Question 1: TDL Wideband and Narrowband Channel Models

Demonstration and Statistical Models for Taps in Wideband Case

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Outline

Introduction

Wireless Communication Channel

- A communication channel is the medium through which the electromagnetic signal propagates.
- It is characterized by three main phenomena:
  - **Attenuation**: Reduction in signal strength over distance.
  - **Time Dispersion**: Signal spreading over time due to multipath propagation.
  - **Fading**: Fluctuations in signal amplitude and phase.

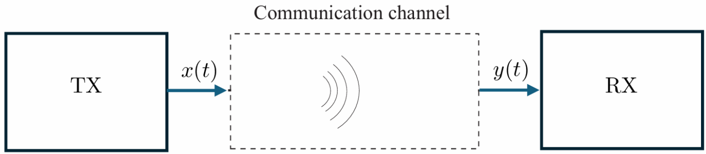


Figure: Basic communication system diagram

Physical Impulse Response of the Channel

- The physical channel is described by its time-varying impulse response, $h(\tau, t)$.
- It is a sum of delayed Dirac delta functions, each representing a multipath component (MPC):

$$h(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)\delta(\tau - \tau_n)$$

Physical Impulse Response of the Channel

- The term $\alpha_n(t)$ is the complex amplitude of the n^{th} MPC:

$$\alpha_n(t) = a_n(t) e^{j\phi_n(t)} e^{-j2\pi f_c \tau_n}$$

  - $a_n(t)$: Real amplitude.
  - $\phi_n(t)$: Phase shift.
  - τ_n: Propagation delay.

Received Signal Formulation

- The received baseband signal $y(t)$ is the convolution of the transmitted signal $x(t)$ with the channel's impulse response:

$$y(t) = \int_0^\infty h(\tau, t)x(t-\tau)\, d\tau$$

- Substituting the physical impulse response gives:

$$y(t) = \sum_{n=1}^{N} \alpha_n(t)x(t-\tau_n)$$

Received Signal Formulation

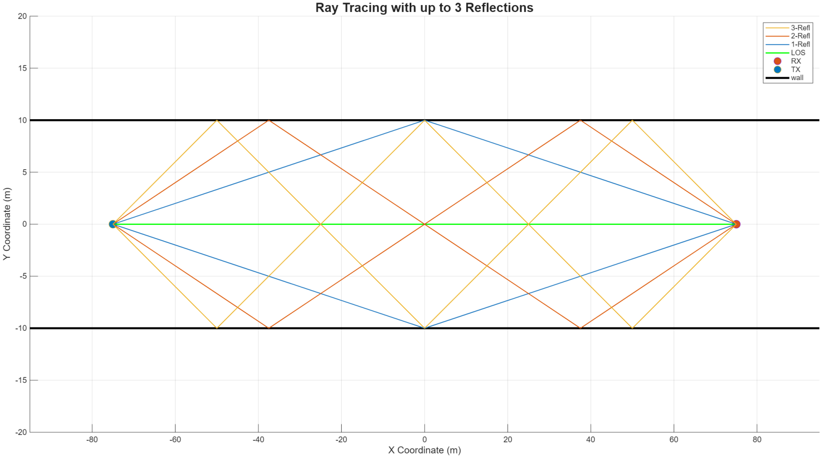


Figure: Diagram of multipath components arriving at the receiver.

Tapped Delay Line Models

The Problem of Limited Bandwidth

Physical devices have a **limited bandwidth** B.

This implies a **limited time resolution** of $\Delta\tau \approx 1/B$.

Unresolvable MPCs are grouped into discrete **taps**.

This leads to the **Tapped Delay Line (TDL) Model** approximation.

Narrowband TDL Model

Condition for Narrowband

- Define the delay spread: $\sigma_\tau = \max_{i,j} |\tau_i - \tau_j|$
- And the coherence bandwidth: $\Delta f_c \approx \frac{1}{\sigma_\tau}$

A channel is **narrowband** if the signal bandwidth $B \ll \Delta f_c$, which implies:

$$\Delta\tau \gg \sigma_\tau$$

Consequence

All multipath components arrive within the same time resolution interval, $\Delta\tau$. They are therefore indistinguishable and combine into a single tap.

Condition for Narrowband

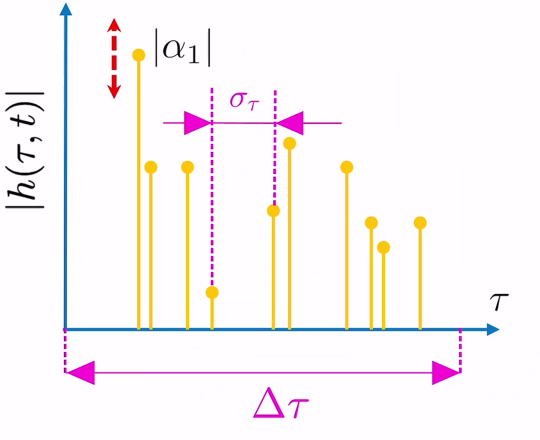


Figure: Diagram showing a large $\Delta\tau$ encompassing all MPCs.

Derivation of the Narrowband Model

- Since all MPCs are unresolvable, the TDL model simplifies to a single tap at $\tau = 0$:

$$h_{TDL}(\tau, t) = h_0(t)\delta(\tau)$$

- The coefficient $h_0(t)$, or simply $h(t)$, is the coherent sum of all multipath components:

$$h(t) \approx \sum_{n=1}^{N} \alpha_n(t) = \sum_{n=1}^{N} a_n(t) e^{j\phi_n(t)} e^{-j2\pi f_c \tau_n}$$

Derivation of the Narrowband Model

- The received signal becomes a simple product:

$$y(t) = \int_0^\infty h(t)\delta(\tau)x(t-\tau)\,d\tau = h(t)x(t)$$

- The channel transfer function $H(f,t)$ is independent of the baseband frequency f:

$$H(f,t) = \int_{-\infty}^\infty h(t)\delta(\tau)e^{-j2\pi f\tau}\,d\tau = h(t)$$

  This is known as **flat fading**.

Wideband TDL Model

Condition for Wideband

A channel is **wideband** when the signal bandwidth B is comparable to or larger than the coherence bandwidth Δf_c, which implies the time resolution $\Delta\tau$ is smaller than the delay spread σ_τ.

$$\Delta\tau < \sigma_\tau$$

Consequence

The system's time resolution is fine enough to resolve groups of multipath components. This leads to a model with multiple taps, causing frequency-selective fading.

Condition for Wideband

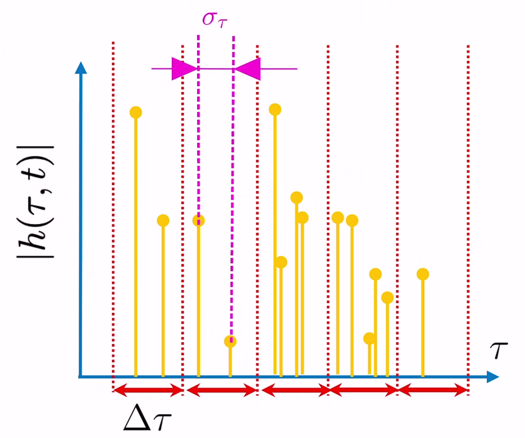


Figure: Diagram showing a small $\Delta\tau$ resolving different MPCs.

Derivation of the Wideband Model (1/2)

- The bandlimited signal $x(t)$ allows us to use the sampling theorem. The delayed signal $x(t-\tau)$ can be written as:

$$x(t-\tau) = \sum_{l=-\infty}^{\infty} x(t - l\Delta\tau)\text{sinc}(B(\tau - l\Delta\tau))$$

- We substitute this into the convolution integral for $y(t)$:

$$y(t) = \int_0^{\infty} h(\tau, t) \sum_l x(t - l\Delta\tau)\text{sinc}(B(\tau - l\Delta\tau))d\tau$$

Derivation of the Wideband Model (2/2)

- Swapping the integral and summation:

$$y(t) = \sum_{l} x(t - l\Delta\tau) \underbrace{\int_0^{\infty} h(\tau, t)\text{sinc}(B(\tau - l\Delta\tau))d\tau}_{\equiv h_l(t)}$$

- This gives the discrete-time convolution model:

$$y(t) = \sum_{l=0}^{L} h_l(t)x(t - l\Delta\tau)$$

The Taps of the Wideband Model

- The resulting impulse response is the Tapped Delay Line (TDL) model:

$$h_{TDL}(\tau, t) = \sum_{l=0}^{L} h_l(t)\delta(\tau - l\Delta\tau)$$

- Substituting the physical model for $h(\tau, t)$, the exact tap gain is:

$$h_l(t) = \sum_{n=1}^{N} \alpha_n(t)\mathrm{sinc}(B(\tau_n - l\Delta\tau))$$

- This can be approximated as the sum of all MPCs whose delays τ_n fall within the l-th time bin:

$$h_l(t) \approx \sum_{\tau_n \in \text{tap l}} \alpha_n(t)$$

Visualizing the Taps

Consequence

Each tap represents the interference of all MPCs arriving around the delay $l\Delta\tau$. The physical, continuous impulse response is sampled into a discrete TDL model.

Visualizing the Taps

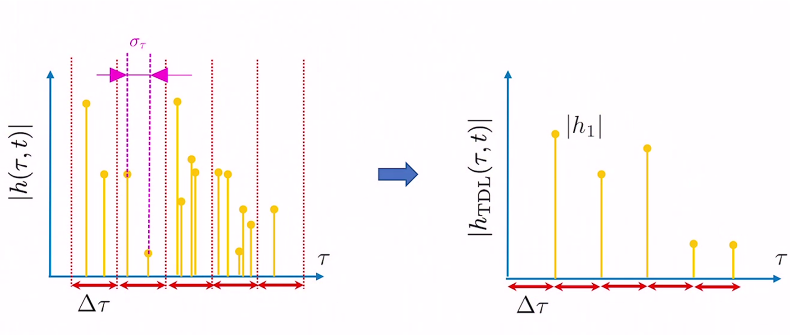


Figure: Grouping of physical MPCs into discrete taps.

Wideband Scenario - Statistical Model

Motivation for a Statistical Approach

- A deterministic model (like ray-tracing) is often impractical. The exact properties of each MPC are highly sensitive to small changes in the environment.
- A statistical model aims to generate a channel with the same statistical properties (e.g., average power, fading distribution) as a real channel.

Motivation for a Statistical Approach

- We model the complex tap coefficients $h_l(t)$ as random variables.
- The phases Φ_n of the constituent MPCs are assumed to be independent and uniformly distributed in $[0, 2\pi)$.

$$h_l(t) = \sum_{\tau_n \in \text{tap } l} a_n e^{j\Phi_n}$$

Rayleigh Fading Taps (NLOS)

- **Hypothesis**: A tap consists of a large number of MPCs, with no single component being significantly stronger than the others (Non-Line-of-Sight condition).
- By the **Central Limit Theorem**, $h_l(t)$ converges to a complex Gaussian random variable with zero mean.

$$h_l(t) = X_l + jY_l \quad \text{where } X_l, Y_l \sim \mathcal{N}(0, \sigma_l^2)$$

- The envelope, $|h_l(t)| = \sqrt{X_l^2 + Y_l^2}$, follows a **Rayleigh distribution**.

The Rayleigh Distribution

- Probability Density Function (PDF):

$$p(|h_l|) = \frac{|h_l|}{\sigma_l^2} \exp\left(-\frac{|h_l|^2}{2\sigma_l^2}\right)$$

- Cumulative Distribution Function (CDF):

$$F(|h_l|) = \int_0^{|h_l|} p(x)dx = 1 - \exp\left(-\frac{|h_l|^2}{2\sigma_l^2}\right)$$

- Expected values:

$$\mathbb{E}[|h_l|] = \sigma_l\sqrt{\frac{\pi}{2}}, \quad \mathbb{E}[|h_l|^2] = 2\sigma_l^2$$

The Rayleigh Distribution

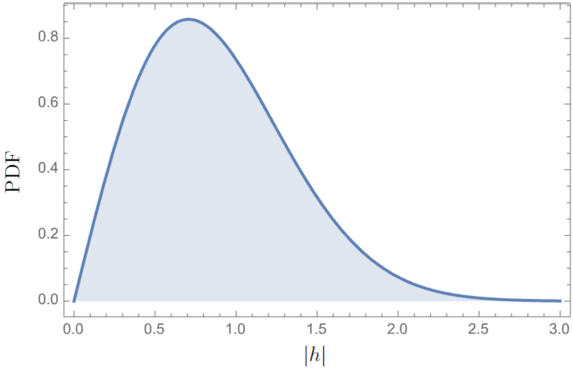


Figure: Plot of the Rayleigh PDF.

Rician Fading Taps (LOS)

- **Hypothesis**: A tap contains a dominant, stable component (typically a Line-of-Sight path), plus many weaker, scattered components.

$$h_l(t) = \underbrace{A_l e^{j\theta_l}}_{\text{Dominant}} + \underbrace{\sum a_n e^{j\Phi_n}}_{\text{Scattered}}$$

- The tap coefficient $h_l(t)$ is a complex Gaussian variable with a **non-zero mean**.
- The envelope $|h_l(t)|$ follows a **Rician distribution**.

The Rician Distribution

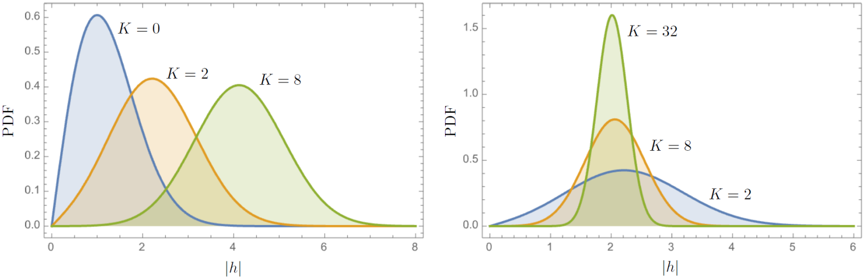
- The PDF is given by:

$$p(|h_l|) = \frac{|h_l|}{\sigma_l^2} \exp\left(-\frac{|h_l|^2 + A_l^2}{2\sigma_l^2}\right) I_0\left(\frac{|h_l|A_l}{\sigma_l^2}\right)$$

- It is characterized by the **Rician K-factor**, the ratio of dominant to scattered power:

$$K_l = \frac{A_l^2}{2\sigma_l^2}$$

The Rician Distribution



σ^2 constant $\qquad$ a_0^2 constant

Figure: Plot of Rician PDFs for different K-factors.

Example: Standardized PDP

ITU Vehicular A model

A standard model for vehicle communications in an urban environment.

Tap	Delay [ns]	Avg. Power [dB]
1	0	0
2	310	-1.0
3	710	-9.0
4	1090	-10.0
5	1730	-15.0
6	2510	-20.0

All taps in this specific model are defined as having Rayleigh fading.

Conclusion

Summary of Models

Narrowband Model

- Condition: $\Delta\tau \gg \sigma_\tau$
- Model: $y(t) = h(t)x(t)$
- Physical Effect: Flat fading

Wideband Model

- Condition: $\Delta\tau < \sigma_\tau$
- Model: $y(t) = \sum h_l(t)x(t - l\Delta\tau)$
- Physical Effect: Frequency-selective fading

Final Conclusion

Statistical Modeling of Taps

The complex nature of multipath propagation makes a statistical approach essential for wideband channels. By modeling taps as Rayleigh (NLOS) or Rician (LOS) random processes, parameterized by a standard Power Delay Profile, we can accurately simulate the behavior of real-world frequency-selective channels for system design and performance analysis.