From now on, we consider only sequences satisfying Rule 1.

When tabulating knot projections, one need use only standard sequences, with the subsequence $a_1, a_3, \ldots, a_{2n-1}$ minimal in lexicographic order when the transformations $y = b + x$ and $y = b - x$, $b = 1, 2, \ldots, 2n$, are applied to i and a_i. However in this paper it will not be assumed that the sequences are standard.

Now let $[0, 2n]$ denote the closed interval in $\mathbb{R}$. A *realization* of the sequence S is a piecewise linear mapping $\rho: [0, 2n] \to \mathbb{R}^2$ such that

(i) $\rho(0) = \rho(2n)$;

(ii) $\rho(i) = \rho(a_i)$ for $i = 1, 2, \ldots, 2n$;

(iii) ρ maps $[0, 2n] \setminus \{0, 1, \ldots, 2n\}$ homeomorphically;

(iv) the arc $\rho([i - \frac{1}{2}, i + \frac{1}{2}])$ crosses the arc $\rho([a_i - \frac{1}{2}, a_i + \frac{1}{2}])$ at $\rho(i)$, where $i - \frac{1}{2}$, $i + \frac{1}{2}$, etc. are mod $2n$.

For each such realization, there exists an alternating knot K which can be parametrized by $[0, 2n]$ so that ρ is the projection of K to $\mathbb{R}^2$. If there exists a realization of S, we say that S is *realizable*.

Let G be the 4-valent graph obtained from $[0, 2n]$ by identifying 0 with $2n$ and then identifying i with a_i for $i = 1, 2, \ldots, 2n$. It follows from Rule 1 that

(α) each edge of G joins two different vertices;

(β) if any two of its edges are cut, G remains connected;

(γ) if any vertex is removed, G remains connected.

A realization of S may be regarded as a piecewise linear embedding of G in $\mathbb{R}^2$ which preserves or reverses the cyclic order of the edges $[i-1, i]$, $[a_i - 1, a_i]$, $[i, i+1]$, $[a_i, a_i + 1]$ at each vertex i. Or more simply, a realization of S is a piecewise linear embedding $\tau: G \to \mathbb{R}^2$ which preserves the unoriented cyclic order at each vertex.

Let S^2 be the 2-sphere consisting of $\mathbb{R}^2$ with a point at ∞. Let G be any finite graph. Two piecewise linear embeddings $\tau_1, \tau_2: G \to \mathbb{R}^2$ are called *equivalent* if there is a homemorphism $h: S^2 \to S^2$ such that $h \circ \tau_1 = \tau_2$. Hence we say that two realizations $\rho_1, \rho_2: [0, 2n] \to \mathbb{R}^2$ of S are *equivalent* if there is a homeomorphism $h: S^2 \to S^2$ such that $h \circ \rho_1 = \rho_2$.

If K_1 is a knot parametrized by $[0, 2n]$ so that ρ_1 is its projection to $\mathbb{R}^2$, and if ρ_2 is equivalent to ρ_1, then there is a parametrized knot K_2, of the same type as K_1, whose projection to $\mathbb{R}^2$ is ρ_2.

Theorem 1. *If S is realizable, any two realizations of S are equivalent.*

This is a consequence of the following lemma.

Lemma 1. *Let G be a finite graph such that (α) each edge of G joins two different vertices; (β) if any two of its edges are cut, G remains connected; (γ) if any vertex is removed, G remains connected. Let there be given an unoriented cyclic order of edges at each vertex of G. If there exists a piecewise linear embedding of G in $\mathbb{R}^2$ which preserves the unoriented cyclic order at each vertex, then any two such embeddings are equivalent.*