Parameter Identification of Racing Vehicle Dynamic Model based on Forgetting Factor-Recursive Least Square

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***Abstract*— The dynamic model plays an important role in automatic driving system. A precise dynamic allows racing car to travel at a higher speed, whereas a normal vehicle dynamic is not accurate enough. In this paper, we design the Tanh tire fitting model, which is more computationally efficient than the traditional empirical model. Besides, based on the bicycle model, static identification and online tracking methods are proposed to identify the model parameters. To implement the algorithm on on-board computer, we introduce the Forgetting Factor Recursive Least Square (FF-RLS) method to reduce the error from constant estimation. The simulation results finally show that the Tanh model well fits the vehicle dynamic for high-speed racing cars, which providing a more accurate dynamic model, while FF-RLS remarkably shortcut the estimation error more than 60%.**

Keywords*— Autonomous Racing, tanh Tire Model, Vehicle Dynamic Identification, Forgetting Factor-RLS

I. INTRODUCTION

Benefit from advanced navigation, prediction and decision-making technology, autonomous driving has developed rapidly in the last decade. Commercial self-driving vehicles such as Waymo[1] has successfully driven on the road and will soon be in people's lives. But the development of autonomous racing technology is just beginning. The Dynamic Design Lab[2, 3] at Stanford has done significant research on car driving decisions and human-computer interaction and has trained cars to drift at extreme speed within a limited track. Roborace, founded in 2017, launched the DEVBOT driverless car[4], which uses sensor fusion technology to automatically drive the car on professional tracks. Moreover, MPC Lab[5] in UC Berkeley focuses on MPC and its improvements to iteratively train racing car on the track.

Although autonomous racing is beginning to take shape, they can only reach the speed up to 80 percent of human experts, so there is still room for improvement. Because the dynamic method of autonomous racing car is mainly borrowed from ordinary vehicles, where there are differences. Therefore, this paper compares the dynamic characteristics of ordinary vehicles and racing cars and identifies the dynamic model of racing cars.

Fig. 1. Dynamic Design Lab Audi and Roborace DEVBOT

A. *Characteristic of Vehicle Dynamic*

The dynamic of ordinary vehicles has been improved over the years. Among them, the most classical bicycle model combines the front and rear wheels, while considering the accuracy of dynamics and the simplicity of mathematical calculation, which is the standard model for vehicle dynamics research. In the bicycle model, both the motion of the body and the lateral force of the tire are calculated. It adopts a simple linear model, that is, the sideslip angle and lateral force have a linear relationship. However, the actual tire saturates[6], and the lateral force will not increase when the sideslip angle reaches saturation. Therefore, the magic formula tire model[7] and tire brush model[8] based on experience fitting were proposed. The two methods can effectively fit the tire force model with verification.

However, the state of a racing car is different from that of an ordinary vehicle, resulting in the particularity of its dynamic. First, the speed of the car is high and change fast. In the track, the average speed of the racing car reaches 150km/h, from 90 to 200 km/h. As the range of velocity is large, the lateral dynamic changes rapidly. Second, the tire usually works in saturation, so the sideslip and lateral force are no longer simply linear. In addition, the lateral stiffness of the tire varies as well due to the hysteresis characteristic caused by the temperature and load change. A fixed parameter cannot truly express the state of the vehicle. Therefore, the dynamic parameters must be updated in real time, otherwise the states cannot be accurately described.

B. Identification Methods

The linear tires adopted in the standard bicycle model are too simplified and do not conform to the real racing tires. Besides, there are too many parameters in the magic formula, which is not suitable for online parameter updating. In addition, the tire brush model[9] requires vertical force and fraction ratio in real that without feasibility. Therefore, a Tanh tire model is proposed in this paper, which can not only accurately express tire dynamic saturation characteristics, but also reduce identification parameters and computational complexity. The model is more suitable to describe racing tire because it has the characteristics of saturation and calculation simplicity.

Tanh model can be identified statically and updated online. Static identification is to make the vehicle run on the predetermined circular track and collect the vehicle motion state data for model identification. On-line updating is to track the changes of model parameters in real time by iteration. In addition, Tanh model identification does not require special sensors, such as tire vertical force information, only according to the vehicle chassis speed and acceleration information, reducing the uncertainty brought by multiple sensors and improving the reliability.

In order to reduce error of lateral force, we adopt forgetting factor recursive least square (FF-RLS) algorithm. This method is not only computational efficient, but also effectively resists the data saturation caused by the increase of data collection, which is very suitable for fast time-varying racing. Even if the speed of the car changes rapidly in a short time, the fitting method can still track the tire force changes to the maximum extent.

The Section 1 of this paper introduces the characteristics of the dynamic model of racing car and main content in this paper. The Section 2 introduces related work about vehicle dynamic model and parameter identification. In the Section 3, we state the expression of Tanh tire model and the static identification results. Besides, the online tracking algorithm FF-RLS of racing car dynamics model is explained. The Section 4 discusses the improvement of Tanh tire dynamics model and its extension. Finally, we summarize the content and contribution of this paper.

II. RELATED WORK

A. Vehicle Dynamic and Identification

Dynamic model for racing car varies according to the different types of accessories. Fortunately, the only one method to manipulate the vehicle is providing force generated by four wheels. Therefore, the essential method to construct an accurate dynamic model should focus on lateral movement and tire performance.

Wesemeier et at. [10] proposed vehicle one track model, which combined multiple unknown parameters and estimated static gain. Russo et al. [11] considered vehicle four wheels dynamic and calculated force on each wheels separately. He adopted Kalman Filter to observe vehicle states. Similarly, J. Kim [12] fitted lateral dynamic model with EKF and analyzed identification of every parameters in the magic formula. He typically took reduce hysteresis area of tire into consideration and adaptively adjusted the saturation according to the vertical force. Akar et al.[13] simultaneously estimated vertical and lateral dynamic model to acquire constantly changing pressure on tire and designed switching roller controller for coupled direction identification. Ahn et al. [14] analyzed the determinant of friction coefficient on different road surface like dry, wet or snow. The identification utilized linear part of tire brush model and considered the interaction with both longitudinal and lateral force.

As we know, the parameter identification for ordinary car with low speed has been almost solved. However, fewer research touched the area of adaptive online identification, especially for high-speed racing cars. Lundquist and Schon [15] proposed gray model dynamic model for longitudinal estimation. They adopted parabolic function for tire lateral model and recursively identified lateral stiffness. Sierra et al.[16] summarize four methods to estimate lateral stiffness, while the hysteretic caused singularity around equilibrium and performed differently at track. Especially, the beta-less algorithm kept stable but react slowly to variation. Hong et al.[17] focused on nonlinear dynamic model to identify the position of center of gravity. They utilized Dual Kalman Filter to reduce the influence vibration on vehicle chassis.

Except for static identification or Kalman Filter, the recursive least square algorithm performs well in unknown system and adjusts adaptively. C.F. Soa et al.[18] proposed gradient based FF-RLS algorithm, which added mean square gradient descend to minimize the error. Leung et al.[19] updated RLS with variable forgetting factor, which renewed the forgetting factor at each sampling. W. Yin and Mehr [20] debated LS identification applied on LPTV system. They analyzed the design of training signal and IIR filter to identify the parameter in the bound.

Hence, we adopted RLS to identify dynamic parameters and adaptively tracked the variation. The identification contains both static experiments and real-time tracking only based on the basic inertial states.

III. DYNAMIC MODEL AND STATIC IDENTIFICATION

A. Dynamics of Bicycle Model

Bicycle model combines two front wheels and two rear wheels. As is shown in **Error! Reference source not found.**, the two front wheels are represented by one wheel at point A and two rear wheels are presented by one wheel at point B.

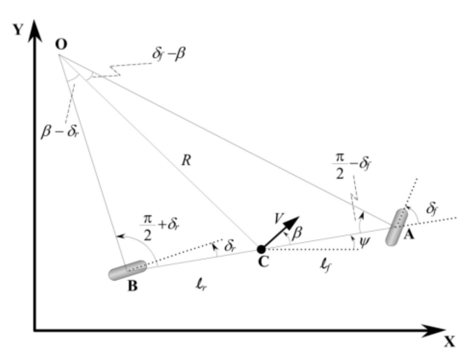

Fig. 2. Bicycle Model

The front and rear steering angle are δ_f and δ_r, while usually cars are front steering and the rear steering angle equals zero. The center of gravity (CG) is at point C and the distance from point A to C is l_f while the distance from point B to C is l_r. The whole wheelbase of the car $L = l_f + l_r$.

In the bicycle model, the vehicle just has planar motion, neglecting vertical movement. Therefore, three degree of freedom (DoF) including X position, Y position and yaw angle ψ are considered, where X and Y are inertial coordinates of the position of CG and the yaw angle ψ represents the orientation of vehicle body. Besides, the velocity at CG is V and the angle between velocity and heading is β, which is called slip angle of the vehicle.

Basically, the lateral model contains both lateral velocity and yaw angular velocity, caused by combination of lateral force and yaw moment. The state-space representation of lateral bicycle model [21] is shown as formula (1).

$$(\dot{V}_y + \dot{\psi} V_x) m = F_f + F_r \tag{1.a}$$

$$I_{zz}\ddot{\psi} = l_f F_f - l_r F_r \tag{1.b}$$

$$F_f = C_{\alpha f}\alpha_f \tag{1.c}$$

$$F_r = C_{\alpha r}\alpha_r \tag{1.d}$$

In the formula (1), $C_{\alpha f}$ and $C_{\alpha r}$ denote lateral stiffness of front and rear tires. I_{zz} represents yaw moment of inertia. The formula (1.a-b) represent lateral movement and yaw spin of chassis, while the formula (1.c-d) mean linear tire model, which can only meet the requirement of small sideslip angle and low velocity. The explicit expression of lateral force F_{y_i} and sideslip angle α_i are expressed in formula (4). As we combine two dynamic function into state-space function, the linear model is shown as formula (2)

$$\frac{d}{dt}\begin{bmatrix} V_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\frac{C_{\alpha f} + C_{\alpha r}}{V_x m} & -V_x - \frac{C_{\alpha f} l_f - C_{\alpha r} l_r}{V_x m} \\ -\frac{C_{\alpha f} l_f - C_{\alpha r} l_r}{V_x I_{zz}} & -\frac{C_{\alpha f} l_f^2 + C_{\alpha r} l_r^2}{V_x I_{zz}} \end{bmatrix} \begin{bmatrix} V_y \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \frac{2C_{\alpha f}}{m} \\ \frac{2C_{\alpha f} l_f}{I_z} \end{bmatrix} \delta_f \tag{2}$$

In the state-space function, the lateral dynamic model is time-varying because of both the changing longitudinal velocity V_x and uncertain lateral stiffness C_{α_i}. If we simply regard C_{α_i} as a constant, the lateral force is extremely inaccurate when sideslip angle reaches saturation. Therefore, we introduce the Tanh tire model to fit the saturating characteristic for racing vehicle.

In this paper, we simulated the racing vehicle by Carsim and Simulink, the worldwide vehicle simulator for dynamic analysis. The basic parameters are shown as TABLE I.

TABLE I. PARAMETERS OF VEHICLE DYNAMIC

Item	Value
Mass (kg)	1207
Wheelbase (m)	2.6
Front Length (m)	1.07
Rear Length (m)	1.53
Moment of Inertia (Nm^2)	2452.4
Height (m)	1.846
Height of CoG (m)	0.54

B. Tanh Tire Model

The Tanh tire model is a model to fit the saturation relationship between sideslip angle and lateral force. Its principle is to use simple hyperbolic tangent function instead of complex magic formula or brush model that requires real-time acquisition of vertical force. The following is the fitting formula of the tanh tire model, shown as formula (3)

$$F_{y_i} = A_i \tanh(k_i \alpha_i) \tag{3.a}$$

$$C_{\alpha_i} = A_i k_i \tag{3.b}$$

Where, F_{y_i} denotes tire lateral force, α_i denotes side slip angle. A_i and k_i are coefficients of tanh tire model and C_{α_i} denotes combined tire lateral stiffness.

In the formula, there are two coefficients determining the lateral force. One is the scale coefficient k_i, which represents the saturation characteristic caused by the increase of sideslip angle and is usually related to the tire type. The other coefficient is the saturate coefficient A_i, which represents the maximum lateral force that can be generated under the current friction and pressure, affected by the speed and acceleration. The product of the two coefficients is defined as the lateral stiffness C_{α_i}, which represents the lateral force relationship in the linear region when the sideslip angle is small (-0.5°~0.5°).

C. Static Identification

The static circular motion test method is used to obtain the static identification for the lateral dynamics model.

First, to drive the vehicle at a constant longitudinal speed, holding the steering wheel at a fixed angle. The vehicle then moves along a fixed circular track for a period and its longitudinal and lateral velocity information is recorded. According to the expression of the bicycle model, the lateral force and sideslip angle can be calculated, as shown in the formula (4)

$$F_{y_f} = \frac{l_r a_y m + l_r \dot{\psi} m V_x + I_{zz}\ddot{\psi}}{L} \tag{4.a}$$

$$F_{y_r} = \frac{l_f a_y m + l_f \dot{\psi} m V_x - I_{zz}\ddot{\psi}}{L} \tag{4.b}$$

$$\alpha_f = \delta_f - \frac{V_y + \dot{\psi} l_f}{V_x} \tag{4.c}$$

$$\alpha_r = -\frac{V_y - \dot{\psi} l_f}{V_x} \qquad (4.d)$$

Where, the expression of sideslip angle and lateral force are applied in the bicycle model. The corresponding lateral force and sideslip angle can be obtained by taking the vehicle dynamic state into the formula. At a constant speed, we gradually change the steering angle of the front wheel (-6°~6°) to make it in the corresponding circular motion. When the lateral force and sideslip angle are calculated, the least square method is adopted to fit the Tanh model. The process is as formula (5)

$$\min_{A_i, k_i} \left\| F_{y_i} - A_i tanh(k_i \alpha_i) \right\|_2 \qquad (5)$$

The fitted Tanh model is shown in the **Error! Reference source not found.** and **Error! Reference source not found.**. There is saturation in the lateral force, and the Tanh model can well simulate the saturation. The linear area is within -1 to 1°of sideslip angle, as is shown by the black line, while the lateral reaches the maximum at 3°. It's easy to find that Tanh model closely represents the trend of saturated lateral force. In **Error! Reference source not found.** the maximum error of F_y is no more than 3%. Therefore, the Tanh model could accurately fit lateral tire dynamic.

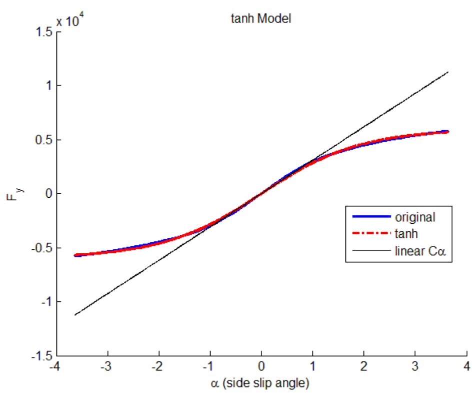

Fig. 3. Fitted Tanh Model

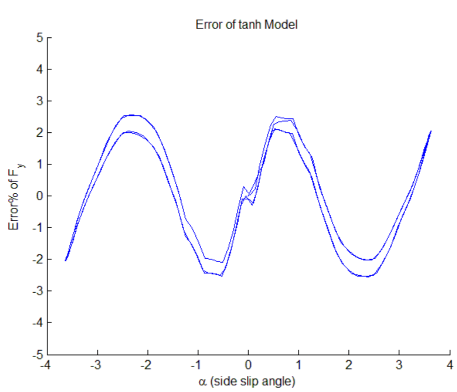

Fig. 4. Error of Tanh Model

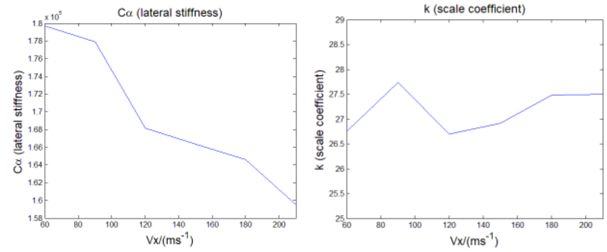

Fig. 5. Coefficient C_{α_i} and k_i at Different Velocity

The **Error! Reference source not found.** shows two coefficients of the Tanh model identified at different velocity. The lateral stiffness C_{α_i} is obviously decreasing as the speed increases, while the scale coefficient k_i roughly remains unchanged. This is because the tire pressure at different speeds and longitudinal forces are different, so that the maximum saturation limit changes. In order to cope with changes in the actual competition, online tracking method is necessary, which is not available in typical magic formulas and brush models. On the contrary, Tanh model well fits the lateral dynamics.

D. Online Tracking with FF-RLS

In the section 3.3, the characteristics of the two fitting coefficients in the Tanh model were analyzed. The variation of the scale parameter k_i was small and only related to the its type. So, it is determined as a fixed constant and the average value is taken in the online tracking. The target of online tracking is to estimate the saturation coefficient A_i during a real race. The greater the lateral force that can be provided, the faster the car will be able to go through the corners, which reduces time to turn. Therefore, it is necessary to keep a fast tracking online.

Forgetting Factor Recursive Least Square (FF-RLS) is proposed to identify the coefficient of saturation A_i and calculate lateral stiffness. FF-RLS adopts iterative update operation, with small calculation amount and fast update speed. The saturation effect of historical data is reduced by introducing forgetting factor. The identification parameters are defined as formula (6)

$$\boldsymbol{y}_N = \begin{bmatrix} F_{y_i}(t_1) \\ F_{y_i}(t_2) \\ \vdots \\ F_{y_i}(t_N) \end{bmatrix}, \theta_N = A_i \, , \qquad (6.a\text{-}b)$$

$$\mathbf{\Phi}_\mathrm{N} = \begin{bmatrix} \tanh\,(k_i \alpha(t_1)) \\ \tanh\,(k_i \alpha(t_2)) \\ \vdots \\ \tanh\,(k_i \alpha(t_N)) \end{bmatrix}, y_N = \mathbf{\Phi}_\mathrm{N} \theta_\mathrm{N} \qquad (6.c\text{-}d)$$

The purpose of identification is to obtain the optimal $\hat{\theta}$ estimation, which minimizes the cost function J as formula (7)

$$\boldsymbol{e} = \boldsymbol{y}_N - \widetilde{\boldsymbol{y}}_N = \boldsymbol{y}_N - \mathbf{\Phi}_\mathrm{N} \hat{\theta}_N \qquad (7.a)$$

$$J = \boldsymbol{e}^T \boldsymbol{e} = \left(\boldsymbol{y}_N - \mathbf{\Phi}_\mathrm{N} \hat{\theta}_N\right)^T \left(\boldsymbol{y}_N - \mathbf{\Phi}_\mathrm{N} \hat{\theta}_N\right) \qquad (7.b)$$

The result of estimation is as formula (8)

$$\hat{\theta}_N = (\mathbf{\Phi}_\mathrm{N}^T \mathbf{\Phi}_\mathrm{N})^{-1} \mathbf{\Phi}_\mathrm{N}^T \boldsymbol{y}_N \qquad (8)$$

Where, matrix is $\mathbf{\Phi}^T \mathbf{\Phi}$ is positive definite.

The least square method can capture static data but is not suitable for online tracking. The state of the car is sampled at a frequency of 60Hz, and large amounts of data are stored over a long period of time. So, the latest state cannot always be tracked. Therefore, the FF-RLS is proposed to deal with the numerous data. Define the state formula (9)

$$\boldsymbol{\Phi}_{N+1} = \begin{bmatrix} \rho\, \boldsymbol{\Phi}_N \\ \Psi_{N+1}^T \end{bmatrix} \tag{9.a}$$

$$\boldsymbol{Y}_{N+1} = \begin{bmatrix} \rho\, \boldsymbol{Y}_N \\ y_{N+1} \end{bmatrix} \tag{9.b}$$

Where, Ψ_{N+1}^T and y_{N+1} are new observed tanh slip angle and lateral force respectively. ρ denotes raw forgetting factor.

After each iteration, the previous state would be attenuated to a certain extent, gradually forgetting the data obtained in the earlier collection. Defined as formula (9), the least squares solution is FF-RLS is shown as formula (10)

$$\hat{\theta}_{N+1} = \hat{\theta}_N + K_{N+1}(\boldsymbol{y}_{N+1} - \Psi_{N+1}^T \hat{\theta}_N) \tag{10.a}$$

$$K_{N+1} = P_N \Psi_{N+1} (\lambda + \Psi_{N+1}^T P_N \Psi_{N+1})^{-1} \tag{10.b}$$

$$P_{N+1} = \frac{1}{\lambda}[P_N - K_{N+1} \Psi_{N+1}^T P_N] \tag{10.c}$$

Initial value

$$P_N = (\boldsymbol{\Phi}_{\mathrm{N}}^T \boldsymbol{\Phi}_{\mathrm{N}})^{-1} \tag{10.d}$$

$$\hat{\theta}_N = (\boldsymbol{\Phi}_{\mathrm{N}}^T \boldsymbol{\Phi}_{\mathrm{N}})^{-1} \Phi_{\mathrm{N}}^T\, \boldsymbol{Y}_N \tag{10.e}$$

Where, K_{N+1} is new weight matrix; P_{N+1} is new square matrix; $\lambda = \rho^2$ denotes forgetting factor in range 0~1, which is usually set within 0.9~1.

In the FF-RLS algorithm, the initial value (8.d-e) need to be set firstly and obtained from the data collected in the static experiment. Then the recursive calculation is performed according to the iterative formula (8.a-c).

Assuming the vehicle is in the track for acceleration through continuous corners and the initial value of the tire model is obtained by static experiment. In the real competition, the front wheel dynamics adopts Tanh model and is updated iteratively by FF-RLS algorithm. The identified coefficient and error are shown as **Error! Reference source not found.** and **Error! Reference source not found.**

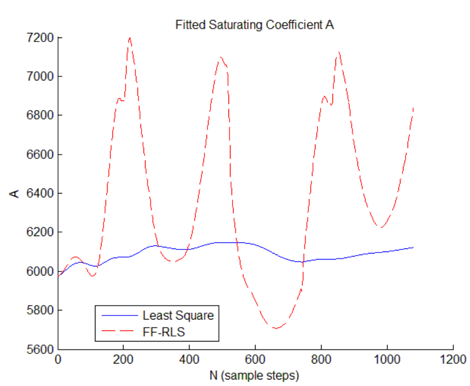

Fig. 6. Fitted Saturate Coefficient A_i

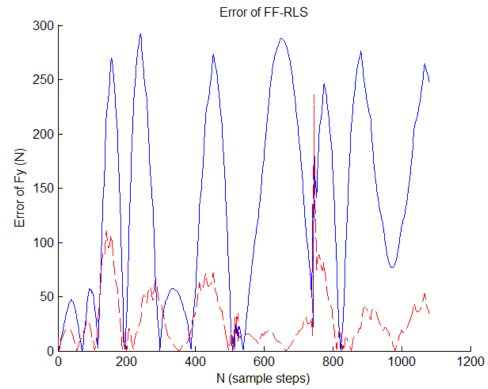

Fig. 7. F_y Error of FF-RLS

The saturated coefficient obtained by FF-RLS updating in the figure is updated quickly and less affected by historical information, so the current state of the system can be identified more accurately. In the lateral force error figure, the simple least square fitting has higher error, while the online tracking FF-RLS can reduce the error. Adjusting forgetting factor can balance the influence of historical data and prevent high frequency vibration caused by saturation effect and overfitting. The average error of RLS (150N) is reduce to only 30~40% of the original value by FF-RLS tracking algorithm (40N).

IV. Discussion

A. *Improvement of Tanh Tire Model*

First, the Tanh tire model is an empirical fit model, which is consistent with the magic formula. But it has a simpler form that's easy to compute. Secondly, there is an assumption simplification in the Tanh tire model, and the scale coefficient k_i is set as a constant, while the actual measurement has a change of 3%-5% at different speeds. However, the variation of the saturation coefficient A_i is more drastic than that of the scale coefficient, and the lateral stiffness in the linear region is multiplied of A_i and k_i. Therefore, tracking with the saturation coefficient can compensate for the slight variation of the scale coefficient to some extent.

The saturate coefficient should be determined by the occupation of longitudinal force F_{x_i}. For the maximum of lateral force is as formula (11)

$$F_{y_{max}} = \sqrt{(\mu g)^2 - F_x^2} \tag{11}$$

The modified Tanh tire model replace the saturate coefficient with maximum of lateral force like formula (12)

$$F_y = B_i F_{y_{max}} \tanh{(k_i \alpha_i)} \tag{12}$$

Where, B_i is the damping coefficient of maximum lateral force. When the racing car accelerates or decelerates, the slip ratio produces longitudinal force, which diminishes available friction. As we know, the modified model couples longitudinal and lateral dynamic, we need to consider the influence from longitudinal force.

B. *Longitudinal Fitting of Tanh Model*

The Tanh tire model can be used not only in lateral force fitting, but also in longitudinal force fitting. The model of longitudinal force consists of tire slip ratio and longitudinal force. The slip ratio κ_i is defined as the ratio of deviation between wheel speed and vehicle speed. As formula (13) shows,

$$\kappa_i = \frac{V_x - \omega_i r_{eff}}{V_x} \tag{13}$$

Where, ω_i denotes wheel angular velocity; r_{eff} represents effective radius of wheel. The fitting formula (14) of longitudinal force is

$$F_{x_i} = A_{x_i} \tanh\left(k_{x_i} \kappa_i\right) \tag{14}$$

The method and principle of fitting are the same lateral dynamic. The difference is that linear acceleration and deceleration experiments are used instead of circular motion experiments for static identification. The result of identification is shown in **Error! Reference source not found.**.

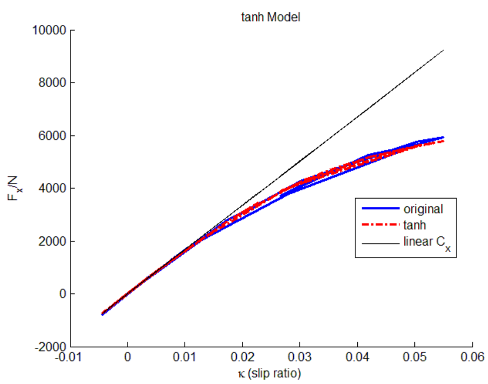

Fig. 8. Longitudinal Tanh Model

The longitudinal force dynamics is similar to the lateral force dynamics, and there is a saturation relationship between the longitudinal force of the tire and the slip ratio. So, we can use the Tanh model to fit the longitudinal tire model.

However, the longitudinal force is not only determined by the slip ratio, but also by the friction coefficient and the tire viscosity effect. At the same time, the wheel speed and effective radius are intermediate variables, which are not easy to measure. Besides, it is difficult to calculate the pure longitudinal force provided by the tire directly with the disturbance of rolling and wind resistance. Therefore, simply using Tanh model for longitudinal force fitting is not as effective as lateral dynamics in actual competition.

V. Conclusion

As the unknown dynamic model of racing vehicle impedes the development of autonomous racing, parameter identification for the complex time-varying dynamic model becomes an essential task. the In order to identify the vehicle dynamic parameter for the racing vehicle, we propose Tanh tire model to fit the saturation with large sideslip angle. By analyzing the bicycle model, we point out the weakness of linear model or complex empirical model like magic formula. Then we design static circle experiments to measure the scale and saturate coefficients at different velocity. Besides, the FF-RLS algorithm for online tracking is presented to adaptively catch up with the changing lateral stiffness in the real match. Finally, we discuss the improvement of Tanh model and the extension on the longitudinal dynamics. From the simulation and experiments by professional vehicle simulator Carsim, the Tanh model well fits the lateral dynamic and consumes less computation. Meanwhile, the FF-RLS algorithm helps tracking the changing parameters, which can be easily applied to the vehicle on-board computer.

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