OF 3 Governing Equations

- Two dimensional Euler Eqns in vector form
  - identify: ~~the~~ $\underline{U}$, $\underline{E}$, $\underline{G}$

$\cancel{P=\rho RT}$

$$\frac{\partial}{\partial t}\underline{U}+\frac{\partial}{\partial x}\underline{E}+\frac{\partial}{\partial y}\underline{G}=0$$

$$P=(\gamma-1)\left(E-\frac{1}{2}\rho\left(u^2+v^2\right)\right)$$

Where

$$\underline{U}=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ E\end{bmatrix},\quad \underline{E}=\begin{bmatrix}\rho u\\ \rho u^2+P\\ \rho uv\\ (E+P)u\end{bmatrix},\quad \underline{G}=\begin{bmatrix}\rho v\\ \rho uv\\ \rho v^2+P\\ (E+P)v\end{bmatrix}$$

where ρ = density
u = velocity component in x-direction
v = velocity component in y-direction
E = total energy per unit volume
p = pressure
γ = specific heat ratio = 1.4

- working definitions of local Mach # & sound speed

$$Ma = \frac{|u|}{a} = \frac{|u|}{\sqrt{\frac{\gamma p}{\rho}}}$$

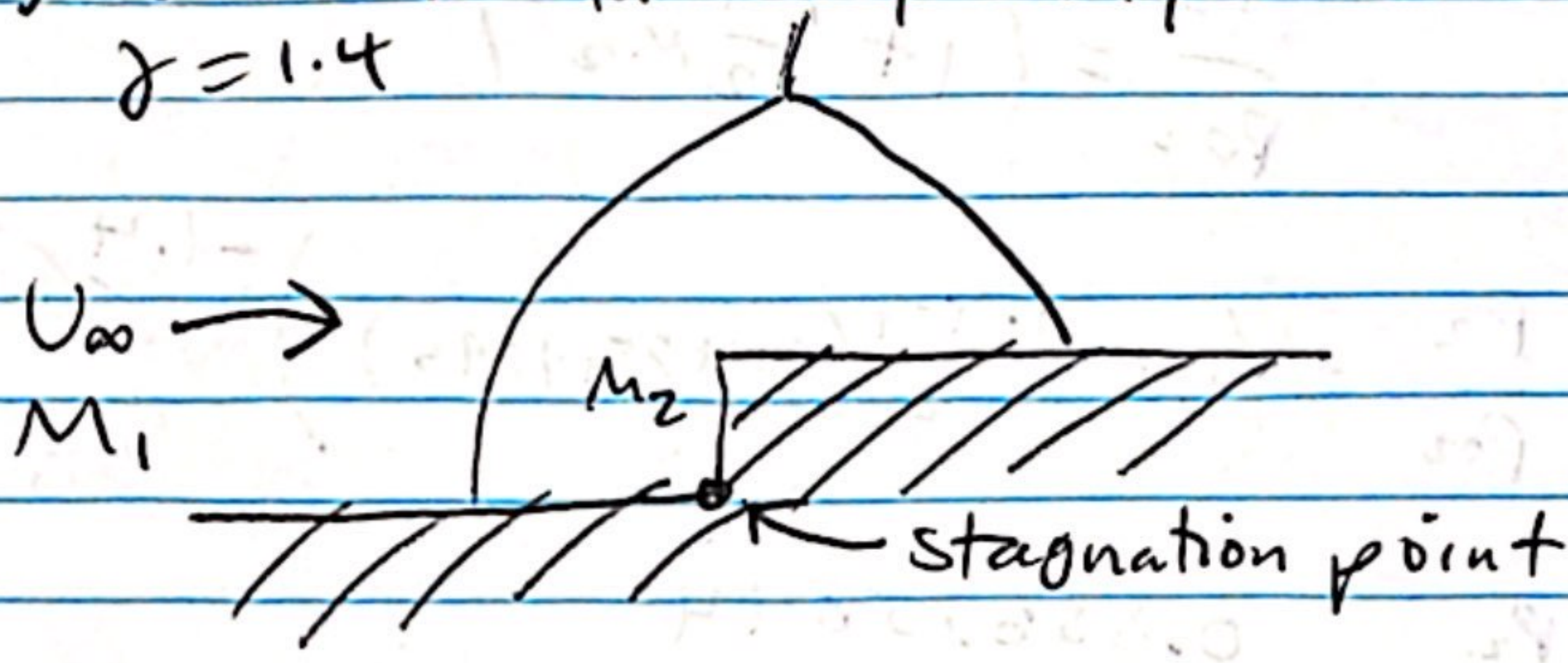
$$a = \sqrt{\frac{\gamma p}{\rho}}$$

- local mach # is ratio of local flow velocity to the local speed of sound
  where $|u|$ is magnitude of flow velocity
  a is local speed of sound

- Local speed of sound is speed at which small disturbances propogate in the fluid (air),
  where γ is specific heat ratio $= 1.4$
  p = pressure
  ρ = density of fluid

• Estimate Pressure at Stagnation of the step at $(x,y) = (0.6, 0)$ for $M_1 = 3$, $P_1 = 1\,Pa$

$\gamma = 1.4$



$M_1 = 3$

using eqn: $$M_1^2 = \frac{(\gamma-1)M_2^2 + 2}{2\gamma M_2^2 - (\gamma-1)}$$

$$3^2 = \frac{(1.4-1)M_2^2 + 2}{2(1.4)M_2^2 - (1.4-1)}$$

$$\Rightarrow M_2 = 0.47519096$$

using eqn: $$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1}$$

$$\frac{P_2}{P_1} = \frac{2(1.4)(3)^2 - (1.4-1)}{1.4+1} = 10.33333$$

$$\Rightarrow P_2 = P_1 \cdot \frac{P_2}{P_1} = (1\,Pa)(10.333) = 10.333\,Pa$$

using eqn:

$$\frac{P_2}{P_{o2}} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{-\gamma/\gamma-1}$$

$$\frac{P_2}{P_{o2}} = \left(1 + \frac{1.4-1}{2}(0.47519096)^2\right)^{-1.4/(1.4-1)}$$

$$\Rightarrow \frac{P_2}{P_{o2}} = 0.85675844$$

$$\Rightarrow P_{o2} = \frac{P_2}{P_2/P_{o2}} = \frac{10.3333\,Pa}{0.85675844}$$

$$\Rightarrow \boxed{P_{o2} = 12.06096\ Pa}$$