Applications of Taylor Polynominals

1. **Quote.** "I have been impressed with the urgency of doing. Knowing is not enough; we must apply."

   (Leonardo da Vinci, Italian polymath, 1452-1519)

2. **Reminder 1.**

   If f has a power series representation at a then

   $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

   and this representation is called the **Taylor series of the function** f **at** a.

3. **Reminder 2 - Taylor's Inequality.**

   If

   $$\left| f^{(n+1)}(x) \right| \leq M \text{ for } |x-a| \leq d$$

   then the remainder of the Taylor series satisfies the inequality

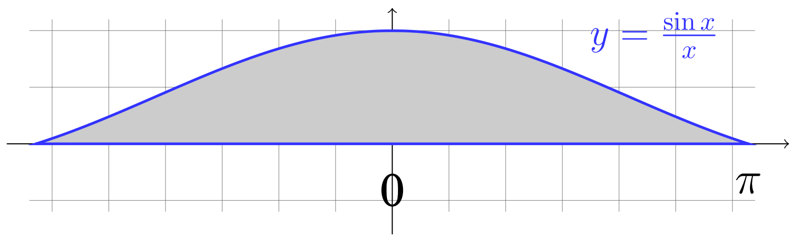
   $$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \text{ for } |x-a| \leq d.$$

4. **Example.**

(a) Approximate $f(x) = x^{2/3}$ by a Taylor polynomial centred at $a = 1$ with degree 3.

(b) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_3(x)$ when $0.8 \leq x \leq 1.2$.

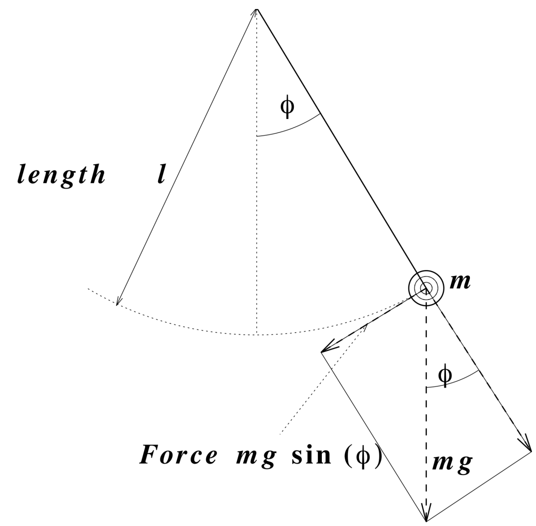
5. **Example.** Approximate the area between the curve $y = \dfrac{\sin x}{x}$ and the x-axis for $-\pi \leq x \leq \pi$.



6. **Example.** A classic example from Physics – the frictionless pendulum.

We consider a pendulum of mass m and length ℓ, and ignore friction. Denote by $\phi(t)$ the angle of displacement at time t, and let $g \approx 9.81ms^{-2}$, the gravitational constant. Newton's law of motion states $\vec{F} = m\vec{a}$, where $\vec{a}$ denotes acceleration. In our case this leads to $m\ell\phi''(t) = -mg\sin(\phi(t))$. Setting $\omega^2 = \frac{g}{l}$ this becomes a differential equation for the function $\phi(t)$,

$$\phi''(t) + \omega^2 \sin(\phi(t)) = 0$$



Taylor approximation: $\sin\phi =$

Let’s try to solve this new differential equation via power series.

Notes.