Question 2: Over-the-Ground Propagation, Path Loss, and Channel Impulse Response

Demonstration and Application to Half-Wave Dipoles

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Outline

The Over-the-Ground Propagation Model

Motivation: Beyond Free Space

- The Friis formula describes propagation in free space, but most terrestrial communications involve reflections, particularly from the ground.
- The **over-the-ground model**, also known as the two-ray model, is a fundamental physical model that accounts for the interference between the direct (LOS) path and the ground-reflected path.

Motivation: Beyond Free Space

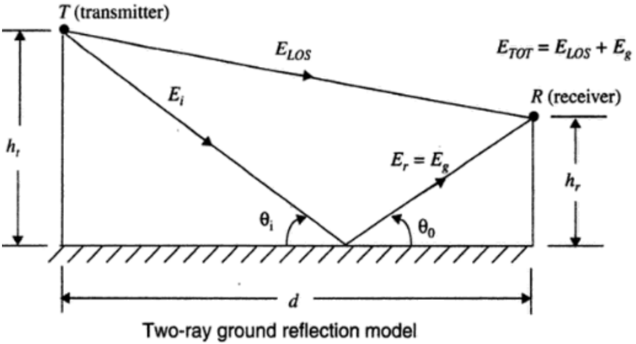


Figure: Geometry of the two-ray (over-the-ground) propagation model.

Model Setup and Assumptions

- We consider two propagation paths:
  - A direct Line-of-Sight (LOS) path of length d_1.
  - A single ground-reflected path of length d_2.
- **Assumptions**:
  - The antennas (TX and RX) are vertically polarized (z-directed) and omnidirectional in the horizontal (xy) plane.
  - The ground is modeled as a smooth, planar surface with a relative permittivity ϵ_r.
  - We use ray-tracing and geometrical optics to determine the paths.

Formulating the Received Power

- The total received electric field $\underline{\vec{E}}_{tot}$ is the coherent sum of the fields from the two paths:

$$\underline{\vec{E}}_{tot} = \underline{\vec{E}}_1 + \underline{\vec{E}}_2$$

- The LOS field $\underline{\vec{E}}_1$ and the reflected field $\underline{\vec{E}}_2$ are given by:

$$\underline{\vec{E}}_1 = E_0 \frac{e^{-j\beta d_1}}{d_1} \vec{1}_z \quad , \quad \underline{\vec{E}}_2 = \Gamma_\perp(\theta_i) E_0 \frac{e^{-j\beta d_2}}{d_2} \vec{1}_z$$

where $\Gamma_\perp(\theta_i)$ is the reflection coefficient for perpendicular (vertical) polarization.

Formulating the Received Power

Reflection Coefficient

The coefficient depends on the angle of incidence θ_i and the ground's permittivity ϵ_r:

$$\Gamma_{\perp}(\theta_i) = \frac{\cos\theta_i - \sqrt{\epsilon_r - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\epsilon_r - \sin^2\theta_i}}$$

Deriving Total Received Power

- The open-circuit voltage V_{oc} at the receiver is the sum of contributions from both paths:

$$\underline{V}_{oc} \propto h_{e\perp}^{TX} h_{e\perp}^{RX} \left(\frac{e^{-j\beta d_1}}{d_1} + \Gamma_\perp \frac{e^{-j\beta d_2}}{d_2} \right)$$

Deriving Total Received Power

- Using the relationship between received power, voltage, and antenna parameters, we can write the total received power P_{RX} as:

$$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi} \right)^2 \left| \frac{e^{-j\beta d_1}}{d_1} + \Gamma_{\perp} \frac{e^{-j\beta d_2}}{d_2} \right|^2$$

Result

This relation is the complete two-ray model. It captures the constructive and destructive interference between the two paths, leading to oscillations in received power as a function of distance.

Large Distance Approximation & Path Loss

Simplifying for Large Distances

- In many macro-cell scenarios, the horizontal distance d is much larger than the antenna heights ($d \gg h_{TX}, h_{RX}$).
- This allows for several simplifications:
  - **Grazing Incidence**: The reflection angle $\theta_i \to \pi/2$, which makes the reflection coefficient $\Gamma_\perp \approx -1$.
  - **Amplitudes**: For the amplitude terms, the path lengths are nearly equal: $d_1 \approx d_2 \approx d$.
  - **Phases**: For the phase terms, the small difference between d_1 and d_2 is critical and must be retained.

Derivation of the d^{-4} Law

- Applying the approximations $\Gamma_{\perp} \approx -1$ and $d_1 \approx d_2 \approx d$ to the amplitude part of the P_{RX} relation gives:

$$P_{RX} \approx P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d} \right)^2 \left| e^{-j\beta d_1} - e^{-j\beta d_2} \right|^2$$

- The magnitude term can be rewritten using Euler's relation:

$$|e^{-j\beta d_1} - e^{-j\beta d_2}|^2 = |e^{-j\beta(d_1+d_2)/2}(e^{j\beta(d_2-d_1)/2} - e^{-j\beta(d_2-d_1)/2})|^2$$

$$= |2j \sin\left(\frac{\beta(d_2 - d_1)}{2} \right)|^2 = 4\sin^2\left(\frac{\beta(d_2 - d_1)}{2} \right)$$

Derivation of the d^{-4} Law

- This yields:

$$P_{RX} \approx P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d} \right)^2 \cdot 4 \sin^2 \left(\frac{\beta(d_2 - d_1)}{2} \right)$$

- The path length difference $\Delta d = d_2 - d_1$ can be approximated using Taylor expansion:

$$d_1 = \sqrt{d^2 + (h_{TX} - h_{RX})^2} \approx d \left(1 + \frac{(h_{TX} - h_{RX})^2}{2d^2} \right)$$

$$d_2 = \sqrt{d^2 + (h_{TX} + h_{RX})^2} \approx d \left(1 + \frac{(h_{TX} + h_{RX})^2}{2d^2} \right)$$

Derivation of the d^{-4} Law

$$\implies \Delta d = d_2 - d_1 \approx \frac{2h_{TX}h_{RX}}{d}$$

- Since Δd is small, we use the small-angle approximation $\sin(x) \approx x$:

$$\sin^2\left(\frac{\beta(d_2 - d_1)}{2}\right) \approx \left(\frac{\beta\Delta d}{2}\right)^2 = \left(\frac{2\pi}{\lambda}\frac{h_{TX}h_{RX}}{d}\right)^2$$

- Substituting this back into the P_{RX} expression gives the result:

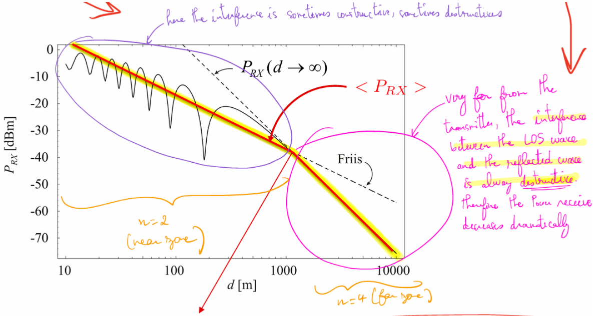
$$P_{RX}(d) \approx P_{TX}G_{TX}G_{RX}\frac{h_{TX}^2h_{RX}^2}{d^4}$$

The Canonical Path Loss Model

- The large distance approximation shows that $P_{RX} \propto d^{-4}$.
- This fits the general **canonical path loss model**:

$$\ll P_{RX}(d) \gg [\text{dBm}] = \ll P_{RX}(d_0) \gg [\text{dBm}] - 10n \log_{10}\left(\frac{d}{d_0}\right)$$

- For the over-the-ground model at large distances, the **path loss exponent** is $n = 4$.



Breakpoint

$$d_{\mathrm{BP}} = 4\pi \frac{h_{TX}\, h_{RX}}{\lambda}$$

n=2 before d_{BP}, and n=4 after d_{BP}

Multi-slope path loss model

Figure: Received power vs. distance, showing the Friis model ($n = 2$), the full two-ray model, and the large distance approximation ($n = 4$).

Impulse Response for Dipole Antennas

Physical Impulse Response

- The physical channel is described by its time-varying impulse response, $h(\tau, t)$. For a static scenario, it is $h(\tau)$.
- It is a sum of delayed Dirac delta functions, each representing a multipath component (MPC):

$$h(\tau) = \sum_{n=1}^{N} \alpha_n \delta(\tau - \tau_n)$$

- For the over-the-ground model, we have $N = 2$ MPCs.
- The complex amplitude $\alpha_n = a_n e^{j\phi_n} e^{-j2\pi f_c \tau_n}$ represents the gain and phase shift of each path at the carrier frequency f_c.

Wideband Impulse Response

- A wideband system has enough time resolution to distinguish the two paths.
- The impulse response consists of two distinct Dirac deltas:

$$h_{WB}(\tau) = \alpha_1 \delta(\tau - \tau_1) + \alpha_2 \delta(\tau - \tau_2)$$

Wideband Impulse Response

- The parameters are:
  - **Delays**: $\tau_1 = d_1/c$ and $\tau_2 = d_2/c$.
  - **Amplitudes**: The α_n terms are derived from the terms in the received voltage expression. Let K be a constant representing transmit power and antenna properties. Then:

$$\alpha_1 = K\frac{e^{-j2\pi f_c \tau_1}}{d_1}$$

$$\alpha_2 = K \cdot \Gamma_\perp(\theta_i)\frac{e^{-j2\pi f_c \tau_2}}{d_2}$$

where $\Gamma_\perp$ is the reflection coefficient for z-directed (vertical) dipoles.

Narrowband Impulse Response

- A system is narrowband if its time resolution $\Delta\tau \approx 1/B$ is much larger than the channel's delay spread $\sigma_\tau = \tau_2 - \tau_1$.
- In this case, the receiver cannot distinguish between the LOS and reflected paths.
- The two MPCs merge into a single, effective tap at $\tau = 0$.

Narrowband Impulse Response

- The narrowband channel coefficient, h_{NB}, is the coherent sum of the complex amplitudes of all MPCs:

$$h_{NB} = \sum_{n=1}^{2} \alpha_n = \alpha_1 + \alpha_2$$

$$h_{NB} = K \left(\frac{e^{-j2\pi f_c \tau_1}}{d_1} + \Gamma_{\perp}(\theta_i) \frac{e^{-j2\pi f_c \tau_2}}{d_2} \right)$$

- The impulse response is then simply:

$$h_{NB}(\tau) = h_{NB} \cdot \delta(\tau)$$

Conclusion

Summary

Over-the-Ground Model

- Provides a more realistic propagation model than free-space by including a ground-reflected path.
- Correctly predicts interference phenomena (fading nulls) as a function of distance.

Summary

Path Loss

- At large distances, the two-ray model simplifies to a power decay of $P_{RX} \propto d^{-4}$.
- This corresponds to a canonical path loss model with an exponent of $n = 4$.

Summary

Channel Impulse Response

- In a wideband view, the channel is represented by two distinct paths with different delays and amplitudes.
- In a narrowband view, these paths are unresolvable and combine into a single complex coefficient, leading to flat fading.

Conclusion

From Physics to Models

The demonstration of the over-the-ground model shows how a simple physical scenario (a direct and a reflected ray) leads to powerful and widely applicable concepts in wireless communications.

It explains the origin of a non-trivial path loss exponent ($n = 4$) and provides a clear physical basis for the distinction between wideband (frequency-selective) and narrowband (flat-fading) channel models.

Understanding this foundation is crucial for designing and predicting the performance of real-world wireless systems.

Thank You