STAT 485/685 Lecture 10
Fall 2017
12 October 2017

- I discussed the MA(q), AR(p) and ARMA(p, q) processes.
- For
$$Y_t - \phi_1 Y_{t-1} - \cdots - \phi_p Y_{t-p} = \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q}$$
I introduced two polynomials.
- For the AR(p) part the polynomial is
$$1 - \phi_1 x - \cdots - \phi_p x^p$$
- This polynomial has roots $x_1, \ldots, x_p$. A stationary Y exists if and only if every root x_i has $|x_i| > 1$.
- For the MA(q) part the polynomial is
$$1 - \theta_1 x - \cdots - \theta_p x^p$$
- We *choose* to make model *identifiable* by assuming all roots of MA polynomial have $|x_i| > 1$.
- Like last time, I wrote the AR(1) process in two ways: as a General Linear Process and as an autoregression:
$$Y_t - \mu = \theta(Y_t - \mu) + \epsilon_t.$$
- I noted that $|\theta| < 1$ was needed to make this work.
- Then I showed how to convert
$$Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2}$$
into a General Linear Process by factoring
$$1 - \phi_1 x - \phi_2 x^2 = (1 - x/x_1)(1 - x/x_2) = (1 - r_1 x)(1 - r_2 x).$$
- Condition $|x_i| > 1$ becomes $|r_i| < 1$.
- Then use geometric formula to "prove" the algebraic fact that
$$(1 - r_1^k B^k)(1 - r_2^k B^k) Y_t = (1 + \cdots + r_1^{k+1} B^{k+1})(1 + \cdots + r_2^{k+1} B^{k+1}) \epsilon_t$$
- Here BY_t just means Y_{t-1}, $B^2 Y_t = Y_{t-2}$ and so on.
- Since both $|r_i| < 1$ we can let $k \to \infty$ to get
$$Y_t = (1 + \cdots + r_1^k B^k + \cdots)(1 + \cdots + r_2^k B^k + \cdots) \epsilon_t$$

- If we set

$$W_t = (1 + \cdots + r_2^k B^k + \cdots)\epsilon_t = \epsilon_t + r_2\epsilon_{t-1} + r_2^2\epsilon_{t-2} + \cdots$$

  then the sum converges and we find

$$Y_t = W_t + r_1 W_{t-1} + \cdots$$

  gives a formula for Y and shows Y is stationary.

- Then I introduced some non-stationary models: non-stationary mean, random walk, non-stationary autoregressions.
- I showed how a random walk plus linear trend can be differenced to produce a stationary process: constant plus white noise.
- I did quadratic trend and used 2 differences to eliminate the trend and end up with an MA(1) (with a *unit* root).
- I ran a variety of `R` code simulating various processes of form

$$Y_t = \phi Y_{t-1} + \epsilon_t$$

  with ϕ bigger than 1 or near 1.

- I analyzed the `electricity` data set from `TSA` as in the book.
- I took logs and differenced.
- Book says result looks reasonably stationary *but* `monthplot` shows the mean depends strongly on the month.
- The code is here.
- In the text I am working on Chapter 5.
- The midterm is next Thursday; details come on Monday.
- You should be reading all of Chapters 1 through 5.
- Handwritten slides.