Scuola universitaria professionale
della Svizzera italiana
SUPSI

Università
della
Svizzera
italiana

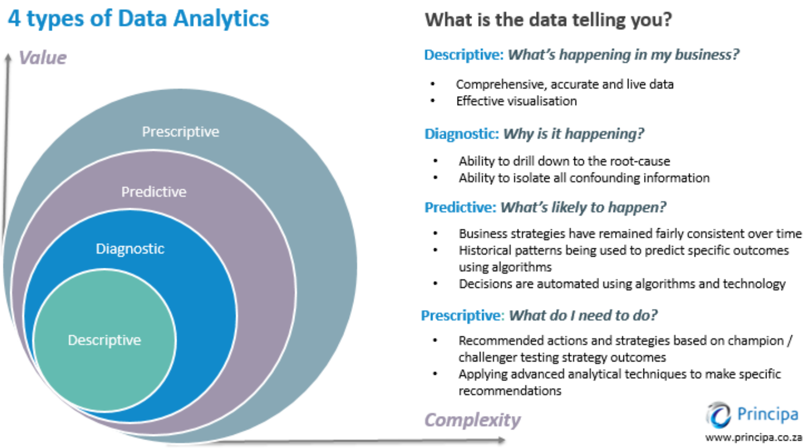
Istituto Dalle Molle di studi sull'intelligenza artificiale

Applied Operations Research (C-B4051E.1)
Bachelor in Data Science and AI

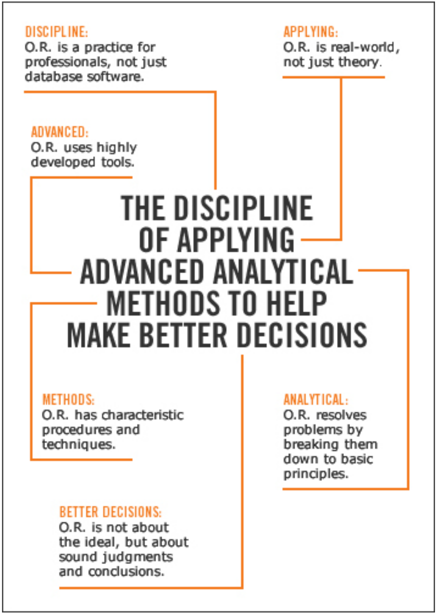
Lecture 1
Introduction

Matteo Salani
matteo.salani@idsia.ch

Data analytics



Operations Research



Operations Research was developed in England in the late 1930s in order to deal with operational aspects of military systems (e.g. coordination and correlation of radar systems). In 1941 the Operational Research Section was created within the Royal Air Force, and the term Operations research was offcially coined.

Operations Research

Also known as decision science

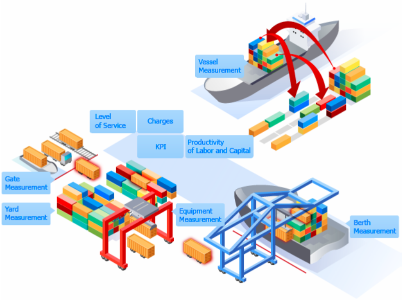
Today OR is applied for complex system planning and decision support when dealing with complex practical problems. Operations research is also simply referred to as optimization.

OR deals with:

- Strategic decisions: long term (e.g. years)
- Tactical decisions: medium term (e.g. months)
- Operational decisions: short term (e.g. seconds to days)

Some practical examples

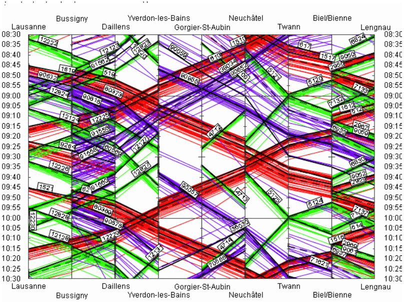
...private transportation (check this out on Youtube)



- Where to place a Container Terminal?
- How big should it be?
- How many quay cranes and yard cranes are needed?
- How many crane/gantry operators and how are they organized?
- Where to place incoming containers and how to stack them?

Some practical examples

...public transportation



- Which lines are needed?
- How many trains of which size?
- What's the best frequency?
- How to reroute in case of delays?

Some practical examples

...energy and smart grids (check this out on Youtube).

- How to size a photovoltaic plant or stationary battery?
- When to charge/discharge batteries and cars?
- When to perform maintenance?

Some practical examples

…economy

- How to build a portfolio?
- How to minimize risk for a given target?
- How to harmonise tax policy and public investments?

Industrial cases - American Airlines

- 250k passengers, 3400 flights, 300k luggages, 239k reservations per day
- 2017, revenues 42b\$, costs 37b\$
- Route planning, fleet assignment, pilots and crew, revenue management, maintenance planning, disruption recovery
- Savings 1.5-2 b\$/year

Industrial cases - UPS

- ORION System for pickup and delivery operations.
- 10 years of development, 35.000 drivers/day, 160 customers/day for each driver
- Reduced consumption by 32 million litres of fuel/year and 85 thousand tons of CO_2
- Cost of the project \$250 millions, estimated savings \$320 millions/year since it was introduced.

Industrial cases - AirLiquide

- Distribution logistics
- Integrated stock management
- Demand forecast

check out OR Analytics Success Stories

Modelling and Optimization

Modelling is a crucial aspect of engineering. It is an art but it must be rigorous at the same time.

Variables: what we can decide, the degree of freedom we have in the system.

$$x \in \mathbb{R}^n$$

x is an n dimensional vector of real numbers (sometimes values are constrained to be integer or just binary).

We speak of a **solution** as an assignment of values to variables. Geometrically, it is a point in an n dimensional space.

Modelling and Optimization

How do we select the best assignment?

Objective function: it is an indicator of how **good** our solution is and we would like either to **minimize** or **maximize** it.

$$f(x) : \mathbb{R}^n \to \mathbb{R}$$

It is normally expressed as a function $f(x)$ that takes variables x as arguments and computes a real value.

The objective function computes the **value** of a solution x which we often refer with z.

In optimization we are looking for an **OPTIMAL** solution x^*, that is a solution with the lowest/highest value, z^*.

Modelling and Optimization

Often, not all possible solutions (i.e. assignments of values to variables) are acceptable for the system or phenomenon we are modelling, we say not all solutions are **feasible**.

In particular, we search solutions within a subset of the n dimensional space

$$x \in X \subseteq \mathbb{R}^n$$

Normally X is expressed as a set of linear/non-linear equations/inequalities called **constraints**.

$$z = \min f(x)$$
$$s.t.\ x \in X \subseteq \mathbb{R}^n$$

s.t. means "subject to"

A simple example: investment choices

Problem: A company is evaluating 5 projects of production development on a 3-year timeline. The annual cost of the projects and the potential profit have been estimated and the available budget is given. Which is the optimal strategy?

	Costs (mioCHF/year)			Exp. Profit
Projects	Y1	Y2	Y3	(mioCHF)
P1	5	1	8	20
P2	4	7	10	40
P3	3	9	2	20
P4	7	4	1	15
P5	8	6	10	30
Budget (mioCHF)	25	25	25	

Decision making choice: Which projects must be financed in order to maximise the profits and respect the annual budget?

Decision problem: for each project we have a *yes/no* decision $\rightarrow$ binary variable: x_j

$$x_j = \begin{cases} 1 & \text{if project } j \text{ gets financed} \\ 0 & \text{otherwise} \end{cases}$$

A simple example: mathematical model

Obj. Function:	*max*	$20x_1 + 40x_2 + 20x_3 + 15x_4 + 30x_5$
Constraints:	*s.t.*	$5x_1 + 4x_2 + 3x_3 + 7x_4 + 8x_5 \leq 25$
		$1x_1 + 7x_2 + 9x_3 + 4x_4 + 6x_5 \leq 25$
		$8x_1 + 10x_2 + 2x_3 + 1x_4 + 10x_5 \leq 25$
Vaiables:		$x_1, x_2, x_3, x_4, x_5 \in \{0, 1\}$

OR for a Data Scientist

Goals of the course:

Provide theoretical and practical tools for the construction and solution of mathematical optimization models, able to represent different classes of decision problems that arise commonly in production, logistics, transport, business organisation, everyday life, $\cdots$

This will be done via practical problems:

- Production of goods
- Resource allocation
- Task scheduling
- Inventory and stock management
- Network design, flows and paths

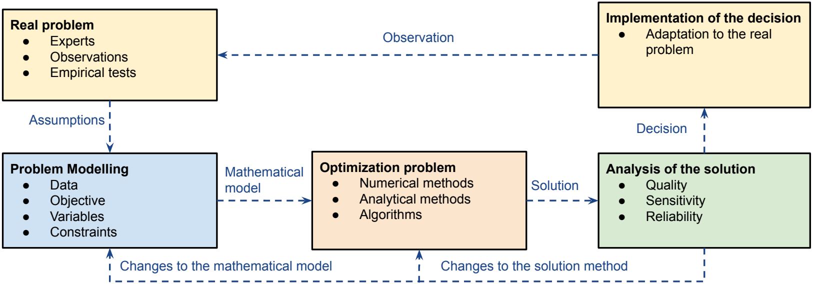
Limitations of the course

We don't have an entire semester just for OR, thus we will cover a fraction of it.

In particular, we will limit to:

- Problems with **one** decision maker that has full control on the system/process/phenomenon.
- Problems with **one** objective function (although we will briefly cover basics of multi-objective optimization).
- Problems with **deterministic** data (although we will briefly cover basics of stochastic and robust optimization).

Prescriptive analytics logical flow



Further examples

Example 1: Determine the production mix which maximizes the revenue, given the quantity of available raw material. The sales price of each product is known.

Example 2: Design a building in such a way that its volume is maximized, while several geometrical constraints are respected.

Example 3: Assign N jobs to N persons, in such a way that the total completion time is minimized.

Example 1: Product mix

N Products
M Raw Materials
$A_{M\times N} = [a_{ji}]$ Quantity of raw material j needed for each unit of product i ($j = 1..M, i = 1..N$)
$P = [p_i]$ Sales price of one unit of product i ($i = 1..N$)
$Q = [q_j]$ Available quantity of product j ($j = 1..M$)
$X = [x_i]$ Quantity of product i to be produced and sold ($i = 1..N$)

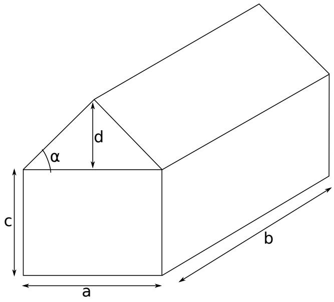
$$z = \max \sum_{i=1}^{N} p_i x_i$$

$$s.t. \ \sum_{i=1}^{N} a_{ij} x_i \leq q_j \ \forall j = 1..M$$

$$x_i \geq 0 \ \forall i = 1..N$$

Example 2: Geometric Optimization

k_1 Maximum ratio between height and width
k_2 Maximum roof slope ($tg\alpha$)
k_3 Maximum depth
k_4 Maximum width
a, b, c, d Building dimensions



$$
\begin{aligned}
z = \max abc + \frac{1}{2}abd \\
s.t.\ k_1 a - c \geq 0 \\
k_2 a - 2d \geq 0 \\
b \leq k_3 \\
a \leq k_4 \\
a, b, c, d \geq 0
\end{aligned}
$$

Example 3: Assignment problem

N Number of persons and jobs
$T_{N\times N} = [t_{ij}]$ time needed by person i to complete job j
$X_{N\times N} = [x_{ij}]$ binary matrix: $x_{ij} = 1$ iif person i is assigned to job j
Goal **assign a single job** to each person so that the completion time is minimized.

$$z = \min \max_{\forall i=1..N, \forall j=1..N} t_{ij} \cdot x_{ij}$$

$$\sum_{i=1}^{N} x_{ij} = 1 \qquad \forall j = 1..N$$

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad \forall i = 1..N$$

$$x_{ij} \in \{0, 1\} \qquad \forall i = 1..N, \forall j = 1..N$$

Example 3: Assignment problem

Let w be a variable that represents the time to complete the last job

$$
\begin{aligned}
z = \min\ & w \\
s.t.\ & w \geq t_{ij} \cdot x_{ij} && \forall i = 1..N, \forall j = 1..N \\
& \sum_{i=1}^{N} x_{ij} = 1 && \forall j = 1..N \\
& \sum_{j=1}^{N} x_{ij} = 1 && \forall i = 1..N \\
& x_{ij} \in \{0, 1\} && \forall i = 1..N, \forall j = 1..N
\end{aligned}
$$

Classification of mathematical programming problems

$$\max_x f(x_1, x_2, \cdots, x_n)$$

$$s.t.\ g_j(x_1, x_2, \cdots, x_n) \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b_j \qquad \forall j = 1, \cdots, m$$

$$x_1, x_2, \cdots, x_n \in X$$

In the simplest case , f is a linear function: $f = \sum_{i=1}^{n} c_i x_i$ and constraints g_j are linear

- Linear Programming (LP): $X \subseteq \mathbb{R}^n$
- Integer Linear Programming (ILP): $X \subseteq \mathbb{Z}^n$
- Binary or 0-1 LP: $X = \{0, 1\}^n$
- Mixed Integer Linear Programming (MILP): $(x_1, \cdots, x_r) \in X_r \subseteq \mathbb{R}^r, (x_{r+1}, \cdots, x_n) \in X_z \subseteq \mathbb{Z}^{n-r}$

Important!

The characteristics of the mathematical programming model have a direct impact on the tractability of the problem and on the choice of the solution method.