princ_math

Mathematical, Statistical and Dynamics Principles

Difference Equations

In mathematics, a basic representation of a process over time is a difference equation:

$$y = y_{t-1} \tag{1}$$

where the value of y is the same at each t, and the emergent behavior would be a flat line across time. In systems theory terms, there would be no trend.

Although equation 1 seems simple, it introduces a fundamental concept in dynamics: memory. The variable now depends on where it was in the past. It is constrained, there are boundaries on where it can go.

As we add terms to this basic difference equation the behavior of the variable becomes more complex. Adding a forcing constant, c in equation 1 produces positive or negative trend depending on whether c is, respectively, positive or negative. For example, the following equation:

$$\begin{aligned} y &= y_{t-1} + c \\ c &= -4 \end{aligned} \tag{2}$$

produces a line that decreases by four units at each time point.

Autoregressive terms represent the extent to which the variable relates to itself over time. Here:

$$\begin{aligned} y &= a y_{t-1} \\ a &= 0.5 \end{aligned} \tag{3}$$

the variable is described over time but it does not retain the same value at each t. Instead, the variable is *similar* over time and the autoregressive term, a, describes the extent of that similarity. The term is 0.5, meaning that the relationship between the variable now and itself at the next time point will be 0.5.

There are fundamental properties of dynamic variables based on their autoregressive terms, and these are shown in figure @ref(fig:dynamics_fig). The top row of figure 1 shows the trajectory of a variable with autoregressive terms that are greater than one in absolute value. These large terms produce explosive behavior – exponential growth when a is positive and oscillating CHAOS? when a is negative. When the autoregressive term falls between zero and one, conversely, the variable converges to an equilibrium value – shown in the bottom two panels. Either the variable oscillates at a decreasing rate until it reaches equilibrium (when a is negative) or it converges there smoothly (when a is positive).

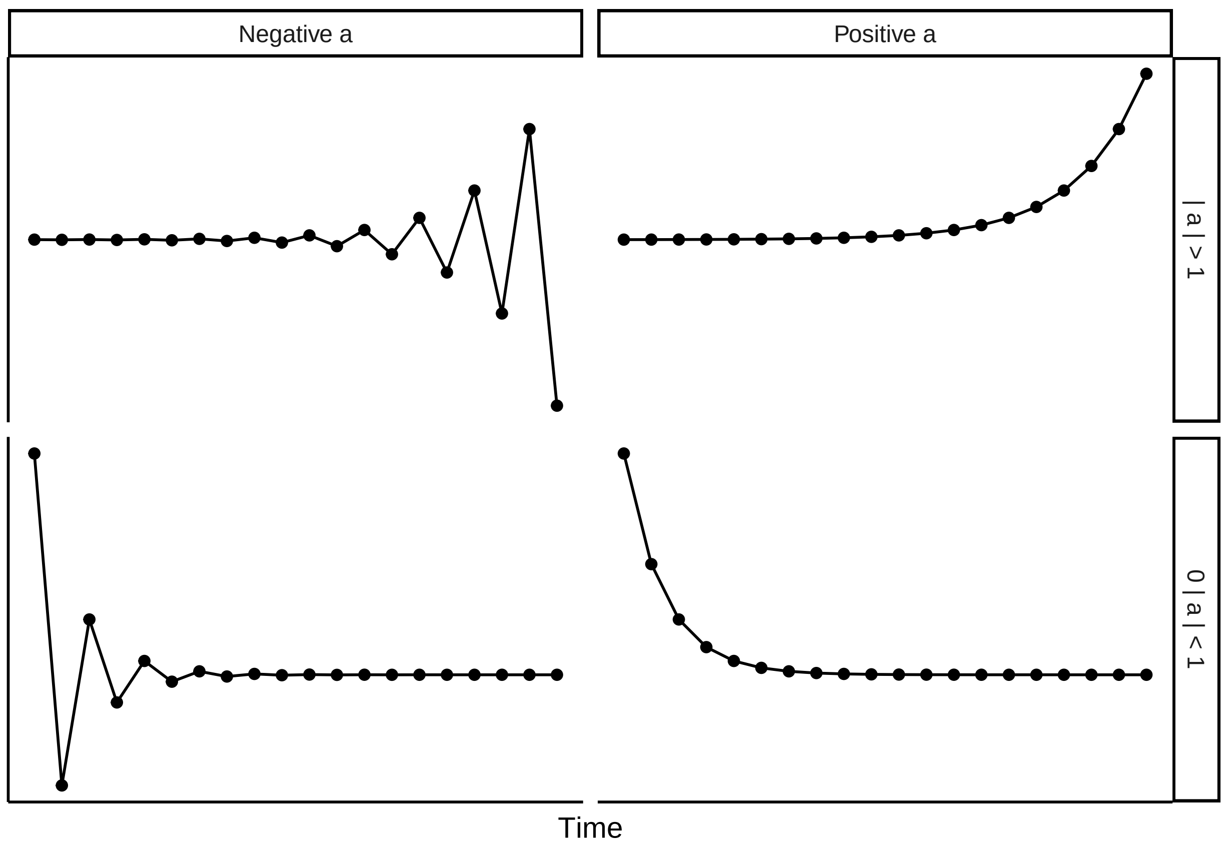

Figure 1: something here

Initial Conditions

A deterministic difference equation

A difference equation with a forcing term

A difference equation with a forcing term an an autoregressive coefficient

Changing the parameters leads to different types of behavior.

Stochastics

Random Walks

Stationarity

Cointegration

Granger Causality and Directionality

X granger causes Y if Y can be better predicted by the histories of both X and Y than the history of Y alone. If lagged values of X help predict current values of Y in a forecast formed from lagged values of both X and Y, then X is said to Granger cause Y. We implement this notion by regressing eggs on lagged eggs and lagged chickens; if the coefficients on lagged chickens are significant as a group, then chickens cause eggs. A symetric regression tests the reverse causality. We perform the Granger causality tests using one to four lags. The number of lags in each equation is the same for eggs and chickens. To conclude that one of the two "came first," we must find unidirectional causality from one to the other. In other words, we must reject the noncausality of the one to the other and at the same time fail to reject the noncausality of the other to the one. If either both cause each other or neigher causes the other, the question will remain unanswered. Results reject that eggs do not Granger cause chickens. They provide no such reject of the hypothesis that chickens do not Granger cause eggs. Therefore, we conclude that eggs cause chickens. A better phrasing might be "temporally related" (Granger & Newbold, p. 225) – Thurman and Fisher 1988 call it temporally ordered.

Diffusion

Damping

Markov Process