

Reductieregels:

$$\neg_L: \frac{\Phi, \neg\alpha \circ \Psi}{\Phi \circ \alpha, \Psi} \quad \neg_R: \frac{\Phi \circ \neg\alpha, \Psi}{\Phi, \alpha \circ \Psi}$$

$$\wedge_L: \frac{\Phi, \alpha \wedge \beta \circ \Psi}{\Phi, \alpha, \beta \circ \Psi} \quad \wedge_R: \frac{\Phi \circ \alpha \wedge \beta, \Psi}{\Phi \circ \alpha, \Psi \quad \Phi \circ \beta, \Psi}$$

$$\vee_L: \frac{\Phi, \alpha \vee \beta \circ \Psi}{\Phi, \alpha \circ \Psi \quad \Phi, \beta \circ \Psi} \quad \vee_R: \frac{\Phi \circ \alpha \vee \beta, \Psi}{\Phi \circ \alpha, \beta, \Psi}$$

$$\rightarrow_L: \frac{\Phi, \alpha \rightarrow \beta \circ \Psi}{\Phi, \beta \circ \Psi \quad \Phi \circ \alpha, \Psi} \quad \rightarrow_R: \frac{\Phi \circ \alpha \rightarrow \beta, \Psi}{\Phi, \alpha \circ \beta, \Psi}$$

$$\leftrightarrow_L: \frac{\Phi, \alpha \leftrightarrow \beta \circ \Psi}{\Phi, \alpha, \beta \circ \Psi \quad \Phi \circ \alpha, \beta, \Psi} \quad \leftrightarrow_R: \frac{\Phi \circ \alpha \leftrightarrow \beta, \Psi}{\Phi, \alpha \circ \beta, \Psi \quad \Phi, \beta \circ \alpha, \Psi}$$

$$\forall_L: \frac{\Phi, \forall x \varphi \circ \Psi}{\Phi, [d/x]\varphi \circ \Psi} \quad \text{voor alle } d \in D \text{ hier aanwezig} \quad \forall_R: \frac{\Phi \circ \forall x \varphi, \Psi}{\Phi \circ [d_{k+1}/x]\varphi, \Psi} \quad \text{voor een nieuwe } d_{k+1}$$

$$\exists_L: \frac{\Phi, \exists x \varphi \circ \Psi}{\Phi, [d_{k+1}/x]\varphi \circ \Psi} \quad \text{voor een nieuwe } d_{k+1} \text{ in het domein} \quad \exists_R: \frac{\Phi \circ \exists x \varphi, \Psi}{\Phi \circ [d/x]\varphi, \Psi} \quad \text{voor alle } d \in D \text{ hier aanwezig}$$

Afleidingsregels:

$$\frac{\Sigma \quad \varphi \wedge \psi}{\varphi} \wedge E \quad \frac{\Sigma \quad \varphi \wedge \psi}{\psi} \wedge E$$

$$\frac{\Sigma \quad \Phi \quad \varphi \quad \psi}{\varphi \wedge \psi} \wedge I$$

$$\frac{\Sigma \quad \varphi \rightarrow \psi \quad \Phi \quad \varphi}{\psi} \rightarrow E$$

$$\frac{\Sigma, \varphi \quad \psi}{\varphi \rightarrow \psi} \rightarrow I, [-\varphi]$$

$$\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ \varphi \vee \psi \end{array} \quad \begin{array}{c} \Phi, \varphi \\ \vdots \\ \alpha \end{array} \quad \begin{array}{c} \Psi, \psi \\ \vdots \\ \alpha \end{array} \\
\hline
\alpha \quad \vee E, [-\varphi, -\psi]
\end{array}
\quad
\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ \varphi \end{array} \quad \vee I \\
\hline
\varphi \vee \psi
\end{array}
\quad
\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ \psi \end{array} \quad \vee I \\
\hline
\varphi \vee \psi
\end{array}$$

$$\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ \varphi \end{array} \quad \begin{array}{c} \Phi \\ \vdots \\ \neg \varphi \end{array} \\
\hline
\psi \quad \neg E
\end{array}
\quad
\begin{array}{c}
\begin{array}{c} \Sigma, \neg \psi \\ \vdots \\ \varphi \end{array} \quad \begin{array}{c} \Phi, \neg \psi \\ \vdots \\ \neg \varphi \end{array} \\
\hline
\psi \quad \neg E^*, [-\neg \psi]
\end{array}
\quad
\begin{array}{c}
\begin{array}{c} \Sigma, \psi \\ \vdots \\ \varphi \end{array} \quad \begin{array}{c} \Phi, \psi \\ \vdots \\ \neg \varphi \end{array} \\
\hline
\neg \psi \quad \neg I, [-\psi]
\end{array}$$

$$\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ \forall x \varphi \end{array} \\
\hline
[t/x]\varphi \quad \forall E
\end{array}
\quad
\begin{array}{c}
t \text{ term zonder} \\ \text{variabelen}
\end{array}
\quad
\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ [d/x]\varphi \end{array} \\
\hline
\forall x \varphi \quad \forall I
\end{array}
\quad
\begin{array}{c}
d \text{ een constante die} \\ \text{niet voorkomt in } \Sigma \\ \text{of in } \forall x \varphi
\end{array}$$

$$\begin{array}{c}
\begin{array}{c} \Phi \\ \vdots \\ \exists x \varphi \end{array} \quad \begin{array}{c} \Sigma, [d/x]\varphi \\ \vdots \\ \psi \end{array} \\
\hline
\psi \quad \exists E \quad [-[d/x]\varphi]
\end{array}
\quad
\begin{array}{c}
d \text{ een constante} \\ \text{die niet voorkomt} \\ \text{in } \Sigma, \psi \text{ of in } \exists x \varphi
\end{array}$$

$$\begin{array}{c}
\begin{array}{c} \Sigma \\ \vdots \\ [t/x]\varphi \end{array} \\
\hline
\exists x \varphi \quad \exists I
\end{array}
\quad
\begin{array}{c}
\text{mits } t \text{ vrij is voor } x \text{ in } \varphi
\end{array}$$