CMPT 419/726 Final Exam, Spring 2020

April. 8, 2020

Group 3

Instructions:

1. Write your first name, last name, and SFU ID clearly.
2. Submit a PDF file including your answers. It can be hand-written or typed. If you choose the hand-written option, write clearly. Answers will be marked incorrect if not legible.
3. This exam has 4 questions worth a total of 100 marks, designed for 60 minutes. You have 1 extra hour to submit your response in PDF format. Submission must be done before Apr 8, 17:30, **no exceptions**!
4. Read questions carefully and write your answers. It is highly recommended that you read all questions first, before starting to answer them.
5. This exam is open book and open internet but *not* open collaboration! **Do the exam on your own.**

Good luck!

1 Classification (20 marks)

Suppose we have the following training set with one real-valued input X and a categorical output Y that has three values.
Assume $\alpha = p(X = 2|Y = A)$ and $\beta = p(X = 2|Y = B)$ and $\gamma = p(X = 2|Y = C)$

X	Y
0	A
2	A
3	B
4	B
5	B
6	B
7	B
8	C
9	C
10	C

a) What is $p(X = 2 \wedge Y = A)$? (Answer in terms of α.)

b) What is $p(X = 2 \wedge Y = B)$? (Answer in terms of β.)

c) What is $p(X = 2 \wedge Y = C)$? (Answer in terms of γ.)

d) What is $p(X = 2)$? (Answer in terms of α, β and γ.)

e) What is $p(Y = A|X = 2)$? (Answer in terms of α, β and γ.)

f) For the data in Figure 1, suppose we use normal distributions as generative models for the 2 classes – that is, the class-conditional densities are Gaussians.

   After training the generative models using maximum likelihood, what class would be predicted for the query location marked with a question mark? Explain Briefly.

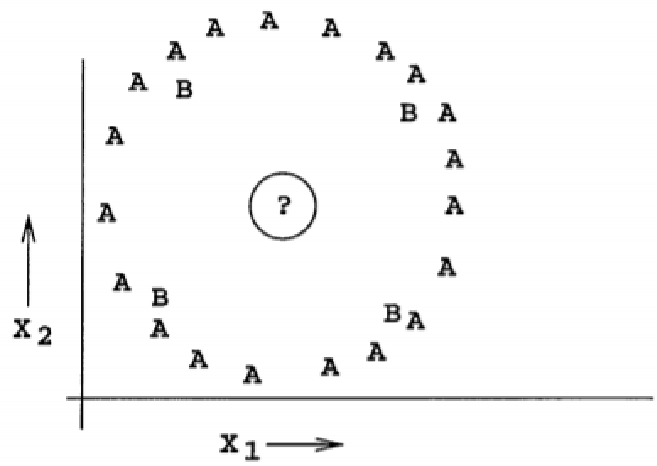
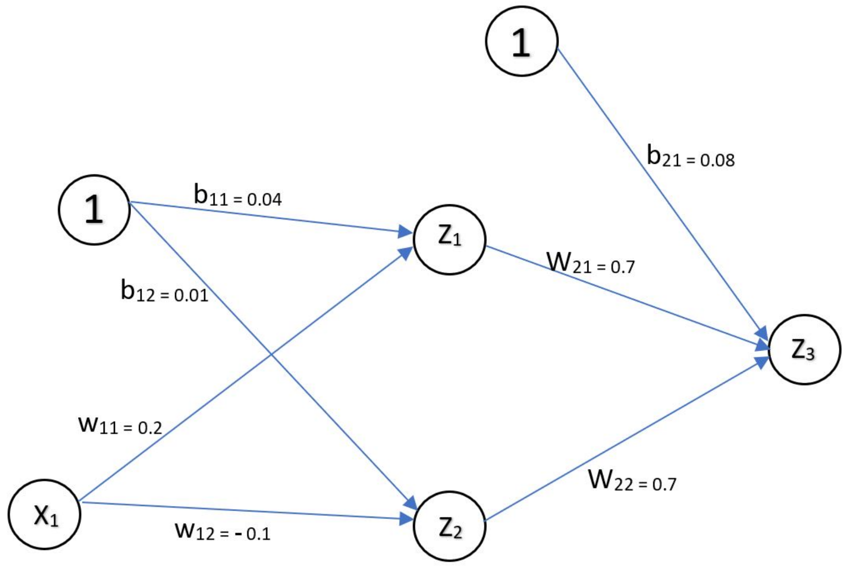


Figure 1

2 Neural Networks (26 marks)



Consider the neural network shown above for a 2-class (labeled 0 and 1) classification problem. The values for weights and biases are shown in the figure. We define the following:

$$
\begin{aligned}
a_1 &= w_{11}x_1 + b_{11}\\
a_2 &= w_{12}x_1 + b_{12}\\
a_3 &= w_{21}z_1 + w_{22}z_2 + b_{21}\\
z_1 &= ReLU(a_1)\\
z_2 &= ReLU(a_2)\\
z_3 &= \sigma(a_3), \quad \sigma(x) = \frac{1}{1+e^{-x}}
\end{aligned}
$$

a) For $x_1 = 0.3$, compute z_3 (in terms of e).

b) Perform back propagation on the bias b_{21} by deriving the expression for the gradient of the loss function $L(y, z_3)$ with respect to the bias term b_{21}, $\frac{\partial L}{\partial b_{21}}$, in terms of the partial derivatives $\frac{\partial \alpha}{\partial \beta}$, where α and β can be any of $L, z_i, a_i, b_{ij}, w_{ij}, x_1$ for all valid values of i, j.

c) Perform back propagation on the bias b_{12} by deriving the expression for the gradient of the loss function $L(y, z_3)$ with respect to the bias term b_{12},$\frac{\partial L}{\partial b_{12}}$, in terms of the partial derivatives $\frac{\partial \alpha}{\partial \beta}$, where α and β can be any of $L, z_i, a_i, b_{ij}, w_{ij}, x_1$ for all valid values of i, j.

Your back propagation algorithm should be as explicit as possible – that is, make sure each partial derivative $\frac{\partial \alpha}{\partial \beta}$ cannot be decomposed further into simpler partial derivatives. Do not evaluate the partial derivatives.

3 Graphical Models (30 marks)

The following Bayesian network represents a joint distribution over the variables Season, Coronavirus, Dehydration, Coughs, Fatigue, Nausea, and Dizziness. The data given in this question is ficticious and fabricated solely for an academic exercise.

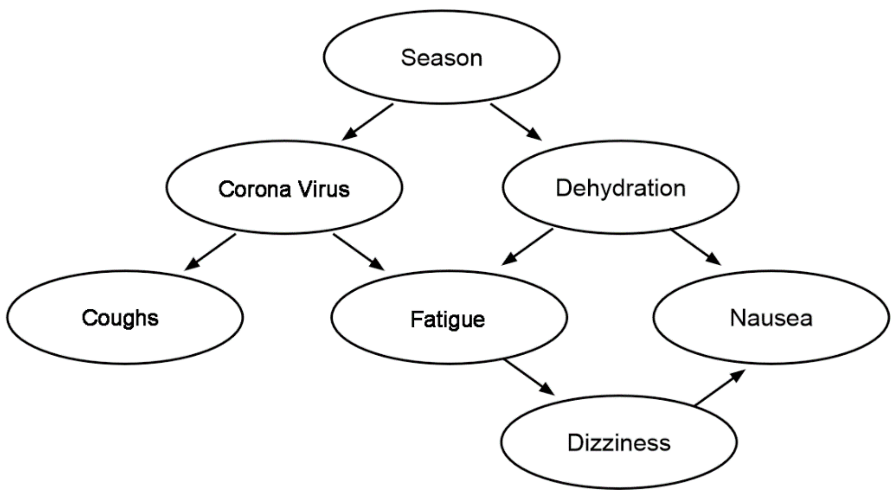


Figure 2: A Bayesian network that represents a joint distribution over the variables Season, Coronavirus, Dehydration, Coughs, Fatigue, Nausea, and Dizziness

Notation

S: Season, **CV**: Coronavirus, **D**: Dehydration, **C**: Coughs, **F**: Fatigue, **N**: Nausea, **Z**: Dizziness

$P(S = \text{winter})$	$P(S = \text{summer})$
0.5	0.5

	$P(\textsf{CV}= \text{true} \mid S)$	$P(\textsf{CV}= \text{false} \mid S)$
$S = \text{winter}$	0.4	0.6
$S = \text{summer}$	0.1	0.9

	$P(D = \text{true} \mid S)$	$P(D = \text{false} \mid S)$
$S = \text{winter}$	0.1	0.9
$S = \text{summer}$	0.3	0.7

	$P(C = \text{true} \mid \textsf{CV})$	$P(C = \text{false} \mid \textsf{CV})$
$\textsf{CV}= \text{true}$	0.8	0.2
$\textsf{CV}= \text{false}$	0.1	0.9

	$P(\mathbf{F} = \text{true} \mid \textsf{CV}, D)$	$P(\mathbf{F} = \text{false} \mid \textsf{CV}, D)$
$\textsf{CV}= \text{true}, D = \text{true}$	0.9	0.1
$\textsf{CV}= \text{true}, D = \text{false}$	0.8	0.2
$\textsf{CV}= \text{false}, D = \text{true}$	0.8	0.2
$\textsf{CV}= \text{false}, D = \text{false}$	0.3	0.7

	$P(Z = \text{true} \mid \mathbf{F})$	$P(Z = \text{false} \mid \mathbf{F})$
$\mathbf{F} = \text{true}$	0.8	0.2
$\mathbf{F} = \text{false}$	0.2	0.8

	$P(N = \text{true} \mid D, Z)$	$P(N = \text{false} \mid D, Z)$
$D = \text{true}, Z = \text{true}$	0.9	0.1
$D = \text{true}, Z = \text{false}$	0.8	0.2
$D = \text{false}, Z = \text{true}$	0.6	0.4
$D = \text{false}, Z = \text{false}$	0.2	0.8

Table 1: Conditional probability tables for the Bayesian network

Given the above conditional probability tables for the model, answer the following.

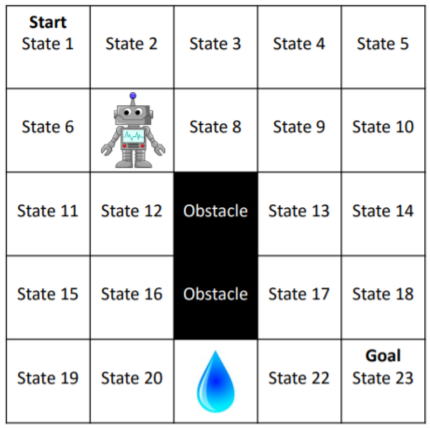
a) Write down the factorized form of the joint distribution over all of the variables, $P(S, CV, D, C, F, N, Z)$.

b) What is the probability that one has the Coronavirus, when no prior information is known?

c) What is the probability that one has the Coronavirus, given that it is winter, that one is fatigued, and that one is dehydrated?

4 Reinforcement Learning (24 marks)

Grid World is a 2D rectangular grid with an agent starting off at one grid square and trying to move to another grid square located elsewhere.



In the above grid world, the position of the robot drawing is for illustration only.

We define the following.

- **State(S_t)**: Position of robot. (The state does not include the direction that it is facing.)
- **Actions (A_t)** : Attempt Up, Attempt Down, Attempt Left, Attempt Right.

  We abbreviate these as AU, AD, AL, and AR respectively.
- **Rewards (R_t)**: The agent receives a reward of -10 for visiting the state with the water and a reward of $+10$ for visiting the goal state. Visiting any other state results in a reward of -1. When the agent enters an obstacle state, it is placed back where it started, and results in a reward of -2.

Suppose that the discount factor is $\gamma = 0.7$, the Q-learning rate is $\alpha = 0.2$ for all calculations, and the target policy is the greedy policy. Given the partial table of initial Q-values, compute the new Q value after one update for the agent attempting right at state 22. You may assume that the Q value for only this state and action pair – and no other state and action pair – is being updated.

s	a	$Q(s,a)$
17	AD	-5
20	AR	3
21	AR	6
21	AD	4
21	AU	-1
21	AL	3
22	AR	2
22	AD	-1
22	AU	1
22	AL	-3
23	AR	-2
23	AD	-7
23	AU	-4
23	AL	-1