Table of Laplace and Z Transforms

Laplace and Z Transforms | **Laplace Properties** | **Z Xform Properties**

Using this table for Z Transforms with Discrete Indices

Shortened 2-page pdf of Laplace Transforms and Properties

Shortened 2-page pdf of Z Transforms and Properties

All time domain functions are implicitly=0 for t<0 (i.e. they are multiplied by unit step).

Entry	Laplace Domain	Time Domain *(Note)*	Z Domain (t=kT)
unit impulse	1	$\delta(t)$ unit impulse	1
unit step	$\Gamma(s)=\frac{1}{s}$	$\gamma(t)$ *(Note)*	$\frac{z}{z-1}$
ramp	$\frac{1}{s^2}$	t	$T\frac{z}{(z-1)^2}$
parabola	$\frac{2}{s^3}$	t^2	$T^2\frac{z(z+1)}{(z-1)^3}$
t^n (n is integer)	$\frac{n!}{s^{(n+1)}}$	t^n	
exponential	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
power		$b^k \quad (b=e^{-aT})$	$\frac{z}{z-b}$
time multiplied exponential	$\frac{1}{(s+a)^2}$	te^{-at}	$T\frac{ze^{-aT}}{(z-e^{-aT})^2}$
Asymptotic exponential	$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
double exponential	$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{(b-a)}$	$\frac{z(e^{-aT}-e^{-bT})}{(b-a)(z-e^{-aT})(z-e^{-bT})}$
asymptotic double exponential	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left(1-\frac{be^{-at}-ae^{-bt}}{(b-a)}\right)$	
asymptotic critically damped	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{(1-e^{-Ta}(1+Ta))z^2+e^{-Ta}(Ta-1+e^{-Ta})z}{a^2(z-e^{-Ta})^2(z-1)}$
differentiated critically damped	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$\frac{z(z-(Ta+1)e^{-Ta})}{(z-e^{-Ta})^2}$
sine	$\frac{\omega_0}{s^2+\omega_0^2}$	$\sin(\omega_0 t)$	$\frac{z\sin(\omega_0 T)}{z^2-2z\cos(\omega_0 T)+1}$
cosine	$\frac{s}{s^2+\omega_0^2}$	$\cos(\omega_0 t)$	$\frac{z(z-\cos(\omega_0 T))}{z^2-2z\cos(\omega_0 T)+1}$
decaying sine	$\frac{\omega_d}{(s+a)^2+\omega_d^2}$	$e^{-at}\sin(\omega_d t)$	$\frac{ze^{-aT}\sin(\omega_d T)}{z^2-2ze^{-aT}\cos(\omega_d T)+e^{-2aT}}$
decaying cosine	$\frac{s+a}{(s+a)^2+\omega_d^2}$	$e^{-at}\cos(\omega_d t)$	$\frac{z(z-e^{-aT}\cos(\omega_d T))}{z^2-2ze^{-aT}\cos(\omega_d T)+e^{-2aT}}$
generic decaying oscillatory	$\frac{Bs+C}{(s+a)^2+\omega_d^2}$	$e^{-at}\left(B\cos(\omega_d t)+\frac{C-aB}{\omega_d}\sin(\omega_d t)\right)$	
generic decaying oscillatory (alternate)	$\frac{Bs+C}{(s+a)^2+\omega_d^2}$	$Me^{-at}\cos(\omega_d t+\phi)$ $M=\sqrt{B^2+\left(\frac{C-aB}{\omega_d}\right)^2}$ $\phi=-\mathrm{atan}\left(\frac{C-aB}{B\omega_d}\right)=-\mathrm{atan2}(C-aB,B\omega_d)$ *(Note)*	
Z-domain generic decaying oscillatory		$\sqrt{\frac{a^2d^2+b^2-2abc}{d^2-c^2}}\cdot d^n\cdot\cos(\omega_d n+\phi)$	$\frac{az^2+bz}{z^2+2cz+d^2}, \quad d>0$

		$\omega_d = \operatorname{acos}\left(-\frac{c}{d}\right), \quad \phi = \operatorname{atan}\left(\frac{ac-b}{a\sqrt{d^2-c^2}}\right)$ *(Note)*	
Prototype Second Order System (ζ<1, underdampded)			
Prototype 2^{nd} order lowpass step response	$\frac{\omega_0^2}{s\left(s^2+2\zeta\omega_0 s+\omega_0^2\right)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}t+\phi\right)$ $\phi=\operatorname{acos}(\zeta)$	$\frac{z}{z-1}-\frac{1}{\sqrt{1-\zeta^2}}\frac{z^2\sqrt{1-\zeta^2}+ze^{-\zeta\omega_0 T}\sin\left(\omega_0\sqrt{1-\zeta^2}T-\phi\right)}{z^2-2ze^{-\zeta\omega_0 T}\cos\left(\omega_0\sqrt{1-\zeta^2}T\right)+e^{-2\zeta\omega_0 T}}$ $\phi=\operatorname{acos}(\zeta)$
Prototype 2^{nd} order lowpass impulse response	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}t\right)$	$\frac{\omega_0}{\sqrt{1-\zeta^2}}\frac{ze^{-\zeta\omega_0 T}\sin\left(\omega_0\sqrt{1-\zeta^2}T\right)}{z^2-2ze^{-\zeta\omega_0 T}\cos\left(\omega_0\sqrt{1-\zeta^2}T\right)+e^{-2\zeta\omega_0 T}}$
Prototype 2^{nd} order bandpass impulse response	$\frac{2\zeta\omega_0 s}{s^2+2\zeta\omega_0 s+\omega_0^2}$	$-\frac{2\zeta\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\cdot\omega_0 t-\phi\right)$ $\phi=\operatorname{acos}(\zeta)$	$-\frac{2\zeta\omega_0 T}{\sqrt{1-\zeta^2}}\frac{-z^2\sqrt{1-\zeta^2}+ze^{-\zeta\omega_0 T}\sin\left(\sqrt{1-\zeta^2}\cdot\omega_0 T+\phi\right)}{z^2-2ze^{-\zeta\omega_0 T}\cos\left(\sqrt{1-\zeta^2}\cdot\omega_0 T\right)+e^{-2\zeta\omega_0 T}}$ $\phi=\operatorname{acos}(\zeta)$

Using this table for Z Transforms with discrete indices

Commonly the "time domain" function is given in terms of a discrete index, k, rather than time. This is easily accommodated by the table. For example if you are given a function:

$$f[k] = k$$

Since t=kT, simply replace k in the function definition by k=t/T. So, in this case,

$$f[k] = \frac{t}{T}$$

and we can use the table entry for the ramp

$$t \xleftrightarrow{\mathfrak{Z}} T\frac{z}{(z-1)^2}$$

The answer is then easily obtained

$$f[k] = k = \frac{t}{T} \xleftrightarrow{\mathfrak{Z}} \frac{1}{T}T\frac{z}{(z-1)^2} = \frac{z}{(z-1)^2} = F(z)$$

References