Question 5: Long-Range Communications

Earth Curvature, LEO Satellite Links, and Tropospheric Refraction

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Outline

Maximal Communication Range due to Earth Curvature

Earth Curvature Effects

- For long-range terrestrial or Earth-to-satellite links, the curvature of the Earth becomes a limiting factor for Line-of-Sight (LOS) propagation.
- The maximum communication range is achieved when the direct path between the transmitter (TX) and receiver (RX) is exactly tangent to the Earth's surface.
- We can model this geometrically to find the maximum possible distance, d, between two antennas of heights h_A and h_B.

Earth Curvature Effects

pictures/earth_curvature_range.png

Demonstration: Maximum LOS Distance (1/2)

- We consider the right-angled triangle formed by the Earth's center, the location of antenna A, and the point of tangency on the Earth's surface.
- The hypotenuse has length $(R_e + h_A)$, where R_e is the Earth's radius (6375 km). The other two sides are R_e and the distance to the horizon, d_A.
- By the Pythagorean theorem:

$$(R_e + h_A)^2 = R_e^2 + d_A^2$$

- Expanding the left side gives:

$$R_e^2 + 2R_e h_A + h_A^2 = R_e^2 + d_A^2$$

Demonstration: Maximum LOS Distance (2/2)

- For typical antenna heights, $h_A \ll R_e$, so the h_A^2 term is negligible compared to the other terms. We can make the approximation:

$$2R_e h_A \approx d_A^2$$

- Solving for the distance to the horizon from antenna A:

$$d_A \approx \sqrt{2R_e h_A}$$

- Similarly, for antenna B, the distance to the horizon is $d_B \approx \sqrt{2R_e h_B}$.
- The total maximum LOS communication range is the sum of these two distances:

$$d = d_A + d_B \approx \sqrt{2R_e}(\sqrt{h_A} + \sqrt{h_B})$$

Application to LEO Satellite Link

LEO Satellite at Horizon: Communication Range

- We apply this model to a communication link between a ground station and a Low Earth Orbit (LEO) satellite.
- **Hypotheses**:
  - The ground station antenna is at sea level, so $h_A = 0$ km.
  - The LEO satellite is at an altitude $h_B = 450$ km.
  - The satellite is at the horizon, meaning the communication path is tangent to the Earth's surface.
- Since $h_A = 0$, the formula for the maximum range simplifies to:

$$d \approx \sqrt{2R_e h_B}$$

- Substituting the values $R_e = 6375$ km and $h_B = 450$ km:

$$d \approx \sqrt{2 \cdot 6375 \cdot 450} = \sqrt{5737500} \approx 2395.3\,\text{km}$$

LEO Satellite at Horizon: Free-Space Loss (1/3)

- The free-space path loss (FSPL) for this link can be calculated using the Friis formula, expressed in decibels:

$$L_{FS}[\text{dB}] = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) = 20\log_{10}\left(\frac{4\pi df}{c}\right)$$

- We can separate the terms that depend on distance and frequency:

$$L_{FS}[\text{dB}] = 20\log_{10}(d) + 20\log_{10}(f) + 20\log_{10}\left(\frac{4\pi}{c}\right)$$

LEO Satellite at Horizon: Free-Space Loss (2/3)

- Using the calculated distance $d = 2.3953 \times 10^6$ m and the speed of light $c = 3 \times 10^8$ m/s, we evaluate the constant terms:

$$20\log_{10}(d) = 20\log_{10}(2.3953 \times 10^6) \approx 127.6\,\text{dB}$$

$$20\log_{10}\left(\frac{4\pi}{c}\right) \approx 20\log_{10}(4.1888 \times 10^{-8}) \approx -147.6\,\text{dB}$$

- Combining these constant terms with the frequency-dependent term:

$$L_{FS}[\text{dB}] \approx 127.6 + 20\log_{10}(f) - 147.6$$

$$L_{FS}[\text{dB}] \approx 20\log_{10}(f) - 20$$

LEO Satellite at Horizon: Free-Space Loss (3/3)

- To make the formula practical, let's express the frequency f in GHz. Let $f_{GHz} = f/10^9$. Then $f = f_{GHz} \cdot 10^9$.

$$L_{FS}[\text{dB}] = 20\log_{10}(f_{GHz} \cdot 10^9) - 20$$

$$L_{FS}[\text{dB}] = 20\log_{10}(f_{GHz}) + 20\log_{10}(10^9) - 20$$

$$L_{FS}[\text{dB}] = 20\log_{10}(f_{GHz}) + 180 - 20$$

Final Result

The free-space loss for the LEO satellite at the horizon, as a function of frequency in GHz, is:

$$L_{FS}[\text{dB}] = 20\log_{10}(f_{GHz}) + 160$$

Tropospheric Refraction and Its Impact

The Concept of Tropospheric Refraction

- The previous calculation assumed that radio waves travel in straight lines (rectilinear propagation).
- However, for microwave links, propagation occurs in the troposphere, the lowest layer of the atmosphere.
- The refractive index of the troposphere, n, is not constant. It decreases slowly with altitude h.

$$n(h) = 1 + 10^{-6} N_t \quad \text{where} \quad N_t \approx 315 \exp(-h/H)$$

- As a wave passes through layers of air with different refractive indices, it is bent or **refracted**.

The Concept of Tropospheric Refraction

pictures/snells_law_atmosphere.png

Demonstration: Ray Bending (1/2)

- According to Snell's Law, as a ray passes from a denser medium (n_1) to a less dense medium ($n_2 < n_1$), it bends away from the normal.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- In the atmosphere, this means the ray is continuously bent towards the Earth, which has a higher refractive index.
- The propagation path is no longer a straight line but a curve. We can find the local radius of curvature of this path, R_r.
- From the differential form of Snell's law ($n \cos \varphi =$ constant, where φ is the angle with the horizontal), we can derive:

$$\frac{1}{R_r} = -\frac{\cos \varphi}{n} \frac{dn}{dh}$$

Demonstration: Ray Bending (2/2)

- For most terrestrial links, the propagation is near-horizontal, so the elevation angle $\varphi \approx 0$ and $\cos\varphi \approx 1$. Also, $n \approx 1$.
- The radius of curvature of the ray path simplifies to:

$$R_r \approx -\frac{1}{dn/dh}$$

- Since the refractive index n decreases approximately linearly with height for the first few kilometers, dn/dh is a negative constant.
- This means R_r is a positive constant. The radio wave follows a circular arc, bending towards the Earth.

The Effective Earth Radius Concept (1/2)

- Dealing with curved ray paths is mathematically complex.
- A clever engineering trick is to model the curved ray path over the real Earth as an equivalent straight-line path over a fictitious Earth with a larger radius. This is the **effective Earth radius**, R_{eff}.
- The height of a ray with elevation angle E at a distance x is the difference between the ray's vertical position y_r and the Earth's surface position y_e.

The Effective Earth Radius Concept (1/2)

- For a curved ray over a curved Earth, the height is:

$$h_r(x) = y_r - y_e = \left(Ex - \frac{x^2}{2R_r}\right) - \left(-\frac{x^2}{2R_e}\right)$$

$$h_r(x) = Ex + \frac{x^2}{2}\left(\frac{1}{R_e} - \frac{1}{R_r}\right)$$

The Effective Earth Radius Concept (2/2)

- This equation has the same form as the height of a straight ray ($R_r \to \infty$) above a modified Earth with radius R_{eff}:

$$h_r(x) = Ex + \frac{x^2}{2R_{eff}}$$

- By comparing the two expressions, we define the effective Earth radius:

$$\frac{1}{R_{eff}} = \frac{1}{R_e} - \frac{1}{R_r} = \frac{1}{R_e} + \frac{dn}{dh}$$

- We introduce the effective Earth radius factor, k_e:

$$R_{eff} = \frac{R_e}{1 + R_e \frac{dn}{dh}} = k_e R_e$$

- Under standard atmospheric conditions, $k_e \approx 4/3 \approx 1.33$.

Impact on Maximal Communication Range

Conclusion

Tropospheric refraction effectively "flattens" the Earth from the perspective of the radio wave, allowing it to travel beyond the geometric horizon.

- To calculate the new, extended maximum communication range, we simply replace the real Earth radius R_e with the effective Earth radius R_{eff} in our original formula.

Impact on Maximal Communication Range

- The maximum range becomes:

$$d_{refracted} \approx \sqrt{2R_{eff}}(\sqrt{h_A} + \sqrt{h_B})$$

$$d_{refracted} \approx \sqrt{2k_e R_e}(\sqrt{h_A} + \sqrt{h_B})$$

- This is an increase of a factor of $\sqrt{k_e} \approx \sqrt{4/3} \approx 1.15$, or about a 15% increase in range compared to the purely geometric calculation.

Thank You