Instructions

- The homework is due on **Friday 2/10 at 5pm ET**.
- No extension will be provided, unless for serious documented reasons.
- **Start early!**
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.
- The code necessary for problem 2 should be written in the Jupyter notebook handed out to you.

1 Probability [25 points]

Solve the following problems:

a. (5pts) Let X, Y be independent random variables with common density function f. Prove that the density function of $Z = \max(X, Y)$ is given by $f_Z(x) = 2f(x)\mathbb{P}(X \leq x)$.

b. (10pts) If U is a uniform random variable in $[0, 1]$, what is the distribution of $\lfloor 100U \rfloor + 1$?

c. (10pts) If U is a uniform random variable in $[0, 1]$ and $0 < q < 1$, prove that $X = 1 + \lfloor \frac{\log U}{\log q} \rfloor$ has a geometric distribution. What is the parameter of the geometric distribution?

2 Bayes rule [20 points]

Let N be a discrete random variable that takes values from the set $\{1, n\}$ with equal probability, i.e., $\mathbf{Pr}[N = 1] = \mathbf{Pr}[N = n] = \frac{1}{2}$. Consider the following process.

- First we draw a value for N.
- Then, we draw N iid uniform RV $\{X_i\}_{i=1,\ldots,N}$ in $[0, 1]$.

Someone tells you the value $Z = \min_{i=1,\ldots,N} X_i = 0.05$, namely the smallest value among the N uniform RVs drawn. However you do not know the value of N.

What is the probability $\mathbf{Pr}[N = 1 | Z = 0.05]$ when: (a) $n = 2$, and (b) $n = 10$.

3 Needles and Probability [55 Points]

The Jupyter notebook in on Git.