

# Question 1: TDL Wideband and Narrowband Channel Models

Demonstration and Statistical Models for Taps in Wideband Case

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# Introduction

# Wireless Communication Channel

- A communication channel is the medium through which the electromagnetic signal propagates.
- It is characterized by three main phenomena:
  - ▶ **Attenuation**: Reduction in signal strength over distance.
  - ▶ **Time Dispersion**: Signal spreading over time due to multipath propagation.
  - ▶ **Fading**: Fluctuations in signal amplitude and phase.

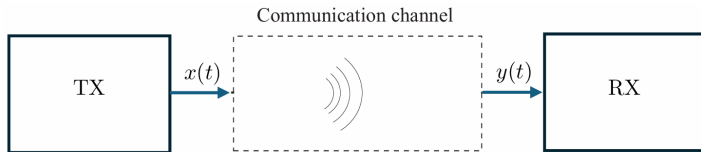


Figure: Basic communication system diagram

# Physical Impulse Response of the Channel

- The physical channel is described by its time-varying impulse response,  $h(\tau, t)$ .
- It is a sum of delayed Dirac delta functions, each representing a multipath component (MPC):

$$h(\tau, t) = \sum_{n=1}^N \alpha_n(t) \delta(\tau - \tau_n)$$

# Physical Impulse Response of the Channel

- The term  $\alpha_n(t)$  is the complex amplitude of the  $n^{\text{th}}$  MPC:

$$\alpha_n(t) = a_n(t)e^{j\phi_n(t)}e^{-j2\pi f_c\tau_n}$$

- ▶  $a_n(t)$ : Real amplitude.
- ▶  $\phi_n(t)$ : Phase shift.
- ▶  $\tau_n$ : Propagation delay.

# Received Signal Formulation

- The received baseband signal  $y(t)$  is the convolution of the transmitted signal  $x(t)$  with the channel's impulse response:

$$y(t) = \int_0^{\infty} h(\tau, t) x(t - \tau) d\tau$$

- Substituting the physical impulse response gives:

$$y(t) = \sum_{n=1}^N \alpha_n(t) x(t - \tau_n)$$

# Received Signal Formulation

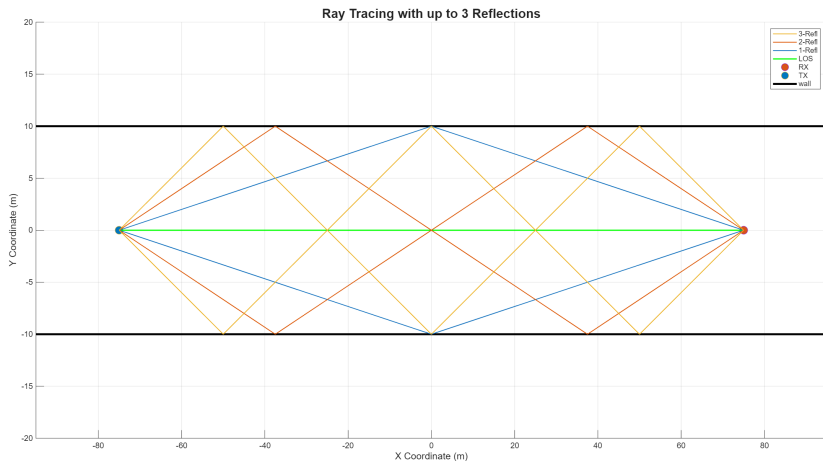


Figure: Diagram of multipath components arriving at the receiver.



# Tapped Delay Line Models

# The Problem of Limited Bandwidth

Physical devices have a **limited bandwidth**  $B$ .

This implies a **limited time resolution** of  $\Delta\tau \approx 1/B$ .

Unresolvable MPCs are grouped into discrete **taps**.

This leads to the **Tapped Delay Line (TDL) Model** approximation.

## Narrowband TDL Model

# Condition for Narrowband

A channel is **narrowband** when the system's time resolution is much larger than the channel's delay spread,  $\sigma_\tau$ .

$$\Delta\tau \gg \sigma_\tau$$

## Consequence

All multipath components arrive within the same time resolution interval,  $\Delta\tau$ . They are therefore indistinguishable and combine into a single tap.

# Condition for Narrowband

placeholder-narrowband-condition.png

# Derivation of the Narrowband Model

- Since all MPCs are unresolvable, the TDL model simplifies to a single tap at  $\tau = 0$ :

$$h_{TDL}(\tau, t) = h_0(t)\delta(\tau)$$

- The coefficient  $h_0(t)$ , or simply  $h(t)$ , is the coherent sum of all multipath components:

$$h(t) \approx \sum_{n=1}^N \alpha_n(t) = \sum_{n=1}^N a_n(t) e^{j\phi_n(t)} e^{-j2\pi f_c \tau_n}$$

# Derivation of the Narrowband Model

- The received signal becomes a simple product:

$$y(t) = \int_0^{\infty} h(t)\delta(\tau)x(t - \tau) d\tau = h(t)x(t)$$

- The channel transfer function  $H(f, t)$  is independent of the baseband frequency  $f$ :

$$H(f, t) = \int_{-\infty}^{\infty} h(t)\delta(\tau)e^{-j2\pi f\tau} d\tau = h(t)$$

This is known as **flat fading**.

# Wideband TDL Model



# Condition for Wideband

A channel is **wideband** when the time resolution  $\Delta\tau$  is smaller than the delay spread  $\sigma_\tau$ .

$$\Delta\tau < \sigma_\tau$$

## Consequence

The system's time resolution is fine enough to resolve groups of multipath components. This leads to a model with multiple taps, causing frequency-selective fading.

# Condition for Wideband

placeholder-wideband-condition.png

# Derivation of the Wideband Model (1/2)

- The bandlimited signal  $x(t)$  allows us to use the sampling theorem. The delayed signal  $x(t - \tau)$  can be written as:

$$x(t - \tau) = \sum_{l=-\infty}^{\infty} x(t - l\Delta\tau) \text{sinc}(B(\tau - l\Delta\tau))$$

- We substitute this into the convolution integral for  $y(t)$ :

$$y(t) = \int_0^{\infty} h(\tau, t) \sum_l x(t - l\Delta\tau) \text{sinc}(B(\tau - l\Delta\tau)) d\tau$$

# Derivation of the Wideband Model (2/2)

- Swapping the integral and summation:

$$y(t) = \sum_l x(t - l\Delta\tau) \underbrace{\int_0^\infty h(\tau, t) \text{sinc}(B(\tau - l\Delta\tau)) d\tau}_{\equiv h_l(t)}$$

- This gives the discrete-time convolution model:

$$y(t) = \sum_{l=0}^L h_l(t) x(t - l\Delta\tau)$$

# The Taps of the Wideband Model

- The resulting impulse response is the Tapped Delay Line (TDL) model:

$$h_{TDL}(\tau, t) = \sum_{l=0}^L h_l(t) \delta(\tau - l\Delta\tau)$$

- The tap coefficient  $h_l(t)$  can be approximated as the sum of all physical MPCs,  $\alpha_n(t)$ , whose delays  $\tau_n$  fall within the  $l$ -th time bin:

$$h_l(t) \approx \sum_{\tau_n \in \text{tap } l} \alpha_n(t)$$

# Visualizing the Taps

## Consequence

Each tap represents the interference of all MPCs arriving around the delay  $/\Delta\tau$ . The physical, continuous impulse response is sampled into a discrete TDL model.

# Visualizing the Taps

placeholder-tdl-grouping.png

## Wideband Scenario - Statistical Model



# Motivation for a Statistical Approach

- A deterministic model (like ray-tracing) is often impractical. The exact properties of each MPC are highly sensitive to small changes in the environment.
- A statistical model aims to generate a channel with the same statistical properties (for example, average power, fading distribution) as a real channel.

# Motivation for a Statistical Approach

- We model the complex tap coefficients  $h_l(t)$  as random variables.
- The phases  $\Phi_n$  of the constituent MPCs are assumed to be independent and uniformly distributed in  $[0, 2\pi)$ .

$$h_l(t) = \sum_{\tau_n \in \text{tap } l} a_n e^{j\Phi_n}$$

# Rayleigh Fading Taps (NLOS)

- **Hypothesis:** A tap consists of a large number of MPCs, with no single component being significantly stronger than the others (Non-Line-of-Sight condition).
- By the **Central Limit Theorem**,  $h_l(t)$  converges to a complex Gaussian random variable with zero mean.

$$h_l(t) = X_l + jY_l \quad \text{where } X_l, Y_l \sim \mathcal{N}(0, \sigma_l^2)$$

- The envelope,  $|h_l(t)| = \sqrt{X_l^2 + Y_l^2}$ , follows a **Rayleigh distribution**.

# The Rayleigh Distribution

- The Probability Density Function (PDF) is:

$$p(|h_I|) = \frac{|h_I|}{\sigma_I^2} \exp\left(-\frac{|h_I|^2}{2\sigma_I^2}\right)$$

- The parameter  $\sigma_I^2$  is half the average power of the tap:  
 $\mathbb{E}[|h_I|^2] = 2\sigma_I^2.$

# The Rayleigh Distribution

placeholder-rayleigh-pdf.png

# Rician Fading Taps (LOS)

- **Hypothesis:** A tap contains a dominant, stable component (typically a Line-of-Sight path), plus many weaker, scattered components.

$$h_l(t) = \underbrace{A_l e^{j\theta_l}}_{\text{Dominant}} + \underbrace{\sum a_n e^{j\phi_n}}_{\text{Scattered}}$$

- The tap coefficient  $h_l(t)$  is a complex Gaussian variable with a **non-zero mean**.
- The envelope  $|h_l(t)|$  follows a **Rician distribution**.

# The Rician Distribution

- The PDF is given by:

$$p(|h_I|) = \frac{|h_I|}{\sigma_I^2} \exp\left(-\frac{|h_I|^2 + A_I^2}{2\sigma_I^2}\right) I_0\left(\frac{|h_I|A_I}{\sigma_I^2}\right)$$

- It is characterized by the **Rician K-factor**, the ratio of dominant to scattered power:

$$K_I = \frac{A_I^2}{2\sigma_I^2}$$

# The Rician Distribution

placeholder-rician-pdf.png



# The Power Delay Profile

- To parameterize these statistical models, we need the average power distribution over the delays.
- This is the **Power Delay Profile**,  $P(\tau_l)$ , which gives the average power  $\mathbb{E}[|h_l|^2]$  for each tap delay  $\tau_l = l\Delta\tau$ .

$$P(\tau_l) = \mathbb{E}[|h_l|^2]$$

# The Power Delay Profile

- The PDP is a fundamental characteristic of a wireless environment (for example., urban, rural).
- Standardized models (for example., from ITU) provide typical PDPs for different scenarios.

# The Power Delay Profile

placeholder-pdp-example.png

# Example: Standardized PDP

## ITU Vehicular A model

A standard model for vehicle communications in an urban environment.

Tap	Delay [ns]	Avg. Power [dB]
1	0	0
2	310	-1.0
3	710	-9.0
4	1090	-10.0
5	1730	-15.0
6	2510	-20.0

All taps in this specific model are defined as having Rayleigh fading.

## Conclusion

# Summary of Models

## Narrowband Model

- Condition:  $\Delta\tau \gg \sigma_\tau$
- Model:  $y(t) = h(t)x(t)$
- Physical Effect: Flat fading

## Wideband Model

- Condition:  $\Delta\tau < \sigma_\tau$
- Model:  $y(t) = \sum h_l(t)x(t - l\Delta\tau)$
- Physical Effect: Frequency-selective fading

## Statistical Modeling of Taps

The complex nature of multipath propagation makes a statistical approach essential for wideband channels. By modeling taps as Rayleigh (NLOS) or Rician (LOS) random processes, parameterized by a standard Power Delay Profile, we can accurately simulate the behavior of real-world frequency-selective channels for system design and performance analysis.