Rigid Body - Cheat Sheet

Philip Trettner, Kersten Schuster

November 29, 2017

Everything in "world space" unless noted otherwise.

1 Static Properties

- Point Masses m_i (at points p_i)
- Total Mass $M := \sum m_i$
- Center of Gravity $c_g := \frac{1}{M} \sum m_i p_i$
- Relative positions $r_i := p_i - c_g$
- Tensor of Inertia $I_b := \sum m_i \begin{pmatrix} r_y^2 + r_z^2 & -r_x r_y & -r_x r_z \\ -r_y r_x & r_z^2 + r_x^2 & -r_y r_z \\ -r_z r_x & -r_z r_y & r_x^2 + r_y^2 \end{pmatrix}$ (in "body space")

2 Dynamic Properties (Motion State)

linear position	$x(t)$	$\in \mathbb{R}^3$	"position"
linear momentum	$P(t)$	$\in \mathbb{R}^3$	"momentum"
angular position	$R(t)$	$\in \mathbb{R}^{3\times3}$	"rotation"
angular momentum	$L(t)$	$\in \mathbb{R}^3$	

3 Derived Properties

world-space inertia	$I_w(t)$	$:= R(t) \cdot I_b \cdot R(t)^T$	$\in \mathbb{R}^{3\times3}$
	$I_w^{-1}(t)$	$:= R(t) \cdot I_b^{-1} \cdot R(t)^T$	$\in \mathbb{R}^{3\times3}$
linear velocity	$v(t)$	$:= \frac{1}{M} P(t)$	$\in \mathbb{R}^3$
angular velocity	$\omega(t)$	$:= I_w^{-1}(t) L(t)$	$\in \mathbb{R}^3$
velocity at point q	$v(t,q)$	$:= v(t) + \omega(t) \times (q - x(t))$	$\in \mathbb{R}^3$

4 Forces

linear force	$F(t)$	$\in \mathbb{R}^3$	"force"
angular force	$\tau(t)$	$\in \mathbb{R}^3$	"torque"

5 Differential Equations

linear part:

- $\frac{\partial}{\partial t} x(t) = v(t)$
- $\frac{\partial}{\partial t} P(t) = F(t)$

angular part:

- $\frac{\partial}{\partial t} R_i(t) = \omega(t) \times R_i(t)$ (R_i is the i-th column of R)
- $\frac{\partial}{\partial t} L(t) = \tau(t)$

  Note: $R(t)$ must always be orthonormal after an update step.

6 Adding Impulse

Adding an impulse j at world-space point q. (An impulse is a change of momentum.)

- linear: $P(t)$ += j
- angular: $L(t)$ += $(q - x(t)) \times j$

7 Adding Force

Adding an force $\widetilde{F}$ at world-space point q.

- linear: $F(t)$ += $\widetilde{F}$
- angular: $\tau(t)$ += $(q - x(t)) \times \widetilde{F}$