<u>Series</u> (left over slides from Wednesday)

13. **Two Useful Results.**

Theorem.

(a) If the series $\sum_{n=1}^{\infty} a_n$ is convergent then $\lim_{n\to\infty} a_n = 0$.

(b) If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

14. **Example.** Show that $\sum_{n=1}^{\infty} n \sin(1/n)$ is divergent.

15. **Theorem.**

If $\sum a_n$ and $\sum b_n$ are convergent series and c is a constant, then $\sum ca_n$, $\sum(a_n + b_n)$, $\sum(a_n - b_n)$ are also convergent, and

(a) $\sum ca_n = c \sum a_n$

(b) $\sum(a_n + b_n) = \sum a_n + \sum b_n$

(c) $\sum(a_n - b_n) = \sum a_n - \sum b_n$

16. **Example.** If $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{2^n} - \frac{6}{(n+1)(n+2)} \right)$ is convergent, find its sum.

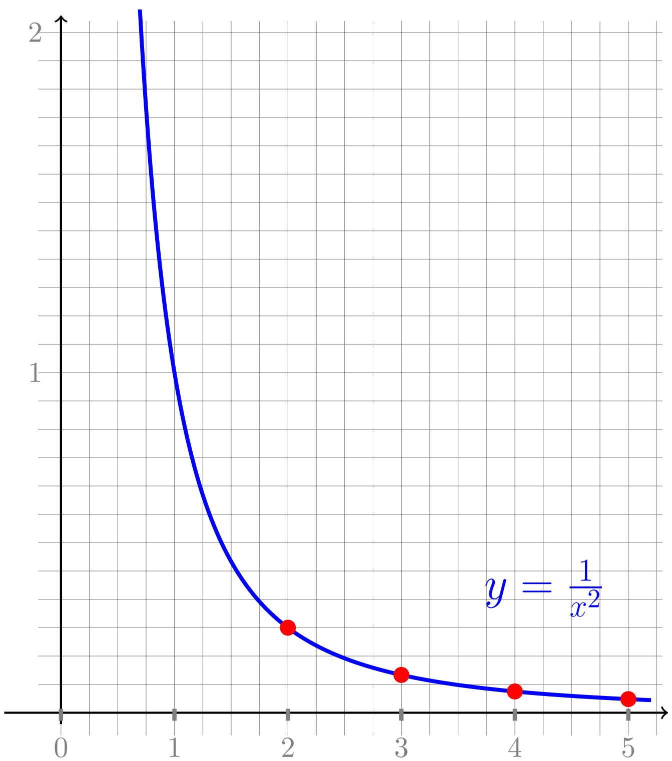
From Examples 7 and 11, we know that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ are convergent, with sums 1 and $\frac{1}{2}$, respectively.

The given series is convergent, since it can be written as

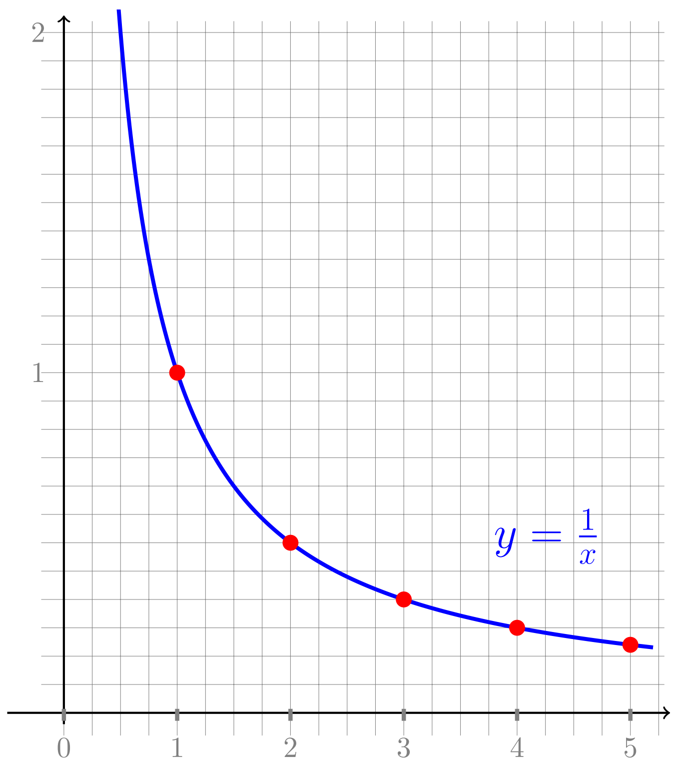
$$5 \sum_{n=1}^{\infty} \frac{1}{2^n} \;-\; 6 \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \;=\; 5(1) - 6(\tfrac{1}{2}) \;=\; 2.$$

1. **Quote.** "Education is what remains after one has forgotten what one has learned in school." (Albert Einstein, Theoretical Physicist, 1879–1955)

2. **Problem.** Compare $\displaystyle\int_1^\infty \frac{dx}{x^2}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.



3. Problem. Compare $\displaystyle\int_1^\infty \frac{dx}{x}$ and $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$.



4. **The Integral Test.**

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)dx$ is convergent. In other words:

(a) If $\displaystyle\int_1^{\infty} f(x)dx$ is convergent, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

(b) If $\displaystyle\int_1^{\infty} f(x)dx$ is divergent, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent.

5. **Example.** Is the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

convergent or divergent?

6. **Example.** Use the integral test to test the p**-series** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ for convergence.

7. **Remainder when using partial sums to estimate a series.**

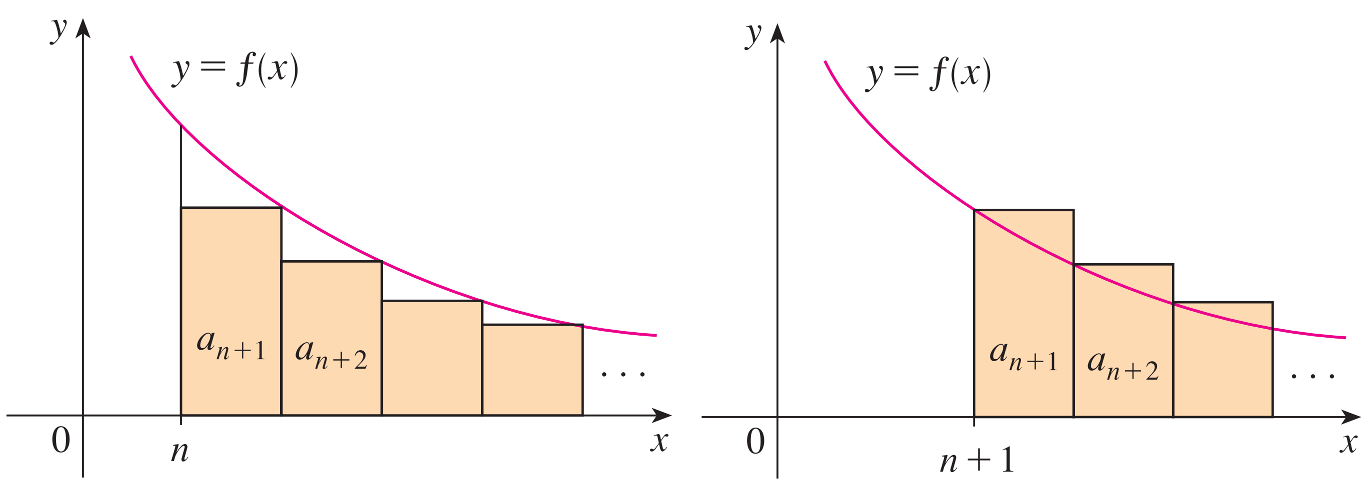
If $\sum_{n=1}^{\infty} a_n = s$ is convergent the the $n^{\textbf{th}}$ remainder is defined as

$$R_n = s - s_n = a_{n+1} + a_{n+2} + a_{n+3} + \ldots$$

8. **Remainder Estimate for the Integral Test.**

Suppose $f(k) = a_k$, where f is continuous, positive, decreasing function for $x \geq n$ and $\sum a_n$ is convergent. If $R_n = s - s_n$, then

$$\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_{n}^{\infty} f(x)dx.$$



9. **Example.** In a previous example we showed that the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

converges. Determine how many terms you would need to add to find the value of this sum accurate to within 0.01. That is, how large must n be for the remainder to satisfy the inequality $R_n < 0.01$?