

Frequency filtering

Semester 2, 2022

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Adversarial images



Network: MobileNet V2
Prediction: English springer (90.08%)



Model: MobileNet V2
Prediction: hot dog (68.88%)

Images: ImageNet, <https://kennysong.github.io/adversarial.js/>

Outline

- Intro to Fourier analysis in 1D
- Fourier analysis of images
- Filtering in frequency
- Applications

Learning outcomes

- Explain at a conceptual level how images are represented in the frequency domain
- Implement filters in the frequency domain
- Explain how frequency representations are used for image compression and analysis

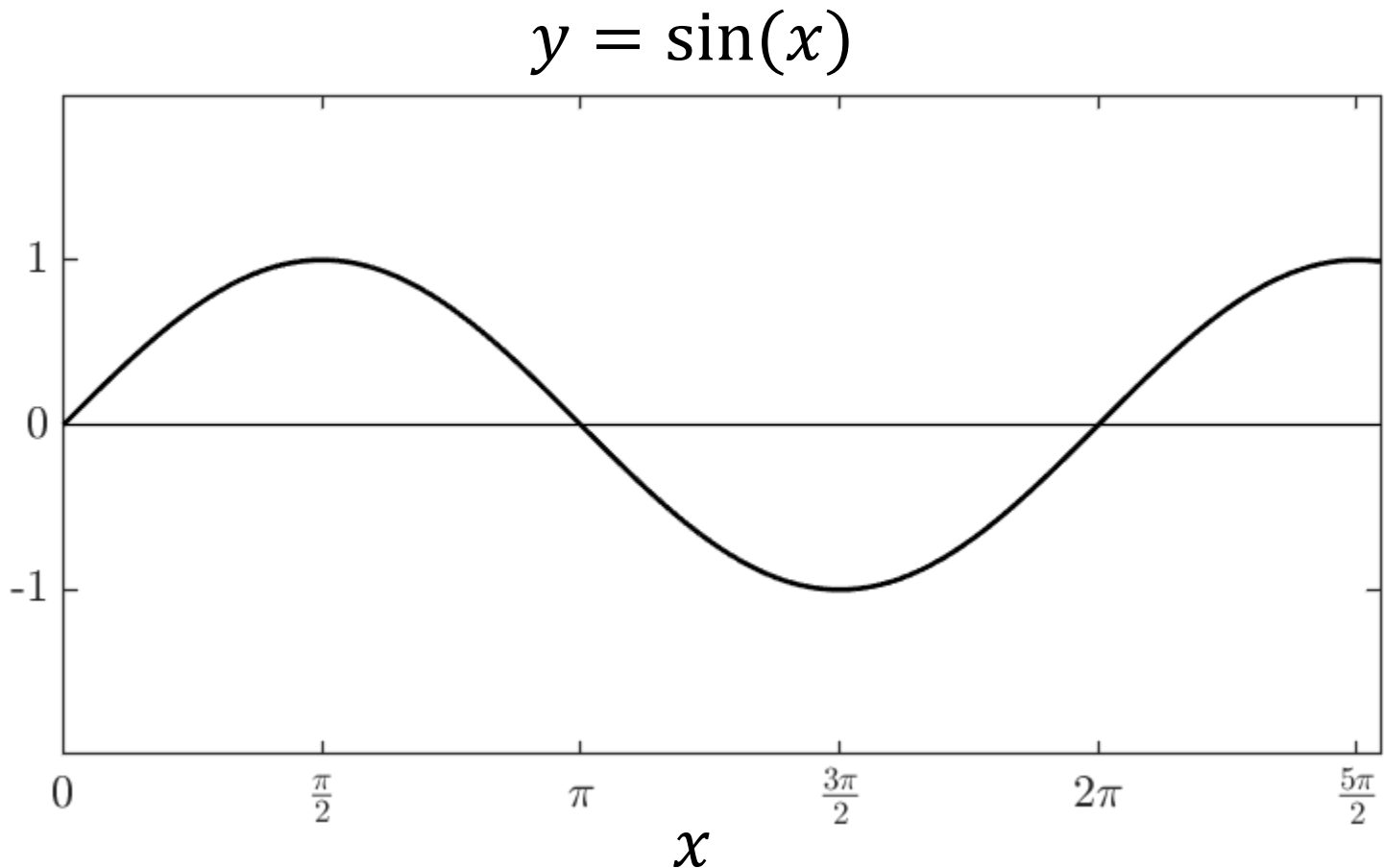
Fourier analysis (in 1D)

Signals

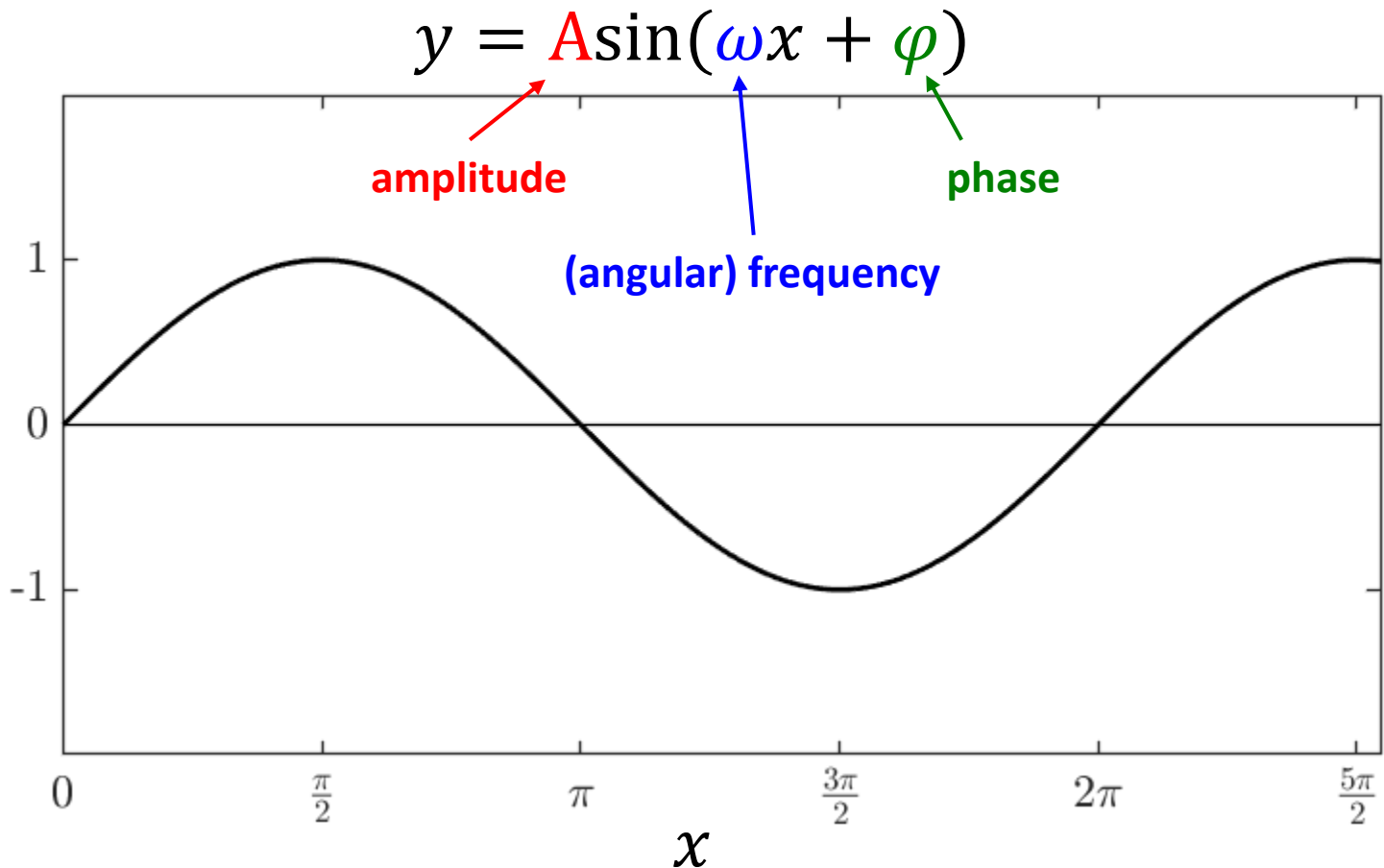
- Any signal or pattern can be described as a sum of sinusoids



Sinusoids

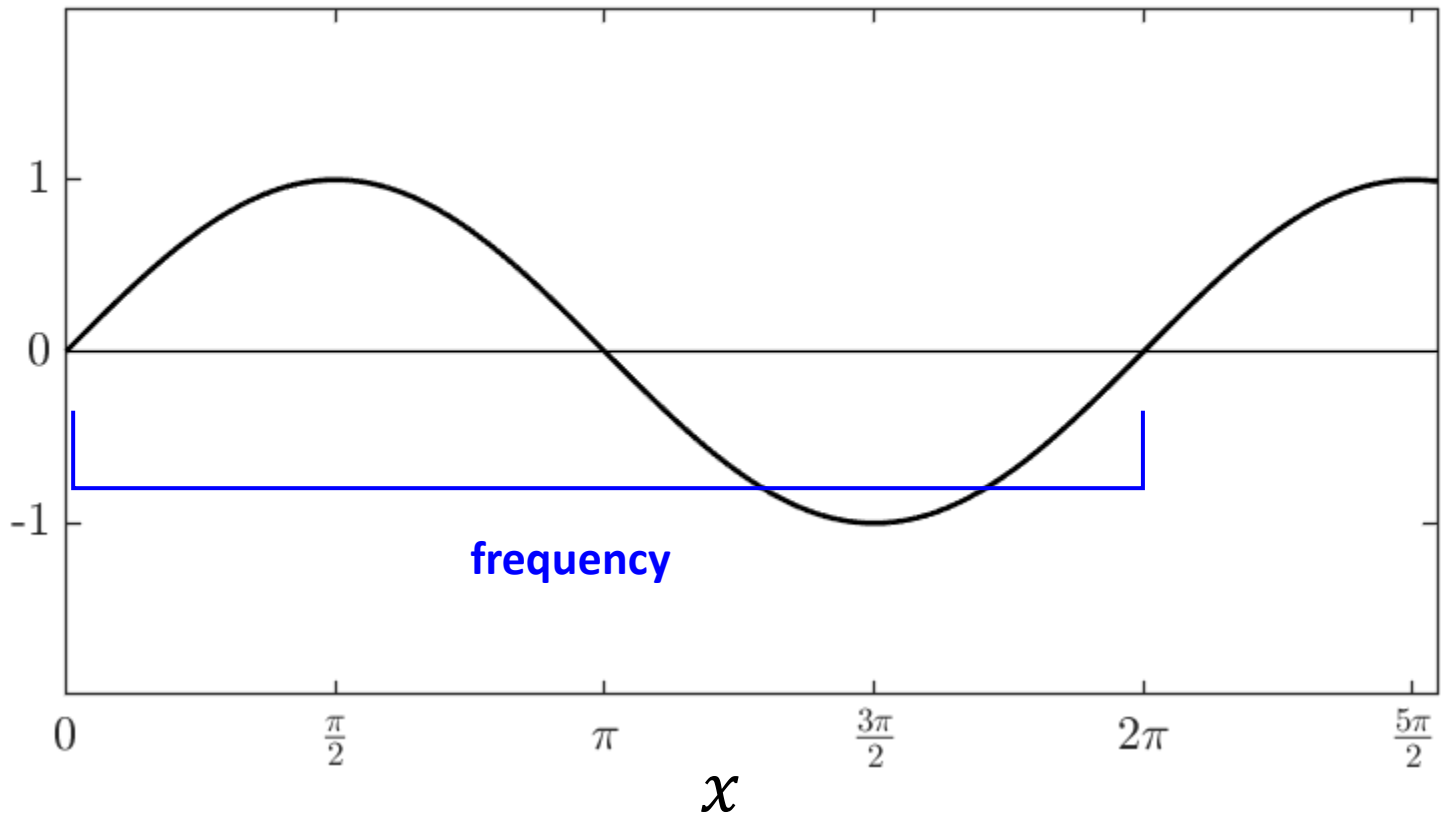


Sinusoids



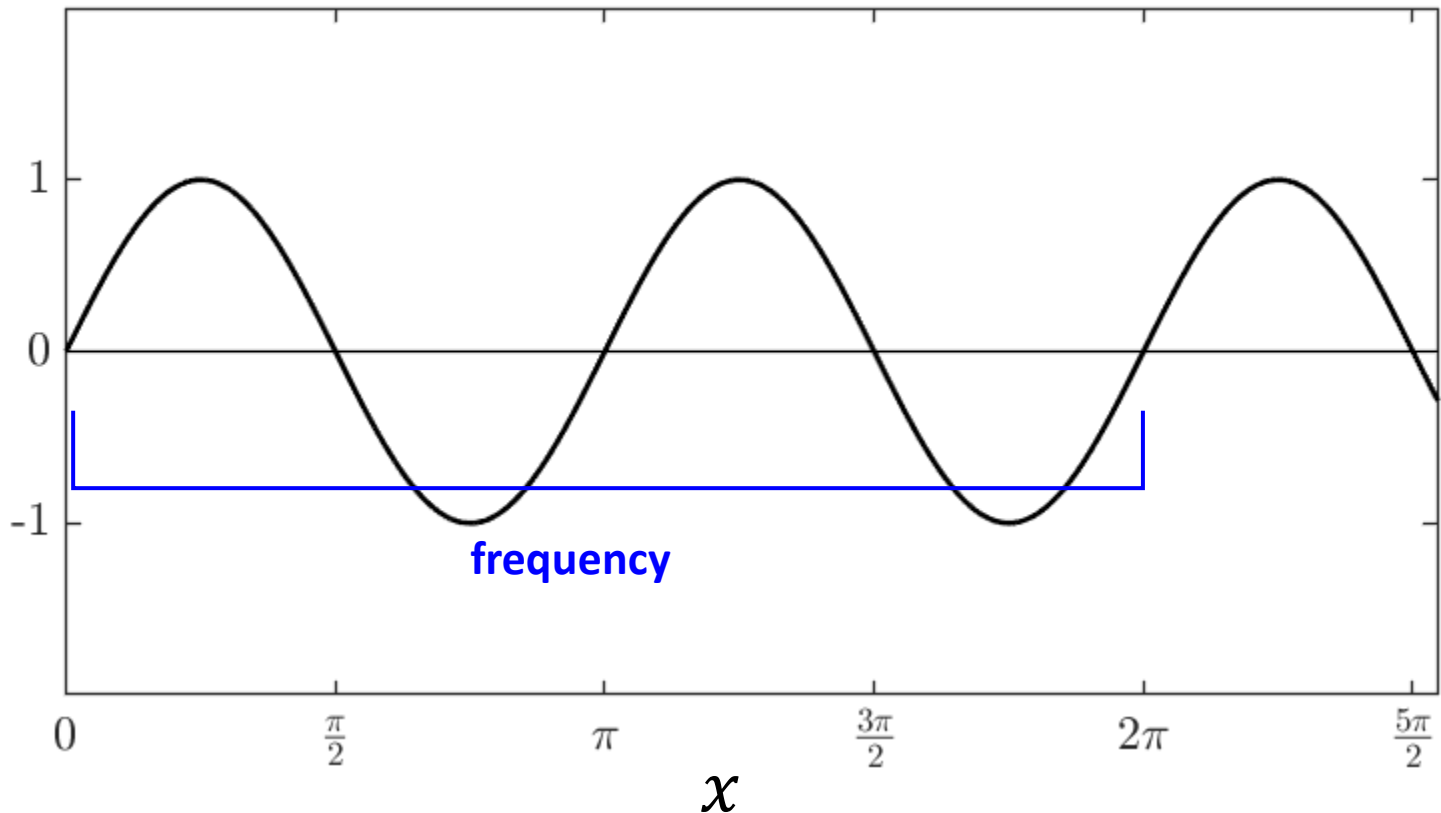
Sinusoids

$$y = \textcolor{red}{1}\sin(\textcolor{blue}{1}x + 0)$$



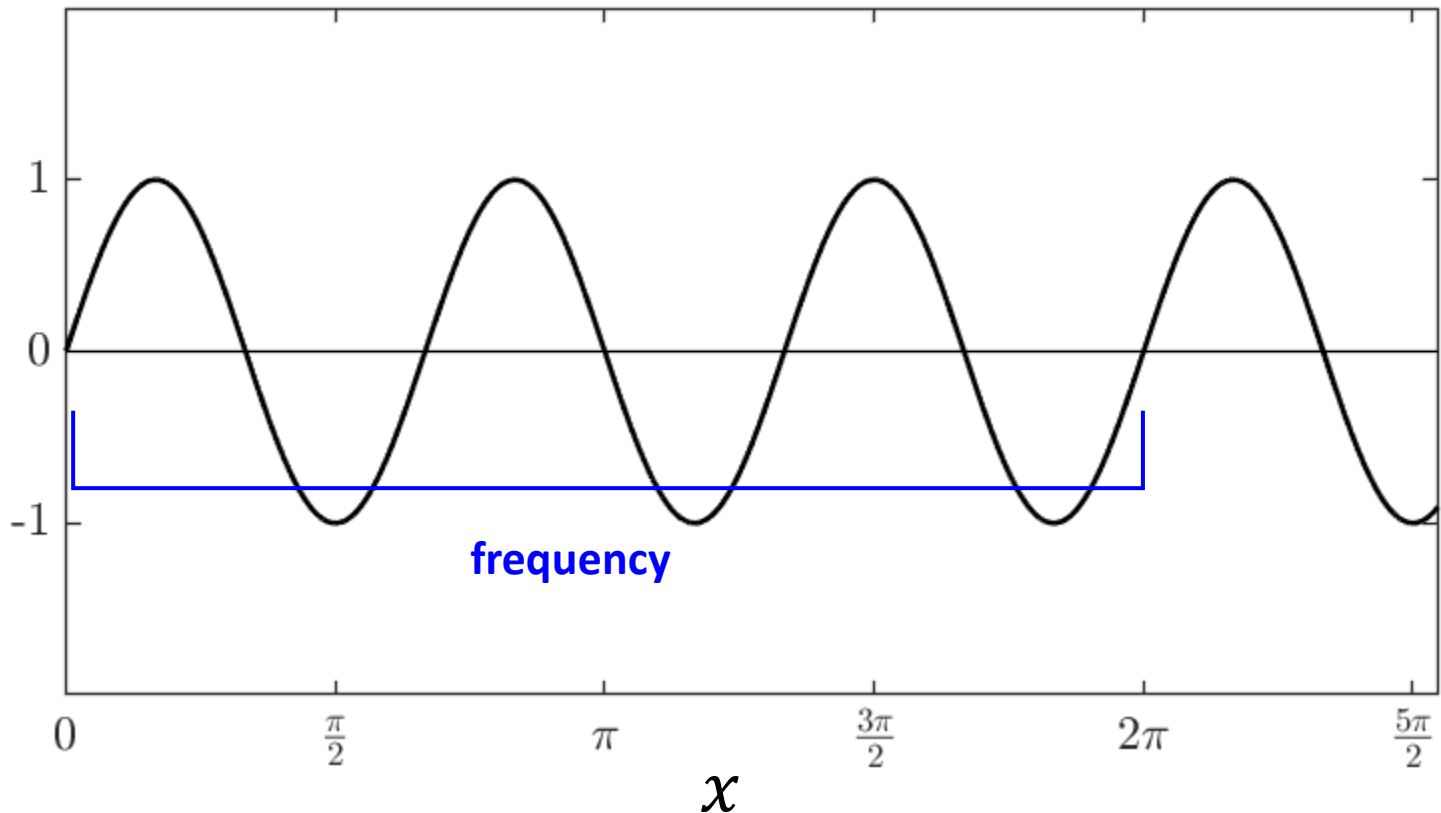
Sinusoids

$$y = 1\sin(2x + 0)$$



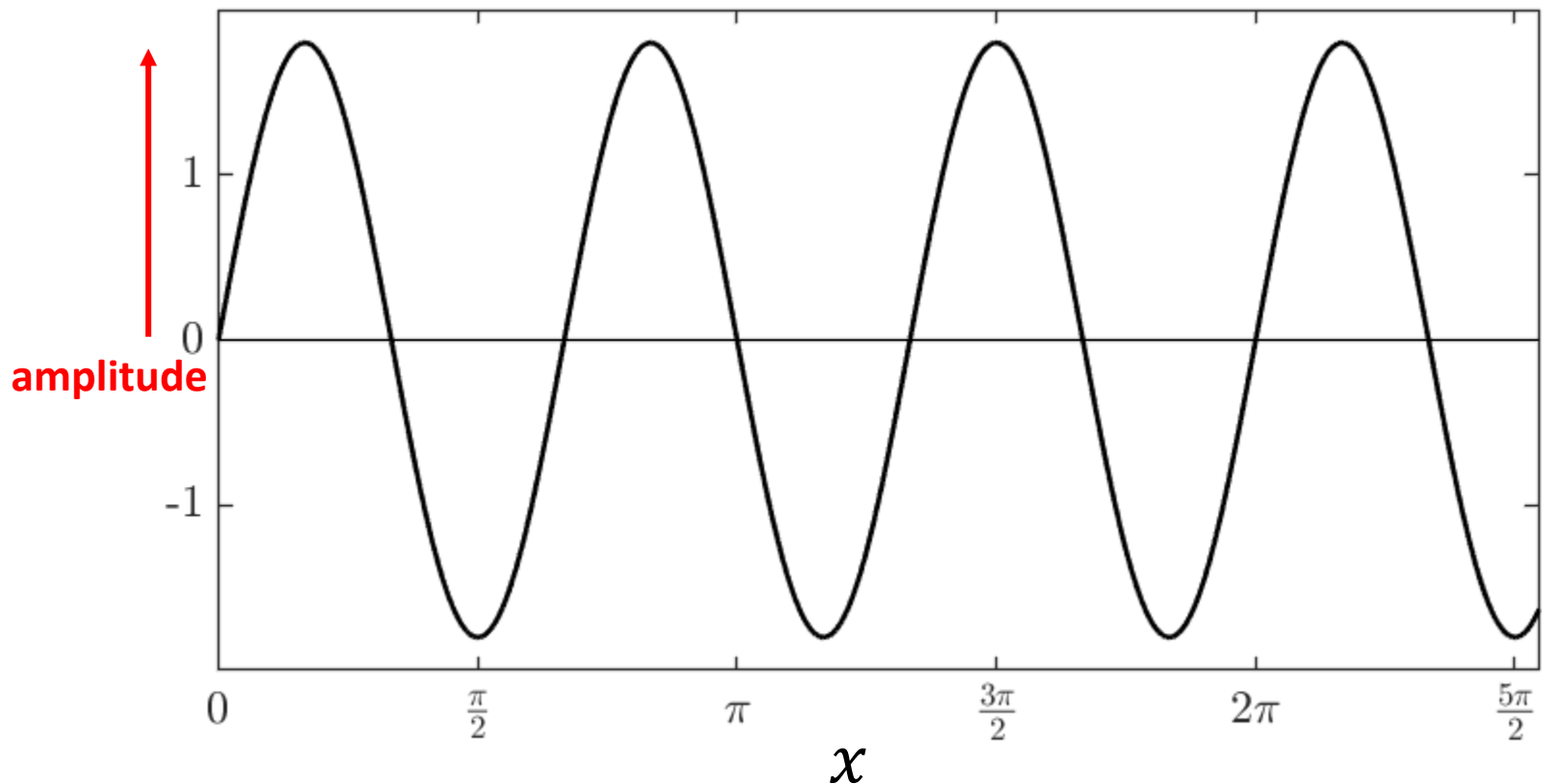
Sinusoids

$$y = 1\sin(3x + 0)$$



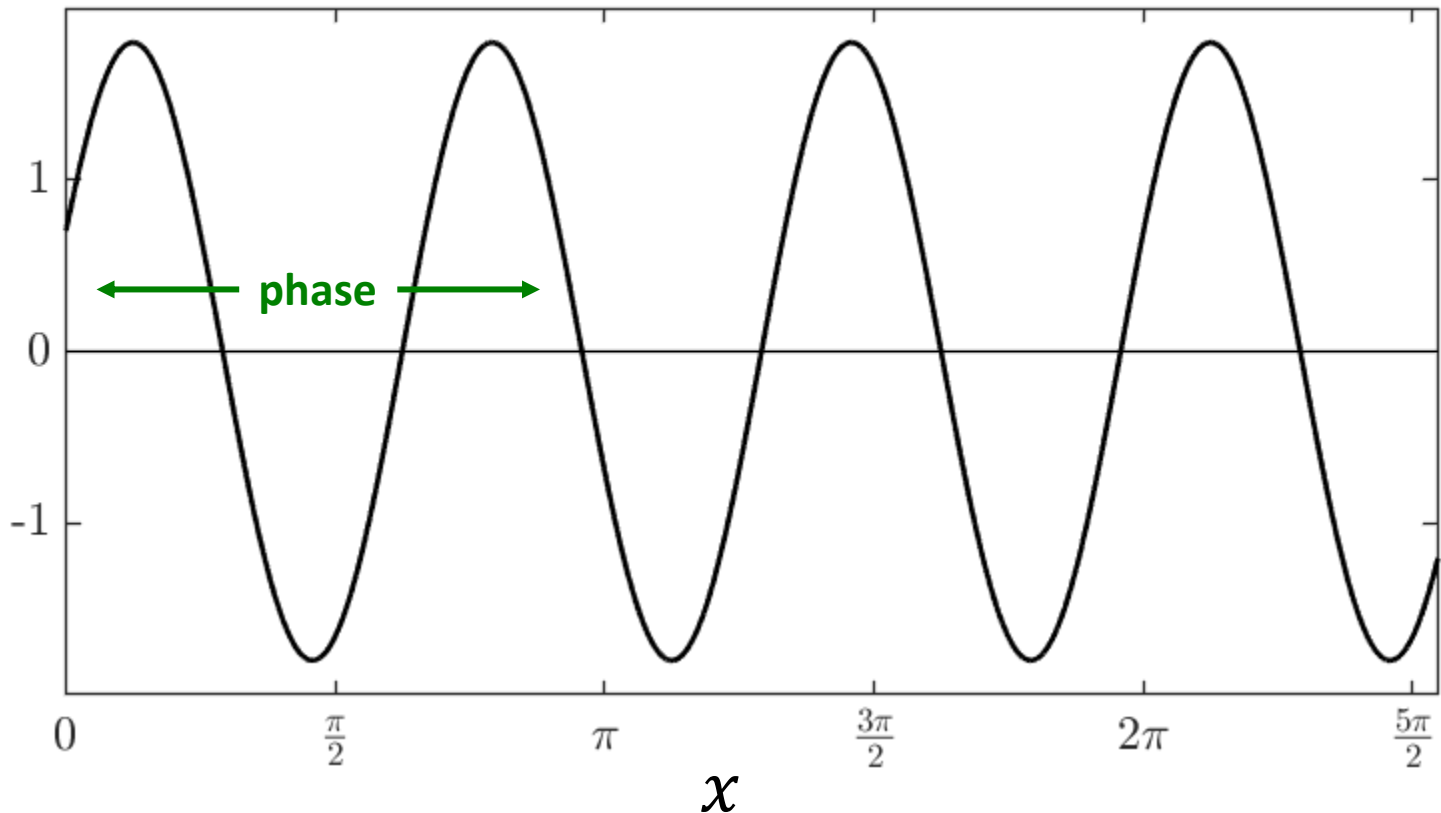
Sinusoids

$$y = 1.8\sin(3x + 0)$$



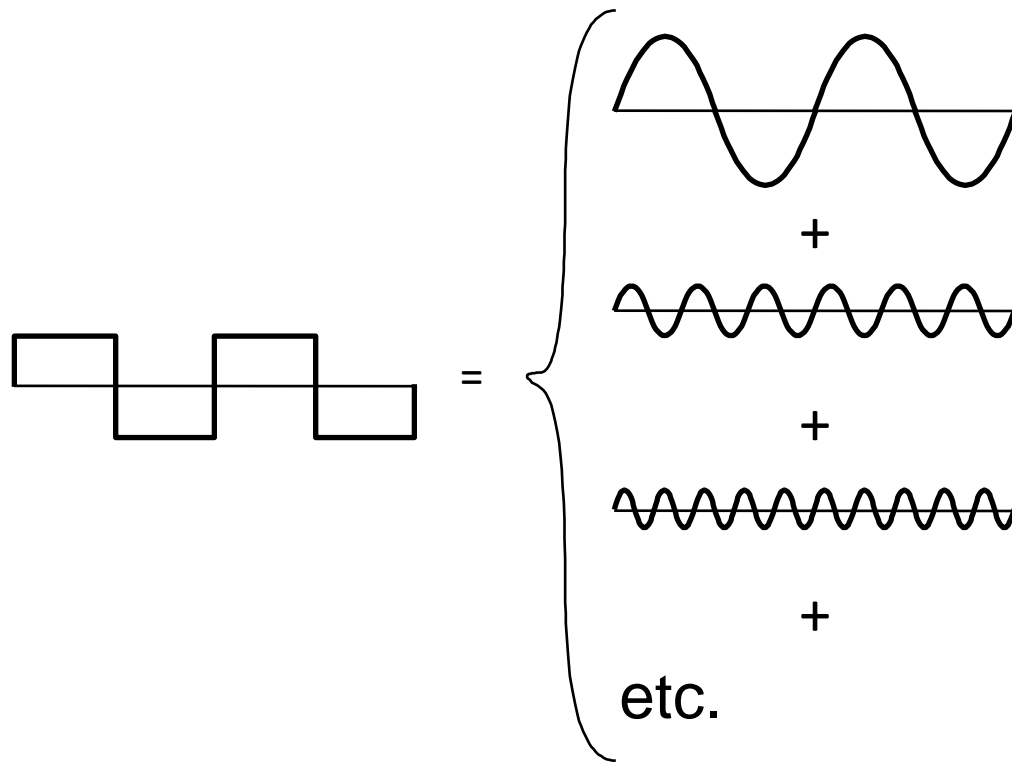
Sinusoids

$$y = 1.8\sin(3x + 0.4)$$

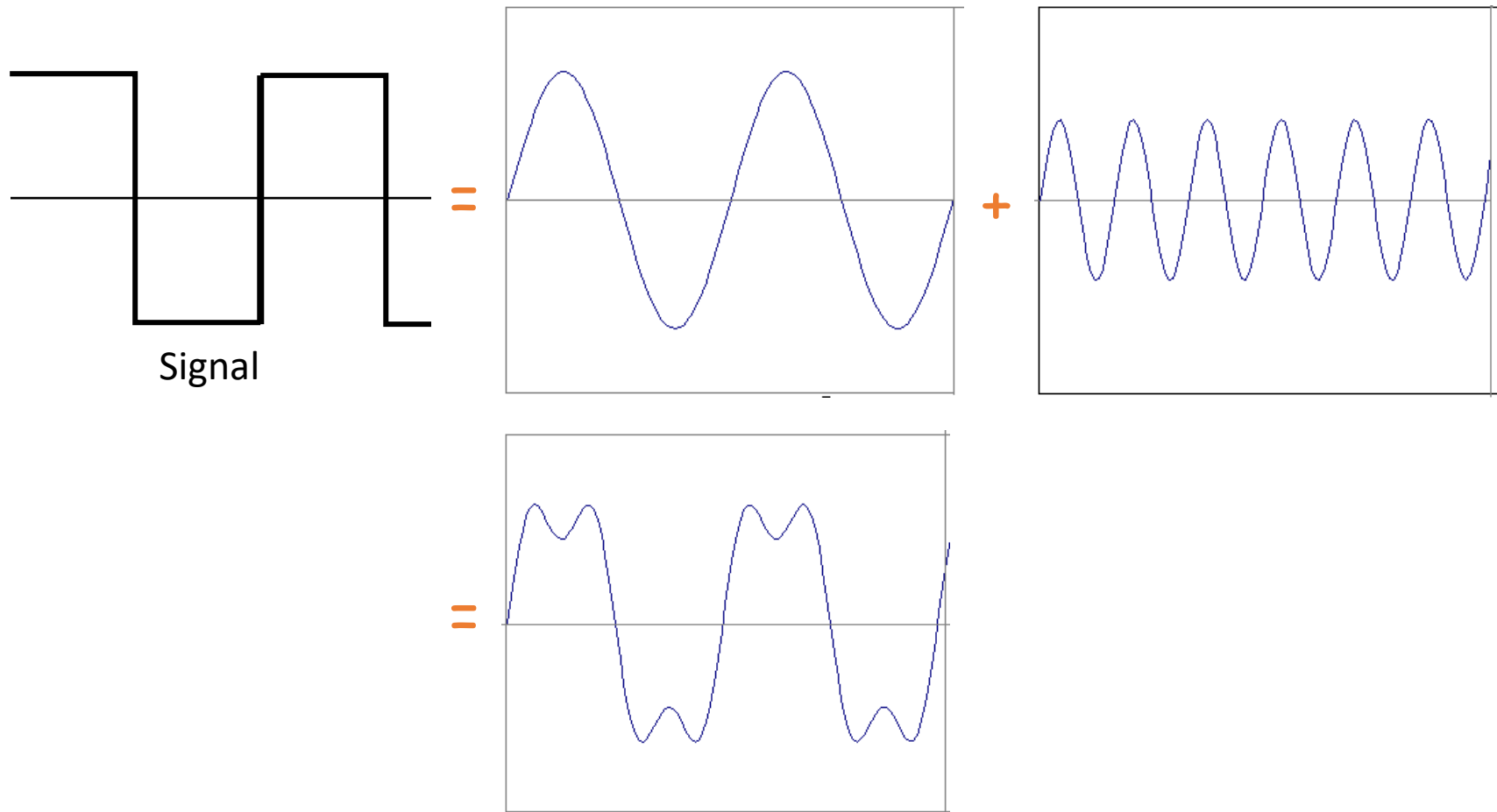


Fourier analysis

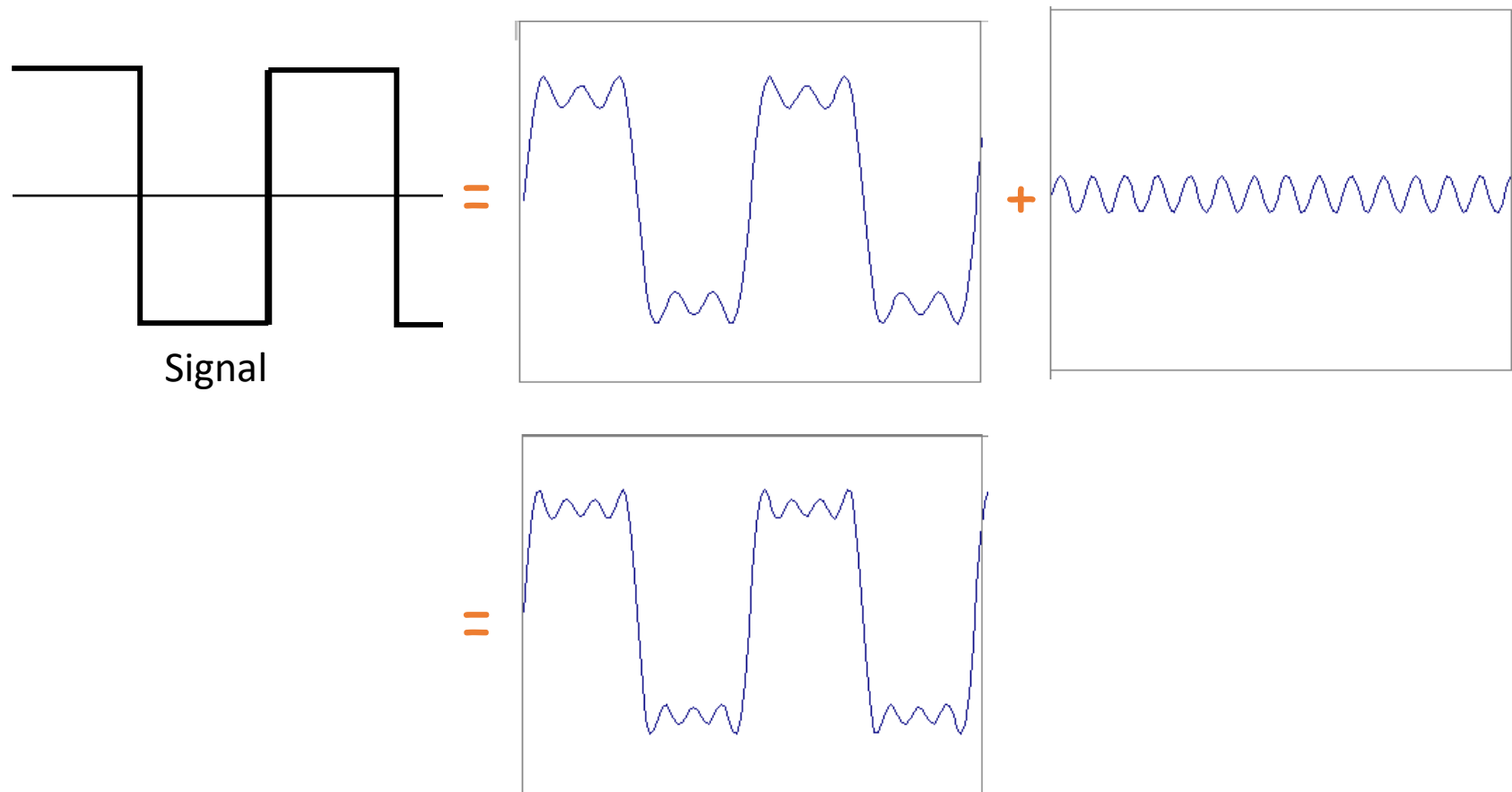
- Any signal can be expressed as a sum of sinusoids



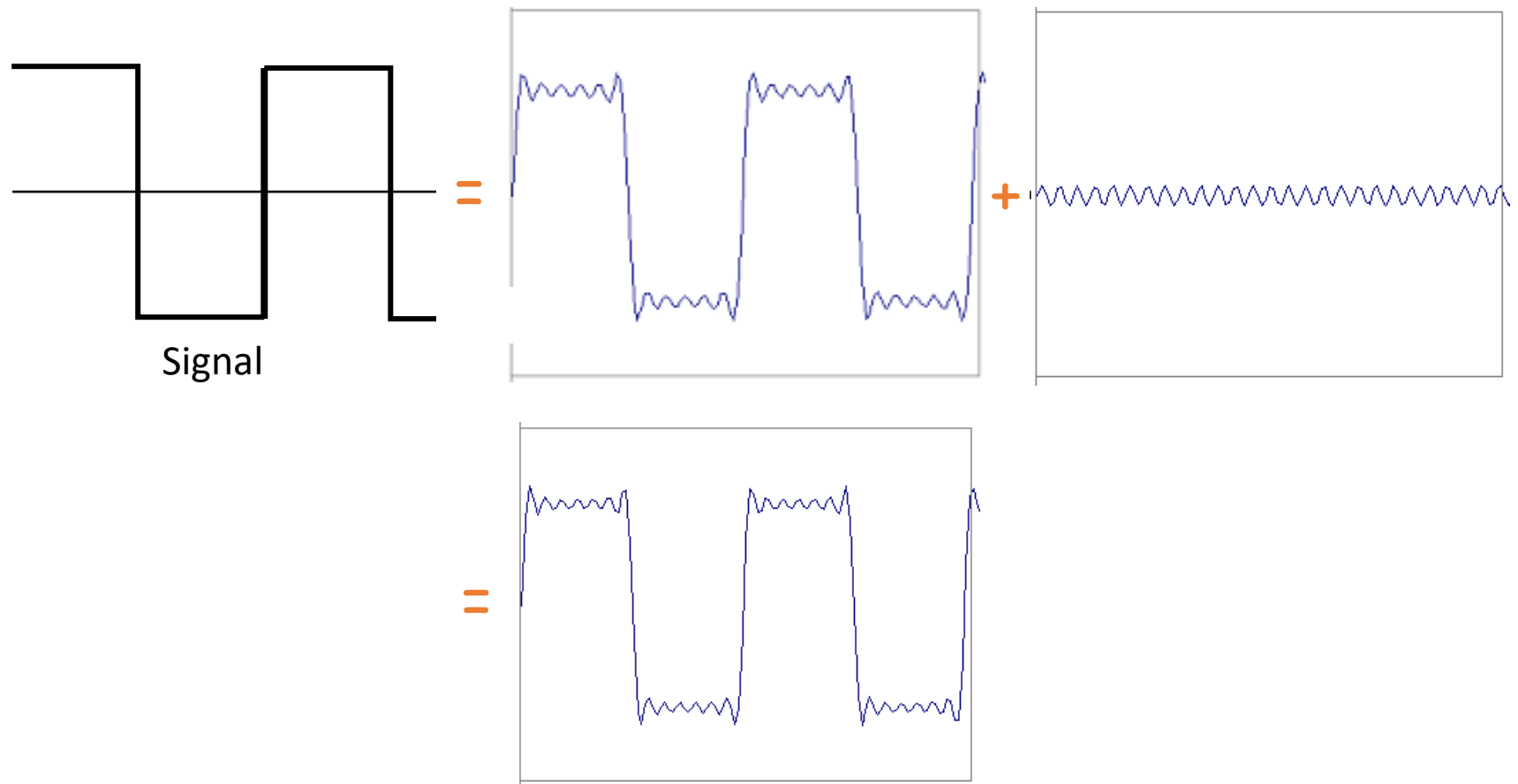
Sum of sinusoids



Sum of sinusoids



Sum of sinusoids

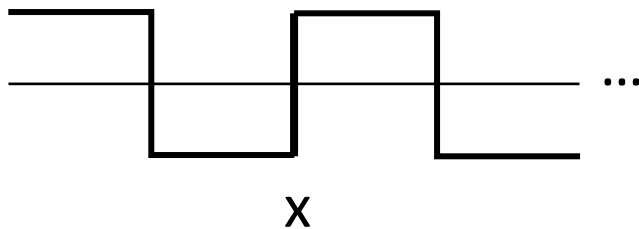


Fourier transform

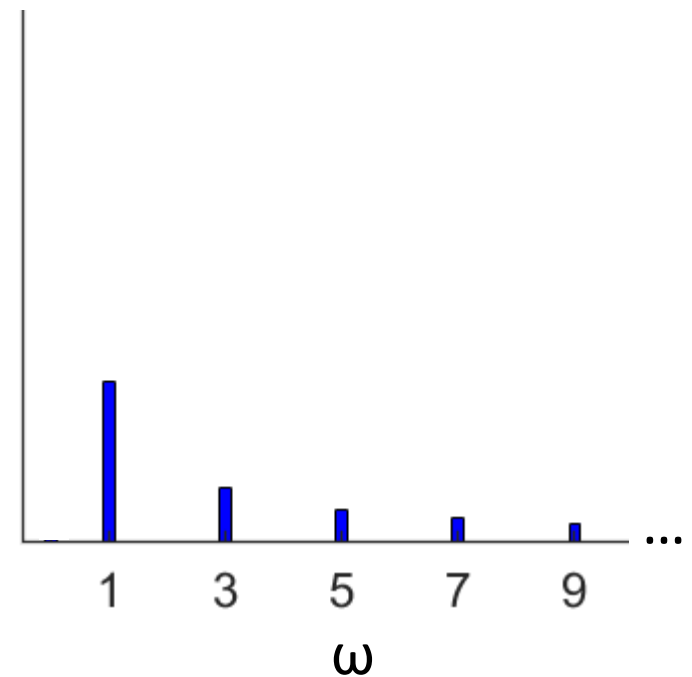
- Fourier transform decomposes signal into component frequencies
 - Values are complex numbers representing amplitude and phase of sinusoids
 - Time domain -> frequency domain (or, for images, spatial domain -> frequency domain)
- $F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2i\pi\omega x} dx$
- `scipy.fft` (1D), `scipy.fft2` (2D), `scipy.fftn` (3D+)
- Inverse Fourier transform converts from frequency domain back to space domain

Frequency spectrum

Signal in spatial
(or time) domain

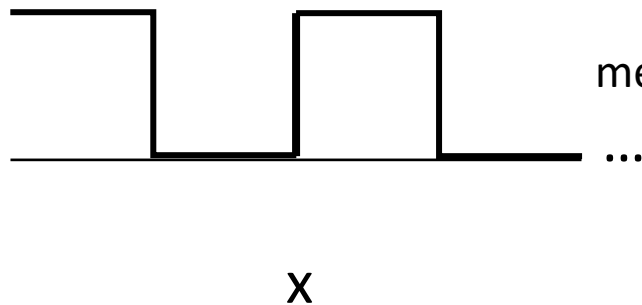


Signal in frequency
domain (magnitude)

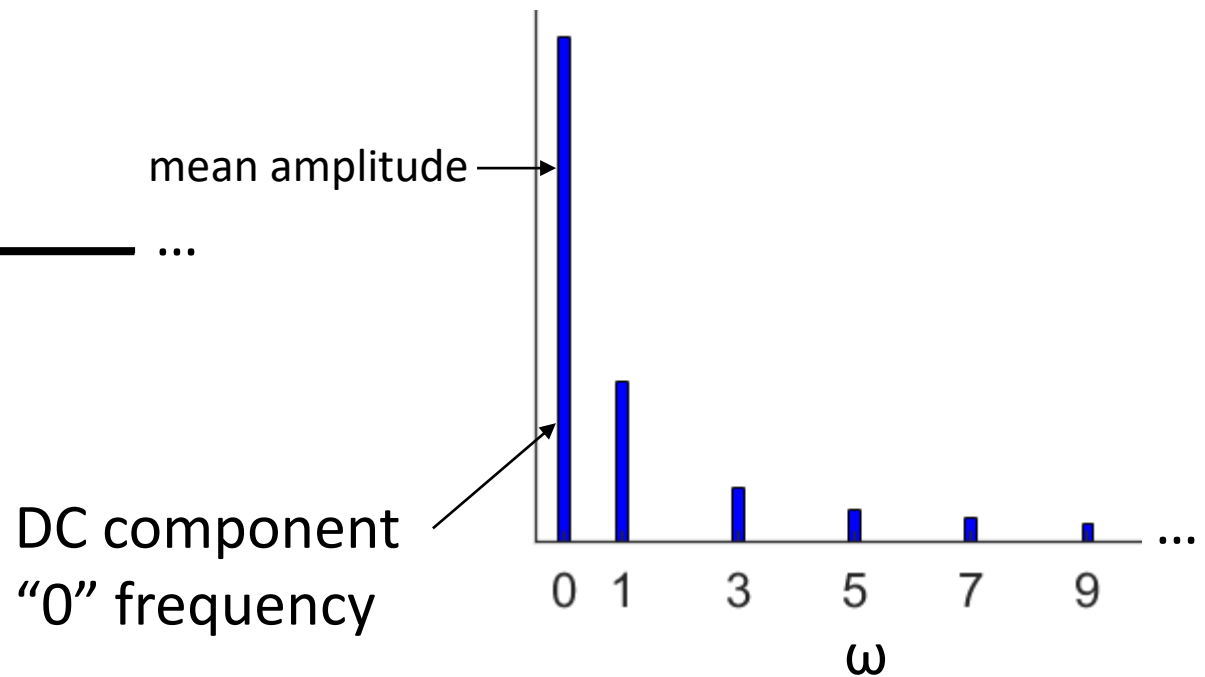


Frequency spectrum

Signal in spatial
(or time) domain

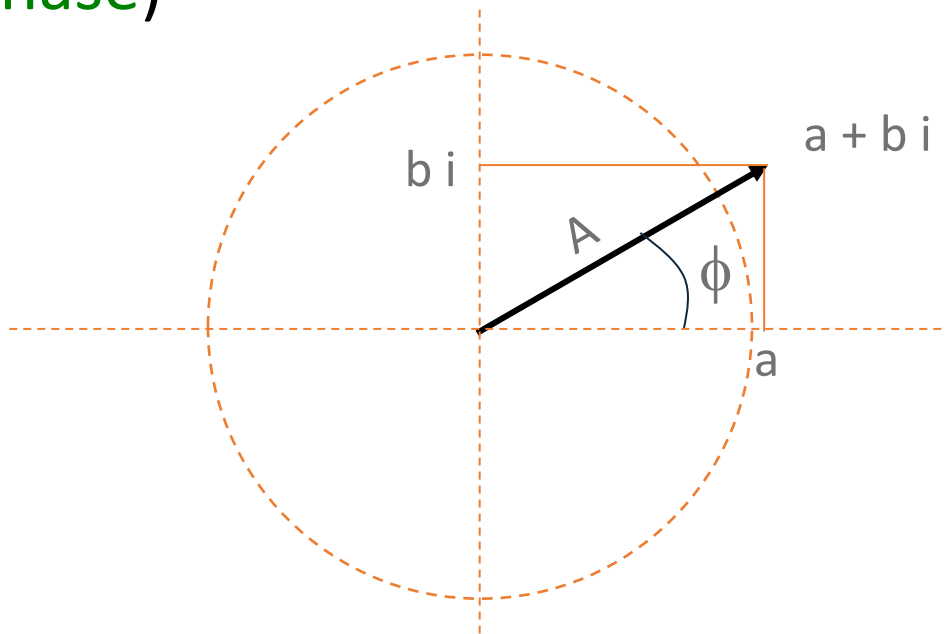


Signal in frequency
domain (magnitude)



Frequency spectrum

- Values in frequency domain are complex numbers
- For each **frequency**: magnitude (=amplitude) and angle (=phase)

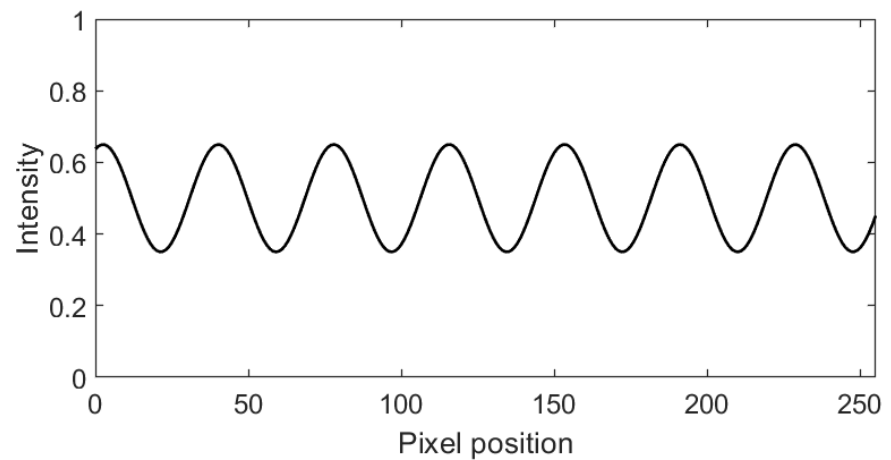
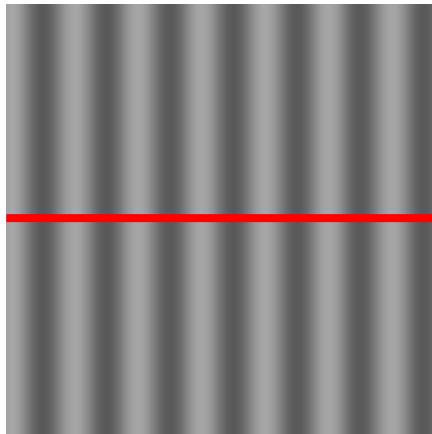
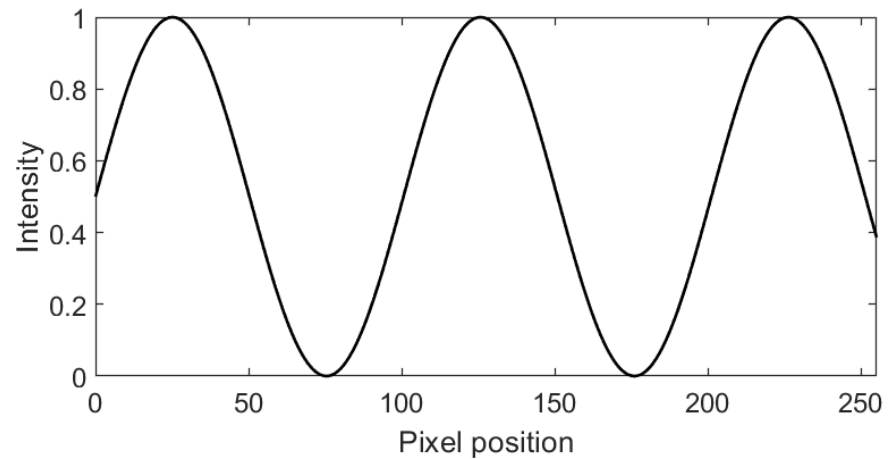
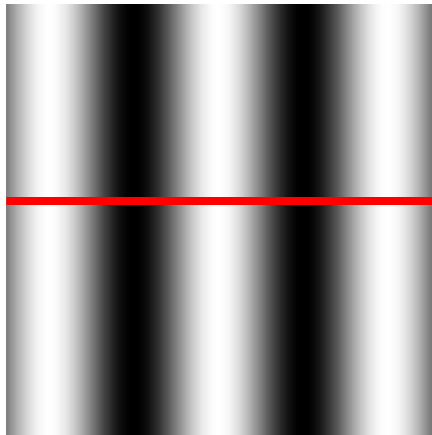


Summary

- Any signal or pattern can be described as a sum of sinusoids
- Fourier transform decomposes a signal into its component sinusoids:
 - The axis is frequency
 - Values are complex numbers
 - Magnitude = amplitude of the sinusoid
 - Angle = phase of the sinusoid

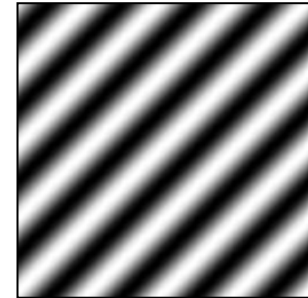
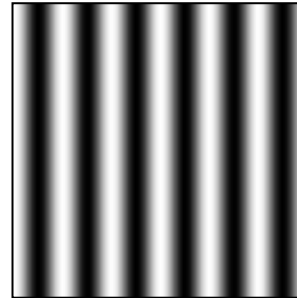
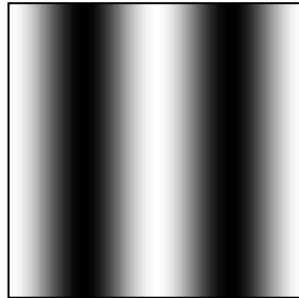
Fourier analysis (images)

Images as sinusoids

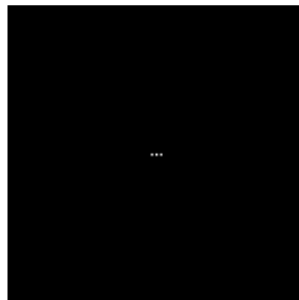


Fourier transforms of images

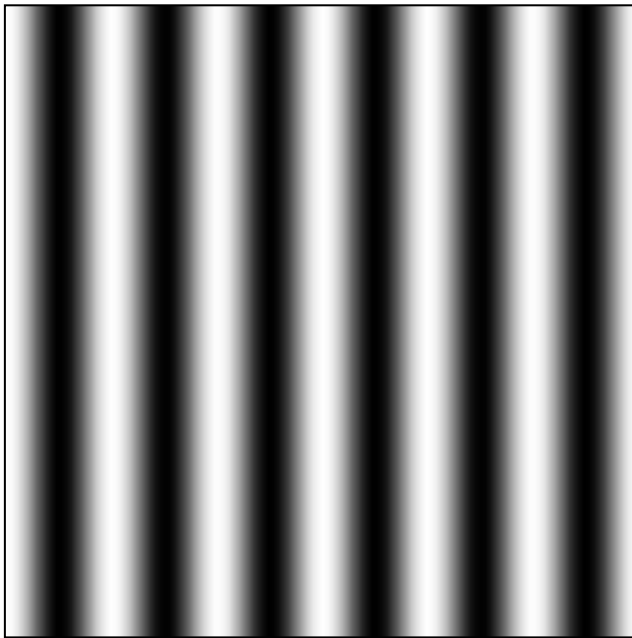
Image



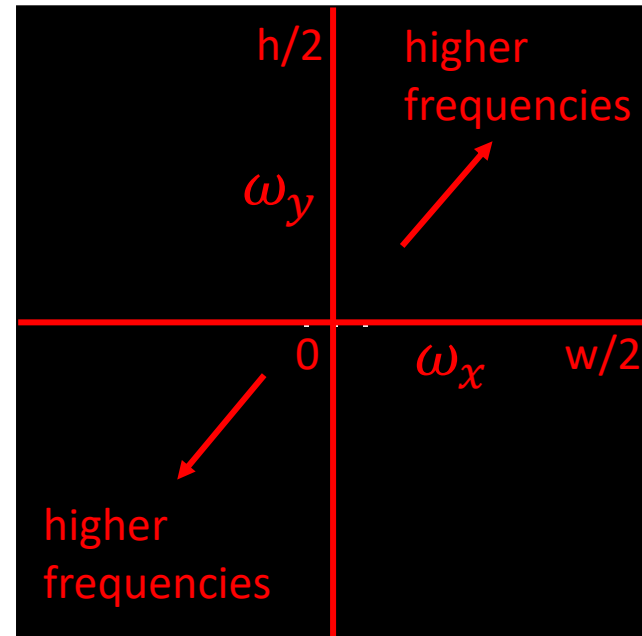
Fourier
transform
(magnitude)



How to interpret Fourier spectra

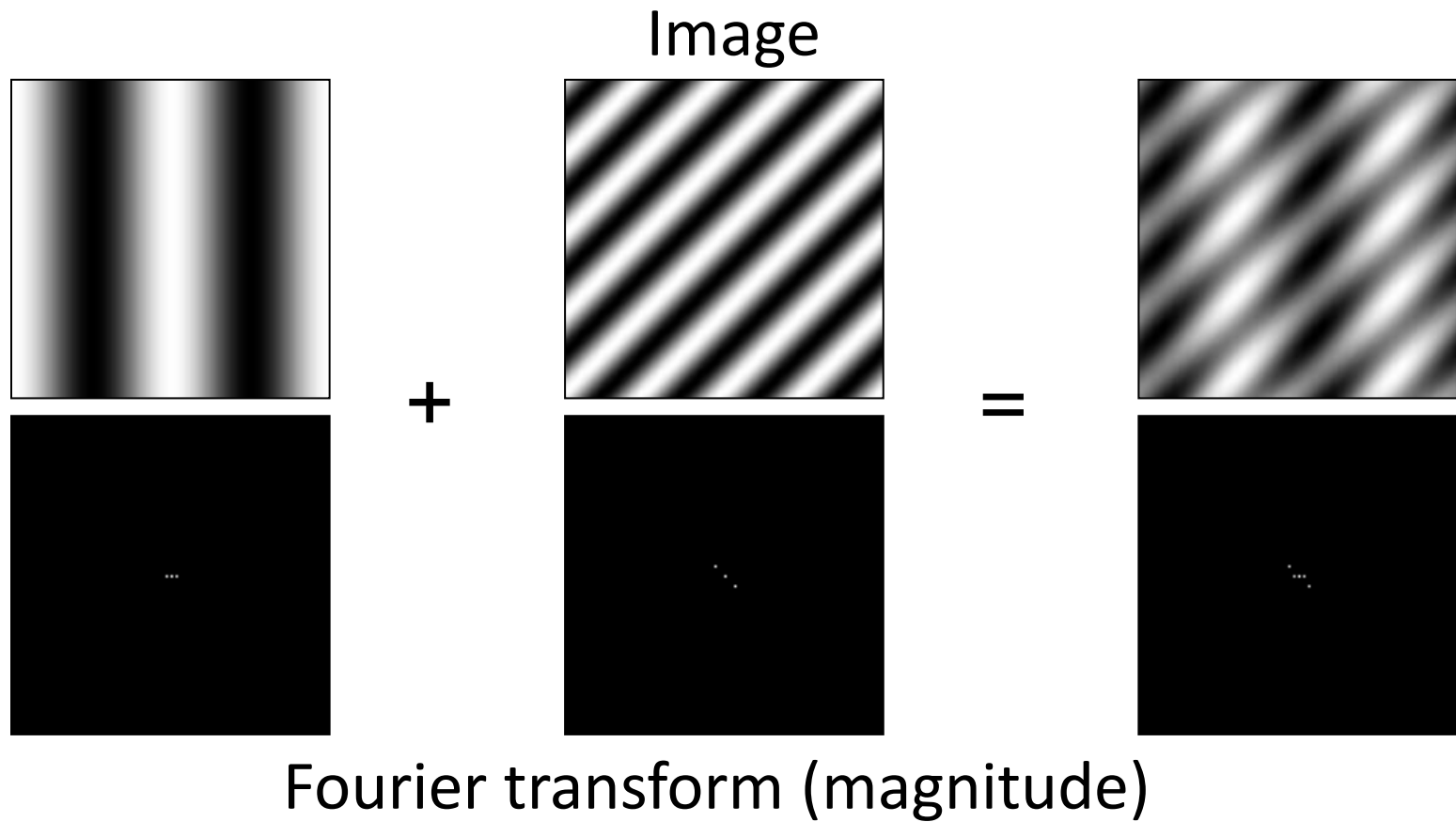


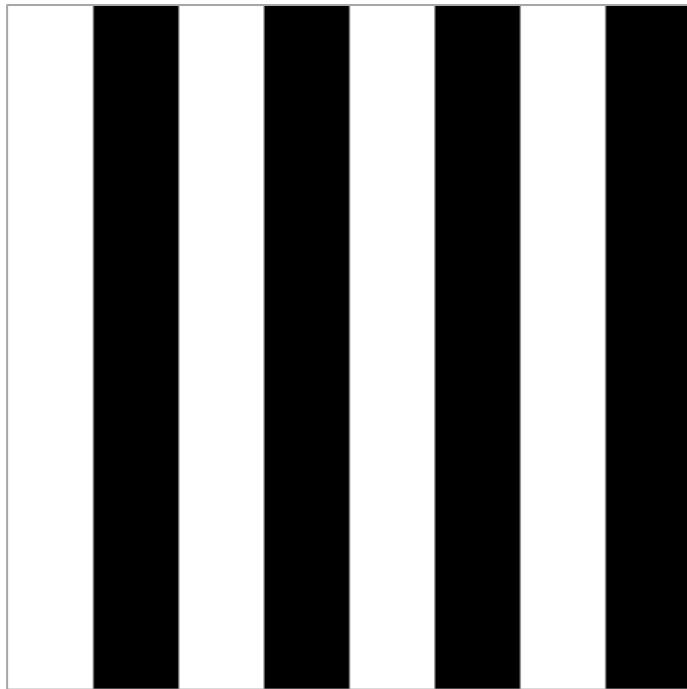
Image



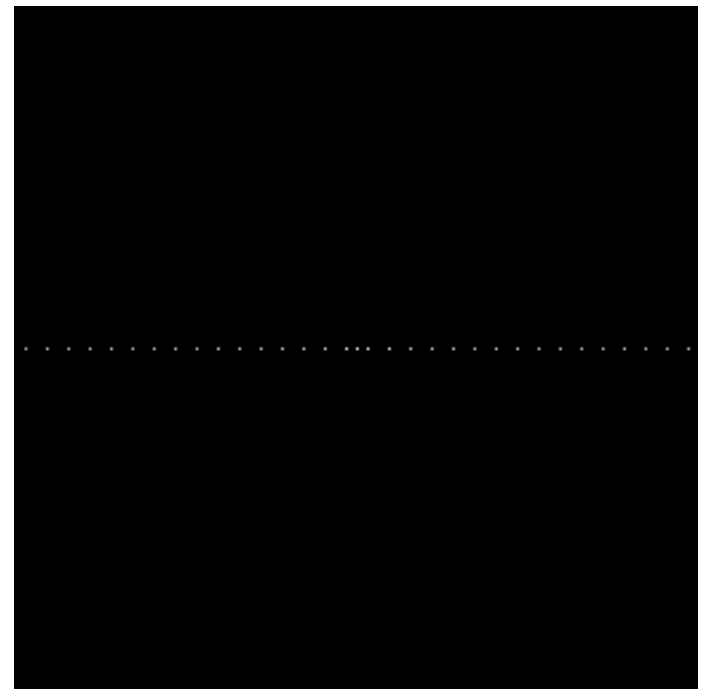
Fourier transform
(magnitude)

Fourier transforms of images

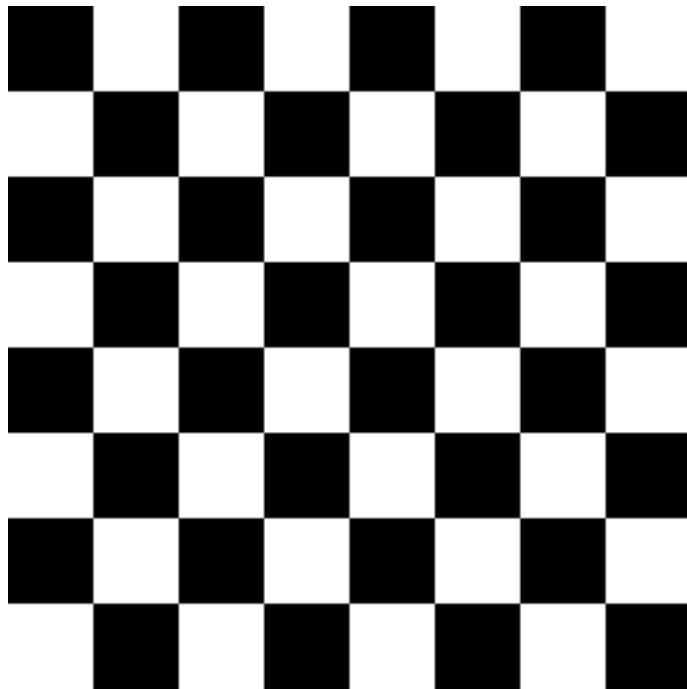




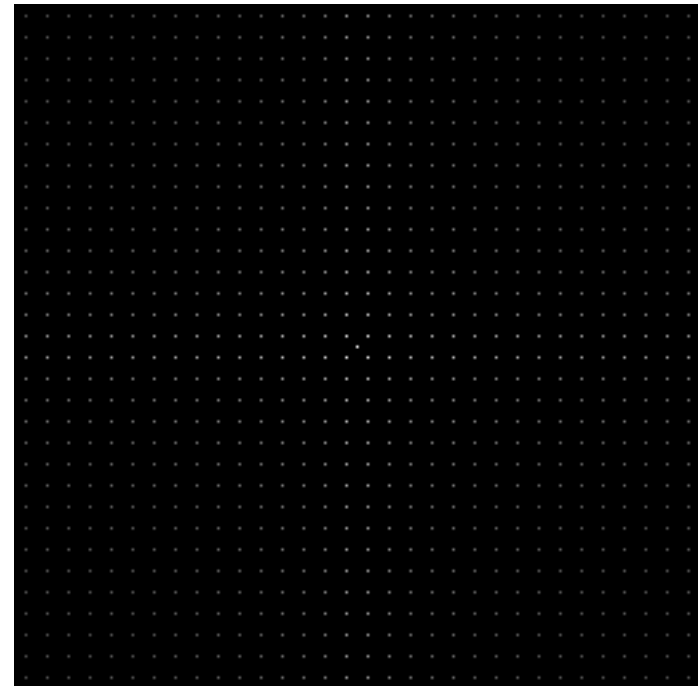
Image



Fourier transform
(magnitude)



Image

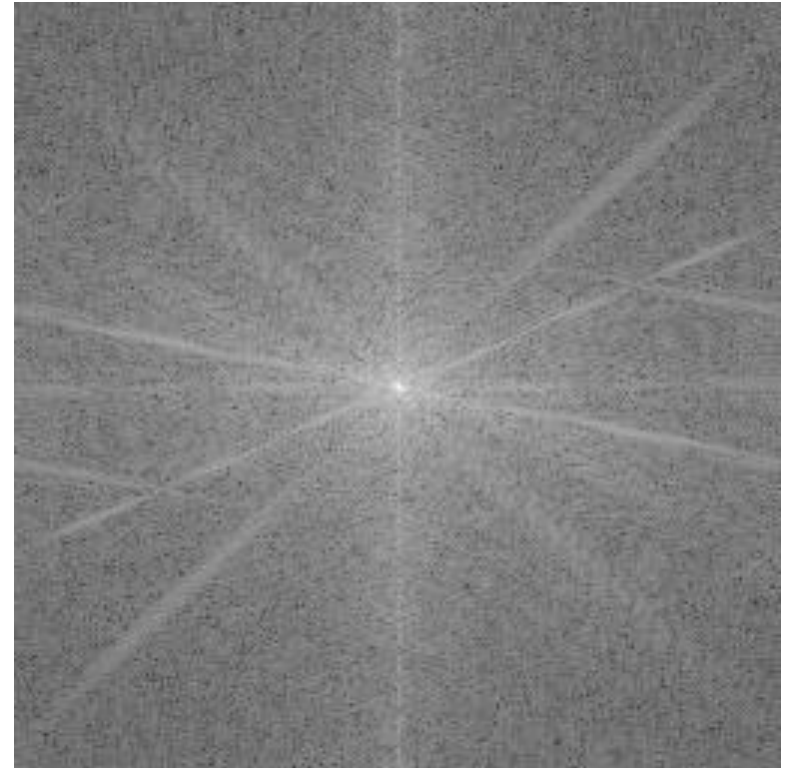


Fourier transform
(magnitude)

Fourier transforms of images

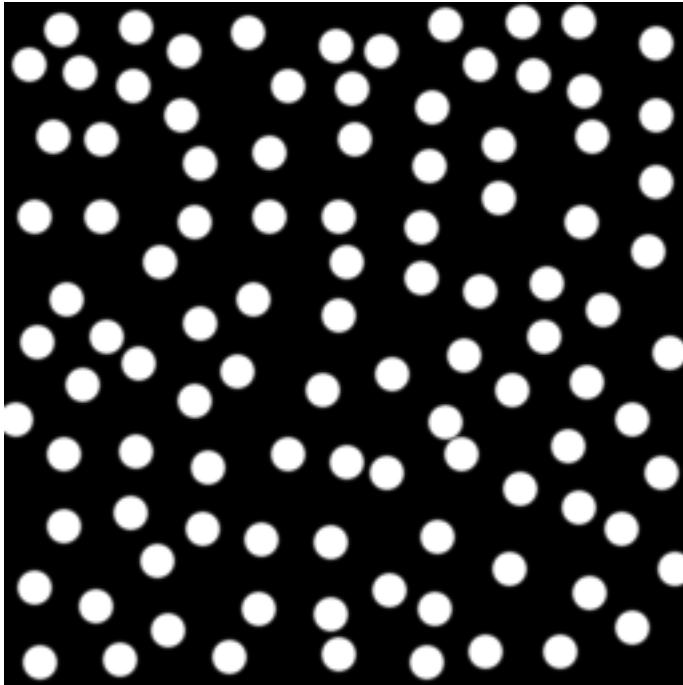


Image

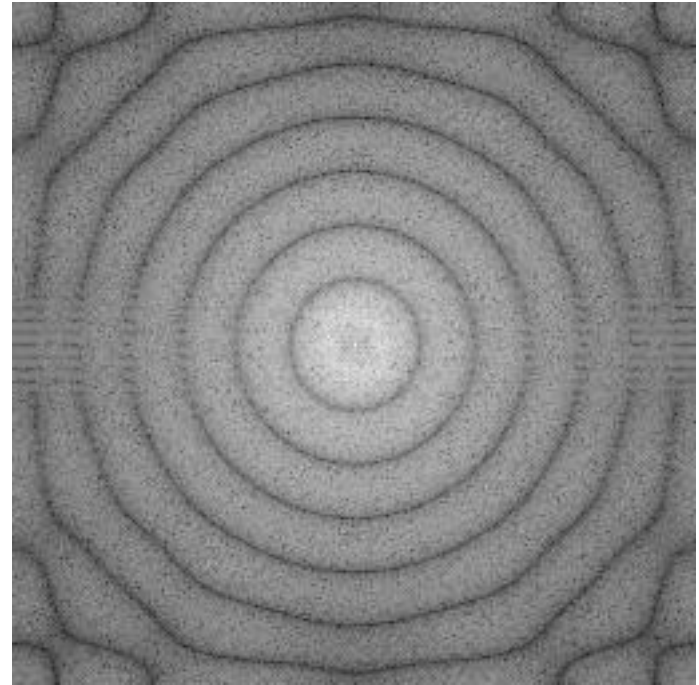


Fourier transform
(magnitude)

Fourier transforms of images

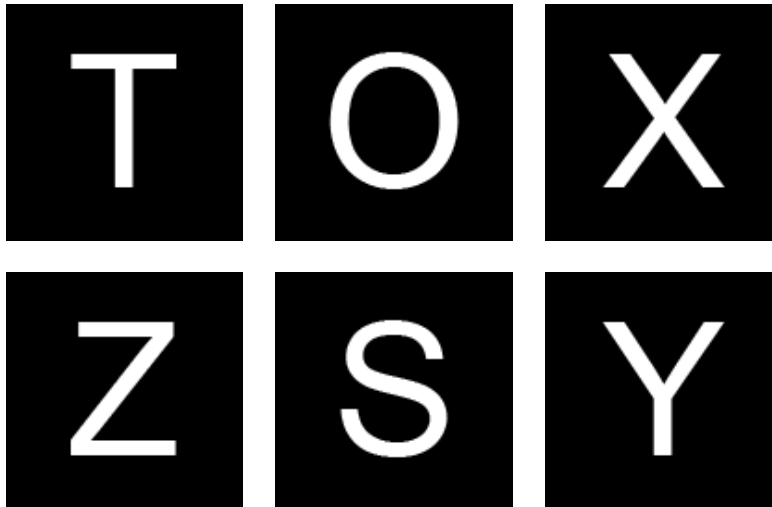


Image

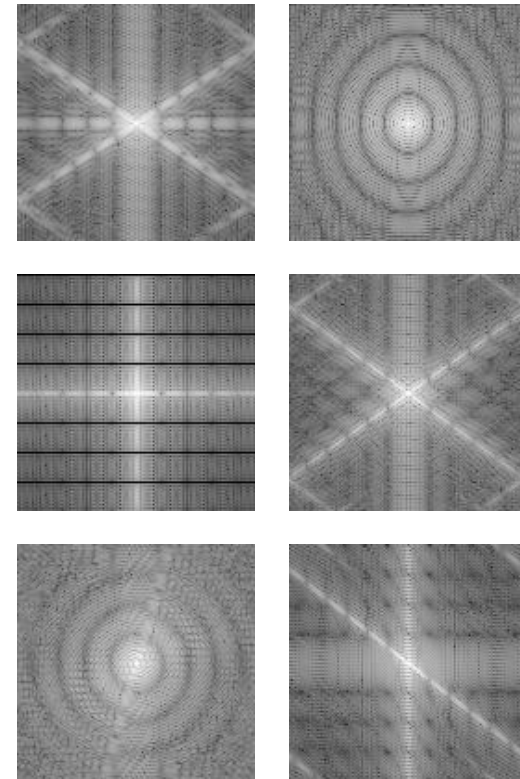


Fourier transform
(magnitude)

Fourier transforms of images



Which transform
corresponds to each letter?

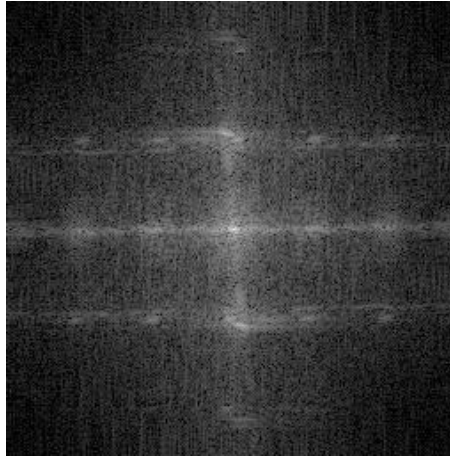


Magnitude and phase

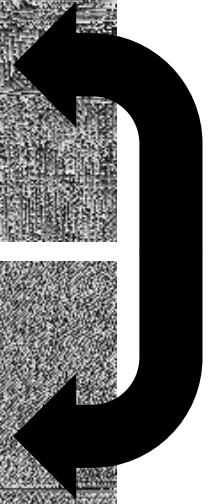
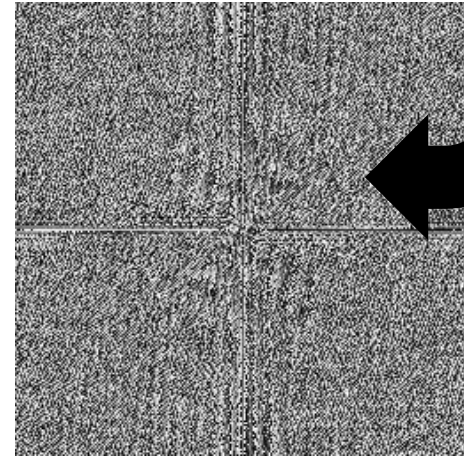
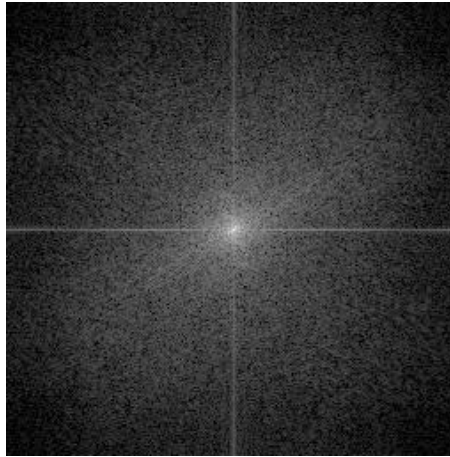
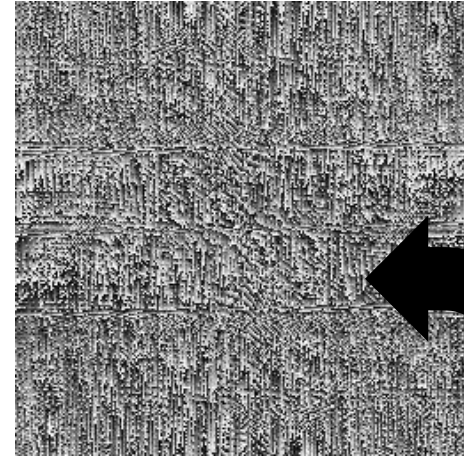
Image



Magnitude



Phase



Fourier analysis of images

- Any image can be represented by its Fourier transform
- Fourier transform = for each frequency, magnitude (amplitude) + phase
- Magnitude captures the holistic “texture” of an image, but the edges are mainly represented by Fourier phase

Frequency filtering

Operations in frequency domain

- Operations in the spatial domain have equivalent operations in frequency domain
- Convolution in spatial domain = multiplication in frequency domain

$$FT[h * f] = FT[h]FT[f]$$

- Inverse:

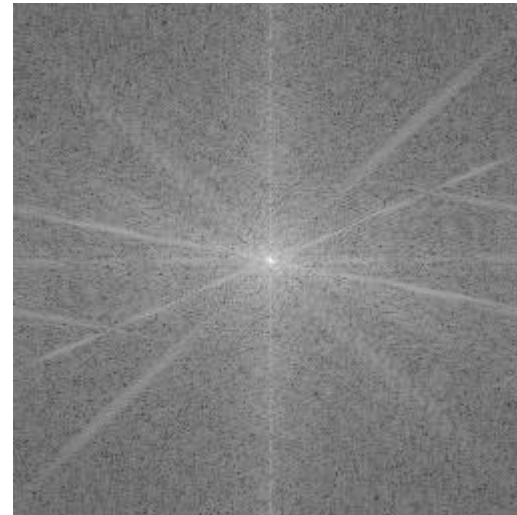
$$FT^{-1}[hf] = FT^{-1}[h] * FT^{-1}[f]$$

Bandpass filter

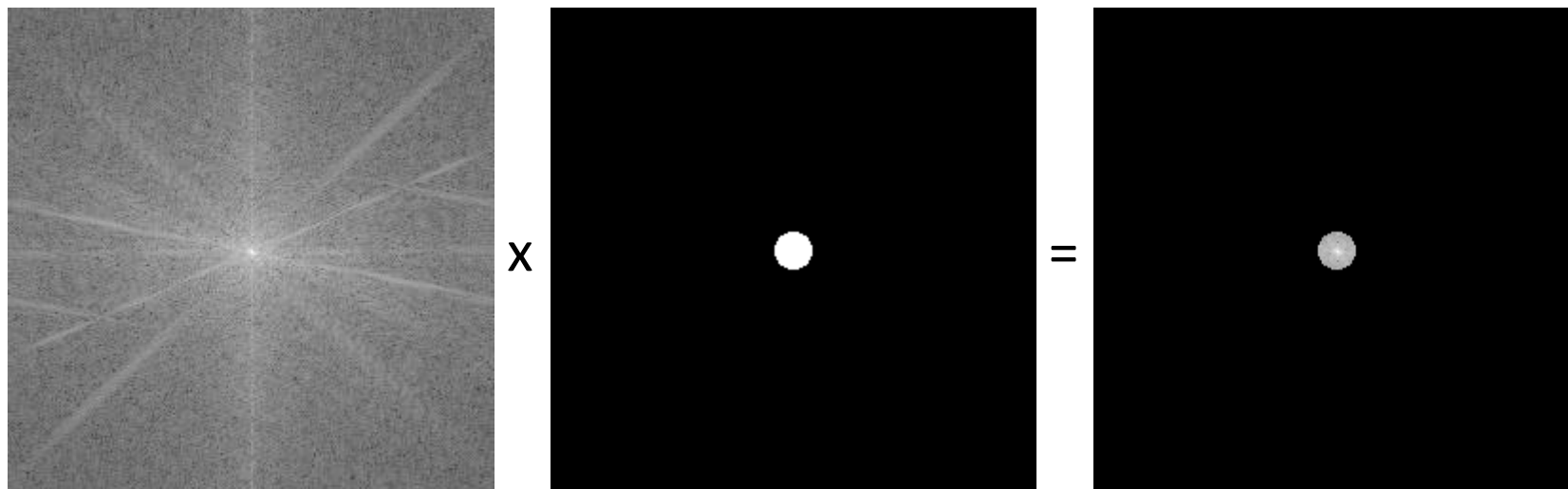
- Bandpass filter = a filter that removes a range of frequencies from a signal

Low pass filter

- Low pass filter = keep low spatial frequencies, remove high frequencies



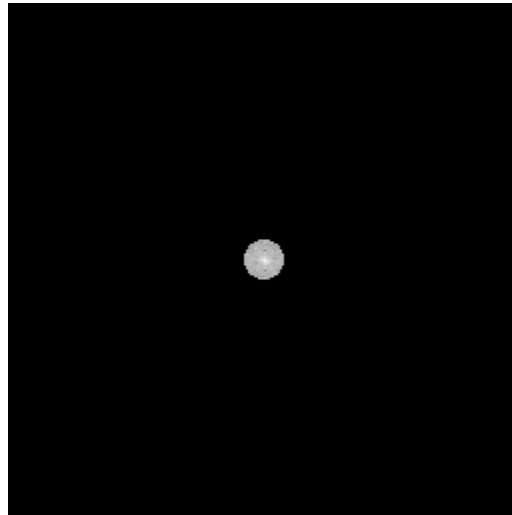
Low pass filter



Element-wise multiplication

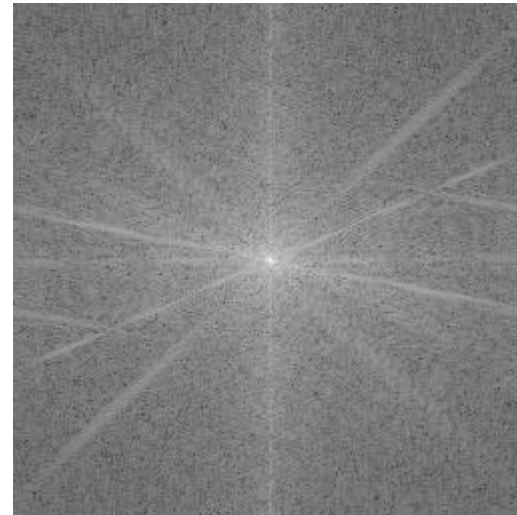
black = 0
white = 1

Low pass filter

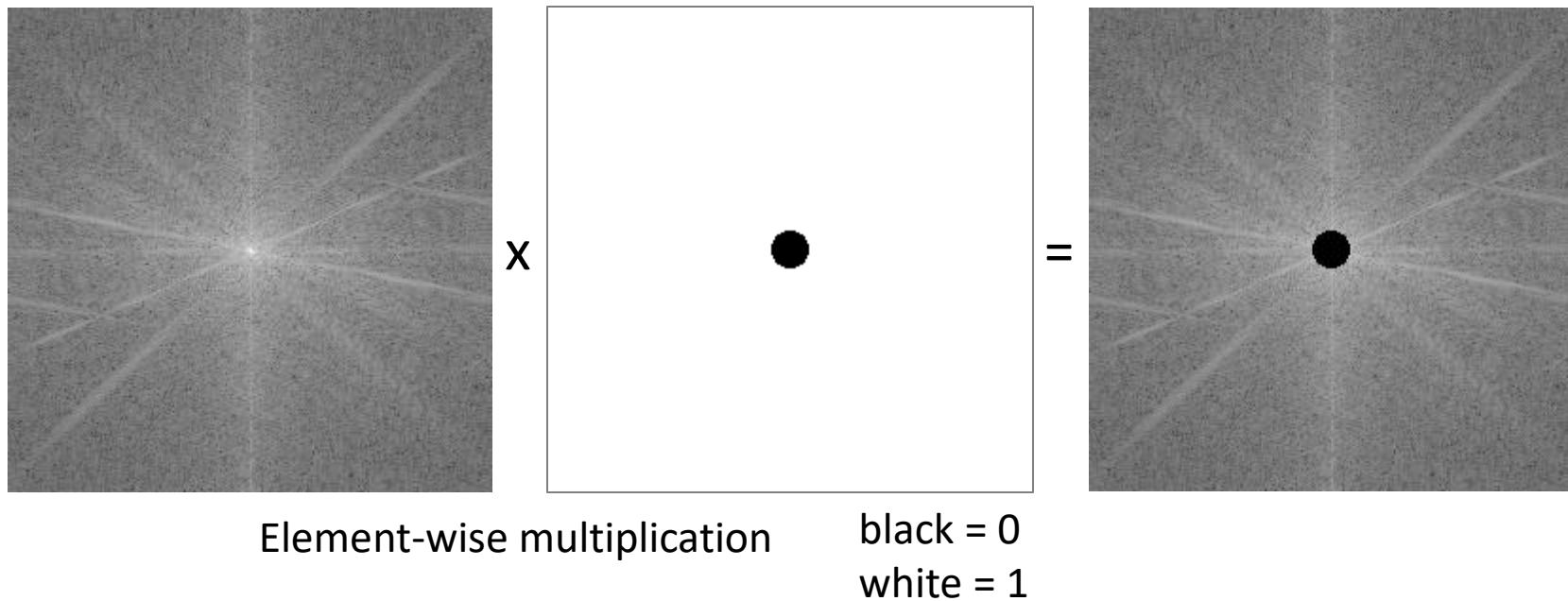


High pass filter

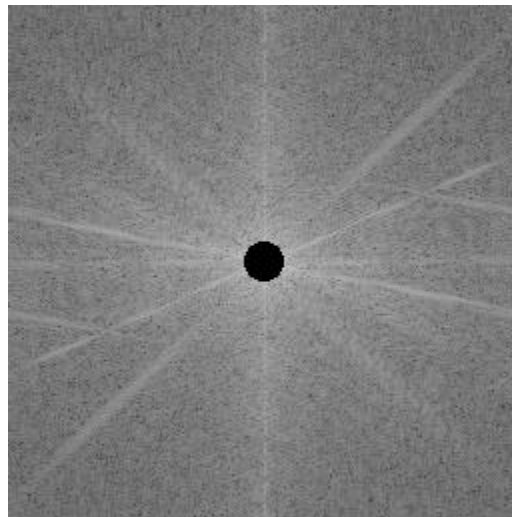
- High pass filter = keep high spatial frequencies, remove low frequencies



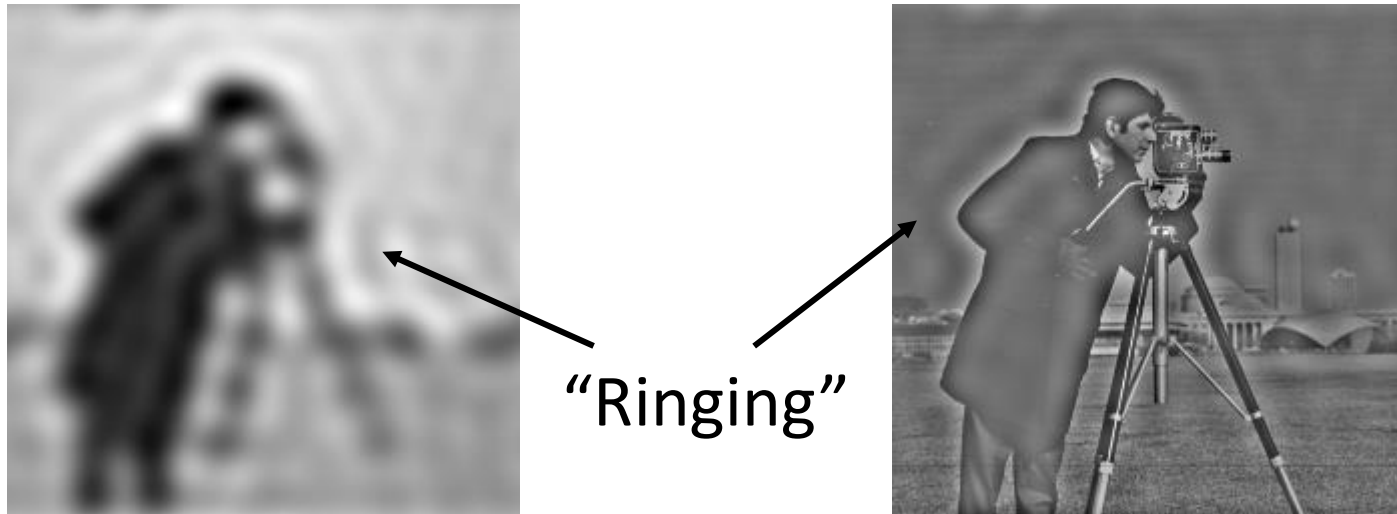
High pass filter



High pass filter



Filter artefacts



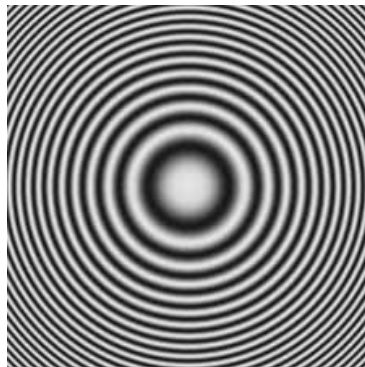
Why does this happen?

Inverse convolution theorem

$$\text{FT}^{-1} \left(\begin{array}{|c|c|} \hline \text{Black square with white dot} & \text{Grayscale image with radial lines} \\ \hline \end{array} \right) = \text{FT}^{-1} \left(\begin{array}{|c|} \hline \text{Black square with white dot} \\ \hline \end{array} \right) * \text{FT}^{-1} \left(\begin{array}{|c|} \hline \text{Grayscale image with radial lines} \\ \hline \end{array} \right)$$

Inverse convolution theorem

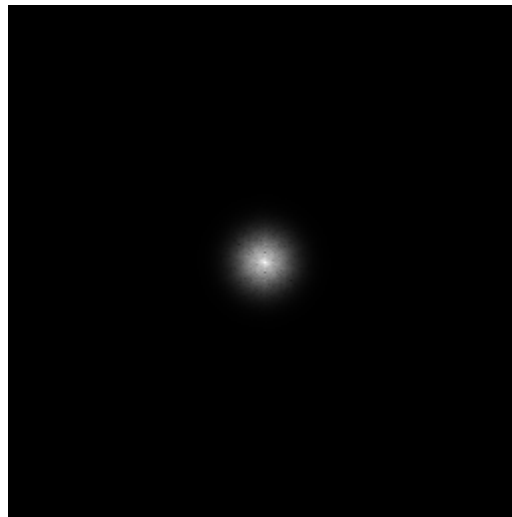
$$\text{FT}^{-1} \left(\begin{array}{c} \text{[Black square with white dot]} \quad \text{[Grayscale image with radial lines]} \end{array} \right) =$$



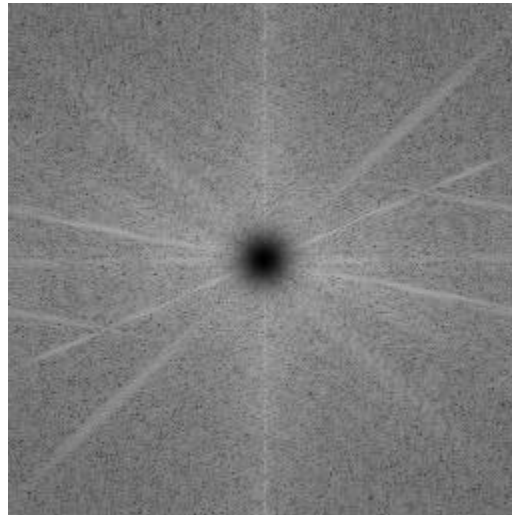
*



Gaussian low pass filter



Gaussian high pass filter



Summary

- Images can be filtered in the spatial domain, or the frequency domain
- Operations in one domain have an equivalent in the other domain
 - Convolution in spatial domain = multiplication in Fourier domain
- Modelling filters in both domains can help understand/debug what a filter is doing

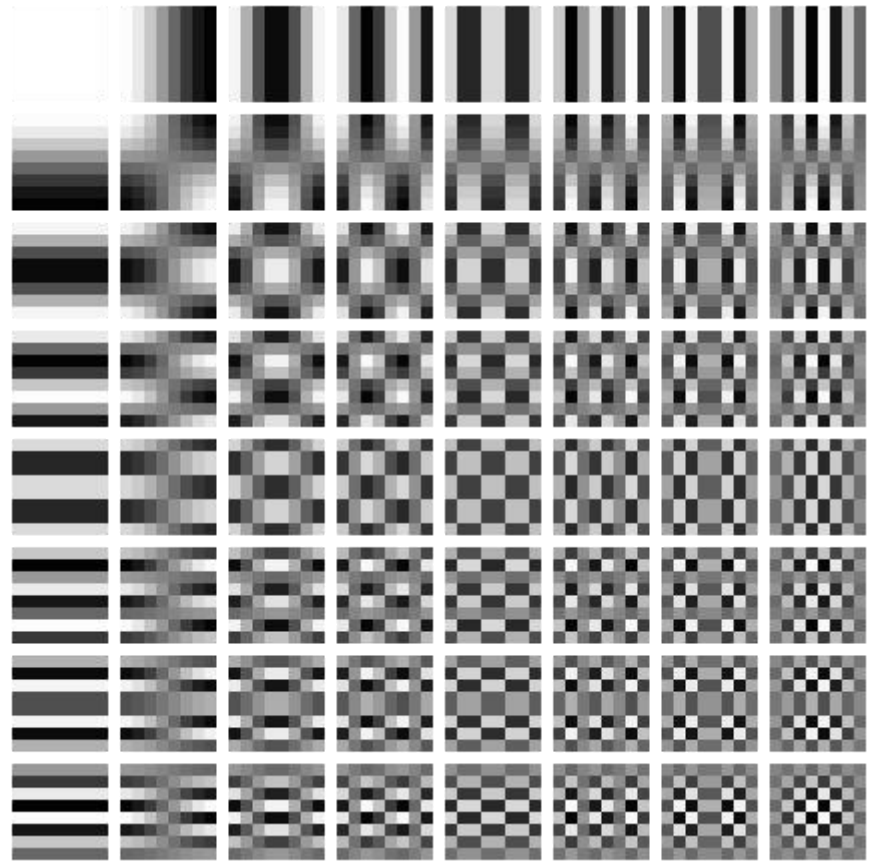
Applications

Image compression

- Frequency domain is a convenient space for image compression
- Why?
- Human visual system is not very sensitive to contrast in high spatial frequencies
- Discarding information in high spatial frequencies doesn't change the “look” of an image

Image compression

- JPEG compression:
break image into 8x8
pixel blocks, each
represented in
frequency space
- Discrete cosine
transform (DCT)
- High spatial frequency
components are
quantised

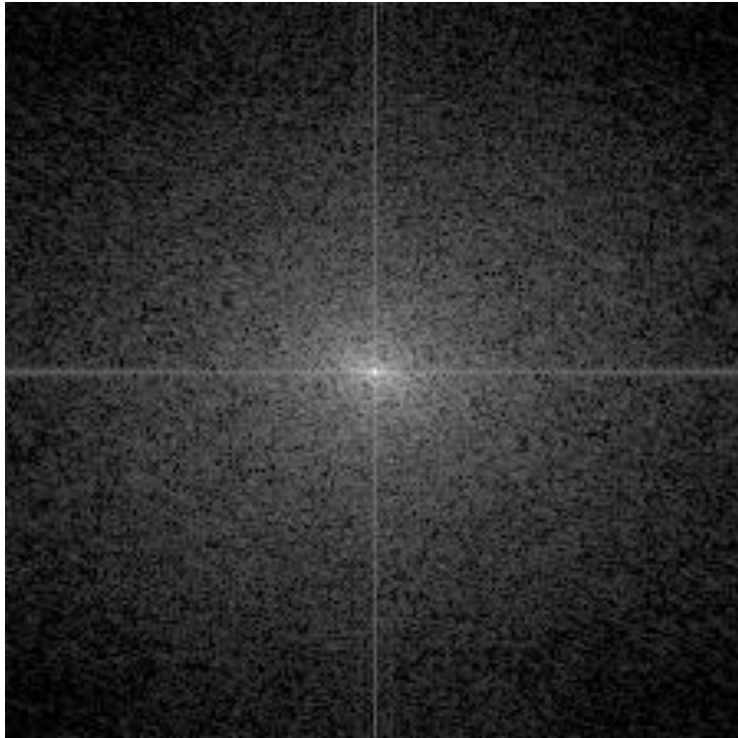


JPEG compression

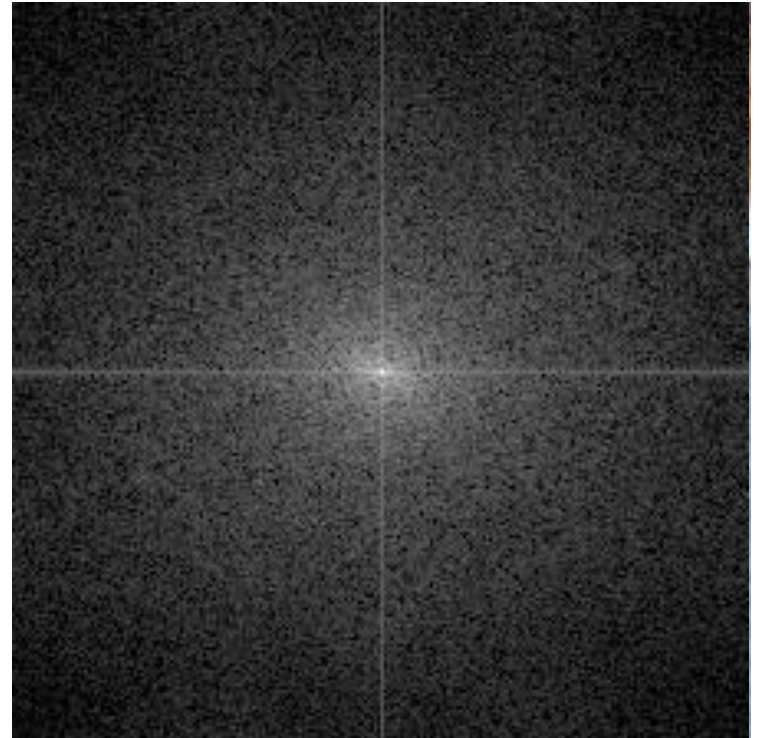


Image: https://en.wikipedia.org/wiki/File:Felis_silvestris_silvestris_small_gradual_decrease_of_quality.png

Image forensics



Network: MobileNet V2
Prediction: English springer (90.08%)

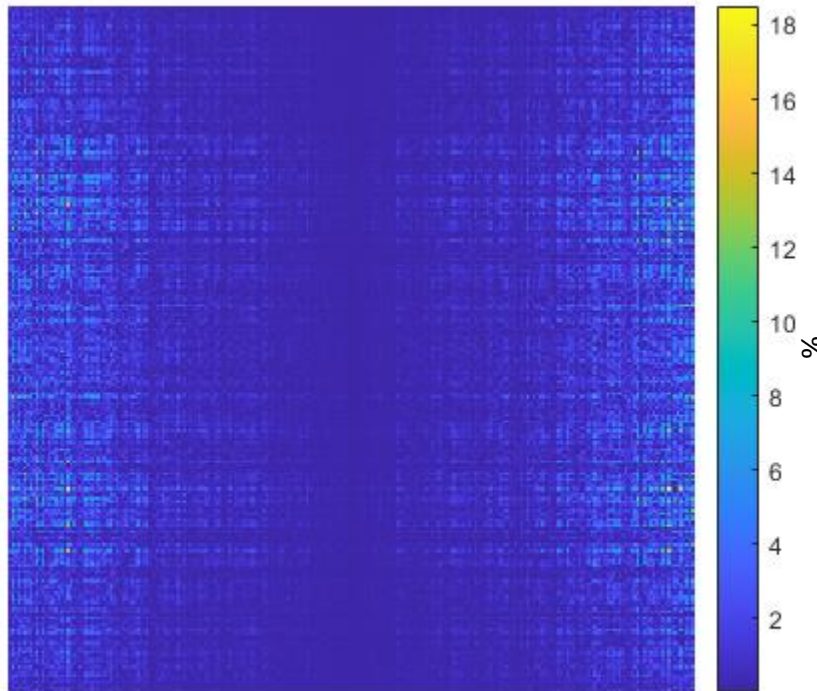


Model: MobileNet V2
Prediction: hot dog (68.88%)

Images: ImageNet, <https://kennysong.github.io/adversarial.js/>

Dog vs. hot dog – what changed?

Absolute difference in magnitude



Absolute difference in phase

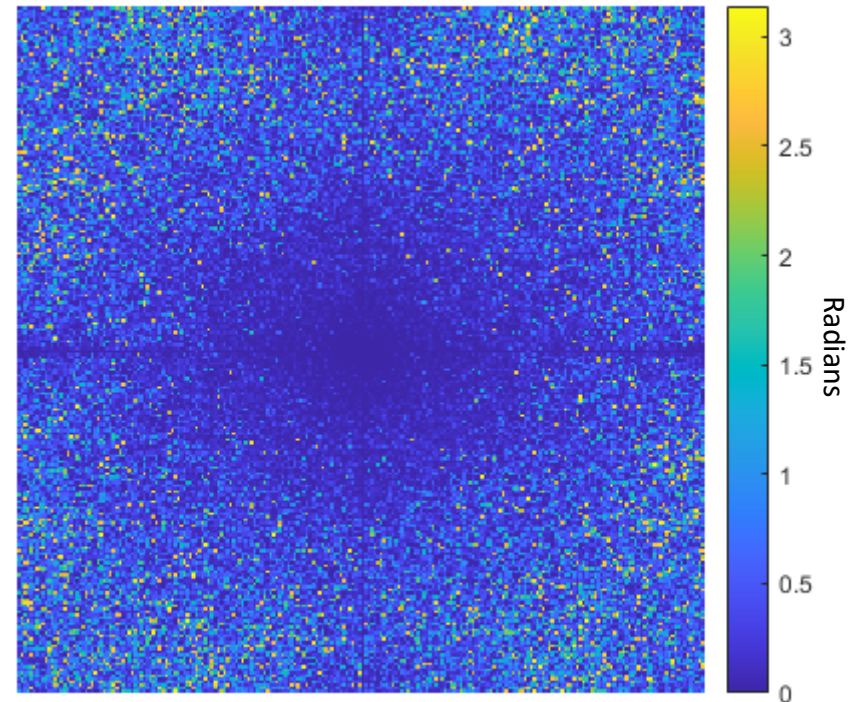


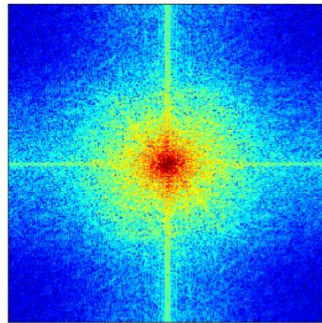
Image forensics



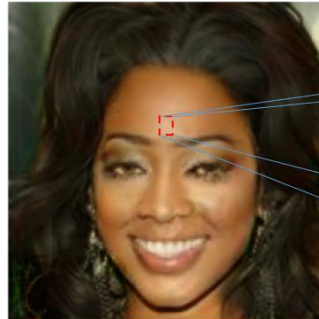
Which of these are real people vs. GAN-generated?



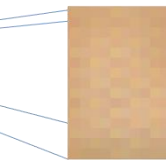
Real



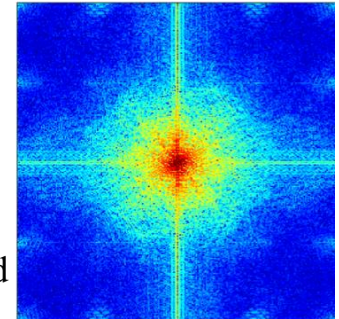
Spectrum



Fake



Checkerboard
Pattern



Spectrum

Dzanic, Shah, & Witherden (2019)
Zhang, Karaman, & Chang (2019)

Summary

- Any image can be represented in either the spatial or the frequency domain
- Frequency domain is a convenient space for many applications:
 - Filtering
 - Compression
 - Forensics
 - Frequency-based features