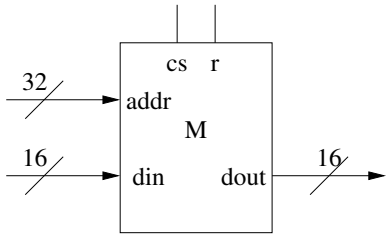
Sample Memory Questions SOLUTIONS

1. Give the behavioural description of a $2^{32} \times 16$ memory.



cs	r	function
0	X	none
1	0	M[addr] $\leftarrow$ din, dout = Z...Z
1	1	dout = M[addr]

2. How many $2^{10} \times 4$ RAM memory chips are required to construct the following larger memories:

   (a) A $2^{16} \times 4$ memory?

   ANSWER: $2^{10} \times 4$ RAM provides 2^{12} bits of storage. $2^{16} \times 4$ memory requires 2^{18} bits.
   Therefore number of RAM's required is $2^{18}/2^{12} = 2^6 = 64$ RAM's.

   (b) A $2^{10} \times 32$ memory?

   ANSWER: $2^{10} \times 32$ RAM requires 2^{15} bits.
   Therefore number of RAM's required is $2^{15}/2^{12} = 2^3 = 8$ RAM's.

   (c) A $2^{12} \times 16$ memory?

   ANSWER: $2^{12} \times 16$ RAM requires 2^{16} bits.
   Therefore number of RAM's required is $2^{16}/2^{12} = 2^4 = 16$ RAM's.

3. A $2^{12} \times 16$ memory system is implemented using a direct mapping strategy, with a blocking factor of 16. The following sequence of <u>blocks</u> of main memory will be retrieved during the execution of a program:

   03, 06, 3F, 0B, 03, 01, 06, 03, 3F, 06

   (a) Assume the cache memory provides storage for 8 blocks and their respective tags. For each distinct block in the retrieval sequence, determine which cache line should be checked and what tag value stored in the cache line is required for a hit to occur. Express your answers in hexadecimal:

block	cache line	tag
03	3	00
06	6	00
3F	7	07
0B	3	01
01	1	00

   (b) Which blocks, if any, cannot be in cache at the same time. Explain your answer.

   ANSWER: Blocks 03 and 0B, since they both have the same cache address (3).

   (c) A cache "miss" can occur in two ways:

      i. When a block if first placed in a previously "empty" cache line.

      ii. When a block replaces one already in a given cache line.

If the number of hits = total number of memory accesses − the number of misses, what is the hit ratio for the program with the sequence of memory requests given?

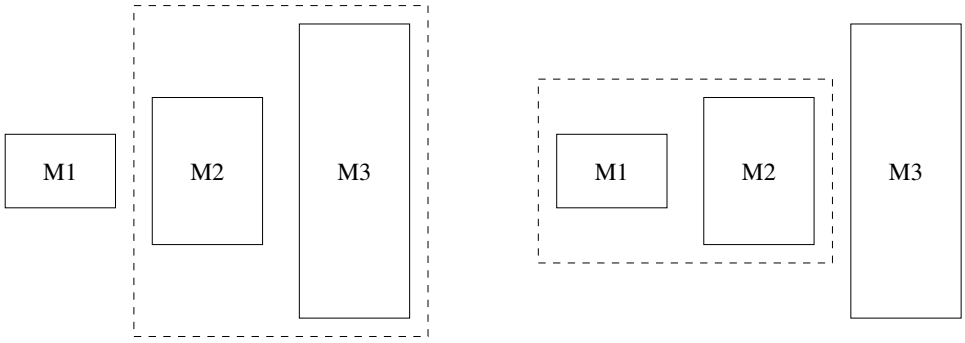
ANSWER:
misses = 4 initial cache line loads + replacement of 03 by 0B, + replacement of OB by 03 = 6
hits = 10 − 6 = 4. hit ratio = 4/10 = 2/5.

Calculate the average memory access time if $t_{sram} = 10$ ns, $t_{dram} = 100$ ns:

ANSWER $t_avg = t_{sram} + (1 - \text{hit-ratio}) * t_{dram} = 10 + 0.6 * 100 = 70$ ns.

4. A design team plans to construct a hierarchical memory system using three types of memory technologies: M1, M2, and M3. M3 will provide the primary (i.e., MAIN) storage. The memory access times satisfy the following inequality: $t_{\mathrm{M1}} < t_{\mathrm{M2}} < t_{\mathrm{M3}}$.

   Two possible architectures are being considered, depending on how the memory components are grouped:



Memory System 1 Memory System 2

   Should either of these architectures be preferred? If so, which one? Support your decision by calculating the average memory access time for a given retrieval, If p is the probability of a hit in M1 and q is the probability of a hit in M2.

   HINT: If the probability of event E_1 occurring is p_1 and the probability of event E_2 occurring is p_2 then the probability of event E_1 or E_2 occurring is $p_1 + p_2 - p_1 \cdot p_2$ (Principle of inclusion/exclusion). **(5 marks)**.

   For memory system 1:

$$\begin{aligned} t_{ms1} &= t_{m1} + (1-p)(t_{m2} + (1-q) \cdot t_{m3} \\ &= t_{m1} + (1-p) \cdot t_{m2} + (1-p)(1-q) \cdot t_{m3} \end{aligned}$$

   For memory system 2:

   Let P be the probability of a hit occurring in the M1-M2 hierarchy.
   By the principle of inclusion/exclusion: $P = p + q - pq$.

$$\begin{aligned} t_{ms2} &= (t_{m1} + (1-p) \cdot t_{m2}) + (1-P) \cdot t_{m3} \\ \text{where } P &= p + q - pq \\ t_{ms2} &= t_{m1} + (1-p) \cdot t_{m2} + (1-p-q+pq) \cdot t_{m3} \\ &= t_{m1} + (1-p) \cdot t_{m2} + (1-p) - q(1-p) \cdot t_{m3} \\ &= t_{m1} + (1-p) \cdot t_{m2} + (1-p)(1-q) \cdot t_{m3} \end{aligned}$$

   Therefore $t_{ms1} = t_{ms2}$ and so the two architectures are equivalent.