CMPT 295: Encoding and Decoding Solutions

1. **Integer Representations**

   (a) Express the decimal number 114.625 as a base-2 number and as a hexadecimal number.

   ANSWER:

$$
\begin{aligned}
114_{10} &= 2^6 + 2^5 + 2^4 + 2 \\
&= 1110010 \\
0.625_{10} &= 2^{-1} + 2^{-3} \\
&= 0.101 \\
\text{Hence } 114.625_{10} &= 1110010.101
\end{aligned}
$$

   (b) Consider the following binary sequence: 11000111.

   i. What decimal number does this sequence represent if it is interpreted as a base 2 integer?

   ANSWER: $11000111_2 = 2^7 + 2^6 + 2^2 + 2^1 + 2^0 = 128 + 64 + 7 = 199_{10}$

   ii. Can it be interpreted as a BCD encoding? If so, then what number is represented; otherwise explain why it does not represent a valid BCD encoding.

   ANSWER: No it is not a valid BCD encoding because 1100 is not a valid BCD codeword.

   iii. If the binary sequence is chosen from the set of all fixed length codewords of the same length, How many distinct fixed length codewords belong to the set?

   ANSWER: 2^8

   iv. An alphabet of 100 symbols is to be encoded with a set of fixed length codewords. What is the minimum length of each codeword?

   ANSWER: $2^6 < 100 \leq 2^7$ so the minimum length is 7.

2. **Floating Point Representation**

A floating point representation for real numbers is to be designed, subject to the following constraints:

- A 16 bit format is to be adopted.
- The value zero will be represented buy 0000_{16}.
- The representation should accommodate eight significant bits of precision, including a most significant implied '1' bit (except for the representation of zero).
- Exponents in the range (-126 to $+126$) should be accommodated.
- An exponent value of $+127$ will represent overflow.
- An exponent value of -127 will represent underflow.
- The significant field will be interpreted as a sign-magnitude value.
- The exponent will be interpreted as a bias-127 encoded value.

(a) Draw a diagram of the floating point format, indicating the bit positions of the exponent and significant fields.

15	14 7	6 0
S	EXPONENT	SIGNIFICAND

(b) What real number is represented by the floating point value $BF80_{16}$?

ANSWER:

BF80 = 1011 1111 1000 0000
exponent : 011 1111 1 = 127 − bias = 0
significand : 1.0000000
sign : 1 = negative Therefore number is $-1 \times 2^0 = -1$.

(c) Convert the number -2^{16} into its corresponding 16-bit floating point representation. Express your answer in hexadecimal.

ANSWER:

$-2^{16} = -1 \times 2^{16}$
exponent : 16 + bias = 16 + 127 = 143 = 1000 1111. significand : 1.0000000
sign : negative = 1 Therefore the floating point representation is 1 1000 1111 000 0000 = $C780_{16}$

3. **Floating Point Representation**

A floating point floating point format is defined as follows:

15 10	9 0
EXPONENT	SIGNIFICAND

(a) What is the 16-bit floating point representation of the decimal number 9.75?

ANSWER:

$9.75 = 1001.11 = 1.00111 \times 2^3$. Exponent $= 31 + 3 = 34 = 100010$. Significand $= 0011100000$
Therefore the encoded number is 100001 0011100000

(b) Express your answer as a VHDL hexadecimal constant.

ANSWER: 1000 1000 1110 0000 = 0x88E0

(c) What base-2 value is represented by the floating point encoding `0x8500`?

ANSWER:

X"8500" = 1000 0101 0000 0000 . Exponent = 100001 - 011111 = 2. Significand = 01 0000 0000
Therefore number is 1.01×2^2.

(d) Suppose that the floating point format is defined as follows:

15 9	8 0
EXPONENT	SIGNIFICAND

i. What is the 16-bit floating point representation of the decimal number 9.75?

ANSWER:

$9.75 = 1001.11 = 1.00111 \times 2^3$. Exponent $= 63 + 3 = 66 = 1000010$. Significand = 001110000
Therefore the encoded number is 1000010 001110000

ii. Express your answer as a VHDL hexadecimal constant.

ANSWER: 1000 0100 1001 0000 = 0x8470