

Statistical Power

Fill In Your Name

27 August, 2021

What is power?

Analytical calculations of power

Simulation-based power calculation

Power with covariate adjustment

Power for cluster randomization

Comparative statics

What is power?

What is power?

- ▶ We want to separate signal from noise.
- ▶ Power = probability of rejecting null hypothesis, given true effect $\neq 0$.
- ▶ In other words, it is the ability to detect an effect given that it exists.
- ▶ Formally: $(1 - \text{Type II})$ error rate.
- ▶ Thus, power $\in (0, 1)$.
- ▶ Standard thresholds: 0.8 or 0.9.

Starting point for power analysis

- ▶ Power analysis is something we do *before* we run a study.
 - ▶ Helps you figure out the sample you need to detect a given effect size.
 - ▶ Or helps you figure out a minimal detectable difference given a set sample size.
 - ▶ May help you decide whether to run a study.
- ▶ It is hard to learn from an under-powered null finding.
 - ▶ Was there an effect, but we were unable to detect it? or was there no effect? We can't say.

Power

- ▶ Say there truly is a treatment effect and you run your experiment many times. How often will you get a statistically significant result?
- ▶ Some guesswork to answer this question.
 - ▶ How big is your treatment effect?
 - ▶ How many units are treated, measured?
 - ▶ How much noise is there in the measurement of your outcome?

Approaches to power calculation

- ▶ Analytical calculations of power
- ▶ Simulation

Power calculation tools

- ▶ Interactive
 - ▶ EGAP Power Calculator
 - ▶ rpsychologist
- ▶ R Packages
 - ▶ pwr
 - ▶ DeclareDesign, see also <https://declaredesign.org/>

Analytical calculations of power

Analytical calculations of power

- ▶ Formula:

$$\text{Power} = \Phi \left(\frac{|\tau| \sqrt{N}}{2\sigma} - \Phi^{-1} \left(1 - \frac{\alpha}{2} \right) \right)$$

- ▶ Components:

- ▶ ϕ : standard normal CDF is monotonically increasing
- ▶ τ : the effect size
- ▶ N : the sample size
- ▶ σ : the standard deviation of the outcome
- ▶ α : the significance level (typically 0.05)

Example: Analytical calculations of power

```
# Power for a study with 80 observations and effect  
# size of 0.25  
library(pwr)  
pwr.t.test(  
  n = 40, d = 0.25, sig.level = 0.05,  
  power = NULL, type = c(  
    "two.sample",  
    "one.sample", "paired"  
  )  
)
```

Two-sample t test power calculation

```
      n = 40  
      d = 0.25  
sig.level = 0.05  
  power = 0.1972  
alternative = two.sided
```

NOTE: n is number in *each* group

Limitations to analytical power calculations

- ▶ Only derived for some test statistics (differences of means)
- ▶ Makes specific assumptions about the data-generating process
- ▶ Incompatible with more complex designs

Simulation-based power calculation

Simulation-based power calculation

- ▶ Create dataset and simulate research design.
- ▶ Assumptions are necessary for simulation studies, but you make your own.
- ▶ For the DeclareDesign approach, see <https://declaredesign.org/>

Steps

- ▶ Define the sample and the potential outcomes function.
- ▶ Define the treatment assignment procedure.
- ▶ Create data.
- ▶ Assign treatment, then estimate the effect.
- ▶ Do this many times.

Examples

- ▶ Complete randomization
- ▶ With covariates
- ▶ With cluster randomization

Example: Simulation-based power for complete randomization

```
# install.packages("randomizr")
library(randomizr)
library(estimatr)

## Y0 is fixed in most field experiments.
## So we only generate it once:
make_Y0 <- function(N) {
  rnorm(n = N)
}

repeat_experiment_and_test <- function(N, Y0, tau) {
  Y1 <- Y0 + tau
  Z <- complete_ra(N = N)
  Yobs <- Z * Y1 + (1 - Z) * Y0
  estimator <- lm_robust(Yobs ~ Z)
  pval <- estimator$p.value[2]
  return(pval)
}
```

Example: Simulation-based power for complete randomization

```
power_sim <- function(N, tau, sims) {  
  Y0 <- make_Y0(N)  
  pvals <- replicate(  
    n = sims,  
    repeat_experiment_and_test(N = N, Y0 = Y0, tau = tau)  
  )  
  pow <- sum(pvals < .05) / sims  
  return(pow)  
}
```

```
set.seed(12345)  
power_sim(N = 80, tau = .25, sims = 100)
```

```
[1] 0.15
```

```
power_sim(N = 80, tau = .25, sims = 100)
```

```
[1] 0.21
```

Example: Using DeclareDesign I

```
library(DeclareDesign)
library(tidyverse)
P0 <- declare_population(N, u0 = rnorm(N))
# declare Y(Z=1) and Y(Z=0)
O0 <- declare_potential_outcomes(Y_Z_0 = 5 + u0, Y_Z_1 = Y_Z_0 + tau)
# design is to assign m units to treatment
A0 <- declare_assignment(Z = conduct_ra(N = N, m = round(N / 2)))
# estimand is the average difference between Y(Z=1) and Y(Z=0)
estimand_ate <- declare_inquiry(ATE = mean(Y_Z_1 - Y_Z_0))
R0 <- declare_reveal(Y, Z)
design0_base <- P0 + A0 + O0 + R0

## For example:
design0_N100_tau25 <- redesign(design0_base, N = 100, tau = .25)
dat0_N100_tau25 <- draw_data(design0_N100_tau25)
head(dat0_N100_tau25)
```

Example: Using DeclareDesign II

| | ID | u0 | Z | Y_Z_0 | Y_Z_1 | Y |
|---|-----|---------|---|-------|-------|-------|
| 1 | 001 | -0.2060 | 0 | 4.794 | 5.044 | 4.794 |
| 2 | 002 | -0.5875 | 0 | 4.413 | 4.663 | 4.413 |
| 3 | 003 | -0.2908 | 1 | 4.709 | 4.959 | 4.959 |
| 4 | 004 | -2.5649 | 0 | 2.435 | 2.685 | 2.435 |
| 5 | 005 | -1.8967 | 0 | 3.103 | 3.353 | 3.103 |
| 6 | 006 | -1.6401 | 1 | 3.360 | 3.610 | 3.610 |

```
with(dat0_N100_tau25, mean(Y_Z_1 - Y_Z_0)) # true ATE
```

```
[1] 0.25
```

```
with(dat0_N100_tau25, mean(Y[Z == 1]) - mean(Y[Z == 0])) # estimate
```

```
[1] 0.5569
```

```
lm_robust(Y ~ Z, data = dat0_N100_tau25)$coef # estimate
```

| (Intercept) | Z |
|-------------|--------|
| 4.8458 | 0.5569 |

Example: Using DeclareDesign III

```
E0 <- declare_estimator(Y ~ Z,
  model = lm_robust, label = "t test 1",
  inquiry = "ATE"
)
t_test <- function(data) {
  test <- with(data, t.test(x = Y[Z == 1], y = Y[Z == 0]))
  data.frame(statistic = test$statistic, p.value = test$p.value)
}
T0 <- declare_test(handler = label_test(t_test), label = "t test 2")
design0_plus_tests <- design0_base + E0 + T0

design0_N100_tau25_plus <- redesign(design0_plus_tests, N = 100, tau = .25)

## Only repeat the random assignment, not the creation of Y0. Ignore warning
names(design0_N100_tau25_plus)

[1] "P0"      "A0"      "00"      "R0"      "t test 1" "t test 2"
design0_N100_tau25_sims <- simulate_design(design0_N100_tau25_plus,
  sims = c(1, 100, 1, 1, 1, 1)
) # only repeat the random assignment
```

Warning: We recommend you choose a higher number of simulations than 1 for the

Example: Using DeclareDesign IV

```
# design0_N100_tau25_sims has 200 rows (2 tests * 100 random assignments)
# just look at the first 6 rows
head(design0_N100_tau25_sims)
```

| | design | N | tau | sim_ID | estimator | term | estimate | std.error | sta |
|---|-------------------------|-------------|------|--------|-----------|------|----------|-----------|-----|
| 1 | design0_N100_tau25_plus | 100 | 0.25 | 1 | t test 1 | Z | 0.1108 | 0.2150 | |
| 2 | design0_N100_tau25_plus | 100 | 0.25 | 1 | t test 2 | <NA> | NA | NA | |
| 3 | design0_N100_tau25_plus | 100 | 0.25 | 2 | t test 1 | Z | 0.2458 | 0.2154 | |
| 4 | design0_N100_tau25_plus | 100 | 0.25 | 2 | t test 2 | <NA> | NA | NA | |
| 5 | design0_N100_tau25_plus | 100 | 0.25 | 3 | t test 1 | Z | 0.5463 | 0.2133 | |
| 6 | design0_N100_tau25_plus | 100 | 0.25 | 3 | t test 2 | <NA> | NA | NA | |
| | step_1_draw | step_2_draw | | | | | | | |
| 1 | 1 | 1 | | | | | | | |
| 2 | 1 | 1 | | | | | | | |
| 3 | 1 | 2 | | | | | | | |
| 4 | 1 | 2 | | | | | | | |
| 5 | 1 | 3 | | | | | | | |
| 6 | 1 | 3 | | | | | | | |

```
# for each estimator, power = proportion of simulations with p.value < 0.5
design0_N100_tau25_sims %>%
  group_by(estimator) %>%
  summarize(pow = mean(p.value < .05), .groups = "drop")
```

Example: Using DeclareDesign V

```
# A tibble: 2 x 2
  estimator    pow
* <chr>      <dbl>
1 t test 1    0.2
2 t test 2    0.2
```

Power with covariate adjustment

Covariate adjustment and power

- ▶ Covariate adjustment can improve power because it mops up variation in the outcome variable.
 - ▶ If prognostic, covariate adjustment can reduce variance dramatically. Lower variance means higher power.
 - ▶ If non-prognostic, power gains are minimal.
- ▶ All covariates must be pre-treatment. Do not drop observations on account of missingness.
 - ▶ See the module on threats to internal validity and the 10 things to know about covariate adjustment.
- ▶ Freedman's bias as n of observations decreases and K covariates increases.

Blocking

- ▶ Blocking: randomly assign treatment within blocks
 - ▶ “Ex-ante” covariate adjustment
 - ▶ Higher precision/efficiency implies more power
 - ▶ Reduce “conditional bias”: association between treatment assignment and potential outcomes
 - ▶ Benefits of blocking over covariate adjustment clearest in small experiments

Example: Simulation-based power with a covariate I

```
## Y0 is fixed in most field experiments. So we only generate it once
make_Y0_cov <- function(N) {
  u0 <- rnorm(n = N)
  x <- rpois(n = N, lambda = 2)
  Y0 <- .5 * sd(u0) * x + u0
  return(data.frame(Y0 = Y0, x = x))
}
## X is moderately predictive of Y0.
test_dat <- make_Y0_cov(100)
test_lm <- lm_robust(Y0 ~ x, data = test_dat)
summary(test_lm)
```

Call:

```
lm_robust(formula = Y0 ~ x, data = test_dat)
```

Standard error type: HC2

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) | CI Lower | CI Upper | DF |
|-------------|----------|------------|---------|-------------|----------|----------|----|
| (Intercept) | 0.11 | 0.1880 | 0.585 | 0.559753653 | -0.263 | 0.483 | 98 |
| x | 0.44 | 0.0814 | 5.413 | 0.000000441 | 0.279 | 0.602 | 98 |

Multiple R-squared: 0.231 , Adjusted R-squared: 0.223

Example: Simulation-based power with a covariate II

F-statistic: 29.3 on 1 and 98 DF, p-value: 0.000000441

```
## now set up the simulation
repeat_experiment_and_test_cov <- function(N, tau, Y0, x) {
  Y1 <- Y0 + tau
  Z <- complete_ra(N = N)
  Yobs <- Z * Y1 + (1 - Z) * Y0
  estimator <- lm_robust(Yobs ~ Z + x, data = data.frame(Y0, Z, x))
  pval <- estimator$p.value[2]
  return(pval)
}

## create the data once, randomly assign treatment sims times
## report what proportion return p-value < 0.05
power_sim_cov <- function(N, tau, sims) {
  dat <- make_Y0_cov(N)
  pvals <- replicate(n = sims, repeat_experiment_and_test_cov(
    N = N,
    tau = tau, Y0 = dat$Y0, x = dat$x
  ))
  pow <- sum(pvals < .05) / sims
  return(pow)
}
```

Example: Simulation-based power with a covariate III

```
set.seed(12345)  
power_sim_cov(N = 80, tau = .25, sims = 100)
```

```
[1] 0.13
```

```
power_sim_cov(N = 80, tau = .25, sims = 100)
```

```
[1] 0.19
```

Power for cluster randomization

Power and clustered designs

- ▶ Recall the randomization module.
- ▶ Given a fixed N , a clustered design is weakly less powered than a non-clustered design.
 - ▶ The difference is often substantial.
- ▶ We have to estimate variance correctly:
 - ▶ Clustering standard errors (the usual)
 - ▶ Randomization inference
- ▶ To increase power:
 - ▶ Better to increase number of clusters than number of units per cluster.
 - ▶ How much clusters reduce power depends critically on the intra-cluster correlation (the ratio of variance within clusters to total variance).

A note on clustering in observational research

- ▶ Often overlooked, leading to (possibly) wildly understated uncertainty.
 - ▶ Frequentist inference based on ratio $\hat{\beta}/\hat{se}$
 - ▶ If we underestimate \hat{se} , we are much more likely to reject H_0 . (Type-I error rate is too high.)
- ▶ Many observational designs much less powered than we think they are.

Example: Simulation-based power for cluster randomization

|

```
## Y0 is fixed in most field experiments. So we only generate it once
make_Y0_clus <- function(n_indivs, n_clus) {
  # n_indivs in number of people per cluster
  # n_clus is number of clusters
  clus_id <- gl(n_clus, n_indivs)
  N <- n_clus * n_indivs
  u0 <- fabricatr::draw_normal_icc(N = N, clusters = clus_id, ICC = .1)
  Y0 <- u0
  return(data.frame(Y0 = Y0, clus_id = clus_id))
}

test_dat <- make_Y0_clus(n_indivs = 10, n_clus = 100)
# confirm that this produces data with 10 in each of 100 clusters
table(test_dat$clus_id)
```

Example: Simulation-based power for cluster randomization

II

| | | | | | | | | | | | | | | | | | | | |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 100 | | | | | | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | | | | | | | | |

```
# confirm ICC
```

```
ICC::ICCbare(y = Y0, x = clus_id, data = test_dat)
```

```
[1] 0.09655
```

Example: Simulation-based power for cluster randomization

III

```
repeat_experiment_and_test_clus <- function(N, tau, Y0, clus_id) {  
  Y1 <- Y0 + tau  
  # here we randomize Z at the cluster level  
  Z <- cluster_ra(clusters = clus_id)  
  Yobs <- Z * Y1 + (1 - Z) * Y0  
  estimator <- lm_robust(Yobs ~ Z,  
    clusters = clus_id,  
    data = data.frame(Y0, Z, clus_id), se_type = "CR2"  
  )  
  pval <- estimator$p.value[2]  
  return(pval)  
}  
  
power_sim_clus <- function(n_indivs, n_clus, tau, sims) {  
  dat <- make_Y0_clus(n_indivs, n_clus)  
  N <- n_indivs * n_clus  
  # randomize treatment sims times  
  pvals <- replicate(  
    n = sims,  
    repeat_experiment_and_test_clus(  
      N = N, tau = tau,  
      Y0 = dat$Y0, clus_id = dat$clus_id  
    )  
  )  
  sum(pvals < .05) / sims
```

Example: Simulation-based power for cluster randomization (DeclareDesign) I

```
P1 <- declare_population(  
  N = n_clus * n_indivs,  
  clusters = gl(n_clus, n_indivs),  
  u0 = draw_normal_icc(N = N, clusters = clusters, ICC = .2)  
)  
O1 <- declare_potential_outcomes(Y_Z_0 = 5 + u0, Y_Z_1 = Y_Z_0 + tau)  
A1 <- declare_assignment(Z = conduct_ra(N = N, clusters = clusters))  
estimand_ate <- declare_inquiry(ATE = mean(Y_Z_1 - Y_Z_0))  
R1 <- declare_reveal(Y, Z)  
design1_base <- P1 + A1 + O1 + R1 + estimand_ate  
  
## For example:  
design1_test <- redesign(design1_base, n_clus = 10, n_indivs = 100, tau = .25)  
test_d1 <- draw_data(design1_test)  
# confirm all individuals in a cluster have the same treatment assignment  
with(test_d1, table(Z, clusters))
```

| | clusters | | | | | | | | | |
|---|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Z | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 100 | 0 | 100 | 100 | 100 | 0 | 0 | 100 | 0 | 0 |
| 1 | 0 | 100 | 0 | 0 | 0 | 100 | 100 | 0 | 100 | 100 |

Example: Simulation-based power for cluster randomization (DeclareDesign) II

```
# three estimators, differ in se_type:
```

```
E1a <- declare_estimator(Y ~ Z,  
  model = lm_robust, clusters = clusters,  
  se_type = "CR2", label = "CR2 cluster t test",  
  inquiry = "ATE"
```

```
)
```

```
E1b <- declare_estimator(Y ~ Z,  
  model = lm_robust, clusters = clusters,  
  se_type = "CRO", label = "CRO cluster t test",  
  inquiry = "ATE"
```

```
)
```

```
E1c <- declare_estimator(Y ~ Z,  
  model = lm_robust, clusters = clusters,  
  se_type = "stata", label = "stata RCSE t test",  
  inquiry = "ATE"
```

```
)
```

```
design1_plus <- design1_base + E1a + E1b + E1c
```

```
design1_plus_tosim <- redesign(design1_plus, n_clus = 10, n_indivs = 100, tau =
```

Example: Simulation-based power for cluster randomization (DeclareDesign) III

```
## Only repeat the random assignment, not the creation of Y0. Ignore warning  
## We would want more simulations in practice.  
set.seed(12355)  
design1_sims <- simulate_design(design1_plus_tosim,  
  sims = c(1, 1000, rep(1, length(design1_plus_tosim) - 2))  
)
```

Warning: We recommend you choose a higher number of simulations than 1 for the

```
design1_sims %>%  
  group_by(estimator) %>%  
  summarize(  
    pow = mean(p.value < .05),  
    coverage = mean(estimand <= conf.high & estimand >= conf.low),  
    .groups = "drop"  
  )
```

Example: Simulation-based power for cluster randomization (DeclareDesign) IV

```
# A tibble: 3 x 3
  estimator      pow coverage
* <chr>          <dbl>    <dbl>
1 CR0 cluster t test 0.155    0.911
2 CR2 cluster t test 0.105    0.936
3 stata RCSE t test  0.131    0.918

library(DesignLibrary)
## This may be simpler than the above:
d1 <- block_cluster_two_arm_designer(
  N_blocks = 1,
  N_clusters_in_block = 10,
  N_i_in_cluster = 100,
  sd_block = 0,
  sd_cluster = .3,
  ate = .25
)
d1_plus <- d1 + E1b + E1c
d1_sims <- simulate_design(d1_plus, sims = c(1, 1, 1000, 1, 1, 1, 1, 1))
```

Example: Simulation-based power for cluster randomization (DeclareDesign) V

```
d1_sims %>%  
  group_by(estimator) %>%  
  summarize(  
    pow = mean(p.value < .05),  
    coverage = mean(estimand <= conf.high & estimand >= conf.low),  
    .groups = "drop"  
  )
```

```
# A tibble: 3 x 3  
  estimator      pow coverage  
* <chr>      <dbl>   <dbl>  
1 CR0 cluster t test 0.209   0.914  
2 estimator      0.143   0.941  
3 stata RCSE t test 0.194   0.925
```


Comparative statics

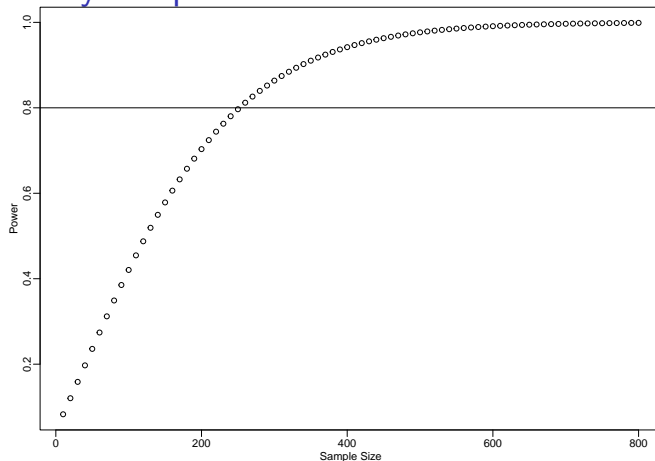
Comparative Statics

- ▶ Power is:
 - ▶ Increasing in N
 - ▶ Increasing in $|\tau|$
 - ▶ Decreasing in σ

Power by sample size I

```
some_ns <- seq(10, 800, by = 10)
pow_by_n <- sapply(some_ns, function(then) {
  pwr.t.test(n = then, d = 0.25, sig.level = 0.05)$power
})
plot(some_ns, pow_by_n,
     xlab = "Sample Size",
     ylab = "Power"
)
abline(h = .8)
```

Power by sample size II

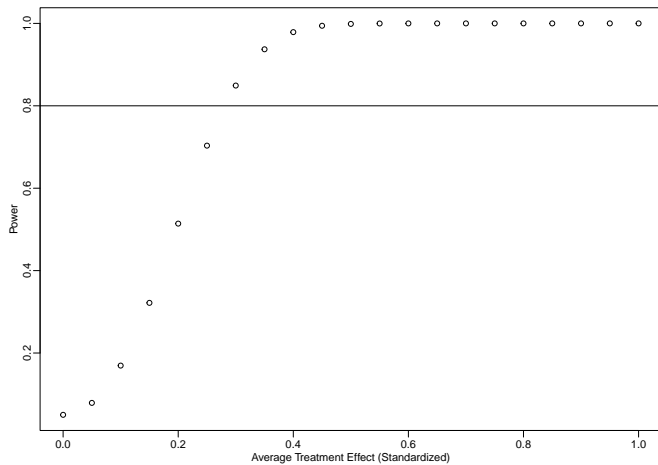


```
## See https://cran.r-project.org/web/packages/pwr/vignettes/pwr-vignette.html  
## for fancier plots  
## ptest <- pwr.t.test(n = NULL, d = 0.25, sig.level = 0.05, power = .8)  
## plot(ptest)
```

Power by treatment effect size I

```
some_taus <- seq(0, 1, by = .05)
pow_by_tau <- sapply(some_taus, function(theta) {
  pwr.t.test(n = 200, d = theta, sig.level = 0.05)$power
})
plot(some_taus, pow_by_tau,
     xlab = "Average Treatment Effect (Standardized)",
     ylab = "Power"
)
abline(h = .8)
```

Power by treatment effect size II



EGAP Power Calculator

- ▶ Try the calculator at: <https://egap.shinyapps.io/power-app/>
- ▶ For cluster randomization designs, try adjusting:
 - ▶ Number of clusters
 - ▶ Number of units per clusters
 - ▶ Intra-cluster correlation
 - ▶ Treatment effect

Comments

- ▶ Know your outcome variable.
- ▶ What effects can you realistically expect from your treatment?
- ▶ What is the plausible range of variation of the outcome variable?
 - ▶ A design with limited possible movement in the outcome variable may not be well-powered.

Conclusion: How to improve your power

1. Increase the N
 - ▶ If clustered, increase the number of clusters if at all possible
2. Strengthen the treatment
3. Improve precision
 - ▶ Covariate adjustment
 - ▶ Blocking
4. Better measurement of the outcome variable