Strategy for Testing Series

1. **Quote.** ”You have to be fast on your feet and adaptive or else a strategy is useless.”

   (Charles de Gaulle, French statesman, 1890-1970)

2. **Quote.** ”Always start at the end before you begin.”

   (Robert Kiyosaki, American businessman, 1947-)

3. **Quote.** ”Failure is nothing more than a chance to revise your strategy.”

   (Anonymous)

4. **Quote.** ”Hope is not a strategy.”

   (Vince Lombardi, American fooball player, 1913-1970)

5. **Quote.** ”The essence of strategy is choosing what not to do.”

   (Michael Porter, American economist, 1947-)

Obviously I did not have a good strategy choosing quotes on strategy.

6. **Given a series, the *Main Question* is: convergent or divergent? .**

To help solve this question, we have the following tests:

(a) Test for Divergence if $a_n \not\to 0$ as $n \to \infty$, then series diverges

(b) If $a_n \geq 0$, then we could use these tests:

- geometric series or p-series
- telescoping series
- integral test
- comparison test
- limit comparison test

(c) If a_n is alternating in sign, try the Alternating Series Test

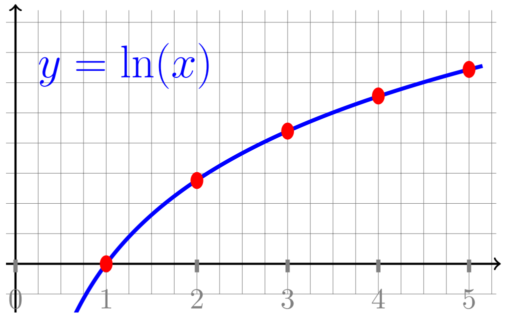
(d) If a_n is any real number, then:

- check absolute convergence
- try the Ratio Test
- try the Root Test

In many of these examples the term $n! = \Gamma(n+1) = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$ (see Assignment 6) appears.

It's useful to have a rough idea of how fast the term $n!$ grows with n, beyond simply listing the first few values, $1, 1, 2, 6, 24, 120, 720, 5040, 40320 \ldots$.

$$\ln(n!) = \sum_{k=1}^{n} \ln k \approx$$



Note, that the precise asymptotic behaviour of $n!$ is given by Stirling's formula

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

7. **Time Machine.** Test for convergence.

(a) **Spring 2002**

i. $\displaystyle\sum_{n=1}^{\infty} n \sin(1/n)$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{n+\sin n}}$

iii. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln \sqrt{n}}$

(b) **Summer 2002**

i. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$

ii. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

iii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$

iv. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(c) **Fall 2002**

  i. $\displaystyle\sum_{n=1}^{\infty}(\arctan(n+1)-\arctan n)$

  ii. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{-3}{\pi}\right)^n$

  iii. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$

(d) **Spring 2003**

  i. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^p}$

  ii. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+1}}$

  iii. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(2n^2+1)^{2/3}}$

  iv. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{3^n-n}$

(e) **Summer 2003**

i. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{1/n}}$

iii. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$

iv. $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$

v. $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{2}-1)^n$

(f) **Fall 2003**

i. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^5+4}}$

iii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n^2+1}$

iv. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+1) 5^n}{n 3^{2n}}$

(g) Spring 2004

i. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(h) Summer 2004

i. $\displaystyle\sum_{n=1}^{\infty} \frac{n^4}{(1+n^2)^3}$

ii. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

iii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-1)!}{2^{2n-1}}$