The Comparison Test

1. **Quote.** ”Comparison is the thief of joy.”

   (Theodore Roosevelt Jr., 26^{th} American President [1901-1909], 1858-1919)

2. **The Comparison Test.**

   Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with $0 \leq a_n \leq b_n$ for all n.

   (a) If $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is also convergent.

   (b) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} b_n$ is also divergent.

3. **Example.** Test if

$$\sum_{n=1}^{\infty} \frac{1}{n^4 + e^n}$$

   is convergent.

4. **Useful tip.**

   When applying the comparison test, you can often use geometric series or p-series.

5. **Example.** Test the series

$$\sum_{n=1}^{\infty} \frac{1}{n!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

   for convergence.

6. **The Limit Comparison Test.**

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

7. **Example.** Test for convergence

(a) $\sum \frac{3n^2+n}{n^4+\sqrt{n}}$

(b) $\sum \frac{1}{2n+\ln n}$

Notes.