

Spline Modeling

2018-05-05

A few spline models (also known as piecewise models). As in previous posts, ‘affect’ is the name given to values of y throughout.

1) Growth and Even More Growth

A model that captures a process that increases initially and then increases at an even greater rate once it reaches time point 5. The data generating process:

$$y_{it} = \begin{cases} 4 + 0.3t + error_t, & \text{if time} < 5 \\ 8 + 0.9t + error_t, & \text{otherwise} \end{cases} \quad (1)$$

The data generating code and plot

```
library(tidyverse)
library(lavaan)
library(ggplot2)
library(MASS)

N <- 400
time <- 10

intercept_1 <- 4
intercept_2 <- 8

growth1 <- 0.3
growth2 <- 0.9

df_matrix <- matrix(, ncol = 3, nrow = N*time)

count <- 0

for(i in 1:N){

  unob_het_y <- rnorm(1,0,1)

  for(j in 1:time){

    count <- count + 1

    if(j < 5){
      df_matrix[count, 1] <- i
      df_matrix[count, 2] <- j
      df_matrix[count, 3] <- intercept_1 + growth1*j + unob_het_y + rnorm(1,0,1)
    } else {
      df_matrix[count, 1] <- i
      df_matrix[count, 2] <- j
      df_matrix[count, 3] <- intercept_2 + growth2*j + unob_het_y + rnorm(1,0,1)
    }
  }
}
```

```

    }else{

      df_matrix[count, 1] <- i
      df_matrix[count, 2] <- j
      df_matrix[count, 3] <- intercept_2 + growth2*j + unob_het_y + rnorm(1,0,1)

    }
  }
}

df <- data.frame(df_matrix)

names(df) <- c('id', 'time', 'affect')

df1 <- df %>%
  filter(time < 5)

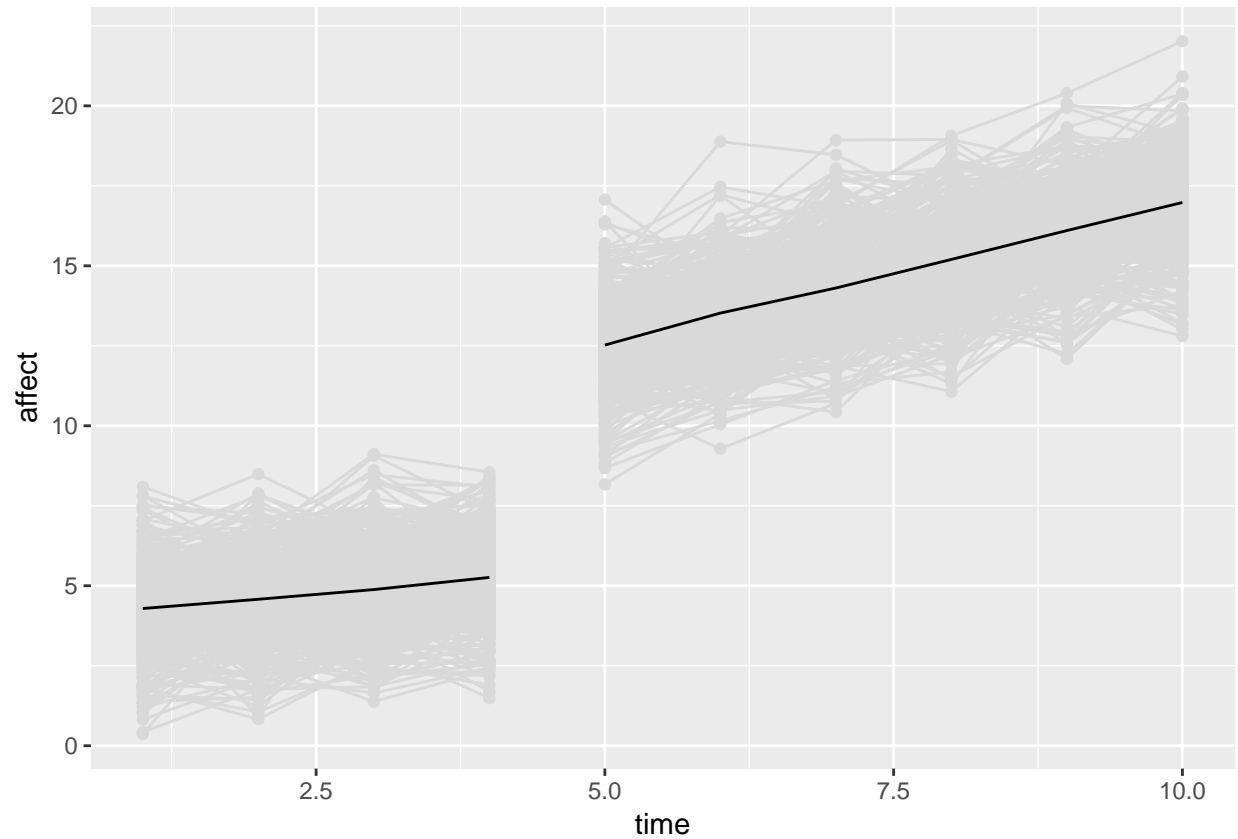
df2 <- df %>%
  filter(time >= 5)

df_sum1 <- df1 %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

df_sum2 <- df2 %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

ggplot() +
  geom_point(data = df1, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df1, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_point(data = df2, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df2, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df_sum1, aes(x = time, y = affect)) +
  geom_line(data = df_sum2, aes(x = time, y = affect))

```



Estimating the parameters using SEM:

```
df_wide <- reshape(df, idvar = 'id', timevar = 'time', direction = 'wide')

spline_string <- '
# latent intercept for first half
level1_affect =~ 1*affect.1 + 1*affect.2 + 1*affect.3 + 1*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7
# latent intercept for second half
level2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 1*affect.5 + 1*affect.6 + 1*affect.7
# latent slope for first half basis coefficients
slope1_affect =~ 1*affect.1 + 2*affect.2 + 3*affect.3 + 4*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7
# latent slope for second half basis coefficients
slope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 5*affect.5 + 6*affect.6 + 7*affect.7
# means and variance of latent factors
level1_affect ~~ level1_affect
```

```

level2_affect ~~ level2_affect
slope1_affect ~~ slope1_affect
slope2_affect ~~ slope2_affect

# covariance between latent factors

level1_affect ~~ level2_affect
level1_affect ~~ slope1_affect
level1_affect ~~ slope2_affect

level2_affect ~~ slope1_affect
level2_affect ~~ slope2_affect

slope1_affect ~~ slope2_affect

# constrain means of indicators to zero across time

affect.1 ~ 0
affect.2 ~ 0
affect.3 ~ 0
affect.4 ~ 0
affect.5 ~ 0
affect.6 ~ 0
affect.7 ~ 0
affect.8 ~ 0
affect.9 ~ 0
affect.10 ~ 0

# constrain residual variance to equality across time

affect.1 ~~ res_var*affect.1
affect.2 ~~ res_var*affect.2
affect.3 ~~ res_var*affect.3
affect.4 ~~ res_var*affect.4
affect.5 ~~ res_var*affect.5
affect.6 ~~ res_var*affect.6
affect.7 ~~ res_var*affect.7
affect.8 ~~ res_var*affect.8
affect.9 ~~ res_var*affect.9
affect.10 ~~ res_var*affect.10

,

spline_model <- growth(spline_string, data = df_wide)
summary(spline_model, fit.measures = T)

```

```

## lavaan 0.6-2 ended normally after 75 iterations
##
##      Optimization method          NLMINB
##      Number of free parameters      24
##      Number of equality constraints    9
##
##      Number of observations          400

```

```

##
## Estimator ML
## Model Fit Test Statistic 41.354
## Degrees of freedom 50
## P-value (Chi-square) 0.803
##
## Model test baseline model:
##
## Minimum Function Test Statistic 1924.393
## Degrees of freedom 45
## P-value 0.000
##
## User model versus baseline model:
##
## Comparative Fit Index (CFI) 1.000
## Tucker-Lewis Index (TLI) 1.004
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0) -6145.492
## Loglikelihood unrestricted model (H1) -6124.815
##
## Number of free parameters 15
## Akaike (AIC) 12320.984
## Bayesian (BIC) 12380.856
## Sample-size adjusted Bayesian (BIC) 12333.260
##
## Root Mean Square Error of Approximation:
##
## RMSEA 0.000
## 90 Percent Confidence Interval 0.000 0.021
## P-value RMSEA <= 0.05 1.000
##
## Standardized Root Mean Square Residual:
##
## SRMR 0.037
##
## Parameter Estimates:
##
## Information Expected
## Information saturated (h1) model Structured
## Standard Errors Standard
##
## Latent Variables:
## Estimate Std.Err z-value P(>|z|)
## level1_affect =~
## affect.1 1.000
## affect.2 1.000
## affect.3 1.000
## affect.4 1.000
## affect.5 0.000
## affect.6 0.000
## affect.7 0.000
## affect.8 0.000

```

```

##      affect.9      0.000
##      affect.10     0.000
## level2_affect =~
##      affect.1      0.000
##      affect.2      0.000
##      affect.3      0.000
##      affect.4      0.000
##      affect.5      1.000
##      affect.6      1.000
##      affect.7      1.000
##      affect.8      1.000
##      affect.9      1.000
##      affect.10     1.000
## slope1_affect =~
##      affect.1      1.000
##      affect.2      2.000
##      affect.3      3.000
##      affect.4      4.000
##      affect.5      0.000
##      affect.6      0.000
##      affect.7      0.000
##      affect.8      0.000
##      affect.9      0.000
##      affect.10     0.000
## slope2_affect =~
##      affect.1      0.000
##      affect.2      0.000
##      affect.3      0.000
##      affect.4      0.000
##      affect.5      5.000
##      affect.6      6.000
##      affect.7      7.000
##      affect.8      8.000
##      affect.9      9.000
##      affect.10     10.000
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|)
## level1_affect ~~
## level2_affect      1.046   0.175   5.966   0.000
## slope1_affect      0.015   0.044   0.333   0.739
## slope2_affect     -0.005   0.019  -0.246   0.806
## level2_affect ~~
## slope1_affect     -0.008   0.047  -0.170   0.865
## slope2_affect     -0.031   0.037  -0.841   0.400
## slope1_affect ~~
## slope2_affect      0.002   0.005   0.326   0.745
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|)
## .affect.1      0.000
## .affect.2      0.000
## .affect.3      0.000
## .affect.4      0.000

```

```
##      .affect.5          0.000
##      .affect.6          0.000
##      .affect.7          0.000
##      .affect.8          0.000
##      .affect.9          0.000
##      .affect.10         0.000
##      level1_affect      3.949    0.078    50.727    0.000
##      level2_affect      8.150    0.108    75.796    0.000
##      slope1_affect      0.322    0.022    14.822    0.000
##      slope2_affect      0.883    0.012    71.418    0.000
##
## Variances:
##              Estimate Std.Err  z-value  P(>|z|)
##      lvl1_ff          0.945    0.177     5.349    0.000
##      lvl2_ff          1.290    0.341     3.786    0.000
##      slp1_ff          -0.009    0.014    -0.611    0.541
##      slp2_ff           0.005    0.005     1.039    0.299
##      .affct.1 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.2 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.3 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.4 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.5 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.6 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.7 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.8 (rs_v)    0.986    0.028    34.641    0.000
##      .affct.9 (rs_v)    0.986    0.028    34.641    0.000
##      .affc.10 (rs_v)    0.986    0.028    34.641    0.000
```

The structure of the basis coefficients is the important piece that allows us to capture the change in slope:

```
'
# latent slope for first half basis coefficients

slope1_affect =~ 1*affect.1 + 2*affect.2 + 3*affect.3 + 4*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10

# latent slope for second half basis coefficients

slope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 5*affect.5 + 6*affect.6 + 7*affect.7 + 8*affect.8 + 9*affect.9 + 10*affect.10
'
```

2) Growth and Negative Growth

A model that captures a process that goes up and then goes down. The data generating process:

$$y_{it} = \begin{cases} 4 + 0.5t + error_t, & \text{if time} < 5 \\ 4 - 0.5t + error_t, & \text{otherwise} \end{cases} \quad (2)$$

The data generating code and plot

```

library(tidyverse)
library(lavaan)
library(ggplot2)
library(MASS)

N <- 400
time <- 10

intercept_1 <- 4
intercept_2 <- 4

growth1 <- 0.8
growth2 <- -0.8

df_matrix_b <- matrix(, ncol = 3, nrow = N*time)

count <- 0

for(i in 1:N){

  unob_het_y <- rnorm(1,0,1)

  for(j in 1:time){

    count <- count + 1

    if(j < 5){
      df_matrix_b[count, 1] <- i
      df_matrix_b[count, 2] <- j
      df_matrix_b[count, 3] <- intercept_1 + growth1*j + unob_het_y + rnorm(1,0,1)
    }else{

      df_matrix_b[count, 1] <- i
      df_matrix_b[count, 2] <- j
      df_matrix_b[count, 3] <- intercept_2 + growth2*j + unob_het_y + rnorm(1,0,1)

    }
  }
}

df_b <- data.frame(df_matrix_b)

names(df_b) <- c('id', 'time', 'affect')

df1_b <- df_b %>%
  filter(time < 5)

df2_b <- df_b %>%

```



```

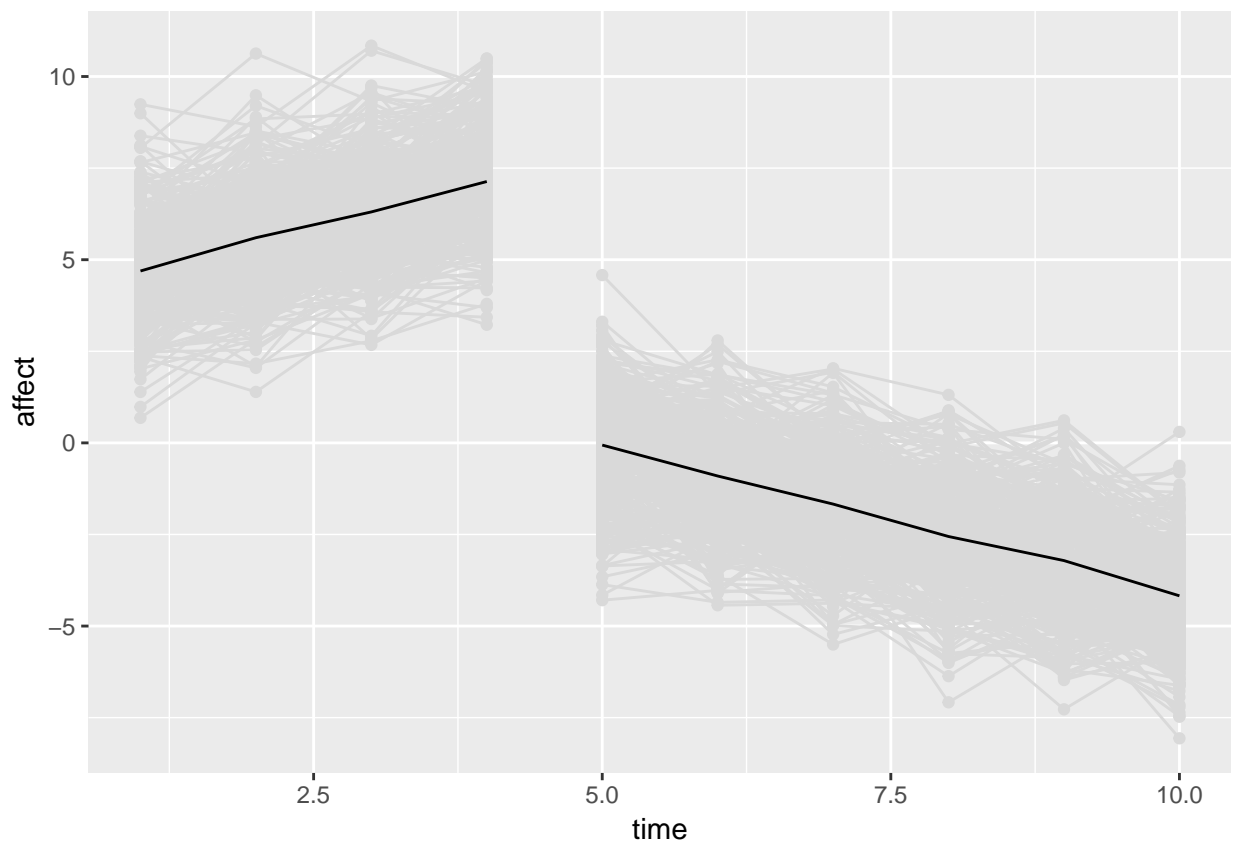
filter(time >= 5)

df_sum1_b <- df1_b %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

df_sum2_b <- df2_b %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

ggplot() +
  geom_point(data = df1_b, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df1_b, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_point(data = df2_b, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df2_b, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df_sum1_b, aes(x = time, y = affect)) +
  geom_line(data = df_sum2_b, aes(x = time, y = affect))

```



Estimating the parameters using SEM:

```

df_wide_b <- reshape(df_b, idvar = 'id', timevar = 'time', direction = 'wide')

spline_string_b <- '

# latent intercept for first half
level1_affect =~ 1*affect.1 + 1*affect.2 + 1*affect.3 + 1*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10

# latent intercept for second half
level2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 1*affect.5 + 1*affect.6 + 1*affect.7 + 1*affect.8 + 1*affect.9 + 1*affect.10

# latent slope for first half basis coefficients
slope1_affect =~ 1*affect.1 + 2*affect.2 + 3*affect.3 + 4*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10

# latent slope for second half basis coefficients
slope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 5*affect.5 + 6*affect.6 + 7*affect.7 + 8*affect.8 + 9*affect.9 + 10*affect.10

# means and variance of latent factors
level1_affect ~~ level1_affect
level2_affect ~~ level2_affect
slope1_affect ~~ slope1_affect
slope2_affect ~~ slope2_affect

# covariance between latent factors
level1_affect ~~ level2_affect
level1_affect ~~ slope1_affect
level1_affect ~~ slope2_affect

level2_affect ~~ slope1_affect
level2_affect ~~ slope2_affect

slope1_affect ~~ slope2_affect

# constrain means of indicators to zero across time
affect.1 ~ 0
affect.2 ~ 0
affect.3 ~ 0
affect.4 ~ 0
affect.5 ~ 0
affect.6 ~ 0
affect.7 ~ 0
affect.8 ~ 0
affect.9 ~ 0
affect.10 ~ 0

# constrain residual variance to equality across time

```

```

affect.1 ~~ res_var*affect.1
affect.2 ~~ res_var*affect.2
affect.3 ~~ res_var*affect.3
affect.4 ~~ res_var*affect.4
affect.5 ~~ res_var*affect.5
affect.6 ~~ res_var*affect.6
affect.7 ~~ res_var*affect.7
affect.8 ~~ res_var*affect.8
affect.9 ~~ res_var*affect.9
affect.10 ~~ res_var*affect.10

'

spline_model_b <- growth(spline_string_b, data = df_wide_b)
summary(spline_model_b, fit.measures = T)

```

```

## lavaan 0.6-2 ended normally after 77 iterations
##
##      Optimization method          NLMINB
##      Number of free parameters      24
##      Number of equality constraints    9
##
##      Number of observations          400
##
##      Estimator                      ML
##      Model Fit Test Statistic        50.635
##      Degrees of freedom              50
##      P-value (Chi-square)            0.448
##
## Model test baseline model:
##
##      Minimum Function Test Statistic  1845.408
##      Degrees of freedom              45
##      P-value                        0.000
##
## User model versus baseline model:
##
##      Comparative Fit Index (CFI)      1.000
##      Tucker-Lewis Index (TLI)        1.000
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)      -6129.204
##      Loglikelihood unrestricted model (H1) -6103.887
##
##      Number of free parameters          15
##      Akaike (AIC)                     12288.408
##      Bayesian (BIC)                   12348.280
##      Sample-size adjusted Bayesian (BIC) 12300.684
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                            0.006

```

```

## 90 Percent Confidence Interval          0.000  0.033
## P-value RMSEA <= 0.05                  1.000
##
## Standardized Root Mean Square Residual:
##
## SRMR                                    0.038
##
## Parameter Estimates:
##
## Information                               Expected
## Information saturated (h1) model          Structured
## Standard Errors                          Standard
##
## Latent Variables:
##      Estimate  Std.Err  z-value  P(>|z|)
## level1_affect =~
##   affect.1      1.000
##   affect.2      1.000
##   affect.3      1.000
##   affect.4      1.000
##   affect.5      0.000
##   affect.6      0.000
##   affect.7      0.000
##   affect.8      0.000
##   affect.9      0.000
##   affect.10     0.000
## level2_affect =~
##   affect.1      0.000
##   affect.2      0.000
##   affect.3      0.000
##   affect.4      0.000
##   affect.5      1.000
##   affect.6      1.000
##   affect.7      1.000
##   affect.8      1.000
##   affect.9      1.000
##   affect.10     1.000
## slope1_affect =~
##   affect.1      1.000
##   affect.2      2.000
##   affect.3      3.000
##   affect.4      4.000
##   affect.5      0.000
##   affect.6      0.000
##   affect.7      0.000
##   affect.8      0.000
##   affect.9      0.000
##   affect.10     0.000
## slope2_affect =~
##   affect.1      0.000
##   affect.2      0.000
##   affect.3      0.000
##   affect.4      0.000
##   affect.5      5.000

```

```

##      affect.6          6.000
##      affect.7          7.000
##      affect.8          8.000
##      affect.9          9.000
##      affect.10         10.000
##
## Covariances:
##              Estimate Std.Err z-value P(>|z|)
## level1_affect ~~
##   level2_affect    1.106   0.181   6.104   0.000
##   slope1_affect     0.003   0.044   0.067   0.947
##   slope2_affect    -0.016   0.020  -0.829   0.407
## level2_affect ~~
##   slope1_affect     0.012   0.050   0.232   0.817
##   slope2_affect    -0.081   0.040  -2.022   0.043
## slope1_affect ~~
##   slope2_affect    -0.002   0.006  -0.294   0.768
##
## Intercepts:
##              Estimate Std.Err z-value P(>|z|)
##   .affect.1         0.000
##   .affect.2         0.000
##   .affect.3         0.000
##   .affect.4         0.000
##   .affect.5         0.000
##   .affect.6         0.000
##   .affect.7         0.000
##   .affect.8         0.000
##   .affect.9         0.000
##   .affect.10        0.000
##   level1_affect     3.925   0.077  50.913   0.000
##   level2_affect     3.986   0.112  35.607   0.000
##   slope1_affect     0.803   0.022  36.203   0.000
##   slope2_affect    -0.811   0.013 -63.422   0.000
##
## Variances:
##              Estimate Std.Err z-value P(>|z|)
##   lvl1_ff           0.905   0.173   5.218   0.000
##   lvl2_ff           1.693   0.367   4.611   0.000
##   slp1_ff           0.000   0.015   0.027   0.978
##   slp2_ff           0.009   0.005   1.899   0.058
##   .affct.1 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.2 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.3 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.4 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.5 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.6 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.7 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.8 (rs_v)   0.982   0.028  34.641   0.000
##   .affct.9 (rs_v)   0.982   0.028  34.641   0.000
##   .affc.10 (rs_v)   0.982   0.028  34.641   0.000

```

Notice that the string syntax is the exact same because the process changes at the same point in time, it does not matter if the process changes to ‘more positive’ or ‘more negative.’

3) Negative Growth, Growth, and Negative Growth

Now a process that goes down, goes up, and then goes back down. The data generating process:

$$y_{it} = \begin{cases} 4 - 0.5t + error_t, & \text{if time} < 5 \\ 4 + 0.5t + error_t, & \text{if } 5 < \text{time} < 10 \\ 4 - 0.5t + error_t, & \text{otherwise} \end{cases} \quad (3)$$

The data generating code and plot

```
library(tidyverse)
library(lavaan)
library(ggplot2)
library(MASS)

N <- 400
time <- 15

intercept_1 <- 4
intercept_2 <- 4
intercept_3 <- 4

growth1 <- -0.5
growth2 <- 0.5
growth3 <- -0.5

df_matrix_c <- matrix(, ncol = 3, nrow = N*time)

count <- 0

for(i in 1:N){

  unob_het_y <- rnorm(1,0,1)

  for(j in 1:time){

    count <- count + 1

    if(j < 5){
      df_matrix_c[count, 1] <- i
      df_matrix_c[count, 2] <- j
      df_matrix_c[count, 3] <- intercept_1 + growth1*j + unob_het_y + rnorm(1,0,1)
    }else if(j >= 5 && j < 10){

      df_matrix_c[count, 1] <- i
      df_matrix_c[count, 2] <- j
      df_matrix_c[count, 3] <- intercept_2 + growth2*j + unob_het_y + rnorm(1,0,1)
    }
  }
}
```

```

    }else{

      df_matrix_c[count, 1] <- i
      df_matrix_c[count, 2] <- j
      df_matrix_c[count, 3] <- intercept_3 + growth3*j + unob_het_y + rnorm(1,0,1)

    }
  }
}

df_c <- data.frame(df_matrix_c)

names(df_c) <- c('id', 'time', 'affect')

df1_c <- df_c %>%
  filter(time < 5)

df2_c <- df_c %>%
  filter(time >= 5 & time < 10)

df3_c <- df_c %>%
  filter(time >= 10)

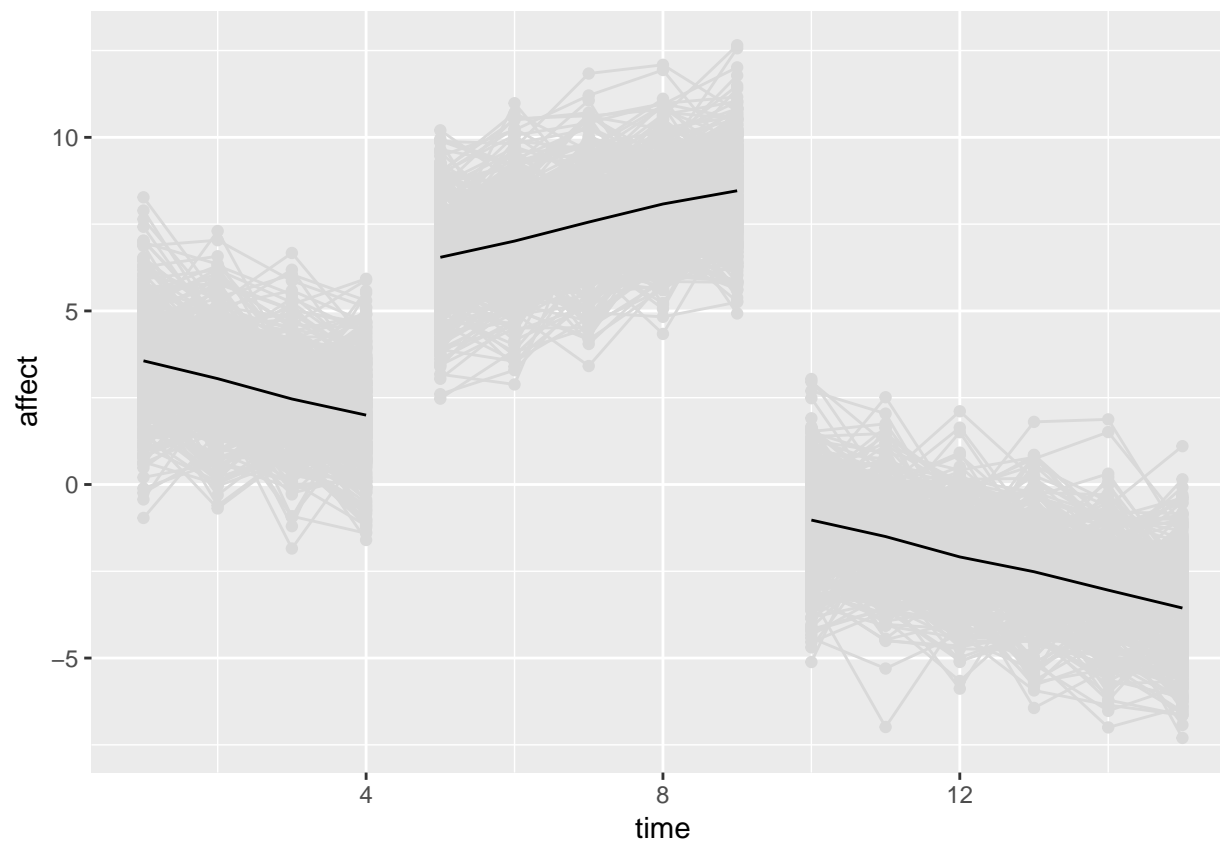
df_sum1_c <- df1_c %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

df_sum2_c <- df2_c %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

df_sum3_c <- df3_c %>%
  group_by(time) %>%
  summarise(
    affect = mean(affect)
  )

ggplot() +
  geom_point(data = df1_c, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df1_c, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_point(data = df2_c, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df2_c, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df_sum1_c, aes(x = time, y = affect)) +
  geom_line(data = df_sum2_c, aes(x = time, y = affect)) +
  geom_point(data = df3_c, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df3_c, aes(x = time, y = affect, group = id), color = 'gray85') +
  geom_line(data = df_sum3_c, aes(x = time, y = affect))

```



Now estimate the parameters using SEM:

```
df_wide_c <- reshape(df_c, idvar = 'id', timevar = 'time', direction = 'wide')

spline_string_c <- '
# latent intercept for first third
level1_affect =~ 1*affect.1 + 1*affect.2 + 1*affect.3 + 1*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7
# latent intercept for second third
level2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 1*affect.5 + 1*affect.6 + 1*affect.7
# latent intercept for final third
level3_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7
# latent slope for first third basis coefficients
slope1_affect =~ 1*affect.1 + 2*affect.2 + 3*affect.3 + 4*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7
# latent slope for second third basis coefficients
slope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 1*affect.5 + 2*affect.6 + 3*affect.7
# latent slope for final third basis coefficients
slope3_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7'
```



```

slope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 5*affect.5 + 6*affect.6 + 7*affect.7 + 8*affect.8 + 9*affect.9 + 10*affect.10

# latent slope for final third basis coefficients

slope3_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10

# means and variance of latent factors

level1_affect ~~ level1_affect
level2_affect ~~ level2_affect
level3_affect ~~ level3_affect
slope1_affect ~~ slope1_affect
slope2_affect ~~ slope2_affect
slope3_affect ~~ slope3_affect

# covariance between latent factors

level1_affect ~~ level2_affect
level1_affect ~~ level3_affect
level1_affect ~~ slope1_affect
level1_affect ~~ slope2_affect
level1_affect ~~ slope3_affect

level2_affect ~~ level3_affect
level2_affect ~~ slope1_affect
level2_affect ~~ slope2_affect
level2_affect ~~ slope3_affect

level3_affect ~~ slope1_affect
level3_affect ~~ slope2_affect
level3_affect ~~ slope3_affect

slope1_affect ~~ slope2_affect
slope1_affect ~~ slope3_affect

slope2_affect ~~ slope3_affect

# constrain means of indicators to zero across time

affect.1 ~ 0
affect.2 ~ 0
affect.3 ~ 0
affect.4 ~ 0
affect.5 ~ 0
affect.6 ~ 0
affect.7 ~ 0
affect.8 ~ 0
affect.9 ~ 0
affect.10 ~ 0

# constrain residual variance to equality across time

```

```

affect.1 ~~ res_var*affect.1
affect.2 ~~ res_var*affect.2
affect.3 ~~ res_var*affect.3
affect.4 ~~ res_var*affect.4
affect.5 ~~ res_var*affect.5
affect.6 ~~ res_var*affect.6
affect.7 ~~ res_var*affect.7
affect.8 ~~ res_var*affect.8
affect.9 ~~ res_var*affect.9
affect.10 ~~ res_var*affect.10

'

spline_model_c <- growth(spline_string_c, data = df_wide_c)
summary(spline_model_c, fit.measures = T)

```

```

## lavaan 0.6-2 ended normally after 146 iterations
##
##      Optimization method          NLMINB
##      Number of free parameters      42
##      Number of equality constraints    9
##
##      Number of observations          400
##
##      Estimator                      ML
##      Model Fit Test Statistic       105.762
##      Degrees of freedom             102
##      P-value (Chi-square)           0.380
##
## Model test baseline model:
##
##      Minimum Function Test Statistic 3075.773
##      Degrees of freedom             105
##      P-value                        0.000
##
## User model versus baseline model:
##
##      Comparative Fit Index (CFI)     0.999
##      Tucker-Lewis Index (TLI)       0.999
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)    -9013.102
##      Loglikelihood unrestricted model (H1) -8960.221
##
##      Number of free parameters        33
##      Akaike (AIC)                    18092.204
##      Bayesian (BIC)                  18223.923
##      Sample-size adjusted Bayesian (BIC) 18119.211
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                          0.010

```

```

## 90 Percent Confidence Interval          0.000  0.028
## P-value RMSEA <= 0.05                  1.000
##
## Standardized Root Mean Square Residual:
##
## SRMR                                   0.036
##
## Parameter Estimates:
##
## Information                          Expected
## Information saturated (h1) model    Structured
## Standard Errors                     Standard
##
## Latent Variables:
##      Estimate  Std.Err  z-value  P(>|z|)
## level1_affect =~
##   affect.1      1.000
##   affect.2      1.000
##   affect.3      1.000
##   affect.4      1.000
##   affect.5      0.000
##   affect.6      0.000
##   affect.7      0.000
##   affect.8      0.000
##   affect.9      0.000
##   affect.10     0.000
##   affect.11     0.000
##   affect.12     0.000
##   affect.13     0.000
##   affect.14     0.000
##   affect.15     0.000
## level2_affect =~
##   affect.1      0.000
##   affect.2      0.000
##   affect.3      0.000
##   affect.4      0.000
##   affect.5      1.000
##   affect.6      1.000
##   affect.7      1.000
##   affect.8      1.000
##   affect.9      1.000
##   affect.10     0.000
##   affect.11     0.000
##   affect.12     0.000
##   affect.13     0.000
##   affect.14     0.000
##   affect.15     0.000
## level3_affect =~
##   affect.1      0.000
##   affect.2      0.000
##   affect.3      0.000
##   affect.4      0.000
##   affect.5      0.000
##   affect.6      0.000

```

```

##      affect.7      0.000
##      affect.8      0.000
##      affect.9      0.000
##      affect.10     1.000
##      affect.11     1.000
##      affect.12     1.000
##      affect.13     1.000
##      affect.14     1.000
##      affect.15     1.000
## slope1_affect =~
##      affect.1      1.000
##      affect.2      2.000
##      affect.3      3.000
##      affect.4      4.000
##      affect.5      0.000
##      affect.6      0.000
##      affect.7      0.000
##      affect.8      0.000
##      affect.9      0.000
##      affect.10     0.000
##      affect.11     0.000
##      affect.12     0.000
##      affect.13     0.000
##      affect.14     0.000
##      affect.15     0.000
## slope2_affect =~
##      affect.1      0.000
##      affect.2      0.000
##      affect.3      0.000
##      affect.4      0.000
##      affect.5      5.000
##      affect.6      6.000
##      affect.7      7.000
##      affect.8      8.000
##      affect.9      9.000
##      affect.10     0.000
##      affect.11     0.000
##      affect.12     0.000
##      affect.13     0.000
##      affect.14     0.000
##      affect.15     0.000
## slope3_affect =~
##      affect.1      0.000
##      affect.2      0.000
##      affect.3      0.000
##      affect.4      0.000
##      affect.5      0.000
##      affect.6      0.000
##      affect.7      0.000
##      affect.8      0.000
##      affect.9      0.000
##      affect.10     10.000
##      affect.11     11.000
##      affect.12     12.000

```

```

##      affect.13      13.000
##      affect.14      14.000
##      affect.15      15.000
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|)
##      level1_affect ~~
##      level2_affect      1.061      0.212      4.992      0.000
##      level3_affect      0.624      0.249      2.511      0.012
##      slope1_affect     -0.015      0.047     -0.309      0.758
##      slope2_affect     -0.011      0.026     -0.402      0.688
##      slope3_affect      0.032      0.019      1.711      0.087
##      level2_affect ~~
##      level3_affect      1.065      0.394      2.705      0.007
##      slope1_affect     -0.001      0.057     -0.024      0.981
##      slope2_affect     -0.042      0.060     -0.701      0.483
##      slope3_affect      0.004      0.030      0.144      0.885
##      level3_affect ~~
##      slope1_affect      0.108      0.069      1.573      0.116
##      slope2_affect     -0.033      0.050     -0.665      0.506
##      slope3_affect      0.036      0.054      0.658      0.510
##      slope1_affect ~~
##      slope2_affect     -0.002      0.007     -0.277      0.782
##      slope3_affect     -0.010      0.005     -1.954      0.051
##      slope2_affect ~~
##      slope3_affect      0.001      0.004      0.203      0.839
##
## Intercepts:
##      Estimate Std.Err z-value P(>|z|)
##      .affect.1          0.000
##      .affect.2          0.000
##      .affect.3          0.000
##      .affect.4          0.000
##      .affect.5          0.000
##      .affect.6          0.000
##      .affect.7          0.000
##      .affect.8          0.000
##      .affect.9          0.000
##      .affect.10         0.000
##      .affect.11         0.000
##      .affect.12         0.000
##      .affect.13         0.000
##      .affect.14         0.000
##      .affect.15         0.000
##      level1_affect      4.088      0.081     50.687      0.000
##      level2_affect      4.100      0.128     32.141      0.000
##      level3_affect      4.044      0.153     26.447      0.000
##      slope1_affect     -0.528      0.022    -23.524      0.000
##      slope2_affect      0.490      0.016     30.253      0.000
##      slope3_affect     -0.507      0.012    -43.660      0.000
##
## Variances:
##      Estimate Std.Err z-value P(>|z|)
##      lvl1_ff          1.078      0.189      5.689      0.000

```

##	lvl2_ff	1.328	0.485	2.738	0.006
##	lvl3_ff	0.397	0.723	0.549	0.583
##	slp1_ff	-0.002	0.015	-0.119	0.906
##	slp2_ff	0.003	0.008	0.426	0.670
##	slp3_ff	-0.002	0.004	-0.514	0.608
##	.affct.1 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.2 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.3 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.4 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.5 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.6 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.7 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.8 (rs_v)	1.016	0.030	33.731	0.000
##	.affct.9 (rs_v)	1.016	0.030	33.731	0.000
##	.affc.10 (rs_v)	1.016	0.030	33.731	0.000
##	.affc.11	0.943	0.073	12.991	0.000
##	.affc.12	1.008	0.075	13.373	0.000
##	.affc.13	0.948	0.071	13.303	0.000
##	.affc.14	0.904	0.071	12.804	0.000
##	.affc.15	0.991	0.084	11.776	0.000

Again, the basis coefficients are the important piece here:

```

'

# latent slope for first third basis coefficients

slope1_affect =~ 1*affect.1 + 2*affect.2 + 3*affect.3 + 4*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10 + 0*affect.11 + 0*affect.12 + 0*affect.13 + 0*affect.14 + 0*affect.15

# latent slope for second third basis coefficients

slope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 5*affect.5 + 6*affect.6 + 7*affect.7 + 8*affect.8 + 9*affect.9 + 10*affect.10 + 11*affect.11 + 12*affect.12 + 13*affect.13 + 14*affect.14 + 15*affect.15

# latent slope for final third basis coefficients

slope3_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10 + 0*affect.11 + 0*affect.12 + 0*affect.13 + 0*affect.14 + 0*affect.15

'

```

```
## [1] "\n\n\n# latent slope for first third basis coefficients\n\nslope1_affect =~ 1*affect.1 + 2*affect.2 + 3*affect.3 + 4*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10 + 0*affect.11 + 0*affect.12 + 0*affect.13 + 0*affect.14 + 0*affect.15\n\nslope2_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 5*affect.5 + 6*affect.6 + 7*affect.7 + 8*affect.8 + 9*affect.9 + 10*affect.10 + 11*affect.11 + 12*affect.12 + 13*affect.13 + 14*affect.14 + 15*affect.15\n\nslope3_affect =~ 0*affect.1 + 0*affect.2 + 0*affect.3 + 0*affect.4 + 0*affect.5 + 0*affect.6 + 0*affect.7 + 0*affect.8 + 0*affect.9 + 0*affect.10 + 0*affect.11 + 0*affect.12 + 0*affect.13 + 0*affect.14 + 0*affect.15\n\n'
```

$\text{Bo}^2_{\text{m}} =)$