SOLUTIONS: Combinational Logic Questions

1. For the function $F(A,B,C) = AB'C + A'C' + AB$ find "algebraic" representations as follows:

   (a) Using only AND and NOT.

   ANSWER:

$$\begin{aligned} F &= AB'C + A'C' + AB \\ &= ((AB'C)'(A'C')'(AB)')' \end{aligned}$$

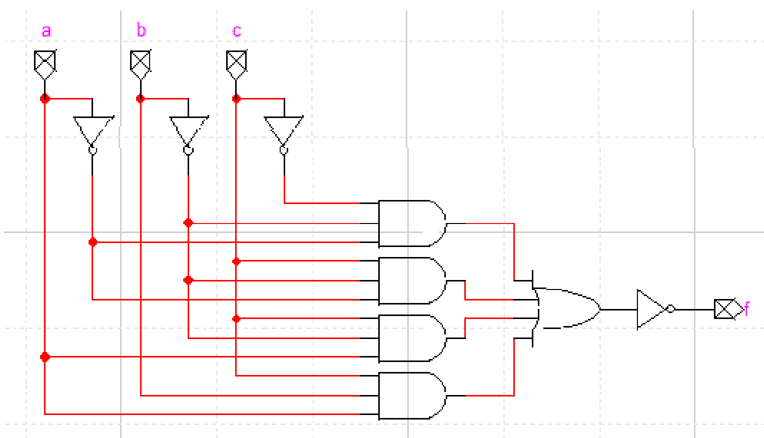
   (b) Using only OR and NOT.

   ANSWER:

$$\begin{aligned} F &= AB'C + A'C' + AB \\ &= (A' + (B')' + C')' + ((A')' + (C')') + (A' + B')' \\ &= ((A' + B + C')' + (A + C)' + (A' + B')' \end{aligned}$$

2. Consider the function: $f(a,b,c) = (a'b'c' + a'b'c + ab'c + abc)'$

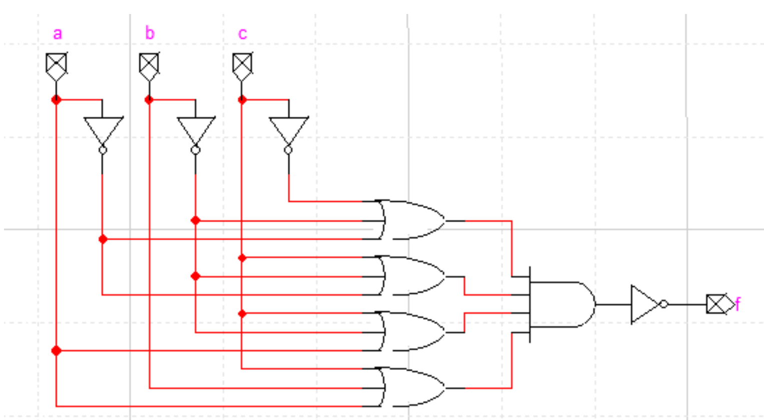
   (a) Schematic:

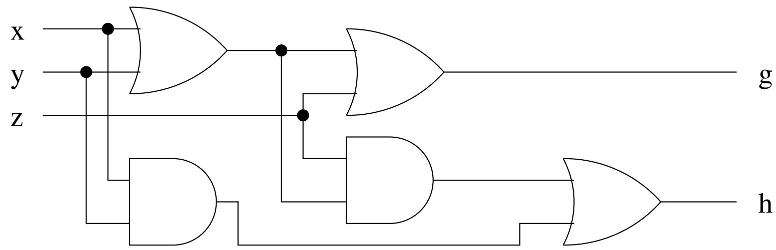


   (b) What is the dual of f?

   ANSWER: $f_{dual}(a,b,c) = ((a' + b' + c') \cdot (a' + b' + c) \cdot (a + b' + c) \cdot (a + b + c))'$

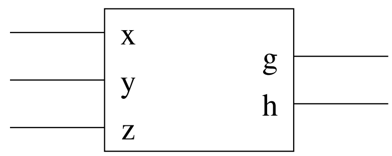
   (c) Schematic:





3. Construct a **behavioral description** for the following circuit, providing your functional specification in 2 ways - algebraically and as a function table:

   ANSWER:



$$
\begin{aligned}
a &= x + y \\
g &= a + z \\
&= x + y + z \\
b &= x \cdot y \\
c &= a \cdot z \\
&= (x + y) \cdot z \\
h &= c + b \\
&= (x + y) \cdot z + x \cdot y
\end{aligned}
$$

x	y	z	g	h
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

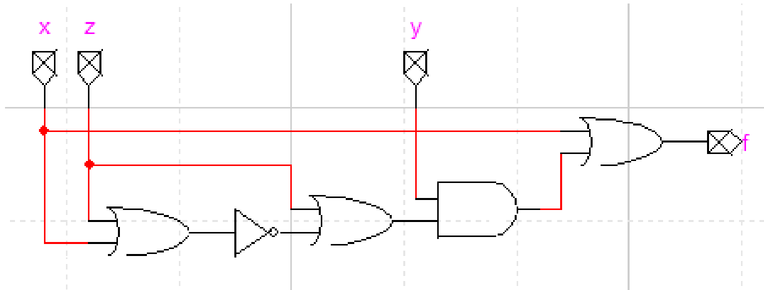
4. Implement the following function using only 3 AND gates and any NOT gates as required.

$$
\begin{aligned}
H(X,Y,Z) &= X' \cdot Y' \cdot Z + X' \cdot Y \cdot Z' + X \cdot Y' \cdot Z + X \cdot Y \cdot Z' \\
&= X' \cdot (Y' \cdot Z + Y \cdot Z') + X \cdot (Y' \cdot Z + Y' \cdot Z') \\
&= (X' + X) \cdot (Y' \cdot Z + Y \cdot Z') \\
&= ((Y' \cdot Z)' \cdot (Y \cdot Z')')'
\end{aligned}
$$

5. Simplify the following to an expression that can be implemented with as few gates as possible.:

$$
\begin{aligned}
\overline{\mathrm{A}} \cdot \overline{\mathrm{C}} + \overline{\mathrm{A}} \cdot B \cdot C + \overline{\mathrm{B}} \cdot C &= \overline{\mathrm{A}} \cdot \overline{\mathrm{C}} + \overline{\mathrm{A}} \cdot B \cdot C + \overline{\mathrm{A}} \cdot (\overline{\mathrm{B}} \cdot C) + \overline{\mathrm{B}} \cdot C \\
&= \overline{\mathrm{A}}(\overline{\mathrm{C}} + B \cdot C + \overline{\mathrm{B}} \cdot C) + \overline{\mathrm{B}} \cdot C \\
&= \overline{\mathrm{A}} \cdot (\overline{\mathrm{C}} + C \cdot (B + \overline{\mathrm{B}}) + \overline{\mathrm{B}} \cdot C \\
&= \overline{\mathrm{A}} \cdot (\overline{\mathrm{C}} + C) + \overline{\mathrm{B}} \cdot C \\
&= \overline{\mathrm{A}} + \overline{\mathrm{B}} \cdot C
\end{aligned}
$$

6. Construct a simpler circuit for a function `f`, currently implemented as follows:

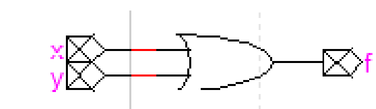


SOLUTION:

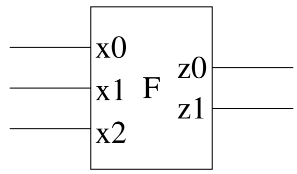
The Boolean expression for the circuit is: $f = x + y \cdot (z + (\overline{x + z}\ )$

SImplifiying:

$$
\begin{aligned}
f &= x + y \cdot (z + (\overline{x + z}\ ) \\
&= x + y \cdot (z + \overline{x} \cdot \overline{z}) \\
&= x + y \cdot (\overline{x} \cdot z + x \cdot z + \overline{x} \cdot \overline{z}) \\
&= x + y \cdot (\overline{x} \cdot z + x \cdot z + \overline{x} \cdot z + \overline{x} \cdot \overline{z}) \\
&= x + y \cdot (z + \overline{x}) \\
&= x + y \cdot z + y \cdot \overline{x} \\
&= y \cdot z + x + \overline{x} \cdot y \\
&= y \cdot z + x + y \text{ (as was done in step 2 through step 5)} \\
&= x + y
\end{aligned}
$$



7. A digital system has the following behavioral description:



x2	x1	x0	z1	z0
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

(a) Construct Boolean expressions for the functional specification of this system.

$$\begin{aligned} z1 &= x2' \cdot x1' \cdot x0 + x2' \cdot x1 \cdot x0' + x2 \cdot x1' \cdot x0 + x2 \cdot x1 \cdot x0' \\ z0 &= x2' \cdot x1' \cdot x0' + x2' \cdot x1 \cdot x0' + x2 \cdot x1' \cdot x0' + x2 \cdot x1 \cdot x0' \end{aligned}$$

(b) Using the laws of Boolean algebra, obtain a simpler but equivalent functional specification using only AND, OR, and NOT.

$$\begin{aligned} z1 &= x2' \cdot x1' \cdot x0 + x2' \cdot x1 \cdot x0' + x2 \cdot x1' \cdot x0 + x2 \cdot x1 \cdot x0' \\ &= x2' \cdot (x1' \cdot x0 + x1 \cdot x0') + x2 \cdot (x1' \cdot x0 + x1 \cdot x0') \\ &= (x2' + x2) \cdot (x1' \cdot x0 + x1 \cdot x0') \\ &= (x1' \cdot x0 + x1 \cdot x0') \end{aligned}$$

$$\begin{aligned} z0 &= x2' \cdot x1' \cdot x0' + x2' \cdot x1 \cdot x0' + x2 \cdot x1' \cdot x0' + x2 \cdot x1 \cdot x0' \\ &= x0' \cdot (x2' \cdot x1' + x2' \cdot x1 + x2 \cdot x1' + x2 \cdot x1) \\ &= x0' \cdot (x2' \cdot (x1' + x1) + x2 \cdot (x1' + x1)) \\ &= x0' \cdot (x2' + x2) \\ &= x0' \end{aligned}$$

(c) Draw the logic diagram from the simplified specification.