

CMPT 295: Encoding and Decoding Solutions

1. Integer Representations

- (a) Express the decimal number 114.625 as a base-2 number and as a hexadecimal number.

ANSWER:

$$\begin{aligned}
 114_{10} &= 2^6 + 2^5 + 2^4 + 2 \\
 &= 1110010 \\
 0.625_{10} &= 2^{-1} + 2^{-3} \\
 &= 0.101 \\
 \text{Hence } 114.625_{10} &= 1110010.101
 \end{aligned}$$

- (b) Consider the following binary sequence: 11000111.

- i. What decimal number does this sequence represent if it is interpreted as a base 2 integer?

ANSWER: $11000111_2 = 2^7 + 2^6 + 2^2 + 2^1 + 2^0 = 128 + 64 + 7 = 199_{10}$

- ii. Can it be interpreted as a BCD encoding? If so, then what number is represented; otherwise explain why it does not represent a valid BCD encoding.

ANSWER: No it is not a valid BCD encoding because 1100 is not a valid BCD codeword.

- iii. If the binary sequence is chosen from the set of all fixed length codewords of the same length, How many distinct fixed length codewords belong to the set?

ANSWER: 2^8

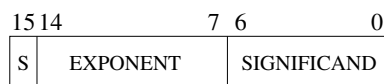
- iv. An alphabet of 100 symbols is to be encoded with a set of fixed length codewords. What is the minimum length of each codeword?

ANSWER: $2^6 < 100 \leq 2^7$ so the minimum length is 7.

2. Floating Point Representation

A floating point representation for real numbers is to be designed, subject to the following constraints:

- A 16 bit format is to be adopted.
 - The value zero will be represented by 0000_{16} .
 - The representation should accommodate eight significant bits of precision, including a most significant implied '1' bit (except for the representation of zero).
 - Exponents in the range $(-126 \text{ to } +126)$ should be accommodated.
 - An exponent value of $+127$ will represent overflow.
 - An exponent value of -127 will represent underflow.
 - The significant field will be interpreted as a sign-magnitude value.
 - The exponent will be interpreted as a bias-127 encoded value.
- (a) Draw a diagram of the floating point format, indicating the bit positions of the exponent and significant fields.



- (b) What real number is represented by the floating point value $BF80_{16}$?

ANSWER:

$BF80 = 1011\ 1111\ 1000\ 0000$

exponent : $011\ 1111\ 1 = 127 - \text{bias} = 0$

significand : 1.0000000

sign : $1 = \text{negative}$ Therefore number is $-1 \times 2^0 = -1$.

- (c) Convert the number -2^{16} into its corresponding 16-bit floating point representation. Express your answer in hexadecimal.

ANSWER:

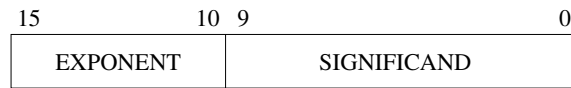
$-2^{16} = -1 \times 2^{16}$

exponent : $16 + \text{bias} = 16 + 127 = 143 = 1000\ 1111$. significand : 1.0000000

sign : negative = 1 Therefore the floating point representation is $1\ 1000\ 1111\ 000\ 0000 = C780_{16}$

3. Floating Point Representation

A floating point floating point format is defined as follows:



- (a) What is the 16-bit floating point representation of the decimal number 9.75?

ANSWER:

$9.75 = 1001.11 = 1.00111 \times 2^3$. Exponent = $31 + 3 = 34 = 100010$. Significand = 0011100000
Therefore the encoded number is 100001 0011100000

- (b) Express your answer as a VHDL hexadecimal constant.

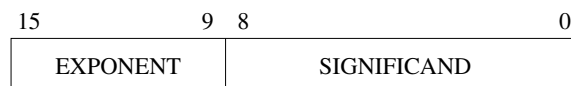
ANSWER: 1000 1000 1110 0000 = 0x88E0

- (c) What base-2 value is represented by the floating point encoding 0x8500?

ANSWER:

X"8500" = 1000 0101 0000 0000 . Exponent = $100001 - 011111 = 2$. Significand = 01 0000 0000
Therefore number is 1.01×2^2 .

- (d) Suppose that the floating point format is defined as follows:



- i. What is the 16-bit floating point representation of the decimal number 9.75?

ANSWER:

$9.75 = 1001.11 = 1.00111 \times 2^3$. Exponent = $63 + 3 = 66 = 1000010$. Significand = 0011100000
Therefore the encoded number is 1000010 0011100000

- ii. Express your answer as a VHDL hexadecimal constant.

ANSWER: 1000 0100 1001 0000 = 0x8470