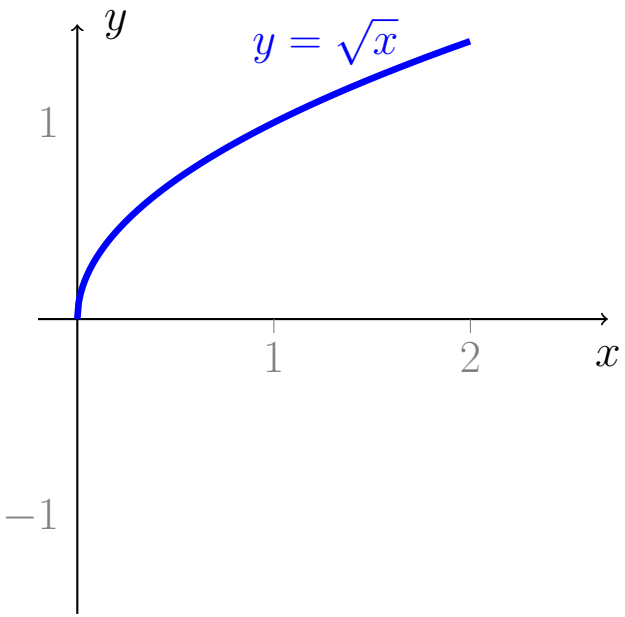
Area of a Surface of Revolution

1. **Lyrics.**

You say you want a revolution
Well, you know
We all want to change the world
.....
You say you got a real solution
Well, you know
We'd all love to see the plan

John Lennon (1940-1980) / Paul McCartney (1942-)

2. **Problem.** Find the surface area of the paraboloid which is obtained by revolving the parabolic arc $y = \sqrt{x}$, $0 \leq x \leq 2$, about the x-axis.



3. **Surface Area.**

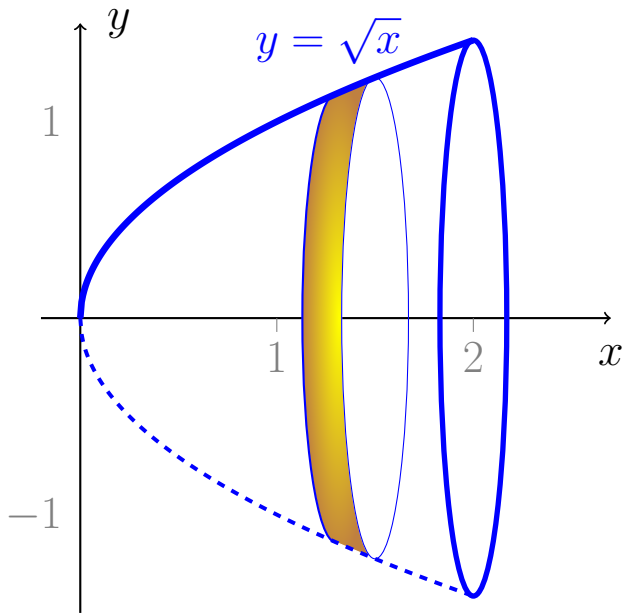
Let a smooth curve C be given by $y = f(x)$, $x \in [a, b]$.

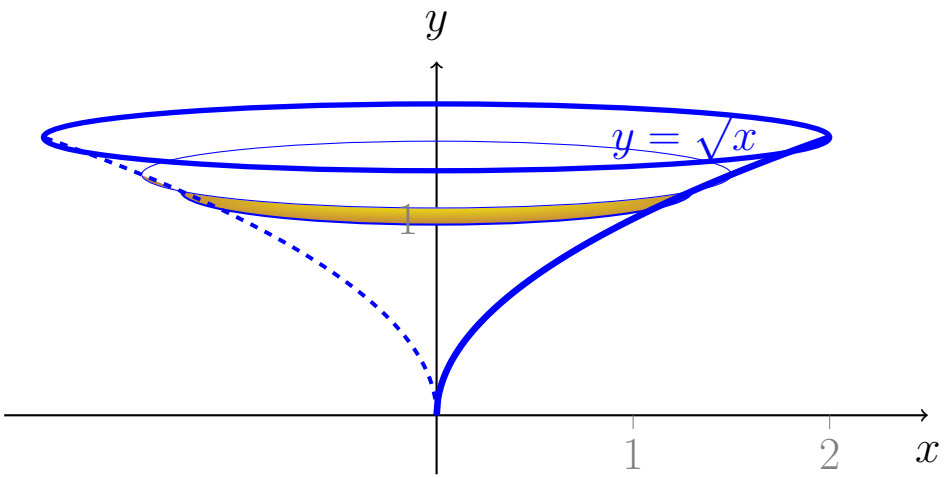
(a) The area of the surface obtained by rotating C about the x-axis is defined as

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'x)]^2}dx.$$

(b) The area of the surface obtained by rotating C about the y-axis is defined as

$$S = \int_a^b 2\pi x\sqrt{1 + [f'(x)]^2}dx.$$





4. **Area formulas for surfaces of revolution.**

Description of curve C	Revolution about x-axis	Revolution about y-axis
$y = f(x)$ $x \in [a, b]$	$\int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx$	$\int_a^b 2\pi x\sqrt{1 + [f'(x)]^2}\, dx$
$x = g(y)$ $y \in [c, d]$	$\int_c^d 2\pi y\sqrt{1 + [g'(y)]^2}\, dy$	$\int_c^d 2\pi g(y)\sqrt{1 + [g'(y)]^2}\, dy$

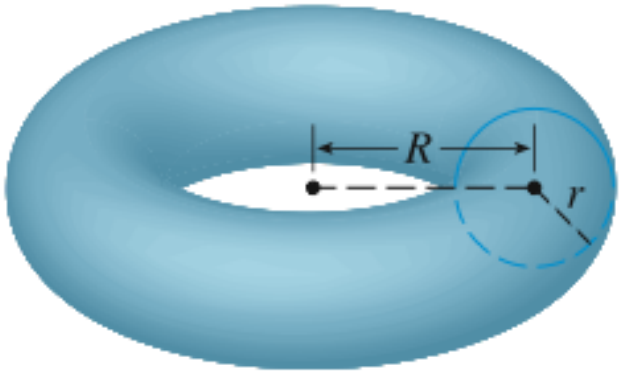
5. **Examples.** Find the area of the surface obtained by rotating the given arc about the corresponding axis.

(a) $y = \sqrt{x}$, $0 \leq x \leq 2$, about the x-axis

(b) $y = x^2$, $0 \leq x \leq \sqrt{2}$, about the y-axis

(c) $y = x^3$, $0 \leq x \leq 2$, **about the** x**-axis**

6. **Example.** Find the surface area of the torus.



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