Question 4: Path Loss, Shadowing, and Small-Scale Fading

Synthesizing a Complete Channel Model

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Outline

Introduction: The Three Layers of Propagation

Deconstructing Received Power Variations

- The power received by a mobile user is the result of three distinct physical phenomena acting on different spatial scales.
- To build a complete and realistic channel model, we must understand and model each of these components separately before combining them.
  - **Path Loss**: Large-scale average power decay with distance.
  - **Shadowing**: Medium-scale variations due to large obstacles.
  - **Small-Scale Fading**: Rapid, small-scale fluctuations from multipath interference.

Components of Received Power Variation

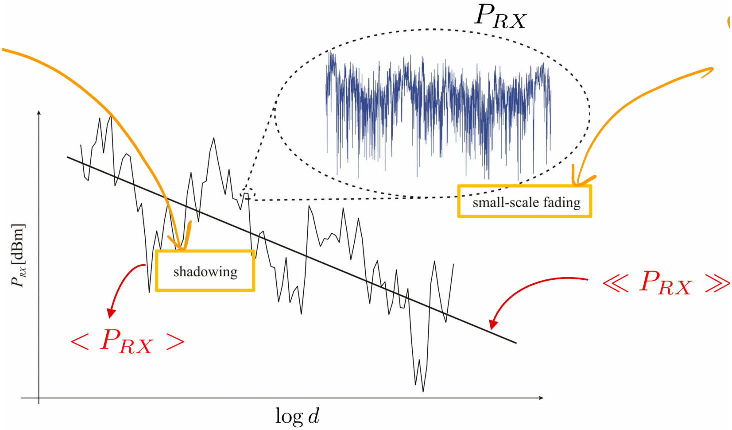

Figure: Illustration of path loss, shadowing, and small-scale fading.

Component 1: Path Loss Canonical Models

Defining the Large-Scale Trend

- Path loss describes the average attenuation of signal power as a function of the distance d between the transmitter and receiver.
- It represents the mean received power, denoted as $\ll P_{RX} \gg$.
- **Hypothesis**: We assume that the environment's large-scale properties are statistically homogeneous.
- This allows us to use a simple, empirically-validated mathematical form known as the **canonical path loss model**.

The Canonical Model Formulation

- The model states that the average received power in dBm decays linearly with the logarithm of the distance.

$$\ll P_{RX}(d) \gg [\text{dBm}] = \ll P_{RX}(d_0) \gg [\text{dBm}] - 10n \log_{10}\left(\frac{d}{d_0}\right)$$

The Canonical Model Formulation

- Key Parameters:
  - d_0: A reference distance, chosen in the far-field of the antenna.
  - n: The **path loss exponent**, which characterizes the environment.
- In terms of path loss $L(d) = P_{TX} - \ll P_{RX}(d) \gg$:

$$L(d)[\text{dB}] = L(d_0)[\text{dB}] + 10n \log_{10}\left(\frac{d}{d_0}\right)$$

Physical Origin of the Path Loss Exponent

- The value of n is determined by the dominant propagation mechanism.
- **Free Space ($n = 2$)**: Derived from the Friis formula, where power density decreases with the surface area of a sphere $(1/d^2)$.

$$P_{RX}(d) = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d}\right)^2 \implies \ll P_{RX} \gg \propto d^{-2}$$

Physical Origin of the Path Loss Exponent

- **Over-the-Ground ($n = 4$)**: At large distances in the two-ray model, destructive interference between the direct and ground-reflected paths leads to a much faster power decay.

$$P_{RX}(d) \approx P_{TX} G_{TX} G_{RX} \frac{h_{TX}^2 h_{RX}^2}{d^4} \implies \ll P_{RX} \gg \propto d^{-4}$$

- Other environments (urban, indoor) have values of n between 2 and 6, determined empirically or through complex physical models.

Component 2: Shadowing Statistical Model

Modeling Medium-Scale Variations

- The path loss model gives the average power over all possible locations at a distance d.
- In reality, large obstacles like buildings or hills cause the *local* average power, $< P_{RX} >$, to deviate from this global average. This is **shadowing** or slow fading.
- **Hypothesis**: The shadowing effect is a random process.
- **Observation**: Numerous measurement campaigns have shown that the variations of $< P_{RX} >$ in dB follow a **Normal (Gaussian) distribution**.

The Log-Normal Model

- The local average received power is modeled as the mean power from the path loss model plus a random variable.

$$< P_{RX} > (d)[\text{dBm}] = \ll P_{RX} \gg [\text{dBm}] - L_{\sigma_L}$$

- L_{σ_L} is a zero-mean Gaussian random variable with standard deviation σ_L.

$$L_{\sigma_L} \sim \mathcal{N}(0, \sigma_L^2)$$

The Log-Normal Model

- The parameter σ_L is the **shadowing variability** (in dB), which depends on the environment's clutter (e.g., 4 dB for open areas, up to 10 dB for dense urban).
- Since the power in dB is Gaussian, the power in linear units (Watts) follows a **log-normal distribution**.

Component 3: Small-Scale Fading Models

Modeling Small-Scale Variations

- Even within a small "local area" where $< P_{RX} >$ is constant, the instantaneous power P_{RX} fluctuates rapidly as the receiver moves over distances of about half a wavelength.
- This **small-scale fading** is caused by the constructive and destructive interference of multiple signal copies (Multipath Components or MPCs) arriving from different directions.

Modeling Small-Scale Variations

- The narrowband channel is modeled by a single complex coefficient $h(t)$:

$$y(t) = h(t)x(t)$$

$$h(t) = \sum_{n=1}^{N} a_n e^{j\Phi_n(t)}$$

where a_n and $\Phi_n(t)$ are the amplitude and phase of the n-th MPC.

The Rayleigh Fading Model (NLOS)

- **Hypothesis**:
  - There is a large number of MPCs ($N \to \infty$).
  - There is no dominant (Line-of-Sight) path; all MPCs have comparable amplitudes.
  - The phases of the MPCs, Φ_n, are independent and uniformly distributed in $[0, 2\pi)$.

The Rayleigh Fading Model (NLOS)

- **Derivation**: By the Central Limit Theorem, the channel coefficient $h(t) = X(t) + jY(t)$ becomes a zero-mean complex Gaussian random variable.
- The envelope $|h(t)| = \sqrt{X^2 + Y^2}$ follows a **Rayleigh distribution**.

$$p(|h|) = \frac{|h|}{\sigma^2} \exp\left(-\frac{|h|^2}{2\sigma^2}\right)$$

where $2\sigma^2 = \mathbb{E}[|h|^2]$ is the average power of the channel, which is given by the local average power $< P_{RX} >$.

The Rician Fading Model (LOS)

- **Hypothesis**:
  - There is one dominant, stable (LOS) component, plus a large number of weaker, scattered components.

$$h(t) = \underbrace{Ae^{j\theta}}_{\text{Dominant}} + \underbrace{\sum_{n=1}^{N} a_n e^{j\Phi_n}}_{\text{Scattered}}$$

The Rician Fading Model (LOS)

- **Derivation**: The channel coefficient $h(t)$ is now a complex Gaussian variable with a **non-zero mean**.
- The envelope $|h(t)|$ follows a **Rician distribution**.

$$p(|h|) = \frac{|h|}{\sigma^2} \exp\left(-\frac{|h|^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{|h|A}{\sigma^2}\right)$$

- This distribution is characterized by the **Rician K-factor**, the ratio of dominant to scattered power:

$$K = \frac{A^2}{2\sigma^2}$$

- As $K \to 0$, the Rician distribution converges to the Rayleigh distribution.

Synthesis: Building the Complete Model

Step-by-Step Construction

- We can now synthesize the complete model shown in Figure 4.8 by combining the three components sequentially.
- **Goal**: Generate a realistic series of received power values $P_{RX}(d)$ as a function of distance d.

Procedure

1. Define the large-scale trend with a path loss model.
2. Add medium-scale variations by introducing shadowing.
3. Superimpose small-scale fluctuations using a fading model.

Step 1: Path Loss Foundation

- First, we calculate the mean received power $\ll P_{RX} \gg$ over the entire distance range using a canonical model.
- We choose an environment, which defines the path loss exponent n. For example, an urban micro-cell with $n = 3.5$.
- We compute the mean power at a reference distance d_0.

$$\ll P_{RX}(d) \gg [\text{dBm}] = \ll P_{RX}(d_0) \gg -10n \log_{10}\left(\frac{d}{d_0}\right)$$

- This gives us the straight line (on a log-log plot) that represents the large-scale average.

Step 1: Path Loss Foundation

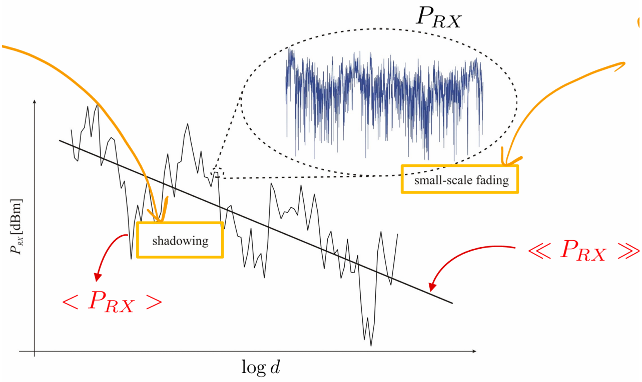

Figure: The path loss model provides the mean trend line.

Step 2: Adding Shadowing

- Next, we generate the local average power $< P_{RX} >$ by adding shadowing.
- We choose a shadowing variability σ_L appropriate for the environment (e.g., $\sigma_L = 8$ dB for urban).
- For each point (or local area) along the path, we draw a random number from a zero-mean Gaussian distribution $\mathcal{N}(0, \sigma_L^2)$ and subtract it from the path loss mean.

$$< P_{RX} > (d)[\text{dBm}] = \ll P_{RX}(d) \gg [\text{dBm}] - L_{\sigma_L}$$

- This creates the slowly varying signal that "rides" on top of the path loss trend.

Step 2: Adding Shadowing

pictures/synthesis-step2.png

Step 3: Superimposing Small-Scale Fading

- Finally, we generate the instantaneous received power P_{RX} by adding small-scale fading.
- At each point, the local average power $< P_{RX} >$ defines the average power of the fading distribution.

$$\mathbb{E}[|h|^2] \propto < P_{RX} > \text{[Watts]}$$

- We draw a random variable $|h|$ from either a Rayleigh (for NLOS) or Rician (for LOS) distribution, scaled by $< P_{RX} >$.
- The instantaneous power is then $P_{RX} = |h|^2 P_{TX}$.
- This process creates the rapid fluctuations that are characteristic of multipath interference.

Step 3: Superimposing Small-Scale Fading

pictures/synthesis-step3.png

Conclusion

Conclusion

A Unified Statistical Model

The total received signal power is a composite of three distinct statistical processes, each modeling a different physical scale of interaction:

- **Path Loss ($L(d)$)**: A deterministic function of distance, setting the mean power level.
- **Shadowing (L_{σ_L})**: A log-normal (Gaussian in dB) random process modeling large-scale blockages.
- **Small-Scale Fading ($|h|^2$)**: A Rayleigh or Rician random process modeling multipath interference.

Conclusion

A Unified Statistical Model

[0.8] By systematically combining these three layers—starting with the path loss trend, adding log-normal shadowing, and finally superimposing Rayleigh/Rician fading—we can construct a comprehensive and statistically accurate model of a wireless channel. This synthesized model is fundamental for simulating and predicting the performance of any real-world communication system.

Thank You