Areas and Distances

The concept of area features prominently in societies with land ownership.

A Section of Land - 640 Acres

80 Chains - 320 Rods - 5280 Feet

Quarter Section

Subdivisions of a quarter section

160 rods - 40 Chains or 2640 Ft

(160 Acres)

Table of Land Measurements

Linear Measure	Square Measure
7.92 inches = 1 link	30 1/4 square yards = 1 square rod
25 links = 16 1/2 feet	16 square rods = 1 square chain
25 links = 1 rod	1 square rod = 272 1/4 square feet
100 links = 1 chain	1 square chain = 4356 square feet
16 1/2 fee t= 1 rod	4840 square yards = 1 acre
5 1/2 yards = 1 rod	640 acres = 1 square mile
4rods = 100 links	1 section = 1 square mile
66 feet = 1 chain	1 Township = 36 square miles
80 chains = 1 mile	1Township = 6 miles square
320 rods = 1 mile	
5280 feet = 1 mile	
1760 yards = 1 mile	

160 rods - 40 Chains or 2640 Ft

20 Chains

(80 Acres)

An Acre is

43,560 square feet
165 feet X 264 feet
198 feet X 220 Feet
5280 feet X 8.25 feet
2640 feet X 16.50 feet
1320 feet X 33 feet
660 feet X 66 feet
330 feet X 132 feet
160 square rods
approx 208 feet 8 1/2 inches square

160 rods - 40 Chains or 2640 Ft

10 Chains

(20 Acres)

40 Rods or 660 feet

(10 Acres)

40 Rods or 660 feet

80 Rods - 20 Chains or 1320 Ft

(40 Acres)

80 Rods - 20 Chains or 1320 Ft

80 Rods - 20 Chains or 1320 Ft

1	31	25	19	13	7	1	31
6	36	30	24	18	12	6	36
5	35	29	23	17	11	5	35
4	34	28	22	16	10	4	34
3	33	27	21	15	9	3	33
2	32	26	20	14	8	2	32
1	31	25	19	13	7	1	31
6	36	30	24	18	12	6	36

6 miles 1920 Rods, 480 chains

NW NW (Lot 4) 40 Acres	NE NW (Lot 3) 40 Acres	NW NE (Lot 2) 40 Acres	NE NE (Lot 1) 40 Acres
SW NW (Lot 5) 40 Acres	SE NW 40 Acres	SW NE 40 Acres	SE NE 40 Acres
NW SW (Lot 6) 40 Acres	NE SW 40 Acres	NW SE 40 Acres	NE SE 40 Acres
SW SW (Lot 7) 40 Acres	SE SW 40 Acres	SW SE 40 Acres	SE SE 40 Acres

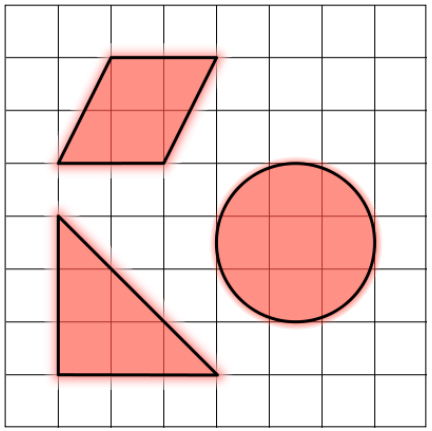
5280 feet, 320 Rods, 80 Chains

Township

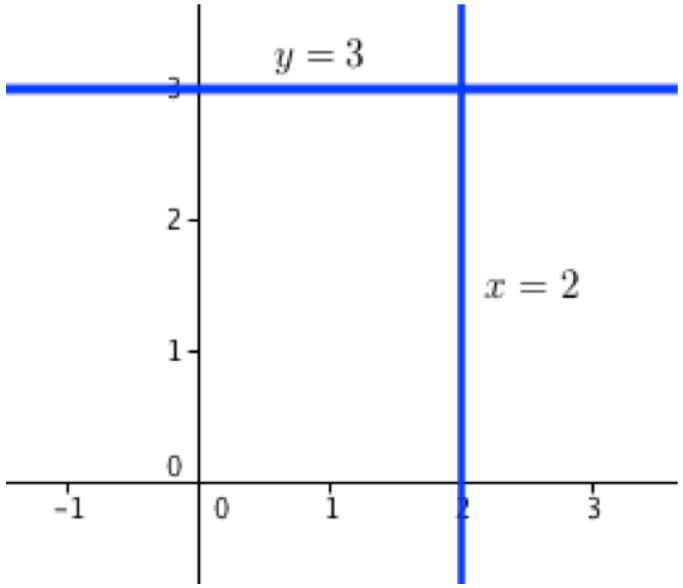
Section

Sectional Map of Townships with Adjoining Sections
A township is 36 square miles (theoretically)
Standard Lots are usually in the north and west sections
of a township and the acreage may be more or less than 40

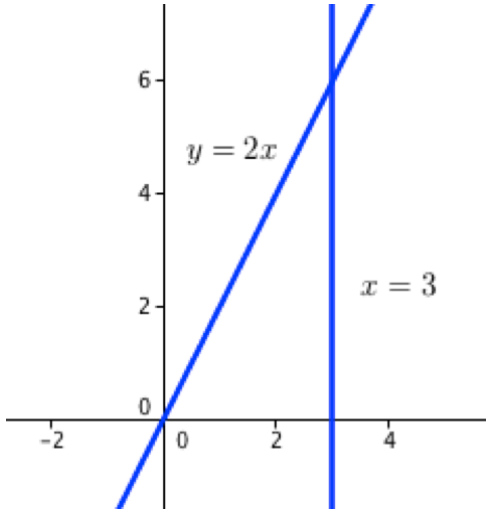
Subdivisions of a Section
A section is 1 mile square (theoretically)



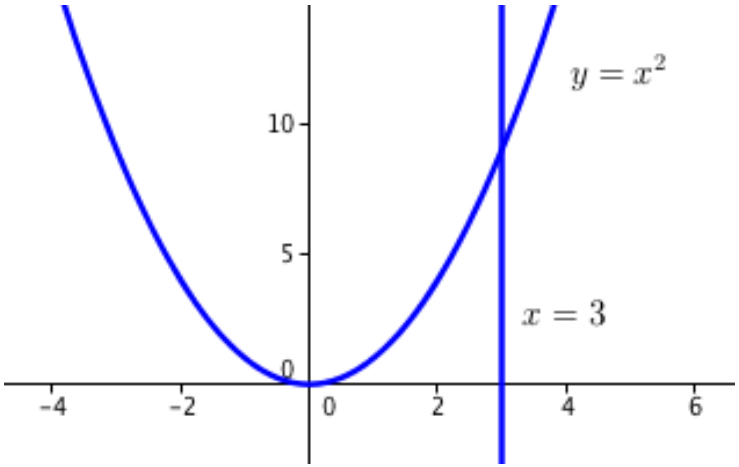
1. **Example.** Find the area of the region in the coordinate plane bounded by the coordinate axes and lines $x = 2$ and $y = 3$.



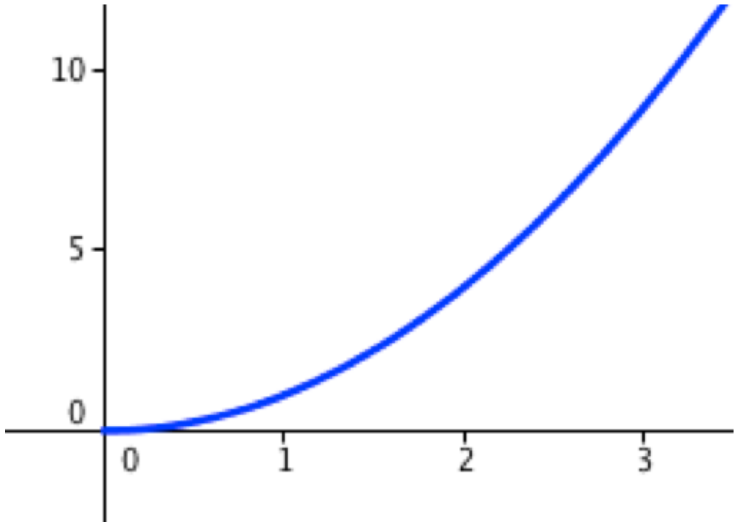
2. **Example.** Find the area of the region in the coordinate plane bounded by the x-axis and lines $y = 2x$ and $x = 3$.



3. **Example.** Find the area of the region in the coordinate plane bounded by the x-axis and lines $y = x^2$ and $x = 3$.



4. **Example.** *Estimate* the area of the region in the coordinate plane bounded by the x-axis and lines $y = x^2$ and $x = 3$.



5. **Example. (Over- and under-estimates.)** In the previous example, show that

$$\lim_{n\to\infty} R_n = 9 \quad \textbf{and} \quad \lim_{n\to\infty} L_n = 9.$$

6. **A more general formulation.**

Ingredients: A function f that is continuous on a closed interval $[a, b]$.

Let $n \in \mathbb{N}$, and define $\Delta x = \frac{b-a}{n}$.

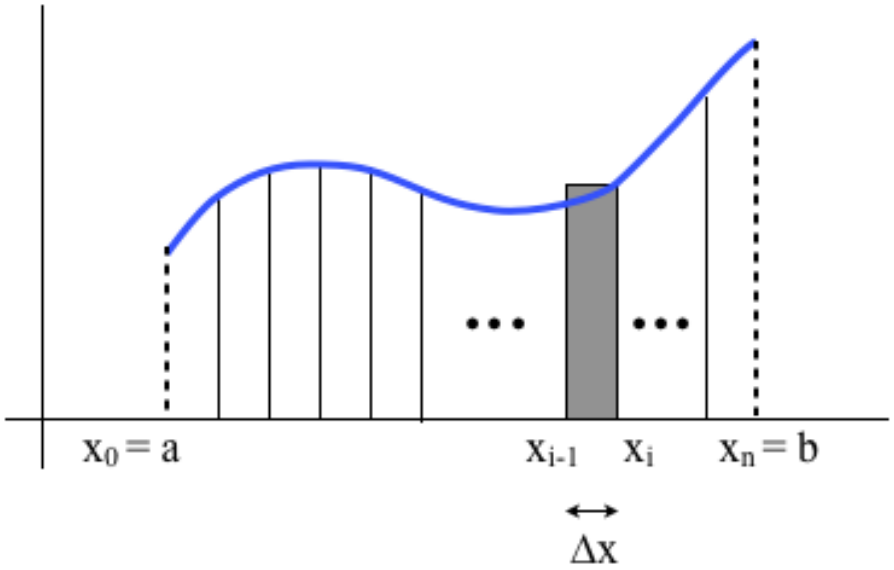
Let

$$\begin{aligned} x_0 &= a \\ x_1 &= a + \Delta x \\ x_2 &= a + 2\Delta x \\ x_3 &= a + 3\Delta x \\ &\vdots \\ x_n &= a + n\Delta x = b. \end{aligned}$$

Define

$$R_n = f(x_1) \cdot \Delta x + f(x_2) \cdot \Delta x + \ldots + f(x_n) \cdot \Delta x.$$

("R" stands for "right-hand", since we are using the right hand endpoints of the little rectangles.)



7. **Definition (Area).** The area A of the region S that lies under the graph of the continuous function f over and interval $[a, b]$ is the limit of the sum of the areas of approximating rectangles R_n. That is,

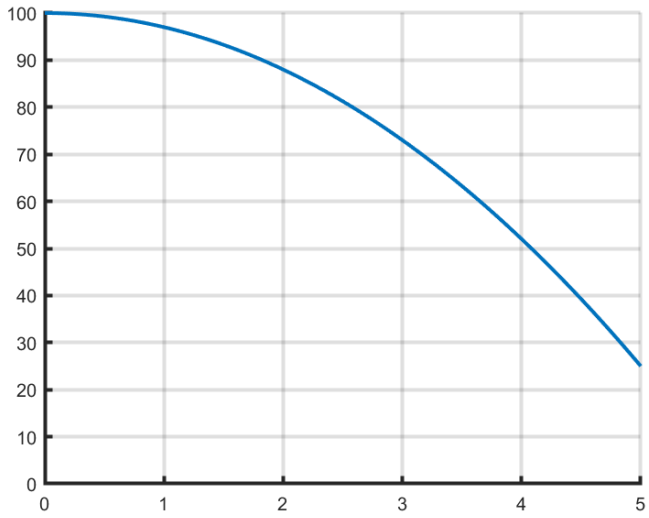
$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1) + f(x_2) + \ldots + f(x_n)]\,\Delta x.$$

The more compact **sigma notation** can be used to write this as

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(x_i) \right) \Delta x.$$

8. **Example.** Find the area under the graph of $f(x) = 100 - 3x^2$ from $x = 1$ to $x = 5$.

From the definition of area, we have $A = \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(x_i) \right) \Delta x$.



9. **Distance Problem.** Find the distance traveled by an object during a certain time period if the velocity of the object is known at all times.

Reminder. **distance = velocity · time**

Estimate the distance traveled during the period of 30 seconds.