mod p; for a factor of degree k, as above, this is p^k. Since each of the n_i in Kronecker's method is at least 4, the number of steps in the program Y is always less than the number in X if $p = 2$ or 3, and often if $p = 5$. On the other hand, the test modulo p may give information of a quite varying nature. The polynomial may be irreducible mod p, in which case it is irreducible. Or it may have factors mod p; then each factor must correspond to a factor mod p or a product of factors mod p. An equation of order 6 cannot have an irreducible factor of order 3 if it decomposes mod p into three irreducible factors of degree 2. It is also possible that the factors mod p_1 do not agree with the factors mod p_2. A polynomial of degree 6 which has three irreducible factors of degree 2 (mod p_1) and two irreducible factors of degree 3 (mod p_2) must itself be irreducible.

Another example occurs on a more global level. It is possible to find the Galois group of an equation mod p by an obvious exhaustive process, since in this case we are searching for the set of all automorphisms of a *finite* field. This must then be a subgroup of the Galois group over the rationals. This calculation may reduce the amount of time taken to calculate the Galois group over the rationals by eliminating possibilities. If the Galois group mod p is the symmetric group, then the Galois group over the rationals must be the symmetric group. If the Galois group mod p contains any odd permutation (such as, for example, a transposition) then the Galois group over the rationals cannot be the alternating group or any subgroup of it. We can calculate roughly how long it will take us to find the Galois group modulo the next prime p, and compare this with the time estimate of the calculation of the Galois group over the rationals by other methods.

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An enumeration of knots and links, and some of their algebraic properties

J. H. Conway

Introduction. In this paper, we describe a notation in terms of which it has been found possible to list (by hand) all knots of 11 crossings or less, and all links of 10 crossings or less, and we consider some properties of their algebraic invariants whose discovery was a consequence of this notation. The enumeration process is eminently suitable for machine computation, and should then handle knots and links of 12 or 13 crossings quite readily. Recent attempts at computer enumeration have proved unsatisfactory mainly because of the lack of a suitable notation, and it is a remarkable consequence that the knot tables used by modern knot theorists derive entirely from those prepared last century by Kirkman, Tait, and Little, which we now describe.

Tait came to the problem via Kelvin's theory of vortex atoms, although his interest outlived that theory, which regarded atoms as (roughly) knots tied in the vortex lines of the ether. His aim was a description of chemistry in terms of the properties of knots. He made little progress with the enumeration problem until the start of a happy collaboration with Kirkman, who provided lists of polyhedral diagrams which Tait grouped into knot-equivalence classes to give his tables **[9]**, **[10]**, **[11]** of alternating knots with at most 10 crossings. Little's tables **[4]**, **[5]**, **[6]** of non-alternating knots to 10 crossings and alternating knots of 10 and 11 crossings were based on similar information supplied by Kirkman.

Tait's and Little's tables overlap in the 120 alternating 10-crossing knots, and Tait was able to collate his version with Little's before publication and so correct its only error. The tables beyond this range are checked here for the first time. Little's table **[6]** of non-alternating knots is complete, but his 1890 table **[5]** of alternating 11-crossing knots has 1 duplication and 11 omissions. It can be shown that responsibility for these errors must be shared between Little and Kirkman, but of course Kirkman should also receive his share of the praise for this mammoth undertaking. (Little tells us that the enumeration of the 54 knots of **[6]** took him 6 years — from 1893 to 1899 — the notation we shall soon describe made this just one afternoon's work!)

Our tables of links, and the list of non-alternating 11-crossing knots, appear here for the first time, so cannot be collated with any earlier table, and for this reason the corresponding enumerations have been performed three times.

The enumeration here described was completed some 9 years ago, and a survey calculation of knot-polynomials was then made before an envisaged computer calculation. However, this survey brought to light certain algebraic relations between these polynomials which made the computer redundant. But we suspect that our table will find its main use as a basis for more sophisticated computer calculations with the many algebraic knot-invariants.

1. Notation for tangles. This paper is an abbreviated form of a longer one in which completeness is proved by means of a process for locating any knot or link within range of the table, but for reasons of space we only sketch this process here. For the same reasons, we describe our ideas rather informally, feeling that most readers will find that this helps rather than hinders their comprehension. Since most of what we say applies to links of 2 or more components as well as knots, we use "knot" as an inclusive term, reserving "proper knot" for the 1-component case.

In the light of these remarks, we define a *tangle* as a portion of knot-diagram from which there emerge just 4 arcs pointing in the compass directions NW, NE, SW, SE, hoping that Fig. 1 clarifies our meaning.

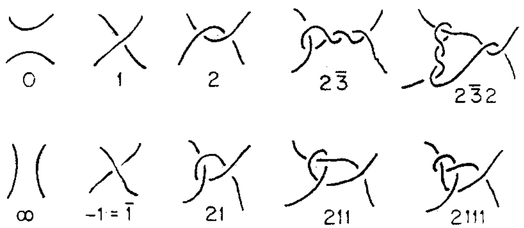


FIG. 1

The NW arc we call the *leading string* of the tangle, and the NW–SE axis its *principal diagonal*. The typical tangle t we represent diagrammatically by a circle containing an L-shaped symbol with the letter t nearby. The 8 tangles obtained from t by rotations and reflections preserving the "front" of t are indicated by making the corresponding operations on the L-shaped symbol, leaving the original letter t outside. The 8 other tangles obtained by reflecting these in the plane of the paper have the corresponding "broken" forms of the L-symbols, with the original letter t appended. Figure 2 shows how we represent the tangles t, t_h, t_v, $t_{hv} = t_r$, $-t$, t_h being the result of rotation in a "horizontal" or E–W axis, t_v that of

rotation in the "vertical" or N–S axis, and $-t$ that of reflection in the plane of the paper.

Tangles can also be combined and modified by the operations of Fig. 3, leading from tangles a and b to new tangles $a+b$, (ab), $a+$, and $a-$. Tangles which can be obtained from the particular tangles 0 and ∞ by these operations we call *algebraic*. In particular, we have the *integral* tangles $n = 1+1+\ldots+1$ and $\bar{n} = -n = \bar{1}+\bar{1}+\ldots+\bar{1}$, both to n terms. If $m, n, p, \ldots, s, t$, are integral tangles, the tangle $mnp\ldots st$, abbreviating

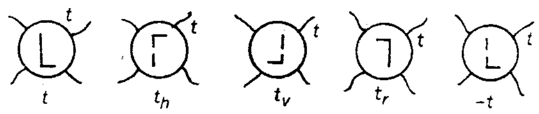


FIG. 2

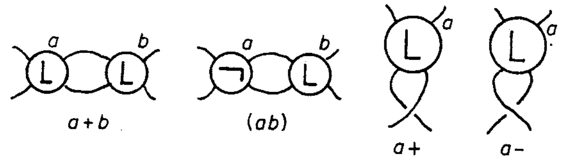


FIG. 3

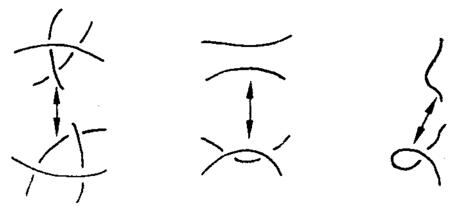


FIG. 4

$((\ldots(mn)p\ldots s)t)$, the brackets being associated to the left, is called a *rational* tangle. Figure 1 shows the step-by-step formation of the particular rational tangles 2 $\bar{3}$ 2 and 2 1 1 1 as examples. In the tables, the "comma" notation $(a, b, \ldots, c) = a0+b0+\ldots+c0$ is preferred to the sum notation, but is only used with 2 or more terms in the bracket. Figure 3 shows that $a0$ is the result of reflecting a in a plane through its principal diagonal, and $ab = a0+b$. The abbreviation $a-b$ denotes $a\bar{b}$ (not $a+\bar{b}$ or $(a-)b$), and outermost brackets are often omitted, in addition to those whose omission is already described above.

The tangles a and b are called *equivalent* (written $a \doteq b$) if they are related by a chain of *elementary knot deformations* (Fig. 4). The importance of the class of rational tangles is that we can show that the rational tangles $ijk\ldots lm$ and $npq\ldots st$ are equivalent if and only if the corresponding continued fractions $m+\frac{1}{l+}\cdots\frac{1}{k+}\frac{1}{j+}\frac{1}{i}$ and $t+\frac{1}{s+}\cdots\frac{1}{q+}\frac{1}{p+}\frac{1}{n}$ have the same value, so that there is a natural $1-1$ correspondence between the equivalence classes of rational tangles and the rational numbers (including $\infty = 1/0$). The continued fractions $2+\frac{1}{-3+}\frac{1}{2}$ and $1+\frac{1}{1+}\frac{1}{1+}\frac{1}{2}$ have the same value $8/5$, and so the tangles $2\,\bar{3}\,2$ and $2\,1\,1\,1$ of Fig. 1 are equivalent. Using this rule, we can reduce any rational tangle to a standard form, either one of 0, ∞, 1, -1, or a form $mnp\ldots st$ in which $|m| \geqslant 2$ and $m, n, \ldots, s, t$ have the same sign except that t might be 0. Each rational tangle other than 0 and ∞ has a definite *sign*, namely the sign of the associated rational number.

2. Notation for knots. An edge-connected 4-valent planar map we shall call a *polyhedron*, and a polyhedron is *basic* if no region has just 2 vertices. The term *region* includes the infinite region, which is regarded in the same light as the others, so that we are really considering maps on the sphere. We can obtain knot diagrams from polyhedra by substituting tangles for their vertices as in Fig. 5 — for instance we could always substitute tangles 1 or -1. Now let us suppose that a knot diagram K can be obtained by substituting algebraic tangles for the vertices of some non-basic polyhedron P. Then there is a polyhedron Q with fewer vertices than P obtained by "shrinking" some 2-vertex region of P, and plainly K can be obtained by substituting algebraic tangles for the vertices of Q, as in Fig. 5. Thus any knot diagram can be obtained by substituting algebraic tangles for the vertices of some *basic* polyhedron P — in fact P and the manner of substitution are essentially unique, but we do not need this.

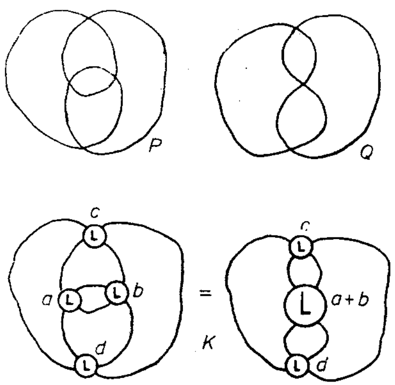


FIG. 5. Derivation of knots by substitution of tangles into polyhedra.

Only 8 basic polyhedra are needed in the range of this table (Fig. 6), although for convenience we have given one of them two distinct names. The notations $(2\times3)^* = 6^*$, $(2\times4)^* = 8^*$, $(2\times5)^* = 10^*$, $(3\times3)^* = 9^*$ extend obviously to $(a\times b)^*$.

The knot obtained from the polyhedron P^* by substituting tangles $a, b, \ldots, k$ in the appropriate places we call $P^*a.\,b.\ldots.\,k$. To save space, we omit substituents of value 1, telescoping the dots which would have separated them so as to show how many have been omitted. Thus 8*2:3.4:.5 abbreviates 8*2.1.3.4.1.1.5.1, the final dots being omitted from the abbreviation. We also omit the prefixes 1*, 6*, 6** from certain knot names — the original form is recovered by prefixing 1* if the abbreviation has no dots, 6** if it has an initial dot, and 6* otherwise. The symbol 10−*** abbreviates $10^{***}\bar{1}.\bar{1}.\bar{1}.\bar{1}.\bar{1}$.

3. Some tangle equivalences. Flyping. The reader should now be able to interpret any knot name taken from our table, but he will not yet appreciate the reasons which make our ragbag of conventions so suspiciously efficient at naming small knots. Much of this efficiency arises from the fact that the notation absorbs Tait's "flyping" operation (Fig. 7), which replaces $1+t$ by t_h+1, or $\bar{1}+t$ by $t_h+\bar{1}$. For rational tangles t we have $t \doteq t_h \doteq t_v \doteq t_r$, and so when $a, b, \ldots, c$ are all rational, the exact positions of the terms 1 or $\bar{1}$ in $(a, b, \ldots, c)$ are immaterial, and we can collect them at the end. Thus $(1, 3, 1, 2) \doteq (3, 1, 1, 2) \doteq (3, 2, 1, 1)$.

Now using another part of our notation, we can also replace a pair of terms t, 1 in such an expression by the single term $t+$, or a pair t, $\bar{1}$ by $t-$. Supposing again that $a, b, \ldots, c$ are rational, this justifies the equivalences

$$(a, b, c, 1) \doteq (a, b, c+) \doteq (a, b+, c) \doteq (a+, b, c)$$

and

$$(a, b, c, \bar{1}) \doteq (a, b, c-) \doteq (a, b-, c) \doteq (a-, b, c),$$

showing that in such expressions the postscripts $+$ and $-$ can be regarded as floating, rather than being attached to particular terms. We therefore collect these postscripts on the rightmost term, cancelling $+$ postscripts with $-$ postscripts. If this process would leave in the bracket only a single tangle c followed by p $+$signs and q $-$ signs, we replace the entire expression by cn, where $n = p-q$.

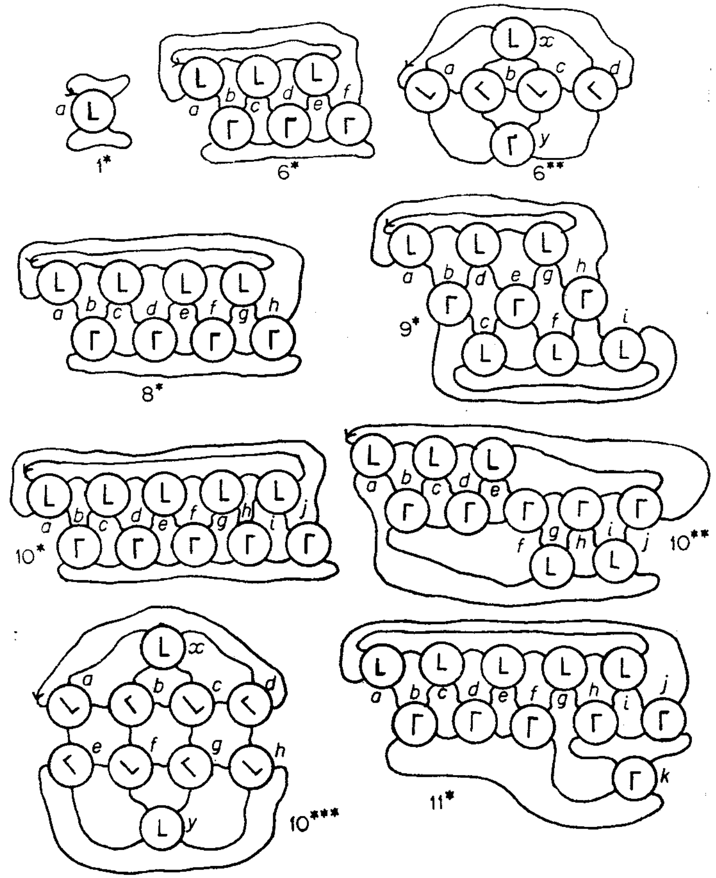
Now from the formula $x- = x\,\bar{1}\,0$ and the continued fraction rule, it follows that we have the equivalences

$$2- \doteq -2,\quad 3- \doteq -2\,1,\quad 2\,1- \doteq -3,\quad 2\,2 \doteq -2\,1\,1,$$

as particular cases of the equivalences

$$mn\ldots pq\,1- \doteq -mn\ldots pr \quad\text{and}\quad mn\ldots pr- \doteq mn\ldots pq\,1,$$

for more general rational tangles, which hold whenever $r = q+1$. This



1*a 6*a.b.c.d.e.f. 6**x.a.b.c.d.y.

8*a.b.c.d.e.f.g.h. 9*a.b.c.d.e.f.g.h.i

10*a.b.c.d.e.f.g.h.i.j 10**a.b.c.d.e.f.g.h.i.j

10***x.a.b.c.d.e.f.g.h.y. 11*a.b.c.d.e.f.g.h.i.j.k

FIG. 6. The basic polyhedra

FIG. 7. Flyping — the equivalence of $1 + t$ and $t_h + 1$

leads to a kind of concealed flyping, instanced by:

$$(2\ 2, 3, 2, -1) \doteq (2\ 2, 3, -2) \doteq (2\ 2, -2\ 1, 2) \doteq (-2\ 1\ 1, 3, 2),$$

as illustrated in Fig. 8. Each of these expressions should be translated at sight into (2 2, 3, 2 $-$) which is regarded as the standard form. The reader should similarly be able to write down 2 2 3 on seeing any of the flyped variants shown in Fig. 9.

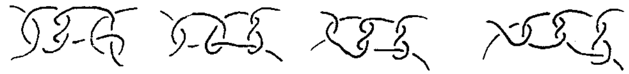


FIG. 8. Concealed flyping

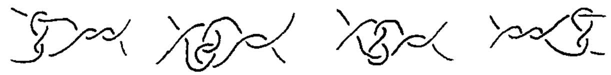


FIG. 9. Flyping variants of 2 2 3

4. Equivalences for knots. The following equivalences refer to the whole knot diagram rather than its component tangles. If two vertices of a triangular region are substituted 1 and -1, then in all cases within range the first deformation of Fig. 10 produces a form with fewer vertices in the

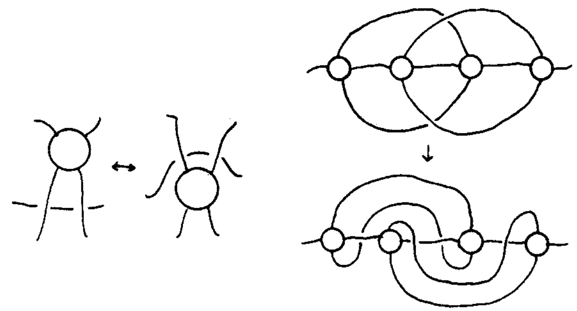


FIG. 10. Two knot reductions

basic polyhedron. If the substituents x and y of 6** $x.a.b.c.d.y$ are both 1, then the second deformation of Fig. 10 produces a form with basic polyhedron 1*. This increases the crossing number by 2, but we can use the continued fraction rule to reduce it by 2 again should any one of a, b, c, d be a negative rational tangle. If instead x and y are -1 and 1, other reduction processes apply to all cases within range.

In the tables, these and other equivalences have been taken into account, so that for example no substituent in the form 6**. $a.b.c.d$ is negative rational (this remark explains our preference for the 6** form rather than the 6* form when two opposite vertices are substituted as 1). Unfortunately the use of such equivalences means that the user might at first fail to locate his knot in our table. He should in these circumstances apply some reasonable transformation and try again — success comes easily after a little experience.

These remarks have probably convinced the reader that our notation has little structural significance, although it might be convenient in practice. The following remarks show that at least it has some structure. Let us call the knot 1^*t a *rational knot* whenever t is a rational tangle. Then the double branched covering space of the rational knot obtained from the rational number p/q is just the lens space with parameters p and q, and in fact the rational knots are precisely the Viergeflechte, long recognized as an interesting class. More generally, if p and q are the determinants of the knots 1^*t and 1^*t0, we call p/q the *determinant fraction* of the (arbitrary) tangle t. Then the determinant fraction of $a+b$ is $(ps+qr)/qs = (p/q)+(r/s)$ and that of (ab) is $(qs+pr)/ps = (p/q)^{-1}+(r/s)$, if those of a and b are p/q and r/s, which explains the continued fraction process. Under more restricted circumstances similar identities hold for the fractions obtained from Alexander polynomials, as we shall see later.

5. Orientation and string-labelling. An *oriented knot* will mean a proper knot with a preferred orientation (arrowhead) on its string. From any such knot we can obtain 3 others by simple geometric operations. Reflecting in a mirror gives us the enantiomorph, or *obverse*, $\neg K$ of K, reversing string orientation the *reverse*, K_r, of K, and doing both the *inverse*, $\neg K_r$, of K. A knot equivalent to its obverse is *amphicheiral*, one equivalent to its reverse is *reversible*, and one equivalent to its inverse is *involutory*. (Our notation is more mnemonic than the usual one — the inverse in our sense is also the inverse in the cobordism group.) For links of more than one component the situation is more complicated, and we need a convention for labelling and orienting strings. The convention we adopt is easy to remember and apply, although it leads occasionally to unexpected labellings.

We orient the leading string of the tangle named a in Fig. 6 so as to point into that tangle, and label this string r_1. We now move along r_1 in the direction of its orientation, labelling the other strings $r_2, r_3, \ldots$ in the order of their first crossing with r_1, either over or under, and orient these strings so that their first crossing with r_1 is *positive* in the sense of Fig. 11. If any strings remain, we proceed along r_2 in the direction of its orientation, labelling the unlabelled strings crossing r_2 in the same way. Repeating this process with r_3, etc., if necessary, we eventually obtain a complete system of labels and orientations. This convention tends to ensure that the homological linking is positive, since the linking number of two strings is half the sum of the ε's of Fig. 11 over all crossings in which both those strings appear.

For the purposes of polynomial calculation, we replace the labels $r_1, r_2, r_3, \ldots$ by $r, s, t, \ldots$. We also need to describe links obtained by relabelling a given one in various ways. Let π be any function from the string labels $r, s, t, \ldots$ to the symbols $r, r^{-1}, s, s^{-1}, t, t^{-1}, \ldots$. Then for any labelled link K we define K_π to be the link obtained from K by relabelling all the r strings of K as s strings in K_π, if $\pi(r) = s$, and as reversely oriented s strings in K_π, if $\pi(r) = s^{-1}$, and so on. We define $|\pi|$ as the total number of strings whose orientation is reversed in this process, and $|K|$ to be the total number of strings in K. Note that two distinct strings can have the same label.

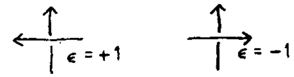


FIG. 11. A positive crossing ($\varepsilon = +1$) and a negative crossing

In the table, we give only one knot from each enantiomorphic pair, and only one from each labelling and orientation class. We indicate the symmetries of proper knots in the column S by writing a for amphicheiral knots, r for reversible knots, i for involutory knots, f for knots with full symmetry (all of these properties), and n for knots for which no symmetry has been observed. For links of 2 components we give the generators of the (observed) symmetry group, r and s being the operations of reversing the r and s strings respectively, t the operation of transposing these strings, and q the operation of reflection in a mirror. The column S is left empty for links of 3 or more components.

6. Polynomials and potentials. Each labelled and oriented knot K has a *potential function* $\nabla_K = \nabla_K(r, s, \ldots)$ which is a rational function with one variable for each string label appearing in K. We shall see in a moment that ∇_K is just a disguised and normalized form of the Alexander polynomial Δ_K, but it is in fact completely defined by the properties given below. We first have the symmetry properties

$$\nabla_K(r, s, \ldots) = \nabla_K(-r^{-1}, -s^{-1}, \ldots) = (-)^{|K|}\cdot\nabla_K(-r, -s, \ldots),$$
$$\nabla_{K_\pi}(r, s, \ldots) = (-)^{|\pi|}\cdot\nabla_K(\pi(r), \pi(s), \ldots).$$
$$\nabla_{\neg K}(r, s, \ldots) = (-)^{|K|+1}\cdot\nabla_K(r, s, \ldots),$$

the first of which makes it appropriate to use the abbreviation $\{f(r, s, \ldots)\}$ for $f(r, s, \ldots)+f(-r^{-1}, -s^{-1}, \ldots)$ in our table of potentials and elsewhere. If L is obtained from K by deleting a string labelled r in K, then

$$\nabla_K(1, s, t, \ldots) = (s^a t^b \ldots - s^{-a}t^{-b} \ldots)\cdot\nabla_L(s, t, \ldots),$$

where $a, b, \ldots$ are the total homological linking numbers of the deleted r string of K with the strings labelled $s, t, \ldots$ respectively. Finally, if

$K \circledast_r L$ is a *product* of K and L, obtained by tying each of them separately in a string labelled r, then

$$\nabla_{K \circledast_r L} = \nabla_K \cdot \{r\} \cdot \nabla_L.$$

Our tables list only knots which are *prime* in the sense of such products, and the assumption of primality is implicit elsewhere in this paper.

Our potential function is related to the Alexander polynomial Δ_K by the identity

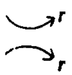
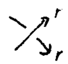
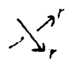
$$\Delta_K(r^2) = \{r\} \cdot \nabla_K(r)$$

if K is a proper knot, and by

$$\Delta_K(r^2, s^2, \ldots) = \nabla_K(r, s, \ldots)$$

otherwise, but it is important to realize that Δ_K is defined to within multiplication by powers of the variables and -1, while ∇_K is defined absolutely.

The most important and valuable properties of the potential function are for this reason not shared by the polynomial. Let K_0 yield K_+ and K_- on replacement of the tangle

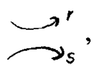
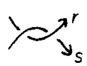
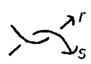
 by  and  respectively,

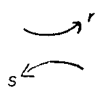
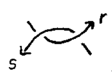
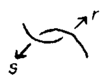
the labellings and orientations being significant. Then we have

$$\nabla_{K_+} = \nabla_{K_-} + \{r\} \cdot \nabla_{K_0},$$

called the *first identity*, which enables us to compute any one of the three potentials from the other two.

The *second identity* relates knots K_{00}, K_{++}, K_{--}, defined as above, but

now using the tangles ,  and 

or alternatively ,  and 

The second identity asserts that

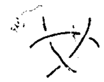
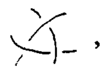
$$\nabla_{K_{++}} + \nabla_{K_{--}} = \{rs\} \cdot \nabla_{K_{00}}$$

in the first case, and

$$\nabla_{K_{++}} + \nabla_{K_{--}} = \{r^{-1}s\} \cdot \nabla_{K_{00}}$$

in the second case.

The *third identity* involves possibly three distinct string labels. If K_1 yields K_2, K_3, and K_4 on the replacement of

 by ,  and 

then we have

$$\nabla_{K_1} + \nabla_{K_2} = \nabla_{K_3} + \nabla_{K_4},$$

where now the labellings are immaterial.

These identities have many consequences which we cannot explore in detail here, although we shall give a few examples. Let t be a tangle whose 4 emerging strings are oriented and labelled as in Fig. 12. Define the *polynomial fraction* of t as the formal fraction

$$\frac{\{r\} \cdot \nabla_K}{\{r\} \cdot \nabla_L}$$

where K and L are the knots $1 * t$ and $1 * t0$. Then the identities which we asserted for determinant fractions in Section 4 hold also for polynomial fractions.

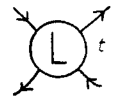


FIG. 12

If we consider generalized tangles with $2n$ emerging arcs instead of 4 (such as, for $2n = 6$, those of the third identity), then we can determine the potential of any knot obtained by joining the emergent arcs of two such tangles in terms of $n!$ potential functions associated with each tangle separately, provided that all the emerging strings have the same label. In the case $n = 2$ the $2!$ potentials are the numerator and denominator of the polynomial fraction. It becomes natural to think of such tangles as being —to within a certain equivalence relation—elements of a certain vector space in which our identities become linear relations, and there are many natural questions we can ask about this space. However, when the emerging arcs may have distinct labels, it is not even known whether the dimension of the tangle space is finite.

We have not found a satisfactory *explanation* of these identities, although we have *verified* them by reference to a "normalized" form of the 'L-matrix' definition of the Alexander polynomial, obtained by associating the rows and columns in a natural way. This normalization is useful in other ways

—thus our symmetry formulae show that a 2-component link can only be amphicheiral if its polynomial vanishes identically. It seems plain that much work remains to be done in this field.

7. Determinants and signature. We define the *reduced polynomial* $D_K(x)$ by the equation

$$D_K(x) = \{x\}\cdot\nabla_K(x, x, \ldots),$$

and the *determinant* $\delta = \delta_K$ as the number $D_K(i)$. Our determinant differs from the usual one only by a power of i. The potential identities of the last section yield determinant identities when we put i for each variable.

Murasugi [7] has defined invariants called the *signature*, σ_K, and *nullity*, n_K, and described some of their properties. These invariants depend on the string orientations of K, but not on its labelling. We shall describe enough of their properties to enable their calculation to proceed in much the same way as that of the potential function.

For any knot K we have the identity

$$\delta_K = \delta_K^0 \cdot i^{\sigma_K},$$

where $\delta_K^0 = |\delta_K|$, and the condition

$$n_K > 1 \quad \text{if and only if} \quad \delta_K = 0,$$

the first of which determines σ_K modulo 4 provided $\sigma_K \neq 0$. But one of Murasugi's results is that

$$|\sigma_{K_+} - \sigma_{K_0}| + |n_{K_+} - n_{K_0}| = 1,$$

whenever K_+ and K_0 are related as in the first identity. These two results determine σ_K completely in almost all cases, and make its calculation very swift indeed. Of course it should be remembered that σ_K and n_K are integers, and $1 \leqslant n_K \leqslant |K|$.

We have the relations

$$\sigma_{\neg K} = -\sigma_K \quad \text{and} \quad \sigma_{K \divideontimes L} = \sigma_K + \sigma_L,$$

concerning obverses and products, and if we define σ_K^0 as $\sigma_K - \lambda_K$, where λ_K is the total linking of K (the sum of the linking numbers of each pair of distinct strings of K), then the reorienting identity is that σ_K^0, like δ_K^0, is an invariant of the unoriented knot K. In the tables, we give only these *residual* invariants, δ^0 being the numerator of the rational fraction which we give for rational knots.

8. Slice knots and the cobordism group. A proper knot which can arise as the central 3-dimensional section of a (possibly knotted) locally flat 2-sphere in 4-space we call a *slice knot*. A natural application of our tables is to the discovery of interesting slice knots, since for slice knots there are simple conditions on the polynomial, signature, and Minkowski units. In particular, we might hope to find a slice knot which is not a ribbon knot [3], since several published proofs that all slice knots are ribbon have been found to be fallacious.

The slice knots with 10 crossings or less were found to be

∞, 4 2, 3 1 1 3, 2 3 1 2, (3, 2 1, 2−), 2 1 2 1 1 2, 2 0:2 0:2 0, (3, 3, 2 1−), 6 4, 3 3 1 3, 2 4 2 2, 2 2 1 1 1 1 2, (4 1, 3, 2), (2 1, 2 1, 2 1+), ·2 2·2 0, ·2·2·2 0·2 0, 1 0*, (3 2, 2 1, 2−), (2 2, 2 1 1, 2−), (4, 3, 2 1−), ((3, 2)−(2 1, 2)), $\bar{3}$: 2 : 2,

together with the composite knots $3 \divideontimes \bar{3}$, $2\,2 \divideontimes \overline{2\,2}$, $5 \divideontimes \bar{5}$, $3\,2 \divideontimes \overline{3\,2}$. The granny knot $3 \divideontimes 3$ and the knot ·2·2·2·2 0 satisfy the polynomial condition but not the signature condition, and so are not slice knots.

Now suppose that in the slab of 4-space bounded by two parallel 3-spaces we have an annulus ($S^1 \times I$) whose boundary circles lie in the two 3-spaces. Then the two knots defined by these circles we call *cobordant*. Cobordism is an equivalence relation, and the cobordism classes form a group under the product operation, the unit class being the class of slice knots, and the inverse of any class is the class of inverse knots to the knots of that class.

A search was made for cobordances between knots of at most 10 crossings and knots of at most 6 crossings, which in addition to the cobordances between slice knots, found only

3 ≅ 3 2 1 2 ≅ 3, 2 1, 2 ≅ 2 2 2 1 1 2 ≅ 2 2, 2 1 1, 2 ≅ 3 1, 3, 2 1 ≅ 2 1 1, 3, 2 1−.

2 2 ≅ 3, 2 1, 2 1 ≅ 3, 2 1, 2+

5 ≅ 3 4 1 2 ≅ 4, 3, 2 1

3 2 ≅ 3 1, 3, 2 1 ≅ 3, 3, 2 1 + ≅ 3, 2 1, 2++

$3 \divideontimes 3$ ≅ .2.2.2.2 0,

all of these being to within sign and orientation.

All slice knots given were found to be ribbon knots. However, the presence of the particular knot 10* = (2×5)* leads us to examine the more general a strand b bight Turk's Head knot $((a-1)\times b)^*$. Andrew Tristram has proved that if a and b are odd and coprime this knot obeys all known algebraic conditions for slice knots, but despite a prolonged attack the only cases definitely known to be slice are the trivial cases with a or $b = 1$, and the cases $a = 3$, $b = 5$, and $a = 5$, $b = 3$. Since most of our methods for proving knots slice would also prove them ribbon, the way is left open for a conjecture that some of least of these are slice knots which are **not** ribbon knots.

9. Notes on the tables. Acknowledgments. The tables (pp. 343–357) list all proper knots of at most 11 crossings, and all links of at most 10 crossings, with various invariants tabulated over parts of this range. Knots listed separately are believed to be distinct, and the symmetries listed under S are believed to be a complete set. (The evidence is very strong—

each knot has been subjected to a reduction procedure which in every known case has been shown to yield all forms with minimal crossing number.) By the same token, all knots listed are believed to be prime.

The columns headed ∇, δ^0, σ^0 give the invariants of §§ 6 and 7, and for proper knots the column headed Δ gives a coded form of the polynomial or equally of the potential, $[a+b+c$ abbreviating the polynomial $a+b(r+r^{-1})+c(r^2+r^{-2})$ or the potential $(a+b\{r^2\}+c\{r^4\})/\{r\}$. The column "units" gives the Minkowski units (for definition see **[8]**, but beware errors!) of K and its obverse, $+p$ meaning that $C_p = +1$ for both K and $\neg K$, $\mp p$ that C_p is -1 for K and 1 for $\neg K$, and so on. The units have been recomputed even in range of the existing tables, since these do not distinguish between a knot and its obverse. Under "λ" we give the linking numbers of pairs of strings, in the order λ_{rs}, λ_{rt}, λ_{ru}, λ_{st}, λ_{su}, λ_{tu}, but omitting linking numbers of non-existent strings.

The tables have been collated with the published tables of Tait (T in the tables), Little† (L), Alexander and Briggs (A&B) **[1]**, and Reidemeister **[8]**, and with some unpublished polynomial tables computed by Anger **[2]** and Seiverson of the Princeton knot theory group. I thank Professor H. F. Trotter for making these available—they have enabled me to correct a number of (related) errors in the 10 crossing knot polynomials. Much of the material of §§ 7 and 8 of this paper arose as the result of some stimulating conversations with Andrew Tristram, whose assistance I gratefully acknowledge here.

Note added in proof.

An idea of Professor Trotter has led me to the discovery of an identity for the Minkowski units like those of the text for the other invariants. In fact we have, if $K = K_0$, $L = K_+$, that

$$C_p(K)\cdot C_p(L) = [i\delta_K/\delta_L(p)]_p,$$

where $[X]_p = \left(\dfrac{p^{-x}X}{p}\right)^x$, and $X(p) = (-1)^x X$ when $p^x \| X$, and $\left(\dfrac{a}{p}\right)$ is the Legendre symbol.

† The 11-crossing knot numbered 400 in the table is the knot which appeared twice in Little's table, as numbers 141 and 142, and the knots 401–411 are those omitted by Little.

Knots to 8 crossings

A&B	T/L	knot	S	σ^0	units	δ^0	Δ
0_1	1	∞	f	0	+	1/1	[1
3_1	1	3	r	+2	±3	3/1	[−1+1
4_1	1	2 2	f	0	−5	5/2	[3−1
5_1	2	5	r	+4	+5	5/1	[1−1+1
5_2	1	3 2	r	+2	∓7	7/3	[−3+2
6_1	3	4 2	r	0	+3	9/4	[5−2
6_2	2	3 1 2	r	+2	±11	11/4	[−3+3−1
6_3	1	2 1 1 2	f	0	−13	13/5	[5−3+1
7_1	7	7	r	+6	±7	7/1	[−1+1−1+1
7_2	6	5 2	r	+2	±11	11/5	[−5+3
7_3	5	4 3	r	−4	+13	13/4	[3−3+2
7_5	3	3 2 2	r	+4	−17	17/7	[5−4+2
7_4	4	3 1 3	r	−2	∓3−5	15/4	[−7+4
7_6	2	2 2 1 2	r	+2	±19	19/7	[−7+5−1
7_7	1	2 1 1 1 2	r	0	∓3∓7	21/8	[9−5+1
8_1	18	6 2	r	0	−13	13/6	[7−3
8_2	15	5 1 2	r	+4	−17	17/6	[3−3+3−1
8_3	17	4 4	f	0	+17	17/4	[9−4
8_4	16	4 1 3	r	+2	±19	19/5	[−5+5−2
8_7	13	4 1 1 2	r	−2	±23	23/9	[−5+5−3+1
8_6	11	3 3 2	r	+2	±23	23/10	[−7+6−2
8_{11}	10	3 2 1 2	r	+2	±3	27/10	[−9+7−2
8_9	12	3 1 1 3	f	0	+5	25/7	[7−5+3−1
8_{13}	8	3 1 1 1 2	r	0	−29	29/11	[11−7+2
8_8	6	2 3 1 2	r	0	+5	25/9	[9−6+2
8_{12}	5	2 2 2 2	f	0	−29	29/12	[13−7+1
8_{14}	2	2 2 1 1 2	r	+2	∓31	31/12	[−11+8−2
8_5	14	3, 3, 2	r	−4	±3∓7	21	[5−4+3−1
8_{10}	9	3, 2 1, 2	r	−2	∓3	27	[−7+6−3+1
8_{15}	3	2 1, 2 1, 2	r	+4	∓3∓11	33	[11−8+3
8_{17}	1	.2.2	i	0	−37	37	[11−8+4−1
8_{16}	4	.2.20	r	+2	−5∓7	35	[−9+8−4+1
8_{18}	7	8*	f	0	−3+5	45	[13−10+5−1
8_{19}	III	3, 3, 2−	r	−6	±3	3	[1+0−1+1
8_{20}	I	3, 2 1, 2−	r	0	+3	9	[3−2+1
8_{21}	II	2 1, 2 1, 2−	r	+2	∓3+5	15	[−5+4−1

2 string links to 8 crossings

Link	λ	S	σ^0	units	δ^0	Δ
0	0	q, r, s, t	0	+	0/1	0
2	+1	qr, qs, t	0	+	2/1	1
4	+2	rs, t	+1	+	4/1	$\{rs\}$
2 1 2	0	r, s, t	+1	+	8/3	$-\{r\}\{s\}$
6	+3	rs, t	+2	∓ 3	6/1	$\{r^2s^2\}+1$
3 3	+3	qr, qs, t	0	+5	10/3	$\{r^2+s^2\}-1$
2 2 2	+2	rs, t	+1	∓ 3	12/5	$\{rs\}+\{r\}\{s\}$
4 1 2	+1	rs, t	+2	∓ 7	14/5	$1-\{r\}\{s\}\{rs\}$
3 1 1 2	+1	rs, t	0	+3	18/7	$1-\{r\}\{s\}\{r^{-1}s\}$
2 3 2	0	r, s, t	+1	+	16/7	$-2\{r\}\{s\}$
3, 2, 2	0	r, s	−3	+	16	$\{r\}\{s\}\{r^2\}$
2 1, 2, 2	+2	rs	+1	−5	20	$\{r^{-3}s\}+2\{r\}^3\{s\}$
.2	0	r, s	+1	± 3	24	$\{r\}\{s\}+\{r\}^3\{s\}$
3, 2, 2−	+2	rs	−3	+	4	$\{r^{-3}s\}$
2 1, 2, 2−	0	r, s	+1	+	8	$-\{r\}\{s\}$
8	+4	rs, t	+3	+	8/1	$\{r^3s^3+rs\}$
5 3	+4	rs, t	−1	+	16/5	$\{r\}\{s\}+\{rs\}\{r^2s^{-2}\}$
4 2 2	+3	rs, t	+2	∓ 11	22/9	$\{2r^2s^2-r^2-s^2\}+3$
3 2 3	+4	rs, t	+1	∓ 3	24/7	$\{rs\}\{r^2+s^2-1\}$
3 1 2 2	+3	rs, t	0	+13	26/11	$\{2r^2+2s^2+r^2-s^2\}-3$
2 4 2	+2	rs, t	+1	+5	20/9	$\{rs\}+2\{r\}\{s\}$
2 1 2 1 2	+1	rs, t	+2	$\mp 3+5$	30/11	$1-\{r\}^2\{s\}^2$
2 1 1 1 1 2	+1	r, s, t	+2	+17	34/13	$1+\{r\}^2\{s\}^2$
2 2, 2, 2	+1	rs	−1	± 7	28	$\{4r^{-1}s-2rs-r^{-3}s\}$
2 1 1, 2, 2	0	r, s	−1	+	32	$-\{r\}^3\{s\}$
3, 2, 2+	+2	rs	−3	∓ 7	28	$\{r^3s\}-2\{r\}\{s\}\{r^2\}$
2 1, 2, 2+	0	r, s	+1	+	32	$\{r\}^3\{s\}$
.21	0	r, s	+1	−5	40	$\{r\}^3\{s\}-\{r\}\{s\}$
.2 : 2	+2	rs	−1	−3	36	$\{r^3s\}-4\{r\}\{s\}$
2 2, 2, 2−	0	r, s	−1	+	8	$\{r\}\{s\}$
2 1 1, 2, 2−	+2	rs	−1	± 3	12	$\{2r^{-1}s-r^{-3}s\}$

9 crossing knots

A&B	T/L	knot	S	σ^0	Δ
9_1	41	9	r	9/1	[1−1+1−1+1
9_2	38	7 2	r	15/7	[−7+4
9_3	40	6 3	r	19/6	[−3+3−3+2
9_4	39	5 4	r	21/5	[5−5+3
9_6	33	5 2 2	r	27/5	[−5+5−4+2
9_5	37	5 1 3	r	23/6	[−11+6
9_9	34	4 2 3	r	31/9	[−7+6−4+2
9_{12}	31	4 2 1 2	r	35/13	[−13+9−2
9_{11}	30	4 1 2 2	r	33/14	[7−7+5−1
9_{14}	28	4 1 1 1 2	r	37/14	[15−9+2
9_7	26	3 4 2	r	29/13	[9−7+3
9_{10}	32	3 3 3	r	33/10	[9−8+4
9_{18}	25	3 2 2 2	r	41/17	[13−10+4
9_{13}	39	3 2 1 3	r	37/10	[11−9+4
9_{20}	21	3 1 2 1 2	r	41/15	[11−9+5−1
9_{21}	18	3 1 1 2 2	r	43/18	[−17+11−2
9_{26}	17	3 1 1 1 1 2	r	47/18	[−13+11−5+1
9_8	16	2 4 1 2	r	31/11	[−11+8−2
9_{15}	15	2 3 2 2	r	39/16	[−15+10−2
9_{19}	14	2 3 1 1 2	r	41/16	[17−10+2
9_{23}	12	2 2 1 2 2	r	45/19	[15−11+4
9_{17}	22	2 1 3 1 2	r	39/14	[−9+9−5+1
9_{27}	8	2 1 2 1 1 2	r	49/19	[15−11+5−1
9_{31}	2	2 1 1 1 1 1 2	r	55/21	[−17+13−5+1
9_{36}	24	2 2, 3, 2	r	37	[9−8+5−1
9_{25}	13	2 2, 2 1, 2	r	47	[−17+12−3
9_{22}	23	2 1 1, 3, 2	r	43	[−11+10−5+1
9_{30}	4	2 1 1, 2 1, 2	r	53	[17−12+5−1
9_{35}	36	3, 3, 3	r	27	[−13+7
9_{37}	19	3, 2 1, 2 1	r	45	[19−11+2
9_{16}	27	3, 3, 2+	r	39	[−9+8−5+2
9_{24}	20	3, 2 1, 2+	r	45	[13−10+5−1
9_{28}	6	2 1, 2 1, 2+	r	51	[−15+12−5+1
9_{33}	3	.2 1. 2	n	61	[19−14+6−1
9_{32}	7	.2 1. 2 0	n	59	[−17+14−6+1
9_{38}	5	.2.2.2	r	57	[19−14+5
9_{29}	11	.2.2 0.2	r	51	[−15+12−5+1
9_{39}	9	2: 2: 2 0	r	55	[−21+14−3
9_{41}	10	2 0: 2 0: 2 0	r	49	[19−12+3
9_{34}	1	8*2 0	r	69	[23−16+6−1
9_{40}	35	9*	r	75	[−23+18−7+1
9_{42}	IV	2 2, 3, 2−	r	7	[−1+2−1
9_{44}	I	2 2, 2 1, 2−	r	17	[7−4+1
9_{43}	V	2 1 1, 3, 2−	r	13	[1−2+3−1
9_{45}	III	2 1 1, 2 1, 2−	r	23	[−9+6−1
9_{46}	VI	3, 3, 2 1−	r	9	[5−2
9_{48}	VII	2 1, 2 1, 2 1−	r	27	[−11+7−1
9_{49}	II	−2 0: −2 0: −2 0	r	25	[7−6+3
9_{47}	VIII	8*−2 0	r	27	[−5+6−4+1

9 crossing 2 string alternating links, with basic polyhedron 1*

link	λ	∇
6 1 2	+2	$\{r^{-1}s\}-\{r\}\{s\}\{r^2s^2\}$
5 1 1 2	+2	$\{rs\}-\{r\}\{s\}\{r^{-1}s\}^2$
4 3 2	+1	$1-2\{r\}\{s\}\{rs\}$
4 1 4	0	$-\{r\}\{s\}(\{r^2+s^2\}+1)$
4 1 1 3	0	$\{r\}\{s\}\{r^{-1}s\}^2$
3 3 1 2	+2	$\{rs\}(1-\{r\}\{s\}\{r^{-1}s\})$
3 2 1 1 2	+2	$\{rs\}+\{r\}\{s\}(1-\{r^2+s^2\})$
3 1 3 2	+1	$1-2\{r\}\{s\}\{r^{-1}s\}$
3 1 1 1 3	0	$\{r\}\{s\}(\{r^2+s^2\}-1)$
2 5 2	0	$-3\{r\}\{s\}$
2 2 2 1 2	+1	$1-\{r\}\{s\}(\{rs\}+\{r\}\{s\})$
2 2 1 1 1 2	+1	$1+\{r\}\{s\}(\{r\}\{s\}-\{r^{-1}s\})$
5, 2, 2	0	$\{r\}\{s\}(\{r^4\}+1)$
4 1, 2, 2	+2	$\{r^{-5}s\}+2\{r\}\{s\}\{r^2\}$
3 2, 2, 2	0	$\{r\}\{s\}(2\{r^2\}-1)$
3 1 1, 2, 2	+2	$\{2r^{-3}s-r^{-1}s\}+4\{r\}\{s\}$
2 3, 2, 2	+2	$\{2r^{-3}s-r^{-1}s\}+3\{r\}\{s\}$
2 2 1, 2, 2	0	$2\{r\}\{s\}(\{r^2\}-1)$
4, 3, 2	+3	$\{s^2\}-1+\{r\}\{s\}\{r^{-1}s\}\{s^2\}$
4, 2 1, 2	+1	$1+\{r^2s^{-4}\}+2\{r\}\{s\}\{r^{-1}s\}$
3 1, 3, 2	+1	$\{s^2\}-1+\{r\}\{s\}\{rs\}\{s^2\}$
3 1, 2 1, 2	+3	$\{r^2s^2\}+1+\{s\}^2(\{r^2+s^2\}-1)$
3, 3, 2 1	+2	$\{r^{-1}s\}(1-\{r\}\{s\}\{r^{-1}s\})$
2 1, 2 1, 2 1	+3	$1+\{r^2s^2\}+\{rs\}\{r\}\{s\}+\{r\}^2\{s\}^2$
2 2, 2, 2+	0	$\{r\}\{s\}(4-\{r^2\})$
2 1 1, 2, 2+	+2	$\{2r^{-1}s-r^{-3}s\}+\{r\}\{s\}(\{r^2\}-3)$
3, 2, 2++	0	$\{r\}\{s\}(2\{r^2\}-1)$
2 1, 2, 2++	+2	$\{r^{-3}s\}+\{r\}\{s\}(3-\{r^2\})$
(3, 2) (2, 2)	+2	$\{r^{-5}s\}+2\{r\}\{s\}\{r^2\}$
(2 1, 2)(2, 2)	+2	$\{r^{-3}s\}+\{r\}^2(\{r^{-1}s\}-\{r\}\{s\})$

9 crossing 2 string links, otherwise

link	λ	∇
.4	0	$\{r\}\{s\}(\{r^4-r^2\}+1)$
.3 1	0	$\{r\}\{s\}(2\{r^2\}-3)$
.2 2	0	$\{r\}\{s\}(3-2\{r^2\})$
.3.2	+1	$1-\{r\}\{s\}\{rs\}(\{r^2\}-1)$
.3.2 0	+1	$1+\{r\}\{s\}\{rs\}(\{r^2\}-1)$
.3 : 2	0	$2\{r\}\{s\}(\{r^2\}-1)$
.3 : 2 0	0	$-\{r\}\{s\}\{r^2\}(\{r^2\}-1)$
.2 1 : 2 0	+2	$\{rs\}+2\{r\}^3\{s\}$
.2.2.20	+1	$(\{s^2\}-1)(1+\{r\}\{s\}\{rs\})$
2 : 2 : 2	+3	$1+\{r^2s^4\}-3\{r\}\{s\}\{rs\}$
2 : 2 0 : 2 0	0	$\{r\}\{s\}(3+\{r\}\{rs^2\})$
8*2	+1	$1-\{r\}\{s\}^3\{rs\}$
5, 2, 2−	+2	$\{r^{-5}s\}$
4 1, 2, 2−	0	$-\{r\}\{s\}\{r^2\}$
3 2, 2, 2−	+2	$\{2r^{-3}s-r^{-1}s\}$
3 1 1, 2, 2−	0	$-2\{r\}\{s\}$
2 3, 2, 2−	0	$-\{r\}\{s\}$
2 2 1, 2, 2−	+2	$\{rs\}+2\{r^{-3}s-r^{-1}s\}$
4, 3, 2−	+3	$1+\{r^{-2}s^4\}$
4, 2 1, 2−	+1	$1+\{s\}\{r^{-2}s\}$
3 1, 3, 2−	+3	$1+\{s^4\}+\{r\}\{rs^{-2}\}$
3 1, 2 1, 2−	+1	$\{s^2\}-1-\{r\}\{s\}\{rs\}$
3, 3, 3−	+4	$\{r^{-3}s^3+rs\}$
3, 2 1, 2 1−	−1	$\{r\}\{s\}\{r^{-1}s\}-1$
(3, 2) (2, 2−)	0	$-\{r\}\{s\}\{r^2\}$
(2 1, 2) (2, 2−)	0	$\{r\}\{s\}+\{r\}^3\{s\}$
(3, 2−) (2, 2)	+2	$\{rs\}-\{r^3\}\{s\}$
(2 1, 2−) (2, 2)	+2	$\{rs\}+\{r\}^3\{s\}$
(3, 2)−(2, 2)	+2	$\{r^5s\}$
(2 1, 2)−(2, 2)	+2	$\{r\}\{r^2s\}+\{r^{-3}s\}$
2 : −2 0 : −2 0	+4	$\{r^3s+rs^{-3}\}$

3 and 4 string links to 9 crossings

Link	λ		∇
2, 2, 2	+1+1+1		$\{rst\}-\{r\}\{s\}\{t\}$
.1	0 0 0		$\{r\}\{s\}\{t\}$
2, 2, 2−	+1+1−1		$-\{r^{-1}st\}$
2, 2, 2+	+1+1−1		$\{r\}\{s\}\{t\}-\{r^{-1}st\}$
4, 2, 2	+2+1+1		$\{r^{-1}s\}(\{rst\}-\{r\}\{s\}\{t\})-\{t\}$
3 1, 2, 2	+2+1+1		$\{rs\}(\{rst\}-\{r\}\{s\}\{t\})-\{t\}$
2, 2, 2+ +	+1+1+1		$\{rst\}-2\{r\}\{s\}\{t\}$
(2, 2) (2, 2)	+2+2 0		$\{r\}\{rs\}\{r^{-1}t\}+\{r\}\{s\}\{t\}$
.3	+1 0 0		$\{rs\}\{r\}\{s\}\{t\}$
.2 : 2 0	0+1−1		$-\{t\}-\{r\}\{s\}\{t\}\{rs\}$
4, 2, 2−	+2+1−1		$-\{r^{-2}s^2t\}$
3 1, 2, 2−	+2+1−1		$\{t\}-\{rs\}\{r^{-1}st\}$
(2, 2) (2, 2−)	+2 0 0		$-\{r\}\{s\}\{t\}$
(2, 2)−(2, 2)	+2+2 0		$\{r\}\{s\}\{t\}+\{r\}\{rs\}\{rt\}$
2, 2, 2, 2	+1+1 0	0+1+1	$\{rstu-rs^{-1}t^{-1}u\}-\{r\}\{s\}\{t\}\{u\}$
2, 2, 2, 2−	+1+1 0	0+1−1	$\{rt^{-1}\}\{su^{-1}\}-\{rs^{-1}\}\{tu\}$
2, 2, 2, 2− −	+1+1 0	0+1+1	$\{rstu-rs^{-1}t^{-1}u\}$
2 1 2, 2, 2	0+1−1		$\{r\}\{s\}(\{rs^{-1}t\}+\{r\}\{s\}\{t\})-\{t\}$
2 1 1 1, 2, 2	0+1−1		$-\{r\}\{s\}(\{rs^{-1}t\}+\{r\}\{s\}\{t\})-\{t\}$
3, 2, 2, 2	+1+1−1		$(1-\{r^2\})\{r^{-1}st\}+\{r\}\{s\}\{t\}\{r^2\}$
2 1, 2, 2, 2	+1+1+1		$\{r^3st\}-\{r\}\{st\}-\{r\}^3\{s\}\{t\}$
4, 2, 2+	+2+1−1		$\{r\}\{s\}\{t\}\{r^{-1}s\}-\{r^{-2}s^2t\}$
3 1, 2, 2+	+2+1−1		$\{rs\}(\{r\}\{s\}\{t\}-\{r^{-1}st\})+\{t\}$
2, 2, 2+ + +	+1+1−1		$2\{r\}\{s\}\{t\}-\{r^{-1}st\}$
(2, 2+) (2, 2)	0+2 0		$-\{r\}\{s\}\{t\}-\{r\}^2\{s\}\{r^{-1}t\}$
(2, 2) 1 (2, 2)	0 0 0		$\{r\}\{s\}\{t\}+\{r\}^3\{s\}\{t\}$
.2 1 1	+1 0 0		$\{r\}^2\{s\}^2\{t\}$
.2 1 : 2	+2+1−1		$-\{r\}^2\{s\}^2\{t\}-\{r^{-2}s^2t\}$
.(2, 2)	0 0 0		$-\{r\}^3\{s\}\{t\}$
2 1 2, 2, 2−	0+1+1		$\{r\}\{s\}\{rst^{-1}\}+\{t\}$
2 1 1 1, 2, 2−	0+1+1		$\{t\}-\{r\}\{s\}\{rst^{-1}\}$
3, 2, 2, 2−	+1+1+1		$\{r^2\}(\{rst\}-\{r\}\{s\}\{t\})-\{rst\}$
2 1, 2, 2, 2−	+1+1−1		$\{r\}\{r^{-2}st\}-\{rst\}-\{r\}\{s\}\{t\}$
3, 2, 2, 2− −	+1+1−1		$-\{r\}\{s^{-1}t\}-\{r^{-3}st\}$
(2, 2+) (2, 2−)	0 0 0		$\{r\}\{s\}\{t\}$
(2, 2+)−(2, 2)	0+2 0		$-\{r\}\{s\}\{r^2t\}$
.(2, 2−)	+2 0 0		$-\{r\}^2\{t\}\{r^{-1}s\}$
.−(2, 2)	0 0 0		0
2, 2, 2, 2+	+1+1 0	0+1−1	$\{rt^{-1}\}\{su^{-1}\}-\{rs^{-1}\}\{tu\}+\{r\}\{s\}\{t\}\{u\}$

10 crossing alternating knots. Basic polyhedron 1*

T	knot	S	δ^0	Δ
120	8 2	r	17/8	[9−4
119	7 1 2	r	23/8	[−3+3−3+3−1
102	6 4	r	25/6	[13−6
122	6 1 3	r	27/7	[−7+7−3
117	6 1 1 2	r	33/13	[5−5+5−3+1
81	5 3 2	r	37/16	[7−7+6−2
78	5 2 1 2	r	43/16	[−15+11−3
121	5 1 4	r	29/6	[5−5+5−2
101	5 1 1 3	r	39/11	[−7+7−5+3−1
108	5 1 1 1 2	r	45/17	[17−11+3
80	4 3 3	r	43/13	[−13+11−4
24	4 3 1 2	r	47/17	[−11+10−6+2
74	4 2 2 2	r	53/22	[23−13+2
68	4 2 1 1 2	r	57/22	[13−12+8−2
106	4 1 3 2	r	43/19	[−9+9−6+2
79	4 1 2 3	r	47/14	[−15+12−4
37	4 1 1 4	f	41/9	[9−7+5−3+1
67	4 1 1 2 2	r	55/23	[−19+14−4
107	4 1 1 1 3	r	51/14	[−11+11−7+2
77	3 5 2	r	35/16	[−11+9−3
66	3 4 1 2	r	45/16	[9−9+7−2
76	3 3 1 3	r	49/13	[13−10+6−2
20	3 3 1 1 2	r	59/23	[−15+13−7+2
65	3 2 3 2	r	55/24	[−19+14−4
61	3 2 2 1 2	r	65/24	[17−14+8−2
71	3 2 1 1 3	r	61/17	[17−13+7−2
16	3 2 1 1 1 2	r	71/27	[−19+16−8+2
105	3 1 3 1 2	r	53/19	[19−13+4
62	3 1 2 2 2	r	63/26	[−17+15−7+1
60	3 1 2 1 1 2	r	67/26	[−25+17−4
21	3 1 1 3 2	r	57/25	[21−14+4
58	3 1 1 1 2 2	r	69/29	[19−15+8−2
22	3 1 1 1 1 3	f	65/18	[25−16+4
104	2 5 1 2	r	37/13	[13−9+3
52	2 4 2 2	r	49/20	[21−12+2
51	2 4 1 1 2	r	51/20	[−19+13−3
15	2 3 3 2	f	53/23	[19−13+4
49	2 3 1 2 2	r	59/25	[−21+15−4
48	2 2 3 1 2	r	61/22	[15−13+8−2
11	2 2 2 1 1 2	r	75/29	[−21+17−8+2
41	2 2 1 2 1 2	r	71/26	[−21+17−7+1
3	2 2 1 1 1 1 2	r	81/31	[27−19+7−1
14	2 1 2 2 1 2	f	73/27	[23−17+7−1
43	2 1 2 1 1 1 2	r	79/30	[−25+19−7+1
1	2 1 1 1 1 1 1 2	f	89/34	[31−21+7−1

10 crossing alternating knots. Basic polyhedron 1*

T	knot	S	δ⁰	Δ
123	5, 3, 2	r	31	[−5+5−4+3−1
116	5, 2 1, 2	r	41	[7−7+6−3+1
36	4 1, 3, 2	r	49	[11−9+6−3+1
70	4 1, 2 1, 2	r	59	[−13+12−8+3
75	3 2, 3, 2	r	53	[13−11+7−2
18	3 2, 2 1, 2	r	67	[−19+15−7+2
109	3 1 1, 3, 2	r	59	[−15+13−7+2
57	3 1 1, 2 1, 2	r	73	[25−18+6
103	2 3, 3, 2	r	47	[−11+10−6+2
50	2 3, 2 1, 2	r	61	[21−15+5
64	2 2 1, 3, 2	r	65	[17−14+8−2
5	2 2 1, 2 1, 2	r	79	[−23+18−8+2
47	2 2, 2 2, 2	r	65	[27−16+3
42	2 2, 2 1, 1, 2	r	75	[−23+18−7+1
39	2 1 1, 2 1 1, 2	r	85	[29−20+7−1
118	4, 3, 3	r	33	[7−6+5−2
115	4, 3, 2 1	r	45	[9−8+6−3+1
69	4, 2 1, 2 1	r	57	[19−14+5
100	3 1, 3, 3	r	51	[−11+10−6+3−1
23	3 1, 3 2 1	r	63	[−17+14−7+2
56	3 1, 2 1, 2 1	r	75	[−19+16−9+3
63	2 2, 3, 2 1	n	63	[−23+16−4
114	2 1 1, 3, 3	r	57	[21−14+4
2	2 1 1, 2 1, 2 1	r	87	[−29+21−7+1
53	2 2, 3, 2+	r	67	[−19+16−7+1
7	2 2, 2 1, 2+	r	77	[25−18+7−1
55	2 1 1, 3, 2+	r	73	[19−16+9−2
4	2 1 1, 2 1, 2+	r	83	[−27+20−7+1
72	3, 3, 2 1+	r	63	[−23+16−4
40	2 1, 2 1, 2 1+	r	81	[27−19+7−1
73	3, 3, 2++	r	57	[15−12+7−2
19	3, 2 1, 2++	r	63	[−17+14−7+2
45	2 1, 2 1, 2++	r	69	[21−16+7−1
35	(3, 2) (3, 2)	i	61	[15−12+7−3+1
59	(3, 2) (2 1, 2)	n	71	[−17+15−9+3
9	(2 1, 2) (2 1, 2)	i	75	[27−20+8−1

10 crossing knots
Alternating, basic polyhedron not 1*

T	knot	S	δ⁰	Δ
99	.4.2	n	63	[−13+12−8+4−1
54	.3 1.2	n	85	[25−19+9−2
6	.2 2.2	n	87	[−25+20−9+2
113	.4.2 0	n	57	[11−10+8−4+1
17	.3 1.2 0	n	83	[−23+19−9+2
46	.2 2.2 0	n	81	[23−18+9−2
13	.2 1.2 1	i	101	[35−24+8−1
12	.2 1.2 1 0	r	99	[−33+24−8+1
95	.3.3.2	n	77	[23−17+8−2
33	.3.2.2 0	n	73	[17−14+9−4+1
44	.2 1.2.2 0	n	89	[25−20+10−2
111	.3.2 0.2	n	67	[−17+15−8+2
97	.3 0.2.2	n	71	[−15+14−9+4−1
8	.2 1 0.2.2	n	91	[−27+21−9+2
84	.2.2 1.2	r	93	[33−22+7−1
86	.2.2 1 0.2	r	87	[−33+22−5
92	.2.2.2.2 0	n	81	[23−18+9−2
31	.2.2.2 0.2 0	f	81	[19−16+10−4+1
112	3 : 2 : 2	r	65	[13−12+9−4+1
90	2 1 : 2 : 2	r	85	[29−21+7
98	3 : 2 : 2 0	n	73	[21−16+8−2
34	3 0 : 2 : 2	r	75	[−21+17−8+2
32	3 : 2 0 : 2 0	r	77	[19−15+9−4+1
83	2 1 : 2 0 : 2 0	r	91	[−29+22−8+1
96	3 0 : 2 : 2 0	n	75	[−17+15−9+4−1
10	2 1 0 : 2 : 2 0	n	93	[31−22+8−1
110	3 0 : 2 0 : 2 0	r	63	[−15+14−8+2
27	2.2.2.2	i	85	[21−17+10−4+1
89	2.2.2.2 0	n	83	[−25+20−8+1
91	2.2.2 0.2	r	37	[21−17+9−2
94	8*3	r	87	[−19+17−11+5−1
30	8*2 1	r	111	[−33+26−11+2
93	8*3 0	r	93	[27−21+10−2
29	8*2 0.2 0	i	109	[37−26+9−1
88	8*2 : 2	r	95	[−21+19−12+5−1
25	8*2 : 2 0	n	103	[−31+24−10+2
26	8*2 : .2	i	97	[23−19+12−5+1
85	8*2 : .2 0	n	101	[31−23+10−2
82	8*2 0 : : 2 0	r	105	[37−26+8
28	9*2 0	r	115	[−35+27−11+2
87	9*.2 0	r	105	[31−24+11−2
38	10*	f	121	[29−24+15−6+1

Non-alternating

L	knot	S	δ^0	Δ
6I	5, 3, 2−	r	1	[−1+1+0−1+1
1I	5, 2 1, 2−	r	11	[−1+2−2+1
3I	4 1, 3, 2−	r	19	[−5+4−2+1
4I	4 1, 2 1, 2−	r	29	[7−6+4−1
6IV	3 2, 3, 2−	r	11	[1+1−3+2
1II	3 2, 2 1, 2−	r	25	[9−6+2
3IV	3 1 1, 3, 2−	r	17	[5−4+2
4II	3 1 1, 2 1, 2−	r	31	[−11+8−2
3V	2 3, 3, 2−	r	5	[1−1+1
4III	2 3, 2 1, 2−	r	19	[−7+5−1
6V	2 2 1, 3, 2−	r	23	[−3+4−4+2
1IV	2 2 1, 2 1, 2−	r	37	[13−9+3
2V	2 2, 2 2, 2−	r	15	[−5+4−1
2IV	2 2, 2 1 1, 2−	r	25	[11−6+1
2VI	2 1 1, 2 1 1, 2−	r	35	[−7+8−5+1
6III	4, 3, 3−	r	3	[−3+2+0−1+1
3II	4, 3, 2 1−	r	9	[3−2+1
2I	4, 2 1, 2 1−	r	21	[5−4+3−1
6II	3 1, 3, 3−	r	15	[−1+2−3+2
3III	3 1, 3, 2 1−	r	27	[−7+6−3+1
2III	3 1, 2 1, 2 1−	r	39	[−13+10−3
5I	2 2, 3, 3−	r	3	[−3+1+1
1III	2 2, 2 1, 2 1−	r	33	[13−8+2
2II	2 1 1, 3, 2 1−	n	27	[−9+7−2
3VII	(3, 2) (3, 2−)	n	31	[−9+7−3+1
4VIII	(3, 2) (2 1, 2−)	n	41	[11−9+5−1
4VI	(2 1, 2) (3, 2−)	n	29	[7−6+4−1
3IX	(2 1, 2) (2 1, 2−)	n	43	[−13+10−4+1
6VIII	(3, 2)−(3, 2)	r	11	[−5+4−1−1+1
1VI	(3, 2)−(2 1, 2)	n	1	[3−1−1+1
6VII	(2 1, 2)−(2 1, 2)	r	17	[7−4+0+1
2VII	−3 : 2 : 2	r	25	[7−5+3−1
3X	−3 : 2 : 2 0	r	35	[−9+8−4+1
4VII	−3 : 2 0 : 2 0	r	49	[13−11+6−1
2VIII	−3 0 : 2 : 2	r	45	[15−10+4−1
3VIII	−3 0 : 2 : 2 0	r	49	[−11+9−4+1
4V	−3 0 : 2 0 : 2 0	r	21	[3−4+4−1
5II	3 : −2 0 : −2 0	r	5	[3−2+0+1
6VI	2 1 : −2 0 : −2 0	r	5	[3−2+0+1
2IX	−3 0 : −2 0 : −2 0	r	35	[−11+9−3
3VI	8*−3 0	r	51	[−15+12−5+1
1V	8*2 : −2 0	r	45	[17−11+3
4IV	8*2 : . −2 0	r	39	[−15+10−2

10 crossing 2 string links

10	3, 3, 2, 2	.2 1.2.2	3, 3, 2, 2−
7 3	3, 2 1, 2, 2	.2 1.2 0.2	3, 2 1, 2, 2−
6 2 2	2 1, 2 1, 2, 2	.2.3.2	2 1, 2 1, 2, 2−
5 5	3, 2, 3, 2	.2.3 0.2	3, 2, 3, 2−
5 2 3	3, 2, 2 1, 2	.2.2 1.2 0	3, 2, 2 1, 2−
5 1 2 2	2 1, 2, 2 1, 2	.2.2.2.2	2 1, 2, 2 1, 2−
4 4 2	5, 2, 2+	.2.2 0.2.2 0	3, 3, 2, 2− −
4 2 4	4 1, 2, 2+	.(3, 2)	3, 2 1, 2, 2− −
4 2 1 3	3 2, 2, 2+	.(2 1, 2)	3, 2, 3, 2− −
4 1 2 1 2	3 1 1, 2, 2+	.(2, 2).2	3, 2, 2 1, 2− −
4 1 1 1 1 2	2 3, 2, 2+	.(2, 2).20	(2 2, 2) (2, 2−)
3 4 3	2 2 1, 2, 2+	2 1 : 2 : 2 0	(2 1 1, 2) (2, 2−)
3 3 2 2	4, 3, 2+	2 1 0 : 2 : 2	(2 2, 2−) (2, 2)
3 2 2 3	4, 2 1, 2+	2 1 0 : 2 0 : 2 0	(2 1 1, 2−) (2, 2)
3 2 1 2 2	3 1, 3, 2+	2.2.2 0.2 0	(3, 2 1) (2, 2−)
3 1 4 2	3 1, 2 1, 2+	2.2 0.2.2 0	(3, 3−) (2, 2)
3 1 2 1 3	3, 3, 3+	2 0.2.2.2 0	(2 1, 2 1−) (2, 2)
3 1 1 2 1 2	3, 2 1, 2 1+	8*2 1 0	(3, 2+) (2, 2−)
3 1 1 1 1 1 2	2 2, 2, 2++	8*2.2 0	(2 1, 2+) (2, 2−)
2 6 2	2 1 1, 2, 2++	8*2 0 : 2 0	(2, 2+)(3, 2−)
2 3 2 1 2	3, 2, 2+++	8*2 0 : .2 0	(2, 2+) (2 1, 2−)
2 3 1 1 1 2	2 1, 2, 2+++	8*2 : : 2 0	(2 2, 2)−(2, 2)
2 2 2 2 2	(2 2, 2) (2, 2)	9*2	(2 1 1, 2)−(2, 2)
2 2 1 1 2 2	(2 1 1, 2) (2, 2)	9*.2	(3, 2 1)−(2, 2)
2 1 4 1 2	(3, 2 1) (2, 2)	10**	(3, 2+)−(2, 2)
2 1 3 1 1 2	(3, 2+) (2, 2)	4 2, 2, 2−	(21, 2+)−(2, 2)
2 1 1 2 1 1 2	(2 1, 2+) (2, 2)	4 1 1, 2, 2−	(2, 2+)−(3, 2)
4 2, 2, 2	(3, 2) (2, 2+)	3 1 2, 2, 2−	(2, 2+)−(2 1, 2)
4 1 1, 2, 2	(2 1, 2) (2, 2+)	3 1 1 1, 2, 2−	.(2, 2−).2
3 1 2, 2, 2	(3, 2) 1 (2, 2)	2 4, 2, 2−	.(2, 2−).2 0
3 1 1 1, 2, 2	(2 1, 2) 1 (2, 2)	2 3 1, 2, 2−	.−(2, 2).2
2 4, 2, 2	.4 1	2 1 3, 2, 2−	.−(2, 2).2 0
2 3 1, 2, 2	.3 1 1	2 1 2 1, 2, 2−	−2 1 0 : 2 : 2
2 1 3, 2, 2	.2 3	2 1 1 2, 2, 2−	−2 1 0 : 2 0 : 2 0
2 1 2 1, 2, 2	.2 1 2	2 1 1 1 1, 2, 2−	−2 1 0 : −2 0 : −20
2 1 1 2, 2, 2	.2 1 1 1	4, 2 2, 2−	2.−2.−2 0.2 0
2 1 1 1 1, 2, 2	.2 1 1.2	4, 2 1 1, 2−	2.−2 0.−2.2 0
4, 2 2, 2	.2 1 1.2 0	3 1, 2 2, 2−	8*2.−2 0
4, 2 1 1, 2	.3.2 1	3 1, 2 1 1, 2−	8*2 0 : −2 0
3 1, 2 2, 2	.3.2 1 0	2 1 2, 3, 2−	8*−2 0 : −2 0
3 1, 2 1 1, 2	.4 : 2	2 1 2, 2 1, 2−	8*2 0 : .−2 0
2 1 2, 3, 2	.3 1 : 2	2 1 1 1, 3, 2−	9*.−2
2 1 2, 2 1, 2	.2 1 1 : 2	2 1 1 1, 2 1, 2−	
2 1 1 1, 3, 2	.2 2 : 2 0	2 2, 3, 21−	
2 1 1 1, 2 1, 2	.2 1 1 : 20	2 1 1, 3, 3−	
2 2, 3, 3	.3 : 2 1	2 1 1, 2 1, 2 1−	
2 2, 2 1, 2 1	.3 : 2 1 0		
2 1 1, 3, 2 1	.2 1 : 2 1		

10 crossing links with 3 or more strings

3 strings		4 strings	5 strings
6, 2, 2	6, 2, 2−	4, 2, 2, 2	2, 2, 2, 2, 2
5 1, 2, 2	5 1, 2, 2−	3 1, 2, 2, 2	2, 2, 2, 2, 2−
3 3, 2, 2	3 3, 2, 2−	2, 2, 2, 2+ +	2, 2, 2, 2, 2− −
3 2 1, 2, 2	3 2 1, 2, 2−	(2, 2, 2) (2, 2)	
2 2 2, 2, 2	2 2 2, 2, 2−	(2, 2) 2 (2, 2)	
2 2 1 1, 2, 2	2 2 1 1, 2, 2−	(2, 2) 1 1 (2, 2)	
4, 4, 2	4, 4, 2−	.(2, 2) 1	
4, 3 1, 2	4, 3 1, 2−	.(2, 2) : 2 0	
3 1, 3 1, 2	3 1, 3 1, 2−	10***	
2 2, 2, 2, 2	2 2, 2, 2, 2−	4, 2, 2, 2−	
2 1 1, 2, 2, 2	2 1 1, 2, 2, 2−	3 1, 2, 2, 2−	
2 1 2, 2, 2+	2 2, 2, 2, 2− −	4, 2, 2, 2− −	
2 1 1 1, 2, 2+	(4, 2) (2, 2−)	(2, 2, 2) (2, 2−)	
3, 2, 2, 2+	(3 1, 2) (2, 2−)	(2, 2, 2−) (2, 2)	
2 1, 2, 2, 2+	(3, 3) (2, 2−)	(2, 2, 2−) (2, 2−)	
4, 2, 2+ +	(2 1, 2 1) (2, 2−)	(2, 2, 2− −) (2, 2)	
3 1, 2, 2+ +	(3, 2 1−) (2, 2)	(2, 2, 2)−(2, 2)	
2, 2, 2+ + + +	(2, 2+ +) (2, 2−)	.(2, 2−) 1	
(4, 2) (2, 2)	(4, 2)−(2, 2)	.(2, 2−) : 2 0	
(3 1, 2) (2, 2)	(3 1, 2)−(2, 2)	. −(2, 2) : 2 0	
(3, 3) (2, 2)	(3, 3)−(2, 2)	10−***	
(2 1, 2 1) (2, 2)	(2 1, 2 1)−(2, 2)		
(2, 2+ +) (2, 2)	(2, 2+ +)−(2, 2)		
(2, 2+) (2, 2+)	(2, 2), 2, (2, 2−)		
(2, 2+) 1 (2, 2)	(2, 2), −2, (2, 2)		
(2, 2), 2, (2, 2)	(2, 2), 2, −(2, 2)		
.5	(2, 2−), 2, (2, 2−)		
.3 2	.(2, 2−) : 2		
.2 2 1	. −(2, 2) : 2		
.3.3	2 0. −2. −2 0.2 0		
.3.3 0			
.3 : 3			
.3 : 3 0			
. 2 1 : 2 1 0			
.4 : 2 0			
.3 1 : 2 0			
.2 2 : 2			
.2.3.2 0			
.(2, 2) : 2			
.(2, 2+)			
2 0.2.2 0.2 0			
8*2.2			
8*2 : : 2			

Alternating 11 crossing knots. Basic polyhedron 1*

L	knot	L	knot	L	knot	L	knot
1	11	278	3 2 1 2 3	109	2 4, 2 1, 2	84	3, 3, 2 1, 2
5	9 2	324	3 2 1 2 1 2	268	2 3 1, 3, 2	357	3, 2 1, 3, 2
13	8 3	320	3 2 1 1 2 2	307	2 3 1, 2 1, 2	225	3, 2 1, 2 1, 2
60	7 4	341	3 2 1 1 1 1 2	31	2 1 3, 3, 2	220	2 1, 3, 2 1, 2
23	7 2 2	37	3 1 4 1 2	131	2 1 3, 2 1, 2	240	2 1, 2 1, 2 1, 2
3	7 1 3	119	3 1 3 2 2	290	2 1 2 1, 3, 2	30	5, 3, 2+
187	6 5	135	3 1 3 1 1 2	329	2 1 2 1, 2 1, 2	93	5, 2 1, 2+
27	6 2 3	311	3 1 2 3 2	113	2 1 1 2, 3, 2	249	4 1, 3, 2+
80	6 2 1 2	138	3 1 2 1 2 2	330	2 1 1 2, 2 1, 2	122	4 1, 2 1, 2+
22	6 1 2 2	298	3 1 2 1 1 3	130	2 1 1 1 1, 3, 2	300	3 2, 3, 2+
20	6 1 1 1 2	284	3 1 1 4 2	345	2 1 1 1 1, 2 1, 2	321	3 2, 2 1, 2+
250	5 4 2	314	3 1 1 3 1 2	17	5, 2 2, 2	124	3 1 1, 3, 2+
81	5 3 3	338	3 1 1 2 1 1 2	14	5, 2 1 1, 2	334	3 1 1, 2 1, 2+
24	5 2 4	134	3 1 1 1 3 2	246	4 1, 2 2, 2	120	2 3, 3, 2+
105	5 2 2 2	401	3 1 1 1 2 1 2	269	4 1, 2 1 1, 2	333	2 3, 2 1, 2+
232	5 2 1 3	128	3 1 1 1 1 1 3	282	3 2, 2 2, 2	319	2 2 1, 3, 2+
4	5 1 5	347	3 1 1 1 1 1 1 2	294	3 2, 2 1 1, 2	·350	2 2 1, 2 1, 2+
79	5 1 2 3	32	2 6 1 2	116	3 1 1, 2 2, 2	317	2 2, 2 2, 2+
34	5 1 2 1 2	118	2 5 2 2	133	3 1 1, 2 1 1, 2	348	2 2, 2 1 1, 2+
90	5 1 1 2 2	117	2 5 1 1 2	126	2 3, 2 2, 2	342	2 1 1, 2 1 1, 2+
104	5 1 1 1 1 2	293	2 4 3 2	132	2 3, 2 1 1, 2	19	4, 3, 3+
251	4 4 3	136	2 4 1 2 2	318	2 2 1, 2 2, 2	91	4, 3, 2 1+
248	4 4 1 2	313	2 3 3 1 2	328	2 2 1, 2 1 1, 2	270	4, 2 1, 2 1+
273	4 3 2 2	335	2 3 2 1 1 2	2	5, 3, 3	235	3 1, 3, 3+
272	4 3 1 1 2	127	2 3 1 3 2	78	5, 2 1, 2 1	299	3 1, 3, 2 1+
103	4 2 3 2	400	2 3 1 2 1 2	230	4 1, 3, 2 1	129	3 1, 2 1, 2 1+
275	4 2 2 3	346	2 3 1 1 1 1 2	85	3 2, 3, 3	108	2 2, 3, 3+
233	4 2 1 4	310	2 2 3 2 2	281	3 2, 2 1, 2 1	325	2 2, 2 1, 2 1+
283	4 2 1 2 2	344	2 2 2 2 1 2	107	3 1 1, 3, 2 1	337	2 1 1, 3, 2 1+
94	4 2 1 1 3	343	2 2 2 1 2 2	100	2 3, 3, 2 1	340	2 2, 3, 2+ +
29	4 1 4 2	349	2 2 2 1 1 1 2	83	2 2 1, 3, 3	353	2 2, 2 1, 2+ +
33	4 1 3 1 2	351	2 2 1 1 2 1 2	301	2 2 1, 2 1, 2 1	339	2 1 1, 3, 2+ +
35	4 1 2 1 3	327	2 2 1 1 1 2 2	26	2 1 2, 3, 3	356	2 1 1, 2 1, 2+ +
123	4 1 2 1 1 2	354	2 2 1 1 1 1 1 2	114	2 1 2, 3, 2 1	77	3, 3, 3+ +
271	4 1 1 3 2	38	2 1 5 1 2	287	2 1 2, 2 1, 2 1	312	3, 2 1, 2 1+ +
297	4 1 1 2 1 2	139	2 1 4 1 1 2	82	2 1 1 1, 3, 3	402	3, 3, 2+ + +
21	4 1 1 1 4	336	2 1 3 2 1 2	306	2 1 1 1, 3, 2 1	322	3, 2 1, 2+ + +
274	4 1 1 1 1 3	143	2 1 3 1 1 1 2	302	2 1 1 1, 2 1, 2 1	140	2 1, 2 1, 2+ + +
137	4 1 1 1 1 1 2	352	2 1 2 2 1 1 2	18	4, 2 2, 3	245	(2 2, 2) (3, 2)
106	3 6 2	332	2 1 1 3 1 1 2	89	4, 2 2, 2 1	289	(2 2, 2) (2 1, 2)
252	3 5 3	355	2 1 1 2 1 1 1 2	16	4, 2 1 1, 3	280	(2 1 1, 2) (3, 2)
296	3 4 2 2	231	4 2, 3, 2	102	4, 2 1 1, 2 1	304	(2 1 1, 2) (2 1, 2)
92	3 4 1 3	247	4 2, 2 1, 2	253	3 1, 2 2, 3	234	(3, 2 1) (3, 2)
279	3 3 2 3	15	4 1 1, 3, 2	292	3 1, 2 2, 2 1	266	(3, 2 1) (2 1, 2)
316	3 3 2 1 2	101	4 1 1, 2 1, 2	277	3 1, 2 1 1, 3	254	(3, 2+) (3, 2)
295	3 3 1 2 2	36	3 1 2, 3, 2	309	3 1, 2 1 1, 2 1	291	(3, 2+) (2 1, 2)
315	3 5 1 1 1 2	115	3 1 2, 2 1, 2	99	2 2, 2 2, 3	112	(2 1, 2+) (3, 2)
121	3 2 4 2	276	3 1 1 1, 3, 2	326	2 2, 2 1 1, 2 1	331	(2 1, 2+) (2 1, 2)
323	3 2 2 2 2	308	3 1 1 1, 2 1, 2	111	2 1 1, 2 1 1, 3	25	(3, 2) 1 (3, 2)
125	3 2 2 1 3	28	2 4, 3, 2	12	3, 3, 3, 2	96	(3, 2) 1 (2 1, 2)
						238	(2 1, 2) 1 (2 1, 2)

Alternating 11 crossing knots. Basic polyhedron not 1*

L	knot	L	knot	L	knot	L	knot
75	.41.2	7	.3.20.2.20	53	20.3.2.2	155	9*2.2
229	.41.20	43	.2 1.2 0.2.2 0	176	2.3.2 0.2 0	44	9*2.2 0
264	.3 1 1.2	50	.(3, 2).2	196	2.2 1.2 0.2 0	149	9*2 0.2
98	.3 1 1.2 0	200	.(2 1, 2).2	8	2 0.3.2 0.2	39	9*2 : 2
263	.2 3.2	51	.2.(3, 2)	64	2 0.2 1.2 0.2	161	9*2 : 2 0
97	.2 3. 2 0	195	.2.(2 1, 2)	167	2.2.2.2.2 0	168	9*2 0 :..2 0
288	.2 1 2.2	178	.(3, 2).2 0	157	2.2.2.2 0.2 0	145	9*.2 : .2
110	.2 1 2.2 0	204	.(2 1, 2).2 0	169	2.2 0.2.2.2 0	153	9*.2 : .2 0
305	.2 1 1 1.2	177	.2 0.(3, 2)	212	8*2 2	147	9* : 2 0.2 0
303	.2 1 1 1.2 0	197	.2 0.(2 1, 2)	191	8*2 1 1	163	9*2 0 : : 2 0
74	.4.2 1	73	2 2 : 2 : 2	61	8*4 0	148	10*2 0
76	.4.2 1 0	65	2 1 1 : 2 : 2	214	8*3 1 0	150	10**2
265	.3 1.2 1	59	4 : 2 : 2 0	188	8*2 1 1 0	410	10**2 0
267	.3 1.2 1 0	226	3 1 : 2 : 2 0	48	8*3.2 0	151	10** : 2
285	.2 2.2 1	262	2 1 1 : 2 : 2 0	190	8*2 1.2 0	144	100** : 2 0
286	.2 2. 2 1 0	170	4 0 : 2 : 2	173	8*3 0.2 0	411	11*
58	.4.2.2	71	3 1 0 : 2 : 2	198	8*2 1 : 2		
219	.3 1.2.2	209	2 1 1 0 : 2 : 2	172	8*3 : 2 0		
256	.2 1 1.2.2	206	2 2 : 2 0 : 2 0	192	8*2 1 0 : 2		
244	.2 2.2.2 0	205	2 1 1 : 2 0 : 2 0	47	8*3 0 : 2 0		
261	.2 1 1.2.2 0	242	2 2 0 : 2 : 2 0	199	8*2 1 0 : 2 0		
11	.4.2 0. 2	257	2 1 1 0 : 2 : 2 0	193	8*2 1 : .2		
228	.3 1.2 0.2	9	4 0 : 2 0 : 2 0	49	8*3 : .2 0		
95	.2 1 1.2 0.2	227	3 1 0 : 2 0 : 2 0	201	8*2 1 0 : .2		
243	.2 2 0.2.2	63	2 1 1 0 : 2 0 : 20	174	8*3 0 : .2 0		
260	.2 1 1 0.2.2	69	3 : 2 1 : 2	189	8*2 1 0 : .2 0		
186	.2.4.2	217	3 : 2 1 : 2 0	171	8*3 : : 2 0		
54	.2.3 1.2	237	2 1 : 2 1 : 2 0	146	8*2 1 : : 2 0		
213	.2.2 2.2	86	3 : 2 1 0 : 2	46	8*3 0 : : 2 0		
10	.2.4 0.2	259	2 1 : 2 1 0 : 2	403	8*2.2 0.2		
184	.2.3 1 0.2	223	3 0 : 2 1 : 2	154	8*2.2 0.2 0		
72	.2.2 2 0.2	221	3 : 2 1 0 : 2 0	166	8*2.2 0 : 2		
66	.3.2 1.2	87	3 0 : 2 1 : 2 0	40	8*2.2 0 : 2 0		
224	.3.2 1.2 0	216	3 0 : 2 1 0 : 2	404	8*2 0.2 : 2		
255	.2 1.2 1.2 0	67	3 0 : 2 1 0 : 2 0	405	8*2 0.2 : 2 0		
70	.3.2 1 0.2	214	2 1 0 : 2 1 0 : 20	156	8*2 0.2 0 : 2 0		
215	.3 0.2 1.2	194	2 1.2.2.2	164	8*2 0.2 : .2		
258	.2 1 0.2 1.2	56	3.2.2.2 0	41	8* 20.2 : .2 0		
218	.3.2.2 1	181	3.2.2 0.2	158	8*2 0.2 0 : .2		
236	.2 1.2.2 1	55	3.2 0.2.2	165	8*2 : 2 : 2 0		
222	.3.2.2 1 0	202	2 1 0.2.2.2	162	8*2 : 2 0 : 2 0		
68	.3.2 0.2 1	180	3 0.2.2.2 0	62	8*2 0 : 2 0 : 2 0		
239	.2 1.2 0.2 1	207	2 1 0.2.2.2 0	42	8*2 : .2 0 : .2		
88	.3 0.2.2 1	6	3 0.2.2 0.2	152	8*2 : .2 : .2 0		
175	.3.2.2.2	45	2 1 0.2.2 0.2	57	9*3		
159	.2 1.2.2.2	179	3 0.2 0.2.2	406	9*2 1		
211	.2 1.2.2.2 0	160	2 1 0.2 0.2.2	185	9*3 0		
182	.3.2.2 0.2	203	2.2 1.2.2	407	9*.3		
52	.3 0.2.2.2	183	2.3.2.2 0	408	9*.2 1		
210	.2 1.2.2 0.20	208	2.2 1.2 0.2	409	9*.3 0		

Non-alternating 11 crossing knots

4 2, 3, 2−	4, 2 2, 3−	.(3, 2−).2	2. −2 1.2.2
4 2, 2 1 2−	4, 2 2, 2 1−	.(2 1, 2−).2	2.2 1. −2.2
4 1 1, 3, 2−	4, 2 1 1, 3−	.2.(3, 2−)	2. −3.2.2 0
4 1 1, 2 1, 2−	4, 2 1 1, 2 1−	.2.(2 1, 2−)	2.3. −2.2 0
3 1 2, 3, 2−	3 1, 2 2, 3−	.(3, 2−).2 0	2 0.3. −2.2
3 1 2, 2 1, 2−	3 1, 2 2, 2 1−	.(2 1, 2−).2 0	2. −3. −2 0.2 0
3 1 1 1, 3, 2−	3 1, 2 1 1, 3−	.2 0.(3, 2−)	2. −2 1. −2 0.2 0
3 1 1 1, 2 1, 2−	3 1, 2 1 1, 2 1−	.2 0.(2 1, 2−)	2 0. −3. −20.2
2 4, 3, 2−	2 2, 2 2, 2 1−	. −(3, 2).2	2 0. −2 1. −2 0.2
2 4, 2 1, 2−	2 2, 2 1 1, 3−	. −(2 1, 2).2	2.2. −2.2.2 0
2 3 1, 3, 2−	2 1 1, 2 1 1, 2 1−	.2. −(3, 2)	2.2. −2.2 0.20
2 3 1, 2 1, 2−	3, 3, 3, 2−	.2. −(2 1, 2)	2.2 0. −2.2.20
2 1 3, 3, 2−	3, 3, 2 1, 2−	.(3, 2).2 0	8*−4 0
2 1 3, 2 1, 2−	3, 2 1, 3, 2−	.2 0. −(3, 2)	8*−3 1 0
2 1 2 1, 3, 2−	3, 2 1, 2 1, 2−	−2 2 : 2 : 2	8*−2 1 1 0
2 1 2 1, 2 1, 2−	2 1, 3, 2 1, 2−	−2 2 0 : 2 : 2 0	8*−3 0.2 0
2 1 1 2, 3, 2−	2 1, 2 1, 2 1, 2−	−2 2 : 2 0 : 2 0	8*3 : −2 0
2 1 1 2, 2 1, 2−	3, 3, 3, 2− −	−2 2 : −2 0 : −2 0	8*−2 1 0 : 2
2 1 1 1 1, 3, 2−	3, 3, 2 1, 2− −	2 2 : −2 0 : −2 0	8*−3 0 : 2 0
2 1 1 1 1, 2 1, 2−	3, 2 1, 3, 2− −	−2 1 1 : 2 : 2	8*3 0 : −2 0
5, 2 2, 2−	(2 2, 2) (3, 2−)	−2 1 1 0 : 2 : 2 0	8*−2 1 0 : 2 0
5, 2 1 1, 2−	(2 2, 2) (2 1, 2−)	−2 1 1 : 2 0 : 2 0	8*−2 1 0 : .2 0
4 1, 2 2, 2−	(2 1 1, 2) (3, 2−)	−2 1 1 : −2 0 : −2 0	8*−3 0 : .2 0
4 1, 2 1 1, 2−	(2 1 1, 2) (2 1, 2−)	−4 0 : 2 : 2	8*−2 1 0 : .2 0
3 2, 2 2, 2−	(3, 2 1) (3, 2−)	−4 : 2 : 2 0	8*3 0 : : −2 0
3 2, 2 1 1, 2−	(3, 2 1) (2 1, 2−)	−4 0 : 2 0 : 2 0	8*3 : : −2 0
3 1 1, 2 2, 2−	(3, 2+) (3, 2−)	−4 0 : −2 0 : −2 0	8*2 1 : : −2 0
3 1 1, 2 1 1, 2−	(3, 2+) (2 1, 2−)	−3 1 0 : 2 : 2	8*2. −2 0.2
2 3, 2 2, 2−	(2 1, 2+) (3, 2−)	−3 1 0 : 2 0 : 2 0	8*2. −2 0.2 0
2 3, 2 1 1, 2−	(2 1, 2+) (2 1, 2−)	−3 1 0 :−2 0 : −2 0	8*2.2 0. −2 0
2 2 1, 2 2, 2−	(2 2, 2−) (3, 2)	−2 1 1 0 : 2 : 2	8*2 : 2 : −2 0
2 2 1, 2 1 1, 2−	(2 2, 2−) (2 1, 2)	−2 1 1 : 2 : 2 0	8*2 : 20 : −2 0
5, 3, 2 1−	(2 1 1, 2−) (3, 2)	−2 1 1 0 : 2 0 : 2 0	8*2 : − 2 0 : 20
4 1, 3, 3−	(2 1 1, 2−) (2 1, 2)	−2 1 1 0 : − 2 0 : −2 0	8*2 0 : −20 : 20
4 1, 2 1, 2 1−	(3, 3−) (3, 2)	−3 0 : 2 1 : 2	8*2 0 : 2 0 : −20
3 2, 3, 2 1−	(3, 3−) (2 1, 2)	−3 0 : 2 1 : −2 0	8*2 : . −2 0 : .2
3 1 1, 3, 3−	(2 1, 2 1−) (3, 2)	−3 0 : 2 1 0 : 2	8*2 : .2 : . −20
3 1 1, 2 1, 2 1−	(2 1, 2 1−) (2 1, 2)	−2 1 0 : 3 0 : 2	9*. −3
2 3, 3, 3−	(2 2, 2)−(3, 2)	−2 1 0 : − 30 : −2 0	9*. −2 1
2 3, 2 1, 2 1−	(2 2, 2)−(2 1, 2)	−2 1 0 : 2 1 : 2	9*2. −2
2 2 1, 3, 2 1−	(2 1 1, 2)−(3, 2)	−2 1 0 : −2 1 0 : −2 0	9*2 0. −2
2 1 2, 3, 3−	(2 1 1, 2)−(2 1, 2)		9*.2 : . −2
2 1 2, 3, 2 1−	(3, 2 1)−(3, 2)		9*. −2 : . −2
2 1 2, 2 1, 2 1−	(3, 2 1)−(2 1, 2)		9*.2 0 : . −2
2 1 1 1, 3, 3−	(3, 2+)−(3, 2)		10*−2 0
2 1 1 1, 3, 2 1−	(3, 2+)−(2 1, 2)		10**−2 0
2 1 1 1, 2 1, 2 1−	(2 1, 2+)−(3, 2)		
	(2 1, 2+)−(2 1, 2)		

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Computations in knot theory

H. F. TROTTER

1. Computer representation of knots. The commonest way of presenting a specific knot to the human eye is by a diagram of the type shown in Fig. 1, which is to be interpreted as the projection of a curve in 3-dimensional space.

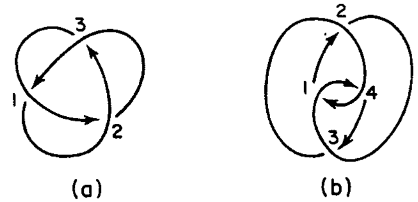

FIG. 1

There are obviously many ways of coding the information in such a diagram for a computer. Conway's notation [2] (which I learned of for the first time at the conference) seems to me much the best both for handwork and (perhaps with some modification) for computer representation. In some work done at Kiel [3, 4, 11] under the direction of Prof. G. Weise, one notation used is based on noting the cyclic order of vertices around the knot, and another is related to Artin's notation for braids. The simple notation described below is what I have actually used for computer input. It has proved reasonably satisfactory for experimental purposes.

To each vertex of the diagram there correspond two points on the knot, which we refer to as the upper and lower nodes. Each node has a successor arrived at by moving along the knot in the direction indicated by the arrows. Each vertex has one of two possible orientations, as indicated in Fig. 2. If the vertices are then numbered in an arbitrary order, a complete descrip-

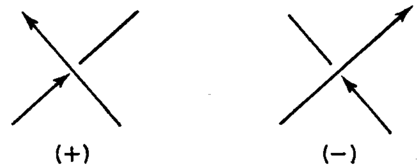

FIG. 2