Homework 1: Review of materials

your name

Due: January 27th at 11:59 PM

Problem 1: Prove that the Binomial distribution arises as a sum of n iid Bernoulli trials each with success probability p.

Problem 2: Let $l(\theta)$ denote a twice continuously differentiable log likelihood corresponding to an iid sample under density f_θ where n is the sample size. The score function is defined as

$$u(\theta) = \frac{\partial l(\theta)}{\partial \theta},$$

and the Fisher information matrix is defined as

$$I(\theta) = -\mathrm{E}\left(\frac{\partial^2 l(\theta)}{\partial \theta^2}\right),$$

where the expectation is over the assumed distribution for the data when the parameter value is θ. Prove that

$$\mathrm{E}(u(\theta)) = 0 \qquad \text{and} \qquad \mathrm{Var}(u(\theta)) = I(\theta).$$

Problem 3: Let $Y \sim \text{binomial}(n, \pi)$ and let $T_n = \hat{\pi} = Y/n$. Use the CLT and the Delta Method to construct an asymptotic confidence interval for $\text{logit}(\pi)$. Note that this recipe does not work when the estimated success probability is on the boundary of its support, ie $\hat{\pi} = 0$ or $\hat{\pi} = 1$. Why?