ELEC-H401

Design and Simulation of a DVB-C Transmission Chain

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Introduction

[TODO: CHANGE THE TENSE FROM PRESENT TO PAST]

This project focuses on designing and simulating a Digital Video Broadcasting Cable transmission chain using Matlab. The main goal is to model and analyze the components of a typical modem. The first part of this project includes establishing an optimal communication chain over an ideal Additive White Gaussian Noise channel, mapping the signal into symbols, Nyquist filtering, and evaluating the performance of the system through Bit Error Rate simulations for Quadrature Amplitude Modulation signals. The second part revolves around the design and assessment of time and frequency synchronization algorithms. This encompasses evaluating the impact of synchronization errors such as carrier phase and frequency offset and sample time shift. The algorithms used for this purpose are the Gardner Algorithm for time recovery, and a differential cross-correlator for joint frame and frequency acquisition. Afterwards, their effectiveness is demonstrated through convergence analysis and residual error evaluation (double check). The study also explores phase interpolation techniques to mitigate remaining phase drifts. Finally, the project aims to validate the simulated chain through real-life experimentation on a Hybrid Fiber-Coax setup using Adalm-Pluto hardware, and to compare the simulated performance with experimental observations.

1 Optimal Communication Chain over the Ideal Channel

[TODO: ADD MORE MATH LATER]

[TODO: MAYBE ADD HOW SOME SIMULATION PARAMETERS AFFECT THE SIMULATION (like taps for example)]

[TODO: MAKE THE AXIS NUMBER OF THE GRAPHS BIGGER]

1.1 Communication Chain

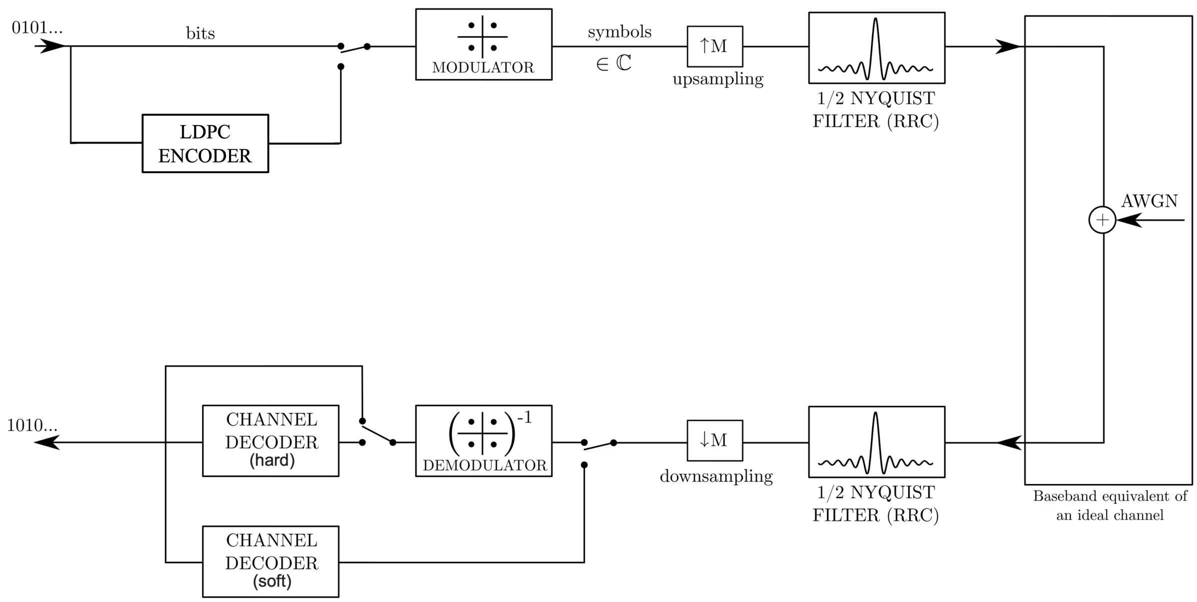

Figure 1: Block diagram of the communication system

The communication chain depicted in Figure 1 models the pipeline of a DVB-C transmitter and receiver at baseband. On the transmitter side, there is a generation of a bit-stream, which is then mapped to complex symbols from a chosen QAM modulation. These symbols are subsequently up-sampled and shaped by a half-root Nyquist filter to limit their bandwidth. On the receiver's side, the signal transmitted is matched-filtered with the same half-root Nyquist filter. The goal of this matched filter is to maximize the signal-to-noise ratio. The output of the matched filter is then down-sampled at the symbols instances, and then demapped to recover an estimate of the bit-stream transmitted.
In this project, the Low Density Parity Check Encoder is not implemented, and the algorithm for symbols mapping and demapping were provided.

1.2 Bit Generation and Symbol Mapping and Demapping

Symbols mapping is the process in which the modulator transforms a sequence of bits into a sequence of complex symbols. This helps improving the spectral efficiency as it allows more bits to be transmitted per symbol over a given bandwidth.
At the receiver end, symbol demapping is performed to convert noisy symbols back into an estimated

sequence of bits. This is done through Maximum Likelihood criterion, which aims to selecting the constellation symbol that is closest to the received sample in minimum Euclidean distance.

$$\tilde{\underline{s}}_m^{ML} = \max_{\underline{s}_m}(\ln p(\underline{r}|\underline{s}_m)) = \min_{\underline{s}_m}\left(\sum_{k=1}^{K}(r_k - s_{mk})^2\right) \quad (1)$$

Where:

- $\tilde{\underline{s}}_m^{ML}$ is the estimated symbol using the Maximum Likelihood criterion.
- $\underline{r} = [r_1, r_2, \ldots, r_K]^T$ is the received vector after demodulation
- $\underline{s}_m = [s_{m1}, s_{m2}, \ldots, s_{mK}]^T$ is the vector representing the m-th possible transmitted symbol.
- $p(\underline{r}|\underline{s}_m)$ is the conditional probability of receiving vector $\underline{r}$ given that symbol $\underline{s}_m$ was sent.
- $\sum_{k=1}^{K}(r_k - s_{mk})^2$ represents the squared Euclidean distance between the received vector $\underline{r}$ and the symbol vector $\underline{s}_m$.

1.3 Nyquist Filtering

After mapping the bit-stream into symbols, the sequence of complex symbols $I[n]$ is up-sampled by a factor of $M > 1$, before passing through a pulse shaping filter $g(t)$. The purpose of this filtering is to

1. To limit the bandwidth of the transmitted signal
2. To control the interference between successive symbols

1.3.1 Half-Root Nyquist Filter Design and Matched Filtering

To achieve optimal performance in terms of ISI cancellation and maximizing the SNR at the receiver, a root-raised cosine filter was used. This filter ensures that the overall desired channel response $h(t)$, from the input symbols at the transmitter to the sampled symbols at the receiver, satisfies the Nyquist criterion for zero ISI. This criterion states that the normalized impulse response of the equivalent discrete-time channel $h(kT_{symb})$ is such that

$$h(kT_{symb}) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \quad (2)$$

The filtering is split between a root-raised cosine (RRC) filter $g(t)$ at the transmitter and its matched version $g^*(-t)$ at the receiver. The convolution in the time domain of these two filters, $h(t) = g(t) \otimes g^*(-t)$, forms the overall channel response that satisfies the Nyquist criterion for zero ISI. To design the root-raised cosine filter $g(t)$, the transfer function of the raised cosine filter $H(f)$ was first defined. A square root was then performed on $H(f)$ to find the frequency response of the RRC filter, $G(f) = \sqrt{H(f)}$. Afterwards, an inverse Fourier transform was computed to find its time-domain equivalent $g(t)$.

The frequency response of the raised cosine filter $H(f)$ is almost equivalent to a rectangular window, but with a slope that is less sharp and characterized by a roll-off factor $\beta = 0.2$. Hence the frequency response of an RC filter is given by:

$$H(f) = \begin{cases} T_{symb} & 0 \leq |f| < \frac{1-\beta}{2T} \\ \frac{T_{symb}}{2}\left(1+\cos\left[\frac{\pi T_{symb}}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right) & \frac{1-\beta}{2T} \leq |f| \leq \frac{1+\beta}{2T} \\ 0 & |f| > \frac{1+\beta}{2T} \end{cases} \tag{3}$$

1.3.2 Filter Properties and Inter Symbol Interference Cancellation

The raised-cosine filter effectively confines the signal energy within a well-defined bandwidth, which is determined by the symbol rate $R_{symb} = 1/T_{symb}$ and the roll-off factor β. The total bandwidth occupied is $B = R_{symb}(1+\beta)/2$. Figure 2 illustrates the frequency response of the raised-cosine filter $H(f)$. It clearly shows a flat passband region, a roll-off region, and a stopband where the response is zero, thus restricting the signal to its allocated spectrum. For the project parameters with a roll-off factor of 0.2, the communication bandwidth is $B = 0.6R_{symb}$

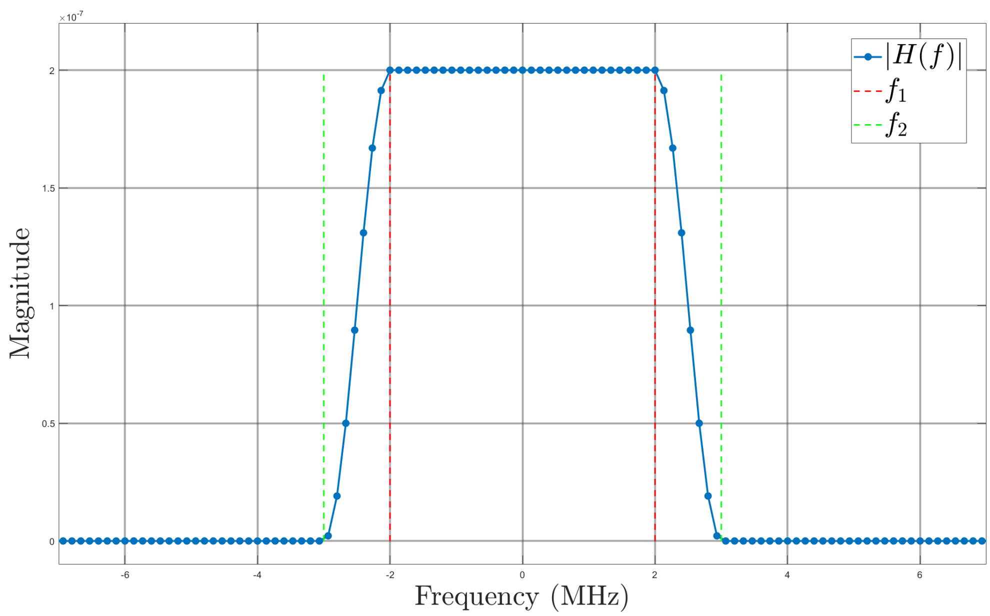

Figure 2: Raised Cosine Filter Frequency Response ($\beta = 0.2$, $taps = 301$, $OSF = 8$)

When the time domain impulse response of the raised-cosine filter is sampled at the symbol rate, it behaves like Dirac's delta pulse.. As shown in Figure 3 by stems representing $h(kT_{sym})$, the normalized RC filter response is unity at $t = 0$ and zero at $t = \pm T_{sym}, \pm 2T_{sym}, \ldots$. This ensures that at the optimal sampling instant for a given symbol, the contributions from all preceding and succeeding symbols are nullified, thus eliminating ISI.

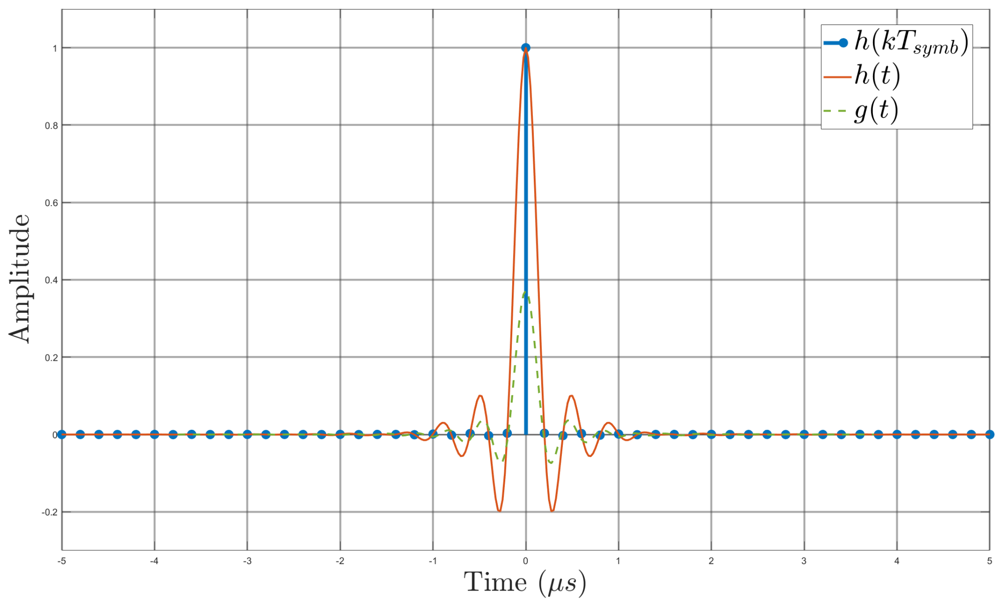

Figure 3: Normalized RC and RCC filter Impulse Responses

1.4 Noise Addition and Performance Evaluation

[LATER]

1.5 Questions and Answers

[LATER]

2 Time and Frequency Synchronization

2.1 The Problem of Synchronization

It is important that the transmitter and receiver are synchronized in order for the demodulation to be accurate. The receiver must precisely determine when to sample the incoming complex signal to capture the symbol values and align its local carrier frequency and phase with that of the received signal.

Physically, synchronization errors occur when the transmitter and receiver operate with independent local oscillator. The local oscillators can be subject to manufacturing imperfection, environmental variation, leading to slight deviations from their target frequencies. Furthermore, the propagation delay of the signal between the transmitter and the receiver introduces an unknown phase shift in the carrier and an unknown time shift in the symbol sequence arrival.

These phenomena give rise to four types of synchronization errors, which can be categorized into carrier synchronization errors and timing synchronization errors.

- Carrier Frequency Offset (CFO): Δ_f, which is the difference in carrier frequency between the transmitted symbols and the received symbols
- Carrier Phase Offset ϕ_0: phase difference between the transmitted symbols and the received symbols
- Sample Clock Offset (SCO): δ. This error occurs when the frequency of the clock driving the Analog-to-Digital Converter at the receiver differs from that of the Digital-to-Analog Converter at the transmitter . This error type was not implemented in the project and was considered negligible.
- Time shift: t_0, represents the receiver's uncertainty about the exact time of arrival of the symbols. The receiver needs to determine the optimal sampling instants, which correspond to the peaks of the Nyquist-filtered pulses.

Figure 4 illustrates these mismatches. The transmitter generates symbols at discrete times nT_{symb}; which are then pulse-shaped and modulated. At the receiver, after down-sampling, the signal is sampled at instances $nT_{symb}(1+\delta)+t_0$ relative to the transmitter.

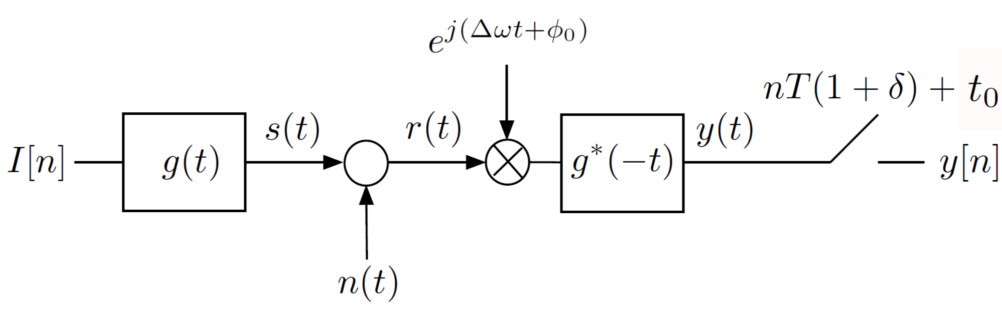

Figure 4: Synchronization mismatches at the receiver

2.2 Impact of Synchronization Errors on Performance

2.2.1 Impact of Carrier Phase Offset (ϕ_0)

A static carrier phase offset arises from the unknown phase difference between the incoming carrier signal and the receiver local oscillator. Assuming perfect timing and no CFO, if the transmitted complex baseband symbol is $I[n]$, the corresponding sample y[n] at the output of the matched filter becomes:

$$y[n] = I[n]e^{j\phi_0} + \text{noise} \tag{4}$$

this equation shows that the phase offset causes a rotation of the entire received symbol constellation by an angle ϕ_0 in the complex plane. Figure **??** illustrate a constellation diagram before and after applying a phase offset.

2.2.2 Impact of Carrier Frequency Offset

If $r(t)$ is the signal before the matched filter, then it becomes $r(t)e^{j(2\pi\Delta ft+\phi_0)} + \text{noise}$, where ϕ_0 is the initial phase offset.

The impacts of CFO are therefore:

1. Phase Drift: After matched filtering and sampling at $t = nT_{symb}$, each symbol $I[n]$ experiences a phase rotation that changes from symbol to symbol: $y[n] = I[n]e^{j(2\pi\Delta ft+\phi_0)} + \text{noise}$. Figure **??** shows the received symbol after CFO as a spiral pattern, due to the progressive phase rotation.
2. Inter-Symbol Interference: If the CFO is significant, the assumption that it only causes a phase rotation at the matched filter output is no longer accurate. The term $e^{2\pi\Delta ft}$ means that the received signal spectrum is shifted. The matched filter $g^*(-t)$ is no longer matched to the incoming because signal $g'(t) = g(t)e^{2\pi\Delta ft}$, leading to ISI.

2.3 Impact of Sample Time Shift

An incorrect sampling instant means that the output of the matched filter is not sampled at the point of maximum signal energy and zero ISI. if $h(t)$ is the impulse response of the overal Nyquist filter (Raised Cosine Filter), and sampling occurs at $nT_{symb} + t_0$, the n-th sample $y[n]$ is given by:

$$y[n] = \sum_m I[m]h((n-m)T_{symb} + t_0) + \text{noie} \tag{5}$$

$$y[n] = I[n]h(t_0) + \sum_{m\neq n} I[m]h((n-m)T_{symb} + t_0) + \text{noise} \tag{6}$$

2.4 Gardner Algorithm for Sampling Time Tracking

2.5 Frame and Frequency Acquisition using Differential Cross-Correlator

2.6 Phase Interpolation

2.7 Questions and Answers

3 Real-life Experimentation on the HFC Setup