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Distributed consensus extended Kalman filter: a variance-constrained approach

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Abstract: This study is concerned with the distributed state estimation problem for non-linear systems over sensor networks. By using the strategy of consensus on prior estimates, a distributed extended Kalman filter (EKF) is developed for each node to guarantee an optimised upper bound on the state estimation error covariance despite consensus terms and linearisation errors. The Kalman gain matrix is derived for each node by solving two Riccati-like difference equations. It is shown that the estimation error is bounded in mean square under certain conditions. The effectiveness of the proposed filter is evaluated on an indoor localisation of a mobile robot with visual tracking systems.

1 Introduction

Distributed estimation over networks has received increasing attention in the signal processing and control communities [1–3]. The aim of distributed estimation is to estimate some parameters of interest from noisy measurements through cooperation between nodes. Consensus has been a widely exploited tool for distributed computations in a scalable way [4–7]. In the consensus-based algorithms, each node in a network shares its information with its immediate neighbours and corrects its own state using the information sent by its neighbours. A large number of works on consensus-based distributed estimation have been reported since they can drastically reduce the utilisation of communication resources [8].

In the past decade, consensus-based distributed Kalman filters (DKFs) have been developed to estimate the state of a dynamical system given measurements provided by a sensor network [9–19]. Most of these works introduce a consensus scheme in a standard Kalman filter, and they can be broadly classified into four categories: consensus on estimates (CE) [12], consensus on measurements (CM) [16], consensus on information (CI) [17] and consensus on pseudo-measurements (CPM) [18]. CE was proposed based on the idea of spreading the available information over the network by performing a consensus averaging of the local state estimates. For instance, a suboptimal Kalman consensus filter (KCF) has been proposed by adding a consensus term of predicted estimates in the standard Kalman filter [12]. The main disadvantage of the suboptimal KCF is that the covariance information is not used in a distributed way. Specifically, the covariance update algorithm of the suboptimal KCF is the same as a standard Kalman filter, although a consensus term has been introduced in a standard Kalman filter. To overcome such limitations, CM has been proposed by implementing consensus on local measurements and innovation covariances in a standard Kalman filter [16]. It should be pointed out that CM requires a large number of times to come to a consensus on the global average of the measurements and innovation covariances, which yields high communication costs. Moreover, CM does not guarantee stability unless the number of consensus steps is sufficiently high [19]. To overcome this problem, the CI was proposed to implement the consensus on information vectors and information matrices [17]. Thus, the CI can be considered as an application of the covariance intersection to distributed estimation. From an information-theoretic viewpoint, the CI can also be interpreted as consensus on probability density functions in the Kullback–Leibler average sense. To develop distributed estimation of unstable dynamic systems, the CPM has been developed by defining a set of pseudo-measurements [18]. It has been shown that the DKF with CPM is asymptotically unbiased with bounded mean-squared error. To improve the estimation performance, hybrid consensus schemes have been proposed by combining CM and CI [19]. Recently, consensus-based distributed extended Kalman filter (EKF) have been proposed for non-linear systems in [20, 21]. Following the lines of the DKF [19], the hybrid CMCI approach has been used in the EKF and it has been shown that the estimation error is bounded under certain conditions. However, the linearisation errors of the EKF are not addressed in the distributed EKF.

In this paper, we attempt to apply the CE strategy to develop a distributed consensus EKF for discrete-time non-linear systems over a network. The proposed filter is derived for each node by applying consensus on prior estimates in the EKF. It should be pointed out that the proposed distributed consensus EKF is not a direct extension of the suboptimal KCF for linear systems in [12]. Instead of neglecting the edge-covariances in [12], the variance-constrained approach is used to provide an upper bound matrix for the state estimation error covariance where the edge-covariances have been bounded by their individual covariances. Moreover, the linearisation errors have been addressed by using the variance-constrained approach. The Kalman gain matrix is determined for each node by minimising the trace of the upper bound matrix. A key contribution of this paper is not only to propose a variance-constrained distributed EKF for non-linear systems but also to present the stability analysis in some detail. The latter task is challenging due to the present of linearisation errors and the interaction between neighbouring nodes. A numerical example is provided to verify the effectiveness of the proposed filter.

This paper is organised as follows: In Section 2, the distributed state estimation problem for non-linear systems is formulated. The state estimation error covariances are derived and the corresponding upper bound matrices are provided to design the gain matrices in Section 3. A numerical example involving an indoor localisation of a mobile robot is shown in Section 4. Conclusions are drawn in Section 5.

2 Problem statement

Consider the following discrete-time non-linear system:

$$x_{k+1} = f(x_k) + w_k \tag{1}$$

$$z_{i,k} = h_i(x_k) + v_{i,k} \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector and $z_{i,k} \in \mathbb{R}^p$ is the measurement vector of the ith sensor. $f(\cdot)$ and $h_i(\cdot)$ are known non-linear functions that are assumed to be twice continuously differentiable. The process noise w_k and the measurement noise $v_{i,k}$ are assumed to be mutually uncorrelated zero-mean white Gaussian with covariances Q_k and $R_{i,k}$, respectively. Generally speaking, the non-linear function in (1) often covers the noise vector. In this paper, we consider the additive case for simplicity.

We are interested in tracking the state of this target using a sensor network with communication topology $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, \ldots, N\}$ and $\mathscr{E} \subset \mathscr{V} \times \mathscr{V}$ denote the nodes and the edges, respectively. Two nodes are said to be connected if they can communicate directly with each other. The set of nodes connected with a certain node i is called the neighbourhood of node i and is denoted by $\mathscr{N}_i$. The number of neighbours of node i is called its degree and is denoted by $d_i = |\mathscr{N}_i|$. We consider a connected undirected network in this paper.

The structure of the EKF is adopted to develop a distributed consensus filter for systems (1) and (2)

$$\hat{x}_{i,k+1|k} = f(\hat{x}_{i,k|k}) \tag{3}$$

(see (4)) where $\hat{x}_{i,k+1|k}$ and $\hat{x}_{i,k+1|k+1}$ denote the predicted and the updated estimates at time instant $k+1$, respectively. $K_{i,k+1}$ is the Kalman gain matrix to be determined and $C_{i,k+1}$ is the predefined consensus gain matrix. The third term on the right hand side of (4) is called as consensus on prior estimates [12]. As stated in [12], adding the consensus term will force local estimators to reach a consensus regarding state estimates. Moreover, the consensus term can be considered as a pseudo-measurement, which helps improve the estimation accuracy for local estimators.

The updated estimation error and the corresponding covariance are defined as

$$e_{i,k+1|k+1} = x_{k+1} - \hat{x}_{i,k+1|k+1} \tag{5}$$

$$P_{i,k+1|k+1} = \mathbb{E}\{e_{i,k+1|k+1} e^{\mathrm{T}}_{i,k+1|k+1}\} \tag{6}$$

The aim of this paper is to design filters described by (3) and (4), such that there exists a sequence of positive-definite matrices $\Phi_{i,k+1|k+1}$ satisfying [22–24]

$$P_{i,k+1|k+1} \le \Phi_{i,k+1|k+1} \tag{7}$$

The gain matrix $K_{i,k+1}$ is determined by minimising the trace of the upper bound matrix $\Phi_{i,k+1|k+1}$ at each time instant.

3 Main results

In this section, the covariances of the predicted and the updated estimation errors are firstly derived. Subsequently, an upper bound matrix is obtained and the Kalman gain matrix is determined by minimising the trace of the upper bound matrix. The stability analysis is also presented under certain conditions.

3.1 Distributed consensus EKF

The distributed consensus EKF is derived based on the following lemmas.

Lemma 1 [25]: Given matrices A, B, C and D with appropriate dimensions such that $CC^{\mathrm{T}} \le I$. Let U be a symmetric positive-definite matrix and $a > 0$ be an arbitrary positive constant such that $a^{-1}I - DUD^{\mathrm{T}} > 0$. Then the following matrix inequality holds

$$(A + BCD)U(A + BCD)^{\mathrm{T}} \le A(U^{-1} - aD^{\mathrm{T}}D)^{-1}A^{\mathrm{T}} + a^{-1}BB^{\mathrm{T}} \tag{8}$$

Lemma 2 [26]: For $0 \le k < n$, suppose that $A = A^{\mathrm{T}} > 0$. Let $\varphi_k(\cdot)$ and $\psi_k(\cdot)$ be two sequences of matrix functions such that

$$\varphi_k(A) = \varphi_k(A^{\mathrm{T}}), \quad \psi_k(A) = \psi_k(A^{\mathrm{T}}) \tag{9}$$

If there exists a matrix $B = B^{\mathrm{T}} > A$ such that

$$\varphi_k(B) \ge \varphi_k(A), \quad \psi_k(B) \ge \varphi_k(B) \tag{10}$$

then the solutions X_k and Y_k to the following difference equations:

$$X_k = \varphi_k(X_{k-1}), \quad Y_k = \psi_k(Y_{k-1}), \quad X_0 = Y_0 > 0 \tag{11}$$

satisfy $X_k \le Y_k$.

Define the predicted estimation error and the corresponding covariance matrix as

$$e_{i,k+1|k} = x_{k+1} - \hat{x}_{i,k+1|k} \tag{12}$$

$$P_{i,k+1|k} = \mathbb{E}\{e_{i,k+1|k} e^{\mathrm{T}}_{i,k+1|k}\} \tag{13}$$

The predicted estimation error can be derived by subtracting (3) from (1)

$$e_{i,k+1|k} = f(x_k) - f(\hat{x}_{i,k|k}) + w_k \tag{14}$$

Expanding the system transition function in a Taylor series about $\hat{x}_{i,k|k}$, we have

$$f(x_{i,k}) = f(\hat{x}_{i,k|k}) + F_{i,k} e_{i,k|k} + o(|e_{i,k|k}|) \tag{15}$$

where $F_{i,k} = \partial f(x)/\partial x|_{x = \hat{x}_{i,k|k}}$. The high-order terms can be represented by [22–24, 27, 28]

$$o(|e_{i,k|k}|) = U_{i,k} \Omega_{i,k} e_{i,k|k} \tag{16}$$

where $U_{i,k}$ is a known problem-dependent scaling matrix and $\Omega_{i,k}$ is an unknown time-varying matrix accounting for the linearisation error satisfying $\Omega_{i,k}\Omega^{\mathrm{T}}_{i,k} \le I$. In this paper, as in [27], the matrices $U_{i,k}$ and $\Omega_{i,k}$ are employed to account for the linearisation errors. For more details, we refer the reader to Appendix C of [27].

Then it follows from (14)–(16) that

$$e_{i,k+1|k} = (F_{i,k} + U_{i,k}\Omega_{i,k}) e_{i,k|k} + w_k \tag{17}$$

The covariance matrix of the predicted estimation error can be obtained with respect to (17)

$$P_{i,k+1|k} = (F_{i,k} + U_{i,k}\Omega_{i,k}) P_{i,k|k} (F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}} + Q_k \tag{18}$$

Similarly, the updated estimation error can be derived by substituting (4) into (5)

$$\hat{x}_{i,k+1|k+1} = \hat{x}_{i,k+1|k} + K_{i,k+1}[z_{i,k+1} - h(\hat{x}_{i,k+1|k})] + C_{i,k+1} \sum_{j \in \mathscr{N}_i} (\hat{x}_{j,k+1|k} - \hat{x}_{i,k+1|k}) \tag{4}$$

$$e_{i,k+1|k+1} = e_{i,k+1|k} - K_{i,k+1}(H_{i,k+1} + V_{i,k+1}\Theta_{i,k+1})e_{i,k+1|k} - C_{i,k+1}\sum_{j\in\mathcal{N}_i}(e_{i,k+1|k} - e_{j,k+1|k}) - K_{i,k+1}v_{i,k+1} = \mathcal{A}_{i,k+1}e_{i,k+1|k} + C_{i,k+1}\sum_{j\in\mathcal{N}_i}e_{j,k+1|k} - K_{i,k+1}v_{i,k+1} \quad (19)$$

where $H_{i,k+1} = \partial h(x)/\partial x|_{x=\hat{x}_{i,k+1|k}}$,

$$\mathcal{A}_{i,k+1} = I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1} - K_{i,k+1}V_{i,k+1}\Theta_{i,k+1} \quad (20)$$

$V_{i,k+1}$ is a known problem-dependent scaling matrix and $\Theta_{i,k+1}$ is an unknown time-varying matrix accounting for the linearisation error satisfying $\Theta_{i,k+1}\Theta_{i,k+1}^{\mathrm{T}} \le I$. Then the covariance matrix of the updated estimation error can be obtained with respect to (19)

$$P_{i,k+1|k+1} = \mathcal{A}_{i,k+1}P_{i,k+1|k}\mathcal{A}_{i,k+1}^{\mathrm{T}} + \sum_{j\in\mathcal{N}_i}\mathbb{E}\{\mathcal{A}_{i,k}e_{i,k+1|k}e_{j,k+1|k}^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}} + C_{i,k+1}e_{j,k+1|k}e_{i,k+1|k}^{\mathrm{T}}\mathcal{A}_{i,k+1}^{\mathrm{T}}\} + K_{i,k+1}R_{i,k+1}K_{i,k+1}^{\mathrm{T}} + \sum_{j\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i}C_{i,k+1}\mathbb{E}\{e_{j,k+1|k}e_{l,k+1|k}^{\mathrm{T}}\}C_{i,k+1}^{\mathrm{T}} \quad (21)$$

So far, we have derived the covariance matrices for the predicted and the updated estimation errors. However, the unknown terms $\Omega_{i,k}$ and $\Theta_{i,k+1}$ are introduced due to the linearisaiton errors. Hence, it is impossible to calculate these covariance matrices directly. On the other hand, the edge-covariance matrices $\mathbb{E}\{e_{j,k+1|k}e_{l,k+1|k}^{\mathrm{T}}\}$ are present due to the consensus on prior estimates. Note that the distributed consensus EKF is not scalable if the edge-covariance matrices are computed explicitly. An alternative way is to find upper bound matrices for $P_{i,k+1|k}$ and $P_{i,k+1|k+1}$ and then determine the gain matrix $K_{i,k+1}$ according to the upper bound matrix.

The main result of this paper is summarised as follows.

Theorem 1: Consider the discrete-time non-linear system described by (1) and (2). Let $\alpha_{i,k}$ and $\beta_{i,k}$ be positive scalars. Let ε be a positive scalar satisfying $0 < \varepsilon < 1/d_{\max}$, where $d_{\max} = \max_{1\le i\le N}\{d_i\}$. If the following two Riccati-like difference equations

$$\Phi_{i,k+1|k} = F_{i,k}(\Phi_{i,k|k}^{-1} - (1+\varepsilon)\alpha_{i,k}I)^{-1}F_{i,k}^{\mathrm{T}} + \varepsilon^{-1}\alpha_{i,k}^{-1}U_{i,k}U_{i,k}^{\mathrm{T}} + Q_k \quad (22)$$

(see (23)) have positive-define solutions $\Phi_{i,k+1|k}$ and $\Phi_{i,k+1|k+1}$ with initial conditions $P_{0|0} \le \Phi_{0|0}$ such that the following inequalities:

$$\Phi_{i,k|k}^{-1} > (1+\varepsilon)\alpha_{i,k}I \quad (24)$$

$$\Phi_{i,k+1|k}^{-1} > \beta_{i,k}I \quad (25)$$

hold for all $k \ge 0$, then the matrix $\Phi_{i,k+1|k+1}$ is an upper bound of $P_{i,k+1|k+1}$. Moreover, the Kalman gain matrix can be determined by minimising the trace of the upper bound matrix $\Phi_{i,k+1|k+1}$ as follows: (see (26))

Proof: According to Lemma 1, an upper bound matrix can be derived for the predicted estimation error covariance

$$\begin{aligned}P_{i,k+1|k} &= (F_{i,k} + U_{i,k}\Omega_{i,k})P_{i,k|k}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}} + Q_k \\ &\le F_{i,k}(P_{i,k|k}^{-1} - (1+\varepsilon)\alpha_{i,k}I)^{-1}F_{i,k}^{\mathrm{T}} + (1+\varepsilon)^{-1}\alpha_{i,k}^{-1}U_{i,k}U_{i,k}^{\mathrm{T}} + Q_k \\ &\le F_{i,k}(P_{i,k|k}^{-1} - (1+\varepsilon)\alpha_{i,k}I)^{-1}F_{i,k}^{\mathrm{T}} + \varepsilon^{-1}\alpha_{i,k}^{-1}U_{i,k}U_{i,k}^{\mathrm{T}} + Q_k\end{aligned} \quad (27)$$

To show the upper bound of the updated estimation error covariance matrix at each time instant, applying Lemma 1 to the first term on the right hand side of (21), we have (see (28)) By using the elementary inequality $x^{\mathrm{T}}y + xy^{\mathrm{T}} \le xx^{\mathrm{T}} + yy^{\mathrm{T}}$, upper bound matrices can be obtained for the second and the third terms on the right hand side of (21)

$$\begin{aligned}&\sum_{j\in\mathcal{N}_i}\mathbb{E}\{\mathcal{A}_{i,k+1}e_{i,k+1|k}e_{j,k+1|k}^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}} + C_{i,k+1}e_{j,k+1|k}e_{i,k+1|k}^{\mathrm{T}}\mathcal{A}_{i,k+1}^{\mathrm{T}}\} \\ &\le \sum_{j\in\mathcal{N}_i}\mathbb{E}\{\mathcal{A}_{i,k+1}e_{i,k+1|k}e_{i,k+1|k}^{\mathrm{T}}\mathcal{A}_{i,k+1}^{\mathrm{T}} + C_{i,k+1}e_{j,k+1|k}e_{j,k+1|k}^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\} \\ &= d_i\mathcal{A}_{i,k+1}P_{i,k+1|k}\mathcal{A}_{i,k+1}^{\mathrm{T}} + \sum_{j\in\mathcal{N}_i}C_{i,k+1}P_{j,k+1|k}C_{i,k+1}^{\mathrm{T}}\end{aligned} \quad (29)$$

$$\begin{aligned}&\sum_{j\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i}C_{i,k+1}\mathbb{E}\{e_{j,k+1|k}e_{l,k+1|k}^{\mathrm{T}}\}C_{i,k+1}^{\mathrm{T}} \\ &\le \frac{1}{2}\sum_{j\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i}C_{i,k+1}(P_{j,k+1|k} + P_{l,k+1|k})C_{i,k+1}^{\mathrm{T}} \\ &= d_i\sum_{j\in\mathcal{N}_i}C_{i,k+1}P_{j,k+1|k}C_{i,k+1}^{\mathrm{T}}\end{aligned} \quad (30)$$

Substituting (28)–(30) into (21) leads to (see (31)) Applying Lemma 2 to (21), (22), (27) and (29), we can draw a conclusion that

$$P_{i,k+1|k+1} \le \Phi_{i,k+1|k+1} \quad (32)$$

We are now ready to determine the Kalman gain matrix by minimising the trace of the matrix $\Phi_{i,k+1|k+1}$. Taking partial derivative of the trace of $\Phi_{i,k+1|k+1}$ with respect to the Kalman gain matrix $K_{i,k+1}$, we have

$$\begin{aligned}\Phi_{i,k+1|k+1} = {}&(1+d_i)[(I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1})(\Phi_{i,k+1|k}^{-1} - \beta_{i,k}I)^{-1} \\ &\times (I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1})^{\mathrm{T}}] + (1+d_i)\sum_{j\in\mathcal{N}_i}C_{i,k+1}\Phi_{j,k+1|k}C_{i,k+1}^{\mathrm{T}} \\ &+ K_{i,k+1}[(1+d_i)\beta_{i,k}^{-1}V_{i,k+1}V_{i,k+1}^{\mathrm{T}} + R_{i,k+1}]K_{i,k+1}^{\mathrm{T}}\end{aligned} \quad (23)$$

$$\begin{aligned}K_{i,k+1} = {}&(I - d_iC_{i,k+1})(\Phi_{i,k+1|k}^{-1} - \beta_{i,k}I)^{-1}H_{i,k+1}^{\mathrm{T}}\Big[H_{i,k+1}(\Phi_{i,k+1|k}^{-1} - \beta_{i,k}I)^{-1}H_{i,k+1}^{\mathrm{T}} \\ &+ \beta_{i,k}^{-1}V_{i,k+1}V_{i,k+1}^{\mathrm{T}} + \frac{1}{1+d_i}R_{i,k+1}\Big]^{-1}\end{aligned} \quad (26)$$

$$\begin{aligned}\mathcal{A}_{i,k+1}P_{i,k+1|k}\mathcal{A}_{i,k+1}^{\mathrm{T}} \le {}&(I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1})(\Phi_{i,k+1|k}^{-1} - \beta_{i,k}I)^{-1} \\ &\times (I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1})^{\mathrm{T}} + \beta_{i,k}^{-1}K_{i,k+1}V_{i,k+1}V_{i,k+1}^{\mathrm{T}}K_{i,k+1}^{\mathrm{T}}\end{aligned} \quad (28)$$

$$\frac{\partial \mathrm{tr}(\Phi_{i,k+1|k+1})}{\partial K_{i,k+1}} = 2K_{i,k+1}H_{i,k+1}(\Phi^{-1}_{i,k+1|k} - \beta_{i,k}I)^{-1}H^{\mathrm{T}}_{i,k+1} \\ -2(I - d_iC_{i,k+1})(\Phi^{-1}_{i,k+1|k} - \beta_{i,k}I)^{-1}H^{\mathrm{T}}_{i,k+1} \\ +2K_{i,k+1}[\beta^{-1}_{i,k}V_{i,k+1}V^{\mathrm{T}}_{i,k+1} + \frac{1}{1+d_i}R_{i,k+1}] \tag{33}$$

The Kalman gain matrix can be determined by setting

$$\frac{\partial \mathrm{tr}(\Phi_{i,k+1|k+1})}{\partial K_{i,k+1}} = 0$$

which leads to the solution as shown in (26). The proof is complete. □

Remark 1: In [12], an optimal KCF has been proposed for discrete-time linear time-variant systems. However, the optimal KCF is not scalable due to computations of edge-covariance matrices in the covariance update algorithm. Then, a suboptimal KCF has been developed for scalable considerations by removing the edge-covariance matrices in the covariance update algorithm. Since the edge-covariance matrices have been neglected in the suboptimal KCF, the covariance update algorithm and the Kalman gain matrix of the suboptimal KCF possess the same forms as those of the standard KF for an isolated node. In other words, the neighbouring information is not used to update the covariance and the Kalman gain matrix. In this paper, the variance-constrained approach has been used to derive upper bound matrices for the edge-covariances instead of removing them in the covariance update algorithm. Compared with the suboptimal KCF in [12], the neighbouring information (i.e. the upper bound of the predicted estimation error covariance $\Phi_{i,k+1|k}$) has been used to derive an upper bound of the updated estimation error covariance. Moreover, the degree of the node and the consensus gain matrix have been used to design the Kalman gain matrix.

Remark 2: It is worth mentioning here that a distributed EKF has been proposed for non-linear systems by applying the hybrid consensus strategy in [21]. The differences between the proposed filter in this note and the filter in [21] are twofold: (i) the linearisation errors have been addressed to compute the covariances in this note while they are not considered in [21], thus the proposed filter possesses robustness with respect to the linearisation errors; (ii) the consensus strategy is applied to the prior estimates in this paper while it is applied to the information pairs and the measurement pairs in [21], thus the communication cost of the proposed filter is one half of that in [21]. (In the proposed filter, the prior estimates $\hat{x}_{i,k+1|k}$ and the corresponding upper bound matrix $\Phi_{i,k+1|k}$ are exchanged between nodes and therefore the communication cost is $(n^2+3n)/2$. In the distributed EKF [21], the information pairs $(q_{i,k}, \Omega_{i,k})$ and the measurement pairs $(\delta q_{i,k}, \delta\Omega_{i,k})$ are exchanged between nodes and therefore the communication cost is n^2+3n.). It should be pointed out that the performance of the proposed filter might be worse than that of the distributed EKF in [21] since less information have been exchanged for consensus.

Remark 3: In [22, 23, 28], the variance-constrained approach has been used to develop the robust EKFs for non-linear systems with stochastic uncertainties and missing measurements. In [24], variance-constrained state estimators have been developed for non-linear complex networks. Although the problem setup of this paper seems close to [24], the determination of the gain matrix is substantially different. In [24], all the estimation errors are formulated in an augmented vector and an overall upper bound matrix is derived for the augmented estimation error covariance. In other words, the augmented approach in [24] cannot be used for developing distributed estimation algorithms.

3.2 Boundedness analysis

A sufficient condition has been presented in [29] to guarantee the boundedness of stochastic processes.

Lemma 3 [29]: Assume there is a stochastic process $V_k(\xi_k)$ as well as real numbers $\underline{\nu}, \bar{\nu}, \mu > 0$ and $0 < \alpha \le 1$ such that

$$\underline{\nu}\|\xi_k\|^2 \le V_k(\xi_k) \le \bar{\nu}\|\xi_k\|^2 \tag{34}$$

and

$$\mathbb{E}\{V_k(\xi_k)|\xi_{k-1}\} \le (1-\alpha)V_{k-1}(\xi_{k-1}) + \mu \tag{35}$$

Then the stochastic process is exponentially bounded in mean square, i.e.

$$\mathbb{E}\{\|\xi_k\|^2\} \le \frac{\bar{\nu}}{\underline{\nu}}\mathbb{E}\{\|\xi_0\|^2\}(1-\alpha)^k + \frac{\mu}{\underline{\nu}}\sum_{i=1}^{k}(1-\alpha)^i \tag{36}$$

To study the stability of the estimation error dynamics (21), the following assumptions are needed.

Assumption 1: The upper bound matrix satisfies $\Phi^{-1}_{i,k|k} \le ((1+d_{\max})/\theta)\alpha_{i,k}I$, where $d_{\max} < \theta < ((1+d_{\max})/(1+\varepsilon))$.

Assumption 2: There exist positive scalars γ, δ such that $\gamma I \le \alpha_{i,k} \le \delta I$ for any i and any k.

Notice that Assumptions 1 and 2 and (24) guarantees that the matrix $\Phi_{i,k|k}$ is bounded. Such assumption for boundedness of the covariance matrix has been used in most papers dealing with the stability of the EKF, e.g. [21, 29, 30]. It should be pointed out that it is difficult to verify Assumptions 1 and 2. However, they might be guaranteed if the estimation error covariance matrix has a lower bounded.

The following theorem provides a sufficient condition under which the estimation error is bounded in mean square.

Theorem 2: Consider the discrete-time non-linear system described by (1) and (2). Under Assumptions 1 and 2, the estimation error $e_{i,k|k}$ is exponentially bounded in mean square provided that the initial estimation error $e_{i,0|0}$ is bounded.

Proof: Define the augmented estimation error $e_k = [e^{\mathrm{T}}_{1,k|k}, \ldots, e^{\mathrm{T}}_{N,k|k}]^{\mathrm{T}}$. To satisfy the conditions of Lemma 3, we define the following Lyapunov function:

$$V_k(e_k) = \sum_{i=1}^{N} e^{\mathrm{T}}_{i,k|k}\Phi^{-1}_{i,k|k}e_{i,k|k} \tag{37}$$

Assumptions 1 and 2 and (24) implies that

$$\gamma(1+\varepsilon)\|e_k\|^2 \le V_k(e_k) \le \delta(1+d_{\max})\theta^{-1}\|e_k\|^2 \tag{38}$$

$$P_{i,k+1|k+1} \le (1+d_i)\mathscr{A}_{i,k+1}P_{i,k+1|k}\mathscr{A}^{\mathrm{T}}_{i,k+1} + (1+d_i)\sum_{j\in\mathscr{N}_i}C_{i,k+1}P_{j,k+1|k}C^{\mathrm{T}}_{i,k+1} + K_{i,k+1}R_{i,k+1}K^{\mathrm{T}}_{i,k+1} \\ \le (1+d_i)[(I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1})(P^{-1}_{i,k+1|k} - \beta_{i,k}I)^{-1} \\ \times (I - d_iC_{i,k+1} - K_{i,k+1}H_{i,k+1})^{\mathrm{T}}] + (1+d_i)\sum_{j\in\mathscr{N}_i}C_{i,k+1}P_{j,k+1|k}C^{\mathrm{T}}_{i,k+1} \\ +K_{i,k+1}[(1+d_i)\beta^{-1}_{i,k}V_{i,k+1}V^{\mathrm{T}}_{i,k+1} + R_{i,k+1}]K^{\mathrm{T}}_{i,k+1} \tag{31}$$

which satisfies (34) in Lemma 3.

To further satisfy the requirements for an application of Lemma 3, we need an upper bound as shown in (35). To this end, substituting the estimation error (19) into (37) and taking expectations yields (see (39))

Similar to the derivations of (29) and (30), applying the elementary inequality to the second and the third terms on the right hand side of (39), we can obtain (see (40) and (41)) (see (41))

Substituting (40) and (41) into (39) yields (see (42))

Applying Lemma 1 to (22) and (23), we can obtain

$$\Phi_{i,k+1|k} \geq (F_{i,k} + U_{i,k}\Omega_{i,k})(\Phi_{i,k|k}^{-1} - \alpha_{i,k}I)^{-1}(F_{i,k} + U_{i,k}\Omega_{i,k}L_{i,k})^{\mathrm{T}} \tag{43}$$

$$\Phi_{i,k+1|k+1} \geq (1+d_i)\mathscr{A}_{i,k+1}\Phi_{i,k+1|k}\mathscr{A}_{i,k+1}^{\mathrm{T}} \tag{44}$$

$$\Phi_{i,k+1|k+1} \geq (1+d_i)C_{i,k+1}\Phi_{j,k+1|k}C_{i,k+1}^{\mathrm{T}} \tag{45}$$

According to (43) and (44) and Assumption 2, the first term on the right hand side of (42) can be bounded by (see (46))

Similarly, an upper bound can be derived for the second term on the right hand side of (42) by using (43) and (45) (see (47))

Substituting (46) and (47) into (42) yields (see (48)) Notice that the second and the third terms on the right hand side of (48) are related with the noise terms, it can be shown that they are bounded as in [29], i.e.

$$\begin{aligned}
\mathbb{E}\{V_{k+1}(e_{k+1})|e_k\} = &\sum_{i=1}^{N} e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}}\mathscr{A}_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}\mathscr{A}_{i,k+1}(F_{i,k} + U_{i,k}\Omega_{i,k})e_{i,k|k} \\
&+2e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}}\mathscr{A}_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}\sum_{j\in\mathcal{N}_i}(F_{j,k} + U_{j,k}\Omega_{j,k})e_{j,k|k} \\
&+\sum_{j\in\mathcal{N}_i} e_{j,k|k}^{\mathrm{T}}(F_{j,k} + U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}\sum_{l\in\mathcal{N}_i}(F_{l,k} + U_{l,k}\Omega_{l,k})e_{l,k|k} \\
&+\mathbb{E}\{w_k^{\mathrm{T}}(\mathscr{A}_{i,k+1}^{\mathrm{T}} + \sum_{j\in\mathcal{N}_i}(F_{j,k} + U_{j,k}\Omega_{j,k}C_{i,k+1}^{\mathrm{T}})\Phi_{i,k+1|k+1}^{-1}(\mathscr{A}_{i,k+1} + C_{i,k+1} \\
&\times\sum_{j\in\mathcal{N}_i}(F_{j,k} + U_{j,k}\Omega_{j,k}))w_k\} + \mathbb{E}\{v_{i,k+1}^{\mathrm{T}}K_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}K_{i,k+1}^{\mathrm{T}}v_{i,k+1}\}
\end{aligned} \tag{39}$$

$$\begin{aligned}
&2e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k}\mathscr{A}_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}\sum_{j\in\mathcal{N}_i}(F_{j,k} + U_{j,k}\Omega_{j,k})e_{j,k|k} \\
&\leq d_i e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}}\mathscr{A}_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}\mathscr{A}_{i,k+1}(F_{i,k} + U_{i,k}\Omega_{i,k})e_{i,k|k} \\
&\quad+\sum_{j\in\mathcal{N}_i} e_{j,k|k}^{\mathrm{T}}(F_{j,k} + U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}(F_{j,k} + U_{j,k}\Omega_{j,k})e_{j,k|k}
\end{aligned} \tag{40}$$

$$\begin{aligned}
&\sum_{j\in\mathcal{N}_i} e_{j,k|k}^{\mathrm{T}}(F_{j,k} + U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}\sum_{l\in\mathcal{N}_i}(F_{l,k} + U_{l,k}\Omega_{l,k})e_{l,k|k} \\
&\leq d_i\sum_{j\in\mathcal{N}_i} e_{j,k|k}^{\mathrm{T}}(F_{j,k} + U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}(F_{j,k} + U_{j,k}\Omega_{j,k})e_{j,k|k}
\end{aligned} \tag{41}$$

$$\begin{aligned}
\mathbb{E}\{V_{k+1}(e_{k+1})|e_k\} = &\sum_{i=1}^{N}(1+d_i)e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}}\mathscr{A}_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1} \\
&\times\mathscr{A}_{i,k+1}(F_{i,k} + U_{i,k}\Omega_{i,k})e_{i,k|k} \\
&+(1+d_i)\sum_{j\in\mathcal{N}_i} e_{j,k|k}^{\mathrm{T}}(F_{j,k} + U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1} \\
&\times C_{i,k+1}(F_{j,k} + U_{j,k}\Omega_{j,k})e_{j,k|k} \\
&+\mathbb{E}\{w_k^{\mathrm{T}}(\mathscr{A}_{i,k+1}^{\mathrm{T}} + \sum_{j\in\mathcal{N}_i}(F_{j,k} + U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}) \\
&\times\Phi_{i,k+1|k+1}^{-1}(\mathscr{A}_{i,k+1} + C_{i,k+1}\sum_{j\in\mathcal{N}_i}(F_{j,k} + U_{j,k}\Omega_{j,k}))w_k\} \\
&+\mathbb{E}\{v_{i,k+1}^{\mathrm{T}}K_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}K_{i,k+1}^{\mathrm{T}}v_{i,k+1}\}
\end{aligned} \tag{42}$$

$$\begin{aligned}
&(1+d_i)e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}}\mathscr{A}_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}\mathscr{A}_{i,k+1}(F_{i,k} + U_{i,k}\Omega_{i,k})e_{i,k|k} \\
&\leq e_{i,k|k}^{\mathrm{T}}(F_{i,k} + U_{i,k}\Omega_{i,k})^{\mathrm{T}}\Phi_{i,k+1|k}^{-1}(F_{i,k} + U_{i,k}\Omega_{i,k})e_{i,k|k} \\
&\leq e_{i,k|k}^{\mathrm{T}}(\Phi_{i,k|k}^{-1} - \alpha_{i,k}I)e_{i,k|k} \\
&\leq \left[1 - \alpha_{i,k}\cdot\frac{\theta}{\alpha_{i,k}(1+d_{\max})}\right]e_{i,k|k}^{\mathrm{T}}\Phi_{i,k|k}^{-1}e_{i,k|k} \\
&\leq \frac{1+d_{\max}-\theta}{1+d_{\max}}e_{i,k|k}^{\mathrm{T}}\Phi_{i,k|k}^{-1}e_{i,k|k}
\end{aligned} \tag{46}$$

$$\sum_{i=1}^{N}\mathbb{E}\{w_k^{\mathrm{T}}(\mathscr{A}_{i,k+1}^{\mathrm{T}}+\sum_{j\in\mathscr{N}_i}(F_{j,k}+U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}) \\ \times\Phi_{i,k+1|k+1}^{-1}(\mathscr{A}_{i,k+1}+C_{i,k+1}\sum_{j\in\mathscr{N}_i}(F_{j,k}+U_{j,k}\Omega_{j,k}))w_k\} \\ +\mathbb{E}\{v_{i,k+1}^{\mathrm{T}}K_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}K_{i,k+1}^{\mathrm{T}}v_{i,k+1}\}\le\mu \tag{49}$$

Summing up, it has been proved that

$$\mathbb{E}\{V_{k+1}(e_{k+1})|e_k\}\le(1+d_{\max}-\theta)V_k(e_k)+\mu$$

Hence, we can conclude that the estimation error is bounded in mean square under Assumption 1. The proof is complete. □

Remark 4: Note that the stability of the proposed filter is not related with the consensus gain matrix $C_{i,k+1}$. This is due to the fact that the consensus gain matrix has been introduced in the computations of the upper bound matrix and the consensus terms have been cancelled in the stability analysis. Similar to the result in [12], an appropriate choice of the consensus gain is to set $C_{i,k+1}=\rho\Phi_{i,k+1|k}$ where ρ is a relative small constant.

4 Numerical study

In this section, the performance of the proposed filter is evaluated via a numerical example involving an indoor localisation of a mobile robot using the visual tracking systems. Although a simple network is used in this paper, the proposed algorithm can be used for large-scale networks since only local state estimates and their corresponding upper bound matrices are exchanged between neighbours. The kinematic model for the robot can be represented by [31]

$$x_{k+1}=x_k+\frac{s_k^{\mathrm{R}}+s_k^{\mathrm{L}}}{2}\cos\theta_k+w_k^x \tag{50}$$

$$y_{k+1}=y_k+\frac{s_k^{\mathrm{R}}+s_k^{\mathrm{L}}}{2}\sin\theta_k+w_k^y \tag{51}$$

$$\theta_{k+1}=\theta_k+\frac{s_k^{\mathrm{R}}-s_k^{\mathrm{L}}}{b}+w_k^\theta \tag{52}$$

where (x_k,y_k) and θ_k denote the position and the orientation, respectively. s_k^{R} and s_k^{L} denote the distances travelled by the right and left driving wheels during the time interval $[k,k+1)$, respectively. b is the distance between the right and left driving wheels. In the robot, the optical encoders are equipped to record the distances s_k^{R} and s_k^{L}. $w_k=(w_k^x,w_k^y,w_k^\theta)$ is zero-mean white Guassian noise with covariance Q_k.

In the visual tracking system, eight cameras are used to generate the measurement of the robot and the measurement model is given by [31]

$$p_{i,k}=\frac{\gamma_u}{z_f^c}[-(x_{i,f}-x_k)\sin\theta_k+(y_{i,f}-y_k)\cos\theta_k-d_2]+p_0+v_{i,k}^p \tag{53}$$

$$q_{i,k}=\frac{\gamma_v}{z_f^c}[-(x_{i,f}-x_k)\cos\theta_k-(y_{i,f}-y_k)\sin\theta_k+d_1]+q_0+v_{i,}^q \tag{54}$$

where $(p_{i,k},q_{i,k})$ denotes the coordinate of the feature in the image plane. (d_1,d_2) is the coordinate in the robot frame. z_f^c is the distance from the optical center of the camera to the robot. γ_u and γ_v are pixel magnification factors. (p_0,q_0) is the image coordinate of the camera's principal point and $(x_{i,f},y_{i,f})$ is the coordinate of the ith camera in the world frame. $v_{i,k}=(v_{i,k}^p,v_{i,k}^q)$ is zero mean white Guassian noise with covariance $R_{i,k}$.

In the simulations, the parameters of the visual tracking system are adopted from the experiments in our lab [31]: $d_1=-0.0668$, $d_2=0.0536$, $z_f^c=2.1050$, $\gamma_u=902.13283$, $\gamma_v=902.50141$, $u_0=347.20436$ and $v_0=284.34705$. The positions of the cameras are taken to be (0.6, 0.6), (0.6, 1.8), (1.2, 1.8), (1.8, 1.8), (2.4, 1.8), (2.4, 0.6), (1.8, 1.2) and (1.2, 1.2). The network topology is shown in Fig. 1. The process noise covariance matrix is $Q_k=\mathrm{diag}\{1,1,1\}$ and the measurement noise covariance matrices are $R_{i,k}=\mathrm{diag}\{25^2,25^2\}$ $(i=1,\cdots,8)$. The scalars in the calculation of the upper bound matrices are taken to be $\alpha_{i,k}=\beta_{i,k}=0.1$ $(i=1,\cdots,8)$. The consensus parameter is taken to be $\rho=0.01$.

To illustrate the tracking performance of the proposed filter, the averaged root mean square errors (RMSE) over the network are

$$(1+d_i)\sum_{j\in\mathscr{N}_i}e_{j,k|k}^{\mathrm{T}}(F_{j,k}+U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}C_{i,k+1}(F_{j,k}+U_{j,k}\Omega_{j,k})e_{j,k|k} \\ \le\frac{1+d_{\max}-\theta}{1+d_{\max}}\sum_{j\in\mathscr{N}_i}e_{j,k|k}^{\mathrm{T}}\Phi_{j,k|k}^{-1}e_{j,k|k} \tag{47}$$

$$\mathbb{E}\{V_{k+1}(e_{k+1})|e_k\}\le\frac{1+d_{\max}-\theta}{1+d_{\max}}\sum_{i=1}^{N}e_{i,k|k}^{T}\Phi_{i,k|k}^{-1}e_{i,k|k} \\ +\frac{1+d_{\max}-\theta}{1+d_{\max}}\sum_{i=1}^{N}\sum_{j\in\mathscr{N}_i}e_{j,k|k}^{T}\Phi_{j,k|k}^{-1}e_{j,k|k} \\ +\sum_{i=1}^{N}\mathbb{E}\Big\{w_k^{\mathrm{T}}(\mathscr{A}_{i,k+1}^{\mathrm{T}}+\sum_{j\in\mathscr{N}_i}(F_{j,k}+U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}) \\ \times\Phi_{i,k+1|k+1}^{-1}(\mathscr{A}_{i,k+1}+C_{i,k+1}\sum_{j\in\mathscr{N}_i}(F_{j,k}+U_{j,k}\Omega_{j,k}))w_k\Big\} \\ +\mathbb{E}\{v_{i,k+1}^{\mathrm{T}}K_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}K_{i,k+1}^{\mathrm{T}}v_{i,k+1}\} \\ \le(1+d_{\max}-\theta)V_k(e_k)+\sum_{i=1}^{N}\mathbb{E}\{w_k^{\mathrm{T}}(\mathscr{A}_{i,k+1}^{\mathrm{T}}+\sum_{j\in\mathscr{N}_i}(F_{j,k}+U_{j,k}\Omega_{j,k})^{\mathrm{T}}C_{i,k+1}^{\mathrm{T}}) \\ \times\Phi_{i,k+1|k+1}^{-1}(\mathscr{A}_{i,k+1}+C_{i,k+1}\sum_{j\in\mathscr{N}_i}(F_{j,k}+U_{j,k}\Omega_{j,k}))w_k\} \\ +\mathbb{E}\{v_{i,k+1}^{\mathrm{T}}K_{i,k+1}^{\mathrm{T}}\Phi_{i,k+1|k+1}^{-1}K_{i,k+1}^{\mathrm{T}}v_{i,k+1}\} \tag{48}$$

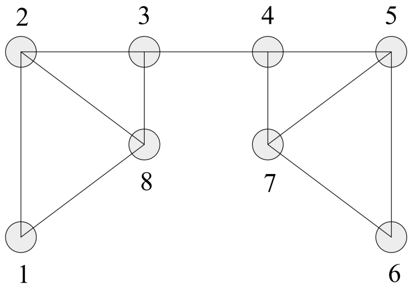

Fig. 1 *Network topology*

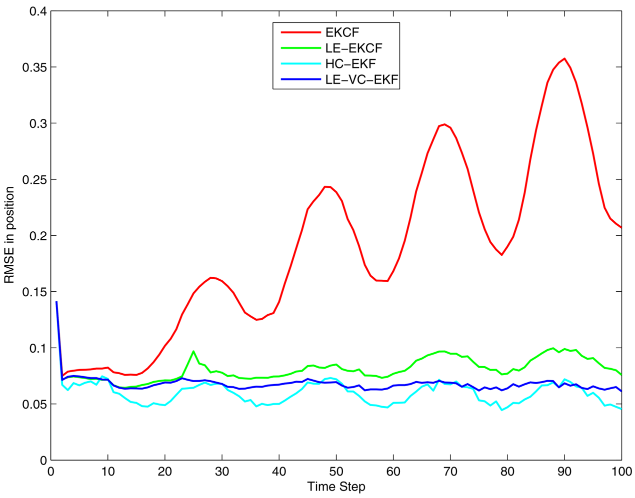

Fig. 2 *RMSE in position versus time instants*

used. They are derived over 100 Monte Carlo runs as shown in Fig. 2. We aim to illustrate the following two aspects: (i) it is necessary to address the linearisation errors for designing the distributed consensus EKF and (ii) the tracking performance can be improved by introducing neighbouring information in the computations of the Kalman gain matrices and the upper bound matrices. The algorithm denoted EKCF corresponds to the case where the EKF is used in the suboptimal KCF [12] (i.e. the consensus terms on prior estimates are introduced to update the state estimates but the Kalman gain matrices and the covariance matrices are derived by using the standard EKF.). The algorithm denoted LE-EKCF is computed based on the EKCF where the linearisation errors have been addressed by using the variance-constrained approach. However, the neighbouring information is not used to compute the Kalman gain matrices and the upper bound matrices in the LE-EKCF. It can be seen that the LE-EKCF outperforms the EKCF. This indicates that the tracking performance can be improved by applying the variance-constrained approach to the linearisation errors. The proposed filter and the distributed EKF using the hybrid consensus strategy in [21] are denoted by LE-VC-EKF and HC-EKF, respectively. Simulation results suggest that the proposed LE-VC-EKF performs better than the LE-EKCF. This is due to the fact that the neighbouring information has been used to design the Kalman gain matrices and the upper bound matrices in the LE-VC-EKF. Notice that the HC-EKF performs the best since more information have been exchanged for consensus including the information pairs and the measurement pairs. A modest conclusion can be drawn that the proposed filter provides a comprise between the tracking performance and the communication cost.

5 Conclusions

In this paper, we have presented a distributed consensus EKF for non-linear systems by using the strategy of consensus on prior estimates. A distinct feature of the proposed filter is that the variance-constrained approach is used to determine the Kalman gain matrix by optimising an upper bound matrix despite the linearisation errors and the edge-covariance matrices. This is the main contribution of this paper with respect to the existing consensus-based non-linear filters. Future work will focus on the stability analysis with collective observability.

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