Bollen & Brand Model - Empirical Example

In this document, we'll apply Bollen and Brand's model to empirical data. The data come from a study conducted by Rowland, Wenzel, and Kubiak (2018) called "A Mind Full of Happiness: How Mindfulness Shapes Affect Dynamics in Daily Life." Find the article here and its corresponding files here. Below, we'll follow the same sequence that we applied in the article: `dp_mod1`, followed by `dp_mod2`, and finally `dp_mod3`.

Study Background & Data

Rowland et al. (2018) assessed the relationship between mindfulness and affect using an undergraduate sample. There were many aspects to their study. Students were randomly assigned to a mindfulness training or control condition, underwent an experience sampling protocol in which they responded to surveys roughly six times per day for forty days, participated in weekly laboratory visits, and completed follow-up assessments. Moreover, the researchers captured both stait and trait measures, and they evaluated patterns across all of these data sources.

To make things simple, we are only going to focus on two variables: anger and satisfaction. Both were measured six times a day (using a beeper protocol) during the forty-day experience sampling procedure. Participants were asked to indicate how they felt at the current moment by adjusting a visual slider from 0 (not at all) to 100 (very much) on two items: satisfied and angry. Below, we aggregate assessments to the day-level, meaning that we take the average of participant i's six anger scores on a given day and save that value. We then analyze data over the first eight days of the experience sampling protocol. Again, we are limiting ourselves to keep this document consistent with the tutorial. Rowland et al. (2018) conducted a superb study, and they were kind enough to post their data on OSF. This document is not a re-analysis of their data, but a tutorial on programming with respect to Bollen and Brand's model. There are additional aspects to their data that more advanced readers may wish to explore.

In sum, we are going to evaluate two variables: anger and satisfaction. They were originally measured six times a day over 40 days. We are going to aggregate that to the day-level and then limit ourselves to the first 8 days. The data set we need to end up with, then, should look something like the following:

id	time	anger	satisfied
1	1	22.36556	19.25526
1	2	21.20902	22.74577
1	3	21.14400	21.03314
1	4	19.84609	18.00563
1	5	22.48112	18.41419
1	6	17.56536	20.49356
1	7	18.63988	20.67641
1	8	22.68277	20.96507
2	1	19.55779	20.67014
2	2	17.54752	20.17154

Participants are listed in the `id` column, periods or days are listed in the `time` column, and the two variables `anger` and `satisfaction` house their respective scores. Let's dig into the data and start cleaning.

Data Cleaning

Load the data, and load necessary libraries.

```r
library(tidyverse)
library(ggplot2)
library(lavaan)
library(kableExtra)
library(tseries)
library(plm)
library(reshape2)

df <- read.csv("Data.csv")
```

Rowland et al.'s item dictionary states the following:

- id: participant number
- dayno: day number
- beep: signal number within day
- sat: satisfaction
- ang: anger

There are of course other variables in the data, but we only need those listed above.

```r
df <- df %>%
  select(id, dayno, beep, sat, ang)
```

Here's a snippet of the data.

```r
head(df, 8) %>% kable() %>% kable_styling()
```

id	dayno	beep	sat	ang
1	1	1	74	19
1	1	2	NA	NA
1	1	3	36	62
1	1	4	29	42
1	1	5	17	35
1	1	6	NA	NA
1	2	1	NA	NA
1	2	2	41	36

We're hoping to get to data with one observation on both variables per day for each student. To get there, we need to aggregate over "beeps."

```r
df <- df %>%
  group_by(id, dayno) %>%
  summarize(
    sat = mean(sat, na.rm = T),
    ang = mean(ang, na.rm = T)
  )

head(df, 8) %>% kable() %>% kable_styling()
```

id	dayno	sat	ang
1	1	39.00000	39.50000
1	2	59.00000	40.50000
1	3	57.83333	46.66667
1	4	84.33333	29.00000
1	5	65.50000	25.66667
1	6	72.40000	43.00000
1	7	87.00000	21.50000
1	8	82.33333	38.33333

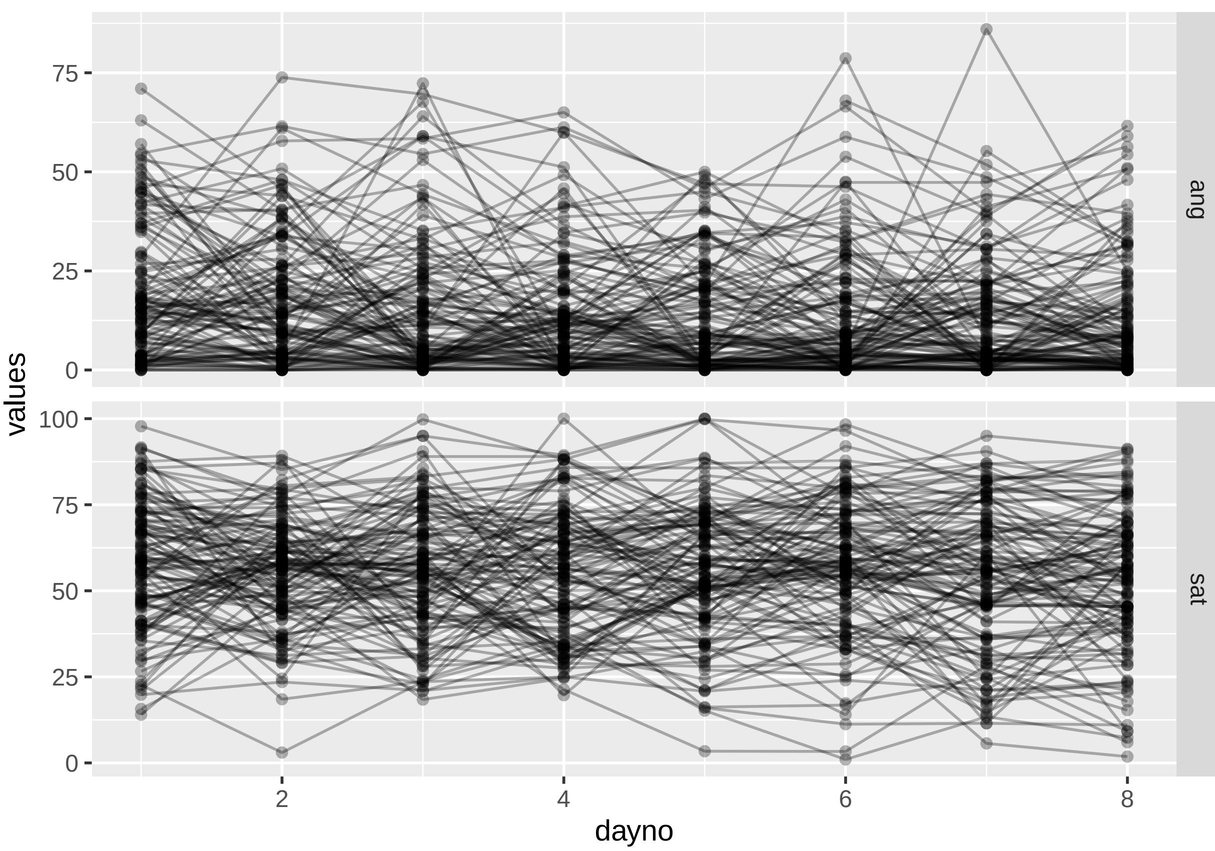
Great. Now we want only the first eight days for each student.

```
df <- df %>%
  filter(dayno %in% c(1:8))
```

The data is ready. Let's plot it before analyzing.

```
plot_df <- df %>%
  pivot_longer(cols = c(sat, ang),
               names_to = "variable",
               values_to = "values")

ggplot(plot_df, aes(x = dayno, y = values, group = id)) +
  geom_point(alpha = 0.3) +
  geom_line(alpha = 0.3) +
  facet_grid(vars(variable), scales = "free")
```



Finally, we can rename it so that the object matches what's written in the article.

```r
rowland_data <- as.data.frame(df)
```

Modeling

Check stationarity for both variables and change to wide format.

```r
data_dickey_structure <- rowland_data %>%
  filter(!is.na(ang) == T) %>%
  filter(!is.na(sat) == T)

data_dickey_structure <- plm.data(data_dickey_structure, index = c("id", "dayno"))

adf.test(data_dickey_structure$sat)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data_dickey_structure$sat
## Dickey-Fuller = -8.3936, Lag order = 9, p-value = 0.01
## alternative hypothesis: stationary
```

```r
adf.test(data_dickey_structure$ang)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data_dickey_structure$ang
## Dickey-Fuller = -8.9603, Lag order = 9, p-value = 0.01
## alternative hypothesis: stationary
```

```r
df_wide <- reshape(rowland_data, timevar = "dayno", idvar = "id", direction = "wide")
```

Both satisfaction and anger are stationary.

Example 1

The first example models a concurrent effect from anger to satisfaction.

```r
dp_mod1 <- "

eta_y =~ 1*sat.2 + 1*sat.3 + 1*sat.4 + 1*sat.5 + 1*sat.6 + 1*sat.7 + 1*sat.8

sat.2 ~ rho_y*sat.1 + b1*ang.2
sat.3 ~ rho_y*sat.2 + b1*ang.3
sat.4 ~ rho_y*sat.3 + b1*ang.4
sat.5 ~ rho_y*sat.4 + b1*ang.5
sat.6 ~ rho_y*sat.5 + b1*ang.6
sat.7 ~ rho_y*sat.6 + b1*ang.7
sat.8 ~ rho_y*sat.7 + b1*ang.8

ang.2 ~~ ang.3 + ang.4 + ang.5 + ang.6 + ang.7 + ang.8
ang.3 ~~ ang.4 + ang.5 + ang.6 + ang.7 + ang.8
ang.4 ~~ ang.5 + ang.6 + ang.7 + ang.8
ang.5 ~~ ang.6 + ang.7 + ang.8
ang.6 ~~ ang.7 + ang.8
ang.7 ~~ ang.8
```

```r
sat.1 ~~ ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7 + ang.8
eta_y ~~ sat.1 + ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7 + ang.8


"

dp_mod1_fit <- sem(dp_mod1, data = df_wide, missing = "FIML")
summary(dp_mod1_fit, fit.measures = T, standardized = T)
```

```
## lavaan 0.6-6 ended normally after 352 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         81
##   Number of equality constraints                    12
##
##   Number of observations                           125
##   Number of missing patterns                        11
##
## Model Test User Model:
##
##   Test statistic                               122.804
##   Degrees of freedom                                66
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              1064.517
##   Degrees of freedom                               105
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.941
##   Tucker-Lewis Index (TLI)                       0.906
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7379.231
##   Loglikelihood unrestricted model (H1)      -7317.829
##
##   Akaike (AIC)                               14896.462
##   Bayesian (BIC)                             15091.616
##   Sample-size adjusted Bayesian (BIC)        14873.426
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.083
##   90 Percent confidence interval - lower         0.060
##   90 Percent confidence interval - upper         0.106
##   P-value RMSEA <= 0.05                          0.012
##
## Standardized Root Mean Square Residual:
```

```
##
##   SRMR                                           0.080
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   eta_y =~
##     sat.2             1.000                               12.087    0.651
##     sat.3             1.000                               12.087    0.610
##     sat.4             1.000                               12.087    0.623
##     sat.5             1.000                               12.087    0.649
##     sat.6             1.000                               12.087    0.653
##     sat.7             1.000                               12.087    0.626
##     sat.8             1.000                               12.087    0.665
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   sat.2 ~
##     sat.1   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.118
##     ang.2     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.414
##   sat.3 ~
##     sat.2   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.113
##     ang.3     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.423
##   sat.4 ~
##     sat.3   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.123
##     ang.4     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.365
##   sat.5 ~
##     sat.4   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.125
##     ang.5     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.355
##   sat.6 ~
##     sat.5   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.121
##     ang.6     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.413
##   sat.7 ~
##     sat.6   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.115
##     ang.7     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.366
##   sat.8 ~
##     sat.7   (rh_y)    0.120    0.037    3.284    0.001    0.120    0.128
##     ang.8     (b1)   -0.472    0.039  -12.075    0.000   -0.472   -0.393
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   ang.2 ~~
##     ang.3           132.748   28.929    4.589    0.000  132.748    0.459
##     ang.4           121.117   24.637    4.916    0.000  121.117    0.496
##     ang.5           106.723   22.693    4.703    0.000  106.723    0.469
##     ang.6           142.027   26.940    5.272    0.000  142.027    0.540
##     ang.7           113.676   24.430    4.653    0.000  113.676    0.467
##     ang.8           126.223   25.221    5.005    0.000  126.223    0.513
##   ang.3 ~~
```

```
##     ang.4           141.884   27.349    5.188    0.000  141.884    0.532
##     ang.5            81.094   23.719    3.419    0.001   81.094    0.326
##     ang.6           102.770   27.540    3.732    0.000  102.770    0.358
##     ang.7            84.925   25.252    3.363    0.001   84.925    0.319
##     ang.8           135.998   27.751    4.901    0.000  135.998    0.506
##   ang.4 ~~
##     ang.5           112.664   21.397    5.265    0.000  112.664    0.537
##     ang.6           110.893   23.958    4.629    0.000  110.893    0.457
##     ang.7           117.683   22.811    5.159    0.000  117.683    0.524
##     ang.8           117.051   22.945    5.101    0.000  117.051    0.515
##   ang.5 ~~
##     ang.6           130.271   23.917    5.447    0.000  130.271    0.576
##     ang.7            91.145   20.779    4.386    0.000   91.145    0.435
##     ang.8           118.402   21.918    5.402    0.000  118.402    0.560
##   ang.6 ~~
##     ang.7            85.797   23.081    3.717    0.000   85.797    0.354
##     ang.8           144.168   25.541    5.644    0.000  144.168    0.589
##   ang.7 ~~
##     ang.8           138.023   24.064    5.736    0.000  138.023    0.609
##   sat.1 ~~
##     ang.2           -47.873   26.909   -1.779    0.075  -47.873   -0.162
##     ang.3           -33.868   29.257   -1.158    0.247  -33.868   -0.105
##     ang.4           -30.518   24.792   -1.231    0.218  -30.518   -0.112
##     ang.5           -43.736   23.442   -1.866    0.062  -43.736   -0.172
##     ang.6           -70.537   27.405   -2.574    0.010  -70.537   -0.239
##     ang.7           -31.918   24.699   -1.292    0.196  -31.918   -0.117
##     ang.8           -28.474   24.877   -1.145    0.252  -28.474   -0.103
##   eta_y ~~
##     sat.1           119.197   24.589    4.848    0.000    9.861    0.541
##     ang.2            34.984   20.054    1.744    0.081    2.894    0.178
##     ang.3            24.219   21.528    1.125    0.261    2.004    0.113
##     ang.4            46.581   18.624    2.501    0.012    3.854    0.257
##     ang.5            31.181   17.218    1.811    0.070    2.580    0.185
##     ang.6            22.605   19.826    1.140    0.254    1.870    0.116
##     ang.7            23.883   18.184    1.313    0.189    1.976    0.132
##     ang.8            39.622   18.809    2.107    0.035    3.278    0.217
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sat.2            57.457    2.706   21.236    0.000   57.457    3.094
##    .sat.3            57.270    2.644   21.661    0.000   57.270    2.890
##    .sat.4            55.546    2.685   20.686    0.000   55.546    2.861
##    .sat.5            55.587    2.594   21.426    0.000   55.587    2.986
##    .sat.6            56.815    2.559   22.202    0.000   56.815    3.070
##    .sat.7            53.257    2.648   20.111    0.000   53.257    2.758
##    .sat.8            52.908    2.529   20.918    0.000   52.908    2.909
##     sat.1            58.366    1.638   35.641    0.000   58.366    3.204
##     ang.2            18.022    1.459   12.354    0.000   18.022    1.108
##     ang.3            16.520    1.606   10.285    0.000   16.520    0.930
##     ang.4            14.765    1.348   10.955    0.000   14.765    0.984
##     ang.5            13.998    1.257   11.139    0.000   13.998    1.001
##     ang.6            14.133    1.450    9.745    0.000   14.133    0.874
##     ang.7            13.693    1.346   10.174    0.000   13.693    0.914
##     ang.8            13.841    1.361   10.173    0.000   13.841    0.915
```

```
##     eta_y            0.000                               0.000    0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sat.2          133.795   19.808    6.755    0.000  133.795    0.388
##    .sat.3          154.613   22.490    6.875    0.000  154.613    0.394
##    .sat.4          179.830   25.997    6.917    0.000  179.830    0.477
##    .sat.5          143.962   20.957    6.869    0.000  143.962    0.415
##    .sat.6          113.739   17.224    6.603    0.000  113.739    0.332
##    .sat.7          155.350   22.425    6.928    0.000  155.350    0.416
##    .sat.8          125.353   18.739    6.689    0.000  125.353    0.379
##     sat.1          331.912   42.119    7.880    0.000  331.912    1.000
##     ang.2          264.684   33.846    7.820    0.000  264.684    1.000
##     ang.3          315.666   40.882    7.721    0.000  315.666    1.000
##     ang.4          225.123   28.577    7.878    0.000  225.123    1.000
##     ang.5          195.439   25.214    7.751    0.000  195.439    1.000
##     ang.6          261.729   33.184    7.887    0.000  261.729    1.000
##     ang.7          224.320   28.625    7.836    0.000  224.320    1.000
##     ang.8          229.030   29.499    7.764    0.000  229.030    1.000
##     eta_y          146.101   24.438    5.978    0.000    1.000    1.000
```

The model fit statistics are $\chi^2(66) = 122.804$; $p < 0.05$; RMSEA = 0.083; CFI = 0.94; SRMR = 0.080, and the standardized coefficient estimates are as follows. Satisfaction had a positive autoregressive effect across time ($B = 0.12$, $SE = 0.03$, $p < 0.05$). The concurrent relationship between anger and satisfaction was negative ($B =$ -0.47, $SE = 0.04$, $p < 0.05$), such that low anger at t was associated with high satisfaction at t.

Example 2

The second example demonstrates a lagged, two-variable dynamic panel. Anger will now be evaluated as a lag-one predictor of satisfaction.

```
dp_mod2 <- "


eta_y =~ 1*sat.2 + 1*sat.3 + 1*sat.4 + 1*sat.5 + 1*sat.6 + 1*sat.7 + 1*sat.8

sat.2 ~ rho_y*sat.1 + b1*ang.1
sat.3 ~ rho_y*sat.2 + b1*ang.2
sat.4 ~ rho_y*sat.3 + b1*ang.3
sat.5 ~ rho_y*sat.4 + b1*ang.4
sat.6 ~ rho_y*sat.5 + b1*ang.5
sat.7 ~ rho_y*sat.6 + b1*ang.6
sat.8 ~ rho_y*sat.7 + b1*ang.7

ang.1 ~~ ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7
ang.2 ~~ ang.3 + ang.4 + ang.5 + ang.6 + ang.7
ang.3 ~~ ang.4 + ang.5 + ang.6 + ang.7
ang.4 ~~ ang.5 + ang.6 + ang.7
ang.5 ~~ ang.6 + ang.7
ang.6 ~~ ang.7

sat.1 ~~ ang.1 + ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7
eta_y ~~ sat.1 + ang.1 + ang.2 + ang.3 + ang.4 + ang.5 + ang.6 + ang.7
```

```r
"

dp_mod2_fit <- sem(dp_mod2, data = df_wide, missing = "FIML")
summary(dp_mod2_fit, fit.measures = T, standardized = T)
```

```
## lavaan 0.6-6 ended normally after 298 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         81
##   Number of equality constraints                    12
##
##   Number of observations                           125
##   Number of missing patterns                        11
##
## Model Test User Model:
##
##   Test statistic                               224.839
##   Degrees of freedom                                66
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              1020.756
##   Degrees of freedom                               105
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.827
##   Tucker-Lewis Index (TLI)                       0.724
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7472.519
##   Loglikelihood unrestricted model (H1)      -7360.100
##
##   Akaike (AIC)                               15083.038
##   Bayesian (BIC)                             15278.192
##   Sample-size adjusted Bayesian (BIC)        15060.001
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.139
##   90 Percent confidence interval - lower         0.119
##   90 Percent confidence interval - upper         0.159
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.095
##
## Parameter Estimates:
##
```

```
##   Standard errors                              Standard
##   Information                                  Observed
##   Observed information based on                 Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   eta_y =~
##     sat.2             1.000                               10.990    0.605
##     sat.3             1.000                               10.990    0.557
##     sat.4             1.000                               10.990    0.536
##     sat.5             1.000                               10.990    0.561
##     sat.6             1.000                               10.990    0.607
##     sat.7             1.000                               10.990    0.560
##     sat.8             1.000                               10.990    0.598
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   sat.2 ~
##     sat.1   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.241
##     ang.1     (b1)    0.208    0.047    4.462    0.000    0.208    0.195
##   sat.3 ~
##     sat.2   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.221
##     ang.2     (b1)    0.208    0.047    4.462    0.000    0.208    0.171
##   sat.4 ~
##     sat.3   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.231
##     ang.3     (b1)    0.208    0.047    4.462    0.000    0.208    0.179
##   sat.5 ~
##     sat.4   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.251
##     ang.4     (b1)    0.208    0.047    4.462    0.000    0.208    0.159
##   sat.6 ~
##     sat.5   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.259
##     ang.5     (b1)    0.208    0.047    4.462    0.000    0.208    0.162
##   sat.7 ~
##     sat.6   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.221
##     ang.6     (b1)    0.208    0.047    4.462    0.000    0.208    0.172
##   sat.8 ~
##     sat.7   (rh_y)    0.240    0.047    5.098    0.000    0.240    0.256
##     ang.7     (b1)    0.208    0.047    4.462    0.000    0.208    0.169
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   ang.1 ~~
##     ang.2           130.141   27.391    4.751    0.000  130.141    0.471
##     ang.3            70.262   28.082    2.502    0.012   70.262    0.233
##     ang.4            97.701   24.648    3.964    0.000   97.701    0.381
##     ang.5           117.179   24.374    4.808    0.000  117.179    0.488
##     ang.6           123.793   27.277    4.538    0.000  123.793    0.447
##     ang.7            98.640   24.592    4.011    0.000   98.640    0.386
##   ang.2 ~~
##     ang.3           128.848   28.259    4.559    0.000  128.848    0.451
##     ang.4           118.121   24.414    4.838    0.000  118.121    0.486
##     ang.5           107.180   22.744    4.713    0.000  107.180    0.470
##     ang.6           141.534   26.880    5.265    0.000  141.534    0.539
##     ang.7           111.509   24.298    4.589    0.000  111.509    0.460
```

```
##   ang.3 ~~
##     ang.4           138.965   27.022    5.143    0.000  138.965    0.525
##     ang.5            78.104   23.608    3.308    0.001   78.104    0.315
##     ang.6            99.993   27.328    3.659    0.000   99.993    0.350
##     ang.7            82.990   25.026    3.316    0.001   82.990    0.315
##   ang.4 ~~
##     ang.5           113.445   21.549    5.264    0.000  113.445    0.537
##     ang.6           111.962   24.069    4.652    0.000  111.962    0.460
##     ang.7           118.567   22.861    5.186    0.000  118.567    0.528
##   ang.5 ~~
##     ang.6           133.347   24.310    5.485    0.000  133.347    0.584
##     ang.7            93.352   20.974    4.451    0.000   93.352    0.443
##   ang.6 ~~
##     ang.7            85.733   23.146    3.704    0.000   85.733    0.353
##   sat.1 ~~
##     ang.1          -126.643   30.258   -4.186    0.000 -126.643   -0.407
##     ang.2           -48.305   26.821   -1.801    0.072  -48.305   -0.164
##     ang.3           -33.502   29.056   -1.153    0.249  -33.502   -0.104
##     ang.4           -31.977   24.844   -1.287    0.198  -31.977   -0.117
##     ang.5           -42.673   23.624   -1.806    0.071  -42.673   -0.166
##     ang.6           -68.648   27.520   -2.494    0.013  -68.648   -0.232
##     ang.7           -33.375   24.749   -1.349    0.177  -33.375   -0.122
##   eta_y ~~
##     sat.1           128.572   24.127    5.329    0.000   11.699    0.642
##     ang.1           -49.849   19.846   -2.512    0.012   -4.536   -0.266
##     ang.2           -60.252   19.280   -3.125    0.002   -5.482   -0.339
##     ang.3           -62.133   20.482   -3.034    0.002   -5.654   -0.321
##     ang.4           -40.161   17.592   -2.283    0.022   -3.654   -0.243
##     ang.5           -48.035   16.706   -2.875    0.004   -4.371   -0.310
##     ang.6           -70.591   19.526   -3.615    0.000   -6.423   -0.396
##     ang.7           -53.217   17.615   -3.021    0.003   -4.842   -0.323
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sat.2            37.839    3.601   10.509    0.000   37.839    2.083
##    .sat.3            39.079    3.499   11.168    0.000   39.079    1.981
##    .sat.4            38.430    3.509   10.953    0.000   38.430    1.876
##    .sat.5            39.253    3.376   11.626    0.000   39.253    2.005
##    .sat.6            40.618    3.297   12.318    0.000   40.618    2.244
##    .sat.7            37.055    3.433   10.795    0.000   37.055    1.887
##    .sat.8            37.178    3.240   11.474    0.000   37.178    2.025
##     sat.1            58.380    1.639   35.621    0.000   58.380    3.202
##     ang.1            19.538    1.535   12.727    0.000   19.538    1.144
##     ang.2            17.957    1.452   12.370    0.000   17.957    1.109
##     ang.3            16.452    1.594   10.319    0.000   16.452    0.933
##     ang.4            14.758    1.348   10.947    0.000   14.758    0.983
##     ang.5            13.980    1.265   11.048    0.000   13.980    0.993
##     ang.6            14.071    1.455    9.671    0.000   14.071    0.867
##     ang.7            13.718    1.346   10.190    0.000   13.718    0.916
##     eta_y             0.000                                0.000    0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sat.2           149.326   22.159    6.739    0.000  149.326    0.452
```

```
##    .sat.3            199.979   28.638    6.983    0.000  199.979    0.514
##    .sat.4            225.789   32.253    7.001    0.000  225.789    0.538
##    .sat.5            178.919   25.707    6.960    0.000  178.919    0.467
##    .sat.6            129.495   19.492    6.643    0.000  129.495    0.395
##    .sat.7            199.779   28.414    7.031    0.000  199.779    0.518
##    .sat.8            143.639   21.258    6.757    0.000  143.639    0.426
##     sat.1            332.444   42.246    7.869    0.000  332.444    1.000
##     ang.1            291.445   37.141    7.847    0.000  291.445    1.000
##     ang.2            262.145   33.312    7.869    0.000  262.145    1.000
##     ang.3            310.685   39.819    7.803    0.000  310.685    1.000
##     ang.4            225.260   28.598    7.877    0.000  225.260    1.000
##     ang.5            198.236   25.789    7.687    0.000  198.236    1.000
##     ang.6            263.230   33.503    7.857    0.000  263.230    1.000
##     ang.7            224.087   28.605    7.834    0.000  224.087    1.000
##     eta_y            120.779   23.006    5.250    0.000    1.000    1.000
```

Model fit statistics are $\chi^2(66) = 224.84$, $p < 0.05$; RMSEA $= 0.14$; CFI $= 0.83$; SRMR $= 0.095$, and the standardized parameter estimates are 0.24 ($SE = 0.05$, $p < 0.05$) for the autoregressive effect of satisfaction and 0.21 ($SE = 0.05$, $p < 0.05$) for the lag-one effect of anger.

Example 3

The last example demonstrates a reciprocal dynamic panel. Both anger and satisfaction will have autoregression, and they will both act as lag-one inputs to the other state.

```
dp_mod3 <- "

eta_y =~ 1*sat.2 + 1*sat.3 + 1*sat.4 + 1*sat.5 + 1*sat.6 + 1*sat.7 + 1*sat.8
eta_x =~ 1*ang.2 + 1*ang.3 + 1*ang.4 + 1*ang.5 + 1*ang.6 + 1*ang.7 + 1*ang.8

sat.2 ~ rho_y*sat.1 + b1*ang.1
sat.3 ~ rho_y*sat.2 + b1*ang.2
sat.4 ~ rho_y*sat.3 + b1*ang.3
sat.5 ~ rho_y*sat.4 + b1*ang.4
sat.6 ~ rho_y*sat.5 + b1*ang.5
sat.7 ~ rho_y*sat.6 + b1*ang.6
sat.8 ~ rho_y*sat.7 + b1*ang.7

ang.2 ~ rho_x*ang.1 + b2*sat.1
ang.3 ~ rho_x*ang.2 + b2*sat.2
ang.4 ~ rho_x*ang.3 + b2*sat.3
ang.5 ~ rho_x*ang.4 + b2*sat.4
ang.6 ~ rho_x*ang.5 + b2*sat.5
ang.7 ~ rho_x*ang.6 + b2*sat.6
ang.8 ~ rho_x*ang.7 + b2*sat.7

sat.1 ~~ sat.1
ang.1 ~~ ang.1
sat.1 ~~ ang.1

eta_x ~~ eta_x
eta_y ~~ eta_y
eta_x ~~ eta_y

sat.1 ~~ eta_x
```

```r
sat.1 ~~ eta_y
ang.1 ~~ eta_x
ang.1 ~~ eta_y

"

dp_mod3_fit <- sem(dp_mod3, data = df_wide, missing = "FIML")
summary(dp_mod3_fit, fit.measures = T, standardized = T)
```

```
## lavaan 0.6-6 ended normally after 200 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         68
##   Number of equality constraints                    24
##
##   Number of observations                           125
##   Number of missing patterns                        11
##
## Model Test User Model:
##
##   Test statistic                               295.849
##   Degrees of freedom                               108
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              1155.444
##   Degrees of freedom                               120
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.819
##   Tucker-Lewis Index (TLI)                       0.798
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7943.818
##   Loglikelihood unrestricted model (H1)      -7795.894
##
##   Akaike (AIC)                               15975.637
##   Bayesian (BIC)                             16100.082
##   Sample-size adjusted Bayesian (BIC)        15960.947
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.118
##   90 Percent confidence interval - lower         0.102
##   90 Percent confidence interval - upper         0.134
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
```

```
##   SRMR                                           0.101
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   eta_y =~
##     sat.2             1.000                               10.965    0.600
##     sat.3             1.000                               10.965    0.552
##     sat.4             1.000                               10.965    0.535
##     sat.5             1.000                               10.965    0.565
##     sat.6             1.000                               10.965    0.608
##     sat.7             1.000                               10.965    0.553
##     sat.8             1.000                               10.965    0.593
##   eta_x =~
##     ang.2             1.000                                9.269    0.584
##     ang.3             1.000                                9.269    0.510
##     ang.4             1.000                                9.269    0.611
##     ang.5             1.000                                9.269    0.634
##     ang.6             1.000                                9.269    0.593
##     ang.7             1.000                                9.269    0.583
##     ang.8             1.000                                9.269    0.644
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   sat.2 ~
##     sat.1   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.246
##     ang.1     (b1)    0.225    0.048    4.657    0.000    0.225    0.209
##   sat.3 ~
##     sat.2   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.227
##     ang.2     (b1)    0.225    0.048    4.657    0.000    0.225    0.180
##   sat.4 ~
##     sat.3   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.239
##     ang.3     (b1)    0.225    0.048    4.657    0.000    0.225    0.199
##   sat.5 ~
##     sat.4   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.260
##     ang.4     (b1)    0.225    0.048    4.657    0.000    0.225    0.176
##   sat.6 ~
##     sat.5   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.265
##     ang.5     (b1)    0.225    0.048    4.657    0.000    0.225    0.182
##   sat.7 ~
##     sat.6   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.224
##     ang.6     (b1)    0.225    0.048    4.657    0.000    0.225    0.177
##   sat.8 ~
##     sat.7   (rh_y)    0.246    0.048    5.111    0.000    0.246    0.264
##     ang.7     (b1)    0.225    0.048    4.657    0.000    0.225    0.193
##   ang.2 ~
##     ang.1   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.167
##     sat.1     (b2)    0.086    0.037    2.303    0.021    0.086    0.098
##   ang.3 ~
```

```
##     ang.2   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.136
##     sat.2     (b2)    0.086    0.037    2.303    0.021    0.086    0.086
##   ang.4 ~
##     ang.3   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.186
##     sat.3     (b2)    0.086    0.037    2.303    0.021    0.086    0.112
##   ang.5 ~
##     ang.4   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.161
##     sat.4     (b2)    0.086    0.037    2.303    0.021    0.086    0.120
##   ang.6 ~
##     ang.5   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.145
##     sat.5     (b2)    0.086    0.037    2.303    0.021    0.086    0.106
##   ang.7 ~
##     ang.6   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.153
##     sat.6     (b2)    0.086    0.037    2.303    0.021    0.086    0.097
##   ang.8 ~
##     ang.7   (rh_x)    0.155    0.045    3.420    0.001    0.155    0.172
##     sat.7     (b2)    0.086    0.037    2.303    0.021    0.086    0.118
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   sat.1 ~~
##     ang.1          -126.160   30.160   -4.183    0.000 -126.160   -0.406
##   eta_y ~~
##     eta_x           -61.029   14.407   -4.236    0.000   -0.600   -0.600
##   eta_x ~~
##     sat.1           -47.457   18.200   -2.608    0.009   -5.120   -0.281
##   eta_y ~~
##     sat.1           127.919   24.076    5.313    0.000   11.667    0.640
##   eta_x ~~
##     ang.1            91.135   18.350    4.966    0.000    9.832    0.576
##   eta_y ~~
##     ang.1           -51.142   19.841   -2.578    0.010   -4.664   -0.273
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .sat.2            37.113    3.707   10.010    0.000   37.113    2.030
##    .sat.3            38.403    3.595   10.682    0.000   38.403    1.935
##    .sat.4            37.758    3.593   10.510    0.000   37.758    1.844
##    .sat.5            38.684    3.459   11.182    0.000   38.684    1.995
##    .sat.6            40.024    3.387   11.818    0.000   40.024    2.219
##    .sat.7            36.436    3.519   10.354    0.000   36.436    1.839
##    .sat.8            36.626    3.322   11.026    0.000   36.626    1.982
##    .ang.2             9.916    3.058    3.243    0.001    9.916    0.624
##    .ang.3             8.933    3.028    2.950    0.003    8.933    0.492
##    .ang.4             7.378    2.856    2.583    0.010    7.378    0.486
##    .ang.5             6.909    2.751    2.512    0.012    6.909    0.473
##    .ang.6             7.136    2.786    2.561    0.010    7.136    0.456
##    .ang.7             6.589    2.846    2.315    0.021    6.589    0.414
##    .ang.8             7.052    2.664    2.647    0.008    7.052    0.490
##     sat.1            58.363    1.638   35.627    0.000   58.363    3.203
##     ang.1            19.564    1.534   12.754    0.000   19.564    1.147
##     eta_y             0.000                               0.000    0.000
##     eta_x             0.000                               0.000    0.000
##
```

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     sat.1          332.088   42.162    7.876    0.000  332.088    1.000
##     ang.1          290.898   37.019    7.858    0.000  290.898    1.000
##     eta_x           85.915   16.006    5.368    0.000    1.000    1.000
##     eta_y          120.222   23.004    5.226    0.000    1.000    1.000
##    .sat.2          153.107   22.382    6.841    0.000  153.107    0.458
##    .sat.3          201.953   28.537    7.077    0.000  201.953    0.513
##    .sat.4          218.767   30.996    7.058    0.000  218.767    0.522
##    .sat.5          178.141   25.357    7.025    0.000  178.141    0.474
##    .sat.6          131.253   19.576    6.705    0.000  131.253    0.403
##    .sat.7          199.999   28.085    7.121    0.000  199.999    0.510
##    .sat.8          144.680   21.245    6.810    0.000  144.680    0.423
##    .ang.2          139.992   19.792    7.073    0.000  139.992    0.555
##    .ang.3          215.727   29.551    7.300    0.000  215.727    0.654
##    .ang.4          113.237   16.434    6.891    0.000  113.237    0.492
##    .ang.5           99.095   14.641    6.768    0.000   99.095    0.464
##    .ang.6          130.718   18.540    7.051    0.000  130.718    0.534
##    .ang.7          138.818   20.191    6.875    0.000  138.818    0.549
##    .ang.8           92.023   13.892    6.624    0.000   92.023    0.445
```

The fit indices for this model are $\chi^2(108) = 295.85$, $p < 0.05$; RMSEA $= 0.12$; CFI $= 0.82$; SRMR $= 0.10$, and the estimated coefficients are, respectively, 0.25 ($SE = 0.05$, $p < 0.05$) for the satisfaction autoregressive effect, 0.16 ($SE = 0.05$, $p < 0.05$) for the anger autoregressive effect, 0.23 ($SE = 0.05$, $p < 0.05$) for the lag-one effect of anger on satisfaction, and 0.09 ($SE = 0.04$, $p < 0.05$) for the lag-one effect of satisfaction on anger.