4

Motion in Two Dimensions

CHAPTER OUTLINE

4.1 The Position, Velocity, and Acceleration Vectors
4.2 Two-Dimensional Motion with Constant Acceleration
4.3 Projectile Motion
4.4 Uniform Circular Motion
4.5 Tangential and Radial Acceleration
4.6 Relative Velocity and Relative Acceleration

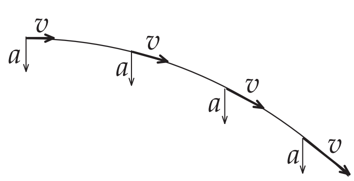
ANSWERS TO QUESTIONS

***Q4.1** The car's acceleration must have an inward component and a forward component: answer (f). Another argument: Draw a final velocity vector of two units west. Add to it a vector of one unit south. This represents subtracting the initial velocity from the final velocity, on the way to finding the acceleration. The direction of the resultant is that of vector (f).

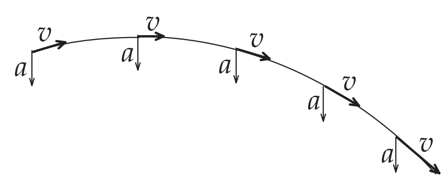
Q4.2 No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

$$\vec{\mathbf{v}}_{avg} = \frac{\Delta \vec{\mathbf{x}}}{\Delta t}$$

Q4.3 (a)



(b)



***Q4.4** (i) The 45° angle means that at point *A* the horizontal and vertical velocity components are equal. The horizontal velocity component is the same at *A*, *B*, and *C*. The vertical velocity component is zero at *B* and negative at *C*. The assembled answer is a = b = c = e > d = 0 > f
(ii) The *x*-component of acceleration is everywhere zero and the *y*-component is everywhere -9.8 m/s^2. Then we have a = c = e = 0 > b = d = f.

Q4.5 A parabola results, because the originally forward velocity component stays constant and the rocket motor gives the spacecraft constant acceleration in a perpendicular direction.

Q4.6 (a) yes (b) no: the escaping jet exhaust exerts an extra force on the plane. (c) no (d) yes (e) no: the stone is only a few times more dense than water, so friction is a significant force on the stone. The answer is (a) and (d).

Q4.7 The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

Q4.8 (a) no (b) yes (c) yes (d) no. Answer: (b) and (c)

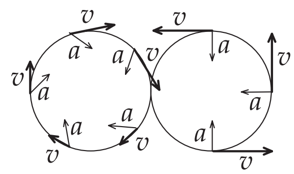
***Q4.9** The projectile on the moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then (i) its range is (d) six times larger and (ii) its maximum altitude is (d) six times larger. *Apollo* astronauts performed the experiment with golf balls.

Q4.10 (a) no. Its velocity is constant in magnitude and direction.
(b) yes. The particle is continuously changing the direction of its velocity vector.

Q4.11 (a) straight ahead (b) either in a circle or straight ahead. The acceleration magnitude can be constant either with a nonzero or with a zero value.

***Q4.12** (i) $a = v^2/r$ becomes $3^2/3 = 3$ times larger: answer (b).
(ii) $T = 2\pi r/v$ changes by a factor of $3/3 = 1$. The answer is (a).

Q4.13



The skater starts at the center of the eight, goes clockwise around the left circle and then counter-clockwise around the right circle.

***Q4.14** With radius half as large, speed should be smaller by a factor of $1/\sqrt{2}$, so that $a = v^2/r$ can be the same. The answer is (d).

***Q4.15** The wrench will hit (b) at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

***Q4.16** Let the positive x direction be that of the girl's motion. The x component of the velocity of the ball relative to the ground is $+5 - 12$ m/s $= -7$ m/s. The x-velocity of the ball relative to the girl is $-7 - 8$ m/s $= -15$ m/s. The relative speed of the ball is $+15$ m/s, answer (d).

SOLUTIONS TO PROBLEMS

Section 4.1 **The Position, Velocity, and Acceleration Vectors**

P4.1

$x(m)$	$y(m)$
0	–3 600
–3 000	0
–1 270	1 270
–4 270 m	–2 330 m

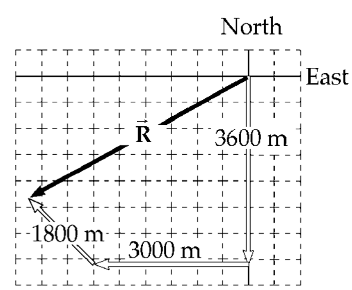


FIG. P4.1

(a) Net displacement $= \sqrt{x^2 + y^2}$ at $\tan^{-1}(y/x)$

$$\vec{\mathbf{R}} = \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) $$\text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \vec{\mathbf{R}}}$$

P4.2 (a) $\vec{\mathbf{r}} = \boxed{18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}}$

(b) $\vec{\mathbf{v}} = \boxed{(18.0 \text{ m/s})\hat{\mathbf{i}} + \left[4.00 \text{ m/s} - \left(9.80 \text{ m/s}^2\right)t\right]\hat{\mathbf{j}}}$

(c) $\vec{\mathbf{a}} = \boxed{\left(-9.80 \text{ m/s}^2\right)\hat{\mathbf{j}}}$

(d) by substitution, $\vec{\mathbf{r}}(3.00 \text{ s}) = \boxed{(54.0 \text{ m})\hat{\mathbf{i}} - (32.1 \text{ m})\hat{\mathbf{j}}}$

(e) $\vec{\mathbf{v}}(3.00 \text{ s}) = \boxed{(18.0 \text{ m/s})\hat{\mathbf{i}} - (25.4 \text{ m/s})\hat{\mathbf{j}}}$

(f) $\vec{\mathbf{a}}(3.00 \text{ s}) = \boxed{\left(-9.80 \text{ m/s}^2\right)\hat{\mathbf{j}}}$

P4.3 The sun projects onto the ground the x component of her velocity:

$$5.00 \text{ m/s } \cos(-60.0°) = \boxed{2.50 \text{ m/s}}$$

P4.4 (a) From $x = -5.00\sin\omega t$, the x component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\sin\omega t) = -5.00\omega\cos\omega t$$

and $a_x = \dfrac{dv_x}{dt} = +5.00\omega^2 \sin\omega t$

similarly, $v_y = \left(\dfrac{d}{dt}\right)(4.00 - 5.00\cos\omega t) = 0 + 5.00\omega\sin\omega t$

and $a_y = \left(\dfrac{d}{dt}\right)(5.00\omega\sin\omega t) = 5.00\omega^2\cos\omega t$

At $t = 0$, $\vec{\mathbf{v}} = -5.00\omega\cos 0\hat{\mathbf{i}} + 5.00\omega\sin 0\hat{\mathbf{j}} = \boxed{\left(5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right) \text{ m/s}}$

and $\vec{\mathbf{a}} = 5.00\omega^2 \sin 0\hat{\mathbf{i}} + 5.00\omega^2\cos 0\hat{\mathbf{j}} = \boxed{\left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right) \text{ m/s}^2}$

(b) $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00 \text{ m})\hat{\mathbf{j}} + (5.00 \text{ m})\left(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}\right)}$

$\vec{\mathbf{v}} = \boxed{(5.00 \text{ m})\omega\left[-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right]}$

$\vec{\mathbf{a}} = \boxed{(5.00 \text{ m})\omega^2\left[\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}\right]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at } (0,\ 4.00 \text{ m})}$.

Section 4.2 **Two-Dimensional Motion with Constant Acceleration**

P4.5 $\vec{\mathbf{v}}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right) \text{ m/s}$ and $\vec{\mathbf{v}}(20.0) = \left(20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right) \text{ m/s}$

(a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20.0 - 4.00}{20.0} \text{ m/s}^2 = \boxed{0.800 \text{ m/s}^2}$

$a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5.00 - 1.00}{20.0} \text{ m/s}^2 = \boxed{-0.300 \text{ m/s}^2}$

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(b) $\theta = \tan^{-1}\left(\dfrac{-0.300}{0.800}\right) = -20.6° = \boxed{339° \text{ from } +x \text{ axis}}$

(c) At $t = 25.0$ s its position is specified by its coordinates and the direction of its motion is specified by the direction angle of its velocity:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360 \text{ m}}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7 \text{ m}}$$

$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24 \text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

P4.6 (a) $\vec{\mathbf{v}} = \dfrac{d\vec{\mathbf{r}}}{dt} = \left(\dfrac{d}{dt}\right)\left(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}\right) = \boxed{-12.0t\,\hat{\mathbf{j}} \text{ m/s}}$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\,\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}} \text{ m/s}^2}$$

(b) by substitution, $\boxed{\vec{\mathbf{r}} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right) \text{ m; } \vec{\mathbf{v}} = -12.0\hat{\mathbf{j}} \text{ m/s}}$

***P4.7** (a) From $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$, we have $\int_i^f d\vec{\mathbf{v}} = \int_i^f \vec{\mathbf{a}}\,dt$

Then $\vec{\mathbf{v}} - 5\,\hat{\mathbf{i}} \text{ m/s} = \int_0^t 6\,t^{1/2}dt\hat{\mathbf{j}} = 6\left.\dfrac{t^{3/2}}{3/2}\right|_0^t \hat{\mathbf{j}} = 4\,t^{3/2}\,\hat{\mathbf{j}}$ so $\vec{\mathbf{v}} = \boxed{5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}}$

(b) From $\vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt$, we have $\int_i^f d\vec{\mathbf{r}} = \int_i^f \vec{\mathbf{v}}\,dt$

Then $\vec{\mathbf{r}} - 0 = \int_0^t \left(5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}\right)dt = \left.\left(5t\hat{\mathbf{i}} + 4\dfrac{t^{5/2}}{5/2}\hat{\mathbf{j}}\right)\right|_0^t = \boxed{5t\hat{\mathbf{i}} + 1.6\,t^{5/2}\,\hat{\mathbf{j}}}$

P4.8 $\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}} \text{ m/s}^2$; $\vec{\mathbf{v}}_i = 5.00\hat{\mathbf{i}} \text{ m/s}$; $\vec{\mathbf{r}}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$

(a) $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \dfrac{1}{2}\vec{\mathbf{a}}t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \dfrac{1}{2}3.00t^2\hat{\mathbf{j}}\right] \text{ m}}$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \boxed{\left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) $t = 2.00$ s, $\vec{\mathbf{r}}_f = 5.00(2.00)\hat{\mathbf{i}} + \dfrac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)$ m

so $x_f = \boxed{10.0 \text{ m}}$, $y_f = \boxed{6.00 \text{ m}}$

$$\vec{\mathbf{v}}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right) \text{ m/s}$$

$$v_f = \left|\vec{\mathbf{v}}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81 \text{ m/s}}$$

Section 4.3 **Projectile Motion**

P4.9 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\text{ m})}{v_{xi}}$$

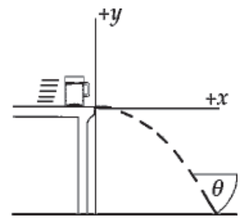

FIG. P4.9

In the same time it falls a distance of 0.860 m with acceleration downward of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2:\ 0 = 0.860\text{ m} + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(\frac{1.40\text{ m}}{v_{xi}}\right)^2$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90\text{ m/s}^2\right)\left(1.96\text{ m}^2\right)}{0.860\text{ m}}} = \boxed{3.34\text{ m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + \left(-9.80\text{ m/s}^2\right)\left(\frac{1.40\text{ m}}{3.34\text{ m/s}}\right) = -4.11\text{ m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9^\circ}$$

P4.10 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \qquad \text{and} \qquad y_f = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2}gt^2$$

which gives the time of impact as

$$t = \sqrt{\frac{2h}{g}}$$

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}t$ becomes $d = v_{xi}\sqrt{\frac{2h}{g}}$ giving the initial velocity as

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

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(b) Just before impact, the x component of velocity is still

$$v_{xf} = v_{xi}$$

while the y component is

$$v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{2h/g}}{d\sqrt{g/2h}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}$$

The answer for v_{xi} indicates that a larger measured value for d would imply larger takeoff speed in direct proportion. A tape measure lying on the floor could be calibrated as a speedometer. A larger value for h would imply a smaller value for speed by an inverse proportionality to the square root of h. That is, if h were nine times larger, v_{xi} would be three times smaller. The answer for θ shows that the impact velocity makes an angle with the horizontal whose tangent is just twice as large as that of the elevation angle α of the edge of the table as seen from the impact point.

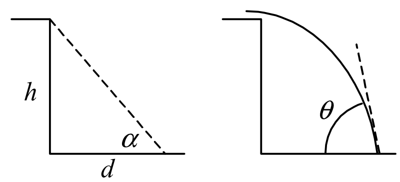

FIG. P4.10

P4.11 $x = v_{xi}t = v_i \cos\theta_i t$

$$x = (300 \text{ m/s})(\cos 55.0°)(42.0 \text{ s})$$

$$x = \boxed{7.23 \times 10^3 \text{ m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300 \text{ m/s})(\sin 55.0°)(42.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(42.0 \text{ s})^2 = \boxed{1.68 \times 10^3 \text{ m}}$$

P4.12 (a) To identify the maximum height we let i be the launch point and f be the highest point:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$
$$0 = v_i^2\sin^2\theta_i + 2(-g)\left(y_{\max} - 0\right)$$
$$y_{\max} = \frac{v_i^2\sin^2\theta_i}{2g}$$

To identify the range we let i be the launch and f be the impact point; where t is not zero:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + v_i\sin\theta_i t + \frac{1}{2}(-g)t^2$$
$$t = \frac{2v_i\sin\theta_i}{g}$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$d = 0 + v_i\cos\theta_i\frac{2v_i\sin\theta_i}{g} + 0$$

For this rock, $d = y_{\max}$

$$\frac{v_i^2\sin^2\theta_i}{2g} = \frac{2v_i^2\sin\theta_i\cos\theta_i}{g}$$
$$\frac{\sin\theta_i}{\cos\theta_i} = \tan\theta_i = 4$$
$$\theta_i = \boxed{76.0^\circ}$$

(b) Since g divides out, the answer is $\boxed{\text{the same on every planet}}$.

(c) The maximum range is attained for $\theta_i = 45^\circ$:

$$\frac{d_{\max}}{d} = \frac{v_i\cos 45^\circ 2v_i\sin 45^\circ g}{gv_i\cos 76^\circ 2v_i\sin 76^\circ} = 2.125$$

So $d_{\max} = \boxed{\dfrac{17d}{8}}$.

P4.13 $h = \dfrac{v_i^2\sin^2\theta_i}{2g}$; $R = \dfrac{v_i^2\left(\sin 2\theta_i\right)}{g}$; $3h = R$

so $\dfrac{3v_i^2\sin^2\theta_i}{2g} = \dfrac{v_i^2\left(\sin 2\theta_i\right)}{g}$

or $\dfrac{2}{3} = \dfrac{\sin^2\theta_i}{\sin 2\theta_i} = \dfrac{\tan\theta_i}{2}$

thus $\theta_i = \tan^{-1}\left(\dfrac{4}{3}\right) = \boxed{53.1^\circ}$

P4.14 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \frac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i \left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2$$

Therefore the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d \tan\theta_i - \frac{gd^2}{2v_i^2 \cos^2\theta_i}}$$

P4.15 (a) $x_f = v_{xi}\, t = 8.00 \cos 20.0°(3.00) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}g t^2$$

$$y_f = 8.00 \sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3 \text{ m}}$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18 \text{ s}}$$

***P4.16** The time of flight of a water drop is given by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$.

$$0 = 2.35 \text{ m} + 0 - \frac{1}{2}(9.8 \text{ m/s}^2)t_1^2$$

For $t_1 > 0$, the root is $t_1 = \sqrt{\frac{2(2.35 \text{ m})}{9.8 \text{ m/s}^2}} = 0.693$ s.

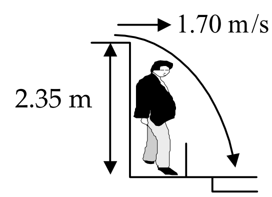

FIG. P4.16

(a) The horizontal range of the font is

$$x_{f1} = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$= 0 + 1.70 \text{ m/s}(0.693 \text{ s}) + 0 = 1.18 \text{ m}$$

This is about the width of a town sidewalk, so $\boxed{\text{there is}}$ space for a walkway behind the waterfall. Unless the lip of the channel is well designed, water may drip on the visitors. A tall or inattentive person may get his head wet.

(b) Now the flight time t_2 is given by $0 = y_2 + 0 - \frac{1}{2}g t_2^2$.

$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2y_1}{g(12)}} = \frac{1}{\sqrt{12}}\sqrt{\frac{2y_1}{g}} = \frac{t_1}{\sqrt{12}}$ From the same equation as in part (a) for horizontal range, $x_2 = v_2 t_2$.

$$\frac{x_1}{12} = v_2 \frac{t_1}{\sqrt{12}} \qquad v_2 = \frac{x_1}{t_1\sqrt{12}} = \frac{v_1}{\sqrt{12}} = \frac{1.70 \text{ m/s}}{\sqrt{12}} = \boxed{0.491 \text{ m/s}}$$

The rule that the scale factor for speed is the square root of the scale factor for distance is Froude's law, published in 1870.

P4.17 (a) We use the trajectory equation:

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2 v_i^2 \cos^2\theta_i}$$

With

$$x_f = 36.0 \text{ m},\ v_i = 20.0 \text{ m/s, and } \theta = 53.0°$$

we find

$$y_f = (36.0 \text{ m})\tan 53.0° - \frac{(9.80 \text{ m/s}^2)(36.0 \text{ m})^2}{2(20.0 \text{ m/s})^2 \cos^2(53.0°)} = 3.94 \text{ m}$$

The ball clears the bar by

$$(3.94 - 3.05) \text{ m} = \boxed{0.889 \text{ m}}$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

The time to travel 36.0 m horizontally is $t_2 = \dfrac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

P4.18 When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km}$$

$$y_f = x_f \tan\theta - \frac{g x_f^2}{2 v_i^2 \cos^2\theta_i}$$

$$-2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - \frac{(9.8 \text{ m/s}^2)(2\,437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2\theta_i}$$

$$\therefore -2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - (371.19 \text{ m})(1 + \tan^2\theta_i)$$

$$\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662,\ \boxed{\theta_i = -33.5°}$$

P4.19 (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24 \text{ m} = 0 + v_i(\cos 53°)(2.2 \text{ s}) + 0$$

$$v_i = \boxed{18.1 \text{ m/s}}$$

continued on next page

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1 \text{ m/s})(\sin 53°)(2.2 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.2 \text{ s})^2 = 8.13 \text{ m}$$

So it clears the parapet by $8.13 \text{ m} - 7 \text{ m} = \boxed{1.13 \text{ m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or

$$6 \text{ m} = (\tan 53°)x_f - \left(\frac{9.8 \text{ m/s}^2}{2(18.1 \text{ m/s})^2\cos^2 53°}\right)x_f^2$$

Solving,

$$(0.041\,2 \text{ m}^{-1})x_f^2 - 1.33x_f + 6 \text{ m} = 0$$

and

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2(0.0412 \text{ m}^{-1})}$$

This yields two results:

$$x_f = 26.8 \text{ m} \text{ or } 5.44 \text{ m}$$

The ball passes twice through the level of the roof.
It hits the roof at distance from the wall

$$26.8 \text{ m} - 24 \text{ m} = \boxed{2.79 \text{ m}}$$

P4.20 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03 \text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(0.900 - 1.85) \text{ m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32 \text{ m/s}$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32 \text{ m/s} = 4.03 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

or $t = \boxed{0.852 \text{ s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80 \text{ m} = v_{xi}(0.852 \text{ s})$$

which yields $v_{xi} = \boxed{3.29 \text{ m/s}}$.

(c) $v_{yi} = \boxed{4.03 \text{ m/s}}$. See above for proof.

continued on next page

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\dfrac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\dfrac{4.03\ \text{m/s}}{3.29\ \text{m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$:

$$0 = v_{yi}^2 + 2\left(-9.80\ \text{m/s}^2\right)(2.50 - 1.20)\ \text{m}$$

so $v_{yi} = 5.04\ \text{m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$ yields

$v_{yf}^2 = 0 + 2\left(-9.80\ \text{m/s}^2\right)(0.700 - 2.50)\ \text{m}$ and $v_{yf} = -5.94\ \text{m/s}$

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94\ \text{m/s} = 5.04\ \text{m/s} + \left(-9.80\ \text{m/s}^2\right)t$ and

$$\boxed{t = 1.12\ \text{s}}$$

P4.21 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0\ \text{m} = 0 + 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$$

$$t = 2.86\ \text{s}$$

The extra time $3.00\ \text{s} - 2.86\ \text{s} = 0.143\ \text{s}$ is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343\ \text{m/s})0.143\ \text{s} = 49.0\ \text{m} = \sqrt{x^2 + (40.0\ \text{m})^2}$$

where x represents the horizontal distance the rock travels.

$$x = 28.3\ \text{m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3\ \text{m}}{2.86\ \text{s}} = \boxed{9.91\ \text{m/s}}$$

***P4.22** We match the given equations

$$x_f = 0 + (11.2\ \text{m/s})\cos 18.5° t$$

$$0.360\ \text{m} = 0.840\ \text{m} + (11.2\ \text{m/s})\sin 18.5° t - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

to the equations for the coordinates of the final position of a projectile

$$x_f = x_i + v_{xi}t$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

For the equations to represent the same functions of time, all coefficients must agree: $x_i = 0$, $y_i = 0.840\ \text{m}$, $v_{xi} = (11.2\ \text{m/s})\cos 18.5°$, $v_{yi} = (11.2\ \text{m/s})\sin 18.5°$ and $g = 9.80\ \text{m/s}^2$.

(a) Then the original position of the athlete's center of mass is the point with coordinates $\left(x_i,\ y_i\right) = \boxed{(0,\ 0.840\ \text{m})}$. That is, his original position has position vector $\vec{\mathbf{r}} = 0\hat{\mathbf{i}} + 0.840\ \text{m}\hat{\mathbf{j}}$.

continued on next page

(b) His original velocity is $\vec{\mathbf{v}}_i = (11.2\ \text{m/s})\cos 18.5°\hat{\mathbf{i}} + (11.2\ \text{m/s})\sin 18.5°\hat{\mathbf{j}} =$

$\boxed{11.2\ \text{m/s at } 18.5°}$ above the x axis.

(c) From $4.90\ \text{m/s}^2\ t^2 - 3.55\ \text{m/s}\ \ t - 0.48\ \text{m} = 0$ we find the time of flight, which must be

positive $t = \dfrac{+3.55\ \text{m/s} + \sqrt{(3.55\ \text{m/s})^2 - 4(4.9\ \text{m/s}^2)(-0.48\ \text{m})}}{2(4.9\ \text{m/s}^2)} = 0.842$ s. Then

$$x_f = (11.2\ \text{m/s})\cos 18.5°(0.8425) = \boxed{8.94\ \text{m}}$$

(d)

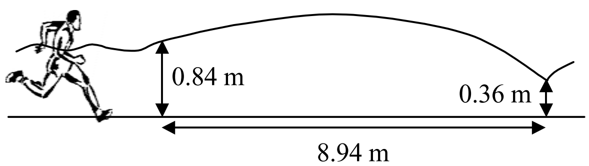

FIG. P4.22

The free-fall trajectory of the athlete is a section around the vertex of a parabola opening downward, everywhere close to horizontal and 48 cm lower on the landing side than on the takeoff side.

P4.23 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)$$

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v,\ v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}$$

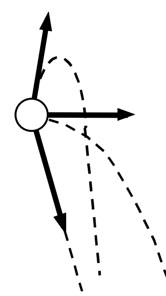

FIG. P4.23

Section 4.4 **Uniform Circular Motion**

P4.24 $a = \dfrac{v^2}{R}$, $T = 24\ \text{h}(3\,600\ \text{s/h}) = 86\,400\ \text{s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times 10^6\ \text{m})}{86\,400\ \text{s}} = 463\ \text{m/s}$$

$$a = \frac{(463\ \text{m/s})^2}{6.37\times 10^6\ \text{m}} = \boxed{0.033\,7\ \text{m/s}^2\ \text{directed toward the center of Earth}}$$

P4.25 $a_c = \dfrac{v^2}{r} = \dfrac{(20.0\ \text{m/s})^2}{1.06\ \text{m}} = \boxed{377\ \text{m/s}^2}$ The mass is unnecessary information.

P4.26 $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{3(9.8 \text{ m/s}^2)(9.45 \text{ m})} = 16.7 \text{ m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45 \text{ m}) = 59.4 \text{ m}$. Then the rotation rate is

$$16.7 \text{ m/s}\left(\frac{1 \text{ rev}}{59.4 \text{ m}}\right) = \boxed{0.281 \text{ rev/s}}$$

P4.27 (a) $v = r\omega$

At 8.00 rev/s, $v = (0.600 \text{ m})(8.00 \text{ rev/s})(2\pi \text{ rad/rev}) = 30.2 \text{ m/s} = 9.60\pi \text{ m/s}$.

At 6.00 rev/s, $v = (0.900 \text{ m})(6.00 \text{ rev/s})(2\pi \text{ rad/rev}) = 33.9 \text{ m/s} = 10.8\pi \text{ m/s}$.

$\boxed{6.00 \text{ rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \frac{v^2}{r} = \frac{(9.60\pi \text{ m/s})^2}{0.600 \text{ m}} = \boxed{1.52 \times 10^3 \text{ m/s}^2}$.

(c) At 6.00 rev/s, acceleration $= \frac{(10.8\pi \text{ m/s})^2}{0.900 \text{ m}} = \boxed{1.28 \times 10^3 \text{ m/s}^2}$. So 8 rev/s gives the higher acceleration.

Section 4.5 Tangential and Radial Acceleration

***P4.28** The particle's centripetal acceleration is $v^2/r = (3 \text{ m/s})^2/2 \text{ m} = 4.50 \text{ m/s}^2$. The total acceleration magnitude can be larger than or equal to this, but not smaller.

> (a) Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude $\sqrt{6^2 - 4.5^2} = 3.97 \text{ m/s}^2$.
>
> (b) No. The magnitude of the acceleration cannot be less than $v^2/r = 4.5 \text{ m/s}^2$.

P4.29 We assume the train is still slowing down at the instant in question.

$$a_c = \frac{v^2}{r} = 1.29 \text{ m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s})}{15.0 \text{ s}} = -0.741 \text{ m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29 \text{ m/s}^2)^2 + (-0.741 \text{ m/s}^2)^2}$$

at an angle of $\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$

$$\vec{\mathbf{a}} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9° \text{ backward}}$$

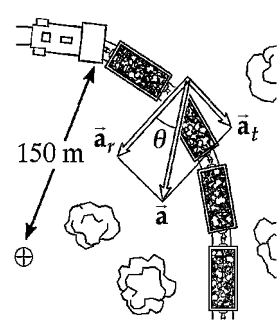

FIG. P4.29

P4.30 (a) See figure to the right.

(b) The components of the 20.2 and the 22.5 $\mathrm{m/s^2}$ along the rope together constitute the centripetal acceleration:

$$a_c = (22.5\ \mathrm{m/s^2})\cos(90.0° - 36.9°) + (20.2\ \mathrm{m/s^2})\cos 36.9° = \boxed{29.7\ \mathrm{m/s^2}}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\ \mathrm{m/s^2}(1.50\ \mathrm{m})} = 6.67\ \mathrm{m/s}$ tangent to circle

$$\vec{\mathbf{v}} = \boxed{6.67\ \mathrm{m/s}\ \text{at } 36.9° \text{ above the horizontal}}$$

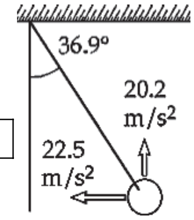

FIG. P4.30

P4.31 $r = 2.50\ \mathrm{m},\ a = 15.0\ \mathrm{m/s^2}$

(a) $a_c = a\cos 30.0° = (15.0\ \mathrm{m/s^2})(\cos 30°) = \boxed{13.0\ \mathrm{m/s^2}}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\ \mathrm{m}(13.0\ \mathrm{m/s^2}) = 32.5\ \mathrm{m^2/s^2}$

$v = \sqrt{32.5}\ \mathrm{m/s} = \boxed{5.70\ \mathrm{m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\,\mathrm{m/s^2})^2 - (13.0\,\mathrm{m/s^2})} = \boxed{7.50\,\mathrm{m/s^2}}$

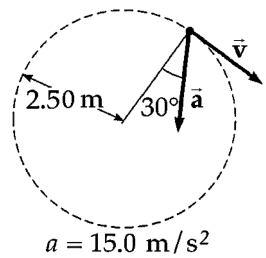

FIG. P4.31

P4.32 Let i be the starting point and f be one revolution later. The curvilinear motion with constant tangential acceleration is described by

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

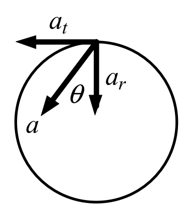

FIG. P4.32

and $v_{xf} = v_{xi} + a_x t,\ v_f = 0 + a_t t = \dfrac{4\pi r}{t}$. The magnitude of the radial acceleration is $a_r = \dfrac{v_f^2}{r} = \dfrac{16\pi^2 r^2}{t^2 r}$.

Then $\tan\theta = \dfrac{a_t}{a_r} = \dfrac{4\pi r t^2}{t^2 16\pi^2 r} = \dfrac{1}{4\pi}$ $\qquad \theta = \boxed{4.55°}$

Section 4.6 **Relative Velocity and Relative Acceleration**

P4.33 $\vec{\mathbf{v}}_{ce}$ = the velocity of the car relative to the earth.

$\vec{\mathbf{v}}_{wc}$ = the velocity of the water relative to the car.

$\vec{\mathbf{v}}_{we}$ = the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

(a) Since $\vec{\mathbf{v}}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0\ \mathrm{km/h}$ or

$\vec{\mathbf{v}}_{wc} = \boxed{57.7\ \mathrm{km/h}\ \text{at } 60.0° \text{ west of vertical}}$.

(b) Since $\vec{\mathbf{v}}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \mathrm{km/h})\cos 60.0° = \boxed{28.9\ \mathrm{km/h}\ \text{downward}}$$

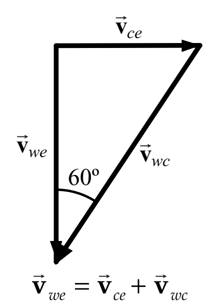

FIG. P4.33

P4.34 (a) $\vec{\mathbf{v}}_H = 0 + \vec{\mathbf{a}}_H t = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\ \text{s})$

$$\vec{\mathbf{v}}_H = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\vec{\mathbf{v}}_J = 0 + \vec{\mathbf{a}}_j t = \left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\ \text{s})$$

$$\vec{\mathbf{v}}_J = \left(5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\vec{\mathbf{v}}_{HJ} = v_H - v_J = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\vec{\mathbf{v}}_{HJ} = \left(10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\left|\vec{\mathbf{v}}_{HJ}\right| = \sqrt{(10.0)^2 + (25.0)^2}\ \ \text{m/s} = \boxed{26.9\ \text{m/s}}$$

(b) $\vec{\mathbf{r}}_H = 0 + 0 + \frac{1}{2}\vec{\mathbf{a}}_H t^2 = \frac{1}{2}\left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\ \text{s})^2$

$$\vec{\mathbf{r}}_H = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \text{m}$$

$$\vec{\mathbf{r}}_J = \frac{1}{2}\left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2\,(5.00\text{s})^2 = \left(12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\vec{\mathbf{r}}_{HJ} = \vec{\mathbf{r}}_H - \vec{\mathbf{r}}_J = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\vec{\mathbf{r}}_{HJ} = \left(25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\left|\vec{\mathbf{r}}_{HJ}\right| = \sqrt{(25.0)^2 + (62.5)^2}\,\text{m} = \boxed{67.3\ \text{m}}$$

(c) $\vec{\mathbf{a}}_{HJ} = \vec{\mathbf{a}}_H - \vec{\mathbf{a}}_J = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$

$$\vec{\mathbf{a}}_{HJ} = \boxed{\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

P4.35 Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = 1.67 \times 10^3$ s.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^3\ \text{s}$$

$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588\ \text{s}$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3\ \text{s}}$.

$\boxed{\text{This is 12.0\% larger than the time in still water.}}$

P4.36 The bumpers are initially $100\ \text{m} = 0.100\ \text{km}$ apart. After time t the bumper of the leading car travels 40.0 t, while the bumper of the chasing car travels $60.0t$. Since the cars are side by side at time t, we have

$$0.100 + 40.0t = 60.0t$$

yielding

$$t = 5.00 \times 10^{-3}\ \text{h} = \boxed{18.0\ \text{s}}$$

P4.37 To guess the answer, think of v just a little less than the speed c of the river. Then poor Alan will spend most of his time paddling upstream making very little progress. His time-averaged speed will be low and Beth will win the race.

Now we calculate: For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$. Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1-v^2/c^2}}$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}$$

Thus, the total time for Beth is $t_2 = \dfrac{2L}{\sqrt{c^2-v^2}} = \boxed{\dfrac{2L/c}{\sqrt{1-v^2/c^2}}}$.

Since $1 - \dfrac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

***P4.38** We can find the time of flight of the can by considering its horizontal motion:

$16\text{ m} = (9.5\text{ m/s})\,t + 0 \qquad t = 1.68\text{ s}$

(a) For the boy to catch the can at the same location on the truck bed, he must throw it $\boxed{\text{straight up, at } 0° \text{ to the vertical}}$.

(b) For the free fall of the can, $y_f = y_i + v_{yi}t + (1/2)a_y t^2$:
$0 = 0 + v_{yi}\,(1.68\text{ s}) - (1/2)(9.8\text{ m/s}^2)(1.68\text{ s})^2 \quad v_{yi} = \boxed{8.25\text{ m/s}}$

(c) The boy sees the can always over his head, traversing $\boxed{\text{a straight line segment upward and then downward}}$.

(d) The ground observer sees the can move as a projectile on $\boxed{\text{a symmetric section of a parabola opening downward}}$. Its initial velocity is
$(9.5^2 + 8.25^2)^{1/2}\text{ m/s} = \boxed{12.6\text{ m/s north at } \tan^{-1}(8.25/9.5) = 41.0° \text{ above the horizontal}}$

P4.39 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

$$\frac{v_y'}{v_x'} = \tan 60.0° = \sqrt{3}$$

Let u represent the speed of S' relative to S. Then because there is no x-motion in S, we can write $v_x = v_x' + u = 0$ so that $v_x' = -u = -10.0\text{ m/s}$. Hence the ball is thrown backwards in S'. Then,

$$v_y = v_y' = \sqrt{3}\left|v_x'\right| = 10.0\sqrt{3}\text{ m/s}$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{\left(10.0\sqrt{3}\text{ m/s}\right)^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{15.3\text{ m}}$$

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

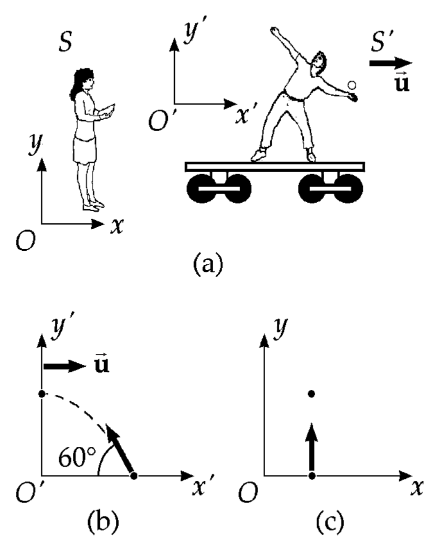

FIG. P4.39

***P4.40** (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50\ \text{m/s})^2 + (9.80\ \text{m/s})^2} = \boxed{10.1\ \text{m/s}^2}$$

$$\tan\theta = \frac{2.50\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.255$$

$$\theta = \boxed{14.3° \text{ to the south from the vertical}}$$

To this observer, the bolt moves as if it were in a gravitational field of 9.80 m/s^2 down + 2.50 m/s^2 south.

(b) $a = \boxed{9.80\ \text{m/s}^2 \text{ vertically downward}}$

(c) If it is at rest relative to the ceiling at release, the bolt moves on a straight line downward and southward at 14.3 degrees from the vertical.

(d) The bolt moves on a parabola with a vertical axis.

P4.41 Choose the x axis along the 20-km distance. The y components of the displacements of the ship and the speedboat must agree:

$$(26\ \text{km/h})t\sin(40° - 15°) = (50\ \text{km/h})t\sin\alpha$$

$$\alpha = \sin^{-1}\frac{11.0}{50} = 12.7°$$

The speedboat should head

$$15° + 12.7° = \boxed{27.7° \text{ east of north}}$$

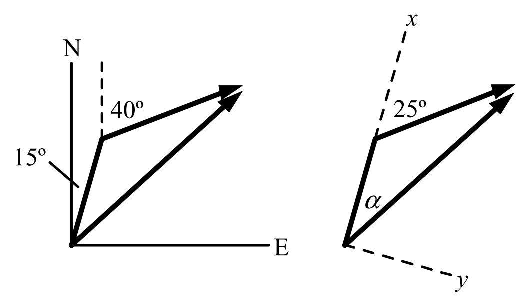

FIG. P4.41

Additional Problems

P4.42 (a) The speed at the top is $v_x = v_i\cos\theta_i = (143\ \text{m/s})\cos 45° = \boxed{101\ \text{m/s}}$.

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (143\ \text{m/s}\sin 45°)^2 + 2(-9.8\ \text{m/s}^2)(y_f - 31\,000\ \text{ft})$$

$$y_f = 31\,000\ \text{ft} + 522\ \text{m}\left(\frac{3.28\ \text{ft}}{1\ \text{m}}\right) = \boxed{3.27 \times 10^4\ \text{ft}}$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$-101\ \text{m/s} = +101\ \text{m/s} - (9.8\ \text{m/s}^2)t$$

$$t = \boxed{20.6\ \text{s}}$$

(d) $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8(9.8\ \text{m/s}^2)4{,}130\ \text{m}} = \boxed{180\ \text{m/s}}$$

***P4.43** (a) At every point in the trajectory, including the top, the acceleration is $\boxed{9.80\text{ m/s}^2\text{ down}}$.

(b) We first find the speed of the ball just before it hits the basket rim.

$$v_{xf}^2 + v_{yf}^2 = v_{xi}^2 + v_{yi}^2 + 2a_y(y_f - y_i)$$

$$v_f^2 = v_i^2 + 2a_y(y_f - y_i) = (10.6\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(3.05\text{ m} - 0) = 52.6\text{ m}^2/\text{s}^2$$

$v_f = 7.25$ m/s. The ball's rebound speed is $v_{yi} = (7.25\text{ m/s})/2 = 3.63$ m/s

Now take the initial point just after the ball leaves the rim, and the final point at the top of its bounce.

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i): \quad 0 = (3.63\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(y_f - 3.05\text{ m})$$

$$y_f = -(3.63\text{ m/s})^2/2(-9.8\text{ m/s}^2) + 3.05\text{ m} = \boxed{3.72\text{ m}}$$

***P4.44** (a) Take the positive x axis pointing east. The ball is in free fall between the point just after it leaves the player's hands, and the point just before it bonks the bird. Its horizontal component of velocity remains constant with the value

$$(10.6\text{ m/s})\cos 55° = 6.08\text{ m/s}$$

We need to know the time of flight up to the eagle. We consider the ball's vertical motion:

$$v_{yf} = v_{yi} + a_y t \quad 0 = (10.6\text{ m/s})\sin 55° = (-9.8\text{ m/s}^2)t \quad t = -(8.68\text{ m/s})/(-9.8\text{ m/s}^2) = 0.886\text{ s}$$

The horizontal component of displacement from the player to the bird is

$$x_f = x_i + v_x t = 0 + (6.08\text{ m/s})(0.886\text{ s}) = 5.39\text{ m}$$

The downward flight takes the same time because the ball moves through the same vertical distance with the same range of vertical speeds, including zero vertical speed at one endpoint. The horizontal velocity component of the ball is $-1.5(6.08\text{ m/s}) = -9.12$ m/s. The final horizontal coordinate of the ball is

$$x_f = x_i + v_x t = 5.39\text{ m} + (-9.12\text{ m/s})(0.886\text{ s}) = 5.39\text{ m} - 8.08\text{ m/s} = -2.69\text{ m}$$

The ball lands a distance of $\boxed{2.69\text{ m behind the player}}$.

(b) The angle could be either positive or negative. Here is a conceptual argument: The horizontal bounce sends the ball 2.69 m behind the player. To shorten this distance, the bird wants to reduce the horizontal velocity component of the ball. It can do this either by sending the ball upward or downward relative to the horizontal.

Here is a mathematical argument: The height of the bird is $(1/2)(9.8\text{ m/s}^2)(0.886\text{ s})^2 = 3.85$ m. The ball's flight from the bird to the player is described by the pair of equations

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad 0 = 3.85\text{ m} + (9.12\text{ m/s})(\sin\theta)\,t + (1/2)(-9.8\text{ m/s}^2)t^2$$

and $$x_f = x_i + v_x t \qquad 0 = 5.39\text{ m} + (-9.12\text{ m/s})(\cos\theta)\,t$$

Eliminating t by substitution gives a quadratic equation in θ. This equation has two solutions.

P4.45 Refer to the sketch. We find it convenient to solve part (b) first.

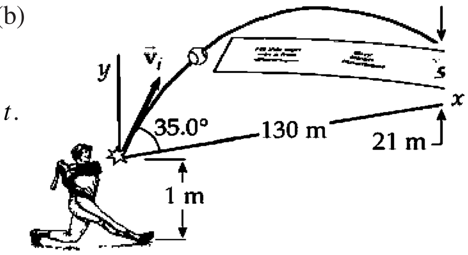

FIG. P4.45

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i \cos 35.0°)t$.

$\Delta y = v_{yi}t + \frac{1}{2}at^2$; substitution yields

$$20.0 = (v_i \sin 35.0°)t + \frac{1}{2}(-9.80)t^2$$

Solving the above by substituting $v_i t = 159$

gives $20 = 91 - 4.9\,t^2$ so $t = \boxed{3.81\text{ s}}$.

(a) substituting back gives $v_i = \boxed{41.7\text{ m/s}}$

(c) $v_{yf} = v_i \sin\theta_i - gt,\ v_x = v_i \cos\theta_i$

At $t = 3.81\text{ s},\ v_{yf} = 41.7\sin 35.0° - (9.80)(3.81) = \boxed{-13.4\text{ m/s}}$

$$v_x = (41.7\cos 35.0°) = \boxed{34.1\text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7\text{ m/s}}$$

P4.46 At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i\cos\theta_i)t = \boxed{2v_i t\cos\theta_i}$$

P4.47 (a) $a_c = \frac{v^2}{r} = \frac{(5.00\text{ m/s})^2}{1.00\text{ m}} = \boxed{25.0\text{ m/s}^2}$

$a_t = g = \boxed{9.80\text{ m/s}^2}$

(b) See figure to the right.

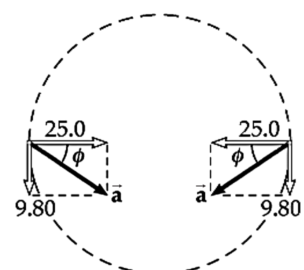

FIG. P4.47

(c) $a = \sqrt{a_c^2 + a_t^2} = \sqrt{(25.0\text{ m/s}^2)^2 + (9.80\text{ m/s}^2)^2} = \boxed{26.8\text{ m/s}^2}$

$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\frac{9.80\text{ m/s}^2}{25.0\text{ m/s}^2} = \boxed{21.4°}$

P4.48 (a) The moon's gravitational acceleration is the probe's centripetal acceleration: (For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}(9.80\text{ m/s}^2) = \frac{v^2}{1.74\times10^6\text{ m}}$$

$$v = \sqrt{2.84\times10^6\text{ m}^2/\text{s}^2} = \boxed{1.69\text{ km/s}}$$

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74\times10^6\text{ m})}{1.69\times10^3\text{ m/s}} = 6.47\times10^3\text{ s} = \boxed{1.80\text{ h}}$$

***P4.49** (a) We find the x coordinate from $x = 12\,t$. We find the y coordinate from $49\,t - 4.9\,t^2$. Then we find the projectile's distance from the origin as $(x^2 + y^2)^{1/2}$, with these results:

***t* (s)**	0	1	2	3	4	5	6	7	8	9	10
***r* (m)**	0	45.7	82.0	109	127	136	138	133	124	117	120

(b) From the table, it looks like the magnitude of r is largest at a bit less than 6 s. The vector $\vec{\mathbf{v}}$ tells how $\vec{\mathbf{r}}$ is changing. If $\vec{\mathbf{v}}$ at a particular point has a component along $\vec{\mathbf{r}}$, then $\vec{\mathbf{r}}$ will be increasing in magnitude (if $\vec{\mathbf{v}}$ is at an angle less than 90° from $\vec{\mathbf{r}}$) or decreasing (if the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ is more than 90°). To be at a maximum, the distance from the origin must be momentarily staying constant, and the only way this can happen is for the angle between velocity and displacement to be a right angle. Then $\vec{\mathbf{r}}$ will be changing in direction at that point, but not in magnitude.

(c) The requirement for perpendicularity can be defined as equality between the tangent of the angle between $\vec{\mathbf{v}}$ and the x direction and the tangent of the angle between $\vec{\mathbf{r}}$ and the y direction. In symbols this is $(9.8t - 49)/12 = 12t/(49t - 4.9t^2)$, which has the solution $t = 5.70$ s, giving in turn $r = 138$ m. Alternatively, we can require $dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]$, which results in the same equation with the same solution.

***P4.50** (a) The time of flight must be positive. It is determined by $y_f = y_i + v_{yi}t - (1/2)a_y t^2$

$0 = 1.2 + v_0 \sin 35°\, t - 4.9t^2$ from the quadratic formula as $t = \dfrac{0.574v_0 + \sqrt{0.329v_0^2 + 23.52}}{9.8}$

Then the range follows from $x = v_{xi}t + 0 = v_0 t$ as

$\boxed{x(v_0) = v_0\sqrt{0.164\,3 + 0.002\,299\,v_0^2} + 0.047\,94\,v_0^2}$ where x is in meters and v_0 is in meters per second.

(b) Substituting $v_0 = 0.1$ gives $x(v_0) = \boxed{0.0410\text{ m}}$

(c) Substituting $v_0 = 100$ gives $x(v_0) = \boxed{961\text{ m}}$

(d) When v_0 is small, v_0^2 becomes negligible. The expression $x(v_0)$ simplifies to

$v_0\sqrt{0.164\,3 + 0} + 0 = \boxed{0.405\,v_0}$ Note that this gives nearly the answer to part (b).

(e) When v_0 is large, v_0 is negligible in comparison to v_0^2. Then $x(v_0)$ simplifies to

$x(v_0) \approx v_0\sqrt{0 + 0.002\,299\,v_0^2} + 0.047\,94\,v_0^2 = \boxed{0.0959\,v_0^2}$ This nearly gives the answer to part (c).

(f) The graph of x versus v_0 starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola $x = (0.095\,9\text{ s}^2/\text{m})\,v_0^2$

P4.51 The special conditions allowing use of the horizontal range equation applies. For the ball thrown at 45°,

$$D = R_{45} = \frac{v_i^2 \sin 90}{g}$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{(v_i/2)^2 \sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$

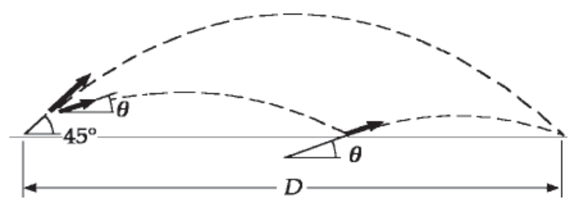

FIG. P4.51

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2.$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2(v_i/2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/g}{2v_i \sin 45.0°/g} = \frac{1.34}{1.41} = \boxed{0.949}$$

P4.52 Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$$y = -\frac{1}{2} g t^2 \quad (3)$$

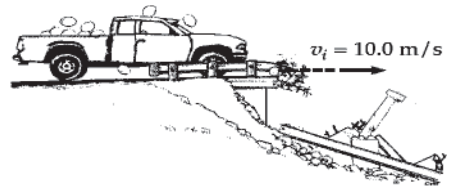

FIG. P4.52

Substitute for t from (2) into (3) $y = -\frac{1}{2} g \left(\frac{x^2}{v_i^2}\right)$. Equate y from the bank equation to y from the equations of motion:

$$16x = \left[-\frac{1}{2} g \left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2 x^4}{4v_i^4} - 16x = x\left(\frac{g^2 x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64 v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$. Also,

$$y = -\frac{1}{2} g \left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2} \frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}$$

P4.53 (a) $\Delta y = -\frac{1}{2} g t^2$; $\Delta x = v_i t$

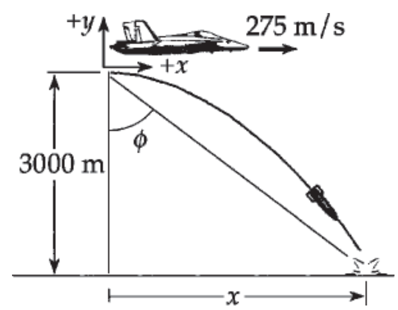

FIG. P4.53

Combine the equations eliminating t:

$$\Delta y = -\frac{1}{2} g \left(\frac{\Delta x}{v_i}\right)^2$$

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right) v_i^2$

thus $\Delta x = v_i \sqrt{\frac{-2\Delta y}{g}} = 275\sqrt{\frac{-2(-3\,000)}{9.80}} = 6.80 \times 10^3 = \boxed{6.80 \text{ km}}$.

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$

therefore, $\phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800}{3\,000}\right) = \boxed{66.2°}$.

P4.54 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \frac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \frac{gR}{v_i^2}$

$$\boxed{v_i > \sqrt{gR}}$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$
or $x = R\sqrt{2}$.

The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}$$

P4.55 (a) From Part (c), the raptor dives for $6.34 - 2.00 = 4.34$ s undergoing displacement 197 m downward and $(10.0)(4.34) = 43.4$ m forward.

$$v = \frac{\Delta d}{\Delta t}\frac{\sqrt{(197)^2 + (43.4)^2}}{4.34} = \boxed{46.5 \text{ m/s}}$$

(b) $\alpha = \tan^{-1}\left(\frac{-197}{43.4}\right) = \boxed{-77.6°}$

(c) $197 = \frac{1}{2}gt^2$, $\boxed{t = 6.34 \text{ s}}$

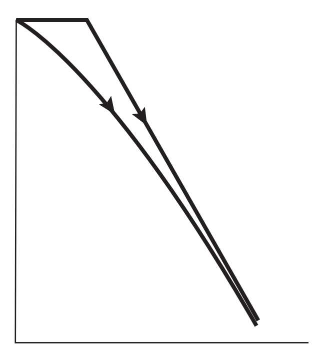

FIG. P4.55

P4.56 (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get

$$v_i = \boxed{22.9 \text{ m/s}} \text{ and } t = 3.06 \text{ s}$$

(b) At the edge of the cliff,

$$v_{xi} = at = (15.0)(3.06) = 45.8 \text{ m/s}$$

Substituting into $\Delta y = \frac{1}{2}a_y t^2$, we find

$$-100 = \frac{1}{2}(-9.80)t^2$$
$$t = 4.52 \text{ s}$$
$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)(4.52 \text{ s}) + \frac{1}{2}(15.0)(4.52 \text{ s})^2$$

Solving,

$$\Delta x = \boxed{360 \text{ m}}$$

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}$$

P4.57 (a) While on the incline

$$v_f^2 - v_i^2 = 2a\Delta x$$
$$v_f - v_i = at$$
$$v_f^2 - 0 = 2(4.00)(50.0)$$
$$20.0 - 0 = 4.00t$$
$$v_f = \boxed{20.0 \text{ m/s}}$$
$$t = \boxed{5.00 \text{ s}}$$

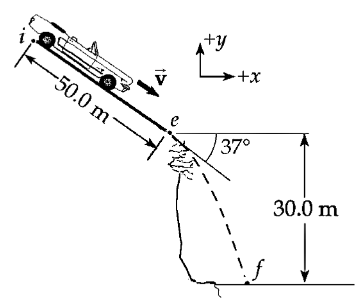

FIG. P4.57

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0\cos 37.0° = 16.0 \text{ m/s}$$

and

$$v_{yi} = -20.0\sin 37.0° = -12.0 \text{ m/s}$$

$$v_{xf} = v_{xi} \text{ since } a_x = 0$$
$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$
$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$$

(c) $t_1 = 5$ s; $t_2 = \dfrac{v_{yf} - v_{yi}}{a_y} = \dfrac{-27.1 + 12.0}{-9.80} = 1.53$ s

$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$

(d) $\Delta x = v_{xi}t_2 = 16.0(1.53) = \boxed{24.5 \text{ m}}$

P4.58 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}} \approx 9\text{ m/s}$$

and its centripetal acceleration is $\dfrac{v^2}{r} \approx \dfrac{(9\text{ m/s})^2}{0.6\text{ m}}\ \boxed{\sim 10^2\text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

P4.59 (a) $\Delta x = v_{xi}t$, $\Delta y = v_{yi}t + \frac{1}{2}gt^2$

$$d\cos 50.0° = (10.0\cos 15.0°)t$$

and

$$-d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)t^2$$

Solving, $d = \boxed{43.2\text{ m}}$ and $t = 2.88$ s.

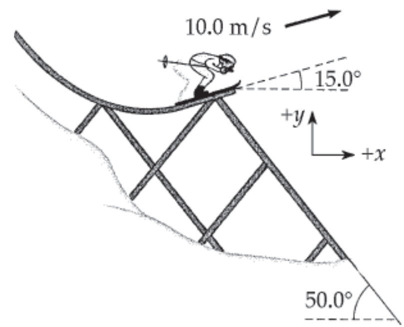

FIG. P4.59

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0\cos 15.0° = \boxed{9.66\text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 10.0\sin 15.0° - 9.80(2.88) = \boxed{-25.6\text{ m/s}}$$

Air resistance would ordinarily decrease the values of the range and landing speed. As an airfoil, he can deflect air downward so that the air deflects him upward. This means he can get some lift and increase his distance.

P4.60 (a) The ice chest floats downstream 2 km in time t, so that 2 km $= v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15$ min. The downstream motion is described by $d + 2\text{ km} = (v + v_w)(t - 15\text{ min})$. We eliminate $t = \dfrac{2\text{ km}}{v_w}$ and d by substitution:

$$(v - v_w)15\text{ min} + 2\text{ km} = (v + v_w)\left(\frac{2\text{ km}}{v_w} - 15\text{ min}\right)$$

$$v(15\text{ min}) - v_w(15\text{ min}) + 2\text{ km} = \frac{v}{v_w}2\text{ km} + 2\text{ km} - v(15\text{ min}) - v_w(15\text{ min})$$

$$v(30\text{ min}) = \frac{v}{v_w}\ 2\text{ km}$$

$$v_w = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w, traveling 2 km. Thus

$$v_w = \frac{\Delta x}{\Delta t} = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}$$

P4.61 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are >45°; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus

$$t = \frac{x_f}{v_i\cos\theta}.$$

Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \frac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$$

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6° \text{ and } \theta_L = 50.1°$$

$$\text{Range }\left(\text{at }\theta_H\right) = \frac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3 \text{ m from enemy ship}$$

$$3.07\times10^3 - 2\,500 - 300 = 270 \text{ m from shore}$$

$$\text{Range }\left(\text{at }\theta_L\right) = \frac{v_i^2\sin 2\theta_L}{g} = 6.28\times10^3 \text{ m from enemy ship}$$

$$6.28\times10^3 - 2\,500 - 300 = 3.48\times10^3 \text{ from shore}$$

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48\times10^3 \text{ m}}$ from the shore.

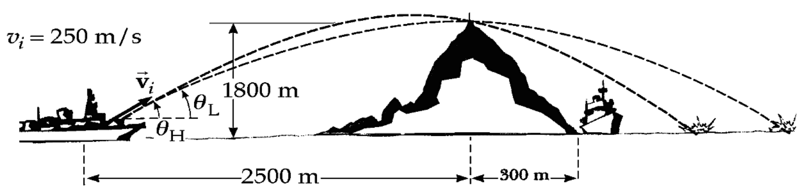

FIG. P4.61

P4.62 We follow the steps outlined in Example 4.7, eliminating $t = \frac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i\sin\theta\, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi$$

Clearing of fractions,

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\frac{dd}{d\theta} = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and

$\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$. Next, $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \frac{1}{\tan 2\theta}$

give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \frac{\phi}{2}$.

ANSWERS TO EVEN PROBLEMS

P4.2 (a) $\vec{\mathbf{r}} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$ (b) $\vec{\mathbf{v}} = 18.0\,\hat{\mathbf{i}} + (4.00 - 9.80t)\,\hat{\mathbf{j}}$ (c) $\vec{\mathbf{a}} = \left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$

(d) $(54.0\ \text{m})\,\hat{\mathbf{i}} - (32.1\ \text{m})\,\hat{\mathbf{j}}$ (e) $(18.0\ \text{m/s})\,\hat{\mathbf{i}} - (25.4\ \text{m/s})\,\hat{\mathbf{j}}$ (f) $\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$

P4.4 (a) $\vec{\mathbf{v}} = \left(-5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right)\ \text{m/s}$; $\vec{\mathbf{a}} = \left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2$

(b) $\vec{\mathbf{r}} = 4.00\ \text{m}\,\hat{\mathbf{j}}\ + 5.00\ \text{m}\left(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}\right)$; $\vec{\mathbf{v}} = 5.00\ \text{m}\ \omega\left(-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right)$;
$\vec{\mathbf{a}} = 5.00\ \text{m}\ \omega^2\left(\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}\right)$
(c) a circle of radius 5.00 m centered at $(0,\ 4.00\ \text{m})$

P4.6 (a) $\vec{\mathbf{v}} = -12.0t\,\hat{\mathbf{j}}\ \text{m/s}$; $\vec{\mathbf{a}} = -12.0\hat{\mathbf{j}}\ \text{m/s}^2$ (b) $\vec{\mathbf{r}} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)\ \text{m}$; $\vec{\mathbf{v}} = -12.0\hat{\mathbf{j}}\ \text{m/s}$

P4.8 (a) $\vec{\mathbf{r}} = \left(5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}\right)\ \text{m}$; $\vec{\mathbf{v}} = \left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \text{m/s}$ (b) $\vec{\mathbf{r}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m}$; $7.81\ \text{m/s}$

P4.10 (a) $d\sqrt{\dfrac{g}{2h}}$ horizontally (b) $\tan^{-1}\left(\dfrac{2h}{d}\right)$ below the horizontal

P4.12 (a) 76.0° (b) the same on every planet. Mathematically, this is because the acceleration of gravity divides out of the answer. (c) $\dfrac{17d}{8}$

P4.14 $d\tan\theta_i - \dfrac{gd^2}{\left(2v_i^2\cos^2\theta_i\right)}$

P4.16 (a) Yes. (b) (1.70 m/s)/$\sqrt{12}$ = 0.491 m/s

P4.18 33.5° below the horizontal

P4.20 (a) 0.852 s; (b) $3.29\ \text{m/s}$; (c) $4.03\ \text{m/s}$;
(d) 50.8°; (e) 1.12 s

P4.22 (a) $\vec{\mathbf{r}}_i = 0\,\hat{\mathbf{i}} + 0.840\ \text{m}\,\hat{\mathbf{j}}$ (b) 11.2 m/s at 18.5° (c) 8.94 m (d) The free-fall trajectory of the athlete is a section around the vertex of a parabola opening downward, everywhere close to horizontal and 48 cm lower on the landing side than on the takeoff side.

P4.24 $0.033\,7\ \text{m/s}^2$ toward the center of the Earth

P4.26 $0.281\ \text{rev/s}$

P4.28 (a) Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude $\sqrt{6^2 - 4.5^2} = 3.97\ \text{m/s}^2$. (b) No. The magnitude of the acceleration cannot be less than $v^2/r = 4.5\ \text{m/s}^2$.

P4.30 (a) see the solution (b) $29.7\ \text{m/s}^2$ (c) 6.67 m/s at 36.9° above the horizontal

P4.32 4.55°

P4.34 (a) $26.9\ \text{m/s}$ (b) 67.3 m (c) $\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$

P4.36 18.0 s

P4.38 (a) 0° (b) 8.25 m/s (c) The can traverses a straight line segment upward and then downward (d) A symmetric section of a parabola opening downward; 12.6 m/s north at 41.0° above the horizontal.

P4.40 (a) $10.1\ \text{m/s}^2$ at 14.3° south from the vertical (b) $9.80\ \text{m/s}^2$ vertically downward (c) The bolt moves on a parabola with its axis downward and tilting to the south. It lands south of the point directly below its starting point. (d) The bolt moves on a parabola with a vertical axis.

P4.42 (a) $101\ \text{m/s}$ (b) $3.27\times 10^4\ \text{ft}$ (c) $20.6\ \text{s}$ (d) $180\ \text{m/s}$

P4.44 (a) 2.69 m (b) The angle could be either positive or negative. The horizontal bounce sends the ball 2.69 m behind the player. To shorten this distance, the bird wants to reduce the horizontal velocity component of the ball. It can do this either by sending the ball upward or downward relative to the horizontal.

P4.46 $2v_i t\cos\theta_i$

P4.48 (a) $1.69\ \text{km/s}$; (b) $6.47\times 10^3\ \text{s}$

P4.50 (a) $x = v_0(0.1643 + 0.002\,299\,v_0^2)^{1/2} + 0.047\,98\ v_0^2$ where x is in meters and v_0 is in meters per second, (b) 0.410 m (c) 961 m (d) $x \approx 0.405\ v_0$ (e) $x \approx 0.095\,9\ v_0^2$ (f) The graph of x versus v_0 starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola $x = (0.095\,9\ \text{s}^2/\text{m})\ v_0^2$.

P4.52 $(18.8\ \text{m};\ -17.3\ \text{m})$

P4.54 (a) $\sqrt{gR}$; (b) $\left(\sqrt{2}-1\right)R$

P4.56 (a) 22.9 m/s (b) 360 m from the base of the cliff (c) $\vec{\mathbf{v}} = (114\ \hat{\mathbf{i}} - 44.3\ \hat{\mathbf{j}})$ m/s

P4.58 Imagine you have a sick child and are shaking down the mercury in an old fever thermometer. Starting with your hand at the level of your shoulder, move your hand down as fast as you can and snap it around an arc at the bottom. $\sim 10^2\ \text{m/s}^2 \sim 10\ g$

P4.60 4.00 km/h

P4.62 see the solution