1. **Problem.** Find the area bounded by parabolas
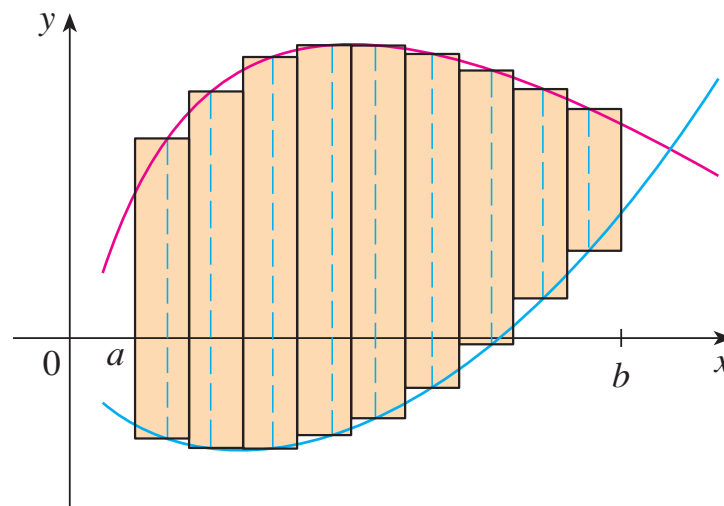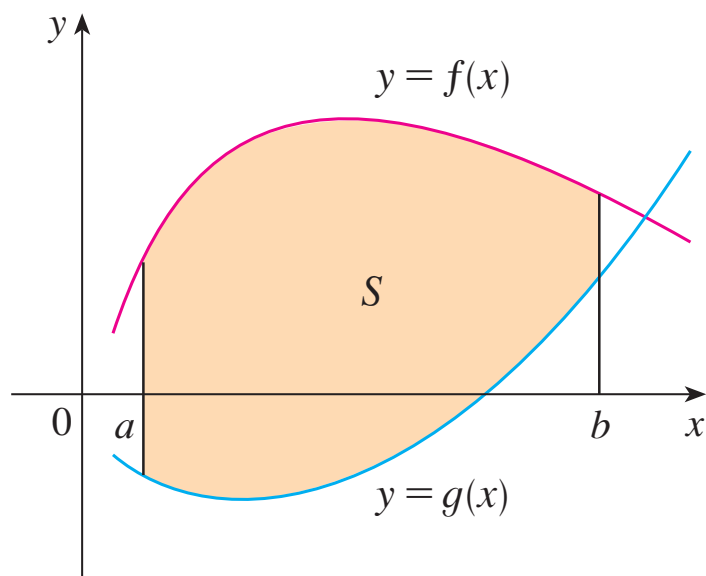
$$y = 2 - x^2 \text{ and } y = x^2.$$

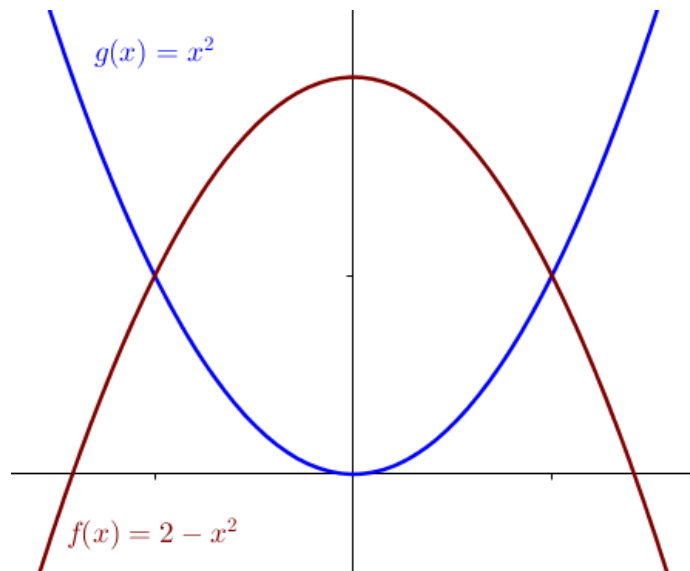$g(x) = x^2$

$f(x) = 2 - x^2$
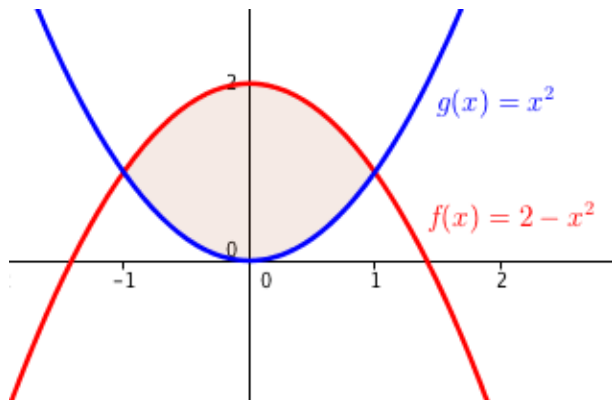
2. Fact. (Area Between Curves).

Suppose f and g are continuous and $f(x) \geq g(x)$ for all $x \in [a, b]$. The area A bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$, is given by
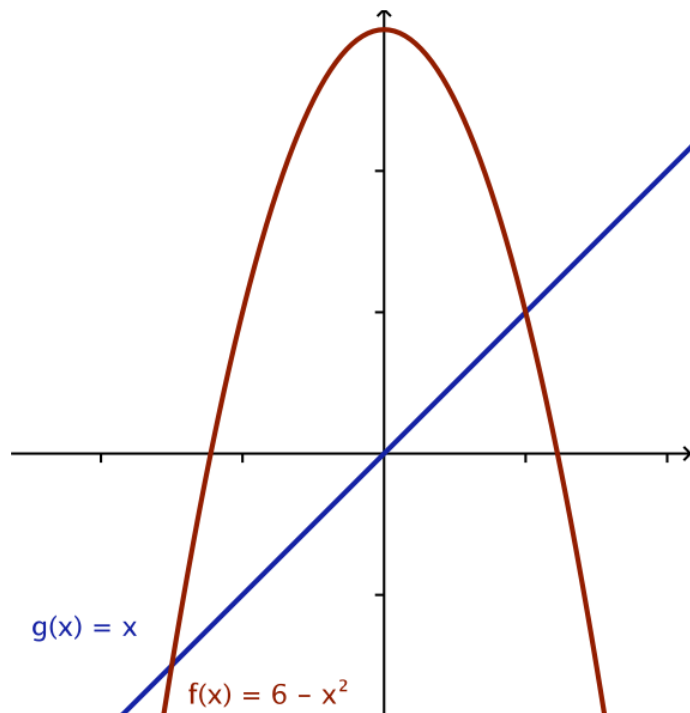
$$A = \int_a^b f(x) - g(x) \ dx.$$

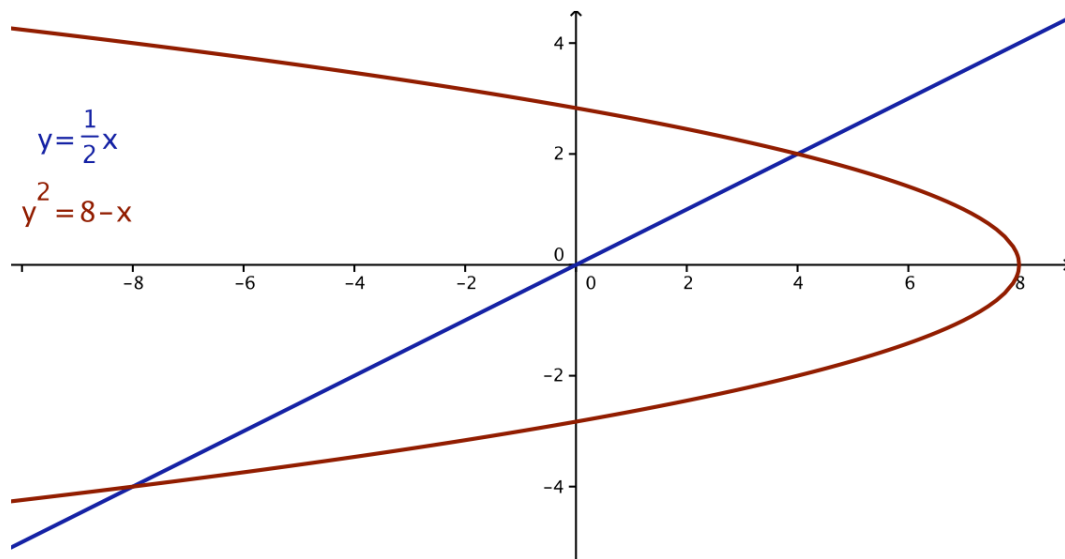3. **Example.** Find the area bounded by parabolas

$$y = 2 - x^2 \text{ and } y = x^2.$$

$g(x) = x^2$

$f(x) = 2 - x^2$

4. **Example.** Find the area of the region bounded by the line $y = x$ and the parabola $y = 6 - x^2$.

g(x) = x

f(x) = 6 − x²

5. **Example.** Find the area of the region bounded by the line $y = x/2$ and the parabola $y^2 = 8 - x$.
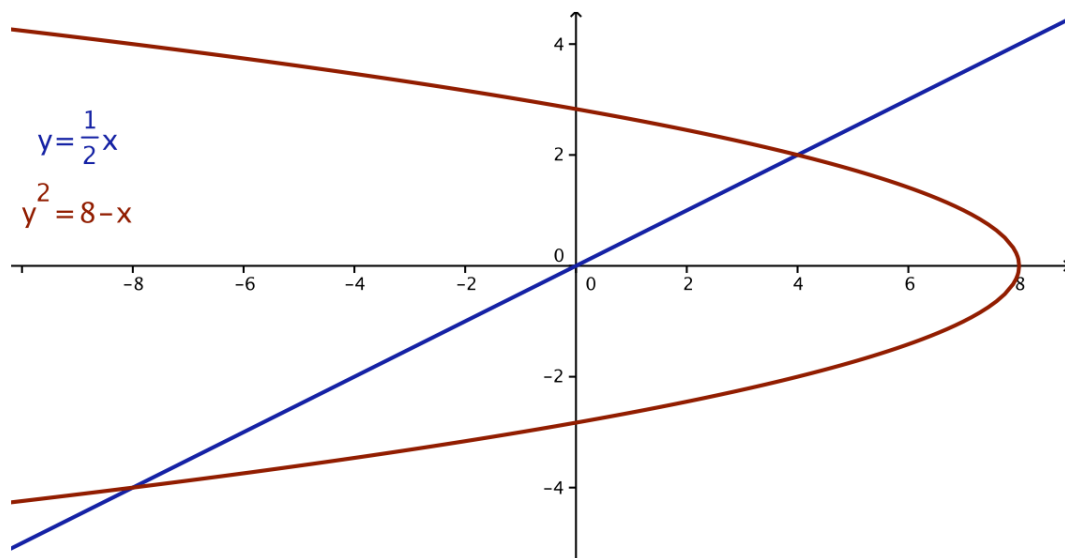
$y = \dfrac{1}{2}x$

$y^2 = 8 - x$

6. **Doing this area calculation along the y-axis...**

Suppose the area A is bounded by the curves $x = f(y)$, $x = g(y)$, and the lines $y = c$, $y = d$, where f and g are continuous and $f(y) \geq g(y)$ for all $y \in [c, d]$. Then the area is given by

$$A = \int_c^d [f(y) - g(y)]dy.$$

7. **Example.** Find the area of the region bounded by the line $y = x/2$ and the parabola $y^2 = 8 - x$.

$y = \dfrac{1}{2}x$

$y^2 = 8 - x$

Notes