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Applied Operations Research (C-B4051E.1)
Bachelor in Data Science and AI

Lecture X
Modelling tricks

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Sets

- **Sets:** the indexing elements on which we define data (parameters) and variables.
- We can find them with some tricks:
  - In case tables are present, often the meaning of each row or each column (title of the rows, title of the columns) are good candidates to be used as sets.
  - Try to answer to the question "which is the index defining the parameters/variables"? If we naturally answer "for each ¡element¿" then we found a good candidate to be used as a set.

Data/Parameters

- **Data/Parameters:** These are "invariants" of the problem.
- We can find them with some tricks:
  - Everywhere we see a number (quite obvious...) or table we need to the fine a parameter.
  - When we encounter a sentence like "let ¡element¿ be known..." we have a parameter that may be implicit (i.e. we have to consider in the modelling and its value will be known later).

Variables

- **Variables:** are the elements with degree of freedom.
- We can find them with some tricks:
  - Find keywords "decide", "determine", "choose" these are concepts linked with variables. Example: "Determine the quantity of products to be acquired..." suggests that Products is a set of items and we need a variable for each product.
  - In linear programming variables are continuous non-negative. Anyway, particular care must be put in the selection of the domain of the variables.

Objective function

- **Objective function:** it is the "mathematical formula " that computes the value we want to be optimized.
- We can find them with some tricks (some are quite obvious):
  - Find keywords "minimize", "maximize".
  - Sentences like "minimize/maximize the worst..." imply formulations using *min max* and *max min* objective functions that must be linearized.
  - Other formulations like "...so that the cost(revenue) is as even as possible..." also imply *min max* (*max min*) objective functions.

Constraints

- **Constraints:** are relationships among sets of variables. They determine the feasibility of a solution.
- We can find them with some tricks (some are quite obvious):
  - Find keywords like "at least", "at most", "exactly". They indicate relationships like $\geq$, $\leq$ o $=$.
  - Find keywords like "such that", "subject to": they suggest the definition of constraints.

Ex-post model checking

Disclaimer

What follows constitutes a set of good practices for model checking.
They are NOT mechanical rules to be used to substitute the understanding the problem at hand.

Model checking

- **Rule 1:** In every equation or inequality(objective function or constraints) at least one variable **MUST** appear. Relationships with just constants do not make sense in optimization.
- **Rule 2:** In every equation or inequality(objective function or constraints) every index of variables and parameters must be defined (in LP either using sums or constraint domains, i.e. $\forall$ "for all"). Examples:
  - $\sum_{c \in C} d_{c,f} \cdot x_c \leq b_f \ \forall f \in F$ seems correctly defined
  - $\sum_{c \in C} d_{c,f,g} \cdot x_c \leq b_{f,g} \ \forall f \in F$ **it is NOT** correctly defined, missing index g
- **Rule 3:** All defined constants and parameters should appear in at least one equation or inequality(objective function or constraints). Otherwise we defined a redundant parameter (less likely) or we forgot a necessary constraint (most likely).

Model checking

- **Rule 4:** When the problem requires the definition of more than one set of variables, then there must exist constraints linking the variables.
  Examples:
  - let x_p the quantity of product p to produce, y_r the quantity of available resource r to be acquired

    $$\sum_{p \in P} a_{r,p} x_p \leq y_r \ \forall r \in R$$

  - let x, y, z three variables, then we should define

    $$x + y \leq (\geq) z$$

    or (in any order)

    $$x \leq (\geq) z$$
    $$y \leq (\geq) z$$

Model checking

- **Rule 5 - not always!:** In LP, how to determine whether indexes are defined with sums or $\forall$, "for all" statements?
When we guess the relationship among variables and data, often the index that appear in both sides is a good candidate to be specified with a $\forall$, "for all" statement while the index appearing on just one side is commonly specified with sums.
Example (Production problem):

$$\sum_{p \in P} a_{r,p} x_p \leq u_r \; \forall r \in R$$

Counter example (Transportation problem):

$$\sum_{i \in O} x_{ij} = 1 \; \forall j \in D$$

Linearizations

Absolute value

$$\min z = |c^T \cdot x|$$

- Introduce an auxiliary variable w.
- Add 2 constraints of the form

$$w \geq c^T \cdot x$$

$$w \geq -c^T \cdot x$$

Example:

$$\min z = |3x_1 - 5x_2 - 4x_3| \quad \rightarrow \quad \begin{aligned} \min z = \; & w \\ w \geq \; & 3x_1 - 5x_2 - 4x_3 \\ w \geq \; & 5x_2 + 4x_3 - 3x_1 \end{aligned}$$

Linearizations

Min Max

$$\min z = \max_{i \in arg} c_i^T \cdot x_i$$

arg is the domain on which the inner objective is defined

- Introduce an auxiliary variable w.
- Add $|arg|$ constraints of the form

$$w \geq c_i^T \cdot x_i \qquad \forall\ i \in arg$$

Example:

$$\min z = \max_{t \in T} c_t \cdot x_t \qquad \rightarrow \qquad \begin{aligned} \min z = \ & w \\ w \geq c_t \cdot x_t & \qquad \forall t \in T \end{aligned}$$

Linearizations

Max Min

$$\max z = \min_{i \in arg} c_i^T \cdot x_i$$

arg is the domain on which the inner objective is defined

- Introduce an auxiliary variable w.
- Add $|arg|$ constraints of the form

$$w \leq c_i^T \cdot x_i \qquad \forall\ i \in arg$$

Example:

$$\max z = \min_{t \in T} c_t \cdot x_t \qquad \rightarrow \qquad \begin{aligned} \max z = \ & w \\ w \leq c_t \cdot x_t & \qquad \forall t \in T \end{aligned}$$