16

Wave Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q16.1 As the pulse moves down the string, the particles of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.

Q16.2 To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.

***Q16.3** (i) Look at the coefficients of the sine and cosine functions: 2, 4, 6, 8, 8, 7. The ranking is d = e > f > c > b > a.

(ii) Look at the coefficients of x. Each is the wave number, $2\pi/\lambda$, so the smallest k goes with the largest wavelength. The ranking is d > a = b = c > e > f.

(iii) Look at the coefficients of t. The absolute value of each is the angular frequency $\omega = 2\pi f$. The ranking is f > e > a = b = c = d.

(iv) Period is the reciprocal of frequency, so the ranking is the reverse of that in part iii: d = c = b = a > e > f.

(v) From $v = f\lambda = \omega/k$, we compute the absolute value of the ratio of the coefficient of t to the coefficient of x in each case. From a to f respectively the numerical speeds are 5, 5, 5, 7.5, 5, 4. The ranking is d > a = b = c = e > f.

***Q16.4** From $v = \sqrt{\dfrac{T}{\mu}}$, we must increase the tension by a factor of 4 to make v double. Answer (b).

***Q16.5** Answer (b). Wave speed is inversely proportional to the square root of linear density.

***Q16.6** (i) Answer (a). Higher tension makes wave speed higher.

(ii) Answer (b). Greater linear density makes the wave move more slowly.

Q16.7 It depends on from what the wave reflects. If reflecting from a less dense string, the reflected part of the wave will be right side up.

Q16.8 Yes, among other things it depends on. The particle speed is described by $v_{y,\max} = \omega A = 2\pi f A = \dfrac{2\pi v A}{\lambda}$. Here v is the speed of the wave.

13794_16_ch16_p427-448.indd 427 12/11/06 5:03:19 PM

***Q16.9** (a) through (d): Yes to all. The maximum particle speed and the wave speed are related by $v_{y,\max} = \omega A = 2\pi f A = \dfrac{2\pi v A}{\lambda}$. Thus the amplitude or the wavelength of the wave can be adjusted to make either $v_{y,\max}$ or v larger.

Q16.10 Since the frequency is 3 cycles per second, the period is $\frac{1}{3}$ second = 333 ms.

Q16.11 Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v = \sqrt{\dfrac{T}{\mu}}$ increases with height.

***Q16.12** Answer (c). If the frequency does not change, the amplitude is increased by a factor of $\sqrt{2}$. The wave speed does not change.

***Q16.13** (i) Answer a. As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v = \sqrt{\dfrac{T}{\mu}}$.

(ii) Answer c. The frequency will remain unchanged. However often crests come up to the boundary they leave the boundary.

(iii) Answer a. Since $v = f\lambda$, the wavelength must increase.

Q16.14 Longitudinal waves depend on the compressibility of the fluid for their propagation. Transverse waves require a restoring force in response to shear strain. Fluids do not have the underlying structure to supply such a force. A fluid cannot support static shear. A viscous fluid can temporarily be put under shear, but the higher its viscosity the more quickly it converts input work into internal energy. A local vibration imposed on it is strongly damped, and not a source of wave propagation.

Q16.15 Let $\Delta t = t_s - t_p$ represent the difference in arrival times of the two waves at a station at distance $d = v_s t_s = v_p t_p$ from the hypocenter. Then $d = \Delta t\left(\dfrac{1}{v_s} - \dfrac{1}{v_p}\right)^{-1}$. Knowing the distance from the first station places the hypocenter on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third non-collinear station will generally limit the possibilities to a point.

Q16.16 The speed of a wave on a "massless" string would be infinite!

SOLUTIONS TO PROBLEMS

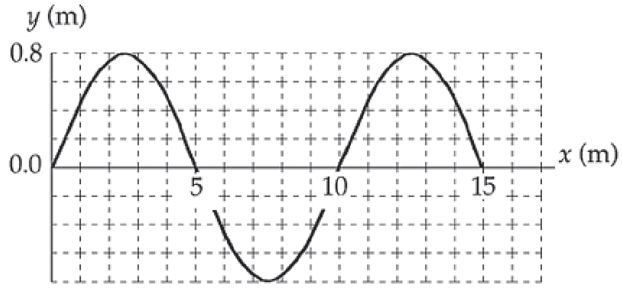
Section 16.1 Propagation of a Disturbance

P16.1 Replace x by $x - vt = x - 4.5t$

to get $\boxed{y = \dfrac{6}{\left[(x-4.5t)^2 + 3\right]}}$

***P16.2**

(a)



(b)

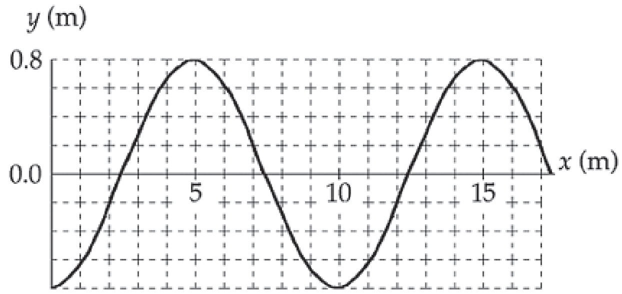


FIG. P16.2

The graph (b) has the same amplitude and wavelength as graph (a). it differs just by being shifted toward larger x by 2.40 m. The wave has traveled 2.40 m to the right.

P16.3 (a) The $\boxed{\text{longitudinal P wave}}$ travels a shorter distance and is moving faster, so it will arrive at point B first.

(b) The wave that travels through the Earth must travel

a distance of $2R\sin 30.0° = 2\left(6.37\times10^6 \text{ m}\right)\sin 30.0° = 6.37\times10^6 \text{ m}$

at a speed of 7 800 m/s

Therefore, it takes $\dfrac{6.37\times10^6 \text{ m}}{7\,800 \text{ m/s}} = 817 \text{ s}$

The wave that travels along the Earth's surface must travel

a distance of $s = R\theta = R\left(\dfrac{\pi}{3}\text{ rad}\right) = 6.67\times10^6 \text{ m}$

at a speed of 4 500 m/s

Therefore, it takes $\dfrac{6.67\times10^6}{4\,500} = 1\,482 \text{ s}$

The time difference is $\boxed{665 \text{ s}} = 11.1 \text{ min}$

P16.4 The distance the waves have traveled is $d = (7.80 \text{ km/s})t = (4.50 \text{ km/s})(t + 17.3 \text{ s})$ where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50 \text{ km/s})(17.3 \text{ s})$

or $t = \dfrac{(4.50 \text{ km/s})(17.3 \text{ s})}{(7.80 - 4.50) \text{ km/s}} = 23.6 \text{ s}$

and the distance is $d = (7.80 \text{ km/s})(23.6 \text{ s}) = \boxed{184 \text{ km}}$.

P16.5 (a) Let $u = 10\pi t - 3\pi x + \dfrac{\pi}{4}$ $\quad \dfrac{du}{dt} = 10\pi - 3\pi\dfrac{dx}{dt} = 0$ at a point of constant phase

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33\ \text{m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction}}$.

(b) $y(0.100,\ 0) = (0.350\ \text{m})\sin\left(-0.300\pi + \dfrac{\pi}{4}\right) = -0.054\,8\ \text{m} = \boxed{-5.48\ \text{cm}}$

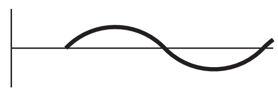
(c) $k = \dfrac{2\pi}{\lambda} = 3\pi$: $\quad \lambda = \boxed{0.667\ \text{m}}$ $\qquad \omega = 2\pi f = 10\pi$: $\quad f = \boxed{5.00\ \text{Hz}}$

(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$ $\qquad v_{y,\,\max} = (10\pi)(0.350) = \boxed{11.0\ \text{m/s}}$

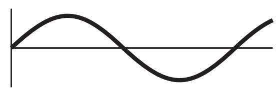
Section 16.2 **The Traveling Wave Model**

***P16.6** (a) a wave

(b) later by $T/4$

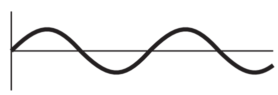


(c) A is 1.5 times larger



(d) λ is 1.5 times larger

(e) λ is 2/3 as large



P16.7 $f = \dfrac{40.0\ \text{vibrations}}{30.0\ \text{s}} = \dfrac{4}{3}\ \text{Hz}$ $\qquad v = \dfrac{425\ \text{cm}}{10.0\ \text{s}} = 42.5\ \text{cm/s}$

$$\lambda = \frac{v}{f} = \frac{42.5\ \text{cm/s}}{\frac{4}{3}\ \text{Hz}} = 31.9\ \text{cm} = \boxed{0.319\ \text{m}}$$

P16.8 Using data from the observations, we have $\lambda = 1.20\ \text{m}$ and $f = \dfrac{8.00}{12.0\ \text{s}}$

Therefore, $\quad v = \lambda f = (1.20\ \text{m})\left(\dfrac{8.00}{12.0\ \text{s}}\right) = \boxed{0.800\ \text{m/s}}$

P16.9 $y = (0.020\,0\ \text{m})\sin(2.11x - 3.62t)$ in SI units $\qquad A = \boxed{2.00\ \text{cm}}$

$k = 2.11\ \text{rad/m}$ $\qquad \lambda = \dfrac{2\pi}{k} = \boxed{2.98\ \text{m}}$

$\omega = 3.62\ \text{rad/s}$ $\qquad f = \dfrac{\omega}{2\pi} = \boxed{0.576\ \text{Hz}}$

$$v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{3.62}{2.11} = \boxed{1.72\ \text{m/s}}$$

P16.10 $v = f\lambda = (4.00\text{ Hz})(60.0\text{ cm}) = 240\text{ cm/s} = \boxed{2.40\text{ m/s}}$

***P16.11** (a) $\omega = 2\pi f = 2\pi\left(5\text{ s}^{-1}\right) = \boxed{31.4\text{ rad/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{20\text{ m/s}}{5\text{ s}^{-1}} = 4.00\text{ m}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{4\text{ m}} = \boxed{1.57\text{ rad/m}}$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take $A = 12$ cm. At $x = 0$ and $t = 0$ we have $y = (12\text{ cm})\sin\phi$. To make this fit $y = 0$, we take $\phi = 0$. Then

$\boxed{y = (12.0\text{ cm})\sin\left((1.57\text{ rad/m})x - (31.4\text{ rad/s})t\right)}$

(d) The transverse velocity is $\dfrac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$

Its maximum magnitude is $A\omega = 12\text{ cm}(31.4\text{ rad/s}) = \boxed{3.77\text{ m/s}}$

(e) $a_y = \dfrac{\partial v_y}{\partial t} = \dfrac{\partial}{\partial t}\left(-A\omega\cos(kx - \omega t)\right) = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.12\text{ m})\left(31.4\text{ s}^{-1}\right)^2 = \boxed{118\text{ m/s}^2}$

P16.12 At time t, the phase of $y = (15.0\text{ cm})\cos(0.157x - 50.3t)$ at coordinate x is $\phi = (0.157\text{ rad/cm})x - (50.3\text{ rad/s})t$. Since $60.0° = \dfrac{\pi}{3}$ rad, the requirement for point B is that $\phi_B = \phi_A \pm \dfrac{\pi}{3}$ rad, or (since $x_A = 0$),

$$(0.157\text{ rad/cm})x_B - (50.3\text{ rad/s})t = 0 - (50.3\text{ rad/s})t \pm \frac{\pi}{3}\text{ rad}$$

This reduces to $x_B = \dfrac{\pm\pi\text{ rad}}{3(0.157\text{ rad/cm})} = \boxed{\pm 6.67\text{ cm}}$.

P16.13 $y = 0.250\sin(0.300x - 40.0t)$ m

Compare this with the general expression $y = A\sin(kx - \omega t)$

(a) $A = \boxed{0.250\text{ m}}$

(b) $\omega = \boxed{40.0\text{ rad/s}}$

(c) $k = \boxed{0.300\text{ rad/m}}$

(d) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.300\text{ rad/m}} = \boxed{20.9\text{ m}}$

(e) $v = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\lambda = \left(\dfrac{40.0\text{ rad/s}}{2\pi}\right)(20.9\text{ m}) = \boxed{133\text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in } +x \text{ direction}}$.

***P16.14** (a) See figure at right.

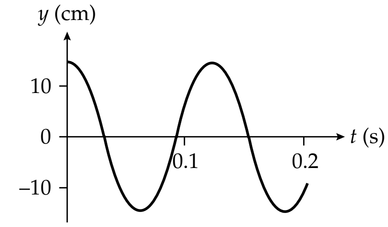


FIG. P16.14

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3} = \boxed{0.125 \text{ s}}$ is the time from one peak to the next one.

$\boxed{\text{This agrees with the period found in the example in the text.}}$

P16.15 (a) $A = y_{max} = 8.00 \text{ cm} = 0.080\,0 \text{ m}$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.800 \text{ m})} = 7.85 \text{ m}^{-1}$

$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi \text{ rad/s}$

Therefore, $y = A\sin(kx + \omega t)$

Or (where $y(0, t) = 0$ at $t = 0$) $\boxed{y = (0.080\,0)\sin(7.85x + 6\pi t) \text{ m}}$

(b) In general, $y = 0.080\,0\sin(7.85x + 6\pi t + \phi)$

Assuming $y(x, 0) = 0$ at $x = 0.100$ m

then we require that $0 = 0.080\,0\sin(0.785 + \phi)$

or $\phi = -0.785$

Therefore, $\boxed{y = 0.080\,0\sin(7.85x + 6\pi t - 0.785) \text{ m}}$

P16.16 (a)

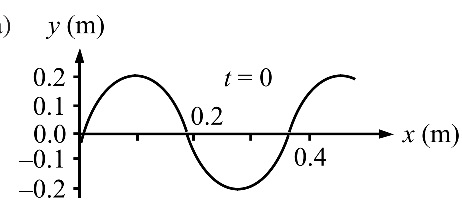


FIG. P16.16(a)

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.350 \text{ m}} = \boxed{18.0 \text{ rad/m}}$

$T = \frac{1}{f} = \frac{1}{12.0/\text{s}} = \boxed{0.083\,3 \text{ s}}$

$\omega = 2\pi f = 2\pi\,12.0/\text{s} = \boxed{75.4 \text{ rad/s}}$

$|v| = f\lambda = (12.0/\text{s})(0.350 \text{ m}) = \boxed{4.20 \text{ m/s}}$

(c) $y = A\sin(kx + \omega t + \phi)$ specializes to

$y = 0.200 \text{ m}\sin(18.0x/\text{m} + 75.4t/\text{s} + \phi)$

at $x = 0$, $t = 0$ we require

$-3.00 \times 10^{-2} \text{ m} = 0.200 \text{ m}\sin(+\phi)$

$\phi = -8.63° = -0.151 \text{ rad}$

so $\quad y(x, t) = \boxed{(0.200 \text{ m})\sin(18.0x/\text{m} + 75.4t/\text{s} - 0.151 \text{ rad})}$

P16.17 $y = (0.120\text{ m})\sin\left(\frac{\pi}{8}x + 4\pi t\right)$

(a) $v = \frac{dy}{dt}$: $\quad v = (0.120)(4\pi)\cos\left(\frac{\pi}{8}x + 4\pi t\right)$

$v(0.200\text{ s},\ 1.60\text{ m}) = \boxed{-1.51\text{ m/s}}$

$a = \frac{dv}{dt}$: $\quad a = (-0.120\text{ m})(4\pi)^2\sin\left(\frac{\pi}{8}x + 4\pi t\right)$

$a(0.200\text{ s},\ 1.60\text{ m}) = \boxed{0}$

(b) $k = \frac{\pi}{8} = \frac{2\pi}{\lambda}$: $\quad \lambda = \boxed{16.0\text{ m}}$

$\omega = 4\pi = \frac{2\pi}{T}$: $\quad T = \boxed{0.500\text{ s}}$

$$v = \frac{\lambda}{T} = \frac{16.0\text{ m}}{0.500\text{ s}} = \boxed{32.0\text{ m/s}}$$

P16.18 (a) Let us write the wave function as $\quad y(x,\ t) = A\sin(kx + \omega t + \phi)$

$$y(0,\ 0) = A\sin\phi = 0.020\,0\text{ m}$$

$$\left.\frac{dy}{dt}\right|_{0,\ 0} = A\omega\cos\phi = -2.00\text{ m/s}$$

Also, $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{0.025\,0\text{ s}} = 80.0\pi/\text{s}$

$$A^2 = x_i^2 + \left(\frac{v_i}{\omega}\right)^2 = (0.020\,0\text{ m})^2 + \left(\frac{2.00\text{ m/s}}{80.0\pi/\text{s}}\right)^2$$

$$A = \boxed{0.021\,5\text{ m}}$$

(b) $\frac{A\sin\phi}{A\cos\phi} = \frac{0.020\,0}{-2/80.0\pi} = -2.51 = \tan\phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19$ rad has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$$\phi = \pi - 1.19\text{ rad} = \boxed{1.95\text{ rad}}$$

(c) $v_{y,\,\max} = A\omega = 0.021\,5\text{ m}(80.0\pi/\text{s}) = \boxed{5.41\text{ m/s}}$

(d) $\lambda = v_x T = (30.0\text{ m/s})(0.025\,0\text{ s}) = 0.750\text{ m}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.750\text{ m}} = 8.38/\text{m} \qquad \omega = 80.0\pi/\text{s}$

$$\boxed{y(x,\ t) = (0.021\,5\text{ m})\sin(8.38x\text{ rad/m} + 80.0\pi t\text{ rad/s} + 1.95\text{ rad})}$$

P16.19 (a) $f = \dfrac{v}{\lambda} = \dfrac{(1.00 \text{ m/s})}{2.00 \text{ m}} = \boxed{0.500 \text{ Hz}}$

$\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14 \text{ rad/s}}$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2.00 \text{ m}} = \boxed{3.14 \text{ rad/m}}$

(c) $y = A\sin(kx - \omega t + \phi)$ becomes

$y = \boxed{(0.100 \text{ m})\sin(3.14x/\text{m} - 3.14t/\text{s} + 0)}$

(d) For $x = 0$ the wave function requires

$\boxed{y = (0.100 \text{ m})\sin(-3.14t/\text{s})}$

(e) $\boxed{y = (0.100 \text{ m})\sin(4.71 \text{ rad} - 3.14t/\text{s})}$

(f) $v_y = \dfrac{\partial y}{\partial t} = 0.100 \text{ m}(-3.14/\text{s})\cos(3.14x/\text{m} - 3.14t/\text{s})$

The cosine varies between +1 and −1, so

$v_y \le \boxed{0.314 \text{ m/s}}$

P16.20 (a) At $x = 2.00$ m, $y = \boxed{(0.100 \text{ m})\sin(1.00 \text{ rad} - 20.0t)}$ Because this disturbance varies sinusoidally in time, it describes simple harmonic motion.

(b) $y = (0.100 \text{ m})\sin(0.500x - 20.0t) = A\sin(kx - \omega t)$

so $\omega = 20.0$ rad/s and $f = \dfrac{\omega}{2\pi} = \boxed{3.18 \text{ Hz}}$

Section 16.3 **The Speed of Waves on Strings**

P16.21 The down and back distance is $4.00 \text{ m} + 4.00 \text{ m} = 8.00 \text{ m}$.

The speed is then $v = \dfrac{d_{\text{total}}}{t} = \dfrac{4(8.00 \text{ m})}{0.800 \text{ s}} = 40.0 \text{ m/s} = \sqrt{\dfrac{T}{\mu}}$

Now, $\mu = \dfrac{0.200 \text{ kg}}{4.00 \text{ m}} = 5.00 \times 10^{-2} \text{ kg/m}$

So $T = \mu v^2 = (5.00 \times 10^{-2} \text{ kg/m})(40.0 \text{ m/s})^2 = \boxed{80.0 \text{ N}}$

P16.22 (a) $\omega = 2\pi f = 2\pi(500) = 3\,140 \text{ rad/s}, \; k = \dfrac{\omega}{v} = \dfrac{3\,140}{196} = 16.0 \text{ rad/m}$

$\boxed{y = (2.00 \times 10^{-4} \text{ m})\sin(16.0x - 3\,140t)}$

(b) $v = 196 \text{ m/s} = \sqrt{\dfrac{T}{4.10 \times 10^{-3} \text{ kg/m}}}$

$T = \boxed{158 \text{ N}}$

P16.23 $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{1\,350\ \text{kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\ \text{kg/m}}} = \boxed{520\ \text{m/s}}$

P16.24 $v = \sqrt{\dfrac{T}{\mu}}$

$$T = \mu v^2 = \rho A v^2 = \rho\pi r^2 v^2$$

$$T = \left(8\,920\ \text{kg/m}^3\right)(\pi)\left(7.50\times10^{-4}\ \text{m}\right)^2(200\ \text{m/s})^2$$

$$T = \boxed{631\ \text{N}}$$

P16.25 $T = Mg$ is the tension; $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg}{m/L}} = \sqrt{\dfrac{MgL}{m}} = \dfrac{L}{t}$ is the wave speed.

Then, $\dfrac{MgL}{m} = \dfrac{L^2}{t^2}$

and $g = \dfrac{Lm}{Mt^2} = \dfrac{1.60\ \text{m}\left(4.00\times10^{-3}\ \text{kg}\right)}{3.00\ \text{kg}\left(3.61\times10^{-2}\ \text{s}\right)^2} = \boxed{1.64\ \text{m/s}^2}$

P16.26 The period of the pendulum is $T = 2\pi\sqrt{\dfrac{L}{g}}$

Let F represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{MgL}{m}}$$

Since it might be difficult to measure L precisely, we eliminate $\sqrt{L} = \dfrac{T\sqrt{g}}{2\pi}$

so

$$v = \sqrt{\frac{Mg}{m}}\frac{T\sqrt{g}}{2\pi} = \boxed{\frac{Tg}{2\pi}\sqrt{\frac{M}{m}}}$$

P16.27 Since μ is constant, $\mu = \dfrac{T_2}{v_2^2} = \dfrac{T_1}{v_1^2}$ and

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{30.0\ \text{m/s}}{20.0\ \text{m/s}}\right)^2(6.00\ \text{N}) = \boxed{13.5\ \text{N}}$$

P16.28 From the free-body diagram $mg = 2T\sin\theta$

$$T = \frac{mg}{2\sin\theta}$$

The angle θ is found from $\cos\theta = \dfrac{3L/8}{L/2} = \dfrac{3}{4}$

$$\therefore \theta = 41.4°$$

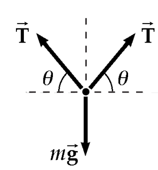


FIG. P16.28

(a) $v = \sqrt{\dfrac{T}{\mu}}$ $\quad v = \sqrt{\dfrac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\dfrac{9.80\ \text{m/s}^2}{2\left(8.00\times10^{-3}\ \text{kg/m}\right)\sin 41.4°}}\right)\sqrt{m}$

or $v = \boxed{\left(30.4\ \dfrac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}}$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89\ \text{kg}}$

P16.29 If the tension in the wire is T, the tensile stress is

$$\text{Stress} = \frac{T}{A} \qquad \text{so} \qquad T = A(\text{stress})$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{m/L}} = \sqrt{\frac{\text{Stress}}{m/AL}} = \sqrt{\frac{\text{Stress}}{m/\text{Volume}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70\times10^{8}\ \text{Pa}}{7\,860\ \text{kg/m}^3}} = \boxed{185\ \text{m/s}}$$

***P16.30** (a) f has units Hz = 1/s, so $T = \frac{1}{f}$ has units of seconds, $\boxed{\text{s}}$. For the other T we have $T = \mu v^2$, with units $\frac{\text{kg}}{\text{m}}\frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \boxed{\text{N}}$.

(b) $\boxed{\text{The first } T \text{ is period of time; the second is force of tension.}}$

P16.31 The total time is the sum of the two times.

In each wire $\quad t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\quad \mu = \rho A = \frac{\pi\rho d^2}{4}$

Thus, $\quad t = L\left(\frac{\pi\rho d^2}{4T}\right)^{1/2}$

For copper, $\quad t = (20.0)\left[\frac{(\pi)(8\,920)(1.00\times10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.137\ \text{s}$

For steel, $\quad t = (30.0)\left[\frac{(\pi)(7\,860)(1.00\times10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.192\ \text{s}$

The total time is $\quad 0.137 + 0.192 = \boxed{0.329\ \text{s}}$

Section 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

P16.32 $f = \frac{v}{\lambda} = \frac{30.0}{0.500} = 60.0\ \text{Hz} \qquad \omega = 2\pi f = 120\pi\ \text{rad/s}$

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\left(\frac{0.180}{3.60}\right)(120\pi)^2(0.100)^2(30.0) = \boxed{1.07\ \text{kW}}$$

P16.33 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source. It is spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{\mathcal{P}}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\mathcal{P}}{2\pi r}}$$

P16.34 T = constant; $v=\sqrt{\frac{T}{\mu}}$; $\mathcal{P}=\frac{1}{2}\mu\omega^2A^2v$

(a) If L is doubled, v remains constant and $\boxed{\mathcal{P} \text{ is constant}}$.

(b) If A is doubled and ω is halved, $\mathcal{P}\propto\omega^2A^2$ $\boxed{\text{remains constant}}$.

(c) If λ and A are doubled, the product $\omega^2A^2\propto\frac{A^2}{\lambda^2}$ remains constant, so $\boxed{\mathcal{P} \text{ remains constant}}$.

(d) If L and λ are halved, then $\omega^2\propto\frac{1}{\lambda^2}$ is quadrupled, so $\boxed{\mathcal{P} \text{ is quadrupled}}$.

(Changing L doesn't affect $\mathcal{P}$.)

P16.35 $A=5.00\times10^{-2}$ m $\quad \mu=4.00\times10^{-2}$ kg/m $\quad \mathcal{P}=300$ W $\quad T=100$ N

Therefore, $\quad v=\sqrt{\frac{T}{\mu}}=50.0$ m/s

$\mathcal{P}=\frac{1}{2}\mu\omega^2A^2v$: $\quad \omega^2=\frac{2\mathcal{P}}{\mu A^2v}=\frac{2(300)}{(4.00\times10^{-2})(5.00\times10^{-2})^2(50.0)}$

$$\omega=346 \text{ rad/s}$$

$$f=\frac{\omega}{2\pi}=\boxed{55.1 \text{ Hz}}$$

P16.36 $\mu=30.0$ g/m $=30.0\times10^{-3}$ kg/m

$\lambda=1.50$ m

$f=50.0$ Hz: $\quad \omega=2\pi f=314$ s^{-1}

$2A=0.150$ m: $\quad A=7.50\times10^{-2}$ m

(a) $y=A\sin\left(\frac{2\pi}{\lambda}x-\omega t\right)$

$$\boxed{y=(7.50\times10^{-2})\sin(4.19x-314t)}$$

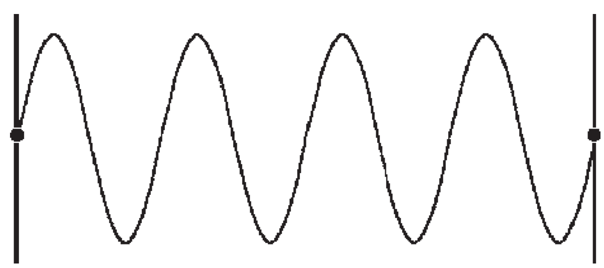


FIG. P16.36

(b) $\mathcal{P}=\frac{1}{2}\mu\omega^2A^2v=\frac{1}{2}(30.0\times10^{-3})(314)^2(7.50\times10^{-2})^2\left(\frac{314}{4.19}\right)$ W $\quad \boxed{\mathcal{P}=625 \text{ W}}$

P16.37 (a) $v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800}\ \text{m/s} = \boxed{62.5\ \text{m/s}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800}\ \text{m} = \boxed{7.85\ \text{m}}$

(c) $f = \frac{50.0}{2\pi} = \boxed{7.96\ \text{Hz}}$

(d) $\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(12.0\times10^{-3}\right)(50.0)^2(0.150)^2(62.5)\ \text{W} = \boxed{21.1\ \text{W}}$

P16.38 Comparing $y = 0.35\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ with $y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$

we have $k = \frac{3\pi}{m}$, $\omega = 10\pi/\text{s}$, $A = 0.35$ m. Then $v = f\lambda = 2\pi f\frac{\lambda}{2\pi} = \frac{\omega}{k} = \frac{10\pi/\text{s}}{3\pi/\text{m}} = 3.33\ \text{m/s}$.

(a) The rate of energy transport is

$$\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)(10\pi/\text{s})^2(0.35\ \text{m})^2\,3.33\ \text{m/s} = \boxed{15.1\ \text{W}}$$

(b) The energy per cycle is

$$E_\lambda = \mathcal{P}T = \frac{1}{2}\mu\omega^2A^2\lambda = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)(10\pi/\text{s})^2(0.35\ \text{m})^2\frac{2\pi\ \text{m}}{3\pi} = \boxed{3.02\ \text{J}}$$

P16.39 Originally,

$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2v$$

$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2\sqrt{\frac{T}{\mu}}$$

$$\mathcal{P}_0 = \frac{1}{2}\omega^2A^2\sqrt{T\mu}$$

The doubled string will have doubled mass-per-length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times.

$$\boxed{\sqrt{2}\mathcal{P}_0} = \frac{1}{2}\omega^2A^2\sqrt{T2\mu}$$

***P16.40** As for a string wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy and the frequency stays constant. As the speed drops the amplitude must increase.

We write $\mathcal{P} = FvA^2$ where F is some constant. With no absorption of energy,

$$Fv_{\text{bedrock}}A^2_{\text{bedrock}} = Fv_{\text{mudfill}}A^2_{\text{mudfill}}$$

$$\sqrt{\frac{v_{\text{bedrock}}}{v_{\text{mudfill}}}} = \frac{A_{\text{mudfill}}}{A_{\text{bedrock}}} = \sqrt{\frac{25v_{\text{mudfill}}}{v_{\text{mudfill}}}} = 5$$

$\boxed{\text{The amplitude increases by 5.00 times.}}$

Section 16.6 **The Linear Wave Equation**

P16.41 (a) $A = (7.00 + 3.00)4.00$ yields $\boxed{A = 40.0}$

(b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal.

Thus, $7.00\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$ requires $\boxed{A = 7.00,\ B = 0,\ \text{and } C = 3.00}$.

(c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs.

In

$$A + B\cos(Cx + Dt + E) = 0 + 7.00 \text{ mm}\cos(3.00x + 4.00t + 2.00)$$

the equality of average values requires that $\boxed{A = 0}$. The equality of maximum values requires $\boxed{B = 7.00 \text{ mm}}$. The equality for the wavelength or periodicity as a function of x requires $\boxed{C = 3.00 \text{ rad/m}}$. The equality of period requires $\boxed{D = 4.00 \text{ rad/s}}$, and the equality of zero-crossings requires $\boxed{E = 2.00 \text{ rad}}$.

P16.42 The linear wave equation is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

If $y = e^{b(x-vt)}$

then $\dfrac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\dfrac{\partial y}{\partial x} = be^{b(x-vt)}$

$\dfrac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)}$ and $\dfrac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$

Therefore, $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution.

P16.43 The linear wave equation is $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2}$

To show that $y = \ln[b(x-vt)]$ is a solution, we find its first and second derivatives with respect to x and t and substitute into the equation.

$$\frac{\partial y}{\partial t} = \frac{1}{b(x-vt)}(-bv) \qquad \frac{\partial^2 y}{\partial t^2} = \frac{-1(-bv)^2}{b^2(x-vt)^2} = -\frac{v^2}{(x-vt)^2}$$

$$\frac{\partial y}{\partial x} = [b(x-vt)]^{-1}b \qquad \frac{\partial^2 y}{\partial x^2} = -\frac{b}{b}(x-vt)^{-2} = -\frac{1}{(x-vt)^2}$$

Then $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{1}{v^2}\dfrac{(-v^2)}{(x-vt)^2} = -\dfrac{1}{(x-vt)^2} = \dfrac{\partial^2 y}{\partial x^2}$ so the given wave function is a solution.

P16.44 (a) From $y = x^2 + v^2t^2$,

evaluate $\dfrac{\partial y}{\partial x} = 2x \qquad \dfrac{\partial^2 y}{\partial x^2} = 2$

$\dfrac{\partial y}{\partial t} = v^2 2t \qquad \dfrac{\partial^2 y}{\partial t^2} = 2v^2$

Does $\dfrac{\partial^2 y}{\partial t^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$?

By substitution, we must test $2 = \dfrac{1}{v^2}2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\dfrac{1}{2}(x+vt)^2 + \dfrac{1}{2}(x-vt)^2 = \dfrac{1}{2}x^2 + xvt + \dfrac{1}{2}v^2t^2 + \dfrac{1}{2}x^2 - xvt + \dfrac{1}{2}v^2t^2$

$= x^2 + v^2t^2$ as required.

So

$\boxed{f(x+vt) = \dfrac{1}{2}(x+vt)^2}$ and $\boxed{g(x-vt) = \dfrac{1}{2}(x-vt)^2}$

(c) $y = \sin x \cos vt$ makes

$\dfrac{\partial y}{\partial x} = \cos x \cos vt \qquad \dfrac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$

$\dfrac{\partial y}{\partial t} = -v \sin x \sin vt \qquad \dfrac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$

Then

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

becomes $-\sin x \cos vt = \dfrac{-1}{v^2}v^2 \sin x \cos vt$ which is true as required.

Note $\sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt$

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$\boxed{f(x+vt) = \dfrac{1}{2}\sin(x+vt)}$ and $\boxed{g(x-vt) = \dfrac{1}{2}\sin(x-vt)}$

Additional Problems

P16.45 Assume a typical distance between adjacent people ~1 m.

Then the wave speed is $v = \dfrac{\Delta x}{\Delta t} \sim \dfrac{1\text{ m}}{0.1\text{ s}} \sim 10\text{ m/s}$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10\text{ m/s}} = 63\text{ s}\ \boxed{\sim 1\text{ min}}$$

***P16.46** (a) From $y = 0.150\text{ m}\sin(0.8x - 50t)$

we compute $dy/dt = 0.150\text{ m}\,(-50)\cos(0.8x - 50t)$

and $a = d^2y/dt^2 = -0.150\text{ m}\,(-50/\text{s})^2 \sin(0.8x - 50t)$

Then $a_{max} = \boxed{375\text{ m/s}^2}$

(b) For the 1-cm segment with maximum force acting on it, $\Sigma F = ma = [12\text{ g}/(100\text{ cm})]$ $1\text{ cm}\ 375\text{ m/s}^2 = \boxed{0.045\,0\text{ N}}$

We find the tension in the string from $v = f\lambda = \omega/k = (50/\text{s})/(0.8/\text{m}) = 62.5\text{ m/s} = (T/\mu)^{1/2}$ $T = v^2\mu = (62.5\text{ m/s})^2(0.012\text{ kg/m}) = 46.9\text{ N}$.

The maximum transverse force is very small compared to the tension, more than a thousand times smaller.

P16.47 The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate)

Thus,

$$v = (19.0 \times 10^{-3}\text{ m/frame})(24.0\text{ frames/s}) = \boxed{0.456\text{ m/s}}$$

P16.48 (a) $0.175\text{ m} = (0.350\text{ m})\sin[(99.6\text{ rad/s})t]$

$\therefore \sin[(99.6\text{ rad/s})t] = 0.5$

The smallest two angles for which the sine function is 0.5 are 30° and 150°, i.e., 0.523 6 rad and 2.618 rad.

$(99.6\text{ rad/s})t_1 = 0.523\,6\text{ rad}$, thus $t_1 = 5.26\text{ ms}$

$(99.6\text{ rad/s})t_2 = 2.618\text{ rad}$, thus $t_2 = 26.3\text{ ms}$

$\Delta t \equiv t_2 - t_1 = 26.3\text{ ms} - 5.26\text{ ms} = \boxed{21.0\text{ ms}}$

(b) Distance traveled by the wave $= \left(\dfrac{\omega}{k}\right)\Delta t = \left(\dfrac{99.6\text{ rad/s}}{1.25\text{ rad/m}}\right)(21.0\times10^{-3}\text{ s}) = \boxed{1.68\text{ m}}$.

P16.49 Energy is conserved as the block moves down distance x:

$$(K + U_g + U_s)_{\text{top}} + \Delta E = (K + U_g + U_s)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$x = \frac{2Mg}{k}$$

(a) $T = kx = 2Mg = 2(2.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{39.2\text{ N}}$

(b) $L = L_0 + x = L_0 + \dfrac{2Mg}{k}$

$L = 0.500\text{ m} + \dfrac{39.2\text{ N}}{100\text{ N/m}} = \boxed{0.892\text{ m}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}}$

$v = \sqrt{\dfrac{39.2\text{ N}\times0.892\text{ m}}{5.0\times10^{-3}\text{ kg}}}$

$v = \boxed{83.6\text{ m/s}}$

P16.50 $Mgx = \frac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \frac{2Mg}{k}}$

(c) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \boxed{\sqrt{\frac{2Mg}{m}\left(L_0 + \frac{2Mg}{k}\right)}}$

***P16.51** (a) The energy a wave crest carries is constant in the absence of absorption. Then the rate at which energy passes a stationary point, which is the power of the wave, is constant. The power is proportional to the square of the amplitude and to the wave speed. The speed decreases as the wave moves into shallower water near shore, so the amplitude must increase.

(b) For the wave described, with a single direction of energy transport, the intensity is the same at the deep-water location ① and at the place ② with depth 9 m. To express the constant intensity we write

$$A_1^2 v_1 = A_2^2 v_2 = A_2^2 \sqrt{gd_2}$$

$$(1.8\text{ m})^2\ 200\text{ m/s} = A_2^2 \sqrt{(9.8\text{ m/s}^2)9\text{ m}}$$

$$= A_2^2 9.39\text{ m/s}$$

$$A_2 = 1.8\left(\frac{200\text{ m/s}}{9.39\text{ m/s}}\right)^{1/2}$$

$$= \boxed{8.31\text{ m}}$$

(c) As the water depth goes to zero, our model would predict zero speed and infinite amplitude. In fact the amplitude must be finite as the wave comes ashore. As the speed decreases the wavelength also decreases. When it becomes comparable to the water depth, or smaller, our formula $\sqrt{gd}$ for wave speed no longer applies.

P16.52 Assuming the incline to be frictionless and taking the positive x-direction to be up the incline:

$\sum F_x = T - Mg\sin\theta = 0$ or the tension in the string is $T = Mg\sin\theta$

The speed of transverse waves in the string is then $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg\sin\theta}{m/L}} = \sqrt{\frac{MgL\sin\theta}{m}}$

The time interval for a pulse to travel the string's length is $\Delta t = \frac{L}{v} = L\sqrt{\frac{m}{MgL\sin\theta}} = \boxed{\sqrt{\frac{mL}{Mg\sin\theta}}}$

***P16.53** (a) In $\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v$ where v is the wave speed, the quantity ωA is the maximum particle speed v_{ymax}. We have $\mu = 0.5 \times 10^{-3}$ kg/m and $v = (T/\mu)^{1/2} = (20\text{ N}/0.5 \times 10^{-3}\text{ kg/m})^{1/2} =$ 200 m/s

Then $\mathcal{P} = \frac{1}{2}(0.5 \times 10^{-3}\text{ kg/m})\, v^2_{ymax}\,(200\text{ m/s}) = \boxed{(0.050\,0\text{ kg/s})v^2_{y,max}}$

(b) $\boxed{\text{The power is proportional to the square of the maximum particle speed.}}$

(c) In time $t = (3\text{ m})/v = (3\text{ m})/(200\text{ m/s}) = 1.5 \times 10^{-2}$ s, all the energy in a 3-m length of string goes past a point. Therefore the amount of this energy is

$E = \mathcal{P}t = (0.05\text{ kg/s})\, v^2_{y,\max}\,(0.015\text{ s}) = 7.5 \times 10^{-4}\text{ kg } v^2_{y,\max}$

The mass of this section is $m_3 = (0.5 \times 10^{-3}\text{ kg/m})3\text{ m} = 1.5 \times 10^{-3}$ kg so $(1/2)m_3 = 7.5 \times 10^{-4}$ kg and $\boxed{E = (1/2)\, m_3 v^2_{y,\max}} = K_{max}$. The string also contains potential energy. We could write its energy as U_{max} or as $U_{avg} + K_{avg}$

(d) $E = \mathcal{P}t = (0.05\text{ kg/s})\, v^2_{y,max}\,(6\text{ s}) = \boxed{0.300\text{ kg } v^2_{y,\max}}$

P16.54 $v = \sqrt{\dfrac{T}{\mu}}$ and in this case $T = mg$; therefore, $m = \dfrac{\mu v^2}{g}$

Now $v = f\lambda$ implies $v = \dfrac{\omega}{k}$ so that

$$m = \frac{\mu}{g}\left(\frac{\omega}{k}\right)^2 = \frac{0.250\text{ kg/m}}{9.80\text{ m/s}^2}\left[\frac{18\pi\text{ s}^{-1}}{0.750\pi\text{ m}^{-1}}\right]^2 = \boxed{14.7\text{ kg}}$$

P16.55 Let M = mass of block, m = mass of string. For the block, $\sum F = ma$ implies $T = \dfrac{mv_b^2}{r} = m\omega^2 r$

The speed of a wave on the string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M\omega^2 r}{m/r}} = r\omega\sqrt{\frac{M}{m}}$$

$$t = \frac{r}{v} = \frac{1}{\omega}\sqrt{\frac{m}{M}}$$

$$\theta = \omega t = \sqrt{\frac{m}{M}} = \sqrt{\frac{0.003\,2\text{ kg}}{0.450\text{ kg}}} = \boxed{0.084\,3\text{ rad}}$$

P16.56 (a) $\mu = \dfrac{dm}{dL} = \rho A\dfrac{dx}{dx} = \rho A$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{[\rho(ax+b)]}} = \sqrt{\frac{T}{\left[\rho\left(10^{-3}x + 10^{-2}\right)\text{cm}^2\right]}}$$

With all SI units,

$$\boxed{v = \sqrt{\frac{T}{\left[\rho\left(10^{-3}x + 10^{-2}\right)10^{-4}\right]}}\text{ m/s}}$$

(b) $$v\big|_{x=0} = \sqrt{\frac{24.0}{\left[(2\,700)\left(0 + 10^{-2}\right)\left(10^{-4}\right)\right]}} = \boxed{94.3\text{ m/s}}$$

$$v\big|_{x=10.0} = \sqrt{\frac{24.0}{\left[(2\,700)\left(10^{-2} + 10^{-2}\right)\left(10^{-4}\right)\right]}} = \boxed{66.7\text{ m/s}}$$

P16.57 $v = \sqrt{\dfrac{T}{\mu}}$ where $T = \mu x g$, to support the weight of a length x, of rope.

Therefore, $v = \sqrt{gx}$

But $v = \dfrac{dx}{dt}$, so that $dt = \dfrac{dx}{\sqrt{gx}}$

and $$t = \int_0^L \frac{dx}{\sqrt{gx}} = \frac{1}{\sqrt{g}} \frac{\sqrt{x}}{\frac{1}{2}}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

P16.58 At distance x from the bottom, the tension is $T = \left(\dfrac{mxg}{L}\right) + Mg$, so the wave speed is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \sqrt{xg + \left(\frac{MgL}{m}\right)} = \frac{dx}{dt}$$

(a) Then $t = \int_0^t dt = \int_0^L \left[xg + \left(\dfrac{MgL}{m}\right)\right]^{-1/2} dx$ $\qquad t = \dfrac{1}{g} \dfrac{\left[xg + (MgL/m)\right]^{1/2}}{\frac{1}{2}}\Bigg|_{x=0}^{x=L}$

$$t = \frac{2}{g}\left[\left(Lg + \frac{MgL}{m}\right)^{1/2} - \left(\frac{MgL}{m}\right)^{1/2}\right] \qquad \boxed{t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m+M} - \sqrt{M}}{\sqrt{m}}\right)}$$

(b) When $M = 0$, as in the previous problem, $t = 2\sqrt{\dfrac{L}{g}}\left(\dfrac{\sqrt{m} - 0}{\sqrt{m}}\right) = \boxed{2\sqrt{\dfrac{L}{g}}}$

(c) As $m \to 0$ we expand $\sqrt{m+M} = \sqrt{M}\left(1 + \dfrac{m}{M}\right)^{1/2} = \sqrt{M}\left(1 + \dfrac{1}{2}\dfrac{m}{M} - \dfrac{1}{8}\dfrac{m^2}{M^2} + \cdots\right)$

to obtain $$t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{M} + \frac{1}{2}\left(m/\sqrt{M}\right) - \frac{1}{8}\left(m^2/M^{3/2}\right) + \cdots - \sqrt{M}}{\sqrt{m}}\right)$$

$$t \approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

P16.59 (a) The speed in the lower half of a rope of length L is the same function of distance (from the bottom end) as the speed along the entire length of a rope of length $\left(\dfrac{L}{2}\right)$.

Thus, the time required $= 2\sqrt{\dfrac{L'}{g}}$ with $L' = \dfrac{L}{2}$

and the time required $= 2\sqrt{\dfrac{L}{2g}} = \boxed{0.707\left(2\sqrt{\dfrac{L}{g}}\right)}$

It takes the pulse more that 70% of the total time to cover 50% of the distance.

(b) By the same reasoning applied in part (a), the distance climbed in τ is given by $d = \dfrac{g\tau^2}{4}$

For $\tau = \dfrac{t}{2} = \sqrt{\dfrac{L}{g}}$, we find the *distance climbed* $= \boxed{\dfrac{L}{4}}$.

In half the total trip time, the pulse has climbed $\dfrac{1}{4}$ of the total length.

P16.60 (a) Consider a short section of chain at the top of the loop. A free-body diagram is shown. Its length is $s = R(2\theta)$ and its mass is $\mu R 2\theta$. In the frame of reference of the center of the loop, Newton's second law is

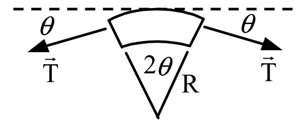


FIG. P16.60(a)

$$\sum F_y = ma_y \qquad 2T\sin\theta \text{ down} = \frac{mv_0^2}{R} \text{ down} = \frac{\mu R 2\theta v_0^2}{R}$$

For a very short section, $\sin\theta = \theta$ and $\boxed{T = \mu v_0^2}$

(b) The wave speed is $v = \sqrt{\dfrac{T}{\mu}} = \boxed{v_0}$

(c) In the frame of reference of the center of the loop, each pulse moves with equal speed clockwise and counterclockwise.

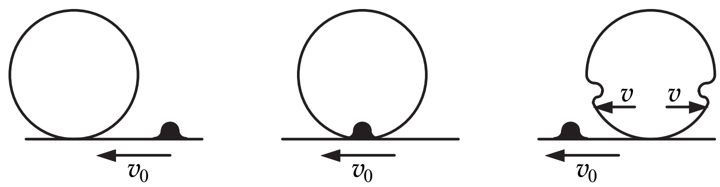


FIG. P16.60(c1)

In the frame of reference of the ground, once pulse moves backward at speed $v_0 + v = 2v_0$ and the other forward at $v_0 - v = 0$.

The one pulse makes two revolutions while the loop makes one revolution and the other pulse does not move around the loop. If it is generated at the six-o'clock position, it will stay at the six-o'clock position.

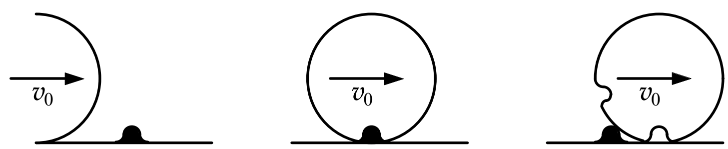


FIG. P16.60(c2)

P16.61 Young's modulus for the wire may be written as $Y = \dfrac{T/A}{\Delta L/L}$, where T is the tension maintained in the wire and ΔL is the elongation produced by this tension. Also, the mass density of the wire may be expressed as $\rho = \dfrac{\mu}{A}$

The speed of transverse waves in the wire is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T/A}{\mu/A}} = \sqrt{\frac{Y(\Delta L/L)}{\rho}}$$

and the strain in the wire is $\dfrac{\Delta L}{L} = \dfrac{\rho v^2}{Y}$

If the wire is aluminum and $v = 100$ m/s, the strain is

$$\frac{\Delta L}{L} = \frac{\left(2.70\times10^3 \text{ kg/m}^3\right)\left(100 \text{ m/s}\right)^2}{7.00\times10^{10} \text{ N/m}^2} = \boxed{3.86\times10^{-4}}$$

P16.62 (a) Assume the spring is originally stationary throughout, extended to have a length L much greater than its equilibrium length. We start moving one end forward with the speed v at which a wave propagates on the spring. In this way we create a single pulse of compression that moves down the length of the spring. For an increment of spring with length dx and mass dm, just as the pulse swallows it up, $\sum F = ma$

becomes $kdx = adm$ or $\dfrac{k}{dm/dx} = a$

But $\dfrac{dm}{dx} = \mu$ so $a = \dfrac{k}{\mu}$

Also, $a = \dfrac{dv}{dt} = \dfrac{v}{t}$ when $v_i = 0$

But $L = vt,$ so $a = \dfrac{v^2}{L}$

Equating the two expressions for a, we have $\dfrac{k}{\mu} = \dfrac{v^2}{L}$ or $\boxed{v = \sqrt{\dfrac{kL}{\mu}}}$

(b) Using the expression from part (a) $v = \sqrt{\dfrac{kL}{\mu}} = \sqrt{\dfrac{kL^2}{m}} = \sqrt{\dfrac{(100 \text{ N/m})(2.00 \text{ m})^2}{0.400 \text{ kg}}} = \boxed{31.6 \text{ m/s}}$

P16.63 (a) $\mathcal{P}(x) = \dfrac{1}{2}\mu\omega^2 A^2 v = \dfrac{1}{2}\mu\omega^2 A_0^2 e^{-2bx}\left(\dfrac{\omega}{k}\right) = \boxed{\dfrac{\mu\omega^3}{2k}A_0^2 e^{-2bx}}$

(b) $\mathcal{P}(0) = \boxed{\dfrac{\mu\omega^3}{2k}A_0^2}$

(c) $\dfrac{\mathcal{P}(x)}{\mathcal{P}(0)} = \boxed{e^{-2bx}}$

P16.64 $v = \dfrac{4\,450 \text{ km}}{9.50 \text{ h}} = 468 \text{ km/h} = \boxed{130 \text{ m/s}}$

$\bar{d} = \dfrac{v^2}{g} = \dfrac{(130 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} = \boxed{1\,730 \text{ m}}$

P16.65 (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x) = mx + b$

To have $\mu(0) = \mu_0$ we require $b = \mu_0$. Then $\mu(L) = \mu_L = mL + \mu_0$

so $m = \dfrac{\mu_L - \mu_0}{L}$

Then $\boxed{\mu(x) = \dfrac{(\mu_L - \mu_0)x}{L} + \mu_0}$

(b) From $v = \dfrac{dx}{dt}$, the time required to move from x to $x + dx$ is $\dfrac{dx}{v}$. The time required to move from 0 to L is

$$\Delta t = \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{T/\mu}} = \frac{1}{\sqrt{T}}\int_0^L \sqrt{\mu(x)}\,dx$$

$$\Delta t = \frac{1}{\sqrt{T}}\int_0^L \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0\right)^{1/2}\left(\frac{\mu_L - \mu_0}{L}\right)dx\left(\frac{L}{\mu_L - \mu_0}\right)$$

$$\Delta t = \frac{1}{\sqrt{T}}\left(\frac{L}{\mu_L - \mu_0}\right)\left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0\right)^{3/2}\frac{1}{\frac{3}{2}}\Bigg|_0^L$$

$$\Delta t = \frac{2L}{3\sqrt{T}(\mu_L - \mu_0)}\left(\mu_L^{3/2} - \mu_0^{3/2}\right)$$

$$\Delta t = \frac{2L\left(\sqrt{\mu_L} - \sqrt{\mu_0}\right)\left(\mu_L + \sqrt{\mu_L\mu_0} + \mu_0\right)}{3\sqrt{T}\left(\sqrt{\mu_L} - \sqrt{\mu_0}\right)\left(\sqrt{\mu_L} + \sqrt{\mu_0}\right)}$$

$$\Delta t = \frac{2L}{3\sqrt{T}}\left(\frac{\mu_L + \sqrt{\mu_L\mu_0} + \mu_0}{\sqrt{\mu_L} + \sqrt{\mu_0}}\right)$$

ANSWERS TO EVEN PROBLEMS

P16.2 See the solution. The graph (b) has the same amplitude and wavelength as graph (a). It differs just by being shifted toward larger x by 2.40 m. The wave has traveled 2.40 m to the right.

P16.4 184 km

P16.6 See the solution

P16.8 0.800 m/s

P16.10 2.40 m/s

P16.12 ±6.67 cm

P16.14 (a) see the solution (b) 0.125 s, in agreement with the example

P16.16 (a) see the solution (b) 18.0/m; 83.3 ms; 75.4 rad/s; 4.20 m/s
(c) $(0.2\text{ m})\sin(18x + 75.4t - 0.151)$

P16.18 (a) 0.021 5 m (b) 1.95 rad (c) 5.41 m/s (d) $y(x, t) = (0.021\,5\text{ m})\sin(8.38x + 80.0\pi t + 1.95)$

P16.20 (a) see the solution (b) 3.18 Hz

P16.22 (a) $y = (0.2\text{ mm})\sin(16x - 3\,140t)$ (b) 158 N

P16.24 631 N

P16.26 $v = \dfrac{Tg}{2\pi}\sqrt{\dfrac{M}{m}}$

P16.28 (a) $v = \left(30.4\ \dfrac{\text{m}}{\text{s}\cdot\sqrt{\text{kg}}}\right)\sqrt{m}$ (b) 3.89 kg

P16.30 (a) s and N (b) The first *T* is period of time; the second is force of tension.

P16.32 1.07 kW

P16.34 (a), (b), (c) $\mathcal{P}$ is a constant (d) $\mathcal{P}$ is quadrupled

P16.36 (a) $y = (0.075\,0)\sin(4.19x - 314t)$ (b) 625 W

P16.38 (a) 15.1 W (b) 3.02 J

P16.40 As for a string wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy and the frequency stays constant. As the speed drops the amplitude must increase. It increases by 5.00 times.

P16.42 see the solution

P16.44 (a) see the solution (b) $\frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2$ (c) $\frac{1}{2}\sin(x+vt) + \frac{1}{2}\sin(x-vt)$

P16.46 (a) 375 m/s^2 (b) 0.0450 N. This force is very small compared to the 46.9-N tension, more than a thousand times smaller.

P16.48 (a) 21.0 ms (b) 1.68 m

P16.50 (a) $2Mg$ (b) $L_0 + \dfrac{2Mg}{k}$ (c) $\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}$

P16.52 $\Delta t = \sqrt{\dfrac{mL}{Mg\sin\theta}}$

P16.54 14.7 Kg

P16.56 (a) $v = \sqrt{\dfrac{T}{\rho(10^{-7}x + 10^{-6})}}$ in SI units (b) 94.3 m/s; 66.7 m/s

P16.58 See the solution.

P16.60 (a) μv_0^2 (b) v_0 (c) One travels 2 rev and the other does not move around the loop.

P16.62 (a) see the solution (b) 31.6 m/s

P16.64 130 m/s; 1.73 km