Implementation for Fast direct sparse solvers

Guanchun Li, Zhe Chen

Courant, NYU

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Intro

- Fast direct sparse solvers(FDSS)[1]: linear or close-to-linear complexity
  Assume $O(m^\alpha)$ for m-by-m Schur complement matrix

$\boxed{\alpha < 2}$ Geometric sum dominated by the *leaves*. Complexity is $O(N)$.

$\boxed{\alpha = 2}$ Every term has the same weight. Complexity is $O(n^2 L) = O(N \log N)$.

$\boxed{\alpha > 2}$ Geometric sum dominated by the *root*. Complexity is $O(n^\alpha) = O(N^{\alpha/2})$.

Elliptic PDEs in a unit square $[0, 1]^2$ in 2D with Dirichlet BC:

1. Laplace Eq. $-\Delta u = 0$, $u = \sin(\pi x) \exp(\pi y)$
2. Helmhotz Eq. $-\Delta u = \kappa^2 u$, $u = \sin(\kappa x) + cos(\kappa y)$

[1] https://github.com/CecilMartin/Fast-Direct-Sparse-Solvers

Sweeping scheme

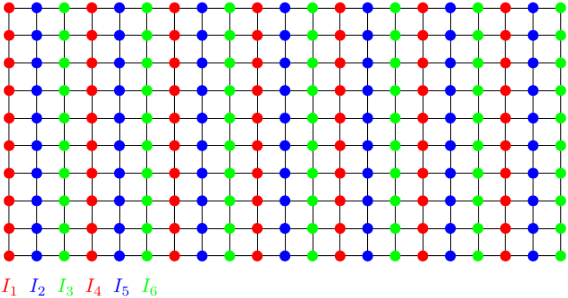

Figure 1: Partition of sweeping scheme, $n_1 = 21, n_2 = 10$

Buffered sweeping scheme

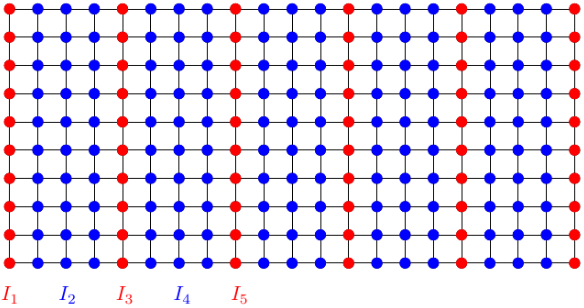

Figure 2: Partition of buffered sweeping scheme, $n_1 = 1 + m(b+1)$

Compressible

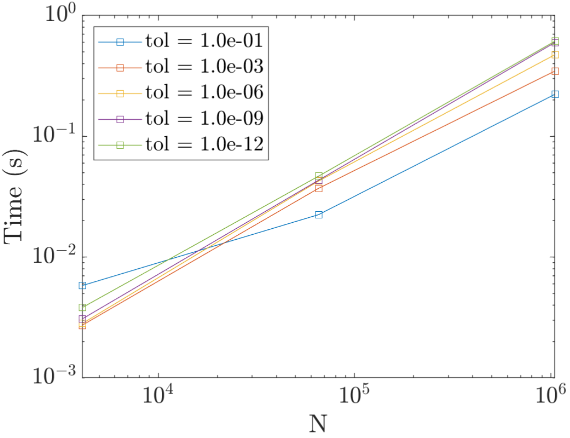

Figure 3: Skeletonization factorization of Schur complement, Laplace 5pt stencil, N=16[2,3,4]

Complexity for Laplace

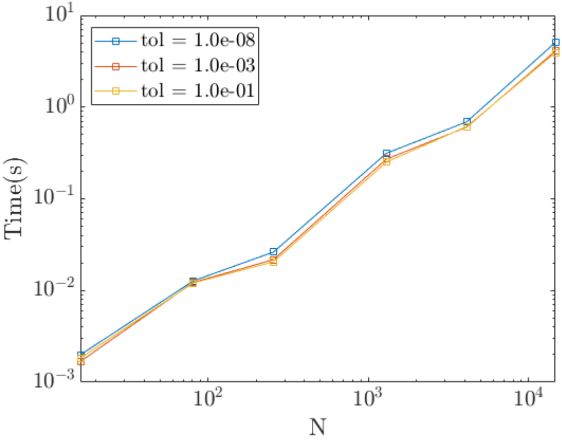

Figure 4: Laplace 5pt stencil, Sweeping scheme, $N=4^{[2,\cdots,7]}$

Complexity for Helmhotz

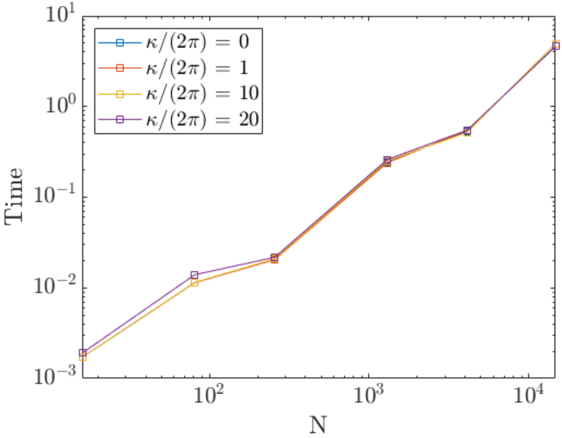

Figure 5: Helmhotz equation, 5pt stencil, $\kappa/(2\pi) = 0, 1, 10, 20$

Complexity for Buffered sweeping

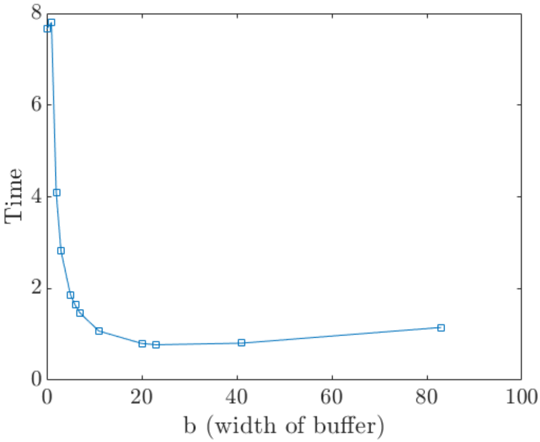

Figure 6: Buffered sweeping scheme, Laplace 5pt stencil, $n_1 = 169$; $b = [0, 1, 2, 3, 5, 6, 7, 11, 20, 23, 41, 83]$