Learning from Observations

Chapter 18, Sections 1–3

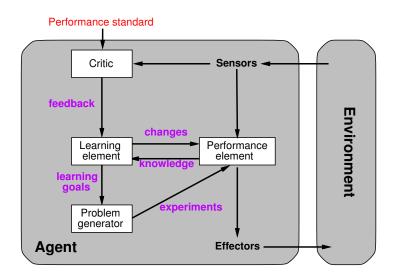
Outline

- Learning agents
- Inductive learning
- Decision tree learning
- Measuring learning performance

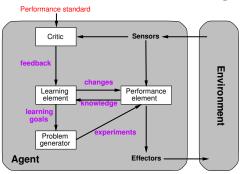
Learning

- Learning modifies the agent's decision mechanisms to improve performance
- Learning is essential for unknown or uncertain environments,
 - I.e., when designer lacks omniscience
- Learning is useful as a system-construction method
 - I.e., expose the agent to reality rather than trying to write it down
 - E.g. neural nets, where learning is essential

Learning agents: General architecture



Learning agents



- Performance element is what we have called the "agent"
- Critic/LearningElement/ProblemGenerator do the "improving"
- Performance standard is fixed
- Learning may require experimentation actions an agent might not normally consider.



- Design of learning element is dictated by
 - what type of performance element is used
 - which functional component is to be learned
 - how that functional component is represented
 - what kind of feedback is available

- Design of learning element is dictated by
 - what type of performance element is used
 - which functional component is to be learned
 - how that functional component is represented
 - what kind of feedback is available
- Example scenarios:

Perf. element	Component	Representation	Feedback	
α/β search	α/β search Eval. function		Win/Loss	

- Design of learning element is dictated by
 - what type of performance element is used
 - which functional component is to be learned
 - how that functional component is represented
 - what kind of feedback is available
- Example scenarios:

Perf. element	Component	Representation	Feedback	
α/β search	Eval. function	Weighted linear function	Win/Loss	
Logical agent	Transition model	Actions	Outcome	

- Design of learning element is dictated by
 - what type of performance element is used
 - which functional component is to be learned
 - how that functional component is represented
 - what kind of feedback is available

Example scenarios:

Perf. element	Component	Representation	Feedback	
α/β search	Eval. function	Weighted linear	Win/Loss	
		function		
Logical agent	Transition	Actions	Outcome	
	model			
Reflex agent	Percept-action	Neural net	Correct	
	function		action	

Learning from Observations: Types of Feedback

- Unsupervised learning:
 - Agent learns patterns in the input; no explicit feedback
 - Example: clustering techniques.

Learning from Observations: Types of Feedback

• Unsupervised learning:

- Agent learns patterns in the input; no explicit feedback
- Example: clustering techniques.

• Reinforcement learning:

- Agent given a series of "rewards" and "punishments"
- Problem: Given a sequence of actions leading to a desired outcome (e.g. winning a game, getting a tip), which actions were responsible?

Learning from Observations: Types of Feedback

• Unsupervised learning:

- Agent learns patterns in the input; no explicit feedback
- Example: clustering techniques.

• Reinforcement learning:

- Agent given a series of "rewards" and "punishments"
- Problem: Given a sequence of actions leading to a desired outcome (e.g. winning a game, getting a tip), which actions were responsible?

• Supervised learning:

- Agent is given examples and their classification
- Requires a "teacher"
- Early example: Winston, with examples and "near misses".



• Induction is basically what science does.

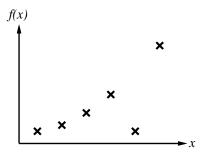
- Induction is basically what science does.
- Simplest form: learn a function from examples
 - f is the target function
 - An example is a pair (x, f(x))
 - E.g.: (customer info, good/bad credit risk?)

- Induction is basically what science does.
- Simplest form: learn a *function* from *examples*
 - f is the target function
 - An example is a pair (x, f(x))
 - E.g.: (customer info, good/bad credit risk?)
- Problem formulation:
 - given a training set of examples
 - find a *hypothesis* h such that $h \approx f$
 - \bowtie h is the "best guess" of f

- Induction is basically what science does.
- Simplest form: learn a *function* from *examples*
 - f is the target function
 - An example is a pair (x, f(x))
 - E.g.: (customer info, good/bad credit risk?)
- Problem formulation:
 - given a training set of examples
 - find a *hypothesis* h such that $h \approx f$
 - h is the "best guess" of f
- This is a *highly* simplified model of real learning:
 - ignores prior knowledge
 - assumes a deterministic, observable "environment"
 - assumes examples are given

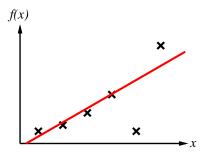
• Construct/adjust h to agree with f on training set

- Construct/adjust h to agree with f on training set
- E.g., curve fitting:



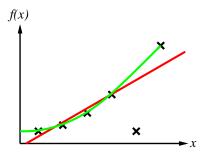
• Q: What's an appropriate function to fit to the known points?

- Construct/adjust h to agree with f on training set
- E.g., curve fitting:



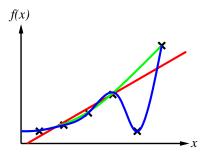
A straight line is the simplest curve.

- Construct/adjust h to agree with f on training set
- E.g., curve fitting:



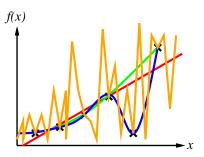
• A nonlinear (e.g. polynomial) function will give a better fit.

- Construct/adjust h to agree with f on training set
- E.g., curve fitting:



• A nonlinear (e.g. polynomial) function will give a better fit.

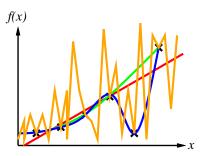
- Construct/adjust h to agree with f on training set
- E.g., curve fitting:



- In fact, any number of functions will fit the data.
- Q: How do we choose between different hypotheses that are consistent with the data?



- Construct/adjust h to agree with f on training set
- E.g., curve fitting:



- Ockham's razor. Choose the simplest hypothesis
 - But defining simplicity isn't easy.

- We'll look at decision tree learning
 - One of the simplest but most successful forms of learning.
 - Idea: Learn $f(\langle attributes \rangle) \rightarrow result$.

- We'll look at decision tree learning
 - One of the simplest but most successful forms of learning.
 - Idea: Learn $f(\langle attributes \rangle) \rightarrow result$.
- Given:
 - A set of attributes (e.g. salary, debt, etc.) and an attribute for classification (e.g. credit risk is high, medium, low).
 - A set of training instances: attributes + outcome.

- We'll look at decision tree learning
 - One of the simplest but most successful forms of learning.
 - Idea: Learn $f(\langle attributes \rangle) \rightarrow result$.
- Given:
 - A set of attributes (e.g. salary, debt, etc.) and an attribute for classification (e.g. credit risk is high, medium, low).
 - A set of training instances: attributes + outcome.
- Construct a tree where:
 - The root is the attribute that discriminates best among the outcomes.
 - I.e. the root is the best *predictor* of the outcome.
 - Descendants are the successive attributes that best discriminate among the remaining instances.

- We'll look at decision tree learning
 - One of the simplest but most successful forms of learning.
 - Idea: Learn $f(\langle attributes \rangle) \rightarrow result$.
- Given:
 - A set of attributes (e.g. salary, debt, etc.) and an attribute for classification (e.g. credit risk is high, medium, low).
 - A set of training instances: attributes + outcome.
- Construct a tree where:
 - The root is the attribute that discriminates best among the outcomes.
 - I.e. the root is the best *predictor* of the outcome.
 - Descendants are the successive attributes that best discriminate among the remaining instances.
- A decision tree is a tree where a node is labelled by an attribute and the edges from the node are labelled with the attribute's values.



Decision tree learning

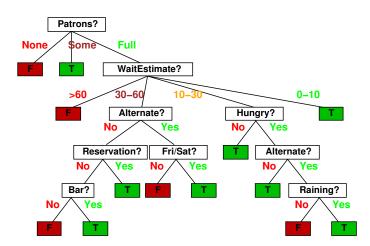
- Examples given by attribute values (Boolean, discrete, continuous, etc.)
 - E.g., situations where I will/won't wait for a restaurant table:

	Attributes							Target:			
Ex.	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Туре	Est	Wait?
X_1	T	F	F	T	Some	\$\$\$	F	T	French	0-10	T
X_2	T	F	F	T	Full	\$	F	F	Thai	30–60	F
X ₃	F	T	F	F	Some	\$	F	F	Burger	0-10	T
X ₄	T	F	T	T	Full	\$	F	F	Thai	10-30	T
X ₅	T	F	T	F	Full	\$\$\$	F	T	French	>60	F
X_6	F	T	F	T	Some	\$\$	T	T	Italian	0-10	T
X ₇	F	T	F	F	None	\$	T	F	Burger	0-10	F
X ₈	F	F	F	T	Some	\$\$	T	T	Thai	0-10	T
X ₉	F	T	T	F	Full	\$	T	F	Burger	>60	F
X ₁₀	T	T	T	T	Full	\$\$\$	F	T	Italian	10-30	F
X ₁₁	F	F	F	F	None	\$	F	F	Thai	0-10	F
X ₁₂	T	T	T	T	Full	\$	F	F	Burger	30–60	Т

• Classification of examples is positive (T) or negative (F)

Decision trees

- One possible representation for hypotheses
- E.g., the author's "personal" tree for deciding whether to wait:



Decision trees

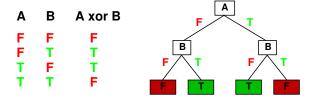
- A decision tree reaches its decision by making a series of tests on an example, beginning at the root.
- Note that all attributes may not be used in the tree.
- An attribute may occur in > 1 place in the tree.
 - An attribute never appears twice along a branch. (Why not?)
- Different attributes may appear at the same depth on the tree
- Issue: There are lots of trees that will classify the same data. How to choose a "good" tree?

Expressiveness

 Decision trees can express any function of the input attributes.

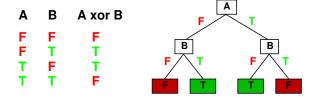
Expressiveness

- Decision trees can express any function of the input attributes.
 - E.g., for Boolean functions, truth table row → path to leaf:



Expressiveness

- Decision trees can express any function of the input attributes.
 - E.g., for Boolean functions, truth table row \rightarrow path to leaf:



- Trivially, there is a consistent DT for any consistent training set, with one path to a leaf for each example
 but it probably won't generalize to new examples
- Prefer to find more *compact* decision trees
- Consider this as a search problem, searching the space of decision trees (next) ...

Hypothesis spaces

• How many distinct decision trees with *n* Boolean attributes?

Hypothesis spaces

How many distinct decision trees with n Boolean attributes?
number of Boolean functions

◆□▶ ◆□▶ ◆■▶ ◆■▶ ■ 900

Hypothesis spaces

- How many distinct decision trees with *n* Boolean attributes?
 - = number of Boolean functions
 - = number of distinct truth tables with 2^n rows

- How many distinct decision trees with *n* Boolean attributes?
 - = number of Boolean functions
 - = number of distinct truth tables with 2^n rows = 2^{2^n}

- How many distinct decision trees with n Boolean attributes?
 - = number of Boolean functions
 - = number of distinct truth tables with 2^n rows = 2^{2^n}
- E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

- How many distinct decision trees with n Boolean attributes?
 - = number of Boolean functions
 - = number of distinct truth tables with 2^n rows = 2^{2^n}
- E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees
- How many purely conjunctive hypotheses (e.g. Hungry ∧ ¬Rain)?

- How many distinct decision trees with n Boolean attributes?
 - = number of Boolean functions
 - = number of distinct truth tables with 2^n rows = 2^{2^n}
- E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees
- How many purely conjunctive hypotheses (e.g. Hungry ∧ ¬Rain)?
- Each attribute can be in (positive), in (negative), or out 3^n distinct conjunctive hypotheses

- How many distinct decision trees with n Boolean attributes?
 - = number of Boolean functions
 - = number of distinct truth tables with 2^n rows = 2^{2^n}
- E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees
- How many purely conjunctive hypotheses (e.g. $Hungry \land \neg Rain$)?
- Each attribute can be in (positive), in (negative), or out 3^n distinct conjunctive hypotheses
- A more expressive hypothesis space
 - increases chance that target function can be expressed (good!)
 - increases number of hypotheses consistent with training set
 - ⇒ makes searching the space harder (bad!)
 - ⇒ may get worse predictions (bad!)

• Aim: Find a small tree consistent with the training examples

- Aim: Find a *small* tree consistent with the training examples
- Idea: Incrementally build a decision tree top down by repeatedly splitting the training data.

- Aim: Find a *small* tree consistent with the training examples
- Idea: Incrementally build a decision tree top down by repeatedly splitting the training data.
- Input is a target attribute, a set of attributes, and a set of examples. (Also optionally a default value.)

- Aim: Find a *small* tree consistent with the training examples
- Idea: Incrementally build a decision tree top down by repeatedly splitting the training data.
- Input is a target attribute, a set of attributes, and a set of examples. (Also optionally a default value.)
- Stop if all examples have the same value (or if you run out of examples).

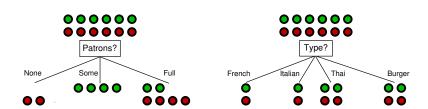
- Aim: Find a small tree consistent with the training examples
- Idea: Incrementally build a decision tree top down by repeatedly splitting the training data.
- Input is a target attribute, a set of attributes, and a set of examples. (Also optionally a default value.)
- Stop if all examples have the same value (or if you run out of examples).
- Otherwise,
 - choose an attribute to split on, and
 - for each of that attribute's values build a subtree for those examples with that attribute value.

- Aim: find a small tree consistent with the training examples
- Idea: (recursively) choose "best" attribute as root of (sub)tree

```
Function DTL(examples, attributes, default) returns a decision tree
 if examples is empty then return default
 if all examples have the same classification
           then return the classification
 if attributes is empty then return Mode(examples)
 best ← Choose-Attribute(attributes, examples)
 tree \leftarrowa new decision tree with root test best
 for each value v; of best do
      examples<sub>i</sub> \leftarrow {elements of examples with best = v_i}
      subtree \leftarrow DTL(examples<sub>i</sub>,attributes – best, Mode(examples))
      add a branch to tree with label v; and subtree subtree
 return tree
```

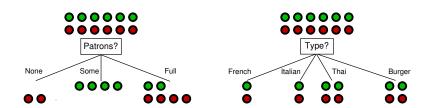
Choosing an attribute

- Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"
- Below, Patrons is a better choice since it gives information about the classification



Choosing an attribute

- Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"
- Below, Patrons is a better choice since it gives information about the classification



• Problem: How to formalise this?

- Information answers questions
- I.e. the more ignorant I am about an answer initially, the more information is contained in the answer

- Information answers questions
- I.e. the more ignorant I am about an answer initially, the more information is contained in the answer
- Information theory measures information content in bits.
- Scale:
 - 1 bit = answer to Boolean question with prior (0.5, 0.5)
- I.e. 1 bit of information is enough to answer a yes/no question about which one has no idea.

- Information answers questions
- I.e. the more ignorant I am about an answer initially, the more information is contained in the answer
- Information theory measures information content in bits.
- Scale:
 - 1 bit = answer to Boolean question with prior (0.5, 0.5)
- I.e. 1 bit of information is enough to answer a yes/no question about which one has no idea.
- Or (the same thing):
 - 1 bit lets you distinguish 2 items.
 - 2 bits lets you distinguish 4 items;
 - n bits lets you distinguish 2^n items.

• The *information* in an answer when the prior is $\langle P_1, \dots, P_n \rangle$ is

$$I(\langle P_1, \ldots, P_n \rangle) = -\sum_{i=1}^n P_i \log_2 P_i$$

- Entropy is a measure of the uncertainty of a random variable.
- A RV with prior $\langle 0.5, 0.5 \rangle$ has lower entropy than one with $\langle 0.25, 0.25, 0.25, 0.25 \rangle$.
- A RV with prior (1.0, 0.0) has no uncertainty
 - Hence it has entropy of 0.

• The *information* in an answer when the prior is $\langle P_1, \dots, P_n \rangle$ is

$$I(\langle P_1, \ldots, P_n \rangle) = -\sum_{i=1}^n P_i \log_2 P_i$$

- Entropy is a measure of the uncertainty of a random variable.
- A RV with prior $\langle 0.5, 0.5 \rangle$ has lower entropy than one with $\langle 0.25, 0.25, 0.25, 0.25 \rangle$.
- A RV with prior $\langle 1.0, 0.0 \rangle$ has no uncertainty
 - Hence it has entropy of 0.
- RV with prior (0.5, 0.5) has entropy 1.0.

• The *information* in an answer when the prior is $\langle P_1, \dots, P_n \rangle$ is

$$I(\langle P_1, \ldots, P_n \rangle) = -\sum_{i=1}^n P_i \log_2 P_i$$

- Entropy is a measure of the uncertainty of a random variable.
- A RV with prior $\langle 0.5, 0.5 \rangle$ has lower entropy than one with $\langle 0.25, 0.25, 0.25, 0.25 \rangle$.
- A RV with prior $\langle 1.0, 0.0 \rangle$ has no uncertainty
 - Hence it has entropy of 0.
- RV with prior $\langle 0.5, 0.5 \rangle$ has entropy 1.0. RV with prior $\langle 0.25, 0.25, 0.25, 0.25 \rangle$ has entropy 2.0.

• The *information* in an answer when the prior is $\langle P_1, \dots, P_n \rangle$ is

$$I(\langle P_1, \ldots, P_n \rangle) = -\sum_{i=1}^n P_i \log_2 P_i$$

- Entropy is a measure of the uncertainty of a random variable.
- A RV with prior $\langle 0.5, 0.5 \rangle$ has lower entropy than one with $\langle 0.25, 0.25, 0.25, 0.25 \rangle$.
- A RV with prior $\langle 1.0, 0.0 \rangle$ has no uncertainty
 - Hence it has entropy of 0.
- RV with prior $\langle 0.5, 0.5 \rangle$ has entropy 1.0. RV with prior $\langle 0.25, 0.25, 0.25, 0.25 \rangle$ has entropy 2.0. RV with prior $\langle 0.95, 0.05 \rangle$ has entropy about .08.

• Suppose we have p +ve and n -ve examples at the root

- Suppose we have p +ve and n -ve examples at the root
- This is an estimate of the probabilities of the possible answers.

- Suppose we have p +ve and n -ve examples at the root
- This is an estimate of the probabilities of the possible answers.
- $I(\langle \frac{p}{p+n}, \frac{n}{p+n} \rangle)$ bits needed to classify a new example

- Suppose we have p +ve and n -ve examples at the root
- This is an estimate of the probabilities of the possible answers.
- $I(\langle \frac{p}{p+n}, \frac{n}{p+n} \rangle)$ bits needed to classify a new example
- I.e.:

$$I(\langle \frac{p}{p+n}, \frac{n}{p+n} \rangle) = -\frac{p}{p+n} \log_2 \left(\frac{p}{p+n} \right) - \frac{n}{p+n} \log_2 \left(\frac{n}{p+n} \right)$$

• E.g., 12 restaurant examples: p = n = 6 so we need 1 bit

• An attribute A with n values splits the training set E into disjoint subsets E_1, \ldots, E_n , according to their values for A.

- An attribute A with n values splits the training set E into disjoint subsets E_1, \ldots, E_n , according to their values for A.
- Let E_i have p_i positive and n_i negative examples

- An attribute A with n values splits the training set E into disjoint subsets E_1, \ldots, E_n , according to their values for A.
- Let E_i have p_i positive and n_i negative examples
 - $I(\langle \frac{p_i}{(p_i+n_i)}, \frac{n_i}{(p_i+n_i)} \rangle)$ bits needed to classify a new example

- An attribute A with n values splits the training set E into disjoint subsets E_1, \ldots, E_n , according to their values for A.
- Let E_i have p_i positive and n_i negative examples
 - $I((\frac{p_i}{(p_i+n_i)}, \frac{n_i}{(p_i+n_i)}))$ bits needed to classify a new example
- The expected number of bits per example over all branches is

Remainder(A) =
$$\sum_{i=1}^{n} \frac{p_i + n_i}{p + n} I\left(\left\langle \frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i} \right\rangle\right)$$

- An attribute A with n values splits the training set E into disjoint subsets E₁,..., E_n, according to their values for A.
- Let E_i have p_i positive and n_i negative examples
 - $I((\frac{p_i}{(p_i+n_i)}, \frac{n_i}{(p_i+n_i)}))$ bits needed to classify a new example
- The expected number of bits per example over all branches is

Remainder(A) =
$$\sum_{i=1}^{n} \frac{p_i + n_i}{p + n} I\left(\left\langle \frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i} \right\rangle\right)$$

• Information Gain (IG) or reduction in entropy from the attribute test:

$$IG(A) = I(\langle \frac{p}{(p+n)}, \frac{n}{(p+n)} \rangle) - Remainder(A)$$

- An attribute A with n values splits the training set E into disjoint subsets E₁,..., E_n, according to their values for A.
- Let E_i have p_i positive and n_i negative examples
 - $I((\frac{p_i}{(p_i+n_i)}, \frac{n_i}{(p_i+n_i)}))$ bits needed to classify a new example
- The expected number of bits per example over all branches is

Remainder(A) =
$$\sum_{i=1}^{n} \frac{p_i + n_i}{p + n} I\left(\left\langle \frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i} \right\rangle\right)$$

• Information Gain (IG) or reduction in entropy from the attribute test:

$$IG(A) = I(\langle \frac{p}{(p+n)}, \frac{n}{(p+n)} \rangle) - Remainder(A)$$

Choose the attribute with the largest information gain.



• For the training set, p = n = 6 and I(6/12, 6/12) = 1 bit.

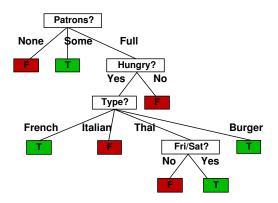
- For the training set, p = n = 6 and I(6/12, 6/12) = 1 bit.
- IG(Patrons) = 1 0.459 = .541

- For the training set, p = n = 6 and I(6/12, 6/12) = 1 bit.
- IG(Patrons) = 1 0.459 = .541IG(Type) = 1 - 1 = 0

- For the training set, p = n = 6 and I(6/12, 6/12) = 1 bit.
- IG(Patrons) = 1 0.459 = .541IG(Type) = 1 - 1 = 0
- Patrons has the highest IG of all attributes and so is chosen by the DTL algorithm as the root.

Example contd.

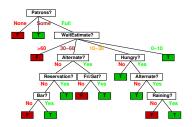
Decision tree learned from the 12 examples:



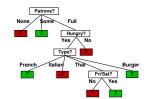
• Much simpler than "true" tree (next slide)

Example contd.

Compare:



VS:

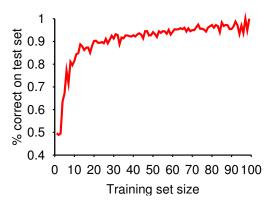


• How do we know if the result of learning is "reasonable"?

- How do we know if the result of learning is "reasonable"?
 - Answer: Test it on other examples

- How do we know if the result of learning is "reasonable"?
 - Answer: Test it on other examples
- Methodology for assessing performance:
 - 1 Collect a large set of examples.
 - 2 Divide into two sets: a training set and a test set.
 - **3** Apply the learning algorithm to the training set.
 - E.g. generate a decision tree
 - Measure the proportion of examples in the test set that are correctly classified.
 - **6** Repeat the above steps for different sizes of training sets and different training sets for each size.
- Intuition: Small training sets will tend to be inaccurate; large training sets will be more work.
 - Try to find a good balance.

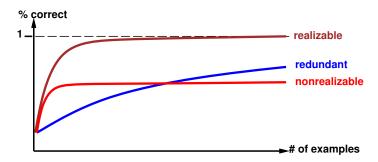
• Plotting the results gives a *learning curve*:



 So big gains when a small training set is increased in size, after which it tapers off.

Performance measurement contd.

- · Learning curve depends on
 - realizable vs. non-realizable learning problem.
 - realizable: can express target function
 - non-realizability can be due to missing attributes or restricted hypothesis class (e.g., thresholded linear function)
 - redundant expressiveness (e.g., loads of irrelevant attributes)



Summary

- Learning needed for unknown environments, "lazy" designers
- Learning agent = performance element + learning element
- Learning method depends on type of performance element, available feedback, type of component to be improved, and its representation
- For supervised learning, the aim is to find a simple hypothesis that is approximately consistent with training examples
- Decision tree learning uses information gain
- Learning performance = prediction accuracy on test sets