LANGARA COLLEGE

MATHEMATICS 1171 - Calculus I
FINAL EXAMINATION
April 2013
Duration: 2 hours

Short answers at back (not available during exam)

Last NAME:____

First NAME:____

STUDENT NUMBER:____________________________________

This exam has *13* pages including the front page.
Please check yours!

INSTRUCTIONS

- Answer questions 1 –13 in the space provided. Indicate if there is work on the back of a page that should be marked.
- Non-graphing calculators are permitted; no other materials are permitted.
- For full marks, all appropriate work must be shown. Exact answers must be given when they can be found.

There are 90 marks in total. Good luck!
____

1		**7 pts**
2		**7 pts**
3		**6 pts**
4		**8 pts**
5		**7 pts**
6		**7 pts**
7		**5 pts**
8		**4 pts**
9		**8 pts**
10		**8 pts**
11		**8 pts**
12		**7 pts**
13		**8 pts**
Total		***90 pts***

Please circle your instructor's name and section number:

Instructors: M. Lavallee, Section 1 D. Lidstone, Section 2

E. Belchev, Section 3 E. Avelino, Section 4

1. (a) [2 points] Define what it means for a function f to be continuous at $x = a$.

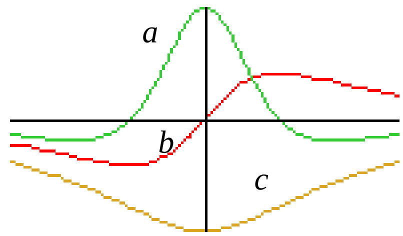
(b) [2 points] If a function f is continuous at $x = a$, must f be differentiable there too? If "yes", give a brief argument why, otherwise, give an example that fails.

(c) [3 points] Let $f(x) = \begin{cases} \dfrac{(x-\pi)\cos x}{1-\sin x} & x \neq \dfrac{\pi}{2} \\ 2 & x = \dfrac{\pi}{2} \end{cases}$

Is f continuous at $x = \dfrac{\pi}{2}$?

2. (a) Shown below are the graphs of a function f, its derivative f', and also an antiderivative F of f.

[3 points] Decide which graph is which (you need not give your reasons).

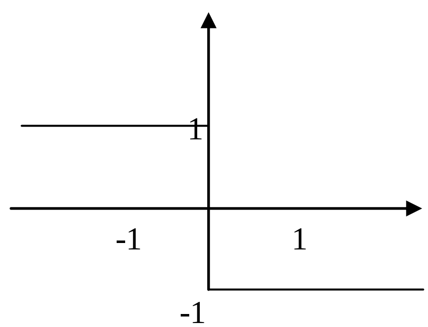


Answer (circle your choice):

The graphs of F, f, f' (in order) are:

$a\ b\ c \quad a\ c\ b \quad b\ a\ c \quad b\ c\ a \quad c\ a\ b \quad c\ b\ a$

(b) [3 points] The graph of $f(x) = -\dfrac{x}{|x|}$ is shown. On the same axes, draw the graph of a continuous function $F(x)$ satisfying $F(0) = 1$ that has derivative $f(x)$ (i.e. $F'(x) = f(x)$ for all $x \neq 0$).



3. (a) [2 points] State the limit definition of $f'(3)$, which is the derivative of a function $f(x)$ at a point $x = 3$.

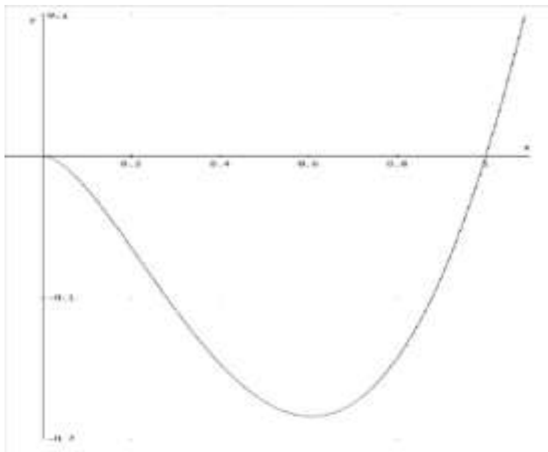
(b) [5 points] Use the limit definition of the derivative to find $f'(a)$ for $f(x) = \dfrac{-3}{x-2}$.

4. For a function $f(x)$, it is known that $f'(x) = 3e^x - x$ and $f(0) = 2$.

(a) [3 points] Use linear approximation at $x = 0$ to estimate $f(1)$.

(b) [5 points] Now use antiderivatives to find a formula for $f(x)$ and then find the exact value of $f(1)$.

5. [7 points] A plot of the graph of $f(x) = x^2 \ln x$ is given below and shows an absolute minimum at about $x = 0.6$ and an inflection point at about $x = 0.3$. Use calculus to find exact values of these locations and explain what happens when $x \to 0^+$.



6. Differentiate each of the following functions

(a) [3.5 points] $f(x) = \arctan(x/2) - 2e^{-\frac{4}{\pi}\sin(x-2)}$. Then, find $f'(2)$.

(b) [3.5 points] $y = (x+1)^{\sin(x+1)}$.

7. The following has been observed on the position $f(t)$ of a particle moving continuously on a line with positive acceleration:

$$f(0) = -1,\ f'(0) = 0; \quad \text{and} \quad f(1) = 2,\ f'(1) = 5.$$

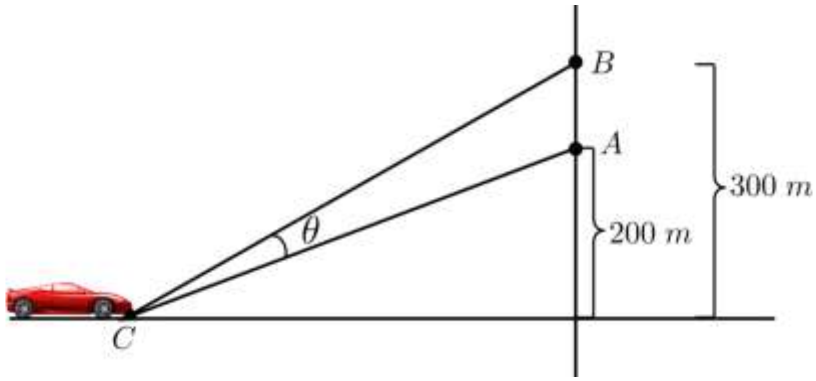
Use the data to estimate:

(a) [3 points] the average speed of the particle over the interval $0 \le t \le 1$

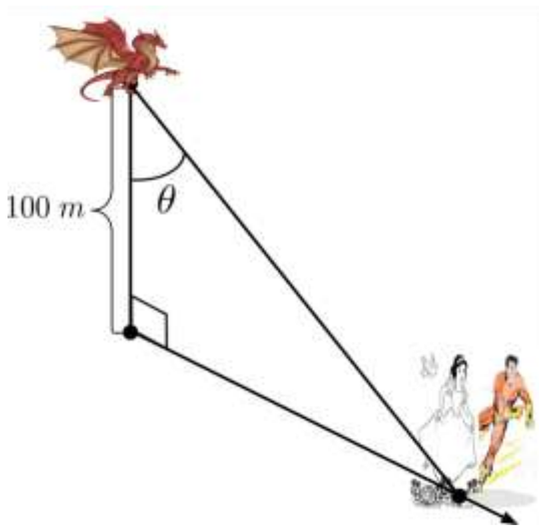
(b) [2 points] the time t when the particle was at the origin, using one iteration of Newton's method. Use an initial approximation of $t_1 = 1$.

8. [4 points] A rock is dropped off of a 98 meters cliff. Its height above the ground t seconds later is given by $s = f(t) = 98 - 4.9t^2$ meters. Find its velocity when it hits the ground.

9. [8 points] Two highways intersect at a right angle. Two recording lights are located at points *A* and *B*, 200 meters and 300 meters away from the intersection on one of the highway. A car, *C*, approaching the intersection along the other highway, is being tracked by the two lights. Let θ be the angle *ACB* (see figure below). Find how far from the intersection the car is when the angle θ is a maximum?



10. [8 points] A Princess and her rescuer, Hero, are running from a castle. On the roof of the castle (100 meters above the ground) an angry (and hungry) dragon watches their escape. Princess and Hero are running at 3 meters per second. When they are 100 meters away how fast is the angle at which the dragon observes them changing?



11. On a college campus of 5000 students, the spread of a flu virus through the student population is modelled by

$$P(t) = \frac{5000}{1 + 4999e^{-0.5t}}$$

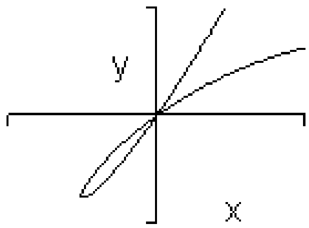
where $P(t)$ is the total number of infected people at time t. Time is measured in days.

(a) [3 points] Find $\lim_{t \to \infty} P(t)$ and interpret the result.

(b) [3 points] Given that $P'(t) = \dfrac{12497500e^{-0.5t}}{\left(1 + 4999e^{-0.5t}\right)^2}$, find $P'(2)$ and interpret the result.

(c) [2 points] Use differentials to approximate $P(2.5)$ - $P(2)$.

12. Given below is a plot of the parametric curve $C : x = t^2 + t, y = t - t^3, -\infty < t < \infty$.



(a) [3 points] Show that C intersects the origin at two different times.

(b) [3 points] Determine the slope of C at the origin the *first* time that it goes through the origin.

(c) [1 point] Show on the plot with an arrow the direction that C is drawn as t increases.

13. A positive function $y = f(x)$ is given implicitly by the equation $\cos(xy) = x\sin(xy)$.

(a) [1 point] Check that the point $\left(1, \frac{\pi}{4}\right)$ is on the graph of the above equation.

(b) [5 points] Find y' at the point $\left(1, \frac{\pi}{4}\right)$ using implicit differentiation.

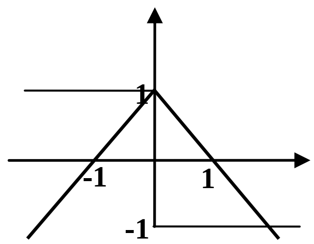
(c) [2 points] Find the equation of the tangent line of this curve at point $\left(1, \frac{\pi}{4}\right)$.

Answers

1. (a) $\lim_{x \to a} f(x) = f(a)$ (b) No. e.g. $f(x) = |x|$ is continuous, but not differentiable at $x = 0$.

   (c) Not continuous. $f(x) \to -\infty$ as $x \to \pi/2$.

2. (a) cba (b)



3. (a) $f'(3) = \lim_{x \to 3} \dfrac{f(x) - f(3)}{x-3} = \lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$ (either of these limits).

   (b) $f'(a) = \dfrac{3}{(a-2)^2}$.

4. (a) $f(1) \approx f(0) + f'(0)(1-0) = 2 + 3 = 5$

   (b) $f(x) = 3e^x - \dfrac{1}{2}x^2 - 1, \quad f(1) = 3e - \dfrac{3}{2}$.

5. Absolute min at $x = \exp(-1/2)$, inflection point at $x = \exp(-3/2)$, $f(x) \to 0$ as $x \to 0^+$ (l'Hosp.)

6. (a) $f'(x) = \dfrac{1}{1 + (x/2)^2} \cdot \dfrac{1}{2} - 2e^{-\frac{4}{\pi}\sin(x-2)} \cdot \dfrac{-4}{\pi}\cos(x-2); \quad f'(2) = \dfrac{1}{4} + \dfrac{8}{\pi}$.

   (b) $y' = (x+1)^{\sin(x+1)}\left(\dfrac{\sin(x+1)}{x+1} + \cos(x+1)\ln(x+1)\right)$

7. (a) $\dfrac{\Delta f}{\Delta t} = 3$, (b) $t = 1 - \dfrac{f(1)}{f'(1)} = 0.6$.

8. Hits ground when $t = \sqrt{20}$ sec. with velocity $f'(\sqrt{20}) = -9.8\sqrt{20} = -19.6\sqrt{5}\, m/s$.

9. θ is maximum when the car is $100\sqrt{6} \approx 245\, m$ from the intersection.

10. 3/200 = 0.015 radians/sec. ≈ 0.86 deg./sec.

11. (a) 5000 students. After enough time all students will be infected.

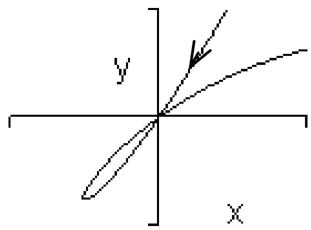
(b) $P'(2) \approx 1.358$ students per day. After 2 days, the rate of spread of the infection is about 1.358 students/day.

(c) $P(2.5) - P(2) \approx 1.358 \times (0.5) = 0.679$ students.

12. (a) $(x, y) = (0,0)$ at times $t = -1,\ t = 0$.

(b) slope = 2

(c)



13. (a) ✓ (b) $y' = -\dfrac{\pi + 2}{4}$ (c) $y - \dfrac{\pi}{4} = -\dfrac{\pi + 2}{4}(x - 1)$