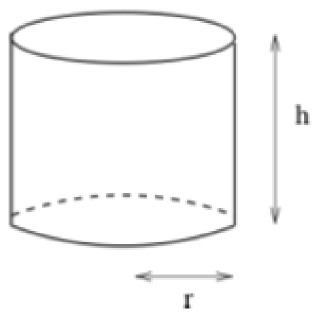
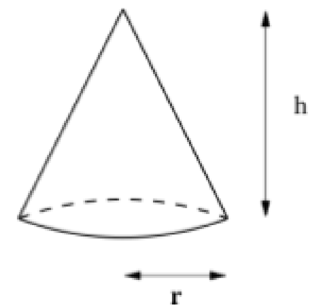
<u>Volumes</u>

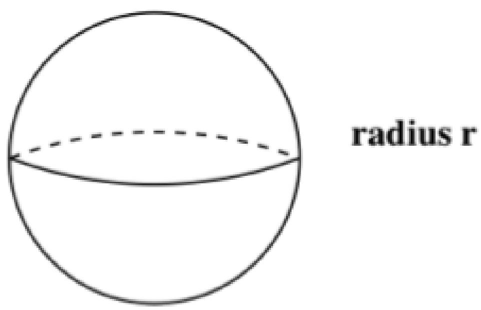
1. **Recall some classic volume formulas:**



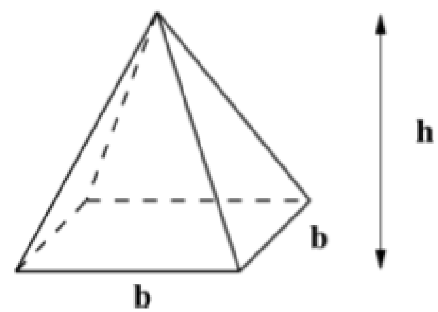
Right circular cylinder, $V = \pi r^2 h$



Cone, $V = \frac{1}{3}\pi r^2 h$



Sphere, $V = \frac{4}{3}\pi r^3$



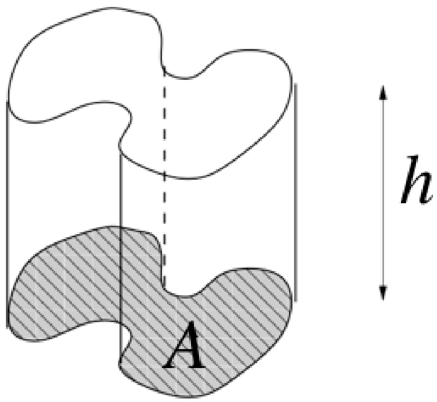
Square pyramid, $V = \frac{1}{3} b^2 h$

Where do the $\frac{1}{3}$ factors come from?

2. **Problem.** Prove these formulas? How do we define the volume of a solid object?

3. **Definition of Volume**... simple beginnings.

(i). The volume of a general cylinder with cross sectional area A and height h is defined to be Ah.



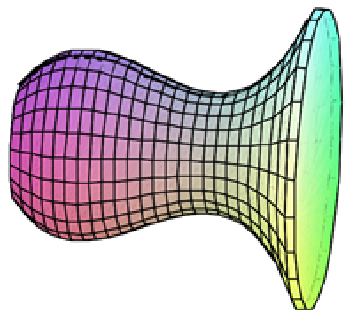
$$V = Ah$$

It turns out (by *Cavalieri's principle*) that these cross-sectional "area slices" can be rearranged and still give the same total volume.

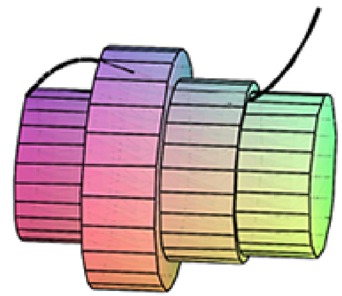
(ii). The volume of a general solid is defined using integrals (calculus).

4. **Definition of Volume**... the technique.

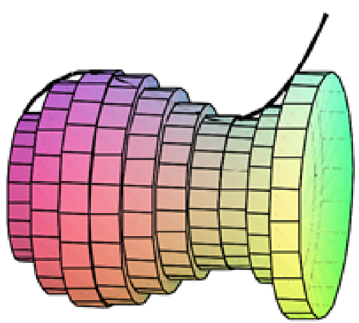
Problem: Find the volume of this solid.



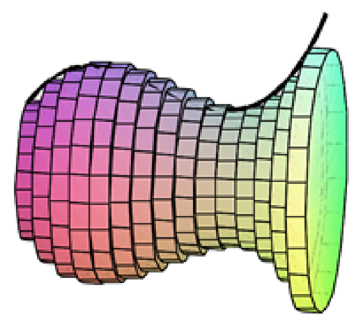
Approximate by 4 cylinders.



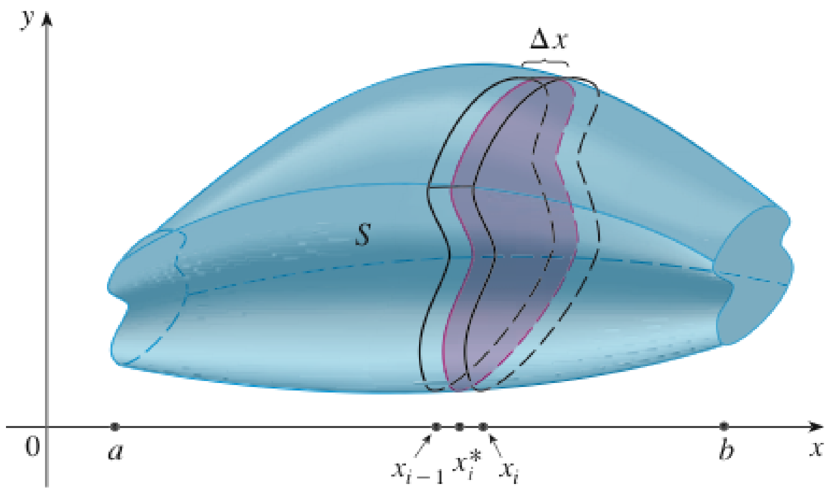
Approximate by 10 cylinders.

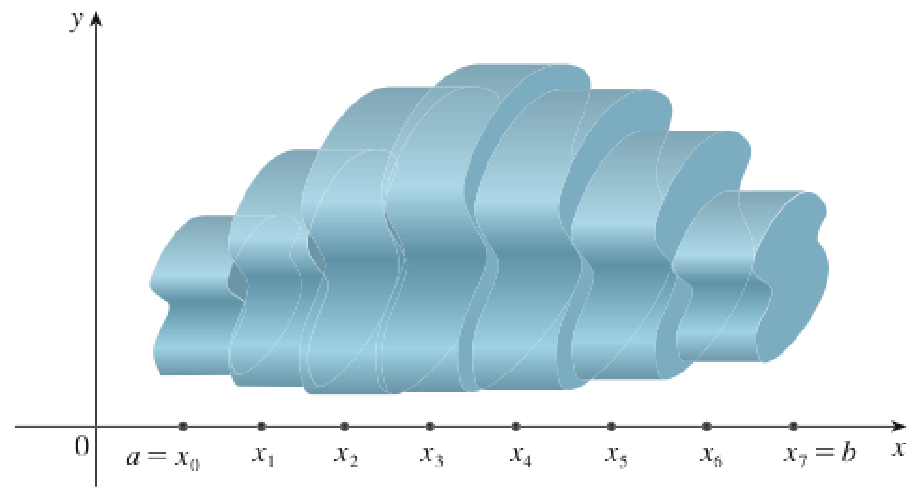


Approximate by 15 cylinders.



5. Computing the volume of a general solid S.





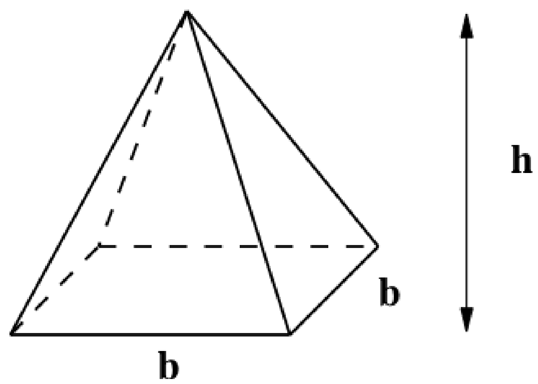
6. **Definition (Volume).**

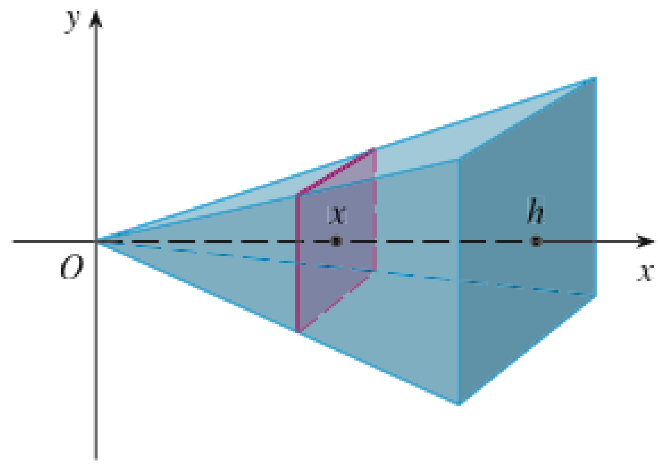
Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \int_a^b A(x)dx.$$

Recall: Our i-clicker question about the volume of the carrot!

7. **Example.** Find the volume of a pyramid whose base is a square with side b and whose height is h.

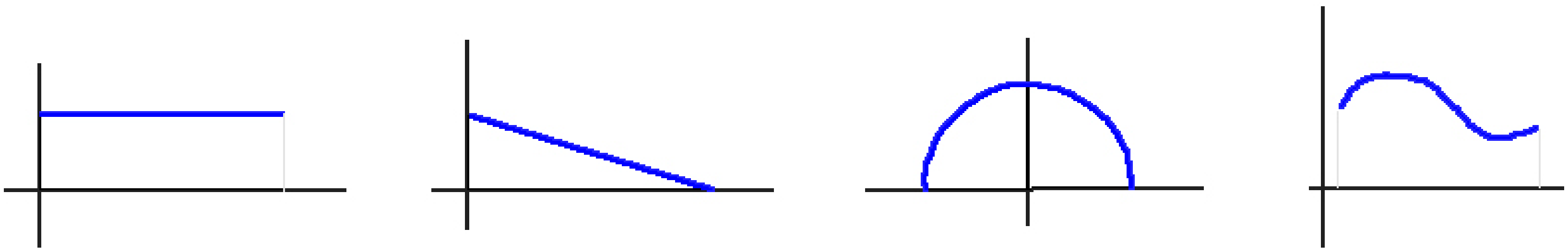




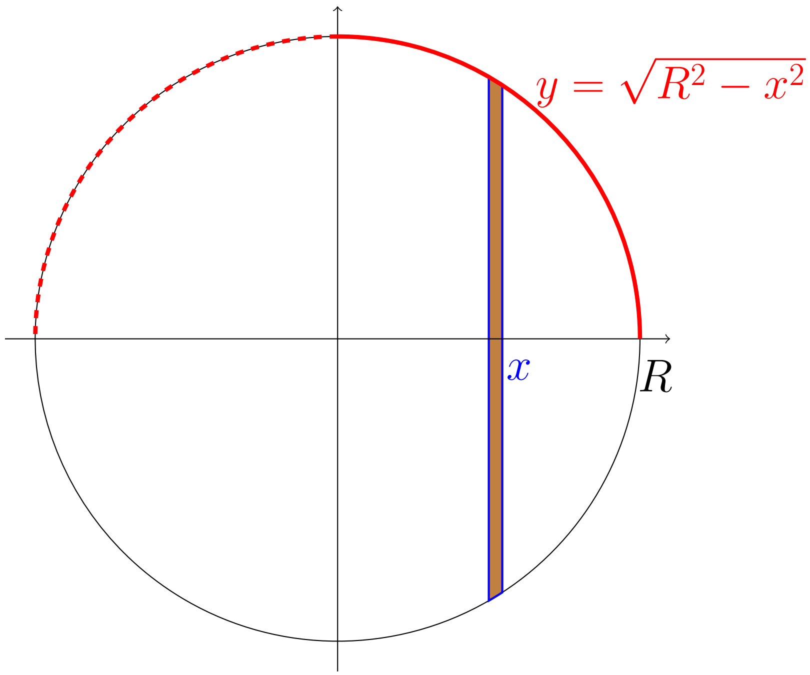
8. **Solid of Revolution.**

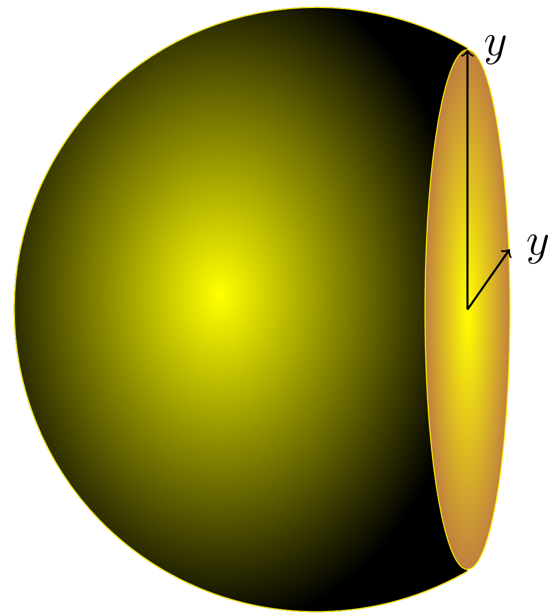
A solid of revolution is a solid (volume) obtained by revolving a region (or area) in the plane about a line.

9. **Example.** Some regions in the plane are shown below.
What solids result if these regions are rotated about the x-axis?



10. **Example.** Find the volume of the sphere with radius R (by rotating a semi-circle about the x-axis).





11. **Example.** Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = \sin x, \quad x = \frac{\pi}{2}, \text{ and } y = 0$$

about the x-axis.

12. **Example.** Find the volume of the solid obtained by rotating the region bounded by the curves

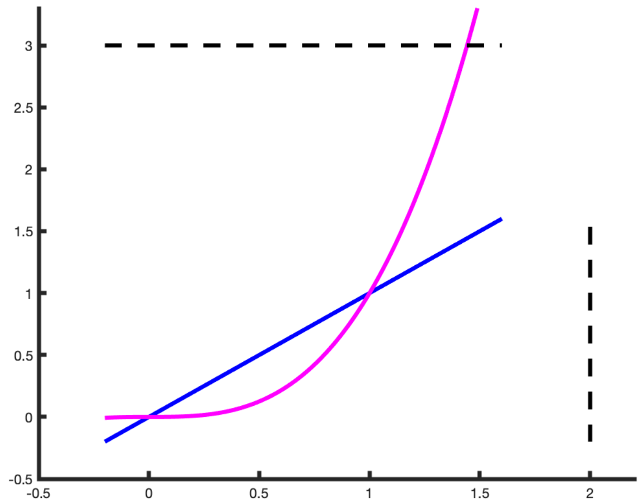
$$y = \sqrt{x}, \quad y = 1, \text{ and } x = 0$$

about the y-axis.

13. **Example.** Find the volume of the solid obtained by rotating the region R, which is enclosed by the curves $y = x$ and $y = x^3$ in the first quadrant, about the line

(a) $y = 3$

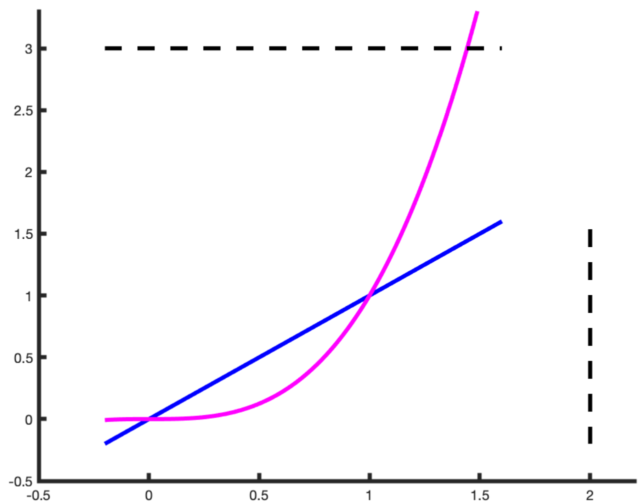
(b) $x = 2$



14. **Example.** Find the volume of the solid obtained by rotating the region R, which is enclosed by the curves $y = x$ and $y = x^3$ in the first quadrant, about the line

(a) $y = 3$

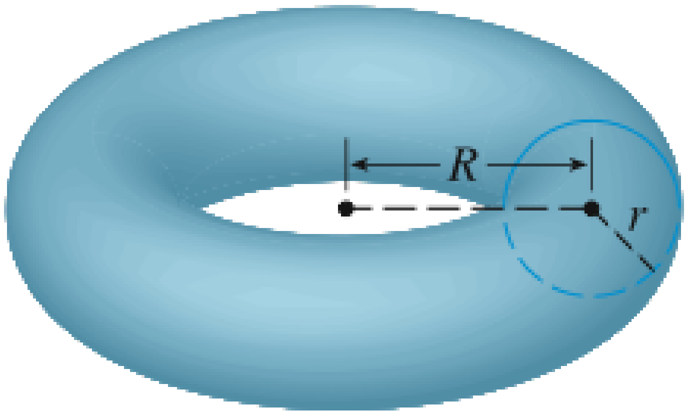
(b) $x = 2$



15. **Example.**

(a) Set up an integral for the volume of a **torus** with inner radius r and outer radius R.

(b) By interpreting the integral as an area, find the volume of the torus.



Notes: