FLASHBACK FRIDAYS

Flashback - previously in Math 152

Test for convergence: conditional, absolute or not.

(-i) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

(-ii) $$\sum_{n=1}^{\infty} \frac{2^n}{3^n - n}$$

(-iii) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

(-iv) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-1)!}{2^{2n-1}}$$

Example. Find the interval of convergence of the following series.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n x^n}{(2n)!}$

2. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

3. $\displaystyle\sum_{n=0}^{\infty} x^n$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

Representations of Functions as Power Series (11.9)

1. **Quote.** ”The aim of art is to represent not the outward appearance of things, but their inward significance.”

   (Aristotle, Ancient Greek philosopher and scientist, 384–322 BC)

2. **Problem.** Can we write a function, for example, $f(x) = \dfrac{1}{1+x}$ as a power series? How about any of those below?

   - $f(x) = \dfrac{1}{1+x^2}$
   - $f(x) = e^x$
   - $f(x) = \sin x$
   - $f(x) = \dfrac{1}{x}$

3. **Representation as a series.**

Let I be the interval of convergence for the power series $\sum_{n=0}^{\infty} c_n x^n$. For each $x \in I$, let $f(x)$ denote the series; that is,

$$f(x) = \sum_{n=0}^{\infty} c_n x^n, \qquad \text{if } x \in I$$

Then we call $\sum_{n=0}^{\infty} c_n x^n$ a **power series representation** of $f(x)$.

4. **Examples.** Find a power series representation of the following functions.

(a) $f(x) = \dfrac{1}{1+4x^2}$

(b) $g(x) = \dfrac{x}{9-x^2}$

5. **Theorem (Term-by-term differentiation or integration).**

Suppose the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ has radius of convergence $R > 0$. Then, the function f defined by $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ is differentiable on the interval $(a-R, a+R)$ and

(a) $f'(x) = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$

(b) $\int f(x)dx = C + \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$

Both these series have radii of convergence equal to R.

6. **Examples.** Find a power series representation of the following functions.

(a) $f(x) = \dfrac{1}{(1-x)^2}$

(b) $g(x) = \ln(1+x)$

(c) $h(x) = \arctan x$

(d) $\ell(x) = \dfrac{1}{x}$

Notes.