LOGICAL AGENTS

CHAPTER 7

Outline

- ♦ Knowledge-based agents
- ♦ Wumpus world
- ♦ Logic in general—models and entailment
- ♦ Propositional (Boolean) logic
- ♦ Equivalence, validity, satisfiability
- \Diamond Inference rules and theorem proving
 - forward chaining
 - backward chaining
 - resolution

Knowledge bases

Inference engine domain-independent algorithms

Knowledge base domain-specific content

Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):

TELL it what it needs to know

Then it can Ask itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

```
function KB-AGENT( percept) returns an action static: KB, a knowledge base t, a counter, initially 0, indicating time  \text{Tell}(KB, \text{Make-Percept-Sentence}(percept, t))   action \leftarrow \text{Ask}(KB, \text{Make-Action-Query}(t))   \text{Tell}(KB, \text{Make-Action-Sentence}(action, t))   t \leftarrow t+1   \text{return } action
```

The agent must be able to:

Represent states, actions, etc.

Incorporate new percepts

Update internal representations of the world

Deduce hidden properties of the world

Deduce appropriate actions

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

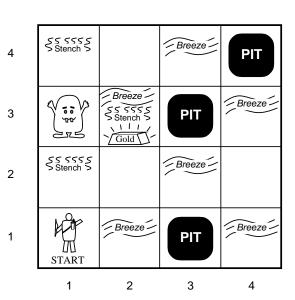
-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly Squares adjacent to pit are breezy Glitter iff gold is in the same square Shooting kills wumpus if you are facing it Shooting uses up the only arrow Grabbing picks up gold if in same square Releasing drops the gold in same square

Actuators Left turn, Right turn, Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell



Observable??

Observable?? No—only local perception

Deterministic??

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic??

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static??

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete??

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??

Observable?? No—only local perception

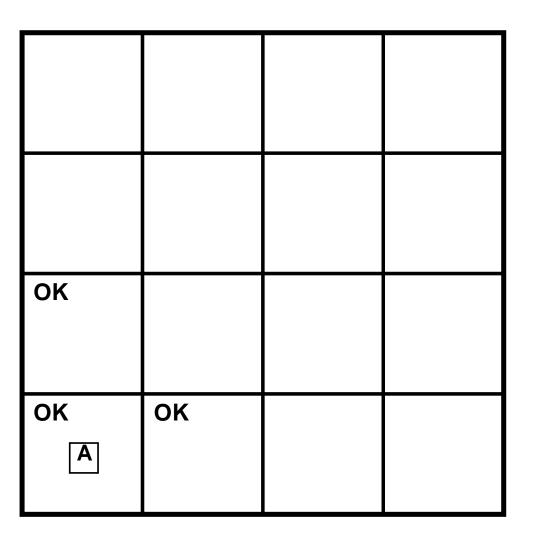
Deterministic?? Yes—outcomes exactly specified

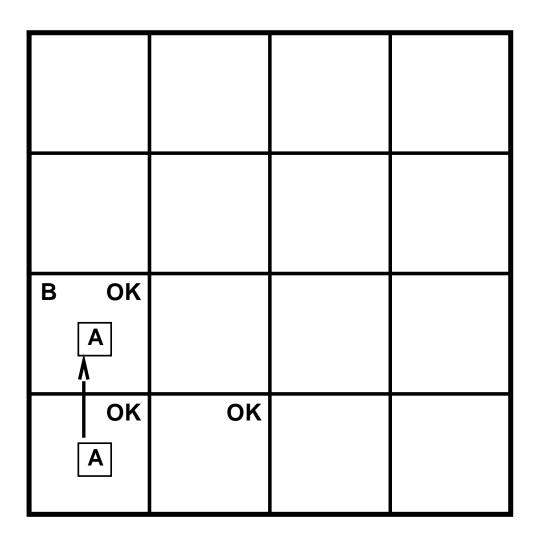
Episodic?? No—sequential at the level of actions

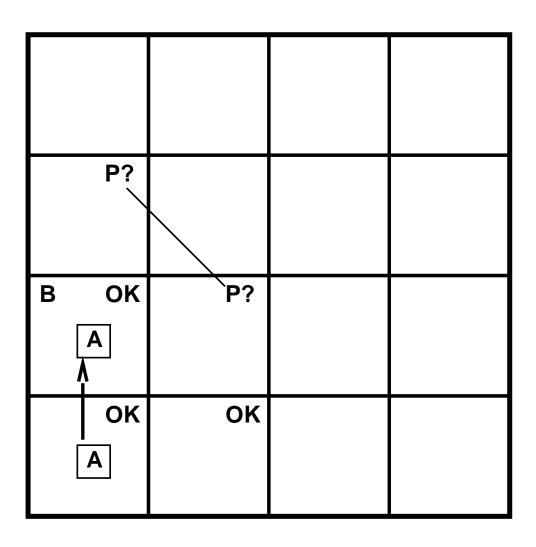
Static?? Yes—Wumpus and Pits do not move

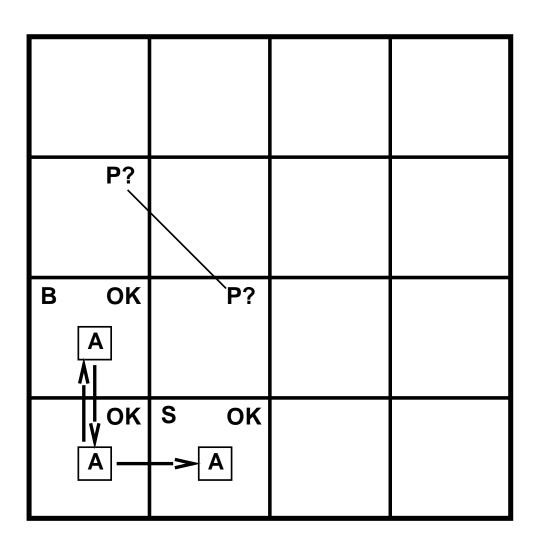
Discrete?? Yes

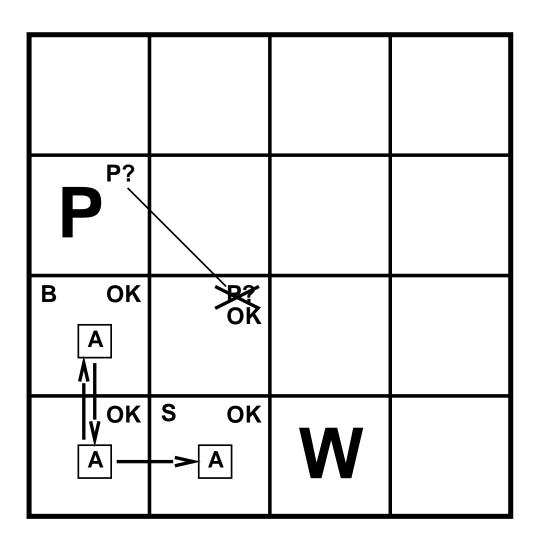
Single-agent?? Yes—Wumpus is essentially a natural feature

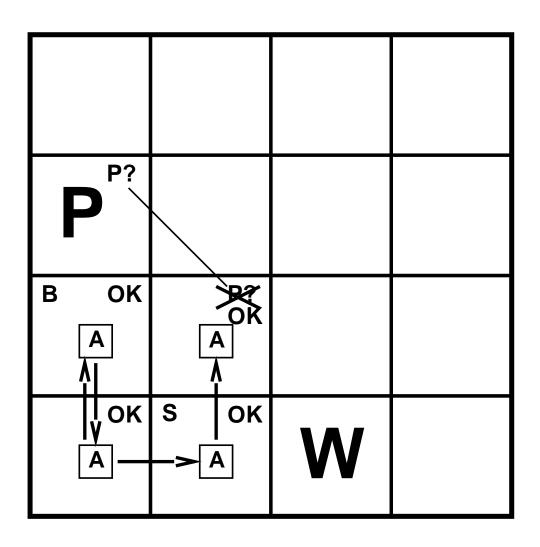


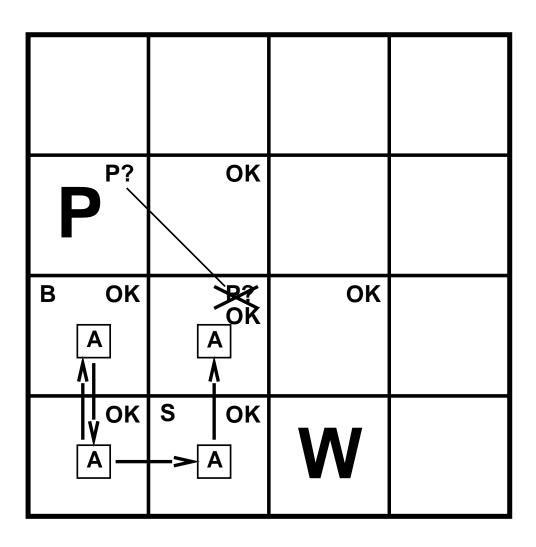


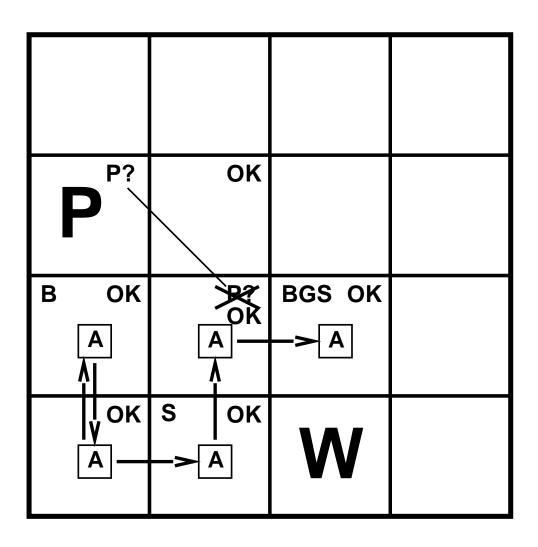




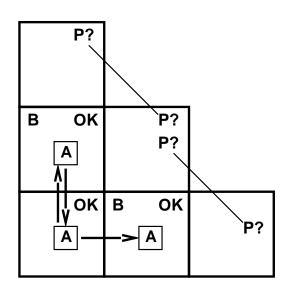






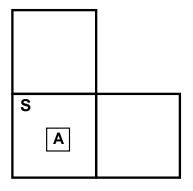


Other tight spots



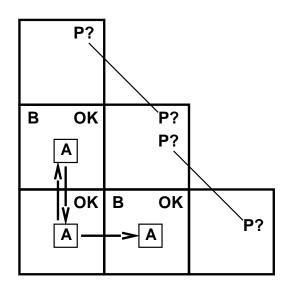
Breeze in (1,2) and (2,1) \Rightarrow no safe actions

Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31



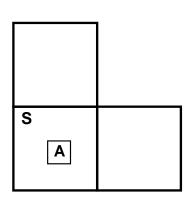
Smell in (1,1) \Rightarrow cannot move

Other tight spots



Breeze in (1,2) and (2,1) \Rightarrow no safe actions

Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1) \Rightarrow cannot move

Can use a strategy of coercion: shoot straight ahead wumpus was there \Rightarrow dead \Rightarrow safe wumpus wasn't there \Rightarrow safe

Logic in general

Logics are formal languages for representing information such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences; i.e., define truth of a sentence in a world

E.g., the language of arithmetic

 $x + 2 \ge y$ is a sentence; x2 + y > is not a sentence

 $x+2 \ge y$ is true iff the number x+2 is no less than the number y

 $x+2 \ge y$ is true in a world where x=7, y=1

 $x + 2 \ge y$ is false in a world where x = 0, y = 6

Entailment

Entailment means that one thing follows from another:

$$KB \models \alpha$$

Knowledge base KB entails sentence α if and only if α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g.,
$$x + y = 4$$
 entails $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Note: brains process syntax (of some sort)

Models

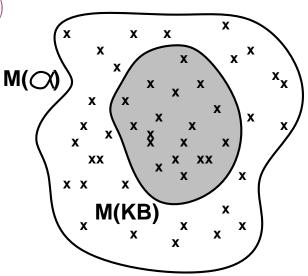
Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

We say m is a model of a sentence α if α is true in m

 $M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. KB= Giants won and Reds won $\alpha=$ Giants won

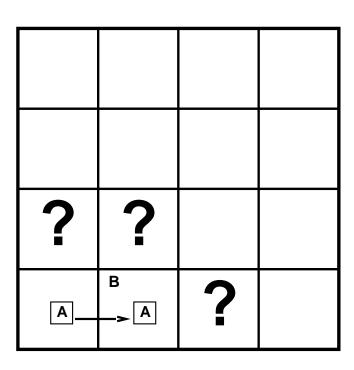


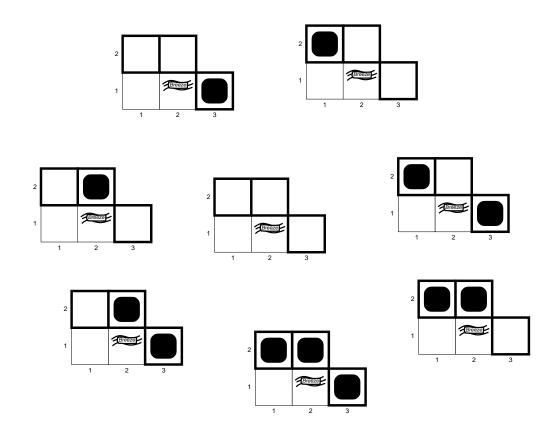
Entailment in the wumpus world

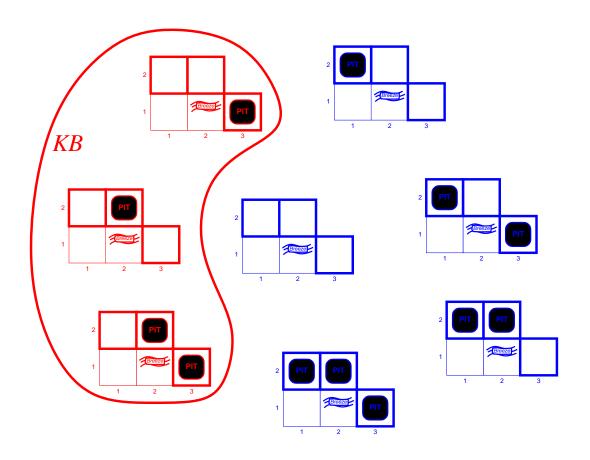
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

Consider possible models for ?s assuming only pits

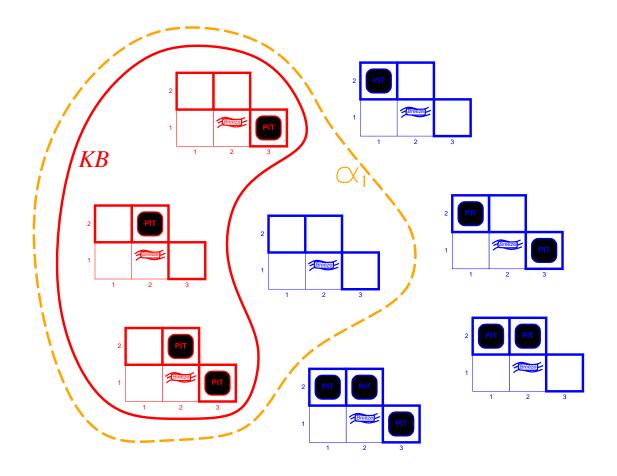
3 Boolean choices \Rightarrow 8 possible models





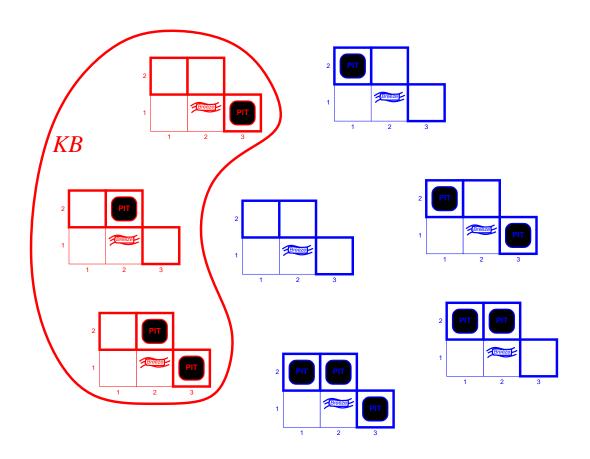


 $KB = {\sf wumpus\text{-}world} \ {\sf rules} + {\sf observations}$

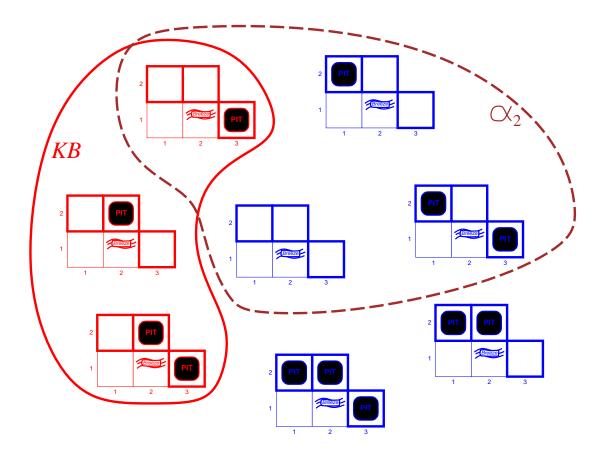


 $KB = {\sf wumpus\text{-}world} \ {\sf rules} + {\sf observations}$

 $\alpha_1=$ "[1,2] is safe", $KB\models\alpha_1$, proved by model checking



 $KB = {\sf wumpus\text{-}world} \ {\sf rules} + {\sf observations}$



KB = wumpus-world rules + observations

$$\alpha_2=$$
 "[2,2] is safe", $KB\not\models\alpha_2$

Inference

 $KB \vdash_i \alpha = \text{sentence } \alpha \text{ can be derived from } KB \text{ by procedure } i$

Consequences of KB are a haystack; α is a needle. Entailment = needle in haystack; inference = finding it

Soundness: i is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1 , P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.
$$P_{1,2}$$
 $P_{2,2}$ $P_{3,1}$ $true true false$

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

```
\neg S
 is true iff S is false S_1 \wedge S_2 is true iff S_1 is true S_1 \vee S_2 is true iff S_1 is true S_1 \vee S_2 is true iff S_1 is false S_2 is true S_1 \Rightarrow S_2 is true iff S_1 is false S_2 is true iff S_1 is false S_2 is false S_1 \Leftrightarrow S_2 is true iff S_1 \Rightarrow S_2 is true S_2 \Rightarrow S_1 is true S_1 \Leftrightarrow S_2 \Rightarrow S_2 is true iff S_1 \Rightarrow S_2 is true S_2 \Rightarrow S_1 is true
```

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \land (P_{2,2} \lor P_{3,1}) = true \land (false \lor true) = true \land true = true$$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i,j]. Let $B_{i,j}$ be true if there is a breeze in [i,j].

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i,j]. Let $B_{i,j}$ be true if there is a breeze in [i,j].

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

 $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

"A square is breezy if and only if there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	true	true	true	true	false	false						
false	false	false	false	false	false	true	true	true	false	true	false	false
:	:	:	:	:	:	:	:	:	:	:	:	:
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	false	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	true	true	true	true	true	true	<u>true</u>
false	true	false	false	true	false	false	true	false	false	true	true	false
:	:	:	:	:	:	:	:	:	:	:	:	:
true	false	true	true	false	true	false						

Enumerate rows (different assignments to symbols), if KB is true in row, check that α is too

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS? (KB, \alpha) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
            \alpha, the query, a sentence in propositional logic
   symbols \leftarrow a list of the proposition symbols in KB and \alpha
   return TT-CHECK-ALL(KB, \alpha, symbols, [])
function TT-CHECK-ALL(KB, \alpha, symbols, model) returns true or false
   if EMPTY?(symbols) then
       if PL-True?(KB, model) then return PL-True?(\alpha, model)
       else return true
   else do
       P \leftarrow \text{First}(symbols); rest \leftarrow \text{Rest}(symbols)
       return TT-CHECK-ALL(KB, \alpha, rest, EXTEND(P, true, model)) and
                  TT-CHECK-ALL(KB, \alpha, rest, Extend(P, false, model))
```

 $O(2^n)$ for n symbols; problem is **co-NP-complete**

Logical equivalence

Two sentences are logically equivalent iff true in same models:

$$\alpha \equiv \beta$$
 if and only if $\alpha \models \beta$ and $\beta \models \alpha$

Validity and satisfiability

A sentence is valid if it is true in all models,

e.g.,
$$True$$
, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$$KB \models \alpha$$
 if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g.,
$$A \vee B$$
, C

A sentence is unsatisfiable if it is true in **no** models

e.g.,
$$A \wedge \neg A$$

Satisfiability is connected to inference via the following:

$$KB \models \alpha \text{ if and only if } (KB \land \neg \alpha) \text{ is unsatisfiable}$$
 i.e., prove α by $reductio~ad~absurdum$

Proof methods

Proof methods divide into (roughly) two kinds:

```
Model checking truth table enumeration (always exponential in n) improved backtracking, e.g., Davis–Putnam–Logemann–Loveland heuristic search in model space, search for model where (KB \land \neg \alpha)
```

satisfied (unsound but complete)

e.g., min-conflicts-like hill-climbing algorithms

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
 Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Horn Clauses

A Horn Clause is a disjunction of literals of which at most one is positive

E.g.,
$$A$$
, $\neg B \lor C$, $\neg D \lor \neg E$

Useful, as they are equivalent to

- ♦ proposition symbol; or
- \Diamond (conjunction of symbols) \Rightarrow symbol; or
- \Diamond (conjunction of symbols) \Rightarrow false

A natural way to describe facts

Forward and backward chaining

A KB is in Horn Form if

KB = conjunction of (restricted) Horn clauses

♦ Restricted to exactly one positive literal

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \dots, \alpha_n, \qquad \alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining. These algorithms are very natural and run in **linear** time

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found

$$P \Rightarrow Q$$

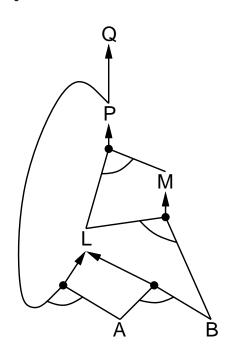
$$L \land M \Rightarrow P$$

$$B \land L \Rightarrow M$$

$$A \land P \Rightarrow L$$

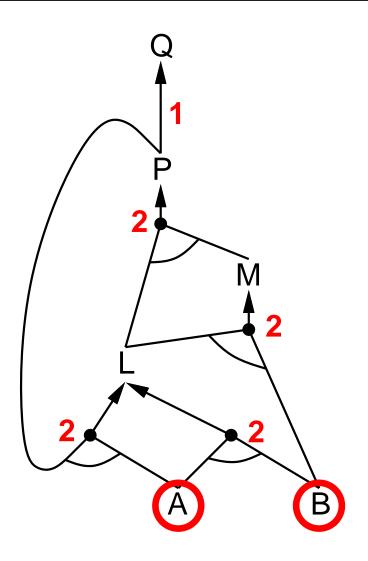
$$A \land B \Rightarrow L$$

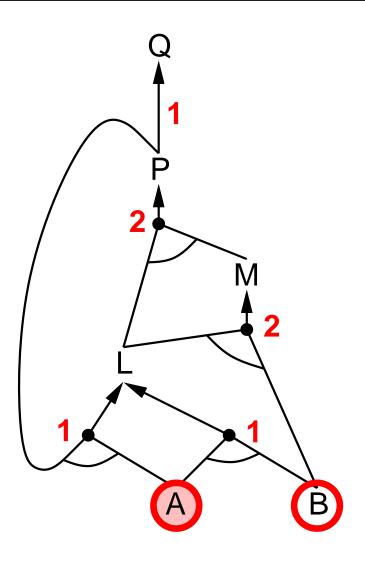
$$A$$

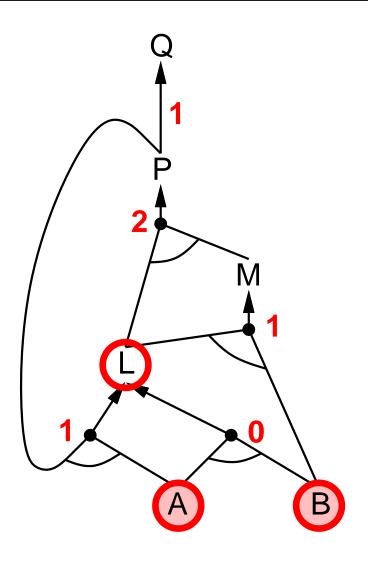


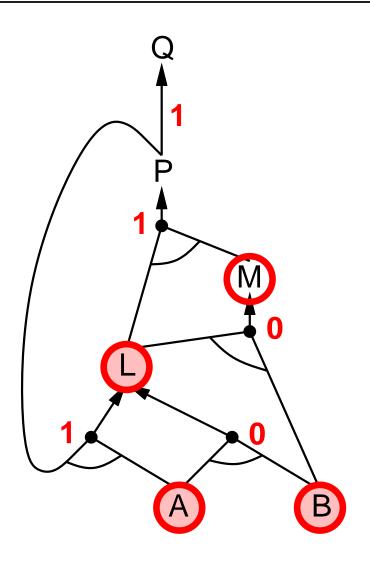
Forward chaining algorithm

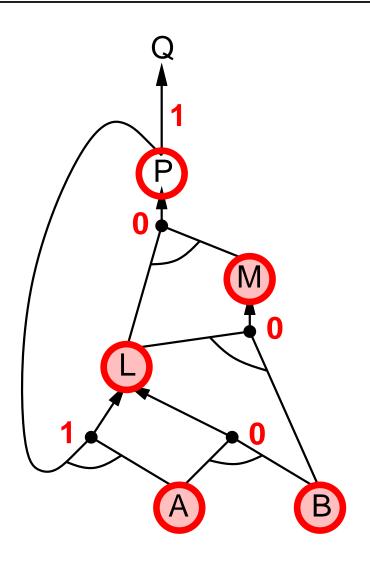
```
function PL-FC-ENTAILS?(KB, q) returns true or false
   inputs: KB, the knowledge base, a set of propositional Horn clauses
            q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, initially the number of premises
                      inferred, a table, indexed by symbol, each entry initially false
                      aqenda, a list of symbols, initially the symbols known in KB
   while agenda is not empty do
       p \leftarrow \text{Pop}(agenda)
       unless inferred[p] do
            inferred[p] \leftarrow true
            for each Horn clause c in whose premise p appears do
                 decrement count[c]
                 if count[c] = 0 then do
                     if HEAD[c] = q then return true
                     Push(Head[c], agenda)
   return false
```

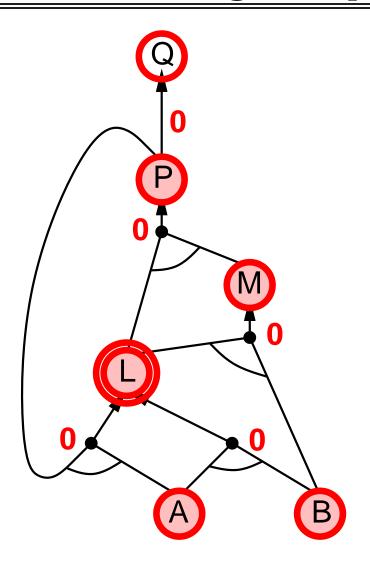


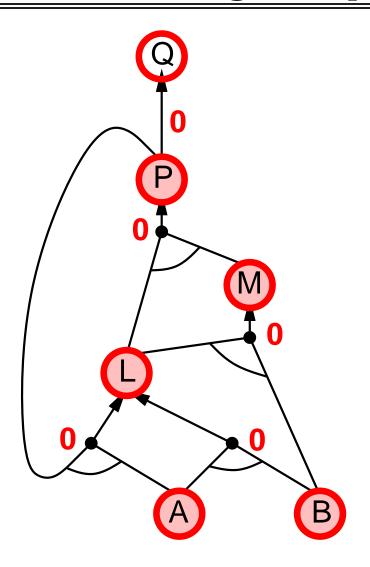


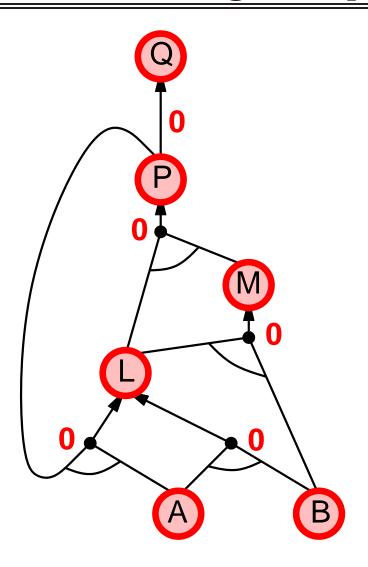












Proof of completeness

FC derives every atomic sentence that is entailed by KB

- 1. FC reaches a fixed point where no new atomic sentences are derived
- 2. Consider the final state as a model m, assigning true/false to symbols
- 3. Every clause in the original KB is true in m **Proof**: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in m Then $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in m Therefore the algorithm has not reached a fixed point!
- 4. Hence m is a model of KB
- 5. If $KB \models q$, q is true in **every** model of KB, including m

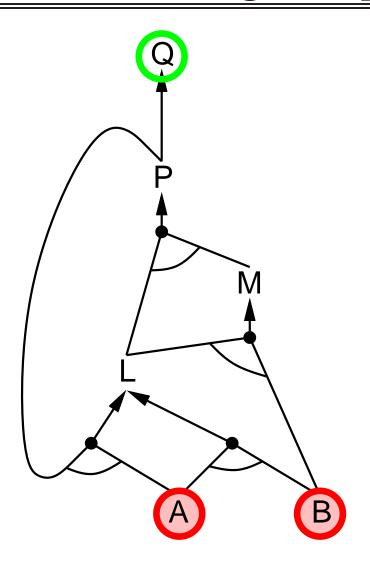
Backward chaining

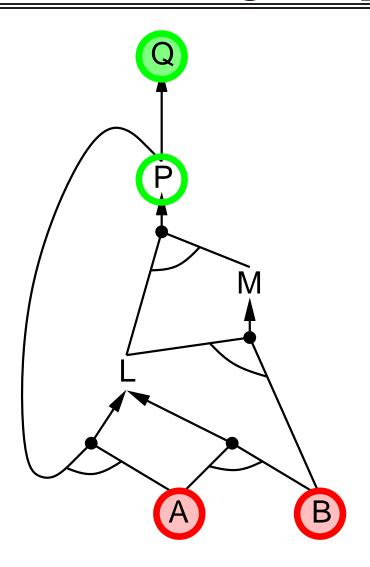
```
Idea: work backwards from the query q: to prove q by BC, check if q is known already, or prove by BC all premises of some rule concluding q
```

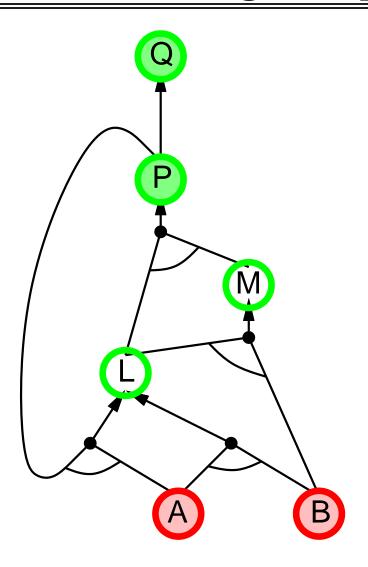
Avoid loops: check if new subgoal is already on the goal stack

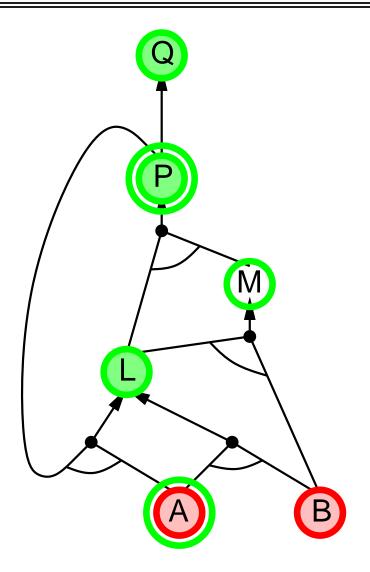
Avoid repeated work: check if new subgoal

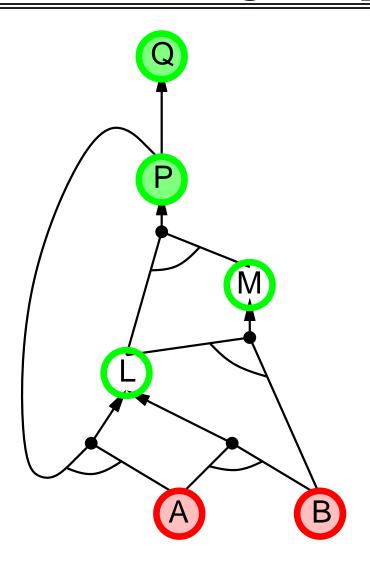
- 1) has already been proved true, or
- 2) has already failed

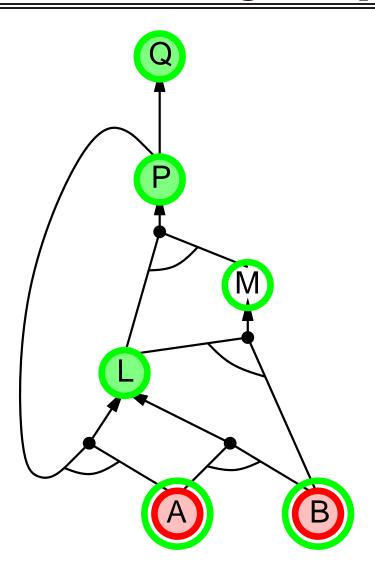


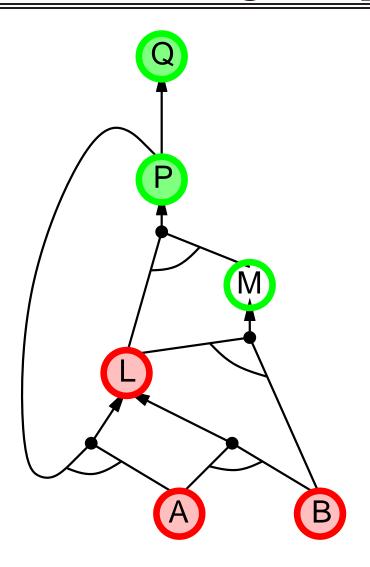


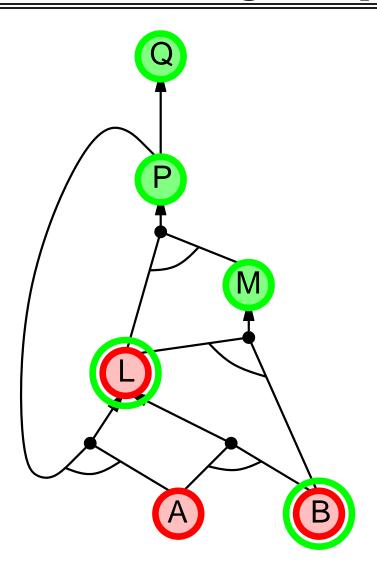


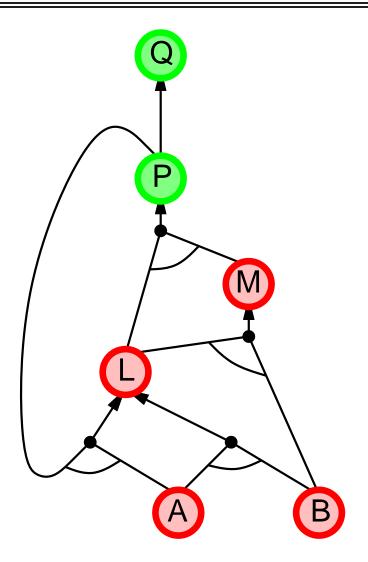


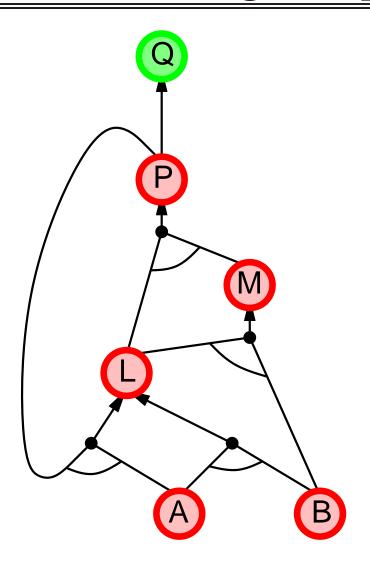


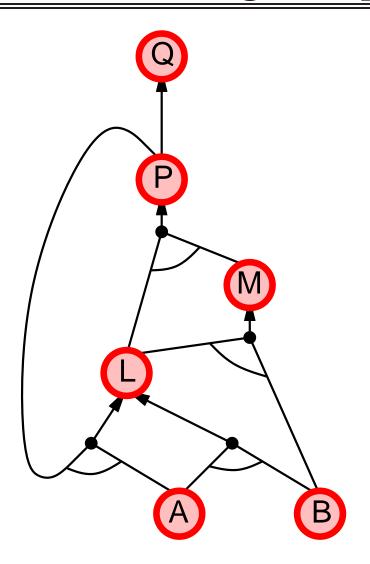












Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing, e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving, e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be much less than linear in size of KB

Resolution

Conjunctive Normal Form (CNF—universal)
conjunction of disjunctions of literals
clauses

E.g.,
$$(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$$

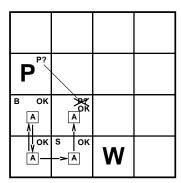
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \quad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for propositional logic



Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \lor \beta$.

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg (P_{1,2} \lor P_{2,1}) \lor B_{1,1})$$

3. Move \neg inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$$

4. Apply distributivity law (\vee over \wedge) and flatten:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$$

Resolution algorithm

Proof by contradiction, i.e., show $KB \wedge \neg \alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, \alpha) returns true or false
inputs: KB, the knowledge base, a sentence in propositional logic
\alpha, the query, a sentence in propositional logic

clauses \leftarrow the set of clauses in the CNF representation of KB \land \neg \alpha

new \leftarrow \{\}
loop do

for each C_i, C_j in clauses do

resolvents \leftarrow \text{PL-RESOLVE}(C_i, C_j)

if resolvents contains the empty clause then return true

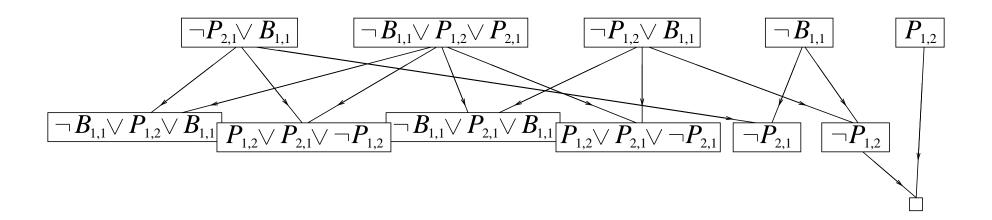
new \leftarrow new \cup resolvents

if new \subseteq clauses then return false

clauses \leftarrow clauses \cup new
```

Resolution example

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \alpha = \neg P_{1,2}$$



Summary

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses Resolution is complete for propositional logic

Propositional logic lacks expressive power