Knab Sampling Window for InSAR Data Interpolation

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***Abstract*—Interferometric synthetic aperture radar processing requires interpolating a slave image onto a master one. Since the signals requiring interpolation are limited in both time and bandwidth, the Knab sampling window provides an almost optimal and viable interpolation kernel. Its performance in terms of coherence preservation and interferometric phase error as a function of the number of retained samples and oversampling factor are shown.**

***Index Terms*—InSAR, interpolation, Knab sampling window.**

I. Introduction

INTERFEROMETRIC systems, particularly, interferometric synthetic aperture radar (InSAR), are very useful tools for environmental remote sensing. An InSAR is an interferometer that is created by two synthetic aperture radar (SAR) antennas that are separated by an across-track distance known as the baseline. Interferometric processing requires accurate coherent processing of the electromagnetic signals that are received at the two antennas to generate two similar but different complex SAR images. The two complex SAR images are coregistered by interpolating one (the slave image) to generate imagery at the same pixel locations as the second (the master image). After registration, the two complex SAR images are multiplied, and the interferometric phase is obtained. The resulting phase is wrapped [i.e., known only to the interval (0, 2π)] and must be unwrapped to its true value to obtain a digital elevation model. Phase unwrapping is an ill-posed inverse problem that is greatly affected by the similarity between the two images, as measured by the coherence function. The coherence is a normalized measure of similarity between two signals, whose modulus takes on values ranging from 0 (indicating no correlation) to 1 (indicating fully correlated signals) [1]. Higher coherence leads to higher quality InSAR products; the coherence is affected by physical phenomena [1]–[3], interferometric configuration [1]–[3], and processing [1], [2], [4], [5].

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In this letter, a numerical study on the effectiveness of the interpolation scheme that is used to interpolate the slave SAR image onto the master SAR image is presented. A parametric study on the effectiveness of the Knab sampling window [5]–[7] as an interpolation kernel is provided, considering as parameters the number of retained samples L and oversampling factor χ. For comparison purposes, the performance of classical interpolation schemes based on a truncated sinc function [8] are also provided, and it is seen that the Knab interpolation kernel provides better exploitation of the oversampling.

II. Background

In this section, we present the relevant background theory that is required to measure the quality of the interpolation procedure. Central to the analysis is coherence C, which can be defined as follows [4]:

$$C = \frac{1}{\sqrt{1+\frac{N}{S}}} \frac{\int |H(f)|^2 K(f) df}{\sqrt{\int |H(f)|^2 df \int |H(f)|^2 |K(f)|^2 df}} \tag{1}$$

where $|H(f)|^2$ is the transfer function of the band-limited SAR system, $K(f)$ is the Fourier transform of interpolating kernel $k(t)$, and S/N is the signal-to-noise ratio due to the interpolation, which is given by the following if, in the resampling process, all interpixel positions are equally probable[1] [4]:

$$\frac{S}{N} = \frac{\int_{-B/2}^{B/2} |H(f)|^2 |K(f)|^2 df}{\sum_{n\neq 0} \int_{nf_s-B/2}^{nf_s+B/2} |H(f-nf_s)|^2 |K(f)|^2 df}. \tag{2}$$

In (2), n is the summation index running over all nonzero integers, and f_s is the sampling frequency (which, in all practical cases, is greater than Nyquist frequency $2B$). The ratio between f_s and the Nyquist frequency is an important factor called oversampling ratio $\chi(> 1)$, which will prove to be important in the performance of interpolation kernels. Coherence degradation causes a corresponding interferometric phase error

[1]This scattering model, as discussed in [5], is not always applicable but allows definition of a theoretical framework.

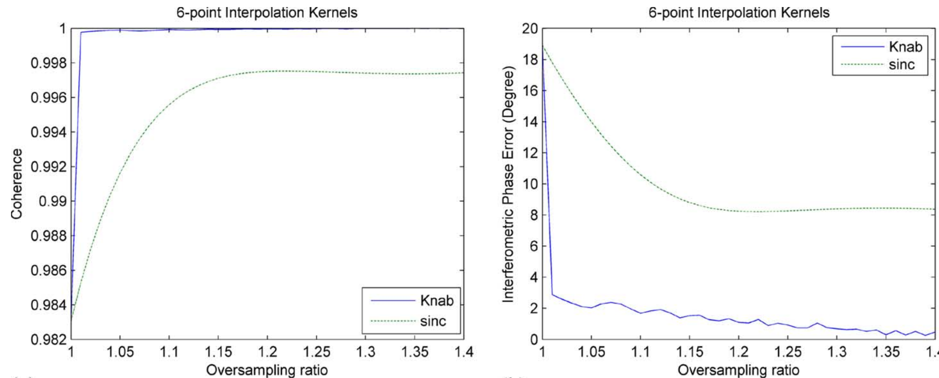

Fig. 1. Interpolation results for six-point interpolation kernels. The solid lines are for the Knab sampling window; the dashed lines are for the truncated sinc function. (a) Modulus of C versus χ. (b) σ_ϕ versus χ.

σ_ϕ in accordance to its probability density function $f(\phi)$ [4]. In fact, $f(\phi)$, in the single-look case, is [4]

$$f(\phi) = \frac{1-|C|^2}{2\pi}\frac{1}{1-|C|^2\cos^2\phi} \cdot \left(1+\frac{|C|\cos\phi \arccos\left(-|C|\cos\phi\right)}{\sqrt{1-|C|^2\cos^2\phi}}\right) \quad (3)$$

and the rms phase error σ_ϕ is given by

$$\sigma_\phi = \sqrt{\int_{-\pi}^{+\pi}\phi^2 f(\phi)d\phi}. \quad (4)$$

From a theoretical viewpoint, the selection of the best interpolation kernel depends on the characteristics of the electromagnetic signal to be interpolated, and when a band-limited signal is in question, the Shannon–Whittaker–Kotel'nikov sampling theorem [8] is to be exploited. It states that a band-limited signal can be completely reconstructed from uniform samples if they are acquired at least at the Nyquist frequency [8]. This statement, over an open domain, implies an infinite number of data samples! In all real cases, only a finite number of samples are available.

From an application viewpoint, it has to be noted that most SAR images are characterized by heterogeneous areas, i.e., the scattering scenarios are different at the macroscopic level, implying that the areas are physically uncorrelated.

For all aforementioned reasons, the problem that is actually in question is the interpolation of truncated band-limited signals. One solution, which we will show provides good performance, is the Knab sampling window kernel [7], i.e.,

$$k(t) = \mathrm{sinc}(t)\frac{\cosh\left[\frac{\pi\nu L}{2}\sqrt{1-\left(\frac{2t}{L}\right)^2}\right]}{\cosh\left(\frac{\pi\nu L}{2}\right)} \quad (5)$$

where $\nu = 1 - 1/\chi$ and the number of retained samples is L. The corresponding $K(f)$ expression is not available in closed analytical form and must be evaluated numerically.

In [5], a comparison of the Knab sampling window interpolation kernel with classical ones, i.e., the nearest neighbor interpolation kernel, as well as piecewise linear truncated sinc, four-point cubic convolution (CC), and six-point CC, is conducted at a given oversampling factor ($\chi = 1.22$). It has been shown that the interpolation scheme based on the Knab sampling window kernel is superior to the classical ones, and it is able to minimize coherence degradation even when a limited number of samples are retained. Physically, this can be simply explained because the Knab sampling kernel adapts to χ [7].

III. Numerical Result

In this section, a numerical study based on the interferometric interpolation scheme that is referred to in Section II is presented and discussed. The study analyzes the interpolation scheme based on the Knab sampling window as a function of oversampling factor χ and of the number of retained samples L. In order to emphasize the relevance of the new approach, a comparative analysis with the classical interpolation scheme based on the truncated sinc is also presented.

Original code has been developed in MatLab 7.1 using the new numerical integration based on the adaptive Lobatto quadrature. In accordance with [4] and [5], the SAR system is modeled as $H(f) = \mathrm{rect}(f/B)$. Interpolation schemes are objectively evaluated by means of the coherence after interpolation C [see, e.g., (1) and (2)] and the corresponding interferometric phase error σ_ϕ for the one-look SAR image case. Within this study, L was varied from 6 to 12, and χ was varied from 1.0 to 1.4, with a step size of 0.01.

In Fig. 1 the six-point case is shown. The modulus of C and σ_ϕ is plotted versus χ in Fig. 1(a) and (b), respectively. The solid line corresponds to the interpolation scheme based on the Knab sampling window, while the dashed line corresponds to the interpolation scheme based on the truncated sinc. The same format is used in the following figures. Analysis of Fig. 1 shows that the two interpolation schemes, as expected, are identical for $\chi = 1$ since the Knab sampling window reduces to a sinc when $\chi = 1$. Otherwise, the two curves are very different. The interpolation scheme based on the Knab sampling window has a monotonic behavior,[2] which indicates its ability to take full benefit of the oversampling. In other words, oversampling per

[2]The ripple is due to numerical integration, which in the Knab case calls for extremely high accuracy.

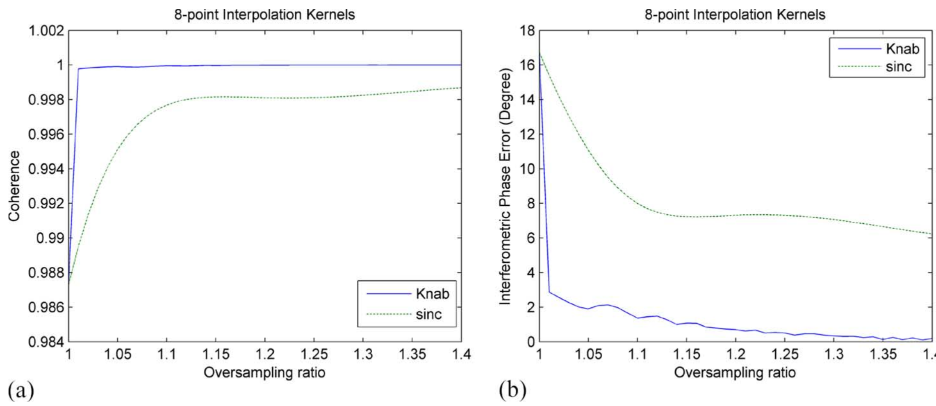

Fig. 2. Same as Fig. 1 but for the eight-point interpolation kernels.

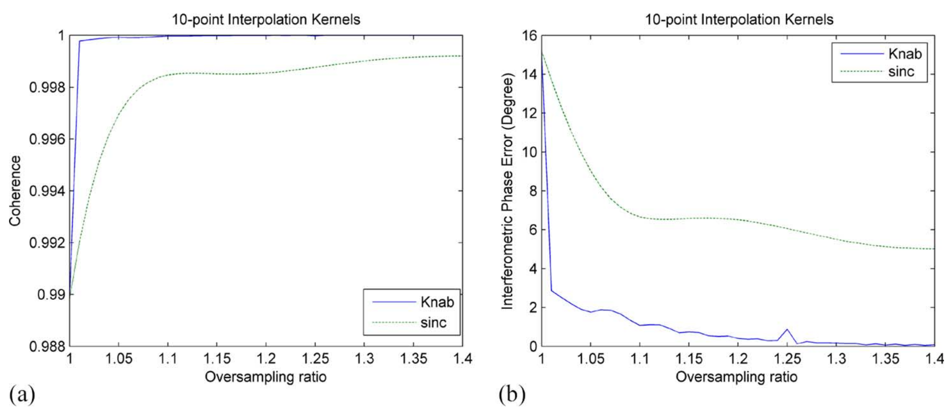

Fig. 3. Same as Fig. 1 but for the ten-point interpolation kernels.

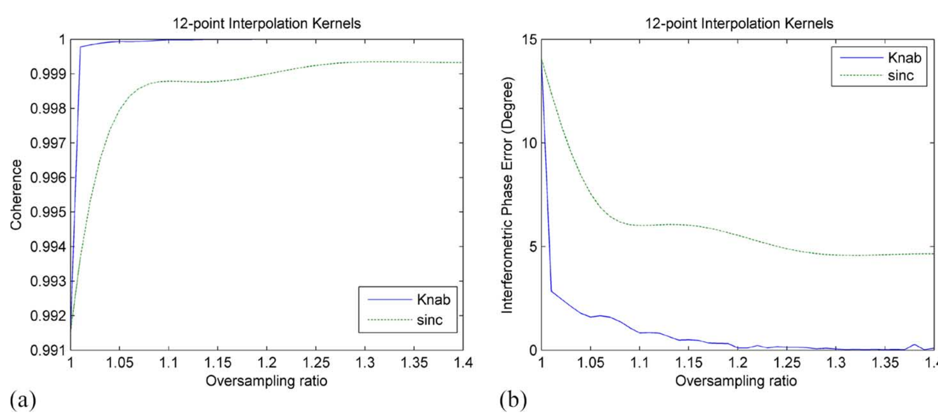

Fig. 4. Same as Fig. 1 but for the 12-point interpolation kernels.

se does not ensure better results. Coherence curve C shows first a rapid increase and then a saturation behavior. This means that the new approach is capable of improving upon classical results even at small χ values.

In Fig. 2, the case that is relevant to eight points is shown. Although all comments that are relevant to Fig. 1 still apply, in this case, a more pronounced oscillatory behavior of the curves that are relevant to the interpolation scheme based on the truncated sinc is experienced. Furthermore, as expected, the interpolation scheme based on the Knab sampling window achieves better results both with respect to the eight-point truncated sinc and the six-point Knab sampling window.

In Figs. 3 and 4, the cases that are relevant to ten and 12 points are shown. These results confirm the preceding statements. Thus, very remarkable results can be achieved by a small oversampling and/or a small number of retained samples. All these are congruent with the study that is presented in [6].

Finally, to provide a simple quantitative summary of all these results, in Table I, the required oversampling factor is listed to provide, in the truncated sinc case and in the Knab sampling window case, an interferometric phase error σ_ϕ that is smaller than 3° and 5°, respectively. This analysis shows that the interpolation scheme based on the truncated sinc is never capable of providing an error that is smaller than 3° in the

TABLE I
REQUIRED OVERSAMPLING RATES TO MAINTAIN INTERFEROMETRIC PHASE ERROR σ_ϕ BELOW 3° AND 5°. THE CASES CORRESPONDING TO L IN THE RANGE OF 6–12 FOR THE TRUNCATED SINC KERNELS AND KNAB SAMPLING WINDOW KERNELS ARE LISTED

L	χ			
	Truncated Sinc		*Knab*	
	$\sigma_\phi < 5°$	$\sigma_\phi < 3°$	$\sigma_\phi < 5°$	$\sigma_\phi < 3°$
6-point	>1.40	>1.40	1.01	1.01
8-point	>1.40	>1.40	1.01	1.01
10-point	>1.40	>1.40	1.01	1.01
12-point	1.25	>1.40	1.01	1.01

investigated range, and only when $L = 12$ is the scheme able to achieve a phase error that is smaller than 5° at $\chi = 1.25$. In contrast, the Knab sampling window achieves an error that is smaller than 3° even when six points are retained and $\chi = 1.01$. This clearly shows that, if χ is given, the number of samples to be retained can also be very limited, and this is very important in real cases because of the scattering heterogeneity.

The 2-D case can be recovered by squaring the 1-D coherence value if the oversampling factor is the same in the azimuth and range channels. In general, the proper values must be considered and multiplied to get the coherence in the 2-D case.

An experiment on real data was performed to confirm the theoretical predictions. SAR data from the Napoli Bay area in Italy, which were acquired by First European Remote Sensing satellite (ERS-1) on August 30, 1995, were processed to a single-look complex image of 20 000 × 4800 samples (not shown to conserve space). The SAR image corresponds to a very heterogeneous area that includes sea and land. The land area of the SAR scene comprises the Phlegrean area, the Vesuvius volcano, the city of Napoli, man-made structures, and agricultural and vegetated areas. To evaluate only the interferometric phase error due to the interpolation scheme, the procedure that is applied over these data follows the rationale that is detailed in [4]: The complex band-limited oversampled ERS signal is filtered by a factor a (here, 10). This signal is the reference signal of [4]. Then, the reference signal is downsampled by the same factor a. Such a signal is interpolated and compared to the reference one. The interferometric phase errors have been evaluated for the six- to 12-point cases (not shown to conserve space). The interpolation scheme based on the truncated sinc and the Knab sampling window kernels are considered. In Table II, the resulting rms phase error σ_ϕ and the theoretical ones[3] are listed. Experimental and theoretical results are congruent but not identical. This can be explained physically because real SAR data are characterized by scattering heterogeneity and the interpixel statistical hypothesis[4] is not strictly applicable.

These results clearly confirm that the interpolation scheme based on the Knab sampling window is better and that the quantitative benefit, as expected, is related to the scattering scenario.

[3]Knab theoretical values show some minor differences with respect to [5] because of the new integration procedure.

[4]See footnote 1.

TABLE II
EXPERIMENTAL AND THEORETICAL INTERFEROMETRIC PHASE ERROR σ_ϕ ($\chi = 1.22$). THE CASES CORRESPONDING TO L IN THE RANGE OF 6–12 FOR THE TRUNCATED SINC KERNELS AND KNAB SAMPLING WINDOW KERNELS ARE LISTED

kernel	Experimental σ_ϕ[deg]	Theoretical σ_ϕ[deg]
6-point truncated sinc	8.8	8.3
8-point truncated sinc	7.7	7.4
10-point truncated sinc	6.4	6.3
12-point truncated sinc	6.2	5.2
6-point Knab	6.3	1.3
8-point Knab	2.4	0.7
10-point Knab	2.2	0.4
12-point Knab	1.5	0.23

The reader should also appreciate that the computational requirements of the new interpolation scheme are, as with any linear interpolation, a function of the kernel length and not of the kernel weights themselves. There is therefore no additional processing power that is required for the Knab sampling window when compared to classical interpolators.

IV. FINAL COMMENT

A study of the use of the Knab sampli ng window for interpolation of InSAR data has been provided. The results show that very good performance is achievable even with a limited oversampling factor and/or with few retained samples.

Knab interpolation is currently implemented in the InSAR processor of the Istituto per il Rilevamento Elettromagnetico dell'Ambiente and also in interferometric synthetic aperture sonars [9].

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