This is shown in Figure 12.3. The distance between the two cameras is called the *baseline distance*, denoted by  $B$ . When  $B$  is close to the separation between the two eyes of the human being, between 2.5 to 3 in., this configuration simulates the human binocular imaging system. If we set the origin of the world coordinate half

<sup>2</sup>Strictly speaking, we should call it *image disparity*, to differentiate it from retinal disparity and parallax, which refers to disparity at the display screen.

<sup>3</sup>Note that we have previously used  $\mathbf{d}$  to represent the motion vector.

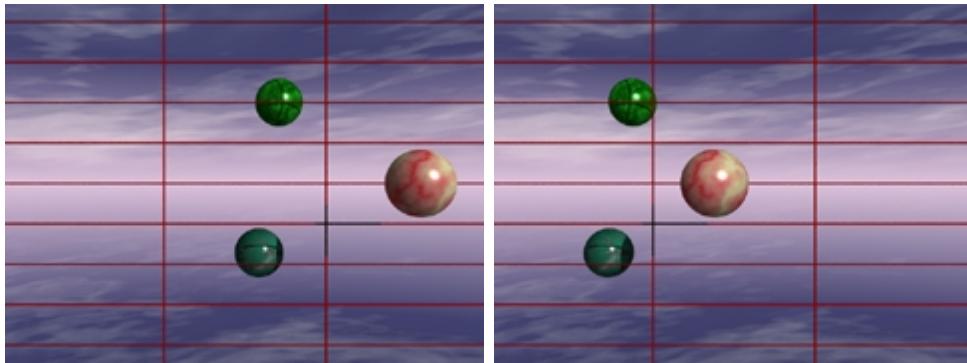


**Figure 12.3.** Paralell camera configuration. (a) the 3-D view; (b) the the X-Z cross-section view ( $Y=0$ ).

way between the two cameras, and assume that the two cameras have the same focal lengths, denoted by  $F$ , the relations in (12.2.1) and (12.2.2) will become:

$$X_l = X + B/2, X_r = X - B/2, Y_l = Y_r = Y, Z_l = Z_r = Z; \quad (12.2.6)$$

$$x_l = F \frac{X + B/2}{Z}, x_r = F \frac{X - B/2}{Z}, y_l = y_r = y = F \frac{Y}{Z}. \quad (12.2.7)$$



**Figure 12.4.** A stereo image pair captured by two parallel cameras. If you try to use your left and right eyes to look at the left and right images separately, you should see three balls at different depths. Courtesy of Rushang Wang.

One can also derive the above relation based on Fig. 12.3(b) directly. The disparity vector has only a horizontal component in this case, and is related to depth by

$$d_x(X, Y, Z) = x_l - x_r = \frac{FB}{Z}. \quad (12.2.8)$$

The 3-D coordinates can be recovered from the image coordinates according to

$$X = \frac{B(x_l + x_r)}{2d_x}, \quad Y = \frac{By}{d_x}, \quad Z = \frac{FB}{d_x}. \quad (12.2.9)$$

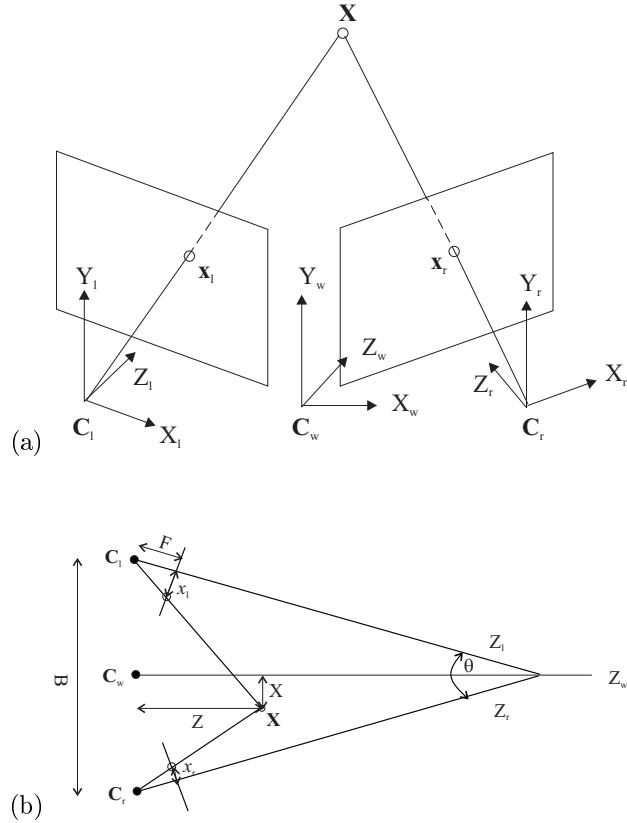
The above relations form the basis for deriving the depth and consequently 3-D structure information from the disparity information.

Equation (12.2.8) plays an important role in stereo sequence analysis. There are several interesting properties that can be derived from this relation. First, the disparity value of a 3-D point  $(X, Y, Z)$  is independent of  $X$  and  $Y$  coordinates and is inversely proportional to the  $Z$  value. The closer is an object point to the camera, the larger is the disparity value. For an object point very far from the camera, the disparity value diminishes. Second, the range of the disparity value increases with the baseline  $B$ , the distance between the two cameras. Finally, with our definition of disparity,  $d_x$  is always positive. This is equivalent to say that the left coordinate is always larger than the right coordinate for the same 3-D point.

Figure 12.4 shows a stereo pair obtained using parallel cameras.<sup>4</sup> It is obvious that corresponding points in the left and right images are on the same horizontal line, and that the closer object (the larger ball) has larger horizontal disparity.

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<sup>4</sup>In fact, the two images here and those in Fig. 12.6 are artificially generated, using a ray tracing technique. Courtesy of Ru-Shang Wang.



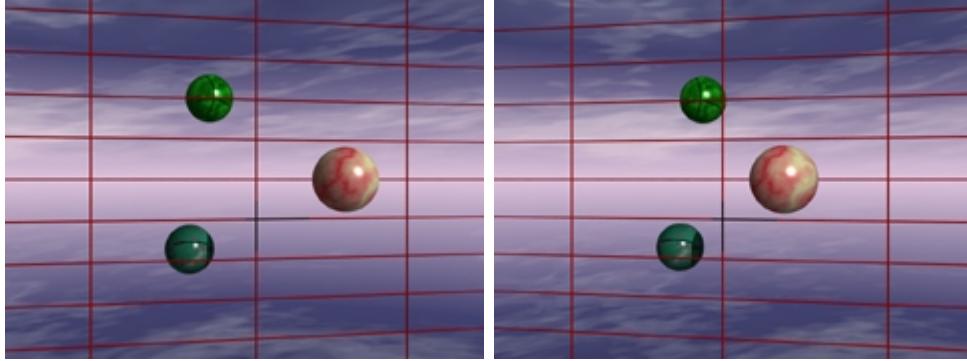
**Figure 12.5.** Converging camera configuration. (a) the 3-D view; (b) the the X-Z cross-section view ( $Y=0$ ).

### 12.2.3 Converging Camera

In a converging stereo configuration, the image planes of the two cameras are tilted with respect to each other so that their focal lines converge to the same point at some distance. This is shown in Figure 12.5. The angle between the two cameras is called the *converging angle*, denoted by  $\theta$ . With this set-up, the left and right camera coordinates are related to the world coordinate by

$$\mathbf{R}_l = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}, \quad \mathbf{t}_l = \begin{bmatrix} \cos \theta B/2 \\ 0 \\ \sin \theta B/2 \end{bmatrix}, \quad (12.2.10)$$

$$\mathbf{R}_r = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}, \quad \mathbf{t}_r = \begin{bmatrix} -\cos \theta B/2 \\ 0 \\ \sin \theta B/2 \end{bmatrix}. \quad (12.2.11)$$



**Figure 12.6.** An example stereo pair obtained by converging cameras. Notice the keystone effect. Courtesy of Rushang Wang.

Substituting the above relation to Eqs. (12.2.2) and (12.2.4) yields

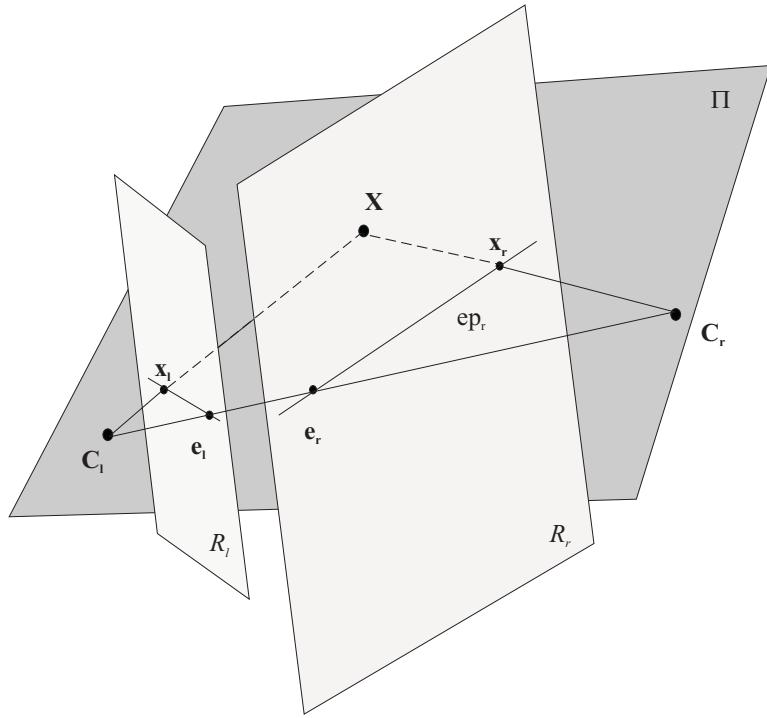
$$\begin{aligned} x_l &= F \frac{\cos \theta (X+B/2) - \sin \theta Z}{\sin \theta (X+B/2) + \cos \theta Z}, & y_l &= F \frac{Y}{\sin \theta (X+B/2) + \cos \theta Z}, \\ x_r &= F \frac{\cos \theta (X-B/2) + \sin \theta Z}{-\sin \theta (X-B/2) + \cos \theta Z}, & y_r &= F \frac{Y}{-\sin \theta (X-B/2) + \cos \theta Z}. \end{aligned} \quad (12.2.12)$$

One can also derive the above relation based on the illustration in Fig. 12.5(b). From the above equations, one can derive the relation between the horizontal (resp. vertical) disparity with the 3-D world coordinate  $(X, Y, Z)$ . The relation is not as straight forward as in the parallel camera case. When  $\theta$  is small (less than  $1^\circ$ ) which is the case with most practical systems, the vertical disparity can be neglected.

With a converging configuration, the imaging system can cover a larger viewing area than a parallel set-up. However, when a stereo pair acquired by a converging stereo camera is directly projected on a screen and viewed by the human eyes, the perceived depth is distorted. One well known artifact is the *keystone distortion*. This refers to the fact that when imaging a rectangular grid with equally spaced lines, with the left camera, the vertical distance between two adjacent lines is larger on the left side than on the right side. The image produced by the right camera has the reverse effect. An example stereo pair obtained by converging cameras is given in Fig. 12.6. When displayed using a parallel projection configuration and viewed by the human eye, such vertical disparity will cause difficulty in depth perception. To display the images acquired by a converging camera system, geometric correction (also known as image rectification) is needed so that the corrected images appear as if they were acquired using a parallel set-up.

#### 12.2.4 Epipolar Geometry

Refer to Fig. 12.7. For an arbitrarily chosen point in 3-D,  $\mathbf{X}$ , the plane  $\Pi$ , defined by this point and the optical centers of the left and right cameras,  $\mathbf{C}_l$  and  $\mathbf{C}_r$  is

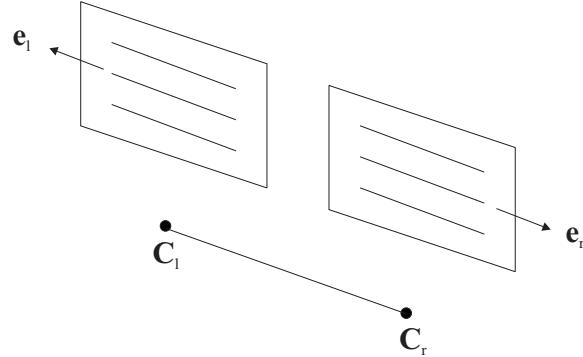


**Figure 12.7.** Epipolar geometry in arbitrary camera configuration.

called the *epipolar plane*. The intersection of this plane with the left and right image planes are called the left (the line connecting  $x_l$  and  $e_l$ ) and right (the line connecting  $x_r$  and  $e_r$ ) epipolar lines, respectively. The image of  $C_l$  in the right image,  $e_r$ , is called the *right epipole*. Similarly, the image of  $C_r$  on the left image,  $e_l$ , is the *left epipole*. These quantities are illustrated in Figure 12.7. We see that, for any imaged point that falls on the left epipolar line, its corresponding pixel in the right image must be on the right epipolar line. This is known as the *epipolar constraint*. This property can be used to constrain the search range for disparity estimation, as discussed in Sec. 12.3.

Note that for each selected imaged point, there is an epipolar plane and two corresponding epipolar lines associated with it. For example, given  $x_l$ , a point in the left image, its left epipolar line is the one connecting this point with the left epipole,  $e_l$ . Its corresponding right epipolar line is the line connecting the image of this point in the right view,  $x_r$  and the right epipole  $e_r$ . All the left epipolar lines pass through the left epipole, and all the right epipolar lines pass through the right epipole.

The relation between an image point and its epipolar line depends on the camera set-up. A very important result in stereo imaging is that this relation can be



**Figure 12.8.** Epipolar geometry for parallel camera configuration: epipoles are at infinity, and epipolar lines are parallel. Modified from [3, Fig. 6.4].

completely characterized by a  $3 \times 3$  matrix known as *fundamental matrix*,  $[\mathbf{F}]$ . Let  $\tilde{\mathbf{x}}_i^T = [\mathbf{x}_i^T, 1]$ ,  $i = l, r$ , where  $\mathbf{x}_l, \mathbf{x}_r$  represent the left and right images of the same 3-D point.<sup>5</sup> They are related by:

$$\tilde{\mathbf{x}}_r^T [\mathbf{F}] \tilde{\mathbf{x}}_l = 0, \quad \tilde{\mathbf{x}}_l^T [\mathbf{F}]^T \tilde{\mathbf{x}}_r = 0. \quad (12.2.13)$$

In our simplified configuration shown in Fig. 12.2, the fundamental matrix only depends on the relative relations of the two camera coordinates,  $[\mathbf{R}]_{rl}, \mathbf{t}_{rl}$ . In general, it will also depend on intrinsic camera parameters. For the derivation of the above result and the relation between  $[\mathbf{F}]$  and camera parameters, the readers are referred to the excellent book by Faugeras [3]. When the camera parameters are unknown, the elements in  $[\mathbf{F}]$  can be estimated from a set of detected corresponding points in the left and right images, based on Eq. (12.2.13). A good reference on this subject is [23].

With parallel camera set-up, the epipoles are at infinity and the epipolar lines are all horizontal, as illustrated in Fig. 12.8. For any given point in, say, the right image,  $\mathbf{x}_r$ , the left and right epipolar lines associated with this point are simply the horizontal lines with the same  $y$ -coordinate as this point. On the other hand, with a converging configuration, the left and right epipolar lines are tilted with respect to each other. This can be easily observed in the example image given in Fig. 12.6. However, when the angle  $\theta$  is sufficiently small, the epipolar lines can be treated as horizontal.

Given a pair of images obtained by two converging cameras, one can rectify them so that the rectified images appear as if they were captured using two parallel cameras. Visually, this has the effect of mapping a pair of images so that the originally tilted set of epipolar lines become horizontal and parallel. Such rectification,

<sup>5</sup>  $\tilde{\mathbf{x}}^T = [\mathbf{x}^T, 1]$  is the representation of  $\mathbf{x}$  in the homogeneous coordinate (also known as projective coordinate). Use of the homogeneous coordinate can convert most non-linear relations encountered in perspective imaging to linear relations.

for example, would greatly simplify the disparity estimation problem. It is also needed for displaying these images by a parallel projection system. Recall from the previous discussion, the epipolar constraint is completely characterized by the fundamental matrix relating the two cameras. Therefore, the key to the rectification process is to determine the fundamental matrix. In practice, the camera configuration parameters may not be known exactly, so that the fundamental matrix must be determined from the corresponding feature points in the two given images. A discussion on camera calibration and rectification techniques is beyond the scope of this chapter. The readers are referred to [19, 3] for a good treatment of this subject.

### 12.3 Disparity Estimation

As introduced in the previous section, disparity refers to the displacement between the 2-D coordinates in the left and right image planes of a 3-D point in the imaged scene. As shown in Sec. 12.2, from the disparity associated with two corresponding points in a stereo image pair, one can derive the depth associated with the underlying 3-D point and consequently its complete 3-D coordinate in the real world. For this reason, disparity estimation is an essential step in any 3-D image/video processing system. It is also important for generating intermediate views between given views.

Disparity estimation problem is similar to the motion estimation problem in that both require establishment of the correspondence between pixels in two images. In non-feature-based approach, for every point in one image, the task is to determine its corresponding point in the other image, or mark it as being occluded in the other image. For feature-based approach, the task is to first detect certain feature points in one image, and then find their corresponding positions in the other. For motion estimation, the two images differ in time; whereas in disparity estimation, the two images are taken at the same time but from different camera view angles.

Disparity estimation tends to be more complex and challenging than motion estimation. First, usually only a limited number of objects move in the scene so that the MV's are zero for most pixels. In stereo imaging, however, almost all pixels have non-zero disparities, except those pixels that are very far away from the image plane. Secondly, the range of MV's is usually quite limited, whereas the DV can be very large for objects close to the camera. For example, with CCIR 601 video, the MV's usually do not exceed a few pixels, whereas the DV's can reach 40-50 pixels in the horizontal direction and up to 5 pixels in the vertical direction (assuming a slightly convergent camera is used) [6]. Obviously if one were to use a search technique similar to the block matching algorithm for motion estimation, the search region must be greatly increased. The blocking artifacts also become more visible because the block-wise constant model does not approximate the disparity variation well.

The general approach to disparity estimation is very similar to motion estimation, except that there are more physically induced constraints in disparity fields, as described below. Basically, one needs to set up an objective function that measures

the error between a given view and its predicted version from the other view using *disparity compensated prediction* (DCP). If we assume that the disparity field is smooth, then some smoothness measures can be added to the objective function. These smoothness constraints should be relaxed at object boundaries or occluded regions. The estimated parameters are further subjected to some physical constraints. The difference between different approaches lie in the parameterization of the disparity field as well as the search method. In the following, we first describe some important constraints that are useful for disparity estimation. We then present several estimation methods.

### 12.3.1 Constraints on Disparity Distribution

The disparity vectors are constrained by the camera geometry and the continuity of the object surface. These constraints, if used judiciously, can be very helpful in disparity estimation.

#### Epipolar Constraint

Given a stereo imaging configuration, the epipolar constraint refers to the fact that the corresponding pixels in a stereo pair always lie on respective epipolar lines. Recall that if the fundamental matrix is known, then, given any point in, say, the right image, one can determine the epipolar line in the left image using Eq. (12.2.13), which defines all possible locations of this point in the left image. Therefore, one only needs to search along this line.

For the parallel camera configuration, the epipolar lines are parallel to horizontal scan lines of the images, so that one can constrain the search within the horizontal line that the right image point is located. When the camera configuration is non-parallel, although one can confine the search to the epipolar lines corresponding to the actual camera set-up, one simpler approach is to first map the left and right images into a parallel configuration, and then apply the procedure for the parallel configuration [21]. In either case, one must first estimate the camera geometry parameters or the fundamental matrix, which is required for determining the epipolar lines or for reprojection.

#### Uni-Directionality with Parallel Camera

As shown in Sec. 12.2.2, with the parallel camera configuration, the disparity vector only has horizontal component and is always positive (i.e., the left horizontal coordinate is always larger than the right one).

#### Ordering Constraint

Assuming opacity of the imaged objects, the relative positions of different points in an object are the same in both views of the scene, i.e., a feature point left of another in the left view lies also on the left in the right view. Let  $x_{r,1}$  and  $x_{r,2}$  represent two points in the right image on the same horizontal line and assume  $x_{r,1} < x_{r,2}$ .

Then by requiring  $x_{l,1} < x_{l,2}$ , we have  $d_{x,2} < d_{x,1} + x_{l,2} - x_{l,1}$ , which constrains the search range for  $d_{x,2}$  given the previously estimated  $d_{x,1}$ .

### 12.3.2 Models for the Disparity Function

Disparity is a deterministic function of the depth, for a given camera configuration. Given a model for the actual surface function, one can derive the corresponding model for the disparity function. Such a model is very helpful for disparity estimation.

Consider the simplest case when the surface of the imaged scene can be approximated by a plane so that

$$Z(X, Y) = aX + bY + c. \quad (12.3.1)$$

Using (12.2.6) and (12.2.7), we can rewrite  $Z$  in terms of the right image coordinates:

$$Z(x_r, y_r) = a \left( \frac{Zx_r}{F} - \frac{B}{2} \right) + b \frac{Zy_r}{F} + c \quad (12.3.2)$$

or

$$Z(x_r, y_r) = \frac{c + aB/2}{1 - ax_r/F - by_r/F}. \quad (12.3.3)$$

We can see that the surface function varies non-linearly in terms of image coordinates. Substituting the above result into (12.2.8) yields:

$$d_x(x_r, y_r) = \frac{1}{c/B + a/2} (F + ax_r + by_r). \quad (12.3.4)$$

Therefore, *the disparity function is affine in the image coordinate when the surface is a plane*.

In reality, the entire surface (i.e., the depth distribution) of the imaged scene cannot be approximated well by a plane. However, it usually can be divided into small patches so that each patch is approximately planar. Using the above result, we can model the disparity function over each patch as affine. The disparity estimation problem can then be turned into estimating three affine parameters for each patch, which is equivalent to estimating the disparity ( $d_x$  only) at three corner points (nodes) of each patch. If we represent the entire surface by a triangular mesh, then the disparity estimation problem can be reduced to estimating the disparity at nodal points. The disparity function within each patch can then be interpolated from the nodal points using the affine model.

Using similar approaches, one can also derive the disparity model for higher order (curved) surfaces. Such parametric models can be very helpful for disparity estimation.

### 12.3.3 Block-Based Approach

This approach assumes that the disparity function over each small block of the reference view can be described by a constant or a low order polynomial. The

constant disparity or the polynomial coefficients are determined by minimizing the error between the two views after warping based on the estimated disparity function (i.e., disparity compensation). Either exhaustive or gradient-descent search methods used for block-based motion estimation can be applied. But the search should be constrained properly, using the constraints described in Sec. 12.3.1. In the special case that the disparity is assumed to be constant, only a single horizontal shift needs to be searched, and only in one direction, positive or negative, depending on which view is used as reference. However, compared to motion estimation, the search range should be much larger, if the depth variation in the imaged scene is relatively large or the baseline separation between cameras is large. To overcome the difficulty associated with the potentially very large disparity value, a hierarchical search strategy, as the one described in Sec. 6.9, should be used. A good review of block-based estimation methods can be found in [20].

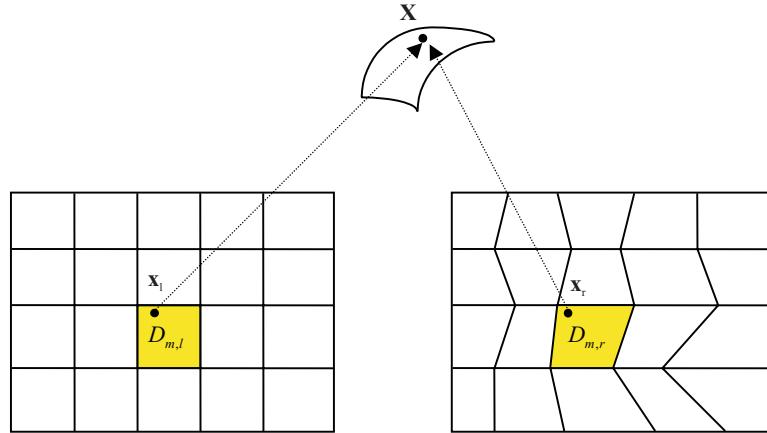
Unlike motion estimation, where the block-wise constant model is fairly accurate for a large number of blocks, this model is only appropriate when the surface patch corresponding to the block is a flat surface that is parallel with the imaging plane, which is rarely the case. On the other hand, the affine model is quite good when the block size is sufficiently small. In this case, the surface patch corresponding to each block is approximately flat, and as shown in Sec. 12.3.2, the disparity function over a planar patch is well described by an affine function.

#### 12.3.4 2-D Mesh-Based Approach

Instead of using a block-wise constant model, the mesh-based approach described for motion estimation can also be applied to disparity estimation. In this case, one first applies a mesh in a reference view, say the left view, and tries to find the corresponding nodal positions in the right view. Note that each pair of corresponding 2-D mesh elements can be considered as the projections of a 3-D surface patch on the left and right images, as shown in Fig. 12.9. As with motion estimation, it is best to parameterize the disparity function by the nodal displacement between the left and right views, rather than polynomial coefficients in affine or bilinear models. One can estimate the nodal disparities by minimizing the disparity compensated prediction error between corresponding elements, similar to the mesh-based motion estimation method (Sec. 6.6). When the underlying images are obtained with a parallel set-up, only horizontal disparities need to be searched.

One such approach is described in [21]. As shown in Fig. 12.9, the left view is described by a rectangular mesh, and the nodes in the right view are shifted horizontally with respect to the corresponding nodes in the left view. The disparity function within each element is modeled as bilinear, so that the 3-D patch corresponding to this element is a curved surface. To determine the horizontal disparity at each node, the disparity compensation error summed over the four elements attached to this node is minimized. Instead of performing exhaustive search, a gradient-based fast algorithm is used.

Figure 12.10 compares the disparity estimation result for a sample stereo image



**Figure 12.9.** Correspondence between the 3-D mesh and 2-D meshes. From [21, Fig. 1].

pair “Man” using the block-based approach and the above mesh-based approach [21]. The image pair is obtained with two parallel cameras with a baseline distance of 50-cm. This large baseline distance leads to very large disparity values between the two images. A global disparity is first estimated and used to compensate the right image before disparity estimation. The block matching algorithm (assuming constant disparity within each block) uses  $16 \times 16$  blocks and a search range of  $\pm 100$  pixels. The mesh-based scheme applies a rectangular mesh on the left image with element size  $32 \times 16$ , and moves the nodal position horizontally in the right image to minimize the disparity compensation error between corresponding elements. Although the block matching algorithm leads to higher PSNR of the disparity compensated image, the mesh-based scheme yields visually more accurate prediction.

In the above described approach, a regular mesh is overlaid over the entire reference image. This will lead to estimation errors near object boundaries and places with depth discontinuity. A better approach, although more complex, is to generate a mesh that follows the object contour and allow discontinuity at object boundaries. For example, if the mesh is generated so that each mesh element corresponds to a surface patch that is approximately flat, then the disparity function over this element can be modeled accurately by an affine function, as shown in Sec. 12.3.2. In this case, triangular meshes would be more appropriate.

A benefit of using mesh-based disparity estimation is that it is easy to construct intermediate views using the resulting disparity information. This can be done by first generating an intermediate mesh by interpolating the nodal positions from their corresponding left and right view positions, and then performing texture warping elements by elements. An element in the interpolated view can be warped from

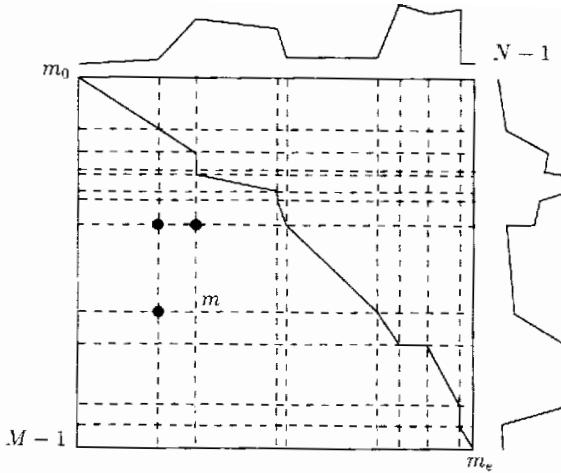


**Figure 12.10.** (a) the original left image; (b) the original right image; (c) the predicted right image obtained by a block matching algorithm (PSNR is 32.03 dB over the foreground) (From [21, Fig. 4d]); (d) the predicted right image obtained by a mesh-based algorithm (PSNR is 27.28 dB over the foreground) (from [21, Fig. 4c]); (e) the predicted right image obtained by the dynamic programming method of [2].

the corresponding element in either the left or right view. This topic is discussed further in Sec. 12.4.

### 12.3.5 Intra-line Edge Matching Using Dynamic Programming

With parallel imaging geometry (or after image rectification), the epipolar lines are horizontal scanlines. Therefore, given a horizontal scan line, a pair of corresponding points in the right and left images should be searched for only within this line. Instead of searching for matches between all the pixels, one can instead find all the edge points on this line, and try to match each edge pixel on the right image with one edge pixel on the left image. For a given scan line, the searching of matching pairs along this line can be converted to a problem of finding a path on a two dimensional search plane whose vertical and horizontal axes are the right and left edge positions, as illustrated in Fig. 12.11. The figure assumes there are  $M$  and  $N$  edge points in the left and right images, respectively. Each node corresponds to the association of a right edge point with a left edge point. In general, not all



**Figure 12.11.** Path-finding analogy of the stereo matching problem. From [3, Fig. 6.24].

edge pixels seen on the right images will also appear in the left image. Also, a single edge pixel in the right image may correspond to several edge points in the left image. Therefore, the matching could be one-to-many, many-to-one, or one-to-one. However, the legitimate matches should satisfy the ordering constraint. Starting from the left most edge pixel in the right image, for each new edge pixel in the right image, its matching edge point in the left image can only be the remaining unmatched edges or the one matched in the previous step. Therefore, the legitimate paths are those which do not go back either horizontally or vertically. If we assign a cost at each possible node in the graph, the problem is to find the best path from the beginning pair to the last pair, so that the total cost is minimized. The cost for a given node can be determined, e.g., as the sum of square or absolute errors between corresponding pixels in a small window surrounding the two edge pixels defining the node. It can also include the errors between the corresponding pixels in the intervals between the current node and the previous node.

The method described above does not take into account the mutual dependency of edge pixels between scanlines. If there is a vertical edge extending across scanlines, the correspondences in one scanline have strong dependency on the correspondences in the neighboring scanlines. Specifically, if two points are on a vertically connected edge in the right image, with a high probability, their corresponding points would also lie on a vertically connected edge in the left image. One way to promote such consistency among the scan lines is by including a penalty term in the cost definition, so that a higher cost is assigned for matches that are not consistent with matches found in the previous line. With this modification, the stereo

matching problem can be cast as that of finding a surface that best satisfies the intra-scanline matches and inter-scanline consistency. Here the matching surface is defined by stacking 2-D matching paths.

Ohta and Kanade [10] first proposed to cast the disparity estimation problem as the surface matching program outlined above. Furthermore, they developed a fast search algorithm using the dynamic programming method. The original work of Ohta and Kanade only considered the constraint imposed by edges along the vertical direction. Falkenhagen extended this scheme to consider more complicated 2-D constraints [2]. Figure 12.10(e) show the predicted right image for “Man” using this method.<sup>6</sup>

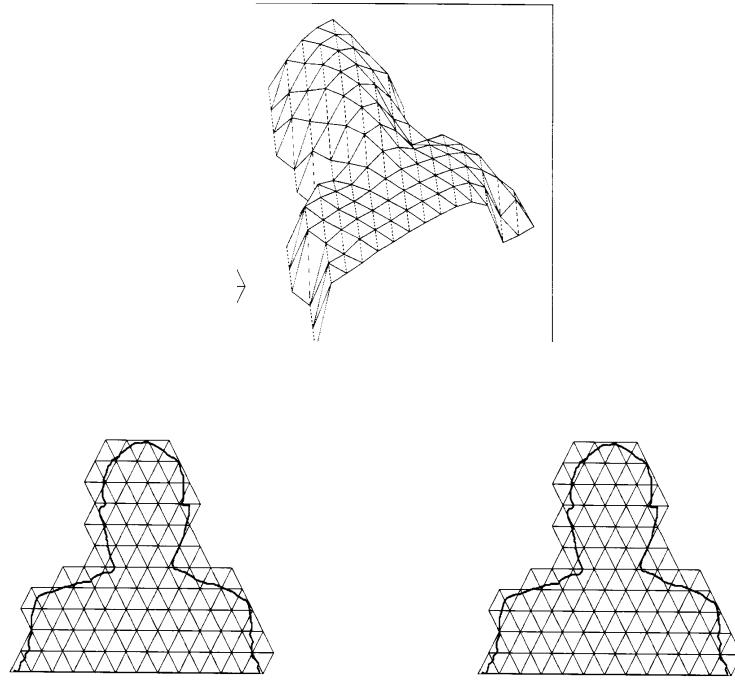
### 12.3.6 Joint Structure and Motion Estimation

The previous discussions have focused on the estimation of the disparity function from a pair of stereoscopic views obtained at the same time. From the disparity function, one can derive the structure (i.e., spatial distribution) of the imaged scene, as described in Sec. 12.2. This would allow one to reconstruct the 3-D surface of the imaged object. With stereo sequences, one is interested in the estimation of not only structure at a given frame, but also 3-D motion between frames. The most challenging problem is to estimate structure and motion jointly.

One common approach to this problem is to model the surface of the imaged object with a 3-D mesh (also known as a wireframe). Then the structure and motion estimation problem is to find the 3-D coordinates of all the nodes in a starting frame, and the 3-D nodal displacements between successive frames. Note that the 3-D mesh will project to 2-D meshes in the left and right images, as illustrated in Fig. 12.12. To determine the initial 3-D mesh, one can first construct a 2-D mesh in one view, and then find the corresponding nodal positions in the other view. This can be solved using the mesh-based disparity estimation method described in Sec. 12.3.4. From the disparities between corresponding nodes, one can then determine the 3-D positions of all the nodes. For the produced 3-D mesh to fit with the actual object surface, the initial 2-D mesh has to be adapted to the object boundary and each element should correspond to a flat patch. This however requires the knowledge of the object structure. Therefore, it is important to obtain a good object segmentation based on the stereo views. Also, an iterative procedure may be necessary, each time starting with a new 2-D mesh adapted based on the previous estimate of the object structure. To determine the 3-D nodal displacements, one can determine 2-D nodal displacements in both views, by minimizing the motion-compensated prediction error between two adjacent frames of the same view, as well as disparity-compensated prediction error between two views at the same frame time. The main difficulty in both structure and motion estimation lies in how to handle occluded regions, including regions seen only in one view at the same frame time, and regions disappearing or newly appearing over time. For research papers on this topic, see [5, 7, 18].

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<sup>6</sup>Courtesy of Fatih Porikli, who implemented the method of [2] with slight modifications.



Note to graphics illustrator: Please perturb the nodes in the right mesh (move only horizontally) so that the left and right meshes are not identical.

**Figure 12.12.** Projection of 3-D surface mesh to 2-D meshes.

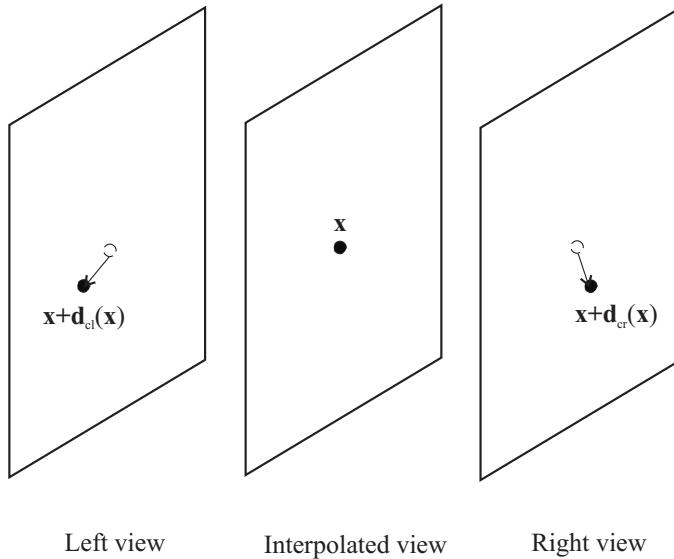
## 12.4 Intermediate View Synthesis

One interesting stereo processing task is to interpolate or synthesize an intermediate view from given views. For example, generating a central view from the left and right views. This is often required in virtual reality displays where the displayed view should be constantly updated based on the viewer preferences. View synthesis is also required in advanced stereo or multiview video coding systems, where the views to be coded are first synthesized from a reference view, and then the synthesis error images are coded. A straight-forward approach to view synthesis is linear interpolation, which generates the interpolated central view  $\psi_c(\mathbf{x})$  from the left and right views,  $\psi_l(\mathbf{x})$  and  $\psi_r(\mathbf{x})$  by

$$\psi_c(\mathbf{x}) = w_l(\mathbf{x})\psi_l(\mathbf{x}) + w_r(\mathbf{x})\psi_r(\mathbf{x}). \quad (12.4.1)$$

If the distance from the central to the left view is  $D_{cl}$  and that to the right view is  $D_{cr}$ , then the weighting factors are determined according to

$$w_l(\mathbf{x}) = \frac{D_{cr}}{D_{cl} + D_{cr}}, \quad w_r(\mathbf{x}) = 1 - w_l(\mathbf{x}). \quad (12.4.2)$$



**Figure 12.13.** Disparity compensated interpolation:  $\mathbf{x}$  is interpolated from  $\mathbf{x} + \mathbf{d}_{cl}(\mathbf{x})$  in the left view and  $\mathbf{x} + \mathbf{d}_{cr}(\mathbf{x})$  in the right view.

Although simple, this approach will not yield satisfactory results because pixels with the same image coordinate in different views correspond to different object points and averaging among them will yield blurred images.

Figure 12.13 illustrates disparity compensated interpolation, which is more appropriate. Let  $\mathbf{d}_{cl}(\mathbf{x})$  and  $\mathbf{d}_{cr}(\mathbf{x})$  represent the disparity field from the central to the left view and that to the right view. The interpolated central view is determined according to

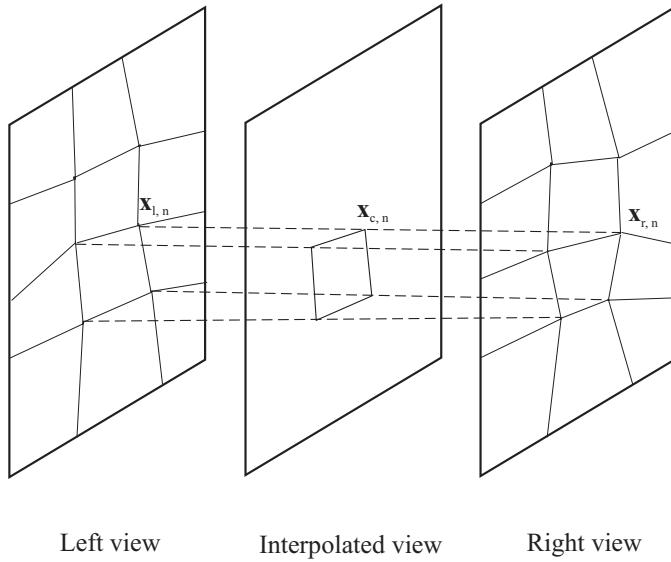
$$\psi_c(\mathbf{x}) = w_l(\mathbf{x})\psi_l(\mathbf{x} + \mathbf{d}_{cl}(\mathbf{x})) + w_r(\mathbf{x})\psi_r(\mathbf{x} + \mathbf{d}_{cr}(\mathbf{x})). \quad (12.4.3)$$

For pixels that are visible in both views, the weighting factors can be chosen as in Eq. (12.4.2). If a pixel is only visible in one of the view, then the weighting factor for the other view should be zero. Specifically,

$$w_l(\mathbf{x}) = \begin{cases} \frac{D_{cr}}{D_{cl}+D_{cr}}, & \text{If } \mathbf{x} \text{ is visible in both views,} \\ 1, & \text{If } \mathbf{x} \text{ is visible only in the left view,} \\ 0, & \text{If } \mathbf{x} \text{ is visible only in the right view.} \end{cases} \quad (12.4.4)$$

For example, if the imaged object is a face, then for the central part of the face, both views will be used for the interpolation. For the left (resp. right) part of the face, only the left (resp. right) view should be used.

The above described method assumes that  $\mathbf{d}_{cl}(\mathbf{x})$  and  $\mathbf{d}_{cr}(\mathbf{x})$  are known. In reality, one can only estimate the disparity, say from the left to the right view,



**Figure 12.14.** The mesh for the central view is generated by linearly interpolating the nodal positions in the left and right views.

$\mathbf{d}_{lr}(\mathbf{x})$ , based on the given left and right images. It is not a straight forward task to generate  $\mathbf{d}_{cl}(\mathbf{x})$  and  $\mathbf{d}_{cr}(\mathbf{x})$  from  $\mathbf{d}_{lr}(\mathbf{x})$ . Note that it is easy to interpolate the disparity field from the left to the central view, by

$$\mathbf{d}_{lc}(\mathbf{x}) = \frac{D_{cl}}{D_{cl} + D_{cr}} \mathbf{d}_{lr}(\mathbf{x}). \quad (12.4.5)$$

But this does not allow one to find the corresponding point in the left view for every pixel in the central view. Rather, it determines the corresponding point in the central view for every pixel in the left view. In general, there will be pixels not covered, or pixels that correspond to more than one pixel in the left view.

The problem is feasible if  $\mathbf{d}_{lr}(\mathbf{x})$  is estimated using the mesh-based approach. In this case, one can easily generate the mesh for the intermediate view from those in the left and right views, as shown in Figure 12.14. Specifically, the nodal positions in the central view,  $\mathbf{x}_{c,n}$  are generated from the nodal positions in the left and right views,  $\mathbf{x}_{l,n}$  and  $\mathbf{x}_{r,n}$  using

$$\mathbf{x}_{c,n} = \frac{D_{cr}}{D_{cl} + D_{cr}} \mathbf{x}_{l,n} + \frac{D_{cl}}{D_{cl} + D_{cr}} \mathbf{x}_{r,n}. \quad (12.4.6)$$

Then for each pixel in an element in the central view, one can determine its corresponding point in the left (resp. right) view, by interpolating from the nodal positions, using the shape functions of the element (cf. Sec. 6.6). In the case of



**Figure 12.15.** An example of intermediate view synthesis: the center image is interpolated from the left and right images. The result is obtained using mesh-based disparity compensated interpolation, with the disparity map between the left and right views estimated using a mesh-based approach. From [21, Fig. 7].

parallel imaging geometry, only horizontal disparities exist, and the problem is even simplified. For an explicit description of this case, see [21]. Figure 12.15 shows the interpolation result for the stereo pair “Man” using this approach [21].

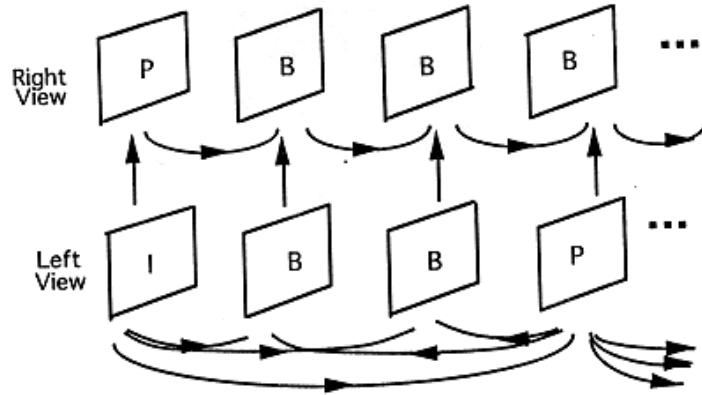
## 12.5 Stereo Sequence Coding

One major obstacle for the application of stereo and multiview sequences including 3DTV is the extremely large amount of data associated with a stereo or multiview sequence. To enable the storage or transmission of a stereo sequence at a reasonable cost, substantial compression of the data must be accomplished. In this section, we present several compression schemes that have been developed for stereo sequences.

### 12.5.1 Block-Based Coding and MPEG-2 Multiview Profile

With this approach, the coder first compresses, say, the left view sequence with a monoscopic video coding algorithm. For the right view sequence, each frame is predicted from the corresponding frame in the left view sequence based on an estimated disparity field, and the disparity field and the prediction error image are coded. The disparity estimation and compensation processes both use a block-based approach, i.e., for each image block in the right view, find a most resembling block in the left view. A better approach is to switch between disparity compensated prediction (DCP) between difference views at the same time and motion compensated prediction (MCP) between different frames in the same view, depending on which gives smaller prediction error.

The above approach has been adopted in the *Multiview Profile* of the MPEG2 standard [4], which can be implemented using the temporal scalability mode of the MPEG2 standard. As will be discussed in Sec. 13.5.3 (cf. Fig. 13.24), with MPEG2 temporal scalability, a sequence is divided into two sets of subsampled frames. The



**Figure 12.16.** MPEG2 Multiview Profile

base layer, consisting of, say, the even frames, is coded using uni-directional motion compensated prediction, where the reference frames are from the previously coded even frames. The enhancement layer, consisting of remaining odd frames, is coded using bidirectional MCP, with one reference frame coming from the previously coded odd frame, and another one from a nearby even frame in the base layer. Figure 12.16 illustrates how to apply the temporal scalability to a pair of stereo views. In this case, the left view is coded as the base layer, and the right view as the enhancement layer. For the left view, a picture is coded in either I-, B-, or P-mode. For B- and P-pictures, the reference frames used for prediction come from the left view only. For the right view, a picture is coded in either the P- or B-mode. In the P-mode, the picture is predictively coded, using the corresponding frame in the left view as the reference. In the B-mode, one reference comes from a previous frame in the right view, and another reference frame is the corresponding frame in the left view. The motion vector used in the latter case is actually a disparity vector, and the prediction process is disparity compensated prediction.

Remember that with bidirectional prediction, the predicted block from either reference frame or the average of both can be used, depending on which provides the best prediction. Usually, the similarity between successive frames within the same view is higher than that between corresponding frames in the left and right views. The insufficiency of the block-based disparity model further limits the accuracy obtainable with DCP. Because of these, MCP is usually preferred over DCP so that the overall performance of the multiview profile is only slightly better than simulcast, where each view is coded independently using MPEG2 main profile [8]. In [13], the multiview profile and simulcast have been applied to several MPEG2 stereo test sequences. An average gain of 1.5 dB is obtained using the multiview profile over simulcast.



**Figure 12.17.** Incomplete 3D representation for sequence “Man”: The augmented texture surface (left) and disparity map (right). The original left and right images are shown in Fig. 12.15. From [8, Fig. 6]. Courtesy of Jens-Rainer Ohm.

### 12.5.2 Incomplete 3-D Representation of Multiview Sequences

In a multiview sequence, the same object region is often visible in multiple views. If we only code the texture map of this region in one of the views (the view that provides the highest resolution representation) and the disparity information between this view and other views, then we can reconstruct its images in other views reasonably well using DCP. For applications not requiring very high reproduction of each view, the error between the predicted images and the original ones do not need to be coded. Instead of coding the texture map of each region separately, one can put the texture maps of all different regions in an augmented image and treat this image as a single texture image. This augmented texture map and the associated region segmentation and disparity information for each region constitute the incomplete 3D (I3D) representation, proposed by Ohm and Müller [9, 8], and adopted in MPEG4. An example of the I3D representation of the sequence “Man” is shown in Fig. 12.17. In this case, the left (resp. right) side of the face texture is obtained from the left (resp.) view. Coding the I3D representation instead of the original two views can lead to significant savings in bitrates. With the coded disparity information, one can also interpolate intermediate views.

### 12.5.3 Mixed Resolution Coding

As described in Sec. 12.1.2, the resolution of one of the two images can be considerably reduced (to approximately half the cut-off frequency of the original using today’s television standards) without causing irritation or deterioration in the subjective impression of sharpness when the image is presented for a short period of time [11]. Based on this asymmetric property of the human vision system, it is not necessary to present two sequences in a stereo pair with the same spatial and temporal resolution. In mixed resolution coding [12, 1, 15], one sequence, say the left, is coded at the highest affordable spatial and temporal resolution, while the other one is first down sampled spatially and temporally, and then coded. It has

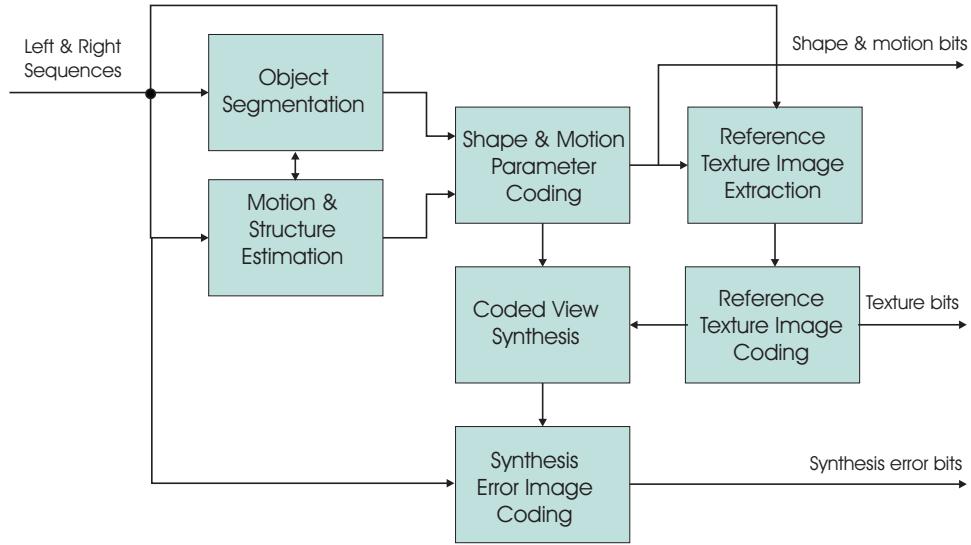
been found [15] that one can subsample one view by a factor of 2 in both horizontal and vertical directions, without incurring degradation in an observer's ability to perceive depth.

#### 12.5.4 3-D Object Based Coding

These approaches go one step further than coders based on DCP and MCP. Instead of deriving 2-D motion and disparity for performing MCP and DCP, 3-D structure and motion parameters are estimated from the stereo or multiple views. The structure, motion, and surface texture (color) are coded, instead of individual image frames. At the decoder, desired views are synthesized based on the structure, motion, and texture information. Such an approach has more potential than a 2-D MCP/DCP combined approach. First, 3-D motion and structure parameters are physically constrained in a more direct and simpler form than 2-D motion and disparity, and proper use of these constraints should lead to more accurate estimation of 3-D motion and structure. In fact, accurate 3-D motion estimation should yield more accurate 2-D motion, which could in turn reduce the number of bits needed to represent each single video. Secondly, with the 3-D information derived from the stereo pair, one can generate any intermediate view. This feature is desired in many applications, e.g., virtual reality, in which continuous views in the 3-D world are required but are too expensive to acquire or transmit. Finally, the coded 3-D information enables manipulation (change of view angle, annotation, animation, etc.) of the imaged object or scene, which is an important feature of interactive multimedia communications.

A general framework for such a coding scheme is shown in Fig. 12.18. Based on the input stereo views, the coder first performs motion and structure estimation as well as object segmentation. As described in Sec. 12.3.6, these three tasks are inter-related and an iterative procedure is usually employed to yield the final solution. Each object is described by a wireframe, so that the motion and structure information can be described by initial nodal positions and nodal displacement vectors. For each object, a reference texture map is also extracted from both views as in the I3D representation described in Sec. 12.5.2. Based on the segmentation map, the structure and motion parameters, one can then synthesize the left and right views from the reference texture map and code the synthesis error image if necessary. Instead of using a reference texture map that is constructed from the given views, one can also choose one of the input views as the reference texture map, and synthesize the other view. The synthesis can be accomplished using disparity compensated texture warping from the the reference texture map (cf. Sec. 12.4). The parameters to be coded include the segmentation map, the structure and motion information, the reference texture map, as well as synthesis error image. Similar methodology can be applied for coding of multiview sequences. Coders that following this general framework can be found in [18, 21].

The success of a 3-D based coder depends critically on its accuracy in 3-D structure and motion estimation. Without knowing what objects and how many



**Figure 12.18.** A general framework for object-based coding of stereoscopic sequences.

objects may exist in an imaged scene, trying to recover the entire 3-D information from a generic stereo sequence is a very difficult, if not impossible task. Although substantial research efforts have been devoted to this problem and many promising approaches have been developed, existing solutions are still limited in their robustness and accuracy, and require very intense computations. Because of these problems, the performance of 3-D object-based coders has yet to compete favorably with non-object based coders.

### 12.5.5 3-D Model-Based Coding

The previous discussion attempts to derive the 3-D structure of the objects in the scene automatically from the observed stereo sequences. Because of the very complex object composition that may exist in the imaged scene, this is a very difficult problem. A less difficult problem is when there are only a few objects in the scene, and the 3-D models of the objects follow some known structure. In this case, a generic model can be built for each potential object. For example, in teleconferencing type of applications, the imaged scene usually consists of one or a few head-and-shoulder type of objects. The coder can use some pre-designed generic face and body models. The structure estimation problem reduces to that of estimating the necessary adaptations of the generic model to fit the captured image. Once the generic model is adapted to the imaged object, only the motion information (nodal movements) needs to be estimated and coded. For more coverage on model-based

coding, see Chap. 10.

## 12.6 Summary

### Stereo imaging principle

With stereo imaging, the images of a 3-D point on two different views are displaced with respect to each other and it is this disparity between the two image positions that enables the recovery of the depth information of the 3-D point. The most general relation between depth and image positions is given by Eq. (12.2.5). For the most popular parallel camera configuration, this is reduced to the famous inverse relationship between disparity and depth (Eq. 12.2.8). The relation between the left and right image positions of the same object point is described by the epipolar geometry (Fig. 12.7, Eq. 12.2.13).

### Disparity Estimation

Disparity estimation is similar to motion estimation in that one needs to find corresponding points in two different images, and hence can be accomplished using similar approaches, including block-based and mesh-based methods. Compared to motion estimation, disparity estimation can be helped with more physical constraints. The most important one is the epipolar constraint, which says for any point in a right image, its corresponding point in the left image must fall on the epipolar line. For parallel set-up, this means that corresponding points in two images are on the same horizontal line.

Disparity estimation is a fundamental step for recovering 3-D structure and motion of imaged objects from a stereo sequence. To achieve this goal, disparity estimation must be done in conjunction with object segmentation, 3-D modeling using wireframes, and detection and treatment of occluded regions.

### Stereo and multiview sequence coding

There are generally two types of approach. The first type is waveform-based: it combines disparity compensated prediction and motion compensated prediction, and codes the prediction error image and the disparity/motion vectors. The second type is object-based: it tries to recover the 3-D structure and motion of the imaged object, and codes these information directly. The second approach has the potential of achieving very high compression ratios, for simply structured scenes. It also facilitates the synthesis of any intermediate view at the decoder.

## 12.7 Problems

- 12.1** What are the pros and cons of parallel vs. converging camera configuration for stereo imaging?

- 12.2** Derive the horizontal and vertical disparity values,  $d_x(x_r, y_r), d_y(x_r, y_r)$ , in the converging camera configuration. Show that the vertical disparity is zero for  $X = 0$  or  $Y = 0$ . Also derive how to determine the 3-D position from  $d_x(x_r, y_r), d_y(x_r, y_r)$ .
- 12.3** What is the fundamental matrix  $[\mathbf{F}]$  for (a) parallel and (b) converging camera configuration?
- 12.4** For a curved surface,  $Z = aX + bY + cXY + d$ , what is the parameterized form of the disparity function?
- 12.5** When the baseline between two parallel cameras is large, the horizontal disparities are very large. In this case, it is better to first estimate a global horizontal disparity value, before applying block-wise or mesh-based disparity estimation. This global disparity can be found by finding a global horizontal shift between the two images that will yield the least error after shifting. Write a program to estimate the global disparity between a stereo pair.
- 12.6** Implement (in C or Matlab) a hierarchical block matching algorithm for disparity estimation for the case of parallel imaging geometry (only horizontal disparity needs to be searched). Evaluate its performance over a stereo image pair. You should first estimate the global disparity and then apply the block matching algorithm to the images after global disparity compensation.
- 12.7** Implement the mesh-based approach for disparity estimation for the case of parallel imaging geometry. For simplicity, use a regular square mesh over the reference view. Compare its performance for the same stereo pair to the block matching algorithm.
- 12.8** Implement the dynamic programming scheme for disparity estimation described in [10]. Evaluate its performance over a stereo image pair. Compare its performance to the block-matching algorithm and the mesh-based approach.
- 12.9** Generate an intermediate view between a stereo pair acquired by parallel cameras. Compare two approaches: a) linear interpolation; and b) disparity-compensated interpolation using the mesh-based approach (Sec. 12.4). Ideally, you should estimate the disparity values using the mesh-based approach. If you choose to use the simpler block-based approach, then you can assume the disparity found for each block is the disparity at the center of the block, and all the block centers are the nodes in the mesh for the reference view.
- 12.10** Implement a stereo image coder using block-based disparity compensation. Code one image (reference view) using the DCT-base method (Sec. 9.1.7). For the other image, for each block, find its prediction using block-based disparity, and then code the error block using the DCT-based approach.

**12.11** Implement a stereo video coder using block-based disparity compensation and motion compensation. Code one view (reference view) using the hybrid coding method (Sec. 9.3.1). Using unidirectional prediction only for simplicity. For the other view, for each block in a frame, it is estimated both using motion compensation from the previous frame in this view, and using disparity compensation from the same frame in the reference view. Use either the average of the two predictions or one of the predictions, depending on the prediction errors. The prediction error is then coded using the DCT-based method.

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## Chapter 13

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# VIDEO COMPRESSION STANDARDS

Digital video communication is a complex and compute intensive process that requires many people to receive video signals from different sources. There are mainly three classes of devices that we use for digital video communications today:

- Digital television sets or settop boxes are mainly designed to receive video signals from different content providers. These devices rely on one fixed video decoding algorithm that is implemented in hardware or in a combination of hardware and a programmable Reduced Instruction Set Computers (RISC) processor or Digital Signal Processor (DSP). Currently, there are no means provided for uploading a new algorithm once the hardware is deployed to the customer.
- Videophones are usually implemented on DSPs with hardware acceleration for some compute intensive parts of the video coding and decoding algorithm such as DCT and motion estimation. Usually, the set of algorithms used in a particular videophone cannot be replaced.
- Personal Computers are the most flexible and most expensive platform for digital video communication. While a PC with a high-end Pentium III processor is able to decode DVDs, the software is usually preinstalled with the operating system in order to avoid hardware and driver problems. Video decoders for streaming video may be updated using automatic software download and installation, as done by commercial software such as Real Player, Windows Media Player, Apple Quicktime and Microsoft Netmeeting.

Digital video communications standards were mainly developed for digital television and video phones in order to enable industry to provide consumers with a bandwidth efficient terminal at an affordable price. We describe the standards organizations, the meaning of compatibility and applications for video coding standards in Section 13.1. We begin the description of actual standards with the ITU video coding standards H.261 and H.263 (Sec. 13.2) for interactive video communications.

In Section 13.3 we present the standards H.323 and H.324 that define multimedia terminals for audiovisual communications. Within ISO, the Moving Picture Experts Group (MPEG) defined MPEG-1 (Sec. 13.4) and MPEG-2 (Sec. 13.5) for entertainment and digital TV. MPEG-4 (Sec. 13.6) is the first international standard that does not only standardize audio and video communications but also graphics for use in entertainment and interactive multimedia services. All standards describe the syntax and semantics of a bitstream. Section 13.7 presents an overview over the organization of a bitstream as used by H.261, H.263, MPEG-1,-2, and MPEG-4. Finally, we give a brief description of the on-going MPEG-7 standardization activity (Sec. 13.8), which intends to standardize the interface for describing the content of an audiovisual document.

## 13.1 Standardization

Developing an International Standard requires collaboration between many partners from different countries, and an organization that is able to support the standardization process as well as to enforce the standards. In Section 13.1.1 we describe the organizations like ITU and ISO. In Section 13.1.2, the meaning of compatibility is defined. Section 13.1.3 briefly describes the workings of a standardization body. In Section 13.1.4 applications for video communications are listed.

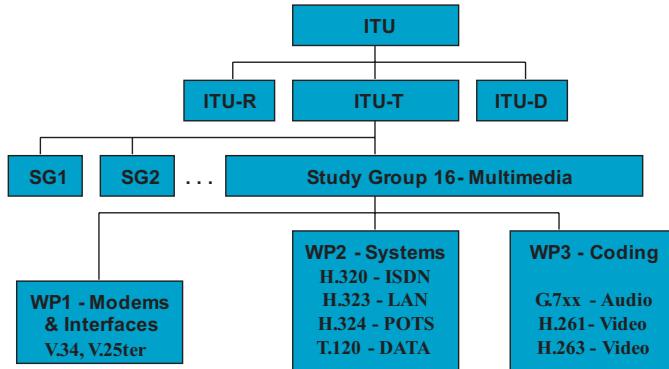
### 13.1.1 Standards Organizations

Standards are required if we want multiple terminals from different vendors to exchange information or to receive information from a common source like a TV broadcast station. Standardization organizations have their roots in the telecom industry creating ITU and trade creating ISO.

#### ITU

The telecom industry has established a long record of setting international standards [7]. At the beginning of electric telegraphy in the 19th century, telegraph lines did not cross national frontiers because each country used a different system and each had its own telegraph code to safeguard the secrecy of its military and political telegraph messages. Messages had to be transcribed, translated and handed over at frontiers before being retransmitted over the telegraph network of the neighboring country. The first International Telegraph Convention was signed in May 1865 and harmonized the different systems used. This event marked the birth of the International Telecommunication Union (ITU).

Following the invention of the telephone and the subsequent expansion of telephony, the Telegraph Union began, in 1885, to draw up international rules for telephony. In 1906 the first International Radiotelegraph Convention was signed. Subsequently, several committees were set up for establishing international standards including the International Telephone Consultative Committee (CCIF) in 1924, the International Telegraph Consultative Committee (CCIT) in 1925, and



**Figure 13.1.** Organization of the ITU with its subgroups relevant to digital video communications. Working Parties (WP) are organized into questions that define the standards.

the International Radio Consultative Committee (CCIR) in 1927. In 1927, the Telegraph Union allocated frequency bands to the various radio services existing at the time. In 1934 the International Telegraph Convention of 1865 and the International Radiotelegraph Convention of 1906 were merged to become the International Telecommunication Union (ITU, <http://www.itu.int>). In 1956, the CCIT and the CCIF were amalgamated to create the International Telephone and Telegraph Consultative Committee (CCITT). In 1989, CCITT published the first digital video coding standard, the CCITT Recommendation H.261 [40], which is still relevant today. In 1992, ITU reformed itself which resulted in renaming CCIR into ITU-R and CCITT into ITU-T. Consequently, the standards of CCITT are now referred to as ITU-T Recommendations. For example, CCITT H.261 is now known as ITU-T H.261.

Fig. 13.1 shows the structural organization of the ITU detailing the parts that are relevant to digital video communications. ITU-T is organized in study groups with Study Group 16 (SG 16) being responsible for multimedia. SG 16 divided its work into different Working Parties (WP) each dealing with several Questions. Here we list some questions that SG 16 worked on in 2001: Question 15 (Advanced video coding) developed the video coding standards ITU-T Recommendation H.261 and H.263 [49]. Question 19 (Extension to existing ITU-T speech coding standards at bit rates below 16 kbit/s) developed speech coding standards like ITU-T Recommendation G.711 [35], G.722 [36] and G.728 [38]. Question numbers tend to change every four years.

ITU is an international organization created by a treaty signed by its member countries. Therefore, countries consider dealings of the ITU relevant to their sovereign power. Accordingly, any recommendation, i.e. standard, of the ITU has to be agreed upon unanimously by the member states. Therefore, the standardiza-

tion process in the ITU is often not able to keep up with the progress in modern technology. Sometimes, the process of reaching unanimous decisions does not work and ITU recommends regional standards like 7 bit or 8 bit representation of digital speech in the USA and Europe, respectively. As far as mobile telephony is concerned ITU did not play a leading role. In the USA there is not even a national mobile telephony standard, as every operator is free to choose a standard of his own liking. This contrasts with the approach adopted in Europe where the GSM standard is so successful that it is expanding all over the world, USA included. With UMTS (so-called 3rd generation mobile) the ITU-T is retaking its original role of developer of global mobile telecommunication standards.

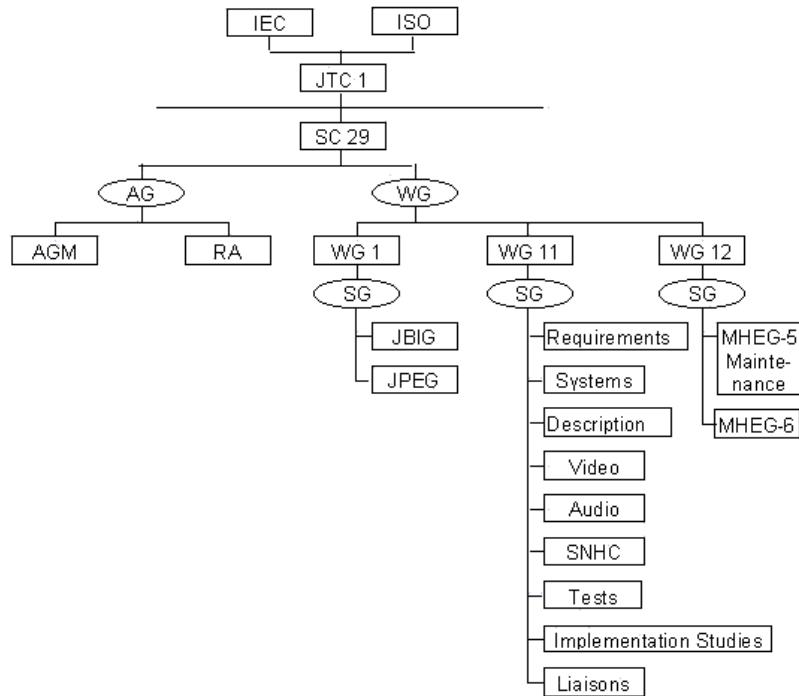
## ISO

The need to establish international standards developed with the growth of trade [7]. The International Electrotechnical Commission (IEC) was founded in 1906 to prepare and publish international standards for all electrical, electronic and related technologies. The IEC is currently responsible for standards of such communication means as "receivers", audio and video recording systems, audio-visual equipment, currently all grouped in TC 100 (Audio, Video and Multimedia Systems and Equipment). International standardization in other fields and particularly in mechanical engineering was the concern of the International Federation of the National Standardizing Associations (ISA), set up in 1926. ISA's activities ceased in 1942 but a new international organization called International Organization for Standardization (ISO) began to operate again in 1947 with the objective "to facilitate the international coordination and unification of industrial standards". All computer-related activities are currently in the Joint ISO/IEC Technical Committee 1 (JTC 1) on Information Technology. This TC has achieved a very large size. About 1/3 of all ISO and IEC standards work is done in JTC 1.

The subcommittees SC 24 (Computer Graphics and Image Processing) and SC 29 (Coding of Audio, Picture, Multimedia and Hypermedia Information) are of interest to multimedia communications. Whereas SC24 defines computer graphics standards like VRML, SC29 developed the well known audiovisual communication standards MPEG-1, MPEG-2 and MPEG-4 (Fig. 13.2). The standards were developed at meetings that between 200 and 400 delegates from industry, research institutes and universities attended.

ISO is an agency of the United Nations since 1947. ISO and IEC have the status of private not-for-profit companies established according to the Swiss Civil Code. Similarly to ITU, ISO requires consensus in order to publish a standard. ISO also fails sometimes to establish truly international standards as can be seen with digital TV. While the same video decoder (MPEG-2 Video) is used worldwide, the audio representation is different in the US and in Europe.

Both ISO ([www.iso.ch](http://www.iso.ch)) and ITU are in constant competition with industry. While ISO and ITU have been very successful in defining widely used audio and video coding standards, they were less successful in defining transport of multimedia



**Figure 13.2.** Audio visual communications standards like MPEG-1, MPEG-2 and MPEG-4 are developed by Working Group (WG) 11 of Subcommittee (SC) 29 under ISO/IEC JTC 1.

signals over the Internet. This is currently handled by the Internet Engineering Task Force (IETF, [www.ietf.org](http://www.ietf.org)) that is a large open international community of network designers, operators, vendors, and researchers concerned with the evolution of the Internet architecture and the smooth operation of the Internet. It is open to any interested individual. Other de-facto standards like JAVA are defined by one or a few companies thus limiting the access of outsiders and newcomers to the technology.

### 13.1.2 Requirements for a Successful Standard

International standards are developed in order to allow interoperation of communications equipment provided by different vendors. This results in the following requirements that enable a successful deployment of audio-visual communications equipment in the market place.

1. Innovation: In order for a standard to distinguish itself from other standards

or widely accepted *industry standards*, it has to provide a significant amount of innovation. Innovation in the context of video coding means that the standard provides new functionalities like broadcast-quality interlaced digital video, video on CDROM or improved compression. If the only distinction of a new standard is better compression, the standard should provide at least an improvement that is visible to the consumer and non-expert, before its introduction makes sense commercially. This usually translates to a gain of 3dB in PSNR of the compressed video at a generally acceptable picture quality.

2. Competition: Standards should not prevent competition between different manufacturers. Therefore, standards specifications need to be open and available to everyone. Free software for encoders and decoders also helps to promote a standard. Furthermore, a standard should only define the syntax and semantics of a bitstream, i.e. *a standard defines how a decoder works*. Bitstream generation is not standardized. Although the development of bitstream syntax and semantics requires to have an encoder and a decoder, the standard does not define the encoder. Therefore, manufacturers of standards-compliant terminals can compete not only about price but also about additional features like postprocessing of the decoded media and more importantly about encoder performance. In video encoding, major differences result from motion estimation, scene change handling, rate control and optimal bit allocation.
3. Transmission and storage media independent: A content provider should be able to transmit or store the digitally encoded content independent of the network or storage media. As a consequence of this requirement, we use audio and video standards to encode the audiovisual information. Then we use a systems standard to format the audio and video bitstreams into a format that is suitable for the selected network or storage system. The systems standard specifies the packetization, multiplexing and packet header syntax for delivering the audio and video bitstreams. The separation of transmission media and media coding usually creates overhead for certain applications. For example, we cannot exploit the advantages of joint source/channel coding.
4. Forward compatibility: A new standard should be able to understand the bitstreams of prior standards, i.e. a new video coding standard like H.263 [49] should be able to decode bitstreams according to the previous video coding standard H.261 [40]. Forward compatibility ensures that new products can be gradually introduced into the market. The new features of the latest standard get only used when terminals conforming to the latest standard communicate. Otherwise, terminals interoperate according to the previous standard.
5. Backward compatibility: A new standard is backward compatible to an older standard, if the old standard can decode bitstreams of the new standard. A very important backward compatible standard was the introduction of analog color TV. Black and white receivers were able to receive the color TV

signal and display a slightly degraded black and white version of the signal. Backward compatibility in today's digital video standards can be achieved by defining reserved bits in the bit stream that a decoder can ignore. A new standard would transmit extra information using these reserved bits of the old standard. Thus old terminals will be able to decode bitstreams according to the new standard. Furthermore, they will understand those parts of the bit stream that comply to the old standard. Backward compatibility can put severe restrictions on the improvements that a new standard may achieve over its predecessor. Therefore, backward compatibility is not always implemented in a new standard.

6. Upward compatibility: A new receiver should be able to decode bitstreams that were made for similar receivers of a previous or cheaper generation. Upward compatibility is important if an existing standard is extended. A new HDTV set should be able to receive standard definition TV since both receivers use the same MPEG-2 standard [19].
7. Downward compatibility: An old receiver should be able to receive and decode bitstreams for the newer generation receivers. Downward compatibility is important if an existing standard is extended. This may be achieved by decoding only parts of the bitstream which is easily possible if the new bitstream is sent as a scalable bitstream (Chapter 11).

Obviously, not all of the above requirements are essential for the wide adoption of a standard. We believe that the most important requirements are innovation, competition, and forward compatibility in this order. Compatibility is most important for devices like TV settop boxes or mobile phones that cannot be upgraded easily. On the other hand, any multimedia PC today comes with more than ten software video codecs installed, relaxing the importance of compatible audio and video coding standards for this kind of terminals.

### 13.1.3 Standard Development Process

All video coding standards were developed in three phases, *competition*, *convergence*, *verification*. Fig. 13.3 shows the process for the video coding standard H.261 [40]. The competition phase started in 1984. The standard was published in December 1990 and revised in 1993.

During the competition phase, the application areas and requirements for the standard are defined. Furthermore, experts gather and demonstrate their best algorithms. Usually, the standardization body issues a *Call for Proposals* as soon as the requirements are defined in order to solicit input from the entire community. This phase can be characterized by independently working competing laboratories.

The goal of the convergence phase is to collaboratively reach an agreement on the coding method. This process starts with a thorough evaluation of the proposals for the standard. Issues like coding efficiency, subjective quality, implementation com-

Subject	1985	1986	1987	1988	1989	1990
nx384 (n=1-5)	divergence	convergence	verification optimization			
	RM 1	2	3	4		
px64 (p=1-30)			convergence		verification optimization	REC. H.261
			RM 6	7	8	
mx64 (m=1,2)			divergence			
			RM 5			

Editing instructions: Replace 'divergence' with 'competition', 2 times.

**Figure 13.3.** Overview of the H.261 standardization process (from [57]). Initially, the target was to develop two standards for video coding at rates of nx384 kbit/s and mx64 kbit/s. Eventually, ITU settled on one standard for a rate of px64 kbit/s.

plexity and compatibility are considered when agreeing on a first common framework for the standard. This framework is implemented at different laboratories and the description is refined until the different implementations achieve identical results. This framework has different names in different standards, such as *Reference Model* or RM for H.261, *Test Model Near term* (TMN) for H.263, *Simulation Model* for MPEG-1, *Test Model* (TM) for MPEG-2, *Verification Model* (VM) in MPEG-4, *Test Model Long term* (TML) for H.26L. After the first version of the framework is implemented, researchers suggest improvements such as new elements for the algorithm or better parameters for existing elements of the algorithm. These are evaluated against the current framework. Proposals that achieve significant improvements are included in the next version of the framework that serves as the new reference for further improvements. This process is repeated until the desired level of performance is achieved.

During the verification phase, the specification is checked for errors and ambiguities. *Conformance bitstreams* and correctly decoded video sequences are generated. A standards compliant decoder has to decode every Conformance bitstream into the correct video sequence.

The standardization process of H.261 can serve as a typical example (Fig. 13.3). In 1985, the initial goal was to develop a video coding standard for bitrates between 384 kbit/s and 1920 kbit/s. Due to the deployment of ISDN telephone lines, another standardization process for video coding at 64 kbit/s up to 128 kbit/s began two years later. In 1988 the two standardization groups realized that one algorithm can be used for coding video at rates between 64 kbit/s and 1920 kbit/s. RM 6 was the first reference model that covered the entire bitrate range. Technical work was finished in 1989 and one year later, H.261 was formally adopted by the ITU.

### 13.1.4 Applications for Modern Video Coding Standards

As mentioned previously, there have been several major initiatives in video coding that have led to a range of video standards for different applications [12].

- Video coding for video teleconferencing, which has led to the ITU standards called H.261 for ISDN videoconferencing [40], H.263 for video conferencing over analog telephone lines, desktop and mobile terminals connected to the Internet [49], and H.262/MPEG-2 video [42][17] for ATM/broadband video-conferencing.
- Video coding for storing movies on CD-ROM and other consumer video applications, with about 1.2 Mbits/s allocated to video coding and 256 kbits/s allocated to audio coding, which led to the initial ISO MPEG-1 standard [16]. Today, MPEG-1 is used for consumer video on CD, Karoke machines, in some digital camcorders and on the Internet. Some Digital Satellites used MPEG-1 to broadcast TV signals prior to the release of MPEG-2.
- Video coding for broadcast and for storing of digital video on DVD, with on the order of 2 – 15 Mbits/s allocated to video and audio coding, which led to the ISO MPEG-2 standard [19] and specifications for DVD operations by the Digital Audio Visual Council (DAVIC) ([www.davic.org](http://www.davic.org)) and the DVD consortium [25]. This work was extended to video coding for HDTV with bitrates ranging from 15 to 400 Mbits/s allocated to video coding. Applications include satellite TV, Cable TV, terrestrial broadcast, video editing and storage. Today, MPEG-2 video is used in every digital settop box. It is also selected as the video decoder for the American HDTV broadcast system.
- Coding of separate audio-visual objects, both natural and synthetic is standardized in ISO MPEG-4 [22]. Target applications are Internet video, interactive video, content manipulation, professional video, 2D and 3D computer graphics, and mobile video communications.

In the following sections, we will first describe H.261. For H.263, we will highlight the differences from H.261 and compare their coding efficiency. Then, we will discuss MPEG-1, MPEG-2 and MPEG-4, again focusing on their differences.

## 13.2 Video Telephony with H.261 and H.263

Video coding at 64 kbit/s was first demonstrated at a conference in 1979 [56]. However, it took more than ten years to be able to define a commercially viable video coding standard at that rate. The standard H.261 was published in 1990 in order to enable video conferencing using between one and thirty ISDN channels. At that time, video conferencing hardware became available from different vendors. Companies like PictureTel that sold video conferencing equipment with a proprietary algorithm quickly offered H.261 as an option. Later, ITU developed the similar

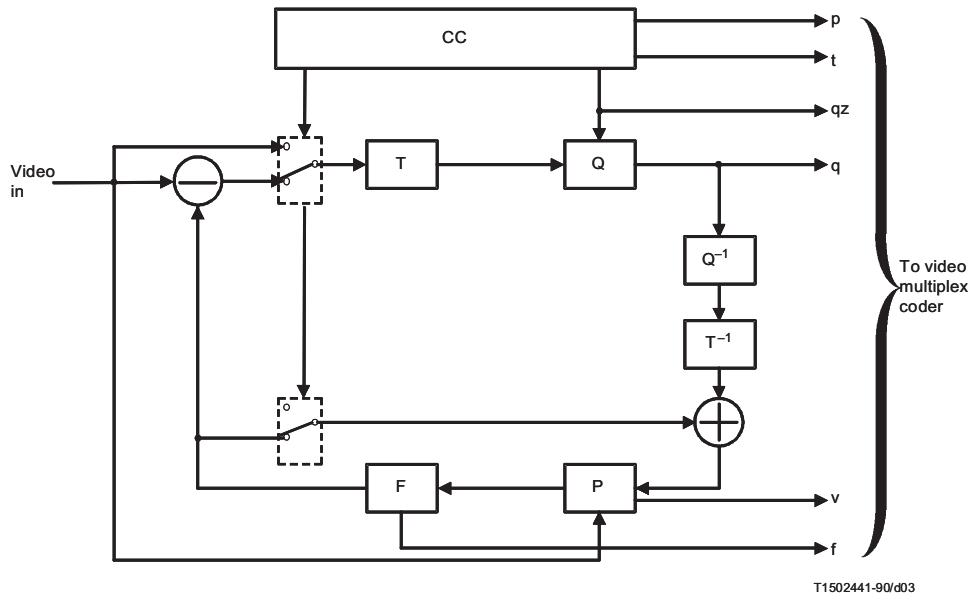
standard H.263 that enables video communications over analog telephone lines. Today, H.263 video encoder and decoder software is installed on every PC with the Windows operating system.

### 13.2.1 H.261 Overview

Figure 13.4 shows the block diagram of an H.261 encoder that processes video in the 4:2:0 sampling format. The video coder is a block-based hybrid coder with motion compensation (Sec. 9.3.1). It subdivides the image into macroblocks (MB) of size 16x16 pels. A MB consists of 4 luminance blocks and 2 chrominance blocks, one for the Cr and another for the Cb component. H.261 uses an 8x8 DCT for each block to reduce spatial redundancy, a DPCM loop to exploit temporal redundancy and unidirectional integer pel forward motion compensation for MBs (box P in Fig. 13.4) to improve the performance of the DPCM loop.

A simple two-dimensional *loop filter* (see Sec. 9.3.5) may be used to lowpass filter the motion compensated prediction signal (box F in Fig. 13.4). This usually decreases the prediction error and reduces the blockiness of the prediction image. The loop filter is separable into one-dimensional horizontal and vertical functions with the coefficients [1/4, 1/2, 1/4]. H.261 uses two quantizers for DCT coefficients. A uniform quantizer with stepsize 8 is used in intra-mode for DC coefficients, a nearly uniform midtread quantizer with the stepsize between 2 and 62 is used for AC coefficients in intra-mode and in inter-mode (Fig. 13.5). The input between  $-T$  and  $T$ , which is called the dead zone, is quantized to 0. Except for the deadzone, the stepsize is uniform. This deadzone avoids coding many small DCT coefficients that would mainly contribute to coding noise.

The encoder transmits mainly two classes of information for each MB that is coded: DCT coefficients resulting from the transform of the prediction error signal ( $q$  in Fig. 13.4) and motion vectors that are estimated by the motion estimator ( $v$  and box P in Fig. 13.4). The motion vector range is limited to  $\pm 16$  pels. The control information that tells the decoder whether and how a MB and its blocks are coded are named *macroblock type* (MTYPE) and *coded block pattern* (CBP). Table 13.1 shows the different MB types. In intra-mode, the bitstream contains transform coefficients for each block. Optionally, a change in the quantizer stepsize of  $\pm 2$  levels (MQUANT) can be signaled. In inter-mode, the encoder has a choice of just sending a differentially coded motion vector (MVD) with or without the loop filter on. Alternatively, a CBP may be transmitted in order to specify the blocks for which transform coefficients will be transmitted. Since the standard does not specify an encoder, it is up to the encoder vendor to decide on an efficient Coding Control (CC in Fig. 13.4) to optimally select MTYPE, CBP, MQUANT, loop filter and motion vectors [69]. As a rough guideline, we can select MTYPE, CBP, and MVD such that the prediction error is minimized. However, since the transmission of motion vectors costs extra bits, we do this only if the prediction error using the motion vector is significantly lower than without it. The quantizer stepsize is varied while coding the picture such that the picture does not require more bits



T	Transform
Q	Quantizer
P	Picture memory with motion compensated variable delay
F	Loop filter
CC	Coding control
p	Flag for INTRA/INTER
t	Flag for transmitted or not
qz	Quantizer indication
q	Quantizing index for transform coefficients
v	Motion vector
f	Switching on/off of the loop filter

FIGURE 3/H.261

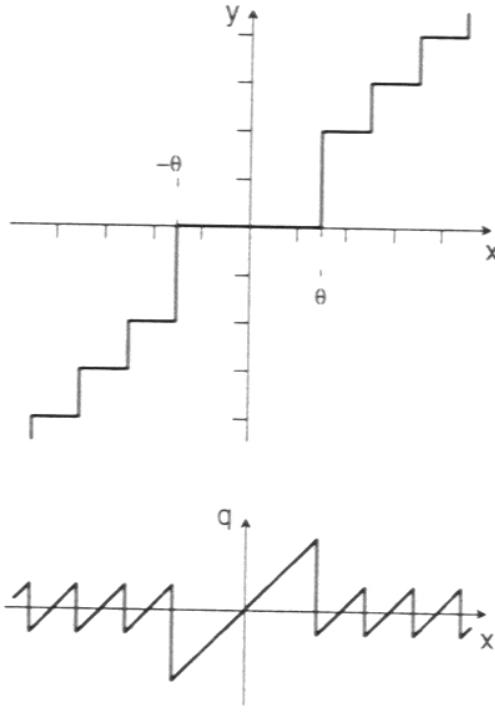
**Source coder**

Editing instructions: P Picture memory with motion estimator and motion compensation unit; Connect lines with a point at first line crossing after 'Video in'; Remove 'FIGURE 3/H.261'; Remove 'Source Coder'; Remove 'T1502441-90/d03'.

**Figure 13.4.** Block diagram of an H.261 encoder [40].

than the coder can transmit during the time between two coded frames. Coding mode selection and parameter selection is previously discussed in Sec. 9.3.3.

Most information within a MB is coded using a variable length code that was derived from statistics of test sequences. Coefficients of the 2D DCT are coded using the runlength coding method discussed in Sec. 9.1.7. Specifically, the quantized DCT coefficients are scanned using a Zigzag scan (Fig. 9.8) and converted into symbols. Each symbol includes the number of coefficients that were quantized to 0



Editing instructions: Replace ' $\theta$ ' with 'T'; replace 'y' with 'Q(x)'; remove 'b)'; replace 'q' by 'e'.

**Figure 13.5.** A midtread quantizer with a dead zone is used in H.261 for all DCT coefficients but the DC coefficient in the intra-mode. The bottom part shows the quantization error  $e = x - Q(x)$  between the input amplitude  $x$  and the output amplitude  $Q(x)$ .

since the last non-zero coefficient together with the amplitude of the current non-zero coefficient. Each symbol is coded using VLC. The encoder sends an End Of Block (EOB) symbol after the last non-zero coefficient of a block (Fig. 9.9).

H.261 does not specify the video encoder capabilities. However, the picture formats that an H.261 decoder has to support are listed in Tab. 13.2. Several standards that setup video conferencing calls, exchange video capabilities between terminals. At a minimum level as defined in H.320, a decoder must be capable of decoding QCIF frames at a rate of 7.5 Hz [45]. An optional level of capability is defined at decoding CIF frames at 15 Hz [45]. The maximum level requires the decoding of CIF frames at 30 Hz (30000/1001 Hz to be precise) [45].

**Table 13.1.** VLC table for macroblock type (MTYPE). Two MTYPEs are used for intra-coded MBs and eight are used for inter-coded MBs. An 'x' indicates that the syntactic element is transmitted for the MB [40].

Prediction	MQUANT	MVD	CBP	TCOEFF	VLC
Intra				x	0001
Intra	x			x	0000 001
Inter			x	x	1
Inter	x		x	x	0000 1
Inter + MC		x			0000 0000 1
Inter + MC		x	x	x	0000 0001
Inter + MC	x	x	x	x	0000 0000 01
Inter + MC + FIL		x			001
Inter + MC + FIL		x	x	x	01
Inter + MC + FIL	x	x	x	x	0000 01

**Table 13.2.** Picture formats supported by H.261 and H.263.

	Sub-QCIF	QCIF	CIF	4CIF	16CIF	Custom sizes
Lum Width (pels)	128	176	352	704	1408	< 2048
Lum Height (pels)	96	144	288	576	1152	< 1152
H.261		✓	Opt	Still pict		
H.263	✓	✓	Opt		Opt	Opt

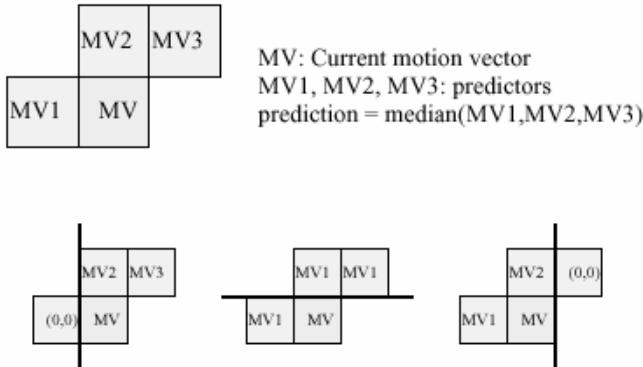
### 13.2.2 H.263 Highlights

The H.263 standard is based on the framework of H.261. Due to progress in video compression technology and the availability of high performance desktop computers at reasonable cost, ITU decided to include more compute intensive and more efficient algorithms in the H.263 standard. The development of H.263 had three phases. The technical work for the initial standard was finished in November 1995. An extension of H.263, nicknamed H.263+, was incorporated into the standard in September 1997. The results of the third phase, nicknamed H.263++, were folded into the standard in 1999 and formally approved in November 2000. In this section, we focus on the differences between H.263 as of 1995 and H.261. We also briefly describe H.263 as of 2000.

#### H.263 Baseline as of 1995 versus H.261

H.263 consists of a baseline decoder with features that any H.263 decoder has to implement. In addition, optional features are defined. The following mandatory features distinguish H.263 as defined in November 1995 from H.261 [6][12]:

1. Half-pixel motion compensation: This feature significantly improves the prediction capability of the motion compensation algorithm in cases where there



**Figure 13.6.** Prediction of motion vectors uses the median vector of MV1, MV2, and MV3. We assume the motion vector is 0 if one MB is outside the picture or group of blocks. If two MBs are outside, we use the remaining motion vector as prediction.

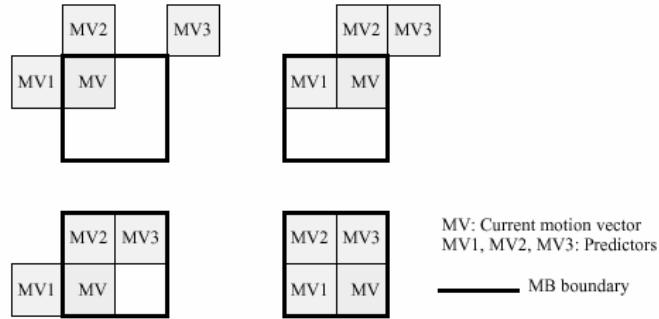
is object motion that needs fine spatial resolution for accurate modeling. Bilinear interpolation (simple averaging) is used to compute the predicted pels in case of non-integer motion vectors. The coding of motion vectors uses the median motion vector of the three neighboring MBs as prediction for each component of the vector (Fig. 13.6).

2. Improved variable-length coding including a 3D VLC for improved efficiency in coding DCT coefficients. Whereas H.261 codes the symbols (run,level) and sends an EOB word at the end of each block, H.263 integrates the EOB word into the VLC. The events to be coded are (last, run, level), where last indicates whether the coefficient is the last non-zero coefficient in the block.
3. Reduced overhead at the Group of Block level as well as coding of MTYPE and CBP.
4. Support for more picture formats (Tab. 13.2).

In addition to the above improvements, H.263 offers a list of *optional* features that are defined in annexes of the standard.

I Unrestricted motion vectors (Annex D) that are allowed to point outside the picture improve coding efficiency in case of camera motion or motion at the picture boundary. The prediction signal for a motion vector that points outside of the image is generated by repeating the boundary pels of the image. The motion vector range is extended to  $[-31.5, 31]$ .

II Syntax-based arithmetic coding (Annex E) may be used in place of the variable length (Huffman) coding resulting in the same decoded pictures at an average



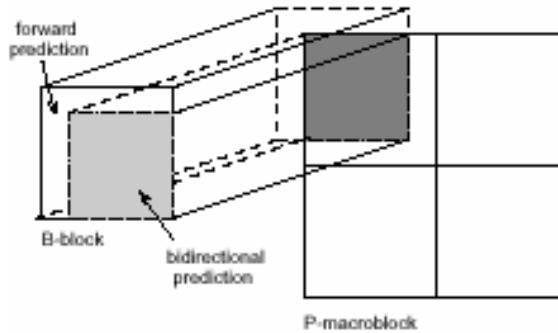
**Figure 13.7.** Prediction of motion vectors in advanced prediction mode with 4 motion vectors in a MB. The predicted value for a component of the current motion vector MV is the median of its predictors.

bitrate saving of 4% for P frames and 10% for I frames. However, decoder computational requirements increase by more than 50% [10]. This will limit the number of manufacturers implementing this annex.

III Advanced prediction mode (Annex F) includes unrestricted motion vector mode. Advanced prediction mode allows for two additional improvements: Overlapped block motion compensation (OBMC) may be used to predict the luminance component of a picture which improves prediction performance and reduces significantly blocking artifacts (see Sec. 9.3.2) [58]. Each pixel in an 8x8 luminance prediction block is a weighted sum of three predicted values computed from the following three motion vectors: Vector of the current MB and vectors of the two MBs that are closest to the current 8x8 block. The weighting coefficients used for motion compensation and the equivalent window function for motion estimation are given previously in Figs. 9.16 and 9.17.

The second improvement of advanced motion prediction is the optional use of four motion vector for a MB, one for each luminance block. This enables better modeling of motion in real images. However, it is up to the encoder to decide in which MB the benefit of four motion vectors is sufficient to justify the extra bits required for coding them. Again, the motion vectors are coded predictively (Fig. 13.7).

IV *PB pictures* (Annex G) is a mode that codes a bidirectionally predicted picture with a normal forward predicted picture. The B-picture temporally precedes the P-picture of the PB picture. In contrast to bidirectional prediction (Sec. 9.2.4, Fig. 9.12) that is computed on a frame by frame basis, PB pictures use bidirectional prediction on a MB level. In a PB frame, the number of blocks per MB is 12 rather than 6. Within each MB, the 6 blocks be-



**Figure 13.8.** Forward prediction can be used for all B-blocks, backward prediction is only used for those pels that the backward motion vector aligns with pels of the current MB (from [10]).

longing to the P-picture are transmitted first, followed by the blocks of the B-picture (Fig. 13.8). Bidirectional prediction is derived from the previous decoded frame and the P-blocks of the current MB. As seen in Fig. 13.8, this limits backward predictions to those pels of the B-blocks that are aligned to pels inside the current P-macroblock in case of motion between the B-picture and the P-picture (light grey area in Fig. 13.8). For the light grey area of the B-block, the prediction is computed by averaging the results of forward and backward prediction. Pels in the white area of the B-block are predicted using forward motion compensation only. An *Improved PB-frame mode* (Annex M) was adopted later that removes this restriction enabling the efficiency of regular B-frames (Sec. 9.2.4).

PB-pictures are efficient for coding image sequences with moderate motion. They tend not to work very well for scenes with fast or complex motion or when coding at low frame rates. Since the picture quality of a B-picture has no effect on the coding of subsequent frames, H.263 defines that the B-picture of a PB-picture set is coded at a lower quality than the P-picture by using a smaller quantizer stepsize for P-blocks than for the associated B-blocks. PB-pictures increase the delay of a coding system, since PB pictures allow the encoder to send bits for the B frame only after the following P-frame has been captured and processed. This limits their usefulness for interactive real-time applications.

Due to the larger number of coding modes, the encoder decisions are more complex than in H.261. A rate distortion optimized H.263 encoder with the options Unrestricted Motion Vector Mode and Advanced Prediction was compared to TMN5, the test model encoder used during the standards development [69]. The optimized encoder increases the PSNR between 0.5 dB and 1.2 dB over TMN5 at

bitrates between 20 and 70 kbit/s.

### H.263 as of 2000

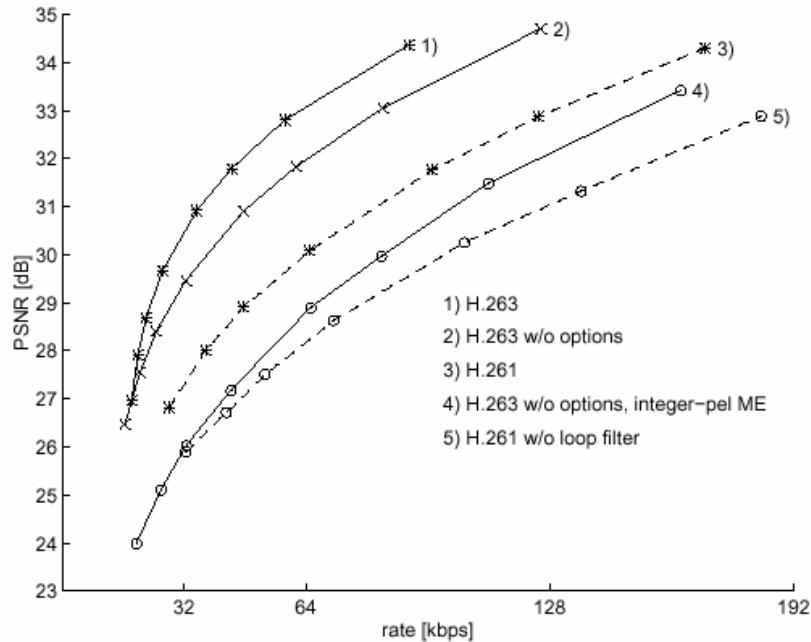
After the initial version of H.263 was approved, work continued with further optional modes being added. However, since more than 15 modes are optional, it is questionable that any manufacturer will implement all these options. ITU recognized that and added recommendations for preferred modes to the standard. The most important preferred modes not mentioned above are listed here.

1. Advanced Intra Coding (Annex I): Intra blocks are coded using the block to the left or above as a prediction provided that block is also coded in intra-mode. This mode increases coding efficiency by 10% to 15% for I pictures.
2. Deblocking filter (Annex J): An adaptive filter is applied across the boundaries of decoded 8x8 blocks to reduce blocking artifacts. This filter affects also the predicted image and is implemented inside the prediction loop of coder and decoder.
3. Supplemental Enhancement Information (Annex L): This information may be used to provide tagging information for external use as defined by an application using H.263. Furthermore, this information can be used to signal enhanced display capabilities like frame freeze, zoom or chroma-keying (see Sec. 10.3).
4. Improved PB-frame Mode (Annex M): As mentioned before, this mode removes the restrictions placed on the backward prediction of Annex G. Therefore, this mode enables regular bidirectional prediction (Sec. 9.2.4).

The above tools are developed for enhancing the coding efficiency. In order to enable transport of H.263 video over unreliable networks such as wireless networks and the Internet, a set of tools are also developed for the purpose of error resilience. These are included in Annex H: Forward Error Correction Using BCH Code, Annex K: Flexible Synchronization Marker Insertion Using the Slice Structured Mode, Annex N and U: Reference Picture Selection, Annex O: Scalability, Annex R: Independent Segment Decoding, Annex V: Data Partitioning and RVLC, Annex W: Header Repetition. These tools are described in Sec. 14.7.1. Further discussion of H.263 can be found in [6] and in the standard itself [49].

#### 13.2.3 Comparison

Fig. 13.9 compares the performance of H.261 and H.263 [10]. H.261 is shown with and without using the filter in the loop (Curves 3 and 5). Since H.261 was designed for data rates of 64 kbit/s and up, we discuss Fig. 13.9 at this rate. Without options, H.263 outperform H.261 by 2 dB (Curves 2 and 3). Another dB is gained if we use the options advanced prediction, syntax-based arithmetic coding, and PB frames (Curve 1). Curve 4 shows that restricting motion vectors in H.263 to integer-pel



**Figure 13.9.** Performance of H.261 and H.263 for the sequence Foreman at QCIF and 12.5 Hz [10].

reduces coding efficiency by 3dB. This is due to the reduced motion compensation accuracy and the lack of the low pass filter that bilinear interpolation brings for half-pel motion vectors. Comparing curves 3 and 5 shows the effect of this lowpass filter on coding efficiency. The differences between curves 4 and 5 are mainly due to the 3D VLC for coding of transform coefficients as well as improvements in coding MTYPE and CBP.

### 13.3 Standards for Visual Communication Systems

In order to enable useful audiovisual communications, the terminals have to establish a common communication channel, exchange their capabilities and agree on the standards for exchanging audiovisual information. In other words, we need much more than just an audio and a video codec in order to enable audiovisual communication. The setup of communication between a server and a client over a network is handled by a *systems* standard. ITU-T developed several system standards including H.323 and H.324 to enable bidirectional multimedia communications over different networks, several audio coding standards for audio communications, and the two important video coding standards H.261 and H.263. Table 13.3 provides an

**Table 13.3.** ITU-T Multimedia Communication Standards. PSTN: Public Switched Telephone Network; N-ISDN: Narrow band ISDN (2x64 kbit/s); B-ISDN: Broadband ISDN; ATM: Asynchronous Transfer Mode; QoS: guaranteed Quality of Service; LAN: Local Area Network; H.262 is identical to MPEG-2 Video [42][17]; G.7xx represents G.711, G.722, and G.728.

Network	System	Video	Audio	Mux	Control
PSTN	H.324	H.261/3	G.723.1	H.223	H.245
N-ISDN	H.320	H.261	G.7xx	H.221	H.242
B-ISDN/ATM	H.321	H.261	G.7xx	H.221	Q.2931
	H.310	H.261/2	G.7xx/MPEG	H.222.0/1	
QoS LAN	H.322	H.261/3	G.7xx	H.221	H.242
Non-QoS LAN	H.323	H.261	G.7xx	H.225.0	H.245

overview of the standards for audio, video, multiplexing and call control that these system standards use [5]. In the following, we briefly describe the functionality of the recent standards H.323 [50] and H.324 [43].

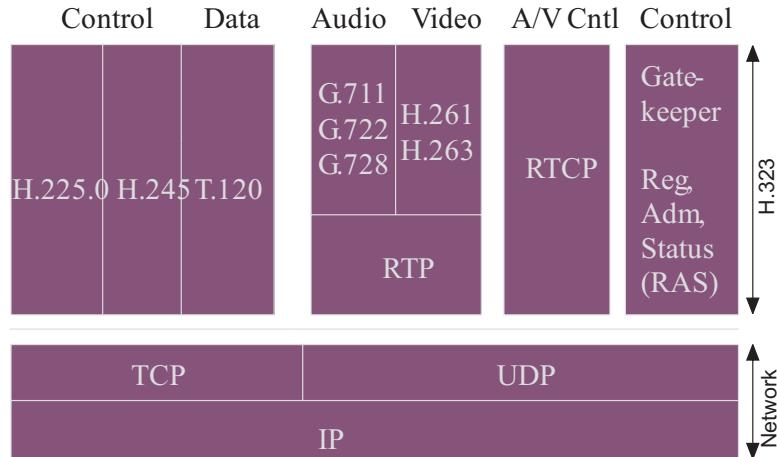
### 13.3.1 H.323 Multimedia Terminals

Recommendation H.323 [50] provides the technical requirements for multimedia communication systems that operate over packet-based networks like the Internet where guaranteed quality of service is usually not available.

Fig. 13.10 shows the different protocols and standards that H.323 requires for video conferencing over packet networks. An H.323 call scenario optionally starts with a gatekeeper admission request (H.225.0 RAS, [47]). Then, call signaling establishes the connection between the communicating terminals (H.225.0, [47]). Next, a communication channel is established for call control and capability exchange (H.245, [48]). Finally, the media flow is established using RTP and its associated control protocol RTCP [64]. A terminal may support several audio and video codecs. However, the support of G.711 audio (64 kbit/s) [35] is mandatory. G.711 is the standard currently used in the Public Switched Telephone Network (PSTN) for digital transmission of telephone calls. If a terminal claims to have video capabilities, it has to include at least an H.261 video codec [40] with a spatial resolution of QCIF. Modern H.323 video terminals usually use H.263 [49] for video communications.

### 13.3.2 H.324 Multimedia Terminals

H.324 [43] differs from H.323 as it enables the same communication over networks with guaranteed quality of service as it is available when using V.34 [41] modems over the PSTN. The standard H.324 may optionally support the media types voice, data and video. If a terminal supports one or more of these media, it uses the same audiovisual codecs as H.323. However, it also supports H.263 for video and G.723.1



**Figure 13.10.** H.323 protocols for multimedia communications over TCP/IP.

[37] for audio at 5.3 kbit/s and 6.3 kbit/s. Audio quality of an G.723.1 codec at 6.3 kbit/s is very close to that of a regular phone call. Call control is handled using H.245. Transmission of these different media types over the PSTN requires the media to be multiplexed (Fig. 13.11) following the multiplexing standard H.223 [44]. The multiplexed data is sent to the PSTN using a V.34 modem and the V.8 or V.8bis procedure [52][51] to start and stop transmission. The modem control V.25ter [46] is used, if the H.324 terminal uses an external modem.

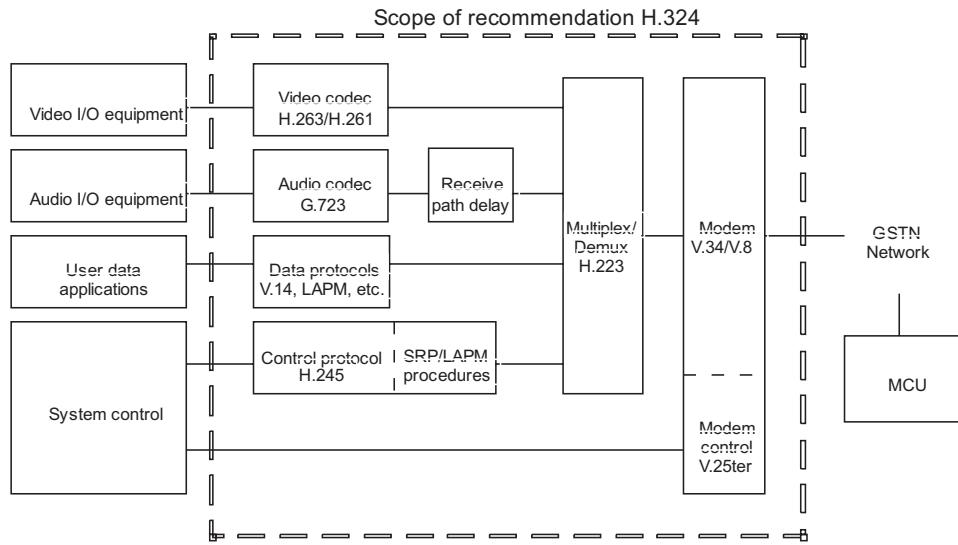
## 13.4 Consumer Video Communications with MPEG-1

The MPEG standards were developed by ISO/IEC JTC1 SC29/WG11, which is chaired by Leonardo Chiariglione. MPEG-1 was designed for progressively scanned video used in multimedia applications, and the target was to produce near VHS quality video at a bit rate of around 1.2 Mb/s (1.5 Mb/s including audio and data). It was foreseen that a lot of multimedia content would be distributed on CD-ROM. At the time of the MPEG-1 development, 1.5 Mbit/s was the access rate of CD-ROM players. The video format is SIF. The final standard supports higher rates and larger image sizes. Another important consideration when developing MPEG-1 were functions that support basic VCR-like interactivity like fast forward, fast reverse and random access into the stored bitstream at every half-second [54].

### 13.4.1 MPEG-1 Overview

The MPEG-1 standard, formally known as ISO 11172 [16], consists of 5 parts, namely Systems, Video, Audio, Conformance, and Software.

MPEG-1 Systems provides a packet structure for combining coded audio and

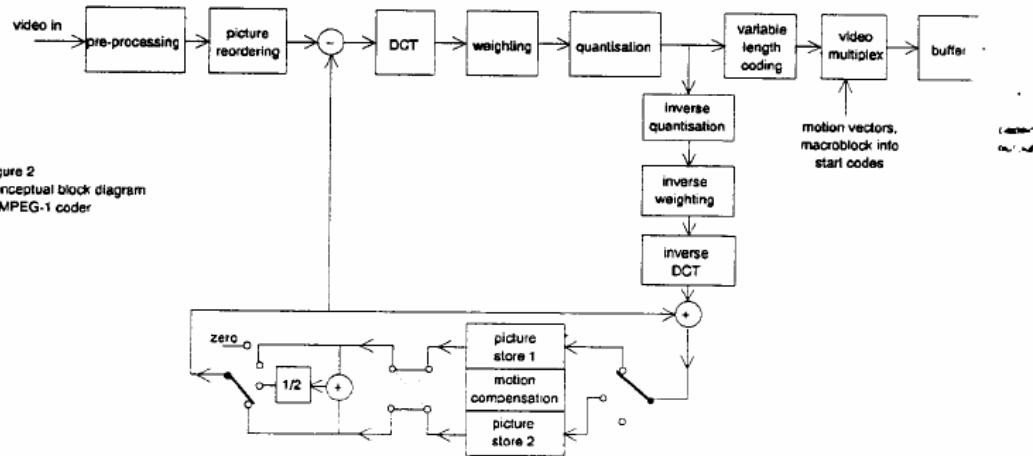


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**Figure 13.11.** Block diagram of an H.324 multimedia communication system over the Public Switched Telephone Network (PSTN).

video data. It enables the system to multiplex several audio and video streams into one stream that allows synchronous playback of the individual streams. This requires all streams to refer to a common system time clock (STC). From this STC, presentation time stamps (PTS) defining the instant when a particular audio or video frame should be presented on the terminal are derived. Since coded video with B-frames required an reordering of decoded images, the concept of Decoding Time Stamps (DTS) is used to indicate by when a certain image has to be decoded.

MPEG-1 audio is a generic standard that does not make any assumptions about the nature of the audio source. However, audio coding exploits perceptual limitations of the human auditory system for irrelevancy reduction. MPEG-1 audio is defined in three layers I, II, and III. Higher layers have higher coding efficiency and require higher resources for decoding. Especially Layer III was very controversial due to its computational complexity at the time of standardization in the early 1990's. However, today it is this Layer III MPEG-1 Audio codec that is known to every music fan as MP3. The reason for its popularity is sound quality, coding efficiency - and, most of all, the fact that for a limited time the proprietary high quality encoder source code was available for download on a company's web site. This started the revolution of the music industry (see Sec. 13.1.2).



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**Figure 13.12.** Block diagram of an MPEG-1 encoder.

### 13.4.2 MPEG-1 Video

The MPEG-1 video convergence phase started after subjective tests in October 1989 and resulted in the standard published in 1993. Since H.261 was published in 1990, there are many similarities between H.261 and MPEG-1. Fig. 13.12 shows the conceptual block diagram of an MPEG-1 coder. Compared to H.261 (Fig. 13.4) we notice the following differences:

1. The loop filter is gone. Since MPEG-1 uses motion vectors with half-pel accuracy, there is no need for the filter (see Sec. 13.2.3). The motion vector range is extended to  $\pm 64$  pels.
2. MPEG-1 uses I-, P-, and B-frames. The use of the B-frames requires a more complex motion estimator and motion compensation unit. Motion vectors for B-frames are estimated with respect to two reference frames, the preceding I- or P-frame and the next I- or P-frame. Hence, we can associate two motion vectors to each MB of a B-frame. For motion compensated prediction, we now need two frame stores for these two reference pictures. The prediction mode for B-frames is decided for each MB. Furthermore, the coding order is different from the scan order (see Fig. 9.12) and therefore, we need a picture reordering unit at the input to the encoder and at the decoder.
3. For I-frames, quantization of DCT coefficients is adapted to the human visual system by dividing the coefficients with a weight matrix. Figure 13.13 shows

$w_{u,v}$	0	1	2	3	4	5	6	8
0	8	16	19	22	26	27	29	34
1	16	16	22	24	27	29	34	37
2	19	22	26	27	29	34	34	38
3	22	22	26	27	29	34	37	40
4	22	26	27	29	32	35	40	48
5	26	27	29	32	35	40	48	58
6	26	27	29	34	38	46	56	69
7	27	29	35	38	46	56	69	83

**Figure 13.13.** Default weights for quantization of I-blocks in MPEG-1. Weights for horizontal and vertical frequencies differ.

the default table. Larger weights results in a coarser quantization of the coefficient. It can be seen that the weights increase with the frequency that a coefficient represents. When comparing a coder with and without the weight matrix at identical bitrates, we notice that the weight matrix reduces the PSNR of decoded pictures but increases the subjective quality.

Another difference from H.261 is that the DC coefficient of an I-block may be predicted from the DCT coefficient of its neighbor to the left. This concept was later extended in JPEG [15][39], H.263, and MPEG-4.

MPEG-1 uses a Group of Picture (GOP) structure (Fig. 9.12). Each GOP starts with an I frame followed by a number of P- and B-frames. This enables random access to the video stream as well as the VCR like functionalities fast forward and reverse.

Because of the large range of the characteristics of bitstreams that is supported by the standard, a special subset of the coding parameters, known as *Constrained Parameters Set* (CPS), has been defined (Tab. 13.4). CPS is a limited set of sampling and bitrate parameters designed to limit computational decoder complexity, buffer size, and memory bandwidth while still addressing the widest possible range of applications. A decoder implemented with the CPS in mind needs only 4 Megabits of DRAM while supporting SIF and CIF. A flag in the bitstream indicates whether or not the bitstream is a CPS.

Compared to an analog consumer quality VCR, MPEG-1 codes video with only half the number of scan lines. At a video bitrate of 1.8 Mbit/s however, it is possible for a good encoder to deliver a video quality that exceeds the quality of a video recorded by an analog consumer VCR onto a used video tape.

### 13.5 Digital TV with MPEG-2

Towards the end of the MPEG-1 standardization process it became obvious that MPEG-1 would not be able to efficiently compress interlaced digital video at broadcast quality. Therefore, the MPEG group issued a call for proposals to submit

**Table 13.4.** Constrained Parameter Set for MPEG-1 video.

Parameter	Maximum value
pixels/line	768 pels
lines/picture	576 lines
number of MBs per picture	396 MBs
number of MBs per second	$396 \cdot 25 = 330 \cdot 30 = 9900$
input buffer size	327680 bytes
motion vector component	$\pm 64$ pels
Bitrate	1.856 Mbps

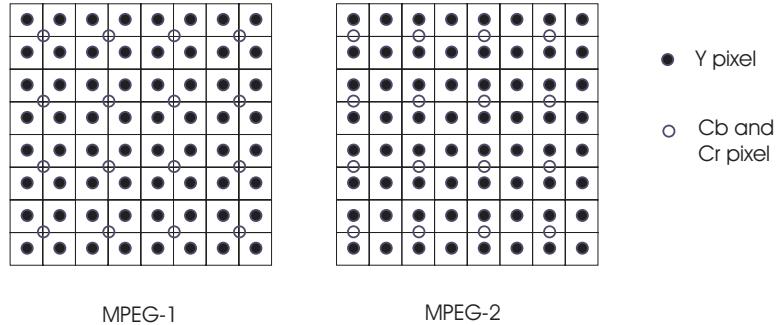
technology for digital coding of audio and video for TV broadcast applications. The best performing algorithms were extensions of MPEG-1 to deal with interlaced video formats. During the collaborative phase of the algorithm development a lot of similarity with MPEG-1 was maintained.

The main purpose of MPEG-2 is to enable MPEG-1 like functionality for interlaced pictures, primarily using the ITU-R BT.601 (formerly CCIR 601) 4:2:0 format [34]. The target was to produce TV quality pictures at data rates of 4 to 8 Mb/s and high quality pictures at 10 to 15 Mb/s. MPEG-2 deals with high quality coding of possibly interlaced video, of either SDTV or HDTV. A wide range of applications, bit rates, resolutions, signal qualities and services are addressed, including all forms of digital storage media, television (including HDTV) broadcasting, and communications [13].

The MPEG-2 standard [19] consists of nine parts: Systems, Video, Audio, Conformance, Software, Digital Storage Media – Command and Control (DSM-CC), Non Backward Compatible (NBC) audio, Real Time Interface, and Digital Storage Media – Command and Control (DSM-CC) Conformance. In this section, we provide a brief overview over MPEG-2 systems, audio and video and the MPEG-2 concept of *Profiles*.

### 13.5.1 Systems

Requirements for MPEG-2 systems are to be somewhat compatible with MPEG-1 systems, be error resilient, support transport over ATM networks and transport more than one TV program in one stream without requiring a common time base for the programs. An MPEG-2 *Program Stream* (PS) is forward compatible with MPEG-1 system stream decoders. A PS contains compressed data from the same program, in packets of variable length usually between 1 and 2 kbytes and up to 64 kbytes. The MPEG-2 *Transport Stream* (TS) is not compatible with MPEG-1. A TS offers error resilience as required for cable TV networks or satellite TV, uses packets of 188 bytes, and may carry several programs with independent time bases that can be easily accessed for channel hopping.



**Figure 13.14.** Luminance and chrominance samples in a 4:2:0 progressive frame.

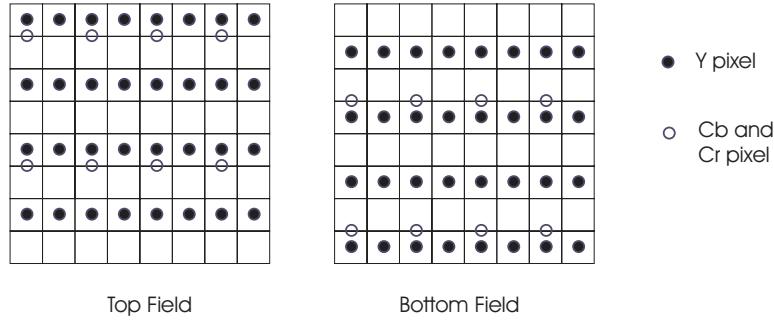
### 13.5.2 Audio

MPEG-2 audio comes in two parts: In part 3 of the standard, MPEG defines a forward and backward compatible audio format that supports five channel surround sound. The syntax is designed such that a MPEG-1 audio decoder is able to reproduce a meaningful downmix out of the five channels of an MPEG-2 audio bitstream [18]. In part 7, the more efficient multi-channel audio decoder, MPEG-2 Advanced Audio Coder (AAC), with sound effects and many other features is defined [20]. MPEG-2 AAC requires 30% less bits than MPEG-1 Layer III Audio for the same stereo sound quality. AAC has been adopted by the Japanese broadcasting industry. AAC is not popular as a format for the internet because no "free" encoder is available.

### 13.5.3 Video

MPEG-2 is targeted at TV studios and TV broadcasting for standard TV and HDTV. As a consequence, it has to support efficiently the coding of interlaced video at bitrates adequate for the applications. The major differences between MPEG-1 and MPEG-2 are the following:

1. Chroma samples in the 4:2:0 format are located horizontally shifted by 0.5 pels compared to MPEG-1, H.261, and H.263 (Fig. 13.14).
2. MPEG-2 is able to code interlaced sequences in the 4:2:0 format (Fig. 13.15).
3. As a consequence, MPEG-2 allows additional scan patterns for DCT coefficient and motion compensation with blocks of size 16x8 pels.
4. Several differences eg 10 bit quantization of the DC coefficient of the DCT, non-linear quantization, better VLC tables improve coding efficiency also for progressive video sequences.



**Figure 13.15.** Luminance and chrominance samples in a 4:2:0 interlaced frame where the top field is temporally first.

5. MPEG-2 supports various modes of scalability. Spatial scalability enables different decoders to get videos of different picture sizes from the same bitstream. MPEG-2 supports temporal scalability such that a bit stream can be decoded into video sequences of different frame rates. Furthermore, SNR scalability provides the ability to extract video sequences with different amplitude resolutions from the bitstream.
6. MPEG-2 defines profiles and levels defining a subset of the MPEG-2 features and their parameter ranges that are signaled in the header of a bitstream (see Sec. 13.5.4). In this way a MPEG-2 compliant decoder knows immediately whether it can decode the bitstream.
7. MPEG-2 allows for much higher bitrates (see Sec. 13.5.4).

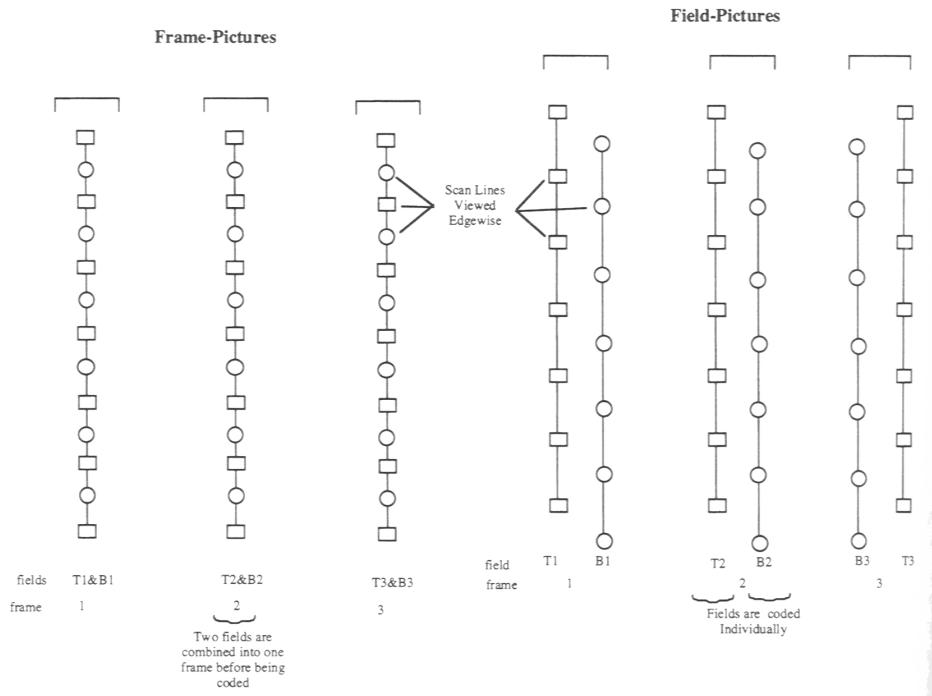
In the following we will discuss on the extensions introduced to support interlaced video and scalability.

### Coding of Interlaced Video

Interlaced video is a sequence of alternating top and bottom fields (see Sec. 1.3.1). Two fields are of identical *parity* if they are both top fields or both bottom fields. Otherwise, two fields are said to have opposite parity. MPEG-2 considers two types of *Picture Structures* for interlaced video (Fig. 13.16). A *Frame picture* consists of lines from the top and bottom fields of an interlaced picture in an interlaced order. This frame picture structure is also used when coding progressive video. A *Field-picture* keeps the top and the bottom field of the picture separate. For each of these pictures, I-, P-, and B-picture coding modes are available.

MPEG-2 adds new prediction modes for motion compensation, all related to interlaced video.

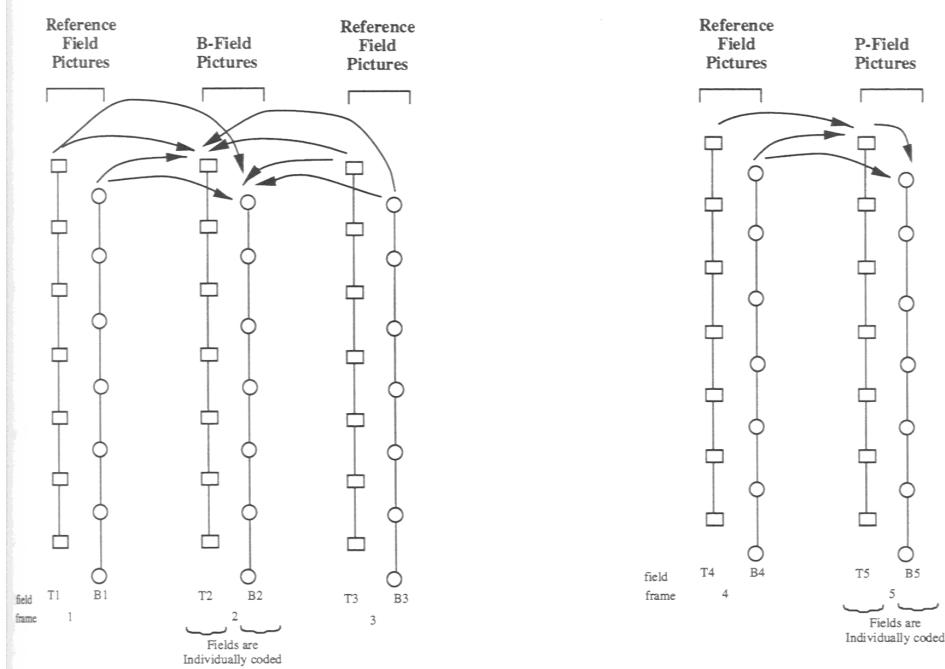
1. *Field prediction for Field-pictures* is used to predict a MB in a Field-picture. For P-fields, the prediction may come from either field of the two most recently



**Figure 13.16.** Frame and Field Picture Structures (side view of the individual fields) : Each frame consists of a top and a bottom field, either one of them may be temporally first.

coded fields. For B-fields, we use the two fields of the two reference pictures (Fig. 13.17).

2. *Field prediction for Frame-pictures* splits a MB of the frame into the pels of the top field and those of the bottom field resulting in two 16x8 Field blocks (Fig. 13.18). Each Field block is predicted independent of the other similar to the method described in item 1 above. This prediction method is especially useful for rapid motion.
3. *Dual Prime for P-pictures* transmits one motion vector per MB that can be used for predicting Field and Frame-pictures from the preceding P- or I-picture. The target MB is represented as two Field blocks. The coder computes two predictions for each field block and averages them. The first prediction of each Field block is computed by doing motion compensation us-



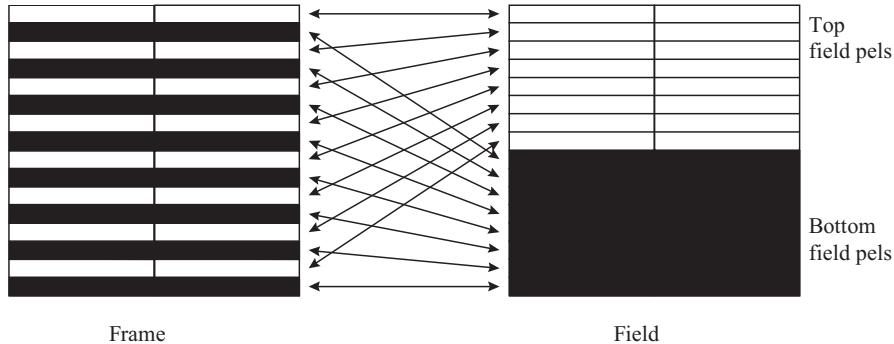
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**Figure 13.17.** Every MB relevant for Field Prediction for Field-pictures is located within one field of the Reference picture. Pictures may have different parity.

ing the transmitted motion vector and the field with the same parity as the reference. The second prediction of each Field block is computed using a *corrected motion vector* and the field with the opposite parity as reference. The corrected motion vector is computed assuming linear motion. Considering the temporal distance between the fields of same parity, the transmitted motion vector is scaled to reflect the temporal distance between the fields of opposite parity. Then we add a transmitted *Differential Motion Vector* (DMV) resulting in the corrected motion vector. For interlaced video, this Dual Prime for P-pictures prediction mode can be as efficient as using B-pictures - without adding the delay of a B-picture.

4. *16X8 MC for Field pictures* corresponds to field prediction for Frame-pictures. Within a MB, the pels belonging to different fields have their own motion vectors for motion compensation, i.e. two motion vectors are transmitted for P-pictures and four for B-pictures.

These many choices for prediction makes the design of an optimal encoder ob-



**Figure 13.18.** Field prediction for Frame-pictures: The MB to be predicted is split into top field pels and bottom field pels. Each 16x8 Field block is predicted separately with its own motion vector (P-frame) or 2 motion vectors (B-frame).

viously very challenging.

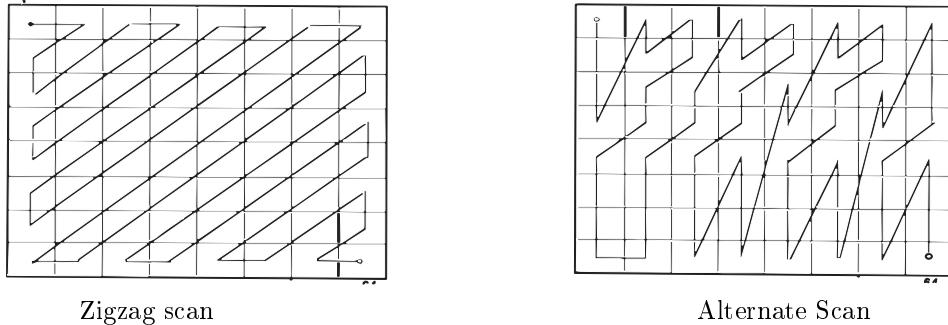
In interlaced video, neighboring rows in a MB come from different fields, thus the vertical correlation between lines is reduced when the underlying scene contains motion with a vertical component. MPEG-2 provides two new coding modes to increase the efficiency of prediction error coding.

1. *Field DCT* reorganizes the pels of a MB into two blocks for the top field and two blocks for the bottom field (Fig. 13.18). This increases the correlation within a block in case of motion and thus increases the coding efficiency.
2. MPEG-2 provides an *Alternate scan* that the encoder may select on a picture-by-picture basis. This scan puts coefficients with high vertical frequencies earlier than the Zigzag scan. Fig. 13.19 compares the new scan to the conventional Zigzag scan.

### Scalability in MPEG-2

The MPEG-2 functionality described so far is achieved with the non scalable syntax of MPEG-2, which is a superset of MPEG-1. The scalable syntax structures the bitstream in layers. The base layer can use the non-scalable syntax and thus be decoded by an MPEG-2 terminal that does not understand the scalable syntax. The basic MPEG-2 scalability tools are *data partitioning*, *SNR scalability*, *spatial scalability* and *temporal scalability* (see Sec. 11.1. Combinations of these basic scalability tools are also supported.

When using scalable codecs, *drift* may occur in a decoder that decodes the base-layer only. Drift is created if the reference pictures used for motion compensation at the encoder and the base-layer decoder differ. This happens if the encoder uses information of the enhancement layer when computing the reference picture for

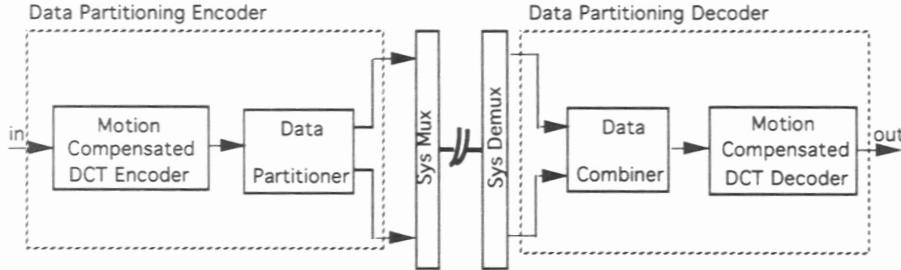


**Figure 13.19.** The Zigzag scan as known from H.261, H.263, and MPEG-1 is augmented by the Alternate scan in MPEG-2 in order to code interlaced blocks that have more correlation in horizontal direction than in vertical direction.

the base layer. The drift is automatically set to zero at every I-frame. Drift does not occur in scalable codecs if the encoder does not use any information of the enhancement layer for coding the base layer. Furthermore, a decoder decoding layers in addition to the base layer may not introduce data from the upper layers into the decoding of the lower layers.

*Data Partitioning:* Data partitioning splits the video bit stream into two or more layers. The encoder decides which syntactic elements are placed into the base layer and which into the enhancement layers. Typically, high frequency DCT coefficients are transmitted in the low priority enhancement layer while all headers, side information, motion vectors, and the first few DCT coefficients are transmitted in the high priority base layer. Data partitioning is appropriate when two transmission channels are available. Due to the data partitioning, the decoder can decode the base layer only if the decoder implements a bitstream loss concealer for the higher layers. This concealer can be as simple as setting to zero the missing higher order DCT coefficients in the enhancement layer. Fig. 13.20 shows a high-level view of the encoder and decoder. The data partitioning functionality may be implemented independent of the encoder and decoder. Data partitioning does not incur any noticeable overhead. However, its performance in an error-prone environment may be poor compared to other methods of scalability [13]. Obviously, we will encounter the drift problem if we decode only the base layer.

*SNR Scalability:* SNR scalability is a frequency domain method where all layers are coded with the same spatial resolution, but with differing picture quality achieved through different MB quantization stepsizes. The lower layer provides the basic video quality while the enhancement layer carry the information which, when added to the lower layer, generate a higher quality reproduction of the input video. Fig. 13.21 shows a SNR scalable coder, which includes a non-scalable base encoder. The base encoder feeds the DCT coefficients after transform and quantization into the SNR enhancement coder. The enhancement coder re-quantizes the quantiza-



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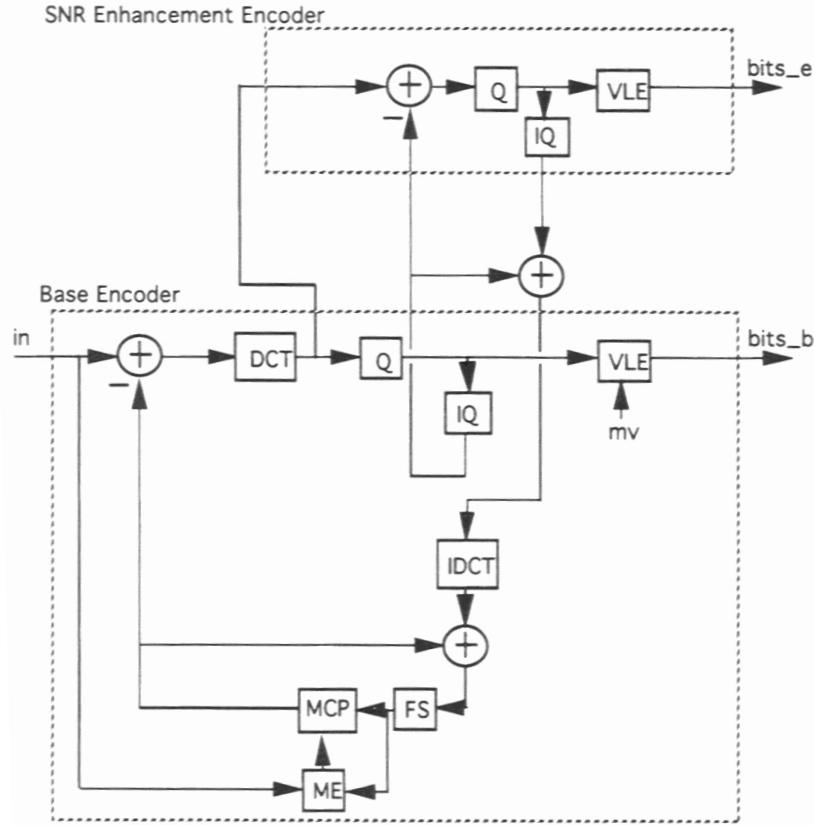
**Figure 13.20.** A data partitioning codec is suited for ATM networks that support two degrees of quality of service.

tion error of the base encoder and feeds the coefficients that it sends to the SNR enhancement decoder, back into the base encoder, which adds them to its dequantized coefficients and to the encoder feedback loop. Due to the feedback of the enhancement layer at the encoder, drift occurs for any decoder that decodes only the base layer.

At a total bitrate between 4 Mbit/s and 9 Mbit/s, the combined picture quality of base and enhancement layers is 0.5 to 1.1 dB less than that obtained with non-scalable coding. Obviously, SNR scalability outperforms data partitioning in terms of picture quality for the base layer [60][13].

*Spatial Scalability:* In MPEG-2, spatial scalability is achieved by combining two complete encoders at the transmitter and two complete decoders at the receiver. The base layer is coded at low spatial resolution using a motion compensated DCT encoder such as H.261, MPEG-1 or MPEG-2 (Fig. 13.22). The image in the frame store of the feedback loop of this base encoder is made available to the spatial enhancement encoder. This enhancement coder is also a motion compensated DCT encoder which codes the input sequence at the high resolution. It uses the upsampled input from the lower layer to enhance its temporal prediction. The prediction image in the enhancement layer coder is the weighted sum of the temporal prediction image of the enhancement coder and the spatial prediction image from the base encoder. Weights may be adapted on a MB level. There are no drift problems with this coder since neither the encoder nor the decoder introduce information of the enhancement layer into the base layer. At a total bitrate of 4 Mbit/s, the combined picture quality of base and enhancement layers is 0.75 to 1.5 dB less than that obtained with nonscalable coding [13].

Compared to simulcast, i.e. sending two independent bitstreams one having the base layer resolution and one having the enhancement layer resolution, spatial scalability is more efficient by 0.5 to 1.25 dB [13][61]. Spatial scalability is the

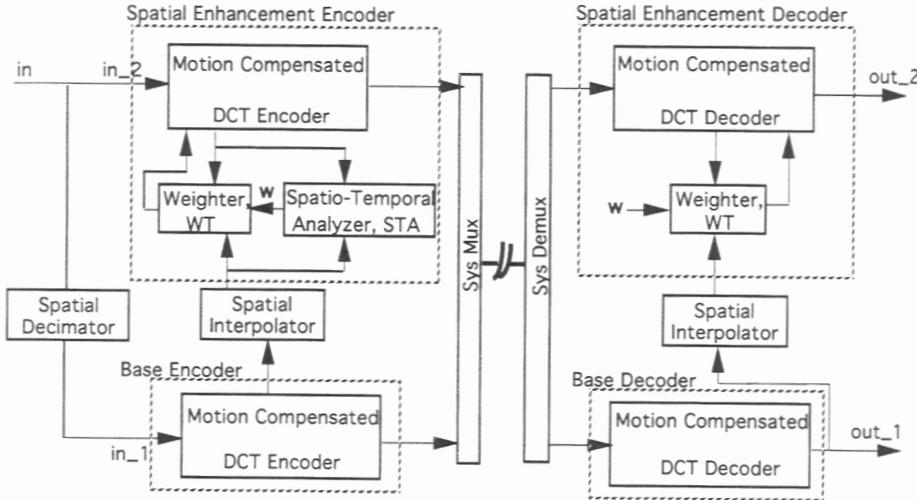


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**Figure 13.21.** A detailed view of the SNR scalability encoder. This encoder defaults to a standard encoder if the enhancement encoder is removed.

appropriate tool to be used in applications where interworking of video standards is necessary and the increased coding efficiency compared to simulcasting is able to offset the extra cost for complexity of encoders and decoders.

*Temporal Scalability:* In temporal scalability, the base layer is coded at a lower frame rate using a nonscalable codec, and the intermediate frames can be coded in a second bitstream using the first bitstream reconstruction as prediction [62]. MPEG-2 defines that only two frames may be used for the prediction of an enhancement layer pictures. Fig. 13.23 and Fig. 13.24 show two typical configurations. If we mentally collapse the images of enhancement layer and base layer in Fig. 13.23, we notice that the resulting sequence of images and the prediction arrangement is



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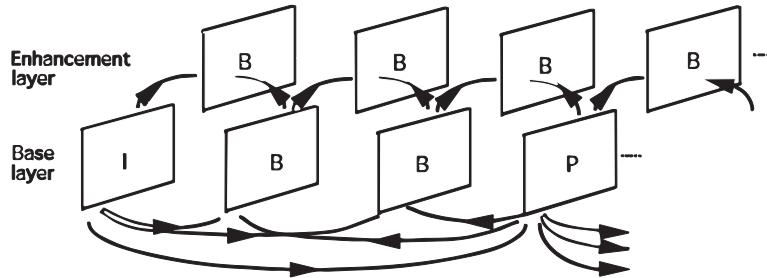
**Figure 13.22.** An encoder with spatial scalability consists of two complete encoders that are connected using a spatial interpolation filter.

similar to a nonscalable coder and identical to a nonscalable coder if the base layer uses only I and P frames. Accordingly, the picture quality of temporal scalability is only 0.2 to 0.3 dB lower than a nonscalable coder [13]. In Fig. 13.25 we see that enhancement and base layer encoders are two complete codecs that both operate at half the rate of the video sequence. Therefore, the computational complexity of temporal scalability is similar to a nonscalable coder operating at the full frequency of the input sequence. There are no drift problems.

Temporal scalability is an efficient means of distributing video to terminals with different computational capabilities like a mobile terminal and a desktop PC. Another application is stereoscopic video transmission where right and left channels are transmitted as the enhancement and base layer, respectively. This is discussed previously in Sec. 12.5.1.

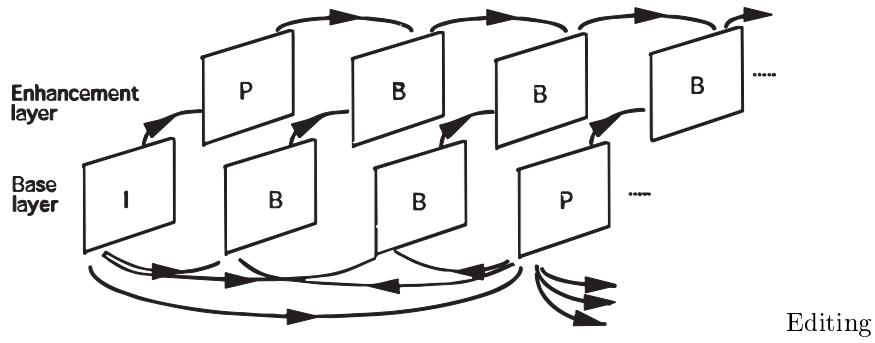
### 13.5.4 Profiles

The full MPEG-2 syntax covers a wide range of features and parameters. Extending the MPEG-1 concept of a constrained parameter set (Tab. 13.4), MPEG-2 defines *Profiles* that describe the tools required for decoding a bitstream and *Levels* that describe the parameter range for these tools. MPEG-2 initially defined five profiles for video, each adding new tools in a hierarchical fashion. Later, two more profiles were added that do not fit the hierarchical scheme:



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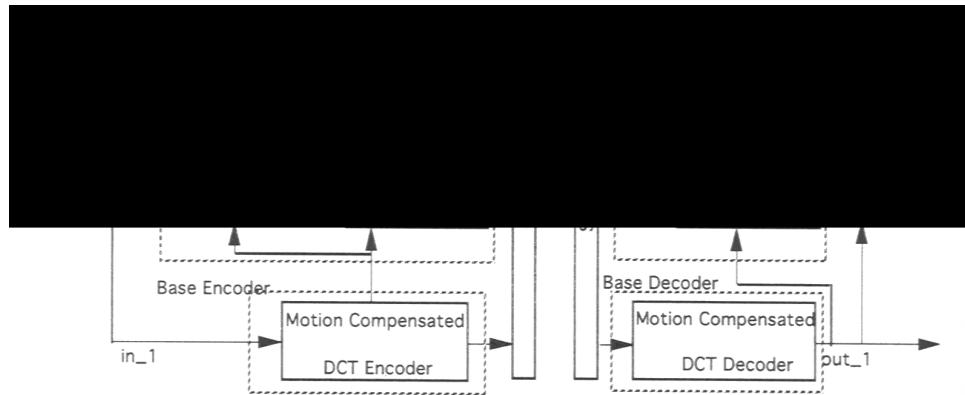
**Figure 13.23.** Temporal scalability may use only the base layer to predict images in the enhancement layer. Obviously, errors in the enhancement layers to do not propagate over time.



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**Figure 13.24.** Temporal scalability may use the base layer and the enhancement layer for prediction. This arrangement is especially useful for coding of stereoscopic video.

1. The *Simple* profile supports I and P frames, the 4:2:0 format, and no scalability. It is currently not used in the market.
2. The *Main* profile adds support for B-frames. The Main profile at Main level (MP@ML) is used for TV broadcasting. This profile is the most widely used.
3. The *SNR* profile supports SNR scalability in addition to the functionality of the Main profile. It is currently not used in the market.
4. The *Spatial* profile supports the functionality of the SNR profile and adds spatial scalability. It is currently not used in the market.
5. Finally, the *High* Profile supports the functionality of the Spatial profile and adds support for the 4:2:2 format. This profile is far too complex to be useful.



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**Figure 13.25.** A temporal scalability encoder consists of two complete encoders with the enhancement encoder using the base layer video as an additional reference for prediction. The Temporal Demux sends pictures alternating to the base encoder and the enhancement encoder.

6. The *4:2:2* profile supports studio post production and high quality video for storage and distribution. It basically extends the main profile to higher bitrates and quality. The preferred frame order in a group of frames is IBIBIBIBI... Equipment with this profile is used in digital studios.
7. The *Multiview* profile enables the transmission of several video streams in parallel thus enabling stereo presentations. This functionality is implemented using temporal scalability thus enabling Main profile decoders to receive one of the video streams. Prototypes exist.

For each profile, MPEG defined levels. Levels essentially define the size of the video frames, the frame rate and picture types thus providing an upper limit for the processing power of a decoder. Table 13.5 shows the levels defined for most profiles. The fact that only two fields in Table 13.5 are used in the market (MP@ML and 4:2:2@ML) is a strong indication that standardization is a consensus process — MPEG had to accommodate many individual desires to get patented technology required in an MPEG profile without burdening the main applications, i.e. TV production and broadcasting.

### 13.6 Coding of Audio Visual Objects with MPEG-4

The MPEG-4 standard is designed to address the requirements of a new generation of highly interactive multimedia applications while supporting traditional applications as well. Such applications, in addition to efficient coding, also require advanced

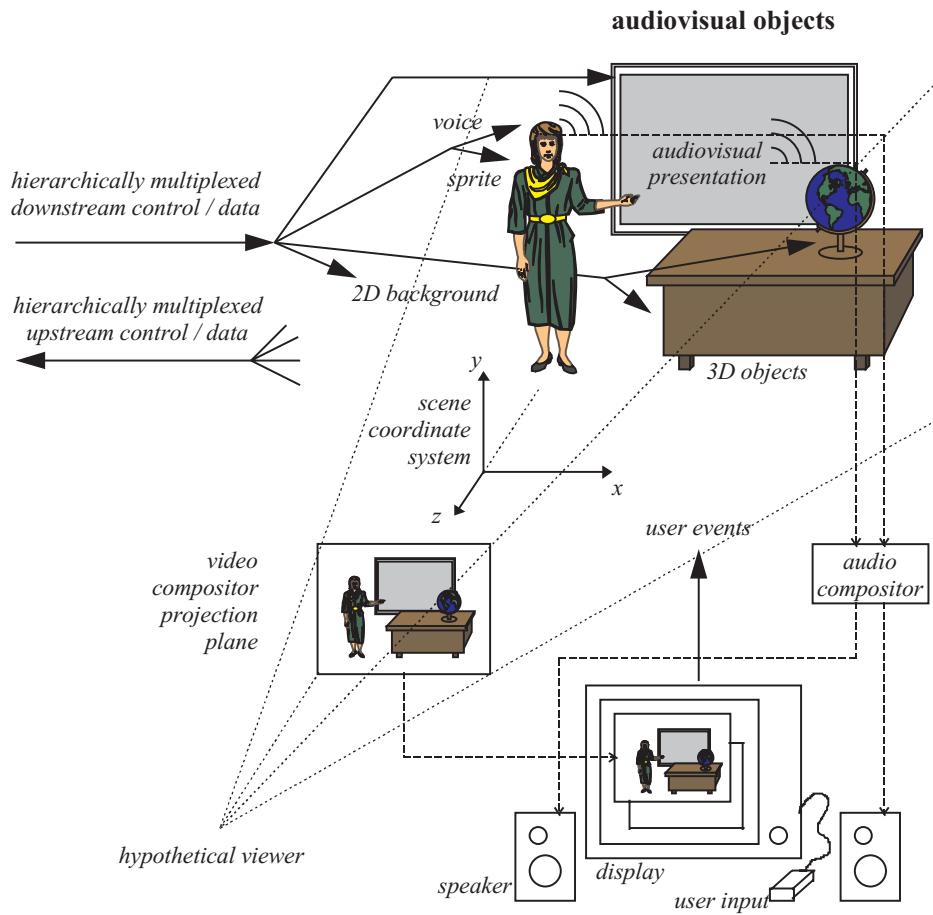
**Table 13.5.** Profiles and levels in MPEG-2 defines allowable picture types (I,P,B), pels/line and lines/picture, picture format, and maximum bitrate (for all layers in case of scalable bitstreams).

Level		Simple (I,P) (4:2:0)	Main (I,P,B) (4:2:0)	SNR (I,P,B) (4:2:0)	Spatial (I,P,B) (4:2:0)	High (I,P,B) (4:2:0; 4:2:2)	Multiview (I,P,B) (4:2:0)	4:2:2 (I,P,B) (4:2:0, 4:2:2)
Low	Pels/line			352	352		352	
	Lines/frame			288	288		288	
	Frames/s			30	30		30	
	Mbit/s			4	4		8	
Main	Pels/line	720	720	720		720	720	720
	Lines/frame	576	576	576		576	576	512/608
	Frames/s	30	30	30		30	30	30
	Mbit/s	15	15	15		20	25	50
High- 1440	Pels/line		1440		1440	1440	1440	
	Lines/frame		1152		1152	1152	1152	
	Frames/s		60		60	60	60	
	Mbit/s		60		60	80	100	
High	Pels/line		1920			1920	1920	1920
	Lines/frame		1152			1152	1152	1152
	Frames/s		60			60	60	60
	Mbit/s		80			100	130	300

functionalities such as interactivity with individual objects, scalability of contents, and a high degree of error resilience. MPEG-4 provides tools for object-based coding of natural and synthetic audio and video as well as graphics. The MPEG-4 standard, similar to its predecessors, consists of a number of parts, the primary parts being systems, visual and audio. The visual part and the audio part of MPEG-4 include coding of both natural and synthetic video and audio, respectively.

### 13.6.1 Systems

MPEG-4 Systems enables the multiplexing of audio-visual objects and their composition into a scene. Fig. 13.26 shows a scene that is composed in the receiver and then presented on the display and speakers. A mouse and keyboard may be provided to enable user input. If we neglect the user input, presentation is as on a regular MPEG-1 or MPEG-2 terminal. However, the audio-visual objects are composited into a scene at the receiving terminal whereas all other standards discussed in this chapter require scene composition to be done prior to encoding. The scene in Fig. 13.26 is composed in a local 3D coordinate system. It consists of a 2D background, a video playing on the screen in the scene, a presenter, coded as a 2D sprite object, with audio, and 3D objects like the desk and the globe. MPEG-4 enables user interactivity by providing the tools to interact with this scene. Obvi-

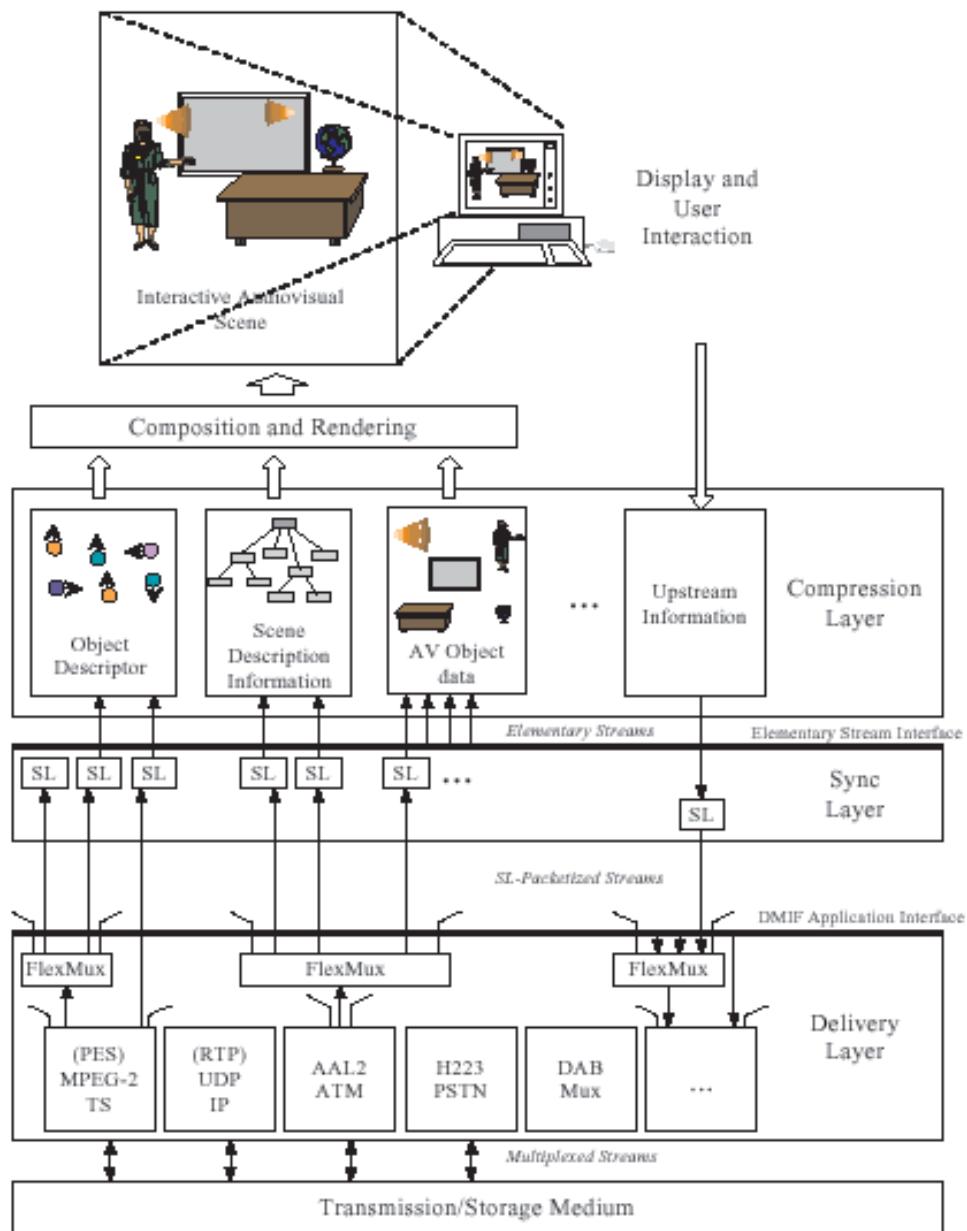


**Figure 13.26.** Audio-visual objects are composed into a scene within the receiver of a MPEG-4 presentation [Courtesy of MPEG-4].

ously, this object-based content description gives tremendous flexibility in creating interactive content and in creating presentations that are customized to a viewer, be it language, text, advertisements, logos, etc..

Fig. 13.27 shows the different functional components of an MPEG-4 terminal [1]:

Media or Compression Layer: This is the component of the system performing the decoding of the media like audio, video, graphics and other suitable media. Media are extracted from the sync layer through the *elementary stream* interface. Specific MPEG-4 media include a Binary Format for Scenes (BIFS)



**Figure 13.27.** An MPEG-4 terminal consists of a delivery layer, a synchronization layer, and a compression layer. MPEG-4 does not standardize the actual composition and rendering [Courtesy of MPEG-4].

for specifying scene compositions and graphics contents. Another specific MPEG-4 media is the Object Descriptor (OD). An OD contains pointers to elementary streams, similar to URLs. Elementary streams are used to convey individual MPEG-4 media. ODs also contain additional information such as Quality of Service parameters. This layer is media aware, but delivery unaware, i.e. it does not consider transmission [66].

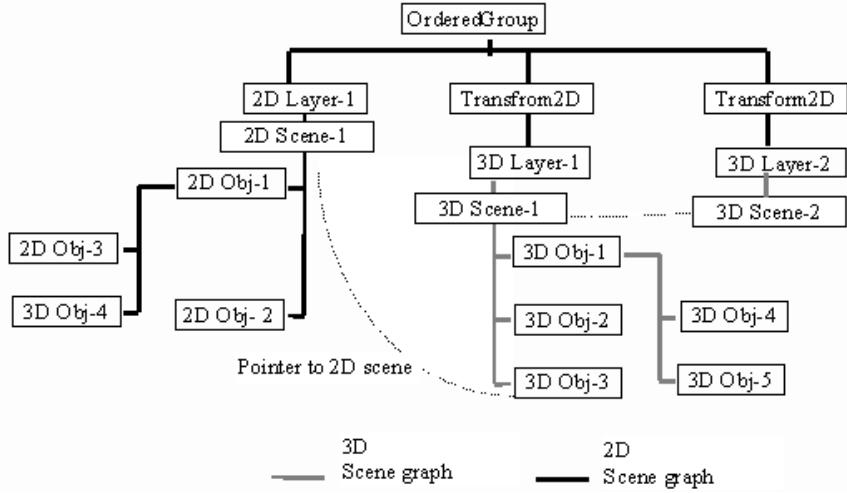
Sync or elementary stream layer: This component of the system is in charge of the synchronization and buffering of individual compressed media. It receives Sync Layer (SL) packets from the Delivery layer, unpacks the elementary streams according to their timestamps and forwards them to the Compression layer. A complete MPEG-4 presentation transports each media in different elementary streams. Some media may be transported in several elementary streams, for instance if scalability is involved. This layer is media unaware and delivery unaware, and talks to the Transport layer through the Delivery Multimedia Integration Framework (DMIF) application interface (DAI). The DAI, additionally to the usual session set up and stream control functions, also enables to set the quality of service requirements for each stream. The DAI is network independent [14].

Transport layer: The transport layer is media unaware and delivery aware. MPEG-4 does not define any specific transport layer. Rather, MPEG-4 media can be transported on existing transport layers such as for instance RTP, MPEG-2 Transport stream, H.223 or ATM using the DAI as specified in [31][2].

### MPEG-4's Binary Format For Scenes (BIFS)

The BIFS scene model is a superset of the *Virtual Reality Modeling Language* (VRML) [21][11]. VRML is a modeling language that allows to describe synthetic 3D objects in a synthetic scene and render it using a virtual camera. MPEG-4 extends VRML in three areas:

- 2-D scene description is defined for placement of 2D audiovisual objects onto a screen. This is important of the coded media are only video streams that do not require the overhead of 3D rendering. 2D and 3D scenes may be mixed. Fig. 13.28 shows a scenegraph that places several 2D objects on the screen. The object position is defined using Transform Nodes. Some of the objects are 3D objects that require 3D rendering. After rendering, these objects are used as 2D objects and placed into the 2D scene.
- BIFS enables the description and animation of scenes and graphics objects using its new compression tools based on arithmetic coders.
- MPEG-4 recognizes the special importance of human faces and bodies. It introduced special tools for very efficient description and animation of virtual humans.

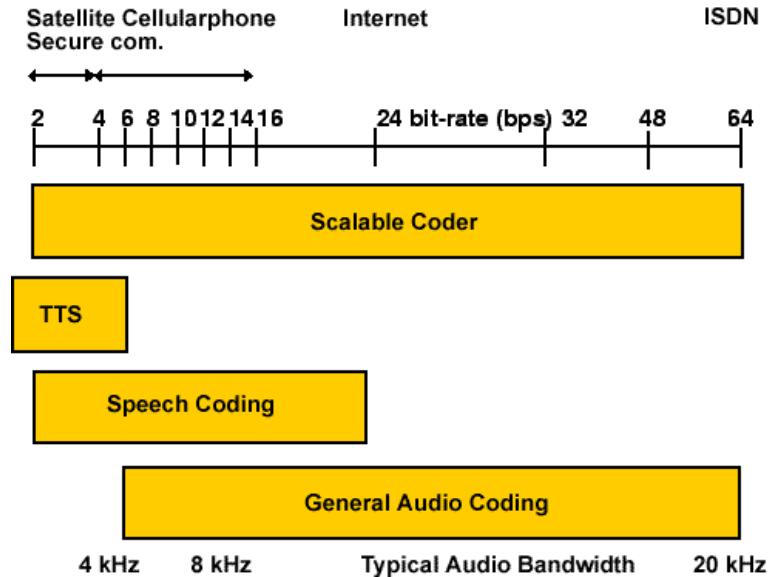


**Figure 13.28.** A scenegraph with 2D and 3D components. The 2D scenegraph requires only simple placement of 2D objects on the image using Transform2D nodes. 3D objects are rendered and then placed on the screen as defined in the 3DLayer nodes. Interaction between objects can be defined using pointers from one node to another (from [65]).

## 13.6.2 Audio

The tools defined by MPEG-4 audio [30] [3] can be combined into different audio coding algorithms. Since no single coding paradigm was found to span the complete range from very low bitrate coding of speech signals up to high quality multi-channel audio coding, a set of different algorithms has been defined to establish optimum coding efficiency for the broad range of anticipated applications (Fig. 13.29). The scalable audio coder can be separated into several components.

- At its lowest rate, a Text-to-Speech (TTS) synthesizer is supported using the MPEG-4 Text-to-Speech Interface (TTSI)
  - Low rate speech coding (3.1 kHz bandwidth) is based on a Harmonic Vector eXcitation Coding (HVXC) coder at 2kbit/s up to 4kbit/s.
  - Telephone speech (8 kHz bandwidth) and wideband speech (16 kHz bandwidth) are coded using a Code Excited Linear Predictive (CELP) coder at rates between 3850 bit/s and 23800 bit/s. This CELP coder can create scalable bitstreams with 5 layers.
  - General audio is coded at 16 kbit/s and up to more than 64 kbit/s per channel



Editing Instructions: move 'bit-rate (bps)' to the left of axis; move 'Typical Audio Bandwidth' to the left of the kHz numbers.

**Figure 13.29.** MPEG-4 Audio supports coding of speech and audio starting at rates below 2kbit/s up to more than 64 kbit/s/channel for multichannel audio coding.

using a more efficient development of the MPEG-2 AAC coder. Transparent audio quality can be achieved.

In addition to audio coding, MPEG-4 audio defines music synthesis at the receiver using a *Structured Audio* toolset that provides a single standard to unify the world of algorithmic music synthesis and to implement scalability and the notion of audio objects [9].

### 13.6.3 Basic Video Coding

Many of the MPEG-4 functionalities require access not only to an entire sequence of pictures, but to an entire object, and further, not only to individual pictures, but also to temporal instances of these objects within a picture. A temporal instance of a video object can be thought of as a snapshot of an arbitrarily shaped object that occurs within a picture. Like a picture, an object is intended to be an access unit, and, unlike a picture, it is expected to have a semantic meaning. MPEG-4 enables content-based interactivity with video objects by coding objects independently using motion, texture and shape. At the decoder, different objects are composed into a scene and displayed. In order to enable this functionality, higher syntactic structures had to be developed. A scene consists of several VideoObjects (VO). The VO has

3 dimensions (2D+time). A VO can be composed of several VideoObjectLayers (VOL). Each VOL (2D+time) represents various instantiations of a VO. A VOL can represent different layers of a scalable bitstream or different parts of a VO. A time instant of a VOL is called VideoObjectPlane (VOP). A VOP is a rectangular video frame or a part thereof. It can be fully described by its texture variations (a set of luminance and chrominance values) and its shape. The video encoder applies the motion, texture and shape coding tools to the VOP using I, P, and B modes similar to the modes of MPEG-2. For editing and random access purposes, consecutive VOPs can be grouped into a GroupOfVideoObjectPlanes (GVOP). A video session, the highest syntactic structure, may consist of several VOs.

The example in Fig. 13.30 shows one VO composed of 2 VOL. VOL1 consists of the tree and the background. VOL2 represents the person. In the example, VOL1 is represented by two separate VOPs, VOP1 and VOP3. Hence, VOL1 may provide content-based scalability in the sense that a decoder may choose not to decode one VOP of VOL1 due to resource limitations. VOL2 contains just one VOP, namely VOP2. VOP2 may be represented using a temporal, spatial or quality scalable bitstream. In this case, a decoder might again decide to decode only the lower layers of VOL 2. The example in Fig. 13.30 shows the complex structures of content-based access and scalability that MPEG-4 supports. However, the given example could also be represented in a straight forward fashion with three VOs. The background, the tree and the person are coded as separate VOs with one layer each, and each layer is represented by one VOP. The VOPs are encoded separately and composed in a scene at the decoder.

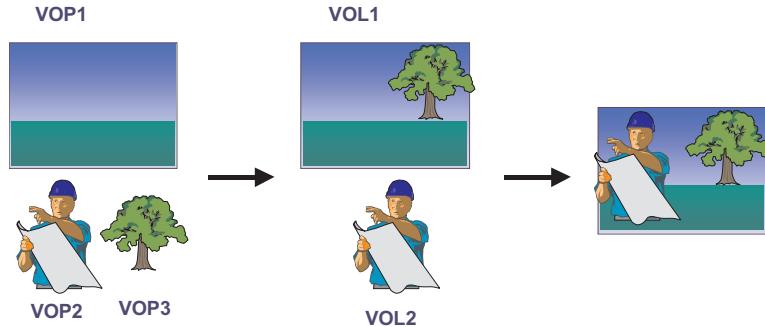
To see how MPEG-4 video coding works, consider a sequence of VOPs. Extending the concept of intra (I-) pictures, predictive (P-) and bidirectionally predictive (B-) pictures of MPEG-1/2 to VOPs, I-VOP, P-VOP and B-VOP result. If, two consecutive B-VOPs are used between a pair of reference VOPs (I- or a P-VOPs), the resulting coding structure is as shown in Fig. 13.31.

### Coding Efficiency Tools

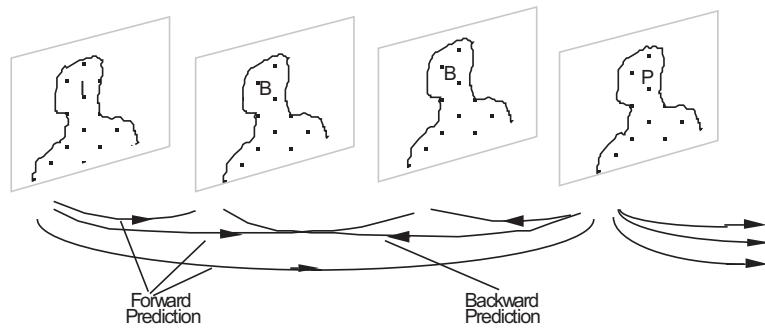
In addition to the obvious changes due to the object-based nature of MPEG-4, the following tools were introduced in order to increase coding efficiency compared to MPEG-1 and MPEG-2:

**DC Prediction:** This is improved compared to MPEG1/2. Either the previous block or the block above the current block can be chosen as predictors to predict the current DC value.

**AC Prediction:** AC prediction of DCT coefficients is new in MPEG-4. The block that was chosen to predict the DC-coefficient is also used for predicting one line of AC coefficients. If the predictor is the previous block, the AC coefficients of its first column are used to predict the co-located AC coefficients of the current block. If the predictor is the block from the previous row, it is used to predict the first row of AC coefficients. AC prediction does not work well for



**Figure 13.30.** Object-based coding requires the decoder to compose different VideoObjectPlanes (VOP) into a scene. VideoObjectLayers (VOL) enable content-based scalability.



**Figure 13.31.** An example prediction structure using I, P and B-VOPs (from [59]).

blocks with coarse texture or diagonal edges or horizontal as well as vertical edges. Switching AC prediction on and off on a block level is desirable but too costly. Consequently, the decision is made on the MB level.

**Alternate Horizontal Scan:** This scan is added to the two scans of MPEG-2 (Fig. 13.19). The Alternate Scan of MPEG-2 is referred to as Alternate Vertical Scan in MPEG-4. The Alternate Horizontal Scan is created by mirroring the Vertical Scan. The scan is selected at the same time as the AC prediction is decided. In case of AC prediction from the previous block, Alternate Vertical Scan is selected. In case of AC prediction from the block above, Alternate Horizontal Scan is used. No AC prediction is coupled to the Zigzag scan.

**3D VLC:** Coding of DCT coefficients is achieved similar to H.263.

**4 Motion Vectors:** Four motion vectors for a MB are allowed. This is done similar to H.263.

**Unrestricted Motion Vectors:** This mode is enabled. Compared to H.263, a much wider motion vector range of  $\pm 2048$  pels may be used.

**Sprite:** A Sprite is basically a large background image that gets transmitted to the decoder. For display, the encoder transmits affine mapping parameters that map a part of the image onto the screen. By changing the mapping, the decoder can zoom in and out of the Sprite, pan to the left or right [8].

**Global Motion Compensation:** In order to compensate for global motion due to camera motion, camera zoom or large moving objects, global motion is compensated according to the eight parameter motion model of Eq. 5.5.14 (see Sec. 5.5.3):

$$\begin{aligned}x' &= \frac{ax+by+c}{gx+hy+1} \\y' &= \frac{dx+ey+f}{gx+hy+1}\end{aligned}. \quad (13.6.1)$$

Global motion compensation is an important tool to improve picture quality for scenes with large global motion. These scenes are difficult to code using block-based motion. In contrast to scenes with arbitrary motion, the human eye is able to track detail in case of global motion. Thus global motion compensation helps to improve the picture quality in the most critical scenes.

**Quarter-pel Motion Compensation:** The main target of quarter-pel motion compensation is to enhance the resolution of the motion compensation scheme with only small syntactical and computational overhead, leading to more accurate motion description and less prediction error to be coded. Quarter-pel motion compensation will only be applied to the luminance pels, chrominance pels are compensated in half pel accuracy.

As pointed out, some tools are similar to those developed in H.263. As in H.263, the MPEG-4 standard describes overlapped motion compensation. However, this tool is not included in any MPEG-4 profile due to its computational complexity for large picture sizes and due to its limited improvements for high quality video, i.e. there is no MPEG-4 compliant decoder that needs to implement overlapped block motion compensation.

**Error Resilience Tools:** Besides the tools developed to enhance the coding efficiency, a set of tools are also defined in MPEG-4 for enhancing the resilience of the compressed bit streams to transmission errors. These are described in Sec. 14.7.2.

#### 13.6.4 Object-based Video Coding

In order to enable object-based functionalities for coded video, MPEG-4 allows the transmission of shapes for video objects. While MPEG-4 does not standardize the method of defining or segmenting the video objects, it defines the decoding

algorithm and implicitly an encoding algorithm for describing the shape. Shape is described using alpha-maps that have the same resolution as the luminance signal. An alpha map is co-located with the luminance picture. MPEG-4 defines the alpha-map as two parts. The binary alpha-map defines pels that belong to the object. In case of grey-scale alpha maps, we have an additional alpha map that defines the transparency using 8 bits/pel. Alpha-maps extend a macroblock. The  $16 \times 16$  binary alpha-map of a MB is called Binary Alpha Block (BAB). In the following, we describe the individual tools that MPEG-4 uses for object-based video coding.

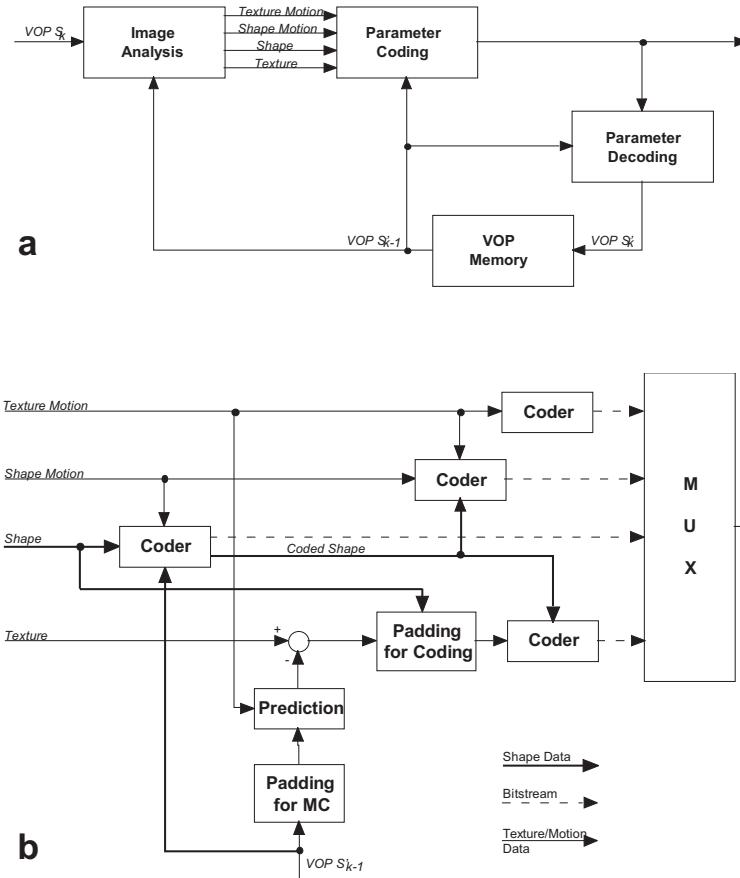
**Binary Shape:** A context-based arithmetic coder as described in Sec. 10.1.1 is used to code boundary blocks of an object. A boundary block contains pels of the object and of the background. It is co-located with a MB. For non-boundary blocks, the encoder just signals whether the MB is part of the object or not. A sequence of alpha-maps may be coded and transmitted without texture. Alternatively, MPEG-4 uses tools like padding and DCT or SA-DCT to code the texture that goes with the object. BABs are coded in intra-mode and inter-mode. Motion compensation may be used in inter mode. Shape motion vector coding uses the motion vectors associated with the texture coding as a predictor.

**Padding:** In order to code the texture of BABs using block-based DCT, the texture of the background may be set to any color. In intra mode, this background color has no effect on the decoded pictures and can be chosen by the encoder. However, for motion compensation, the motion vector of the current block may refer to a boundary block in the previous reference picture. Part of the background pels of the reference picture might be located in the area of the current object - hence the value of these background pictures influences the prediction loop. MPEG-4 uses padding as described in Sec. 10.2.1 to define the background pels used in prediction.

**Shape Adaptive DCT:** The encoder may choose to use SA-DCT for coding the texture of BABs (Sec. 10.2.2). However, padding of the motion compensated prediction image is still required.

**Greyscale Shape Coding:** MPEG-4 allows the transmission of arbitrary alpha-maps. Since the alpha-maps are defined with 8 bits, they are coded the same way as the luminance signal.

Fig. 13.32a shows the block diagram of the object-based MPEG-4 video coder. MPEG-4 uses two types of motion vectors: In Fig. 13.32, we name the conventional motion vectors used to compensate the motion of texture *Texture Motion*. Motion vectors describing the shift of the object shape are called *Shape Motion*. A shape motion vector may be associated to a BAB. Image analysis estimates texture and shape motion of the current VOP  $S_k$  with respect to the reference VOP  $S'_{k-1}$ . Parameter coding encodes the parameters predictively. The parameters get transmitted, decoded and the new reference VOP is stored in the VOP memory. The



**Figure 13.32.** Block diagram of the video encoder (a) and the parameter coder (b) for coding of arbitrarily shaped video objects.

increased complexity due to the coding of arbitrarily shaped video objects becomes evident in Fig. 13.32b. First, shape motion vectors and shape pels are encoded. The shape motion coder knows which motion vectors to code by analyzing the potentially lossily encoded shape parameters. For texture prediction, the reference VOP is padded as described above. The prediction error is padded using the original shape parameters to determine the area to be padded. Then, each MB is encoded using DCT.

### 13.6.5 Still Texture Coding

One of the functionalities supported by MPEG-4 is the mapping of static textures onto 2-D or 3-D surfaces. MPEG-4 visual supports this functionality by providing

a separate mode for encoding static texture information. It is envisioned that applications involving interactivity with texture mapped synthetic scenes require continuous scalability.

For coding static texture maps, Discrete Wavelet Transform (DWT) coding was selected for the flexibility it offered in spatial and quality scalability while maintaining good coding performance (Sec. 11.3.1). In DWT coding, a texture map image is first decomposed using a 2D separable decomposition using Daubechies (9,3)-tap biorthogonal filters. Next, the coefficients of the lowest band are quantized, coded predictively using implicit prediction (similar to that used in intra DCT coding) and arithmetic coding. This is followed by coding of coefficients of higher bands by use of multilevel quantization, zero-tree scanning and arithmetic coding. The resulting bitstream is flexibly arranged allowing a large number of layers of spatial and quality scalability to be easily derived.

This algorithm was extended to code arbitrarily shaped texture maps. In order to adapt a scan line of the shape to the coding with DWT, MPEG-4 uses leading and trailing boundary extensions that mirror the image signal (Sec. 11.3.1).

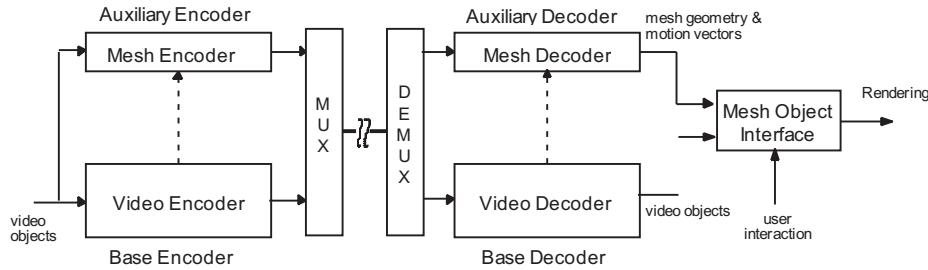
### 13.6.6 Mesh Animation

Mesh based representation of an object is useful for a number of functionalities such as animation, content manipulation, content overlay, merging natural and synthetic video and others [67].

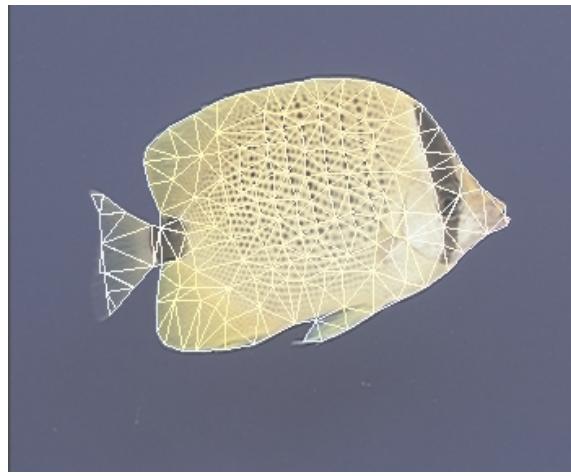
Fig. 13.33 shows a mesh coder and its integration with a texture coder. The mesh encoder generates a 2D mesh based representation of a natural or synthetic video object at its first appearance in the scene. The object is tessellated with triangular patches resulting in an initial 2D mesh (Fig. 13.34). The node points of this initial mesh are then animated in 2D as the VOP moves in the scene. Alternatively, the motion of the node point can be animated from another source. The 2D motion of a video object can thus be compactly represented by the motion vectors of the node points of the mesh. Motion compensation can be achieved by warping of texture map corresponding to patches by affine transform from one VOP to the next. Texture used for mapping on to object mesh models or facial wireframe models are either derived from video or still images. Whereas mesh analysis is not part of the standard, MPEG-4 defines how to encode 2D meshes and the motion of its node points. Furthermore, the mapping of a texture onto the mesh may be described using MPEG-4.

### 13.6.7 Face and Body Animation

An MPEG-4 terminal supporting face and body animation is expected to include a default face and body model. The systems part of MPEG-4 provides means to customize this face or body model by means of face and body definition parameters (FDP, BDP) or to replace it with one downloaded from the encoder. The definition of a scene including 3D geometry and of a face/body model can be sent to the receiver using BIFS [23]. Fig. 13.35 shows a scenegraph that a decoder built



**Figure 13.33.** Simplified architecture of an encoder/decoder supporting the 2D-mesh object. The video encoder provides the texture map for the mesh object (from [67]).



**Figure 13.34.** A content-based mesh designed for the "Bream" video object (from [67]).

according to the BIFS stream. The *Body* node defines the location of the Body. It's child BDP describes the look of the body using a skeleton with joints, surfaces and surface properties. The *bodyDefTable* node describes how the model is deformed as a function of the body animation parameters. The *Face* node is a descendent of the body node. It contains the face geometry as well as the geometry for defining the face deformation as a function of the face animation parameters (FAP). The visual part of MPEG-4 defines how to animate these models using FAPs and body animation parameters (BAP)[24].

Fig. 13.36 shows two phases of a left eye blink (plus the neutral phase) which have been generated using a simple animation architecture [67]. The dotted half circle in Fig. 13.36 shows the ideal motion of a vertex in the eyelid as it moves down

according to the amplitude of FAP 19. In this example, the faceDefTable for FAP 19 approximates the target trajectory with two linear segments on which the vertex actually moves as FAP 19 increases.

### Face Animation

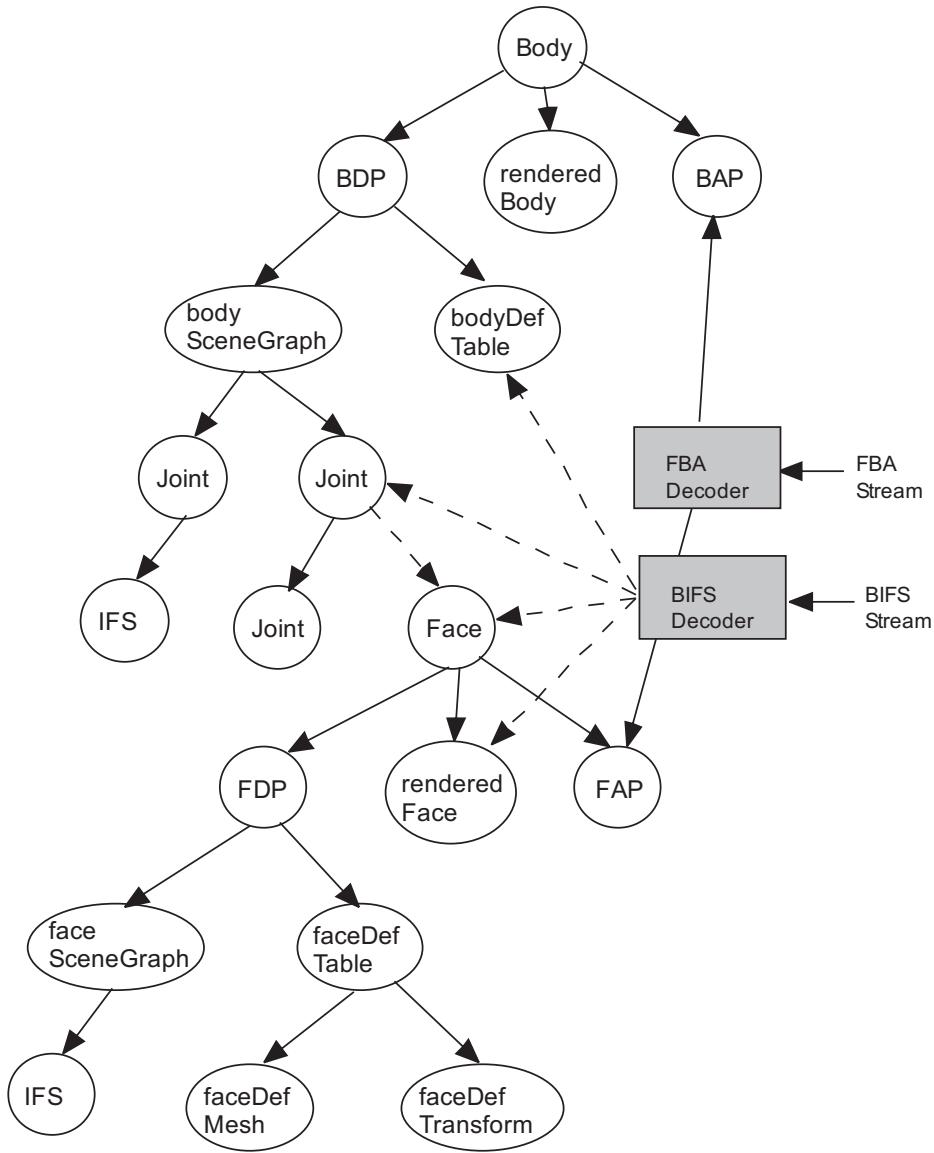
Three groups of facial animation parameters (FAP) are defined [67]. First, for low-level facial animation, a set of 66 FAPs is defined. These include head and eye rotations as well as motion of feature points on mouth, ear, nose and eyebrow deformation (Fig.10.20). Since these parameters are model independent their amplitudes are scaled according to the proportions of the actual animated model. Second, for high-level animation, a set of primary facial expressions like joy, sadness, surprise and disgust are defined. Third, for speech animation, 14 visemes define mouth shapes that correspond to phonemes. Visemes are transmitted to the decoder or are derived from the phonemes of the Text-to-Speech synthesizer of the terminal.

The FAPs are linearly quantized and entropy coded using arithmetic coding. Alternatively, a time sequence of 16 FAPs can also be DCT coded. Due to efficient coding, it takes only about 2 kbit/s to achieve lively facial expressions.

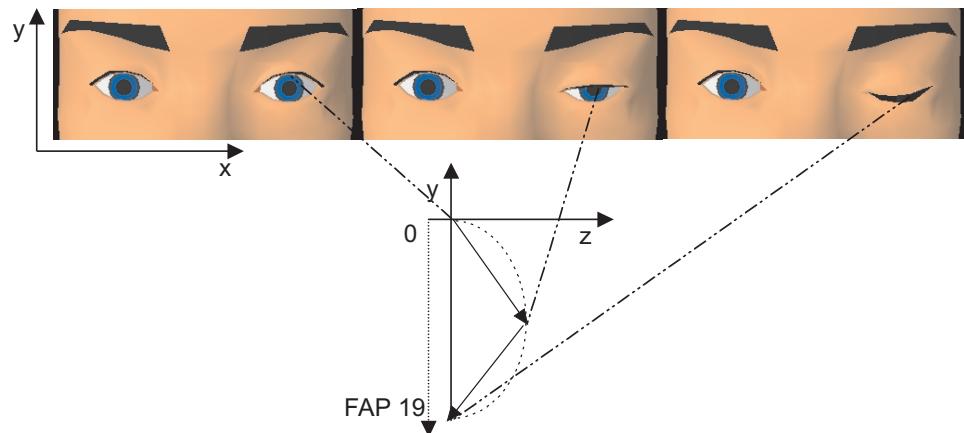
### Body Animation

BAPs manipulate independent degrees of freedom in the skeleton model of the body to produce animation of the body parts [4]. Similar to the face, the remote manipulation of a body model in a terminal with BAPs can accomplish lifelike visual scenes of the body in real-time without sending pictorial and video details of the body every frame. The BAPs will produce reasonably similar high level results in terms of body posture and animation on different body models, also without the need to transmit a model to the decoder. There are a total of 186 predefined BAPs in the BAP set, with an additional set of 110 user-defined extension BAPs. Each predefined BAP corresponds to a degree of freedom in a joint connecting two body parts. These joints include toe, ankle, knee, hip, spine, shoulder, clavicle, elbow, wrist, and the hand fingers. Extension BAPs are provided to animate additional features than the standard ones in connection with body deformation tables [1], e.g. for cloth animation or body parts that are not part of the human skeleton.

The BAPs are categorized into groups with respect to their effect on the body posture. Using this grouping scheme has a number of advantages. First, it allows us to adjust the complexity of the animation by choosing a subset of BAPs. For example, the total number of BAPs in the spine is 72, but significantly simpler models can be used by choosing only a predefined subset. Secondly, assuming that not all the motions contain all the BAPs, only the active BAPs can be transmitted to decrease required bit rate significantly. BAPs are coded similar to FAPs using arithmetic coding.



**Figure 13.35.** The scenegraph describing a human body is transmitted in a BIFS stream. The nodes *Body* and *Face* are animated using the FAP's and BAP's of the FBA stream. The *BDP* and *FDP* nodes and their children describe the virtual human (from [4]).

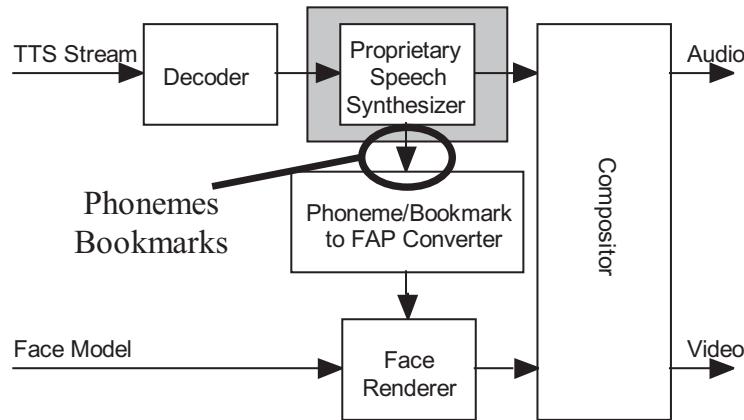


**Figure 13.36.** Neutral state of the left eye (left) and two deformed animation phases for the eye blink (FAP 19). The FAP definition defines the motion of the eyelid in negative y-direction; the faceDefTable defines the motion in one of the vertices of the eyelid in x and z direction.

### Integration of Speech Synthesis

MPEG-4 acknowledges the importance of TTS for multimedia applications by providing a text-to-speech synthesizer interface (TTSI) to a proprietary TTS. A TTS stream contains text in ASCII and optional prosody in binary form. The decoder decodes the text and prosody information according to the interface defined for the TTS synthesizer. The synthesizer creates speech samples that are handed to the compositor. The compositor presents audio and if required video to the user.

Fig. 13.37 shows the architecture for speech driven face animation that allows synchronized presentation of synthetic speech and talking heads. A second output interface of the TTS sends the phonemes of the synthesized speech as well as start time and duration information for each phoneme to a Phoneme/Bookmark-to-FAP-Converter. The converter translates the phonemes and timing information into FAPs that the face renderer uses in order to animate the face model. In addition to the phonemes, the synthesizer identifies bookmarks in the text that convey non-speech related FAPs like joy to the face renderer. The timing information of the bookmarks is derived from their position in the synthesized speech. Since the facial animation is driven completely from the text input to the TTS, transmitting an FAP stream to the decoder is optional. Furthermore, synchronization is achieved since the talking head is driven by the asynchronous proprietary TTS synthesizer.



Editing Instructions: Replace 'Proprietary Speech Synthesizer' with 'Text to Speech Synthesizer'; Add 'FAPs' to arrow at output of the block 'Phoneme/Bookmark ...'.

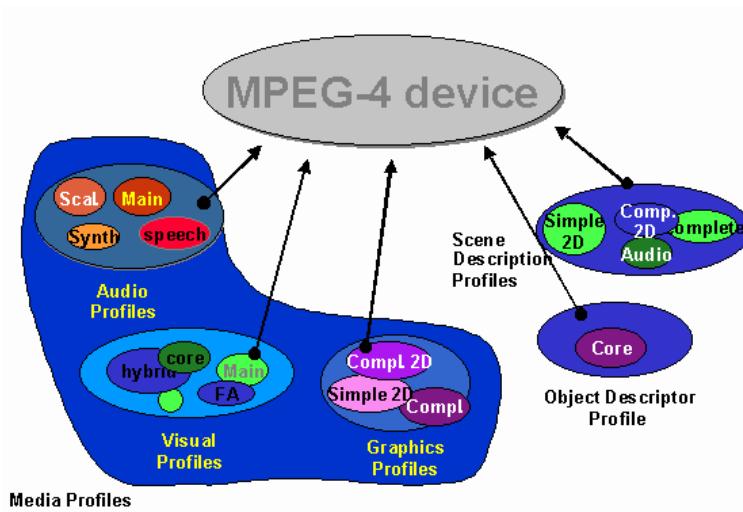
**Figure 13.37.** MPEG-4 architecture for face animation allowing synchronization of facial expressions and speech generated by a proprietary text-to-speech synthesizer.

### 13.6.8 Profiles

MPEG-4 developed an elaborate structure of profiles. As indicated in Fig. 13.38, a MPEG-4 terminal has to implement several profiles. An object descriptor profile is required to enable the transport of MPEG-4 streams and identify these streams in the terminal. A scene description profile provides the tools, to compose the audio, video or graphics objects into a scene. A 2D scene description profile enables just the placement of 2D video objects, higher profiles provide more functionality. A media profile needs to be implemented in order to present actual content on the terminal. MPEG-4 supports audio, video and graphics as media. Several video profiles are defined. Here, we list only a subset of them and mention their main functionalities.

**Simple Profile:** The Simple profile was created with low complexity applications in mind. The first usage is mobile use of (audio)visual services, and the second is putting very low complexity video on the Internet. It supports up to four objects in the scene with, at the lowest level, a maximum total size of a QCIF picture. There are 3 levels for the Simple Profile with bitrates from 64 to 384 kbit/s. It provides the following tools: I-, P-VOPs, AC/DC prediction, 4 motion vectors, unrestricted motion vectors, slice resynchronization, data partitioning and reversible VLC. This profile is able to decode H.263 video streams that do not use any of the optional annexes of H.263.

**Simple Scalable Profile:** This profile adds support for B-frames, temporal scalability and spatial scalability to the Simple profile. This profile is useful for



**Figure 13.38.** An MPEG-4 terminal has to implement at least one profile of Object descriptor, scene description, and media profiles. Not all profiles within a group are listed (from [53]).

applications which provide services at more than one level of quality due to bit-rate or decoder resource limitations, such as Internet use and software decoding.

**Advanced Real-Time Simple (ARTS) Profile:** This profile extends the capabilities of the Simple profile and provides more sophisticated error protection of rectangular video objects using a back-channel, which signals transmission errors from the decoder to the encoder such that the encoder can transmit video information in intra mode for the affected parts of the newly coded images. It is suitable for real-time coding applications such as videophone, tele-conferencing and remote observation.

**Core Profile:** In addition to the tools of the Simple profile, it enables scalable still textures, B-frames, binary shape coding and temporal scalability for rectangular as well as arbitrarily shaped objects. It is useful for higher quality interactive services, combining good quality with limited complexity and supporting arbitrarily shaped objects. Also mobile broadcast services could be supported by this profile. The maximum bitrate is 384 kbit/s at Level 1 and 2 Mbit/s at Level 2.

**Core Scalable Visual Profile:** This adds to the Core profile object based SNR as well as spatial/temporal scalability.

**Main Profile:** The Main profile adds support for interlaced video, grayscale alpha maps, and sprites. The Main profile was created with broadcast services in mind, addressing progressive as well as interlaced material. It combines the highest quality with the versatility of arbitrarily shaped object using grayscale coding. The highest level accepts up to 32 objects for a maximum total bitrate of 38 Mbit/s.

**Advanced Coding Efficiency (ACE):** This profile targets transmission of entertainment videos at bitrates less than 1 MBit/s. However, in terms of specification, it adds to the Main Profile by extending the range of bitrates and adding the tools quarter pel motion compensation, global motion compensation and shape adaptive DCT.

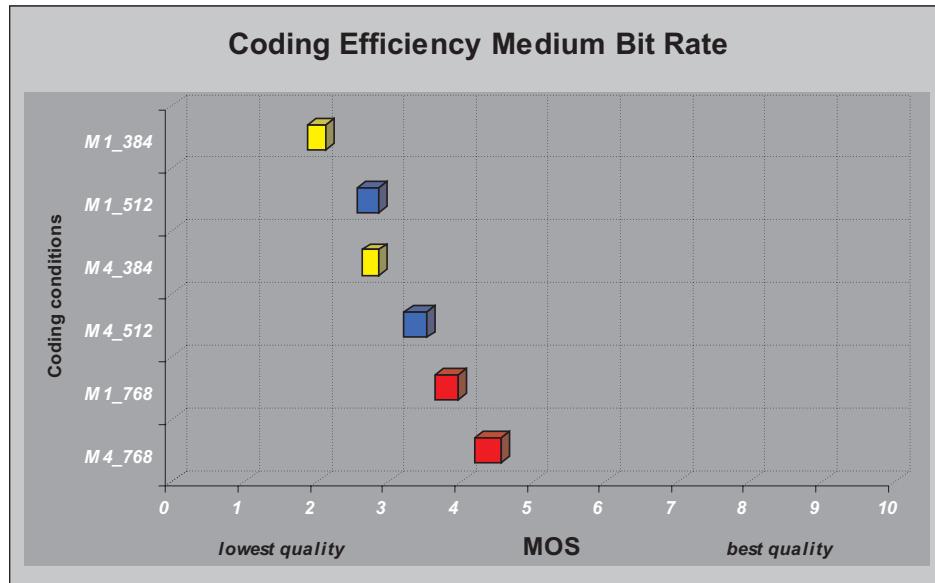
More profiles are defined for face, body and mesh animation. The definition of a Studio profile is in progress supporting bit rates up-to 600 Mb/sec for HDTV and arbitrarily shaped video objects in 4:0:0, 4:2:2 and 4:4:4 formats. At the time of this writing, it is still too early to know what profiles will eventually be implemented in products. First generation prototypes implement only the Simple profile and they target applications in the area of mobile video communications.

### 13.6.9 Evaluation of Subjective Video Quality

MPEG-4 introduces new functionalities like object-based coding and claims to improve coding efficiency. These claims were verified by means of subjective tests. Fig. 13.39 shows results of subjective coding efficiency tests comparing MPEG-4 video and MPEG-1 video at bitrates between 384 kbit/s and 768 kbit/s indicating that MPEG-4 outperforms MPEG-1 significantly in these bitrates. MPEG-4 was coded using the tools of the main profile (Sec. 13.6.8). In Fig. 13.40, we see the improvements in coding efficiency due to the additional tools of the Advanced Coding Efficiency (ACE) profile (Sec. 13.6.8). The quality provided by the ACE Profile at 768 Kbps equals the quality provided by Main Profile at 1024 Kbps. This makes the ACE profile very attractive for delivering movies over cable modem or digital subscriber lines (DSL) to the home. Further subjective tests showed that the object-based functionality of MPEG-4 does not decrease the subjective quality of the coded video object when compared to coding the video object using frame-based video, i.e. the bits spent on shape coding are compensated by saving bits for not coding pels outside of the video object. Hence, the advanced tools of MPEG-4 enable content-based video representation without increasing the bitrate for video coding.

## 13.7 Video Bitstream Syntax

As mentioned earlier, video coding standards define the syntax and semantics of the video bitstream, instead of the actual encoding scheme. They also specify how the bitstream has to be parsed and decoded to produce the decompressed video signal.

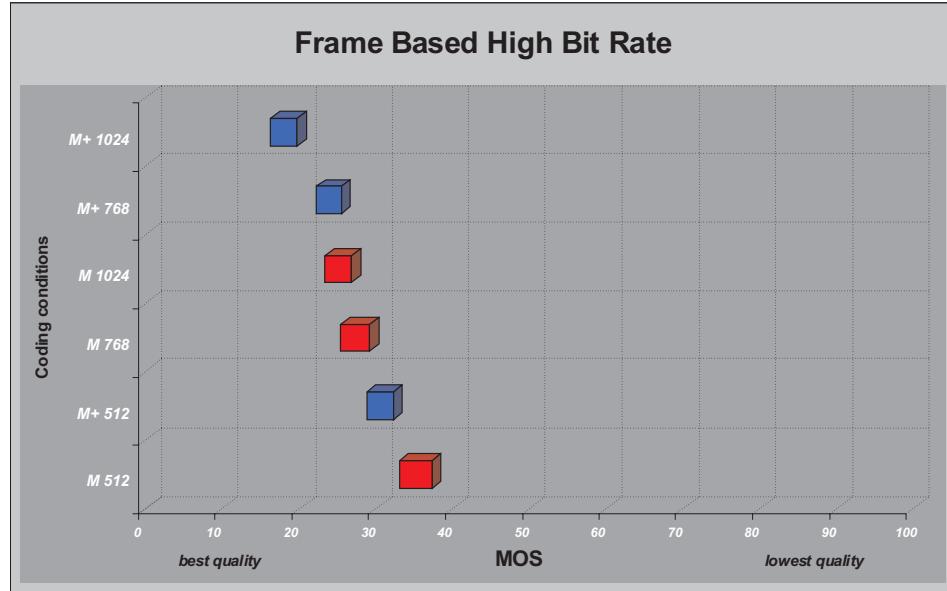


**Figure 13.39.** Subjective quality of MPEG-4 versus MPEG-1. M4\_\* is an MPEG-4 coder operating at the rate of \* kbit/s, M1\_\* is an MPEG-1 encoder operating at the given rate [27].

In order to support different applications, the syntax has to be flexible. This is achieved by having a hierarchy of different layers that each start with a *Header*. Each layer performs a different logical function (Tab. 13.6). Most headers can be uniquely identified in the bitstream because they begin with a *Start Code* that is long sequence of zeros (23 for MPEG-2) followed by a '1' and a start code identifier. Fig. 13.41 visualizes the hierarchy for MPEG-2.

**Sequence:** A video sequence commences with a sequence header and may contain additional sequence headers. It includes one or more groups of pictures and ends with an end-of-sequence code. The sequence header and its extensions contain basic parameters such as picture size, image aspect ratio, picture rate and other global parameters. The *Video Object Layer* header has the same functionality, however it carries additional information that an MPEG-4 decoder needs in order to compose several arbitrarily shape video sequences into one sequence to be displayed.

**Group of Pictures (GOP):** A GOP is a header followed by a series of one or more pictures intended to allow random access into the sequence, fast search and editing. Therefore, the first picture in a GOP is an intra-coded picture (I-picture). This is followed by an arrangement of forward-predictive coded pic-



**Figure 13.40.** Subjective quality of MPEG-4 ACE versus MPEG-4 Main profile.  $M_-*$  is an MPEG-4 coder according to the Main profile operating at the rate  $*$  kbit/s,  $M_+*$  is an MPEG-4 encoder according to the ACE profile operating at the given rate [26].

tures (P-pictures) and optional bidirectionally predicted pictures (B-pictures). This GOP header also contains a time code for synchronization and editing. A GOP is the base unit for editing and random access since it is coded independent of previous and consecutive GOPs. In MPEG-4, the function of GOP is achieved by a *Group of Video Object Planes (GVOP)*. Since H.261 and H.263 were designed mainly for interactive applications, they do not use the concept of GOP. However, the encoder may choose at any time to send an I-picture thus enabling random access and simple editing.

Picture: A picture is the primary coding unit of a video sequence. A picture consists of three rectangular matrices representing luminance (Y) and two chrominance (Cb and Cr) values. The picture header indicates the picture type (I, P, B), picture structure (field/frame) and perhaps other parameters like motion vector ranges. A *VOP* is the primary coding unit in MPEG-4. It has the size of the bounding box of the video object.

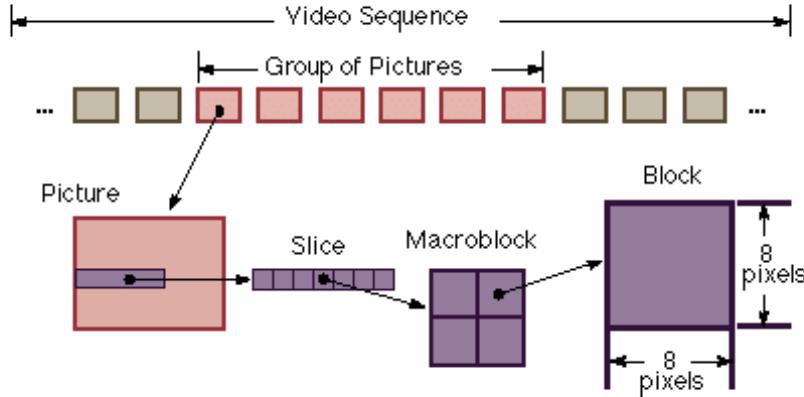
Each standard divides a picture into groups of MBs. Whereas H.261 and H.263 use a fixed arrangement of MBs, MPEG-1 and MPEG-2 allow for a flexible arrangement. MPEG-4 arranges a variable number of MBs into one group.

**Table 13.6.** Syntax hierarchy as used in different video coding standards. Each layer starts with a header. An SC in a syntax layer indicates that the header of that layer starts with a Start Code. VOP = Video Object Plane, GOB = Group of Blocks (adapted from [13]).

Syntax Layer	Functionality	Standard
Sequence (SC)	Definition of entire video sequence	H.261/3, MPEG-1/2
Video Object Layer (SC)	Definition of entire video object	MPEG-4
Group of Pictures (SC)	Enables random access in video stream	MPEG-1/2; MPEG-4
Group of VOP (SC)	Enables random access in video stream	MPEG-4
Picture (SC)	Primary coding unit	H.261/3, MPEG-1/2
VOP (SC)	Primary coding unit	MPEG-4
GOB (SC)	Resynchronization, refresh, and error recovery in a picture	H.261/3
Slice (SC)	Resynchronization, refresh, and error recovery in a picture	MPEG-1/2
Video Packet (SC)	Resynchronization error recovery in a picture	MPEG-4
MB	Motion compensation and shape coding unit	H.261/3, MPEG-1/2/4
Block	Transform and compensation unit	H.261/3, MPEG-1/2/4

GOB: H.261 and H.263 divide the image into GOBs of 3 lines of MBs with 11 MBs in one GOB line. The GOB headers define the position of the GOB within the picture. For each GOB, a new quantizer stepsize may be defined. GOBs are important in the handling of errors. If the bitstream contains an error, the decoder can skip to the start of the next GOB thus limiting the extent of biterrors to within one GOB of the current frame. However, error propagation may occur when predicting the following frame.

Slice: MPEG-1, MPEG-2 and H.263 Annex K extend the concept of GOBs to a variable configuration. A slice groups several consecutive MBs into one unit. Slices may vary in size. In MPEG-1, a slice may be as big as one picture. In MPEG-2 however, at least each row of MBs in a picture starts with a new slice. Having more slices in the bitstream allows better error concealment, but uses bits that could otherwise be used to improve picture quality.



**Figure 13.41.** Visualization of the hierarchical structure of a MPEG-2 bit stream from a video sequence layer down to the block level shown for the luminance component. Each layer has also two chrominance components associated with it.

**Video Packet Header:** The video packet approach adopted by MPEG-4 is based on providing periodic resynchronization markers throughout the bitstream. In other words, the length of the video packets are not based on the number of MBs, but instead on the number of bits contained in that packet. If the number of bits contained in the current video packet exceeds a threshold as defined by the encoder, then a new video packet is created at the start of the next MB. This way, a transmission error causes less damage to regions with higher activity than to regions that are stationary when compared to the more rigid slice and GOB structures. The video packet header carries position information and repeats information of the picture header that is necessary to decode the video packet.

**Macroblock:** A MB is a 16x 16 pixel block in a picture. Using the 4:2:0 format, each chrominance component has one-half the vertical and horizontal resolution of the luminance component. Therefore a MB consists of four Y, one Cr, and one Cb block. Its header carries relative position information, quantizer scale information, MTYPE information (I, P, B), and a CBP indicating which and how the 6 blocks of a MB are coded. As with other headers, other parameters may or may not be present in the header depending on MTYPE. Since MPEG-4 also needs to code the shape of video objects, it extends the MB by a *binary alpha block* (BAB) that defines for each pel in the MB whether it belongs to the VO. In the case of grey-scale alpha maps, the MB also contains four blocks for the coded alpha maps.

Block: A block is the smallest coding unit in standardized video coding algorithms.

It consists of 8x8 pixels and can be one of three types: Y, Cr, or Cb. The pixels of a block are represented by their DCT coefficients coded using a Huffman code that codes the number of '0's before the next non-zero coefficient and the amplitude of this coefficient.

The different headers in the bitstream allow a decoder to recover from errors in the bitstream and start decoding as soon as it receives a start code. The behaviour of a decoder when receiving an erroneous bitstream is not defined in the standard. Different decoders may behave very differently, some decoders crash and require rebooting of the terminal, others recover within a picture, yet others wait until the next I-frame before they start decoding again.

## 13.8 Multimedia Content Description Using MPEG-7

With the ubiquitous use of video, the problem of indexing and searching for video sequences becomes an important capability. MPEG-7 is an on-going standardization effort for content description of audio-visual (AV) documents [32, 63]. In principle, MPEG-1, -2, and -4 are designed to represent the information itself, while MPEG-7 is meant to represent information about the information. Looking from another perspective: *MPEG-1/2/4 make content available, while MPEG-7 allows you to find the content you need* [63]. MPEG-7 is intended to provide complementary functionality to other MPEG standards: representing information about the content, not the content itself (“the bits about the bits”). While MPEG-4 enables to attach limited textual meta information to its streams, the MPEG-7 standard will provide a full set of indexing and search capabilities such that we can search for a movie not only with text keys but also with keys like color histograms, motion trajectory, etc. MPEG-7 will be an international standard by the end of 2001.

In this section, we first provide an overview of the elements standardized by MPEG-7. We then describe multimedia description schemes, with focus on content description. We explain how MPEG-7 decomposes an AV document to arrive at both structural and semantic descriptions. Finally we describe visual descriptors used in these descriptions. The descriptors and description schemes presented below assume that semantically meaningful regions and objects can be segmented and that the shape and motion parameters, and even semantic labels of these regions/objects can be accurately extracted. We would like to note that generation of such information remains an unsolved problem and may need manual assistance. The MPEG-7 standard only defines the syntax that can be used to specify these information, but not algorithms that can be used to extract them.

### 13.8.1 Overview

The main elements of the MPEG-7 standard are [32]:

- Descriptors (D): The MPEG-7 descriptors are designed to represent features,

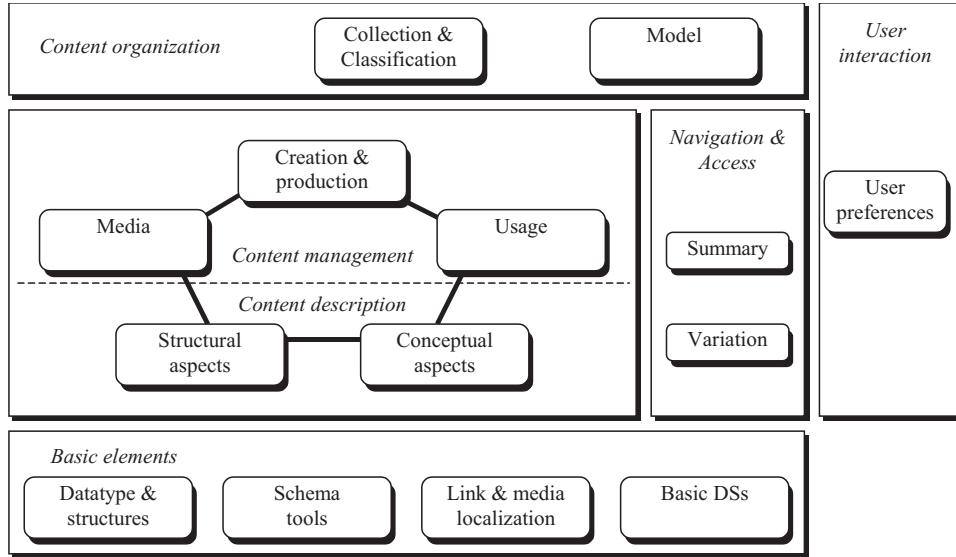
including low-level audio-visual features; high-level features of semantic objects, events and abstract concepts; information about the storage media, and so on. Descriptors define the syntax and the semantics of each feature representation.

- Description Schemes (DS): The MPEG-7 DSs expand on the MPEG-7 descriptors by combining individual descriptors as well as other DSs within more complex structures and by defining the relationships among the constituent descriptors and DSs.
- A Description Definition Language (DDL): It is a language that allows the creation of new DSs and, possibly, new descriptors. It also allows the extension and modification of existing DSs. The XML Schema Language has been selected to provide the basis for the DDL.
- System tools: These are tools that are needed to prepare MPEG-7 descriptions for efficient transport and storage, and to allow synchronization between content and descriptions, and to manage and protect intellectual property.

### 13.8.2 Multimedia Description Schemes

In MPEG-7, the DSs are categorized as pertaining specifically to the audio or visual domain, or pertaining generically to the description of multimedia. The multimedia DSs are grouped into the following categories according to their functionality (Fig. 13.42):

- Basic elements: These deal with basic datatypes, mathematical structures, schema tools, linking and media localization tools as well as basic DSs, which are elementary components of more complex DSs;
- Content description: These DSs describe the structural and conceptual aspects of an AV document;
- Content management: These tools specify information about the storage media, the creation and the usage of an AV document;
- Content organization: These tools address the organization of the content by classification, by the definition of collections of AV documents, and by modeling;
- Navigation and access: These include summaries for browsing and variations of the same AV content for adaptation to capabilities of the client terminals, network conditions or user preferences;
- User interaction: These DSs specify user preferences pertaining to the consumption of the multimedia material.



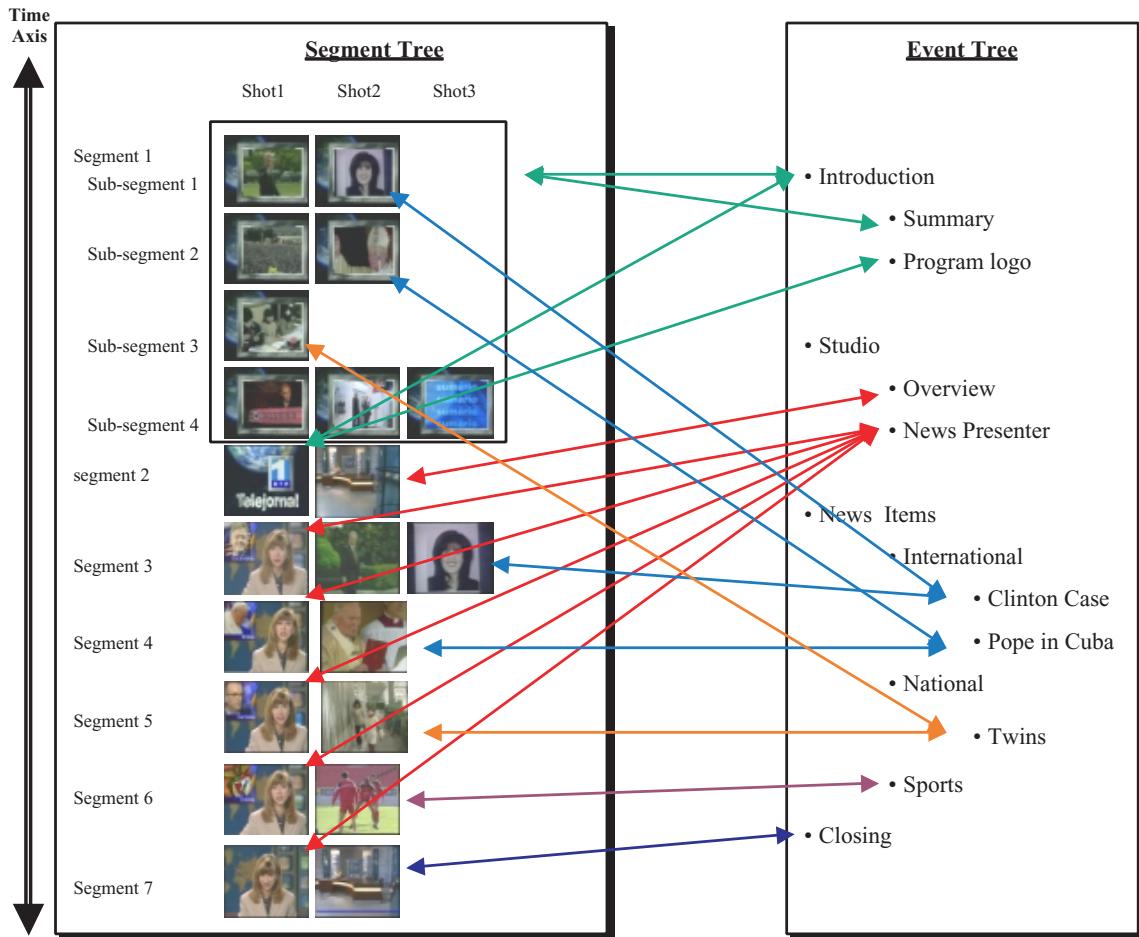
**Figure 13.42.** Overview of MPEG-7 Multimedia Description Schemes. [Courtesy of MPEG-7].

### Content Description

In the following, we briefly describe the DSs for content description. More detailed information can be found in [29]. The DSs developed for content description fall in two categories: those describing the structural aspects of an AV document, and those describing the conceptual aspects.

**Structural Aspects:** These DSs describe the syntactic structure of an AV document in terms of segments and regions. An AV document (e.g., a video program with audio tracks) is divided into a hierarchy of segments, known as a *segment-tree*. For example, the entire document is segmented into several story units, each story unit is then divided into different scenes, and finally each scene is split into many camera shots. A segment at each level of the tree can be further divided into video segment and audio segment, corresponding to the video frames and the audio waveform, respectively. In addition to using a video segment that contains a set of complete video frames (may not be contiguous in time), a still or moving region can also be extracted. A region can be recursively divided into sub-regions, to form a *region-tree*. The concept of the segment tree is illustrated on the left side of Fig. 13.43.

**Conceptual Aspects:** These DSs describe the semantic content of an AV document in terms of events, objects, and other abstract notions. The semantic DS describes events and objects that occur in a document, and attach corresponding “semantic



**Figure 13.43.** Description of an AV document (a news program in this case) based on segment tree and event tree. The segment tree is like the table of contents in the beginning of a book, whereas the event tree is like the index at the end of the book. [Courtesy of MPEG-7].

“labels” to them. For example, the event type could be a news broadcast, a sports game, etc. The object type could be a person, a car, etc. As with the structure description, MPEG-7 also uses hierarchical decomposition to describe the semantic content of an AV document. An event can be further broken up into many sub-events to form an event-tree (right side of Fig. 13.43). Similarly, an object-tree can be formed. An event-object relation graph describes the relation between events and objects.

**Relation between Structure and Semantic DSs:** An event is usually associated with a segment, and an object with a region. Each event or object may occur multiple times in a document, and their actual locations (which segment or region) are described by a set of links, as shown in Fig. 13.43. In this sense, the syntactic structure, represented by the segment-tree and the region-tree, is like the *table of contents* in the beginning of a book, whereas the semantic structure, i.e., the event-tree and the object-tree, is like the *index* at the end of the book.

### 13.8.3 Visual Descriptors and Description Schemes

For each segment or region at any level of the segment- or region-tree, a set of audio and visual descriptors and DSs are used to characterize this segment or region. In this section, we briefly describe the visual descriptors and DSs that have been developed to describe the color, texture, shape, motion, and location of a video segment or object. More complete descriptions can be found in [28, 33].

#### Color

These descriptors describe the color distributions in a video segment, a moving region or a still region.

- Color space: Five color spaces are defined, RGB, YCrCb, HSV, HMMD, or monochrome. Alternatively, one can specify an arbitrary linear transformation matrix from the RGB coordinate.
- Color quantization: This descriptor is used to specify the quantization parameters, including the number of quantization levels and starting values for each color component. Only uniform quantization is considered.
- Dominant color: This descriptor specifies the dominant colors in the underlying segment, including the number of dominant colors, a value indicating the spatial coherence of the dominant color (i.e., whether the dominant color is scattered over the segment or form a cluster), and for each dominant color, the percentage of pixels taking that color, the color value and its variance.
- Color histogram: The color histogram is defined in the HSV space. Instead of the color histogram itself, the Haar transform is applied to the histogram and the Haar coefficients are specified using variable precision depending on the available bit rate. Several types of histograms can be specified. The common color histogram, which includes the percentage of each quantized color among all pixels in a segment or region, is called ScalableColor. The GoF/GoP Color refers to the average, median, or intersection (minimum percentage for each color) of conventional histograms over a group of frames or pictures.
- Color layout: This descriptor is used to describe in a coarse level the color pattern of an image. An image is reduced to  $8 \times 8$  blocks with each block

represented by its dominant color. Each color component (Y/Cb/Cr) in the reduced image is then transformed using DCT, and first few coefficients are specified.

- Color structure: This descriptor is intended to capture the spatial coherence of pixels with the same color. The counter for a color is incremented as long as there is at least one pixel with this color in a small neighborhood around each pixel, called the structuring element. Unlike the color histogram, this descriptor can distinguish between two images in which a given color is present in identical amounts but where the structure of the groups of pixels having that color is different in the two images.

### Texture

This category is used to describe the texture pattern of an image.

- Homogeneous texture: This is used to specify the energy distribution in different orientations and frequency bands (scales). The first two components are the mean value and the standard deviation of the pixel intensities. The following 30 components are obtained through a Gabor transform with 6 orientation zones and 5 scale bands.
- Texture browsing: This descriptor specifies the texture appearances in terms of regularity, coarseness and directionality, which are in-line with the type of descriptions that a human may use in browsing/retrieving a texture pattern. In addition to regularity, up to two dominant directions and the coarseness along each direction can be specified.
- Edge histogram: This descriptor is used to describe the edge orientation distribution in an image. Three types of edge histograms can be specified, each with five entries, describing the percentages of directional edges in four possible orientations and non-directional edges. The global edge histogram is accumulated over every pixel in an image; the local histogram consists of 16 sub-histograms, one for each block in an image divided into  $4 \times 4$  blocks; the semi-global histogram consists of 13 sub-histograms, one for each sub-region in an image.

### Shape

These descriptors are used to describe the spatial geometry of still and moving regions.

- Contour-based descriptor: This descriptor is applicable to a 2D region with a closed boundary. MPEG-7 has chosen to use the peaks in the curvature scale space (CSS) representation [55] to describe a boundary, which has been found to reflect human perception of shapes, i.e., similar shapes have similar parameters in this representation. The CSS representation of a boundary

is obtained by recursively blurring the original boundary using a smoothing filter, computing the curvature along each filtered curve, and finally determining zero-crossing locations of the curvature after successive blurring. The descriptor specifies the number of curvature peaks in the CSS, the global eccentricity and circularity of the boundary, the eccentricity and circularity of the prototype curve, which is the curve leading to the highest peak in the CSS, the prototype filter, and the positions of the remaining peaks.

- Region-based shape descriptor: The region-based shape descriptor makes use of all pixels constituting the shape, and thus can describe any shape, i.e. not only a simple shape with a single connected region but also a complex shape that consists of several disjoint regions. Specifically, the original shape represented by an alpha map is projected onto Angular Radial Transform (ART) basis functions, and the descriptor includes 35 normalized and quantized magnitudes of the ART coefficients.
- Shape 3D: This descriptor provides an intrinsic description of 3D mesh models. It exploits some local attributes of the 3D surface. To derive this descriptor, the so-called shape index is calculated at every point on the mesh surface, which depends on the principle curvature at the point. The descriptor specifies the shape spectrum, which is the histogram of the shape indices calculated over the entire mesh. Each entry in the histogram essentially specifies the relative area of all the 3D mesh surface regions with a shape index lying in a particular interval. In addition, the descriptor includes the relative area of planar surface regions of the mesh, for which the shape index is not defined, and the relative area of all the singular polygonal components, which are regions for which reliable estimation of the shape index is not possible.

### Motion

These descriptors describe the motion characteristics of a video segment or a moving region as well as global camera motion.

- Camera motion: Seven possible camera motions are considered: panning, tracking (horizontal translation), tilting, booming (vertical translation), zooming, dollying (translation along the optical axis), and rolling (rotation around the optical axis) (cf. Fig. 5.4). For each motion, two moving directions are possible. For each motion type and direction, the presence (i.e. duration), speed and the amount of motion are specified. The last term measures the area that is covered or uncovered due to a particular motion.
- Motion trajectory: This descriptor is used to specify the trajectory of a non-rigid moving object, in terms of the 2D or 3D coordinates of certain key points at selected sampling times. For each key point, the trajectory between two adjacent sampling times is interpolated by a specified interpolation function (either linear or parabolic).

- Parametric object motion: This descriptor is used to specify the 2D motion of a rigid moving object. Five types of motion models are included: translation, rotation/scaling, affine, planar perspective, and parabolic. The planar perspective and parabolic motions refer to the projective mapping defined in Eq. 5.5.14, and the biquadratic mapping defined in Eq. 5.5.19, respectively. In addition to the model type and model parameters, the coordinate origin and time duration need to be specified.
- Motion activity: This descriptor is used to describe the intensity and spread of activity over a video segment (typically at the shot level). Four attributes can be specified: i) intensity of activity, measured by the standard deviation of the motion vector magnitudes; ii) direction of activity, which specifies the dominant or average direction of all motion vectors; iii) spatial distribution of activity, derived from the run-lengths of blocks with motion magnitudes lower than the average magnitude, iv) spatial localization of activity, and v) temporal distribution of activity, described by a histogram of quantized activity levels over individual frames in the shot.

### Localization

These descriptors and DSs are used to describe the location of a still or moving region.

- Region locator: This descriptor specifies the location of a region by a brief and scalable representation of a bounding box or polygon.
- Spatial-temporal locator: This DS describes a moving region. It decomposes the entire duration of the region into a few sub-segments, with each segment being specified by the shape of the region in the beginning of the segment, known as a reference region, and the motion between this region and the reference region of the next segment. For a non-rigid object, a FigureTrajectory DS is developed, which defines the reference region by a bounding rectangle, ellipse, or polygon, and specifies the motion between reference regions using the MotionTrajectory descriptor, which specifies the coordinates of selected key points over successive sampling times. For a rigid region, a Parameter-Trajectory DS is used, which uses the RegionLocator descriptor to specify a reference region, and the parametric object motion descriptor to describe the motion.

## 13.9 Summary

Video communications requires standardization in order to build reasonably-priced equipment that interoperates and caters to a large market. Personal video telephony was the first application that was targeted by a digital video compression standard. H.261 was published in 1990, 101 years after Jules Verne wrote down the idea of

a video telephone and 899 years earlier than he predicted [68]. The subsequent important video compression standards H.263, MPEG-1, MPEG-2, and MPEG-4 were established in 1993, 1995, 1995, and 1999, respectively.

Whereas the H.261 and H.263 standards describe only video compression, MPEG-1/2/4 standards describe also the representation of audio as well as a system that enables the joint transmission of audiovisual signals. H.261 is a block-based hybrid coder with integer-pel motion compensation. The main applications for H.261 is video coding for video conferencing over ISDN lines at rates between 64 kbit/s and 2 Mbit/s. H.263 extends H.261 and adds many features including half-pel motion compensation thus enabling video coding for transmission over analog telephone lines at rates below 56 kbit/s.

MPEG-1 is also derived from H.261. It added half-pel motion compensation, bidirectional prediction for B-pictures and other improvements in order to meet the requirements for coding video at rates around 1.2 Mbit/s for consumer video on CD-ROM at CIF resolution. MPEG-2 is the first standard that is able to code interlaced video at full TV and HDTV resolution. It extended MPEG-1 to include new prediction modes for interlaced video. Its main applications are TV broadcasting at rates around 4 Mbit/s and 15 Mbit/s for high-quality video. MPEG-4 video, based on MPEG-2 and H.263, is the latest video coding standard that introduces object-based functionalities describing video objects not only with motion and texture but also by their shape. Shape information is co-located with the luminance signal and coded using a context-based arithmetic coder.

MPEG-2 and MPEG-4 define profiles that require a decoder to implement a subset of the tools that the standard defines. This enables to build standard decoders that are to some extend tailored towards certain application areas.

Whereas MPEG-1/2/4 standards are developed to enable the exchange of audiovisual data, MPEG-7 aims to enable the searching and browsing of such data. MPEG-7 can be used independent of the other MPEG standards — an MPEG-7 description might even be attached to an analog movie. MPEG-7 descriptions could be used to improve the functionalities of previous MPEG standards, but will not replace MPEG-1, MPEG-2 or MPEG-4.

Since the computational power in terminals increases every year, standardization bodies try to improve on their standards. ITU currently works on the video coding standard H.26L that promises to outperform H.263 and MPEG-4 by more than 1 dB for the same bitrate or reducing the bitrate by more than 20% for the same picture quality when coding video at rates above 128 kbit/s.

## 13.10 Problems

**13.1** What kinds of compatibility do you know?

**13.2** What are the most compute intensive parts of an H.261 video encoder. What are the most compute intensive parts of a decoder?

- 13.3** What is a loop filter? Why is H.261 the only standard implementing it?
- 13.4** What are the tools that improve the coding efficiency of H.263 over H.261?
- 13.5** What is the main difference between MPEG-1 B-frames and H.263 PB-frames according to the Improved PB-frame mode?
- 13.6** What is the purpose of the standards H.323 and H.324?
- 13.7** Why does MPEG-2 have more than one scan mode?
- 13.8** What effect does the perceptual quantization of I frames have on the PSNR of a coded picture? How does perceptual quantization affect picture quality. What is a good guideline for choosing the coefficients of the weight matrix?
- 13.9** Explain the concept of profiles and levels in MPEG-2.
- 13.10** Which of the MPEG-2 profiles are used in commercial products? Why do the others exist?
- 13.11** What kind of scalability is supported by MPEG-2?
- 13.12** What is drift? When does it occur?
- 13.13** Discuss the error resilience tools that H.261, H.263, MPEG-1/2/4 provide? Why are MPEG-4 error resilience tools best suited for lossy transmission channels?
- 13.14** What are the differences between MPEG-1 Layer III audio coding and MPEG-2 NBC audio?
- 13.15** MPEG-4 allows to encode shape signals. In case of binary shape, how many blocks are associated with a macroblock? What is their size? What about greyscale shape coding?
- 13.16** Why does MPEG-4 video according to the ACE profile outperform MPEG-1 video?
- 13.17** What part of an MPEG-4 terminal is not standardized?
- 13.18** Why do video bitstreams contain start codes?
- 13.19** What is meta information?
- 13.20** Which standard uses a wavelet coder and for what purpose?
- 13.21** Why is the definition of FAP's as done in MPEG-4 important for content creation?

- 13.22** How is synchronization achieved between a speech synthesizer and a talking face?
- 13.23** What is the functionality and purpose of MPEG-4 mesh animation?
- 13.24** What is the difficulty with video indexing and retrieval? How can a standardized content description interface such as MPEG-7 simplify video retrieval?
- 13.25** How does the segment-tree in MPEG-7 describe the syntactic structure of a video sequence? How does the event-tree in MPEG-7 describe the semantic structure of a video sequence? What are their relations?
- 13.26** What are the visual descriptors developed by MPEG-7? Assuming these descriptors are attached to every video sequence in a large video database. Describe ways that you may use them to retrieve certain type of sequences.

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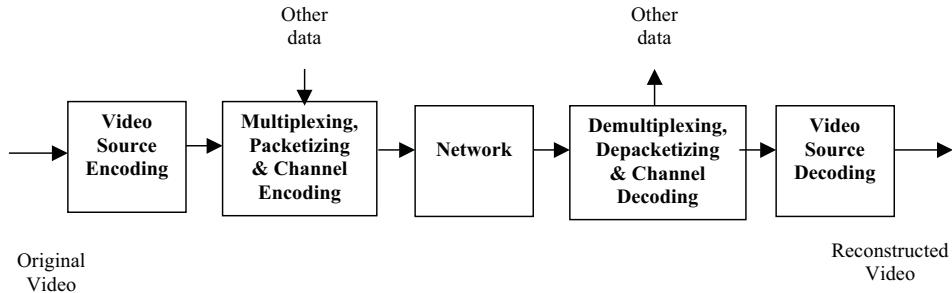
# ERROR CONTROL IN VIDEO COMMUNICATIONS

We have discussed various video coding techniques and standards in previous chapters. For effective video communications, reduction of raw video data rates is only one of the necessary steps. Another equally important task is how to handle errors and losses in a communication network. In contrast with data communications, which are not usually subject to strict delay constraints and can therefore be handled using network protocols that use retransmission to ensure error-free delivery, real-time video is delay sensitive and cannot easily make use of retransmission. The extensive use of predictive coding and variable length coding (VLC) in video coding renders compressed video especially vulnerable to transmission errors, and successful video communication in the presence of errors requires careful designs of the encoder, decoder, and other system layers.

In this chapter, we present approaches that have been developed for error control in video communications. We start by describing the necessity and the challenges in error control for video communications and providing an overview of various approaches that have been developed (Sec. 14.1). To help understand the issues in error control in video communications, we describe in Sec. 14.2 the QoS requirements of various video services and characteristics of different networks. Section 14.3 presents error control mechanisms at the transport level. Sections 14.4 and 14.5 review techniques for error-resilient encoding and error concealment. Finally, Sec. 14.6 describes techniques that rely on encoder and decoder interactions. Finally, Sec. 14.7 summarizes error resilience tools adopted by the H.263 and MPEG-4 standards.

### 14.1 Motivation and Overview of Approaches

A video communication system typically involves five steps, as shown in Fig. 14.1. The video is first compressed by a video encoder to reduce the data rate and the compressed bit stream is then segmented into fixed or variable length packets and multiplexed with other data types, such as audio. The packets might be sent directly

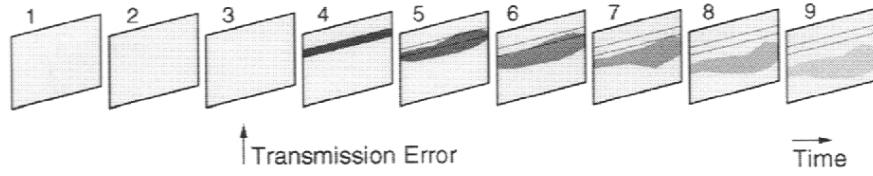


**Figure 14.1.** A typical video communication system.

over the network, if the network guarantees bit error free transmission. Otherwise, they usually undergo a channel encoding stage, typically using forward error correction (FEC), to protect them from transmission errors. At the receiver end, the received packets are FEC decoded and unpacked, and the resulting bitstream is then input to the video decoder to reconstruct the original video. In practice, many applications embed packetization and channel encoding in the source coder as an adaptation layer to the network.

Transmission errors can be roughly classified into two categories: random bit errors and erasure errors. *Random bit errors* are caused by the imperfections of physical channels which results in bit inversion, bit insertion and bit deletion. Depending on the coding methods and the affected information content, the impact of random bit errors can range from negligible to objectionable. When fixed length coding is used, a random bit error will only affect one codeword, and the caused damage is generally acceptable. But if VLC (e.g., Huffman coding) is used, random bit errors can desynchronize the coded information such that many following bits are undecodable until the next synchronization codeword appears. *Erasure errors*, on the other hand, can be caused by packet loss in packet networks such as the Internet, burst errors in storage media due to physical defects or system failures for a short time. Random bit errors in VLC coded streams can also cause effective erasure errors since a single bit error can lead to many following bits undecodable, hence useless. The effect of erasure errors (including those due to random bit errors) is much more destructive than random bit errors due to the loss or damage of a contiguous segment of bits.

Error control in video communications is very challenging for several reasons. First, compressed video streams are extremely vulnerable to transmission errors because of the use of predictive coding and VLC by the source coder. Due to the use of spatiotemporal prediction, a single erroneously recovered sample can lead to errors in the following samples in the same and following frames, as illustrated in Fig. 14.2. Likewise, as hinted before, with VLC, the effect of a bit error is equivalent



**Figure 14.2.** Illustration of spatiotemporal error propagation. From [26].

to that of an erasure error, causing damage over a large portion of a video frame. Typically, in a block-based hybrid coder (Sec. 9.3.1), each group of blocks (GOB) (typically a row of macroblocks) is led by a resynchronization marker, which helps the decoder to regain synchronization. If a transmission error is detected in the middle of a GOB, the entire GOB is discarded. The decoder resumes decoding in the next GOB. To illustrate the visual artifacts caused by transmission errors, Fig. 14.3 shows a typical example of reconstructed frames in the presence of packet loss. In this example, a damaged GOB is recovered using the so-called motion compensated temporal interpolation method (Sec. 14.5.1). It can conceal the error somewhat when the packet loss rate is not too high (see the case with 3% and 5% packet loss rate), but breaks down when too many packets are lost (see the 10% loss case). Note that the artifacts in the frame shown is due to not only packet losses in data for this frame, but also those in previous frames.

To make the compressed bit stream resilient to transmission errors, one must add redundancy into the stream, so that it is possible to detect and correct errors. Typically, this is done at the channel by using FEC, which operates over coded bit streams generated by a source coder. The classical Shannon information theory states that one can separately design the source and channel coders, to achieve error-free delivery of a compressed bit stream, as long as the source is represented by a rate below the channel capacity. Therefore, the source coder should compress a source as much as possible (to below the channel capacity) for a specified distortion, and then the channel coder can add redundancy through FEC to the compressed stream to enable the correction of transmission errors. However, such ideal error-free delivery can be achieved only with infinite delays in implementing FEC and are not acceptable in practice. Therefore, joint source and channel coding is often a more viable scheme, which allocates a total amount of redundancy between the source and channel coding. All the *error-resilient encoding* methods essentially work under this premise, and intentionally make the source coder less efficient than it can be, so that the erroneous or missing bits in a compressed stream will not have a disastrous effect in the reconstructed video quality. This is usually accomplished by carefully designing both the predictive coding loop and the variable length coder, to limit the extent of error propagation.

When an image sample or a block of samples are missing due to transmission



**Figure 14.3.** Effect of transmission errors to a compressed video stream using the H.263 standard for a selected frame. Upper left: no transmission errors were present and the picture quality is as high as the bitrate allows; Upper right: 3% packet loss; lower left: 5% packet loss; lower right: 10% packet loss. From [81, Fig. 2]. Courtesy of Stephan Wenger.

errors, the decoder can estimate them based on surrounding received samples, by making use of inherent correlation among spatially and temporally adjacent samples. Such techniques are known as *error concealment*. This is possible because real source coders do not completely eliminate the redundancy in a signal in the encoding process. Error concealment has, in contrast to error-resilient source coding, the advantage of not employing any additional bitrate,<sup>1</sup> but adds computational complexity at the decoder.

Finally, for the embedded redundancy in the source coder to be useful, and to facilitate error concealment in the decoder, the codec and the network transmission protocol must cooperate with each other. For example, if the bitstream is such that some bits are more important than others, then the important part should be assigned a more stringent set of quality of service (QoS) parameters for delivery over a network. To suppress error propagation, the network may also provide a feedback channel, so that the encoder knows which part of the reconstructed signal at the

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<sup>1</sup>To facilitate error concealment, a small amount of redundancy is usually inserted in the encoder, in the form of restricted prediction and data interleaving.

decoder is damaged, and does not use this part for prediction of future samples.

To summarize, error control mechanisms devised for video transport can be categorized into four groups: i) those exercised at the transport level, including channel coder, packetizer/multiplexer, and transport protocol; ii) those introduced at the source encoder, to make the bit-stream more resilient to potential errors; iii) those invoked at the decoder upon detection of errors, to conceal the effect of errors; and finally iv) those that require interactions between the source encoder and decoder, so that the sender can adapt its operations based on the loss conditions detected at the decoder. We will describe techniques in these four categories separately in Secs. 14.3 to 14.6.

## 14.2 Typical Video Applications and Communication Networks

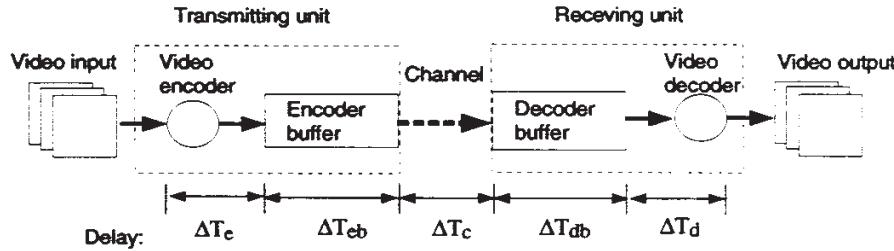
In this section, we describe different types of video applications and characteristics of practical networks. These are important factors to consider, because the necessity for error control and the effectiveness of a technique depends on the type of application as well as the underlying channel characteristics and network protocols.

### 14.2.1 Categorization of Video Applications

When considering error control in video transport, it is important to know whether the underlying application requires real-time delivery, and what is the maximally allowed average end-to-end delay (known as *latency*) and delay variation (known as *jitter*). The name “real-time” means that the compressed data are transferred at a speed that matches with the coded video source rate. For example, if the source video is coded at 10 Mbps, then the throughput<sup>2</sup> of the communication channel devoted for this source should be at least 10 Mbps. Note that the source can be generated in real-time (as in video conferencing applications) or off-line (as in video streaming applications). For all video applications except plain-old downloading, real-time delivery is required. But some applications (e.g., streaming) can afford a relatively large play-out delay. In this case, the receiver can use a large buffer to smooth out the jitter, so that the decoded video can be played out at a constant frame rate, after an initial play-out delay. For such applications, a limited number of retransmissions can be used to handle transmission errors. In the following, we classify video applications based on their requirements in terms of real-time, latency, and jitter.

Note that the end-to-end delay from a video source to a destination can be contributed by several factors (See Fig. 14.4): encoder processing delay (including acquiring the data and encoding), encoder buffer delay (to smooth the rate variation in the compressed bit stream), transmission delay (delay caused by the transmission itself, which is usually very small), and that due to queuing and perhaps retransmis-

<sup>2</sup> “Throughput” means the effective end-to-end delivery rate. For example, a network may have a raw bandwidth of 10 Mbps, but some data have to be retransmitted due to packet loss or timeout, so that the actual throughput is lower than 10 Mbps.



**Figure 14.4.** The contributing factors to the end-to-end delay in a video communication system. From [58].

sion in packet-based network), decoder buffer (to smooth out transmission jitter), and decoder processing delay (including both decoding and display buffer for constant frame play-out). Of course, when the source is pre-compressed, as in video broadcast and streaming applications, the encoding delay is not of concern. In this chapter, we focus on transmission delay, assuming the delay caused by video encoding/decoding and that by encoder/decoder buffer are acceptable by the underlying application and that these delays are relatively fixed.

### Interactive Two-Way Visual Communications

Examples in this category include tele-conferencing, video telephony, virtual classroom, etc. Such applications have very stringent delay requirements. For effective communications, the latency and jitter must be within a certain limit. For example, for inter-continental telephone conversations, the ITU-T G.114 standard recommends a maximum round trip delay of 150 ms for excellent quality, and 400 ms for acceptable quality [37]. The same numbers apply for video conferencing applications. Delay over 500 ms is intolerable for effective communications. Also, to maintain lip synchronization when playing audio and video, the video delay from the audio should be limited. Usually, audio is intentionally delayed slightly in the sender and receiver, to maintain lip synchronization.

For such applications, encoding and decoding have to be accomplished in real-time. Otherwise, incoming frames may build up at the encoder, or received bits at the decoder may have to be thrown away. To satisfy the stringent delay requirement, encoder and decoder buffers are typically very small. Retransmission is usually unacceptable. Jitter in the network also has to be limited. Because of these constraints, interactive applications are considered the most demanding among different types of video communications.

Fortunately, only low to intermediate video quality in terms of both spatial and temporal resolution is expected. For example, QCIF at 5-10 fps is considered acceptable for video phone applications, CIF at 10-20 fps is satisfactory for

most video conferencing scenarios. Furthermore, a moderate amount of compression/transmission artifacts is often tolerable.

### One-Way Video Streaming

In this type of application, a live or pre-compressed video source is delivered to one or multiple destinations. Instead of waiting for the entire video file to arrive, the receiving terminal starts to decode and play the video after an initial playout delay, which can be up to a few seconds. Depending on the number of recipients, the application can be further classified as *broadcast*, *multicast*, and *unicast*. Television broadcast is the most widely deployed video application. Video multicast, such as the Mbone application, which allows any user on the Internet to view the video of a live conference or seminar, is another important application. An application that is becoming increasingly popular is streaming video from a pre-compressed video database over the Internet, typically by a single user.

Obviously, the bandwidth requirements depend on the type of video material. For digital TV broadcast over terrestrial, cable, or direct broadcast satellite (DBS) channels, typically the video programs are in the BT.601 resolution, compressed using the MPEG-2 format with bit rates between 3-10 Mbps. For HDTV, the bit rates could be up to 20 Mbps. For multicasting and unicasting over the Internet or wireless network, because of the limited bandwidth that can be devoted to each user and the variability of the bandwidth and delay, lower quality video (CIF or lower, compressed in MPEG-1 or MPEG4 or other proprietary formats) are typically used.

Because there is no interactive communication between the sender and receiver, the latency could be fairly large, but the jitter must be limited, so that the received stream can be decoded and played out smoothly at a constant frame rate, after an initial delay. TV broadcast uses dedicated channels, which have guaranteed bandwidth and very low jitter. For video multicast or unicast over the Internet, where jitter could be large due to variability of the network conditions, a large smoothing buffer is typically installed in the receiver to absorb the jitter. The larger the buffer, the longer will be the playout delay, but the smoother will be the displayed video.

In the case of multicast or unicast over the Internet, the potential recipients may be connected to the network with different access links (from 100 Mbps Fast-Ethernet to a slow wireless modem link). Also, the receiving terminals may have very different computing powers (from a powerful workstation to a handheld battery powered device). The video server must take these into account. For example, by using scalable coding (Chap. 11), users with different bandwidths and computing powers may choose to extract different portions of a compressed video stream. Video streaming over the Internet and wireless IP networks are discussed in more detail in Chap. 15.

The major difference between live and pre-compressed sources is, obviously, that the live source has to be compressed in real-time, which significantly increase the complexity of the system at the transmission end. The encoding process and encoder

buffer will also contribute to additional delays.

### One-Way Video Downloading

In this case, a pre-recorded video (and audio) is downloaded to a destination. The receiving terminal does not start to play the material until the entire video has arrived. This application is the least demanding in terms of delay requirement, although excessive delay (because of the very low transmission bandwidth) may cause the receiver to abort the downloading in the middle of a communication session. Scalable coding and progressive transmission is desirable so that a low resolution version of the video can be delivered relatively quickly. Because of the relaxed delay requirement, retransmission can be used to handle any lost or erroneously delivered part of the data. No special error control mechanisms are necessary, beyond those typically used for data transfer.

## 14.2.2 Communication Networks

In this subsection, we describe different types of networks and associated protocols that may be used for video transport. For each network, we describe its characteristics in terms of bandwidth, delay, delay variation (jitter) and loss rate (including bit error and packet loss), as well as typical video applications carried over such network.

### Public Switched Telephone Network (PSTN)

The PSTN refers to the plain old telephone system (POTS). The best thing about PSTN is that it is accessible by almost anyone in the US and, to a large extent, in the world. However, the copper wire between the home and central office has a limited bandwidth. The fastest transmission rate by the most advanced modem technology has now reached 56 Kbps. But this is still too low for carrying video with pleasing quality. The ITU-T H.324 standard series (Sec. 13.3.2) has been developed for multimedia communications over PSTN [36]. In H.324-based systems, the tradeoff between bit error rate and bitrate of the modem link is adjusted via modem control mechanisms. Most systems employ strong error control coding to achieve very low error rates, at some expense of bitrate [81].

Even at 56 kbps (typically the achievable transmission rate for the payload on a 56K modem is much lower), only a small window video (QCIF or lower) can be provided. Although this may be adequate for visual communication purposes, it is not nearly as pleasing as most people would like. Higher rates, up to 6 Mbps in the downlink direction (from a central video server to home), are possible with ADSL (Asymmetric Digital Subscriber Loop) modems [54]. ADSL is one of the main transport media for streaming MPEG-1 and MPEG-2 movies through video-on-demand services. A good discussion on issues involved in multimedia transport over ADSL can be found in [96].

### Integrated Service Digital Network (ISDN)

ISDN is the first public network using digital transmission [66]. The data rate can be multiples of basic channels (known as B channel) of 64 Kbps each, with the multiplying factor  $p$  ranging from 1 to 24, corresponding to an aggregated bandwidth from 64 to 1,536 Kbps. The basic subscription of ISDN, which is known as basic rate interface, comes with 2B+D channels, where the D channel is 16 Kbps and is used for return signaling. At 128 Kbps ( $p=2$ ) for the signal delivery (64 Kbps in each direction), only very low quality video (e.g., QCIF at 10 fps) can be achieved. For a better video quality (i.e., CIF at 15 to 30 fps), at least 384 Kbps ( $p=6$ ) is required.

Like the phone line, communication over ISDN is based on *circuit switching*, i.e., an ISDN connection is dedicated to a particular conversation once the connection is established. For this reason, the connection is very reliable. The first ITU-T recommendation series for audiovisual conferencing, H.320, was developed for ISDN [40]. H.320 is currently employed by the vast majority of video-conferencing and video telephony equipment. The multiplex protocol H.221 [39] used by H.320 systems offers a bit oriented, practically error free video transmission channel with a fixed video bitrate. Beyond the mandatory intra MB refresh mechanisms of the video coding standard, no other error control tools are necessary [81].

### Broadband ISDN (B-ISDN)

The above described ISDN service with the basic channel rate being 64 Kbps is more accurately known as narrow-band ISDN or N-ISDN. This can be offered over existing twisted pair local loop wiring. The term broad-band ISDN or B-ISDN [66] describes ISDN services that offer channels in excess of 64 Kbps, including H\_0 channel with a rate of 384 Kbps, H\_11 channel with a rate of 1.536 Mbps, and H\_12 channels with a rate of 1.920 Mbps. This requires the use of higher bandwidth coaxial cables or optical fibers. The bandwidth of B-ISDN is also specified in multiples of 64 Kbps, with the multiplying factor ranges from 1 to 65535.

For efficient usage of the bandwidth, the B-ISDN uses Asynchronous Transfer Mode (ATM) packet switching technology with fixed size packets (known as cells). No dedicated links are created for an established connection, which makes it less reliable (i.e. with more delay variation and potential cell loss) than the circuit switched ISDN. The very short cell size (53 Bytes with 48 bytes payload) makes it suitable for real-time applications with low delay requirements. In such networks, cell loss can occur due to traffic congestion, although the loss ratio is quite low, typically in the range of  $10^{-6}$  to  $10^{-4}$  for video services [3]. Generally, cell loss can be considered as a sub-form of packet loss, whereby cells are extremely small packets. The mechanisms to cope with cell losses, however, are different, because it is not effective to add synchronization markers at the beginning of each cell, from a resilience/overhead tradeoff perspective.

The ATM network provides four types of services: constant bit rate (CBR), variable bit rate (VBR), available bit rate (ABR) and unspecified bit rate (UBR).

Typically, video is carried in the VBR service, where the compressed bit stream has to satisfy pre-set constraints in terms of average rate and peak rate. The allowance of rate variation makes it possible to choose the video coding parameters to maintain a relatively constant video quality, which is a desirable feature. Good reviews of ATM networks and issues related to video transport over ATM can be found in [3, 5].

### Internet

The Internet is composed of multiple physical networks interconnected by computers known as routers. The most basic building block is known as a *local area network* (LAN), which typically has bandwidth from 10 Mbps to 100 Mbps. Interconnected LANs in a local region form a *metropolitan area network* (MAN), and finally several interconnected MANs form a *wide area network* (WAN). Communications on the Internet is based on *packet-switching*. That is, the data to be delivered are truncated into packets, with each packet given a header that specifies the source and destination addresses. Each packet is delivered independently. Packets may be discarded due to buffer overflow at intermediate network nodes such as switches or routers or considered as lost due to excessive long queuing delay. For real-time video applications, any packet arriving after the allowed delay time is also considered lost. The transport of the packets is governed by the TCP/IP protocol suite, which includes three important protocols: the Internet Protocol (IP), the User Datagram Protocol (UDP) for unreliable connectionless packet delivery service, and the Transmission Control Protocol (TCP) for reliable stream services. IP is the lower layer protocol on top of which UDP and TCP operate. TCP guarantees delivery by using retransmission with acknowledgement [16].

The Internet and its associated TCP/IP protocol were originally designed for data communications, which are not delay sensitive. For real-time services such as video conferencing and video streaming, the retransmission mechanism in TCP is not appropriate. To support such applications, Real-time Transport Protocol (RTP) and its companion Real-Time Control Protocol (RTCP) have been developed. RTP is built on top of UDP, but has added information in the header, including sequence number and timing and synchronization information. RTP allows the detection of lost packets based on sequence numbers as well as discarding of late packets at the receiver end. RTCP is designed to provide QoS feedback (such as packet loss rate) to the participants of a RTP session, so that the sender can adjust its operation correspondingly (e.g., adjust the sending rate or change the error control mechanism). A good discussion on the Internet protocols for supporting continuous media services, including RTP/RTCP, real-time streaming protocol (RTSP) for medium-on-demand, session initiation protocol (SIP) for internet telephony, session announcement protocol (SAP) for broadcast applications, and session description protocol (SDP) can be found in [62].

Typically, headers added at different network layers will contribute to about 40 bytes in a packet. To gain a reasonable payload/overhead relationship, large data

**Table 14.1.** Different types of wireless networks

Network	Data Rate	Mobility	Range	Channel Quality
Cellular network	Low ( $\leq 20$ Kbps)	High (vehicular)	Long (2500 meters)	Poor
Wireless data network	Intermediate (64-384 Kbps)	High (vehicular)	Long (2500 meters)	Poor
Wireless LAN	High (2-25 Mbps)	Low (indoor)	Short (50 meters)	location-dependent
Wireless IP network	High (1-600 Mbps, downlink)	Low (indoor)	Long 3-30 miles	Good

packets of around 1500 bytes are usually employed. By using a parity check sum, bit errors in a packet can be detected and a packet is discarded completely if any bit error is detected. Thus any received packets can be considered bit-error free. The packet loss rate depends on the network conditions and can be as low as 0% in case of a highly over provisioned private IP network or as high as 30% and more for long distance connections during peak time on the Internet [6, 81]. With RTP, lost packets can be easily identified by the use of the sequence number. This information can be conveyed to the video decoder, to enable decoder-based error concealment.

The most widely used physical network for supporting Internet traffic is Ethernet, which has a bandwidth of 10 Mbps. More advanced networks include fast-Ethernet and fiber distributed data interface (FDDI); both support data rates up to 100 Mbps. Such faster networks are typically used in a corporate Intranet environment.

Because of its ubiquity, the Internet has been envisioned as the future platform for carrying various video services. However, because the Internet is a best effort network that does not guarantee lossless nor timely delivery, error control in video communication over the Internet is a challenging research field. More substantial discussion on the Internet and the RTP/RTCP protocol can be found in Chapter 15, which focuses on video streaming applications over the Internet.

### Wireless Networks

There are a variety of wireless networks targeting different environments and applications. These systems have very different capacities, coverage, and error characteristics. In the following several paragraphs, we briefly summarize different wireless services. We would like to note that wireless communication is a fast evolving technology and that newer, more reliable and faster services are likely to become

available by the time this book is published.

The *cellular networks* [27] are traditionally developed for mobile voice communication services but have now progressed to allow data transfer through wireless modems. The data rate is limited to under 20 Kbps (the payload rate is much lower). Alternatively, the cellular network can be connected to a packet-based *wireless data network*, which enables one to transfer data between a mobile user and a wired terminal over the Internet (such as in web browsing applications). For example, the basic GPRS (general packet radio service) wireless access network offers a payload bit rate ranging from 9 to 21.4 Kbps using single time slot, while the enhanced GRPS wireless access technology (known as EDGE) can provide bit rates ranging from 8.8 to around 59.2 Kbps [9]. By using multiple time slots, the raw data rate can be as high as 170 Kbps. The available data rate is location-dependent, with higher rates when the user is closer to the base station. The upcoming *third generation (3G) wireless systems* promise to provide higher data rates and ubiquitous coverage by using higher carrier frequencies, wider bandwidth and more sophisticated techniques for multiple access, error control, and signal detection. The outdoor data rates range from 144 Kbps to 384 Kbps depending on the mobile speed and distances between terminals, and the indoor data rates will be at least 2 Mbps. The BER for data transmission will be below  $10^{-6}$ . Real-time audiovisual communication will be an integral part of the 3G services. In general, any wireless channel is quite noisy, with high bit error rates. But the use of rate adaptation, FEC and ARQ yields an almost error-free environment for data transmission. For video transmission, where ARQ has to be limited, one has to cope with quite high bit error rates and packet loss rates. This is discussed further when we consider wireless video transmission using the H.223 multiplex protocol.

A *wireless LAN* refers to a set of information devices (typically indoor, stationary or slowly moving) connected into a LAN using wireless connections. Either IP or ATM transport protocols can be used, which are referred as mobile IP and wireless ATM, respectively. The main challenge in adapting conventional IP and ATM protocols to wireless environments is how to provide continuous network connectivity to mobile nodes and how to handle hand-offs, when the mobile terminal is switching from the coverage area of one access point to another. Achievable bit rates depend on the carrier frequencies. For example, LANs conforming to the IEEE 802.11 standard can have bit rates up to 11 Mbps (with lower payload rate). The connection in a wireless LAN is either very good (e.g. BER lower than  $10^{-5}$ ), when the user is close to the access point, or very bad (e.g. BER higher than  $10^{-2}$ ), when the user is far from the access point. The packet loss rates depend on the packet size and error detection and correction methods incorporated within the packet.

*Broadband Wireless IP networks* refer to the access to the Internet by home or business users through small roof-top antennas communicating to a base station using microwave radio. This includes *multichannel multipoint distribution service* (MMDS) and *local multipoint distribution service* (LMDS). LMDS uses higher carrier frequencies than MMDS, and thus enables a higher data rate (typically up to 100 Mbps, but can be as high as 600 Mbps) than MMDS (up to 1 Mbps). However,

LMDS is effective within a much shorter range between the base-station and the client site (3 miles vs. 30 miles).

Table 14.1 summarizes the characteristics of the networks discussed above. A good review of various wireless networks can be found in [9].

Difficulties for video transport over wireless networks lie in the low bandwidth, high error rate (random bit error as well as long bursts because of the multipath fading effect), and most importantly, the *fluctuation* of available bandwidth and error characteristics. Such variability is particularly acute when mobile stations are involved. A wireless video communication system therefore must be adaptive both in rate and error resilience. To meet such requirements, most proposals for wireless video transport use layered coding with unequal error protection. It is worth noting that, for real-world applications, quite sophisticated FEC codes are used to reduce the bit error rates significantly. On top of FEC, real-world systems generally employ certain forms of channel multiplexers that often include transport protocol functionality as well, thereby reducing the error rates further.

Most wireless interactive multimedia communication systems employ H.223 [38] and its “mobile extensions” as the transport/multiplex protocol on top of the bit-oriented channel [81]. The mobile extensions form a hierarchy with five different levels, which allows for a scalable tradeoff between the robustness against bit errors of the multiplexer itself and the overhead incurred by that multiplexer. Most research for video coding on top of the H.223 transport is performed with average burst lengths in the neighborhood of 16 bits, and BERs between  $10^{-5}$  and  $10^{-3}$ . H.223 conveys media data including compressed video in the form of packets of variable size. Typical packet sizes are around 100 bytes to ensure good delay characteristics. If bit errors damage the protocol structures of H.223 beyond the repair facilities of the employed level (a condition known as multiplex errors), then whole packets can get lost as well. Therefore, video transmission on top of H.223 has to cope with packet losses.

For a good discussion on the issues involved in video transport over wireless systems, see [76, 25]. Video streaming over wireless IP networks is further considered in Chap. 15.

### Broadcast Channels: Terrestrial, Cable, Satellite

This transport environment is used for broadcasting of digital TV including HDTV programs using the MPEG-2 video coding and transport streams. The compressed data are carried over fixed size transport packets of 188 bytes each. Regardless of the underlying wireline or wireless physical layer, the channel coders and the MPEG-2 transport layer ensure an almost error free environment. For SDTV, the available bandwidth is divided into program channels that can carry between 3-10 Mbps each. For HDTV, 20 Mbps is typically allocated. A good reference on digital TV services using direct broadcast satellite (DBS) is [55].

Table 14.2 summarizes the characteristics of major video communications applications, including the standards used, the target network, and transmission char-

**Table 14.2.** Characteristics of Major Video Communications Applications. N/A: not applicable; H.262 is identical to MPEG-2 video, and H.222 is identical to MPEG-2 systems. From [81]

Application and standard family	Multiplex protocol	Video coding standard	Typical video bitrate	Packet size	Error characteristics
ISDN Video-phone (H.320)	H.221 H.263	H.261 and H.263	64-384 kbps	N/A	Error free
PSTN Video-phone (H.324)	H.223	H.263	20 kbps	100 bytes	Very few bit errors and packet losses
Mobile Video-phone (H.324 wireless)	H.223 w/ mobile exten- sions	H.263	10-300 kbps	100 bytes	BER=10 <sup>-5</sup> to 10 <sup>-3</sup> , occasional packet loss
Videophone over Packet network (H.323)	H.225/ RTP/ UDP/IP	H.261, H.263, H.262	10-1000 kbps	≤1500 bytes	BER = 0, 0-30% packet losses
Terrestrial/ Cable/ Satellite TV	MPEG-2 System	MPEG-2 video	6-12 mbps	188 bytes	Almost error free
Videoconferencing over ‘Native’ ATM (H.310, H.321)	H.222	H.262	1-12 mbps	53 bytes (ATM cell)	Almost error free

acteristics, such as packet sizes and typical error rates.

### 14.3 Transport-Level Error Control

Starting from this section, we describe various error control mechanism that have been developed for video communications. We start with transport-level error control mechanisms, because this is typically the most important part of error control. It provides a basic QoS level that can be further improved upon by additional error control mechanisms at the encoder and decoder. Transport-level error control can be exercised at the channel coder, packetizer/multiplexer, and transport protocol level. We discuss these separately in this section.

### 14.3.1 Forward Error Correction (FEC)

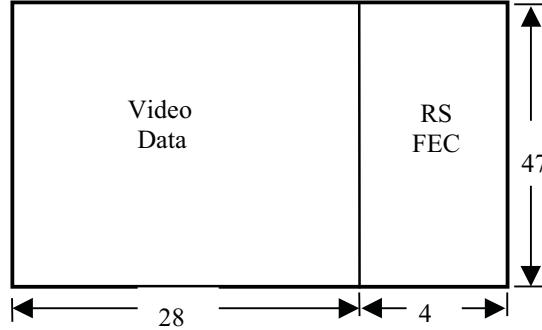
FEC is well known for both error detection and error correction in data communications [52]. However, since FEC has the effect of increasing transmission overhead and therefore reducing usable bandwidth for the payload data, it must be used judiciously in video services, which are very demanding in bandwidth but can tolerate a certain degree of loss.

When applied to the compressed bits directly, FEC is effective only for channels where bit errors dominate. For example, in the H.261 standard for video conferencing over ISDN [35], an 18-bit error correction code, BCH (511,493), is computed and appended to 493 video bits (1 fill bit, 492 coded bits). Together with an additional framing bit, the resulting data are grouped into a 512-bit frame. The FEC code is able to correct single bit errors and to detect double bit errors in each frame. The same FEC mechanism can also be used in H.263 [42, Annex H] (Sec. 14.7.1). However, when H.263 is applied for transporting video over wireless networks or the Internet, where the error burst is usually longer than 2 bits, this method is not very useful and hence seldom used.

For packet-based transmission, it is much more difficult to apply error correction because several hundred bits have to be recovered when a packet loss occurs. Usually, FEC is applied across data packets, so that a packet loss will lead to the loss of only one byte in a FEC block. For example, in the approach of Lee [51], Reed-Solomon (RS) codes are combined with block interleaving to recover lost ATM cells. As shown in Figure 14.5, a RS (32,28,5) code is applied to every block of 28 bytes of data to form a block of 32 bytes. After applying the RS code row by row in the memory up to 47th row, the payload of 32 ATM cells is formed by reading column by column from the memory with the attachment of one byte indicating the sequence number. In this way, a detected cell loss at the decoder corresponds to one byte erasure in each row of 32 bytes after deinterleaving. Up to 2 lost cells out of 32 cells can be recovered. The Grand-Alliance HDTV broadcast system has adopted a similar technique for combating transmission errors [10]. In [2], Ayanoglu *et al.* explored the use of FEC for MPEG-2 video in a wireless ATM. FEC is employed at the byte level for random bit error correction and at the ATM cell level for cell loss recovery. These FEC techniques are applied to both single layer and two-layer MPEG data.

### 14.3.2 Error-Resilient Packetization and Multiplexing

Depending on how compressed data are packetized, a packet loss can have different effects on the reconstructed video. It is important to perform packetization in such a way that the error can be isolated within a small region. This means that compressed bits cannot be packetized blindly into equal sized packets, rather the packets should be constructed according to the underlying encoding algorithm, so that a packet contains one or several independently coded data blocks. For example, for MPEG-2 coded video, a packet may contain one or several slices, possibly with repetition of the picture header in the beginning of each packet. Similarly, for H.263



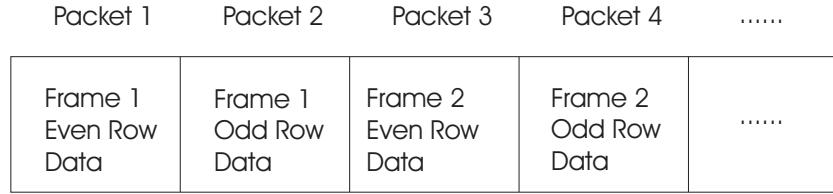
**Figure 14.5.** Illustration of packet-level FEC for ATM cell loss recovery. The numbers in the figure are in bytes. From [82, Fig. 6].

code video, a packet may include one or several GOB's. Such application aware packetization is supported in the Internet, known as *application layer framing* (ALF) [13]. In networks supporting variable size packets, it is desirable to use the largest possible packets to minimize the reduction in effective data rates by packet headers and the packet processing overheads. For example, with the Internet, the maximum transmission unit (MTU) is around 1500 bytes. This size is sufficiently large to pack data from several slices or GOB's. For low-bit-rate interactive applications, the acceptable delay may limit the largest packet size to a value much smaller than the MTU. In such cases, header compression may be used to reduce the overhead [13].

As described previously, for video transport over the Internet, the RTP/UDP/IP protocol stack is typically used. RTP provides support for different video coding formats through the definition of RTP payload formats [73, 32, 15, 7]. A detailed description of the payload formats for H.261, H.263, and MPEG-1 and MPEG-2 can be found in [13].

In order to prevent the loss of contiguous blocks because of a single packet loss, *interleaved packetization* can be used, by which data from adjacent blocks or rows are put into separate packets, as illustrated in Fig. 14.6. This way, a damaged block will usually be surrounded by undamaged blocks, which will ease the error concealment task at the decoder. Note that the use of interleaved packetization in the transport layer requires the source coder to perform block level prediction only within blocks that are to be packetized sequentially. This will reduce the prediction gain slightly. Also, certain headers may have to be repeated so that each received packet is self-decodable. For example, in the packetization format shown in Fig. 14.6, the header for each frame should be repeated in the packets containing both even and odd row data.

Another factor that influence the error characteristics of video communication



**Figure 14.6.** An example of interleaved packetization.

is how are the video and other data (audio, control) multiplexed and how is the header protected. An error in the multiplex header can cause the video data be mis-delivered, causing large chunks of data to disappear at the receiver [76]. To prevent such events from happening, typically, each packet is led by a long synchronization flag, and the header is heavily protected using FEC. A good example is the multiplexing standard H.223 [38], developed under the umbrella of the H.324 system for low-bit rate multimedia communications. It is designed to support multiplexing of data from multiple sources on a circuit-switched network (e.g., wireline and wireless modems). H.223 offers a hierarchical, multilevel multiplexing structure, allowing for a scalable tradeoff between the robustness against bit errors of the multiplexer itself and the overhead incurred by that multiplexer. Level 0, which is targeted for wireline environments, use a short sync flag and no FEC on the header. Higher levels, which are targeted for more error prone environments such as a mobile wireless modem, use increasingly longer sync flags and stronger protection for the header.

### 14.3.3 Delay-Constrained Retransmission

Beyond the low level error control mechanism such as FEC, error resilient packetization and multiplexing, the upper layer transport protocol can exercise error control, typically in the form of automatic retransmission request (ARQ), which requests retransmission upon the detection of lost or overly delayed packets, as is done in TCP. Retransmission has been used very successfully for non real-time data transmission, but it has been generally considered as unacceptable for real-time video applications because of the delay incurred. This is in fact not always the case. For example, for a coast-to-coast interactive service, one retransmission adds only about 70 ms delay, which can be acceptable [59]. For one-way real-time video applications such as Internet video streaming and broadcast, the delay allowance can be further relaxed to a few seconds so that several retransmissions are possible. Retransmission has also been considered inappropriate for multipoint video conferencing because the retransmission requests from a large number of decoders can overwhelm the encoder. However, when a multipoint control unit (MCU) is used in a multipoint conference, the paths between the encoder and the MCU, and between the MCU and the decoders are simply point-to-point. Retransmission can be applied in these paths separately. Another concern about using retransmission is that retransmis-

sion may worsen the problem, because it will add more traffic on the network and thus further increase the packet loss rate. However, if retransmission is controlled appropriately, the end-to-end quality can be improved. For example, the encoder can reduce its current output rate so that the sum of the encoder output and the retransmitted data is kept below a given total data rate.

Obviously, for real-time applications, retransmission must be constrained so that the incurred delay is within the acceptable range. Instead of trying retransmission indefinitely to recover a lost packet, as done in TCP, the number of retransmission trials can be determined by the desired delay [53]. One can also combine layered coding (Sec. 14.4.4) with prioritized retransmission, by placing the base layer packets of a layered coder in the front of the transmission queue to increase the number of retransmission trials for the base layer [64]. Finally, one can send multiple copies of a lost packet in each single retransmission trial, hoping at least one get through in time [98]. The number of retransmission trials and the number of retransmission copies for a lost packet can be determined according to its importance. For example, a base layer packet can be retransmitted several times, each time with multiple copies, while an enhancement layer packet can be simply dropped. Another constrained retransmission scheme specifically designed for video streaming application is described in Chap. 15.

#### 14.3.4 Unequal Error Protection

The binary bits in a compressed video bitstream are not equally important. For example, the picture header and other side information are much more important than the block data, in block-based hybrid coder. These important bits should be protected so that they can be delivered with a much lower error rate. When layered coding is used at the source coder (Sec. 14.4.4), the transport controller must assign appropriate priority to different layers, which is a form of transport level control.

Different networks may implement transport prioritization using different means. In ATM networks, there is one bit in the ATM cell header that signals its priority. When traffic congestion occurs, a network node can choose to discard the cells having low priority first. Transport prioritization can also be implemented by using different levels of power to transmit the substreams in a wireless network. In the Internet, the latest RTP specification defines generic mechanisms for transporting layered video bitstreams [63]. There are also network support for prioritized delivery through differentiated services [4]. In either wireless networks or the Internet, prioritization can be realized with different error control treatments to various layers. For example, retransmission and strong FEC can be applied for the base layer while no retransmission and weaker FEC may be applied to the enhancement layers. In the extreme case, the most important information can be duplicated. For example, dual transmission for picture header information and quantization matrix has been proposed for MPEG video [70]. Another way to provide different levels of protection is by using different transport protocols. For example, in the video-on-demand system described in [14], TCP is used for transmission of a very

small amount of high-priority data (session control data) before a service session and UDP is employed for the remaining low-priority data (video data) during the session.

## 14.4 Error-Resilient Encoding

The previous section described various error control mechanisms that can be exercised at the transport level. These methods are applied to coded video streams, attempting to detect, correct, and if necessary and feasible, retransmit damaged data. Such methods, even when applicable, may not completely correct the error, so that the received bitstream at the decoder still contain errors (bit error as well as packet loss). In this section, we describe source coding methods that can produce a bit stream that is robust to transmission errors, so that an error will not adversely affect the decoder operation and lead to unacceptable distortions in the reconstructed video quality.

Compared to source coders that are optimized for coding efficiency, such coders typically are less efficient in that they use more bits to obtain the same video quality in the absence of any transmission errors. These extra bits are called *redundancy bits*,<sup>3</sup> and they are introduced to enhance the video quality when the bitstream is corrupted by transmission errors. *The design goal in error-resilient coding is to achieve the best decoded video quality for a given amount of redundancy, or minimize the incurred redundancy while maintaining a prescribed quality level, both under an assumed channel environment.* There are many ways to introduce redundancy in the bitstream. Some of the techniques help to isolate the error (Sec. 14.4.1), while others enable the decoder to perform better error concealment upon detection of errors (Secs. 14.4.2 and 14.4.3). Yet another group of techniques are aimed at guaranteeing a basic level of quality and providing a graceful degradation upon the occurrence of transmission errors (Secs. 14.4.4 and 14.4.5).

### 14.4.1 Error Isolation

One main cause for the sensitivity of a compressed video stream to transmission errors is that a video coder uses VLC to represent various symbols. Any bit errors or lost bits in the middle of a codeword can not only make this codeword undecodable but also make the following codewords undecodable or decoded to wrong symbols, even if they are received correctly. Error isolation techniques, as the name implies, try to isolate the effect of a transmission error within a limited region. This is often achieved by placing “resynchronization markers” in the compressed bitstream, and by a technique known as “data partitioning”. Both have been incorporated into MPEG-4 and H.263 standards.

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<sup>3</sup>Such bits are also referred to as “overhead”.

### Inserting Resynchronization Markers

One simple and effective approach for enhancing encoder error-resilience is by inserting resynchronization markers periodically. These markers are designed such that they can be easily distinguished from all other codewords and small perturbation of these codewords. Usually some header information (regarding the spatial and temporal locations or other information that is needed to decode subsequent bits) is attached immediately after the resynchronization information. This way, the decoder can resume proper decoding upon the detection of a resynchronization marker. Obviously, insertion of resynchronization markers will reduce the coding efficiency. First, the longer and more frequent are such markers, the more bits will be used for them. Second, the use of synchronization markers typically interrupts in-picture prediction mechanisms, such as MV or DC coefficient prediction, which adds even more bits. But longer and frequently inserted markers would also enable the decoder to regain synchronization more quickly, so that a transmission error affects a smaller region in the reconstructed frame. Hence in practical video coding systems, relatively long synchronization codewords are used.

### Data Partitioning

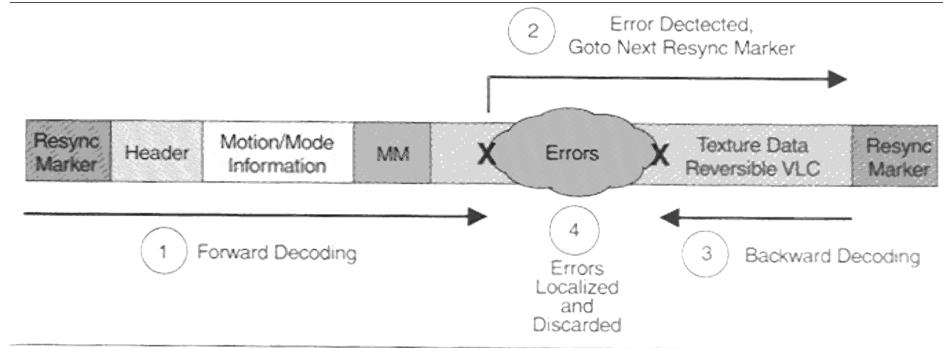
In the absence of any other error resilience tools, the data between the error location and the first following resynchronization marker would have to be discarded. To achieve better error isolation, data between two synchronization points can be further partitioned into finer logic units, with secondary markers in between. This way, logic units before an error location can still be decoded. Such markers can be shorter than the primary markers, because they only need to be free from emulation by data in the logic units that immediately proceed them. This approach is used in the error-resilience modes of MPEG-4 and H.263 standards, by which MB headers, motion vectors and DCT coefficients of all MBs in a slice or GOB are put into separate logic units. This way, if an error happens, say in the logic unit containing DCT coefficients, the header and motion information of the MBs contained in the previous logic units are still decodable.

#### 14.4.2 Robust Entropy Coding

In addition to error isolation by inserting synchronization codewords or partitioning the data into independent segments, one can also directly modify entropy coding methods so that the resulting bitstream is more robust to transmission errors. We present two such techniques below.

##### Reversible Variable Length Coding (RVLC)

In the above discussion, we have assumed that once an error occurs, the decoder discards all the bits until a resynchronization codeword is identified. With RVLC [72], the decoder can not only decode bits after a resynchronization codeword, but also decode the bits before the next resynchronization codeword, from the backward



**Figure 14.7.** RVLC codewords can be parsed in both the forward and backward direction, making it possible to recover more data from a corrupted data stream. MPEG-4 syntax is assumed in the figure, but the basic principle holds true also for other RVLC-coded data. From [8].

direction, as shown in Fig. 14.7. Thus, with RVLC, fewer correctly received bits will be discarded, and the affected area by a transmission error will be reduced. By providing the capability of cross-checking between the output of the forward and backward decoder, at a modest increase in complexity, RVLC can also help the decoder to detect errors that are not detectable when non-reversible VLC is used, or provide more information on the position of the errors, and thus decrease the amount of data unnecessarily discarded. RVLC has been adopted in both MPEG-4 and H.263, in conjunction with insertion of synchronization markers and data partitioning.

In spite of the “reversibility” constraint in the designing of the variable length code, the application of RVLC does not necessarily lead to reduction in coding efficiency. In fact, it has been shown that compressed video data (e.g. quantized and run-length coded DCT coefficients, MV differences), which can be very well modeled by the so-called “generalized Gaussian distribution” (GGD), can be entropy coded with the Golomb-Rice (GR) and exp-Golomb (EG) codes at near perfect efficiency. The EG code is more attractive than the GR code because its performance is very stable even when there is a mismatch between the model distribution and actual data. Therefore it is possible to achieve near optimal efficiency with a fixed EG code table for most video sequences. Both the GR and the EG codes are very well structured. This makes it possible to perform entropy encoding and decoding without look-up-tables, and to design reversible versions of GR and EG codes with exactly the same coding efficiency. More detailed descriptions on how to design RVLC tables with identical coding efficiency as GR and EG codes can be found in [86]. In addition to providing cross-checking capability of forward and backward decoded results, RVLC can provide better error detection and in some cases even error correction than non-reversible VLC. More detailed analysis and comparison

of the decoder operation for reversible and non-reversible VLC can be found in [85].

### Error-Resilient Entropy Coding (EREC)

Instead of using synchronization codewords to enable the decoder to resume decoding, the EREC method employs a way of reordering coded bits so that the decoder can regain synchronization at the start of each block [60]. Specifically, variable length bit streams from individual blocks are distributed into slots of equal sizes. Initially, the coded data for each image block are placed into the designated slot for the block either fully or partially. Then, a predefined offset sequence is used to search for empty slots to place any remaining bits of blocks that are bigger than the slot size. This is done until all the bits are packed into one of the slots. Because the size of each slot is fixed, the decoder can regain synchronization at the start of each block. It also ensures that the beginning of each block is more immune to error propagation than those at the end. Because EREC does not insert any bits just for synchronization purpose, the redundancy introduced is negligible. Application of EREC in MPEG-2 and H.263 coders have been considered in [71, 48].

#### 14.4.3 Error-Resilient Prediction

As we have seen, the use of VLC makes a compressed bitstream very sensitive to transmission errors. Another major cause for the sensitivity is the use of temporal prediction. Once an error occurs so that a reconstructed frame at the decoder differs from that assumed at the encoder, the reference frames used in the decoder from there onwards will differ from those used at the encoder, and consequently all subsequent reconstructed frames will be erroneous. This error propagation effect has been shown previously in Fig. 14.2. The use of spatial prediction for the DC coefficients and MVs will also cause error propagation, although it is confined within the same frame. Error-resilient prediction refer to techniques that constrain the prediction loop so as to confine error propagation within a short time interval.

##### Insertion of Intra-Blocks or Frames

One way to stop temporal error propagation is by periodically inserting intra-coded pictures or MBs. Using an I-frame can cause a surge in the output bit rate, and thus is typically not acceptable for interactive applications due to delay constraints. For such applications, using a sufficiently high number of intra-coded MBs is an efficient and highly scalable tool for error resilience.

When employing intra MBs for error resilience purposes, both the number of such MBs and their spatial placement have to be determined. The number of necessary intra MBs is dependent on the quality of the channel and the error control mechanism applied at the transport level. Many practical systems provide information about the network quality, or heuristic means to gain such information. Examples include antenna signal strength in wireless environments, or RTCP receiver reports on Internet connections. The study in [67] analyzed the dependence

of the overall distortion at the decoder on the intra-MB rate at the encoder, the channel coding rate (assuming Reed-Solomon coding), the channel error parameters (random loss rate and burst length). Based on this relation, one can find the optimal intra-MB rate and/or channel coding rate, for given channel error characteristics.

For the spatial placement of intra MBs, the approaches proposed can be categorized as either heuristic or rate-distortion (RD) optimized. Heuristic methods include random placement, placement in the area of high activity. Hybrid schemes that additionally consider the time of the last intra update of a given MB were also considered. These methods are simple and in general perform well. The RD optimization approach can further improve the performance at the expense of increased encoder complexity.

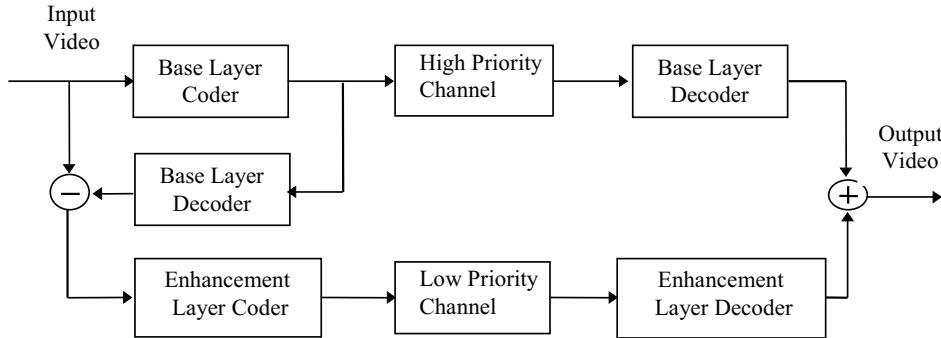
Recall that in the absence of transmission errors, the coding mode for each MB can be determined based on the RD trade-off obtained by different modes (Sec. 9.3.3). Ideally, the mode that leads to the maximum reduction in the distortion per bit spent should be used. For error resilience purpose, the same RD optimization approach can be taken, but the encoder must take into account of the fact that the current MB and previous MB's may be lost, when calculating the distortion associated with each coding mode. Several approaches have been developed based on this framework, and they differ in the way the expected distortion in the decoder is calculated, and the way the channel error is modeled [17, 94, 89].

For example, the algorithm in [17] takes as input a mid-term prediction of the packet loss rate,  $p$ . Each MB is coded in the intra, inter, and skip modes respectively, and for each mode, the resulting rate  $R(\text{mode}_i)$  and distortion  $D_1(\text{mode}_i)$  assuming lossless transmission of the MB are measured. Then, for the same set of coding modes, another set of distortions,  $D_2(\text{mode}_i)$ , is calculated under the assumption that the coded MB got lost during transmission. The error propagation effect in an inter-coded MB as well as error concealment in the decoder for a lost MB are taken into account in the above distortion measurement. For each coding mode, the expected distortion  $D(\text{mode}_i) = (1-p)D_1(\text{mode}_i) + pD_2(\text{mode}_i)$  is determined. The coding mode that leads to the best rate-distortion trade-off is determined. This is accomplished by determining the Lagrangian  $D(\text{mode}_i) + \lambda R(\text{mode}_i)$  for different modes, for a fixed  $\lambda$ , and finding the mode that has the minimal Lagrangian. The Lagrangian multiplier  $\lambda$  is chosen based on the target bit rate.

Finally, when a feedback channel is available, it can be used to convey information about missing or damaged MB data to trigger intra coding at the sender. These schemes are discussed in Sec. 14.6.

### Independent Segment Prediction

Another approach to limit the extent of error propagation is to split the data domain into several segments, and perform temporal/spatial prediction only within the same segment. This way, the error in one segment will not affect another segment. For example, a frame can be divided into multiple regions (e.g., a region can be a GOB or slice), and region one can only be predicted from region one in the previous



**Figure 14.8.** Block diagram of a system using layered coding and prioritized transport.

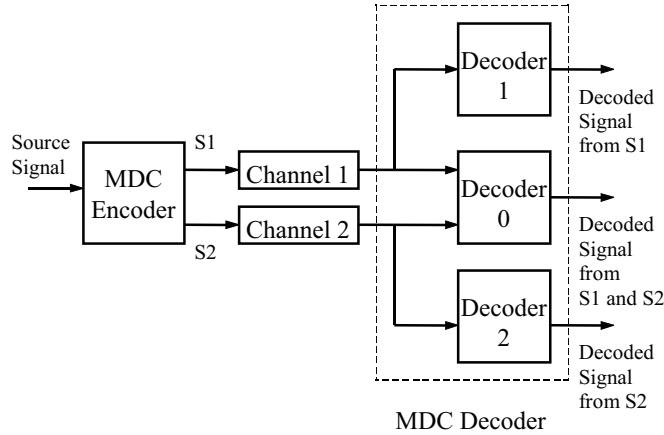
frame. This is known as independent segment decoding (ISD) in H.263.

Another approach in this category is to include even-indexed frames in one segment and odd-indexed frames into another segment. This way, even frames are only predicted from even frames. This approach is called *video redundancy coding* and has been incorporated in H.263 [87, 88]. It can also be considered as an approach for accomplishing multiple description coding, to be described in Sec. 14.4.5.

#### 14.4.4 Layered Coding with Unequal Error Protection

Layered coding refers to coding a video into a base layer and one or several enhancement layers. The base layer provides a low but acceptable level of quality, and each additional enhancement layer will incrementally improve the quality. As described in Chap. 11, layered coding is a special case of scalable coding, which enables receivers with different bandwidth capacities or decoding powers to access the same video at different quality levels. Typical implementations include SNR scalability, spatial scalability and temporal scalability (cf. Sec. 11.1). To serve as an error resilience tool, layered coding must be paired with unequal error protection (Sec. 14.3.4) in the transport system, so that the base layer is protected more strongly. Figure 14.8 shows the block diagram of a generic two-layer coding and transport system.

The redundancy in layered coding mainly comes from two sources. First, in order to avoid error propagation, the enhancement layer may choose to use only the base layer frames as reference frames for temporal prediction. In this case, the prediction gain and consequently the coding efficiency will be reduced. Secondly, similar side information (e.g., header information, coding modes and MVs) has to be transmitted in each layer. A good study on the trade-off between the redundancy and error resilience of different scalability modes in MPEG-2 is described in [1]. Layered coding plus unequal error protection has been explored extensively for video transport over wireless networks. See e.g., [95, 46, 31, 24].



**Figure 14.9.** Illustration of multiple description coding and decoding.

#### 14.4.5 Multiple Description Coding

Layered coding described in the previous subsection can offer error resilience when the base layer is transmitted in an essentially error-free channel, realized via strong FEC and retransmission. However, in certain applications, it may not be feasible or cost effective to guarantee lossless transmission of any data. In this case, a loss in the base layer can lead to a disastrous effect in the decoded visual quality. An alternative approach to combat transmission errors is by using *multiple description coding* (MDC). With this coding scheme, several bit streams (referred to as descriptions) of the same source signal are generated and transmitted over separate channels. Each channel may be temporarily down or suffering from long burst errors. At the destination, depending on which descriptions are received correctly, different reconstruction schemes (or decoders) will be invoked. The MDC coder and decoder are designed such that the quality of the reconstructed signal is acceptable with any one description, and that incremental improvement is achievable with more descriptions. A conceptual schematic for a two-description coder is shown in Fig. 14.9. In this case, there are three decoders at the destination, and only one operates at a time.

With MDC, the channels carrying different descriptions could be physically distinct paths between the source and destination in, for example, an ad-hoc wireless network or a packet switched network such as the Internet. Even when only a single physical path exists between the source and destination, the path can be divided into several virtual channels by using time interleaving, frequency division multiplexing, etc. For example, in the Internet, if the packet size is relatively large and two descriptions are put into alternating packets, then the loss characteristics on the two descriptions will be close to independent.

For each description to provide an acceptable level of quality, all the descriptions

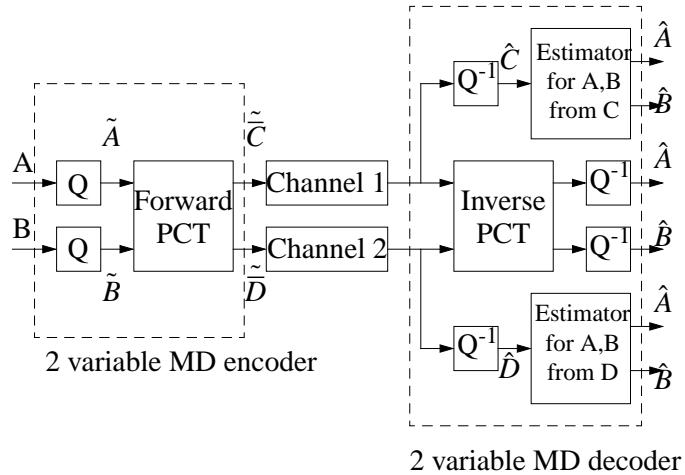
must share some fundamental information about the source, and thus must be correlated. This correlation is what enables the decoder to estimate a missing description from a received one, and thus provide an acceptable quality level from any description. On the other hand, this correlation is also the source of redundancy in MDC. An advantage of MDC over LC is that it does not require special provisions in the network to provide a reliable sub-channel. For example, in a very lossy network, many retransmissions have to be invoked or a lot of redundancy has to be added in FEC to realize error-free transmission. In this case, it may be more effective to use MDC.

To accomplish their respective goals, LC uses a hierarchical, decorrelating decomposition, whereas MDC uses a non-hierarchical, correlating decomposition. Some approaches that have been proposed for yielding such decompositions include overlapping quantization [74, 44, 21], correlated predictors [34], correlating linear transforms [80, 28], block-wise correlating lapped orthogonal transforms [11, 12], correlating filter-banks [91, 65, 45], and interleaved spatial-temporal sampling [79, 87]. The interleaved temporal sampling approach is known as video redundancy coding in the H.263 standard. In the following, we describe the method using correlating linear transform in more detail.

### Multiple Description Transform Coding

In this approach, a linear transform is applied to original signal samples to produce groups of correlated coefficients. Ideally, the transform should be such that the transform coefficients can be divided into multiple groups so that the coefficients between different groups are correlated. This way, if some coefficient groups are lost during transmission, they can be estimated from the received groups. To minimize the loss of coding efficiency, the coefficients within the same group should be uncorrelated. To simplify the design process for a source signal with memory, one can assume the presence of a pre-whitening transform so that the correlation-inducing transform can operate on uncorrelated samples.

To simplify the design of the transform, in the approach of [80], a pairwise correlating transform (PCT) is applied to each pair of uncorrelated variables obtained from the Karhunen-Loeve transform (KLT). The two coefficients resulting from the PCT are split into two streams that are then coded independently. If both streams are received, then an inverse PCT is applied to each pair of transformed coefficients and the original variables can be recovered exactly, up to the quantization error. If only one stream is received, the coefficients in the missing stream are estimated from those in the received stream based on the correlation between the two sets of coefficients. Figure 14.10 shows the block-diagram of this coding scheme for a single pair of variables. The overhead introduced by this approach can be controlled by the number of coefficients that are paired, the pairing scheme, and the transform parameters for the paired coefficients. To achieve the best reconstruction quality (in terms of MSE) from a single description at a fixed redundancy, the transform for each pair of variables should be a special type of generally non-orthogonal transform.



**Figure 14.10.** Multiple description transform coding using pair-wise correlating transform.

This optimal transform is parameterized by a single parameter, which controls the amount of redundancy introduced for coding each pair of variables. Given  $N \geq 2$  variables to code, there exists an optimal pairing strategy that, together with optimal redundancy allocation among the chosen pairs, can minimize the total single description distortion for a given total redundancy.

The MDTC method has been incorporated into a JPEG-like image coder, where the DCT coefficients are split into two streams using PCTs. Figure 14.11 show the reconstructed images from a single description (i.e., half of the bits are lost) at two different redundancies, defined as the extra number of bits per pixel required over the reference single description coder (in this case the JPEG coder) to obtain the same distortion in the absence of transmission loss. We can see that satisfactory images can be obtained from a single description at relatively low redundancies.

The MDTC method has also integrated into a block-based hybrid video coder, in which the motion compensated prediction error is coded using MDTC to produce two descriptions. The challenging problem is how to control the mismatch between the reference frames used in the source coder and those at the decoder, which may have received only one description. For specific implementation options, see [61].

Given the relatively large overhead associated with MDC, this approach is appropriate only for channels that have relatively high loss rates. When the channel loss rate is small, the reconstruction performance in the error-free case dominates and therefore single description coding is more appropriate. On the other hand, when the loss rate is very high, the reconstruction quality in the presence of loss is more critical, so that the MDC approach becomes more suitable. A challenging task is how to design the MDC coder that can automatically adapt the amount of



**Figure 14.11.** Reconstructed images from a single description using the multiple description transform coding method: (a) redundancy=0.088 bpp (15%), PSNR=28.81 dB; and (b) redundancy=0.133 bpp (22%), PSNR=29.63. The reconstruction PSNR=35.78 dB when both descriptions are received. The reference bit rate required by the single description coder (JPEG) is 0.60 bpp.

added redundancy according to underlying channel error characteristics.

#### 14.4.6 Joint Source and Channel Coding

In layered coding and MDC introduced previously, the interaction between the source and channel coders is exercised at a high level. In layered coding, the source coder produces a layered stream assuming that the channel coder can guarantee the delivery of the base layer. On the other hand, with MDC, the source coder assumes that all coded bits will be treated equally and all are subject to loss. The source-channel interaction can also happen at a lower level, i.e., the quantizer and entropy coder design at the source coder, and the design of FEC and modulation schemes at the channel coder. This type of approach is traditionally referred as joint source and channel coding, although in a broader sense, layered coding and MDC can also be considered as belonging to this category.

A popular FEC scheme for compressed video is the rate-compatible punctured convolutional (RCPC) code [29], with which one can easily and precisely control the incurred redundancy rate. By jointly designing RCPC codes and bit rate assignments in the source coder, a desired trade-off between the source and channel rates can be achieved. It is also quite easy to realize unequal error protection with RCPC codes [30].

Another approach to joint source and channel coding is by jointly designing

the quantizer and the binary encoder for given channel error characteristics, to minimize the effect of transmission errors [49, 19, 56, 57, 75, 20, 78]. One can also carefully design the mapping between quantized source codewords and their modulation symbols. By assigning close-by points in the modulation constellation space to close-by source codewords, the effect of channel error can be reduced [93, 22].

Note that techniques in this category are mainly targeted at bit errors and are not very effective in communication systems where compressed data are packetized and packet loss is the dominant contributor to transmission errors.

## 14.5 Decoder Error Concealment

As mentioned in the previous sections, due to various channel/network errors, compressed video data can be damaged or lost during transmission or storage. In Sec. 14.3, we described the mechanisms that can be invoked at the transport layer, to minimize transmission loss. Unless retransmission can be used indefinitely, which is unacceptable for real-time applications, there will be remaining errors and losses in the received data. In Sec. 14.4, we presented various ways that the encoder can use to suppress the effect of such errors. Still, a transmission error may lead to objectionable visual distortion in the reconstructed signal at the decoder. Depending on many factors such as source coding, transport protocol, the amount and type of information loss, etc, the introduced distortion can range from momentary degradation to a completely unusable image or video signal. In this section, we describe techniques that can be employed at the decoder to conceal the effect of transmission errors, so as to make the decoded signal much less visually objectionable.

Error concealment is possible because, due to various constraints such as coding delay, implementation complexity and availability of a good source model, a compressed video bitstream still possesses a certain degree of statistical redundancy, despite the tremendous research effort that has been devoted towards achieving the highest possible compression gain. Also, the source coder may incorporate some of the error-resilient mechanisms, to intentionally introduce redundancy in the coded bits, so as to facilitate the estimation of lost data. In addition, the human perception system can tolerate a limited degree of signal distortion. All these factors can be exploited for error concealment at the decoder.

Error concealment belongs to the general problem of image recovery or restoration. However, because errors are incurred at the compressed bit level, the resulting error patterns in the pixel domain are very peculiar and special measures are usually needed to handle such errors. As described before, because of the use of predictive coding and VLC, a bit error alone can cause damage to a large region. To contain the error effect, various measures can be taken in the encoder to make the compressed stream more error-resilient, at the expense of a certain degree of coding gain and/or complexity. Here we assume synchronization codewords are inserted periodically within a picture (cf. Sec. 14.4.1) and the prediction loop is periodically reset (cf. Sec. 14.4.3), so that a bit error or packet loss will only cause damage to

a limited region in a picture.

Depending on the frequency of synchronization codewords, transport packet size and bitrate, a damaged region can range from part of a macroblock to an entire picture. For high bitrate transmission over small packets as in ATM networks, a lost packet may only damage part of a MB or several adjacent MBs. In this case, a damaged MB is typically surrounded by multiple undamaged MBs in the same frame. When this is the case, we can use spatial interpolation to recover the damaged MB. On the other hand, for low-bitrate application over relatively large packets (for example, 128 kb/s over IP networks), a lost packet is likely to damage a large portion of a frame or an entire frame. Under such situations, we have to rely on the previous and possibly following video frames for concealment of the damaged frame. Obviously, information from adjacent frames can also be incorporated even if neighboring MBs are available in the same frame.

Given the block-based hybrid coding paradigm, there are three types of information that may need to be estimated in a damaged MB: the texture information, including the pixel or DCT coefficient values for either an original image block or a prediction error block, the motion information, consisting of MV(s) for a MB coded in either P- or B-mode, and finally the coding mode of the MB. The recovery techniques for these information types are somewhat different.

It is well-known that images of natural scenes have predominantly low frequency components, i.e., the color values of spatially and temporally adjacent pixels vary smoothly, except in regions with edges. *All the techniques that have been developed for recovering texture information make use of the above smoothness property of image/video signals, and essentially they all perform some kind of spatial/temporal interpolation.* The MV field, to a lesser extent, also shares the smoothness property, and can also be recovered by using spatial/temporal interpolation. For the coding mode information, the methods developed tend to be more driven by heuristics. In the following subsections, we review some representative techniques for each category. More extensive coverage of methods discussed here as well as other methods can be found in [82, 99, 47]. We will assume that error locations have been detected via other means, such as lost packet detection based on sequence numbers in services using RTP. For a coverage of various error detection methods, see [82].

Note that in a typical (non-error-resilient) encoder implementation, bits for these three types of data are stored sequentially for each MB, so that they are all lost in a damaged block. With the use of data partitioning (Sec. 14.4.1), it is possible that the coding mode, MV(s), and possibly first few DCT coefficients are still available in a damaged block. As can be seen from below, the knowledge of the coding mode and MV can help the recovery of the texture information greatly.

### 14.5.1 Recovery of Texture Information

#### Motion Compensated Temporal Interpolation

The simplest approach to recover a damaged MB is by copying the corresponding MB in the previously decoded frame. This however will give unsatisfactory results

if there is a large motion in the scene. A more effective solution is to use the MB in the previous frame pointed to by the MV for the damaged MB. The recovery performance by this approach is critically dependent on the availability of the MV. When the MV is also missing, it must first be estimated, which is discussed below. To reduce the impact of the error in the estimated MVs, temporal prediction may be combined with spatial interpolation (see below).

### Spatial Interpolation

Another simple approach is to interpolate pixels in a damaged block from pixels in adjacent correctly received blocks in the same frame. Usually, all blocks (or MBs) in the same row are put into the same packet, so that a packet loss typically leads to the loss of all blocks in a row. In this situation, the only available neighboring blocks for a damaged block are those above and below. Because most pixels in these blocks are too far away from the missing samples, usually only the boundary pixels in neighboring blocks are used for interpolation. Instead of interpolating individual pixels, a simpler approach is to estimate the DC coefficient (i.e. the mean value) of a damaged block and replace the damaged block by a constant equal to the estimated DC value. The DC value can be estimated by averaging the DC values of surrounding blocks. One way to facilitate such spatial interpolation is by using interleaved packetization (Sec. 14.3.2) so that the loss of one packet will damage only alternating blocks or MBs.

### Maximally Smooth Recovery (MSR)

A problem with spatial interpolation approaches is how to determine appropriate interpolation filter. Another shortcoming is that it ignores received DCT coefficients, if any. These problems are resolved in the MSR approach of [100, 83] by requiring the recovered pixels in a damaged block to be smoothly connected with its neighboring pixels both spatially in the same frame and temporally in the previous/following frames. If some but not all DCT coefficients are received for this block, then the estimation should be such that the recovered block be as smooth as possible, subject to the constraint that the DCT on the recovered block would produce the same values for the received coefficients. The above objectives can be formulated as an optimization problem, and the solutions under different loss patterns correspond to different interpolation filters in the spatial, temporal, and frequency domains.

### Spatial Interpolation Using Projection onto Convex Sets (POCS)

Another way of accomplishing spatial interpolation is by using the POCS method [69, 92]. The general idea behind POCS-based estimation methods is to formulate each constraint about the unknowns as a convex set. The optimal solution is the intersection of all the convex sets, which can be obtained by recursively projecting a previous solution onto individual convex sets. When applying POCS for recovering an image block, the spatial smoothness criterion is formulated in the frequency

domain, by requiring the discrete Fourier transform (DFT) of the recovered block to have energy only in several low frequency coefficients. If the damaged block is believed to contain an edge in a particular direction, then one can require the DFT coefficients to be distributed along a narrow stripe orthogonal to the edge direction, i.e., low-pass along the edge direction, and all-pass in the orthogonal direction. The requirement on the range of each DFT coefficient magnitude can also be converted into a convex set, so is the constraint imposed by any received DCT coefficient. Because the solution can only be obtained through an iterative procedure, this approach may not be suitable for real-time applications.

#### 14.5.2 Recovery of Coding Modes and Motion Vectors

As already indicated, some of the above algorithms are contingent upon the knowledge of the coding mode and MVs for P- or B-mode MBs. To facilitate decoder error concealment, the encoder may perform data partition, to pack the mode and MV information in a separate partition and transmit them with stronger error protection (cf. Sec. 14.4.1). This for example is an error resilience mode in both H.263 and MPEG-4. Still, the mode and MV information can be damaged.

One way to estimate the coding mode for a damaged MB is by collecting the statistics of the coding mode pattern of adjacent MBs, and find a most likely mode given the modes of surrounding MBs [68]. A simple and conservative approach, when the coding mode is lost, is to assume that the MB is coded in the intra-mode, and use only spatial interpolation for recovering the underlying blocks.

For estimating lost MVs, there are several simple operations [50]: (a) assuming the lost MVs to be zeros, which works well for video sequences with relatively small motion; (b) using the MVs of the corresponding MB in the previous frame; (c) using the average of the MVs from spatially adjacent MB's; (d) using the median of MVs from the spatially adjacent MB's; (e) re-estimating the MVs [33]. Typically, when a MB is damaged, its horizontally adjacent MBs are also damaged, and hence the average or mean is taken over the MVs above and below. A simpler approach is to use the MV for the MB above the damaged MB. Research has shown that no significant improvement in picture quality can be achieved when using mean or median values of more than one MV [81]. To facilitate the estimation of lost MVs, interleaved packetization (Sec. 14.3.2) can be used. For example, if even and odd rows of MB's are put into alternating packets, then the loss of a packet will damage only alternating rows.

#### 14.5.3 Syntax-Based Repair

The previous error concealment techniques work in the signal domain, assuming that the error in the transmitted stream has been detected, “questionable” data units have been discarded, and the remaining bits have been decoded into signal variables (e.g., side information, MV's, DCT coefficients) by the VLC decoder. Syntax-based repair, on the other hands, tries to detect and repair errors in the bitstream directly. This is possible, because, for any finite length data packet,

there is only a relatively small number of valid combinations of codewords. In addition, a compressed bitstream is never an i.i.d. white binary sequence. The distribution of 1's and 0's in the bitstream is often location dependent [81]. Based on such information, the decoder can repair the bitstream so that the repaired stream conforms to the syntax dictated by the encoding scheme. The use of data partition and RVLC in H.263 and MPEG-4 helps to perform syntax-based repair, as they allow more “check points” and syntax constraints, which makes the total number of valid combinations of codewords smaller than the number of all possible binary strings of the same length. One such repair scheme, known as “soft-decoding”, is described in [84]. Syntax-based repair is an effective error concealment method for a channel where bit errors dominate. For packet lossy network, where many complete packets may be lost, syntax-based repair is only used to detect errors in the received packets, but cannot be used to reconstruct the mission data.

## 14.6 Encoder and Decoder Interactive Error Control

In the techniques presented thus far, the encoder and decoder operate independently as far as combating transmission errors is concerned. Conceivably if a backward channel from the decoder to the encoder is available, better performance can be achieved if the sender and the receiver cooperate in the process of error control. This cooperation can be realized at either the source coding or transport level. At the source coder, coding parameters can be adapted based on the feedback information from the decoder. At the transport level, the feedback information can be employed to change the percentage of the total bandwidth used for FEC or retransmission. We have described transport-level adaptations in Sec. 14.3. In this section, we describe several techniques that adapt the source coding strategy based on the feedback information from the decoder. *These techniques are developed under the assumption that it is often acceptable to have errors as long as they do not last too long. Therefore, even if one cannot afford to correct the errors whenever they occur, it is important to limit the propagation scope of such errors.*

Before we proceed to describe these techniques, it is worthwhile to note how is the feedback information delivered. Typically, the feedback message is not part of the video syntax but transmitted in a different layer of the protocol stack where control information is exchanged. For example, in the H.323 standards for multimedia communications over unreliable channels [43], the control protocol H.245 [41] allows reporting of the temporal and spatial location of damaged MB's. Usually, such messages are delivered error-free, by the use of retransmission when necessary.

### 14.6.1 Coding Parameter Adaptation based on Channel Conditions

In a channel with varying bandwidth and error characteristics, it is important to match the encoding rate with the available channel bandwidth and to embed appropriate error resilience in the coded bit stream. When a source is coded at a rate

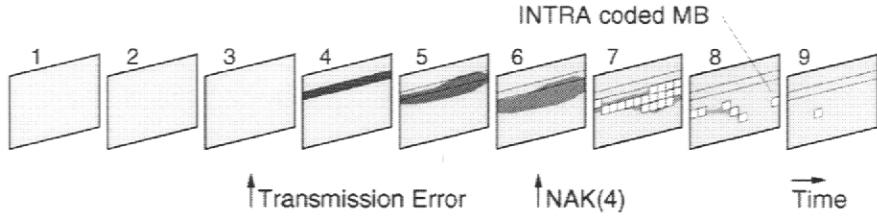
higher than the underlying channel can deliver in time, some data will be thrown away at the discretion of the network layer, typically leading to more disturbing artifacts than the source coding distortion incurred by encoding the source at a lower rate. Furthermore, when a channel is very noisy, it is better to represent a source with lower quality, leaving more bits for error protection either in the form of FEC or error resilience in the encoded stream.

There are two tasks that must be solved: First the transport controller must be able to periodically estimate/update the QoS parameters (e.g. bandwidth, delay, packet loss rates) of an established connection based on the feedback information or other transport level interactions; Secondly, given the available bandwidth and error characteristics, the encoder must determine the coding parameters (such as intra block rate, frequency of synchronization markers, range of prediction, etc. in a block-based hybrid coder) appropriately so as to meet the target bit rate and desired amount of error resilience. We have discussed the rate control issue in Sec. 9.3.4, without considering the error resilience factors. In Sec. 14.4.3, we described several approaches to coding mode selection (intra. vs. inter) that take into account loss characteristics of the network. The paper by Wu *et al.* considers both rate estimation and rate adaptation for video delivery over Internet [90]. More on how to adjust the target encoding rate based on channel feedback for the application of video streaming over the Internet is discussed in Sec. 15.3.1.

#### 14.6.2 Reference Picture Selection based on Feedback Information

One way of taking advantage of an available feedback channel is to employ reference picture selection (RPS). If the encoder learns through a feedback channel about damaged parts of a previously coded frame, it can decide to code the next P-frame not relative to the most recent, but to an older reference picture, which is known to be available in the decoder. This requires that the encoder and decoder both store multiple past decoded frames. Information about the reference picture to be used is conveyed in the bitstream. Compared to coding the current picture as an I-frame, the reduction in the coding efficiency due to use of an older reference picture is significantly lower, if the reference picture is not too far away. A study on the delay and overhead incurred by using RPS can be found in [26].

Note that using RPS does not necessarily mean extra delay in the encoder. The encoder does not have to wait for the arrival of the feedback information about the previous frame to code a current frame. Rather, it can choose to use as reference a frame before the damaged frame whenever it receives the feedback information. For example, if the information about the damage of frame  $n$  does not arrive at the encoder until the time to code frame  $n + d$ , the decoded frames between  $n + 1$  to  $n + d - 1$  would all have errors, because the decoder uses different reference frames than the encoder for these frames. By selecting frame  $n - 1$  as the reference frame to code frame  $n + d$ , error propagation will be stopped from frame  $n + d$  onwards. Of course, the longer it takes to generate and deliver the feedback information, the



Note to graphics illustrator: change "NAK(4)" to "NACK"

**Figure 14.12.** Illustration of error propagation when error tracking is used and damaged blocks are intra-coded upon the receipt of the NACK (negative acknowledgement) information. Modified from [26].

greater will be the loss in the coding efficiency.

### 14.6.3 Error Tracking based on Feedback Information

Instead of using an earlier, undamaged frame as the reference frame, the encoder can track how the damaged areas in frame  $n$  would have affected decoded blocks in frames  $n + 1$  to  $n + d - 1$ , and then perform one of the following when coding frame  $n + d$ . The encoder can (a) code the blocks that are severely damaged in frame  $n + d - 1$  using the intra-mode; (b) code the blocks in frame  $n + d$  that would have used for prediction damaged pixels in frame  $n + d - 1$  using the intra-mode; (c) avoid using the affected area in frame  $n + d - 1$  for prediction in coding frame  $n + d$ ; and (d) perform the same type of error concealment as the decoder for frames  $n + 1$  to  $n + d - 1$ , so that the encoders reference picture matches with that at the decoder, when coding frame  $n + d$ . Figure 14.12 illustrates option (a). The first three approaches only require the encoder to track the locations of damaged pixels or blocks, whereas the last approach requires the duplication of the decoder operation for frames  $n + 1$  to  $n + d - 1$ , which is more complicated. Except option (a), the decoder will recover from errors completely at frame  $n + d$ . Option (a) is the simplest to implement, but may not stop error propagation immediately at frame  $n + d$ . It also suffer higher coding gain loss than the other options. More information on error tracking, correction and fast algorithms can be found in [77, 18, 26].

### 14.6.4 Retransmission without Waiting

In order to make use of the retransmitted data, a typical implementation of the decoder will have to wait for the arrival of the requested retransmission data before processing subsequently received data. This is in fact not necessary. It is possible to use retransmission for recovering lost information without introducing delay. In the approach of Zhu [97] and Ghanbari [23], when a video data unit is damaged, say in

frame  $n$ , a retransmission request is sent to the encoder for recovering the damaged data. Instead of waiting for the arrival of retransmitted data, the damaged video part is concealed by a chosen error concealment method. Then normal decoding is continued, while a trace of the affected pixels and their associated coding information (coding mode and motion vectors) is recorded (similar to the error tracking method described in Sec. 14.6.3). Upon the arrival of the retransmitted data, say at frame  $n + d$ , the affected pixels are corrected, so that they are reproduced as if no transmission loss had occurred. The correction signal is obtained from the retransmitted data and the recorded trace.

This method can achieve lossless recovery except during the time between the information loss and the arrival of the retransmission data. During that interval, any error concealment technique (Sec. 14.5) can be applied to the damaged regions. This scheme eliminates the delay associated with conventional retransmission schemes without compromising the video quality. The price paid is the relatively high implementation complexity. Compared to the error tracking approach in Sec. 14.6, which adapts encoding operation to stop error propagation, the current approach makes use of retransmitted data to correct and stop error at the decoder.

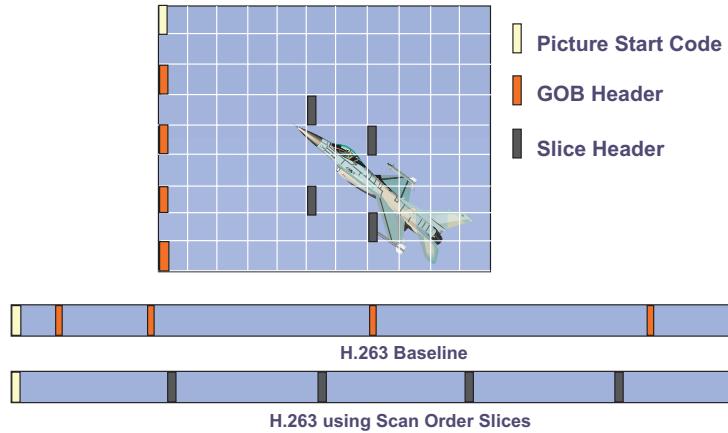
## 14.7 Error Resilience Tools in H.263 and MPEG-4

In this section, we summarize the error resilience tools defined in the H.263 and MPEG-4 standards as options. These tools fall in the error-resilient encoding category, even though the standards only define the bitstream syntax. It is worth noting that, as with any error-resilient encoding methods, these tools tend to reduce the coding efficiency slightly. However, they give a good decoder the tools to localize errors in the bitstream, to limit the image area that gets affected by these errors, and to apply appropriate error concealment methods (Sec. 14.5). How these tools are used to help the decoder operation is outside the scope of the standard, which enables competition between different decoder vendors.

### 14.7.1 Error Resilience Tools in ITU-T H.263

The initial target of the H.263 standard was to serve as a video coding standard for the H.324 system, which aims to enable video telephony over wireline and wireless modems. As described in Sec. 14.2, by applying appropriate FEC and multiplexing, both the wireline and wireless modem channels can be considered error-free. Therefore the initial focus of the H.263 development was on enhancing the coding efficiency only. As the H.323 standard develops, which targets primarily at video telephony and conferencing over the best effort Internet, the error resilience issues become a major concern. A set of annexes were developed to augment the previous H.263 standard. The following are the error resilience tools included in Version 3 of the H.263 standard (i.e., H.263++), which is approved November 2000.

**Forward Error Correction Using BCH Code (Annex H):** This tool allows including, for blocks of 492 coded video bits, a 19 bit BCH (511, 492) FEC parity



**Figure 14.13.** Scan order slice vs. GOB in H.263. With the GOB structure, the GOBs containing the moving airplane (the 3rd and 4th GOB) are more likely to be hit by errors because they contain more bits. A damaged GOB will lead to visually unacceptable distortion. With the slice structure, the same region is covered by more slices. Error resilience is improved, because a larger number of synchronization markers appear in this region and a lost slice affects a smaller region. From [81].

information in the bitstream. Together with a single additional bit to allow for resynchronization to the resulting 512 bit block structure, Annex H introduces an overhead of roughly 4% of the bitrate. The BCH (511, 492) FEC code is able to correct single, and to reliably detect double bit errors in a 512 bit block. For highly bit error prone mobile channels, Annex H is not efficient since an error burst longer than 2 bits is neither correctable, nor reliably detectable. Furthermore, the fixed structure of 492 video bit blocks does not allow to precisely align the block boundaries to synchronization markers. For these reasons, Annex H is not very useful for combating errors in the Internet or wireless networks [81].

**Flexible Synchronization Marker Insertion Using the Slice Structured Mode (Annex K):** The slice structure, when used, replaces the GOB structure. Slice headers serve as synchronization markers and interrupt in-picture prediction of MVs and DC coefficients. Whereas the GOB structure contains a fixed number of MBs, each scan order slice has about the same number of bits. If the number of bits contained in the current slice exceeds a predetermined threshold, then a new slice is created at the start of the next MB. As illustrated in Fig. 14.13, the active area in a video (in which each MB requires more bits) will have more slices and consequently more synchronization markers than in the stationary area. This facilitates error concealment at the decoder in the active region.

A slice in H.263 can also correspond to a rectangular area in a frame, aligned to MB boundaries. This enables independent segment decoding, to be discussed later.

**Reference Picture Selection (RPS) (Annex N and Annex U):** As introduced in Sec. 14.6.2, RPS enables the encoder to use, for motion compensation, an older reference picture that is known to be correctly received at the decoder's site. In H.263, RPS can be used on whole pictures, picture segments (slices or GOBs), or on individual MBs. The former two mechanisms are defined in Annex N and were introduced as an error resilience tool only, whereas the latter, defined in Annex U, was designed with both error resilience and coding efficiency in mind.<sup>4</sup> In H.263, RPS can be used with or without feedback information. Without feedback information, the video redundancy coding method described in Sec. 14.4.3 can be used, which employs a prefixed interleaved reference picture selection scheme. This technique is, however, much less efficient than feedback-based mechanisms [87, 88].

**Scalability (Annex O):** Annex O defines temporal, spatial, and SNR scalability similar to those used in MPEG-2. As described in Sec. 14.4.4, these tools can be used to enhance the error resilience if multiple transport paths with different QoS characteristics are available between the source and the destination.

**Independent Segment Decoding (ISD) (Annex R):** ISD forces encoder and decoder to treat segment (slice or GOB) boundaries like picture boundaries, thereby preventing the propagation of corrupted data in one segment to another. This method is a special case of error-resilient prediction discussed in Sec. 14.4.3. ISD in conjunction with rectangular slices has been shown to improve error resilience [88]. The overhead of ISD is roughly inversely proportional to the picture size, and is impractically high for picture sizes smaller than CIF [81].

**Data Partitioning and RVLC (Annex V):** As already introduced, with data partitioning, MB header, MV, and DCT information are no longer interleaved on a MB by MB basis, rather, they are grouped into separate partitions separated with specially designed markers. Header and motion information is coded with RVLC, whereas the DCT coefficients are coded using the non-reversible VLC table used in the baseline H.263.

**Header Repetition (Annex W):** This allows the repetition of the header of the previous frame in the header of the current frame. This enables, with a frame delay, the decoding of a frame for which the frame header is lost. Note that H.263 contains no syntax element that allows including redundant picture header information at the slice level, as is available in MPEG-4's Header Extension Code (see Sec. 14.7.2).

For a discussion on how the above tools can be incorporated in a video codec for video transmission over the Internet and wireless networks, see [81].

### 14.7.2 MPEG-4

One of the primary objectives of the MPEG-4 standard is to enable universal access. Therefore, error resilience is an important issue to consider from the very beginning.

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<sup>4</sup>By allowing an MB to choose the best matching MB among a set of past frames, the coding efficiency can be increased at the expense of motion estimation complexity.

Resync Marker	MB Index	Quant Info	HEC and Header Repetition (If Present)	Motion Vector	Motion Marker	Texture (DCT)	Rsync. Marker	...
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**Figure 14.14.** Syntactic Structure of Video Packets in MPEG-4 Error Resilient Mode with Data Partitioning. From [81].

Because of the overlapping in the development time, the set of error resilience tools defined in MPEG-4 are quite similar to those in H.263. It incorporates many of the error isolation, synchronization, data recovery and resilient entropy coding tools described in previous sections.

**Resynchronization Tools:** MPEG-4 defines several approaches that enable quick resynchronization after the occurrence of a transmission error. The *video packet* approach is very similar to the adaptive slice of MPEG-2 and Slice Structured Mode of H.263. A video packet functions in the same way as the scan order slice in H.263. The length of a video packet is no longer based on the number of MBs as in the case of the non-error resilient mode of MPEG-4 or baseline H.263, but instead on the number of bits contained in that packet (cf. Fig. 14.13). It is aimed at providing periodic resynchronization throughout the bitstream.

A *resynchronization marker* is placed at the start of the header of each video packet. This marker is distinguishable from all other possible VLC codewords. The header contains the information necessary to restart the decoding process, including the address of the first MB contained in this packet and the QP for the first MB. These are followed by a single bit *Header Extension Code* (HEC). If the HEC is equal to one, then the following information that is already specified in the VOP header is duplicated in this packet header: timing information, temporal reference, VOP prediction type, and some other information. The header extension feature enables the decoder to correctly utilize data contained in the current packet, even if the packet containing the VOP header is lost. It also enables cross-checking since all packets in the same VOP should share the same QP, time stamp, etc.

In addition to using variable length video packets, a method called *fixed interval synchronization* has also been adopted by MPEG-4. This method requires that the start of a video packet appears only at allowable, fixed interval locations in the bitstream. This helps to avoid the problems associated with start code emulation.

**Data Partitioning:** To achieve better error isolation in the video packet and fixed interval synchronization approaches, MPEG-4 also uses data partitioning as an error resilience tool. Data partitioning within a video packet reorganizes the data such that motion vectors and related syntactic elements are first transmitted, followed by syntactic elements like CBP and the DCT coefficients. Figure 14.14 illustrates the syntactic structure of a video packet with data partitioning. Note that the texture (DCT) partition in the structure may be coded with RVLC.

**NEWPRED mode:** This tool is very similar to the H.263 RPS Mode (Annex N) and the Slice Structured Mode (Annex K). When the NEWPRED mode is turned on, the reference used for inter prediction by the encoder will be updated adaptively according to the feedback messages from the decoder. These messages indicate which NEWPRED (ND) segments (which can either be an entire frame, or in MPEG-4 language, a VOP, or the content of a packet, typically one slice) have been successfully decoded, and which ND segments have not. Based on the feedback information the encoder will either use the most recent ND segment, or a spatially corresponding but older ND segment for prediction.

**Data Recovery in Conjunction with RVLC and Data Partitioning:** The MPEG-4 error resilience mode utilizes RVLC for better recovery of DCT data. The usage of RVLC is signaled at the Video Object Layer (VOL). When RVLC is used for DCT data, the bitstream is decoded in the forward direction first. If no errors are detected, the bitstream is assumed to be valid. If an error is detected however, two-way decoding is applied, and the portion of the packet between the first MB in which an error was detected in both the forward and backward directions should not be used.

## 14.8 Summary

In this chapter, we described various error-control mechanisms that can be used to combat transmission errors in real-time video communications. We focused on techniques developed for block-based hybrid coders, especially those that have been adopted in the H.263 and MPEG-4 standards. The key features of different categories of error-control techniques are summarized below.

### Transport-Level Error Control

Transport-level error control mechanisms are most important and guarantees a basic level of quality. Error control can be applied through FEC, interleaved packetization, and when feasible, properly constrained retransmission. Different levels of protection can be applied to parts of coded bits that are of varying importance. Such unequal error protection is an effective way of employing a limited amount of redundancy to achieve acceptable quality.

### Error-Resilient Encoding

Techniques in this category achieve error resilience by adding a certain amount of redundancy in the coded bit streams at the source coder. Some are aimed at guaranteeing a basic level of quality and providing a graceful degradation upon the occurrence of transmission errors (e.g. layered coding and MDC), some help to prevent error propagation (e.g. error-resilient prediction), while others help to detect or recover from bit errors (e.g. error isolation and robust entropy coding). Some techniques require close interaction between the source coder and transport

layer (e.g., layered coding with unequal error protection, error resilient prediction with interleaved packetization), while others assume that different substreams are treated equally in the transport layer (e.g., MDC). Note that some techniques (error isolation, RVLC, joint source and channel coding as described in Sec. 14.4.6) are only useful for combating bit errors and are not helpful for recovering from packet loss.

### **Decoder Error Concealment**

All the error concealment techniques for texture recover the lost information by making use of the temporal and spatial smoothness property of image and video signals. The MSR technique enforces the smoothness constraint by minimizing the roughness of the reconstructed signal. The motion-compensated temporal interpolation method and spatial interpolation method can be considered as a special case of the MSR approach when only the temporal or spatial difference measure is minimized. The POCS method, on the other hand, iteratively projects the reconstructed image block onto the convex sets determined by the received coefficients and the smoothness constraint determined from the estimated edge direction of the block. Although generally giving more accurate results than the MSR, the POCS method is computationally more intensive, as it requires many iterations. Simple and yet effective, motion-compensated temporal interpolation has been employed in commercial systems. More complex methods that exploit spatial correlation (spatial interpolation, MSR and POCS) can offer additional improvement only when the underlying scene has very complex motion or is experiencing scene changes.

Recovery of coding mode and motion information is more difficult than the texture information, because there are less correlation among the coding modes and motion vectors of adjacent MB's. When possible, such information should be more heavily protected than the texture information, through the use of data partitioning and unequal error protection.

### **Encoder and Decoder Interactive Error Control**

Such techniques are applicable when there is a backward channel from the decoder to the encoder so that the decoder can inform the encoder which part of the coded information is lost. The three techniques presented, reference picture selection, error tracking, and retransmission without waiting, all aim to stop error propagation after the feedback information is received. The first two methods, reference picture selection and error tracking, avoid using damaged regions for future prediction at the encoder, while the third, retransmission without waiting, correct the error based on the retransmitted information. When only a general information about, say packet loss rate, is available, but not which part of the information is lost, one can try to adjust the coding parameters so as to reduce data loss and to suppress the effect of any lost data.

**Choice of Different Techniques**

Transport-level error control mechanisms are most important and guarantees a basic level of quality. Error resilient encoding and decoder error concealment are necessary to further improve the quality in very lossy environments such as the Internet and wireless networks. Depending on channel error characteristics, system configuration and requirements, some techniques are more effective than others. The burstiness of transmission errors has a significant impact on the choice of algorithms. For a channel with very long bursts, error-resilient encoding techniques may not be appropriate. This is because the redundancy introduced in such coding methods is wasted when the channel is error-free, and such redundancy is not very helpful when a burst error occurs. Retransmission, when properly constrained, may be more suitable since it only introduces the overhead when needed. The existence of a backward channel from the decoder to the encoder also affects the deployment of some schemes. In applications such as broadcast where there is no backward channel, none of the interactive error control techniques can be applied. Similarly, in video streaming applications, videos are usually pre-compressed and stored in a server, so that it is not possible to adapt encoder operations based on network conditions. The error concealment techniques can be applied in any circumstances. However, the effectiveness of such techniques is limited by the available information. Also some techniques may be either too complicated for cost effective implementation or introduce unacceptable processing delay for real-time applications. Aside from the delay and complexity issues, one important criterion for comparing different schemes is the required redundancy in the source coder and/or the transport layer, to achieve the same degree of error protection.

**14.9 Problems**

- 14.1** What are the main differences in the requirements for transporting audio and video vs. data files?
- 14.2** What is the typical delay requirement for an interactive video conferencing application? What about video streaming?
- 14.3** What are the contributing factors to the end-to-end delay between a video source and the displayed video at the receiver?
- 14.4** We divided the approaches for error control in video communications into four categories. What are they? Describe the advantages and limitations of each.
- 14.5** What are the major differences between circuit-switching networks and packet-based networks? What would you choose as your error control mechanisms in either case?
- 14.6** What are the major difference between LC and MDC? How do they differ in terms of network requirements? Describe network conditions under which LC

may be preferred over MDC, and vice versa.

- 14.7** What would you do differently in the implementation for motion compensated prediction, if the network changed from reliable (almost error free) to unreliable (with high packet losses)?
- 14.8** In principle, lost information is un-recoverable. What make error concealment techniques feasible for video?
- 14.9** What are the major assumptions in error concealment techniques for recovering damaged texture data?
- 14.10** Describe the principle of maximally smooth recovery method and the POCS method for error concealment. What are the pros and cons of each?
- 14.11** Consider an Internet video conferencing application. Assume the compressed video streams are packetized, and that a packet is either lost (due to over long delay or mis-delivery or detected bit errors within a packet) or received intact. Assume that the effective packet loss rate is quite high, about 10%, and that there is no feedback channel available. The average end-to-end delay between the source and destination is about 50 ms. Further assume that the maximum end-to-end delay allowed between the source and destination is 150 ms. What would you propose as mechanisms to control and conceal the effect of packet loss? What approaches described in this chapter would not be applicable or effective?
- 14.12** Consider the previous scenario again, but assume that the network has a lower packet loss rate, say 1%. Would your solution be different?
- 14.13** Again, consider the scenario in Prob. 14.11, but assume this time that a feedback channel is available between the destination and the source and that it usually takes 50 ms to deliver a feedback message. What are the solutions you propose in this case? What would you send on the feedback channel?
- 14.14** Now consider video transport over a wireless network with high bit error rates and bursty errors and narrow bandwidth (say 64 Kbps). You have the choice to further examine a packet even if the network or decoder has detected bit errors within a packet based on the FEC code. What are the solutions you propose in this case? Compared to the Internet case, would you use a shorter or longer packet?

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# STREAMING VIDEO OVER THE INTERNET AND WIRELESS IP NETWORKS

Recent developments in computing technology, compression technology, high bandwidth storage devices, and high-speed networks have made it feasible to provide real-time multimedia services over the Internet. Real-time multimedia, as the name implies, has timing constraints. For example, audio and video data must be played out continuously. If the data does not arrive in time, the playout process will pause, which is annoying to human ears and eyes.

Real-time transport of live video or stored video is the predominant part of real-time multimedia. In this chapter, we are concerned with video streaming, which refers to real-time transmission of stored video.<sup>1</sup> Video streaming typically has bandwidth, delay and loss requirements. However, the current best-effort Internet does not offer any QoS guarantees to streaming video over the Internet. In addition, the heterogeneity of the Internet makes it difficult to efficiently support video multicast while providing service flexibility to meet a wide range of QoS requirements from the users. Furthermore, for streaming video over wireless IP networks, fluctuations of wireless channel conditions tend to greatly degrade video quality. Thus, streaming video over the Internet and wireless IP networks poses many challenges.

To deal with the challenges, extensive efforts have been contributed. With the aim of providing a global view on this field, we cover seven areas regarding streaming video, namely, video compression, application-layer QoS control for streaming video, continuous media distribution services, streaming servers, media synchronization mechanisms, protocols for streaming media, and streaming video over wireless IP networks. For each area, we address the particular issues and review representative approaches and mechanisms.

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<sup>1</sup> Video streaming implies that the video content need not be downloaded in full, but is being played out while parts of the content is being received and decoded.

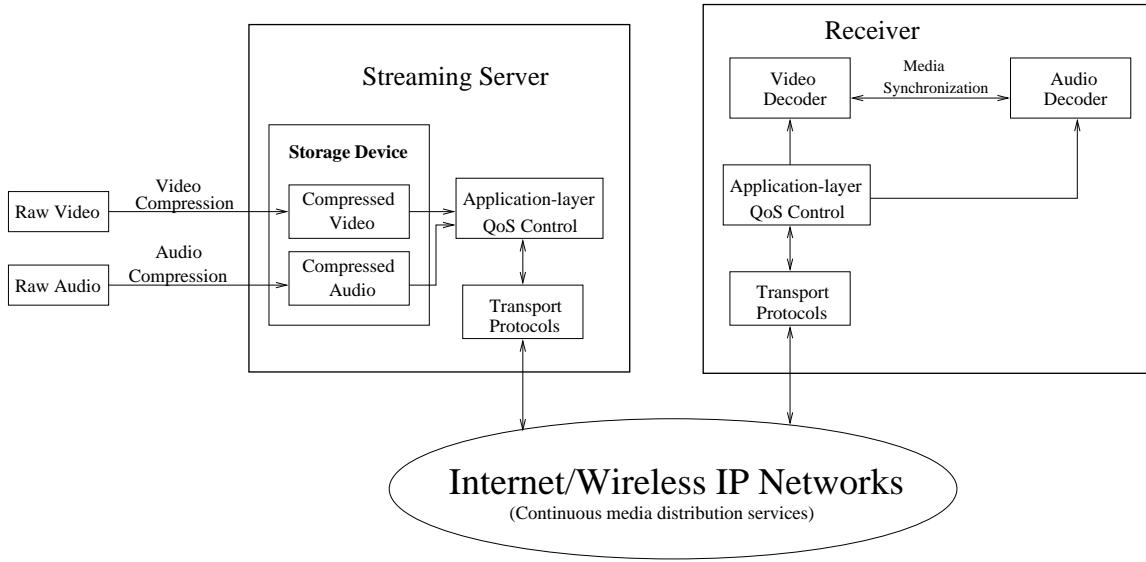


Figure 15.1. An architecture for video streaming.

## 15.1 Architecture for Video Streaming Systems

A video streaming system typically consists of seven building blocks, as illustrated in Fig. 15.1. In Fig. 15.1, raw video and audio data are pre-compressed by *video compression* and audio compression algorithms and then saved in storage devices. Upon the client's request, a *streaming server* retrieves compressed video/audio data from storage devices and then the *application-layer QoS control* module adapts the video/audio bit-streams according to the network status and QoS requirements. After the adaptation, the *transport protocols* packetize the compressed bit-streams and send the video/audio packets to the Internet or *wireless IP networks*. Packets may be dropped or experience excessive delay inside the Internet due to congestion; on wireless IP segments, packets may be damaged by bit errors. To improve the quality of video/audio transmission, *continuous media distribution services* are deployed in the Internet. For packets that are successfully delivered to the receiver, they first pass through the transport layers and then are processed by the application layer before being decoded at the video/audio decoder. To achieve synchronization between video and audio presentations, *media synchronization mechanisms* are required. From Fig. 15.1, it can be seen that the seven areas are closely related and they are coherent constituents of the video streaming architecture.

Next, we briefly describe the seven areas, respectively.

1. **Video compression:** Raw video needs to be compressed before transmission so that efficiency can be achieved. Video compression schemes can be

classified into two categories: scalable and non-scalable video coding. Since scalable video is capable of gracefully coping with the bandwidth fluctuations in the Internet [45], we are primarily concerned with scalable video coding techniques. We have covered this topic previously in Chap. 11, and thus will only provide a brief review of this topic here.

2. **Application-layer QoS control for streaming video:** To cope with changing network conditions and changing presentation quality requested by the users, various application-layer QoS control techniques have been proposed [19, 69, 77]. The application-layer techniques include congestion control and error control. Their respective functions are as follows. Congestion control is employed to prevent packet loss and reduce delay. Error control, on the other hand, is to improve video presentation quality in presence of packet loss. Techniques for error control for general video communication applications have been described in Chapter 14. Here, we will focus on those that are effective for video streaming applications.
3. **Continuous media distribution services:** In order to provide quality multimedia presentations, the support from the network is important. This is because network support can reduce transport delay and packet loss ratio. Built on top of the Internet (IP protocol), continuous media distribution services are able to achieve QoS and efficiency for streaming video/audio over the best-effort Internet. Continuous media distribution services include network filtering, application-level multicast, and content replication.
4. **Streaming servers:** Streaming servers play a key role in providing streaming services. To offer quality streaming services, streaming servers are required to process multimedia data under timing constraints and support interactive control operations such as pause/resume, fast forward and fast backward. Furthermore, streaming servers need to retrieve media components in a synchronous fashion. A streaming server typically consists of three subsystems, namely, a communicator (e.g., transport protocols), a operating system, and a storage system.
5. **Media synchronization mechanisms:** Media synchronization is a major feature that distinguishes multimedia applications from other traditional data applications. With media synchronization mechanisms, the application at the receiver side can present various media streams in the same way as they were originally captured. An example of media synchronization is that the movements of the lips of a speaker correspond to the presented audio.
6. **Protocols for streaming media:** Protocols are designed and standardized for communication between clients and streaming servers. Protocols for streaming media provide such services as network addressing, transport, and session control. According to their functionalities, the protocols can be classified in three categories: (1) network-layer protocol such as Internet Protocol

(IP), (2) transport protocol such as User Datagram Protocol (UDP), and (3) session control protocol such as Real-Time Streaming Protocol (RTSP).

7. **Streaming video over wireless IP networks:** Fluctuations of wireless channel conditions pose many challenges to provide QoS for video transmission over wireless IP networks. To address this problem, an adaptive framework has been proposed [85]. The adaptive framework includes: (1) scalable video representations, (2) network-aware video applications, and (3) adaptive services.

The remainder of this chapter is organized as follows. Section 15.2 discusses video compression techniques. In Section 15.3, we present application-layer QoS control mechanisms for streaming video. Section 15.4 describes continuous media distribution services. In Section 15.5, we discuss key issues on design of streaming servers. Section 15.6 presents various media synchronization mechanisms. In Section 15.7, we overview the key protocols for streaming video. Section 15.8 presents an adaptive framework for streaming video over wireless IP networks. Finally, a summary is given in Section 15.9.

## 15.2 Video Compression

Since raw video consumes large amount of bandwidth, raw video needs to be compressed before transmission so that efficiency can be achieved. Video compression schemes can be classified into two categories: scalable and non-scalable video coding. As described in Chap. 11, a scalable video coding scheme produces a compressed bit-stream, parts of which are decodable. Compared with decoding the complete bit-stream, decoding part of the compressed bit-stream produces pictures with degraded quality, or smaller image size, or smaller frame rate. It has been shown that scalable video is capable of gracefully coping with the bandwidth fluctuations in the Internet [40, 45]. In contrast, non-scalable video is more susceptible to bandwidth variations since it cannot adapt its video representation to bandwidth fluctuations [45]. Furthermore, scalable video representation is an effective means to achieve efficiency and flexibility for multicasting video over heterogeneous networks (e.g., networks with different access link bandwidth) [40, 45]. For these reasons, all the streaming video services employ scalable video coding techniques. For a more detailed discussion on various scalable coding methods, see Chap. 11. Next, we present the application-layer QoS control mechanisms, which adapt the video bit-streams according to the network status and QoS requirements.

## 15.3 Application-layer QoS Control for Streaming Video

Application-layer QoS Control is to maximize video quality in the presence of packet loss and the changes in available bandwidth. The application-layer QoS control techniques include congestion control and error control. These techniques are employed by the end systems and do not require the QoS support from the routers/networks.

We organize the rest of this section as follows. In Section 15.3.1, we survey the approaches for congestion control. Section 15.3.2 describes mechanisms for error control.

### 15.3.1 Congestion Control

Bursty loss and excessive delay have devastating effect on video presentation quality and they are usually caused by network congestion. One solution to reduce packet loss and delay is by applying congestion control at the source and/or receiver.

There are two mechanisms for congestion control: rate control and rate shaping. Rate control attempts to minimize network congestion and the amount of packet loss by matching the rate of the video stream to the available network bandwidth. On the other hand, rate shaping is used to force the source to send the video stream at the rate dictated by the rate control algorithm [19]. We present the approaches for rate control and rate shaping in Sections 15.3.1 and 15.3.1, respectively.

#### Rate Control

Rate control is a technique that determines the sending rate of video traffic based on the estimated available bandwidth in the network.<sup>2</sup> Existing rate control schemes can be classified into three categories: source-based, receiver-based, and hybrid rate control, which are presented as follows.

**Source-based Rate Control:** Under the source-based rate control, the sender is responsible for adapting the video transmission rate. Typically, feedback is employed by source-based rate control mechanisms. Based upon the feedback information about the network, the sender could regulate the rate of the video stream. The source-based rate control can be applied to both unicast [81] and multicast [8].

For unicast video, the existing source-based rate control mechanisms follow two approaches: probe-based and model-based approach.

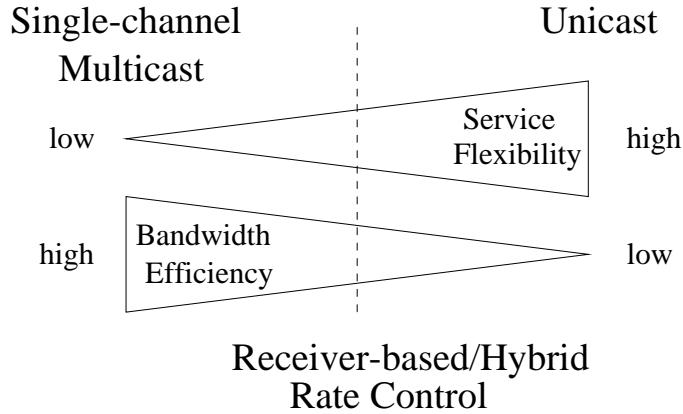
The probe-based approach is based on probing experiments. Specifically, the source probes for the available network bandwidth by adjusting the sending rate in a way that could maintain the packet loss ratio  $p$  below a certain threshold  $P_{th}$  [81]. There are two ways to adjust the sending rate: (1) additive increase and multiplicative decrease [81], and (2) multiplicative increase and multiplicative decrease [74].

The model-based approach is based on a throughput model of a TCP connection. Specifically, the throughput of a TCP connection can be characterized by the following formula [22]:

$$\lambda = \frac{1.22 \times MTU}{RTT \times \sqrt{p}}, \quad (15.3.1)$$

---

<sup>2</sup>Earlier in Sec. 9.3.4, we have defined rate control problem as i) determining the appropriate encoding rate and ii) adjusting the coding parameters to meet the target rate. Rate control in this section refers to the first task only.



**Figure 15.2.** Trade-off between efficiency and flexibility.

where  $\lambda$  is the throughput of a TCP connection,  $MTU$  (Maximum Transmission Unit) is the packet size used by the connection,  $RTT$  is the round trip time for the connection,  $p$  is the packet loss ratio experienced by the connection. Under the model-based rate control, Eq. (15.3.1) is used to determine the sending rate of the video stream. Thus, the video connection could avoid congestion in a similar way to that of TCP and it can compete fairly with TCP flows. For this reason, the model-based rate control is also called “TCP-friendly” rate control [22].

For multicast under the source-based rate control, the sender uses a single channel to transport video to the receivers. Such multicast is called “single-channel multicast”. For single-channel multicast, only the probe-based rate control can be employed [8].

Single-channel multicast is efficient since all the receivers share one channel. However, single-channel multicast is unable to provide flexible services to meet the different demands from receivers with various access link bandwidth. In contrast, if multicast video were to be delivered through individual unicast streams, the bandwidth efficiency is low but the services could be differentiated since each receiver can negotiate the parameters of the services with the source. Unicast and single-channel multicast are two extreme cases shown in Fig. 15.2. To achieve good trade-off between bandwidth efficiency and service flexibility for multicast video, receiver-based and hybrid rate control are proposed which will be described next.

**Receiver-based Rate Control:** Under the receiver-based rate control, the receivers regulate the receiving rate of video streams by adding/dropping channels while the sender does not participate in rate control. Typically, receiver-based rate control is only applied to layered multicast video rather than unicast video.

Similar to the source-based rate control, the existing receiver-based rate control mechanisms follow two approaches: probe-based and model-based approach. The basic probe-based rate control consists of two parts [45]:

1. When no congestion is detected, a receiver probes for the available bandwidth by joining a layer, resulting in increase of its receiving rate. If no congestion is detected after the joining, the join-experiment is successful. Otherwise, the receiver drops the newly added layer.
2. When congestion is detected, a receiver drops a layer, resulting in reduction of its receiving rate.

Unlike the probe-based approach which implicitly estimates the available network bandwidth through probing experiments, the model-based approach uses explicit estimation for the available network bandwidth. The model-based approach is based on the throughput model of a TCP connection (i.e., Eq. 15.3.1), which is described in the source-based rate control. Thus, the model-based rate control is also “TCP-friendly”.

**Hybrid Rate Control:** Under the hybrid rate control, the receivers regulate the receiving rate of video streams by adding/dropping channels while the sender also adjusts the transmission rate of each channel based on feedback from the receivers. An example is Destination Set Grouping [13].

### Rate Shaping

Rate shaping is a technique, through which the rate of pre-compressed video bitstreams can be adapted to the target rate constraint. A rate shaper is an interface (or filter) between the compression layer and the network transport layer or between two network segments, with which the video stream can be matched to the available network bandwidth.

There are many types of rate shapers or filters, which include [87]:

1. **Codec filter:** A codec filter is to decompress and compress a video stream. It is commonly used to perform transcoding between different compression schemes. Depending on the compression scheme used, transcoding could be simplified without full decompression and recompression.
2. **Frame-dropping filter:** The frame-dropping filter can distinguish the frame types (e.g., I-, P-, and B-frame in MPEG) and drop frames according to importance. For example, the dropping order would be first B-frames, then P-frames, and finally I-frames. The frame-dropping filter is used to reduce the data rate of a video stream by discarding a number of frames and transmitting the remaining frames at a lower rate. The frame-dropping filter could be used at the source [90] or used in the network (see Section 15.4.1).
3. **Layer-dropping filter:** The layer-dropping filter can distinguish the layers and drop layers according to importance. The dropping order is from the highest enhancement layer down to the base layer.
4. **Frequency filter:** A frequency filter performs operations on the compression layer. Specifically, it operates in the frequency domain (i.e., DCT coefficients).

Frequency filtering mechanisms include low-pass filtering, color reduction filtering and color-to-monochrome filtering.

Low-pass filtering is to discard the DCT coefficients of the higher frequencies. A color reduction filter performs the same operation as a low-pass filter except that it only operates on the chrominance information in the video stream. A color-to-monochrome filter removes all color information from the video stream. In MPEG, this is done by replacing each chrominance block by an empty block.

Unlike the frame-dropping filter, the frequency filter reduces the bandwidth without affecting the frame rate. Its cost is reduction in presentation quality of the resulting frame.

5. **Re-quantization filter:** The re-quantization filter performs operations on the compression layer (i.e., DCT coefficients). The filter first extracts the DCT coefficients from the compressed video stream through dequantization. Then the filter re-quantizes the DCT coefficients with a larger quantizer stepsize, resulting in rate reduction.

As a summary, the purpose of congestion control is to prevent packet loss. The fact that packet loss is inevitable in the Internet and may have significant impact on perceptual quality prompts the need to design mechanisms to maximize video presentation quality in presence of packet loss. Error control is such a mechanism, which will be presented next.

### 15.3.2 Error Control

Techniques for error control in video communication have been discussed in Chap. 14. There, we have classified error control techniques into four categories: transport level error control, including FEC and delay-constrained retransmission, encoder error-resilient coding, decoder error concealment, and encoder and decoder interactive error control. Here, we describe several techniques that are effective for video streaming applications.

#### FEC

The principle of FEC is to add redundant information so that original message can be reconstructed in presence of packet loss. For Internet applications, block codes are typically used for channel coding. Specifically, a video stream is first chopped into segments, each of which is packetized into  $k$  packets; then for each segment, a block code (e.g., Tornado code [1]) is applied to the  $k$  packets to generate a  $n$ -packet block, where  $n > k$ . To perfectly recover a segment, a user only needs to receive any  $k$  packets in the  $n$ -packet block.

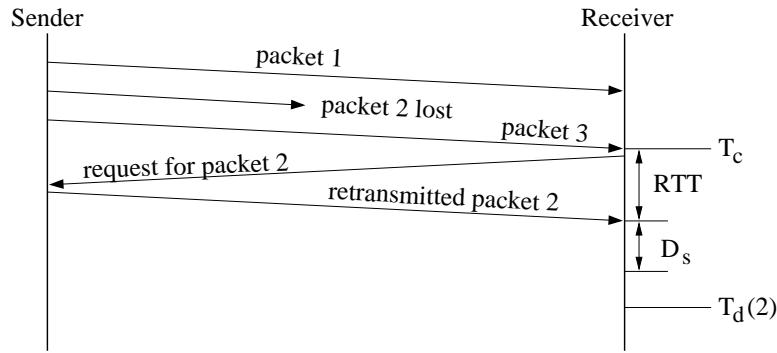


Figure 15.3. Timing diagram for receiver-based control.

### Delay-constrained Retransmission

Retransmission is usually dismissed as a method for transporting real-time video since a retransmitted packet may miss its play-out time. However, for streaming applications, if the one-way trip time is short with respect to the maximum allowable delay, *delay-constrained retransmission* is a viable option for error control (cf. Sec. 14.3.3).

For unicast, based on who determines whether to send and/or respond to a retransmission request, three delay-constrained retransmission mechanisms have been proposed: receiver-based, sender-based, and hybrid control.

The objective of the receiver-based control is to minimize the request of retransmissions that will not arrive timely for display. Under the receiver-based control, the receiver executes the following algorithm.

When the receiver detects the loss of packet  $N$ :

```
if ( $T_c + RTT + D_s < T_d(N)$ )
    send the request for packet  $N$  to the sender
```

where  $T_c$  is the current time,  $RTT$  is an estimated round trip time,  $D_s$  is a slack term, and  $T_d(N)$  is the time when packet  $N$  is scheduled for display. The slack term  $D_s$  could include tolerance of error in estimating  $RTT$ , the sender's response time, and the receiver's decoding delay. The timing diagram for receiver-based control is shown in Fig. 15.3, where  $D_s$  is only the receiver's decoding delay.

The objective of the sender-based control is to suppress retransmission of the packets that will miss their display time at the receiver. Under the sender-based control, the sender executes the following algorithm.

When the sender receives a request for packet  $N$ :

```
if ( $T_c + RTT/2 + D_s < T'_d(N)$ )
    retransmit packet  $N$  to the receiver
```

where  $T'_d(N)$  is an estimate of  $T_d(N)$ .

The hybrid control is a simple combination of the sender-based control and the receiver-based control.

In the multicast case, retransmission has to be restricted within closely located multicast members. This is because one-way trip times between these members tend to be small, making retransmissions effective. Typically, a logical tree is configured to limit the number/scope of retransmission requests and to achieve recovery among closely located multicast members [86]. In addition, to address heterogeneity problem, a receiver-initiated mechanism for error recovery can be adopted [86].

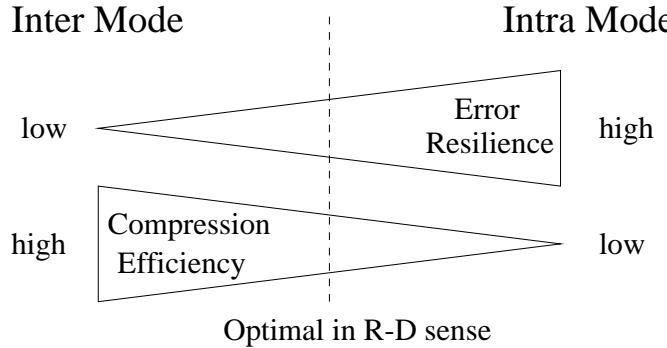
### Error Resilient Encoding

As described in Sec. 14.4, error-resilient schemes address loss recovery from the compression perspective. Specifically, they attempt to prevent error propagation or limit the scope of the damage (caused by packet losses) on the compression layer. The standardized error-resilient tools include re-synchronization marking, data partitioning, and reversible variable length coding (RVLC) [35, 68] (cf. Sec. 14.4). These techniques are targeted at bit-error dominant environments like wireless channels. For Internet video applications, which uses packet-based transport, the above techniques may not be useful since a packet loss may cause the loss of all the motion data and its associated shape/texture data. Furthermore, the boundary of a packet already provides a synchronization point in the variable-length coded bit-stream at the receiver side, making the re-synchronization marking somewhat redundant.

The two techniques that are more promising for robust Internet video transmission are *optimal mode selection* and *multiple description coding*.

**Optimal Intra/Inter Mode Selection:** The effect of lost packets on the video presentation quality depends on the coding scheme used at the source, the network congestion status and the error concealment scheme used at the receiver. High-compression coding algorithms usually employ inter-coding (i.e., prediction) to achieve efficiency. With these coding algorithms, loss of a packet may degrade video quality over a large number of frames, until the next intra-coded frame is received. Intra-coding can effectively stop error propagation at the cost of efficiency while inter-coding can achieve compression efficiency at the risk of error propagation. Therefore, a good mode selection between intra mode and inter mode should be in place to enhance the robustness of the video compressed by both intra- and inter-coding (see Fig. 15.4).

For video communication over a network, a coding algorithm such as H.263 or MPEG-4 [35] usually regulates its output rate so as to match the output rate to the available bandwidth. The objective of rate-regulated compression algorithms is to maximize the video quality under the constraint of a given bit budget. This can be achieved by choosing a mode that minimizes the quantization distortion between the original frame or macroblock (MB) and the reconstructed one under a given bit budget [52], which is the so-called R-D optimized mode selection (cf. Sec. 9.3.3). We refer such R-D optimized mode selection as the classical approach. The classical approach is not able to achieve global optimality under the error-prone environment



**Figure 15.4.** Illustration of optimal mode selection.

since it does not consider the network congestion status and the receiver behavior.

To address this problem, Wu et al. [83] proposed an end-to-end approach to R-D optimized mode selection. It considers all three factors that have impact on the video presentation quality at the receiver: (1) the source behavior, (2) the path characteristics, and (3) the receiver behavior. Due to the consideration of the network congestion status and the receiver behavior, this approach is shown to be capable of offering superior performance over the classical approach for Internet video applications [83]. Other approaches for mode selection with consideration of transmission error effects have been previously introduced in Sec. 14.4.3.

**Multiple Description Coding:** Multiple description coding (MDC) (Sec. 14.4.5) is another way of trading off compression efficiency with robustness to packet loss. Compared to other error-resilient techniques described in Sec. 14.4, it is more effective at handling packet losses. To carry MDC streams over the Internet, bits from different descriptions should be packetized separately, and preferably transmitted through different routes. Alternatively, the packets containing different descriptions should be properly interleaved, so that traffic congestion will not lead to the loss of both descriptions.

### Error Concealment

When packet loss is detected, the receiver can employ error concealment to conceal the lost data and make the presentation more pleasing to human eyes. Since human eyes can tolerate a certain degree of distortion in video signals, error concealment is a viable technique to handle packet loss [80].

As described in Sec. 14.5, there are two basic approaches for error concealment, namely, spatial and temporal interpolation. In spatial interpolation, missing pixel values are reconstructed using neighboring spatial information, whereas in temporal interpolation, the lost data is reconstructed from data in the previous frames. Typically, spatial interpolation is used to reconstruct missing data in intra-coded

frames while temporal interpolation is used to reconstruct missing data in inter-coded frames.

For streaming applications, which require real-time decoding, most of the techniques described in Sec. 14.5 are too complex. The most feasible and effective technique is motion-compensated temporal interpolation (cf. Sec. 14.5.1), by which the receiver replaces the corrupted block with the block from the previous frame pointed by a motion vector. The motion vector is copied from its neighboring block when available, otherwise the motion vector is set to zero.

In sum, we have reviewed various application-layer QoS Control techniques. These techniques are employed by the end systems and do not require the QoS support from the routers/networks. If the networks support QoS for video streaming, the performance will be further enhanced. Next, we present network QoS support mechanisms, which are based on the best-effort Internet.

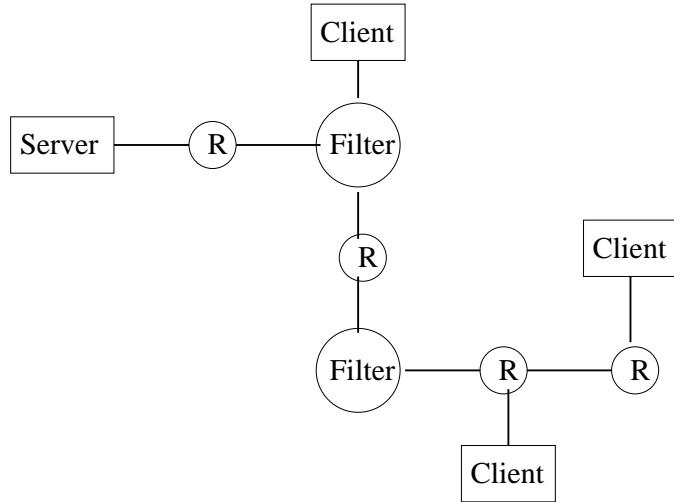
## 15.4 Continuous Media Distribution Services

In order to provide quality multimedia presentations, the support from the network is important. This is because network support can reduce transport delay and packet loss ratio. Streaming video and audio are classified as continuous media because they consist of a sequence of media quanta (such as audio samples or video frames), which convey meaning only when presented in time. Built on top of the Internet (IP protocol), continuous media distribution services are designed to provide QoS and achieve efficiency for streaming video/audio over the best-effort Internet. Continuous media distribution services include network filtering, application-level multicast, and content replication, which are presented in Sections 15.4.1 to 15.4.3, respectively.

### 15.4.1 Network Filtering

As a congestion control technique, network filtering is aimed at maximizing video quality during network congestion. As described in Section 15.3.1, the filter at the video server can adapt the rate of video streams according to the network congestion status. However, the video server may be too busy to handle the computation required to adapt each unicast video stream. Hence, the service providers may like to place filters in the network [32]. Figure 15.5 illustrates an example of placing filters in the network. In Fig. 15.5, the nodes labeled “R” denote routers that have no knowledge of the format of the media streams and may randomly discard packets. The “Filter” nodes receive the client’s requests and adapt the stream sent by the server accordingly. This solution allows the service provider to place the filter on the nodes that connect to network bottlenecks, and there can be multiple filters along the path from a server to a client.

To illustrate the operations of filters, a system model is presented in Fig. 15.6 [32]. The model consists of the server, the client, at least one filter, and two virtual channels between them. Of the two virtual channels, one is for control and the other



**Figure 15.5.** Filters placed inside the network.



**Figure 15.6.** A system model of network filtering.

is for data. The same channels exist between any pair of filters. The control channel is bidirectional, which can be realized by TCP connections. The model shown in Fig. 15.6 allows the client to communicate with only one host (the last filter), which will either forward the requests or act upon them. The operations of a filter on the data plane include: (1) receiving video stream from server or previous filter, and (2) sending video to client or next filter at the target rate. The operations of a filter on the control plane include: (1) receiving requests from client or next filter, (2) acting upon requests, and (3) forwarding the requests to its previous filter.

Typically, frame-dropping filters (see Section 15.3.1) are used as network filters. The receiver can change the bandwidth of the media stream by sending requests to the filter to increase or decrease the frame dropping rate. To facilitate decisions on whether the filter should increase or decrease the bandwidth, the receiver continuously measures the packet loss ratio  $p$ . Based on the packet loss ratio, a rate shaping mechanism is designed as follows [32]. If the packet loss ratio is higher than a threshold  $\alpha$ , the client will ask the filter to increase the frame dropping rate. If the packet loss ratio is less than another threshold  $\beta$  ( $\beta < \alpha$ ), the receiver will ask the filter to reduce the frame dropping rate.

The advantages of using frame-dropping filters inside the network include:

(1) Improved video quality. For example, when a video stream flows from an upstream link with larger available bandwidth to a downstream link with smaller available bandwidth, use of a frame-dropping filter at the connection point (between the upstream link and the downstream link) could help improve the video quality. This is because the filter understands the format of the media stream and can drop packets in a way that gracefully degrades the stream's quality instead of corrupting the flow outright.

(2) Bandwidth efficiency. This is because the filtering can help to save network resources by discarding frames that are late or have been corrupted by loss of constituent packets.

Note that network filtering is targeted at unicast (i.e., point-to-point) media streams. Next, we present a multicast mechanism for streaming media.

### 15.4.2 Application-level Multicast

The Internet's original design, while well suited for point-to-point applications like e-mail, file transfer, and Web browsing, fails to effectively support large-scale content delivery like streaming-media multicast. In an attempt to address this shortcoming, a technology called "IP multicast" was proposed ten years ago. As an extension to the IP layer, IP multicast is capable of providing efficient multipoint packet delivery. To be specific, the efficiency is achieved by the means that ensures that one and only one copy of an original IP packet (sent by the multicast source) is transported along any physical path in the IP multicast tree. However, with a decade's worth of research and development, IP Multicast is still plagued with concerns pertaining to scalability, network management, deployment and support for higher layer functionality such as error, flow and congestion control. To address these concerns, an application-level multicast mechanism was proposed [21]. The application-level multicast is aimed to build a multicast service on top of the Internet and traditional IP networks.

The application-level multicast mechanism would allow independent content delivery service providers (CSPs), Internet service providers (ISPs), or enterprises to build their own Internet multicast networks and interconnect them into larger, world-wide "media multicast networks". That is, the media multicast networks could support "peering relationships" at the application level or the streaming-media/content layer, where "content backbones" interconnect service providers. Hence, much as the Internet is built from an interconnection of networks enabled through IP-level peering relationships among ISPs, the media multicast networks can be built from an interconnection of content-distribution networks enabled through application-level peering relationships among various sorts of service providers, e.g., traditional ISPs, Content Service Providers (CSPs), and Application Service Providers (ASPs).

The operations of the media multicast networks are described as follows. In the media multicast networks, each multicast-capable node (called MediaBridge [21]) performs routing at the application layer. In addition, each MediaBridge is

interconnected with one or more neighboring MediaBridge through explicit configuration, which defines the application-level overlay topology. Collectively, the MediaBridges in a media multicast network employ a distributed application-level multicast routing algorithm to determine the optimal virtual paths for propagating content throughout the network. When the underlying network fails or becomes overly congested, the media multicast network automatically and dynamically re-routes content via alternate paths according to application-level routing policies. In addition, MediaBridges dynamically subscribe to multicast content when and only when a downstream client requests it. This capability ensures that one and only one copy of the multicast content flows across any physical or virtual path independent of the number of downstream clients, resulting in conserving network bandwidth.

The advantage of the application-level multicast is that it breaks the barriers such as scalability, network management, support for congestion control, which have prevented ISPs from establishing “IP multicast” peering arrangements.

### 15.4.3 Content Replication

A fundamental technique for improving scalability of the media delivery system is content/media replication. The content replication takes two forms, namely, caching and mirroring, which are deployed by publisher, Content Service Providers and ISPs. Both caching and mirroring seek to place content closer to the clients and both share the following advantages:

1. reduced bandwidth consumption on network links,
2. reduced load on streaming servers,
3. reduced latency for clients,
4. increased availability.

Mirroring is to place copies of the original multimedia files on several machines scattered around the Internet. That is, the original multimedia files are stored on the main server while copies of the original multimedia files are placed on the duplicate servers. In this way, clients can retrieve multimedia data from the nearest duplicate server, which gives the clients the best performance (e.g., lowest latency). Mirroring has several advantages as follows:

- all contents are replicated;
- the publisher is able to view server access logs and perform user tracking.

On the other hand, there are a number of disadvantages with mirroring. Currently, mechanisms for establishing dedicated mirrors are expensive, ad hoc, and slow. In addition, establishing a mirror on an existing server, while cheaper, is still an ad hoc and administratively complex process. Finally, there is no standard way to make scripts and server setup easily transferable from one server to another.

Caching, which is based on the belief that different clients will load many of the same contents, makes local copies of contents that the clients retrieve. Typically, clients in a single organization retrieve all contents from a single local machine, called a cache. The cache retrieves a video file from the streaming server, storing a copy locally and then passing it on to the client who requests it. If a client asks for a video file which the cache has already stored, the cache will return the copy rather than going all the way back to the streaming server on which the video file resides. In addition, cache sharing and cache hierarchies allow each cache to access files stored at other caches so that the load on the main server can be reduced and network bottlenecks can be alleviated [11, 20].

Caching has the following advantages:

- caches can form a shared infrastructure which works for all streaming servers;
- caches are incrementally deployable.

For these reasons, ISPs have been the largest proponent of caching. However, caches cannot supply the services that publishers need: support for quality of service and security. Firstly, when a cache stores a publisher's document, it makes no promises about how well the document will be treated. The publisher has no way of requesting that its document be delivered within certain time bounds, or that the cache verify the freshness of its copy of the document. Secondly, today's caches trust everyone, so attacks such as inserting fake versions of documents into the cache are extremely easy to accomplish. Finally, caches do not provide feedback to publishers so that publishers are unable to perform user tracking.

Most of the techniques on caching are targeted at generic web objects. Some recent work demonstrated that caching strategies that are specific of particular types of objects can help improve the overall performance [46]. For this reason, a great deal of efforts have been contributed along this direction [46, 58, 62, 91]. A trivial extension of caching techniques to video is to store complete video sequences in the cache. However, such an approach may not be applicable due to the large scale of video data size and possibly limited cache space on a proxy server. Instead, it was shown that only a few cached frames can also contribute to significant improvement in performance [46]. Miao and Ortega proposed two video caching strategies, initial caching and selective caching, which store part of the video stream onto the cache [46]. In particular, it was shown that selective caching can maximize the robustness of the video stream against network congestion while not violating the limited decoder buffer size.

In this section, we have described three network support mechanisms for streaming media. Next, we discuss key issues on design of streaming servers.

## 15.5 Streaming Servers

Streaming servers play a key role in providing streaming services. To offer quality streaming services, streaming servers are required to process multimedia data un-

der timing constraints in order to prevent artifacts (e.g., jerkiness in video motion and pops in audio) during playback at the clients. In addition, streaming servers also need to support VCR-like control operations such as stop, pause/resume, fast forward and fast reverse. Furthermore, streaming servers have to retrieve media components in a synchronous fashion. For example, retrieving a lecture presentation requires synchronizing video and audio with lecture slides.

A streaming server typically consists of the following three subsystems:

1. **Communicator:** A communicator involves the application layer and transport protocols implemented on the server (shown in Fig. 15.1). Through a communicator, the clients can communicate with a server and retrieve multi-media contents in a continuous and synchronous manner. We have addressed the application layer in Section 15.3 and will address transport protocols in Section 15.7.
2. **Operating system:** Different from traditional operating systems, an operating system for streaming services needs to satisfy real-time requirements for streaming applications.
3. **Storage system:** A storage system for streaming services has to support continuous media storage and retrieval.

In this section, we are primarily concerned with operating system support and storage systems for streaming media, which will be presented in Sections 15.5.1 and 15.5.2, respectively.

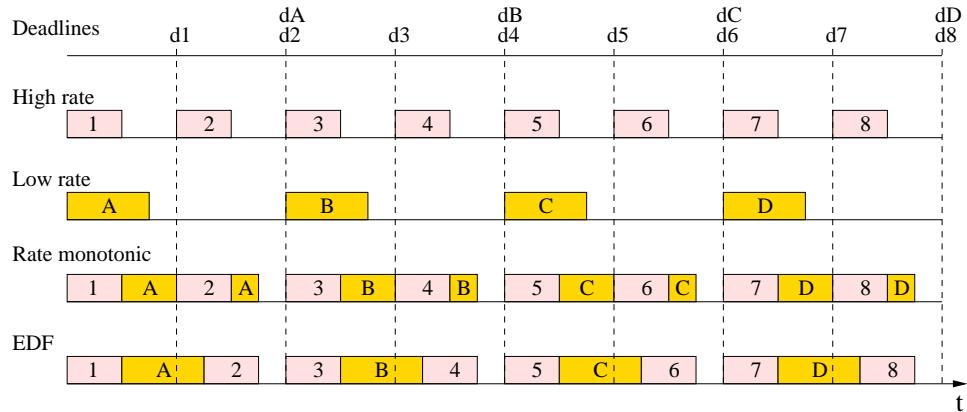
### 15.5.1 Real-time Operating System

The operating system shields the computer hardware from all other software. The operating system offers various services related to the essential resources, such as the CPU, main memory, storage, and all input and output devices. In the following sections, we discuss the unique issues of real-time operating systems and review the associated approaches to the problems introduced by streaming services. We first show how process management takes into account the timing requirements imposed by streaming media and apply appropriate scheduling methods; We then describe how to manage resources to accommodate timing requirements; Finally, we discuss issues on file management.

#### Process Management

Process management deals with the main processor resource. The process manager maps single processes onto the CPU resource according to a specified scheduling policy such that all processes can meet their requirements.

To fulfill the timing requirements of continuous media, the operating system must use real-time scheduling techniques. Most attempts to solve real-time scheduling problems are just variations of two basic algorithms for multimedia systems: ear-



**Figure 15.7.** EDF versus rate-monotonic scheduler.

liest deadline first (EDF) [42] and rate-monotonic scheduling [14]. In EDF scheduling, each task is assigned a deadline and the tasks are processed in the order of increasing deadlines. In rate-monotonic scheduling, each task is assigned a static priority according to its request rate.<sup>3</sup> Specifically, the task with the shortest period (or the highest rate) gets the highest priority, and the task with the longest period (or the lowest rate) gets the lowest priority. And then the tasks are processed in the order of priorities.

Both EDF and rate-monotonic scheduling are preemptive, that is, the schedulers can preempt the running task and schedule the new task for the processor based on its deadline/priority. The execution of the interrupted task will resume at a later time. The difference between EDF and rate-monotonic scheduling is as follows. EDF scheduler is based on a single priority task queue and the processor runs the task with the earliest deadline. On the other hand, rate-monotonic scheduler is a static-priority scheduler with multiple-priority task queues, that is, the tasks in the lower-priority queue can not be executed until all the tasks in the higher-priority queues are served. In the example of Fig. 15.7, there are two task sequences. The high rate sequence is Task 1 to Task 8; the low rate sequence is Task A to Task D. As shown in Fig. 15.7, in rate-monotonic scheduling, Task 2 preempts Task A since Task 2 has a higher priority; on the other hand, in EDF, Task 2 does not preempt Task A since Task A and Task 2 have the same deadlines ( $d_A = d_2$ ). It can be seen that a rate-monotonic scheduler is more prone to task switching than EDF. In summary, the rate-monotonic algorithm ensures that all deadlines will be met if the processor utilization is under 69 percent [14]; the EDF algorithm can achieve 100 percent utilization of processor but EDF may not guarantee the processing of some tasks during overload periods.

<sup>3</sup> Assume that each task is periodic.

### Resource Management

Resources in a multimedia server include CPUs, memories, storage devices, etc. Since resources are limited, a multimedia server can only service a limited number of clients with requested QoS. Therefore, resource management is required to manage resources to accommodate timing requirements. Resource management involves admission control and resource allocation. Specifically, before admitting a new client, a multimedia server must perform admission control test to decide whether a new connection can be admitted without violating performance guarantees already provided to existing connections. If a connection is accepted, the resource manager allocates resources required to meet the QoS for the new connection.

Admission control algorithms can be classified into two categories: deterministic admission control [24] and statistical admission control [76]. Deterministic admission control algorithms provide hard guarantees to clients while statistical admission control algorithms provide statistical guarantees to clients (i.e., the continuity requirements of at least a fixed percentage of media units are ensured to be met). The advantages of deterministic admission control are simplicity and strict assurance of quality; its limitation is lower utilization of server resources. In contrast to this, statistical admission control improves the utilization of server resources by exploiting the human perceptual tolerances as well as the differences between the average and the worst-case performance characteristics of a multimedia server [76].

Corresponding to admission control algorithms, resource allocation schemes can be either deterministic or statistical. Deterministic admission control algorithms require deterministic resource allocation schemes while statistical admission control algorithms require statistical resource allocation schemes. Deterministic resource allocation schemes make reservations for the worst case, e.g., reserving bandwidth for the longest processing time and the highest rate that a task might ever need. On the other hand, statistical resource allocation schemes achieve higher utilization by allowing temporary overload, resulting in a small percentage of QoS violations.

### File Management

The file system provides access and control functions for file storage and retrieval. There are two basic approaches to supporting continuous media in file systems. In the first approach, the organization of files on disks remains as it is for discrete data (i.e., a file is not scattered across several disks), with the necessary real-time support provided through special disk-scheduling algorithms and enough buffer capacity to avoid jitter. The second approach is to organize audio and video files on distributed storage like disk arrays. Under the second approach, the disk throughput can be improved by scattering/stripping each audio/video file across several disks; disk seek-times can be reduced by disk-scheduling algorithms.

Traditional disk-scheduling algorithms such as First-Come-First-Server and SCAN [16, 70] do not provide real-time guarantees. Hence, many disk-scheduling algorithms have been proposed in support of continuous media. These include SCAN-EDF [56], Grouped Sweeping Scheduling (GSS) [88], and Dynamic Circular SCAN

(DC-SCAN) [31], which are described as follows.

- The SCAN-EDF combines the seek optimization of the traditional disk-scheduling method SCAN [16] and the real-time guarantees of the EDF mechanism. Note that the EDF mechanism in disk scheduling is non-preemptive, which is different from the preemptive EDF scheme used in process management.
- The Grouped Sweeping Scheduling divides the set of  $n$  streams into  $g$  groups; groups can be formed in such a way that all streams belonging to the same group have similar deadlines; individual streams within a group are served according to SCAN.
- DC-SCAN employs a Circular SCAN [64] service order to minimize disk seek overhead and variations in inter-service time, resulting in high throughput; it reduces start-up delay by dynamically adapting the Circular SCAN service order.

As a result, the three algorithms, SCAN-EDF, GSS and DC-SCAN, can improve continuous media data throughput and meet real-time requirements imposed by continuous media.

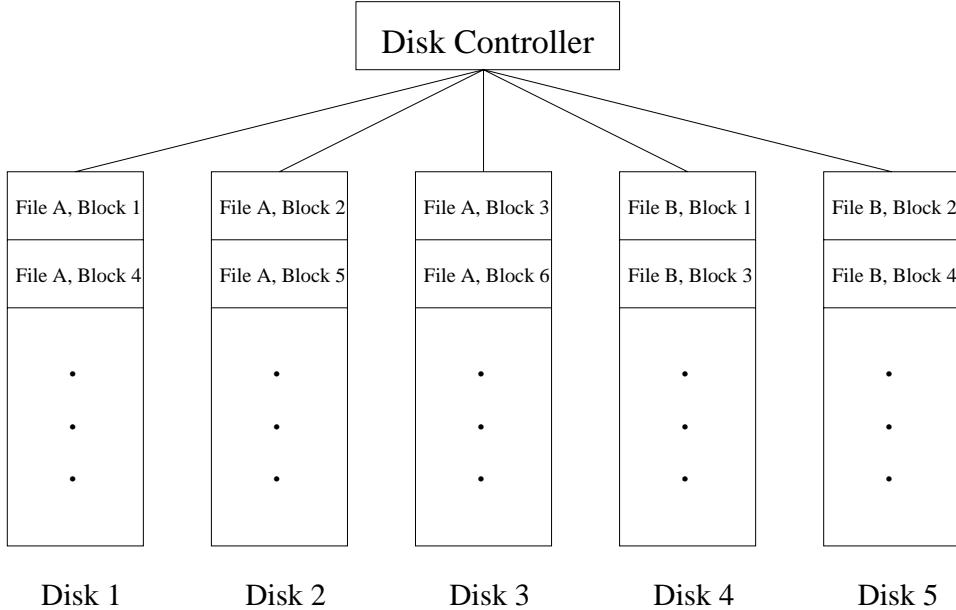
Another function that needs to be supported by file management is interactive control such as pause/resume, fast forward and fast reverse. First, the pause/resume operations pose a significant challenge to the design of efficient buffer management schemes because they interfere with the sharing of a multimedia stream among different viewers. This issue is still under study. Secondly, the fast-forward and fast-reverse operations can be implemented either by playing back media at a higher rate than normal or by continuing playback at the normal rate while skipping some data. Since the former approach can significantly increase the data rate, its direct implementation is impractical. The latter approach, on the other hand, needs to be carefully designed if inter-data dependencies are present (for example, P frames and B frames depend on I frames in MPEG) [12]. As a result, for streaming MPEG video, entire Group of Pictures (GOPs) have to be skipped during fast-forward operations, and the viewer sees normal resolution video with gaps, which is acceptable.

### 15.5.2 Storage System

The challenging issues of designing storage systems for multimedia are high throughput, large capacity and fault-tolerance, which are addressed as follows.

#### **Data striping: a method to increase throughput**

If an entire video file is stored on one disk, the number of concurrent accesses to that file are limited by the throughput of that disk. This dictates the number of clients that are viewing the same video file. To overcome this limitation, data striping was proposed [63]. Under data striping schemes, a multimedia file is scattered across multiple disks and the disk array can be accessed in parallel. An example of data



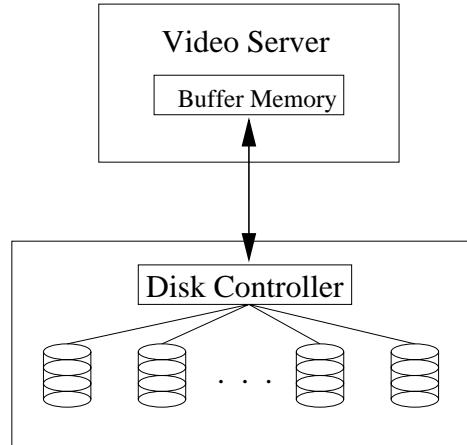
**Figure 15.8.** Data striped in multiple disks and accessed in parallel.

striping is shown in Fig. 15.8. As shown in Fig. 15.8, Block 1, 2 and 3 of File A can be read in parallel, resulting in increased throughput. An important issue in design of a data striping scheme is to balance the load of most heavily loaded disks to avoid overload situations while keeping latency small. The designers have to trade off load balance with low latency since load balance and low latency are two conflicting objectives [63]. Note that data striping is different from file replication (an expensive way to increase throughput) in that data striping allows only one copy of a video file stored on disks while file replication allows multiple copies of a video file stored on disks.

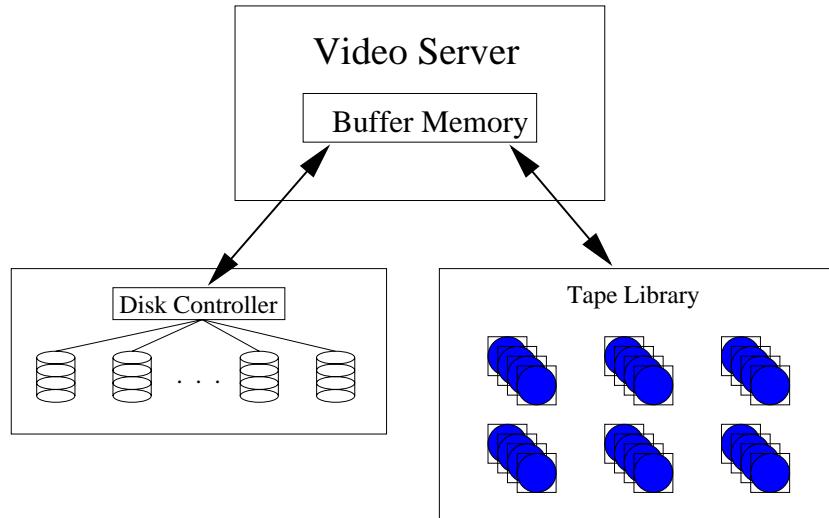
#### Tertiary and hierarchical storage: a method to increase capacity

The introduction of multiple disks can increase the storage capacity as shown in Fig. 15.9. However, the cost for large archives (e.g., with 40 tera-byte storage requirement) is prohibitively high if a large number of disks are used for storage. To keep the storage cost down, tertiary storage (e.g., an automated tape library or CD-ROM jukebox) must be added.

To reduce the overall cost, a hierarchical storage architecture (shown in Fig. 15.10) is typically used. Under the hierarchical storage architecture, only a fraction of the total storage is kept on disks while the major remaining portion is kept on a tertiary tape system. Specifically, frequently requested video files are kept on disks for quick



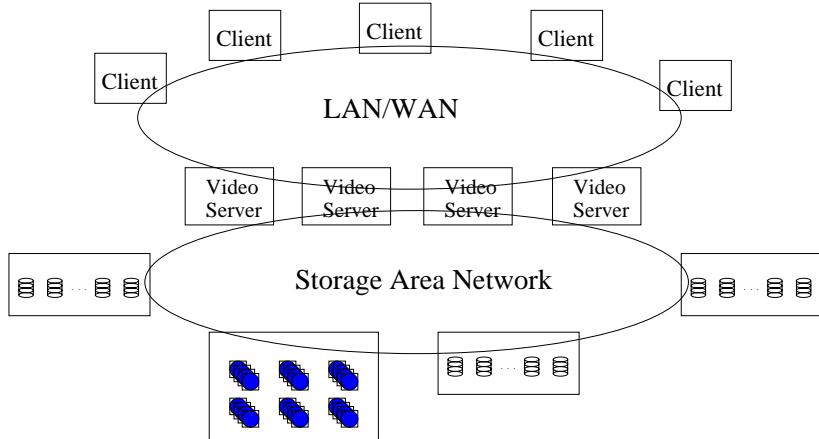
**Figure 15.9.** Disk-based video storage.



**Figure 15.10.** Hierarchical storage.

access; the remainder resides in the automated tape library.

To deploy streaming services on a large scale, a storage area network (SAN) architecture needs to be employed (shown in Fig. 15.11) [17, 29]. An SAN can provide high-speed data pipes between storage devices and hosts at far greater distances than conventional host-attached SCSI (Small Computer Systems Interface).



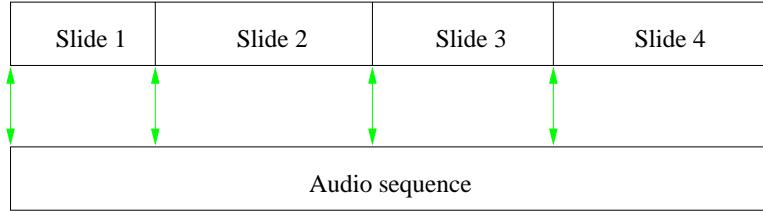
**Figure 15.11.** An SAN-based server and storage architecture for large-scale deployment.

The connections in an SAN can be direct links between specific storage devices and individual hosts, through Fibre Channel Arbitrated Loop (FC-AL) connections; or the connections in an SAN can form a matrix through a Fibre Channel switch. With these high-speed connections, an SAN is able to provide a many-to-many relationship between heterogeneous storage devices (e.g., disk arrays, tape libraries, and optical storage arrays), and multiple servers and storage clients.

### Fault tolerance

In order to ensure uninterrupted service even in the presence of disk failures, a server must be able to reconstruct lost information. This can be achieved by using redundant information. The redundant information could be either parity data generated by error-correcting codes like FEC or duplicate data on separate disks. That is, there are two fault-tolerant techniques: error-correcting (i.e., parity-encoding) [4, 53, 72] and mirroring [48]. Parity data adds a small storage overhead but it requires synchronization of reads and additional processing time to decode lost information. In contrast, mirroring does not require synchronization of reads or additional processing time to decode lost information, which significantly simplifies the design and the implementation of video servers. However, mirroring incurs at least twice as much storage volume as in the non-fault-tolerant case. As a result, there is a trade-off between reliability and complexity (cost). A recent study [23] showed that, for the same degree of reliability, mirroring-based schemes always outperform parity-based schemes in terms of per stream cost as well as restart latency after disk failure.

As a summary, we have addressed various issues in streaming server design and presented important techniques for efficient, scalable and reliable storage and re-



**Figure 15.12.** Synchronization between the slides and the narration audio stream.

trieval of multimedia files. Next, we discuss synchronization mechanisms for streaming media.

## 15.6 Media Synchronization

A major feature that distinguishes multimedia applications from other traditional data applications is the integration of various media streams that have to be presented in a synchronized fashion. For example, in distance learning, the presentation of slides should be synchronized with the narration audio stream (see Fig. 15.12). Otherwise, the current slide being displayed on the screen may not correspond to the lecturer's explanation heard by the students, which is problematic. With media synchronization, the application at the receiver side can present the media in the same way as they were originally captured.

Media synchronization refers to the preservation of the temporal relationships within one data stream and between various media streams. There are three levels of synchronization, namely, intra-stream, inter-stream and inter-object synchronization. The three levels of synchronization correspond to three semantic layers of multimedia data as follows [66].

1. Intra-stream synchronization: The lowest layer of continuous media or time-dependent data (such as video and audio) is the media layer. The unit of the media layer is Logical Data Unit (LDU) such as a video/audio frame, which adheres to strict temporal constraints to ensure acceptable user perception at playback. Synchronization at this layer is referred to as intra-stream synchronization, which maintains the continuity of Logical Data Units. Without intra-stream synchronization, the presentation of the stream may be interrupted by pauses or gaps.
2. Inter-stream synchronization: The second layer of time-dependent data is the stream layer. The unit of the stream layer is a whole stream. Synchronization at this layer is referred to as inter-stream synchronization, which maintains temporal relationships among different continuous media. Without inter-stream synchronization, skew between the streams may become intolerant. For example, users could be annoyed if they notice that the movements

of the lips of a speaker do not correspond to the presented audio.

3. Inter-object synchronization: The highest layer of a multimedia document is the object layer, which integrates streams and time-independent data such as text and still images. Synchronization at this layer is referred to as inter-object synchronization. The objective of inter-object synchronization is to start and stop the presentation of the time-independant data within a tolerable time interval, if some previously defined points of the presentation of a time-dependent media object are reached. Without inter-object synchronization, for example, the audience of a slide show could be annoyed if the audio is commenting one slide while another slide is being presented.

Media streams may lose synchronization after moving from the server to the client. As shown in Fig. 15.1, there are many components in the path transporting data from its storage site to the user. Specifically, the server retrieves data from the storage device and sends that data into the network; the network transports the data to the client; the client reads the data from its network interface and presents it to the user; operating systems and protocols allow these systems to run and do their work. Each of these components on the transport path performs a certain task and affects the data in a different way. They all inevitably introduce delays and delay variations in either predictable or unpredictable manners. In particular, the delay introduced in the network is typically unpredictable due to the best-effort nature of the Internet. The incurred delays and delay variations could disrupt intra-media, inter-media, and inter-object synchronization. Therefore, media synchronization mechanisms are required to ensure proper rendering of the multimedia presentation at the client.

The essential part of any media synchronization mechanism is the specifications of the temporal relations within a medium and between the media. The temporal relations can be specified either automatically or manually. In the case of audio/video recording and playback, the relations are specified automatically by the recording device. In the case of presentations that are composed of independently captured or otherwise created media, the temporal relations have to be specified manually (with human intervention). The manual specification can be illustrated by the design of a slide show: the designer selects the appropriate slides, creates an audio object and defines the segments of the audio stream where the slides have to be presented (see Fig. 15.12).

The methods that are used to specify the temporal relations include interval-based, axes-based, control flow-based, and event-based specifications [7]. A widely-used specification method for continuous media is axes-based specifications or timestamping: at the source, a stream is time-stamped to keep temporal information within the stream and with respect to other streams; at the destination, the application presents the streams according to their temporal relation.

Besides specifying the temporal relations, it is desirable that synchronization is supported by each component on the transport path. For example, the servers store large amount of data in such a way that retrieval is quick and efficient to

reduce delay; the network provides sufficient bandwidth, and delay and jitter introduced by the network are tolerable to the multimedia applications; the operating systems and the applications provide real-time data processing (e.g., retrieval, re-synchronization, and display). However, real-time support from the network is not available in the current Internet. Hence, most synchronization mechanisms are based on the end systems. These synchronization mechanisms can be either preventive or corrective [36].

- Preventive mechanisms are designed to minimize synchronization errors as data are transported from the server to the user. In other words, preventive mechanisms attempt to minimize latencies and jitters. These mechanisms involve disk-reading scheduling algorithms, network transport protocols, operating systems and synchronization schedulers. Disk-reading scheduling is the process of organizing and coordinating the retrieval of data from the storage devices. Network transport protocols provide means for preserving synchronization during data transmission over the Internet. Operating systems achieve the precise control of timing constraints by using EDF or rate monotonic scheduling. A synchronization scheduler can use the synchronization specifications for a presentation to create a schedule for the delivery of the media streams to the client by the servers (delivery schedule) and the presentation of these media streams to the user by the client application (presentation schedule). This scheduler can be centralized (entirely located at the client) or distributed (the delivery scheduling functionalities are shared among the servers and the client).
- Corrective mechanisms are designed to recover synchronization in presence of synchronization errors. Synchronization errors are unavoidable since the Internet introduces random delay. Random delay destroys the continuity of the media stream by producing gaps and jitters during the data transmission. Therefore, certain compensations (i.e., corrective mechanisms) at the receiver are necessary when synchronization errors occur.

An example of corrective mechanisms is the Stream Synchronization Protocol (SSP) [25]. In SSP, the concept of an “intentional delay” is used by the various streams in order to adjust their presentation time to recover from network delay variations. The operations of the Stream Synchronization Protocol are described as follows. At the client side, units that control and monitor the client-end of the data connections compare the real arrival times of data with the ones predicted by the presentation schedule and notify the scheduler of any discrepancies. These discrepancies are then compensated by the scheduler, which delays the display of data that are “ahead” of other data, allowing the late data to “catch up”.

In sum, media synchronization is one of the key issues in the design of media streaming services. A great deal of effort has been contributed in the synchronization area. As an overview of this area, we have described the synchronization

concepts, requirements and approaches. For more information on media synchronization, please refer to [7, 66] and references therein. Next, we describe the protocols for streaming video.

## 15.7 Protocols for Streaming Video

Protocols are designed and standardized for communication between clients and streaming servers. According to their functionalities, the protocols directly related to streaming video over the Internet can be classified into three categories:

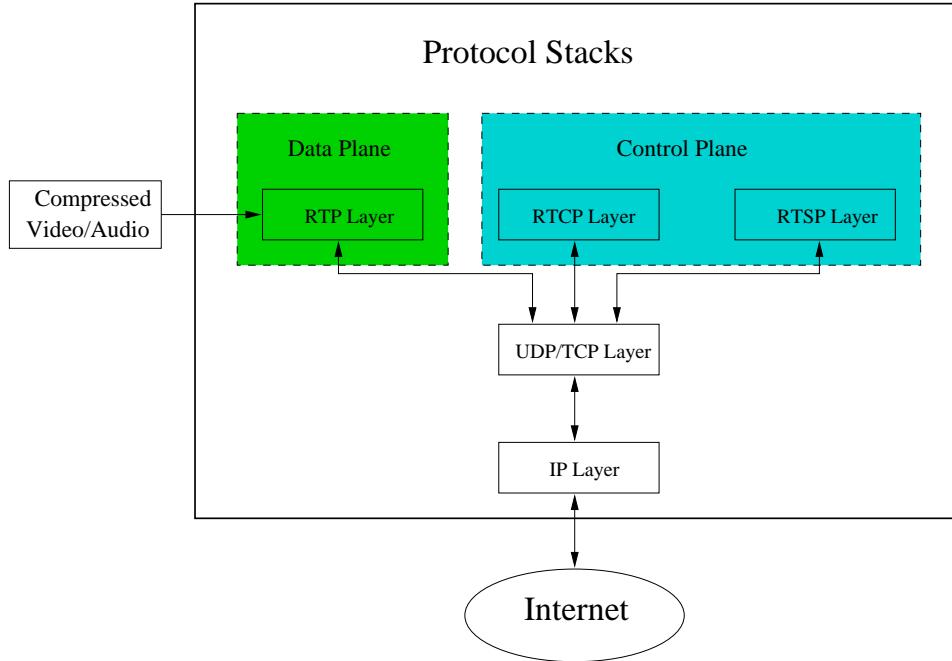
1. **Network-layer protocol:** The network-layer protocol provides basic network service support such as network addressing. The Internet Protocol (IP) serves as the network-layer protocol for Internet video streaming.
2. **Transport protocol:** Transport protocols provide end-to-end network transport functions for streaming applications. These protocols include User Datagram Protocol (UDP), Transmission Control Protocol (TCP), Real-time Transport Protocol (RTP), and Real-Time Control Protocol (RTCP). UDP and TCP are lower-layer transport protocols while RTP and RTCP [59] are upper-layer transport protocols, which are implemented on top of UDP/TCP (see Fig. 15.13).
3. **Session control protocol:** A session control protocol defines the messages and procedures to control the delivery of the multimedia data during an established session. The Real-Time Streaming Protocol (RTSP) [61] is such a session control protocol.

To illustrate the relations among the three types of protocols, we depict the protocol stacks for media streaming in Fig. 15.13. As shown in Fig. 15.13, at the sending side, the compressed video/audio data is retrieved and packetized at the RTP layer. The RTP-packetized streams provide timing and synchronization information, as well as sequence numbers. The RTP-packetized streams are then passed to the UDP/TCP layer and the IP layer. The resulting IP packets are transported over the Internet. At the receiver side, the media streams are processed in the reversed manner before their presentations. This is the case for the data plane. For the control plane, RTCP packets and RTSP packets are multiplexed at the UDP/TCP layer and move to the IP layer for transmission over the Internet.

The rest of this section is organized as follows. In Section 15.7.1, we discuss transport protocols for streaming media. Section 15.7.2 describes the session control protocol, i.e., RTSP.

### 15.7.1 Transport Protocols

The transport protocol family for media streaming includes UDP, TCP, RTP, and RTCP protocols. UDP and TCP provide basic transport functions while RTP and RTCP run on top of UDP/TCP.



**Figure 15.13.** Protocol stacks for media streaming.

UDP and TCP protocols support such functions as multiplexing, error control, or flow control. These functions can be briefly described as follows. Firstly, UDP and TCP can multiplex data streams from different applications running on the same machine with the same IP address. Secondly, for the purpose of error control, TCP and most UDP implementations employ checksum to detect bit errors. If a single bit error or bit errors are detected in the incoming packet, the TCP/UDP layer discards the packet so that the upper layer (e.g., RTP) will not receive the corrupted packet. On the other hand, in contrast to UDP, TCP uses retransmission to recover lost packets. Therefore, TCP provides reliable transmission while UDP does not. Thirdly, TCP employs flow control to adapt the transmission rate according to network congestion status. This is another feature that distinguishes TCP from UDP.

Since TCP retransmission introduces delays that are not acceptable for streaming applications with stringent delay requirements, UDP is typically employed as the transport protocol for video streams. In addition, since UDP does not guarantee packet delivery, the receiver needs to rely on upper layer (i.e., RTP) to detect packet loss.

RTP is an Internet standard protocol designed to provide end-to-end transport functions for supporting real-time applications [59]. RTCP is a companion protocol

to RTP. RTCP is designed to provide QoS feedback to the participants of an RTP session. In other words, RTP is a data transfer protocol while RTCP is a control protocol.

RTP does not guarantee QoS or reliable delivery, but rather, provides the following functions in support of media streaming:

- **Time-stamping:** RTP provides time-stamping so that time-stamps can be used to synchronize different media streams. Note that RTP itself is not responsible for the synchronization. This has to be done by the applications.
- **Sequence numbering:** Since packets arriving at the receiver may be out of sequence (UDP does not deliver packets in sequence), RTP employs sequence numbering to place the incoming RTP packets in the correct order. The sequence numbers are also used for packet loss detection.
- **Payload type identification:** The type of the payload contained in an RTP packet is indicated by an RTP-header field called payload type identifier. The receiver interprets the content of the packet based on the payload type identifier. Certain common payload types such as MPEG-1/2 audio and video have been assigned payload type numbers [60]. For other payloads, this assignment can be done with session control protocols.
- **Source identification:** The source of each RTP packet is identified by an RTP-header field called Synchronization SouRCe identifier (SSRC). SSRC provides a means for the receiver to distinguish different sources.

RTCP is the control protocol designed to work in conjunction with RTP [59]. In an RTP session, participants periodically send RTCP packets to convey feedback on quality of data delivery and information of membership. Basically, RTCP provides the following services:

- **QoS feedback:** This is the primary function of RTCP. RTCP provides feedback to an application about the quality of data distribution. The control information is useful to the senders, the receivers and third-party monitors. The sender can adjust its transmission rate based on the receiver report feedback (see Section 15.3.1). The receivers can determine whether a congestion is local, regional or global. Network managers can evaluate the network performance for multicast distribution.

RTCP provides QoS feedback through the use of Sender Reports and Receiver Reports at the source and destination, respectively. The reports can contain information on the quality of reception such as (1) fraction of the lost RTP packets since the last report, (2) cumulative number of lost packets since the beginning of reception, (3) packet interarrival jitter, and (4) delay since receiving the last sender's report.

- **Participant identification:** A source can be identified by the SSRC field in the RTP header. But the SSRC identifier is not convenient for human

users. To remedy this problem, RTCP provides a human-friendly mechanism for source identification. Specifically, RTCP SDES (source description) packets contain textual information called canonical names as globally unique identifiers of the session participants. It may include user's name, telephone number, email address and other information.

- **Control packets scaling:**

To scale the RTCP control packet transmission with the number of participants, a control mechanism is designed as follows. The control mechanism keeps the total control packets to 5% of the total session bandwidth. Among the control packets, 25% are allocated to the Sender Reports and 75% to the Receiver Reports. To prevent control packet starvation, at least 1 control packet is sent within 5 seconds at the sender or receiver.

- **Inter-media synchronization:** RTCP Sender Reports contain an indication of real time and the corresponding RTP timestamp. This can be used in inter-media synchronization like lip synchronization in video.
- **Minimal session control information:** This optional functionality can be used for transporting session information such as names of the participants.

### 15.7.2 Session Control Protocol: RTSP

The Real-Time Streaming Protocol (RTSP) is a session control protocol for streaming media over the Internet [61]. One of the main functions of RTSP is to support VCR-like control operations such as stop, pause/resume, fast forward and fast reverse. In addition, RTSP also provides means for choosing delivery channels (e.g., UDP, multicast UDP or TCP), and delivery mechanisms based upon RTP. RTSP works for multicast as well as unicast.

Another main function of RTSP is to establish and control streams of continuous audio and video media between the media servers and the clients. Specifically, RTSP provides the following operations:

1. **Media retrieval:** The client can request a presentation description, and ask the server to setup a session to send the requested media data.
2. **Inviting a media server to a conference:** The media server can be invited to the conference to play back media or to record a presentation.
3. **Adding media to an existing session:** The server or the client can notify each other about any additional media becoming available to the established session.

RTSP is intended to provide the same services on streamed audio and video as HTTP does for text and graphics (HTTP: HyperText Transfer Protocol). It is designed to have similar syntax and operations so that most extension mechanisms to HTTP can be added to RTSP.

In RTSP, each presentation and media stream is identified by an RTSP URL (URL: Universal Resource Locator). The overall presentation and the properties of the media are defined in a presentation description file, which may include the encoding, language, RTSP URLs, destination address, port, and other parameters. The presentation description file can be obtained by the client using HTTP, email or other means.

In sum, RTSP is designed to initiate and direct delivery of streaming media data from media servers; RTP is a transport protocol for streaming media data; RTCP is a protocol for monitoring delivery of RTP packets; UDP and TCP are lower-layer transport protocols for RTP/RTCP/RTSP packets; IP provides a means for delivering UDP/TCP packets over the Internet. The combination of these protocols provides a complete streaming service over the Internet.

## 15.8 Streaming Video over Wireless IP Networks

Recently, the emergence of broadband wireless networks has brought great interest in real-time video communications over wireless IP networks. However, delivering quality video over wireless networks in real-time is a challenging task. This is primarily because of the following problems.

**Bandwidth fluctuations:** First, the throughput of a wireless channel may be reduced due to multipath fading, co-channel interference, and noise disturbances. Second, the capacity of a wireless channel may fluctuate with the changing distance between the base station and the mobile host. Third, when a mobile terminal moves between different networks (e.g., from wireless local area network to wireless wide area network), the available bandwidth may vary drastically (e.g., from a few megabits per second to a few kilobits per second). Finally, when a handoff takes place, a base station may not have enough unused radio resource to meet the demand of a newly joined mobile host. As a result, bandwidth fluctuation is a serious problem for real-time video transmission over wireless networks.

**High bit error rate:** Compared with the wired links, wireless channels are typically much more noisy and have both small-scale (multipath) and large-scale (shadowing) fades, making the bit error rate (BER) very high. The resulting bit errors can have devastating effect on video presentation quality. Therefore, there is a critical need for robust transmission of video over wireless channels.

**Heterogeneity:** In a multicast scenario, receivers may have different requirements and properties in terms of latency, visual quality, processing capabilities, power limitations (wireless vs. wired) and bandwidth limitations. The heterogeneous nature of receivers' requirements and properties make it difficult to design an efficient multicast mechanism.

It has been shown that scalable video is capable of coping gracefully with the variability of bandwidth [2, 45]. A scalable video coding scheme produces a com-

pressed bit-stream, parts of which are decodable. Compared with decoding the complete bit-stream, decoding part of the compressed bit-stream produces pictures with degraded quality, or smaller image size, or smaller frame rate [18]. In contrast, non-scalable video is more susceptible to bandwidth fluctuations since it cannot adapt its video representation to bandwidth variations [45]. Thus, scalable video is more suitable for use in a wireless environment to cope with the fluctuation of wireless channels. Furthermore, scalable video representation is a good solution to the heterogeneity problem in multicast case [45].

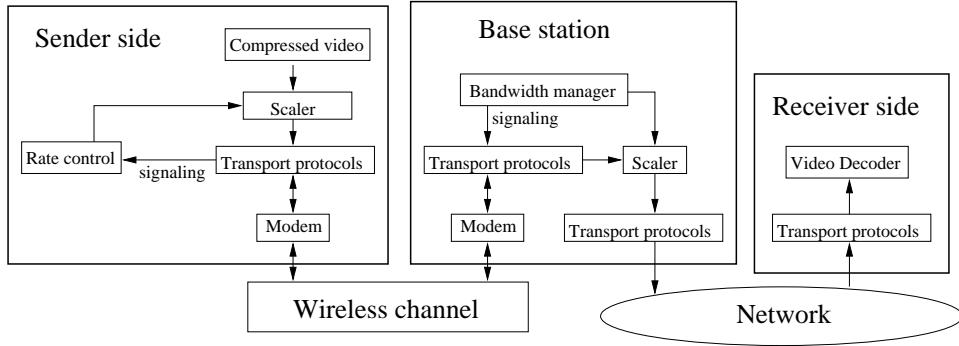
Recently, application-aware adaptive services have been demonstrated to be able to effectively mitigate fluctuations of resource availability in wireless networks [2]. Scalable video representation naturally fits unequal error protection, which can effectively combat bit errors induced by the wireless medium. This motivates us to present an adaptive framework to support quality video communication over wireless IP networks.

For transporting video over wireless, there have been many proposals of adaptive approaches and services in the literature, which include an “adaptive reserved service” framework [38], an adaptive service based on QoS bounds and revenue [44], an adaptive framework targeted at end-to-end QoS provisioning [49], a utility-fair adaptive service [6], a framework for soft QoS control [57], a teleservice model based on an adaptive QoS paradigm [33], an adaptive QoS management architecture [37], and an adaptive framework for scalable video over wireless IP networks [84].

In this section, we present an adaptive framework [85] for future QoS-enabled wireless IP networks. The adaptive framework consists of (1) scalable video representations, each of which has its own specified QoS requirement, (2) network-aware applications, which are aware of network status, and (3) adaptive services, which make network elements support the QoS requirements of scalable video representations. Under this framework, as wireless channel conditions change, the mobile terminal and network elements can scale the video streams and transport the scaled video streams to receivers with acceptable perceptual quality. The adaptive framework has the following key features.

1. *Graceful quality degradation*: Different from non-scalable video, scalable video can adapt its video representation to bandwidth variations and the network can drop packets with awareness of the video representations. As a result, perceptual quality is gracefully degraded under severe channel conditions.
2. *Efficiency*: When there is excess bandwidth (excluding reserved bandwidth), the excess bandwidth will be efficiently used in a way that maximizes the perceptual quality or revenue.
3. *Fairness*: The resources can be shared in either a utility-fair manner [6] or a max-min fair manner [44].

The remainder of this section is organized as follows. Section 15.8.1 describes network-aware applications. In Section 15.8.2, we present the adaptive services for transporting scalable video over wireless IP networks.



**Figure 15.14.** An architecture for transporting scalable video from a mobile terminal to a wired terminal.

### 15.8.1 Network-aware Applications

The use of network-aware applications is motivated by the following facts: (1) the bit error rate is very high when the channel status is poor, and (2) packet loss is unavoidable if the available bandwidth is less than required. If a sender attempts to transmit each layer without any awareness of the channel status, all layers may get corrupted with equal probability, resulting in very poor picture quality. To address this problem, Wu et al. [84] proposed to use network-aware applications, which preemptively discard enhancement layers at the sender in an intelligent manner by considering network status.

For the purpose of illustration, we present an architecture including a network-aware mobile sender, an application-aware base station, and a receiver in Fig. 15.14. The architecture in Fig. 15.14 is applicable to both live and stored video. In Fig. 15.14, at the sender side, the compressed video bit-stream is first filtered by the scaler, the operation of which is to select certain video layers to transmit. Then the selected video representation is passed through transport protocols. Before being transmitted to the base station, the bit-stream has to be modulated by a modem (i.e., modulator/demodulator). Upon receipt of the video packets, the base station transmits them to the destination through the Internet.

Note that a scaler can distinguish the video layers and drop layers according to their significance. The dropping order is from the highest enhancement layer down to the base layer. A scaler only performs two operations: (1) scale down the received video representation, that is, drop the enhancement layer(s); (2) transmit what is received, i.e., do not scale the received video representation.

Under the architecture, a bandwidth manager is maintained at the base station. One function of the bandwidth manager is to notify the sender about the available bandwidth of the wireless channel through signaling channel [50]. Upon receiving this information, the rate control module at the sender conveys the bandwidth parameter to the scaler. Then, the scaler regulates the output rate of the video

stream so that the transmission rate is less than or equal to the available bandwidth.

Another scenario is that the base station notifies the sender about the channel quality (i.e., BER) [3]. Upon receiving this information, the rate control module at the sender commands the scaler to perform the following (suppose that the video is compressed into two layers): (1) if the BER is above a threshold, discard the enhancement layer so that the bandwidth allocated for the enhancement layer can be utilized by forward error correction (FEC) to protect the base layer; (2) otherwise transmit both layers.

The network-aware application has two advantages. Firstly, by taking the available bandwidth into account, the sender can make the best use of network resources by selectively discarding enhancement layers in order to minimize the likelihood of more significant layers being corrupted, thereby increasing the perceptual quality of the video delivered. Secondly, by considering the channel error status, the sender can discard the enhancement layers and FEC can utilize the bandwidth allocated for the enhancement layer to protect the base layer, thereby maximizing the possibility of the base layer being correctly received.

Note that adaptive techniques at the physical/link layer are required to support network-aware applications. Such adaptive techniques include a combination of variable spreading, coding, and code aggregation in Code Division Multiple Access (CDMA) systems, adaptive coding and modulation in Time Division Multiple Access (TDMA) systems, channel quality estimation, and measurement feedback channel [50]. In addition, the feedback interval is typically constrained to be on the order of tens to hundreds of milliseconds [50].

### 15.8.2 Adaptive Service

A scalable video encoder can generate multiple layers or substreams to the network. The adaptive service is to provide scaling of the substreams based on the resource availability conditions in the fixed and wireless network. Specifically, the adaptive service includes the following functions.

- Reserve a minimum bandwidth to meet the demand of the base layer. As a result, the perceptual quality can always be achieved at an acceptable level.
- Adapt the enhancement layers based on the available bandwidth and the fair policy. In other words, it scales the video streams based on resource availability.

Advantages of using scaling inside the network include:

(1) *Adaptivity to network heterogeneity*. For example, when an upstream link with larger bandwidth feeds a downstream link with smaller bandwidth, use of a scaler at the connection point could help improve the video quality. This is because the scaler can selectively drop substreams instead of randomly dropping packets.

(2) *Low latency and low complexity*. Scalable video representations make the operation at a scaler very simple, i.e., only discarding enhancement layers. Thus, the processing is fast, compared with processing on non-scalable video.

(3) *Lower call blocking and handoff dropping probability.* The adaptability of scalable video at base stations can translate into lower call blocking probability and handoff dropping probability.

The adaptive service can be deployed in the whole network (i.e., end-to-end provisioning) or only at base stations (i.e., local provisioning). Since local provisioning of the adaptive service is just a subset of end-to-end provisioning, we will focus on end-to-end provisioning.

The required components of the end-to-end adaptive service include: (1) service contract, (2) call admission control and resource reservation, (3) mobile multicast mechanism, (4) substream scaling, (5) substream scheduling, and (6) link-layer error control. These are described in more detail below.

### Service contract

The service contract between the application and the network could consist of multiple subcontracts, each of which corresponds to one or more substreams with similar QoS guarantees. Each subcontract has to specify traffic characteristics and QoS requirements of the corresponding substream(s). A typical scenario is that a subcontract for the base layer specifies reserved bandwidth while a subcontract for the enhancement layers does not specify any QoS guarantee.

At a video source, substreams must be generated according to subcontracts used by the application and shaped at the network access point. In addition, a substream is assigned a priority according to its significance. For example, the base layer is assigned the highest priority. The priority can be used by routing, scheduling, scaling, and error control components of the adaptive network.

#### 15.8.3 Call admission control and resource reservation

Call admission control (CAC) and resource reservation are two of the major components in end-to-end QoS provisioning. The function of CAC is to check whether admitting the incoming connection would reduce the service quality of existing connections, and whether the incoming connection's QoS requirements can be met. If a connection request is accepted, resources need to be reserved for this connection in two parts. First of all, in order to maintain the specified QoS over a long time-scale, the network must reserve some resources along the current path of a mobile connection. Second, in order to seamlessly achieve the QoS at a short time-scale, some duplication must be done in the transport of the connection to neighboring base stations of a connection so that in the event of a handoff, an outage in the link can be avoided.

The scalable video representation (i.e., substream) concept provides a very flexible and efficient solution to the problem of CAC and resource reservation. First, there is no need to reserve bandwidth for the complete stream since typically only base-layer substream needs QoS guarantee. As a result, CAC is only based on the requirement of the base layer and resource is reserved only for the base-layer substream. Second, the enhancement layer substream(s) of one connection could share

the leftover bandwidth with the enhancement-layer substreams of other connections. The enhancement-layer substreams are subject to scaling under bandwidth shortage and/or severe error conditions, which will be discussed in Section 15.8.3.

### Mobile multicast mechanism

CAC and resource reservation can provide connection-level QoS guarantee. To seamless guarantee QoS at packet level, mobile multicast mechanism has to be used. That is, while being transported along its current path, the base-layer stream is also multicast to its neighboring base stations so that QoS at a small time-scale can be seamlessly achieved.

To support seamless QoS, the mobile routing protocol needs to be proactive and anticipatory in order to match the delay, loss, and jitter constraints of a substream. According to the requirements of a substream, multicast paths might need to be established. The multicast paths terminate at base stations that are potential access-point candidates of a mobile terminal. The coverage of such a multicast path depends on the QoS requirements and the mobility as well as handoff characteristics of a mobile receiver. As a mobile station hands off from a base station to another, new paths are added and old paths are deleted [49].

### Substream scaling

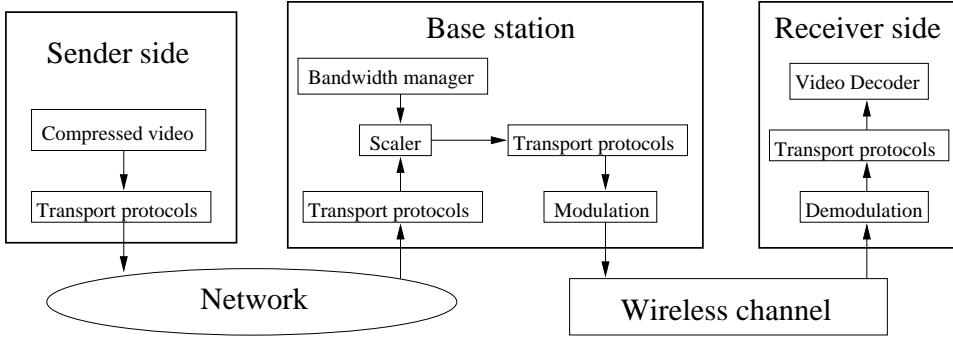
Scaling is employed during bandwidth fluctuations and/or under poor channel conditions. As the available bandwidth on a path decreases due to mobility or fading, lower-priority substreams are dropped by the scaler(s) on the path and substreams with higher priority are transmitted. As more bandwidth becomes available, lower-priority substreams are passed through the scaler, and the perceptual quality at the receivers increases. Figure 15.14 shows an architecture for transporting scalable video from a mobile terminal to a wired terminal. Figure 15.15 depicts an architecture for transporting scalable video from a wired terminal to a mobile terminal. We do not show the case of transporting scalable video from a mobile terminal to a mobile terminal since it is a combination of Fig. 15.14 and Fig. 15.15.

The scaling decision is made by a bandwidth manager. When there is no excess bandwidth (excluding reserved bandwidth), the bandwidth manager instructs the scaler to drop the enhancement layers. If there is excess bandwidth, the excess bandwidth can be shared in either a utility-fair manner [6] or a max-min fair manner [44].

### Substream scheduling

The substream scheduler is used in mobile terminals as well as base stations. Its function is to schedule the transmission of packets on the wireless medium according to their substream QoS specifications and priorities.

When a short fading period is observed, a mobile terminal tries to prioritize the transmission of its substreams in order to achieve a minimum QoS. Here, depending on channel conditions, a substream might be dropped for a period of time in order



**Figure 15.15.** An architecture for transporting scalable video from a wired terminal to a mobile terminal.

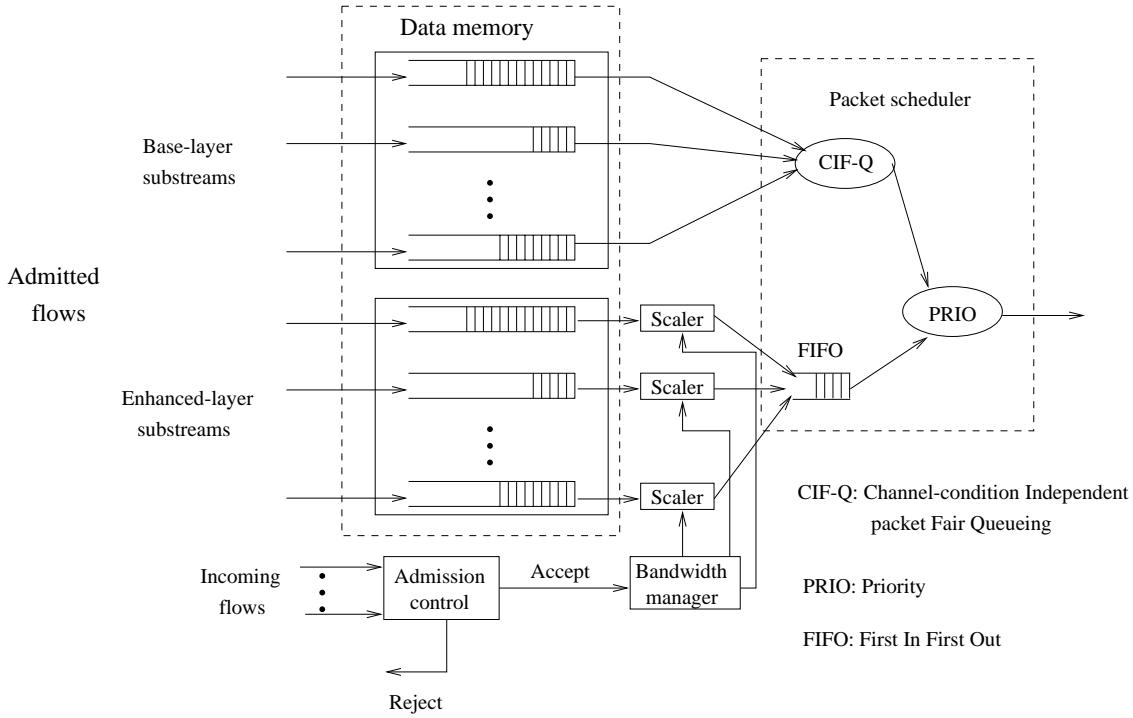
to accommodate higher-priority substreams. To determine the transmission time of any packet in a specific substream (or its position in the transmission queue), the scheduler takes two factors into account: (1) the relative importance of the substream compared to other substreams, and (2) wireless channel conditions.

To achieve both QoS (e.g., bounded delay and reserved bandwidth) and fairness, algorithms like packet fair queueing have to be employed [5]. While the existing packet fair queueing algorithms provide both bounded delay and fairness in wired networks, they cannot be applied directly to wireless networks. The key difficulty is that in wireless networks sessions can experience location-dependent channel errors. This may lead to situations in which a session receives significantly less service than it is supposed to receive, while another receives more. This results in large discrepancies between the sessions' virtual times, making it difficult to provide both delay-guarantees and fairness simultaneously.

To apply packet fair queueing algorithms, Ng et al., [51] identified a set of properties, called Channel-condition Independent Fair (CIF), that a packet fair queueing algorithm should have in a wireless environment: (1) delay and throughput guarantees for error-free sessions, (2) long term fairness for error sessions, (3) short term fairness for error-free sessions, and (4) graceful degradation for sessions that have received excess service time. Then they presented a methodology for adapting packet fair queueing algorithms for wireless networks and applied the methodology to derive an algorithm based on the start-time fair queueing [28], called Channel-condition Independent packet Fair Queueing (CIF-Q), that achieves all the above properties [51].

As an example, we consider two-layer video. Suppose that a subcontract for the base layer specifies reserved bandwidth while a subcontract for the enhancement layer does not specify any QoS guarantee, which is a typical case. An architecture [84] for substream scheduling is shown in Fig. 15.16.

Under the architecture, the buffer pool (i.e., data memory in Fig. 15.16) is



**Figure 15.16.** An architecture for substream scheduling at a base station.

divided into two parts: one for base-layer substreams, and one for enhancement layer substreams. Within the same buffer partition for base or enhancement layer, per-flow queueing for each substream is employed. Furthermore, substreams within the same buffer partition share the buffer pool of that partition while there is no buffer sharing across partitions. It is believed that this approach offers an excellent balance between traffic isolation and buffer sharing [84].

Under the above buffering architecture, Wu et al. [84] designed per-flow based traffic management algorithms to achieve requested QoS and fairness. The first part of the traffic management is CAC and bandwidth allocation. Video connections are admitted by CAC based on their base-layer QoS requirements. And bandwidth reservations for the admitted base-layer substreams are made accordingly. For admitted enhancement layer substreams, their bandwidth will be dynamically allocated by a bandwidth manager, which has been addressed in Section 15.8.3. The scaled enhancement layer substreams enter a shared buffer and are scheduled by a First-In-First-Out (FIFO) scheduler. The second part of the traffic management is packet scheduling. Shown in Fig. 15.16 is a hierarchical packet scheduling architecture where a priority link scheduler is shared among a CIF-Q scheduler for base-layer substreams, and an FIFO scheduler for enhancement layer substreams.

Service priority is first given to the CIF-Q scheduler and then to the FIFO scheduler.

### Link-layer error control

To provide quality video over wireless channels, link-layer error control is required. Basically, there are two kinds of error control mechanisms, namely, forward error correction (FEC) and automatic repeat request (ARQ). The disadvantage of FEC is that it is not adaptive to varying channel conditions and it works only when the BER is below the FEC code's recovery capability. The disadvantage of ARQ is unbounded delay; that is, in the worst case, a packet may be retransmitted an unlimited number of times to recover from bit errors.

To address the problems associated with FEC and ARQ, truncated type-II hybrid ARQ schemes [41, 89] and delay-constrained hybrid ARQ [82] have been proposed. These hybrid ARQ schemes combine the good features of FEC and ARQ: bounded delay and adaptiveness. On the other hand, unequal error protection [30] naturally fit the hierarchical structure of scalable video. Specifically, the base layer can be better protected against transmission errors than the enhancement layers. This form of unequal error protection is much more desirable than having to protect all the substreams. An open issue is how to combine unequal error protection with the hybrid ARQ schemes [85].

## 15.9 Summary

In this chapter, we have surveyed major approaches and mechanisms for Internet video streaming and have presented an adaptive framework for video over wireless IP. The objective is not to provide an exhaustive review of existing approaches and mechanisms, but instead to give the reader a perspective on the range of options available and the associated trade-offs among performance, functionality, and complexity.

We would like to stress that the seven areas (i.e., video compression, application-layer QoS control, continuous media distribution services, streaming servers, media synchronization mechanisms, protocols, video over wireless IP) are basic building blocks for a streaming video architecture. Built with the seven basic building blocks, the streaming video architecture ties together a broad range of signal processing, networking and server technologies. Therefore, a thorough understanding of the whole architecture for streaming video is beneficial for developing signal processing techniques (e.g., scalable video compression) suitable for streaming video. In addition, in-depth knowledge on both signal processing and networking helps to make effective design and use of application-layer QoS control, continuous media distribution services, protocols and services for video over wireless IP. Furthermore, the architectural understanding is instrumental in the design of efficient, scalable, and/or fault-tolerant streaming servers.

## 15.10 Bibliography

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## Appendix A

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# DETERMINATION OF SPATIAL-TEMPORAL GRADIENTS

To implement the various gradient descent methods required for many optimization problems presented in this book, one must be able to determine the spatial and/or temporal gradient of the underlying video signal. This appendix presents several methods for approximating the continuous gradient operation using differential operators.

### A.1 First and Second Order Gradient

The simplest method for approximating the first order gradient along a particular direction is to use the difference between two pixel values along that direction. This yields:

$$\begin{aligned}\left. \frac{\partial \psi}{\partial x} \right|_{(x,y,t)} &= \psi(x, y, t) - \psi(x - 1, y, t), \\ \left. \frac{\partial \psi}{\partial y} \right|_{(x,y,t)} &= \psi(x, y, t) - \psi(x, y - 1, t), \\ \left. \frac{\partial \psi}{\partial t} \right|_{(x,y,t)} &= \psi(x, y, t) - \psi(x, y, t - 1).\end{aligned}$$

Using the above approximation, one can further derive the approximations for the second order spatial gradient operators:

$$\begin{aligned}\left. \frac{\partial^2 \psi}{\partial x^2} \right|_{(x,y,t)} &= \left. \frac{\partial \psi}{\partial x} \right|_{(x+1,y,t)} - \left. \frac{\partial \psi}{\partial x} \right|_{(x,y,t)} = \psi(x + 1, y, t) - 2\psi(x, y, t) + \psi(x - 1, y, t), \\ \left. \frac{\partial^2 \psi}{\partial y^2} \right|_{(x,y,t)} &= \left. \frac{\partial \psi}{\partial y} \right|_{(x,y+1,t)} - \left. \frac{\partial \psi}{\partial y} \right|_{(x,y,t)} = \psi(x, y + 1, t) - 2\psi(x, y, t) + \psi(x, y - 1, t),\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \psi}{\partial xy} \Big|_{(x,y,t)} &= \frac{\partial \psi}{\partial x} \Big|_{(x+1,y+1,t)} - \frac{\partial \psi}{\partial x} \Big|_{(x+1,y,t)} \\ &= \psi(x+1, y+1, t) - \psi(x+1, y, t) - \psi(x, y+1, t) + \psi(x, y, t), \\ \frac{\partial^2 \psi}{\partial t^2} \Big|_{(x,y,t)} &= \frac{\partial \psi}{\partial t} \Big|_{(x,y,t+1)} - \frac{\partial \psi}{\partial t} \Big|_{(x,y,t)} = \psi(x, y, t+1) - 2\psi(x, y, t) + \psi(x, y, t-1).\end{aligned}$$

## A.2 Sobel Operator

The digital approximations presented above are very sensitive to noise in the video data. Furthermore, it is unsymmetric. In image processing, the first order spatial gradient is more often approximated by the Sobel operator, which performs smoothing along the tangent direction, before taking a central difference in the gradient direction. Specifically,

$$\begin{aligned}\frac{\partial \psi}{\partial x} \Big|_{(x,y,t)} &= \psi(x+1, y-1, t) + 2\psi(x+1, y, t) + \psi(x+1, y+1, t) \\ &\quad - \psi(x-1, y-1, t) - 2\psi(x-1, y, t) - \psi(x-1, y+1, t), \\ \frac{\partial \psi}{\partial y} \Big|_{(x,y,t)} &= \psi(x-1, y+1, t) + 2\psi(x, y+1, t) + \psi(x+1, y+1, t) \\ &\quad - \psi(x-1, y-1, t) - 2\psi(x, y-1, t) - \psi(x+1, y-1, t).\end{aligned}$$

Note that the above operations can be simply implemented with a  $3 \times 3$  filter.

## A.3 Difference of Gaussian Filters

A more noise robust implementation of the first (or second) order gradient is to first apply a Gaussian smoothing filter to the original data, and then apply the first (or second) order differential operator to the smoothed signal. Because the smoothing operator (a convolution operator) and the differential operator are interchangeable, the above operation is equivalent to convolving the original signal with a filter that is the first (or second) order differential of the Gaussian function. To apply this filter to digital signals, the above continuous domain filter needs to be sampled and truncated. The resulting first order gradient of the Gaussian is known as the DoG filter, while the second order gradient or Laplacian of the Gaussian is called the LoG filter.

Let  $g(x, y) = \exp(-\frac{x^2+y^2}{2\sigma^2})$  represent the Gaussian function. Then

$$\begin{aligned}\frac{\partial g}{\partial x} &= -\frac{x}{\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right), \\ \frac{\partial g}{\partial y} &= -\frac{y}{\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right),\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 g}{\partial x^2} &= -\frac{1}{\sigma^2}(1 - \frac{x^2}{\sigma^2}) \exp(-\frac{x^2 + y^2}{2\sigma^2}), \\ \frac{\partial^2 g}{\partial y^2} &= -\frac{1}{\sigma^2}(1 - \frac{y^2}{\sigma^2}) \exp(-\frac{x^2 + y^2}{2\sigma^2}), \\ \frac{\partial^2 g}{\partial xy} &= \frac{xy}{\sigma^4} \exp(-\frac{x^2 + y^2}{2\sigma^2}).\end{aligned}$$

Obviously, all the above filters are separable. To obtain the digital approximation of any of the above functions, one can truncate it to about  $2 \sim 3\sigma$ , and sample the truncated function at integer samples. The factor  $\sigma$  should be defined in the units of pixel, and it should be chosen based on the amount of noise present in the data. A popular approximation is to set  $\sigma = 1$  pixel and truncate the function at  $2\sigma$ . The resulting filters are  $5 \times 5$  filters, which are given below:

$$[\mathbf{G}_x] = \begin{bmatrix} 0.0366 & 0.0821 & 0 & -0.0821 & -0.0366 \\ 0.1642 & 0.3679 & 0 & -0.3679 & -0.1642 \\ 0.2707 & 0.6065 & 0 & -0.6065 & -0.2707 \\ 0.1642 & 0.3679 & 0 & -0.3679 & -0.1642 \\ 0.0366 & 0.0821 & 0 & -0.0821 & -0.0366 \end{bmatrix}$$

$$[\mathbf{G}_{xx}] = \begin{bmatrix} 0.0549 & 0 & -0.1353 & 0 & 0.0549 \\ 0.2463 & 0 & -0.6065 & 0 & 0.2463 \\ 0.4060 & 0 & -1.0000 & 0 & 0.4060 \\ 0.2463 & 0 & -0.6065 & 0 & 0.2463 \\ 0.0549 & 0 & -0.1353 & 0 & 0.0549 \end{bmatrix}$$

$$[\mathbf{G}_{xy}] = \begin{bmatrix} 0.0733 & 0.1642 & 0 & -0.1642 & -0.0733 \\ 0.1642 & 0.3679 & 0 & -0.3679 & -0.1642 \\ 0 & 0 & 0 & 0 & 0 \\ -0.1642 & -0.3679 & 0 & 0.3679 & 0.1642 \\ -0.0733 & -0.1642 & 0 & 0.1642 & 0.0733 \end{bmatrix}$$

$$[\mathbf{G}_y] = [\mathbf{G}_x]^T; \quad [\mathbf{G}_{yy}] = [\mathbf{G}_{xx}]^T.$$

## Appendix B

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# GRADIENT DESCENT METHODS

In this appendix, we review some basic gradient descent methods for minimizing an objective function. We will consider the general case when the function is multivariable with dimension  $K$ .

Let  $J(\mathbf{x})$  represent the objective function, where  $\mathbf{x} = [x_1, x_2, \dots, x_k]^T$  is a  $K$ -dimensional vector. Gradient descent methods are iterative methods for determining the point  $\mathbf{x}^*$  that achieves the minimum of  $J(\mathbf{x})$ .

### B.1 First Order Gradient Descent Method

Let  $\mathbf{x}^l$  represent the solution at the  $l$ -th iteration. The first order gradient descent method updates the solution proportional to the first order gradient of the objective function, i.e.,

$$\mathbf{x}^{l+1} = \mathbf{x}^l - \alpha \left. \frac{\partial J}{\partial \mathbf{x}} \right|_{\mathbf{x}^l}. \quad (\text{B.1.1})$$

The rationale behind the first order gradient descent method is that  $\frac{\partial J}{\partial \mathbf{x}}$  represents the direction along which the function  $J(\mathbf{x})$  increases most rapidly, and consequently,  $-\frac{\partial J}{\partial \mathbf{x}}$  is the direction along which  $J(\mathbf{x})$  decreases the fastest.

The constant  $\alpha$  is known as the *step-size*. It has to be chosen properly to guarantee the convergence of the above iteration process to the desired minimum  $\mathbf{x}^*$ . If  $\alpha$  is too large, the solution may oscillate around  $\mathbf{x}^*$ ; whereas if  $\alpha$  is too small, it may take too many iterations to converge. Obviously, if the function  $J(\mathbf{x})$  has multiple local minima, then the solution upon convergence will be the local minimum that is closest to the initial solution. Therefore, it is extremely important to select an appropriate initial solution. In order to obtain the global minimum, one can also start with several different initial solutions that are sufficiently apart, and run the above iteration scheme starting from each initial solution. By comparing the function values at all the local minima obtained from different initial solutions, one can identify the one with the minimum value. However, there is no guarantee

that this will be the global minimum. A more sophisticated approach is *simulated annealing*, which allows the iteration process to jump out of the local minima [1].

## B.2 Steepest Descent Method

In the previous method, the step-size  $\alpha$  is set to a small constant. To speed up the convergence, one can also vary  $\alpha$  at each new iteration, so that the maximum reduction of  $J(\mathbf{x})$  is achieved. This is possible, because for fixed  $\mathbf{x}^l$  and  $\frac{\partial J}{\partial \mathbf{x}}|_{\mathbf{x}^l}$ , the new function value  $J(\mathbf{x}^{l+1})$  is a scalar function of  $\alpha$ . A numerical search method can be used to determine the optimal step-size at each iteration. The first order gradient descent method that uses such a variable step-size is known as the steepest descent method. This method will converge in fewer iterations than the method using a constant step-size, but each iteration will take more time to determine the optimal step-size.

## B.3 Newton's Method

The necessary condition for a function to achieve a local minimum is that its gradient at that point is zero. Given a previous solution  $\mathbf{x}^l$  at which the gradient is not yet zero, one way to determine the update is by requiring the gradient at the updated position be zero, i.e.,

$$\left. \frac{\partial J}{\partial \mathbf{x}} \right|_{\mathbf{x}^l + \Delta \mathbf{x}} = 0. \quad (\text{B.3.1})$$

Using Taylor's expansion for the above gradient function for up to the first term, we obtain

$$\left. \frac{\partial J}{\partial \mathbf{x}} \right|_{\mathbf{x}^l} + [\mathbf{H}(\mathbf{x}^l)] \Delta \mathbf{x} = 0, \quad (\text{B.3.2})$$

where

$$[\mathbf{H}(\mathbf{x})] = \left. \frac{\partial^2 J}{\partial \mathbf{x}^2} \right|_{\mathbf{x}} = \begin{bmatrix} \frac{\partial^2 J}{\partial x_1^2 \partial x_1} & \frac{\partial^2 J}{\partial x_1^2 \partial x_2} & \cdots & \frac{\partial^2 J}{\partial x_1^2 \partial x_K} \\ \frac{\partial^2 J}{\partial x_2 \partial x_1} & \frac{\partial^2 J}{\partial x_2^2 \partial x_2} & \cdots & \frac{\partial^2 J}{\partial x_2 \partial x_K} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 J}{\partial x_K \partial x_1} & \frac{\partial^2 J}{\partial x_K \partial x_2} & \cdots & \frac{\partial^2 J}{\partial x_K \partial x_K} \end{bmatrix}$$

is the second order derivative of  $J(\mathbf{x})$ , known as the Hessian matrix. Solving Eq. (B.3.2), we obtain

$$\Delta \mathbf{x} = - [\mathbf{H}(\mathbf{x}_l)]^{-1} \left. \frac{\partial J}{\partial \mathbf{x}} \right|_{\mathbf{x}^l},$$

or

$$\mathbf{x}^{l+1} = \mathbf{x}^l - [\mathbf{H}(\mathbf{x}_l)]^{-1} \left. \frac{\partial J}{\partial \mathbf{x}} \right|_{\mathbf{x}^l}.$$

The above iterative algorithm is known as the Newton method, which requires the second order derivative, in addition to the first order derivative. The Newton method can converge much faster, but each iteration requires significantly more calculation, and the algorithm is not guaranteed to converge. The use of the second order derivative also makes this algorithm more prone to noise and numerical errors.

In reality, the following update is more often used

$$\mathbf{x}^{l+1} = \mathbf{x}^l - \alpha [\mathbf{H}(\mathbf{x}_l)]^{-1} \frac{\partial J}{\partial \mathbf{x}} \Big|_{\mathbf{x}^l}, \quad (\text{B.3.3})$$

where the step-size  $\alpha$  is a constant smaller than 1, which needs to be chosen properly to reach a good compromise between guaranteeing the convergence and the convergence rate.

## B.4 Newton-Ralphson Method

When the objective function is a sum of squared errors, i.e.,

$$J(\mathbf{x}) = \frac{1}{2} \sum_k e_k^2(\mathbf{x}), \quad (\text{B.4.1})$$

we have

$$\frac{\partial J}{\partial \mathbf{x}} = \sum \frac{\partial e_k}{\partial \mathbf{x}} e_k(\mathbf{x}), \quad (\text{B.4.2})$$

and

$$[\mathbf{H}] = \frac{\partial^2 J}{\partial \mathbf{x}^2} = \sum \frac{\partial e_k}{\partial \mathbf{x}} \left( \frac{\partial e_k}{\partial \mathbf{x}} \right)^T + \frac{\partial^2 e_k}{\partial \mathbf{x}^2} e_k(\mathbf{x}) \approx \sum \frac{\partial e_k}{\partial \mathbf{x}} \left( \frac{\partial e_k}{\partial \mathbf{x}} \right)^T. \quad (\text{B.4.3})$$

The last approximation is obtained by assuming the term with the second order derivative is significantly smaller than that with the first order derivative only. Using the above approximation in Eq. (B.3.3) yields the Newton-Ralphson method. Because almost all the error minimization problems encountered in practice uses a sum of error squares as an objective function, the Newton-Ralphson method is widely used.

## B.5 Bibliography

- [1] P. J. M. Van Laarhoven and E. H. Aarts. *Simulated Annealing: Theory and Applications*. Reidel, Dordrecht, Holland, 1987.

## Appendix C

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# GLOSSARY OF MATHMATICAL NOTATIONS

Video formation	
$\mathbf{X} = (X, Y, Z)^T$	The continuous coordinate in the 3-D space: $X$ , $Y$ , and $Z$ represent the horizontal, vertical, and depth positions, respectively.
$\mathbf{x} = (x, y)^T$	The continuous coordinate in the 2-D image plane: $x$ and $y$ represent the horizontal and vertical positions, respectively.
$t$	The continuous time index.
$\lambda$	The wavelength of a light.
$C(\mathbf{X}, t, \lambda)$	The radiant intensity of a light with wavelength $\lambda$ at location $\mathbf{X}$ and time $t$ .
$r(\mathbf{X}, t, \lambda)$	The reflection coefficient for a light with wavelength $\lambda$ at location $\mathbf{X}$ and time $t$ .
$\bar{\psi}(\mathbf{X}, t)$	The light intensity “visible” to the camera at position $\mathbf{X}$ and time $t$ in the imaged 3-D scene.
$\psi(\mathbf{x}, t)$	The video signal representing the image value at pixel location $\mathbf{x}$ and frame time $t$ . The image value could be a scalar representing the luminance or a vector representing three color components. This notation is used to refer a video signal in general, defined over either a continuous or discrete space.
$\mathcal{P}$	The projection function from 3-D to 2-D, $\mathbf{x} = \mathcal{P}(\mathbf{X})$ . The video signal is related to the 3-D scene by $\psi(\mathbf{x}, t) = \bar{\psi}(\mathbf{X}, t)$ .
Analog raster video	
$f_l$	Line rate, with a unit of lines/s.
$T_l$	Line interval, $T_l = 1/f_l$ .
$T_h$	Horizontal retrace times.
$T_v$	Vertical retrace time.

Digital video	
$\mathbf{m} = (m, n)^T$	The discrete coordinate in a 2-D digital image: $m$ and $n$ represent the column and row indices of pixels, respectively.
$k$	The discrete time index or frame number.
$\psi(\mathbf{m}, k)$	The digital video signal representing the image value at pixel location $\mathbf{m}$ and frame time $k$ .
$d_x, d_y, \Delta_t$	Sampling interval in horizontal, vertical, and temporal directions.
$f_{s,t}$	Sampling frequency in temporal direction or frame rate, $f_{s,t} = 1/\Delta_t$ , measured in frame/s (fps) or Hz.
$f_{s,x}, f_{s,y}$	Sampling frequencies in horizontal and vertical directions, $f_{s,x} = 1/d_x, f_{s,y} = 1/d_y$ , commonly measured in pixels/picture-width and pixels/picture-height.
$f_s$	Generally used to represent the sampling frequency. For video signals, $f_s = f_{s,t} f_{s,y} f_{s,x}$ .
$N_b$	Number of bits used to represent each pixel.
$R$	Data rate in bits/s (bps), $R = f_{s,t} f_{s,y} f_{s,x} N_b$ for a uncompressed video.
$\mathcal{R}^K$	$K$ -D real space.
$\mathcal{C}^K$	$K$ -D complex space.
$\Lambda$	Sampling lattice.
$\Lambda^*$	Reciprocal lattice of $\Lambda$ .
$\rho(\Lambda)$	Sampling density of lattice $\Lambda$ .
Frequency domain representations	
$f_x, f_y$	Horizontal and vertical frequencies, usually measured in cycles/degree (cpd).
$f_t$	Temporal frequency, with unit of cycles/s or Hz.
$\mathbf{s}$	The frequency index in a multidimensional space. For video signals, $\mathbf{s} = [f_x, f_y, f_t]^T$ .
$\mathbf{s}_c, \mathbf{s}_d$	Frequencies in continuous and discrete spaces, respectively.
$\Psi_c(f_x, f_y, f_t)$	Continuous space Fourier transform (CSFT) of $\psi(x, y, t)$ .
$\Psi_s(f_x, f_y, f_t)$	Sampled space Fourier transform (SSFT) of $\psi(x, y, t)$ .
$\Psi_d(f_x, f_y, f_t)$	Discrete space Fourier transform (DSFT) of $\psi(m, n, k)$ .
$\Psi(f_x, f_y, f_t)$	Generally used to refer to the Fourier transform of a video signal, can be either CSFT, SSFT, or DSFT.

Motion characterization	
$\mathbf{D}(\mathbf{X})$	3-D motion vector of point $\mathbf{X}$ at time $t$ to its position at time $t + \delta_t$ , $\mathbf{D}(\mathbf{X}) = (D_x(\mathbf{X}), D_y(\mathbf{X}), D_z(\mathbf{X})) = \mathbf{X}(t + \delta_t) - \mathbf{X}(t)$ . $\psi(\mathbf{x}, t)$ is called the anchor frame, $\psi(\mathbf{x}, t + \Delta_t)$ the tracked frame. When the underlying $\mathbf{X}$ is clear, we simply write $\mathbf{D}$ or $(D_x, D_y, D_z)$ .
$\mathbf{d}(\mathbf{x})$	2-D motion vector of pixel $\mathbf{x}$ at time $t$ to its position at time $t + \delta_t$ , $\mathbf{d}(\mathbf{x}) = (d_x(\mathbf{x}), d_y(\mathbf{x})) = \mathbf{x}(t + \delta_t) - \mathbf{x}(t)$ . When the underlying $\mathbf{x}$ is clear, we simply write $\mathbf{d}$ or $(d_x, d_y)$ . Note that the same notation is also used to represent the disparity vector in a stereo sequence.
$\mathbf{v}(\mathbf{x})$	2-D velocity vector of pixel $\mathbf{x}$ , $\mathbf{v}(\mathbf{x}) = (v_x(\mathbf{x}), v_y(\mathbf{x}))$ .
$\mathbf{w}(\mathbf{x})$	The mapping function between two image frames, $\mathbf{w}(\mathbf{x}) = \mathbf{x} + \mathbf{d}(\mathbf{x})$ .
Multiview video signals	
$\psi_v(\mathbf{x}, t)$	Image function in view $v$ in a multiview system.
$\mathbf{d}_{v_1, v_2, t_1, t_2}$	The displacement between points in view $v_1$ at time $t_1$ and its corresponding point in view $v_2$ at time $t_2$ .
$\mathbf{d}(\mathbf{x})$	The disparity vector between a point $\mathbf{x}$ in the right view and its corresponding point in the left view in a stereo sequence. Note that the same notation is also used for motion vectors within the same view.
Color representation	
$Y$	The luminance component of a color signal.
$C_b, C_r$	The color difference components in the YCbCr coordinate, $C_b = Y - B, C_r = Y - R$ .
$R, G, B$	The red, green, blue tristimulus values in the RGB coordinate.
Camera characterization	
$\theta_x, \theta_y, \theta_z$	Camera rotation angles with respect to $x, y$ , and $z$ axes, respectively, of a predefined world coordinate.
$t_x, t_y, t_z$	Camera translation along $x, y$ , and $z$ axes, respectively.
$F$	Camera focal length.
$\gamma_c$	Gamma value of a camera.
$\gamma_d$	Gamma value of a display monitor.

## Appendix D

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# GLOSSARY OF ACRONYMS

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2D	Two-Dimensional
3D	Three-Dimensional
3DTV	Three-dimensional TV
K-D	K-Dimensional
AAC	Advanced Audio Coder (the MPEG-2 audio coder)
AC	Alternating Current (refer to transformed coefficients corresponding to non-zero frequencies)
ADSL	Asymmetric Digital Subscriber Loop
ARQ	Automatic Retransmission reQuest
AU	Action Unit (used in FACS)
AV	audiovisual
ATM	Asynchronous Transfer Mode
BAB	Binary Alpha Block
BDP	body definition parameters
BAP	body animation parameters
BER	Bit Error Rate
BIFS	Binary Format for Scenes
B-ISDN	Broadband ISDN
BMA	Block Matching Algorithm
bps	bits per second
BT.601	A digital video format defined by ITU-R. Formerly known as CCIR601
CAC	Call Admission Control
CBP	coded block pattern (specified for each MB in MPEG-1/2/4 standards)
CCIR	the International Radio Consultative Committee
CCIR601	See BT.601.
CCITT	International Telephone and Telegraph Consultative Committee. Renamed to ITU-T
CDMA	Code Division Multiple Access
CELP	Code Excited Linear Prediction (used for speech coding)

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CIE	Commission Internationale de L'Eclairage, an international body of color scientists
CIF	Common Intermediate Format
CMY	The color coordinate using Cyan, Magenta, and Yellow as primary colors
CMYK	The color coordinate using Cyan, Magenta, Yellow, and Black as primary colors
codec	coder and decoder
cpd	Cycles/s, the unit of the spatial frequency in either horizontal or vertical directions.
CRT	Cathode Ray Tube
CSFT	Continuous space Fourier transform
DAVIC	Digital Audio Visual Council
DBMA	Deformable BMA
DBS	direct broadcast satellite
DC	Direct Current (refer to the transformed coefficient corresponding to zero frequency )
DCP	disparity compensated prediction
DCT	Discrete Cosine Transform
DFT	Discrete Fourier Transform
DSFT	Sampled space Fourier transform
DTV	Digital Television
DFD	Displaced Frame Difference
DPCM	Differential Pulse Coded Modulation
DSL	Digital Subscriber Loop
DVD	Digital Video Disk
DVTR	Digital Video Tape Recorder
DWT	Discrete Wavelet Transform
EBMA	Exhaustive Search BMA
EOB	End of Block (a symbol used in coding a DCT block)
EZW	Embedded Zero-Tree Wavelet (a wavelet-based coding method)
FACS	Facial Action Coding System
FAP	Facial Animation Parameter
FDP	Facial Definition Parameter
FEC	Forward Error Correction
FGS	Fine Granularity Scalability
fps	Frames/s, the unit of the frame rate or temporal sampling frequency. Also represented as Hz.
GOB	Group of Blocks
GOP	Group of Pictures
GSM	Global System for Mobile communication
GVOP	Group of Video Object Planes

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HBMA	Hierarchical BMA
HDTV	High Definition Television
HSI	The color coordinate using Hue, Saturation and Intensity (also known as HSV, where V stands for Value)
HTTP	HyperText Transfer Protocol
HVS	Human visual system
IAR	Image aspect ratio
IEC	International Electrotechnical Commission
IETF	Internet Engineering Task Force
IP	Internet Protocol
ISDN	Integrated Service Digital Network (usually refer to narrow-band ISDN)
ISA	International Federation of the National Standardizing Associations
ISO	International Standard Organization
ISP	Independent Service Provider
ITU	International Telecommunications Union
ITU-R	International Telecommunications Union - Radio Sector
ITU-T	International Telecommunications Union - Telecommunications Sector
JPEG	Joint Photographic Expert Group
KBASC	Knowledge-Based Analysis-Synthesis Coding
kbps	kilobits per second
KLT	Karhunen-Loeve Transform
LAN	Local Area Network
LCD	Liquid Crystal Display
LMMSE	Linear Minimal Mean Square Error
LTI	Linear Time Invariant
MAD	Mean Absolute Difference
MB	Macroblock (the coding unit in ITU and MPEG video coding standards)
mbps	megabits per second
MC	Motion Compensation
MCI	Motion Compensated Interpolation
MCP	Motion Compensated Prediction
MCU	Multipoint Control Unit
MDC	Multiple Description Coding
MDTC	Multiple Description Transform Coding
MMSE	Minimum Mean Square Error
MPEG	Motion Picture Expert Group
MQUANT	Quantizer Stepsize over a MB in video coding standards
ms	millisecond
MSE	Mean Square Error
MTYPE	Coding mode for a MB in video coding standards
MTU	Maximum transmission unit
MV	Motion Vector
MVD	Differentially Coded Motion Vector in video coding standards
MZTE	Multi-scale ZTE (a wavelet-based coding method)

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nm	nanometer
NTSC	the analog TV system used mostly in North America, defined by American National Television System Committee
OBASC	Object-Based Analysis-Synthesis Coding
OBMC	Overlapping Block Motion Compensation
PAL	Phase Alternating Line, the analog TV system used mostly in Western Europe
PAR	Pixel aspect ratio
PCM	Pulse Coded Modulation
pdf	probability density function
pmf	probability mass function
POCS	Projection Onto Convex Sets
PSD	Power Spectral Density
PSNR	Peak Signal to Noise Ratio
PSTN	Public Switched Telephone Network
QCIF	Quarter CIF
QoS	Quality of Service
QP	Quantization Parameter
RCPC	Rate-Compatible Punctured Convolutional code
RGB	The color coordinate using Red, Green, and Blue as primary colors
RPS	Reference Picture Selection
RTCP	Real-Time Control Protocol
RTP	Real-time Transport Protocol
RTSP	Real-Time Streaming Protocol
RV	Random Variable
RVLC	Reversible Variable Length Coding
SA-DCT	Shape-Adaptive DCT
SA-DWT	Shape-Adaptive DWT
SA-ZTE	Shape-Adaptive ZTE
SECAM	Sequential Couleur avec Memoire, the analog TV system used mostly in Eastern Europe
SMPTE	Society of Motion Picture and Television Engineers
SNR	Signal to Noise Ratio
SPIHT	set partitioning in hierarchical trees (a wavelet-based coding method)
SQ	Scalar Quantization
SSFT	Discrete space Fourier transform
S-VHS	an enhanced version of VHS

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TC	Transform Coding
TCP	Transmission Control Protocol
TDMA	Time Division Multiple Access
TTS	Text-To-Speech
TTSI	Text-To-Speech Interface
TV	Television
UDP	User Datagram Protocol
URL	Universal Resource Locator
UMTS	Universal Mobile Telecommunications System
VCR	Video Cassette Recorder
VHS	an analog video tape recording format
VLC	Variable Length Coding
VO	Video Object (a terminology in MPEG-4)
VOL	Video Object Layer (a terminology in MPEG-4)
VOP	Video Object Plane (a terminology in MPEG-4)
VRML	Virtual Reality Modeling Language
VQ	Vector Quantization
YCbCr	The color coordinate used in most digital video formats, consisting of Luminance (Y) and two color difference signals (Cb and Cr)
YIQ	The color coordinate used by the NTSC system, consisting of Luminance (Y) and two color components I and Q
YUV	The color coordinate used by the PAL and SECAM systems, consisting of Luminance (Y) and two color components U and V
ZTE	Zero Tree Entropy coding (a wavelet-based coding method)

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**Errata for**  
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**Symbols Used**

$T_i$  = i-th line from top;  $B_i$  = i-th line from bottom;  $F_i$  = Figure i,  $TA_i$  = Table i,  
 $P_i$ =Problem i,  $E(i)$ =Equation(i),  $X \rightarrow Y$  = replace X with Y

Page	Line/Fig/Tab	Corrections
16	F1.5	Add an output from the demultiplexing box to a microphone at the bottom of the figure.
48	B6, E(2.4.4)-E(2.4.6)	Replace “ $v_x$ ”, “ $v_y$ ” by “\tilde v_x”, “\tilde v_y”
119	E(5.2.7)	$C(X) \rightarrow C(X,t), r(X) \rightarrow r(X,t), E(N) \rightarrow E(N,t)$
125	F5.11	Caption: “cameras”-> “a camera”, “diffuse”-> “ambient”
126	T7	“diffuse illumination”-> “ambient illumination”
133	B10	$T_x, T_y, T_z \rightarrow T_x, T_y, T_z$ , and Z
	B4	Delete “when there is no translational motion in the Z direction, or”
	B2	“ $aX+bY+cZ=1$ ” -> “ $Z=aX+bY+c$ ”
	Before E(5.5.13)	Add “(see Problem 5.3)” after “before and after the motion”
138	P5.3	“a planar patch” -> “any 3-D object”, “projective mapping”->Equation (5.5.13)”
	P5.4	“Equation 5.5.14”-> “Equation (5.5.14)”, “ $aX+bY+cZ=1$ ”-> “ $Z=aX+bY+c$ ”
143	T4	After “true 2-D motion.” Add “Optical flow depends on not only 2-D motion, but also illumination and object surface texture.”
159	T6	After “block size is 16x16” add “, and the search range is 16x16”
189	P6.1	“global”->”global-based”
190	P6.12	Add at the end “Choose two frames that have sufficient motion in between, so that it is easier to observe the effect of motion estimation inaccuracy. If necessary, choose frames that are not immediate neighbors.”
199	T9	“Equation (7.1.11) defines a linear dependency ... straight line.” -> “Equation (7.1.11) says that the possible positions $x'$ of a point $x$ after motion lie on a straight line. The actual position depends on the Z-coordinate of the original 3-D point.”
200	B8	“[A]” -> “[A] <sup>T</sup> [A]”
214	P7.5	“Derive”-> “Equation (7.1.5) describes”
		Add at the end “(assuming F=1)”
	P7.6	Replace “\delta” with “\bf \delta”
218	F8.1	“Parameter statistics” -> “Model parameter statistics”
247	F8.9	Add a box with words “Update previous distortion \ D_0=D_1” in the line with the word “No”.

255	F8.14	Same as for F8.9
261	P8.13(a)	“B_l={f_k, k=1,2,...,K_l}” -> “B_l, which consists of K_l vectors in {\cal F}”
416	TA13.2	Item “4CIF/H.263” should be “Opt.”
421	TA13.3	Item “Video/Non-QoS LAN” should be “H.261/3”
436	T13	“MPEG-2, defined” -> “MPEG-2 defined”
443	T10	“I-VOP”->”I-VOPs”, “B-VOP”-> “B-VOPs”
575	P1.3	“red+green=blue”-> “red+green=black”
	P1.4	“(1.4.4)” -> “(1.4.3)”, “(1.4.2)” -> “(1.4.1)”