STAT 485/685 Lecture 15
Fall 2017
2 November 2017

- I discussed the sample autocorrelation function
$$r_k = \frac{\sum_{t=k+1}^{T}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{T}(Y_t - \bar{Y})^2}.$$
- I reminded you that the acf of an MA(q) is 0 at lags $k > q$.
- The equivalent for AR(p) is provided by the partial autocorrelation function.
- I discussed $\text{Cov}(U, V|W)$: the covariance of U and V adjusted for V. This is a *partial* correlation.
- If U, V, W are Gaussian and have covariance matrix
$$\begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v & \rho_{uw}\sigma_u\sigma_w \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 & \rho_{vw}\sigma_v\sigma_w \\ \rho_{uw}\sigma_u\sigma_w & \rho_{vw}\sigma_v\sigma_w & \sigma_w^2 \end{bmatrix}$$
then we split that 3 by 3 matrix up into
$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$
Then
$$\text{Cov}(U, V|W) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$
- I illustrated this for $U = Y_t$, $V = Y_{t-2}$ and $W = Y_{t-1}$ and found the correlation between Y_t and Y_{t-2} adjusted for Y_{t-1} is
$$\frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$
- This is 0 for an AR(1).
- In general for an AR(p): if $k > p$
$$\text{Corr}(Y_t, Y_{t-k}|Y_{t-1}, \ldots, Y_{t-(k-1)}) = 0$$
In other words the partial acf is 0 for lags $k > p$.
- We estimate the partial acf doing the same arithmetic to the sample acf.
- Then I went back to model comparisons for the electricity data in `TSA`. We saw strong seasonal effects; an interaction between month and time; that differencing at lag 12 could help.
- Then I started a computing demonstration. I used the electricity data from `TSA`. Code is at `R` code.
- Handwritten slides.