CMPT 308 - Computability and Complexity
Homework 1
Due: Sep 22

1. Give the state diagrams for the DFA accepting the following languages. Assume the input alphabet $\Sigma = \{0, 1\}$.
   (a) $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$.
   (b) $\{w \mid w \text{ doesn't contain the substring 110}\}$.
   (c) $\{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$.
2. Argue that the class of regular languages is closed under complementation. That is, show that if L is a regular language over an alphabet Σ, then its complement $\bar{L} = \Sigma^* \setminus L$ is also regular.
3. Give an NFA accepting the language $(01 \cup 001 \cup 010)^*$. Next, convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.
4. For a string $w = a_1 \dots a_n$, its reverse is $w^R = a_n \dots a_1$. For a language L, its reverse is $L^R = \{w^R \mid w \in L\}$. Show that if L is a regular language, then so is L^R.
5. Let $B_n = \{a^k \mid \text{where } k \text{ is a multiple of n}\}$. Show that for each $n \geq 1$, the language B_n is regular.
6. Let $\Sigma = \{0, 1\}$, and let

   $$D = \{w \mid w \text{ contains an equal number of occurrences of the substrings 01 and 10}\}.$$

   (So $101 \in D$, but $1010 \notin D$.)

   Show that D is regular.
7. Let $B = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that B is regular.
8. Show that each of the following languages is *not* regular.
   (a) $\{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$,
   (b) $\{0^n 1^m 0^n \mid m, n \geq 0\}$,
   (c) $\{www \mid w \in \{0, 1\}^*\}$.