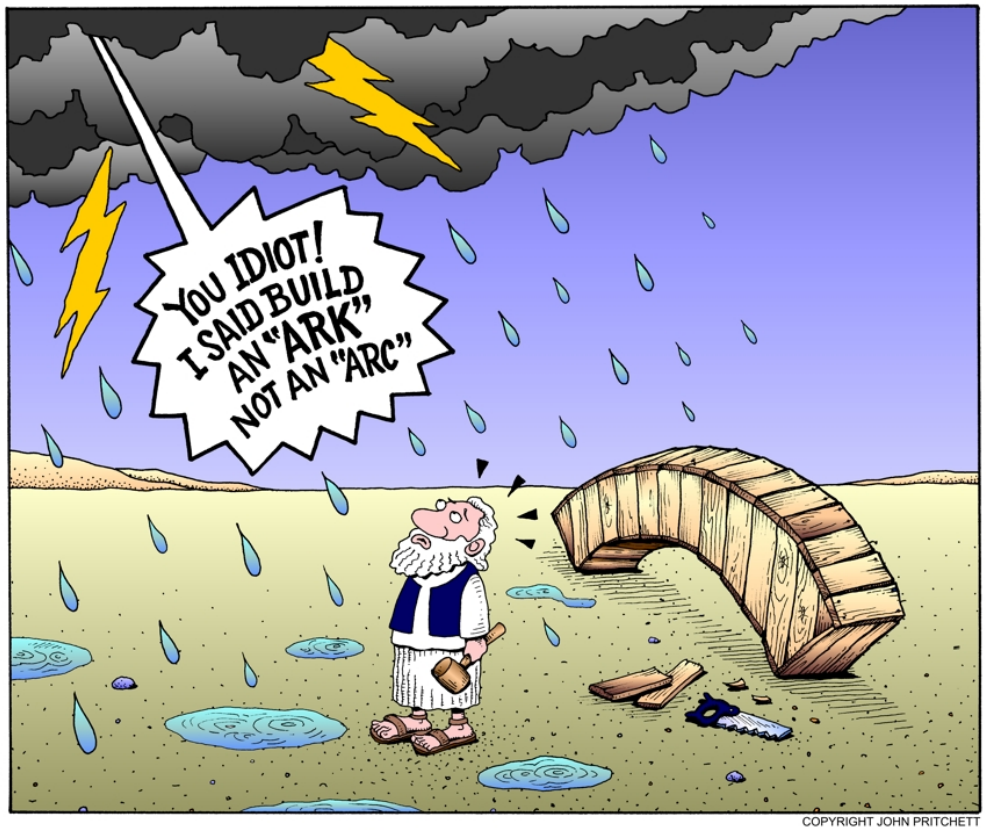
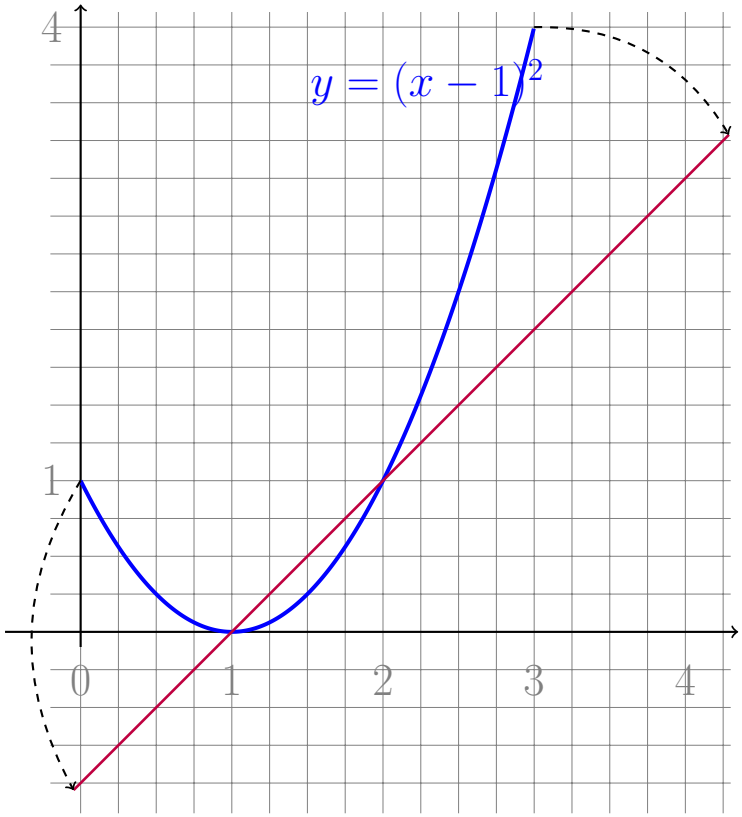
Arc Length





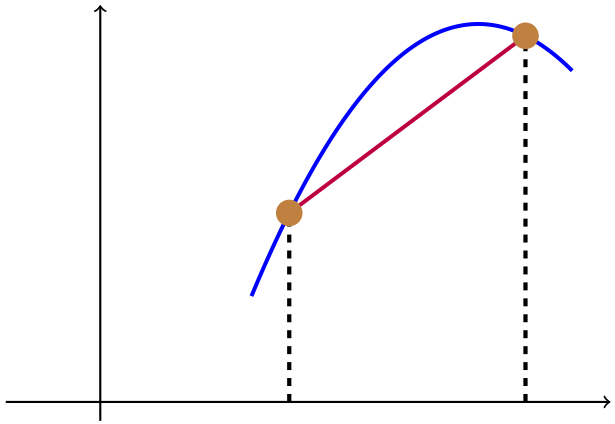
1. **Problem.** Find the length of the arc of the parabola $y = (x-1)^2$ between the points $(0, 1)$ and $(3, 4)$.

   - The formulas are important, but the integrals often cannot be evaluated analytically (because of the square root), and must be computed numerically.
   - Here we talk about curves which are the graphs of a function $y = f(x)$.
   - Parametric curves come later.

2. **The Arc Length Formula.**

If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$ is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx.$$



3. **Example.** Find the length of the following arcs.

(a) $y = (x-1)^2$ **between the points** $(0,1)$ **and** $(3,4)$

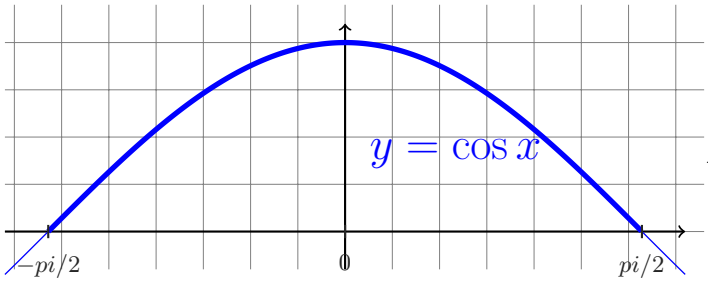
(b) $x = \frac{1}{6}y^3 + \frac{1}{2y}$, $1 \leq y \leq 2$

(c) $y = x^3$, $0 \leq x \leq 5$

(d) $y = x^{\frac{3}{2}}$, **from** $(0, 0)$ **to** $(4, 8)$

4. **Revisit approximate integration**

What is the length of the arch of the cosine function form $-\pi/2$ to $\pi/2$?

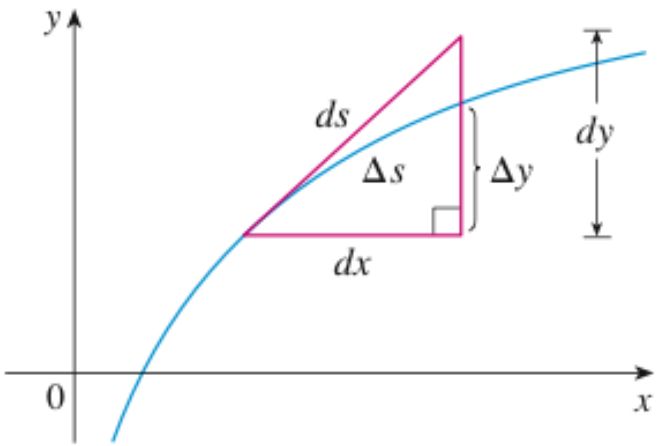


$$L = \int_{-\pi/2}^{\pi/2}$$

5. **The Arc Length Function.** Let C be a smooth curve determined by the equation $y = f(x)$, $a \le x \le b$.
Let $s(x)$ be the distance along C from the initial point $P_0(a, f(a))$ to the point $Q(x, f(x))$.

(a) Find the formula for $s(x)$.

(b) Find $\dfrac{ds}{dx}$, as well as the differential ds.



6. **Example.** Find the arc length function for the curve $y = 2x^{3/2}$ with starting point $P_0(1, 2)$.