# Rotational Maximum Section (Prose-Only)

Version: v1 — November 17, 2025

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## Rotational Maximum in m_phi,crit(k_rot)

A concise prose-only section for direct inclusion in repositories or appendices

### Overview

In our phase-bifurcation framework for Siamese Universes, the effective phase Δφ obeys a damped, nonlinear oscillator with a transient rotational drive. When we scan the rotation parameter k_rot, the "critical phase mass" (the value at which the synchrony probability P_A crosses 0.5) shows a shallow maximum near k_rot ≈ 0.33 and remains nearly vertical in the (m_phi, k_rot) plane. This section explains, in plain language, why that mild peak appears and why it is robust rather than a numerical artefact.

### Empirical facts from the sweep

- P_A(m_phi) rises monotonically with m_phi, allowing a well-defined m_phi,crit by P_A = 0.5.
- In P_A(m_phi, k_rot), the transition band is almost vertical: m_phi controls the fate; rotation only nudges it.
- The derived curve m_phi,crit(k_rot) shows a modest crest near k_rot ≈ 0.33 with amplitude of only a few ×10^-2.
- Phase-portrait diagnostics at the crest reveal a single stable spiral around Δφ = 0 and a smooth separatrix.

### Intuition: why a shallow peak appears

Rotation injects a short-lived torque early on. For small k_rot, the drive is too weak to matter; trajectories behave as in the undriven case. For very large k_rot, the drive quickly dephases but also increases early-time kinetic "leakage," which makes synchrony slightly easier again. Between those extremes there is an intermediate k_rot where the rotational impulse best counters Hubble damping, keeping borderline trajectories hovering near the separatrix longer; that raises the mass threshold a little. The result is a shallow, broad maximum rather than a sharp resonance.

### Minimal model sketch (plain text; no LaTeX required)

Core evolution (in e-folds N):

    d2Δφ/dN2 + (3 + ξ(N)) dΔφ/dN + m_phi^2 sin(Δφ) = S_rot(N; k_rot).

Transient rotation:

    S_rot(N; k_rot) = k_rot * f(N) * exp(-γ N), with γ ~ 2 and f(N) localized at early times.

Near the fixed point Δφ ≈ 0, sin(Δφ) ≈ Δφ so the system reduces to a driven, damped linear oscillator with natural frequency ~ m_phi and friction ~ (3+ξ). The impulse-like S_rot briefly boosts the effective initial velocity. The optimal boost that most delays capture ("critical slowing down" along the separatrix) occurs at an intermediate k_rot, producing the observed shallow maximum in m_phi,crit(k_rot).

### Why it's robust

- The effect persists under modest changes of thresholds used to define sectors A/B/C.
- It appears with different time windows (ΔN) used for late-time averages.
- It is insensitive to the exact shape of f(N) as long as rotation decays well before settling.
- The peak is shallow: shifting numerical details changes its height by ~10^-3–10^-2 but not its existence.

### Implications

1) The verticality of the transition band confirms m_phi as the primary control knob of phase selection.
2) The rotational peak is a secondary modulation: real but modest. It refines, rather than overturns, the physical picture.
3) For cosmological links (baryogenesis efficiency, anisotropy forecasts), m_phi,crit can be treated as a stable scale with a small k_rot-dependent correction.

### Reproducibility note (what to check)

- Re-run the (m_phi, k_rot) grid with N_Δ = 40 (or more) uniformly sampled initial phases in [0, π), zero initial velocity, and a radiation-like background (ξ ≈ 0).
- Classify sectors via late-time means of Δφ and dΔφ/dN with thresholds (ε_φ, ε_π, ε_φ′) in the ballpark of (0.1, 0.1, 0.05).
- Extract m_phi,crit(k_rot) by linear interpolation of P_A(m_phi)|_krot at P_A = 0.5. Fit a quadratic if desired; the vertex typically lands near k_rot ≈ 0.33 with m_phi,peak ≈ 1.99.

### One-line takeaway

Rotation gives an early "nudge" that is most effective at an intermediate strength, producing a small, smooth crest in m_phi,crit(k_rot); otherwise, m_phi sets the fate.