Math 1171 Midterm 2 Practice Questions

1. Compute the following derivatives. You do not need to simplify your answers.

   (a) $f'(x)$ if $f(x) = (2x^6 - 4x + 3)^4$.

   (b) $g'(x)$ if $g(x) = \dfrac{\sec x}{xe^x}$.

   (c) $h'(x)$ if $h(x) = \dfrac{3 + 2\sin x}{x^3 + 1}$

   (d) y' if $y = x^2 \log_3 \left(x^{2/3}\right)$

(e) $\dfrac{ds}{dt}$ if $s = 2^{t^2}$

(f) $h^{(51)}(t)$ if $h(t) = \ln(t^2)$. (Compute the first few derivatives to find a pattern.)

(g) $\left.\dfrac{dy}{dx}\right|_{x=0}$ if $2\left(\dfrac{x}{y}\right) - \ln(x+y) = 0$

(h) y' if $y = x^{\cos x}$.

2. **True or False.** Justify your answers.

(a) If f and g are differentiable then the derivative of $f(x)g(x)$ is $f'(x)g'(x)$.

(b) The function $f(x) = |x|$ is differentiable for all real numbers.

(c) If f is differentiable, then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{x}}$.

(d) $\frac{d}{dx}(10^x) = x10^{x-1}$.

3. Without computing any derivatives, demonstrate that $f(x) = 2^x + x$ has a point $c \in (0, 3)$ such that $f'(c) = \frac{10}{3}$.

4. Find the linearization of $f(x) = \sqrt[3]{1+3x}$ about $x = 0$. Use it to approximate $\sqrt[3]{1.03}$.

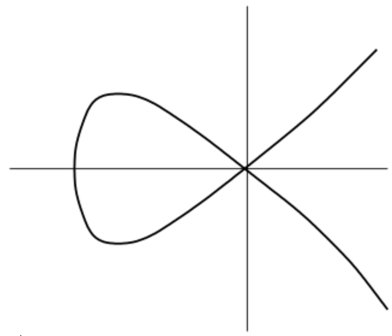
5. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05cm thick to a hemispherical dome with radius 25m.

6. Consider the function $g(x) = (x^2 - 1)^3$. Find the absolute max and absolute min of $g(x)$ on $x \in [-2, 3]$.

7. Use logarithmic differentiation to find the derivative y' of the following function (you do not need to simplify your answer)

$$y = \frac{\sqrt{x^2+1}\,(3-4x)^5}{2(3x-1)^{1/4}\,(x-2)^4}$$

8. Consider the curve defined by $y^2 = x^3 + 5x^2$. The graph of the curve is shown below.



(a) Show that the point $(-1, 2)$ is on the curve.

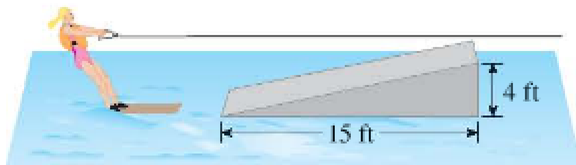
(b) Use implicit differentiation to find $\dfrac{dy}{dx}$.

(c) Find the equation of the tangent line to the curve at the point $(-1, 2)$.

9. A girl facing North is standing next to a river which flows East. She tosses a stick into the water exactly 4 meters North of where she stands. The river carries the stick East at the constant rate of 3 m/s. How fast is the stick moving away from the girl after 2 seconds?
Note - solution is $\frac{9}{\sqrt{13}} \approx 2.5 \ m/s$

10. A waterskier skis over the ramp shown in the figure at a speed of 30 ft/s. How fast is she rising as she leaves the ramp?

Note: Solution is $\frac{120}{\sqrt{241}}$ *ft/s*



11. Suppose we have the parameterizations $x = f(t)$ and $y = g(t)$ on $t \in [0, \infty)$, where

$$f(t) = e^{t-1}, \; g(t) = e^{2t}.$$

Sketch the parametric curve. Include features in your sketch. Then, find the equation of the tangent line at $t = 1$.

12. Consider the parameterizations $x = f(t)$ and $y = g(t)$, where

$$f(t) = t^2 - 1, \;\; g(t) = t^2 + 2t + 2.$$

(a) Find when the function crosses the x-axis, and when it crosses the y-axis.

(b) Find when the function crosses the origin.

(c) Find the point(s) where the tangent line is horizontal, and point(s) where the tangent line is vertical.