STAT 485/685
Some Time Series Models

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Purposes of These Notes

- Discuss Linear Processes.
- Discuss MA(q) processes.
- Discuss AR(p) processes.
- Discuss relationships
- Discuss ARMA(p, q) processes.
- Conditions for stationarity and invertibility.

Linear Processes

- All our processes are built up from white noise: $\epsilon_t, t = 0, \pm 1, \ldots$.
- Most general:

$$Y_t = \mu + \cdots + a_2\epsilon_{t-2} + a_1\epsilon_{t-1} + \epsilon_t + a_{-1}\epsilon_{t+1} + \cdots$$

- Usually require them to make physical sense: no future errors!
- So (using book's notation with parameters "psi"):

$$Y_t = \mu + \epsilon_t + \psi_1\epsilon_{t-1} + \psi_2\epsilon_{t-2} + \cdots$$

- Does the infinite sum make sense?
- Yes if $\sum_{j=1}^{\infty} \psi_j^2 < \infty$.
- Take coefficient of ϵ_t to be 1 because variance of errors not specified.

Autocorrelation

- Can we compute γ_k?
- Yes:

$$\mathrm{Cov}(Y_t, Y_{t-k}) = \mathrm{Cov}(\epsilon_t + \psi_1\epsilon_{t-1} + \cdots, \epsilon_{t-k} + \psi_1\epsilon_{t-k-1} + \cdots)$$

- Expand this out; most terms are 0.
- Exceptions are

$$\mathrm{Cov}(\psi_k\epsilon_{t-k}, \epsilon_{t-k}) + \mathrm{Cov}(\psi_{k+1}\epsilon_{t-(k+1)}, \psi_1\epsilon_{t-(k+1)}) + \cdots$$

- These are just

$$\psi_k + \psi_{k+1}\psi_1 + \psi_{k+2}\psi_2 + \cdots .$$

- Sometimes there is a simpler formula.