9

Linear Momentum and Collisions

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q9.1** (a) No. Impulse, $\vec{\mathbf{F}}\Delta t$, depends on the force and the time for which it is applied.

(b) No. Work depends on the force and on the distance over which it acts.

***Q9.2** The momentum magnitude is proportional to the speed and the kinetic energy is proportional to the speed squared.

(i) The speed of the constant-mass object becomes 4 times larger and the kinetic energy 16 times larger. Answer (a).

(ii) The speed and the momentum become two times larger. Answer (d).

***Q9.3** (i) answer (c). For example, if one particle has 5 times larger mass, it will have 5 times smaller speed and 5 times smaller kinetic energy.

(ii) answer (d). Momentum is a vector.

***Q9.4** (i) Equal net work inputs imply equal kinetic energies. Answer (c).

(ii) Imagine one particle has four times more mass. For equal kinetic energy it must have half the speed. Then this more massive particle has 4(1/2) = 2 times more momentum. Answer (a).

Q9.5 (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

***Q9.6** Mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration on the order of 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved. Answer (d).

Q9.7 (a) Linear momentum is conserved since there are no external forces acting on the system. The fragments go off in different directions and their vector momenta add to zero.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

Q9.8 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is much smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

***Q9.9** (i) During the short time the collision lasts, the total system momentum is constant. Whatever momentum one loses the other gains. Answer (c).

(ii) When the car overtakes the manure spreader, the faster-moving one loses more energy than the slower one gains. Answer (a).

Q9.10 The rifle has a much lower speed than the bullet and much less kinetic energy. Also, the butt distributes the recoil force over an area much larger than that of the bullet.

***Q9.11** (i) answer (a). The ball gives more rightward momentum to the block when the ball reverses its momentum.

(ii) answer (b). In case (a) there is no temperature increase because the collision is elastic.

Q9.12 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q9.13 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended, more for a faster missile, so that the force stopping it is never too large.

***Q9.14** Think about how much the vector momentum of the Frisbee changes in a horizontal plane. This will be the same in magnitude as your momentum change. Since you start from rest, this quantity directly controls your final speed. Thus f is largest and d is smallest. In between them, b is larger than c and c is larger than g and g is larger than a. Also a is equal to e, because the ice can exert a normal force to prevent you from recoiling straight down when you throw the Frisbee up. The assembled answer is f > b > c > g > a = e > d.

Q9.15 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Next step: Try a rod with a nonuniform mass distribution.

Next step: Wear a piece of sandpaper as a ring on one finger to change its coefficient of friction.

***Q9.16** (a) No: mechanical energy turns into internal energy in the coupling process.

(b) No: the Earth feeds momentum into the boxcar during the downhill rolling process.

(c) Yes: total energy is constant as it turns from gravitational into kinetic.

(d) Yes: If the boxcar starts moving north the Earth, very slowly, starts moving south.

(e) No: internal energy appears.

(f) Yes: Only forces internal to the two-car system act.

Q9.17 The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little closed loop with a parabolic top and likely a circular bottom, making three revolutions for every one revolution that one ball makes. Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J\,0.60T = 3F_gT$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

Q9.18 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. The rocket body does accelerate as it blows exhaust containing momentum out the back.

According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q9.19 To generalize broadly, around 1740 the English favored position (a), the Germans position (b), and the French position (c). But in France Emilie de Chatelet translated Newton's *Principia* and argued for a more inclusive view. A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All the theories are equally correct. Each is useful for giving a mathematically simple and conceptually clear solution for some problems. There is another comprehensive mechanical theory, the angular impulse—angular momentum theorem, which we will glimpse in Chapter 11. It identifies the product of the torque of a force and the time it acts as the cause of a change in motion, and change in angular momentum as the effect. We have here an example of how scientific theories are different from what people call a theory in everyday life. People who think that different theories are mutually exclusive should bring their thinking up to date to around 1750.

SOLUTIONS TO PROBLEMS

Section 9.1 **Linear Momentum and Its Conservation**

P9.1 $m = 3.00\text{ kg}, \qquad \vec{\mathbf{v}} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right)\text{ m/s}$

(a) $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}$

Thus,

$$\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$$

and

$$\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0\text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

***P9.2** (a) Whomever we consider the aggressor, brother and sister exert equal-magnitude oppositely-directed forces on each other, to give each other equal magnitudes of momentum. We take the eastward component of the equation

total original momentum = total final momentum for the two-sibling system

$0 = 65\text{ kg }(-2.9\text{ m/s}) + 40\text{ kg }v \qquad v = 4.71\text{ m/s}$, meaning $\boxed{\text{she moves at 4.71 m/s east}}$

continued on next page

(b) original chemical energy in girl's body = total final kinetic energy

$U_{chemical} = (1/2)(65\text{ kg})(2.9\text{ m/s})^2 + (1/2)(40\text{ kg})(4.71\text{ m/s})^2 = \boxed{717\text{ J}}$

(c) System momentum is conserved with the value zero. The net forces on the two siblings are of equal magnitude in opposite directions. Their impulses add to zero. Their final momenta are of equal magnitude in opposite directions, to add as vectors to zero.

P9.3 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a\left(x_f - x_i\right): \qquad 0 - v_i^2 = 2\left(-9.80\text{ m/s}^2\right)(0.250\text{ m})$$

$$v_i = 2.20\text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the planet down and myself up:

$$0 = \left(5.98\times10^{24}\text{ kg}\right)(-v_e) + (85.0\text{ kg})(2.20\text{ m/s})$$

$$v_e \sim \boxed{10^{-23}\text{ m/s}}$$

***P9.4** (a) For the system of two blocks $\Delta p = 0$,

or

$$p_i = p_f$$

Therefore,

$$0 = Mv_m + (3M)(2.00\text{ m/s})$$

Solving gives $v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

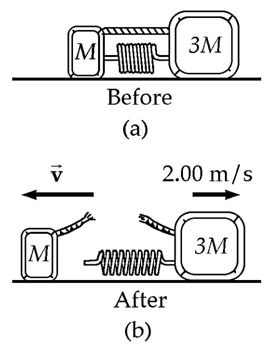

FIG. P9.4

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40\text{ J}}$

(c) The original energy is in the spring. A force had to be exerted over a distance to compress the spring, transferring energy into it by work. The cord exerts force, but over no distance.

(d) System momentum is conserved with the value zero. The forces on the two blocks are of equal magnitude in opposite directions. Their impulses add to zero. The final momenta of the two blocks are of equal magnitude in opposite directions.

P9.5 (a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$

Section 9.2 **Impulse and Momentum**

***P9.6** From the impulse-momentum theorem, $F(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$F = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\text{ kg})(0 - 60\text{ mi/h})}{0.050\text{ s} - 0}\left(\frac{1\text{ m/s}}{2.237\text{ mi/h}}\right) = -6.44 \times 10^3\text{ N}$$

> In trying to hang onto the child, he would have to exert a force of 6.44 kN (over 1 400 lb) toward the back of the car, to slow down the child's forward motion. He is not strong enough to exert so large a force. If he were belted in and his arms were firmly tied around the child, the child would exert this size force on him toward the front of the car. A person cannot safely exert or feel a force of this magnitude and a safety device should be used.

P9.7 (a) $I = \int F dt = \text{area under curve}$

$$I = \frac{1}{2}(1.50 \times 10^{-3}\text{ s})(18\,000\text{ N}) = \boxed{13.5\text{ N} \cdot \text{s}}$$

(b) $F = \dfrac{13.5\text{ N} \cdot \text{s}}{1.50 \times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$

(c) From the graph, we see that $F_{\max} = \boxed{18.0\text{ kN}}$

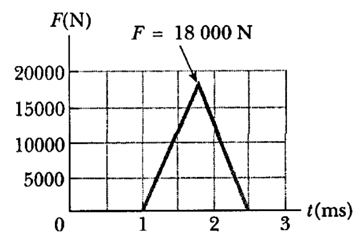

FIG. P9.7

P9.8 The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse the floor imparts to the ball is the change in the ball's momentum,

$$\begin{aligned} mv_2 \text{ up} - mv_1 \text{ down} &= m(v_2 + v_1)\text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)\text{ up} \\ &= 0.15\text{ kg}\sqrt{2(9.8\text{ m/s}^2)}\left(\sqrt{0.960\text{ m}} + \sqrt{1.25\text{ m}}\right)\text{ up} \\ &= \boxed{1.39\text{ kg} \cdot \text{m/s upward}} \end{aligned}$$

P9.9

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$$

$$\Delta p_y = m(v_{fy} - v_{iy}) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\begin{aligned} \Delta p_x &= m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0° \\ &= -2(3.00\text{ kg})(10.0\text{ m/s})(0.866) \\ &= -52.0\text{ kg} \cdot \text{m/s} \end{aligned}$$

$$F_{\text{avg}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\text{ kg} \cdot \text{m/s}}{0.200\text{ s}} = \boxed{-260\text{ N}}$$

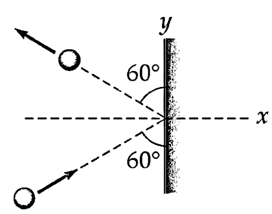

FIG. P9.9

P9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)(-\hat{\mathbf{i}}) = \boxed{5.40\hat{\mathbf{i}}\text{ N} \cdot \text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\text{ J}}$

***P9.11** (a) The impulse is to the right and equal to the area under the F-t graph:

$$I = [(0 + 4\text{ N})/2](2\text{ s} - 0) + (4\text{ N})(3\text{ s} - 2\text{ s}) + (2\text{ N})(2\text{ s}) = \boxed{12.0\text{ N}\cdot\text{s }\hat{\mathbf{i}}}$$

(b) $m\vec{\mathbf{v}}_i + \vec{\mathbf{F}}t = m\vec{\mathbf{v}}_f$ $\quad (2.5\text{ kg})(0) + 12\ \hat{\mathbf{i}}\ \text{N}\cdot\text{s} = (2.5\text{ kg})\ \vec{\mathbf{v}}_f$ $\quad \vec{\mathbf{v}}_f = \boxed{4.80\ \hat{\mathbf{i}}\ \text{m/s}}$

(c) From the same equation, $(2.5\text{ kg})(-2\ \hat{\mathbf{i}}\text{ m/s}) + 12\ \hat{\mathbf{i}}\ \text{N}\cdot\text{s} = (2.5\text{ kg})\ \vec{\mathbf{v}}_f$ $\quad \vec{\mathbf{v}}_f = \boxed{2.80\ \hat{\mathbf{i}}\ \text{m/s}}$

(d) $\vec{\mathbf{F}}_{avg}\Delta t = 12.0\hat{\mathbf{i}}\ \text{N}\cdot\text{s} = \vec{\mathbf{F}}_{avg}(5\text{ s})$ $\quad \vec{\mathbf{F}}_{avg} = \boxed{2.40\hat{\mathbf{i}}\ \text{N}}$

***P9.12** (a) A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.8 s interval.

$$I = \int_0^{0.8\text{s}} F dt = \int_0^{0.8\text{s}} \left(9200\ t\ \text{N/s}\ - 11500\ t^2\ \text{N/s}^2\right) dt$$

$$= \left[(9200\text{ N/s})t^2/2 - \left(11500\text{ N/s}^2\right)t^3/3\right]_0^{0.8\text{s}}$$

$$= (9200\text{ N/s})(0.8\text{ s})^2/2 - \left(11500\text{ N/s}^2\right)(0.8\text{ s})^3/3$$

$$= 2944\text{ N}\cdot\text{s} - 1963\text{ N}\cdot\text{s} = 981\text{ N}\cdot\text{s}$$

The athlete imparts downward impulse to the platform, so the platform imparts $\boxed{\text{981 N·s of upward impulse}}$ to her.

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of energy calculation: $mgy_{top} = (1/2)mv_{impact}^2$

$v_{impact} = (2gy_{top})^{1/2} = [2(9.8\text{ m/s}^2)0.6\text{ m}]^{1/2} = \boxed{3.43\text{ m/s down}}$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform.

$mv_i + I_{platform} + mgt = mv_f$

$(65\text{ kg})(-3.43\text{ m/s}) + 981\text{ N}\cdot\text{s} - (65\text{ kg})(9.8\text{ m/s}^2)(0.8\text{ s}) = 65\text{ kg}\ v_f$

$-223\text{ N}\cdot\text{s} + 981\text{ N}\cdot\text{s} - 510\text{ N}\cdot\text{s} = 65\text{ kg}\ v_f$ $\quad v_f = 249\text{ N}\cdot\text{s}/65\text{ kg} = \boxed{3.83\text{ m/s up}}$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight. $\quad (1/2)mv_{takeoff}^2 = mgy_{top}$

$y_{top} = v_{takeoff}^2/2g = (3.83\text{ m/s})^2/2(9.8\text{ m/s}^2) = \boxed{0.748\text{ m}}$

P9.13 (a) Energy is conserved for the spring-mass system:

$$K_i + U_{si} = K_f + U_{sf}: \qquad 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{\frac{k}{m}}$ larger.

(c) $I = \left|\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i\right| = mv_f - 0 = mx\sqrt{\frac{k}{m}} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

***P9.14** After 3 s of pouring, the bucket contains (3s)(0.25 L/s) = 0.75 liter of water, with mass 0.75 L(1 kg/1 L) = 0.75 kg, and feeling gravitational force 0.75 kg(9.8 m/s^2) = 7.35 N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.75-kg bucket itself.

Water is entering the bucket with speed given by $mgy_{top} = (1/2)mv_{impact}{}^2$

$$v_{impact} = (2gy_{top})^{1/2} = [2(9.8\text{ m/s}^2)2.6\text{ m}]^{1/2} = 7.14\text{ m/s downward}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{impact} + F_{extra}\,t = mv_f$$

The rate of change of momentum is the force itself: $(dm/dt)\,v_{impact} + F_{extra} = 0$

$$F_{extra} = -(dm/dt)\,v_{impact} = -(0.25\text{ kg/s})(-7.14\text{ m/s}) = +1.78\text{ N}$$

Altogether the scale must exert 7.35 N + 7.35 N + 1.78 N = $\boxed{16.5\text{ N}}$

Section 9.3 **Collisions in One Dimension**

P9.15 Momentum is conserved for the bullet-block system

$$\left(10.0\times10^{-3}\text{ kg}\right)v = (5.01\text{ kg})(0.600\text{ m/s})$$

$$v = \boxed{301\text{ m/s}}$$

P9.16 (a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50\times10^4$ kg

$$v_f = \frac{4.00+3(2.00)}{4} = \boxed{2.50\text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right] = \left(2.50\times10^4\right)(12.5-8.00-6.00) = -3.75\times10^4\text{ J}$$

$$K_i = K_f + \Delta E_{int} \qquad \Delta E_{int} = \boxed{+37.5\text{ kJ}}$$

P9.17 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

$$(4m)v_i = (3m)(2.00\text{ m/s}) + m(4.00\text{ m/s})$$

$$v_i = \frac{6.00\text{ m/s}+4.00\text{ m/s}}{4} = \boxed{2.50\text{ m/s}}$$

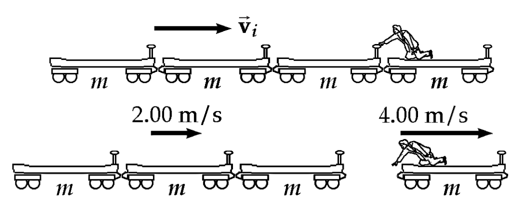

FIG. P9.17

(b) $$W_{\text{actor}} = K_f - K_i = \frac{1}{2}\left[(3m)(2.00\text{ m/s})^2 + m(4.00\text{ m/s})^2\right] - \frac{1}{2}(4\text{ m})(2.50\text{ m/s})^2$$

$$W_{\text{actor}} = \frac{\left(2.50\times10^4\text{ kg}\right)}{2}(12.0+16.0-25.0)(\text{m/s})^2 = \boxed{37.5\text{ kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.18 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

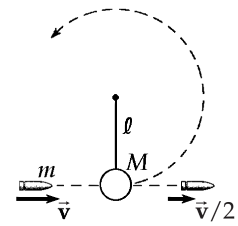

FIG. P9.18

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.19 First we find v_1, the speed of m_1 at B before collision.

$$\frac{1}{2}m_1v_1^2 = m_1gh$$

$$v_1 = \sqrt{2(9.80)(5.00)} = 9.90 \text{ m/s}$$

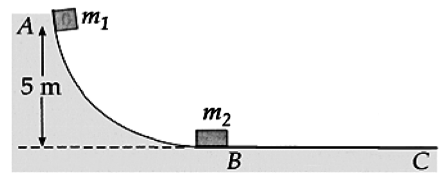

FIG. P9.19

Now we use the text's analysis of one-dimensional elastic collisions to find v_{1f}, the speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90) \text{ m/s} = -3.30 \text{ m/s}$$

Now the 5-kg block bounces back up to its highest point after collision according to

$$m_1gh_{\max} = \frac{1}{2}m_1(-3.30)^2 \qquad h_{\max} = \frac{(-3.30 \text{ m/s})^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.556 \text{ m}}$$

***P9.20** (a) We assume that energy is conserved in the fall of the basketball and the tennis ball. Each reaches its lowest point with a speed given by

$$\left(K + U_g\right)_{\text{release}} = \left(K + U_g\right)_{\text{bottom}}$$

$$0 + mgy_i = \frac{1}{2}mv_b^2 + 0$$

$$v_b = \sqrt{2gy_i} = \sqrt{2\left(9.8 \text{ m/s}^2\right)(1.20 \text{ m})} = \boxed{4.85 \text{ m/s}}$$

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(b) The two balls exert no forces on each other as they move down. They collide with each other after the basketball has its velocity reversed by the floor. We choose upward as positive. Momentum conservation:

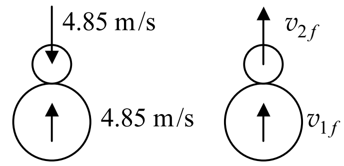

FIG. P9.20(b)

$$(57\text{ g})(-4.85\text{ m/s})+(590\text{ g})(4.85\text{ m/s})=(57\text{ g})v_{2f}+(590\text{ g})v_{1f}$$

To describe the elastic character of the collision, we use the relative velocity equation

$$4.85\text{ m/s}-(-4.85\text{ m/s})=v_{2f}-v_{1f}$$

we solve by substitution

$$v_{1f}=v_{2f}-9.70\text{ m/s}$$

$$2\,580\text{ gm/s}=(57\text{ g})v_{2f}+(590\text{ g})\left(v_{2f}-9.70\text{ m/s}\right)$$

$$=(57\text{ g})v_{2f}+(590\text{ g})v_{2f}-5\,720\text{ gm/s}$$

$$v_{2f}=\frac{8\,310\text{ m/s}}{647}=12.8\text{ m/s}$$

Now the tennis ball-Earth system keeps constant energy as the ball rises:

$$\frac{1}{2}(57\text{ g})(12.8\text{ m/s})^2=(57\text{ g})\left(9.8\text{ m/s}^2\right)y_f$$

$$y_f=\frac{165\text{ m}^2/\text{s}^2}{2\left(9.8\text{ m/s}^2\right)}=\boxed{8.41\text{ m}}$$

P9.21 (a), (b) Let v_g and v_p be the *x*-components of velocity of the girl and the plank relative to the ice surface. Then we may say that v_g-v_p is the velocity of the girl relative to the plank, so that

$$v_g-v_p=1.50 \qquad (1)$$

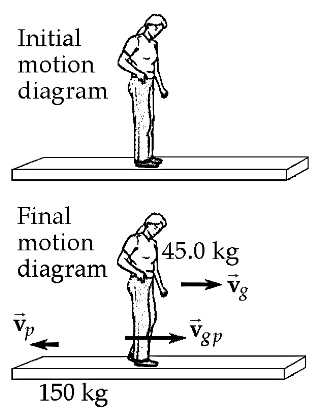

FIG. P9.21

But also we must have $m_gv_g+m_pv_p=0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g+150v_p=0,\text{ or } v_g=-3.33v_p$$

Putting this into the equation (1) above gives

$$-3.33v_p-v_p=1.50\text{ or } v_p=\boxed{-0.346\,\hat{\mathbf{i}}\text{ m/s}}\text{ (answer b)}$$

$$\text{Then } v_g=-3.33(-0.346)=\boxed{1.15\,\hat{\mathbf{i}}\text{ m/s}}\text{ (answer a)}$$

P9.22 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first,

$$K_i + \Delta E_{\text{mech}} = K_f \qquad \frac{1}{2}\left(7.00\times10^{-3}\ \text{kg}\right)v^2 - F\left(8.00\times10^{-2}\ \text{m}\right) = 0$$

For the second,

$$p_i = p_f \qquad \left(7.00\times10^{-3}\ \text{kg}\right)v = (1.014\ \text{kg})v_f$$

$$v_f = \frac{\left(7.00\times10^{-3}\right)v}{1.014}$$

Again,

$$K_i + \Delta E_{\text{mech}} = K_f: \qquad \frac{1}{2}\left(7.00\times10^{-3}\ \text{kg}\right)v^2 - Fd = \frac{1}{2}(1.014\ \text{kg})v_f^2$$

Substituting for v_f,

$$\frac{1}{2}\left(7.00\times10^{-3}\ \text{kg}\right)v^2 - Fd = \frac{1}{2}(1.014\ \text{kg})\left(\frac{7.00\times10^{-3}v}{1.014}\right)^2$$

$$Fd = \frac{1}{2}\left(7.00\times10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00\times10^{-3}\right)^2}{1.014}v^2$$

Substituting for v,

$$Fd = F\left(8.00\times10^{-2}\ \text{m}\right)\left(1 - \frac{7.00\times10^{-3}}{1.014}\right) \qquad d = \boxed{7.94\ \text{cm}}$$

P9.23 (a) From the text's analysis of a one-dimensional elastic collision with an originally stationary target, the x-component of the neutron's velocity changes from v_i to $v_{1f} = (1-12)v_i/13 = -11v_i/13$. The x-component of the target nucleus velocity is $v_{2f} = 2v_i/13$.

The neutron started with kinetic energy $(1/2)\ m_1 v_i^2$

The target nucleus ends up with kinetic energy $(1/2)\ (12\ m_1)(\ 2v_i/13)^2$

Then the fraction transferred is

$$\frac{\frac{1}{2}12m_1(2v_i/13)^2}{\frac{1}{2}m_1 v_i^2} = \frac{48}{169} = \boxed{0.284}$$ Because the collision is elastic, the other 71.6% of the original energy stays with the neutron. The carbon is functioning as a *moderator* in the reactor, slowing down neutrons to make them more likely to produce reactions in the fuel.

(b) $$K_n = (0.716)\left(1.6\times10^{-13}\ \text{J}\right) = \boxed{1.15\times10^{-13}\ \text{J}}$$

$$K_C = (0.284)\left(1.6\times10^{-13}\ \text{J}\right) = \boxed{4.54\times10^{-14}\ \text{J}}$$

P9.24 (a) Using conservation of momentum, $\left(\sum \vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum \vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.0\ \text{kg})(5.0\ \text{m/s}) + (10\ \text{kg})(3.0\ \text{m/s}) + (3.0\ \text{kg})(-4.0\ \text{m/s}) = [(4.0+10+3.0)\ \text{kg}]v$$

Therefore,

$$v = +2.24\ \text{m/s, or } \boxed{2.24\ \text{m/s toward the right}}$$

(b) $\boxed{\text{No.}}$ For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\ \text{kg})v_1 = (10\ \text{kg})(3.0\ \text{m/s}) + (3.0\ \text{kg})(-4.0\ \text{m/s}),\ \text{or}\ v_1 = +1.38\ \text{m/s}$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\ \text{kg})v = (13\ \text{kg})(1.38\ \text{m/s}) + (4.0\ \text{kg})(5.0\ \text{m/s}),\ \text{and}\ v = +2.24\ \text{m/s}$$

just as in part (a). $\boxed{\text{Coupling order makes no difference to the final velocity.}}$

P9.25 During impact, momentum of the clay-block system is conserved:

$$mv_1 = (m_1 + m_2)v_2$$

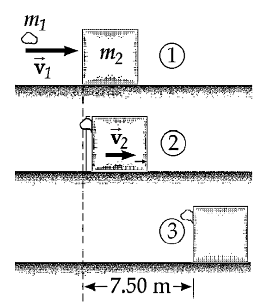

FIG. P9.25

During sliding, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112\ \text{kg})v_2^2 = 0.650(0.112\ \text{kg})(9.80\ \text{m/s}^2)(7.50\ \text{m})$$

$$v_2^2 = 95.6\ \text{m}^2/\text{s}^2 \qquad v_2 = 9.77\ \text{m/s}$$

$$(12.0 \times 10^{-3}\ \text{kg})v_1 = (0.112\ \text{kg})(9.77\ \text{m/s}) \qquad v_1 = \boxed{91.2\ \text{m/s}}$$

Section 9.4 **Two-Dimensional Collisions**

***P9.26** (a) Over a very short time interval, outside forces have no time to impart significant impulse—thus the interaction is a collision. The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is completely inelastic.

continued on next page

(b) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s})+0=(185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s})+0=(185\text{ kg})(V\sin\theta)$$

which gives

$$V\sin\theta = 1.54\text{ m/s} \qquad (2)$$

Divide equation (2) by (1)

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which

$$\boxed{\theta = 32.3°}$$

Then, either (1) or (2) gives

$$V = \boxed{2.88\text{ m/s}}$$

(c) $K_i = \frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2 = 1.55\times10^3\text{ J}$

$K_f = \frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = 7.67\times10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{786 J into internal energy}}$.

P9.27 By conservation of momentum for the system of the two billiard balls (with all masses equal), in the x and y directions separately,

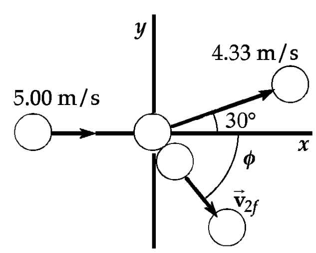

FIG. P9.27

$$5.00\text{ m/s}+0=(4.33\text{ m/s})\cos 30.0°+v_{2fx}$$
$$v_{2fx} = 1.25\text{ m/s}$$
$$0=(4.33\text{ m/s})\sin 30.0°+v_{2fy}$$
$$v_{2fy} = -2.16\text{ m/s}$$
$$\vec{\mathbf{v}}_{2f} = \boxed{2.50\text{ m/s at }-60.0°}$$

Note that we did not need to explicitly use the fact that the collision is perfectly elastic.

P9.28 We use conservation of momentum for the system of two vehicles for both northward and eastward components, to find the original speed of car number 2.

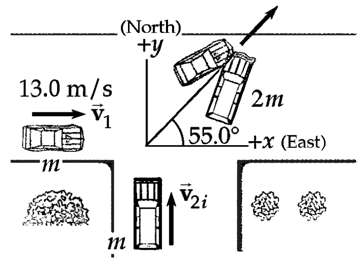

FIG. P9.28

For the eastward direction:

$$M(13.0\ \text{m/s}) = 2MV_f\cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f\sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\ \text{m/s})\tan 55.0° = 18.6\ \text{m/s} = 41.5\ \text{mi/h}$$

$\boxed{\text{Thus, the driver of the north bound car was untruthful. His original speed was more than 35 mi/h.}}$

P9.29 $p_{xf} = p_{xi}$

$$mv_{\text{O}}\cos 37.0° + mv_{\text{Y}}\cos 53.0° = m(5.00\ \text{m/s})$$

$$0.799v_{\text{O}} + 0.602v_{\text{Y}} = 5.00\ \text{m/s} \qquad (1)$$

$p_{yf} = p_{yi}$

$$mv_{\text{O}}\sin 37.0° - mv_{\text{Y}}\sin 53.0° = 0$$

$$0.602v_{\text{O}} = 0.799v_{\text{Y}} \qquad (2)$$

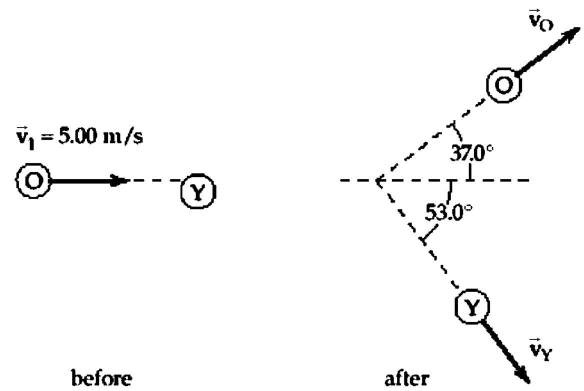

FIG. P9.29

Solving (1) and (2) simultaneously,

$\boxed{v_{\text{O}} = 3.99\ \text{m/s}}$ and $\boxed{v_{\text{Y}} = 3.01\ \text{m/s}}$

P9.30 $p_{xf} = p_{xi}$: $\quad mv_{\text{O}}\cos\theta + mv_{\text{Y}}\cos(90.0° - \theta) = mv_i$

$$v_{\text{O}}\cos\theta + v_{\text{Y}}\sin\theta = v_i \qquad (1)$$

$p_{yf} = p_{yi}$: $\quad mv_{\text{O}}\sin\theta - mv_{\text{Y}}\sin(90.0° - \theta) = 0$

$$v_{\text{O}}\sin\theta = v_{\text{Y}}\cos\theta \qquad (2)$$

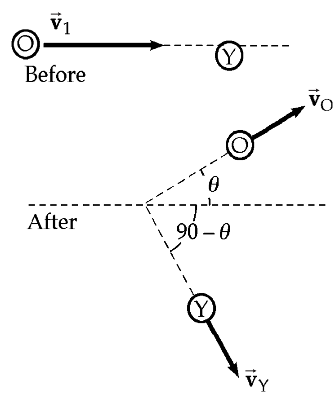

FIG. P9.30

From equation (2),

$$v_{\text{O}} = v_{\text{Y}}\left(\frac{\cos\theta}{\sin\theta}\right) \qquad (3)$$

Substituting into equation (1),

$$v_{\text{Y}}\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_{\text{Y}}\sin\theta = v_i$$

so

$$v_{\text{Y}}\left(\cos^2\theta + \sin^2\theta\right) = v_i\sin\theta, \text{ and } \boxed{v_{\text{Y}} = v_i\sin\theta}$$

Then, from equation (3), $\boxed{v_O = v_i\cos\theta}$.

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

P9.31 $m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f$: $\quad 3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\vec{\mathbf{v}}$

$$\vec{\mathbf{v}} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right)\ \text{m/s}}$$

P9.32 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $\quad -mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $\quad 0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $\quad +\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$

we have

$$v_{2x} = \frac{2v_i}{3}$$

also

$$v_{1y} = 3v_{2y}$$

So the energy equation becomes

$$4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or

$$v_{2y} = \frac{\sqrt{2}v_i}{3}$$

(a) The object of mass m has final speed

$$v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$$

and the object of mass 3 m moves at

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right) \qquad \theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$

P9.33 $m_0 = 17.0 \times 10^{-27}$ kg $\qquad \vec{\mathbf{v}}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\qquad \vec{\mathbf{v}}_1 = 6.00 \times 10^6\,\hat{\mathbf{j}}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\qquad \vec{\mathbf{v}}_2 = 4.00 \times 10^6\,\hat{\mathbf{i}}$ m/s

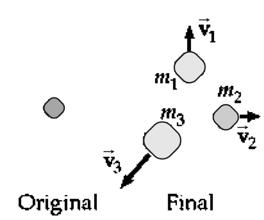

FIG. P9.33

(a) $m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + m_3\vec{\mathbf{v}}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6\,\hat{\mathbf{j}}\right) + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6\,\hat{\mathbf{i}}\right) + \left(3.60 \times 10^{-27}\right)\vec{\mathbf{v}}_3 = 0$$

$$\vec{\mathbf{v}}_3 = \boxed{\left(-9.33 \times 10^6\,\hat{\mathbf{i}} - 8.33 \times 10^6\,\hat{\mathbf{j}}\right)\ \text{m/s}}$$

continued on next page

(b) $E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$

$$E = \frac{1}{2}\Big[\left(5.00\times10^{-27}\right)\left(6.00\times10^{6}\right)^2 + \left(8.40\times10^{-27}\right)\left(4.00\times10^{6}\right)^2 + \left(3.60\times10^{-27}\right)\left(12.5\times10^{6}\right)^2\Big]$$

$$\boxed{E = 4.39\times10^{-13}\text{ J}}$$

P9.34 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\text{ m/s})$$

and

$$v_{Bi} = 8.33\text{ m/s}$$

$$K_i = \frac{1}{2}m(10.0\text{ m/s})^2 + \frac{1}{2}(1.20m)(8.33\text{ m/s})^2 = \frac{1}{2}m\left(183\text{ m}^2/\text{s}^2\right)$$

$$K_f = \frac{1}{2}m\left(v_G\right)^2 + \frac{1}{2}(1.20m)\left(v_B\right)^2 = \frac{1}{2}\left(\frac{1}{2}m\left(183\text{ m}^2/\text{s}^2\right)\right)$$

or

$$v_G^2 + 1.20v_B^2 = 91.7\text{ m}^2/\text{s}^2 \qquad (1)$$

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or

$$v_G = 1.20v_B \qquad (2)$$

Solving (1) and (2) simultaneously, we find

$$(1.20v_B)^2 + 1.20v_B^2 = 91.7\text{ m}^2/\text{s}^2 \qquad v_B = (91.7\text{ m}^2/\text{s}^2\,/\,2.64)^{1/2}$$

$\boxed{v_B = 5.89\text{ m/s}}$ (speed of blue puck after collision)

and

$\boxed{v_G = 7.07\text{ m/s}}$ (speed of green puck after collision)

Section 9.5 **The Center of Mass**

P9.35 The x-coordinate of the center of mass is

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{(2.00\text{ kg}+3.00\text{ kg}+2.50\text{ kg}+4.00\text{ kg})}$$

$$\boxed{x_{\text{CM}} = 0}$$

and the y-coordinate of the center of mass is

$$y_{\text{CM}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00\text{ kg})(3.00\text{ m})+(3.00\text{ kg})(2.50\text{ m})+(2.50\text{ kg})(0)+(4.00\text{ kg})(-0.500\text{ m})}{2.00\text{ kg}+3.00\text{ kg}+2.50\text{ kg}+4.00\text{ kg}}$$

$$\boxed{y_{\text{CM}} = 1.00\text{ m}}$$

P9.36 Let the x axis start at the Earth's center and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{5.98\times10^{24}\text{ kg }0 + 7.36\times10^{22}\text{ kg}\left(3.84\times10^{8}\text{ m}\right)}{6.05\times10^{24}\text{ kg}}$$

$$= \boxed{4.67\times10^{6}\text{ m from the Earth's center}}$$

The center of mass is within the Earth, which has radius 6.37×10^{6} m. It is 1.7 Mm below the point on the Earth's surface where the Moon is straight overhead.

P9.37 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

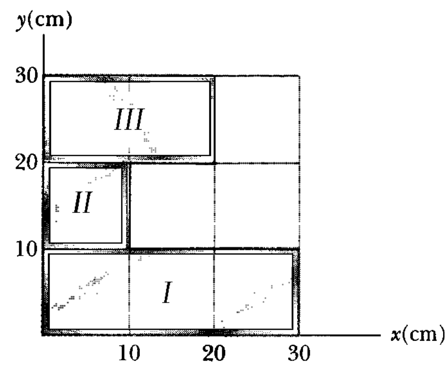

FIG. P9.37

$A = A_1 + A_2 + A_3$

$M = M_1 + M_2 + M_3$

$\frac{M_1}{A_1} = \frac{M}{A}$

$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,$
$A = 600\text{ cm}^2$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$

$$x_{CM} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$

$$x_{CM} = \boxed{11.7\text{ cm}}$$

$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$

$$y_{CM} = \boxed{13.3\text{ cm}}$$

P9.38 (a) Represent the height of a particle of mass dm within the object as y. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g = \int_{\text{all mass}} gy\,dm = g\int y\,dm$. For the center of mass we have $y_{CM} = \frac{1}{M}\int y\,dm$, so

$$U_g = gMy_{CM}$$

(b) The volume of the ramp is $\frac{1}{2}(3.6\text{ m})(15.7\text{ m})(64.8\text{ m}) = 1.83\times10^{3}\text{ m}^3$. Its mass is $\rho V = \left(3\,800\text{ kg/m}^3\right)\left(1.83\times10^{3}\text{ m}^3\right) = 6.96\times10^{6}\text{ kg}$. As shown in the chapter, its center of mass is above its base by one-third of its height, $y_{CM} = \frac{1}{3}15.7\text{ m} = 5.23\text{ m}$. Then

$$U_g = Mgy_{CM} = 6.96\times10^{6}\text{ kg}\left(9.8\text{ m/s}^2\right)5.23\text{ m} = \boxed{3.57\times10^{8}\text{ J}}$$

P9.39 This object can be made by wrapping tape around a light stiff uniform rod.

(a) $M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right] dx$

$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$

(b) $x_{\text{CM}} = \dfrac{\int_{\text{all mass}} x\,dm}{M} = \dfrac{1}{M}\int_0^{0.300\text{ m}} \lambda x\,dx = \dfrac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right] dx$

$x_{\text{CM}} = \dfrac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \dfrac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$

P9.40 Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{r d\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L} d\theta$ where we have used the definition of radian measure. Now

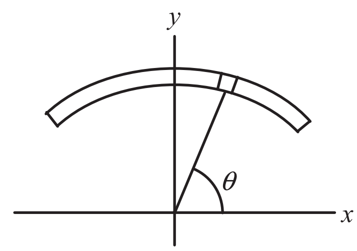

FIG. P9.40

$$y_{\text{CM}} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta \frac{Mr}{L} d\theta = \frac{r^2}{L}\int_{45°}^{135°} \sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2 \frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

Section 9.6 **Motion of a System of Particles**

P9.41 (a) $\vec{\mathbf{v}}_{\text{CM}} = \dfrac{\sum m_i \vec{\mathbf{v}}_i}{M} = \dfrac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{M}$

$$= \frac{(2.00\text{ kg})\left(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s}\right) + (3.00\text{ kg})\left(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s}\right)}{5.00\text{ kg}}$$

$\vec{\mathbf{v}}_{\text{CM}} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s}}$

(b) $\vec{\mathbf{p}} = M\vec{\mathbf{v}}_{\text{CM}} = (5.00\text{ kg})\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s} = \boxed{\left(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$

***P9.42** $\vec{\mathbf{r}}_{CM}=\dfrac{m_1\vec{\mathbf{r}}_1+m_2\vec{\mathbf{r}}_2}{m_1+m_2}=\dfrac{3.5\left[(3\hat{\mathbf{i}}+3\hat{\mathbf{j}})t+2\hat{\mathbf{j}}t^2\right]+5.5\left[3\hat{\mathbf{i}}-2\hat{\mathbf{i}}t^2-6\hat{\mathbf{j}}t\right]}{3.5+5.5}$

$$=\left(1.83+1.17t-1.22t^2\right)\hat{\mathbf{i}}+\left(-2.5t+0.778t^2\right)\hat{\mathbf{j}}$$

(a) At $t = 2.5$ s, $\vec{\mathbf{r}}_{CM}=(1.83+1.17\cdot 2.5-1.22\cdot 6.25)\hat{\mathbf{i}}+(-2.5\cdot 2.5+0.778\cdot 6.25)\hat{\mathbf{j}}=$

$\boxed{(-2.89\,\hat{\mathbf{i}}-1.39\,\hat{\mathbf{j}})\text{cm}}$ We can conveniently do part (c) on the way to part (b):

$$\vec{\mathbf{v}}_{CM}=\frac{d\vec{\mathbf{r}}_{CM}}{dt}=(1.17-2.44t)\hat{\mathbf{i}}+(-2.5+1.56t)\hat{\mathbf{j}}$$

at $t = 2.5$ s, $\vec{\mathbf{v}}_{CM}=(1.17-2.44\cdot 2.5)\hat{\mathbf{i}}+(-2.5+1.56\cdot 2.5)\hat{\mathbf{j}}$

$$=\boxed{(-4.94\,\hat{\mathbf{i}}+1.39\,\hat{\mathbf{j}})\text{ cm/s}}$$

(b) Now the total linear momentum is the total mass times the velocity of the center of mass:

$$(9\text{ g})(-4.94\,\hat{\mathbf{i}}+1.39\,\hat{\mathbf{j}})\text{ cm/s}=\boxed{(-44.5\,\hat{\mathbf{i}}+12.5\,\hat{\mathbf{j}})\text{ g}\cdot\text{cm/s}}$$

(d) Differentiating again, $\vec{\mathbf{a}}_{CM}=\dfrac{d\vec{\mathbf{v}}_{CM}}{dt}=(-2.44)\hat{\mathbf{i}}+1.56\hat{\mathbf{j}}$

The center of mass acceleration is $\boxed{(-2.44\,\hat{\mathbf{i}}+1.56\,\hat{\mathbf{j}})\text{ cm/s}^2}$ at $t = 2.5$ s and at all times.

(e) The net force on the system is equal to the total mass times the acceleration of the center of mass:

$$(9\text{ g})(-2.44\,\hat{\mathbf{i}}+1.56\,\hat{\mathbf{j}})\text{ cm/s}^2=\boxed{(-220\,\hat{\mathbf{i}}+140\,\hat{\mathbf{j}})\,\mu\text{N}}$$

P9.43 Let x = distance from shore to center of boat

ℓ = length of boat

x' = distance boat moves as Juliet moves toward Romeo

The center of mass stays fixed.

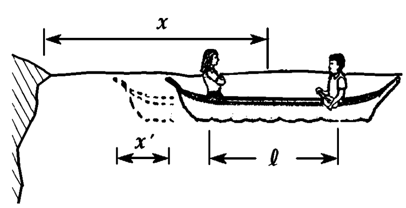

FIG. P9.43

Before: $x_{\text{CM}}=\dfrac{\left[M_b x+M_J\left(x-\frac{\ell}{2}\right)+M_R\left(x+\frac{\ell}{2}\right)\right]}{\left(M_B+M_J+M_R\right)}$

After: $x_{\text{CM}}=\dfrac{\left[M_B(x-x')+M_J\left(x+\frac{\ell}{2}-x'\right)+M_R\left(x+\frac{\ell}{2}-x'\right)\right]}{\left(M_B+M_J+M_R\right)}$

$$\ell\left(-\frac{55.0}{2}+\frac{77.0}{2}\right)=x'(-80.0-55.0-77.0)+\frac{\ell}{2}(55.0+77.0)$$

$$x'=\frac{55.0\ell}{212}=\frac{55.0(2.70)}{212}=\boxed{0.700\text{ m}}$$

P9.44 (a) Conservation of momentum for the two-ball system gives us:

$$0.200\text{ kg}\left(1.50\text{ m/s}\right)+0.300\text{ kg}\left(-0.400\text{ m/s}\right)=0.200\text{ kg }v_{1f}+0.300\text{ kg }v_{2f}$$

Relative velocity equation:

$$v_{2f}-v_{1f}=1.90\text{ m/s}$$

Then

$$0.300-0.120=0.200v_{1f}+0.300\left(1.90+v_{1f}\right)$$

$$v_{1f}=-0.780\text{ m/s}\qquad v_{2f}=1.12\text{ m/s}$$

$$\boxed{\vec{\mathbf{v}}_{1f}=-0.780\hat{\mathbf{i}}\text{ m/s}}\qquad\boxed{\vec{\mathbf{v}}_{2f}=1.12\hat{\mathbf{i}}\text{ m/s}}$$

continued on next page

(b) Before,

$$\vec{\mathbf{v}}_{CM} = \frac{(0.200\ \text{kg})(1.50\ \text{m/s})\hat{\mathbf{i}} + (0.300\ \text{kg})(-0.400\ \text{m/s})\hat{\mathbf{i}}}{0.500\ \text{kg}}$$

$$\boxed{\vec{\mathbf{v}}_{CM} = (0.360\ \text{m/s})\hat{\mathbf{i}}}$$

Afterwards, the center of mass must move at the same velocity, because the momentum of the system is conserved.

Section 9.7 **Deformable Systems**

***P9.45** (a) $\boxed{\text{Yes.}}$ The only horizontal force on the vehicle is the frictional force exerted by the floor, so it gives the vehicle all of its final momentum, (6 kg)(3 $\hat{\mathbf{i}}$ m/s) = $\boxed{18.0\ \hat{\mathbf{i}}\ \text{kg}\cdot\text{m/s}}$

(b) $\boxed{\text{No.}}$ The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work.

(c) $\boxed{\text{Yes,}}$ we could say that the final momentum of the cart came from the floor or from the planet through the floor, because the floor imparts impulse.

(d) $\boxed{\text{No.}}$ The floor does no work. The final kinetic energy came from $\boxed{\text{the original gravitational energy of the elevated load}}$, in amount $(1/2)(6\ \text{kg})(3\ \text{m/s})^2 = 27.0$ J.

(e) Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the caterpillar tracks from slipping backward.

***P9.46** (a) $\boxed{\text{Yes.}}$ The floor exerts a force, larger than the person's weight over time as he is taking off.

(b) $\boxed{\text{No.}}$ The work by the floor on the person is zero because the force exerted by the floor acts over zero distance.

(c) He leaves the floor with a speed given by $(1/2)mv^2 = mgy_f$
$v = [2(9.8\ \text{m/s}^2)0.15\ \text{m}]^{1/2} = 1.71$ m/s,
so his momentum immediately after he leaves the floor is (60 kg)(1.71 m/s up) =
$\boxed{103\ \text{kg}\cdot\text{m/s up}}$

(d) Yes. You could say that it came from the planet, that gained momentum 103 kg·m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact.

(e) $(1/2)(60\ \text{kg})(1.71\ \text{m/s})^2 = \boxed{88.2\ \text{J}}$

(f) No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.

***P9.47** (a) When the cart hits the bumper it immediately stops, and the hanging particle keeps moving with its original speed v_i. The particle swings up as a pendulum on a fixed pivot, keeping constant energy. Measure elevations from the pivot:

$(1/2)mv_i^2 + mg(-L) = 0 + mg(-L\cos\theta)$ Then $\boxed{v_i = [2gL(1-\cos\theta)]^{1/2}}$

(b) $v_i = [2gL(1-\cos\theta)]^{1/2} = [2(9.8\text{ m/s}^2)(1.2\text{ m})(1-\cos 35°)]^{1/2} = \boxed{2.06\text{ m/s}}$

(c) $\boxed{\text{Yes.}}$ The bumper must provide the horizontal force to the left to slow down the swing of the particle to the right, to reverse its rightward motion, and to make it speed up to the left.

$\boxed{\text{When the particle passes its straight-down position moving to the left, the bumper stops exerting force.}}$

It is at this moment that the cart-particle system momentarily has zero horizontal acceleration for its center of mass.

***P9.48** Depending on the length of the cord and the time interval Δt for which the force is applied, the sphere may have moved very little when the force is removed, or we may have x_1 and x_2 nearly equal, or the sphere may have swung back, or it may have swung back and forth several times. Our solution applies equally to all of these cases.

(a) The applied force is constant, so the center of mass of the glider-sphere system moves with constant acceleration. It starts, we define, from $x = 0$ and moves to $(x_1 + x_2)/2$. Let v_1 and v_2 represent the horizontal components of velocity of glider and sphere at the moment the force stops. Then the velocity of the center of mass is $v_{CM} = (v_1 + v_2)/2$ and because the acceleration is constant we have $(x_1 + x_2)/2 = [(v_1 + v_2)/2]\Delta t/2$ $\quad \Delta t = 2(x_1 + x_2)/(v_1 + v_2)$

The impulse-momentum theorem for the glider-sphere system is

$F\Delta t = mv_1 + mv_2$ $\quad F2(x_1 + x_2)/(v_1 + v_2) = m(v_1 + v_2)$

$F2(x_1 + x_2)/m = (v_1 + v_2)^2$ $\quad F2(x_1 + x_2)/4m = (v_1 + v_2)^2/4 = v_{CM}^2$

Then $\boxed{v_{CM} = [F(x_1 + x_2)/2m]^{1/2}}$

(b) The applied force does work that becomes, after the force is removed, kinetic energy of the constant-velocity center-of-mass motion plus kinetic energy of the vibration of the glider and sphere relative to their center of mass. The applied force acts only on the glider, so the work-energy theorem for the pushing process is

$Fx_1 = (1/2)(2\,m)\ v_{CM}^2 + E_{vib}$

Substitution gives $Fx_1 = (1/2)(2\,m)F(x_1 + x_2)/2m + E_{vib} = Fx_1/2 + Fx_2/2 + E_{vib}$

Then $E_{vib} = Fx_1/2 - Fx_2/2$

When the cord makes its largest angle with the vertical, the vibrational motion is turning around. No kinetic energy is associated with the vibration at this moment, but only gravitational energy:

$mgL(1-\cos\theta) = F(x_1 - x_2)/2$ Solving gives $\boxed{\theta = \cos^{-1}[1 - F(x_1 - x_2)/2mgL]}$

P9.49 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\dfrac{\Delta p_x}{\Delta t} = \dfrac{(5.00\text{ kg})(0.750\text{ m/s})}{1.00\text{ s}} = \boxed{3.75\text{ N}}$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi} + f\Delta t = p_{xf}$ this is $f = \dfrac{\Delta p_x}{\Delta t} = \boxed{3.75\text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x$: $\quad +F_{\text{ext}} - f = 0$ and $F_{\text{ext}} = \boxed{3.75\text{ N}}$

continued on next page

(d) $W = F\Delta r\cos\theta = 3.75\text{ N}(0.750\text{ m})\cos 0° = \boxed{2.81\text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2 = \frac{1}{2}5.00\text{ kg}(0.750\text{ m/s})^2 = \boxed{1.41\text{ J}}$

(f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

Section 9.8 Rocket Propulsion

P9.50 (a) The fuel burns at a rate $\frac{dM}{dt} = \frac{12.7\text{ g}}{1.90\text{ s}} = 6.68\times10^{-3}\text{ kg/s}$

Thrust $= v_e\frac{dM}{dt}$: $5.26\text{ N} = v_e(6.68\times10^{-3}\text{ kg/s})$

$v_e = \boxed{787\text{ m/s}}$

(b) $v_f - v_i = v_e\ln\left(\frac{M_i}{M_f}\right)$: $v_f - 0 = (787\text{ m/s})\ln\left(\frac{53.5\text{ g}+25.5\text{ g}}{53.5\text{ g}+25.5\text{ g}-12.7\text{ g}}\right)$

$v_f = \boxed{138\text{ m/s}}$

P9.51 (a) Thrust $= \left|v_e\frac{dM}{dt}\right|$ Thrust $= (2.60\times10^3\text{ m/s})(1.50\times10^4\text{ kg/s}) = \boxed{3.90\times10^7\text{ N}}$

(b) $\sum F_y = \text{Thrust} - Mg = Ma$: $3.90\times10^7 - (3.00\times10^6)(9.80) = (3.00\times10^6)a$

$a = \boxed{3.20\text{ m/s}^2}$

P9.52 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e\ln\left(\frac{M_i}{M_f}\right) = -v_e\ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e\ln\left(\frac{M_i - kt}{M_i}\right) = -v_e\ln\left(1-\frac{k}{M_i}t\right)$

With the definition $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e\ln\left(1-\frac{t}{T_p}\right)}$$

(b) With $v_e = 1\,500\text{ m/s}$, and $T_p = 144\text{ s}$, $v = -(1\,500\text{ m/s})\ln\left(1-\frac{t}{144\text{ s}}\right)$

$t(s)$	$v(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

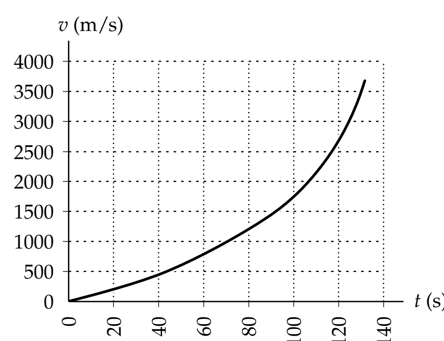

FIG. P9.52(b)

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(c) $a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1-\frac{t}{T_p}\right)\right]}{dt} = -v_e\left(\frac{1}{1-\frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1-\frac{t}{T_p}}\right)$, or

$$a(t) = \boxed{\frac{v_e}{T_p - t}}$$

(d) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a = \frac{1\,500 \text{ m/s}}{144 \text{ s} - t}$

$t(s)$	$a(\text{m/s}^2)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

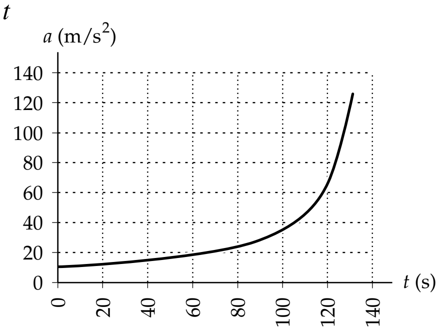

FIG. P9.52(d)

(e) $x(t) = 0 + \int_0^t v\,dt = \int_0^t \left[-v_e \ln\left(1-\frac{t}{T_p}\right)\right]dt = v_e T_p \int_0^t \ln\left[1-\frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$

$$x(t) = v_e T_p \left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right) - \left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t) = \boxed{v_e\left(T_p - t\right)\ln\left(1-\frac{t}{T_p}\right) + v_e t}$$

(f) With $v_e = 1\,500$ m/s $= 1.50$ km/s, and $T_p = 144$ s,

$$x = 1.50(144-t)\ln\left(1-\frac{t}{144}\right) + 1.50t$$

$t(\text{s})$	$x(\text{km})$
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

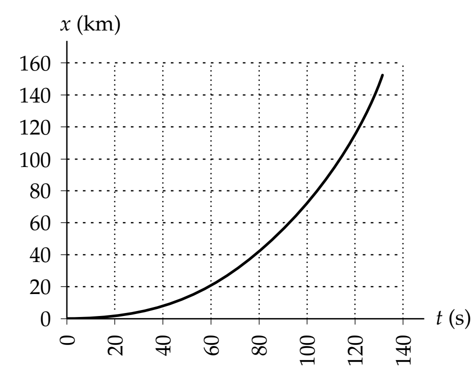

FIG. P9.52(f)

P9.53 In $v = v_e \ln \dfrac{M_i}{M_f}$ we solve for M_i.

(a) $M_i = e^{v/v_e} M_f \qquad M_i = e^5\left(3.00\times10^3 \text{ kg}\right) = 4.45\times10^5 \text{ kg}$

The mass of fuel and oxidizer is $\Delta M = M_i - M_f = (445 - 3.00)\times10^3 \text{ kg} = \boxed{442 \text{ metric tons}}$

(b) $\Delta M = e^2(3.00 \text{ metric tons}) - 3.00 \text{ metric tons} = \boxed{19.2 \text{ metric tons}}$

This is much less than the suggested value of 442/2.5. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed, by counting again and again in the speed the body attains second after second during its burn. Because of the exponential, a relatively small increase in engine efficiency causes a large change in the amount of fuel and oxidizer required.

Additional Problems

P9.54 (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f: \qquad m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = (m_1+m_2)\vec{\mathbf{v}} \qquad \boxed{\vec{\mathbf{v}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1+m_2}}$$

(b) Only conservative forces act; therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1+m_2)m_1v_1^2 + (m_1+m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1+m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2+v_2^2-2v_1v_2\right)}{k(m_1+m_2)}} = \boxed{(v_1-v_2)\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}}$$

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(c) $m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$

Conservation of momentum: $m_1\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{1f}\right) = m_2\left(\vec{\mathbf{v}}_{2f} - \vec{\mathbf{v}}_2\right)$ (1)

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$

Factoring gives

$$m_1\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{1f}\right)\cdot\left(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_{1f}\right) = m_2\left(\vec{\mathbf{v}}_{2f} - \vec{\mathbf{v}}_2\right)\cdot\left(\vec{\mathbf{v}}_{2f} + \vec{\mathbf{v}}_2\right)$$

and with the use of the momentum equation (equation (1)),

this reduces to $\left(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_{1f}\right) = \left(\vec{\mathbf{v}}_{2f} + \vec{\mathbf{v}}_2\right)$

or $\vec{\mathbf{v}}_{1f} = \vec{\mathbf{v}}_{2f} + \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$ (2)

Substituting equation (2) into equation (1) and simplifying yields:

$$\vec{\mathbf{v}}_{2f} = \boxed{\left(\frac{2m_1}{m_1+m_2}\right)\vec{\mathbf{v}}_1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)\vec{\mathbf{v}}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\vec{\mathbf{v}}_{1f} = \boxed{\left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{\mathbf{v}}_1 + \left(\frac{2m_2}{m_1+m_2}\right)\vec{\mathbf{v}}_2}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

P9.55 We hope the momentum of the wrench provides enough recoil so that the astronaut can reach the ship before he loses life support! We might expect the elapsed time to be on the order of several minutes based on the description of the situation.

No external force acts on the system (astronaut plus wrench), so the total momentum is constant. Since the final momentum (wrench plus astronaut) must be zero, we have final momentum = initial momentum = 0.

$$m_{\text{wrench}}v_{\text{wrench}} + m_{\text{astronaut}}v_{\text{astronaut}} = 0$$

Thus

$$v_{\text{astronaut}} = -\frac{m_{\text{wrench}}v_{\text{wrench}}}{m_{\text{astronaut}}} = -\frac{(0.500\text{ kg})(20.0\text{ m/s})}{80.0\text{ kg}} = -0.125\text{ m/s}$$

At this speed, the time to travel to the ship is

$$t = \frac{30.0\text{ m}}{0.125\text{ m/s}} = \boxed{240\text{ s}} = 4.00\text{ minutes}$$

The astronaut is fortunate that the wrench gave him sufficient momentum to return to the ship in a reasonable amount of time! In this problem, we did not think of the astronaut as drifting away from the ship when he threw the wrench. However slowly, he must be drifting away since he did not encounter an external force that would reduce his velocity away from the ship. There is no air friction beyond earth's atmosphere. In a real-life situation, the astronaut would have to throw the wrench hard enough to overcome his momentum caused by his original push away from the ship.

***P9.56** Proceeding step by step, we find the real actor's speed just before collision, using energy conservation in the swing-down process: $m_a g y_i = (1/2)\, m_a v_i^2$ $\quad [2(9.8\text{ m/s}^2)(1.8\text{ m})]^{1/2} = v_i = 5.94\text{ m/s}$
Now for the elastic collision with a stationary target we use the specialized equation from the chapter text $v_{2f} = (2\,m_1 v_{1i})/(m_1 + m_2) = 2(80\text{ kg})(5.94\text{ m/s})/(80\text{ kg} + m) = (950\text{ kg·m/s})/(80\text{ kg} + m)$
The time for the clone's fall into the ocean is given by

$$\Delta y = v_{yi}t + (1/2)a_y t^2 - 36\text{ m} = 0 + (1/2)(-9.8\text{ m/s}^2)t^2 \qquad t = 2.71\text{ s}$$

so his horizontal range is

$$R = v_{2f}\,t = (2.71\text{ s})(950\text{ kg}\cdot\text{m/s})/(80\text{ kg} + m) = \boxed{2.58\times 10^3\text{ kg}\cdot\text{m}/(80\text{ kg} + m)}$$

(b) By substitution, $2576\text{ kg}\cdot\text{m }(80\text{ kg} + 79\text{ kg})^{-1} = \boxed{16.2\text{ m}}$

(c) A little heavier and he does not go so far: $2576\text{ kg}\cdot\text{m }(80\text{ kg} + 81\text{ kg})^{-1} = \boxed{16.0\text{ m}}$

(d) We solve $30\text{ m} = 2580\text{ kg·m }(80\text{ kg} + m)^{-1}$ $\quad 80\text{ kg} + m = 85.87\text{ kg}$ $\quad m = \boxed{5.87\text{ kg}}$

(e) The maximum value for R is $2576/80 = \boxed{32.2\text{ m}}$, obtained in the limit as

(f) we make m go to $\boxed{\text{zero}}$.

(g) The minimum value of R is approaching $\boxed{\text{zero}}$, obtained in the limit as

(h) we make m go to $\boxed{\text{infinity}}$.

(i) Yes, mechanical energy is conserved until the clone splashes down. This principle is not sufficient to solve the problem. We need also conservation of momentum in the collision.

(j) Yes, but it is not useful to include the planet in the analysis of momentum. We use instead momentum conservation for the actor-clone system while they are in contact.

(k) In symbols we have $v_i = [2\,g\,(1.8\text{ m})]^{1/2}$
$v_{2f} = 2(80\text{ kg})\,[2\,g\,(1.8\text{ m})]^{1/2}/(80\text{ kg} + m)$
$t = [2(36\text{ m})/g]^{1/2}$
and $R = [2(36\text{ m})/g]^{1/2}\, 2(80\text{ kg})\,[2\,g\,(1.8\text{ m})]^{1/2}/(80\text{ kg} + m)$

Here g divides out. At a location with weaker gravity, the actor would be moving more slowly before the collision, but the clone would follow the same trajectory, moving more slowly over a longer time interval.

P9.57 Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or

$$v_i = \left(\frac{M+m}{m}\right)v_f \qquad (1)$$

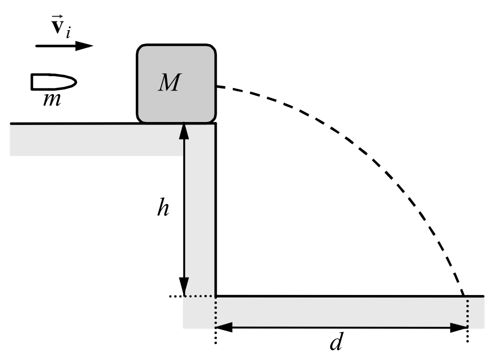

FIG. P9.57

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus,

$$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into (1) from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

P9.58 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or

$$(3.00\text{ kg})v_{\text{wedge}} + (0.500\text{ kg})(+4.00\text{ m/s}) = 0$$

so

$$v_{\text{wedge}} = \boxed{-0.667\text{ m/s}}$$

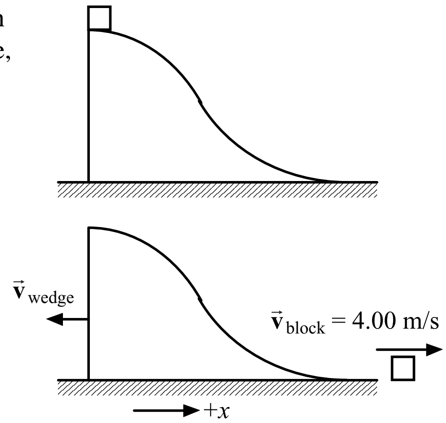

FIG. P9.58

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or

$$\left[0 + m_1 gh\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2 \text{ which gives } \boxed{h = 0.952\text{ m}}$$

P9.59 (a) Conservation of momentum:

$$0.5\text{ kg}\left(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\text{ m/s} + 1.5\text{ kg}\left(-1\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\text{ m/s}$$
$$= 0.5\text{ kg}\left(-1\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}\right)\text{ m/s} + 1.5\text{ kg }\vec{\mathbf{v}}_{2f}$$
$$\vec{\mathbf{v}}_{2f} = \frac{\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} + \left(0.5\hat{\mathbf{i}} - 1.5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}} = \boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}0.5\text{ kg}\left(2^2 + 3^2 + 1^2\right)\text{ m}^2/\text{s}^2 + \frac{1}{2}1.5\text{ kg}\left(1^2 + 2^2 + 3^2\right)\text{ m}^2/\text{s}^2 = 14.0\text{ J}$$

The final kinetic energy is $\frac{1}{2}0.5\text{ kg}\left(1^2 + 3^2 + 8^2\right)\text{ m}^2/\text{s}^2 + 0 = 18.5\text{ J}$ different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} = 0.5\text{ kg}\left(-0.25\hat{\mathbf{i}} + 0.75\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)\text{ m/s} + 1.5\text{ kg }\vec{\mathbf{v}}_{2f}$$
$$\vec{\mathbf{v}}_{2f} = \frac{\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} + \left(0.125\hat{\mathbf{i}} - 0.375\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$
$$= \boxed{\left(-0.250\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}} - 2.00\hat{\mathbf{k}}\right)\text{ m/s}}$$

We see $\vec{\mathbf{v}}_{2f} = \vec{\mathbf{v}}_{1f}$, so the collision is $\boxed{\text{perfectly inelastic}}$.

continued on next page

(c) Conservation of momentum:

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}=0.5\ \text{kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\ \text{m/s}+1.5\ \text{kg}\ \vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}-0.5a\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}}{1.5\ \text{kg}}$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\ \text{m/s}}$$

Conservation of energy:

$$14.0\ \text{J}=\frac{1}{2}0.5\ \text{kg}\left(1^2+3^2+a^2\right)\ \text{m}^2/\text{s}^2+\frac{1}{2}1.5\ \text{kg}(2.67+0.333a)^2\ \ \text{m}^2/\text{s}^2$$

$$=2.5\ \text{J}+0.25a^2+5.33\ \text{J}+1.33a+0.083\,3a^2$$

$$0=0.333a^2+1.33a-6.167$$

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

$a=2.74$ or -6.74. Either value is possible.

with $\boxed{a=2.74}$, $\quad \vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\ \text{m/s}=\boxed{-3.58\hat{\mathbf{k}}\ \text{m/s}}$

with $\boxed{a=-6.74}$, $\quad \vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\ \text{m/s}=\boxed{-0.419\hat{\mathbf{k}}\ \text{m/s}}$

P9.60 Consider the motion of the firefighter during the three intervals: (1) before, (2) during, and (3) after collision with the platform.

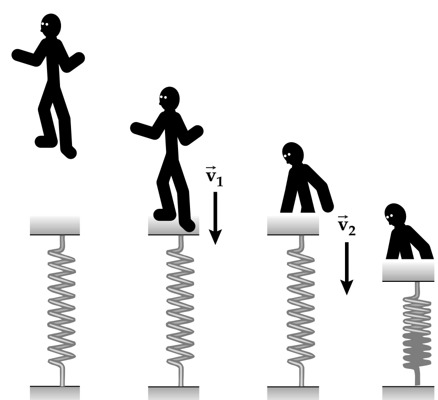

FIG. P9.60

(a) While falling a height of 4.00 m, her speed changes from $v_i=0$ to v_1 as found from

$$\Delta E=\left(K_f+U_f\right)-\left(K_i-U_i\right),\ \text{or}$$

$$K_f=\Delta E-U_f+K_i+U_i$$

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2=fh\cos(180°)-0+0+mgh$$

Solving for v_1 gives

$$v_1=\sqrt{\frac{2(-fh+mgh)}{m}}=\sqrt{\frac{2(-300(4.00)+75.0(9.80)4.00)}{75.0}}=\boxed{6.81\ \text{m/s}}$$

continued on next page

(b) During the inelastic collision, momentum is conserved; and if v_2 is the speed of the fire-fighter and platform just after collision, we have $mv_1 = (m+M)v_2$ or

$$v_2 = \frac{m_1 v_1}{m+M} = \frac{75.0(6.81)}{75.0+20.0} = 5.38 \text{ m/s}$$

Following the collision and again solving for the work done by non-conservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform):

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}, \text{ or}$$

$$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0 \text{ or } \boxed{s = 1.00 \text{ m}}$$

P9.61 (a) Each primate swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P9.62 (a) Utilizing conservation of momentum,

$$m_1 v_{1A} = (m_1 + m_2) v_B$$

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29\ \text{m/s}}$$

(b) Utilizing the two equations

$\frac{1}{2}gt^2 = y$ and $x = v_{1A}t$

we combine them to find

$$v_{1A} = \frac{x}{\sqrt{2y/g}} = x\sqrt{\frac{g}{2y}}$$

From the data, $v_{1A} = \boxed{6.16\ \text{m/s}}$

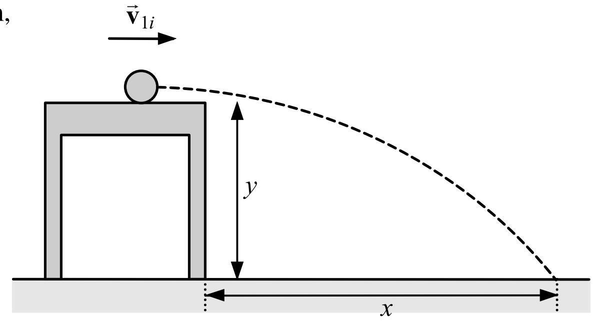

FIG. P9.62

> Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\frac{0.01}{8.68} + \frac{0.1}{68.8} + \frac{1}{263} + \frac{1}{257} + \frac{0.1}{85.3} = 1.1\%$.

***P9.63** (a) In the same symbols as in the text's Example, the original kinetic energy is $K_A = (1/2)m_1 v_{1A}^2$. The example shows that the kinetic energy immediately after latching together is $K_B = (1/2)\, m_1^2 v_{1A}^2/(m_1 + m_2)$ so the fraction of kinetic energy remaining as kinetic energy is $\boxed{K_B/K_A = m_1/(m_1 + m_2)}$

(b) $K_B/K_A = 9.6\text{ kg}/(9.6\text{ kg} + 214\text{ kg}) = \boxed{0.0429}$

(c) Momentum is conserved in the collision so momentum after divided by momentum before is $\boxed{1.00}$.

> (d) Energy is an entirely different thing from momentum. A comparison: When a photographer's single-use flashbulb flashes, a magnesium filament oxidizes. Chemical energy disappears. (Internal energy appears and light carries some energy away.) The measured mass of the flashbulb is the same before and after. It can be the same in spite of the 100% energy conversion, because energy and mass are totally different things in classical physics. In the ballistic pendulum, conversion of energy from mechanical into internal does not upset conservation of mass or conservation of momentum.

***P9.64** (a) The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the *x* direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the seat relative to the ground after your action, and the velocity of the sleigh.

(b) We substitute $v_{1f} = 8\ \text{m/s} - v_{2f}$

$$270\ \text{kg}(7.5\ \text{m/s}) = 15\ \text{kg}\left(-8\ \text{m/s} + v_{2f}\right) + (255\ \text{kg})v_{2f}$$

$$2\,025\ \text{kg}\cdot\text{m/s} = -120\ \text{kg}\cdot\text{m/s} + (270\ \text{kg})v_{2f}$$

$$v_{2f} = \frac{2\,145\ \text{m/s}}{270} = 7.94\ \text{m/s}$$

$$v_{1f} = 8\ \text{m/s} - 7.94\ \text{m/s} = 0.055\,6\ \text{m/s}$$

The final velocity of the seat is $-0.055\,6\ \text{m/s}\,\hat{\mathbf{i}}$. That of the sleigh is $7.94\ \text{m/s}\,\hat{\mathbf{i}}$.

(c) You do work on both the sleigh and the seat, to change their kinetic energy according to

$$K_i + W = K_{1f} + K_{2f}$$

$$\frac{1}{2}(270\ \text{kg})(7.5\ \text{m/s})^2 + W = \frac{1}{2}(15\ \text{kg})(0.055\,6\ \text{m/s})^2 + \frac{1}{2}(255\ \text{kg})(7.94\ \text{m/s})^2$$

$$7\,594\ \text{J} + W = 0.023\,1\ \text{J} + 8\,047\ \text{J}$$

$$W = \boxed{453\ \text{J}}$$

***P9.65** The force exerted by the spring on each block is in magnitude $|F_s| = kx = (3.85\ \text{N/m})(0.08\ \text{m}) = 0.308\ \text{N}$.

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring.

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$\frac{1}{2}(3.85\ \text{N/m})(0.08\ \text{m})^2 = \frac{1}{2}(0.25\ \text{kg})v_{1f}^2 + \frac{1}{2}(0.50\ \text{kg})v_{2f}^2$$

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = (0.25\ \text{kg})v_{1f}\left(-\hat{\mathbf{i}}\right) + (0.50\ \text{kg})v_{2f}\hat{\mathbf{i}}$$

$$v_{1f} = 2v_{2f}$$

$$0.012\,3\ \text{J} = \frac{1}{2}(0.25\ \text{kg})\left(2v_{2f}\right)^2 + \frac{1}{2}(0.50\ \text{kg})v_{2f}^2 = \frac{1}{2}(1.5\ \text{kg})v_{2f}^2$$

$$v_{2f} = \left(\frac{0.123\ \text{J}}{0.75\ \text{kg}}\right)^{1/2} = 0.128\ \text{m/s} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 0.128\ \text{m/s}\,\hat{\mathbf{i}}}$$

$$v_{1f} = 2(0.128\ \text{m/s}) = 0.256\ \text{m/s} \qquad \boxed{\vec{\mathbf{v}}_{1f} = 0.256\ \text{m/s}\left(-\hat{\mathbf{i}}\right)}$$

(b) For the lighter block, $\sum F_y = ma_y$, $\quad n - 0.25\ \text{kg}\left(9.8\ \text{m/s}^2\right) = 0$, $\quad n = 2.45\ \text{N}$, $f_k = \mu_k n = 0.1(2.45\ \text{N}) = 0.245\ \text{N}$. We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: $f_k = 0.490\ \text{N}$. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compression becomes x_f given by $|F_s| = kx$, $\quad 0.245\ \text{N} = (3.85\ \text{N/m})x_f$, $\quad 0.063\,6\ \text{m} = x_f$. Now for the energy of the lighter block as it moves to this maximum-speed point we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\,3\ \text{J} - 0.245\ \text{N}(0.08 - 0.063\,6\ \text{m}) = \frac{1}{2}(0.25\ \text{kg})v_f^2 + \frac{1}{2}(3.85\ \text{N/m})(0.063\,6\ \text{m})^2$$

$$0.012\,3\ \text{J} - 0.004\,01\ \text{J} = \frac{1}{2}(0.25\ \text{kg})v_f^2 + 0.007\,80\ \text{J}$$

$$\left(\frac{2(0.000\,515\ \text{J})}{0.25\ \text{kg}}\right)^{1/2} = v_f = 0.064\,2\ \text{m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter

$\boxed{0.064\,2\ \text{m/s}\left(-\hat{\mathbf{i}}\right)}$.

(c) For the lighter block, $f_k = 0.462(2.45\ \text{N}) = 1.13\ \text{N}$. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P9.66 The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi 1.496\times10^{11}\ \text{m}}{3.156\times10^{7}\ \text{s}} = 2.98\times10^{4}\ \text{m/s}$$

In six months the Earth reverses its direction, to undergo momentum change

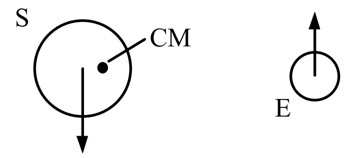

FIG. P9.66

$$m_E\left|\Delta\vec{\mathbf{v}}_E\right| = 2m_Ev_E = 2\left(5.98\times10^{24}\ \text{kg}\right)\left(2.98\times10^{4}\ \text{m/s}\right) = 3.56\times10^{29}\ \text{kg}\cdot\text{m/s}$$

Relative to the center of mass, the sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S\left|\Delta\vec{\mathbf{v}}_S\right| = 3.56\times10^{29}\ \text{kg}\cdot\text{m/s}$.

Then $\left|\Delta\vec{\mathbf{v}}_S\right| = \dfrac{3.56\times10^{29}\ \text{kg}\cdot\text{m/s}}{1.991\times10^{30}\ \text{kg}} = \boxed{0.179\ \text{m/s}}$

P9.67 (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops.

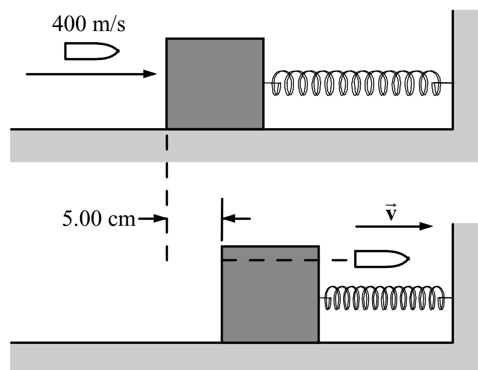

FIG. P9.67

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\ \text{N/m})\left(5.00\times10^{-2}\ \text{m}\right)^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{\left(5.00\times10^{-3}\ \text{kg}\right)(400\ \text{m/s}) - (1.00\ \text{kg})(1.50\ \text{m/s})}{5.00\times10^{-3}\ \text{kg}}$$

$$v = \boxed{100\ \text{m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}\left(5.00\times10^{-3}\ \text{kg}\right)(100\ \text{m/s})^2 - \frac{1}{2}\left(5.00\times10^{-3}\ \text{kg}\right)(400\ \text{m/s})^2 + \frac{1}{2}(900\ \text{N/m})\left(5.00\times10^{-2}\ \text{m}\right)^2$$

$\Delta E = -374$ J, or there is a mechanical energy loss of $\boxed{374\ \text{J}}$.

P9.68 (a) $\vec{\mathbf{p}}_i + \vec{\mathbf{F}}t = \vec{\mathbf{p}}_f$: $(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + (12.0\text{ N}\hat{\mathbf{i}})(5.00\text{ s}) = (3.00\text{ kg})\vec{\mathbf{v}}_f$

$$\vec{\mathbf{v}}_f = \boxed{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $\vec{\mathbf{a}} = \dfrac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t}$: $\vec{\mathbf{a}} = \dfrac{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}\right)\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(c) $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m}$: $\vec{\mathbf{a}} = \dfrac{12.0\text{ N}\hat{\mathbf{i}}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(d) $\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$: $\Delta\vec{\mathbf{r}} = \left(7.00\text{ m/s}\,\hat{\mathbf{j}}\right)(5.00\text{ s}) + \frac{1}{2}\left(4.00\text{ m/s}^2\,\hat{\mathbf{i}}\right)(5.00\text{ s})^2$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}\right)\text{ m}}$$

(e) $W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$: $W = \left(12.0\text{ N}\hat{\mathbf{i}}\right)\cdot\left(50.0\text{ m}\hat{\mathbf{i}} + 35.0\text{ m}\hat{\mathbf{j}}\right) = \boxed{600\text{ J}}$

(f) $\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\cdot\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})\left(449\text{ m}^2/\text{s}^2\right) = \boxed{674\text{ J}}$$

(g) $\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$

(h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.69 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

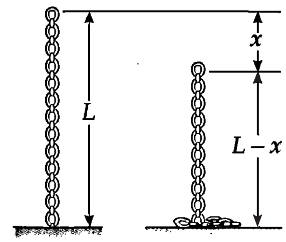

FIG. P9.69

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant*.

ANSWERS TO EVEN PROBLEMS

P9.2 (a) She moves at 4.71 m/s east. (b) 717 J (c) System momentum is conserved with the value zero. The forces on the two siblings are of equal magnitude in opposite directions. Their impulses add to zero. Their final momenta are of equal magnitude in opposite directions.

P9.4 (a) $6.00\,(-\hat{\mathbf{i}})$ m/s (b) 8.40 J (c) The original energy is in the spring. A force had to be exerted over a distance to compress the spring, transferring energy into it by work. The cord exerts force, but over no distance. (d) System momentum is conserved with the value zero. The forces on the two blocks are of equal magnitude in opposite directions. Their impulses add to zero. The final momenta of the two blocks are of equal magnitude in opposite directions.

P9.6 In trying to hang onto the child, he would have to exert a force of 6.44 kN toward the back of the car, to slow down the child's forward motion. He is not strong enough to exert so large a force. If he were solidly belted in and tied to the child, the child would exert this size force on him toward the front of the car.

P9.8 1.39 $kg \cdot m/s$ upward

P9.10 (a) 5.40 $N \cdot s$ toward the net (b) -27.0 J

P9.12 (a) 981 N·s up (b) 3.43 m/s (c) 3.83 m/s (d) 0.748 m

P9.14 16.5 N

P9.16 (a) 2.50 m/s (b) 3.75×10^4 J

P9.18 $v = \frac{4M}{m}\sqrt{g\ell}$

P9.20 (a) 4.85 m/s (b) 8.41 m

P9.22 7.94 cm

P2.24 (a) 2.24 m/s toward the right (b) No, coupling order makes no difference

P9.26 (a) Over a very short time interval, outside forces have no time to impart significant impulse—thus the interaction is a collision. The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is completely inelastic. (b) 2.88 m/s at 32.3° (c) 783 J becomes internal energy.

P9.28 No; his speed was 41.5 mi/h

P9.30 $v_Y = v_i \sin\theta;\ v_O = v_i \cos\theta$

P9.32 (a) $\sqrt{2}v_i$; $\sqrt{\frac{2}{3}}v_i$ (b) 35.3°

P9.34 $v_{Blue} = 5.89$ m/s and $v_{Green} = 7.07$ m/s

P9.36 4.67×10^6 m from the Earth's center

P9.38 (a) see the solution (b) 3.57×10^8 J

P9.40 0.063 5L

P9.42 (a) $(-2.89\,\hat{\mathbf{i}} - 1.39\,\hat{\mathbf{j}})$cm (b) $(-44.5\,\hat{\mathbf{i}} + 12.5\,\hat{\mathbf{j}})$ g·cm/s (c) $(-4.94\,\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})$ cm/s (d) $(-2.44\,\hat{\mathbf{i}} + 1.56\,\hat{\mathbf{j}})$ cm/s^2 (e) $(-220\,\hat{\mathbf{i}} + 140\,\hat{\mathbf{j}})$ μN

P9.44 (a) $-0.780\hat{\mathbf{i}}$ m/s; $1.12\hat{\mathbf{i}}$ m/s (b) $0.360\hat{\mathbf{i}}$ m/s

P9.46 (a) Yes (b) No. The work by the floor on the person is zero. (c) 103 kg·m/s up (d) Yes. You could say that it came from the planet, that gained momentum 103 kg·m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact. (e) 88.2 J (f) No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.

P9.48 (a) $[F(x_1 + x_2)/2m]^{1/2}$ (b) $\cos^{-1}[1 - F(x_1 - x_2)/2mgL]$

P9.50 (a) 787 m/s (b) 138 m/s

P9.52 see the solution

P9.54 (a) $\dfrac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$ (b) $(v_1 - v_2)\sqrt{\dfrac{m_1 m_2}{k(m_1 + m_2)}}$ (c) $\vec{\mathbf{v}}_{1f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{\mathbf{v}}_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{\mathbf{v}}_2$;

$\vec{\mathbf{v}}_{2f} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{\mathbf{v}}_1 + \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{\mathbf{v}}_2$

P9.56 (a) $R = 2580\ \text{kg}\cdot\text{m}\ (80\ \text{kg} + m)^{-1}$ (b) 16.2 m (c) 16.0 m (d) 5.87 kg (e) 32.2 m (f) $m \to 0$ (g) 0 (h) $m \to \infty$ (i) Yes, until the clone splashes down. No; we need also conservation of momentum in the collision. (j) Yes, but it is not useful to include the planet in the analysis of momentum. We use instead momentum conservation for the actor-clone system while they are in contact. (k) At a location with weaker gravity, the actor would be moving more slowly before the collision, but the clone would follow the same trajectory, moving more slowly over a longer time interval.

P9.58 (a) -0.667 m/s (b) 0.952 m

P9.60 (a) 6.81 m/s (b) 1.00 m

P9.62 (a) 6.29 m/s (b) 6.16 m/s (c) Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\dfrac{0.01}{8.68} + \dfrac{0.1}{68.8} + \dfrac{1}{263} + \dfrac{1}{257} + \dfrac{0.1}{85.3} = 1.1\%$

P9.64 (a) The mass of the sleigh plus you is 270 kg and your velocity is 7.50 m/s in the x direction. You unbolt a 15.0-kg seat and throw it back at the wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the seat relative to the ground afterward, and the velocity of the sleigh afterward. (b) 0.055 6 m/s in the $-x$ direction; 7.94 m/s in the $+x$ direction (c) 453 J

P9.66 0.179 m/s

P9.68 (a) $(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})$ m/s (b) $4.00\hat{\mathbf{i}}$ m/s^2 (c) $4.00\hat{\mathbf{i}}$ m/s^2 (d) $(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}})$ m (e) 600 J (f) 674 J (g) 674 J (h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.