ELEC-H415

Modeling A Vehicle-to-Vehicle Communication Channel in an Urban Environment

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Introduction

For their first year of master in Electrical Engineering and Information Technology in the Bruface program, students were asked to do a project for their communication channels course. The project consists of modelling a vehicle-to-vehicle wireless communication channel. The analysis is grounded in an urban canyon scenario where two vehicles, equipped with vertical $\lambda/2$ dipole antennas, travel along the center of a 20-meter wide street surrounded by building with a relative permittivity of $\epsilon_r = 4$. The distance d between the vehicles is variable and can be maximum $d_{max} = 1km$

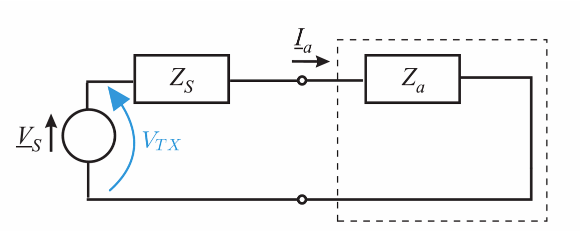
The communication system operated at a carrier frequency of, $f_c = 5.9GHz$ with a bandwidth of $B_{RF} = 1000MHz$ and a transmitter power of $P_{TX} = 0.1W$. This report develops the channel model from fundamental principles, progressing through narrowband and wideband analyses of both Line-of-Sight (LOS) and full multipath conditions, with an emphasis on the mathematical derivations and physical interpretation of the results.

This Goes further by ****Put the new thing that the teacher talked about th have extra points****

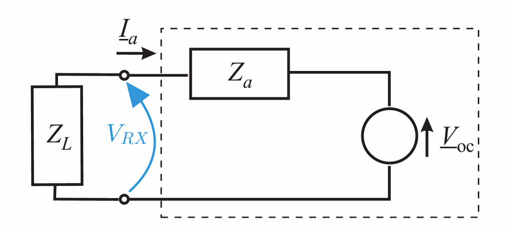
1 Theoretical Preliminaries

1.1 Theoretical Background

An antenna is an electrical component that acts as a transducer between a guided electrical signal and a propagating electromagnetic wave. To analyze its behavior within an electrical system, an equivalent circuit was drawn. In transmission mode, a signal source feeds the antenna, which has an impedance Z_a. This impedance consists of a radiation resistance R_{ar}, representing the power radiated into space, and a loss resistance R_{al}, representing ohmic losses. In reception mode, an incoming electromagnetic wave induces an open-circuit voltage V_{oc} at the antenna's terminals, which then delivers a signal to the receiver's load impedance Z_L.



(a) Transmit antenna equivalent circuit, showing the source voltage V_S, source impedance Z_S, transmit voltage V_{TX}, and antenna current I_a.



(b) Receive antenna equivalent circuit, showing the induced open-circuit voltage V_{oc}, the load impedance Z_L, and the received voltage V_{RX}.

Figure 1.1: Equivalent circuits for the transmit and receive antennas

The equivalent circuits for the transmitter and receiver are drawn in Figure 1.1. In Figure 1.1a, can be seen the equivalent circuit of the transmitter. The voltage source V_S with its internal impedance Z_S drives the antenna, resulting in a current I_a and a voltage V_{TX} at the antenna's input terminals. At the receiver (Figure 1.1b), the incoming wave induces an open-circuit voltage V_{oc}, which in turn produces the received voltage V_{RX} across the load impedance Z_L.

For this project, we consider the electronics to be perfectly matched to the antennas. This implies that for the transmitter, the source impedance is the complex conjugate of the antenna impedance:

$$Z_S = Z_a^* \tag{1.1}$$

and for the receiver, the load impedance is the complex conjugate of the antenna impedance:

$$Z_L = Z_a^* \tag{1.2}$$

This consideration ensures maximum power transfer.

1.1.1 Antenna Effective Height

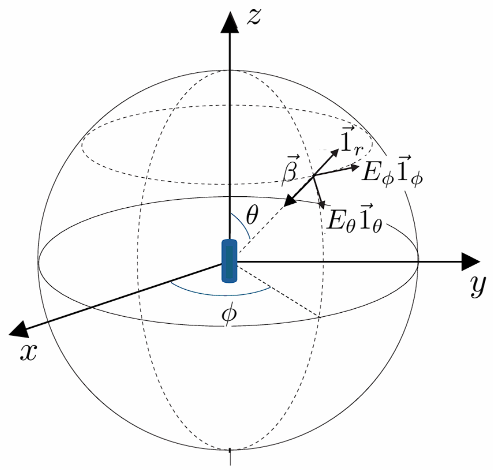


Figure 1.2: Illustration of the vertical dipole antenna and coordinate axes.

The effective height $\vec{h}_e$ of an antenna links the circuit domain to the electromagnetic wave domain. It is derived from the current distribution $\vec{J}(\vec{r}\,')$ on the antenna when transmitting with an input current $\underline{I}_a$:

$$\vec{h}_e(\theta, \phi) = \frac{1}{\underline{I}_a} \int_{\mathcal{D}} \vec{J}(\vec{r}\,') e^{j\beta(\vec{r}\,' \cdot \vec{1}_r)} dV' \tag{1.3}$$

where $\mathcal{D}$ is the volume of the antenna, $\vec{1}_r$ is the unit vector in the direction of radiation (θ, ϕ), and β is the wavenumber:

$$\beta = \frac{2\pi}{\lambda} \tag{1.4}$$

For a thin, vertical half-wave dipole antenna of length $L = \frac{\lambda}{2}$ oriented along the z-axis and centered at the origin (Figure 1.2), the current flows only in the z-direction. The current distribution is given by:

$$\vec{J}(\vec{r}\,') = \underline{I}_a \cos(\beta z') \delta(x') \delta(y') \vec{1}_z, \quad \text{for } -\frac{\lambda}{4} \leq z' \leq \frac{\lambda}{4} \tag{1.5}$$

where $\delta(x')$ and $\delta(y')$ are Dirac's deltas.

The volume integral reduces to a line integral along the z-axis. The dot product in the exponent simplifies to:

$$\vec{r}\,' \cdot \vec{1}_r = z' \cos\theta \tag{1.6}$$

Substituting this into Equation 1.3 gives:

$$\vec{h}_e(\theta, \phi) = \left(\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos(\beta z') e^{j\beta z' \cos\theta} dz' \right) \vec{1}_z \tag{1.7}$$

The integral in Equation 1.7 is solved using Euler's formula:

$$\cos(\beta z') = \frac{1}{2}(e^{j\beta z'} + e^{-j\beta z'}) \tag{1.8}$$

$$\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos(\beta z') e^{j\beta z' \cos\theta} dz' = \frac{1}{2} \int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \left(e^{j\beta z'(1+\cos\theta)} + e^{j\beta z'(\cos\theta - 1)} \right) dz' \tag{1.9}$$

$$= \frac{1}{2j\beta} \left[\frac{e^{j\beta z'(1+\cos\theta)}}{1+\cos\theta} + \frac{e^{j\beta z'(\cos\theta-1)}}{\cos\theta - 1} \right]_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \tag{1.10}$$

With $\beta = \frac{2\pi}{\lambda}$, the term $\frac{\beta\lambda}{4}$ simplifies to $\frac{\pi}{2}$. Evaluating at the limits yields:

$$= \frac{1}{2j\beta} \left[\frac{e^{j\frac{\pi}{2}(1+\cos\theta)} - e^{-j\frac{\pi}{2}(1+\cos\theta)}}{1+\cos\theta} - \frac{e^{j\frac{\pi}{2}(1-\cos\theta)} - e^{-j\frac{\pi}{2}(1-\cos\theta)}}{1-\cos\theta} \right] \tag{1.11}$$

Using the definition of sine: $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$$= \frac{1}{\beta} \left[\frac{\sin\left(\frac{\pi}{2}(1+\cos\theta)\right)}{1+\cos\theta} - \frac{\sin\left(\frac{\pi}{2}(\cos\theta - 1)\right)}{1-\cos\theta} \right] \tag{1.12}$$

$$= \frac{1}{\beta} \left[\frac{\sin(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta)}{1+\cos\theta} + \frac{\sin(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta)}{1-\cos\theta} \right] \tag{1.13}$$

Applying the identities $\sin(\frac{\pi}{2} + x) = \cos(x)$ and $\sin(\frac{\pi}{2} - x) = \cos(x)$:

$$= \frac{1}{\beta} \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{1+\cos\theta} + \frac{\cos(\frac{\pi}{2}\cos\theta)}{1-\cos\theta} \right] \tag{1.14}$$

$$= \frac{\cos(\frac{\pi}{2}\cos\theta)}{\beta} \left[\frac{(1-\cos\theta) + (1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \right] = \frac{2\cos(\frac{\pi}{2}\cos\theta)}{\beta\sin^2\theta} \tag{1.15}$$

Substituting $\beta = \frac{2\pi}{\lambda}$, the final result of the integral is:

$$\frac{\lambda}{\pi} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \tag{1.16}$$

The effective height for the vertical half-wave dipole is therefore:

$$\vec{h}_e(\theta,\phi) = \frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\vec{1}_z \tag{1.17}$$

This vector is oriented along the z-axis. For reception, the induced voltage depends on the component of the effective height that is transverse (perpendicular) to the direction of wave propagation, $\vec{1}_r$. This transverse component is denoted $\vec{h}_{e\perp}$. The Cartesian unit vector $\vec{1}_z$ is expressed in spherical coordinates as:

$$\vec{1}_z = \cos\theta\vec{1}_r - \sin\theta\vec{1}_\theta \tag{1.18}$$

The transverse part, $\vec{h}_{e\perp}$, consists of the components in the $\vec{1}_\theta$ and $\vec{1}_\phi$ directions. Substituting the spherical representation of $\vec{1}_z$ into Equation 1.17 and retaining only the transverse component gives:

$$\vec{h}_{e\perp}(\theta,\phi) = -\frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}(\sin\theta\vec{1}_\theta) = -\frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\vec{1}_\theta \tag{1.19}$$

This is the general expression for the transverse effective height of a vertical $\frac{\lambda}{2}$ dipole. In the horizontal plane, where $\theta = \frac{\pi}{2}$, the expression simplifies significantly. The transverse effective height in the horizontal plane is therefore:

$$\vec{h}_{e\perp}\left(\theta = \frac{\pi}{2},\phi\right) = -\frac{\lambda}{\pi}\vec{1}_\theta \tag{1.20}$$

This final expression indicates that in the horizontal plane, the antenna's effective height has a constant magnitude of $\frac{\lambda}{\pi}$, is constant for all ϕ, and is oriented in the $\vec{1}_\theta$ direction.

1.1.2 Emitted Electric Field in Free-Space

The electric field radiated by an antenna is given by:

$$\underline{\vec{E}}(\vec{r}) = -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\vec{h}_{e\perp}(\theta,\phi) \tag{1.21}$$

Substituting the transverse effective height for the horizontal plane (Eq. 1.20) into the general expression gives:

$$\underline{\vec{E}}(r,\pi/2,\phi) = -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\left(-\frac{\lambda}{\pi}\vec{1}_\theta\right) = j\underline{I}_a\frac{\omega\mu_0\lambda}{4\pi^2}\frac{e^{-j\beta r}}{r}\vec{1}_\theta \tag{1.22}$$

Expressing this field in terms of circuit parameters involves the substitutions $\omega = 2\pi f_c$, $\lambda = \frac{c}{f_c}$, $\mu_0 = \frac{Z_0}{c}$, and propagation delay $\tau = \frac{r}{c}$. The exponential term becomes:

$$e^{-j\beta r} = e^{-j\frac{2\pi}{\lambda}c\tau} = e^{-j2\pi f_c\tau} \tag{1.23}$$

The electric field is then:

$$\underline{\vec{E}} = j\underline{I}_a \frac{(2\pi f_c)(\frac{Z_0}{c})(\frac{c}{f_c})}{4\pi^2} \frac{e^{-j2\pi f_c \tau}}{c\tau} \vec{1}_\theta = j\underline{I}_a \frac{Z_0}{2\pi c\tau} e^{-j2\pi f_c \tau} \vec{1}_\theta \tag{1.24}$$

A half-wave dipole at its resonant frequency has an almost purely real impedance, meaning its reactance is negligible ($X_a \approx 0$), so $Z_a = R_a + jX_a \approx R_a$. Under the perfect matching condition ($Z_S = Z_a^*$), the total impedance in the transmitter circuit is $Z_S + Z_a \approx Z_a^* + Z_a \approx 2R_a$. The antenna current is then related to the transmit voltage by:

$$\underline{I}_a = \frac{\underline{V}_{TX}}{2R_a} \tag{1.25}$$

Substituting this for $\underline{I}_a$ yields:

$$\underline{\vec{E}} = j\left(\frac{\underline{V}_{TX}}{2R_a}\right) \frac{Z_0}{2\pi c\tau} e^{-j2\pi f_c \tau} \vec{1}_\theta = j\frac{Z_0}{4\pi R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \vec{1}_\theta \tag{1.26}$$

1.1.3 Received Voltage in Free-Space

The open-circuit voltage $\underline{V}_{oc}$ induced at the terminals of a receiving antenna is given by the dot product of its effective height and the incident electric field:

$$\underline{V}_{oc} = -\vec{h}_{e\perp}^{RX} \cdot \underline{\vec{E}} \tag{1.27}$$

The receiving antenna is also a vertical $\frac{\lambda}{2}$ dipole, so its transverse effective height in the horizontal plane is given by Eq. 1.20. The incident electric field is given by Eq. 1.26. The dot product is:

$$\underline{V}_{oc} = -\left(-\frac{\lambda}{\pi}\vec{1}_\theta\right) \cdot \left(j\frac{Z_0}{4\pi R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \vec{1}_\theta\right) \tag{1.28}$$

$$= j\frac{\lambda Z_0}{4\pi^2 R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \tag{1.29}$$

The voltage $\underline{V}_{RX}$ across the receiver load Z_L is found using a voltage divider on the receiver equivalent circuit (Figure 1.1b):

$$\underline{V}_{RX} = \underline{V}_{oc} \frac{Z_L}{Z_a + Z_L} \tag{1.30}$$

With perfect matching ($Z_L = Z_a^*$) and a resonant dipole ($Z_a \approx R_a$), the total impedance in the receiver circuit is $Z_a + Z_L \approx R_a + R_a = 2R_a$. The expression for $\underline{V}_{RX}$ simplifies to:

$$\underline{V}_{RX} \approx \frac{\underline{V}_{oc}}{2} \tag{1.31}$$

Substituting the expression for $\underline{V}_{oc}$ gives the final relationship between the received and transmitted voltages:

$$\underline{V}_{RX} = j\frac{\lambda Z_0}{8\pi^2 R_a c\tau}\underline{V}_{TX}e^{-j2\pi f_c \tau} \tag{1.32}$$

Chapter 2

Chapter 3