Phil 320

<u>**Chapter 5, §5.2: Every abacus-computable function is Turing-computable**</u>

Method of proof: show how the graph of any abacus-computable f can be transformed into the flow graph of a Turing machine that computes the same f.

1. Definition of an abacus-computable function

Suppose *A* is any abacus. Then *A* defines (computes) f as follows:

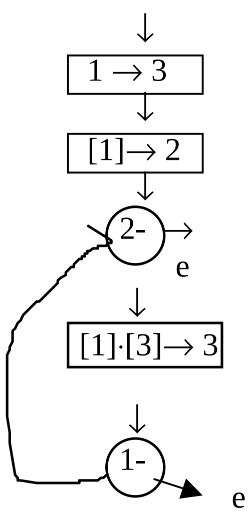
(1) Start with $x_1 = [1], \ldots, x_r = [r]$, and $0 = [r+1] = [r+2] = \ldots$

(2) Specify a solution register n. If the computation halts with $y = [n]$, then $f(x_1, \ldots, x_r) = y$. (**Note:** the other registers don't have to be empty when machine halts.)

(3) If the computation never halts, then $f(x_1,\ldots, x_r)$ is undefined.

Given an abacus *A*, there are two parameters needed to determine a function: *r* (the number of arguments) and *n*(the index of the solution register). Write A_n^r for the function computed by *A* that has *r* arguments and solution in register *n*.

Example: Let *A* be the following definite version of the factorial machine.



A_3^1 is the factorial function: $A_3^1(x) = x!$.
$A_4^1(x) = 0$ for all x, and similarly $A_5^1(x) = 0\ldots$
$A_3^2(x, y) = x!$, $A_3^3(x, y, z) = x!$, and so on.

2. Outline of Solution

Problem: Given an abacus A_n, with solution register specified as R_n, find a Turing machine M such that for each *r* (# of arguments), M defines the same function of *r* arguments as A_n^r.

A) Register/Block Correspondence.

The registers, taken in order, correspond to blocks on the tape separated by a single blank, taken in order. For instance:

BB11B1B11111B1B111BB

corresponds to

R_1	R_2	R_3	R_4	R_5	
1	0	4	0	2	(and 0 elsewhere);

- If $n \neq 0$, a register containing *n* is represented by a block of *n+1* 1's
- If $n = 0$, a register containing *n* is represented by a blank or by a single 1. The single 1 is mandatory if there are any 1's further to the right
- Two blanks in a row signify no further 1's on the tape.

B) Three Procedures

1. Replace n+ nodes with the following TM graph (always starting in std. Config.):

move to the blank to the right of block n, replacing B with 1 for empty registers along the way

↓

write a 1

↓

shift any remaining blocks over to the right by 1

↓

return to std. final position

N.B. The replacement is needed in case n is beyond any of the registers where initial non-zero information was stored.

Special cases:
- there are no remaining blocks
- 0 arguments

Turing machine: Figs. 5-9, 5-10.

2. Replace n- / e nodes with the following TM graph

Go to first 1 in nth block, replacing B by 1 for empty registers as needed

↓

If [n] = 0, return to std. position

↓

If [n] ≠ 0, go to rightmost 1 in nth block and erase it.

↓

Skip over the blank and shift any remaining blocks back one square. If there are remaining blocks, repeat the procedure; if not, return to std. position.

Turing machine: Fig. 5-11, 5-12

Result of steps 1 and 2: you get the right number of 1's in the n'th block if the machine halts. But there may also be lots of other blocks of 1's on the tape.

3. [After replacing all n+ and n- routines] Point all **loose arrows** to the initial node in a 'mop-up' graph that erases all but the n'th block of 1's.

N.B. B,B&J ensure that the n-th block is also re-positioned so that the machine halts at the same square where it started. We won't bother with this, but it is possible to do it.

To do this:

If n=1, erase all but the first block and halt in std. position.

Move to end of first block.

Erase each subsequent block until there are two blanks in a row; then return to std. position.

If $n \neq 1$:

Go to leftmost 1 of block n, erasing every 1 on the way
Move to end of block n
Erase subsequent blocks
Return to std. position.

Conclusion: Every abacus-computable function is Turing-computable.