ELEC-H415

Modeling A Vehicle-to-Vehicle Communication Channel in an Urban Environment

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Introduction

For their first year of master in Electrical Engineering and Information Technology in the Bruface program, students were asked to do a project for their communication channels course. The project consists of modelling a vehicle-to-vehicle wireless communication channel. The analysis is grounded in an urban canyon scenario where two vehicles, equipped with vertical $\lambda/2$ dipole antennas, travel along the center of a 20-meter wide street surrounded by building with a relative permittivity of $\epsilon_r = 4$. The distance d between the vehicles is variable and can be maximum $d_{max} = 1km$

The communication system operated at a carrier frequency of, $f_c = 5.9GHz$ with a bandwidth of $B_{RF} = 1000MHz$ and a transmitter power of $P_{TX} = 0.1W$. This report develops the channel model from fundamental principles, progressing through narrowband and wideband analyses of both Line-of-Sight (LOS) and full multipath conditions, with an emphasis on the mathematical derivations and physical interpretation of the results.

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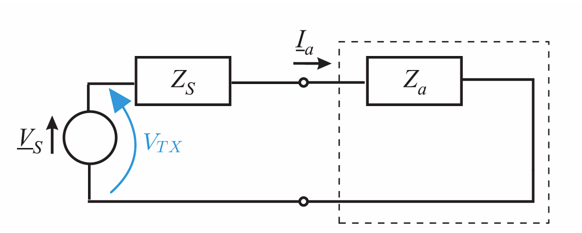
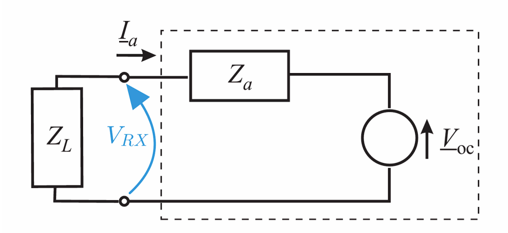
1 Theoretical Preliminaries

1.1 Theoretical Background

An antenna is an electrical component that acts as a transducer between a guided electrical signal and a propagating electromagnetic wave. To analyze its behavior within an electrical system, an equivalent circuit was drawn. In transmission mode, a signal source feeds the antenna, which has an impedance Z_a. This impedance consists of a radiation resistance R_{ar}, representing the power radiated into space, and a loss resistance R_{al}, representing ohmic losses. In reception mode, an incoming electromagnetic wave induces an open-circuit voltage V_{oc} at the antenna's terminals, which then delivers a signal to the receiver's load impedance Z_L.



(a) Transmit antenna equivalent circuit, showing the source voltage V_S, source impedance Z_S, transmit voltage V_{TX}, and antenna current I_a.



(b) Receive antenna equivalent circuit, showing the induced open-circuit voltage V_{oc}, the load impedance Z_L, and the received voltage V_{RX}.

Figure 1.1: Equivalent circuits for the transmit and receive antennas

The equivalent circuits for the transmitter and receiver are drawn in Figure 1.1. In Figure 1.1a, can be seen the equivalent circuit of the transmitter. The voltage source V_S with its internal impedance Z_S drives the antenna, resulting in a current I_a and a voltage V_{TX} at the antenna's input terminals. At the receiver (Figure 1.1b), the incoming wave induces an open-circuit voltage V_{oc}, which in turn produces the received voltage V_{RX} across the load impedance Z_L.

For this project, it was considered that the electronics to be perfectly matched to the antennas. This implies that for the transmitter, the source impedance is the complex conjugate of the antenna impedance:

$$Z_S = Z_a^* \tag{1.1}$$

and for the receiver, the load impedance is the complex conjugate of the antenna impedance:

$$Z_L = Z_a^* \tag{1.2}$$

This consideration ensures maximum power transfer.

1.1.1 Antenna Effective Height

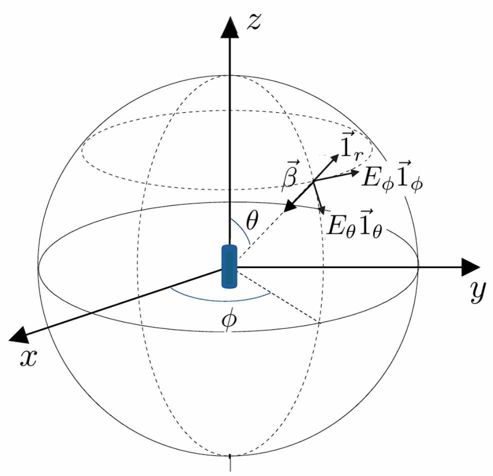


Figure 1.2: Illustration of the vertical dipole antenna and coordinate axes.

The effective height $\vec{h}_e$ of an antenna links the circuit domain to the electromagnetic wave domain. It is derived from the current distribution $\vec{J}(\vec{r}\,')$ on the antenna when transmitting with an input current $\underline{I}_a$:

$$\vec{h}_e(\theta, \phi) = \frac{1}{\underline{I}_a} \int_{\mathcal{D}} \vec{J}(\vec{r}\,') e^{j\beta(\vec{r}\,' \cdot \vec{1}_r)} dV' \tag{1.3}$$

where $\mathcal{D}$ is the volume of the antenna, $\vec{1}_r$ is the unit vector in the direction of radiation (θ, ϕ), and β is the wavenumber:

$$\beta = \frac{2\pi}{\lambda} \tag{1.4}$$

For a thin, vertical half-wave dipole antenna of length $L = \frac{\lambda}{2}$ oriented along the z-axis and centered at the origin (Figure 1.2), the current flows only in the z-direction. The current distribution is given by:

$$\vec{J}(\vec{r}\,') = \underline{I}_a \cos(\beta z')\delta(x')\delta(y')\vec{1}_z, \quad \text{for } -\frac{\lambda}{4} \leq z' \leq \frac{\lambda}{4} \tag{1.5}$$

where $\delta(x')$ and $\delta(y')$ are Dirac's deltas.

The volume integral reduces to a line integral along the z-axis. The dot product in the exponent simplifies to:

$$\vec{r}\,' \cdot \vec{1}_r = z' \cos\theta \tag{1.6}$$

Substituting this into Equation 1.3 gives:

$$\vec{h}_e(\theta, \phi) = \left(\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos(\beta z') e^{j\beta z' \cos\theta} dz' \right) \vec{1}_z \tag{1.7}$$

The integral in Equation 1.7 is solved using Euler's formula:

$$\cos(\beta z') = \frac{1}{2}(e^{j\beta z'} + e^{-j\beta z'}) \tag{1.8}$$

$$\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos(\beta z') e^{j\beta z' \cos\theta} dz' = \frac{1}{2} \int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \left(e^{j\beta z'(1+\cos\theta)} + e^{j\beta z'(\cos\theta - 1)} \right) dz' \tag{1.9}$$

$$= \frac{1}{2j\beta} \left[\frac{e^{j\beta z'(1+\cos\theta)}}{1+\cos\theta} + \frac{e^{j\beta z'(\cos\theta-1)}}{\cos\theta - 1} \right]_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \tag{1.10}$$

With $\beta = \frac{2\pi}{\lambda}$, the term $\frac{\beta\lambda}{4}$ simplifies to $\frac{\pi}{2}$. Evaluating at the limits yields:

$$= \frac{1}{2j\beta} \left[\frac{e^{j\frac{\pi}{2}(1+\cos\theta)} - e^{-j\frac{\pi}{2}(1+\cos\theta)}}{1+\cos\theta} - \frac{e^{j\frac{\pi}{2}(1-\cos\theta)} - e^{-j\frac{\pi}{2}(1-\cos\theta)}}{1-\cos\theta} \right] \tag{1.11}$$

Using the definition of sine: $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$$= \frac{1}{\beta} \left[\frac{\sin\left(\frac{\pi}{2}(1+\cos\theta)\right)}{1+\cos\theta} - \frac{\sin\left(\frac{\pi}{2}(\cos\theta - 1)\right)}{1-\cos\theta} \right] \tag{1.12}$$

$$= \frac{1}{\beta} \left[\frac{\sin(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta)}{1+\cos\theta} + \frac{\sin(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta)}{1-\cos\theta} \right] \tag{1.13}$$

Applying the identities $\sin(\frac{\pi}{2} + x) = \cos(x)$ and $\sin(\frac{\pi}{2} - x) = \cos(x)$:

$$= \frac{1}{\beta} \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{1+\cos\theta} + \frac{\cos(\frac{\pi}{2}\cos\theta)}{1-\cos\theta} \right] \tag{1.14}$$

$$= \frac{\cos(\frac{\pi}{2}\cos\theta)}{\beta} \left[\frac{(1-\cos\theta) + (1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \right] = \frac{2\cos(\frac{\pi}{2}\cos\theta)}{\beta\sin^2\theta} \tag{1.15}$$

Substituting $\beta = \frac{2\pi}{\lambda}$, the final result of the integral is:

$$\frac{\lambda}{\pi} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \tag{1.16}$$

The effective height for the vertical half-wave dipole is therefore:

$$\vec{h}_e(\theta,\phi) = \frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\vec{1}_z \tag{1.17}$$

This vector is oriented along the z-axis. For reception, the induced voltage depends on the component of the effective height that is transverse (perpendicular) to the direction of wave propagation, $\vec{1}_r$. This transverse component is denoted $\vec{h}_{e\perp}$. The Cartesian unit vector $\vec{1}_z$ is expressed in spherical coordinates as:

$$\vec{1}_z = \cos\theta\vec{1}_r - \sin\theta\vec{1}_\theta \tag{1.18}$$

The transverse part, $\vec{h}_{e\perp}$, consists of the components in the $\vec{1}_\theta$ and $\vec{1}_\phi$ directions. Substituting the spherical representation of $\vec{1}_z$ into Equation 1.17 and retaining only the transverse component gives:

$$\vec{h}_{e\perp}(\theta,\phi) = -\frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}(\sin\theta\vec{1}_\theta) = -\frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\vec{1}_\theta \tag{1.19}$$

This is the general expression for the transverse effective height of a vertical $\frac{\lambda}{2}$ dipole. In the horizontal plane, where $\theta = \frac{\pi}{2}$, the expression simplifies significantly. The transverse effective height in the horizontal plane is therefore:

$$\vec{h}_{e\perp}\left(\theta = \frac{\pi}{2},\phi\right) = -\frac{\lambda}{\pi}\vec{1}_\theta \tag{1.20}$$

This final expression indicates that in the horizontal plane, the antenna's effective height has a constant magnitude of $\frac{\lambda}{\pi}$, is constant for all ϕ, and is oriented in the $\vec{1}_\theta$ direction.

1.1.2 Emitted Electric Field in Free-Space

The electric field radiated by an antenna is given by:

$$\underline{\vec{E}}(\vec{r}) = -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\vec{h}_{e\perp}(\theta,\phi) \tag{1.21}$$

Substituting the transverse effective height for the horizontal plane (Equation 1.20) into the general expression gives:

$$\underline{\vec{E}}(r,\pi/2,\phi) = -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\left(-\frac{\lambda}{\pi}\vec{1}_\theta\right) = j\underline{I}_a\frac{\omega\mu_0\lambda}{4\pi^2}\frac{e^{-j\beta r}}{r}\vec{1}_\theta \tag{1.22}$$

Expressing this field in terms of circuit parameters involves the substitutions $\omega = 2\pi f_c$, $\lambda = \frac{c}{f_c}$, $\mu_0 = \frac{Z_0}{c}$, and propagation delay $\tau = \frac{r}{c}$. The exponential term becomes:

$$e^{-j\beta r} = e^{-j\frac{2\pi}{\lambda}c\tau} = e^{-j2\pi f_c\tau} \tag{1.23}$$

The electric field is then:

$$\underline{\vec{E}} = j\underline{I}_a \frac{(2\pi f_c)(\frac{Z_0}{c})(\frac{c}{f_c})}{4\pi^2} \frac{e^{-j2\pi f_c \tau}}{c\tau} \vec{1}_\theta = j\underline{I}_a \frac{Z_0}{2\pi c\tau} e^{-j2\pi f_c \tau} \vec{1}_\theta \tag{1.24}$$

A half-wave dipole at its resonant frequency has an almost purely real impedance, meaning its reactance is negligible ($X_a \approx 0$), so $Z_a = R_a + jX_a \approx R_a$. Under the perfect matching condition ($Z_S = Z_a^*$), the total impedance in the transmitter circuit is $Z_S + Z_a \approx Z_a^* + Z_a \approx 2R_a$. The antenna current is then related to the transmit voltage by:

$$\underline{I}_a = \frac{\underline{V}_{TX}}{2R_a} \tag{1.25}$$

Substituting this for $\underline{I}_a$:

$$\underline{\vec{E}} = j\left(\frac{\underline{V}_{TX}}{2R_a}\right) \frac{Z_0}{2\pi c\tau} e^{-j2\pi f_c \tau} \vec{1}_\theta = j\frac{Z_0}{4\pi R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \vec{1}_\theta \tag{1.26}$$

1.1.3 Received Voltage in Free-Space

The open-circuit voltage $\underline{V}_{oc}$ induced at the terminals of a receiving antenna is given by the dot product of its effective height and the incident electric field:

$$\underline{V}_{oc} = -\vec{h}_{e\perp}^{RX} \cdot \underline{\vec{E}} \tag{1.27}$$

The receiving antenna is also a vertical $\frac{\lambda}{2}$ dipole, so its transverse effective height in the horizontal plane is given by Eq. 1.20. The incident electric field is given by Eq. 1.26. The dot product is:

$$\underline{V}_{oc} = -\left(-\frac{\lambda}{\pi}\vec{1}_\theta\right) \cdot \left(j\frac{Z_0}{4\pi R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \vec{1}_\theta\right) \tag{1.28}$$

$$= j\frac{\lambda Z_0}{4\pi^2 R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \tag{1.29}$$

The voltage $\underline{V}_{RX}$ across the receiver load Z_L is found using a voltage divider on the receiver equivalent circuit (Figure 1.1b):

$$\underline{V}_{RX} = \underline{V}_{oc} \frac{Z_L}{Z_a + Z_L} \tag{1.30}$$

With perfect matching ($Z_L = Z_a^*$) and a resonant dipole ($Z_a \approx R_a$), the total impedance in the receiver circuit is $Z_a + Z_L \approx R_a + R_a = 2R_a$. The expression for $\underline{V}_{RX}$ simplifies to:

$$\underline{V}_{RX} \approx \frac{\underline{V}_{oc}}{2} \tag{1.31}$$

Substituting the expression for $\underline{V}_{oc}$ gives the final relationship between the received and transmitted voltages:

$$\underline{V}_{RX} = j\frac{\lambda Z_0}{8\pi^2 R_a c\tau}\underline{V}_{TX}e^{-j2\pi f_c \tau} \tag{1.32}$$

2

Narrowband Analysis - LOS Channel

The analysis begins with the simplest communication scenario: a direct Line-of-Sight (LOS) path between the transmitter TX and the receiver RX. A narrowband analysis is conducted, which assumes that the signal's bandwidth is much smaller than the channel's coherence bandwidth. This simplification allows the channel to be characterized by one complex coefficient.

2.1 Theoretical Background

The physical channel can be described by its time-variant impulse response, which for a set of N multipath components is:

$$h(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)\delta(\tau - \tau_n) \tag{2.1}$$

where $\alpha_n(t)$ and τ_n are the complex amplitude and propagation delay of the n-th path, respectively.

A practical communication system has a finite bandwidth B, which limits its ability to resolve paths arriving at different times. The system's time resolution is $\Delta\tau = 1/B$. This physical limitation leads to the Tapped Delay Line (TDL) model. The impulse response of the TDL model is a discrete-time representation of the physical channel:

$$h_{TDL}(\tau, t) = \sum_{l=0}^{L} h_l(t)\delta(\tau - l\Delta\tau) \tag{2.2}$$

where $h_l(t)$ is the complex gain of the l-th tap.

The condition for a narrowband channel is that the signal bandwidth B is much smaller than the channel's coherence bandwidth Δf_c. The coherence bandwidth is inversely proportional to the channel's delay spread, σ_τ, which is the maximum time difference between arriving paths. The narrowband condition is thus expressed as:

$$B \ll \Delta f_c \approx \frac{1}{\sigma_\tau} \implies \Delta\tau \gg \sigma_\tau \tag{2.3}$$

This inequality means that the system's time resolution is much larger than the delay spread. From the receiver's perspective, all MPCs arrive at effectively the same time, as their delay differences are too small to be resolved. Consequently, all MPCs fall into the first tap (with index $l = 0$) of the TDL model. The summation in Equation 2.2 therefore reduces to one term for $l = 0$:

$$h_{TDL}(\tau, t) = h_0(t)\delta(\tau - 0 \cdot \Delta\tau) = h_0(t)\delta(\tau) \tag{2.4}$$

where the tap gain $h_0(t)$ is the sum of all individual path gains:

$$h_0(t) = \sum_{n=1}^{N} \alpha_n(t) \tag{2.5}$$

The channel's transfer function, $H(f, t)$, is the Fourier transform of this simplified TDL impulse response. The derivation proceeds as follows:

$$H(f, t) = \mathcal{F}\{h_{TDL}(\tau, t)\} = \int_{-\infty}^{\infty} h_{TDL}(\tau, t)e^{-j2\pi f\tau} d\tau \tag{2.6}$$

Substituting the expression for the narrowband TDL impulse response:

$$H(f, t) = \int_{-\infty}^{\infty} h_0(t)\delta(\tau)e^{-j2\pi f\tau} d\tau \tag{2.7}$$

To solve this integral, the sifting property of the Dirac delta function is used, which states that $\int g(\tau)\delta(\tau)d\tau = g(0)$. In this case, the function $g(\tau)$ is $h_0(t)e^{-j2\pi f\tau}$. Evaluating this function at $\tau = 0$ gives:

$$g(0) = h_0(t)e^{-j2\pi f(0)} = h_0(t) \tag{2.8}$$

Therefore, the transfer function simplifies to:

$$H(f, t) = h_0(t) \tag{2.9}$$

Since $H(f, t)$ does not depend on the baseband frequency f, the channel is described as frequency-flat. The narrowband transfer function, h_{NB}, is this constant complex gain. For a time-invariant channel, this becomes:

$$h_{NB} = \sum_{n=1}^{N} \alpha_n \tag{2.10}$$

In the case of a unique LOS path, this sum reduces to a single term, $h_{NB} = \alpha_1$. The benchmark for received power in this scenario is the Friis formula:

$$P_{RX} = P_{TX}G_{TX}G_{RX}\left(\frac{\lambda}{4\pi d_1}\right)^2 \tag{2.11}$$

where d_1 is the distance between the antennas.

2.2 Analysis and Mathematical Derivations

2.2.1 Impulse Response $h(\tau)$

The expression for α_1 for the LOS path is found by relating the system-level Friis formula to the complex amplitude. The received power is proportional to the squared magnitude of this coefficient:

$$P_{RX} = |\alpha_1|^2 P_{TX} \tag{2.12}$$

By comparing this with the Friis formula (Equation 2.11), the magnitude of α_1, denoted as $|a_1|$, is:

$$|\alpha_1|^2 = G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d_1} \right)^2 \implies |a_1| = \sqrt{G_{TX} G_{RX}} \frac{\lambda}{4\pi d_1} \tag{2.13}$$

For this project, both antennas are identical vertical $\lambda/2$ dipoles. The gain of a lossless half-wave dipole is maximal in the horizontal plane ($\theta = \pi/2$) and is given by

$$G_{TX} = G_{RX} = G\left(\theta = \frac{\pi}{2}\right) = \frac{16}{3\pi} \approx 1.6976 \tag{2.14}$$

Assuming the additional phase shift ϕ_1 is zero for the direct LOS path, the complex amplitude is $\alpha_1 = |a_1| e^{-j2\pi f_c \tau_1}$. The full impulse response for the LOS path is then:

$$h(\tau) = \left(\sqrt{G_{TX} G_{RX}} \frac{\lambda}{4\pi d_1} e^{-j2\pi f_c \tau_1} \right) \delta(\tau - \tau_1) \tag{2.15}$$

where $\tau_1 = d_1/c$ is the propagation delay.

2.2.2 Transfer Function $H(f)$

The transfer function $H(f)$ is obtained by taking the Fourier transform of the impulse response in Equation 2.15. The explicit integral is:

$$H(f) = \int_{-\infty}^{\infty} \left(\sqrt{G_{TX} G_{RX}} \frac{\lambda}{4\pi d_1} e^{-j2\pi f_c \tau_1} \delta(\tau - \tau_1) \right) e^{-j2\pi f \tau} d\tau \tag{2.16}$$

Using the sifting property of the Dirac delta function, which states that $\int g(\tau)\delta(\tau - \tau_1) d\tau = g(\tau_1)$, the integral simplifies to:

$$H(f) = \sqrt{G_{TX} G_{RX}} \frac{\lambda}{4\pi d_1} e^{-j2\pi f_c \tau_1} e^{-j2\pi f \tau_1} \tag{2.17}$$

$$= \sqrt{G_{TX} G_{RX}} \frac{\lambda}{4\pi d_1} e^{-j2\pi (f + f_c) \tau_1} \tag{2.18}$$

This function shows that the channel introduces a phase shift that is linear with the baseband frequency f, which corresponds to the time delay τ_1. The magnitude $|H(f)|$ is constant across all frequencies.

2.2.3 Narrowband Transfer Function h_{NB}

As established in the theoretical background, the narrowband assumption holds perfectly for one LOS path because the delay spread σ_τ is zero. The narrowband transfer function h_{NB} is therefore the frequency-independent complex gain obtained by evaluating the full transfer function (Equation 2.18) at the baseband center, $f = 0$:

$$h_{NB} = H(f=0) = \sqrt{G_{TX} G_{RX}} \frac{\lambda}{4\pi d_1} e^{-j2\pi f_c \tau_1} = \alpha_1 \tag{2.19}$$

The channel is thus represented by one complex number, which scales and rotates the transmitted signal.

2.2.4 Received Power P_{RX}

The received power is derived from the circuit-level relationships established in the preliminaries, demonstrating its equivalence to the Friis formula. The received power P_{RX} and transmitted power P_{TX} are given by:

$$P_{RX} = \frac{1}{8R_a} |\underline{V}_{oc}|^2 \tag{2.20}$$

$$P_{TX} = \frac{1}{2} R_a |\underline{I}_a|^2 \implies |\underline{I}_a|^2 = \frac{2P_{TX}}{R_a} \tag{2.21}$$

The open-circuit voltage $\underline{V}_{oc}$ is induced by the incident electric field $\underline{\vec{E}}$: $\underline{V}_{oc} = -\vec{h}_{e\perp}^{RX} \cdot \underline{\vec{E}}$. For the LOS path in the horizontal plane, this becomes:

$$\underline{V}_{oc} = -\left(-\frac{\lambda}{\pi} \vec{1}_\theta\right) \cdot \left(j \frac{Z_0 |\underline{I}_a|}{2\pi d_1} e^{-j\beta d_1} \vec{1}_\theta\right) = j \frac{\lambda Z_0 |\underline{I}_a|}{2\pi^2 d_1} e^{-j\beta d_1} \tag{2.22}$$

Taking the magnitude squared:

$$|\underline{V}_{oc}|^2 = \left(\frac{\lambda Z_0}{2\pi^2 d_1}\right)^2 |\underline{I}_a|^2 \tag{2.23}$$

Substitute this and Equation 2.21 into Equation 2.20:

$$P_{RX} = \frac{1}{8R_a} \left(\frac{\lambda Z_0}{2\pi^2 d_1}\right)^2 \left(\frac{2P_{TX}}{R_a}\right) \tag{2.24}$$

$$= \frac{\lambda^2 Z_0^2}{16\pi^4 R_a^2 d_1^2} P_{TX} \tag{2.25}$$

Rearranging the terms reveals the antenna gains. The gain of a $\lambda/2$ dipole in the horizontal plane is $G = Z_0/(\pi R_a)$.

$$P_{RX} = \left(\frac{Z_0}{\pi R_a}\right)\left(\frac{Z_0}{\pi R_a}\right)\left(\frac{\lambda^2}{16\pi^2 d_1^2}\right) P_{TX} \tag{2.26}$$

$$= G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d_1}\right)^2 P_{TX} \tag{2.27}$$

This result is identical to the Friis formula.

Table 2.1: Comparison of Derived Power Equation with Friis' Formula

Component	**Derived Result**	**Friis' Formula**
Transmit Power	P_{TX}	P_{TX}
Transmit Gain	$G_{TX} = \frac{Z_0}{\pi R_a}$	G_{TX}
Receive Gain	$G_{RX} = \frac{Z_0}{\pi R_a}$	G_{RX}
Path Loss Factor	$\left(\frac{\lambda}{4\pi d_1}\right)^2$	$\left(\frac{\lambda}{4\pi d_1}\right)^2$

2.3 Code Implementation and Methodology

(Placeholder) In the accompanying script, the derived equation for P_{RX} (Equation 2.27) is implemented to calculate the received power at various distances d_1. The script defines variables for the transmitted power (P_{TX}), antenna gains (G_{TX}, G_{RX}), wavelength (λ), and the distance vector (d_1). The final result is plotted as received power in dBm versus distance.

2.4 Interpretation of Results

The derivations confirm the fundamental principles of a simple LOS communication link.

- **Frequency-Flat Channel:** The single propagation path results in a transfer function $|H(f)|$ that is constant with frequency. This means the channel does not distort the signal's spectrum, a condition known as flat fading. This is a direct consequence of having no time dispersion (zero delay spread).
- **Validation of Friis' Formula:** The derivation of received power starting from the electromagnetic and circuit principles yields a result identical to the well-known Friis' formula. This demonstrates a powerful consistency between the low-level physical model (fields and circuits) and the high-level system model (power and gains). It validates the assumptions made, such as perfect matching and the use of lossless $\lambda/2$ dipole antennas, as the basis for the Friis link budget in this ideal scenario. The comparison highlights that the antenna gains and free-space path loss are not just abstract parameters but are directly rooted in the physical properties of the antennas and wave propagation.

3

Chapter 3