12

Static Equilibrium and Elasticity

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q12.1** The force exerts counterclockwise torque about pivot D. The line of action of the force passes through C, so the torque about this axis is zero. In order of increasing negative (clockwise) values come the torques about F, E and B essentially together, and A. The answer is then $\tau_D > \tau_C > \tau_F > \tau_E = \tau_B > \tau_A$

Q12.2 When you bend over, your center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.

Q12.3 Yes, it can. Consider an object on a spring oscillating back and forth. In the center of the motion both the sum of the torques and the sum of the forces acting on the object are (separately) zero. Again, a meteoroid flying freely through interstellar space feels essentially no forces and keeps moving with constant velocity.

Q12.4 (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.

(b) An object in free fall has a non-zero net force acting on it, but a net torque of zero about its center of mass.

***Q12.5** Answer (a). Our theory of rotational motion does not contradict our previous theory of translational motion. The center of mass of the object moves as if the object were a particle, with all of the forces applied there. This is true whether the object is starting to rotate or not.

Q12.6 A V-shaped boomerang, a barstool, an empty coffee cup, a satellite dish, and a curving plastic slide at the edge of a swimming pool each have a center of mass that is not within the bulk of the object.

Q12.7 Suspend the plywood from the nail, and hang the plumb bob from the nail. Trace on the plywood along the string of the plumb bob. Now suspend the plywood with the nail through a different point on the plywood, not along the first line you drew. Again hang the plumb bob from the nail and trace along the string. The center of gravity is located halfway through the thickness of the plywood under the intersection of the two lines you drew.

***Q12.8** In cases (a) and (c) the center of gravity is above the base by one-half the height of the can. So (b) is the answer. In this case the center of gravity is above the base by only a bit more than one-quarter of the height of the can.

***Q12.9** Answer (b). The skyscraper is about 300 m tall. The gravitational field (acceleration) is weaker at the top by about 900 parts in ten million, by on the order of 10^{-4} times. The top half of the uniform building is lighter than the bottom half by about $(1/2)(10^{-4})$ times. Relative to the center of mass at the geometric center, this effect moves the center of gravity down, by about $(1/2)(10^{-4})(150\text{ m}) \sim 10$ mm.

Q12.10 She can be correct. If the dog stands on a relatively thick scale, the dog's legs on the ground might support more of its weight than its legs on the scale. She can check for and if necessary correct for this error by having the dog stand like a bridge with two legs on the scale and two on a book of equal thickness—a physics textbook is a good choice.

***Q12.11** Answer (b). Visualize the hatchet as like a balanced playground seesaw with one large-mass person on one side, close to the fulcrum, and a small-mass person far from the fulcrum on the other side. Different masses are on the two sides of the center of mass. The mean position of mass is not the median position.

Q12.12 The free body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. If there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip, from the equilibrium conditions alone.

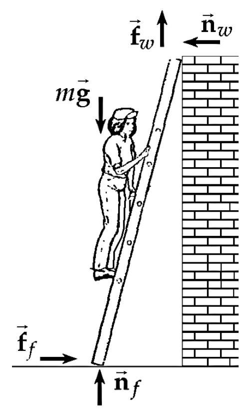


FIG. Q12.12

***Q12.13** Answer (g). In the problems we study, the forces applied to the object lie in a plane, and the axis we choose is a line perpendicular to this plane, so it appears as a point on the free-body diagram. It can be chosen anywhere. The algebra of solving for unknown forces is generally easier if we choose the axis where some unknown forces are acting.

***Q12.14** (i) Answer (b). The extension is directly proportional to the original dimension, according to $F/A = Y\Delta L/L_i$.
(ii) Answer (e). Doubling the diameter quadruples the area to make the extension four times smaller.

Q12.15 Shear deformation.

SOLUTIONS TO PROBLEMS

Section 12.1 **The Conditions for Equilibrium**

P12.1 Take torques about P.

$$\sum \tau_p = -n_0\left[\frac{\ell}{2}+d\right] + m_1 g\left[\frac{\ell}{2}+d\right] + m_b g d - m_2 g x = 0$$

We want to find x for which $n_0 = 0$.

$$x = \frac{\left(m_1 g + m_b g\right)d + m_1 g \frac{\ell}{2}}{m_2 g} = \boxed{\frac{\left(m_1 + m_b\right)d + m_1 \frac{\ell}{2}}{m_2}}$$

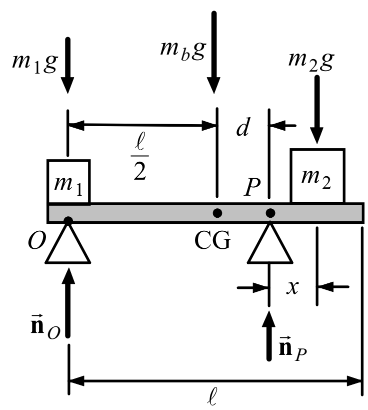


FIG. P12.1

P12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0}$$

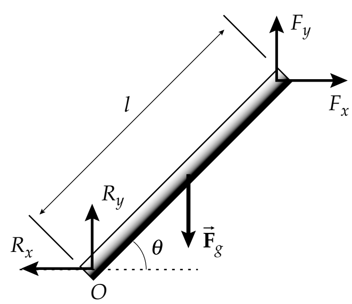


FIG. P12.2

Section 12.2 **More on the Center of Gravity**

P12.3 The coordinates of the center of gravity of piece 1 are

$$x_1 = 2.00 \text{ cm} \quad \text{and} \quad y_1 = 9.00 \text{ cm}$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm} \quad \text{and} \quad y_2 = 2.00 \text{ cm}$$

The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \quad \text{and} \quad A_2 = 32.0 \text{ cm}^2$$

And the mass of each piece is proportional to the area. Thus,

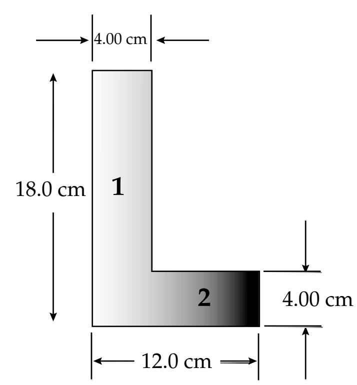


FIG. P12.3

$$x_{\text{CG}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left(72.0 \text{ cm}^2\right)\left(2.00 \text{ cm}\right) + \left(32.0 \text{ cm}^2\right)\left(8.00 \text{ cm}\right)}{72.0 \text{ cm}^2 + 32.0 \text{ cm}^2} = \boxed{3.85 \text{ cm}}$$

and

$$y_{\text{CG}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left(72.0 \text{ cm}^2\right)\left(9.00 \text{ cm}\right) + \left(32.0 \text{ cm}^2\right)\left(2.00 \text{ cm}\right)}{104 \text{ cm}^2} = \boxed{6.85 \text{ cm}}$$

P12.4 The hole we can count as negative mass

$$x_{CG} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{CG} = \frac{\sigma\pi R^2 0 - \sigma\pi\left(\frac{R}{2}\right)^2\left(-\frac{R}{2}\right)}{\sigma\pi R^2 - \sigma\pi\left(\frac{R}{2}\right)^2}$$

$$x_{CG} = \frac{R/8}{3/4} = \boxed{\frac{R}{6}}$$

P12.5 Let the fourth mass (8.00 kg) be placed at (x, y), then

$$x_{CG} = 0 = \frac{(3.00)(4.00) + m_4(x)}{12.0 + m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50\text{ m}}$$

Similarly,

$$y_{CG} = 0 = \frac{(3.00)(4.00) + 8.00(y)}{12.0 + 8.00}$$

$$y = \boxed{-1.50\text{ m}}$$

P12.6 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $\frac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma(x-3.00)^2\,dx}{9}$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2\,dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right)\int_0^{3.00}\left(x^2 - 6x + 9\right)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^{3.00} = \sigma$$

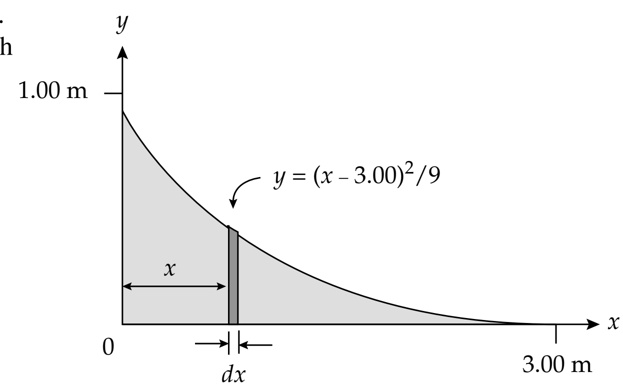


FIG. P12.6

The x-coordinate of the center of gravity is

$$x_{CG} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00}\sigma x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00}\left(x^3 - 6x^2 + 9x\right)dx$$

$$= \frac{1}{9}\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$

P12.7 In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see the Example on the center of mass of a triangle in Chapter 9).

The coordinates of the center of gravity of the three-object system are then:

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00 \text{ kg})(5.50 \text{ m}) + (3.00 \text{ kg})(6.67 \text{ m}) + (5.00 \text{ kg})(-3.50 \text{ m})}{(6.00 + 3.00 + 5.00) \text{ kg}}$$

$$x_{CG} = \frac{35.5 \text{ kg}\cdot\text{m}}{14.0 \text{ kg}} = \boxed{2.54 \text{ m}} \text{ and}$$

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00 \text{ kg})(7.00 \text{ m}) + (3.00 \text{ kg})(2.33 \text{ m}) + (5.00 \text{ kg})(+3.50 \text{ m})}{14.0 \text{ kg}}$$

$$y_{CG} = \frac{66.5 \text{ kg}\cdot\text{m}}{14.0 \text{ kg}} = \boxed{4.75 \text{ m}}$$

Section 12.3 **Examples of Rigid Objects in Static Equilibrium**

P12.8 (a) For rotational equilibrium of the lowest rod about its point of support, $\sum \tau = 0$.

$$+12.0 \text{ g } g \text{ 3 cm} - m_1 g 4 \text{ cm} \qquad \boxed{m_1 = 9.00 \text{ g}}$$

(b) For the middle rod,

$$+m_2 2 \text{ cm} - (12.0 \text{ g} + 9.0 \text{ g})5 \text{ cm} = 0 \qquad \boxed{m_2 = 52.5 \text{ g}}$$

(c) For the top rod,

$$(52.5 \text{ g} + 12.0 \text{ g} + 9.0 \text{ g})4 \text{ cm} - m_3 6 \text{ cm} = 0 \qquad \boxed{m_3 = 49.0 \text{ g}}$$

P12.9 $\sum \tau = 0 = mg(3r) - Tr$

$$2T - Mg\sin 45.0° = 0$$

$$T = \frac{Mg\sin 45.0°}{2} = \frac{1\,500 \text{ kg}(g)\sin 45.0°}{2}$$

$$= (530)(9.80) \text{ N}$$

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177 \text{ kg}}$$

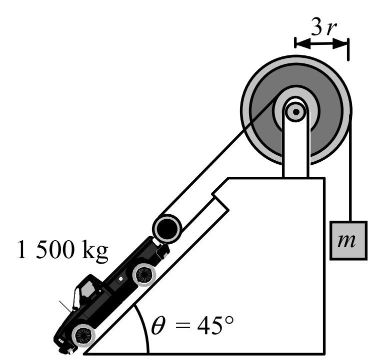


FIG. P12.9

P12.10 (a) Taking moments about P,

$$(R\sin 30.0°)0 + (R\cos 30.0°)(5.00 \text{ cm}) - (150 \text{ N})(30.0 \text{ cm}) = 0$$

$$R = 1\,039.2 \text{ N} = 1.04 \text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

$\boxed{\text{1.04 kN at 60° upward and to the right.}}$

(b) $f = R\sin 30.0° - 150 \text{ N} = 370 \text{ N}$

$$n = R\cos 30.0° = 900 \text{ N}$$

$$\boxed{\vec{\mathbf{F}}_{\text{surface}} = (370 \text{ N})\hat{\mathbf{i}} + (900 \text{ N})\hat{\mathbf{j}}}$$

P12.11 (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800\text{ N} - 500\text{ N} = 0$

Taking torques about an axis at the foot of the ladder,

$$(800\text{ N})(4.00\text{ m})\sin 30.0° + (500\text{ N})(7.50\text{ m})\sin 30.0° - n_w(15.0\text{ cm})\cos 30.0° = 0$$

Solving the torque equation,

$$n_w = \frac{[(4.00\text{ m})(800\text{ N}) + (7.50\text{ m})(500\text{ N})]\tan 30.0°}{15.0\text{ m}} = 268\text{ N}$$

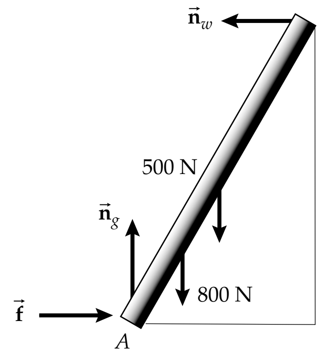


FIG. P12.11

Next substitute this value into the F_x equation to find

$$f = n_w = \boxed{268\text{ N}} \text{ in the positive } x \text{ direction.}$$

Solving the equation $\sum F_y = 0$,

$$n_g = \boxed{1\,300\text{ N}} \text{ in the positive } y \text{ direction.}$$

(b) In this case, the torque equation $\sum \tau_A = 0$ gives:

$$(9.00\text{ m})(800\text{ N})\sin 30.0° + (7.50\text{ m})(500\text{ N})\sin 30.0° - (15.0\text{ m})(n_w)\sin 60.0° = 0$$

or

$$n_w = 421\text{ N}$$

Since $f = n_w = 421$ N and $f = f_{\max} = \mu n_g$, we find

$$\mu = \frac{f_{\max}}{n_g} = \frac{421\text{ N}}{1\,300\text{ N}} = \boxed{0.324}$$

P12.12 (a) $\sum F_x = f - n_w = 0$ (1)

$\sum F_y = n_g - m_1 g - m_2 g = 0$ (2)

$$\sum \tau_A = -m_1 g\left(\frac{L}{2}\right)\cos\theta - m_2 g x\cos\theta + n_w L\sin\theta = 0$$

From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

Then, from equation (1): $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from equation (2): $n_g = \boxed{(m_1 + m_2)g}$

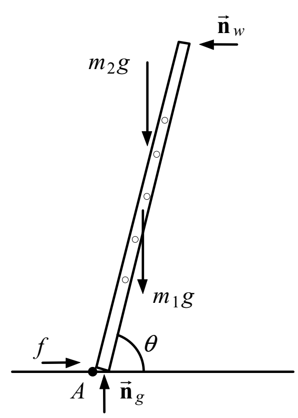


FIG. P12.12

(b) If the ladder is on the verge of slipping when $x = d$,

then

$$\mu = \frac{f|_{x=d}}{n_g} = \boxed{\frac{(m_1/2 + m_2 d/L)\cot\theta}{m_1 + m_2}}$$

P12.13 Torque about the front wheel is zero.

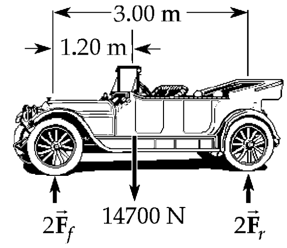


FIG. P12.13

$$0 = (1.20\text{ m})(mg) - (3.00\text{ m})(2F_r)$$

Thus, the force at each rear wheel is

$$F_r = 0.200mg = \boxed{2.94\text{ kN}}$$

The force at each front wheel is then

$$F_f = \frac{mg - 2F_r}{2} = \boxed{4.41\text{ kN}}$$

***P12.14** (a) The gravitational force on the floodlight is (20 kg)(9.8 m/s^2) = 196 N
We consider the torques acting on the beam, about an axis perpendicular to the page and through the left end of the horizontal beam.

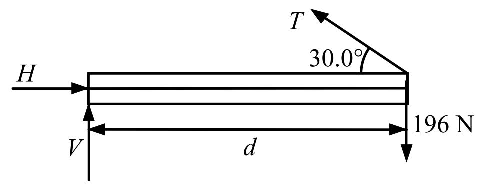


FIG. P12.14

$$\sum \tau = +(T \sin 30.0°)d - (196\text{ N})d = 0$$

giving $T = \boxed{392\text{ N}}$.

(b) From $\sum F_x = 0$, $H - T\cos 30.0° = 0$, or $H = (392\text{ N})\cos 30.0° = \boxed{339\text{ N to the right}}$.

(c) From $\sum F_y = 0$, $V + T\sin 30.0° - 196\text{ N} = 0$, or $V = 196\text{ N} - (392\text{ N})\sin 30.0° = \boxed{0}$.

(d) From the same free-body diagram with the axis chosen at the right-hand end, we write

$\sum \tau = H(0) - Vd + T(0) + 196\text{ N}(0) = 0$ so $\boxed{V = 0}$

(e) From $\sum F_y = 0$, $V + T\sin 30.0° - 196\text{ N} = 0$, or $T = 0 + 196\text{ N}/\sin 30.0° = \boxed{392\text{ N}}$.

(f) From $\sum F_x = 0$, $H - T\cos 30.0° = 0$, or $H = (392\text{ N})\cos 30.0° = \boxed{339\text{ N to the right}}$.

(g) The two solutions agree precisely. They are equally accurate. They are essentially equally simple. But note that many students would make a mistake on the negative (clockwise) sign for the torque of the upward force V in the equation in part (d).

Taking together the equations we have written, we appear to have four equations but we cannot determine four unknowns. Only three of the equations are independent, so we can determine only three unknowns.

P12.15 (a) Vertical forces on one half of the chain: $T_e \sin 42.0° = 20.0\text{ N}$ $\boxed{T_e = 29.9\text{ N}}$

(b) Horizontal forces on one half of the chain: $T_e \cos 42.0° = T_m$ $\boxed{T_m = 22.2\text{ N}}$

P12.16 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00 \text{ m})\cos 20.0° = 4.70 \text{ m}$ and vertically down a distance $y = (5.00 \text{ m})\sin 20.0° =$ 1.71 m. The cable then makes the following angle with the horizontal:

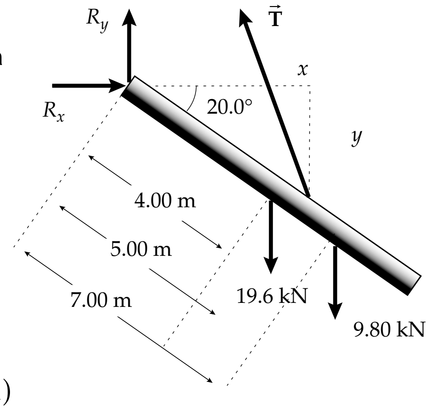


FIG. P12.16

$$\theta = \tan^{-1}\left[\frac{(12.0+1.71) \text{ m}}{4.70 \text{ m}}\right] = 71.1°$$

(a) Take torques about the hinge end of the bridge:

$$R_x(0)+R_y(0)-19.6 \text{ kN}(4.00 \text{ m})\cos 20.0°$$
$$-T\cos 71.1°(1.71 \text{ m})+T\sin 71.1°(4.70 \text{ m})$$
$$-9.80 \text{ kN}(7.00 \text{ m})\cos 20.0° = 0$$

which yields $T = \boxed{35.5 \text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.1° = 0$

or

$$R_x = (35.5 \text{ kN})\cos 71.1° = \boxed{11.5 \text{ kN (right)}}$$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6 \text{ kN} + T\sin 71.1° - 9.80 \text{ kN} = 0$

Thus,

$$R_y = 29.4 \text{ kN} - (35.5 \text{ kN})\sin 71.1° = -4.19 \text{ kN}$$
$$= \boxed{4.19 \text{ kN down}}$$

P12.17 (a) We model the horse as a particle. The drawbridge will fall out from under the horse.

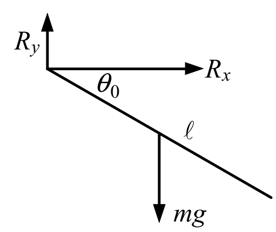


FIG. P12.17(a)

$$\alpha = mg\frac{\frac{1}{2}\ell\cos\theta_0}{\frac{1}{3}m\ell^2} = \frac{3g}{2\ell}\cos\theta_0$$
$$= \frac{3(9.80 \text{ m/s}^2)\cos 20.0°}{2(8.00 \text{ m})} = \boxed{1.73 \text{ rad/s}^2}$$

(b) $\frac{1}{2}I\omega^2 = mgh$

$$\therefore \frac{1}{2}\cdot\frac{1}{3}m\ell^2\omega^2 = mg\cdot\frac{1}{2}\ell(1-\sin\theta_0)$$

Solving,

$$\omega = \sqrt{\frac{3g}{\ell}(1-\sin\theta_0)} = \sqrt{\frac{3(9.80 \text{ m/s}^2)}{8.00 \text{ m}}(1-\sin 20°)} = \boxed{1.56 \text{ rad/s}}$$

continued on next page

(c) The linear acceleration of the center of mass of the bridge is

$$a = \frac{1}{2}\ell\alpha = \frac{1}{2}(8.0\text{ m})\left(1.73\text{ rad/s}^2\right) = 6.907\text{ m/s}^2$$

The force at the hinge plus the gravitational force produce the acceleration $a = 6.907\text{ m/s}^2$ at right angles to the bridge.

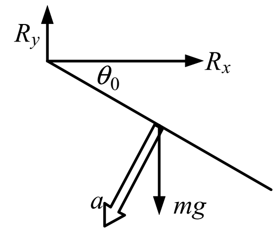


FIG. P12.17(c)

$$R_x = ma_x = (2\,000\text{ kg})\left(6.907\text{ m/s}^2\right)\cos 250° = -4.72\text{ kN}$$

$$R_y - mg = ma_y$$

Solving,

$$R_y = m\left(g + a_y\right) = (2\,000\text{ kg})\left[9.80\text{ m/s}^2 + \left(6.907\text{ m/s}^2\right)\sin 250°\right] = 6.62\text{ kN}$$

Thus

$$\boxed{\vec{\mathbf{R}} = \left(-4.72\hat{\mathbf{i}} + 6.62\hat{\mathbf{j}}\right)\text{ kN}}$$

(d) $R_x = 0$

$$a = \omega^2\left(\frac{1}{2}\ell\right) = (1.56\text{ rad/s})^2(4.0\text{ m}) = 9.67\text{ m/s}^2$$

$$R_y - mg = ma$$

$$\therefore R_y = (2\,000\text{ kg})\left(9.8\text{ m/s}^2 + 9.67\text{ m/s}^2\right) = 38.9\text{ kN}$$

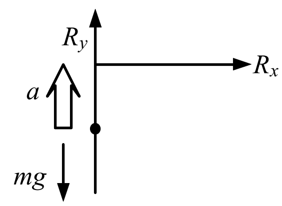


FIG. P12.17(d)

Thus:

$$\boxed{R_y = 38.9\hat{\mathbf{j}}\text{ kN}}$$

P12.18 Call the required force F, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.

Just as the wheel leaves the ground, the ground exerts no force on it.

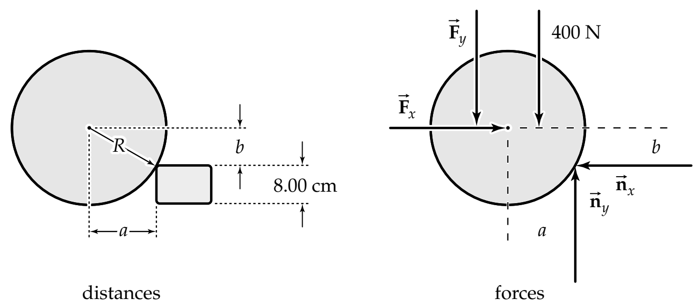


FIG. P12.18

$$\sum F_x = 0:\ F\cos 15.0° - n_x \qquad (1)$$

$$\sum F_y = 0:\ -F\sin 15.0° - 400\text{ N} + n_y = 0 \qquad (2)$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00\text{ cm} = (20.0 - 8.00)\text{ cm} = 12.0\text{ cm}$$

$$a = \sqrt{R^2 - b^2} = 16.0\text{ cm}$$

(a) $\sum \tau = 0:\ -F_x b + F_y a + (400\text{ N})a = 0$, or

$$F\left[-(12.0\text{ cm})\cos 15.0° + (16.0\text{ cm})\sin 15.0°\right] + (400\text{ N})(16.0\text{ cm}) = 0$$

so

$$F = \frac{6\,400\text{ N}\cdot\text{cm}}{7.45\text{ cm}} = \boxed{859\text{ N}}$$

continued on next page

(b) Then, using Equations (1) and (2),

$n_x = (859\text{ N})\cos 15.0° = 830\text{ N}$ and

$n_y = 400\text{ N} + (859\text{ N})\sin 15.0° = 622\text{ N}$

$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04\text{ kN}}$

$\theta = \tan^{-1}\left(\dfrac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$

P12.19 When $x = x_{\min}$, the rod is on the verge of slipping, so

$$f = (f_s)_{\max} = \mu_s n = 0.50n$$

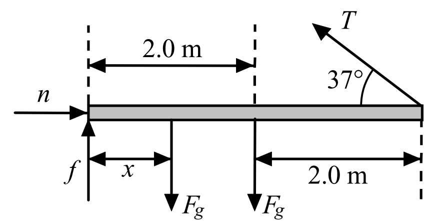


FIG. P12.19

From $\sum F_x = 0,\ n - T\cos 37° = 0$, or $n = 0.799T$.

Thus,

$$f = 0.50(0.799T) = 0.399T$$

From $\sum F_y = 0,\ f + T\sin 37° - 2F_g = 0$,

or

$$0.399T - 0.602T - 2F_g = 0, \text{ giving } T = 2.00F_g$$

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives

$-F_g \cdot x_{\min} - F_g(2.0\text{ m}) + \left[(2F_g)\sin 37°\right](4.0\text{ m}) = 0$, which reduces to $x_{\min} = \boxed{2.82\text{ m}}$

P12.20 Consider forces and torques on the beam.

$\sum F_x = 0$: $\quad R\cos\theta - T\cos 53° = 0$

$\sum F_y = 0$: $\quad R\sin\theta + T\sin 53° - 800\text{ N} = 0$

$\sum \tau = 0$: $\quad (T\sin 53°)8\text{ m} - (600\text{ N})x - (200\text{ N})4\text{ m} = 0$

(a) Then $T = \dfrac{600\text{ N}x + 800\text{ N}\cdot\text{m}}{8\text{ m}\sin 53°} = (93.9\text{ N/m})x + 125\text{ N}$

As x increases from 2 m, this expression grows larger.

(b) From substituting back,

$$R\cos\theta = [93.9x + 125]\cos 53°$$
$$R\sin\theta = 800\text{ N} - [93.9x + 125]\sin 53°$$

Dividing,

$$\tan\theta = \frac{R\sin\theta}{R\cos\theta} = -\tan 53° + \frac{800\text{ N}}{(93.9x + 125)\cos 53°}$$
$$\tan\theta = \tan 53°\left(\frac{32}{3x+4} - 1\right)$$

As x increases the fraction decreases and $\boxed{\theta \text{ decreases}}$.

continued on next page

(c) To find R we can work out $R^2\cos^2\theta+R^2\sin^2\theta=R^2$. From the expressions above for $R\cos\theta$ and $R\sin\theta$,

$$R^2=T^2\cos^2 53°+T^2\sin^2 53°-1\,600\text{ N}T\sin 53°+(800\text{ N})^2$$
$$R^2=T^2-1\,600T\sin 53°+640\,000$$
$$R^2=(93.9x+125)^2-1\,278(93.9x+125)+640\,000$$
$$R=\left(8\,819x^2-96\,482x+495\,678\right)^{1/2}$$

At $x=0$ this gives $R=704$ N. At $x=2$ m, $R=581$ N. At $x=8$ m, $R=537$ N. Over the range of possible values for x, the negative term $-96\,482x$ dominates the positive term $8\,819x^2$, and $\boxed{R \text{ decreases}}$ as x increases.

P12.21 To find U, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U=29.4(2.25) \qquad \boxed{U=88.2\text{ N}}$$

To find D, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D=(1.50)(29.4) \qquad \boxed{D=58.8\text{ N}}$$

Also, notice that $U=D+F_g$, so $\sum F_y=0$.

Section 12.4 **Elastic Properties of Solids**

P12.22 The definition of $Y=\dfrac{\text{stress}}{\text{strain}}$ means that Y is the slope of the graph:

$$Y=\frac{300\times10^6\text{ N/m}^2}{0.003}=\boxed{1.0\times10^{11}\text{ N/m}^2}$$

P12.23 $\dfrac{F}{A}=Y\dfrac{\Delta L}{L_i}$

$$\Delta L=\frac{FL_i}{AY}=\frac{(200)(9.80)(4.00)}{\left(0.200\times10^{-4}\right)\left(8.00\times10^{10}\right)}=\boxed{4.90\text{ mm}}$$

P12.24 (a) stress $=\dfrac{F}{A}=\dfrac{F}{\pi r^2}$

$$F=(\text{stress})\pi\left(\frac{d}{2}\right)^2$$

$$F=\left(1.50\times10^8\text{ N/m}^2\right)\pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2$$

$$F=\boxed{73.6\text{ kN}}$$

(b) stress $=Y(\text{strain})=\dfrac{Y\Delta L}{L_i}$

$$\Delta L=\frac{(\text{stress})L_i}{Y}=\frac{\left(1.50\times10^8\text{ N/m}^2\right)(0.250\text{ m})}{1.50\times10^{10}\text{ N/m}^2}=\boxed{2.50\text{ mm}}$$

P12.25 From the defining equation for the shear modulus, we find Δx as

$$\Delta x = \frac{hf}{SA} = \frac{\left(5.00\times10^{-3}\text{ m}\right)(20.0\text{ N})}{\left(3.0\times10^{6}\text{ N/m}^2\right)\left(14.0\times10^{-4}\text{ m}^2\right)} = 2.38\times10^{-5}\text{ m}$$

or $\Delta x = \boxed{2.38\times10^{-2}\text{ mm}}$

P12.26 Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\text{ kN}}{0.200\text{ kN}} = 100$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\text{ mm})\sqrt{100}\ \boxed{\sim 1\text{ cm}}$$

P12.27 (a)

$$\begin{aligned} F &= (A)(\text{stress}) \\ &= \pi\left(5.00\times10^{-3}\text{ m}\right)^2\left(4.00\times10^{8}\text{ N/m}^2\right) \\ &= \boxed{3.14\times10^{4}\text{ N}} \end{aligned}$$

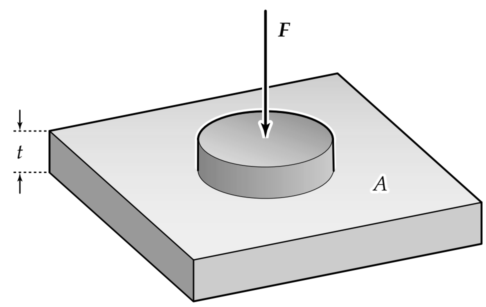


FIG. P12.27(b)

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$\begin{aligned} A &= (2\pi r)t = 2\pi\left(5.00\times10^{-3}\text{ m}\right)\left(5.00\times10^{-3}\text{ m}\right) \\ &= 1.57\times10^{-4}\text{ m}^2 \end{aligned}$$

So,

$$\begin{aligned} F &= (A)\text{Stress} = \left(1.57\times10^{-4}\text{ m}^2\right)\left(4.00\times10^{8}\text{ N/m}^2\right) \\ &= \boxed{6.28\times10^{4}\text{ N}} \end{aligned}$$

P12.28 The force acting on the hammer changes its momentum according to

$$mv_i + \bar{F}(\Delta t) = mv_f \text{ so } \left|\bar{F}\right| = \frac{m\left|v_f - v_i\right|}{\Delta t}$$

Hence,

$$\left|\bar{F}\right| = \frac{30.0\text{ kg}\left|-10.0\text{ m/s} - 20.0\text{ m/s}\right|}{0.110\text{ s}} = 8.18\times10^{3}\text{ N}$$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress} = \frac{F}{A} = \frac{8.18\times10^{3}\text{ N}}{\pi(0.023\,0\text{ m})^2/4} = 1.97\times10^{7}\text{ N/m}^2$$

and the strain is: $\text{strain} = \frac{\text{stress}}{Y} = \frac{1.97\times10^{7}\text{ N/m}^2}{20.0\times10^{10}\text{ N/m}^2} = \boxed{9.85\times10^{-5}}$

P12.29 Consider recompressing the ice, which has a volume $1.09V_0$.

$$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = \frac{-\left(2.00\times10^9\ \text{N/m}^2\right)(-0.090)}{1.09} = \boxed{1.65\times10^8\ \text{N/m}^2}$$

P12.30 Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \qquad (1) \qquad \text{and} \qquad m_2 a = m_2 g - T \qquad (2)$$

where T is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \dfrac{T}{m_1} - g$

and substituting this into equation (2) yields: $\dfrac{m_2}{m_1}T - m_2 g = m_2 g - T$

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00\text{ kg})(5.00\text{ kg})\left(9.80\ \text{m/s}^2\right)}{8.00\text{ kg}} = 36.8\text{ N}$$

From the definition of Young's modulus, $Y = \dfrac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8\text{ N})(2.00\text{ m})}{\left(2.00\times10^{11}\ \text{N/m}^2\right)\pi\left(2.00\times10^{-3}\ \text{m}\right)^2} = \boxed{0.029\,3\text{ mm}}$$

P12.31 Part of the load force extends the cable and part compresses the column by the same distance $\Delta\ell$:

$$F = \frac{Y_A A_A \Delta\ell}{\ell_A} + \frac{Y_s A_s \Delta\ell}{\ell_s}$$

$$\Delta\ell = \frac{F}{Y_A A_A/\ell_A + Y_s A_s/\ell_s}$$

$$= \frac{8\,500\text{ N}}{7\times10^{10}\pi\left(0.162\,4^2 - 0.161\,4^2\right)/4(3.25) + 20\times10^{10}\pi(0.012\,7)^2/4(5.75)}$$

$$= \boxed{8.60\times10^{-4}\ \text{m}}$$

P12.32 $B = -\dfrac{\Delta P}{\Delta V/V_i} = -\dfrac{\Delta P V_i}{\Delta V}$

(a) $\Delta V = -\dfrac{\Delta P V_i}{B} = -\dfrac{\left(1.13\times10^8\ \text{N/m}^2\right)1\text{ m}^3}{0.21\times10^{10}\ \text{N/m}^2} = \boxed{-0.053\,8\text{ m}^3}$

(b) The quantity of water with mass 1.03×10^3 kg occupies volume at the bottom $1\text{ m}^3 - 0.053\,8\text{ m}^3 = 0.946\text{ m}^3$. So its density is $\dfrac{1.03\times10^3\text{ kg}}{0.946\text{ m}^3} = \boxed{1.09\times10^3\ \text{kg/m}^3}$

(c) With only a 5% volume change in this extreme case, liquid water is indeed nearly incompressible.

Additional Problems

P12.33 Let n_A and n_B be the normal forces at the points of support.

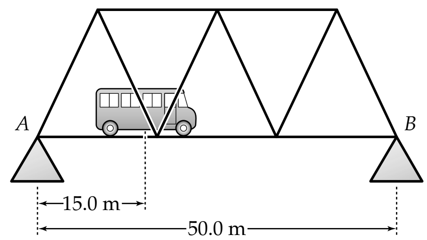


FIG. P12.33

Choosing the origin at point A with $\sum F_y = 0$ and $\sum \tau = 0$, we find:

$$n_A + n_B - (8.00\times 10^4)g - (3.00\times 10^4)g = 0 \quad \text{and}$$

$$-(3.00\times 10^4)(g)15.0 - (8.00\times 10^4)(g)25.0 + n_B(50.0) = 0$$

The equations combine to give $n_A = \boxed{5.98\times 10^5 \text{ N}}$ and $b_B = \boxed{4.80\times 10^5 \text{ N}}$.

***P12.34** Model the stove as a uniform 68 kg box. Its center of mass is at its geometric center, $\frac{28}{2} = 14$ inches behind its feet at the front corners. Assume that the light oven door opens to be horizontal and that a person stands on its outer end, $46.375 - 28 = 18.375$ inches in front of the front feet.

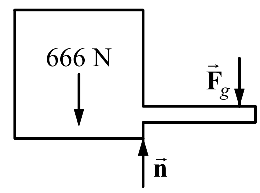


FIG. P12.34

We find the weight F_g of a person standing on the oven door with the stove balanced on its front feet in equilibrium: $\sum \tau = 0$

$$(68 \text{ kg})(9.8 \text{ m/s}^2)(14 \text{ in.}) + n(0) - F_g(18.375 \text{ in.}) = 0$$

$$F_g = \boxed{508 \text{ N}}$$

If the weight of the person is greater than this, the stove can tip forward. This weight corresponds to mass 51.8 kg, so the person could be a child. If the oven door is heavy (compared to the backsplash) or if the front feet are significantly far behind the front corners, the maximum weight will be significantly less than 508 N.

P12.35 With ℓ as large as possible, n_1 and n_2 will both be large. The equality sign in $f_2 \le \mu_s n_2$ will be true, but the less-than sign will apply in $f_1 < \mu_s n_1$. Take torques about the lower end of the pole.

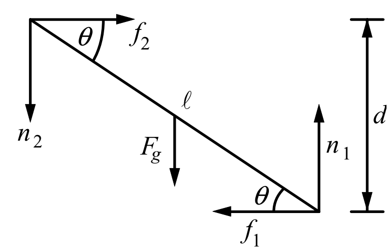


FIG. P12.35

$$n_2\ell\cos\theta + F_g\left(\frac{1}{2}\ell\right)\cos\theta - f_2\ell\sin\theta = 0$$

Setting $f_2 = 0.576 n_2$, the torque equation becomes

$$n_2(1 - 0.576\tan\theta) + \frac{1}{2}F_g = 0$$

Since $n_2 > 0$, it is necessary that

$$1 - 0.576\tan\theta < 0$$

$$\therefore \tan\theta > \frac{1}{0.576} = 1.736$$

$$\therefore \theta > 60.1°$$

$$\therefore \ell = \frac{d}{\sin\theta} < \frac{7.80 \text{ ft}}{\sin 60.1°} = \boxed{9.00 \text{ ft}}$$

P12.36 When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2, the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00 \times 10^6 \text{ N/m}^2 = Y \cdot (\text{strain}) = Y\left(\frac{\Delta L}{L_i}\right)$

Thus,

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00 \times 10^6 \text{ N/m}^2)(1.50 \text{ m})}{30.0 \times 10^9 \text{ N/m}^2}$$

or

$$\Delta L = 4.00 \times 10^{-4} \text{ m} = \boxed{0.400 \text{ mm}}$$

(b) In the concrete: stress $= \frac{T_2}{A_c} = 8.00 \times 10^6 \text{ N/m}^2$, so

$$T_2 = (8.00 \times 10^6 \text{ N/m}^2)(50.0 \times 10^{-4} \text{ m}^2) = \boxed{40.0 \text{ kN}}$$

(c) For the rod: $\frac{T_2}{A_R} = \left(\frac{\Delta L}{L_i}\right) Y_{\text{steel}}$ so $\Delta L = \frac{T_2 L_i}{A_R Y_{\text{steel}}}$

$$\Delta L = \frac{(4.00 \times 10^4 \text{ N})(1.50 \text{ m})}{(1.50 \times 10^{-4} \text{ m}^2)(20.0 \times 10^{10} \text{ N/m}^2)} = 2.00 \times 10^{-3} \text{ m} = \boxed{2.00 \text{ mm}}$$

(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40 \text{ mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right) Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right) A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40 \times 10^{-3} \text{ m}}{1.50 \text{ m}}\right)(1.50 \times 10^{-4} \text{ m}^2)(20.0 \times 10^{10} \text{ N/m}^2) = \boxed{48.0 \text{ kN}}$$

P12.37 (a) See the diagram.

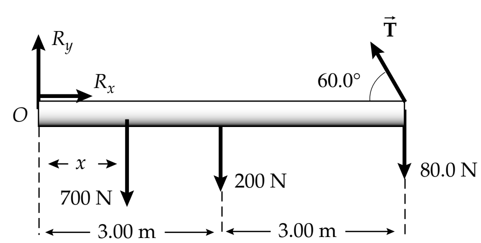


FIG. P12.37

(b) If $x = 1.00$ m, then

$$\sum \tau_O = (-700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m}) + (T \sin 60.0°)(6.00 \text{ m}) = 0$$

Solving for the tension gives: $T = \boxed{343 \text{ N}}$.

From $\sum F_x = 0$, $R_x = T \cos 60.0° = \boxed{171 \text{ N}}$.

From $\sum F_y = 0$, $R_y = 980 \text{ N} - T \sin 60.0° = \boxed{683 \text{ N}}$.

(c) If $T = 900$ N:

$$\sum \tau_O = (-700 \text{ N})x - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m}) + \left[(900 \text{ N})\sin 60.0°\right](6.00 \text{ m}) = 0$$

Solving for x gives: $x = \boxed{5.13 \text{ m}}$.

***P12.38** The 392 N is the weight of the uniform gate, which is 3 m wide. The hinges are 1.8 m apart. They exert horizontal forces A and C. Only one hinge exerts a vertical force. We assume it is the upper hinge.

(a) Free body diagram:

Statement:

A uniform 40.0-kg farm gate, 3.00 m wide and 1.80 m high, supports a 50.0-N bucket of grain hanging from its latch as shown. The gate is supported by hinges at two corners. Find the force each hinge exerts on the gate.

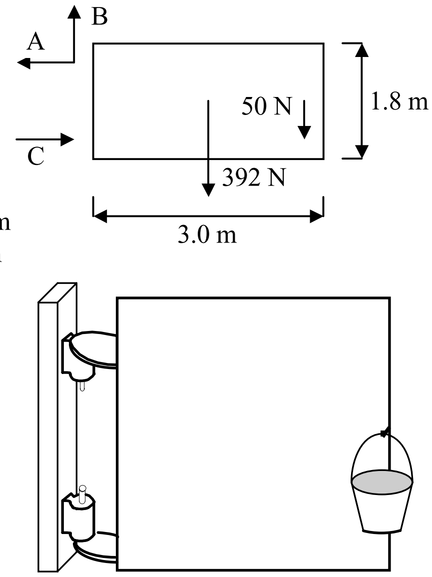


FIG. P12.38

(b) From the torque equation,

$$C = \frac{738\ \text{N}\cdot\text{m}}{1.8\ \text{m}} = 410\ \text{N}$$

Then $A = 410$ N. Also $B = 442$ N.

$$\boxed{\begin{array}{l}\text{The upper hinge exerts 410 N to the left and 442 N up.}\\ \text{The lower hinge exerts 410 N to the right.}\end{array}}$$

P12.39 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

$$\sum F_x = R_x - T\cos\theta = 0$$
$$\sum F_y = R_y + T\sin\theta - F_g = 0$$

and $\sum \tau = -F_g(L+d) + T\sin\theta(2L+d) = 0.$

Solving these equations, we find:

(a) $T = \boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$

(b) $R_x = \boxed{\dfrac{F_g(L+d)\cot\theta}{2L+d}}$ $\qquad R_y = \boxed{\dfrac{F_g L}{2L+d}}$

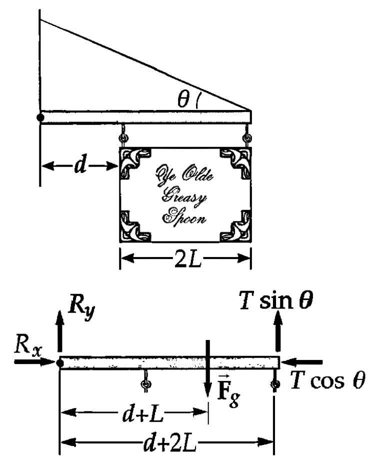


FIG. P12.39

P12.40 $\sum \tau_{\text{point 0}} = 0$ gives

$$(T\cos 25.0°)\left(\frac{3\ell}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3\ell}{4}\cos 65.0°\right)$$
$$= (2\,000\ \text{N})(\ell\cos 65.0°) + (1\,200\ \text{N})\left(\frac{\ell}{2}\cos 65.0°\right)$$

From which, $T = 1\,465\ \text{N} = \boxed{1.46\ \text{kN}}$

From $\sum F_x = 0$,

$H = T\cos 25.0° = 1\,328\ \text{N}(\text{toward right}) = \boxed{1.33\ \text{kN}}$

From $\sum F_y = 0$,

$V = 3\,200\ \text{N} - T\sin 25.0° = 2\,581\ \text{N}(\text{upward}) = \boxed{2.58\ \text{kN}}$

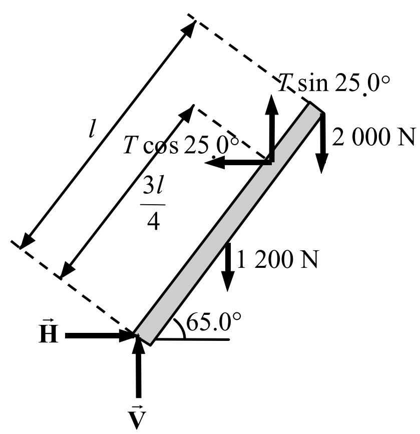


FIG. P12.40

P12.41 We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the x direction and

$$\boxed{F_{By}=0}$$

$$\sum F_x = F_{Bx} - F_{Ax} = 0$$

$$F_{Ay} - (3\,000 + 10\,000)g = 0$$

and

$$\sum \tau = -(3\,000g)(2.00) - (10\,000g)(6.00) + F_{Bx}(1.00) = 0$$

These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47\times10^5\text{ N}}$$

$$F_{Ay} = \boxed{1.27\times10^5\text{ N}}$$

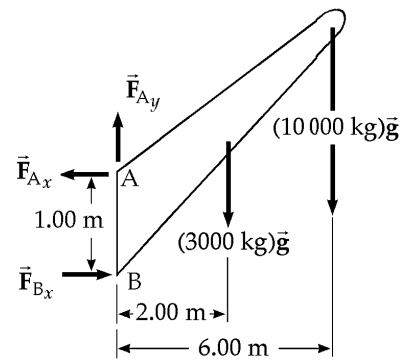


FIG. P12.41

***P12.42** Choosing torques about the hip joint, $\sum \tau = 0$ gives

$$-\frac{L}{2}(350\text{ N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\text{ N})L = 0$$

From which, $T = \boxed{2.71\text{ kN}}$.

Let R_x = compression force along spine, and from $\sum F_x = 0$

$$R_x = T_x = T\cos 12.0° = \boxed{2.65\text{ kN}}$$

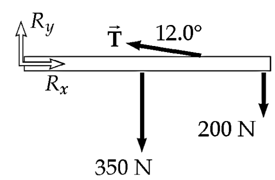


FIG. P12.42

(c)	You should lift "with your knees" rather than "with your back." In this situation, with a load weighing only 200 N, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

P12.43 From the free-body diagram, the angle that the string tension makes with the rod is

$$\theta = 60.0° + 20.0° = 80.0°$$

and the perpendicular component of the string tension is $T\sin 80.0°$. Summing torques around the base of the rod gives

$$\sum \tau = 0:\quad -(4.00\text{ m})(10\,000\text{ N})\cos 60° + T(4.00\text{ m})\sin 80° = 0$$

$$T = \frac{(10\,000\text{ N})\cos 60.0°}{\sin 80.0°} = \boxed{5.08\times10^3\text{ N}}$$

$$\sum F_x = 0:\quad F_H - T\cos 20.0° = 0$$

$$F_H = T\cos 20.0° = \boxed{4.77\times10^3\text{ N}}$$

$$\sum F_y = 0:\quad F_V + T\sin 20.0° - 10\,000\text{ N} = 0$$

and $\quad F_V = (10\,000\text{ N}) - T\sin 20.0° = \boxed{8.26\times10^3\text{ N}}$

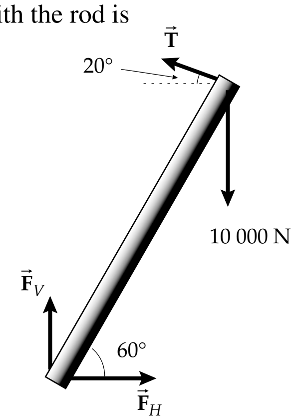


FIG. P12.43

P12.44 (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of A and B will intersect at a point above the rod. They will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in Figure 12.52(a), and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in Figure 12.52(b). All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the trough and the normal forces, and the rod's center of gravity is vertically above the bottom of the trough.

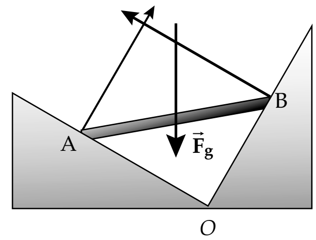


FIG. P12.44(a)

(b) In Figure (b), $\overline{AO}\cos 30.0° = \overline{BO}\cos 60.0°$ and

$$L^2 = \overline{AO}^2 + \overline{BO}^2 = \overline{AO}^2 + \overline{AO}^2\left(\frac{\cos^2 30.0°}{\cos^2 60.0°}\right)$$

$$\overline{AO} = \frac{L}{\sqrt{1+\left(\dfrac{\cos 30°}{\cos 60°}\right)^2}} = \frac{L}{2}$$

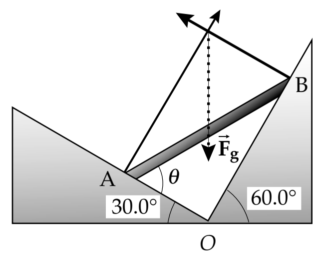


FIG. P12.44(b)

So

$$\cos\theta = \frac{\overline{AO}}{L} = \frac{1}{2} \text{ and } \theta = \boxed{60.0°}$$

P12.45 (a) Locate the origin at the bottom left corner of the cabinet and let x = distance between the *resultant normal force* and the *front of the cabinet.* Then we have

$$\sum F_x = 200\cos 37.0° - \mu n = 0 \qquad (1)$$

$$\sum F_y = 200\sin 37.0° + n - 400 = 0 \qquad (2)$$

$$\sum \tau = n(0.600 - x) - 400(0.300) + 200\sin 37.0°(0.600)$$
$$-200\cos 37.0°(0.400) = 0 \qquad (3)$$

From (2),

$$n = 400 - 200\sin 37.0° = 280 \text{ N}$$

From (3),

$$x = \frac{72.2 - 120 + 280(0.600) - 64.0}{280}$$

$$x = \boxed{20.1 \text{ cm}} \text{ to the left of the front edge}$$

From (1), $\mu_k = \dfrac{200\cos 37.0°}{280} = \boxed{0.571}$

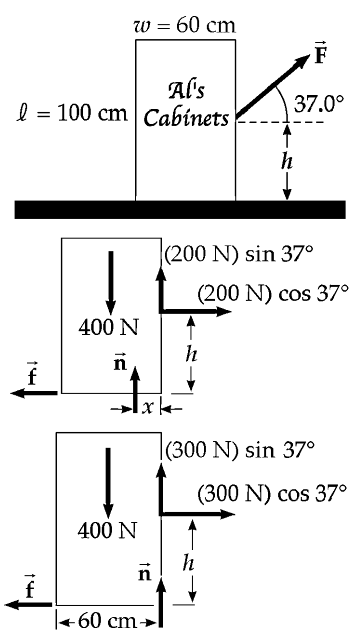


FIG. P12.45

(b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum \tau = 0$ to find h:

$$\sum \tau = 400(0.300) - (300\cos 37.0°)h = 0 \qquad h = \frac{120}{300\cos 37.0°} = \boxed{0.501 \text{ m}}$$

P12.46 (a), (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.

$$\sum \tau = 0 \Rightarrow -F(1.00\text{ m}) + (400\text{ N})(0.300\text{ m}) = 0$$

yielding $F = \dfrac{(400\text{ N})(0.300\text{ m})}{1.00\text{ m}} = \boxed{120\text{ N}}$

$$\sum F_x = 0 \Rightarrow -f + 120\text{ N} = 0, \quad \text{or} \quad f = 120\text{ N}$$

$$\sum F_y = 0 \Rightarrow -400\text{ N} + n = 0, \quad \text{so} \quad n = 400\text{ N}$$

Thus,

$$\mu_s = \frac{f}{n} = \frac{120\text{ N}}{400\text{ N}} = \boxed{0.300}$$

(c) Apply F' at the upper rear corner and directed so $\theta + \phi = 90.0°$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00\text{ m}}{0.600\text{ m}}\right) = 59.0°$$

Thus,

$$\phi = 90.0° - 59.0° = 31.0°$$

Sum the torques about the lower front corner of the cabinet:

$$-F'\sqrt{(1.00\text{ m})^2 + (0.600\text{ m})^2} + (400\text{ N})(0.300\text{ m}) = 0$$

so

$$F' = \frac{120\text{ N}\cdot\text{m}}{1.17\text{ m}} = 103\text{ N}$$

Therefore, the minimum force required to tip the cabinet is

$\boxed{\text{103 N applied at 31.0° above the horizontal at the upper left corner}}$.

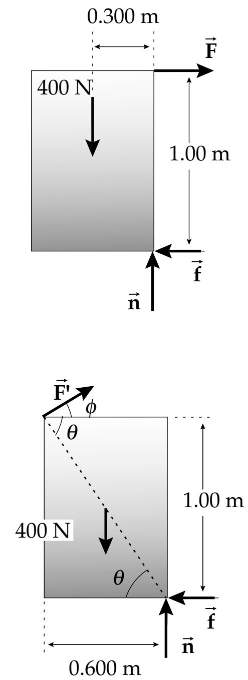


FIG. P12.46

P12.47 (a) We can use $\sum F_x = \sum F_y = 0$ and $\sum \tau = 0$ with pivot point at the contact on the floor.

Then

$$\sum F_x = T - \mu_s n = 0$$

$$\sum F_y = n - Mg - mg = 0, \text{ and}$$

$$\sum \tau = Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0$$

Solving the above equations gives

$$M = \boxed{\frac{m}{2}\left(\frac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

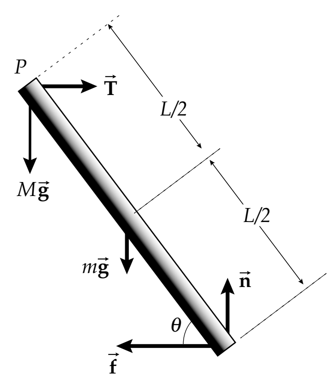


FIG. P12.47

This answer is the maximum value for M if $\mu_s < \cot\theta$. If $\mu_s \geq \cot\theta$, the mass M can increase without limit. It has no maximum value.

(b) At the floor, we have the normal force in the y-direction and frictional force in the x-direction. The reaction force then is

$$R = \sqrt{n^2 + (\mu_s n)^2} = \boxed{(M+m)g\sqrt{1+\mu_s^2}}$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M+m)^2}}$$

***P12.48** Suppose that a bar exerts on a pin a force not along the length of the bar. Then, the pin exerts on the bar a force with a component perpendicular to the bar. The only other force on the bar is the pin force on the other end. For $\sum \vec{\mathbf{F}} = 0$, this force must also have a component perpendicular to the bar. Then each of the forces produces a torque about the center of the bar in the same sense. The total torque on the bar is not zero. The contradiction proves that the bar can only exert forces along its length.

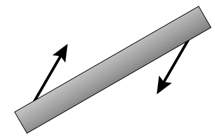


FIG. P12.48

***P12.49** (a) The height of pin B is

$$(10.0 \text{ m})\sin 30.0° = 5.00 \text{ m}$$

The length of bar BC is then

$$\overline{BC} = \frac{5.00 \text{ m}}{\sin 45.0°} = 7.07 \text{ m}$$

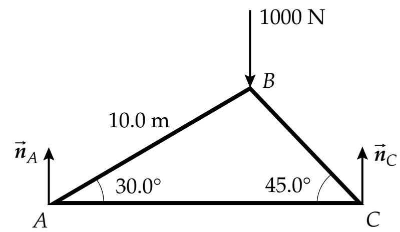


FIG. P12.49(a)

Consider the entire truss:

$$\sum F_y = n_A - 1\,000 \text{ N} + n_C = 0$$

$$\sum \tau_A = -(1\,000 \text{ N})10.0\cos 30.0° + n_C[10.0\cos 30.0° + 7.07\cos 45.0°] = 0$$

Which gives $\boxed{n_C = 634 \text{ N}}$.

Then,

$$\boxed{n_A = 1\,000 \text{ N} - n_C = 366 \text{ N}}$$

(b) Joint A:

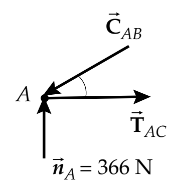
$$\sum F_y = 0: \; -C_{AB}\sin 30.0° + 366 \text{ N} = 0$$

so

$$C_{AB} = \boxed{732 \text{ N}}$$

$$\sum F_x = 0: \; -C_{AB}\cos 30.0° + T_{AC} = 0$$

$$T_{AC} = (732 \text{ N})\cos 30.0° = \boxed{634 \text{ N}}$$



Joint B:

$$\sum F_x = 0: \; (732 \text{ N})\cos 30.0° - C_{BC}\cos 45.0° = 0$$

$$C_{BC} = \frac{(732 \text{ N})\cos 30.0°}{\cos 45.0°} = \boxed{897 \text{ N}}$$

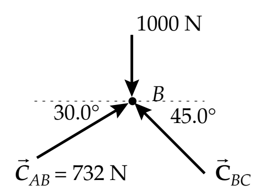


FIG. P12.49(b)

***P12.50** Considering the torques about the point at the bottom of the bracket yields:

$$W(0.050\,0\text{ m}) - F(0.060\,0\text{ m}) = 0 \text{ so } F = 0.833W$$

(a) With $W = 80.0$ N, $F = 0.833(80\text{ N}) = \boxed{66.7\text{ N}}$.

(b) Differentiate with respect to time: $dF/dt = 0.833\ dW/dt$

The force exerted by the screw is increasing at the rate $dF/dt = 0.833(0.15\text{ N/s}) = \boxed{0.125\text{ N/s}}$

P12.51 From geometry, observe that

$$\cos\theta = \frac{1}{4} \qquad \text{and} \qquad \theta = 75.5°$$

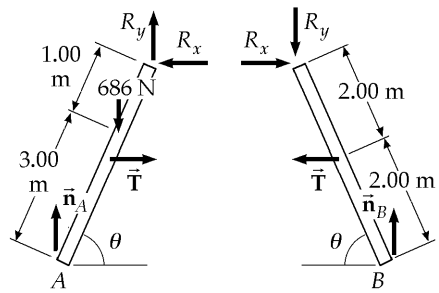


FIG. P12.51

For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \qquad (1)$$

$$\sum F_y = R_y + n_A - 686\text{ N} = 0 \qquad (2)$$

$$\sum \tau_{\text{top}} = 686\text{ N}(1.00\cos 75.5°) + T(2.00\sin 75.5°)$$

$$-n_A(4.00\cos 75.5°) = 0 \qquad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$

$$\sum F_y = n_B - R_y = 0 \qquad (4)$$

$$\sum \tau_{\text{top}} = n_B(4.00\cos 75.5°) - T(2.00\sin 75.5°) = 0 \qquad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $\boxed{T = 133\text{ N}}$

(b) $\boxed{n_A = 429\text{ N}}$ and $\boxed{n_B = 257\text{ N}}$

(c) $\boxed{R_x = 133\text{ N}}$ and $\boxed{R_y = 257\text{ N}}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

P12.52 Imagine gradually increasing the force P. This will make the force of static friction at the bottom increase, so that the normal force at the wall increases and the friction force at the wall *can* increase. As P reaches its maximum value, the cylinder will turn clockwise microscopically to stress the welds at both contact points and make both forces of friction increase to their maximum values. A comparison: To make a four-legged table start to slide across the floor, you must push on it hard enough to counterbalance the maximum static friction forces on all four legs together.

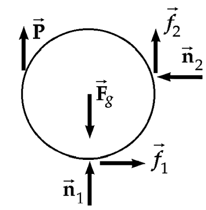


FIG. P12.52

When it is on the verge of slipping, the cylinder is in equilibrium.

$\sum F_x = 0$: $\quad f_1 = n_2 = \mu_s n_1 \quad$ and $\quad f_2 = \mu_s n_2$

$\sum F_y = 0$: $\quad P + n_1 + f_2 = F_g$

$\sum \tau = 0$: $\quad P = f_1 + f_2$

As P grows so do f_1 and f_2.

Therefore, since $\mu_s = \frac{1}{2}$, $\quad f_1 = \frac{n_1}{2} \quad$ and $\quad f_2 = \frac{n_2}{2} = \frac{n_1}{4}$

then $\quad P + n_1 + \frac{n_1}{4} = F_g \quad (1) \quad$ and $\quad P = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4} n_1 \quad (2)$

So $\quad P + \frac{5}{4} n_1 = F_g \quad$ becomes $\quad P + \frac{5}{4}\left(\frac{4}{3} P\right) = F_g \quad$ or $\quad \frac{8}{3} P = F_g$

Therefore,

$$P = \boxed{\frac{3}{8} F_g}$$

P12.53 (a) $|F| = k(\Delta L)$, Young's modulus is $Y = \frac{F/A}{\Delta L/L_i} = \frac{FL_i}{A(\Delta L)}$

Thus,

$$Y = \frac{kL_i}{A} \quad \text{and} \quad k = \boxed{\frac{YA}{L_i}}$$

(b) $$W = -\int_0^{\Delta L} F dx = -\int_0^{\Delta L} (-kx) dx = \frac{YA}{L_i} \int_0^{\Delta L} x dx = \boxed{YA \frac{(\Delta L)^2}{2L_i}}$$

P12.54 (a) Take both balls together. Their weight is 3.33 N and their CG is at their contact point.

$$\sum F_x = 0:\ +P_3 - P_1 = 0$$

$$\sum F_y = 0:\ +P_2 - 3.33\text{ N} = 0 \qquad P_2 = \boxed{3.33\text{ N}}$$

$$\sum \tau_A = 0:\ -P_3 R + P_2 R - 3.33\text{ N}\,(R + R\cos 45.0°)$$

$$+P_1(R + 2R\cos 45.0°) = 0$$

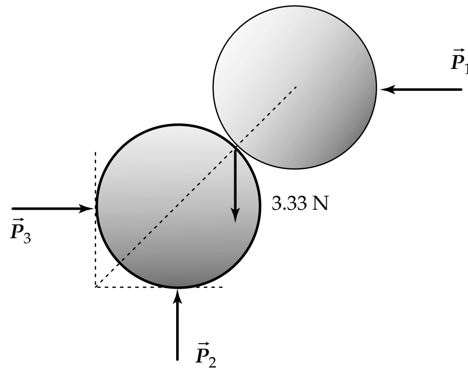


FIG. P12.54(a)

Substituting,

$$-P_1 R + (3.33\text{ N})R - (3.33\text{ N})R(1+\cos 45.0°)$$

$$+P_1 R(1 + 2\cos 45.0°) = 0$$

$$(3.33\text{ N})\cos 45.0° = 2P_1\cos 45.0°$$

$$P_1 = \boxed{1.67\text{ N}} \text{ so } P_3 = \boxed{1.67\text{ N}}$$

(b) Take the upper ball. The lines of action of its weight, of P_1, and of the normal force n exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

$$\sum F_x = 0:\ n\cos 45.0° - P_1 = 0$$

$$n = \frac{1.67\text{ N}}{\cos 45.0°} = \boxed{2.36\text{ N}}$$

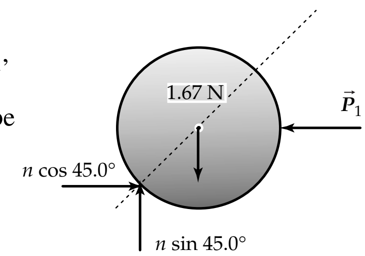


FIG. P12.54(b)

$\sum F_y = 0:\ n\sin 45.0° - 1.67\text{ N} = 0$ gives the same result.

P12.55 $\sum F_y = 0: \quad +380\text{ N} - F_g + 320\text{ N} = 0$

$$F_g = 700\text{ N}$$

Take torques about her feet:

$$\sum \tau = 0: \quad -380\text{ N}(2.00\text{ m}) + (700\text{ N})x + (320\text{ N})0 = 0$$

$$x = \boxed{1.09\text{ m}}$$

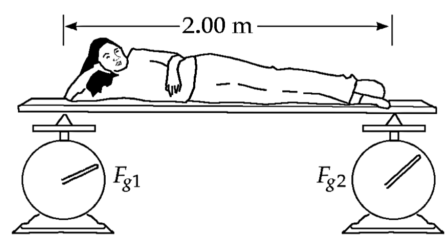


FIG. P12.55

P12.56 The tension in this cable is not uniform, so this becomes a fairly difficult problem.

$$\frac{dL}{L} = \frac{F}{YA}$$

At any point in the cable, F is the weight of cable below that point. Thus, $F = \mu g y$ where μ is the mass per unit length of the cable.

Then,

$$\Delta y = \int_0^{L_i}\left(\frac{dL}{L}\right)dy = \frac{\mu g}{YA}\int_0^{L_i} y\,dy = \frac{1}{2}\frac{\mu g L_i^2}{YA}$$

$$\Delta y = \frac{1}{2}\frac{(2.40)(9.80)(500)^2}{(2.00\times 10^{11})(3.00\times 10^{-4})} = 0.049\,0\text{ m} = \boxed{4.90\text{ cm}}$$

P12.57 (a) $F = m\left(\dfrac{\Delta v}{\Delta t}\right) = (1.00\text{ kg})\dfrac{(10.0-1.00)\text{ m/s}}{0.002\text{ s}} = \boxed{4\,500\text{ N}}$

(b) $\text{stress} = \dfrac{F}{A} = \dfrac{4\,500\text{ N}}{(0.010\text{ m})(0.100\text{ m})} = \boxed{4.50\times10^{6}\text{ N/m}^2}$

(c) $\boxed{\text{Yes.}}$ This is more than sufficient to break the board.

P12.58 Let θ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = (0.850\text{ m})\sin\theta$.
For the mass:

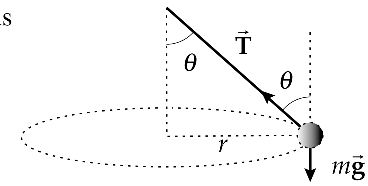


FIG. P12.58

$$\sum F_r = ma_r = m\frac{v^2}{r} = mr\omega^2$$

$$T\sin\theta = m\left[(0.850\text{ m})\sin\theta\right]\omega^2$$

Further, $\dfrac{T}{A} = Y\cdot(\text{strain})$ or $T = AY\cdot(\text{strain})$

Thus, $AY\cdot(\text{strain}) = m(0.850\text{ m})\omega^2$, giving

$$\omega = \sqrt{\frac{AY\cdot(\text{strain})}{m(0.850\text{ m})}} = \sqrt{\frac{\pi\left(3.90\times10^{-4}\text{ m}\right)^2\left(7.00\times10^{10}\text{ N/m}^2\right)\left(1.00\times10^{-3}\right)}{(1.20\text{ kg})(0.850\text{ m})}}$$

or $\omega = \boxed{5.73\text{ rad/s}}$

P12.59 (a) If the acceleration is a, we have $P_x = ma$ and $P_y + n - F_g = 0$. Taking the origin at the center of gravity, the torque equation gives

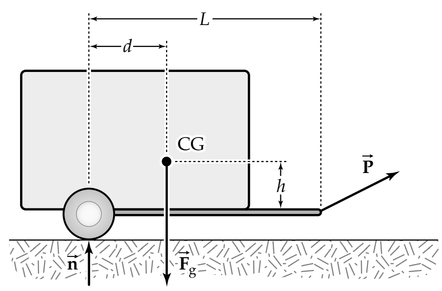


FIG. P12.59

$$P_y(L-d) + P_x h - nd = 0$$

Solving these equations, we find

$$P_y = \boxed{\frac{F_g}{L}\left(d - \frac{ah}{g}\right)}$$

(b) If $P_y = 0$, then $d = \dfrac{ah}{g} = \dfrac{\left(2.00\text{ m/s}^2\right)(1.50\text{ m})}{9.80\text{ m/s}^2} = \boxed{0.306\text{ m}}$.

(c) Using the given data, $P_x = -306$ N and $P_y = 553$ N.

Thus, $\boxed{\vec{\mathbf{P}} = \left(-306\hat{\mathbf{i}} + 553\hat{\mathbf{j}}\right)\text{ N}}$.

P12.60 When the car is on the point of rolling over, the normal force on its inside wheels is zero.

$$\sum F_y = ma_y: \qquad n - mg = 0$$

$$\sum F_x = ma_x: \qquad f = \frac{mv^2}{R}$$

Take torque about the center of mass: $fh - n\dfrac{d}{2} = 0$.

Then by substitution $\dfrac{mv_{\max}^2}{R}h - \dfrac{mgd}{2} = 0 \qquad v_{\max} = \sqrt{\dfrac{gdR}{2h}}$

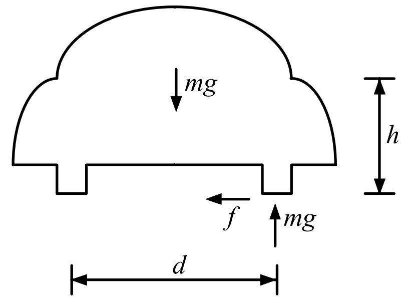


FIG. P12.60

A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.

ANSWERS TO EVEN PROBLEMS

P12.2 $F_y + R_y - F_g = 0; \quad F_x - R_x = 0; \quad F_y \ell\cos\theta - F_g\left(\dfrac{\ell}{2}\right)\cos\theta - F_x \ell\sin\theta = 0$

P12.4 see the solution

P12.6 0.750 m

P12.8 (a) 9.00 g (b) 52.5 g (c) 49.0 g

P12.10 (a) 1.04 kN at 60° upward and to the right. (b) $\vec{\mathbf{F}}_{\text{surface}} = (370\text{ N})\hat{\mathbf{i}} + (900\text{ N})\hat{\mathbf{j}}$

P12.12 (a) $f = \left[\dfrac{m_1 g}{2} + \dfrac{m_2 g x}{L}\right]\cot\theta$; $n_g = (m_1 + m_2)g$ (b) $\mu = \dfrac{(m_1/2 + m_2 d/L)\cot\theta}{m_1 + m_2}$

P12.14 (a) 392 N (b) 339 N to the right (c) 0 (d) 0 (e) 392 N (f) 339 N to the right (g) Both are equally accurate and essentially equally simple. We appear to have four equations, but only three are independent. We can determine three unknowns.

P12.16 (a) 35.5 kN (b) 11.5 kN to the right (c) 4.19 kN down

P12.18 (a) 859 N (b) 104 kN at 36.9° above the horizontal to the left

P12.20 (a) see the solution (b) θ decreases (c) R decreases

P12.22 1.0×10^{11} N/m^2

P12.24 (a) 73.6 kN (b) 2.50 mm

P12.26 ~ 1 cm

P12.28 9.85×10^{-5}

P12.30 0.029 3 mm

P12.32 (a) −0.053 8 m^3 (b) 1.09×10^3 kg/m^3 (c) With only a 5% change in volume in this extreme case, liquid water is indeed nearly incompressible in biological and student-laboratory situations.

P12.34 The weight must be 508 N or more. The person could be a child. We assume the stove is a uniform box with feet at its corners. We ignore the masses of the backsplash and the oven door. If the oven door is heavy, the minimum weight for the person might be somewhat less than 508 N.

P12.36 (a) 0.400 mm (b) 40.0 kN (c) 2.00 mm (d) 2.40 mm (e) 48.0 kN

P12.38 (a) See the solution. The weight of the uniform gate is 392 N. It is 3.00 m wide. The hinges are separated vertically by 1.80 m. The bucket of grain weighs 50.0 N. One of the hinges, which we suppose is the upper one, supports the whole weight of the gate. Find the components of the forces that both hinges exert on the gate. (b) The upper hinge exerts $A = 410$ N to the left and $B = 442$ N up. The lower hinge exerts $C = 410$ N to the right.

P12.40 1.46 kN; $\left(1.33\hat{\mathbf{i}} + 2.58\hat{\mathbf{j}}\right)$ kN

P12.42 (a) 2.71 kN (b) 2.65 kN (c) You should lift "with your knees" rather than "with your back." In this situation, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

P12.44 (a) see the solution (b) 60.0°

P12.46 (a) 120 N (b) 0.300 (c) 103 N at 31.0° above the horizontal to the right

P12.48 Assume a strut exerts on a pin a force with a component perpendicular to the length of the strut. Then the pin must exert a force on the strut with a perpendicular component of this size. For translational equilibrium, the pin at the other end of the strut must also exert the same size force on the strut in the opposite direction. Then the strut will feel two torques about its center in the same sense. It will not be in equilibrium, but will start to rotate. The contradiction proves that we were wrong to assume the existence of the perpendicular force. The strut can exert on the pin only a force parallel to its length.

P12.50 (a) 66.7 N (b) increasing at 0.125 N/s

P12.52 If either static friction force were at less than its maximum value, the cylinder would rotate by a microscopic amount to put more stress on some welds and to bring that friction force to its maximum value. $P = 3F_g/8$

P12.54 (a) $P_1 = 1.67$ N; $P_2 = 3.33$ N; $P_3 = 1.67$ N (b) 2.36 N

P12.56 4.90 cm

P12.58 5.73 rad/s

P12.60 See the solution. A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.