Question 3: Shadowing, Cell Coverage, and Link Budgets

Statistical Model and Practical Implications

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Outline

1. The Shadowing Statistical Model
2. Impact of Shadowing on Cell Coverage
3. Shadowing in Link Budgets
4. Conclusion

The Shadowing Statistical Model

Components of Received Power Variation

- The received power P_{RX} is not constant but varies due to several effects.
- It is modeled as the combination of three main components:
  1. **Path Loss**: The average power decay with distance, $\ll P_{RX} \gg$.
  2. **Shadowing (or Slow Fading)**: Large-scale variations around the path loss mean, caused by obstacles like buildings and hills, $< P_{RX} >$.
  3. **Small-Scale Fading (or Fast Fading)**: Rapid fluctuations due to multipath interference, P_{RX}.

Components of Received Power Variation

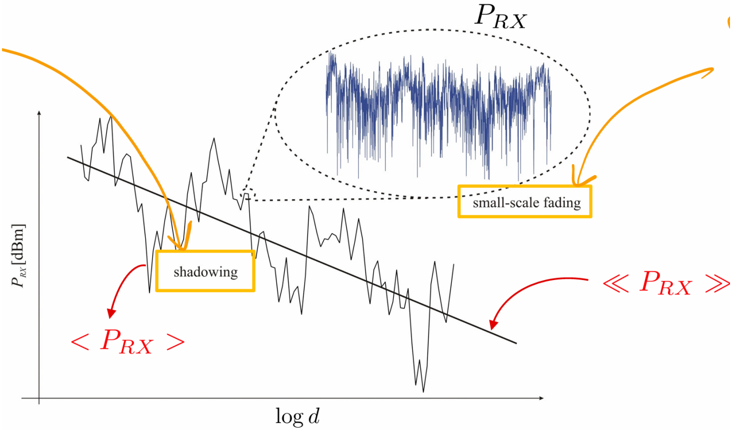

Figure: Illustration of path loss, shadowing, and small-scale fading.

Defining the Statistical Model

- Shadowing describes the random variations of the locally averaged received power, $< P_{RX} >$, around the mean power, $\ll P_{RX} \gg$, predicted by a path loss model.
- Experimental data shows that when the received power is expressed in decibels (dB or dBm), the variations due to shadowing follow a **Normal (Gaussian) distribution**.
- This means the power in linear units (Watts) follows a **log-normal distribution**.

Mathematical Formulation

- The locally averaged received power in dBm is modeled as:

$$< P_{RX} > (d)[\text{dBm}] = \ll P_{RX} \gg [\text{dBm}] - L_{\sigma_L}$$

- Where:
  - $\ll P_{RX} \gg$ [dBm] is the mean power at distance d from the path loss model.
  - L_{σ_L} is a zero-mean Gaussian random variable with standard deviation σ_L.
- The parameter σ_L, known as the **shadowing variability** or standard deviation, is determined empirically and typically ranges from 4 dB to 10 dB depending on the environment.

Mathematical Formulation

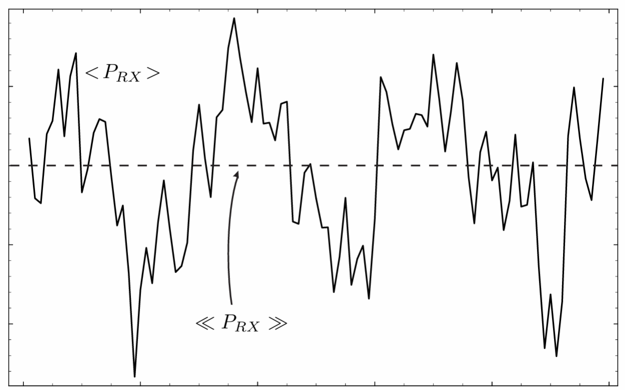

Figure: Example of shadowing variations around the mean predicted by path loss.

Impact of Shadowing on Cell Coverage

The Challenge of Defining Cell Radius

- A cell's boundary is defined by the distance at which the received power drops to the receiver's minimum required level, known as its **sensitivity**.
- If we ignore shadowing and define the cell radius R as the distance where the *mean* received power equals the sensitivity, we encounter a problem.

$$\ll P_{RX}(R) \gg = \text{sensitivity}$$

The Challenge of Defining Cell Radius

- Since shadowing is a zero-mean Gaussian process, this definition implies that at the cell edge, the actual received power $< P_{RX} >$ will be below the sensitivity threshold for 50% of the locations.

Conclusion

A 50% service reliability at the cell edge is unacceptable for most communication systems.

The Challenge of Defining Cell Radius

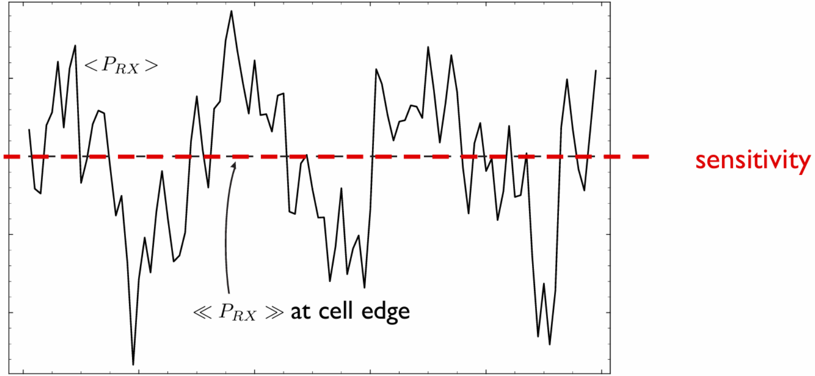

Figure: At the cell edge (R), 50% of locations fall below the sensitivity threshold.

The Challenge of Defining Cell Radius

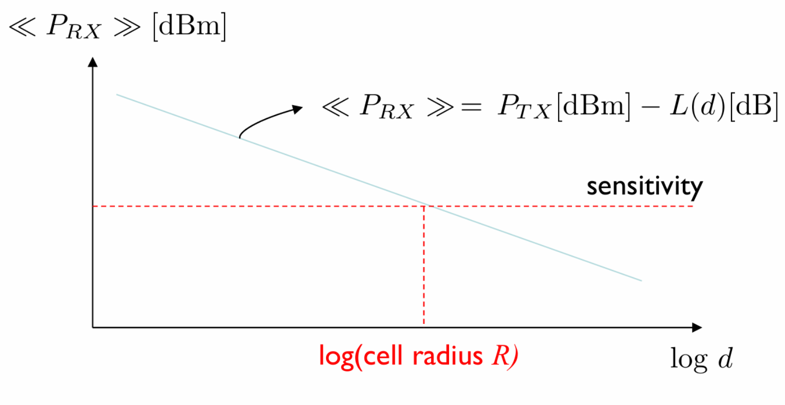

Figure: At the cell edge (R), 50% of locations fall below the sensitivity threshold.

The Solution: The Fade Margin

- To ensure a higher reliability, we must design the system so that the mean received power at the cell edge is *greater* than the sensitivity.
- This buffer is called the **Fade Margin**, M.
- The cell radius R is now defined by the condition:

$$\ll P_{RX}(R) \gg [\text{dBm}] = \text{sensitivity} + M$$

- This is equivalent to reducing the maximum allowed path loss:

$$L(R) = L_{max} - M$$

The Solution: The Fade Margin

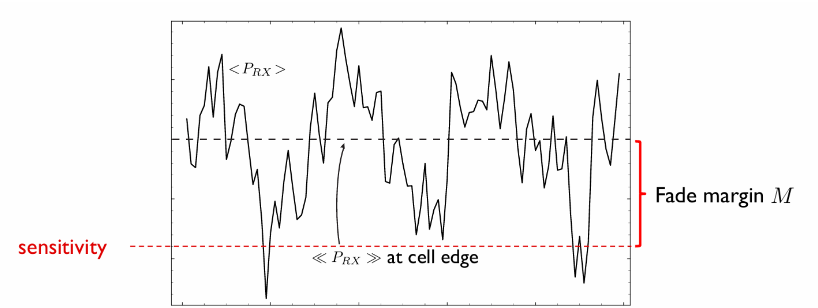

Figure: Introducing a fade margin M increases the mean power at the new, smaller cell edge, improving reliability.

The Solution: The Fade Margin

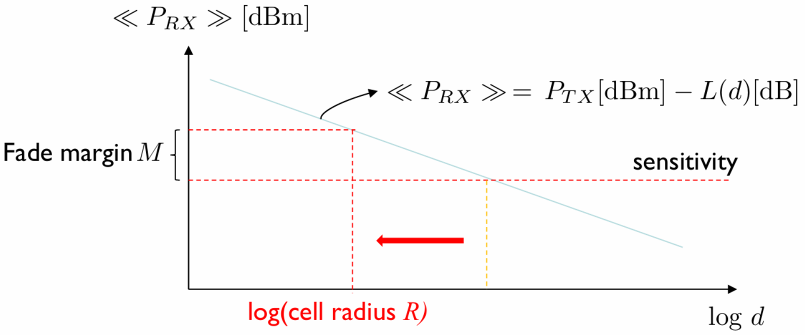

Figure: Introducing a fade margin M increases the mean power at the new, smaller cell edge, improving reliability.

Demonstration: Overall Cell Coverage (1/3)

- We want to find the overall service probability, F_u, within a cell of radius R. This is the probability that the received power is above the sensitivity.
- The probability of an outage (connection failure) at a distance r is the probability that the total path loss exceeds the maximum allowed loss, L_{max}:

$$P_{\text{outage}}(r) = \Pr[L(r) + L_{\sigma_L} > L_{max}] = \Pr[L_{\sigma_L} > L_{max} - L(r)]$$

- Let $l(r) = L_{max} - L(r)$. The outage probability is then:

$$P_{\text{outage}}(r) = \Pr[L_{\sigma_L} > l(r)] = \frac{1}{2}\text{erfc}\left(\frac{l(r)}{\sigma_L\sqrt{2}}\right)$$

Demonstration: Overall Cell Coverage (2/3)

- To find the average outage probability over the entire cell, we integrate over the cell area:

$$\bar{P}_{\text{outage}} = \frac{1}{\pi R^2} \int_0^R P_{\text{outage}}(r)\, 2\pi r\, dr$$

- The overall coverage probability is $F_u = 1 - \bar{P}_{\text{outage}}$:

$$F_u = 1 - \frac{2}{R^2} \int_0^R \frac{1}{2} \text{erfc}\left(\frac{I(r)}{\sigma_L \sqrt{2}}\right) r\, dr$$

- We use the canonical path loss model: $L(r) = L(R) + 10n \log_{10}(r/R)$.
- Therefore,
  $I(r) = L_{max} - L(R) - 10n \log_{10}(r/R) = M - 10n \log_{10}(r/R)$.

Demonstration: Overall Cell Coverage (3/3)

- The integral becomes:

$$F_u = 1 - \frac{1}{R^2} \int_0^R \operatorname{erfc}\left(\frac{M - 10n \log_{10}(r/R)}{\sigma_L \sqrt{2}}\right) r \, dr$$

- Let $a = \frac{M}{\sigma_L \sqrt{2}}$ and $b = \frac{10n \log_{10}(e)}{\sigma_L \sqrt{2}}$. The argument of erfc is $(a - b \ln(r/R))$.

Demonstration: Overall Cell Coverage (3/3)

- Solving this integral yields the final result for cell coverage probability:

$$F_u = 1 - \frac{1}{2}\mathrm{erfc}(a) + \frac{1}{2}e^{2a/b+1/b^2}\mathrm{erfc}\left(a + \frac{1}{b}\right)$$

Conclusion

The overall cell coverage is a direct function of the chosen fade margin (M), the environment's path loss exponent (n), and its shadowing variability (σ_L).

Demonstration: Overall Cell Coverage

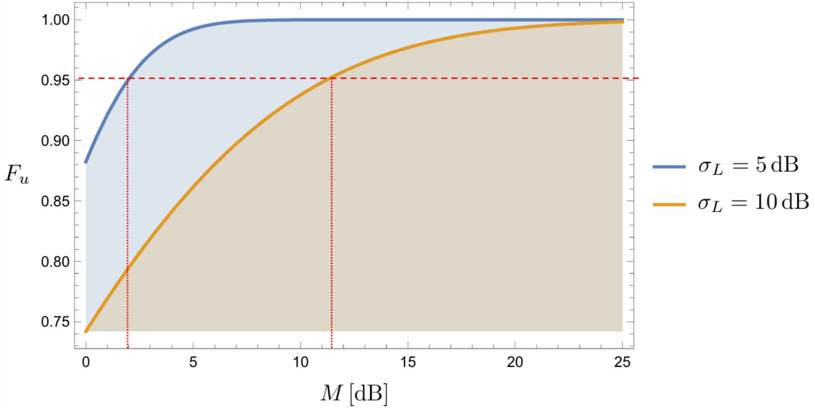

Figure: Cell coverage probability (F_u) vs. Fade Margin (M) for different shadowing variabilities (σ_L).

Shadowing in Link Budgets

The Role of a Link Budget

- A link budget is a systematic accounting of all gains and losses in a communication system.
- Its primary goal is to calculate the maximum allowed path loss (L_{max}) that the system can tolerate while still meeting a target performance metric (e.g., a minimum Signal-to-Noise Ratio, SNR).
- Shadowing is a critical "loss" that must be accounted for to ensure reliable communication.

Example: Cellular Downlink Budget

- Let's analyze how shadowing is incorporated into a typical link budget.
- The goal is to find the maximum path loss. We start with the transmitter's power and subtract all losses and required margins until we reach the receiver's sensitivity.
- The **Shadowing Margin** (or Fade Margin) is explicitly included as one of these required margins.

Example: Cellular Downlink Budget

Parameter	Symbol	Calculation	Value
Transmitter Side (Gains)			
TX Power	P_{TX}	Given	43 dBm
TX Antenna Gain	G_{TX}	Given	15 dBi
Effective Isotropic Radiated Power	**EIRP**	$P_{TX} + G_{TX}$	58 dBm
Receiver Side (Requirements)			
Thermal Noise Power ($T_0 = 290\,\mathrm{K}$, $B = 10\,\mathrm{MHz}$)		$10\log_{10}(kT_0B/1\,\mathrm{mW})$	−104 dBm
RX Noise Figure	F_{dB}	Given	7 dB
Receiver Noise Floor	N		−97 dBm
Target Signal-to-Noise Ratio	SNR	Required for service	1 dB
Receiver Sensitivity		$N + \mathrm{SNR}$	−96 dBm
Margins (Losses)			
Shadowing (Fade) Margin	**M**	**For 95% coverage**	7 dB
Interference Margin		For other-cell interference	4 dB
Indoor Penetration Margin		Loss from walls	10 dB
Total Margin		Sum of margins	21 dB
Result			
RX Antenna Gain	G_{RX}	Given	0 dB
Maximum Allowed Path Loss	$\mathbf{L}_{\mathbf{max}}$	$\mathrm{EIRP} + G_{RX} - \mathrm{Sens.} - \mathrm{Margins}$	133 dB

Interpretation of the Link Budget

- In the example, a 7 dB margin is reserved specifically for shadowing.
- This means the system is designed to work even if the channel is 7 dB worse than the average predicted by the path loss model.
- The final Maximum Allowed Path Loss of 133 dB is the value that should be used with a propagation model (e.g., Okumura-Hata) to find the reliable cell radius R.
- Without the shadowing margin, the allowed path loss would be 140 dB, leading to a much larger, but unreliable, calculated cell radius.

Conclusion

Summary and Conclusion

Shadowing Model

- Shadowing represents large-scale signal variations due to obstacles.
- It is statistically modeled as a log-normal process (i.e., Gaussian in dB) characterized by a standard deviation σ_L.

Summary and Conclusion

Impact and Mitigation

- Shadowing's random nature makes deterministic cell planning impossible and necessitates a probabilistic approach to guarantee service reliability.
- The engineering solution is the **Fade Margin**, a power buffer explicitly included in the link budget.
- This margin ensures a high probability of coverage throughout the cell, at the cost of a slightly reduced cell radius compared to an idealized, non-shadowed scenario.

Thank You