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CLASSIFICATION OF KNOT PROJECTIONS

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The first step in tabulating the non-composite knots with n crossings is the tabulation of the non-singular plane projections of such knots, where two (piecewise linear) projections are regarded as equivalent, or in the same class, if they agree up to homeomorphism of the extended plane, i.e. two-sphere. This first step is here reduced to a simple algorithm suitable for computer use.

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knot	classification of knots
algorithm	tabulation of knots

The nineteenth century knot tables were based on a prior classification of knot projections. The knot projections with 8 to 11 crossings were found, mainly by Kirkman (see e.g. [2]), using geometrical methods. But Tait [3, Part 1] tabulated the knots with up to 7 crossings using mainly combinatorial methods. We have modified Tait's notation and methods to make them convenient for computer programming. This paper contains the theoretical justification of the methods.

Although none of the proofs needs diagrams, the reader will find the paper much easier to follow if he makes sketches.

Let L be a regular projection [1] of a knot k. Each crossing in L is the image of two points, the upper crossing and lower crossing, in K. If there are n crossings in L, and hence $2n$ crossing points in K, we number the crossing points $1, 2, \ldots, 2n$ consecutively, starting from any one of them, and going in a chosen direction along K.

There is an involutory function a from $\{1, 2, \ldots, 2n\}$ to itself, i.e. $a(a(i)) = i$, for which $a(i)$ is the other crossing point of K having the same projection as i. We shall usually write a_i for $a(i)$. The involution a is parity reversing: if i is odd then $a(i)$ is even, and vice versa. We write S for the sequence $a_1, a_2, \ldots, a_{2n}$. The subsequence $a_1, a_3, \ldots, a_{2n-1}$ determines S and a.

There are $4n$ ways of numbering the crossing points of K, corresponding to $2n$ possible starting points and 2 possible directions, and the corresponding sequences need not be all the same. The *standard* sequence corresponding to the knot