28

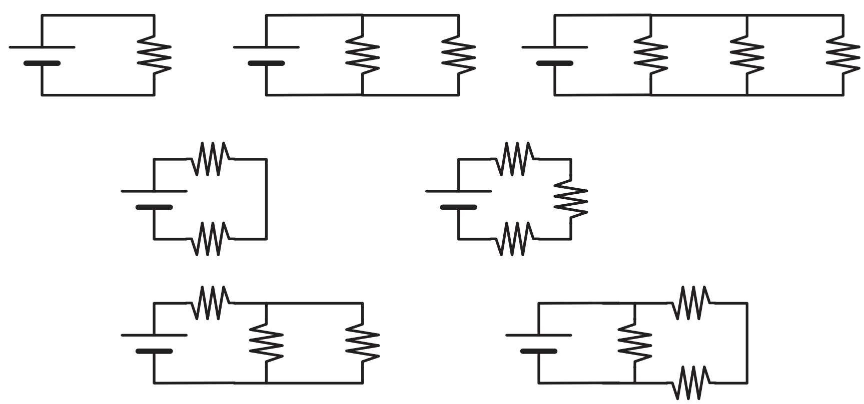
Direct Current Circuits

ANSWERS TO QUESTIONS

Q28.1 No. If there is one battery in a circuit, the current inside it will be from its negative terminal to its positive terminal. Whenever a battery is delivering energy to a circuit, it will carry current in this direction. On the other hand, when another source of emf is charging the battery in question, it will have a current pushed through it from its positive terminal to its negative terminal.

***Q28.2** The terminal potential difference is $\mathcal{E} - Ir$ where I is the current in the battery in the direction from its negative to its positive pole. So the answer to (i) is (d) and the answer to (ii) is (b). The current might be zero or an outside agent might push current backward through the battery from positive to negative terminal.

***Q28.3**



***Q28.4** Answers (b) and (d), as described by Kirchhoff's junction rule.

***Q28.5** Answer (a).

Q28.6 The whole wire is very nearly at one uniform potential. There is essentially zero *difference* in potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than the resistance through the wire between the same two points.

***Q28.7** Answer (b). Each headlight's terminals are connected to the positive and negative terminals of the battery. Each headlight can operate if the other is burned out.

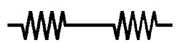
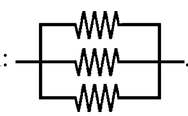
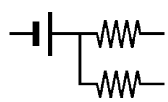
Q28.8 Answer their question with a challenge. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you patch past the first bulb to short it out, the second gets brighter.

***Q28.9** Answer (a). When the breaker trips to off, current does not go through the device.

***Q28.10** (i) For both batteries to be delivering electric energy, currents are in the direction g to a to b, h to d to c, and so e to f. Points f, g, and h are all at zero potential. Points b, c, and e are at the same higher voltage, d still higher, and a highest of all. The ranking is a > d > b = c = e > f = g = h.
(ii) The current in ef must be the sum of the other two currents. The ranking is e = f > g = a = b > h = d = c.

***Q28.11** Closing the switch removes lamp C from the circuit, decreasing the resistance seen by the battery, and so increasing the current in the battery. (i) Answer (a). (ii) Answer (d). (iii) Answer (a).
(iv) Answer (a). (v) Answer (d). (vi) Answer (a).

***Q28.12** Closing the switch lights lamp C. The action increases the battery current so it decreases the terminal voltage of the battery. (i) Answer (b). (ii) Answer (a). (iii) Answer (a).
(iv) Answer (b). (v) Answer (a). (vi) Answer (a).

Q28.13 Two runs in series:  . Three runs in parallel: . Junction of one lift and two runs:  .

Gustav Robert Kirchhoff, Professor of Physics at Heidelberg and Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving. A loop rule: the total change in altitude must be zero for any skier completing a closed path.

Q28.14 The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all. If the value of RC is small, this whole process might occupy a very short time interval.

Q28.15 The hospital maintenance worker is right. A hospital room is full of electrical grounds, including the bed frame. If your grandmother touched the faulty knob and the bed frame at the same time, she could receive quite a jolt, as there would be a potential difference of 120 V across her. If the 120 V is DC, the shock could send her into ventricular fibrillation, and the hospital staff could use the defibrillator you read about in Chapter 26. If the 120 V is AC, which is most likely, the current could produce external and internal burns along the path of conduction. Likely no one got a shock from the radio back at home because her bedroom contained no electrical grounds—no conductors connected to zero volts. Just like the bird in Question 28.6, granny could touch the "hot" knob without getting a shock so long as there was no path to ground to supply a potential difference across her. A new appliance in the bedroom or a flood could make the radio lethal. Repair it or discard it. Enjoy the news from Lake Wobegon on the new plastic radio.

Q28.16 Both 120-V and 240-V lines can deliver injurious or lethal shocks, but there is a somewhat better safety factor with the lower voltage. To say it a different way, the insulation on a 120-V line can be thinner. On the other hand, a 240-V device carries less current to operate a device with the same power, so the conductor itself can be thinner. Finally, as we will see in Chapter 33, the last step-down transformer can also be somewhat smaller if it has to go down only to 240 volts from the high voltage of the main power line.

SOLUTIONS TO PROBLEMS

Section 28.1 **Electromotive Force**

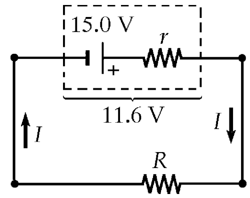


FIG. P28.1

P28.1 (a) $\mathcal{P} = \dfrac{(\Delta V)^2}{R}$

becomes $\quad 20.0 \text{ W} = \dfrac{(11.6 \text{ V})^2}{R}$

so $\quad R = \boxed{6.73\ \Omega}$

(b) $\Delta V = IR$

so $\quad 11.6 \text{ V} = I(6.73\ \Omega)$

and $\quad I = 1.72 \text{ A}$

$\mathcal{E} = IR + Ir$

so $\quad 15.0 \text{ V} = 11.6 \text{ V} + (1.72 \text{ A})r$

$r = \boxed{1.97\ \Omega}$

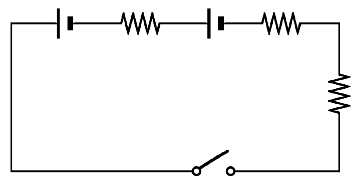


FIG. P28.2

P28.2 The total resistance is $R = \dfrac{3.00 \text{ V}}{0.600 \text{ A}} = 5.00\ \Omega$.

(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\dfrac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$

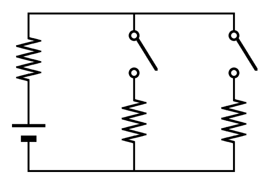


FIG. P28.3

P28.3 (a) Here $\mathcal{E} = I(R + r)$, so $I = \dfrac{\mathcal{E}}{R + r} = \dfrac{12.6 \text{ V}}{(5.00\ \Omega + 0.0800\ \Omega)} = 2.48 \text{ A}$.

Then, $\Delta V = IR = (2.48 \text{ A})(5.00\ \Omega) = \boxed{12.4 \text{ V}}$.

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

Then, $\quad I_1 = I_2 + 35.0 \text{ A}$, and $\mathcal{E} - I_1 r - I_2 r = 0$

so $\quad \mathcal{E} = (I_2 + 35.0 \text{ A})(0.0800\ \Omega) + I_2(5.00\ \Omega) = 12.6 \text{ V}$

giving $\quad I_2 = 1.93 \text{ A}$

Thus, $\quad \Delta V_2 = (1.93 \text{ A})(5.00\ \Omega) = \boxed{9.65 \text{ V}}$

***P28.4** (a) At maximum power transfer, $r = R$. Equal powers are delivered to r and R. The efficiency is $\boxed{50.0\%}$.

(b) For maximum fractional energy transfer to R, we want zero energy absorbed by r, so we want $r = \boxed{0}$.

(c) High efficiency. The electric company's economic interest is to minimize internal energy production in its power lines, so that it can sell a large fraction of the energy output of its generators to the customers.

(d) High power transfer. Energy by electric transmission is so cheap compared to the sound system that she does not spend extra money to buy an efficient amplifier.

Section 28.2 **Resistors in Series and Parallel**

P28.5 (a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$

$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

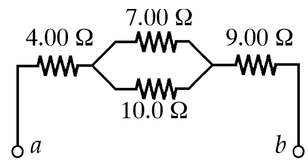


FIG. P28.5

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for $4.00\ \Omega$, $9.00\ \Omega$ resistors

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

$8.18\ \text{V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\ \text{A}}$ for $7.00\ \Omega$ resistor

$8.18\ \text{V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\ \text{A}}$ for $10.0\ \Omega$ resistor

***P28.6** (a) The conductors in the cord have resistance. There is a potential difference across each when current is flowing. The potential difference applied to the light bulb is less than 120 V, so it will carry less current than it is designed to, and will operate at lower power than 75 W.

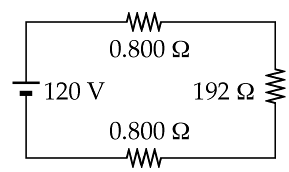


FIG. P28.6

(b) If the temperature of the bulb does not change much between the design operating point and the actual operating point, we can take the resistance of the filament as constant.
For the bulb in use as intended,

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{75.0\ \text{W}}{120\ \text{V}} = 0.625\ \text{A}$$

and $$R = \frac{\Delta V}{I} = \frac{120\ \text{V}}{0.625\ \text{A}} = 192\ \Omega$$

Now, presuming the bulb resistance is unchanged,

$$I = \frac{120\ \text{V}}{193.6\ \Omega} = 0.620\ \text{A}$$

Across the bulb is $\Delta V = IR = 192\ \Omega(0.620\ \text{A}) = 119\ \text{V}$

so its power is $\mathcal{P} = I\Delta V = 0.620\ \text{A}(119\ \text{V}) = \boxed{73.8\ \text{W}}$

P28.7 If we turn the given diagram on its side, we find that it is the same as figure (a). The $20.0\ \Omega$ and $5.00\ \Omega$ resistors are in series, so the first reduction is shown in (b). In addition, since the $10.0\ \Omega$, $5.00\ \Omega$, and $25.0\ \Omega$ resistors are then in parallel, we can solve for their equivalent resistance as:

$$R_{\text{eq}} = \frac{1}{\left(\dfrac{1}{10.0\ \Omega} + \dfrac{1}{5.00\ \Omega} + \dfrac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega$$

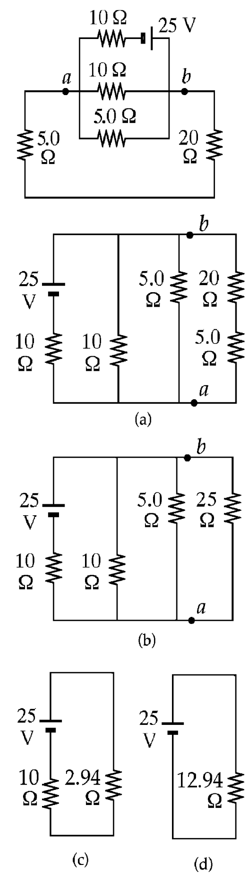


FIG. P28.7

This is shown in figure (c), which in turn reduces to the circuit shown in figure (d).

Next, we work backwards through the diagrams applying $I = \dfrac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The $12.94\ \Omega$ resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In figure (c), this 1.93 A goes through the $2.94\ \Omega$ equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From figure (b), we see that this potential difference is the same across ΔV_{ab}, the 10 Ω resistor, and the 5.00 Ω resistor.

Thus we have first found the answer to part (b), which is $\Delta V_{ab} = \boxed{5.68\ \text{V}}$.

(a) Since the current through the $20.0\ \Omega$ resistor is also the current through the $25.0\ \Omega$ line *ab*,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

P28.8 We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is $1.00\ \text{M}\Omega + R_{\text{shoes}}$. The current through both resistors is $\dfrac{50.0\ \text{V}}{1.00\ \text{M}\Omega + R_{\text{shoes}}}$. The voltmeter displays

$$\Delta V = I(1.00\ \text{M}\Omega) = \frac{50.0\ \text{V}(1.00\ \text{M}\Omega)}{1.00\ \text{M}\Omega + R_{\text{shoes}}} = \Delta V$$

(a) We solve to obtain $50.0\ \text{V}(1.00\ \text{M}\Omega) = \Delta V(1.00\ \text{M}\Omega) + \Delta V\left(R_{\text{shoes}}\right)$

$$R_{\text{shoes}} = \frac{1.00\ \text{M}\Omega(50.0 - \Delta V)}{\Delta V}$$

(b) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$$\frac{50.0\ \text{V}}{1.00\ \text{M}\Omega} = 50.0\ \mu\text{A} \qquad \boxed{\text{The current will never exceed } 50\ \mu\text{A.}}$$

P28.9 (a) Since all the current in the circuit must pass through the series 100 Ω resistor, $\mathcal{P} = I^2 R$

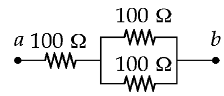


FIG. P28.9

$$\mathcal{P}_{\max} = RI_{\max}^2$$

so $$I_{\max} = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{25.0\text{ W}}{100\ \Omega}} = 0.500\text{ A}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{\text{eq}} I_{\max} = \boxed{75.0\text{ V}}$$

(b) $\mathcal{P}_1 = I\Delta V = (0.500\text{ A})(75.0\text{ V}) = \boxed{37.5\text{ W}}$ total power

$$\mathcal{P}_1 = \boxed{25.0\text{ W}}$$

$$\mathcal{P}_2 = \mathcal{P}_3 = RI^2 (100\ \Omega)(0.250\text{ A})^2 = \boxed{6.25\text{ W}}$$

P28.10 Using 2.00-Ω, 3.00-Ω, and 4.00-Ω resistors, there are 7 series, 4 parallel, and 6 mixed combinations:

Series		Parallel	Mixed
2.00 Ω	6.00 Ω	0.923 Ω	1.56 Ω
3.00 Ω	7.00 Ω	1.20 Ω	2.00 Ω
4.00 Ω	9.00 Ω	1.33 Ω	2.22 Ω
5.00 Ω		1.71 Ω	3.71 Ω
			4.33 Ω
			5.20 Ω

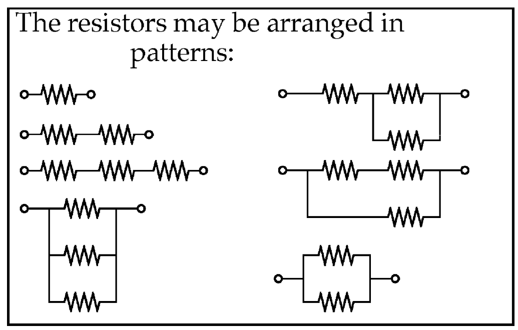


FIG. P28.10

P28.11 *When S is open*, R_1, R_2, R_3 are in series with the battery. Thus:

$$R_1 + R_2 + R_3 = \frac{6\text{ V}}{10^{-3}\text{ A}} = 6\text{ k}\Omega \tag{1}$$

When S is closed in position 1, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus:

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\text{ V}}{1.2\times 10^{-3}\text{ A}} = 5\text{ k}\Omega \tag{2}$$

When S is closed in position 2, R_1 and R_2 are in series with the battery. R_3 is shorted. Thus:

$$R_1 + R_2 = \frac{6\text{ V}}{2\times 10^{-3}\text{ A}} = 3\text{ k}\Omega \tag{3}$$

From (1) and (3): $R_3 = 3\text{ k}\Omega$.

Subtract (2) from (1): $R_2 = 2\text{ k}\Omega$.

From (3): $R_1 = 1\text{ k}\Omega$.

Answers: $\boxed{R_1 = 1.00\text{ k}\Omega,\ R_2 = 2.00\text{ k}\Omega,\ R_3 = 3.00\text{ k}\Omega}$.

P28.12 Denoting the two resistors as x and y,

$$x+y=690, \quad \text{and} \quad \frac{1}{150}=\frac{1}{x}+\frac{1}{y}$$

$$\frac{1}{150}=\frac{1}{x}+\frac{1}{690-x}=\frac{(690-x)+x}{x(690-x)}$$

$$x^2-690x+103\,500=0$$

$$x=\frac{690\pm\sqrt{(690)^2-414\,000}}{2}$$

$$x=\boxed{470\ \Omega} \qquad y=\boxed{220\ \Omega}$$

***P28.13** The resistance between a and b $\boxed{\text{decreases}}$. Closing the switch opens a new path with resistance of only 20 Ω. The original resistance is $R+\dfrac{1}{\dfrac{1}{90+10}+\dfrac{1}{10+90}}=R+50\ \Omega$.

The final resistance is $R+\dfrac{1}{\dfrac{1}{90}+\dfrac{1}{10}}+\dfrac{1}{\dfrac{1}{10}+\dfrac{1}{90}}=R+18\ \Omega$.

We require $R+50\ \Omega=2(R+18\ \Omega)$ so $R=\boxed{14.0\ \Omega}$

***P28.14** (a) The resistors 2, 3, and 4 can be combined to a single $2R$ resistor. This is in series with resistor 1, with resistance R, so the equivalent resistance of the whole circuit is $3R$. In series, potential difference is shared in proportion to the resistance, so resistor 1 gets $\frac{1}{3}$ of the battery voltage and the 2-3-4 parallel combination gets $\frac{2}{3}$ of the battery voltage. This is the potential difference across resistor 4, but resistors 2 and 3 must share this voltage. In this branch $\frac{1}{3}$ goes to 2 and $\frac{2}{3}$ to 3. The ranking by potential difference is $\boxed{\Delta V_4>\Delta V_3>\Delta V_1>\Delta V_2}$.

(b) Based on the reasoning above the potential differences are $\boxed{\Delta V_1=\dfrac{\mathcal{E}}{3},\ \Delta V_2=\dfrac{2\mathcal{E}}{9},\ \Delta V_3=\dfrac{4\mathcal{E}}{9},\ \Delta V_4=\dfrac{2\mathcal{E}}{3}}$.

(c) All the current goes through resistor 1, so it gets the most. The current then splits at the parallel combination. Resistor 4 gets more than half, because the resistance in that branch is less than in the other branch. Resistors 2 and 3 have equal currents because they are in series. The ranking by current is $\boxed{I_1>I_4>I_2=I_3}$.

(d) Resistor 1 has a current of I. Because the resistance of 2 and 3 in series is twice that of resistor 4, twice as much current goes through 4 as through 2 and 3. The currents through the resistors are $\boxed{I_1=I,\ I_2=I_3=\dfrac{I}{3},\ I_4=\dfrac{2I}{3}}$.

(e) Increasing resistor 3 increases the equivalent resistance of the entire circuit. The current in the battery, which is the current through resistor 1, decreases. This decreases the potential difference across resistor 1, increasing the potential difference across the parallel combination. With a larger potential difference the current through resistor 4 is increased. With

continued on next page

more current through 4, and less in the circuit to start with, the current through resistors 2 and 3 must decrease. To summarize, $\boxed{I_4 \text{ increases and } I_1,\ I_2, \text{ and } I_3 \text{ decrease}}$.

(f) If resistor 3 has an infinite resistance it blocks any current from passing through that branch, and the circuit effectively is just resistor 1 and resistor 4 in series with the battery. The circuit now has an equivalent resistance of $4R$. The current in the circuit drops to $\frac{3}{4}$ of the original current because the resistance has increased by $\frac{4}{3}$. All this current passes through resistors 1 and 4, and none passes through 2 or 3. Therefore $\boxed{I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}}$.

P28.15 $R_p = \left(\dfrac{1}{3.00} + \dfrac{1}{1.00}\right)^{-1} = 0.750\ \Omega$

$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$

$I_{\text{battery}} = \dfrac{\Delta V}{R_s} = \dfrac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$

$\mathcal{P}_2 = I^2 R$: $\quad \mathcal{P}_2 = (2.67\ \text{A})^2 (2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\ \text{W}}$ in $2.00\ \Omega$

$\mathcal{P}_4 = (2.67\ \text{A})^2 (4.00\ \text{A}) = \boxed{28.4\ \text{W}}$ in $4.00\ \Omega$

$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V},$

$\Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$

$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V} \left(= \Delta V_3 = \Delta V_1\right)$

$\mathcal{P}_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}$ in $3.00\ \Omega$

$\mathcal{P}_1 = \dfrac{(\Delta V_1)}{R_1} = \dfrac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}$ in $1.00\ \Omega$

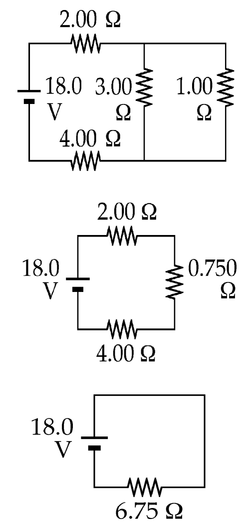


FIG. P28.15

Section 28.3 **Kirchhoff's Rules**

P28.16 $+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$

$5.00 = 7.00\ I_1$ so $\boxed{I_1 = 0.714\ \text{A}}$

$I_3 = I_1 + I_2 = 2.00\ \text{A}$

$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29\ \text{A}}$

$+\mathcal{E} - 2.00(1.29) - 5.00(2.00) = 0$ $\quad \boxed{\mathcal{E} = 12.6\ \text{V}}$

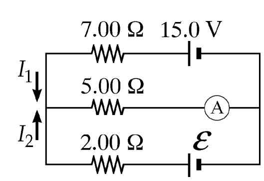


FIG. P28.16

P28.17 We name currents I_1, I_2, and I_3 as shown.

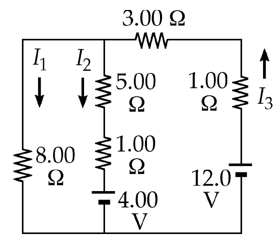


FIG. P28.17

From Kirchhoff's current rule, $I_3 = I_1 + I_2$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$$
$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0 \qquad (8.00)I_1 = 4.00 + (6.00)I_2$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

$$I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A} \qquad \text{Then} \qquad I_2 = 1.33(0.846\text{ A}) - 0.667$$

and $\quad I_3 = I_1 + I_2 \qquad$ give $\quad \boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$

All currents are in the directions indicated by the arrows in the circuit diagram.

P28.18 The solution figure is shown to the right.

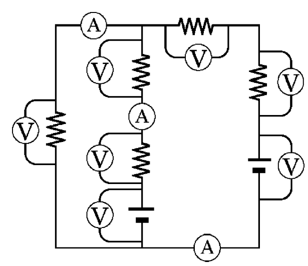


FIG. P28.18

***P28.19** We use the results of Problem 28.17.

(a) By the 4.00-V battery: $\Delta U = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})120\text{ s} = \boxed{-222\text{ J}}$

By the 12.0-V battery: $(12.0\text{ V})(1.31\text{ A})120\text{ s} = \boxed{1.88\text{ kJ}}$

(b) By the 8.00-Ω resistor: $I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)120\text{ s} = \boxed{687\text{ J}}$

By the 5.00-Ω resistor: $(0.462\text{ A})^2(5.00\ \Omega)120\text{ s} = \boxed{128\text{ J}}$

By the 1.00-Ω resistor: $(0.462\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{25.6\text{ J}}$

By the 3.00-Ω resistor: $(1.31\text{ A})^2(3.00\ \Omega)120\text{ s} = \boxed{616\text{ J}}$

By the 1.00-Ω resistor: $(1.31\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{205\text{ J}}$

(c) $-222\text{ J} + 1.88\text{ kJ} = \boxed{1.66\text{ kJ}}$ from chemical to electrically transmitted. Like a child counting his lunch money twice, we can count the same energy again, $687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$, as it is transformed from electrically transmitted to internal. The net energy transformation is $\boxed{\text{from chemical to internal}}$.

***P28.20** (a) The first equation represents Kirchhoff's loop theorem. We choose to think of it as describing a clockwise trip around the left-hand loop in a circuit; see Figure (a). For the right-hand loop see Figure (b). The junctions must be between the 5.8 V and the $370\ \Omega$ and between the $370\ \Omega$ and the $150\ \Omega$. Then we have Figure (c). This is consistent with the third equation,

$$I_1 + I_3 - I_2 = 0$$
$$I_2 = I_1 + I_3$$

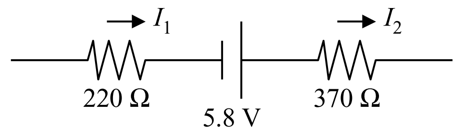


Figure (a)

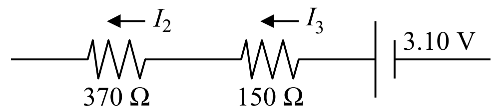


Figure (b)

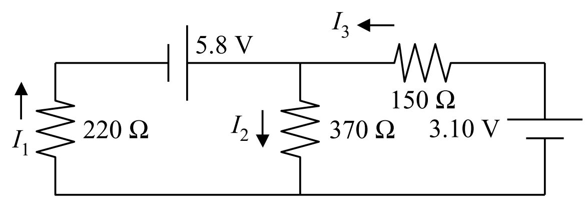


Figure (c)

FIG. P28.20

(b) We substitute:

$$-220I_1 + 5.8 - 370I_1 - 370I_3 = 0$$
$$+370I_1 + 370I_3 + 150I_3 - 3.1 = 0$$

Next

$$I_3 = \frac{5.8 - 590I_1}{370}$$
$$370I_1 + \frac{520}{370}(5.8 - 590I_1) - 3.1 = 0$$
$$370I_1 + 8.15 - 829I_1 - 3.1 = 0$$
$$I_1 = \frac{5.05\text{ V}}{459\ \Omega} = \boxed{\text{11.0 mA in the 220-}\Omega\text{ resistor and out of the positive pole of the 5.8-V battery}}$$
$$I_3 = \frac{5.8 - 590(0.011\,0)}{370} = -1.87\text{ mA}$$

$\boxed{\text{The current is 1.87 mA in the 150-}\Omega\text{ resistor and out of the negative pole of the 3.1-V battery.}}$

$$I_2 = 11.0 - 1.87 = \boxed{9.13\text{ mA in the 370-}\Omega\text{ resistor}}$$

***P28.21** Let I_6 represent the current in the ammeter and the top 6-Ω resistor. The bottom 6-Ω resistor has the same potential difference across it, so it carries an equal current. For the top loop we have

$$6\text{ V} - 10\ \Omega\, I_{10} - 6\ \Omega\, I_6 = 0$$

For the bottom loop, $4.5 - 5\,I_5 - 6\,I_6 = 0$.

For the junctions on the left side, taken together, $+I_{10} + I_5 - I_6 - I_6 = 0$.

We eliminate $I_{10} = 0.6 - 0.6\,I_6$ and $I_5 = 0.9 - 1.2\,I_6$ by substitution:

$0.6 - 0.6\,I_6 + 0.9 - 1.2\,I_6 - 2\,I_6 = 0 \quad I_6 = 1.5/3.8 = \boxed{0.395\text{ A}}$

The loop theorem for the little loop containing the voltmeter gives

$+6\text{ V} - \Delta V - 4.5\text{ V} = 0 \qquad \Delta V = \boxed{1.50\text{ V}}$

P28.22 Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

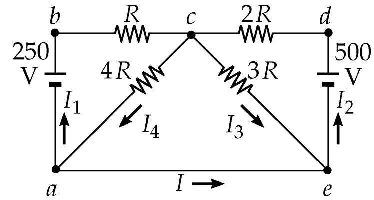


Figure (a)

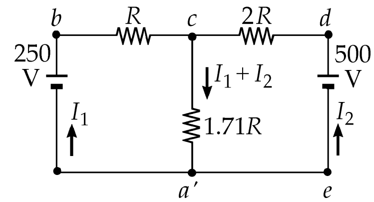


Figure (b)

FIG. P28.22

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $$(1.71R)I_1 + (3.71R)I_2 = 500$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\text{ mA}$$

and $$I_2 = 130.0\text{ mA}$$

From Figure (b), $$V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}$$

Thus, from Figure (a), $$I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4\,000\ \Omega} = 60.0\text{ mA}$$

Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA}$$

or $$I = \boxed{50.0\text{ mA from point } a \text{ to point } e}$$

P28.23 Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are

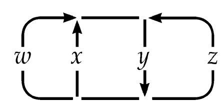


FIG. P28.23

$$-200w - 40.0 + 80.0x = 0$$
$$-80.0x + 40.0 + 360 - 20.0y = 0$$
$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate y by substitution. $$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate x. $$\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$$

Eliminate $z = 17.5 - 13.5w$ to obtain $$430 - 70.0w - 1\,575 + 1\,215w = 0$$

$$w = \frac{70.0}{70.0} = \boxed{1.00\text{ A upward in } 200\ \Omega}$$

Now $$z = \boxed{4.00\text{ A upward in } 70.0\ \Omega}$$

$$x = \boxed{3.00\text{ A upward in } 80.0\ \Omega}$$

$$y = \boxed{8.00\text{ A downward in } 20.0\ \Omega}$$

and for the 200 Ω, $$\Delta V = IR = (1.00\text{ A})(200\ \Omega) = \boxed{200\text{ V}}$$

P28.24 Using Kirchhoff's rules,

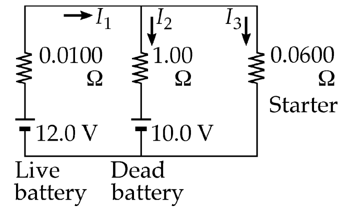


FIG. P28.24

$$12.0-(0.010\,0)I_1-(0.060\,0)I_3=0$$
$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

and $\quad I_1=I_2+I_3$

$$12.0-(0.010\,0)I_2-(0.070\,0)I_3=0$$
$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

Solving simultaneously,

$I_2=\boxed{0.283\text{ A downward}}$ in the dead battery

and $\quad I_3=\boxed{171\text{ A downward}}$ in the starter

The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

P28.25 We name the currents I_1, I_2, and I_3 as shown.

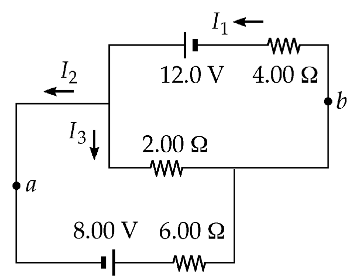


FIG. P28.25

(a) $\quad I_1=I_2+I_3$

Counterclockwise around the top loop,

$$12.0\text{ V}-(2.00\ \Omega)I_3-(4.00\ \Omega)I_1=0$$

Traversing the bottom loop,

$$8.00\text{ V}-(6.00\ \Omega)I_2+(2.00\ \Omega)I_3=0$$

$$I_1=3.00-\frac{1}{2}I_3,\quad I_2=\frac{4}{3}+\frac{1}{3}I_3,\ \text{and}\ \boxed{I_3=909\text{ mA}}$$

(b) $\quad V_a-(0.909\text{ A})(2.00\ \Omega)=V_b$

$$V_b-V_a=\boxed{-1.82\text{ V}}$$

P28.26

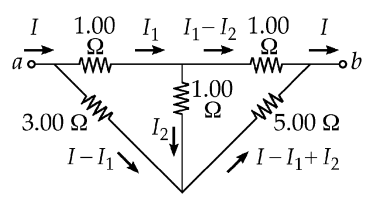


FIG. P28.26

$$\Delta V_{ab}=(1.00)I_1+(1.00)(I_1-I_2)$$
$$\Delta V_{ab}=(1.00)I_1+(1.00)I_2+(5.00)(I-I_1+I_2)$$
$$\Delta V_{ab}=(3.00)(I-I_1)+(5.00)(I-I_1+I_2)$$

Let $I=1.00$ A, $I_1=x$, and $I_2=y$.

Then, the three equations become:

$$\Delta V_{ab}=2.00x-y,\ \text{or}\ y=2.00x-\Delta V_{ab}$$
$$\Delta V_{ab}=-4.00x+6.00y+5.00$$

and $\Delta V_{ab}=8.00-8.00x+5.00y$

Substituting the first into the last two gives:

$$7.00\Delta V_{ab}=8.00x+5.00\quad\text{and}\quad 6.00\Delta V_{ab}=2.00x+8.00$$

Solving these simultaneously yields $\Delta V_{ab}=\frac{27}{17}$ V.

Then, $R_{ab}=\dfrac{\Delta V_{ab}}{I}=\dfrac{\frac{27}{17}\text{ V}}{1.00\text{ A}}$ $\quad$ or $\quad \boxed{R_{ab}=\frac{27}{17}\ \Omega}$

Section 28.4 ***RC* Circuits**

P28.27 (a) $RC = \left(1.00 \times 10^6\ \Omega\right)\left(5.00 \times 10^{-6}\ \text{F}\right) = \boxed{5.00\ \text{s}}$

(b) $Q = C\mathcal{E} = \left(5.00 \times 10^{-6}\ \text{C}\right)(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$

(c) $I(t) = \dfrac{\mathcal{E}}{R} e^{-t/RC} = \left(\dfrac{30.0}{1.00 \times 10^6}\right)\exp\left[\dfrac{-10.0}{\left(1.00 \times 10^6\right)\left(5.00 \times 10^{-6}\right)}\right]$

$= \boxed{4.06\ \mu\text{A}}$

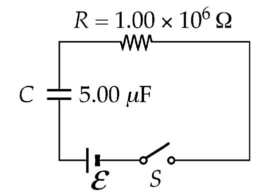


FIG. P28.27

P28.28 The potential difference across the capacitor $\Delta V(t) = \Delta V_{\text{max}}\left(1 - e^{-t/RC}\right)$

Using 1 Farad $= 1\ \text{s}/\Omega$, $4.00\ \text{V} = (10.0\ \text{V})\left[1 - e^{-(3.00\ \text{s})/\left[R\left(10.0 \times 10^{-6}\ \text{s}/\Omega\right)\right]}\right]$

Therefore, $0.400 = 1.00 - e^{-\left(3.00 \times 10^5\ \Omega\right)/R}$

Or $e^{-\left(3.00 \times 10^5\ \Omega\right)/R} = 0.600$

Taking the natural logarithm of both sides, $-\dfrac{3.00 \times 10^5\ \Omega}{R} = \ln(0.600)$

and $R = -\dfrac{3.00 \times 10^5\ \Omega}{\ln(0.600)} = +5.87 \times 10^5\ \Omega = \boxed{587\ \text{k}\Omega}$

P28.29 (a) $I(t) = -I_0 e^{-t/RC}$

$$I_0 = \frac{Q}{RC} = \frac{5.10 \times 10^{-6}\ \text{C}}{(1\,300\ \Omega)\left(2.00 \times 10^{-9}\ \text{F}\right)} = 1.96\ \text{A}$$

$$I(t) = -(1.96\ \text{A})\exp\left[\frac{-9.00 \times 10^{-6}\ \text{s}}{(1\,300\ \Omega)\left(2.00 \times 10^{-9}\ \text{F}\right)}\right] = \boxed{-61.6\ \text{mA}}$$

(b) $q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\dfrac{-8.00 \times 10^{-6}\ \text{s}}{(1\ 300\ \Omega)\left(2.00 \times 10^{-9}\ \text{F}\right)}\right] = \boxed{0.235\ \mu\text{C}}$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\ \text{A}}$.

P28.30 We are to calculate

$$\int_0^\infty e^{-2t/RC}\,dt = -\frac{RC}{2}\int_0^\infty e^{-2t/RC}\left(-\frac{2dt}{RC}\right) = -\frac{RC}{2}e^{-2t/RC}\Big|_0^\infty = -\frac{RC}{2}\left[e^{-\infty} - e^0\right] = -\frac{RC}{2}[0-1] = \boxed{+\frac{RC}{2}}$$

P28.31 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.

$V_L = 8.00$ V because of voltage divider:

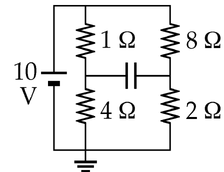


FIG. P28.31(a)

$$I_L = \frac{10.0\text{ V}}{5.00\ \Omega} = 2.00\text{ A}$$

$$V_L = 10.0\text{ V} - (2.00\text{ A})(1.00\ \Omega) = 8.00\text{ V}$$

Likewise, $$V_R = \left(\frac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right)(10.0\text{ V}) = 2.00\text{ V}$$

or $$I_R = \frac{10.0\text{ V}}{10.0\ \Omega} = 1.00\text{ A}$$

$$V_R = (10.0\text{ V}) - (8.00\ \Omega)(1.00\text{ A}) = 2.00\text{ V}$$

Therefore, $$\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\text{ V}}$$

(b) Redraw the circuit $$R = \frac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$$

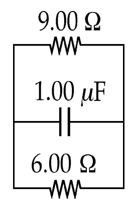


FIG. P28.31(b)

$$RC = 3.60 \times 10^{-6}\text{ s}$$

and $$e^{-t/RC} = \frac{1}{10}$$

so $$t = RC \ln 10 = \boxed{8.29\ \mu\text{s}}$$

P28.32 (a) $\tau = RC = (1.50 \times 10^5\ \Omega)(10.0 \times 10^{-6}\text{ F}) = \boxed{1.50\text{ s}}$

(b) $\tau = (1.00 \times 10^5\ \Omega)(10.0 \times 10^{-6}\text{ F}) = \boxed{1.00\text{ s}}$

(c) The battery carries current $$\frac{10.0\text{ V}}{50.0 \times 10^3\ \Omega} = 200\ \mu\text{A}$$

The 100 kΩ carries current of magnitude $$I = I_0 e^{-t/RC} = \left(\frac{10.0\text{ V}}{100 \times 10^3\ \Omega}\right) e^{-t/1.00\text{ s}}$$

So the switch carries downward current $$\boxed{200\ \mu\text{A} + (100\ \mu\text{A}) e^{-t/1.00\text{ s}}}$$

Section 28.5 **Electrical Meters**

P28.33 $\Delta V = I_g r_g = (I - I_g) R_p$, or $R_p = \dfrac{I_g r_g}{(I - I_g)} = \dfrac{I_g (60.0\ \Omega)}{(I - I_g)}$

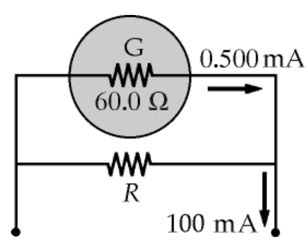


FIG. P28.33

Therefore, to have $I = 0.100\text{ A} = 100\text{ mA}$ when $I_g = 0.500\text{ mA}$:

$$R_p = \frac{(0.500\text{ mA})(60.0\ \Omega)}{99.5\text{ mA}} = \boxed{0.302\ \Omega}$$

P28.34 *Ammeter:* $I_g r = (0.500\text{ A} - I_g)(0.220\ \Omega)$

or $I_g(r + 0.220\ \Omega) = 0.110\text{ V}$ (1)

Voltmeter: $2.00\text{ V} = I_g(r + 2\,500\ \Omega)$ (2)

Solve (1) and (2) simultaneously to find:

$$I_g = \boxed{0.756\text{ mA}} \quad\text{and}\quad r = \boxed{145\ \Omega}$$

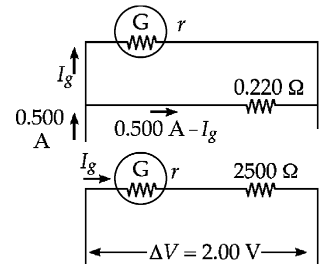


FIG. P28.34

P28.35 Series Resistor → Voltmeter

$\Delta V = IR$: $25.0 = 1.50 \times 10^{-3}(R_s + 75.0)$

Solving, $\boxed{R_s = 16.6\text{ k}\Omega}$

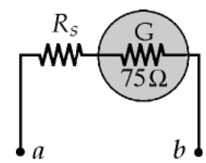


FIG. P28.35

***P28.36** (a) In Figure (a), the emf sees an equivalent resistance of 200.00 Ω.

$$I = \frac{6.000\,0\text{ V}}{200.00\ \Omega} = \boxed{0.030\,000\text{ A}}$$

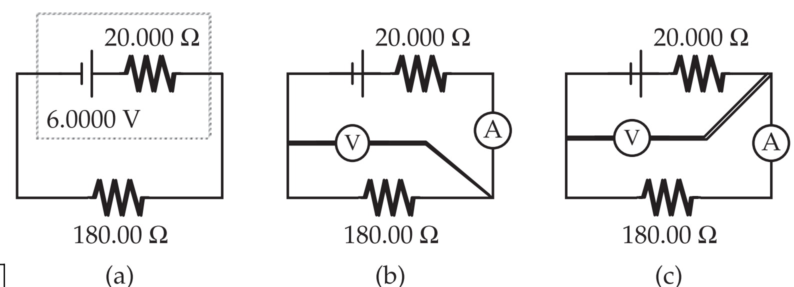


FIG. P28.36

The terminal potential difference is $\Delta V = IR = (0.030\,000\text{ A})(180.00\ \Omega) = \boxed{5.400\,0\text{ V}}$

(b) In Figure (b), $R_{eq} = \left(\frac{1}{180.00\ \Omega} + \frac{1}{20\,000\ \Omega}\right)^{-1} = 178.39\ \Omega$

The equivalent resistance across the emf is $178.39\ \Omega + 0.500\,00\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The ammeter reads $I = \frac{\mathcal{E}}{R} = \frac{6.000\,0\text{ V}}{198.89\ \Omega} = \boxed{0.030\,167\text{ A}}$

and the voltmeter reads $\Delta V = IR = (0.030\,167\text{ A})(178.39\ \Omega) = \boxed{5.381\,6\text{ V}}$

(c) In Figure (c), $\left(\frac{1}{180.50\ \Omega} + \frac{1}{20\,000\ \Omega}\right)^{-1} = 178.89\ \Omega$

Therefore, the emf sends current through $R_{tot} = 178.89\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The current through the battery is but not all of this goes through the ammeter. $I = \frac{6.000\,0\text{ V}}{198.89\ \Omega} = 0.030\,168\text{ A}$

The voltmeter reads $\Delta V = IR = (0.030\,168\text{ A})(178.89\ \Omega) = \boxed{5.396\,6\text{ V}}$

The ammeter measures current $I = \frac{\Delta V}{R} = \frac{5.396\,6\text{ V}}{180.50\ \Omega} = \boxed{0.029\,898\text{ A}}$.

continued on next page

(d) Both circuits are good enough for some measurements. The connection in Figure (c) gives data leading to value of resistance that is too high by only about 0.3%. Its value is more accurate than the value, 0.9% too low, implied by the data from the circuit in part (b).

Section 28.6 Household Wiring and Electrical Safety

P28.37 (a) $\mathcal{P} = I\Delta V$: So for the Heater, $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,500 \text{ W}}{120 \text{ V}} = \boxed{12.5 \text{ A}}$

For the Toaster, $I = \dfrac{750 \text{ W}}{120 \text{ V}} = \boxed{6.25 \text{ A}}$

And for the Grill, $I = \dfrac{1\,000 \text{ W}}{120 \text{ V}} = \boxed{8.33 \text{ A}}$

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1 \text{ A}}$

The current draw is greater than 25.0 amps, so a circuit with this circuit breaker would not be sufficient.

P28.38 (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 mm^2 and thickness 1 mm. Its resistance is

$$R = \frac{\rho \ell}{A} \approx \frac{(10^{13} \ \Omega \cdot \text{m})(10^{-3} \text{ m})}{4 \times 10^{-6} \text{ m}^2} \approx 2 \times 10^{15} \ \Omega$$

The current will be driven by 120 V through total resistance (series)

$$2 \times 10^{15} \ \Omega + 10^4 \ \Omega + 2 \times 10^{15} \ \Omega \approx 5 \times 10^{15} \ \Omega$$

It is: $I = \dfrac{\Delta V}{R} \sim \dfrac{120 \text{ V}}{5 \times 10^{15} \ \Omega} \boxed{\sim 10^{-14} \text{ A}}$.

(b) The resistors form a voltage divider, with the center of your hand at potential $\dfrac{V_h}{2}$, where V_h is the potential of the "hot" wire. The potential difference between your finger and thumb is $\Delta V = IR \sim (10^{-14} \text{ A})(10^4 \ \Omega) \sim 10^{-10} \text{ V}$. So the points where the rubber meets your fingers are at potentials of

$$\boxed{\sim \frac{V_h}{2} + 10^{-10} \text{ V}} \quad \text{and} \quad \boxed{\sim \frac{V_h}{2} - 10^{-10} \text{ V}}$$

Additional Problems

***P28.39** Several seconds is many time constants, so the capacitor is fully charged and (d) the current in its branch is $\boxed{\text{zero}}$.

Center loop: $+8 \text{ V} + 3 \ \Omega \, I_2 - 5 \ \Omega \, I_1 = 0$

Right loop: $+4 \text{ V} - 3 \ \Omega \, I_2 - 5 \ \Omega \, I_3 = 0$

Top junction: $+I_1 + I_2 - I_3 = 0$

Now we will eliminate $I_1 = 1.6 + 0.6 I_2$ and $I_3 = 0.8 - 0.6 I_2$

by substitution: $1.6 + 0.6 I_2 + I_2 - 0.8 + 0.6 I_2 = 0$ Then $I_2 = -0.8/2.2 = -0.3636$

So (b) $\boxed{\text{the current in } 3 \ \Omega \text{ is } 0.364 \text{ A down}}$.

Now (a) $I_3 = 0.8 - 0.6(-0.364) = \boxed{1.02 \text{ A down in } 4 \text{ V and in } 5 \ \Omega}$.

(c) $I_1 = 1.6 + 0.6(-0.364) = \boxed{1.38 \text{ A up in the } 8 \text{ V battery}}$

(e) For the left loop $+3 \text{ V} - Q/6 \ \mu\text{F} + 8 \text{ V} = 0$ so $Q = 6 \ \mu\text{F} \ 11 \text{ V} = \boxed{66.0 \ \mu\text{C}}$

***P28.40** The current in the battery is $\dfrac{15\text{ V}}{10\ \Omega+\dfrac{1}{1/5\ \Omega+1/8\ \Omega}}=1.15\text{ A.}$

The voltage across 5 Ω is 15 V – 10 Ω 1.15 A = 3.53 V.

(a) The current in it is 3.53 V/5 Ω = 0.706 A.

(b) $\mathcal{P}=3.53\text{ V }0.706\text{ A}=\boxed{2.49\text{ W}}$

(c) Only the circuit in Figure P28.40c requires the use of Kirchhoff's rules for solution. In the other circuits the 5-Ω and 8-Ω resistors are still in parallel with each other.

(d) The power is lowest in Figure P28.40c. The circuits in Figures P28.40b and P28.40d have in effect 30-V batteries driving the current.

P28.41 The set of four batteries boosts the electric potential of each bit of charge that goes through them by $4\times 1.50\text{ V}=6.00\text{ V}$. The chemical energy they store is

$$\Delta U=q\Delta V=(240\text{ C})(6.00\text{ J/C})=1\,440\text{ J}$$

The radio draws current $I=\dfrac{\Delta V}{R}=\dfrac{6.00\text{ V}}{200\ \Omega}=0.030\,0\text{ A}$

So, its power is $\mathcal{P}=(\Delta V)I=(6.00\text{ V})(0.030\,0\text{ A})=0.180\text{ W}=0.180\text{ J/s}$

Then for the time the energy lasts,
we have $\mathcal{P}=\dfrac{E}{\Delta t}$: $\Delta t=\dfrac{E}{\mathcal{P}}=\dfrac{1\,440\text{ J}}{0.180\text{ J/s}}=8.00\times 10^{3}\text{ s}$

We could also compute this from $I=\dfrac{Q}{\Delta t}$: $\Delta t=\dfrac{Q}{I}=\dfrac{240\text{ C}}{0.030\,0\text{ A}}=8.00\times 10^{3}\text{ s}=\boxed{2.22\text{ h}}$

***P28.42** $I=\dfrac{\mathcal{E}}{R+r}$, so $\mathcal{P}=I^2R=\dfrac{\mathcal{E}^2R}{(R+r)^2}$ or $(R+r)^2=\left(\dfrac{\mathcal{E}^2}{\mathcal{P}}\right)R$

Let $x\equiv\dfrac{\mathcal{E}^2}{\mathcal{P}}$, then $(R+r)^2=xR$ or $R^2+(2r-x)R-r^2=0$

With $r=1.20\ \Omega$, this becomes $R^2+(2.40-x)R-1.44=0$

which has solutions of $R=\dfrac{-(2.40-x)\pm\sqrt{(2.40-x)^2-5.76}}{2}$

(a) With $\mathcal{E}=9.20\text{ V}$ and $\mathcal{P}=12.8\text{ W}$, $x=6.61$:

$$R=\frac{+4.21\pm\sqrt{(4.21)^2-5.76}}{2}=\boxed{3.84\ \Omega}\text{ or }\boxed{0.375\ \Omega}.$$ Either external resistance extracts the same power from the battery.

(b) For $\mathcal{E}=9.20\text{ V}$ and $\mathcal{P}=21.2\text{ W}$, $x\equiv\dfrac{\mathcal{E}^2}{\mathcal{P}}=3.99$

$$R=\frac{+1.59\pm\sqrt{(1.59)^2-5.76}}{2}=\frac{1.59\pm\sqrt{-3.22}}{2}$$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R=r=1.20\ \Omega$, and that maximum is $\mathcal{P}_{\max}=\dfrac{\mathcal{E}^2}{4r}=17.6\text{ W.}$

P28.43 Using Kirchhoff's loop rule for the closed loop, $+12.0-2.00I-4.00I=0$, so $I=2.00\text{ A}$

$V_b-V_a=+4.00\text{ V}-(2.00\text{ A})(4.00\ \Omega)-(0)(10.0\ \Omega)=-4.00\text{ V}$

Thus, $|\Delta V_{ab}|=\boxed{4.00\text{ V}}$ and $\boxed{\text{point } a \text{ is at the higher potential}}$.

P28.44 (a) $R_{\text{eq}}=3R$ $\quad I=\dfrac{\mathcal{E}}{3R}$ $\quad \mathcal{P}_{\text{series}}=\mathcal{E}I=\boxed{\dfrac{\mathcal{E}^2}{3R}}$

(b) $R_{\text{eq}}=\dfrac{1}{(1/R)+(1/R)+(1/R)}=\dfrac{R}{3}$ $\quad I=\dfrac{3\mathcal{E}}{R}$ $\quad \mathcal{P}_{\text{parallel}}=\mathcal{E}I=\boxed{\dfrac{3\mathcal{E}^2}{R}}$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

***P28.45** The charging current is given by $14.7\text{ V}-13.2\text{ V}-I\,0.85\ \Omega=0$ $\quad I=1.76\text{ A}$

The energy delivered by the 14.7 V supply is $\Delta VIt=14.7\text{ V}\ 1.76\text{ A}\ 1.80\text{ h}\ (3\,600\text{ s/h})=168\,000\text{ J}$

The energy stored in the battery is $13.2\text{ V}\ 1.76\text{ A}\ 1.8\ (3\,600\text{ s})=151\,000\text{ J}$

The same energy is released by the emf of the battery: $13.2\text{ V}\ (I)\ 7.3\ (3\,600\text{ s})=151\,000\text{ J}$

so the discharge current is $I=0.435\text{ A}$

The load resistor is given by $13.2\text{ V}-(0.435\text{ A})\,R-(0.435\text{ A})\,0.85\ \Omega=0$

$R=(12.8\text{ V})/0.435\text{ A}=29.5\ \Omega$

The energy delivered to the load is $\Delta VIt=I^2R\,t=(0.435\text{ A})^2\ 29.5\ \Omega\ (7.3)\ 3\,600\text{ s}=147\,000\text{ J}$

The efficiency is 147 000 J/168 000 J = $\boxed{0.873}$

P28.46 (a) $\mathcal{E}-I\left(\sum R\right)-(\mathcal{E}_1+\mathcal{E}_2)=0$

$40.0\text{ V}-(4.00\text{ A})[(2.00+0.300+0.300+R)\Omega]-(6.00+6.00)\text{ V}=0;$ so $R=\boxed{4.40\ \Omega}$

(b) Inside the supply, $\mathcal{P}=I^2R=(4.00\text{ A})^2(2.00\ \Omega)=\boxed{32.0\text{ W}}$

Inside both batteries together, $\mathcal{P}=I^2R=(4.00\text{ A})^2(0.600\ \Omega)=\boxed{9.60\text{ W}}$

For the limiting resistor, $\mathcal{P}=(4.00\text{ A})^2(4.40\ \Omega)=\boxed{70.4\text{ W}}$

(c) $\mathcal{P}=I(\mathcal{E}_1+\mathcal{E}_2)=(4.00\text{ A})[(6.00+6.00)\text{ V}]=\boxed{48.0\text{ W}}$

P28.47 Let the two resistances be x and y.

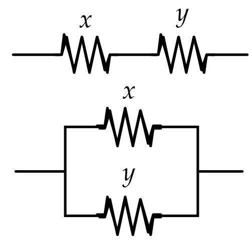


FIG. P28.47

Then, $R_s = x + y = \frac{\mathcal{P}_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega \qquad y = 9.00\ \Omega - x$

and $R_p = \frac{xy}{x+y} = \frac{\mathcal{P}_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$

so $\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega \qquad x^2 - 9.00x + 18.0 = 0$

Factoring the second equation, $(x - 6.00)(x - 3.00) = 0$

so $x = 6.00\ \Omega$ or $x = 3.00\ \Omega$

Then, $y = 9.00\ \Omega - x$ gives $y = 3.00\ \Omega$ or $y = 6.00\ \Omega$

There is only one physical answer: The two resistances are $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

P28.48 Let the two resistances be x and y.

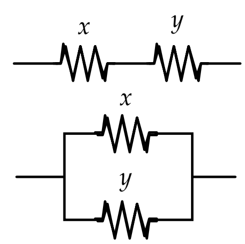


FIG. P28.48

Then, $R_s = x + y = \frac{\mathcal{P}_s}{I^2}$ and $R_p = \frac{xy}{x+y} = \frac{\mathcal{P}_p}{I^2}$.

From the first equation, $y = \frac{\mathcal{P}_s}{I^2} - x$, and the second

becomes $\frac{x\left(\mathcal{P}_s/I^2 - x\right)}{x + \left(\mathcal{P}_s/I^2 - x\right)} = \frac{\mathcal{P}_p}{I^2}$ or $x^2 - \left(\frac{\mathcal{P}_s}{I^2}\right)x + \frac{\mathcal{P}_s\mathcal{P}_p}{I^4} = 0.$

Using the quadratic formula, $x = \frac{\mathcal{P}_s \pm \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$.

Then, $y = \frac{\mathcal{P}_s}{I^2} - x$ gives $y = \frac{\mathcal{P}_s \mp \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$.

The two resistances are $\boxed{\frac{\mathcal{P}_s + \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}}$ and $\boxed{\frac{\mathcal{P}_s - \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}}$.

P28.49 (a) $\Delta V_1 = \Delta V_2 \qquad I_1R_1 = I_2R_2$

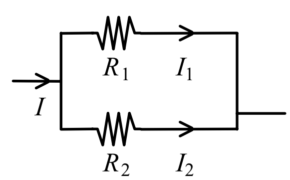


FIG. P28.49(a)

$$I = I_1 + I_2 = I_1 + \frac{I_1R_1}{R_2} = I_1\frac{R_2 + R_1}{R_2}$$

$$\boxed{I_1 = \frac{IR_2}{R_1 + R_2}} \qquad I_2 = \frac{I_1R_1}{R_2} = \boxed{\frac{IR_1}{R_1 + R_2} = I_2}$$

(b) The power delivered to the pair is $\mathcal{P} = I_1^2R_1 + I_2^2R_2 = I_1^2R_1 + \left(I - I_1\right)^2 R_2$. For minimum power we want to find I_1 such that $\frac{d\mathcal{P}}{dI_1} = 0$.

$$\frac{d\mathcal{P}}{dI_1} = 2I_1R_1 + 2\left(I - I_1\right)(-1)R_2 = 0 \qquad I_1R_1 - IR_2 + I_1R_2 = 0$$

$I_1 = \frac{IR_2}{R_1 + R_2}$ This is the same condition as that found in part (a).

***P28.50** (a) When the capacitor is fully charged, no current exists in its branch. The current in the left resistors is 5 V/83 Ω = 0.060 2 A. The current in the right resistors is 5 V/(2 Ω + R). Relative to the negative side of the battery, the left capacitor plate is at voltage 80 Ω (0.060 2 A) = 4.82 V. The right plate is at R (5 V)/(2 Ω + R). The voltage across the capacitor is 4.82 V – R (5 V)/(2 Ω + R). The charge on the capacitor is

$$Q = 3\ \mu\text{F}\,[4.82\text{ V} - R\,(5\text{ V})/(2\ \Omega + R)] = \boxed{(28.9\ \Omega - 0.542\,R)\ \mu\text{C}/(2\ \Omega + R)}$$

(b) With $R = 10\ \Omega$, $Q = (28.9 - 5.42)\ \mu\text{C}/(2 + 10) = \boxed{1.96\ \mu\text{C}}$

(c) $\boxed{\text{Yes.}}$ $Q = 0$ when $28.9\ \Omega - 0.542\,R = 0$ $R = \boxed{53.3\ \Omega}$

(d) The maximum charge occurs for $R = 0$. It is $28.9/2 = \boxed{14.5\ \mu\text{C}}$.

(e) $\boxed{\text{Yes. Taking } R = \infty \text{ corresponds to disconnecting a wire to remove the branch containing } R.}$

In this case $|Q| = 0.542\,R/R = \boxed{0.542\ \mu\text{C}}$.

P28.51 Let R_m = measured value, R = actual value,

I_R = current through the resistor R

I = current measured by the ammeter

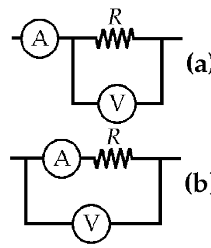


FIG. P28.51

(a) When using circuit (a), $I_R R = \Delta V = 20\,000\left(I - I_R\right)$ or

$$R = 20\,000\left[\frac{I}{I_R} - 1\right]$$

But since $I = \dfrac{\Delta V}{R_m}$ and $I_R = \dfrac{\Delta V}{R}$, we have $\dfrac{I}{I_R} = \dfrac{R}{R_m}$

and $$R = 20\,000\frac{\left(R - R_m\right)}{R_m} \qquad (1)$$

When $R > R_m$, we require $\dfrac{\left(R - R_m\right)}{R} \le 0.050\,0$

Therefore, $R_m \ge R\left(1 - 0.050\,0\right)$ and from (1) we find $\boxed{R \le 1\,050\ \Omega}$

(b) When using circuit (b), $I_R R = \Delta V - I_R\left(0.5\ \Omega\right)$

But since $I_R = \dfrac{\Delta V}{R_m}$, $$R_m = \left(0.500 + R\right) \qquad (2)$$

When $R_m > R$, we require $\dfrac{\left(R_m - R\right)}{R} \le 0.050\,0$

From (2) we find $\boxed{R \ge 10.0\ \Omega}$

P28.52 The battery supplies energy at a changing rate $\frac{dE}{dt} = \mathcal{P} = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-1/RC}\right)$

Then the total energy put out by the battery is $\int dE = \int_{t=0}^{\infty} \frac{\mathcal{E}^2}{R}\exp\left(-\frac{t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty}\exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\mathcal{E}^2 C\exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\mathcal{E}^2 C[0-1] = \mathcal{E}^2 C$$

The power delivered to the resistor is $\frac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$

So the total internal energy appearing in the resistor is $\int dE = \int_0^{\infty}\frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty}\exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$. Thus, energy of the circuit is conserved $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$ and resistor and capacitor share equally in the energy from the battery.

P28.53 (a) $q = C\Delta V\left(1 - e^{-t/RC}\right)$

$$q = \left(1.00\times10^{-6}\text{ F}\right)(10.0\text{ V})\left[1 - e^{-10.0/\left[\left(2.00\times10^{6}\right)\left(1.00\times10^{-6}\right)\right]}\right] = \boxed{9.93\ \mu\text{C}}$$

(b) $I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$

$$I = \left(\frac{10.0\text{ V}}{2.00\times10^{6}\ \Omega}\right)e^{-5.00} = 3.37\times10^{-8}\text{ A} = \boxed{33.7\text{ nA}}$$

(c) $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \left(\frac{q}{C}\right)\frac{dq}{dt} = \left(\frac{q}{C}\right)I$

$$\frac{dU}{dt} = \left(\frac{9.93\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ C/V}}\right)\left(3.37\times10^{-8}\text{ A}\right) = 3.34\times10^{-7}\text{ W} = \boxed{334\text{ nW}}$$

(d) $\mathcal{P}_{\text{battery}} = I\mathcal{E} = \left(3.37\times10^{-8}\text{ A}\right)(10.0\text{ V}) = 3.37\times10^{-7}\text{ W} = \boxed{337\text{ nW}}$

The battery power could also be computed as the sum of the instantaneous powers delivered to the resistor and to the capacitor:

$$I^2R + dU/dt = (33.7\times10^{-9}\text{ A})^2\ 2\times10^{6}\ \Omega + 334\text{ nW} = 337\text{ nW}$$

***P28.54** (a) We find the resistance intrinsic to the vacuum cleaner:

$$\mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{535\text{ W}} = 26.9\ \Omega$$

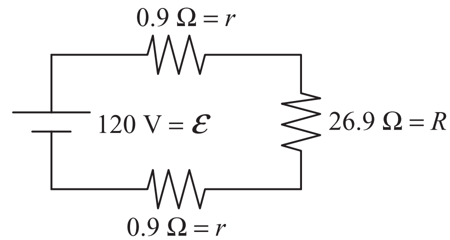


FIG. P28.54

with the inexpensive cord, the equivalent resistance is $0.9\ \Omega + 26.9\ \Omega + 0.9\ \Omega = 28.7\ \Omega$ so the current throughout the circuit is

$$I = \frac{\mathcal{E}}{R_{\text{Tot}}} = \frac{120\text{ V}}{28.7\ \Omega} = 4.18\text{ A}$$

and the cleaner power is

$$\mathcal{P}_{\text{cleaner}} = I(\Delta V)_{\text{cleaner}} = I^2 R = (4.18\text{ A})^2(26.9\ \Omega) = \boxed{470\text{ W}}$$

In symbols, $R_{\text{Tot}} = R + 2r$, $I = \dfrac{\mathcal{E}}{R+2r}$ and $\mathcal{P}_{\text{cleaner}} = I^2 R = \dfrac{\mathcal{E}^2 R}{(R+2r)^2}$

(b) $$R + 2r = \left(\frac{\mathcal{E}^2 R}{\mathcal{P}_{\text{cleaner}}}\right)^{1/2} = 120\text{ V}\left(\frac{26.9\ \Omega}{525\text{ W}}\right)^{1/2} = 27.2\ \Omega$$

$$r = \frac{27.2\ \Omega - 26.9\ \Omega}{2} = 0.128\ \Omega = \frac{\rho\ell}{A} = \frac{\rho\ell 4}{\pi d^2}$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(15\text{ m})}{\pi(0.128\ \Omega)}\right)^{1/2} = \boxed{1.60\text{ mm or more}}$$

(c) Unless the extension cord is a superconductor, it is impossible to attain cleaner power 535 W. To move from 525 W to 532 W will require a lot more copper, as we show here:

$$r = \frac{\mathcal{E}}{2}\left(\frac{R}{\mathcal{P}_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\text{ V}}{2}\left(\frac{26.9\ \Omega}{532\text{ W}}\right)^{1/2} - \frac{26.9\ \Omega}{2} = 0.037\,9\ \Omega$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(15\text{ m})}{\pi\left(0.037\,9\ \Omega\right)}\right)^{1/2} = \boxed{2.93\text{ mm or more}}$$

P28.55 (a) First determine the resistance of each light bulb: $\mathcal{P} = \dfrac{(\Delta V)^2}{R}$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega$$

We obtain the equivalent resistance R_{eq} of the network of light bulbs by identifying series and parallel equivalent resistances:

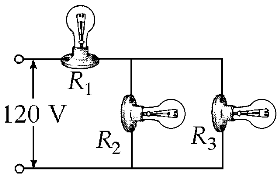


FIG. P28.55

$$R_{eq} = R_1 + \frac{1}{(1/R_2)+(1/R_3)} = 240\ \Omega + 120\ \Omega = 360\ \Omega$$

The total power dissipated in the 360 Ω is $\mathcal{P} = \dfrac{(\Delta V)^2}{R_{eq}} = \dfrac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}$

(b) The current through the network is given by $\mathcal{P} = I^2 R_{eq}$: $I = \sqrt{\dfrac{\mathcal{P}}{R_{eq}}} = \sqrt{\dfrac{40.0\text{ W}}{360\ \Omega}} = \dfrac{1}{3}\text{ A}$

The potential difference across R_1 is $\Delta V_1 = IR_1 = \left(\dfrac{1}{3}\text{ A}\right)(240\ \Omega) = \boxed{80.0\text{ V}}$

The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23} = IR_{23} = \left(\frac{1}{3}\text{ A}\right)\left(\frac{1}{(1/240\ \Omega)+(1/240\ \Omega)}\right) = \boxed{40.0\text{ V}}$$

P28.56 (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

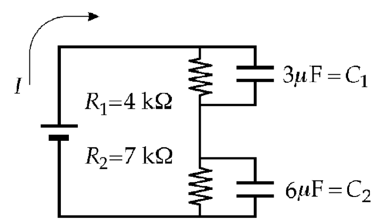


FIG. P28.56(a)

$$\mathcal{P} = I^2 R_2 \qquad I = \sqrt{\frac{\mathcal{P}}{R_2}} = \sqrt{\frac{2.40\text{ V}\cdot\text{A}}{7\,000\text{ V/A}}} = 18.5\text{ mA}$$

The potential difference across R_1 and C_1 is

$$\Delta V = IR_1 = (1.85\times10^{-2}\text{ A})(4\,000\text{ V/A}) = 74.1\text{ V}$$

The charge on C_1

$$Q = C_1\Delta V = (3.00\times10^{-6}\text{ C/V})(74.1\text{ V}) = \boxed{222\ \mu\text{C}}$$

The potential difference across R_2 and C_2 is

$$\Delta V = IR_2 = (1.85\times10^{-2}\text{ A})(7\,000\ \Omega) = 130\text{ V}$$

The charge on C_2

$$Q = C_2\Delta V = (6.00\times10^{-6}\text{ C/V})(130\text{ V}) = 778\ \mu\text{C}$$

The battery emf is

$$IR_{eq} = I(R_1+R_2) = 1.85\times10^{-2}\text{ A}(4\,000+7\,000)\text{ V/A} = 204\text{ V}$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge on C_2 is

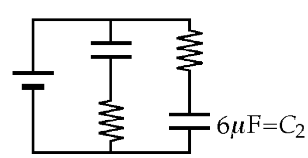


FIG. P28.56(b)

$$Q = C_2\Delta V = (6.00\times10^{-6}\text{ C/V})(204\text{ V}) = 1\,222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C} - 778\ \mu\text{C} = \boxed{444\ \mu\text{C}}$.

***P28.57** (a) The emf of the battery is $\boxed{9.30\text{ V}}$. Its internal resistance is given by

$9.30\text{ V} - 3.70\text{ A}\, r = 0 \qquad r = \boxed{2.51\ \Omega}$

(b) Total emf = $20(9.30\text{ V}) = \boxed{186\text{ V}}$. The maximum current is given by

$20(9.30\text{ V}) - 20(2.51\ \Omega)\, I - 0\, I = 0 \qquad I = \boxed{3.70\text{ A}}$

(c) For the circuit $20(9.30\text{ V}) - 20(2.51\ \Omega)\, I - 120\ \Omega\, I = 0 \quad I = 186\text{ V}/170\ \Omega = \boxed{1.09\text{ A}}$

(d) $\mathcal{P} = I^2 R = (1.09\text{ A})^2\, 120\ \Omega = \boxed{143\text{ W}}$. This is a potentially deadly situation.

(e) The potential difference across his body is $120\ \Omega\, (0.005\,00\text{ A}) = 0.600\text{ V}$.

This must be the terminal potential difference of the bank of batteries:

$186\text{ V} - I_{tot}\, 20(2.51\ \Omega) = 0.6\text{ V} \qquad I_{tot} = 185.4\text{ V}/50.3\ \Omega = 3.688\text{ A}$

For the copper wire we then have $\;0.6\text{ V} = (3.688\text{ A} - 0.005\text{ A})\, R \quad R = \boxed{0.163\ \Omega}$

(f) For the experimenter's body, $\mathcal{P} = I\Delta V = 0.005\text{ A}\; 0.6\text{ V} = \boxed{3.00\text{ mW}}$.

(g) For the wire $\mathcal{P} = I\Delta V = 3.683\text{ A}\; 0.6\text{ V} = \boxed{2.21\text{ W}}$.

(h) The power output of the emf depends on the resistance connected to it. A question about "the rest of the power" is not meaningful when it compares circuits with different currents. The net emf produces more current in the circuit where the copper wire is used. The net emf delivers more power when the copper wire is used, 687 W rather than 203 W without the wire. Nearly all of this power results in extra internal energy in the internal resistance of the batteries, which rapidly rise to a high temperature. The circuit with the copper wire is unsafe because the batteries overheat. The circuit without the copper wire is unsafe because it delivers an electric shock to the experimenter.

***P28.58** The battery current is

$$(150 + 45 + 14 + 4)\text{ mA} = 213\text{ mA}$$

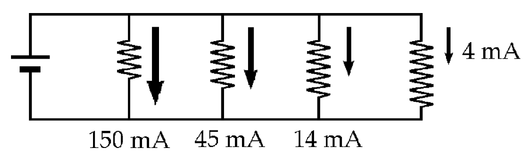
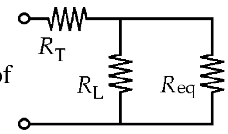


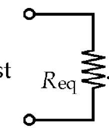
FIG. P28.58

(a) The resistor with highest resistance is that carrying 4 mA. Doubling its resistance will reduce the current it carries to 2 mA. Then the total current is $(150 + 45 + 14 + 2)\text{ mA} = 211\text{ mA}$, nearly the same as before. The ratio is $\dfrac{211}{213} = \boxed{0.991}$.

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to $(75 + 45 + 14 + 4)\text{ mA} = 138\text{ mA}$. The ratio is $\dfrac{138}{213} = \boxed{0.648}$, representing a much larger reduction (35.2% instead of 0.9%).

(c) This problem is precisely analogous. As a battery maintained a potential difference in parts (a) and (b), a furnace maintains a temperature difference here. Energy flow by heat is analogous to current and takes place through thermal resistances in parallel. Each resistance can have its "*R*-value" increased by adding insulation. Doubling the thermal resistance of the attic door will produce only a negligible (0.9%) saving in fuel. The ceiling originally has the smallest thermal resistance. Doubling the thermal resistance of $\boxed{\text{the ceiling}}$ will produce a much larger saving.

P28.59 From the hint, the equivalent resistance of 

is just 

.

That is, $$R_T + \frac{1}{1/R_L + 1/R_{\text{eq}}} = R_{\text{eq}}$$

$$R_T + \frac{R_L R_{\text{eq}}}{R_L + R_{\text{eq}}} = R_{\text{eq}}$$

$$R_T R_L + R_T R_{\text{eq}} + R_L R_{\text{eq}} = R_L R_{\text{eq}} + R_{\text{eq}}^2$$

$$R_{\text{eq}}^2 - R_T R_{\text{eq}} - R_T R_L = 0$$

$$R_{\text{eq}} = \frac{R_T \pm \sqrt{R_T^2 - 4(1)(-R_T R_L)}}{2(1)}$$

Only the + sign is physical:

$$R_{\text{eq}} = \frac{1}{2}\left(\sqrt{4R_T R_L + R_T^2} + R_T\right)$$

For example, if $R_T = 1\ \Omega$

And $R_L = 20\ \Omega,\ R_{\text{eq}} = 5\ \Omega$

P28.60 (a) First let us flatten the circuit on a 2-D plane as shown; then reorganize it to a format easier to read. Notice that the two resistors shown in the top horizontal branch carry the same current as the resistors in the horizontal branch second from the top. The center junctions in these two branches are at the same potential. The vertical resistor between these two junctions has no potential difference across it and carries no current. This middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance

$$R_{\text{eq}} = \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{10}\right)^{-1} = \boxed{5.00\ \Omega}$$

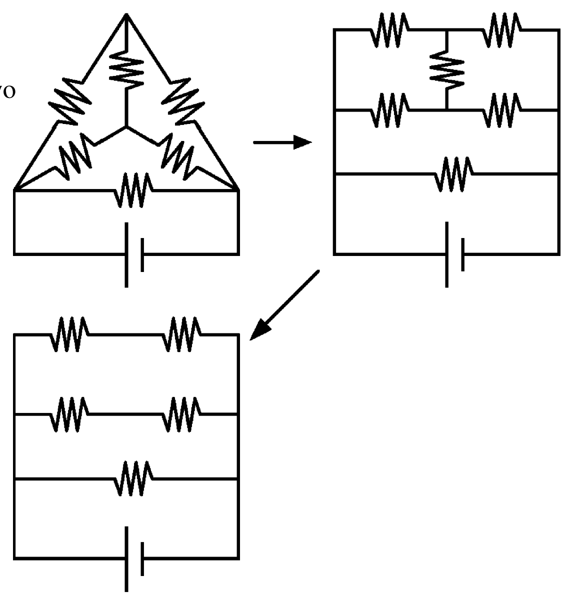


FIG. P28.60(a)

(b) So the current through the battery is

$$\frac{\Delta V}{R_{\text{eq}}} = \frac{12.0\ \text{V}}{5.00\ \Omega} = \boxed{2.40\ \text{A}}$$

P28.61 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3: $\boxed{I_{R_3} = 0 \text{ (steady-state)}}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2: $$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1+R_2} = \frac{9.00\text{ V}}{(12.0\text{ k}\Omega + 15.0\text{ k}\Omega)} = \boxed{333\ \mu\text{A (steady-state)}}$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\text{ k}\Omega)$$
$$= \boxed{50.0\ \mu\text{C}}$$

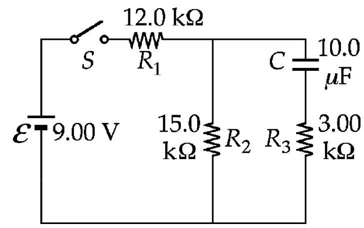


FIG. P28.61(b)

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of $(R_2 + R_3)C = (15.0\text{ k}\Omega + 3.00\text{ k}\Omega)(10.0\ \mu\text{F}) = 0.180$ s. The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

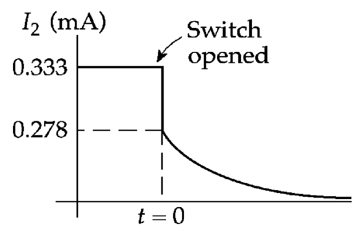


FIG. P28.61(c)

$$I_i = \frac{(\Delta V)_C}{(R_2+R_3)} = \frac{IR_2}{(R_2+R_3)} = \frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega + 3.00\text{ k}\Omega)} = 278\ \mu\text{A}$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from $333\ \mu\text{A}$ (downward) to $278\ \mu\text{A}$ (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})} \text{ (for } t > 0)}$$

(d) The charge q on the capacitor decays from Q_i to $\frac{Q_i}{5}$ according to

$$q = Q_i e^{-t/(R_2+R_3)C}$$
$$\frac{Q_i}{5} = Q_i e^{(-t/0.180\text{ s})}$$
$$5 = e^{t/0.180\text{ s}}$$
$$\ln 5 = \frac{t}{180\text{ ms}}$$
$$t = (0.180\text{ s})(\ln 5) = \boxed{290\text{ ms}}$$

P28.62 $\Delta V = \mathcal{E} e^{-t/RC}$

so $\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = \left(\frac{1}{RC}\right)t$

A plot of $\ln\left(\frac{\mathcal{E}}{\Delta V}\right)$ versus t should be a straight line with slope equal to $\frac{1}{RC}$.

Using the given data values:

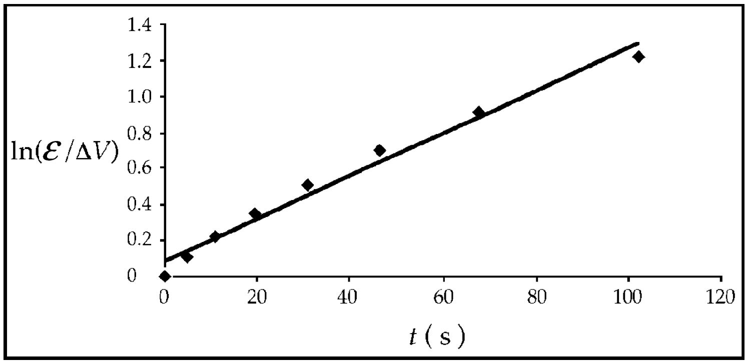


FIG. P28.62

$t(s)$	$\Delta V(V)$	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

(a) A least-square fit to this data yields the graph above.

$\sum x_i = 282, \qquad \sum x_i^2 = 1.86 \times 10^4,$

$\sum x_i y_i = 244, \qquad \sum y_i = 4.03, \qquad N = 8$

$$\text{Slope} = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.011\,8$$

$$\text{Intercept} = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.088\,2$$

The equation of the best fit line is: $\boxed{\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = (0.011\,8)t + 0.088\,2}$

(b) Thus, the time constant is $\tau = RC = \frac{1}{\text{slope}} = \frac{1}{0.011\,8} = \boxed{84.7 \text{ s}}$

and the capacitance is $C = \frac{\tau}{R} = \frac{84.7 \text{ s}}{10.0 \times 10^6 \ \Omega} = \boxed{8.47\ \mu\text{F}}$

P28.63 (a) For the first measurement, the equivalent circuit is as shown in Figure 1.

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $\qquad R_y = \frac{1}{2}R_1 \qquad (1)$

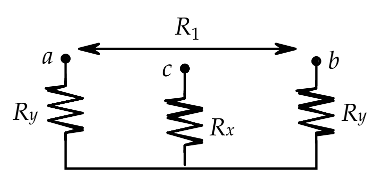


Figure 1

For the second measurement, the equivalent circuit is shown in Figure 2.

Thus, $\qquad R_{ac} = R_2 = \frac{1}{2}R_y + R_x \qquad (2)$

Substitute (1) into (2) to obtain:

$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x, \text{ or } \boxed{R_x = R_2 - \frac{1}{4}R_1}$$

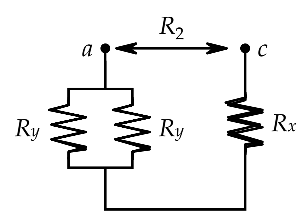


Figure 2

FIG. P28.63

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$.

$\boxed{\text{The antenna is inadequately grounded}}$ since this exceeds the limit of $2.00\ \Omega$

P28.64 Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C

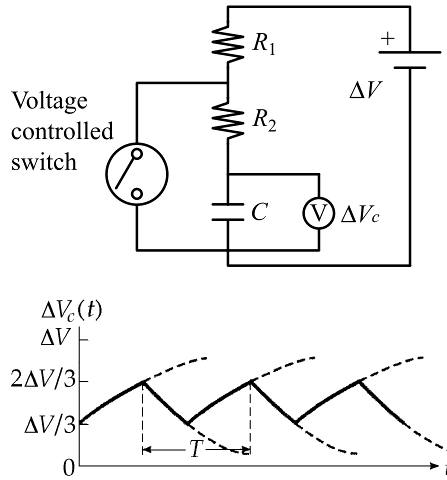


FIG. P28.64

$$\Delta V_C(t) = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.

Therefore, $\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$

or $e^{-t/R_2C} = \frac{1}{2}$

or $t_1 = R_2C\ln 2$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1+R_2)C$:

$$\Delta V_C(t) = \Delta V - \left[\frac{2}{3}\Delta V\right]e^{-t/(R_1+R_2)C}$$

When $\Delta V_C(t) = \frac{2}{3}\Delta V$

$\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/(R_1+R_2)C}$ or $e^{-t/(R_1+R_2)C} = \frac{1}{2}$

So $t_2 = (R_1+R_2)C\ln 2$ and $T = t_1 + t_2 = \boxed{(R_1+2R_2)C\ln 2}$

P28.65 A certain quantity of energy $\Delta E_{\text{int}} = \mathcal{P}(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $\mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$ where (ΔV) is a constant. Thus, comparing coils 1 and 2, we have for the energy $\frac{(\Delta V)^2\Delta t}{R_1} = \frac{(\Delta V)^2 2\Delta t}{R_2}$. Then $R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$$R_p = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/R_1 + 1/2R_1} = \frac{2R_1}{3}.\ \text{Now}\ \frac{(\Delta V)^2\Delta t}{R_1} = \frac{(\Delta V)^2\Delta t_p}{2R_1/3} \qquad \Delta t_p = \boxed{\frac{2\Delta t}{3}}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and $\frac{(\Delta V)^2\Delta t}{R_1} = \frac{(\Delta V)^2\Delta t_s}{3R_1}$

$\Delta t_s = \boxed{3\Delta t}$

P28.66 (a) We model the person's body and street shoes as shown. For the discharge to reach 100 V,

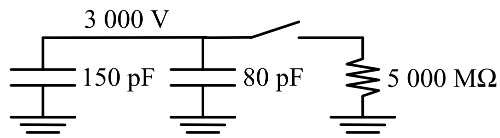


FIG. P28.66(a)

$$q(t) = Qe^{-t/RC} = C\Delta V(t) = C\Delta V_0 e^{-t/RC}$$

$$\frac{\Delta V}{\Delta V_0} = e^{-t/RC} \qquad \frac{\Delta V_0}{\Delta V} = e^{+t/RC}$$

$$\frac{t}{RC} = \ln\left(\frac{\Delta V_0}{\Delta V}\right)$$

$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = 5\,000\times10^6\ \Omega\left(230\times10^{-12}\ \text{F}\right)\ln\left(\frac{3\,000}{100}\right) = \boxed{3.91\ \text{s}}$$

(b) $t = \left(1\times10^6\ \text{V/A}\right)\left(230\times10^{-12}\ \text{C/V}\right)\ln 30 = \boxed{782\ \mu\text{s}}$

ANSWERS TO EVEN PROBLEMS

P28.2 (a) 4.59 Ω (b) 8.16%

P28.4 (a) 50.0% (b) $r = 0$ (c) High efficiency. The electric company's economic interest is to minimize internal energy production in its power lines, so that it can sell a large fraction of the energy output of its generators to the customers. (d) High power transfer. Energy by electric transmission is so cheap compared to the sound system that she does not spend extra money to buy an efficient amplifier.

P28.6 (a) The 120-V potential difference is applied across the series combination of the two conductors in the extension cord and the light bulb. The potential difference across the light bulb is less than 120 V and its power is less than 75 W. (b) We assume the bulb has constant resistance—that is, that its temperature does not change much from the design operating point. See the solution. 73.8 W

P28.8 (a) See the solution. (b) no

P28.10 See the solution.

P28.12 470 Ω and 220 Ω

P28.14 (a) $\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2$ (b) $\Delta V_1 = \mathcal{E}/3$, $\Delta V_2 = 2\mathcal{E}/9$, $\Delta V_3 = 4\mathcal{E}/9$, $\Delta V_4 = 2\mathcal{E}/3$ (c) $I_1 > I_4 > I_2 = I_3$ (d) $I_1 = I$, $I_2 = I_3 = I/3$, $I_4 = 2I/3$ (e) Increasing the value of resistor 3 increases the equivalent resistance of the entire circuit. The current in the battery, which is also the current in resistor 1, therefore decreases. Then the potential difference across resistor 1 decreases and the potential difference across the parallel combination increases. Driven by a larger potential difference, the current in resistor 4 increases. This effect makes the current in resistors 2 and 3 decrease. In summary, I_4 increases while I_1, I_2, and I_3 decrease. (f) $I_1 = 3I/4$, $I_2 = I_3 = 0$, $I_4 = 3I/4$

P28.16 $I_1 = 714$ mA $I_2 = 1.29$ A $\mathcal{E} = 12.6$ V

P28.18 See the solution.

P28.20 (a) See the solution. (b) The current in the 220-Ω resistor and the 5.80-V battery is 11.0 mA out of the positive battery pole. The current in the 370-Ω resistor is 9.13 mA. The current in the 150-Ω resistor and the 3.10-V battery is 1.87 mA out of the negative battery pole.

P28.22 50.0 mA from a to e

P28.24 starter 171 A downward in the diagram; battery 0.283 A downward

P28.26 See the solution.

P28.28 587 kΩ

P28.30 See the solution.

P28.32 (a) 1.50 s (b) 1.00 s (c) $200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}$

P28.34 145 Ω, 0.756 mA

P28.36 (a) 30.000 mA, 5.400 0 V (b) 30.167 mA, 5.381 6 V (c) 29.898 mA, 5.396 6 V (d) Both circuits are good enough for some measurements. The circuit in part (c) gives data leading to a value of resistance that is too high by only about 0.3%. Its value is more accurate than the value, 0.9% too low, implied by the data from the circuit in part (b).

P28.38 (a) $\sim 10^{-14}$ A (b) $V_h/2 + \sim 10^{-10}$ V and $V_h/2 - \sim 10^{-10}$ V, where V_h is the potential of the live wire, $\sim 10^2$ V

P28.40 (a) 0.706 A (b) 2.49 W (c) Only the circuit in Figure P28.40c requires the use of Kirchhoff's rules for solution. In the other circuits the 5-Ω and 8-Ω resistors are still in parallel with each other. (d) The power is lowest in Figure P28.40c. The circuits in Figures P28.40b and P28.40d have in effect 30-V batteries driving the current.

P28.42 (a) either 3.84 Ω or 0.375 Ω (b) No load resistor can extract more than 17.6 W from this battery.

P28.44 (a) $\mathcal{E}^2/3R$ (b) $3\mathcal{E}^2/R$ (c) in the parallel connection

P28.46 (a) 4.40 Ω (b) 32.0 W, 9.60 W, 70.4 W (c) 48.0 W

P28.48 $\dfrac{\mathcal{P}_s + \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$ and $\dfrac{\mathcal{P}_s - \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$

P28.50 (a) $15.0\ \mu\text{C}\dfrac{160\ \Omega - 3R}{166\ \Omega + 83R}$ (b) 1.96 μC (c) Yes; 53.3 Ω (d) 14.5 μC for $R = 0$ (e) Yes; it corresponds to disconnecting the wire; 0.542 μC

P28.52 See the solution.

P28.54 (a) 470 W (b) 1.60 mm or more (c) 2.93 mm or more

P28.56 (a) 222 μC (b) increase by 444 μC

P28.58 (a) 0.991 (b) 0.648 (c) The energy flows are precisely analogous to the currents in parts (a) and (b). The ceiling has the smallest R-value of the thermal resistors in parallel, so increasing its thermal resistance will produce the biggest reduction in the total energy flow.

P28.60 (a) 5.00 Ω (b) 2.40 A

P28.62 (a) $\ln(\mathcal{E}/\Delta V) = 0.011\,8\,t + 0.088\,2$ (b) 84.7 s, 8.47 μF

P28.64 $(R_1 + 2R_2)C\ln 2$

P28.66 (a) 3.91 s (b) 0.782 ms