EAS 2025

Discovering optimal analytic dark matter halo profiles with deep RL

Wassim Tenachi

With Yashar Hezaveh, Laurence Perault-Levasseur, Jonathan Freundlich, Jenna Karcheski & Rodrigo Ibata

June 24, 2025

Background

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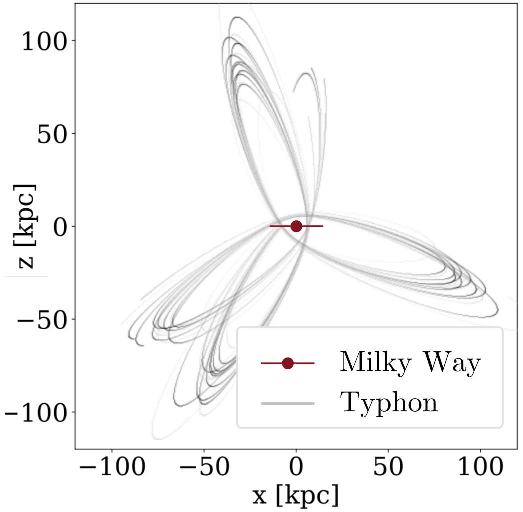
Detection of a dwarf galaxy remnant from the outer halo passing through the solar neighborhood



esentation of Typhon (abyss) in Greek mythology

[Tenachi et al 2022]

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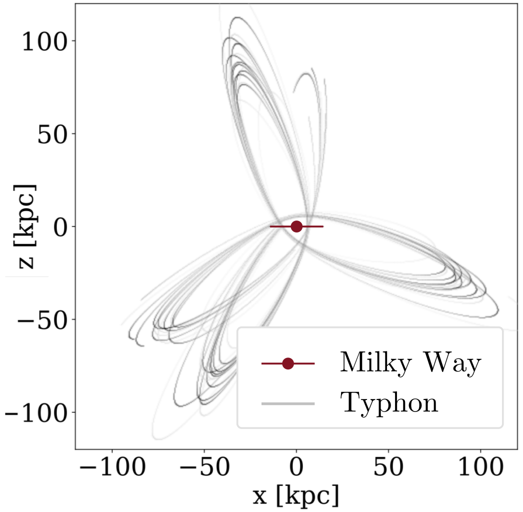

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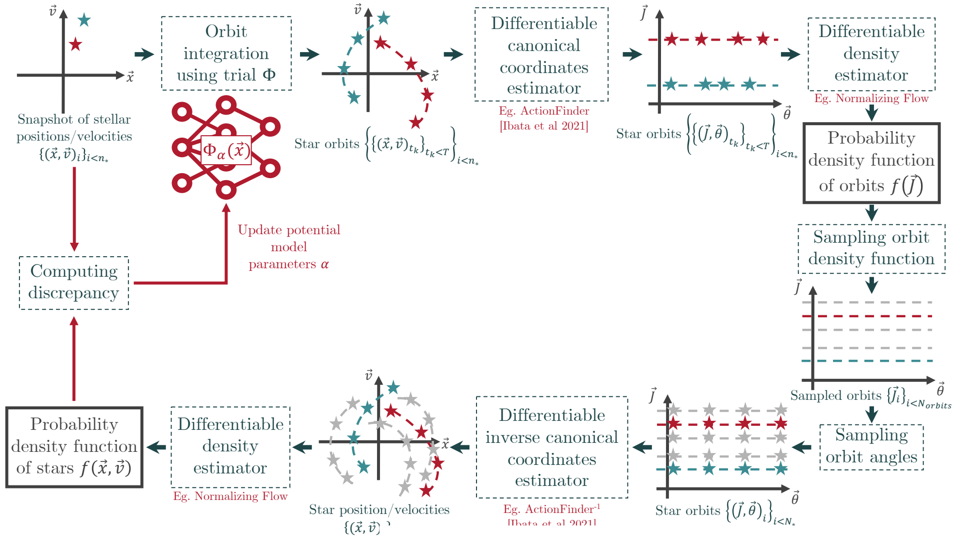
Agnostically mapping the Galactic acceleration field with unsupervised learning



[Tenachi et al 2023c] (in proc.)

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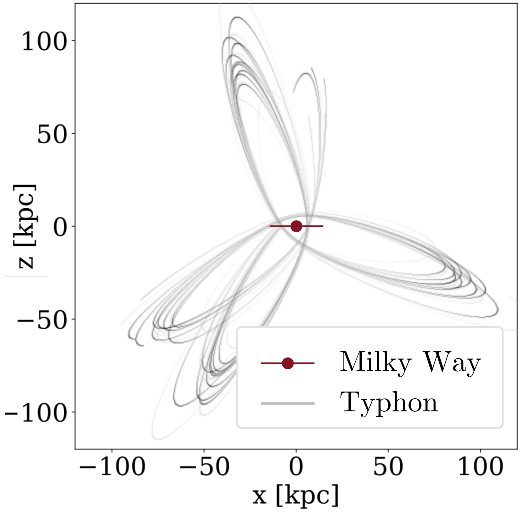

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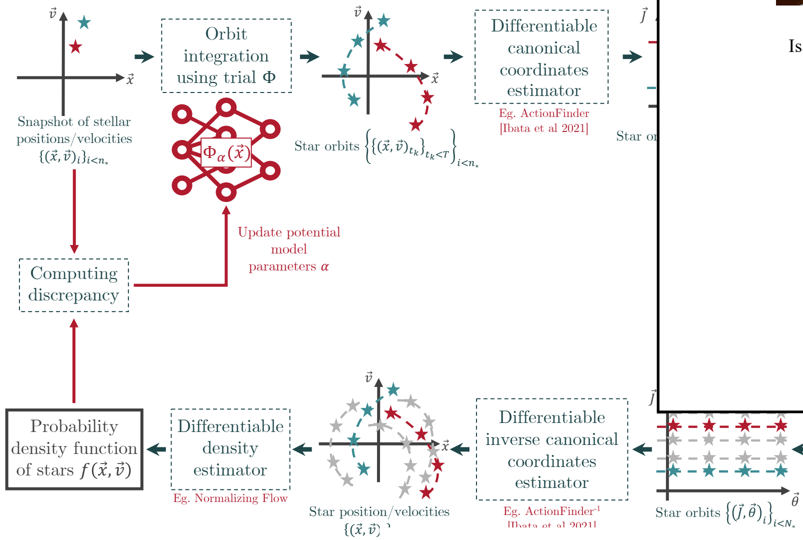

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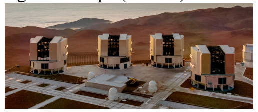
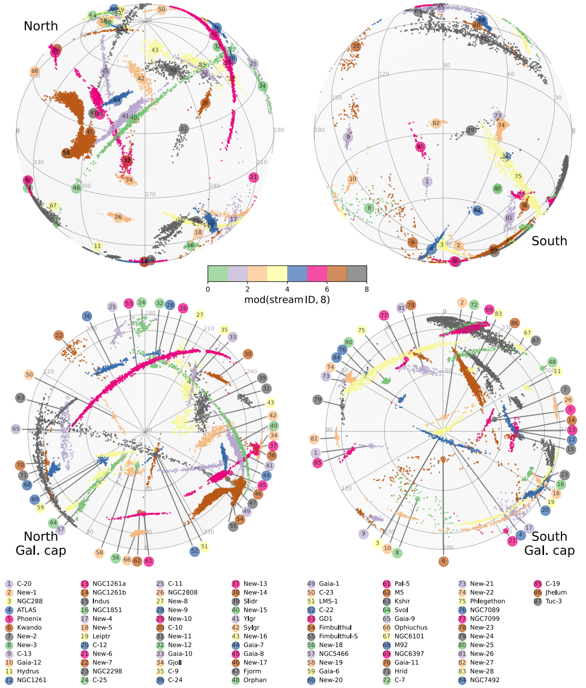
Constraining the dark matter distribution of our galaxy

Very Large Telescope (UVES)



Isaac Newton Telescope



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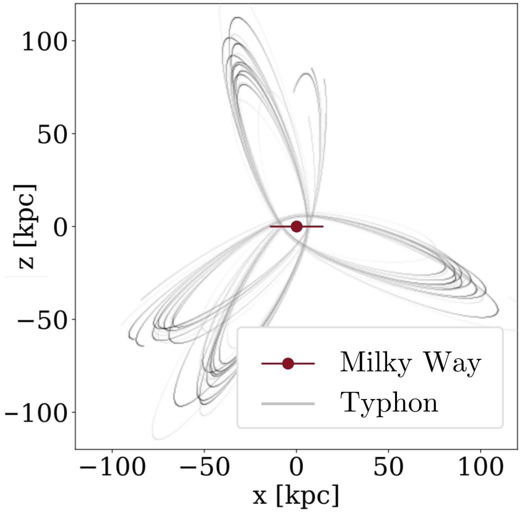

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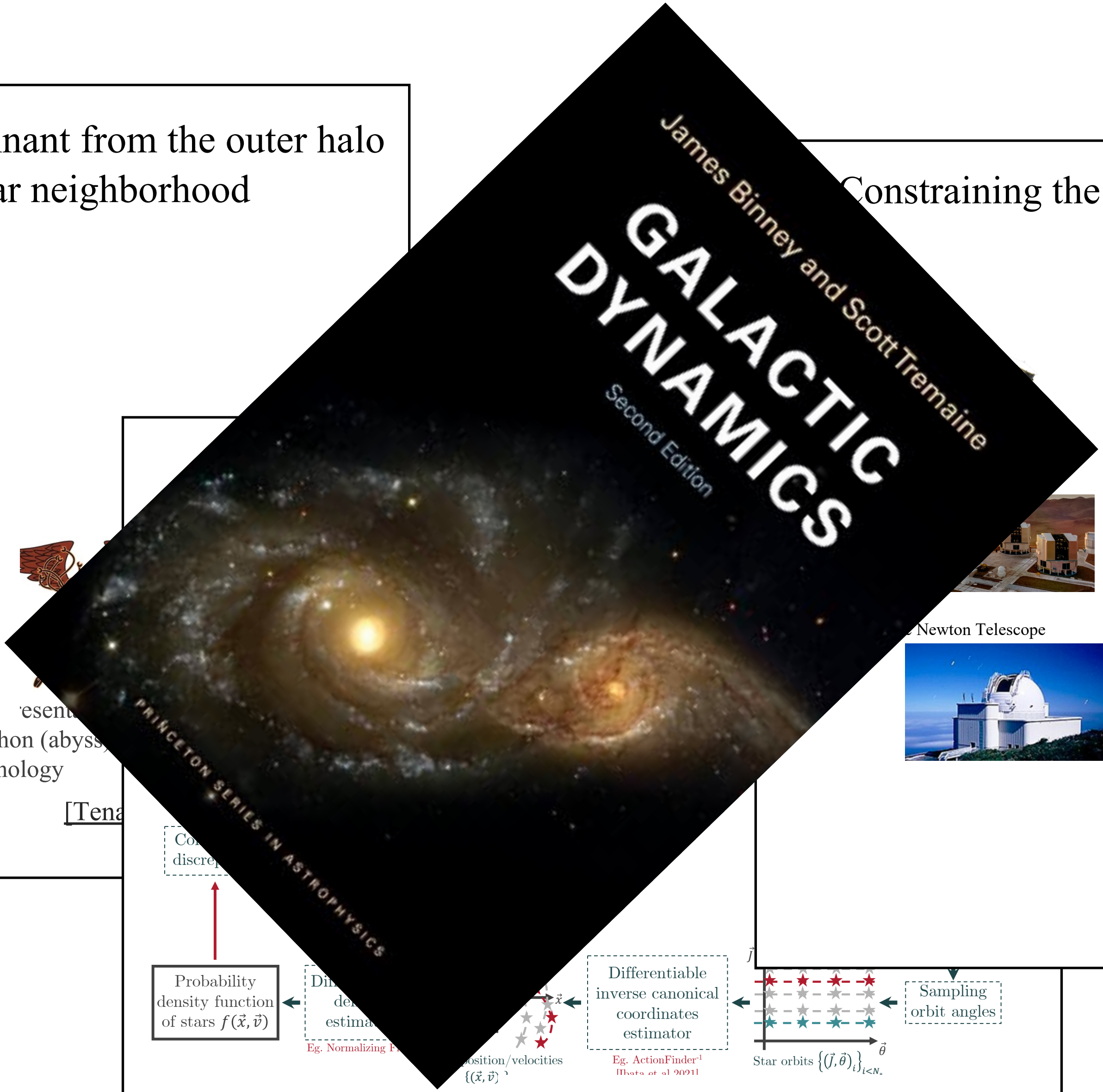

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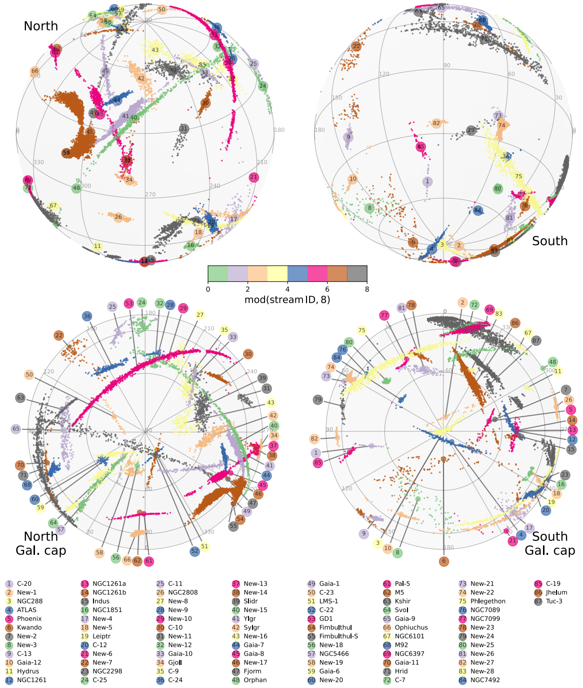

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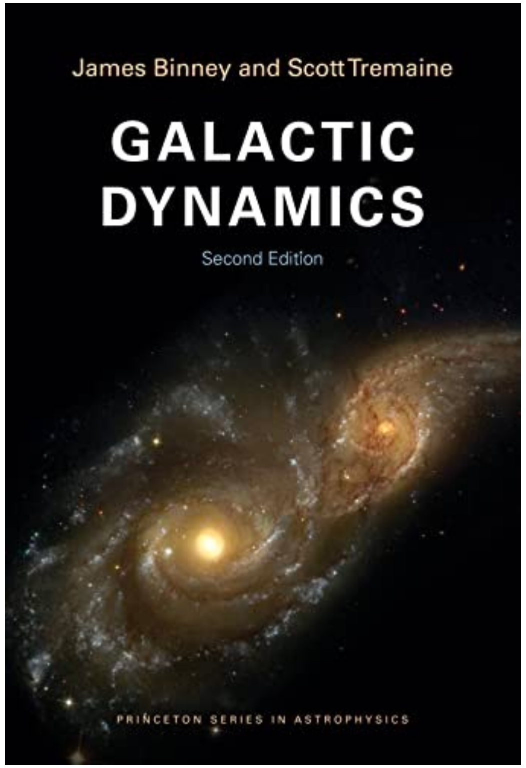
A zoo of DM profiles

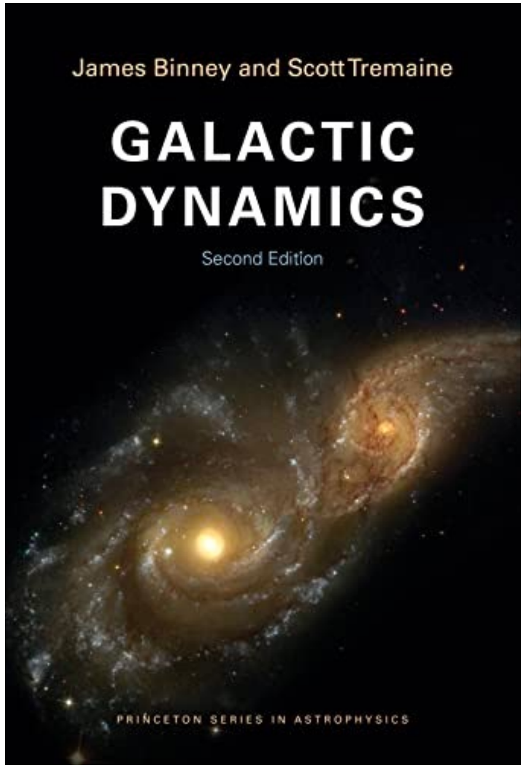


A zoo of DM profiles

NFW $$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$$

SuperNFW $$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^{5/2}}$$

(cusps)



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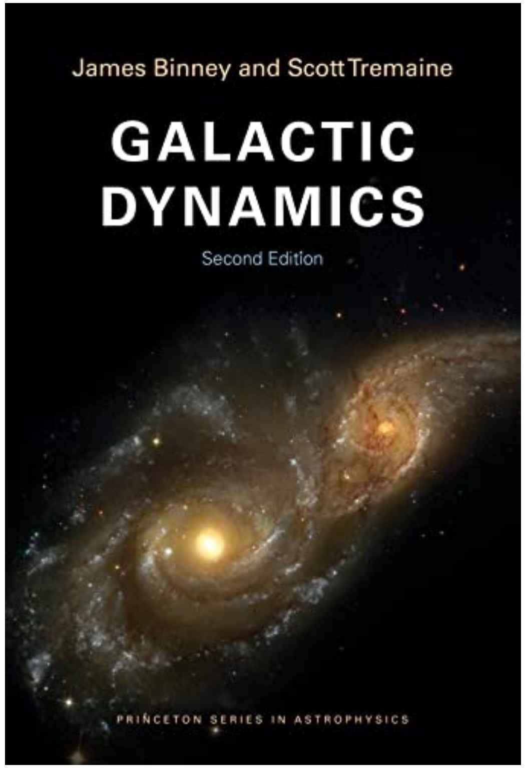

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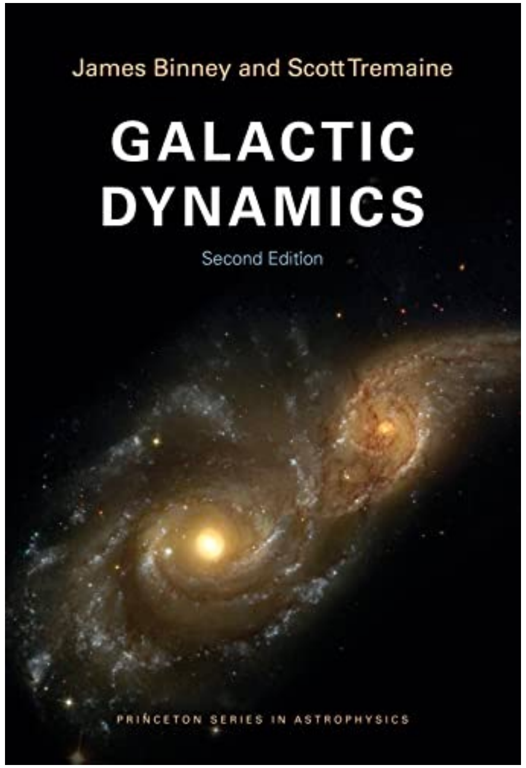

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Einasto $$\rho(r) = \rho_0 \exp\left(-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha} - 1\right]\right)$$

gNFW $$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^{\alpha}\left(1+\left(\frac{r}{r_s}\right)\right)^{3-\alpha}}$$

(variable inner slope)

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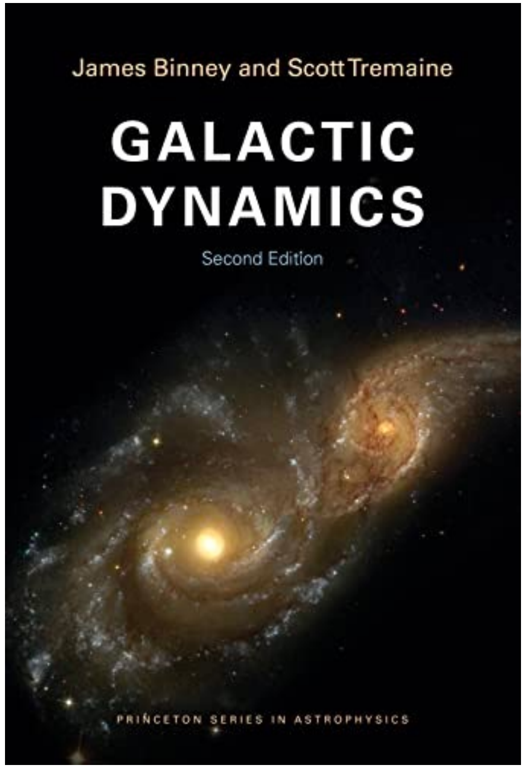

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(cores)

→ Let's explore the space of analytical equations for DM profiles

Symbolic regression (SR)

$(\boldsymbol{x}, y)$ data

x_1	x_2	y
0.75582	0.25850	0.02674
0.36786	0.42401	0.06278
0.69507	0.38057	0.74014
0.96493	0.33398	0.81558
0.07139	0.16604	0.07716
0.86413	0.41952	0.87872
0.18012	0.40581	0.63637

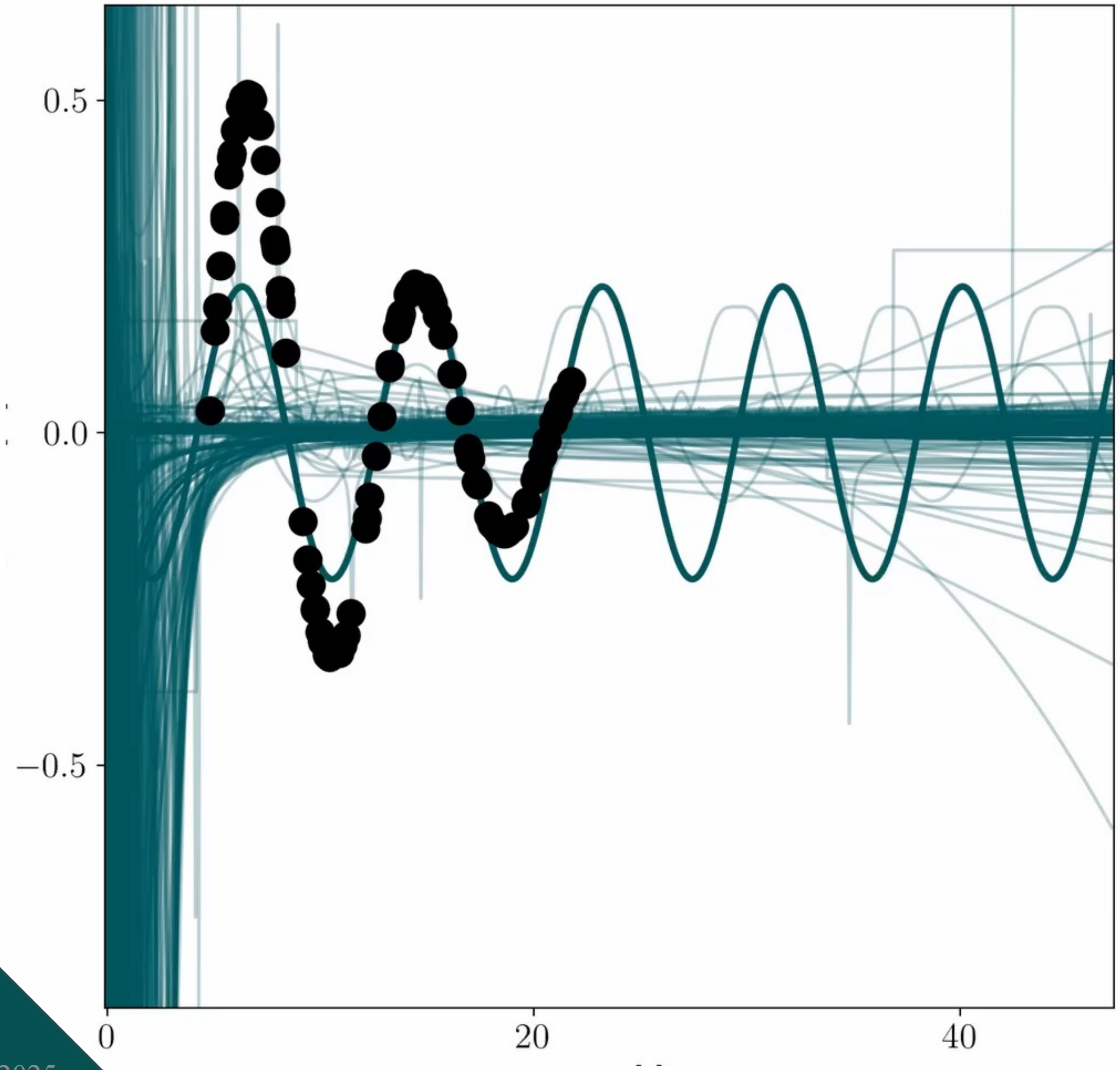
$\longrightarrow$

$$f : \mathbb{R}^n \to \mathbb{R} \text{ such that } y = f(\boldsymbol{x})$$

Symbolic regression (SR)

Φ-SO

Physical Symbolic Optimization.



Best fit:

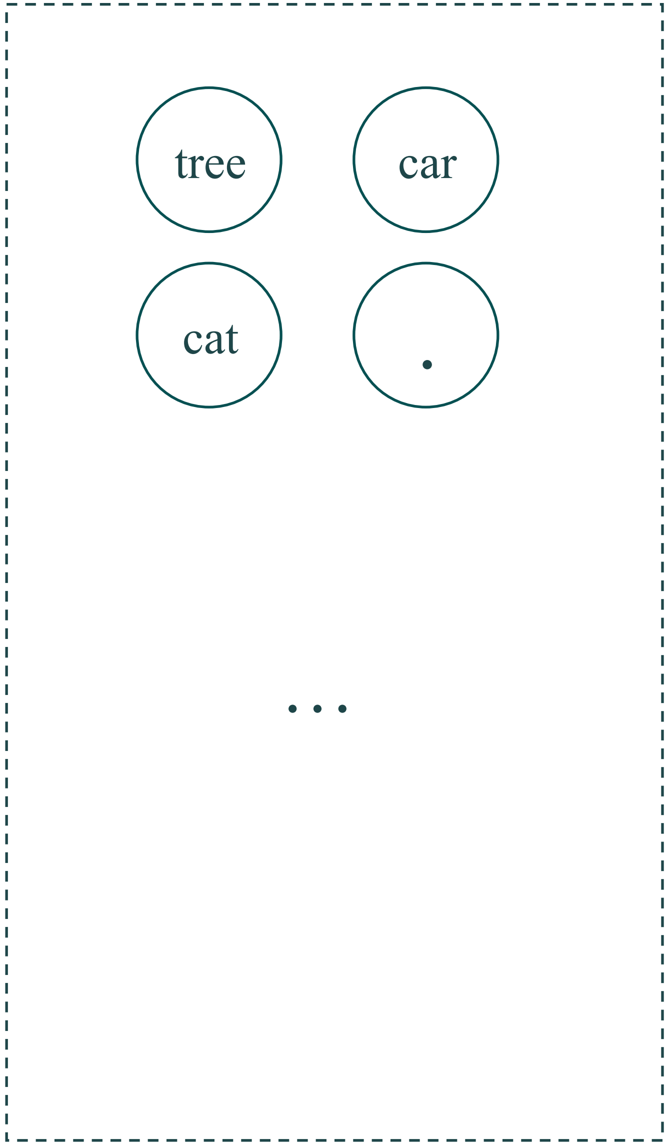
$$x(t) = \sin(\alpha t)\cos(\phi^2)$$

Trying:

$$x(t) = \sin(\alpha t)\cos(\phi^2)$$

https://youtu.be/wubzZMkoTUY

Language processing for symbolic mathematics



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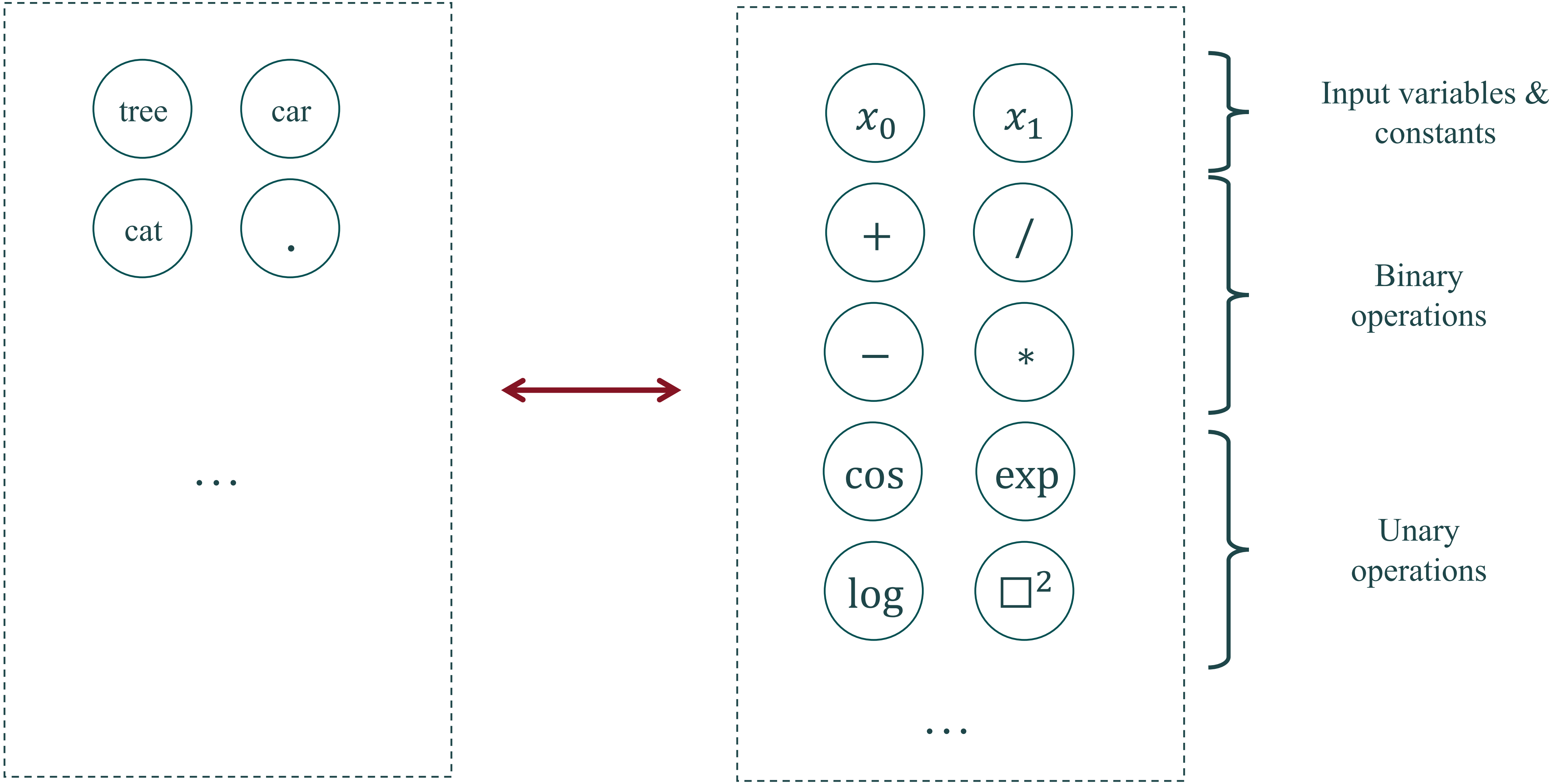


Embedding (1) : how to go from numbers to symbols ?

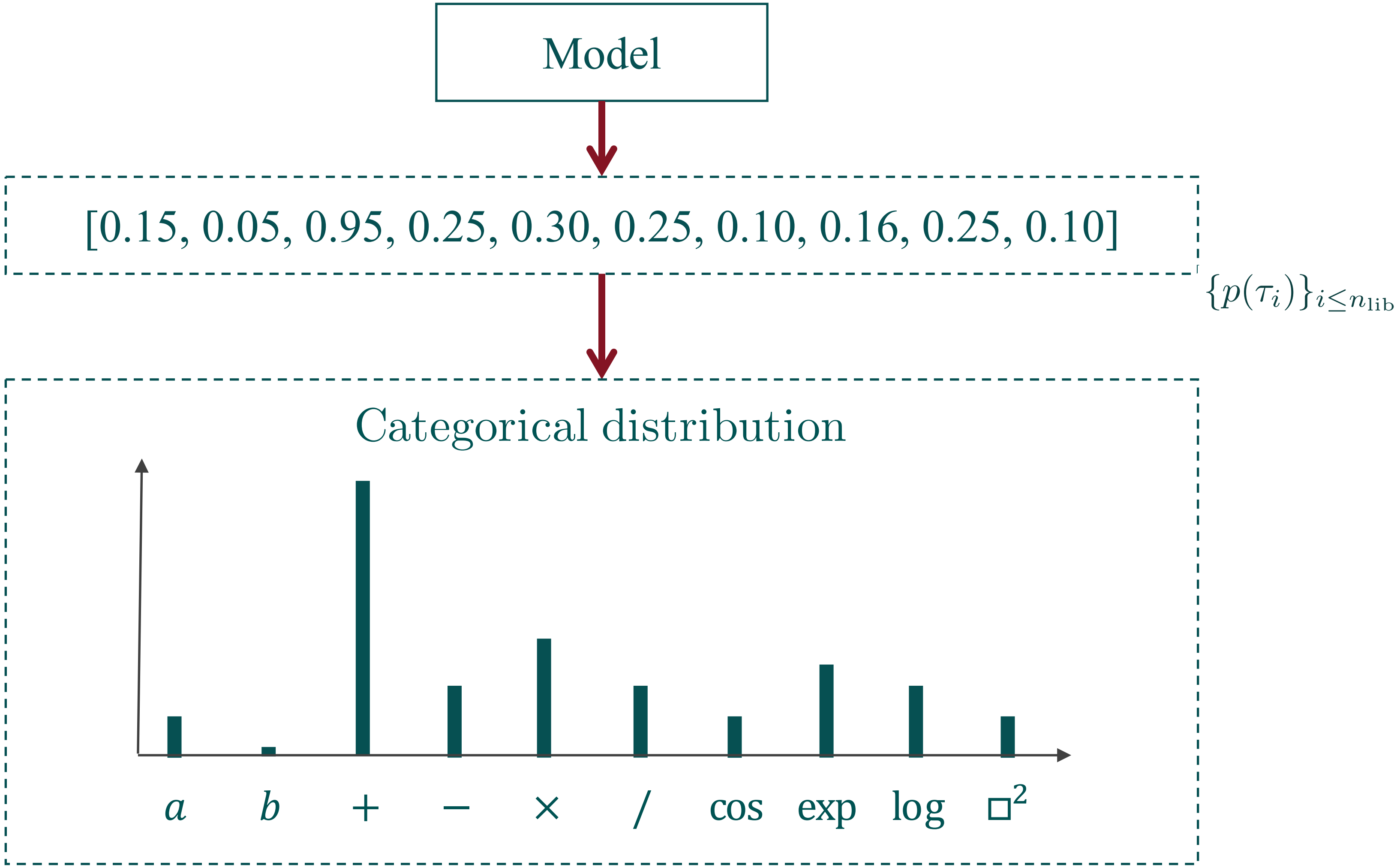
Model

$\downarrow$

[0.15, 0.05, 0.95, 0.25, 0.30, 0.25, 0.10, 0.16, 0.25, 0.10]

$\{p(\tau_i)\}_{i \leq n_{\text{lib}}}$

Embedding (1) : how to go from numbers to symbols ?



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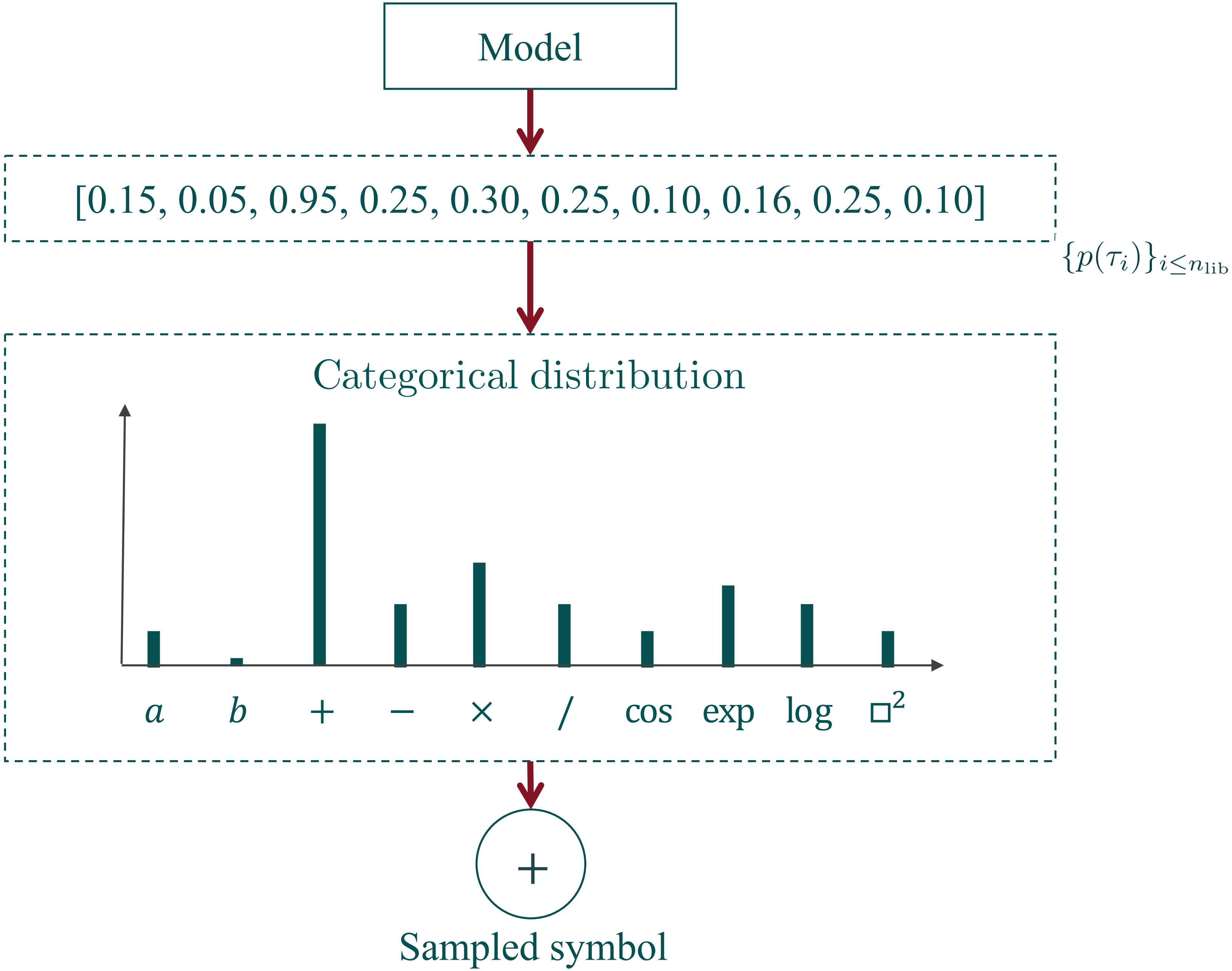

Embedding (2) : how to go from vector of symbols to expressions ?

Infix notation

$$a + \cos(b)$$

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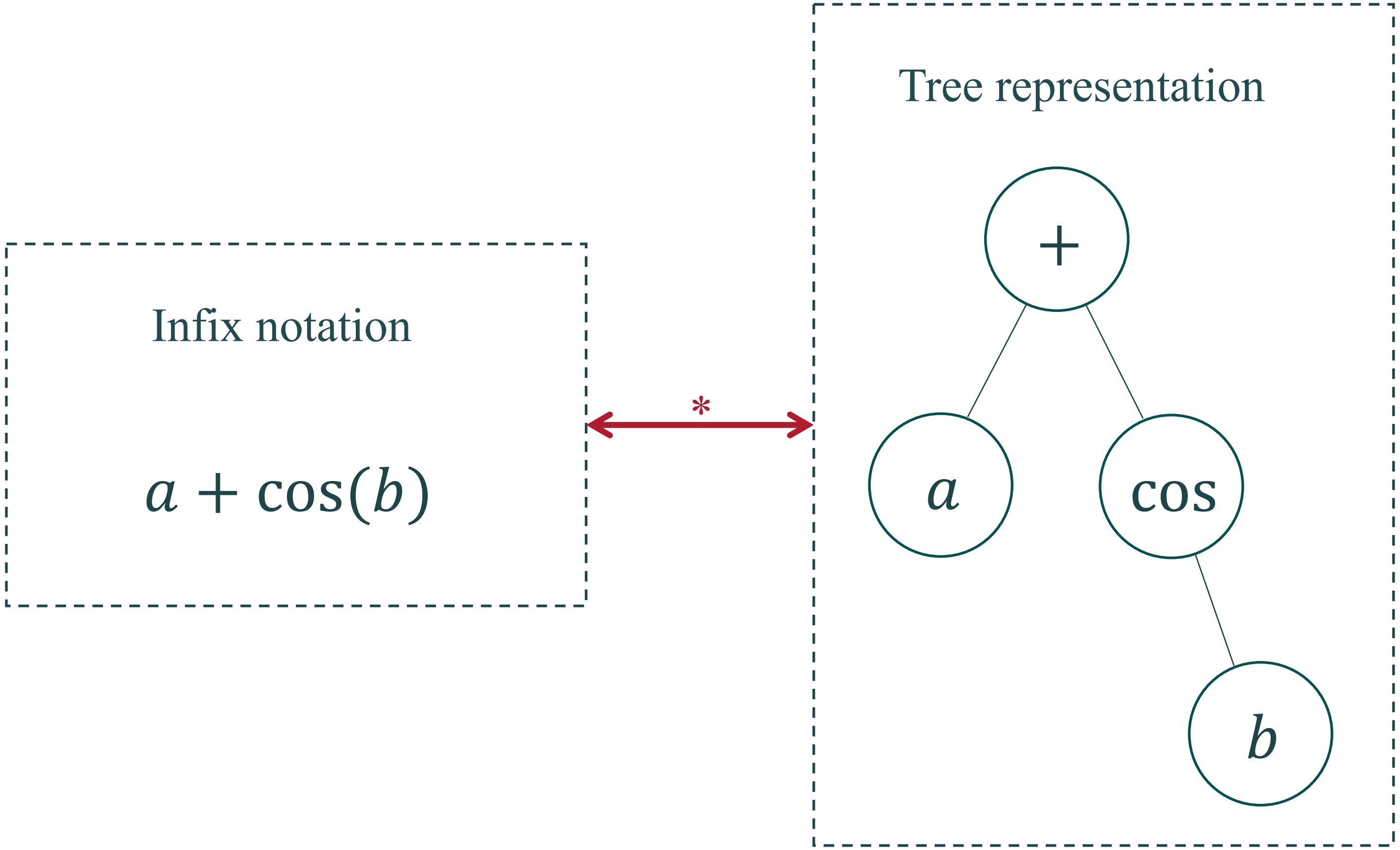

* 1:1 equivalence

Embedding (2) : how to go from vector of symbols to expressions ?

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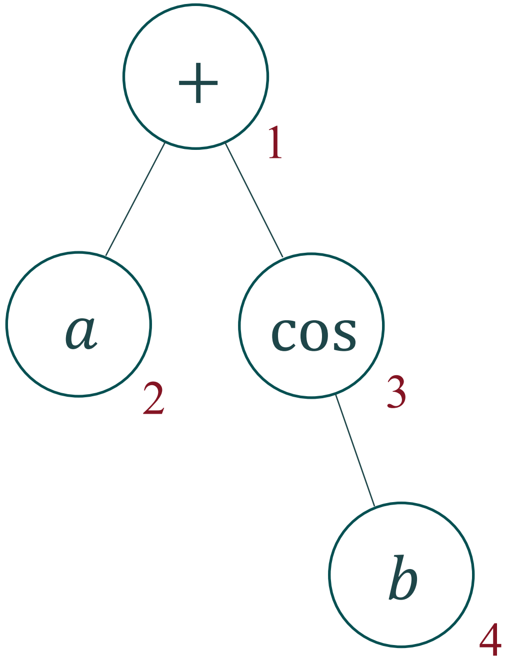
$\xleftrightarrow{*}$

Tree representation



Listing each node first in depth and then left to right

* 1:1 equivalence

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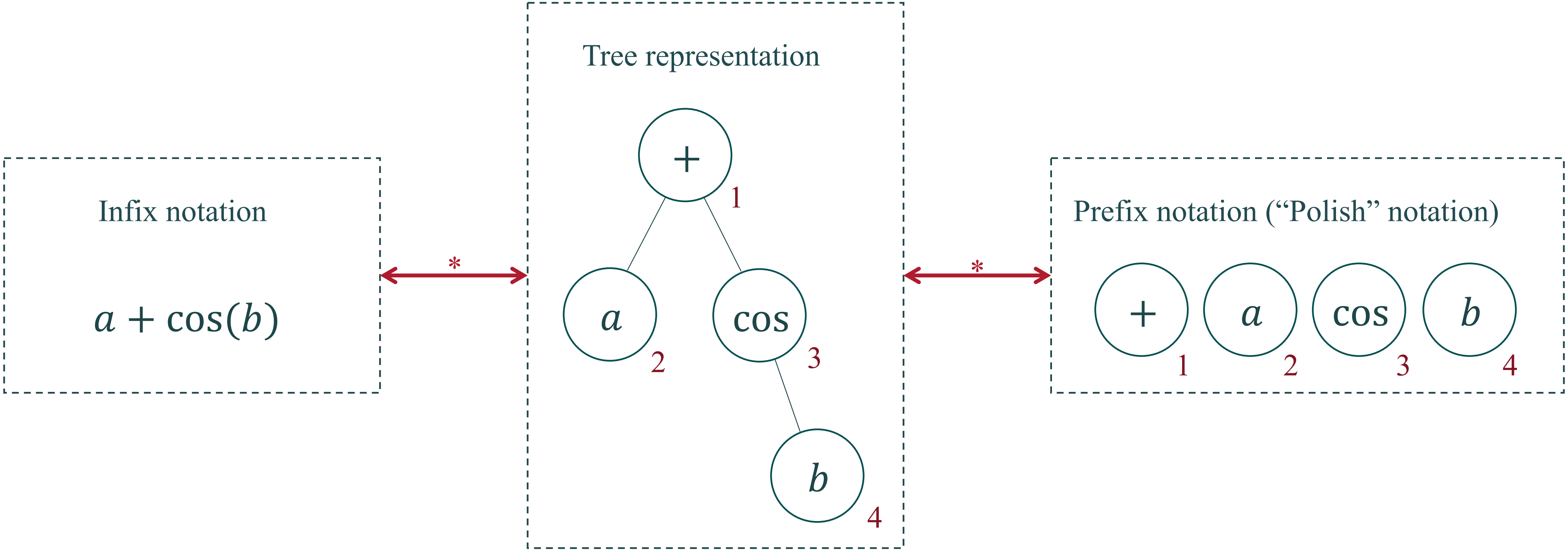

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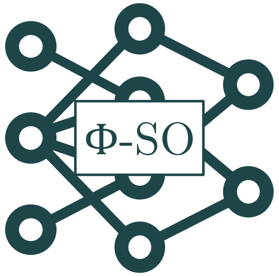
Reinforcement learning & risk seeking policy



Categorical distributions

$$v = v_0 + x/t$$

$$v = v_0$$

$$v = v_0 - x/t$$

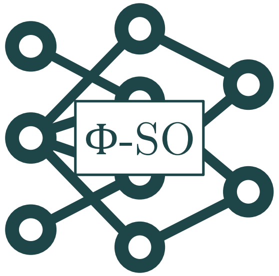
$$v = x/t$$

$$\vdots$$

$$v = \frac{v_0^2}{x/t}$$

Wassim Tenachi - w.tenachi@gmail.com

Reinforcement learning & risk seeking policy



Categorical distributions

Candidates vs data → reward

$v = v_0 + x/t$ → r = 0.99

$v = v_0$ → r = 0.93

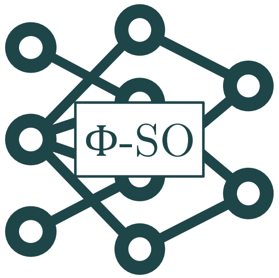
$v = v_0 - x/t$ → r = 0.73

$v = x/t$ → r = 0.28

$\vdots$

$v = \frac{v_0^2}{x/t}$ → r = 0.64

Reinforcement learning & risk seeking policy



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⋮

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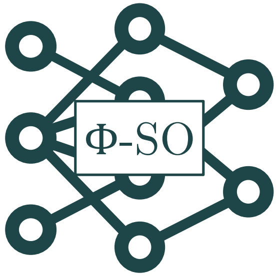
Reinforce on the 5% best candidates

Not punished/rewarded on 95% of candidates → "risk seeking"

Reinforcement learning & risk seeking policy

→ Black box reward function



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Not punished/rewarded on 95% of candidates → "risk seeking"

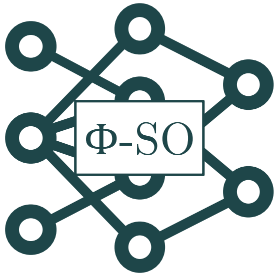
Reward → no auto-differentiation

(unlike most machine learning methods used in physics)

→ We can apply any physical constraints we want even if it is not differentiable

Reinforcement learning & risk seeking policy

↳ Black box reward function



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Reinforce on the 5% best candidates

Not punished/rewarded on 95% of candidates → "risk seeking"

Reward → no auto-differentiation
(unlike most machine learning methods used in physics)

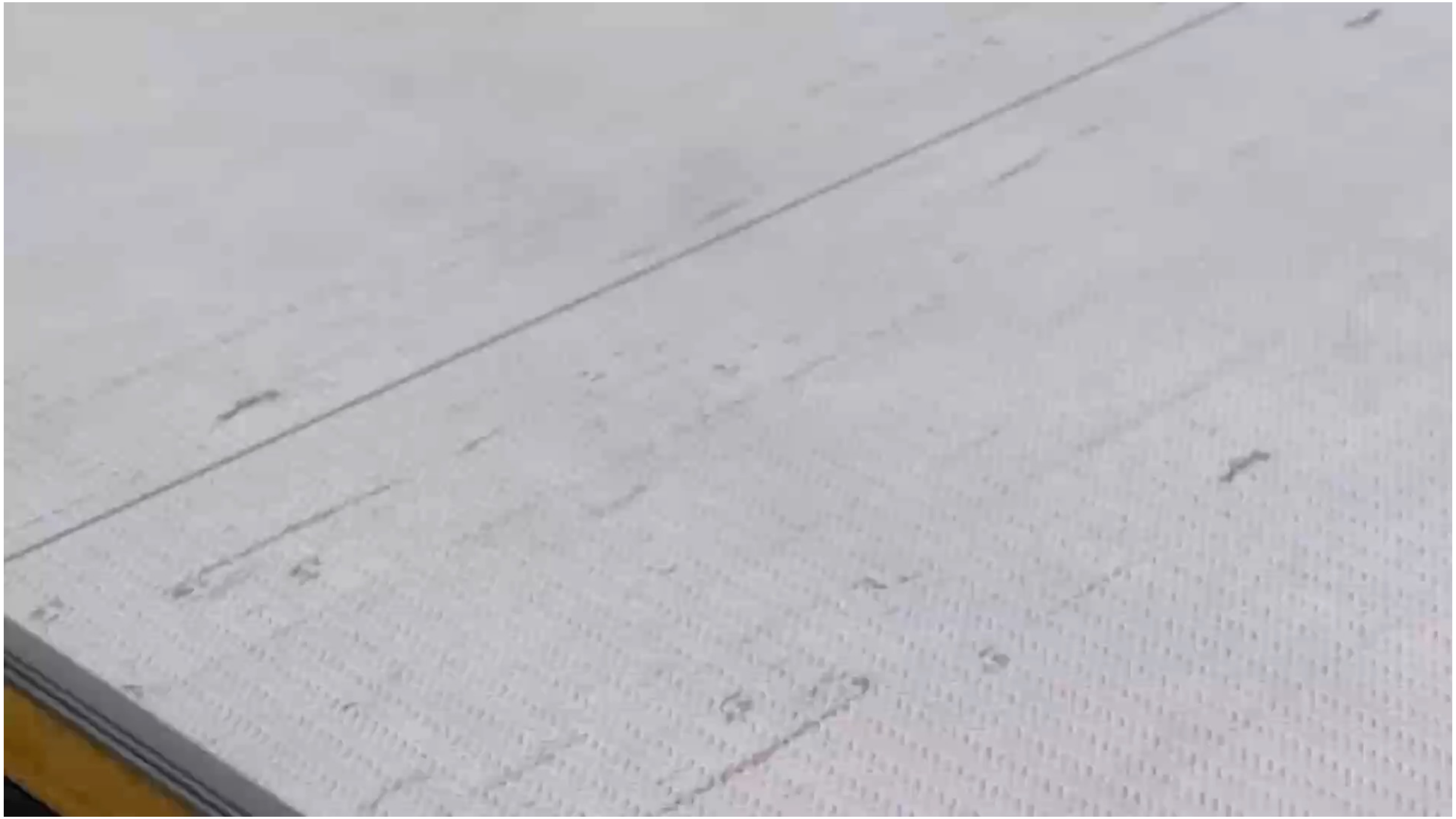
→ We can apply any physical constraints we want even if it is not differentiable

→Complexity (Occam's razor)
→Symmetries
→Constraints on derivatives/primitives
→Symbolic computing using Sympy/Mathematica
→Fitness in ODEs, limits values
→Behavior in N-body simulation
…

Reinforcement learning in a nutshell (1)

https://youtu.be/spfpBrBjntg

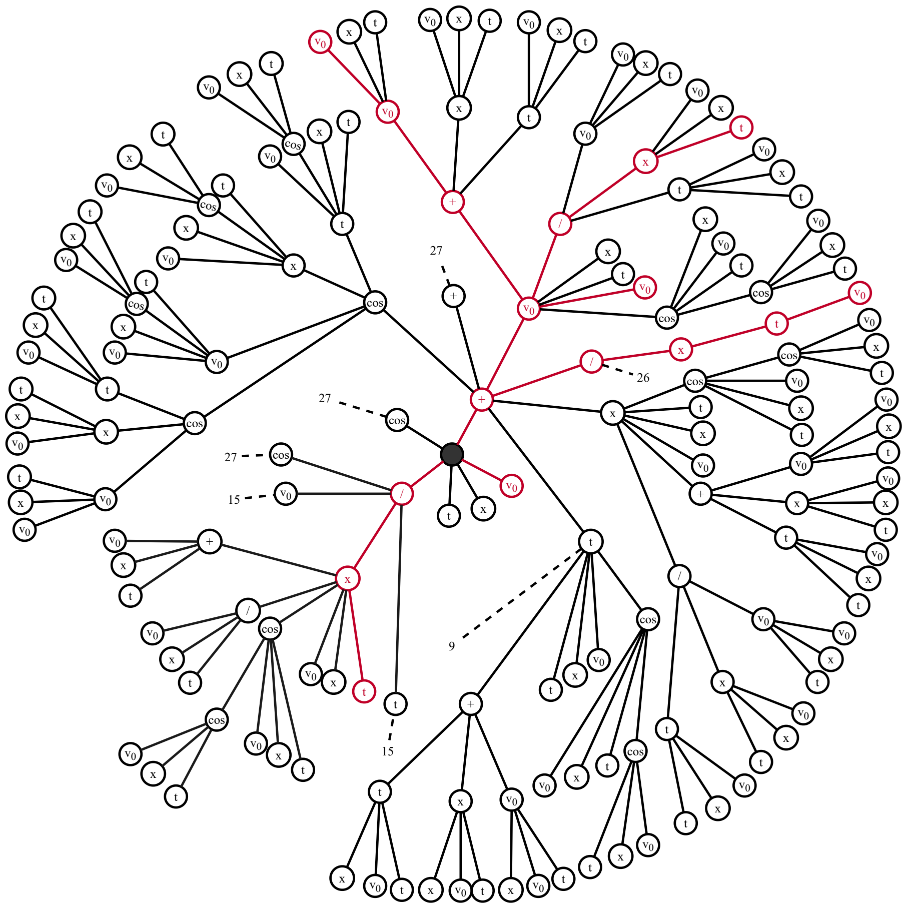
Reinforcement learning in a nutshell (2)



https://youtu.be/igZ6IPQimjQ

Search space reduction using physical units constraints

268 expressions



Search space

Prefix notation paths for expressing a velocity v using a library of symbols {+, /, cos, v_0, x, t} where v_0 is a velocity, x is a length, and t is a time (length < 5 for readability).

[Tenachi et al 2023a,b]

Search space reduction using physical units constraints

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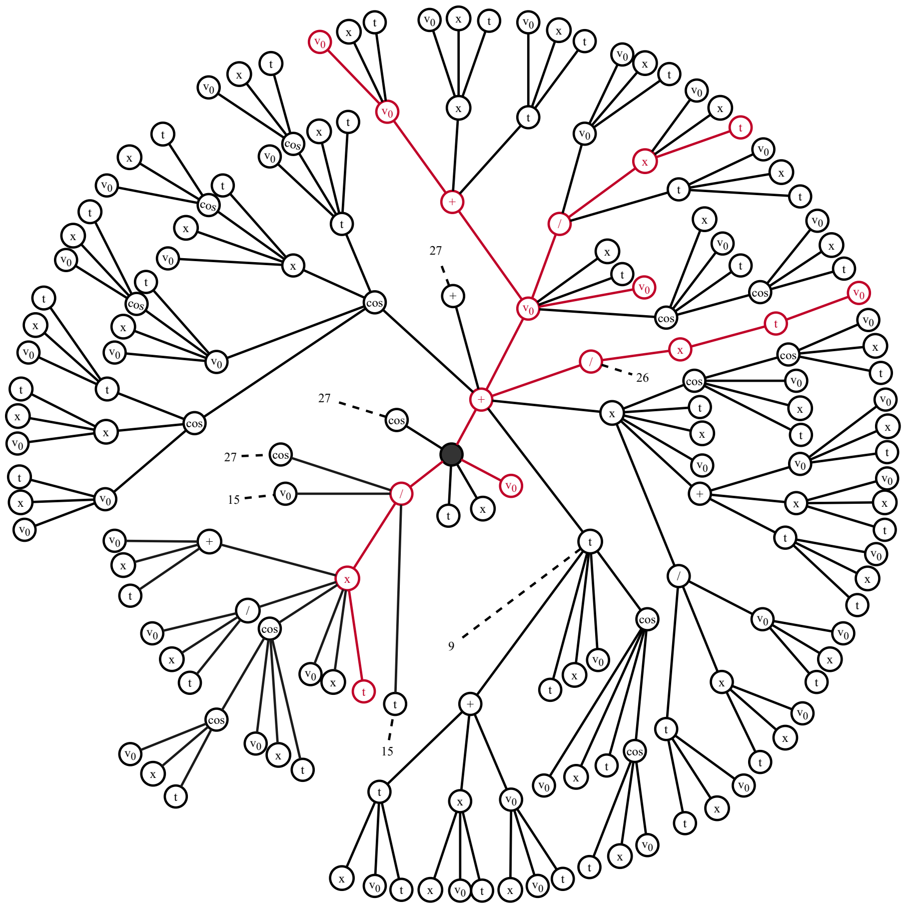
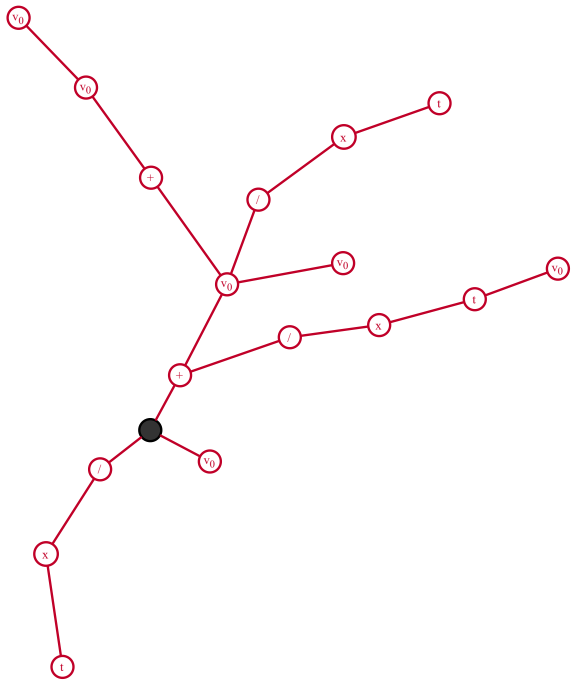


Search space

6 expressions



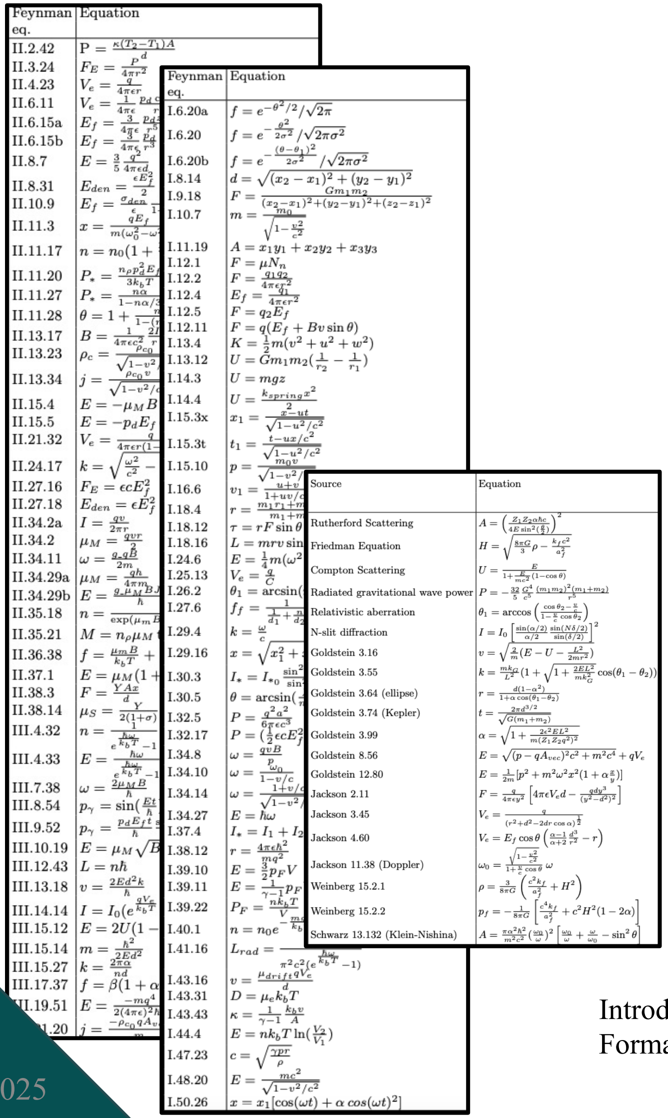
Search space with our *in situ* physical units prior

Prefix notation paths for expressing a velocity v using a library of symbols {+, /, cos, v_0, x, t} where v_0 is a velocity, x is a length, and t is a time (length < 5 for readability).

[Tenachi et al 2023a,b]

Feynman benchmark

120 equations from the Feynman Lectures on Physics (+ other physics textbooks)



Source	Equation
Rutherford Scattering	$A = \left(\frac{Z_1 Z_2 \alpha \hbar c}{4E\sin^2(\frac{\theta}{2})}\right)^2$
Friedman Equation	$H = \sqrt{\frac{8\pi G}{3}\rho - \frac{k_f c^2}{a_f^2}}$
Compton Scattering	$U = \frac{E}{1+\frac{E}{mc^2}(1-\cos\theta)}$
Radiated gravitational wave power	$P = -\frac{32}{5}\frac{G^4}{c^5}\frac{(m_1 m_2)^2(m_1+m_2)}{r^5}$
Relativistic aberration	$\theta_1 = \arccos\left(\frac{\cos\theta_2 - \frac{v}{c}}{1-\frac{v}{c}\cos\theta_2}\right)$
N-slit diffraction	$I = I_0\left[\frac{\sin(\alpha/2)}{\alpha/2}\frac{\sin(N\delta/2)}{\sin(\delta/2)}\right]^2$
Goldstein 3.16	$v = \sqrt{\frac{2}{m}(E - U - \frac{L^2}{2mr^2})}$
Goldstein 3.55	$k = \frac{mk_G}{L^2}(1+\sqrt{1+\frac{2EL^2}{mk_G^2}}\cos(\theta_1-\theta_2))$
Goldstein 3.64 (ellipse)	$r = \frac{d(1-\alpha^2)}{1+\alpha\cos(\theta_1-\theta_2)}$
Goldstein 3.74 (Kepler)	$t = \frac{2\pi d^{3/2}}{\sqrt{G(m_1+m_2)}}$
Goldstein 3.99	$\alpha = \sqrt{1+\frac{2\epsilon^2 EL^2}{m(Z_1 Z_2 q^2)^2}}$
Goldstein 8.56	$E = \sqrt{(p-qA_{vec})^2c^2+m^2c^4}+qV_e$
Goldstein 12.80	$E = \frac{1}{2m}[p^2+m^2\omega^2x^2(1+\alpha\frac{x}{y})]$
Jackson 2.11	$F = \frac{q}{4\pi\epsilon y^2}\left[4\pi\epsilon V_e d - \frac{qdy^3}{(y^2-d^2)^2}\right]$
Jackson 3.45	$V_e = \frac{q}{(r^2+d^2-2dr\cos\alpha)^{\frac{1}{2}}}$
Jackson 4.60	$V_e = E_f\cos\theta\left(\frac{\alpha-1}{\alpha+2}\frac{d^3}{r^2}-r\right)$
Jackson 11.38 (Doppler)	$\omega_0 = \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cos\theta}\omega$
Weinberg 15.2.1	$\rho = \frac{3}{8\pi G}\left(\frac{c^2 k_f}{a_f^2}+H^2\right)$
Weinberg 15.2.2	$p_f = -\frac{1}{8\pi G}\left[\frac{c^4 k_f}{a_f^2}+c^2H^2(1-2\alpha)\right]$
Schwarz 13.132 (Klein-Nishina)	$A = \frac{\pi\alpha^2\hbar^2}{m^2c^2}(\frac{\omega_0}{\omega})^2\left[\frac{\omega_0}{\omega}+\frac{\omega}{\omega_0}-\sin^2\theta\right]$

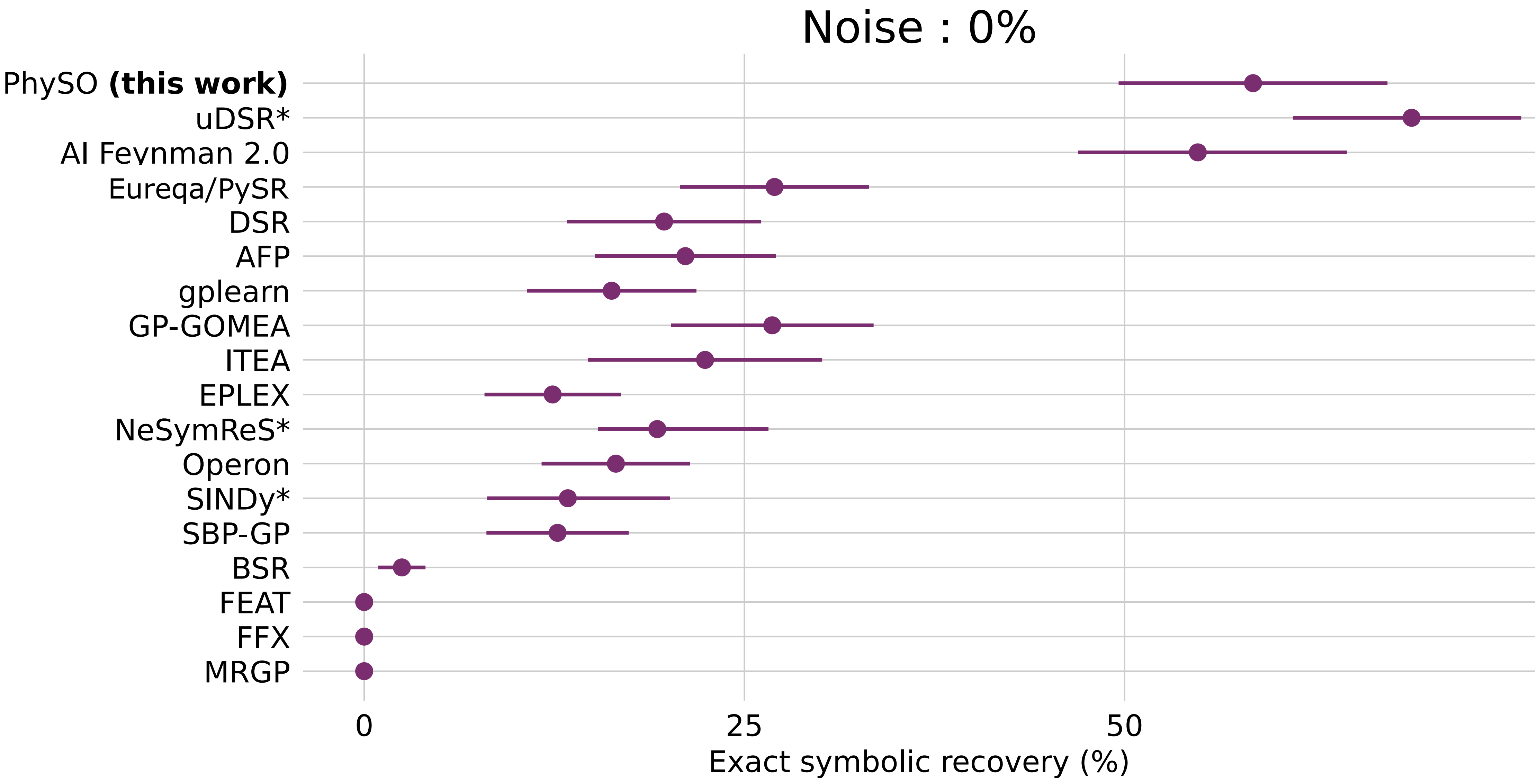
Against 17 other algorithms

Method	Technique(s)	Description	Reference
`PhySO`	RL, DA	Physical Symbolic Optimization	This work
`uDSR`	RL, GP, Simp., Sup.	A Unified Framework for Deep Symbolic Regression	Landajuela et al. (2022)
`AIFeynman 2.0`	Simp., DA	Symbolic regression exploiting graph modularity	Udrescu et al. (2020)
`AFP_FE`	GP	AFP with co-evolved fitness estimates, Eureqa-esque	Schmidt & Lipson (2009)
`DSR`	RL	Deep Symbolic Regression	Petersen et al. (2021a)
`AFP`	GP	Age-fitness Pareto Optimization	Schmidt & Lipson (2011)
`gplearn`	GP	Koza-style symbolic regression in Python	Stephens (2015)
`GP-GOMEA`	GP	GP-Optimal Mixing Evolutionary Algorithm	Virgolin et al. (2021)
`ITEA`	GP	Interaction-Transformation EA	de Franca & Aldeia (2021)
`EPLEX`	GP	ϵ-lexicase selection	La Cava et al. (2019)
`NeSymReS`	Sup.	Neural Symbolic Regression that Scales	Biggio et al. (2021)
`Operon`	GP	SR with Non-linear least squares	Kommenda et al. (2020)
`SINDy`	NeuroSym	Sparse identification of non-linear dynamics	Brunton et al. (2016)
`SBP-GP`	GP	Semantic Back-propagation Genetic Programming	Virgolin et al. (2019)
`BSR`	MCMC	Bayesian Symbolic Regression	Jin et al. (2019)
`FEAT`	GP	Feature Engineering Automation Tool	Cava et al. (2019)
`FFX`	Rand.	Fast function extraction	McConaghy (2011)
`MRGP`	GP	Multiple Regression Genetic Programming	Arnaldo et al. (2014)

Table 3. Summary of baseline symbolic regression methods along with the the underlying techniques they rely on: reinforcement learning (RL), genetic programming (GP), problem simplification schemes (Simp.), end-to-end supervised learning (Sup.), dimensional analysis (DA), neuro-symbolic / auto-differentiation based sparse fitting techniques (NeuroSym), Markov chain Monte Carlo (MCMC) and random search (Rand.).

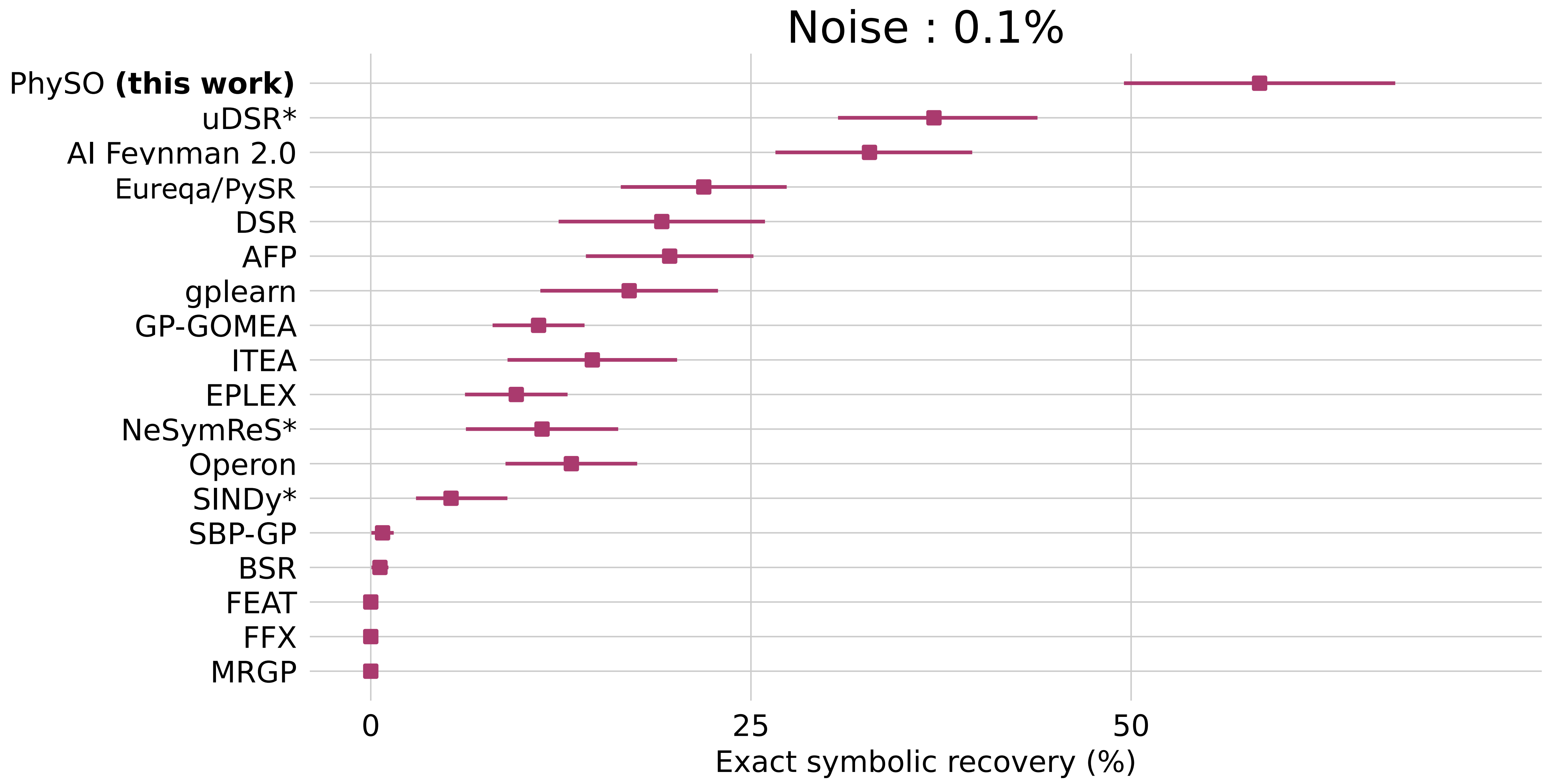
Introduced in [Udrescu & Tegmark 2020]
Formalized benchmark in SRBench [LaCava et al 2021]

Feynman benchmark



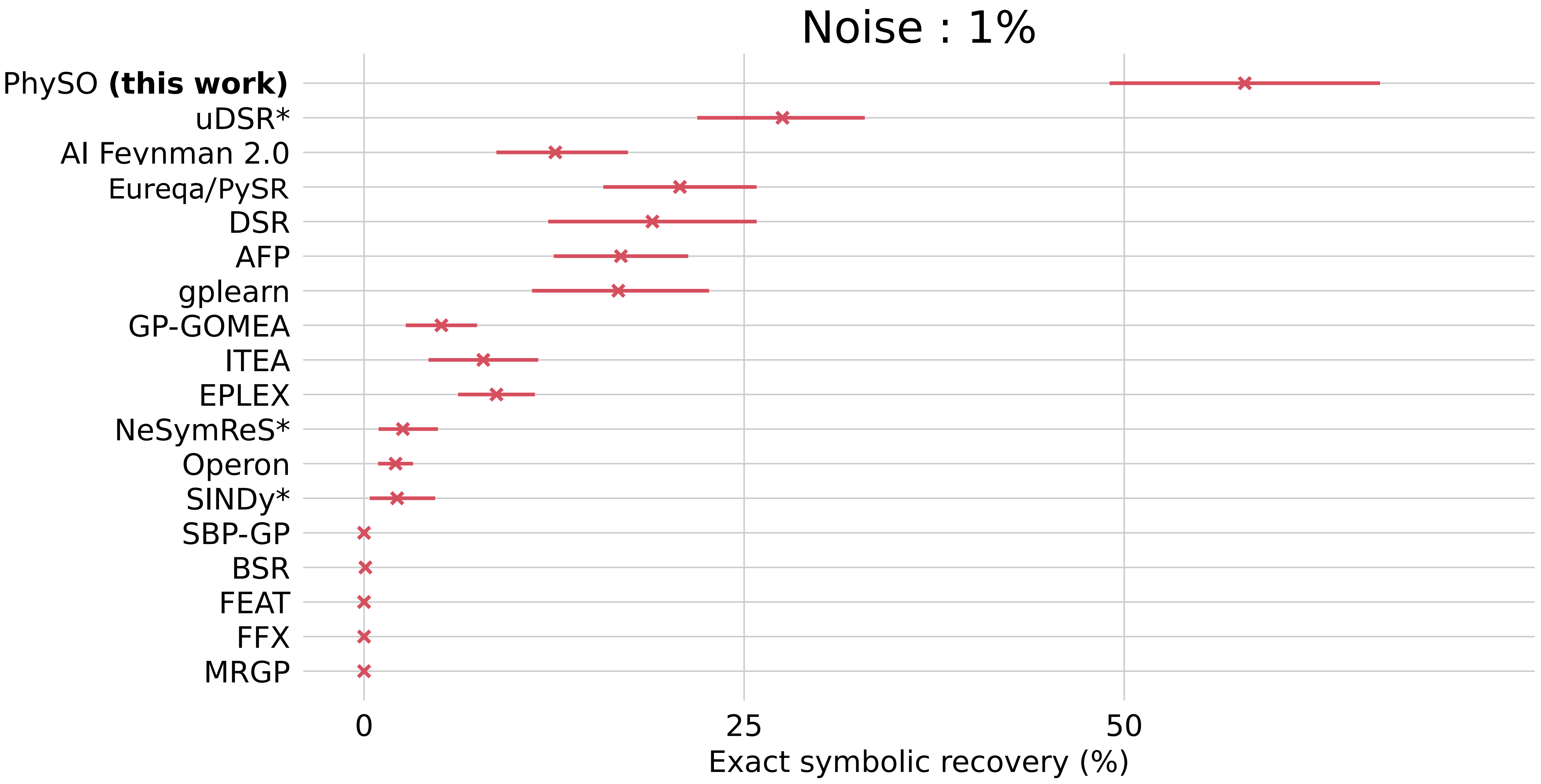
[Tenachi et al 2023a,b]

Feynman benchmark



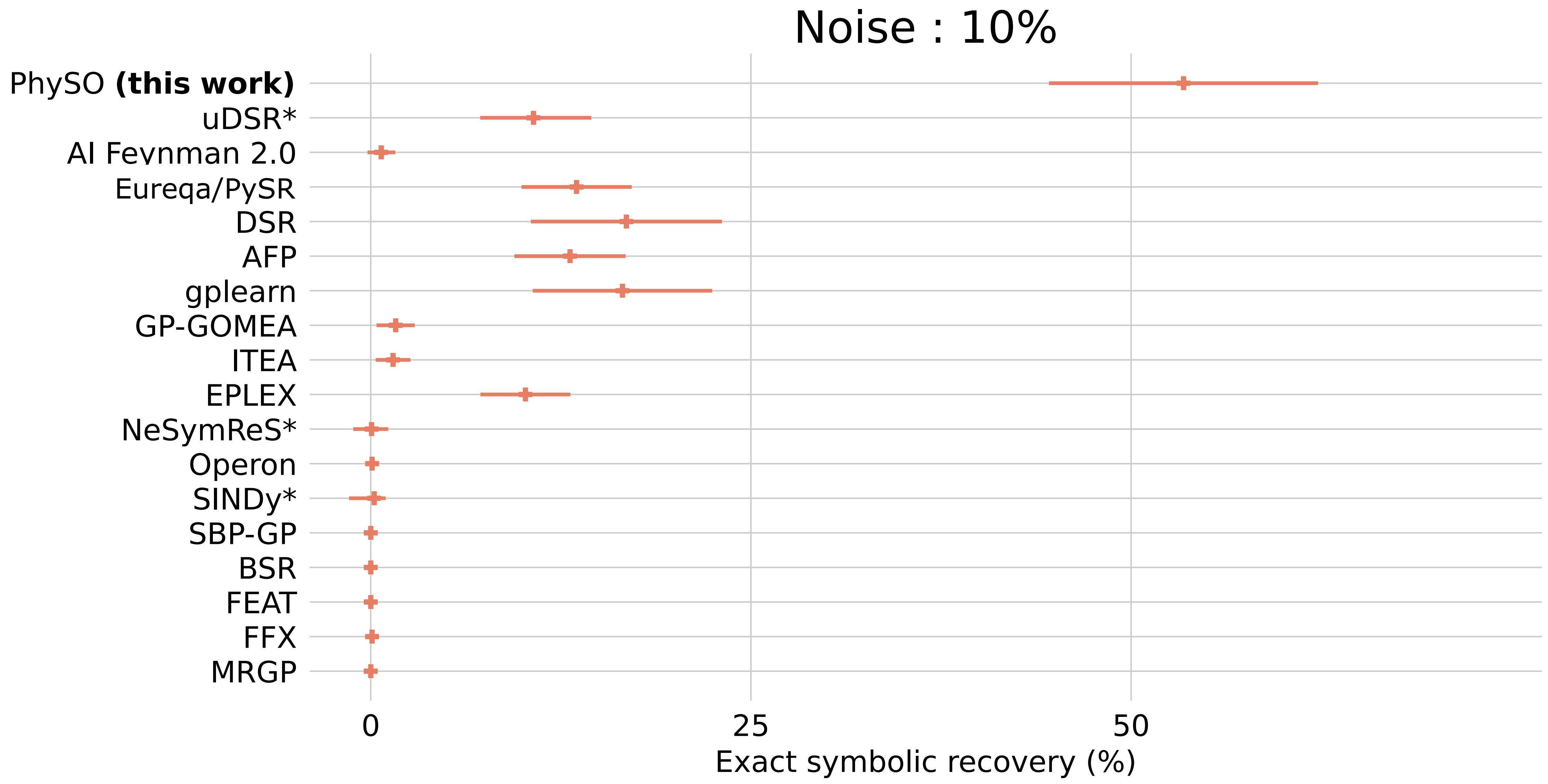
[Tenachi et al 2023a,b]

Feynman benchmark



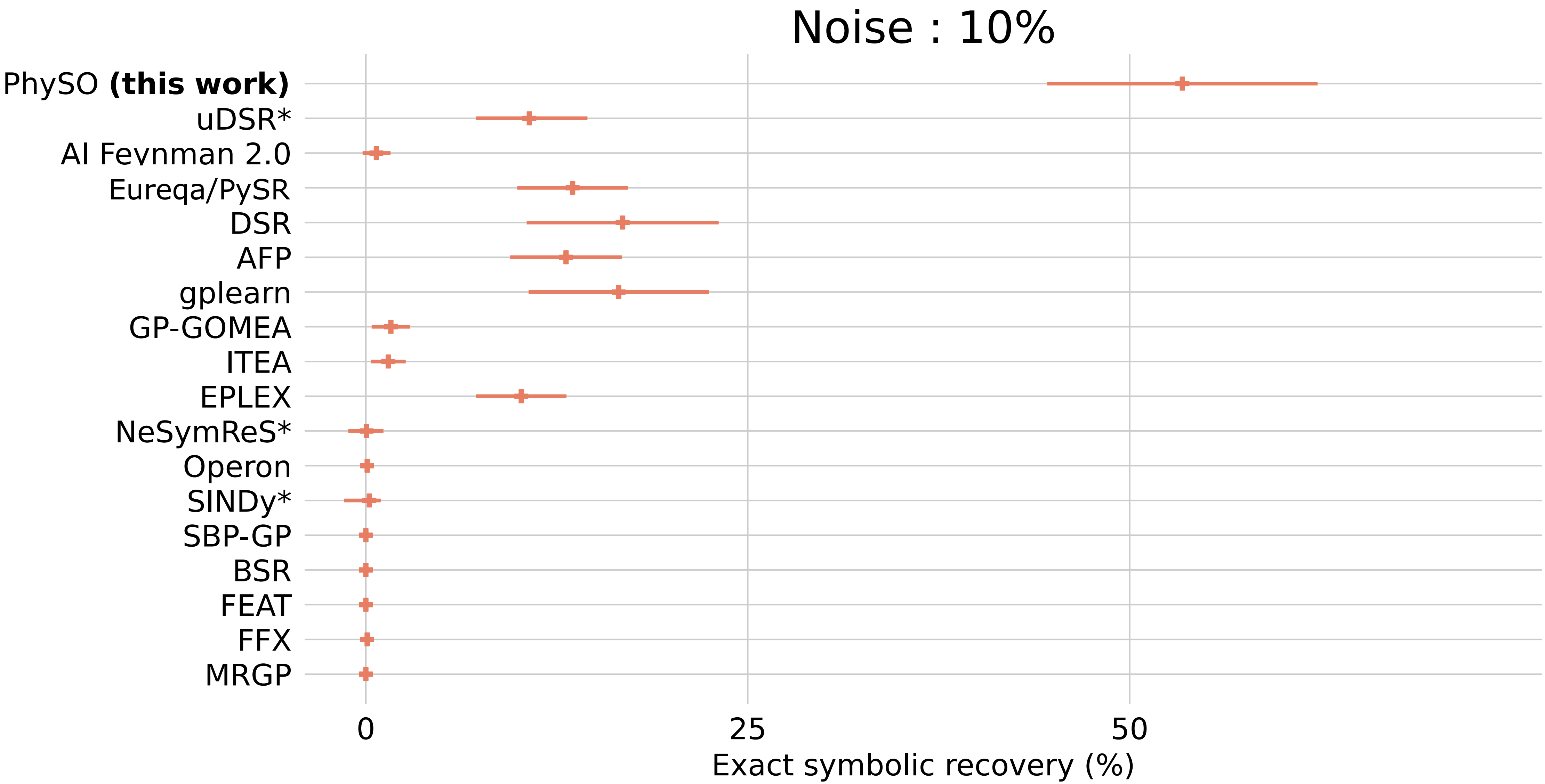
[Tenachi et al 2023a,b]

Feynman benchmark



[Tenachi et al 2023a,b]

Feynman benchmark

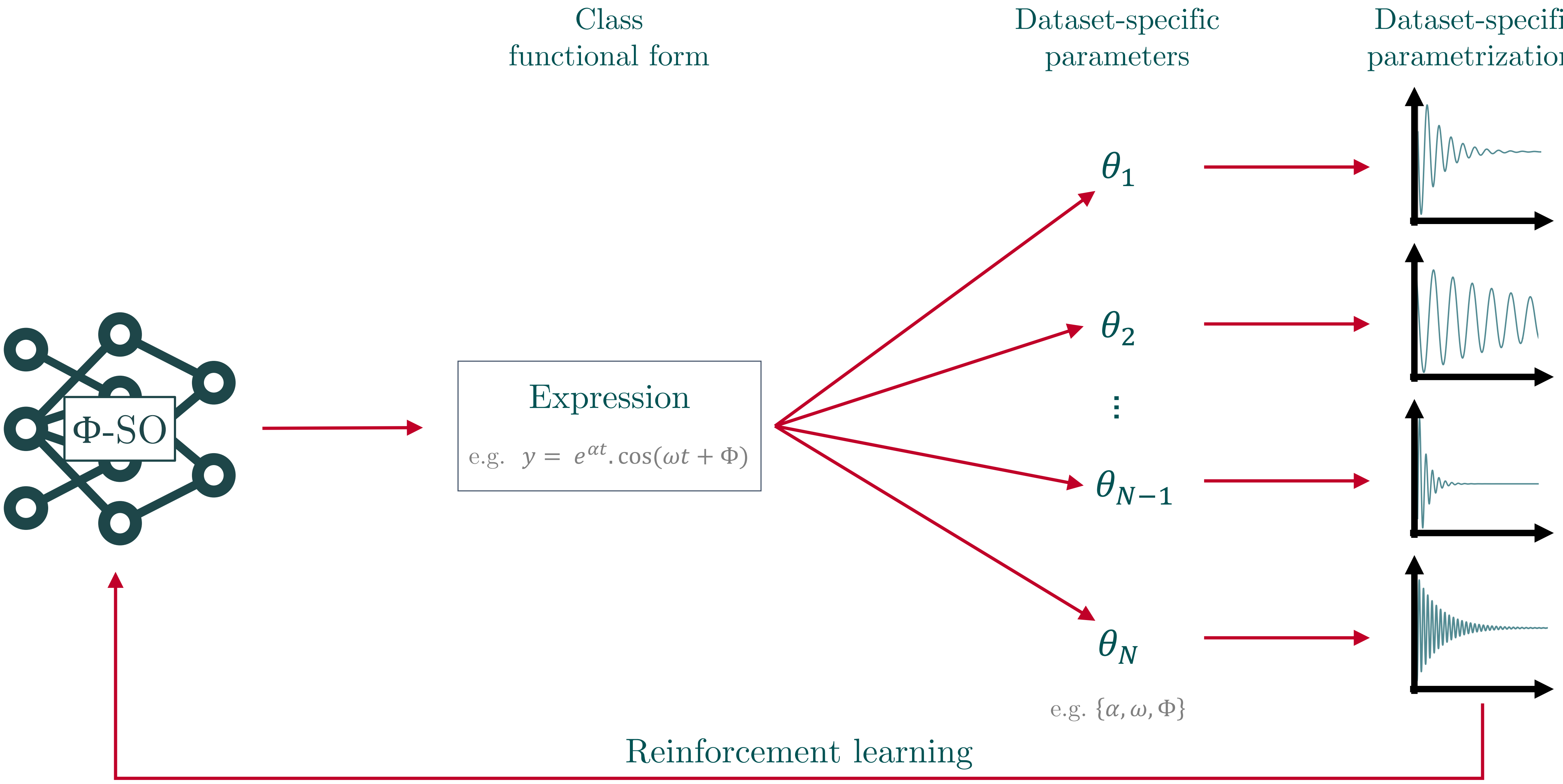


→ Φ-SO is the state-of-the-art in noisy scenarios

→ Neural net is better than no neural net (e.g. PySR, Operon etc.)

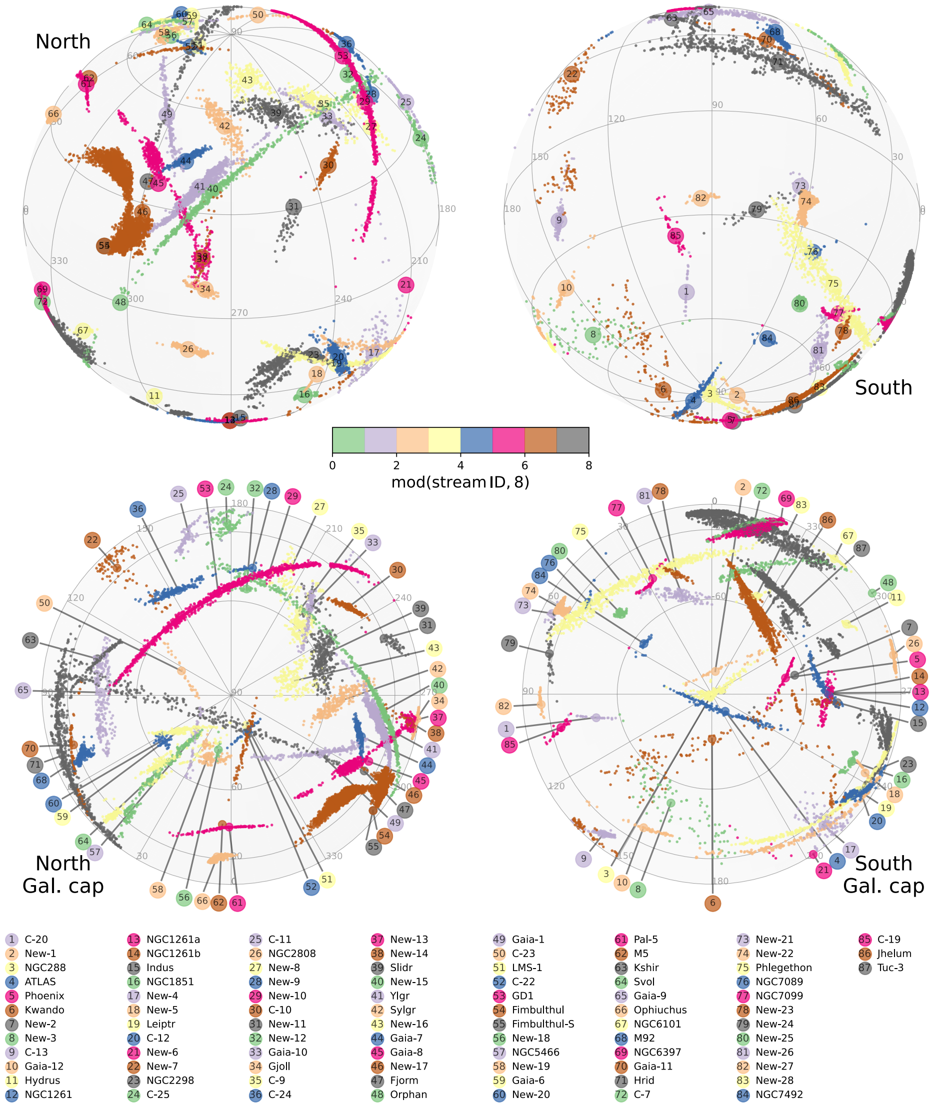
[Tenachi et al 2023a,b]

Class symbolic regression



[Tenachi et al 2024]

Class SR : Milky Way stellar streams

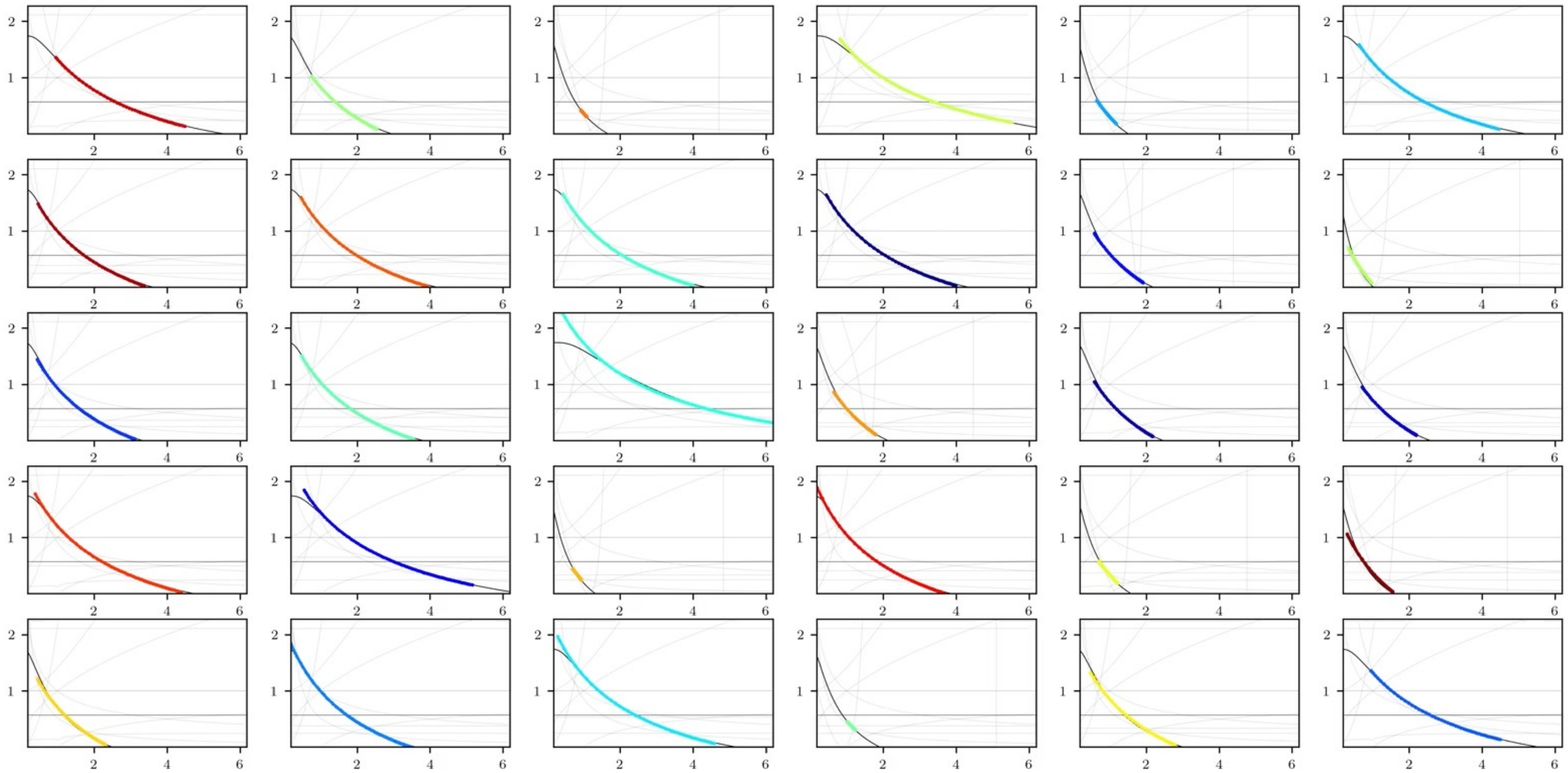


→ Constraining the dark matter distribution of our galaxy

Gaia

[Ibata, Malhan, Tenachi et al 2023]
[Tenachi et al 2024] (Class SR)

Class SR : Milky Way stellar streams



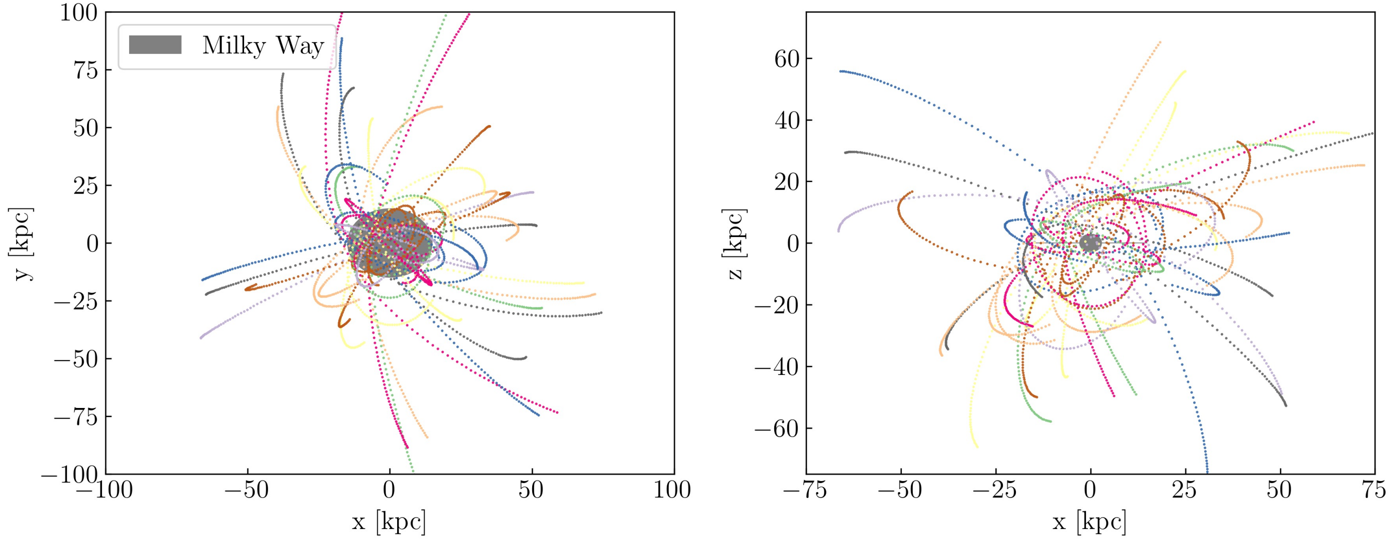
Best fit: $$E(r) = \frac{-Ace^{\frac{R}{r}} - E_t c e^{\frac{R}{r}} + E_t}{ce^{\frac{R}{r}} - 1}$$

Trying: $$E(r) = \frac{-Ace^{\frac{R}{r}} - E_t c e^{\frac{R}{r}} + E_t}{ce^{\frac{R}{r}} - 1}$$

https://youtu.be/iZKuCTG0YA4

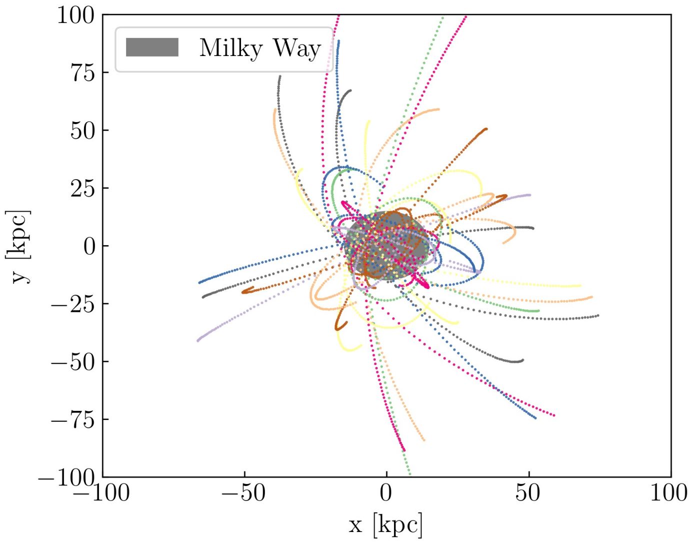
[Tenachi et al 2024] (Class SR)

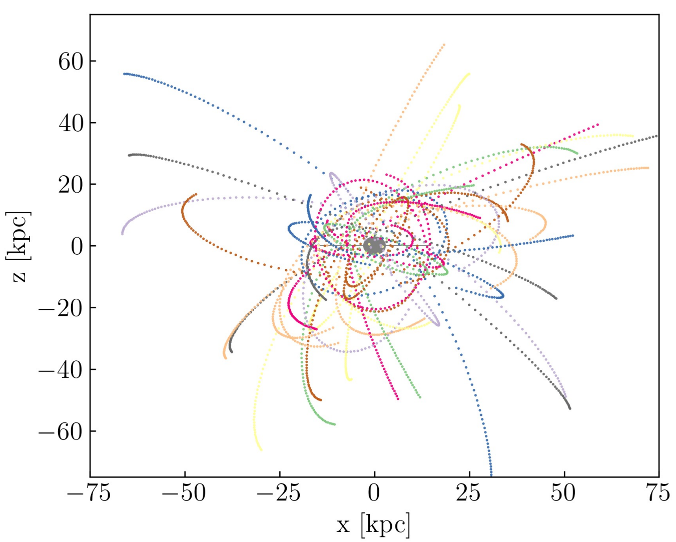
Class SR : Milky Way stellar streams

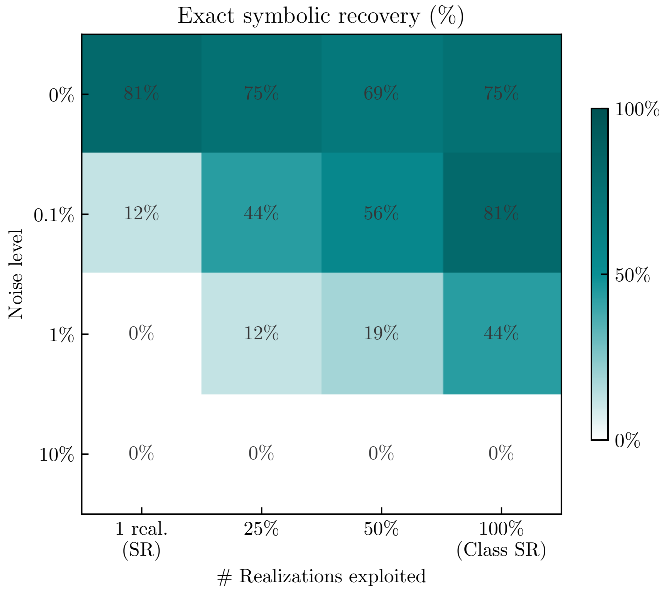


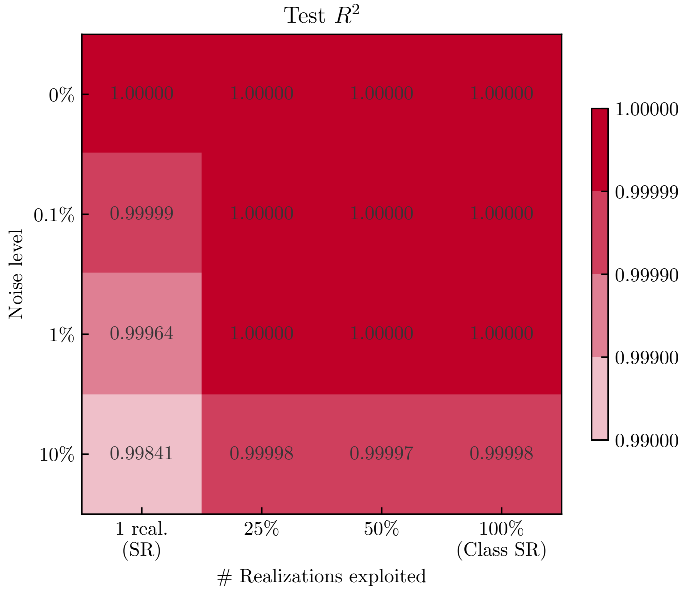
[Tenachi et al 2024] (Class SR)

Class SR : Milky Way stellar streams







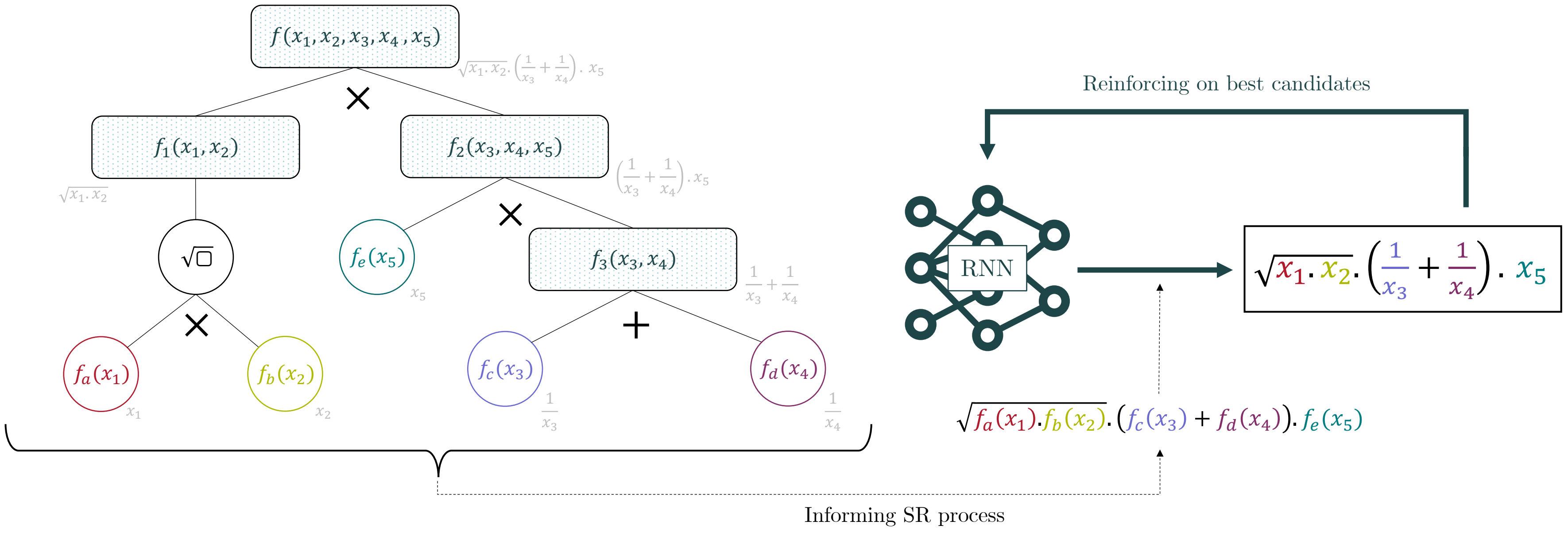


[Tenachi et al 2024] (Class SR)

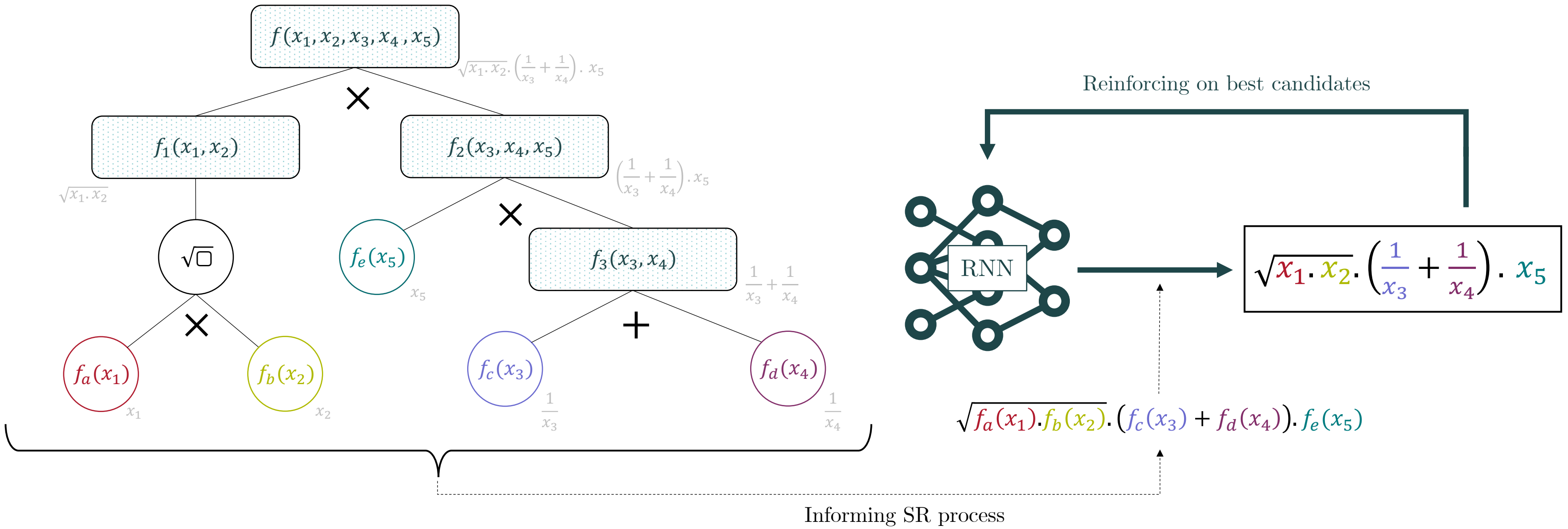
PhySO development plan

PhySO extension #1 – Dimensional analysis ✅
PhySO extension #2 – Class SR ✅
PhySO extension #3 – Structure analysis 🏗️
PhySO extension #4 – Differential equations 🏗️
PhySO extension #5 – Hindsight relabeling RL 🏗️
PhySO extension #6 – Theory Φ-SO (planned)

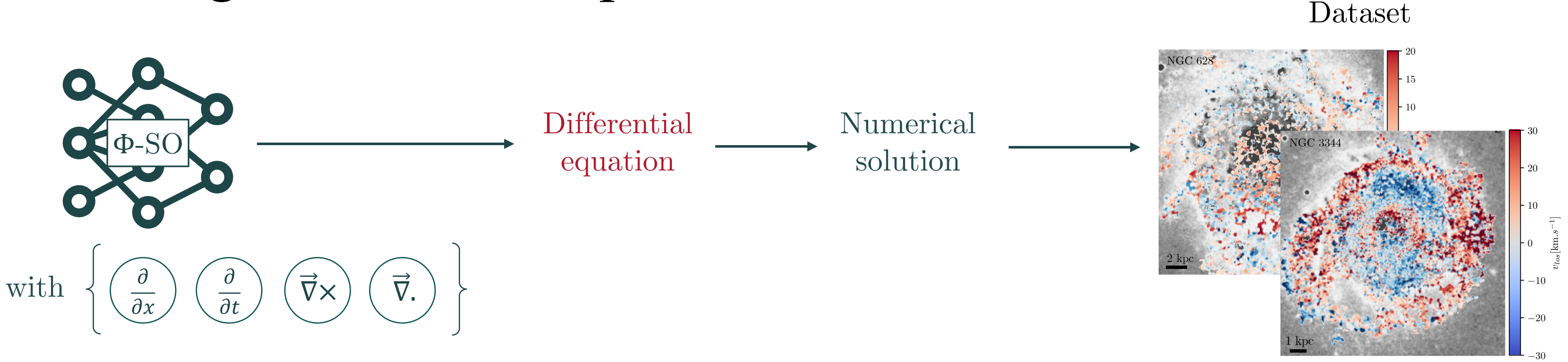
Unveiling graph structures by detecting separabilities in data



Unveiling graph structures by detecting separabilities in data

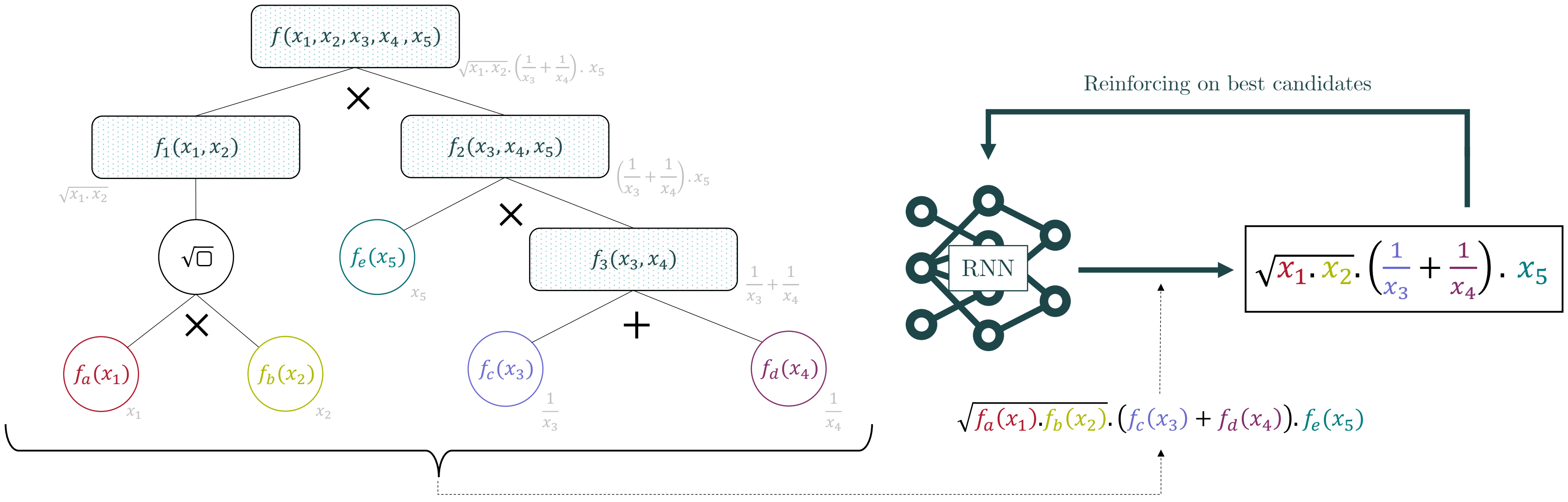


Uncovering differential equations from data

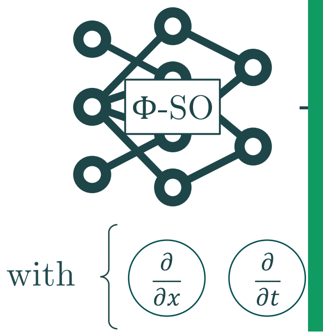


Adapted from [Urrejola-Mora et al 2022]

Unveiling graph structures by detecting separabilities in data



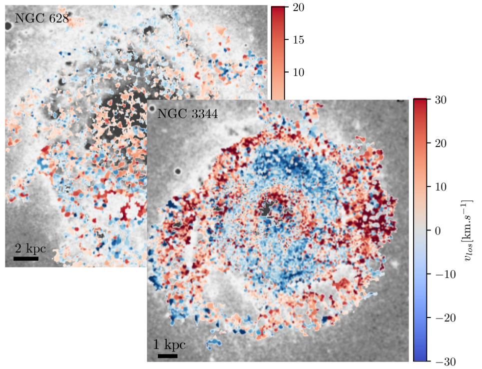
Uncovering d…



S15b: The changing macrocosm of astroinformatics : big data, catalogues, machine learning, challenges, and more!

Thu. 26th Jun. 15:00 - 16:30 Boole 2

Dataset



Adapted from [Urrejola-Mora et al 2022]

Physical Symbolic Optimization

Φ-SO

Physical Symbolic Optimization.

An open source …

Github repository: WassimTenachi/PhySO

PhySO Public

Physical Symbolic Optimization

Python 1.7k 240

[Tenachi et al 2023a,b]

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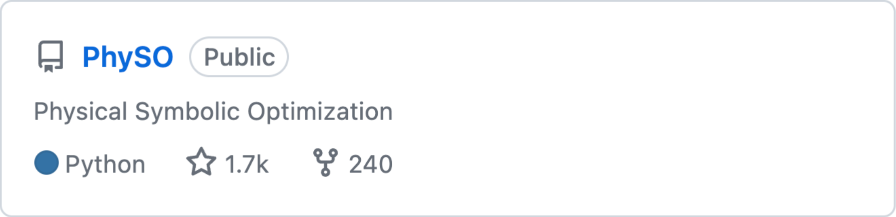
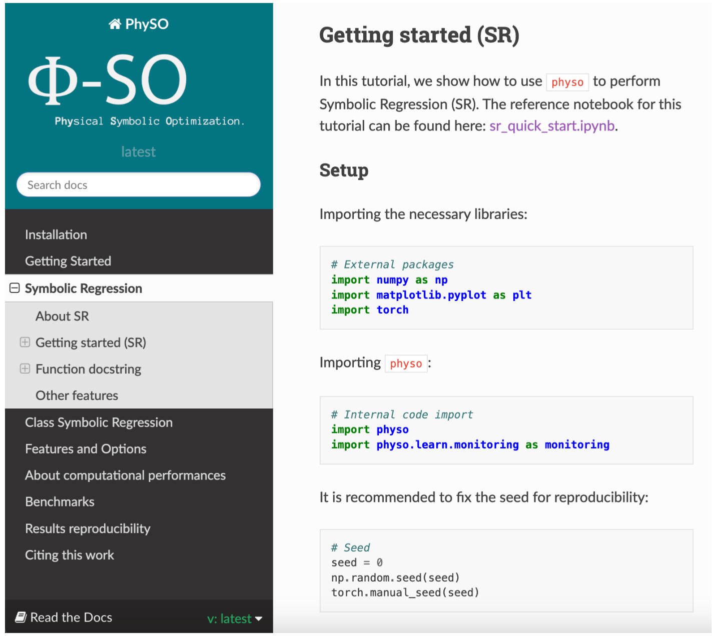


… and documented package

Documentation: physo.readthedocs.io



[Tenachi et al 2023a,b]

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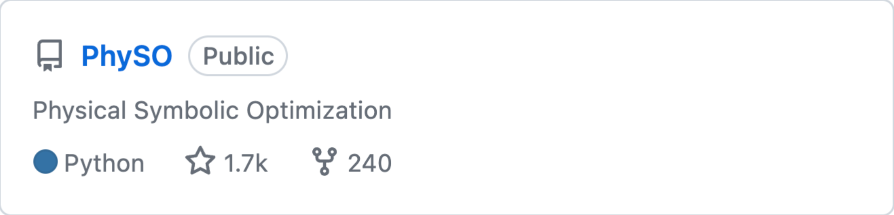

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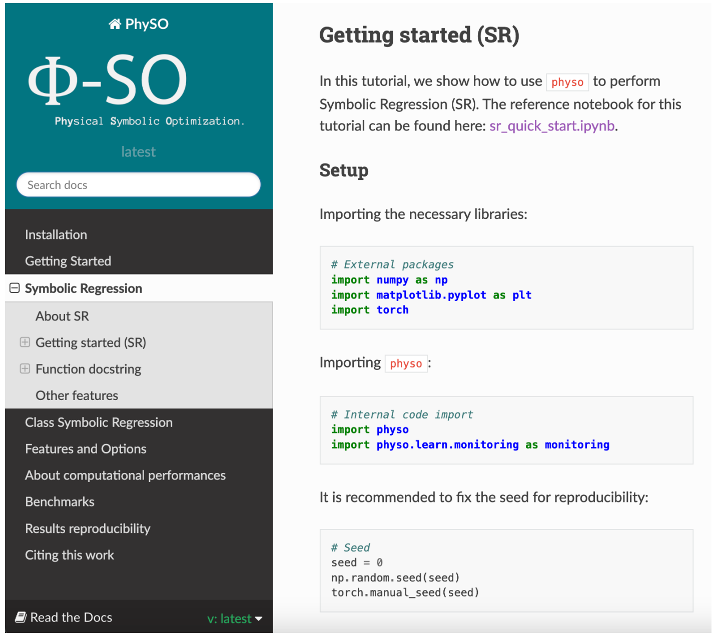
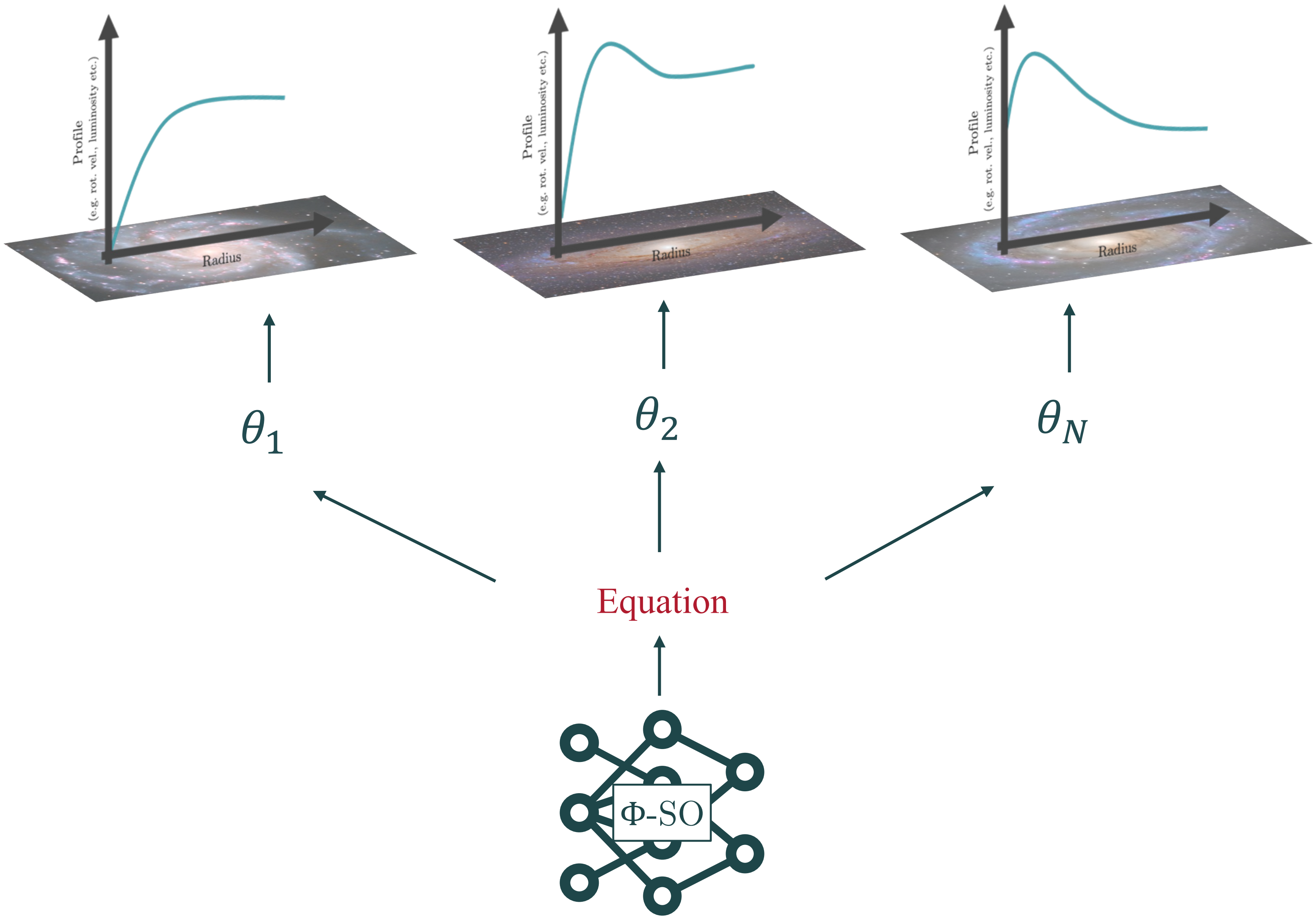


Creating a physical free-form symbolic analytical function $f: \mathbb{R}^n \rightarrow \mathbb{R}$ fitting $y = f(\boldsymbol{X})$ given $(\boldsymbol{X}, y)$ data:

```
expression, logs = physo.SR(X, y,
                            X_units = [ [1, 0, 0] , [1, -1, 0] ],
                            y_units = [2, -2, 1],
                            fixed_consts       = [ 1.      ],
                            fixed_consts_units = [ [0,0,0] ],
                            free_consts_units  = [ [0, 0, 1] , [1, -2, 0] ],
)
```

[Tenachi et al 2023a,b]

Class SR for galactic profiles



Wassim Tenachi - w.tenachi@gmail.com

Learning a dark matter profile

Numerical Investigation of a Hundred Astrophysical Objects (NIHAO) [Butsky 2015]

[Tenachi et al 2025] (in prep)

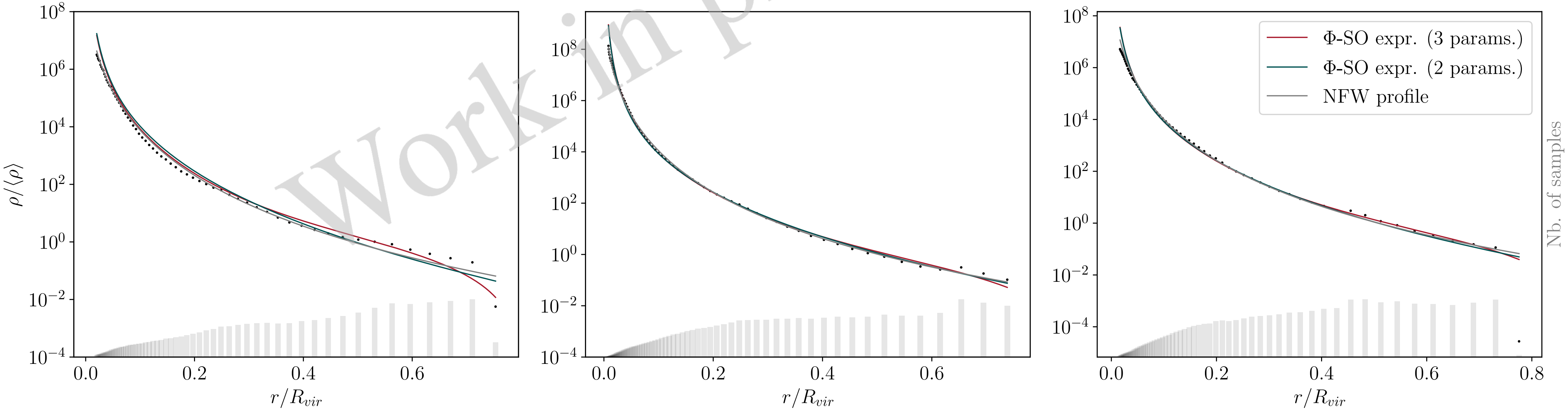
Learning a dark matter profile

Numerical Investigation of a Hundred Astrophysical Objects (NIHAO) [Butsky 2015]

NFW profile $\dfrac{\rho_0 r_s^3}{r\,(r+r_s)^2}$

Φ-SO expr. (2 params.) $\dfrac{\rho_0 r_s^2 \left(c_0 + e^{\frac{c_0 r_s}{r}}\right)}{c_0^2 r^2}$

Φ-SO expr. (3 params.) $\rho_0 - \dfrac{\rho_0 \left(c_0 r_{s_0} - r_{s_1} e^{\frac{c_0 r_{s_1}}{r}}\right)^2}{r^2}$



[Tenachi et al 2025] (in prep)

Derived quantities

Density profile $\rho(r)$

Derived quantities

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Assumptions:
- Spherical symmetry
- Isotropic velocity distribution
- Steady-state (time-independent)

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Enclosed mass $M(r) = 4\pi \int_0^r \rho(r')\, r'^2\, dr'$

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Circular velocity $V_c(r) = \sqrt{\dfrac{GM(r)}{r}}$

Assumptions:
- Spherical symmetry
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Wassim Tenachi - w.tenachi@gmail.com

Derived quantities

Assumptions:
- Spherical symmetry
- Isotropic velocity distribution
- Steady-state (time-independent)

Density profile $\rho(r)$

Enclosed mass $$M(r) = 4\pi \int_0^r \rho(r')\, r'^2\, dr'$$

Circular velocity $$V_c(r) = \sqrt{\frac{GM(r)}{r}}$$

Potential $$\Phi(r) = -G \int_r^\infty \frac{M(r')}{r'^2} dr'$$

$$\nabla^2\Phi = 4\pi G\rho$$

(Poisson)

Derived quantities

Assumptions:
- Spherical symmetry
- Isotropic velocity distribution
- Steady-state (time-independent)

Density profile $\rho(r)$

Enclosed mass $M(r) = 4\pi \int_0^r \rho(r')\, r'^2 \, dr'$

Circular velocity $V_c(r) = \sqrt{\frac{GM(r)}{r}}$

Potential $\Phi(r) = -G \int_r^\infty \frac{M(r')}{r'^2} dr'$

Radial velocity dispersion $\sigma_r^2(r) = \frac{1}{\rho(r)} \int_r^\infty \rho(r') \frac{GM(r')}{r'^2} dr'$

$$\frac{d(\rho\sigma_r^2)}{dr} + \frac{2\beta}{r}\rho\sigma_r^2 = -\rho\frac{GM(r)}{r^2}$$

$(\beta = 0)$

(Jeans)

Derived quantities of DM profiles

	Profile	Density	$M(r)$	$V_c(r)$	$\Phi(r)$	$\sigma^2(r)$
Cusps	NFW	$\rho(r) = \dfrac{\rho_0}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$	✓	✓	✓	✗
Cusps	SuperNFW	$\rho(r) = \dfrac{\rho_0}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^{5/2}}$	✓	✓	✓	✓
Cores	pISO	$\rho(r) = \dfrac{\rho_0}{1+\left(\frac{r}{r_s}\right)^2}$	✓	✓	✓	✓
Cores	Burkert	$\rho(r) = \dfrac{\rho_0}{\left(1+\frac{r}{r_s}\right)\left(1+\left(\frac{r}{r_s}\right)^2\right)}$	✓	✓	✓	✗
Cores	Lucky13	$\rho(r) = \dfrac{\rho_0}{\left(1+\frac{r}{r_s}\right)^3}$	✓	✓	✓	✗
Variable slope	Einasto	$\rho(r) = \rho_0 \exp\left(-\dfrac{2}{\alpha}\left[\left(\dfrac{r}{r_s}\right)^{\alpha} - 1\right]\right)$	✗	✗	✗	✗
Variable slope	gNFW	$\rho(r) = \dfrac{\rho_0}{\left(\frac{r}{r_s}\right)^{\alpha}\left(1+\left(\frac{r}{r_s}\right)\right)^{3-\alpha}}$	✗	✗	✗	✗

✗ Non-elementary functions

Adapted from [Freundlich et al 2020]

Learning a profile with analytic constraints

$$\rho(r) \qquad -c_0\rho_0 - \rho_0 \log\left(-c_0^2\right) + \frac{\rho_0}{c_0}$$

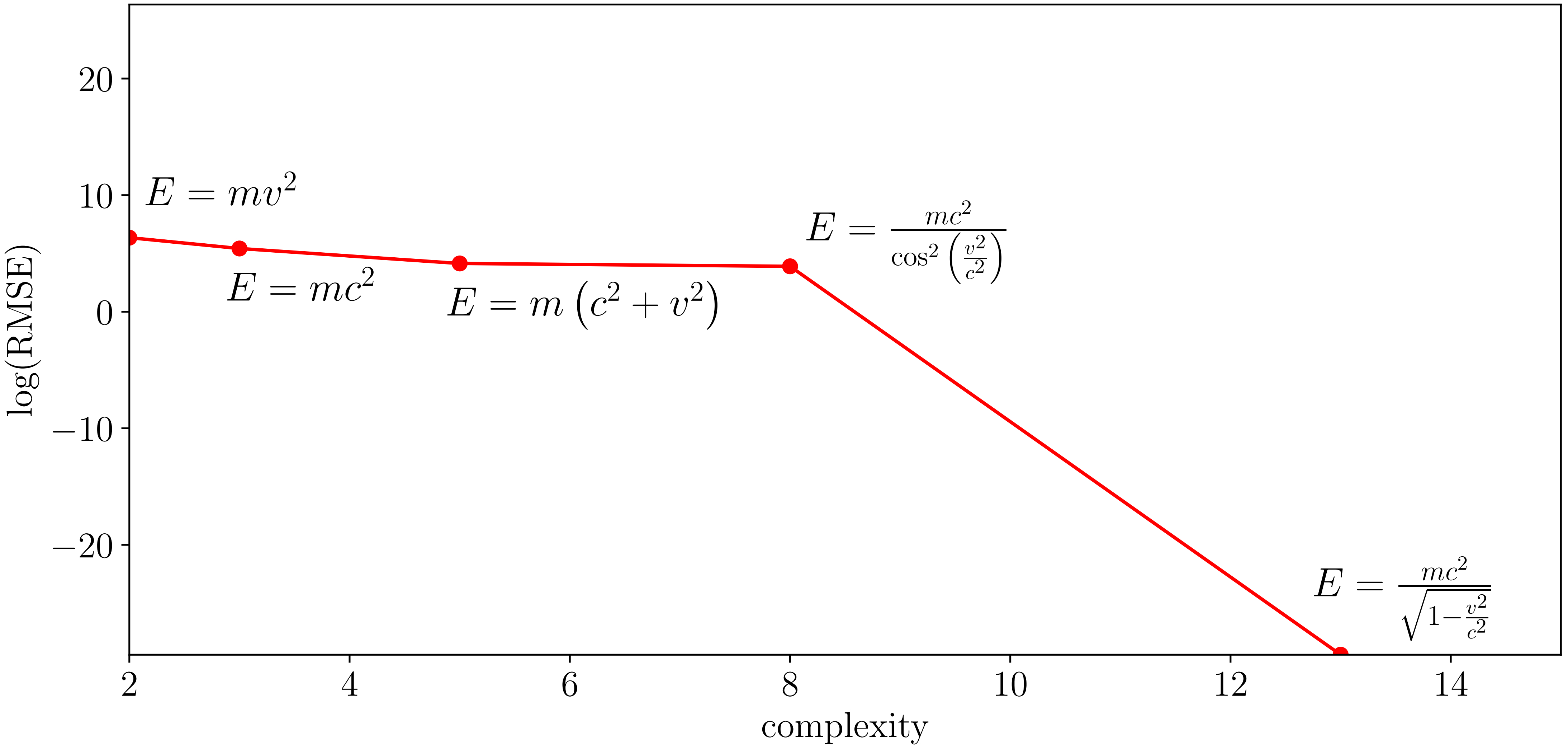
$$M(r) \qquad r^3\left(-\frac{4\pi c_0\rho_0}{3} - \frac{8\pi\rho_0\log\left(c_0\right)}{3} - \frac{4i\pi^2\rho_0}{3} + \frac{4\pi\rho_0}{3c_0}\right)$$

$$V_c(r) \qquad \frac{2\sqrt{3}\sqrt{\pi}\sqrt{G}r\sqrt{\rho_0}\sqrt{-c_0(c_0+2\log\left(c_0\right)+i\pi)+1}}{3\sqrt{c_0}}$$

$$\Phi(r) \qquad r^2\left(-\frac{2\pi G c_0\rho_0}{3} - \frac{4\pi G\rho_0\log\left(c_0\right)}{3} - \frac{2i\pi^2 G\rho_0}{3} + \frac{2\pi G\rho_0}{3c_0}\right)$$

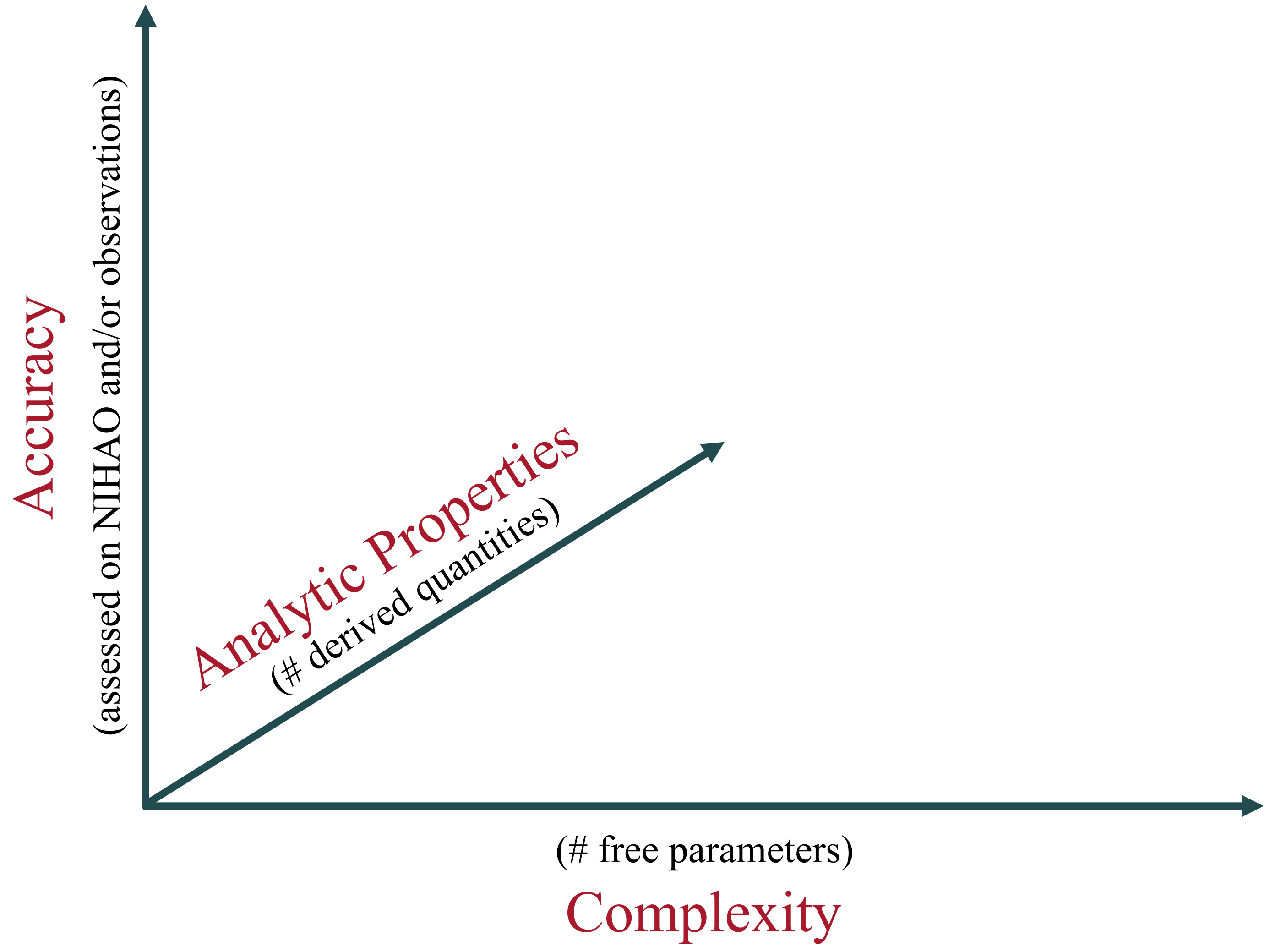
https://youtu.be/Js3VbbmEaIo

Pareto fronts: accuracy vs complexity optima



[Tenachi et al 2023a]

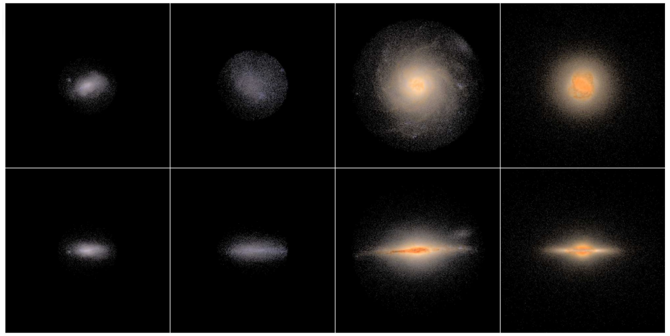
Toward the ultimate DM profile(s)



Perspectives

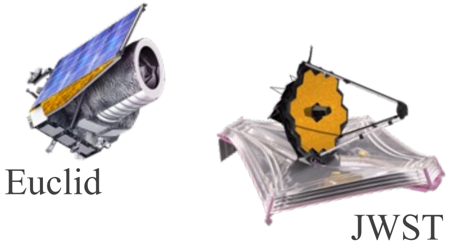
- Exploring anisotropic distributions

- Impact of the hydrodynamics on the functional form



Numerical Investigation of a Hundred Astrophysical Objects (NIHAO) [Butsky 2015]

- Including observational constraints via velocity-related quantities



- Other analytic constraints
(eg. converging enclosed mass – unlike NFW)

$$\exists M_{\rm tot} \in \mathbb{R}^+ : \lim_{r\to\infty} M(r) = M_{\rm tot}$$

Thank you for your attention !

Wassim Tenachi - w.tenachi@gmail.com