Stochastic Finance (FIN 519)
Homework Solutions

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1. (3 points) **HW 1-1** Consider the gambler's fortune with an unfair coin:

$$S_n = X_1 + X_2 + \cdots X_n \quad \text{where} \quad X_n = \begin{cases} 1 & (\text{probability } p) \\ -1 & (\text{probability } q) \end{cases}.$$

   (a) Prove that $M_n = (q/p)^{S_n}$ is a martingale.

   (b) If τ is the first time n that S_n hits A or $-B$, find $\text{Prob}(S_\tau = A)$ using the martingale property,

$$1 = M_0 = E(M_{n\wedge\tau}) \text{ for all } n = E(M_\tau).$$

2. (2 points) **HW 1-2** Prove that, if B_t is a standard BM, the inverted process,

$$Y_0 = 0 \quad \text{and} \quad Y_t = t\, B_{1/t} \quad \text{for} \quad t > 0,$$

   is also a standard BM.