Signal Analog Demodulation with GNU Radio

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1 Part I

In this part we will study the theory of signal modulation in order to be able to demodulate several signals in Parts II and III.

1.1 Expression of the signals

Let's express the signals $r_R(t)$ and $r_I(t)$.

We know that :
(A) $r_{RF}(t) = s_{RF}(t) = s_R(t)cos(2\pi f_0 t) - s_I(t)sin(2\pi f_0 t)$
(1) $s_{RF}(t) = A(t)cos(2\pi f_0 t + \phi(t))$
(2) $s_{RF}(t) = s_R(t)cos(2\pi f_0 t) - s_I(t)sin(2\pi f_0 t)$
(B) $2cos\alpha cos\beta = cos(\alpha - \beta) + cos(\alpha + \beta)$
(C) $2sin\alpha cos\beta = sin(\alpha - \beta) + sin(\alpha + \beta)$
(D) $2sin\alpha sin\beta = cos(\alpha\beta)cos(\alpha + \beta)$

Let's express $\widetilde{r}_R(t)$:

$\widetilde{r}_R(t) = r_{RF}(t)cos(2\pi f_c t)$

$\widetilde{r}_R(t) = (s_R(t)cos(2\pi f_0 t) - s_I(t)sin(2\pi f_0 t))cos(2\pi f_c t)$

$\widetilde{r}_R(t) = s_R(t)cos(2\pi f_0 t)cos(2\pi f_c t) - s_I(t)sin(2\pi f_0 t)cos(2\pi f_c t)$

$(B)(C) : \widetilde{r}_R(t) = s_R(t)\frac{1}{2}[cos(2\pi f_0 t - 2\pi f_c t) + cos(2\pi f_0 t + 2\pi f_c t)] - s_I(t)\frac{1}{2}[sin(2\pi f_0 t - 2\pi f_c t) + sin(2\pi f_0 t + 2\pi f_c t)]$

$\widetilde{r}_R(t) = s_R(t)\frac{1}{2}[cos(2\pi(f_0 - f_c)t + cos(2\pi(f_0 + f_c)t)] - s_I(t)\frac{1}{2}[sin(2\pi(f_0 - f_c)t) + sin(2\pi(f_0 + f_c)t)]$

In a same way:

$\widetilde{r}_I(t) = -r_{RF}(t)sin(2\pi f_c t)$

$\widetilde{r}_I(t) = (s_I(t)sin(2\pi f_0 t) - s_R(t)cos(2\pi f_0 t))sin(2\pi f_c t)$

$\widetilde{r}_I(t) = s_I(t)sin(2\pi f_0 t)sin(2\pi f_c t - s_R(t)cos(2\pi f_0 t)sin(2\pi f_c t)$

$(C)(D) : \widetilde{r}_I(t) = s_I(t)\frac{1}{2}[cos(2\pi f_0 t - 2\pi f_c t) - cos(2\pi f_0 t + 2\pi f_c t)] - s_R(t)\frac{1}{2}[sin(2\pi f_0 t - 2\pi f_c t) + sin(2\pi f_0 t + 2\pi f_c t)]$

$\widetilde{r}_I(t) = s_I(t)\frac{1}{2}[cos(2\pi(f_0 - f_c)t) - cos(2\pi(f_0 + f_c)t)] - s_R(t)\frac{1}{2}[sin(2\pi(f_0 - f_c)t) + sin(2\pi(f_0 + f_c)t)]$

1.2 Taking f_0 as central frequency

We take $f_c = f_0$ and we want to know the characteristics of the filter h located just before the analog-to-digital converter in order to get $r_R(t) = s_R(t)$ and $r_I(t) = s_I(t)$.

$f_c = f_0$ leads to:

$\widetilde{r}_R(t) = \frac{1}{2}s_R(t)[1 + cos(2\pi 2f_0t)] - \frac{1}{2}s_I(t)sin(2\pi 2f_0t)]$

$\widetilde{r}_I(t) = \frac{1}{2}s_I(t)[1 - cos(2\pi 2f_0t)] - \frac{1}{2}s_R(t)sin(2\pi 2f_0t)]$

Let's pass in frequency domain by using Fourier transform:

$$\widetilde{R}_R(f) = \frac{1}{2}[S_R(f) + \frac{S_R(f)}{2} * (\delta(f - 2f_0) + \delta(f + 2f_0))\frac{S_I(f)}{2j} * (\delta(f - 2f_0) - \delta(f + 2f_0))$$

Which has this allure: We need to delete both sides and double the central signal, which leads to this function for our filter H(f):

$$H(f) = \begin{cases} 2 & \text{if} \quad |f| \leq B \\ 0 & \text{if} \quad |f| \geq 2f_0 \end{cases}$$

1.3 What use for this filter ?

The receptor we are studying cannot be used for wide band signals because if $\frac{B}{2} > f_0$, we get a covering and we cannot isolate the signal we want. It can only be used for narrow band signals.
Wide band signal are quite unusual but do exist (i.e acoustic signals or underwater communications). They need a special filter called Hilbert filter.

1.4 Sampling frequency F_e

Shannon Theory : $F_e > 2F_{max}$
Here, $F_{max} = \frac{B}{2}$ so we need $F_e > B$.

1.5 Analog-to-digital converter

We need to sample the signal after having filtered it because if we don't, the frequency spectrum will be wider and it will need a lot of symbols to proceed to the analog-to-digital conversion.
Therefore, there are more and more signal digitalizing near the reception to avoid errors from the hardware, which needs more power.

1.6 Analytic signal

An analytic signal is a complex-valued function that has no negative frequency components. The real and imaginary parts of an analytic signal are real-valued functions related to each other by the Hilbert transform.
This analytic is got within 3 steps:

- Its amplitude is doubled.
- The negative frequencies are cut.
- The signal is centered around 0.

Since we want to get a real signal to be playable and listen to it, we need to express the analytic signal and its complex envelop with f_0.

We know that:

- $S_{RF}(f) = S *_{R\ F}(-f)$ because the Fourier transform of a real signal is even.
- $s_{RF}(t) = s_R(t)cos(2\pi f_0t) - s_I(t)sin(2\pi f_0t)$
- The expression of the analytic signal is: $s_a(t) = s_{RF}(t) + jHs_{RF}(t)$
- And its Fourier tranform is: $S_a(f) = S_{RF}(f) + sgn(f)S_{RF}(f)$.

We need to express the Fourier transform of $s_{RF}(t)$ ($S_{RF}(f)$) first:

$S_{RF}(f) = \frac{1}{2}[S_R(f) * (\delta(f - f_0) + \delta(f + f_0))] + j\frac{1}{2}[S_I(f) * (\delta(f - f_0) - \delta(f + f_0))]$

$S_{RF}(f) = \frac{1}{2}[S_R(f - f_0) + S_R(f + f_0)] + j\frac{1}{2}[S_I(f - f_0) - S_I(f + f_0)]$

Because we only want the positive frequencies, we got: $S_{RF}(f) = \frac{1}{2}(S_R(f - f_0) + jS_I(f - f_0))$

Let's express the Fourier transform of $s_a(t)$:
$S_a(f) = S_{RF}(f) + sgn(S_{RF}(f))$

$S_a(f) = 2S_{RF}(f)$ because the frequencies are positive.

$S_a(f) = S_R(f - f_0) + jS_I(f - f_0)$

Finally, $S_F(f) = S_a(f + f_0) = S_R(f) + jS_I(f)$

Which leads to $s(t) = s_R(t) + js_I(t)$ when applying the inverse of Fourier tranform.

2 Part II

2.1 Frequency analysis of the radio signal record

We have been given a record of a radio signal, and we will use the software GNURadio to perform an analysis of it. First of all we want to display the specter of the signal.

To do so, we need three different blocks in the system:

- File source: This block is used to import the the record of the radio signal
- Throttle: By setting the sample rate, we limit the data throughput of the signal
- QT GUI Frequency Sink: This block is used to visualize the specter of the signal.

In addition to these blocks we have also defined two variables, one for the sampling frequency and one for the centering frequency. Both these values are given, with $F_e = 1,5MHz$ and $F_c = 99,5MHz$. The variables are used in the other blocks where the sample rate and centering frequency is demanded. In addition to this the Frequency Sink Bandwidth is also equal to the sampling frequency. According to the Shanon Theorem, we have a signal with frequencies going from $-\frac{F_e}{2}$ to $+\frac{F_e}{2}$ witch gives us a bandwidth of F_e.

When we display the spectre of the signal we can clearly see two frequency channels that distinct themselves from the noise. They are centered at approximately $99,106MHz$ and $99,99Mhz$. By using a table over the FM radio channels in Toulouse, we can deduct that the two channels detected and recorded by the antenna are RFM at $99,10MHz$ and SkyRock at $100MHz$.

The power of the two radio signal are approximately $-53,49dB$, and the power of the noise is $-92,98dB$. This gives a signal to noise ratio of: $SNR = S(dB) - N(dB) = -53,49 - (-92,98) = 40,13dB$. This gives us a power of the signal that is approximately 10 000 times greater than the power of the noise, which is sufficient for the demodulation of the signal.

We have also measured the bandwidth of the channels, which is approximately $200kHz$.

2.2 Extraction of one channel by frequency translation followed by a low-pass filter

To extract one of the channels there are two steps that we need to do:

1. Translate the signal in frequency so that it is centered around the frequency of interest
2. Filter the signal with a low-pass filter

We use a complex envelop, where the signal is translated in function of the given frequency. $r_l[k] = r[k] \exp -j2\pi \frac{f_l}{F_e} k$

In order to implement this in the GNURadio software, we have to add three block to our system.

- A Signal Source to generate the complex envelop
- A QT GUI Range so that we can vary the f_l frequency of the complex envelop
- A Multiply so that we can multiply the radio signal with the envelop

2.2.1 Offset frequency

The offset needed to center one of the channels, is equal to the frequency of the channel minus the center frequency, $f_1 - f_c$. For the two channels in question, the offset frequency will be:

- RFM: $f_l = 99,1MHz - 99,5Mhz = -400kHz$
- SkyRock: $f_l = 100MHz - 99,5MHz = 500kHz$

We limit the range of the f_l to the value of F_e, if we have an offset that is greater than this it will introduce parts of a copy of the signal seeing that it is periodical. At this point we have centered the signal around zero instead of the centering frequency $f_c = 99,5MHz$. We will therefore limit the range of f_l to go from $-\frac{F_e}{2}$ to $+\frac{F_e}{2}$

2.2.2 Low-pass filter

The last step is to use a low-pass filter to remove the noise from our signal. We added a block to our system with the following parameters:

- Decimation equal to 6 in order to reduce the calculation charge
- Sample rate of 1,5MHz.
- Cut Off Frequency equal to 100kHz (because the signals bandwidth is 200kHz, the signal is centered around zero and a low-pass filter is symmetrical.
- Transition width of 10% of the cut off frequency, i.e. 10kHz

In the end we use the following system to extract one of the radio channels:

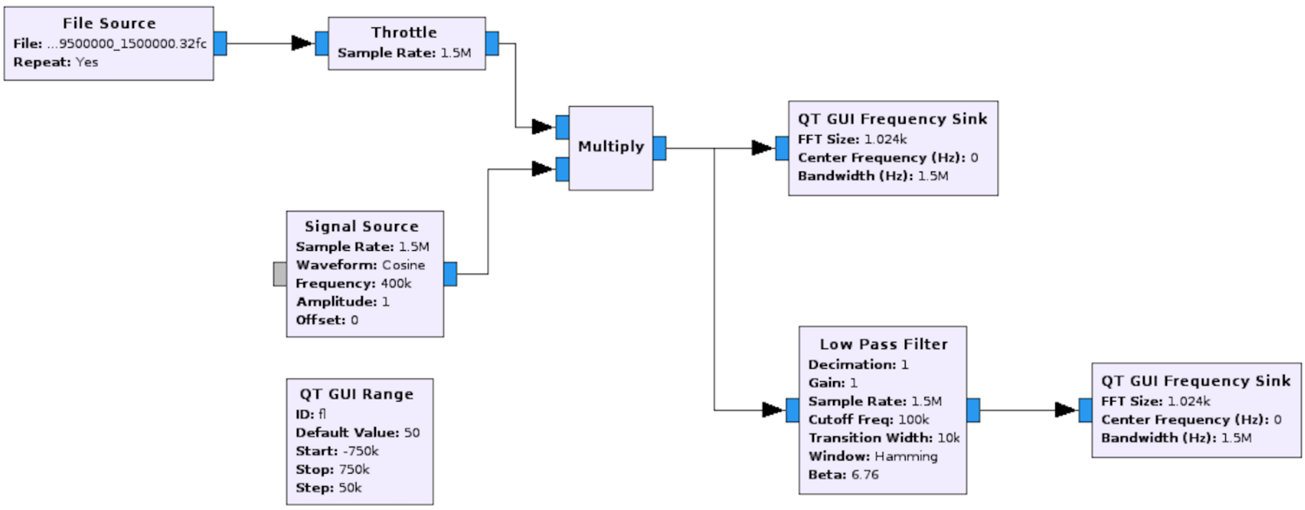

Figure 1: System to extract one channel

We tested the system for both the SkyRock and the RFM channel. By using the QT GUI Range we could change the offset frequency, and therefor which of the two channels that were centered at all time. After the low-pass filter the only part of the signal that remains is the channel that is centered.

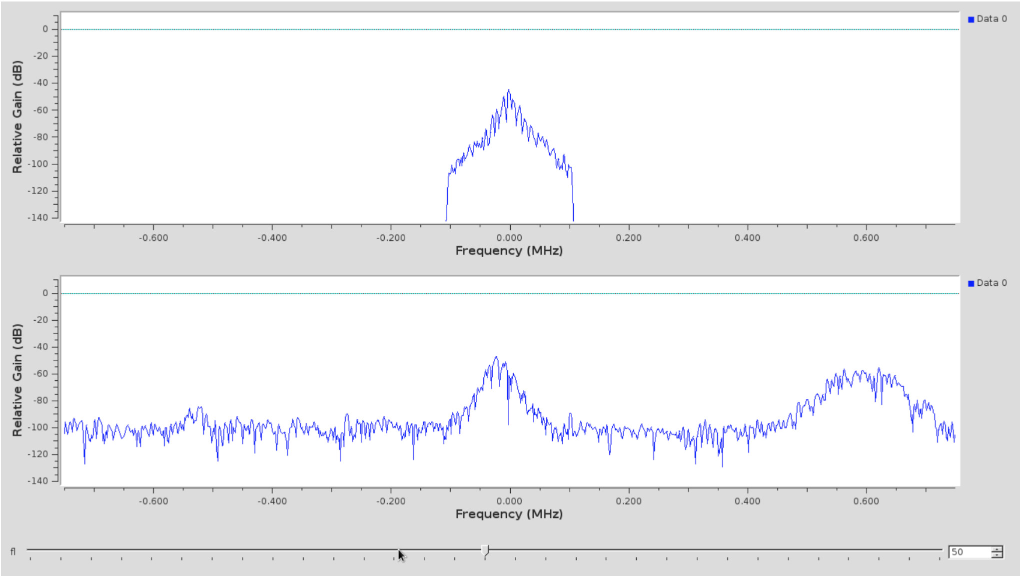

Figure 2: Extraction of the SkyRock channel

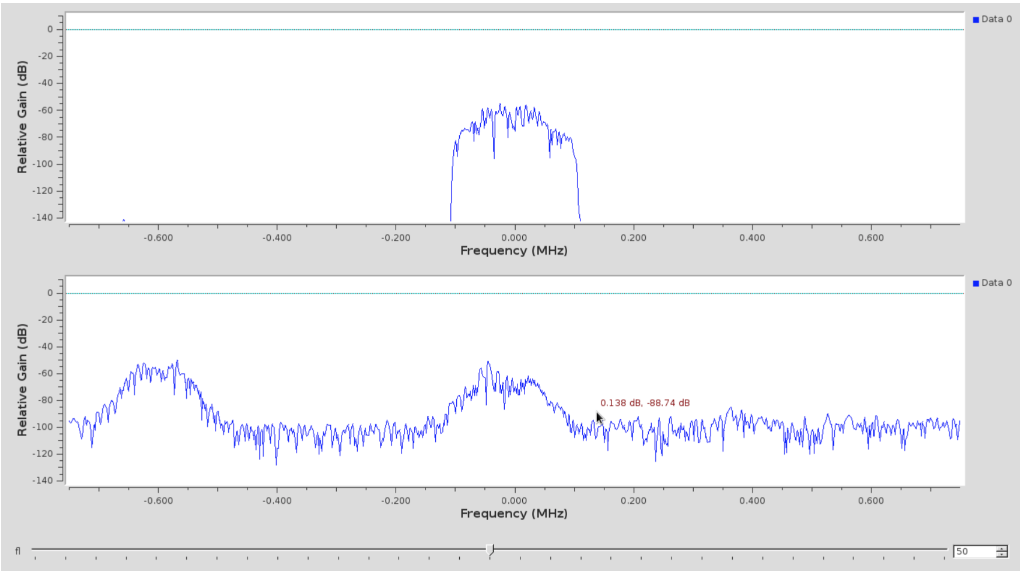

Figure 3: Extraction of the RFM channel

We can observe on both graphs that the signal that is left after the low-pass filter contains only the radio channel signal, and that it has been filtered at $\pm 100kHz$ equal to the cut off frequency parameter.

2.3 Frequency demodulation and restitution of the signal

2.3.1 Carson bandwidth rule

After the Carson bandwidth rule, the value of the bandwidth can be approached by the following formula: $B_{FM} \approx 2(\Delta f + f_m)$. Where f_m is the max frequency and Δf is the max frequency deviation. We applied the Carson bandwidth rule on our signal.

Our signal is composed of stereophonic channels with a max frequency of 15kHz, and the frequency deviation is limited to 75kHz. After the rule the bandwidth of the two radio channels measured previously should be the following: $B_{FM} = 2 * (75kHz + 15kHz) = 180kHz$
We measured the bandwidth of the two radio channels to be 200kHz, and with the uncertainty of the measure done on the graph we can deduct that the measured bandwidth confirms the theory.

2.3.2 Expression of the signals $y_l[k]$

We want to demonstrate that:

$$y_l[k] = b[k] + Ae^{jk_f \sum_{i=0}^{k} m[i]}$$

We saw previously that this signal...

$$s_{RF}(t) = Acos(2\pi f_0 t + \phi(t))$$

...has its complex envelop such as:

$$s(t) = Aej\phi(t)$$

By identifying ϕ in (16), we got:

$$s(t) = Ae^{jk_f \frac{\Delta F}{max|m|}} \int_{-\infty}^{t} m(u)\,\mathrm{d}u$$

With:

$$t = kT_e$$

and

$$u = iT_e$$

We can replace this integral with the sum of the sampled discrete values of the message m:

$$s[k] = Ae^{jk_f \frac{\Delta F}{max|m|} \sum_{i=0}^{k} m[i]}$$

And because of the periodicity:

$$s[k] = s[kT_e]$$

Then by getting m[k] using the equation (19), we show that the frequency demodulation can be done:

$$\widetilde{m}_f[k] = arg(y[k]y*[k-1])$$

$$\widetilde{m}_f[k] = arg(e^{j(\sum_{i=0}^{k} m[i] - \sum_{i=0}^{k-1} m[i])})$$

$$\widetilde{m}_f[k] = arg(e^{jm[k])})$$

$$\widetilde{m}_f[k] = m[k]$$

2.3.3 Frequency Demodulation

After having successfully extracted one channel, we wanted to demodulate it so that we could listen to the record. We added a WBFM Receive block to our system in order to do so. Seeing that we introduced a block that decimates the signal by a factor $\frac{1}{6}$ earlier in the system, we will use a Quadrature Rate (sampling rate) of $\frac{F_e}{6}$ for the WBFM Receive.

We also need to adapt the signal to the sampling rates supported by the sound card. Due to the Shanon/Nyquist theorem we know that the smallest frequency possible not to distort the signal will be $F_e > 2 \times f_{max} = 2 \times 15kHz = 30kHz$. In our case the minimal sampling rate will be 30kHz. For the sampling rates supported by the sound card we can chose between 32kHz, 44kHz and 48kHz. We chose to procede with a sampling rate of 48kHz (which is also the default value for applications that uses the sound card in question).

After demodulation, we got the spectrum as shows in figure 4. We can see that we managed to isolate, center and double the middle part of the signal and delete both sides. We can now listen to this demodulated signal just the same way as we would listen to the radio.

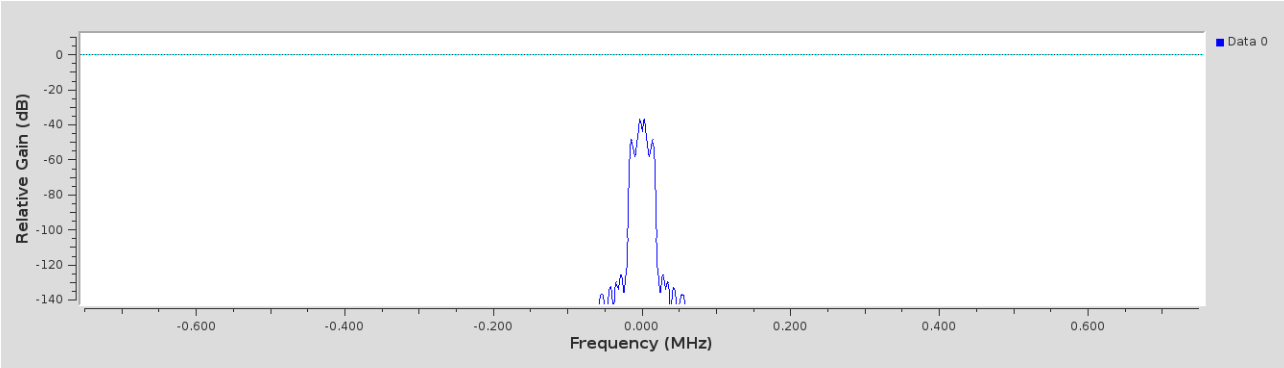

Figure 4: Capture of our signal's spectrum after demodulation

2.3.4 Restitution

After having successfully restituted the signal, we could listen to the record of the radio signal from SkyRock and confirm that the winner of the Sam Smith album was Jordi.

2.4 Real-time receiving of radio signal

In this part, we do not want to use a recorded sample anymore, but listen in real time to the radio we choose at the moment. To do so, we need to be able to change our RLC components to adapt the receiving circuit to the channel we want to listen to. That is why we use an USRP, which embeds an FPGA card inside and enables a software-based radio reception. By changing our f_l frequency on the GNURadio window, we can adapt the electronic circuit embedded in the USRP to listen to different frequency, depending on our current mood, France Classique or Radio Campus...

3 Part III

In this part we will study the High Frequencies (HF) band. The HF band goes from $3MHz$ to $30MHz$, and we will focus on the frequencies between $11,175MHz$ and $11,4MHz$, that is reserved for VOLMET messages.

3.1 Frequency analysis of the signal

We display the spectre from the recording by using a QT GUI Frequency Sink with a centering frequency equal to $f_c = 11,2965MHz$ and bandwidth equal to $F_e = 250kHz$. This will display the spectre between $f_c - \frac{F_e}{2}$ and $f_c + \frac{F_e}{2}$.

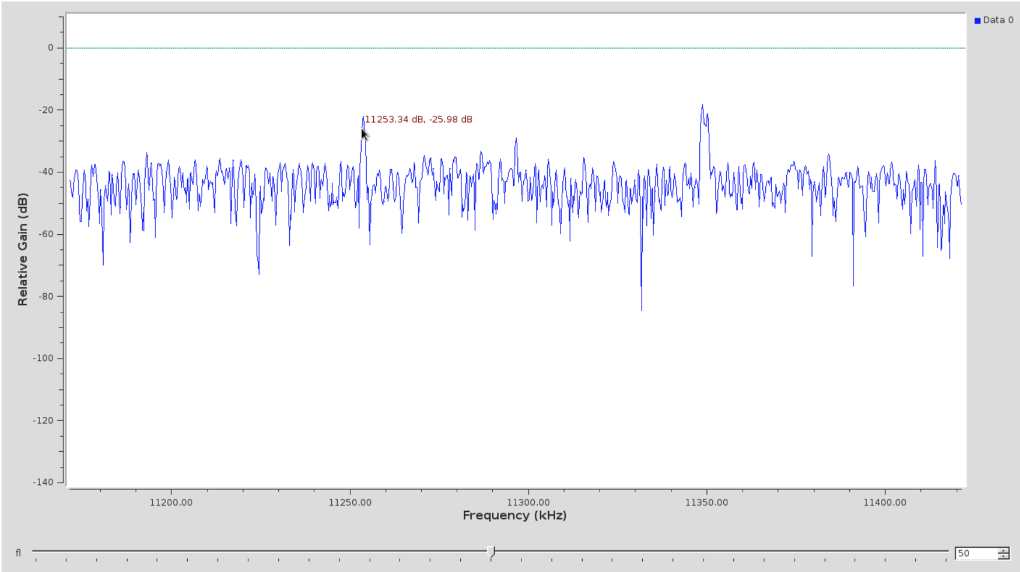

Figure 5: Display of messages in the HF band

A table of the different VOLMET stations shows that the St Eval base transmit with a frequency equal to $11,253MHz$. We can see that there is a spike in the spectre of the signal at that frequency, so we have received messages from the St Eval base.

11 MHz									
11.247	*, 35*	MTS	FLK	MOUNT PLEASANT	*VIPER*		-51 50 14	-58 28 14	IRREGULAR HOURS
11.253	*Cont*	MKL	GBR	ST. EVAL	*MILITARY ONE*		50 28 58	-5 00 00	
11.297	*25 , 55*	RLAP	RUS	ROSTOV-ON-DON	*ROSTOV METEO*	*RR*	47 15 12	39 49 02	DAY
11.318	*00 , 30*	UBB-2	RUS	SYKTYVKAR	*SIVKAR METEO*	*RR*	61 38 17	50 31 49	DAY
	10 , 40	UNNN	RUS	NOVOSIBIRSK	*NOVOSIBIRSK METEO*	*RR*	55 00 16	82 33 44	DAY
	15 , 45	RQCI	RUS	SAMARA	*SAMARA METEO*	*RR*	53 11 00	49 46 00	DAY
11.369	*01 ,*	LWB	ARG	BUENOS AIRES	*EZEIZA AERADIO*	*SS*	-34 49 59	-58 31 55	
11.387	*00 , 30*	AXQ-421	AUS	BRISBANE	*AUSTRALIAN VOLMET*		-27 04 06	153 03 17	
	05 , 35	AWC	IND	CALCUTTA	*KOLKATA RADIO*		22 38 00	88 27 00	0305 - 1240 Z
	10 , 40	HSD	THA	BANGKOK	*BANGKOK RADIO*		13 44 00	100 30 00	2310 - 1145 Z
	15 , 45	ARA	PAK	KARACHI	*KARACHI RADIO*		25 54 00	67 09 00	0145 - 1450 Z
	20 , 50	9VA-43	SNG	SINGAPORE	*SINGAPORE RADIO*		1 20 11	103 41 10	2250 - 1225 Z
	25 , 55	AWB	IND	BOMBAY	*MUMBAI RADIO*		19 05 15	72 51 09	0325 - 1300 Z
11.393	*15 , 45*	EPD	IRN	TEHRAN	*TEHRAN RADIO*		35 41 00	51 16 00	OUT OF SERVICE
	20 , 50	EQP	IRN	SHIRAZ			29 32 00	52 35 00	PLANNED
	25 , 55	TCB	TUR	ISTANBUL	*YELISKOY RADIO*		40 58 00	28 50 00	OUT OF SERVICE

Figure 6: Table of VOLMET stations in the 11MHz band

3.2 Frequency translation

We use a complex envelop to translate the spectre in frequency, the system in GNURadio is built up the same way as previously.
The offset frequency that is necessary in order for the St Eval base station signal to be centered around zero is equal to $f_c - f_1 = 11,2965MHz - 11,253MHz = 43,5kHz$.

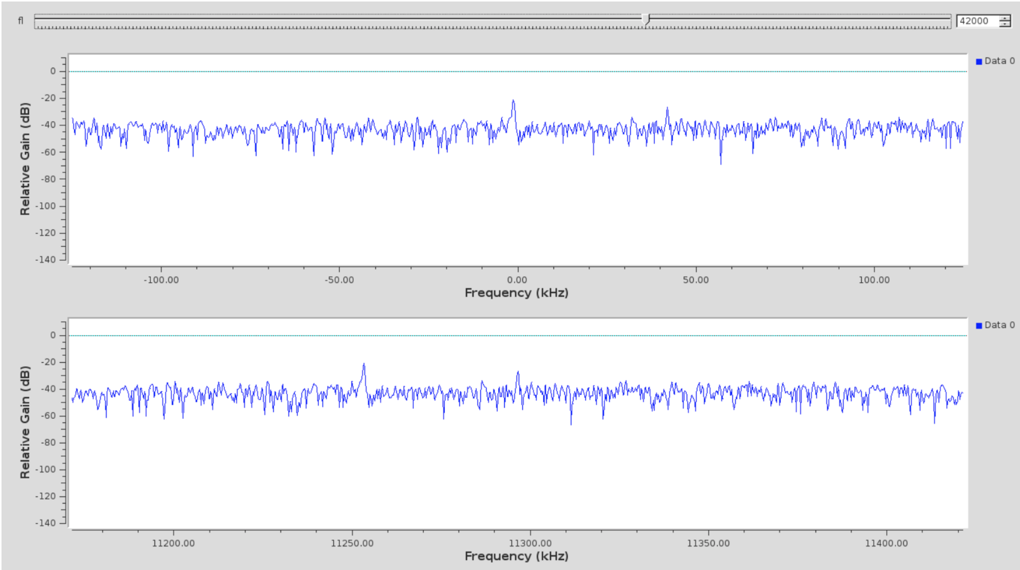

Figure 7: Frequency translation to center the signal from St Eval

By preforming a frequency translation of $\sim 43,5kHz$ we can observe that the spike that was previously centered around 11,253MHz, with 11,2965MHz as the centering frequency of the spectre, has now been translated to the zero frequency.