ELEC-H415

Modeling A Vehicle-to-Vehicle Communication Channel in an Urban Environment

Author :
Cédric Sipakam

Teacher :
Philippe de Doncker

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Introduction

For their first year of master in Electrical Engineering and Information Technology in the Bruface program, students were asked to do a project for their communication channels course. The project consists of modelling a vehicle-to-vehicle wireless communication channel. The analysis is grounded in an urban canyon scenario where two vehicles, equipped with vertical $\lambda/2$ dipole antennas, travel along the center of a 20-meter wide street surrounded by building with a relative permittivity of $\epsilon_r = 4$. The distance d between the vehicles is variable and can be maximum $d_{max} = 1km$

The communication system operated at a carrier frequency of, $f_c = 5.9GHz$ with a bandwidth of $B_{RF} = 1000MHz$ and a transmitter power of $P_{TX} = 0.1W$. This report develops the channel model from fundamental principles, progressing through narrowband and wideband analyses of both Line-of-Sight and full multipath conditions, with an emphasis on the mathematical derivations and physical interpretation of the results.

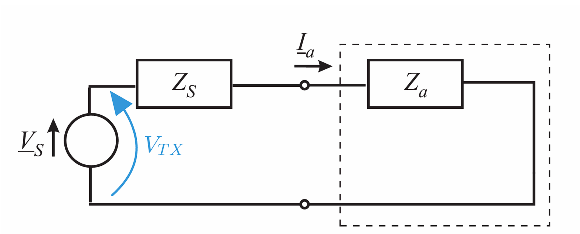
This Goes further by ****Put the new thing that the teacher talked about to have extra points****

1

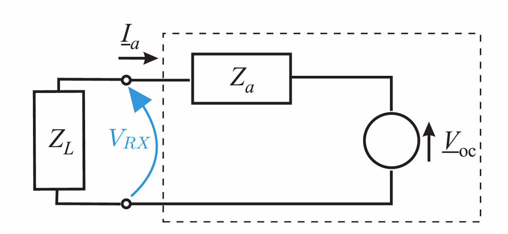
Theoretical Preliminaries

1.1 Theoretical Background

An antenna is an electrical component that acts as a transducer between a guided electrical signal and a propagating electromagnetic wave. To analyze its behavior within an electrical system, an equivalent circuit was drawn. In transmission mode, a signal source feeds the antenna, which has an impedance Z_a. This impedance consists of a radiation resistance R_{ar}, representing the power radiated into space, and a loss resistance R_{al}, representing ohmic losses. In reception mode, an incoming electromagnetic wave induces an open-circuit voltage V_{oc} at the antenna's terminals, which then delivers a signal to the receiver's load impedance Z_L.



(a) Transmit antenna equivalent circuit, showing the source voltage V_S, source impedance Z_S, transmit voltage V_{TX}, and antenna current I_a.



(b) Receive antenna equivalent circuit, showing the induced open-circuit voltage V_{oc}, the load impedance Z_L, and the received voltage V_{RX}.

Figure 1.1: Equivalent circuits for the transmit and receive antennas

The equivalent circuits for the transmitter and receiver are drawn in Figure 1.1. In Figure 1.1a, can be seen the equivalent circuit of the transmitter. The voltage source V_S with its internal impedance Z_S drives the antenna, resulting in a current I_a and a voltage V_{TX} at the antenna's input terminals. At the receiver (Figure 1.1b), the incoming wave induces an open-circuit voltage V_{oc}, which in turn produces the received voltage V_{RX} across the load impedance Z_L.

For this project, it was considered that the electronics are perfectly matched to the antennas. This implies that for the transmitter, the source impedance is the complex conjugate of the antenna impedance:

$$Z_S = Z_a^* \tag{1.1}$$

and for the receiver, the load impedance is the complex conjugate of the antenna impedance:

$$Z_L = Z_a^* \tag{1.2}$$

This consideration ensures maximum power transfer.

1.1.1 Antenna Effective Height

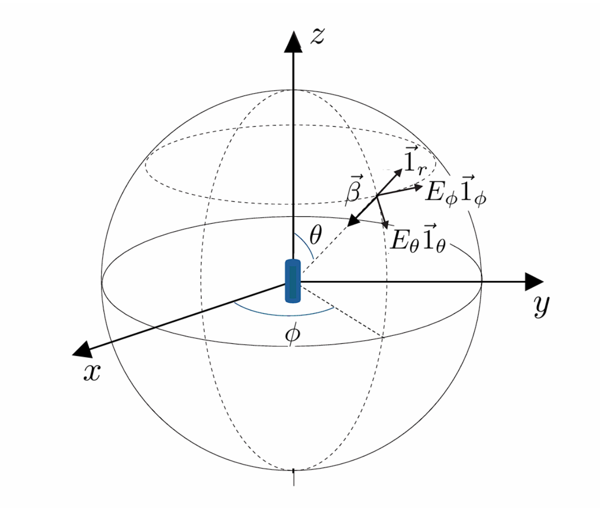


Figure 1.2: Illustration of the vertical dipole antenna and coordinate axes.

The effective height $\vec{h}_e$ of an antenna links the circuit domain to the electromagnetic wave domain. It is derived from the current distribution $\vec{J}(\vec{r}\,')$ on the antenna when transmitting with an input current $\underline{I}_a$:

$$\vec{h}_e(\theta,\phi) = \frac{1}{\underline{I}_a}\int_{\mathcal{D}} \vec{J}(\vec{r}\,')e^{j\beta(\vec{r}\,'\cdot\vec{1}_r)}dV' \tag{1.3}$$

where $\mathcal{D}$ is the volume of the antenna, $\vec{1}_r$ is the unit vector in the direction of radiation (θ, ϕ), and β is the wavenumber:

$$\beta = \frac{2\pi}{\lambda} \tag{1.4}$$

For a thin, vertical half-wave dipole antenna of length $L = \frac{\lambda}{2}$ oriented along the z-axis and centered at the origin (Figure 1.2), the current flows only in the z-direction. The current distribution is given by:

$$\vec{J}(\vec{r}\,') = \underline{I}_a \cos(\beta z')\delta(x')\delta(y')\vec{1}_z, \quad \text{for } -\frac{\lambda}{4} \leq z' \leq \frac{\lambda}{4} \tag{1.5}$$

where $\delta(x')$ and $\delta(y')$ are Dirac's deltas.

The volume integral reduces to a line integral along the z-axis. The dot product in the exponent simplifies to:

$$\vec{r}\,' \cdot \vec{1}_r = z' \cos\theta \tag{1.6}$$

Substituting this into Equation 1.3 gives:

$$\vec{h}_e(\theta, \phi) = \left(\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos(\beta z') e^{j\beta z' \cos\theta} dz' \right) \vec{1}_z \tag{1.7}$$

The integral in Equation 1.7 is solved using Euler's formula:

$$\cos(\beta z') = \frac{1}{2}(e^{j\beta z'} + e^{-j\beta z'}) \tag{1.8}$$

$$\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos(\beta z') e^{j\beta z' \cos\theta} dz' = \frac{1}{2} \int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \left(e^{j\beta z'(1+\cos\theta)} + e^{j\beta z'(\cos\theta - 1)} \right) dz' \tag{1.9}$$

$$= \frac{1}{2j\beta} \left[\frac{e^{j\beta z'(1+\cos\theta)}}{1+\cos\theta} + \frac{e^{j\beta z'(\cos\theta-1)}}{\cos\theta - 1} \right]_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \tag{1.10}$$

With $\beta = \frac{2\pi}{\lambda}$, the term $\frac{\beta\lambda}{4}$ simplifies to $\frac{\pi}{2}$. Evaluating at the limits yields:

$$= \frac{1}{2j\beta} \left[\frac{e^{j\frac{\pi}{2}(1+\cos\theta)} - e^{-j\frac{\pi}{2}(1+\cos\theta)}}{1+\cos\theta} - \frac{e^{j\frac{\pi}{2}(1-\cos\theta)} - e^{-j\frac{\pi}{2}(1-\cos\theta)}}{1-\cos\theta} \right] \tag{1.11}$$

Using the definition of sine: $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$$= \frac{1}{\beta} \left[\frac{\sin\left(\frac{\pi}{2}(1+\cos\theta)\right)}{1+\cos\theta} - \frac{\sin\left(\frac{\pi}{2}(\cos\theta - 1)\right)}{1-\cos\theta} \right] \tag{1.12}$$

$$= \frac{1}{\beta} \left[\frac{\sin(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta)}{1+\cos\theta} + \frac{\sin(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta)}{1-\cos\theta} \right] \tag{1.13}$$

Applying the identities $\sin(\frac{\pi}{2} + x) = \cos(x)$ and $\sin(\frac{\pi}{2} - x) = \cos(x)$:

$$= \frac{1}{\beta} \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{1+\cos\theta} + \frac{\cos(\frac{\pi}{2}\cos\theta)}{1-\cos\theta} \right] \tag{1.14}$$

$$= \frac{\cos(\frac{\pi}{2}\cos\theta)}{\beta} \left[\frac{(1-\cos\theta) + (1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \right] = \frac{2\cos(\frac{\pi}{2}\cos\theta)}{\beta \sin^2\theta} \tag{1.15}$$

Substituting $\beta = \frac{2\pi}{\lambda}$, the final result of the integral is:

$$\frac{\lambda}{\pi} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \tag{1.16}$$

The effective height for the vertical half-wave dipole is therefore:

$$\vec{h}_e(\theta,\phi) = \frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\vec{1}_z \tag{1.17}$$

This vector is oriented along the z-axis. For reception, the induced voltage depends on the component of the effective height that is transverse (perpendicular) to the direction of wave propagation, $\vec{1}_r$. This transverse component is denoted $\vec{h}_{e\perp}$. The Cartesian unit vector $\vec{1}_z$ is expressed in spherical coordinates as:

$$\vec{1}_z = \cos\theta\vec{1}_r - \sin\theta\vec{1}_\theta \tag{1.18}$$

The transverse part, $\vec{h}_{e\perp}$, consists of the components in the $\vec{1}_\theta$ and $\vec{1}_\phi$ directions. Substituting the spherical representation of $\vec{1}_z$ into Equation 1.17 and retaining only the transverse component gives:

$$\vec{h}_{e\perp}(\theta,\phi) = -\frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}(\sin\theta\vec{1}_\theta) = -\frac{\lambda}{\pi}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\vec{1}_\theta \tag{1.19}$$

This is the general expression for the transverse effective height of a vertical $\frac{\lambda}{2}$ dipole. In the horizontal plane, where $\theta = \frac{\pi}{2}$, the expression simplifies significantly. The transverse effective height in the horizontal plane is therefore:

$$\boxed{\vec{h}_{e\perp}\left(\theta = \frac{\pi}{2},\phi\right) = -\frac{\lambda}{\pi}\vec{1}_\theta} \tag{1.20}$$

This final expression indicates that in the horizontal plane, the antenna's effective height has a constant magnitude of $\frac{\lambda}{\pi}$, is constant for all ϕ, and is oriented in the $\vec{1}_\theta$ direction.

1.1.2 Emitted Electric Field in Free-Space

The electric field radiated by an antenna is given by:

$$\underline{\vec{E}}(\vec{r}) = -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\vec{h}_{e\perp}(\theta,\phi) \tag{1.21}$$

Substituting the transverse effective height for the horizontal plane (Equation 1.20) into the general expression gives:

$$\underline{\vec{E}}(r,\pi/2,\phi) = -j\omega\underline{I}_a\frac{\mu_0}{4\pi}\frac{e^{-j\beta r}}{r}\left(-\frac{\lambda}{\pi}\vec{1}_\theta\right) = j\underline{I}_a\frac{\omega\mu_0\lambda}{4\pi^2}\frac{e^{-j\beta r}}{r}\vec{1}_\theta \tag{1.22}$$

Expressing this field in terms of circuit parameters involves the substitutions $\omega = 2\pi f_c$, $\lambda = \frac{c}{f_c}$, $\mu_0 = \frac{Z_0}{c}$, and propagation delay $\tau = \frac{r}{c}$. The exponential term becomes:

$$e^{-j\beta r} = e^{-j\frac{2\pi}{\lambda}c\tau} = e^{-j2\pi f_c\tau} \tag{1.23}$$

The electric field is then:

$$\underline{\vec{E}} = j\underline{I}_a \frac{(2\pi f_c)(\frac{Z_0}{c})(\frac{c}{f_c})}{4\pi^2} \frac{e^{-j2\pi f_c \tau}}{c\tau} \vec{1}_\theta = j\underline{I}_a \frac{Z_0}{2\pi c\tau} e^{-j2\pi f_c \tau} \vec{1}_\theta \tag{1.24}$$

A half-wave dipole at its resonant frequency has an almost purely real impedance, meaning its reactance is negligible ($X_a \approx 0$), so $Z_a = R_a + jX_a \approx R_a$. Under the perfect matching condition ($Z_S = Z_a^*$), the total impedance in the transmitter circuit is $Z_S + Z_a \approx Z_a^* + Z_a \approx 2R_a$. The antenna current is then related to the transmit voltage by:

$$\underline{I}_a = \frac{\underline{V}_{TX}}{2R_a} \tag{1.25}$$

Substituting this for $\underline{I}_a$:

$$\boxed{\underline{\vec{E}} = j\frac{Z_0}{4\pi R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \vec{1}_\theta} \tag{1.26}$$

1.1.3 Received Voltage in Free-Space

The open-circuit voltage $\underline{V}_{oc}$ induced at the terminals of a receiving antenna is given by the dot product of its effective height and the incident electric field:

$$\underline{V}_{oc} = -\vec{h}_{e\perp}^{RX} \cdot \underline{\vec{E}} \tag{1.27}$$

The receiving antenna is also a vertical $\frac{\lambda}{2}$ dipole, so its transverse effective height in the horizontal plane is given by Equation 1.20. The incident electric field is given by Eq. 1.26. The dot product is:

$$\underline{V}_{oc} = -\left(-\frac{\lambda}{\pi}\vec{1}_\theta\right) \cdot \left(j\frac{Z_0}{4\pi R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \vec{1}_\theta\right) \tag{1.28}$$

$$= j\frac{\lambda Z_0}{4\pi^2 R_a c\tau} \underline{V}_{TX} e^{-j2\pi f_c \tau} \tag{1.29}$$

The voltage $\underline{V}_{RX}$ across the receiver load Z_L is found using a voltage divider on the receiver equivalent circuit (Figure 1.1b):

$$\underline{V}_{RX} = \underline{V}_{oc} \frac{Z_L}{Z_a + Z_L} \tag{1.30}$$

With perfect matching ($Z_L = Z_a^*$) and a resonant dipole ($Z_a \approx R_a$), the total impedance in the receiver circuit is $Z_a + Z_L \approx R_a + R_a = 2R_a$. The expression for $\underline{V}_{RX}$ simplifies to:

$$\underline{V}_{RX} \approx \frac{\underline{V}_{oc}}{2} \tag{1.31}$$

Substituting the expression for $\underline{V}_{oc}$ gives the final relationship between the received and transmitted voltages:

$$\boxed{\underline{V}_{RX} = j\frac{\lambda Z_0}{8\pi^2 R_a c\tau}\underline{V}_{TX}e^{-j2\pi f_c \tau}} \tag{1.32}$$

2

Narrowband Analysis - Line-of-Sight Channel

The analysis begins with the simplest communication scenario: a direct Line-of-Sight (LOS) path between the transmitter TX and the receiver RX. A narrowband analysis is conducted, which assumes that the signal's bandwidth is much smaller than the channel's coherence bandwidth. This simplification allows the channel to be characterized by one complex coefficient.

2.1 Theoretical Background

The physical channel can be described by its time-variant impulse response, which for a set of N multipath components is:

$$h(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)\delta(\tau - \tau_n) \tag{2.1}$$

where $\alpha_n(t)$ and τ_n are the complex amplitude and propagation delay of the n-th path, respectively.

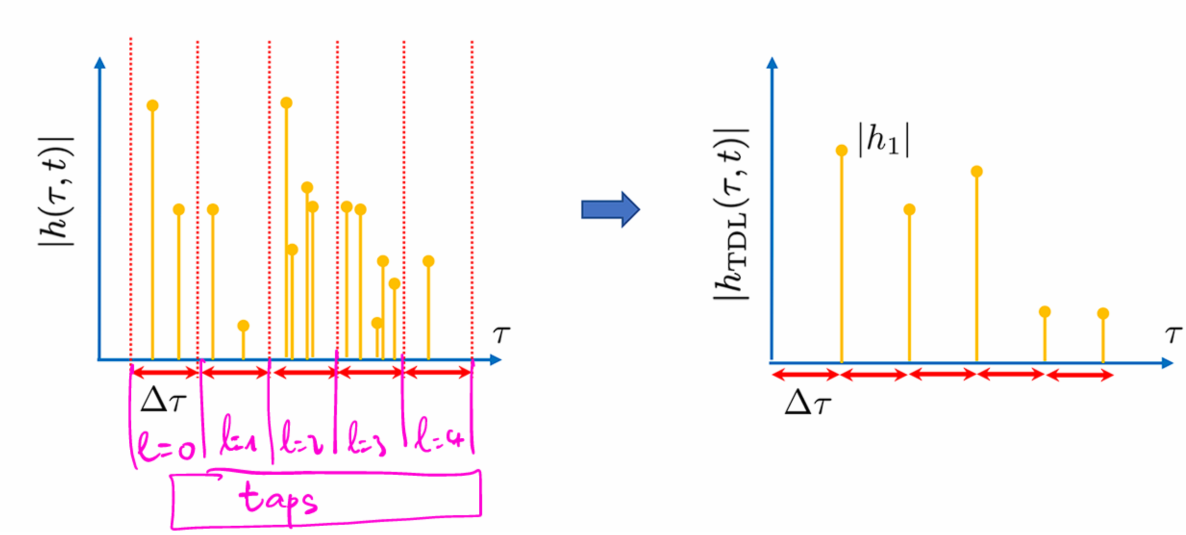


Figure 2.1: Physical impulse response and TDL model under the Uncorrelated Scattering assumption

A practical communication system has a finite bandwidth B, which limits its ability to resolve paths arriving at different times. The system's time resolution is $\Delta\tau = 1/B$.

The received signal is thus defined by:

$$y(t) = \sum_{l} x(t - l\Delta\tau) \int_{0}^{\infty} h(\tau, t)\,\mathrm{sinc}(B(\tau - l\Delta\tau))d\tau \tag{2.2}$$

From there, can be defined the complex gain of the l-th tap:

$$\begin{align}
h_l(t) &= \int_{0}^{\infty} h(\tau, t)\,\mathrm{sinc}(B(\tau - l\Delta\tau))d\tau \tag{2.3}\\
&= \int_{0}^{\infty} (\sum_{n=1}^{N} \alpha_n(t)\delta(\tau - \tau_n)) \cdot \mathrm{sinc}(B(\tau - l\Delta\tau))d\tau \tag{2.4}\\
&= \sum_{n=1}^{N} \alpha_n(t)\,\mathrm{sinc}\left(B\left(\tau_n - l\Delta\tau\right)\right) \tag{2.5}
\end{align}$$

$$\implies \boxed{h_l(t) \approx \sum_{\tau_n \in \mathrm{tap}\, l} \alpha_n(t)} \tag{2.6}$$

Because, sinc(.) maximizes the amplitude of the components of the l-th tap, and the components of other taps are drastically attenuated. The components of the l-th tap appear to not be delayed between each other.

The physical limitation of the finite bandwidth of the system leads to the Tapped Delay Line model. The impulse response of the TDL model is a discrete-time representation of the physical channel as shown in Figure 2.1:

$$h_{TDL}(\tau, t) = \sum_{l=0}^{L} h_l(t)\delta(\tau - l\Delta\tau) \tag{2.7}$$

The condition for a narrowband channel is that the signal bandwidth B is much smaller than the channel's coherence bandwidth Δf_c. The coherence bandwidth is inversely proportional to the channel's delay spread, $\sigma_\tau = max|\tau_i - \tau_j|$. The narrowband condition is thus expressed as:

$$B \ll \Delta f_c \approx \frac{1}{\sigma_\tau} \implies \Delta\tau \gg \sigma_\tau \tag{2.8}$$

This inequality means the system's time resolution is much larger than the delay spread. From the receiver's perspective, all MPCs arrive at effectively the same time. Consequently, all MPCs fall into the first tap ($l = 0$) of the TDL model. The summation in Equation 2.7 therefore reduces to one term for $l = 0$:

$$h_{TDL}(\tau, t) = h_0(t)\delta(\tau) \tag{2.9}$$

where the tap gain $h_0(t)$ is the sum of all individual path gains:

$$\boxed{h_0(t) = \sum_{n=1}^{N} \alpha_n(t)} \quad N \text{ being the total number of MPCs} \tag{2.10}$$

2.2 Analysis and Mathematical Derivations

2.2.1 Antenna Gain

The gain of an antenna, $G(\theta, \phi)$, quantifies its ability to concentrate radiated power in a specific direction. It is defined by the general formula:

$$G(\theta, \phi) = \frac{\pi Z_0}{R_a} \frac{|\vec{h}_{e\perp}(\theta, \phi)|^2}{\lambda^2} \tag{2.11}$$

where Z_0 is the impedance of free space, R_a is the antenna's radiation resistance, λ is the wavelength, and $\vec{h}_{e\perp}(\theta, \phi)$ is the transverse component of the antenna's effective height.

As derived earlier (Equation 1.20), the transverse effective height for a vertical half-wave dipole in the horizontal plane ($\theta = \frac{\pi}{2}$) is:

$$\vec{h}_{e\perp}\left(\frac{\pi}{2}, \phi\right) = -\frac{\lambda}{\pi}\vec{1}_\theta \tag{2.12}$$

The magnitude squared of this vector is therefore:

$$\left|\vec{h}_{e\perp}\left(\frac{\pi}{2}, \phi\right)\right|^2 = \left|-\frac{\lambda}{\pi}\vec{1}_\theta\right|^2 = \frac{\lambda^2}{\pi^2} \tag{2.13}$$

Substituting this result into the general gain formula (Equation 2.11) provides the expression for the gain of a lossless half-wave dipole in the horizontal plane:

$$G = G\left(\frac{\pi}{2}, \phi\right) = \frac{\pi Z_0}{R_a}\frac{1}{\lambda^2}\left(\frac{\lambda^2}{\pi^2}\right) = \frac{\pi Z_0 \lambda^2}{\pi^2 R_a \lambda^2} \tag{2.14}$$

Simplifying this expression yields the final result:

$$G = \frac{Z_0}{\pi R_a} \tag{2.15}$$

2.2.2 Impulse Response $h(\tau)$

For a single, time-invariant Line-of-Sight path, the channel impulse response is characterized by a single complex amplitude, α_1, and a propagation delay, τ_1. The impulse response is thus expressed as:

$$h(\tau) = \alpha_1 \delta(\tau - \tau_1) \tag{2.16}$$

The central task is to determine the complex amplitude α_1. This can be achieved by relating the circuit-level voltages at the transmitter and receiver. The relationship between the transmitted power P_{TX} and the received power P_{RX} is defined by the squared magnitude of the complex amplitude:

$$P_{RX} = |\alpha_1|^2 P_{TX} \tag{2.17}$$

The transmitted and received powers can be expressed in terms of the terminal voltages $\underline{V}_{TX}$ and $\underline{V}_{RX}$ and the antenna radiation resistance R_a, assuming perfectly matched conditions:

$$P_{TX} = \frac{|\underline{V}_{TX}|^2}{8R_a} \quad \text{and} \quad P_{RX} = \frac{|\underline{V}_{RX}|^2}{2R_a} \tag{2.18}$$

Substituting these power definitions into the power relationship gives:

$$\frac{|\underline{V}_{RX}|^2}{2R_a} = |\alpha_1|^2 \frac{|\underline{V}_{TX}|^2}{8R_a} \tag{2.19}$$

Solving for $|\alpha_1|^2$ yields:

$$|\alpha_1|^2 = \frac{8R_a}{2R_a}\frac{|\underline{V}_{RX}|^2}{|\underline{V}_{TX}|^2} = 4\frac{|\underline{V}_{RX}|^2}{|\underline{V}_{TX}|^2} \tag{2.20}$$

This implies a relationship between the magnitudes: $|\alpha_1| = 2\frac{|\underline{V}_{RX}|}{|\underline{V}_{TX}|}$. This motivates defining the complex amplitude α_1 directly from the complex voltage ratio:

$$\alpha_1 = 2\frac{\underline{V}_{RX}}{\underline{V}_{TX}} \tag{2.21}$$

The relationship between the received and transmitted voltages for a free-space LOS path was derived previously (Equation 1.32) as:

$$\underline{V}_{RX} = j\frac{\lambda Z_0}{8\pi^2 R_a c\tau_1}\underline{V}_{TX} e^{-j2\pi f_c \tau_1} \tag{2.22}$$

Substituting this into Equation 2.21 gives the expression for α_1:

$$\alpha_1 = 2\left(j\frac{\lambda Z_0}{8\pi^2 R_a c\tau_1} e^{-j2\pi f_c \tau_1}\right) = j\frac{\lambda Z_0}{4\pi^2 R_a c\tau_1} e^{-j2\pi f_c \tau_1} \tag{2.23}$$

Replacing the product of the speed of light c and the delay τ_1 with the distance $d_1 = c\tau_1$, the complex amplitude is:

$$\alpha_1 = j\frac{\lambda Z_0}{4\pi^2 R_a d_1} e^{-j2\pi f_c \tau_1} \tag{2.24}$$

The channel impulse response for the LOS path is therefore:

$$h(\tau) = \left(j\frac{\lambda Z_0}{4\pi^2 R_a d_1} e^{-j2\pi f_c \tau_1}\right)\delta(\tau - \tau_1) \tag{2.25}$$

where $\tau_1 = d_1/c$.

2.2.3 Transfer Function $H(f)$

The transfer function $H(f)$ is obtained by taking the Fourier transform of the impulse response $h(\tau)$. The defining integral is:

$$H(f) = \mathcal{FT}\{h(\tau)\} = \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau = \int_{-\infty}^{\infty} (\alpha_1 \delta(\tau - \tau_1))\, e^{-j2\pi f\tau} d\tau \tag{2.26}$$

Applying the sifting property of the Dirac delta function, which states that $\int g(x)\delta(x-a)dx = g(a)$, the integral simplifies to:

$$H(f) = \alpha_1 e^{-j2\pi f\tau_1} \tag{2.27}$$

Substituting the derived expression for α_1 from Equation 2.24:

$$H(f) = \left(j\frac{\lambda Z_0}{4\pi^2 R_a d_1} e^{-j2\pi f_c\tau_1}\right) e^{-j2\pi f\tau_1} \tag{2.28}$$

$$\Rightarrow H(f) = j\frac{\lambda Z_0}{4\pi^2 R_a d_1} e^{-j2\pi(f_c+f)\tau_1} \tag{2.29}$$

This function shows that the channel introduces a phase shift that is linear with the baseband frequency f, which corresponds to the time delay τ_1. The magnitude $|H(f)|$ is constant across all frequencies.

2.2.4 Narrowband Transfer Function h_{NB}

As said mentioned in Equation 2.9, the narrowband is defined such that all MPCs fall into a single tap:

$$h_{NB} = \mathcal{FT}\{h_{TDL}(t,\tau)\} = \mathcal{FT}\{h_0(t)\delta(\tau)\} \tag{2.30}$$

$$= \int_{-\infty}^{\infty} h_0(t)e^{-j2\pi f\tau}\delta(\tau)d\tau \tag{2.31}$$

$$= h_0(t)e^{-j2\pi f\cdot 0} \tag{2.32}$$

$$\implies h_{NB} = h_0(t) \tag{2.33}$$

As found in Equation 2.10, for a single LOS path, the narrowband channel transfer function is equal to the complex amplitude α_1:

$$h_{NB} = \sum_{n=1}^{N=1} \alpha_n(t) = \alpha_1 \tag{2.34}$$

Using the result from Equation 2.24, the narrowband transfer function is:

$$\boxed{h_{NB} = j\frac{\lambda Z_0}{4\pi^2 R_a d_1} e^{-j2\pi f_c\tau_1}} \tag{2.35}$$

The channel is thus represented by one complex number, which scales and rotates the transmitted signal.

2.2.5 Received Power P_{RX}

The received power can now be calculated using the derived complex amplitude α_1 and its relationship to the power gain, $P_{RX} = |\alpha_1|^2 P_{TX}$. First, the magnitude squared of α_1 is computed:

$$|\alpha_1|^2 = \left| j \frac{\lambda Z_0}{4\pi^2 R_a d_1} e^{-j2\pi f_c \tau_1} \right|^2 = \left(\frac{\lambda Z_0}{4\pi^2 R_a d_1} \right)^2 \tag{2.36}$$

Substituting this into the power equation gives the received power as a function of the transmitted power:

$$P_{RX} = \left(\frac{\lambda Z_0}{4\pi^2 R_a d_1} \right)^2 P_{TX} \tag{2.37}$$

To demonstrate that this result is equivalent to the well-known Friis formula, the terms are rearranged. The expression is factored to isolate terms corresponding to the antenna gains:

$$P_{RX} = \frac{\lambda^2 Z_0^2}{16\pi^4 R_a^2 d_1^2} P_{TX} \tag{2.38}$$

$$= \left(\frac{Z_0^2}{\pi^2 R_a^2} \right) \left(\frac{\lambda^2}{16\pi^2 d_1^2} \right) P_{TX} \tag{2.39}$$

$$= \left(\frac{Z_0}{\pi R_a} \right) \left(\frac{Z_0}{\pi R_a} \right) \left(\frac{\lambda}{4\pi d_1} \right)^2 P_{TX} \tag{2.40}$$

Using the expression for the gain of a lossless half-wave dipole in the horizontal plane as derived in Equation 2.15, $G = Z_0/(\pi R_a)$, and assuming identical transmit and receive antennas ($G_{TX} = G_{RX} = G$):

$$P_{RX} = G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d_1} \right)^2 P_{TX} \tag{2.41}$$

This result is identical to the Friis transmission formula. This validates the entire derivation, confirming that the complex amplitude α_1 derived from the voltage ratio correctly predicts the power relationship under free-space LOS conditions.

2.3 Code Implementation and Methodology

(Placeholder) In the accompanying script, the derived equation for P_{RX} (Equation 2.41) is implemented to calculate the received power at various distances d_1. The script defines variables for the transmitted power (P_{TX}), antenna gains (G_{TX}, G_{RX}), wavelength (λ), and the distance vector (d_1). The final result is plotted as received power in dBm versus distance.

2.4 Interpretation of Results

The derivations confirm the principles of a simple LOS communication link.

- **Frequency-Flat Channel:** The single propagation path results in a transfer function $|H(f)|$ that is constant with frequency. This means the channel does not distort the signal's spectrum, a condition known as flat fading. This is a direct consequence of having no time dispersion (zero delay spread).

- **Validation of Friis' Formula:** The derivation, starting from the circuit-level voltage relationship to find the complex amplitude α_1, and then using it to find the received power, shows a result identical to the Friis' formula. This demonstrates a consistency between the low-level physical model (fields and circuits) and the high-level system model (power and gains).

Chapter 3