1

Physics and Measurement

ANSWERS TO QUESTIONS

** An asterisk indicates an item new to this edition.*

Q1.1 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

Q1.2 (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms

***Q1.3** In the base unit we have (a) 0.032 kg (b) 0.015 kg (c) 0.270 kg (d) 0.041 kg (e) 0.27 kg. Then the ranking is c = e > d > a > b

Q1.4 No: A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct.
Yes: If an equation is not dimensionally correct, it cannot be correct.

***Q1.5** The answer is yes for (a), (c), and (f). You cannot add or subtract a number of apples and a number of jokes. The answer is no for (b), (d), and (e). Consider the gauge of a sausage, 4 kg/2 m, or the volume of a cube, $(2\text{ m})^3$. Thus we have (a) yes (b) no (c) yes (d) no (e) no (f) yes

***Q1.6** 41 € ≈ 41 € (1 L/1.3 €)(1 qt/1 L)(1 gal/4 qt) ≈ (10/1.3) gal ≈ 8 gallons, answer (c)

***Q1.7** The meterstick measurement, (a), and (b) can all be 4.31 cm. The meterstick measurement and (c) can both be 4.24 cm. Only (d) does not overlap. Thus (a) (b) and (c) all agree with the meterstick measurement.

***Q1.8** 0.02(1.365) = 0.03. The result is $(1.37 \pm 0.03) \times 10^7$ kg. So (d) 3 digits are significant.

SOLUTIONS TO PROBLEMS

Section 1.1 Standards of Length, Mass, and Time

P1.1 Modeling the Earth as a sphere, we find its volume as $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37\times10^6\text{ m}\right)^3 = 1.08\times10^{21}\text{ m}^3$.
Its density is then $\rho = \frac{m}{V} = \frac{5.98\times10^{24}\text{ kg}}{1.08\times10^{21}\text{ m}^3} = \boxed{5.52\times10^3\text{ kg/m}^3}$. This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2 000 to 3 000 kg/m³. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.2 With $V = (\text{base area})(\text{height})$ $V = (\pi r^2)h$ and $\rho = \frac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\text{ kg}}{\pi(19.5\text{ mm})^2(39.0\text{ mm})}\left(\frac{10^9\text{ mm}^3}{1\text{ m}^3}\right)$$

$$\rho = \boxed{2.15\times10^4\text{ kg/m}^3}.$$

P1.3 Let V represent the volume of the model, the same in $\rho = \frac{m}{V}$ for both. Then $\rho_{\text{iron}} = 9.35\text{ kg}/V$ and $\rho_{\text{gold}} = \frac{m_{\text{gold}}}{V}$. Next, $\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35\text{ kg}}$ and $m_{\text{gold}} = 9.35\text{ kg}\left(\frac{19.3\times10^3\text{ kg/m}^3}{7.86\times10^3\text{ kg/m}^3}\right) = \boxed{23.0\text{ kg}}$.

***P1.4** $\rho = m/V$ and $V = (4/3)\pi r^3 = (4/3)\pi(d/2)^3 = \pi d^3/6$ where d is the diameter.

Then $\rho = 6m/\pi d^3 = \frac{6(1.67\times10^{-27}\text{ kg})}{\pi(2.4\times10^{-15}\text{ m})^3} = \boxed{2.3\times10^{17}\text{ kg/m}^3}$

$2.3\times10^{17}\text{ kg/m}^3/(11.3\times10^3\text{ kg/m}^3) = \boxed{\text{it is }20\times10^{12}\text{ times the density of lead}}$.

P1.5 For either sphere the volume is $V = \frac{4}{3}\pi r^3$ and the mass is $m = \rho V = \rho\frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s} = \frac{\rho 4\pi r_\ell^3 3}{\rho 4\pi r_s^3 3} = \frac{r_\ell^3}{r_s^3} = 5.$$

Then $r_\ell = r_s\sqrt[3]{5} = 4.50\text{ cm}(1.71) = \boxed{7.69\text{ cm}}$.

Section 1.2 **Matter and Model-Building**

P1.6 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance $L = 0.200$ nm, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\text{ nm}}$.

Section 1.3 **Dimensional Analysis**

P1.7 (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are m^2/s^2, while the units of $[v]$ are m/s.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in m^{-1}.

P1.8 (a) Circumference has dimensions of L.

(b) Volume has dimensions of L^3.

(c) Area has dimensions of L^2.

Expression (i) has dimension $\text{L}(\text{L}^2)^{1/2} = \text{L}^2$, so this must be area (c).

Expression (ii) has dimension L, so it is (a).

Expression (iii) has dimension $\text{L}(\text{L}^2) = \text{L}^3$, so it is (b). Thus, $\boxed{(a) = ii;\ (b) = iii; (c) = i}$.

P1.9 Inserting the proper units for everything except G,

$$\left[\frac{\text{kg m}}{\text{s}^2}\right]=\frac{G[\text{kg}]^2}{[\text{m}]^2}.$$

Multiply both sides by $[\text{m}]^2$ and divide by $[\text{kg}]^2$; the units of G are $\boxed{\dfrac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$.

Section 1.4 **Conversion of Units**

P1.10 Apply the following conversion factors:

$$1\text{ in}=2.54\text{ cm},\ 1\text{ d}=86\,400\text{ s},\ 100\text{ cm}=1\text{ m, and }10^9\text{ nm}=1\text{ m}$$

$$\left(\frac{1}{32}\ \text{in/day}\right)\frac{(2.54\ \text{cm/in})\left(10^{-2}\ \text{m/cm}\right)\left(10^9\ \text{nm/m}\right)}{86\,400\ \text{s/day}}=\boxed{9.19\ \text{nm/s}}.$$

This means the proteins are assembled at a rate of many layers of atoms each second!

P1.11 *Conceptualize*: We must calculate the area and convert units. Since a meter is about 3 feet, we should expect the area to be about $A\approx(30\text{ m})(50\text{ m})=1500\text{ m}^2$.

Categorize: We model the lot as a perfect rectangle to use Area = Length × Width. Use the conversion: $1\text{ m}=3.281\text{ ft}$.

Analyze: $A=LW=(100\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)(150\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)=1\,390\text{ m}^2=\boxed{1.39\times10^3\ \text{m}^2}$.

Finalize: Our calculated result agrees reasonably well with our initial estimate and has the proper units of m^2. Unit conversion is a common technique that is applied to many problems.

P1.12 (a) $V=(40.0\text{ m})(20.0\text{ m})(12.0\text{ m})=9.60\times10^3\ \text{m}^3$

$V=9.60\times10^3\ \text{m}^3(3.28\ \text{ft/1 m})^3=\boxed{3.39\times10^5\ \text{ft}^3}$

(b) The mass of the air is

$$m=\rho_{\text{air}}V=\left(1.20\ \text{kg/m}^3\right)\left(9.60\times10^3\ \text{m}^3\right)=1.15\times10^4\ \text{kg}.$$

The student must look up weight in the index to find

$$F_g=mg=\left(1.15\times10^4\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)=1.13\times10^5\ \text{N}.$$

Converting to pounds,

$$F_g=\left(1.13\times10^5\ \text{N}\right)(1\text{ lb}/4.45\text{ N})=\boxed{2.54\times10^4\ \text{lb}}.$$

***P1.13** The area of the four walls is (3.6 + 3.8 + 3.6 + 3.8)m (2.5 m) = 37 m^2. Each sheet in the book has area (0.21 m) (0.28 m) = 0.059 m^2. The number of sheets required for wallpaper is 37 m^2/0.059 m^2 = 629 sheets = 629 sheets(2 pages/1 sheet) = 1260 pages.

$\boxed{\text{The pages from volume one are inadequate, but the full version has enough pages.}}$

P1.14 (a) Seven minutes is 420 seconds, so the rate is

$$r = \frac{30.0 \text{ gal}}{420 \text{ s}} = \boxed{7.14 \times 10^{-2} \text{ gal/s}}.$$

(b) Converting gallons first to liters, then to m^3,

$$r = \left(7.14 \times 10^{-2} \text{ gal/s}\right)\left(\frac{3.786 \text{ L}}{1 \text{ gal}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$$

$$r = \boxed{2.70 \times 10^{-4} \text{ m}^3/\text{s}}.$$

(c) At that rate, to fill a 1-m^3 tank would take

$$t = \left(\frac{1 \text{ m}^3}{2.70 \times 10^{-4} \text{ m}^3/\text{s}}\right)\left(\frac{1 \text{ h}}{3\,600}\right) = \boxed{1.03 \text{ h}}.$$

P1.15 From Table 14.1, the density of lead is $1.13 \times 10^4 \text{ kg/m}^3$, so we should expect our calculated value to be close to this number. This density value tells us that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as mass per volume, in $\rho = \frac{m}{V}$. We must convert to SI units in the calculation.

$$\rho = \frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)\left(\frac{1\,000\,000 \text{ cm}^3}{1 \text{ m}^3}\right) = \boxed{1.14 \times 10^4 \text{ kg/m}^3}$$

At one step in the calculation, we note that *one million* cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference in the two values is probably due to measurement uncertainty and should not be a concern. One important common-sense check on density values is that objects which sink in water must have a density greater than 1 g/cm^3, and objects that float must be less dense than water.

P1.16 The weight flow rate is $1\,200 \frac{\text{ton}}{\text{h}}\left(\frac{2\,000 \text{ lb}}{\text{ton}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{667 \text{ lb/s}}$.

P1.17 (a) $\left(\frac{8 \times 10^{12} \text{ \$}}{1\,000 \text{ \$/s}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{1 \text{ day}}{24 \text{ h}}\right)\left(\frac{1 \text{ yr}}{365 \text{ days}}\right) = \boxed{250 \text{ years}}$

(b) The circumference of the Earth at the equator is $2\pi\left(6.378 \times 10^3 \text{ m}\right) = 4.01 \times 10^7 \text{ m}$. The length of one dollar bill is 0.155 m so that the length of 8 trillion bills is $1.24 \times 10^{12} \text{ m}$. Thus, the 8 trillion dollars would encircle the Earth

$$\frac{1.24 \times 10^{12} \text{ m}}{4.01 \times 10^7 \text{ m}} = \boxed{3.09 \times 10^4 \text{ times}}.$$

P1.18

$$V = \frac{1}{3}Bh = \frac{\left[(13.0 \text{ acres})\left(43\,560 \text{ ft}^2/\text{acre}\right)\right]}{3}(481 \text{ ft})$$

$$= 9.08 \times 10^7 \text{ ft}^3,$$

or

$$V = \left(9.08 \times 10^7 \text{ ft}^3\right)\left(\frac{2.83 \times 10^{-2} \text{ m}^3}{1 \text{ ft}^3}\right)$$

$$= \boxed{2.57 \times 10^6 \text{ m}^3}$$

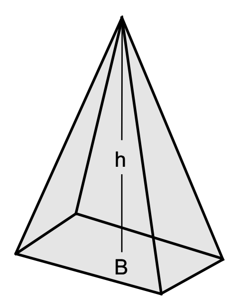

FIG. P1.18

P1.19 $F_g = (2.50 \text{ tons/block})(2.00\times10^6 \text{ blocks})(2\,000 \text{ lb/ton}) = \boxed{1.00\times10^{10} \text{ lbs}}$

P1.20 (a) $d_{\text{nucleus, scale}} = d_{\text{nucleus, real}}\left(\dfrac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right) = (2.40\times10^{-15} \text{ m})\left(\dfrac{300 \text{ ft}}{1.06\times10^{-10} \text{ m}}\right) = 6.79\times10^{-3} \text{ ft}$, or

$$d_{\text{nucleus, scale}} = (6.79\times10^{-3} \text{ ft})(304.8 \text{ mm/1 ft}) = \boxed{2.07 \text{ mm}}$$

(b) $$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = \frac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3 = \left(\frac{1.06\times10^{-10} \text{ m}}{2.40\times10^{-15} \text{ m}}\right)^3$$
$$= \boxed{8.62\times10^{13} \text{ times as large}}$$

P1.21 $V = At$ so $t = \dfrac{V}{A} = \dfrac{3.78\times10^{-3} \text{ m}^3}{25.0 \text{ m}^2} = \boxed{1.51\times10^{-4} \text{ m (or } 151\ \mu\text{m)}}$

P1.22 (a) $$\frac{A_{\text{Earth}}}{A_{\text{Moon}}} = \frac{4\pi r_{\text{Earth}}^2}{4\pi r_{\text{Moon}}^2} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^2 = \left(\frac{(6.37\times10^6 \text{ m})(100 \text{ cm/m})}{1.74\times10^8 \text{ cm}}\right)^2 = \boxed{13.4}$$

(b) $$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{4\pi r_{\text{Earth}}^3/3}{4\pi r_{\text{Moon}}^3/3} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^3 = \left(\frac{(6.37\times10^6 \text{ m})(100 \text{ cm/m})}{1.74\times10^8 \text{ cm}}\right)^3 = \boxed{49.1}$$

P1.23 To balance, $m_{\text{Fe}} = m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}} = \rho_{\text{Al}}V_{\text{Al}}$

$$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}^3 = \rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}^3$$

$$r_{\text{Al}} = r_{\text{Fe}}\left(\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3} = (2.00 \text{ cm})\left(\frac{7.86}{2.70}\right)^{1/3} = \boxed{2.86 \text{ cm}}.$$

P1.24 The mass of each sphere is

$$m_{\text{Al}} = \rho_{\text{Al}}V_{\text{Al}} = \frac{4\pi\rho_{\text{Al}}r_{\text{Al}}^3}{3}$$

and

$$m_{\text{Fe}} = \rho_{\text{Fe}}V_{\text{Fe}} = \frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}^3}{3}.$$

Setting these masses equal,

$$\frac{4\pi\rho_{\text{Al}}r_{\text{Al}}^3}{3} = \frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}^3}{3} \text{ and } \boxed{r_{\text{Al}} = r_{\text{Fe}}\sqrt[3]{\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}}}.$$

The resulting expression shows that the radius of the aluminum sphere is directly proportional to the radius of the balancing iron sphere. The sphere of lower density has larger radius. The fraction $\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}$ is the factor of change between the densities, a number greater than 1. Its cube root is a number much closer to 1. The relatively small change in radius implies a change in volume sufficient to compensate for the change in density.

Section 1.5 Estimates and Order-of-Magnitude Calculations

P1.25 Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m, and each ping-pong ball as a sphere of diameter 0.038 m. The volume of the room is $4\times4\times3=48\ \text{m}^3$, while the volume of one ball is

$$\frac{4\pi}{3}\left(\frac{0.038\ \text{m}}{2}\right)^3=2.87\times10^{-5}\ \text{m}^3.$$

Therefore, one can fit about $\dfrac{48}{2.87\times10^{-5}}\sim\boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2}=0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67\times10^6\times0.740\sim10^6$.

P1.26 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000\ \text{mi})(5\,280\ \text{ft/mi})(1\ \text{rev}/8\ \text{ft})=3\times10^7\ \text{rev}\sim\boxed{10^7\ \text{rev}}$.

P1.27 Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V=(0.5)(1.3\ \text{m})(0.5\ \text{m})(0.3\ \text{m})=0.10\ \text{m}^3.$$

The mass of this volume of water is

$$m_{\text{water}}=\rho_{\text{water}}V=(1\,000\ \text{kg/m}^3)(0.10\ \text{m}^3)=100\ \text{kg}\boxed{\sim10^2\ \text{kg}}.$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}}=\rho_{\text{copper}}V=(8\,920\ \text{kg/m}^3)(0.10\ \text{m}^3)=892\ \text{kg}\boxed{\sim10^3\ \text{kg}}.$$

***P1.28** The time required for the task is

$$10^9\$\left(\frac{1\ \text{s}}{1\ \$}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{working day}}{16\ \text{h}}\right)\left(\frac{1\ \text{bad yr}}{300\ \text{working days}}\right)=58\ \text{yr}$$

Since you are already around 20 years old, you would have a miserable life and likely die before accomplishing the task. You have better things to do. Say no.

P1.29 Assume: Total population $=10^7$; one out of every 100 people has a piano; one tuner can serve about 1 000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$\#\ \text{tuners}\sim\left(\frac{1\ \text{tuner}}{1\,000\ \text{pianos}}\right)\left(\frac{1\ \text{piano}}{100\ \text{people}}\right)(10^7\ \text{people})=\boxed{100\ \text{tuners}}.$$

Section 1.6 Significant Figures

P1.30 METHOD ONE

We treat the best value with its uncertainty as a binomial (21.3 ± 0.2) cm (9.8 ± 0.1) cm,

$$A=[21.3(9.8)\pm21.3(0.1)\pm0.2(9.8)\pm(0.2)(0.1)]\ \text{cm}^2.$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A=\boxed{209\ \text{cm}^2\pm4\ \text{cm}^2}.$$

METHOD TWO

We add the fractional uncertainties in the data.

$$A=(21.3\ \text{cm})(9.8\ \text{cm})\pm\left(\frac{0.2}{21.3}+\frac{0.1}{9.8}\right)=209\ \text{cm}^2\pm2\%=209\ \text{cm}^2\pm4\ \text{cm}^2$$

P1.31 (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

P1.32 $r = (6.50 \pm 0.20)\text{ cm} = (6.50 \pm 0.20) \times 10^{-2}\text{ m}$

$m = (1.85 \pm 0.02)\text{ kg}$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also, $\dfrac{\delta\rho}{\rho} = \dfrac{\delta m}{m} + \dfrac{3\delta r}{r}$.

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103,$$

$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5 \times 10^{-2}\text{ m}\right)^3} = \boxed{1.61 \times 10^3\ \text{kg/m}^3}$$

and

$$\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17) \times 10^3\ \text{kg/m}^3} = (1.6 \pm 0.2) \times 10^3\ \text{kg/m}^3.$$

P1.33 (a)

$$\begin{array}{r} 756.?? \\ 37.2? \\ 0.83 \\ +\ \ 2.5? \\ \hline 796./5/3 \end{array} = \boxed{797}$$

(b) $0.003\,2(2\text{ s.f.}) \times 356.3(4\text{ s.f.}) = 1.140\,16 = (2\text{ s.f.})\ \boxed{1.1}$

(c) $5.620(4\text{ s.f.}) \times \pi(>4\text{ s.f.}) = 17.656 = (4\text{ s.f.})\ \boxed{17.66}$

P1.34 We work to nine significant digits:

$$1\text{ yr} = 1\text{ yr}\left(\frac{365.242\,199\text{ d}}{1\text{ yr}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{31\,556\,926.0\text{ s}}.$$

***P1.35** The tax amount is \$1.36 – \$1.25 = \$0.11. The tax rate is \$0.11/\$1.25 = 0.088 0 = $\boxed{8.80\%}$

***P1.36** (a) We read from the graph a vertical separation of 0.3 spaces = $\boxed{0.015\text{ g}}$.

(b) Horizontally, 0.6 spaces = $\boxed{30\text{ cm}^2}$.

(c) Because the graph line goes through the origin, the same percentage describes the vertical and the horizontal scatter: 30 cm^2/380 cm^2 = $\boxed{8\%}$.

(d) Choose a grid point on the line far from the origin: slope = 0.31 g/600 cm^2 = 0.000 52 g/cm^2 = (0.000 52 g/cm^2)(10 000 cm^2/1 m^2) = $\boxed{5.2\text{ g/m}^2}$.

(e) For any and all shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is 5.2 g/m^2 ± 8%, where the uncertainty is estimated.

(f) This result should be expected if the paper has thickness and density that are uniform within the experimental uncertainty. The slope is the areal density of the paper, its mass per unit area.

***P1.37** 15 players = 15 players (1 shift/1.667 player) = $\boxed{\text{9 shifts}}$

***P1.38** Let o represent the number of ordinary cars and s the number of trucks. We have $o = s + 0.947s = 1.947s$, and $o = s + 18$. We eliminate o by substitution: $s + 18 = 1.947s$ $0.947s = 18$ and $s = 18/0.947 = \boxed{19}$.

***P1.39** Let s represent the number of sparrows and m the number of more interesting birds. We have $s/m = 2.25$ and $s + m = 91$. We eliminate m by substitution: $m = s/2.25$

$s + s/2.25 = 91 \qquad 1.444s = 91 \qquad s = 91/1.444 = \boxed{63}$.

***P1.40** For those who are not familiar with solving equations numerically, we provide a detailed solution. It goes beyond proving that the suggested answer works.

The equation $2x^4 - 3x^3 + 5x - 70 = 0$ is quartic, so we do not attempt to solve it with algebra. To find how many real solutions the equation has and to estimate them, we graph the expression:

x	–3	–2	–1	0	1	2	3	4
$y = 2x^4 - 3x^3 + 5x - 70$	158	–24	–70	–70	–66	–52	26	270

We see that the equation $y = 0$ has two roots, one around $x = -2.2$ and the other near $x = +2.7$. To home in on the first of these solutions we compute in sequence: When $x = -2.2$, $y = -2.20$. The root must be between $x = -2.2$ and $x = -3$. When $x = -2.3$, $y = 11.0$. The root is between $x = -2.2$ and $x = -2.3$. When $x = -2.23$, $y = 1.58$. The root is between $x = -2.20$ and $x = -2.23$. When $x = -2.22$, $y = 0.301$. The root is between $x = -2.20$ and -2.22. When $x = -2.215$, $y = -0.331$. The root is between $x = -2.215$ and -2.22. We could next try $x = -2.218$, but we already know to three-digit precision that the root is $x = -2.22$.

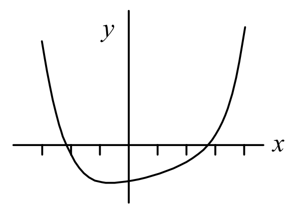

FIG. P1.40

***P1.41** We require $\sin\theta = -3\cos\theta$, or $\dfrac{\sin\theta}{\cos\theta} = -3$, or $\tan\theta = -3$.

For $\tan^{-1}(-3) = \arctan(-3)$, your calculator may return –71.6°, but this angle is not between 0° and 360° as the problem requires. The tangent function is negative in the second quadrant (between 90° and 180°) and in the fourth quadrant (from 270° to 360°). The solutions to the equation are then $360° - 71.6° = \boxed{288°}$ and $180° - 71.6° = \boxed{108°}$.

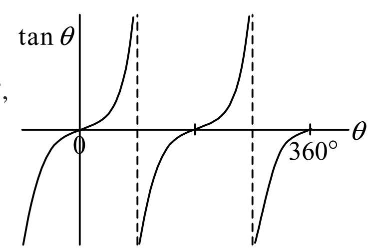

FIG. P1.41

***P1.42** We draw the radius to the initial point and the radius to the final point. The angle θ between these two radii has its sides perpendicular, right side to right side and left side to left side, to the 35° angle between the original and final tangential directions of travel. A most useful theorem from geometry then identifies these angles as equal: $\theta = 35°$. The whole circumference of a 360° circle of the same radius is $2\pi R$. By proportion, then $\dfrac{2\pi R}{360°} = \dfrac{840\text{ m}}{35°}$.

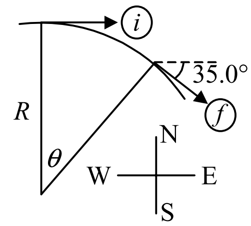

FIG. P1.42

$$R = \frac{360°}{2\pi}\frac{840\text{ m}}{35°} = \frac{840\text{ m}}{0.611} = \boxed{1.38\times10^{3}\text{ m}}$$

We could equally well say that the measure of the angle in radians is

$$\theta = 35° \;= 35°\left(\frac{2\pi\text{ radians}}{360°}\right) = 0.611\text{ rad} = \frac{840\text{ m}}{R}.$$

Solving yields $R = 1.38$ km.

***P1.43** Mass is proportional to cube of length: $m = k\ell^3 \quad m_f/m_i = (\ell_f/\ell_i)^3$.

Length changes by 15.8%: $\ell_f = \ell_i + 0.158\,\ell_i = 1.158\,\ell_i$.

Mass increase: $m_f = m_i + 17.3$ kg.

Eliminate by substitution: $\dfrac{m_f}{m_f - 17.3\text{ kg}} = 1.158^3 = 1.553$

$m_f = 1.553\,m_f - 26.9$ kg $\qquad$ $26.9\text{ kg} = 0.553\,m_f$ $\qquad$ $m_f = 26.9\text{ kg}/0.553 = \boxed{48.\,6\text{ kg}}$.

***P1.44** We use substitution, as the most generally applicable method for solving simultaneous equations. We substitute $p = 3q$ into each of the other two equations to eliminate p:

$$\begin{cases} 3qr = qs \\ \frac{1}{2}3qr^2 + \frac{1}{2}qs^2 = \frac{1}{2}qt^2 \end{cases}.$$

These simplify to $\begin{cases} 3r = s \\ 3r^2 + s^2 = t^2 \end{cases}$. We substitute to eliminate s: $\begin{matrix} 3r^2 + (3r)^2 = t^2 \\ 12r^2 = t^2 \end{matrix}$. We solve for the combination $\dfrac{t}{r}$:

$$\frac{t^2}{r^2} = 12.$$

$$\frac{t}{r} = \boxed{\text{either } 3.46 \text{ or } -3.46}$$

***P1.45** Solve the given equation for Δt: $\quad \Delta t = 4QL/k\pi d^2(T_h - T_c) = [4QL/k\pi\,(T_h - T_c)]\,[1/d^2]$.

(a) Making d three times larger with d^2 in the bottom of the fraction makes Δt $\boxed{\text{nine times smaller}}$.

(b) $\boxed{\Delta t \text{ is inversely proportional to the square of } d.}$

(c) $\boxed{\text{Plot } \Delta t \text{ on the vertical axis and } 1/d^2 \text{ on the horizontal axis.}}$

(d) From the last version of the equation, the slope is $\boxed{4QL/k\pi(T_h - T_c)}$. Note that this quantity is constant as both Δt and d vary.

Additional Problems

P1.46 It is desired to find the distance x such that

$$\frac{x}{100\text{ m}} = \frac{1\,000\text{ m}}{x}$$

(i.e., such that x is the same multiple of 100 m as the multiple that 1 000 m is of x). Thus, it is seen that

$$x^2 = (100\text{ m})(1\,000\text{ m}) = 1.00\times10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00\times10^5\text{ m}^2} = \boxed{316\text{ m}}.$$

***P1.47** (a) The mass is equal to the mass of a sphere of radius 2.6 cm and density 4.7 g/cm³, minus the mass of a sphere of radius a and density 4.7 g/cm³ plus the mass of a sphere of radius a and density 1.23 g/cm³.

$$m = \rho_1 4\pi r^3/3 - \rho_1 4\pi a^3/3 + \rho_2 4\pi a^3/3$$

$$= (4.7\text{ g/cm}^3)4\pi(2.6\text{ cm})^3/3 - (4.7\text{ g/cm}^3)4\pi(a)^3/3 + (1.23\text{ g/cm}^3)4\pi(a)^3/3$$

$$\boxed{m = 346\text{ g} - (14.5\text{ g/cm}^3)a^3}$$

(b) For $\boxed{a = 0}$ the mass is a maximum, (c) $\boxed{346\text{ g}}$. (d) $\boxed{\text{Yes}}$. This is the mass of the uniform sphere we considered in the first term of the calculation.

(e) For $\boxed{a = 2.60\text{ cm}}$ the mass is a minimum, (f) $346 - 14.5(2.6)^3 = \boxed{90.6\text{ g}}$. (g) $\boxed{\text{Yes}}$. This is the mass of a uniform sphere of density 1.23 g/cm³.

(h) $(346\text{ g} + 90.6\text{ g})/2 = \boxed{218\text{ g}}$ (i) $\boxed{\text{No}}$. The result of part (a) gives $346\text{ g} - (14.5\text{ g/cm}^3)(1.3\text{ cm})^3 = 314\text{ g}$, not the same as 218 g.

(j) We should expect agreement in parts b-c-d, because those parts are about a uniform sphere of density 4.7 g/cm³. We should expect agreement in parts e-f-g, because those parts are about a uniform liquid drop of density 1.23 g/cm³. The function $m(a)$ is not a linear function, so a halfway between 0 and 2.6 cm does not give a value for m halfway between the minimum and maximum values. The graph of m versus a starts at $a = 0$ with a horizontal tangent. Then it curves down more and more steeply as a increases. The liquid drop of radius 1.30 cm has only one eighth the volume of the whole sphere, so its presence brings down the mass by only a small amount, from 346 g to 314 g.

(k) No change, so long as the wall of the shell is unbroken.

***P1.48** (a) We have $B + C(0) = 2.70\text{ g/cm}^3$ and $B + C(14\text{ cm}) = 19.3\text{ g/cm}^3$. We know $\boxed{B = 2.70\text{ g/cm}^3}$ and we solve for C by subtracting: $C(14\text{ cm}) = 16.6\text{ g/cm}^3$ so $\boxed{C = 1.19\text{ g/cm}^4}$.

(b)
$$m = \int_0^{14\text{ cm}} (2.70\text{ g/cm}^3 + 1.19\text{ g/cm}^4 x)(9\text{ cm}^2)dx$$

$$= 24.3\text{ g/cm}\int_0^{14\text{ cm}} dx + 10.7\text{ g/cm}^2\int_0^{14\text{ cm}} x\,dx$$

$$= (24.3\text{ g/cm})(14\text{ cm} - 0) + (10.7\text{ g/cm}^2)[(14\text{ cm})^2 - 0]/2$$

$$= 340\text{ g} + 1046\text{ g} = \boxed{1.39\text{ kg}}$$

P1.49 The scale factor used in the "dinner plate" model is

$$S = \frac{0.25 \text{ m}}{1.0 \times 10^5 \text{ lightyears}} = 2.5 \times 10^{-6} \text{ m/lightyears}.$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}} S = \left(2.0 \times 10^6 \text{ lightyears}\right)\left(2.5 \times 10^{-6} \text{ m/lightyears}\right) = \boxed{5.0 \text{ m}}.$$

***P1.50** The rate of volume increase is

$$\frac{dV}{dt} = \frac{d}{dt}\frac{4}{3}\pi r^3 = \frac{4}{3}\pi 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

(a) $dV/dt = 4\,\pi(6.5 \text{ cm})^2(0.9 \text{ cm/s}) = \boxed{478 \text{ cm}^3\text{/s}}$

(b) $\dfrac{dr}{dt} = \dfrac{dV/dt}{4\pi r^2} = \dfrac{478 \text{ cm}^3/\text{s}}{4\pi(13 \text{ cm})^2} = \boxed{0.225 \text{ cm}^3/\text{s}}$

(c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.51 One month is

$$1 \text{ mo} = (30 \text{ day})(24 \text{ h/day})(3\,600 \text{ s/h}) = 2.592 \times 10^6 \text{ s}.$$

Applying units to the equation,

$$V = \left(1.50 \text{ Mft}^3/\text{mo}\right)t + \left(0.008\,00 \text{ Mft}^3/\text{mo}^2\right)t^2.$$

Since $1 \text{ Mft}^3 = 10^6 \text{ ft}^3$,

$$V = \left(1.50 \times 10^6 \text{ ft}^3/\text{mo}\right)t + \left(0.008\,00 \times 10^6 \text{ ft}^3/\text{mo}^2\right)t^2.$$

Converting months to seconds,

$$V = \frac{1.50 \times 10^6 \text{ ft}^3/\text{mo}}{2.592 \times 10^6 \text{ s/mo}}t + \frac{0.008\,00 \times 10^6 \text{ ft}^3/\text{mo}^2}{\left(2.592 \times 10^6 \text{ s/mo}\right)^2}t^2.$$

Thus, $\boxed{V\ [\text{ft}^3] = \left(0.579 \text{ ft}^3/\text{s}\right)t + \left(1.19 \times 10^{-9} \text{ ft}^3/\text{s}^2\right)t^2}$.

***P1.52**

α'(deg)	α(rad)	$\tan(\alpha)$	$\sin(\alpha)$	difference between α and $\tan\alpha$
15.0	0.262	0.268	0.259	2.30%
20.0	0.349	0.364	0.342	4.09%
30.0	0.524	0.577	0.500	9.32%
33.0	0.576	0.649	0.545	11.3%
31.0	0.541	0.601	0.515	9.95%
31.1	0.543	0.603	0.516	10.02%

We see that α in radians, $\tan(\alpha)$ and $\sin(\alpha)$ start out together from zero and diverge only slightly in value for small angles. Thus $\boxed{31.0°}$ is the largest angle for which $\dfrac{\tan\alpha - \alpha}{\tan\alpha} < 0.1$.

P1.53 $2\pi r = 15.0 \text{ m}$

$$r = 2.39 \text{ m}$$

$$\frac{h}{r} = \tan 55.0°$$

$$h = (2.39 \text{ m})\tan(55.0°) = \boxed{3.41 \text{ m}}$$

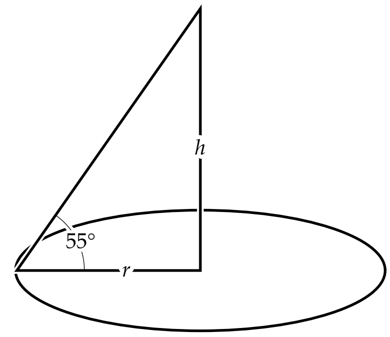

FIG. P1.53

P1.54 Let d represent the diameter of the coin and h its thickness. The mass of the gold is

$$m = \rho V = \rho A t = \rho\left(\frac{2\pi d^2}{4} + \pi d h\right)t$$

where t is the thickness of the plating.

$$m = 19.3\left[2\pi\frac{(2.41)^2}{4} + \pi(2.41)(0.178)\right](0.18\times10^{-4})$$

$$= 0.003\,64 \text{ grams}$$

$$\text{cost} = 0.003\,64 \text{ grams} \times \$10/\text{gram} = \$0.036\,4 = \boxed{3.64 \text{ cents}}$$

This is negligible compared to \$4.98.

P1.55 The actual number of seconds in a year is

$$(86\,400 \text{ s/day})(365.25 \text{ day/yr}) = 31\,557\,600 \text{ s/yr}.$$

The percent error in the approximation is

$$\frac{\left|(\pi\times10^7 \text{ s/yr}) - (31\,557\,600 \text{ s/yr})\right|}{31\,557\,600 \text{ s/yr}} \times 100\% = \boxed{0.449\%}.$$

P1.56 $v = \left(5.00\dfrac{\text{furlongs}}{\text{fortnight}}\right)\left(\dfrac{220 \text{ yd}}{1 \text{ furlong}}\right)\left(\dfrac{0.914\,4 \text{ m}}{1 \text{ yd}}\right)\left(\dfrac{1 \text{ fortnight}}{14 \text{ days}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ hrs}}\right)\left(\dfrac{1 \text{ hr}}{3\,600 \text{ s}}\right) = \boxed{8.32\times10^{-4} \text{ m/s}}$

This speed is almost 1 mm/s; so we might guess the creature was a snail, or perhaps a sloth.

P1.57 (a) The speed of rise may be found from

$$v = \frac{(\text{Vol rate of flow})}{(\text{Area: } \pi D^2/4)} = \frac{16.5 \text{ cm}^3/\text{s}}{\pi(6.30 \text{ cm})^2/4} = \boxed{0.529 \text{ cm/s}}.$$

(b) Likewise, at a 1.35 cm diameter,

$$v = \frac{16.5 \text{ cm}^3/\text{s}}{\pi(1.35 \text{ cm})^2/4} = \boxed{11.5 \text{ cm/s}}.$$

P1.58 The density of each material is $\rho = \dfrac{m}{V} = \dfrac{m}{\pi r^2 h} = \dfrac{4m}{\pi D^2 h}$.

Al: $\rho = \dfrac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})} = \boxed{2.75\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(2.70\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{2\%}$ smaller.

Cu: $\rho = \dfrac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})} = \boxed{9.36\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(8.92\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{5\%}$ smaller.

Brass: $\rho = \dfrac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})} = \boxed{8.91\dfrac{\text{g}}{\text{cm}^3}}$

Sn: $\rho = \dfrac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})} = \boxed{7.68\dfrac{\text{g}}{\text{cm}^3}}$

Fe: $\rho = \dfrac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})} = \boxed{7.88\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(7.86\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{0.3\%}$ smaller.

P1.59 $V_{20\text{ mpg}} = \dfrac{(10^8\text{ cars})(10^4\text{ mi/yr})}{20\text{ mi/gal}} = 5.0\times10^{10}\text{ gal/yr}$

$V_{25\text{ mpg}} = \dfrac{(10^8\text{ cars})(10^4\text{ mi/yr})}{25\text{ mi/gal}} = 4.0\times10^{10}\text{ gal/yr}$

Fuel saved $= V_{25\text{ mpg}} - V_{20\text{ mpg}} = \boxed{1.0\times10^{10}\text{ gal/yr}}$

P1.60 The volume of the galaxy is

$$\pi r^2 t = \pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right) \sim 10^{61}\text{ m}^3.$$

If the distance between stars is 4×10^{16} m, then there is one star in a volume on the order of

$$\left(4\times10^{16}\text{ m}\right)^3 \sim 10^{50}\text{ m}^3.$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}} \sim \boxed{10^{11}\text{ stars}}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P1.2 2.15×10^4 kg/m^3

P1.4 2.3×10^{17} kg/m^3 is twenty trillion times larger than the density of lead.

P1.6 0.141 nm

P1.8 (a) ii (b) iii (c) i

P1.10 9.19 nm/s

P1.12 (a) 3.39×10^5 ft^3 (b) 2.54×10^4 lb

P1.14 (a) 0.071 4 gal/s (b) 2.70×10^{-4} m^3/s (c) 1.03 h

P1.16 667 lb/s

P1.18 $2.57 \times 10^6 \text{ m}^3$

P1.20 (a) 2.07 mm (b) 8.57×10^{13} times as large

P1.22 (a) 13.4; (b) 49.1

P1.24 $r_{\text{Al}} = r_{\text{Fe}} \left(\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}} \right)^{1/3}$

P1.26 $\sim 10^7$ rev

P1.28 No. There is a strong possibility that you would die before finishing the task, and you have much more productive things to do.

P1.30 $(209 \pm 4) \text{ cm}^2$

P1.32 $(1.61 \pm 0.17) \times 10^3 \text{ kg/m}^3$

P1.34 31 556 926.0 s

P1.36 (a) 0.015 g (b) 30 cm^2 (c) 8% (d) 5.2 g/m^2 (e) For any and all shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is 5.2 $\text{g/m}^2 \pm 8\%$, where the uncertainty is estimated. (f) This result is to be expected if the paper has thickness and density that are uniform within the experimental uncertainty. The slope is the areal density of the paper, its mass per unit area.

P1.38 19

P1.40 see the solution

P1.42 1.38 km

P1.44 either 3.46 or −3.46

P1.46 316 m

P1.48 (a) $\rho = 2.70 \text{ g/cm}^3 + 1.19 \text{ g/cm}^4\, x$ (b) 1.39 kg

P1.50 (a) 478 cm^3/s (b) 0.225 cm/s (c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one increment of time in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.52 0.542 rad

P1.54 3.64 cents; no

P1.56 8.32×10^{-4} m/s; a snail

P1.58 see the solution

P1.60 $\sim 10^{11}$ stars