ELEC-H401

Design and Simulation of a DVB-C Transmission Chain

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Academic Year
2024 - 2025

Faculty
Electrical Engineering

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Introduction

[TODO: CHANGE THE TENSE FROM PRESENT TO PAST]

This project focuses on designing and simulating a Digital Video Broadcasting Cable transmission chain using Matlab. The main goal is to model and analyze the components of a typical modem. The first part of this project includes establishing an optimal communication chain over an ideal Additive White Gaussian Noise channel, mapping the signal into symbols, Nyquist filtering, and evaluating the performance of the system through Bit Error Rate simulations for Quadrature Amplitude Modulation signals. The second part revolves around the design and assessment of time and frequency synchronization algorithms. This encompasses evaluating the impact of synchronization errors such as carrier phase and frequency offset and sample time shift. The algorithms used for this purpose are the Gardner Algorithm for time recovery, and a differential cross-correlator for joint frame and frequency acquisition. Afterwards, their effectiveness is demonstrated through convergence analysis and residual error evaluation (double check). The study also explores phase interpolation techniques to mitigate remaining phase drifts. Finally, the project aims to validate the simulated chain through real-life experimentation on a Hybrid Fiber-Coax setup using Adalm-Pluto hardware, and to compare the simulated performance with experimental observations.

1 Optimal Communication Chain over the Ideal Channel

[TODO: ADD MORE MATH LATER]

[TODO: MAYBE ADD HOW SOME SIMULATION PARAMETERS AFFECT THE SIMULATION (like taps for example)]

[TODO: MAKE THE AXIS NUMBER OF THE GRAPHS BIGGER]

1.1 Communication Chain

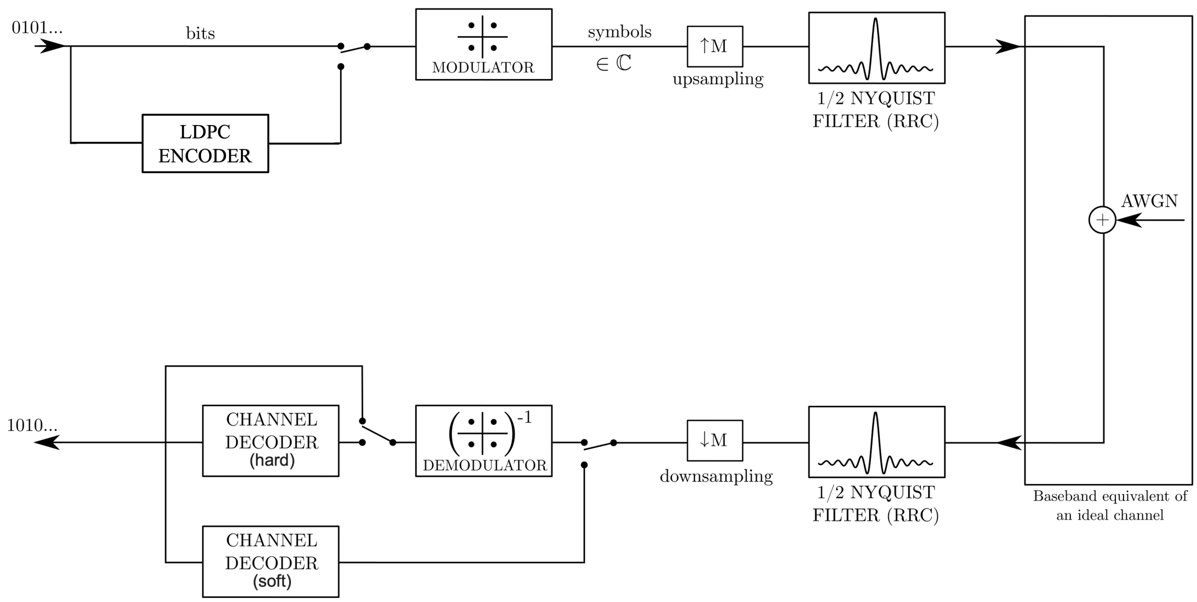

Figure 1: Block diagram of the communication system

The communication chain depicted in Figure 1 models the pipeline of a DVB-C transmitter and receiver at baseband. On the transmitter side, a bit-stream is generated, then mapped to complex symbols from a chosen QAM modulation. These symbols are subsequently up-sampled and shaped by a half-root Nyquist filter to limit their bandwidth. On the receiver's side, the transmitted signal is matched-filtered with the same half-root Nyquist filter to maximize the signal-to-noise ratio. The output of the matched filter is then down-sampled at the symbol instances and demapped to recover an estimate of the transmitted bit-stream.

In this project, the Low Density Parity Check Encoder is not implemented, and the algorithms for symbol mapping and demapping were provided.

1.2 Bit Generation and Symbol Mapping and Demapping

The mapping of bit-streams to complex symbols enhances spectral efficiency by allowing more bits to be transmitted per symbol over a given bandwidth. At the receiver, symbol demapping converts the noisy received symbols back into an estimated sequence of bits. This is achieved using the Maximum Likelihood

(ML) criterion, which selects the constellation symbol closest to the received sample in terms of minimum Euclidean distance:

$$\tilde{\underline{s}}_m^{ML} = \arg\min_{\underline{s}_m} \left(\sum_{k=1}^{K} (r_k - s_{mk})^2 \right) \tag{1}$$

Where:

- $\tilde{\underline{s}}_m^{ML}$ is the estimated symbol using the ML criterion.
- $\underline{r} = [r_1, r_2, \ldots, r_K]^T$ is the received vector after demodulation.
- $\underline{s}_m = [s_{m1}, s_{m2}, \ldots, s_{mK}]^T$ is the vector representing the m-th possible transmitted symbol.
- This selection minimizes the squared Euclidean distance, which is equivalent to maximizing $\ln p(\underline{r}|\underline{s}_m)$ for an AWGN channel, where $p(\underline{r}|\underline{s}_m)$ is the conditional probability of receiving vector $\underline{r}$ given that symbol $\underline{s}_m$ was sent.

1.3 Nyquist Filtering

The sequence of complex symbols $I[n]$, after mapping, is up-sampled by a factor of $M > 1$ and then passed through a pulse shaping filter $g(t)$. This filtering is essential for:

1. Limiting the bandwidth of the transmitted signal.
2. Controlling interference between successive symbols.

1.3.1 Half-Root Nyquist Filter Design and Matched Filtering

To achieve optimal performance in terms of ISI cancellation and maximizing the SNR at the receiver, a root-raised cosine (RRC) filter is utilized. This involves employing an RRC filter $g(t)$ at the transmitter and its matched version $g^*(-t)$ at the receiver. The convolution of these two filters, $h(t) = g(t) \otimes g^*(-t)$, forms the overall channel response. This response $h(t)$ is designed to satisfy the Nyquist criterion for zero ISI, which states that for symbols sampled at intervals T_{symb}:

$$h(kT_{symb}) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \tag{2}$$

To design the RRC filter $g(t)$, the transfer function of a raised-cosine (RC) filter, $H(f)$, is first defined. The frequency response of the RRC filter is then $G(f) = \sqrt{H(f)}$, and its time-domain equivalent $g(t)$ is found via an inverse Fourier transform.
The frequency response of the RC filter, characterized by a roll-off factor $\beta = 0.2$, is given by:

$$H(f) = \begin{cases} T_{symb} & 0 \leq |f| < \frac{1-\beta}{2T_{symb}} \\ \frac{T_{symb}}{2}\left(1 + \cos\left[\frac{\pi T_{symb}}{\beta}\left(|f| - \frac{1-\beta}{2T_{symb}}\right)\right]\right) & \frac{1-\beta}{2T_{symb}} \leq |f| \leq \frac{1+\beta}{2T_{symb}} \\ 0 & |f| > \frac{1+\beta}{2T_{symb}} \end{cases} \tag{3}$$

1.3.2 Filter Properties and Inter-Symbol Interference Cancellation

The RC filter effectively confines the signal energy within a bandwidth $B = R_{symb}(1+\beta)/2$, where $R_{symb} = 1/T_{symb}$ is the symbol rate. Figure 2 illustrates this, showing a flat passband, a roll-off region, and a stopband, restricting the signal to its allocated spectrum. For project parameters ($R_{symb} =$!!!!!!!!!! and $\beta = 0.2$), the communication bandwidth is $B = 0.6R_{symb}$.

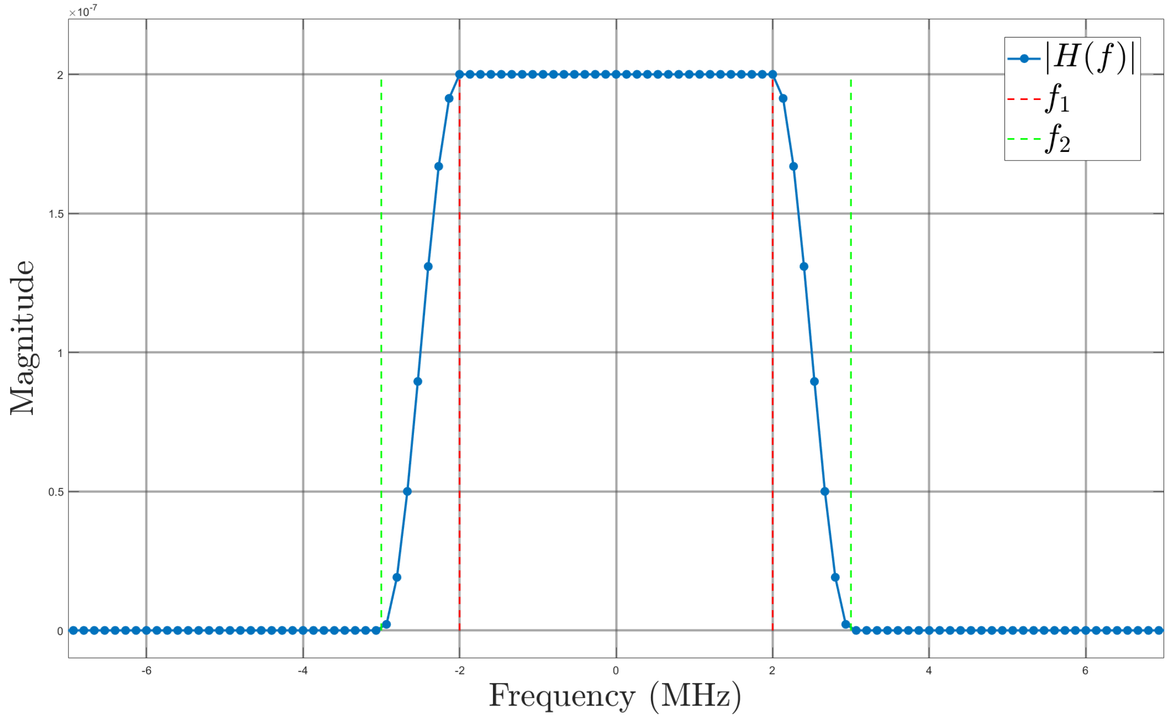

Figure 2: Raised Cosine Filter Frequency Response ($\beta = 0.2$, $taps = 301$, $OSF = 8$)

In the time domain, the impulse response of the overall RC filter $h(t)$, when sampled at the symbol rate, approximates a Dirac delta function. As shown by the stems representing $h(kT_{symb})$ in Figure 3, the normalized response is unity at $t = 0$ and zero at $t = \pm T_{symb}, \pm 2T_{symb}, \ldots$. This property ensures that at the optimal sampling instant for a given symbol, contributions from all other symbols are nullified, thereby eliminating ISI.

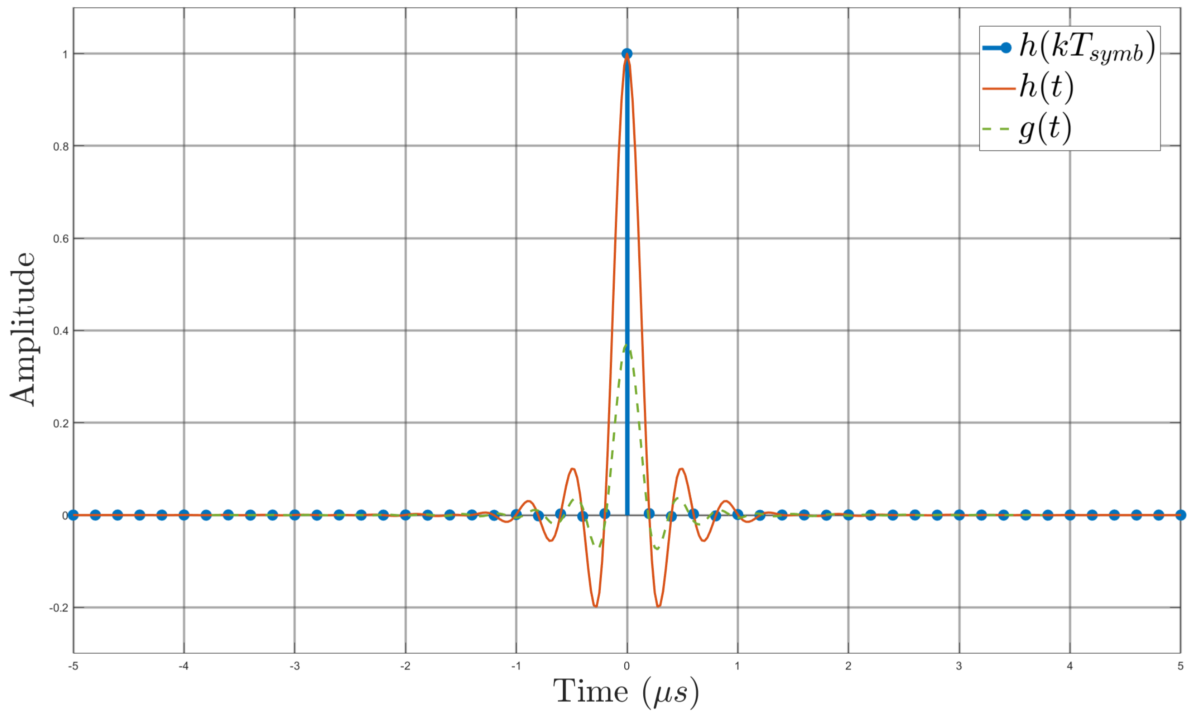

Figure 3: Normalized RC and RRC filter Impulse Responses

1.4 Noise Addition and Performance Evaluation

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1.5 Questions and Answers

[LATER]

2 Time and Frequency Synchronization

2.1 The Problem of Synchronization

Accurate demodulation requires the transmitter and receiver to be synchronized. The receiver must precisely determine symbol sampling instances and align its local carrier frequency and phase with the received signal. Synchronization errors primarily arise because transmitter and receiver operate with independent local oscillators, which can have slight frequency deviations due to manufacturing imperfections or environmental factors. Additionally, signal propagation delay introduces an unknown carrier phase shift and symbol timing offset.

These phenomena lead to four main types of synchronization errors:

- Carrier Frequency Offset (CFO), Δf: The difference between the transmitter's and receiver's carrier frequencies.
- Carrier Phase Offset, ϕ_0: The phase difference between the incoming carrier and the receiver's local oscillator.
- Sample Clock Offset (SCO), δ: The frequency mismatch between the transmitter's DAC clock and the receiver's ADC clock. This error was considered negligible and not implemented in this project.
- Timing Shift, t_0: The receiver's uncertainty about the exact arrival time of symbols, necessitating determination of optimal sampling instants.

Figure 4 conceptually illustrates these mismatches, where transmitter symbols generated at nT_{symb} are effectively sampled at $nT_{symb}(1+\delta)+t_0$ by the receiver.

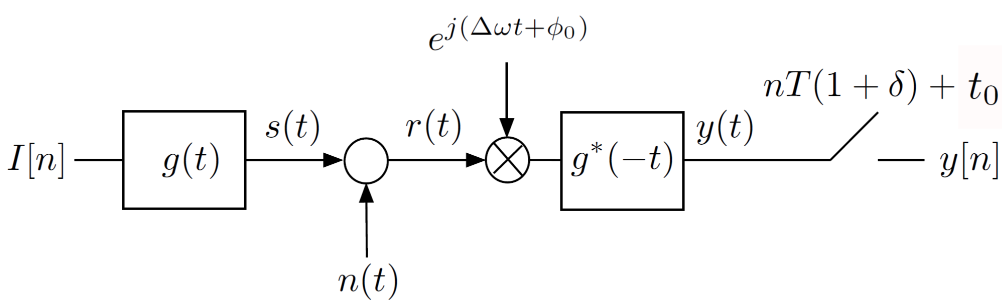


Figure 4: Synchronization mismatches at the receiver

2.2 Impact of Synchronization Errors on Performance

[TODO: COMPLETE LATER THE CONSEQUENCES WITH LECTURE'S MATH]

2.2.1 Impact of Carrier Phase Offset (ϕ_0)

A static carrier phase offset, ϕ_0, resulting from the phase difference between the incoming carrier and the receiver's local oscillator, causes a rotation of the entire received symbol constellation by an angle ϕ_0 in the complex plane. Assuming perfect timing, no CFO, and no noise, a transmitted complex baseband symbol $I[n]$ becomes at the matched filter output:

$$y[n] = I[n]e^{j\phi_0} \tag{4}$$

2.2.2 Impact of Carrier Frequency Offset (Δf)

A carrier frequency offset Δf means the received signal before matched filtering can be modeled as $r(t) = s(t)e^{j(2\pi\Delta ft}$, (assuming no phase offset and no noise). The impacts of CFO are:

1. Phase Drift: After matched filtering and sampling at $t = nT_{symb}$, each symbol $I[n]$ experiences a progressively changing phase rotation: $y[n] = I[n]e^{j(2\pi\Delta fnT_{symb})}$. This causes the received constellation points to form a spiral pattern if uncorrected, as depicted in Figure 5.
2. Inter-Symbol Interference (ISI): If the CFO is significant, the received signal's spectrum is shifted. Consequently, the receiver's filter $g^*(-t)$ is no longer perfectly matched to the incoming signal component $g(t)e^{j2\pi\Delta ft}$, leading to a loss in SNR and the introduction of ISI.

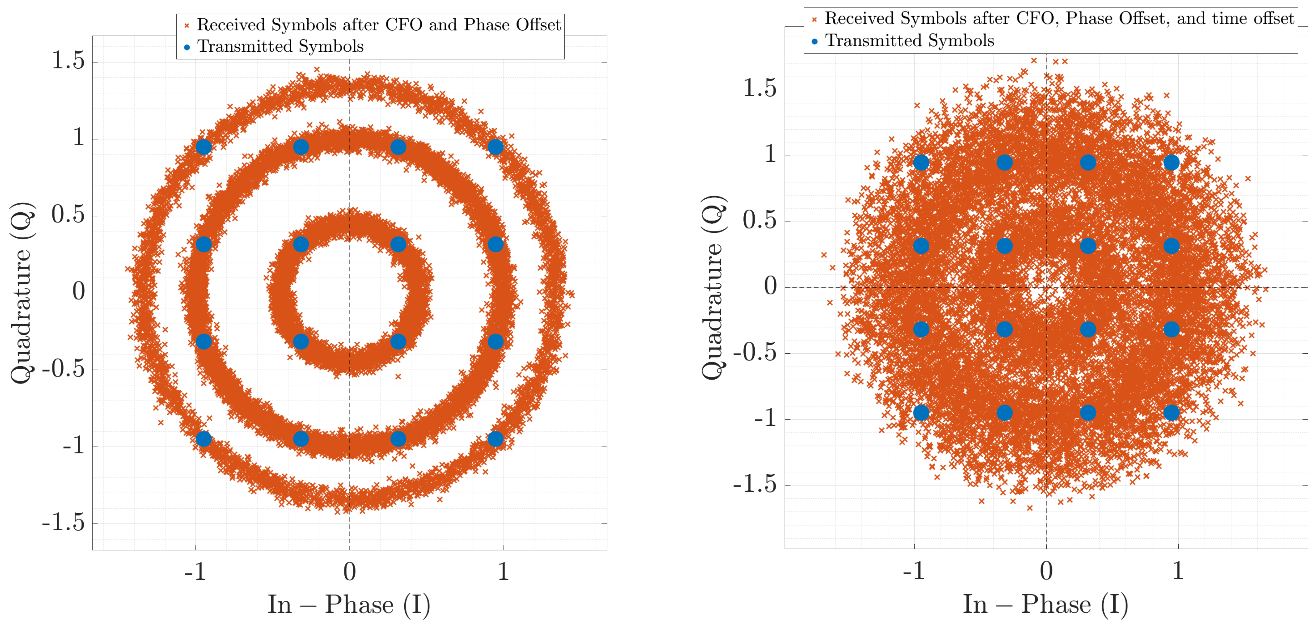
2.2.3 Impact of Sample Time Shift (t_0)

An incorrect sampling instant $t_0 \neq 0$ (relative to the optimal point) means the matched filter output is not sampled at maximum signal energy and zero ISI. If $h(t)$ is the overall Nyquist filter response, sampling at $nT_{symb} + t_0$ (assuming no noise) yields:

$$y[n] = \sum_m I[m]h((n-m)T_{symb} + t_0) \tag{5}$$

$$= I[n]h(t_0) + \sum_{m\neq n} I[m]h((n-m)T_{symb} + t_0) \tag{6}$$

The first term, $I[n]h(t_0)$, represents the attenuated desired symbol (since $h(t_0) < h(0)$ for $t_0 \neq 0$). The second term is the ISI, as $h(kT_{symb} + t_0)$ is non-zero for $k \neq 0$ when $t_0 \neq 0$.



(a) 16-QAM Constellation diagram after CFO and Phase Offset

(b) 16-QAM Constellation diagram after CFO, Phase Offset and time offset

Figure 5: Constellation Diagrams of symbols transmitted and received after adding AWGN noise such that $\frac{E_b}{N_0} = 20$ dB.

2.3 Gardner Algorithm for Sampling Time Tracking

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2.4 Frame and Frequency Acquisition using Differential Cross-Correlator

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2.5 Phase Interpolation

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2.6 Questions and Answers

[LATER]

3 Real-life Experimentation on the HFC Setup