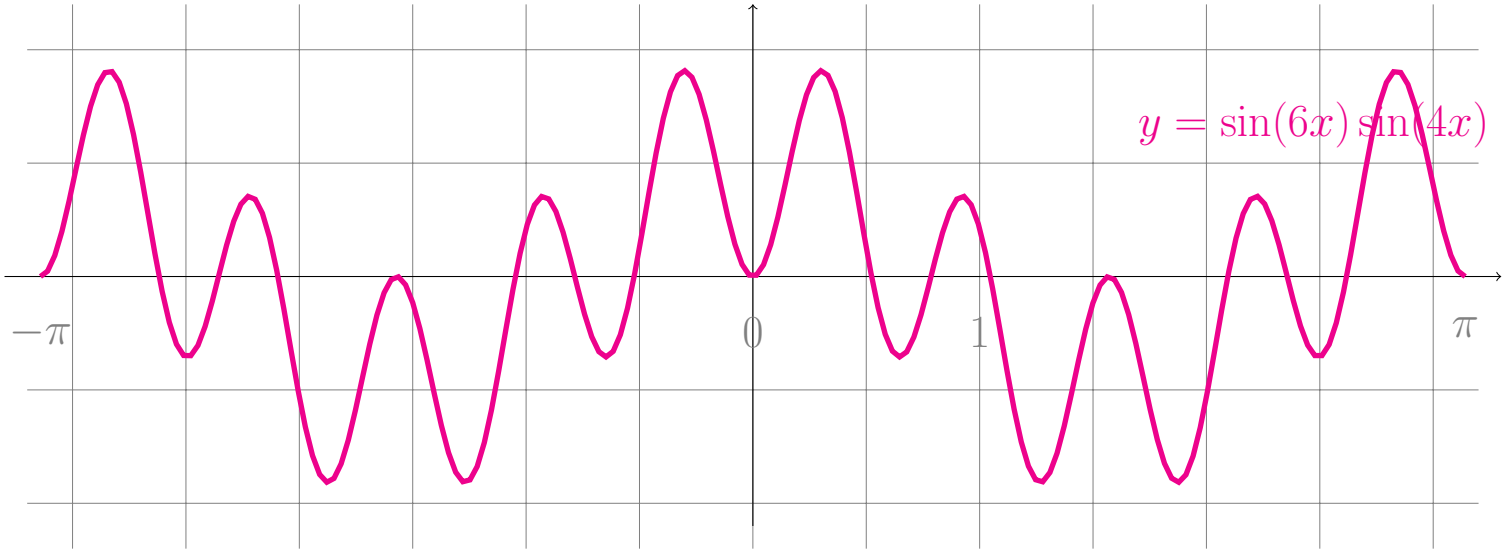
Trigonometric Integrals

1. **Quote.** "I'm an artist; I'm not going to use trigonometry."

(*Taylor Momsen*)



2. **Example.** Integrate $\int \sin^2(x)dx$

(a) Integration by parts. $\int \sin^2(x)dx = \int \underbrace{\sin(x)}_{\uparrow}\underbrace{\sin(x)}_{\downarrow}\, dx$

(b) Using the angle doubling formula $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$:

(c) The results look different, but $\sin(2x) = 2$_____.

3. **Example.** Integrate $\int \cos^2(x)dx$

We could do this all over (either via integration by parts or with $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$**), or use**

$$\underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} = 1.$$

4. **Examples.** Integrate the following:

(a) $\int \sin^2(3x)dx =$

(b) $\int \cot^2(3x)dx =$

5. **Products of Sines and Cosines.**

To evaluate $\int \sin^n x \cos^m x dx$, there are only two possibilities:

(a) At least one of the numbers n and m is **odd**. For example,

$$\int \sin^3 x \cos^2 x \, dx =$$

(b) Both n and m are **even**. For example,

$$\int \sin^2 x \cos^2 x \, dx =$$

6. **Example.** Integrate $\int \cos^5 x \, dx$

7. **Example.** Important definite integrals. $m > 0$, $n > 0$.

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\ dx =$$

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Use:

$$\sin(mx)\sin(nx) = \frac{1}{2}\cdot[\cos((m-n)x) - \cos((m+n)x)]$$
$$\sin(mx)\cos(nx) = \frac{1}{2}\cdot[\sin((m-n)x) + \sin((m+n)x)]$$
$$\cos(mx)\cos(nx) = \frac{1}{2}\cdot[\cos((m-n)x) + \cos((m+n)x)]$$

8. **Integrating Other Trig Functions:** Tangent, Cotangent, Secant, and Cosecant.

(a) $\int \tan x \; dx =$

(b) $\int \cot x \; dx =$

(c) $\int \sec x \; dx =$

(d) $\int \csc x \; dx =$

Notes.