37

Interference of Light Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q37.1 (a) Two waves interfere constructively if their path difference is zero, or an integral multiple of the wavelength, according to $\delta = m\lambda$, with $m = 0, 1, 2, 3, \ldots$.

(b) Two waves interfere destructively if their path difference is a half wavelength, or an odd multiple of $\frac{\lambda}{2}$, described by $\delta = \left(m + \frac{1}{2}\right)\lambda$, with $m = 0, 1, 2, 3, \ldots$.

Q37.2 The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no *coherence* between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

***Q37.3** (i) The angles in the interference pattern are controlled by λ/d, which we estimate in each case: (a) $0.45\ \mu\text{m}/400\ \mu\text{m} \approx 1.1 \times 10^{-3}$ (b) $0.7\ \mu\text{m}/400\ \mu\text{m} \approx 1.6 \times 10^{-3}$ (c) and (d) $0.7\ \mu\text{m}/800\ \mu\text{m} \approx 0.9 \times 10^{-3}$. The ranking is b > a > c = d.

(ii) Now we consider $L\lambda/d$: (a) $4\text{ m}(0.45\ \mu\text{m}/400\ \mu\text{m}) \approx 4.4\text{ mm}$ (b) $4\text{ m}(0.7\ \mu\text{m}/400\ \mu\text{m}) \approx 7\text{ mm}$ (c) $4\text{ m}(0.7\ \mu\text{m}/800\ \mu\text{m}) \approx 3\text{ mm}$ (d) $8\text{ m}(0.7\ \mu\text{m}/800\ \mu\text{m}) \approx 7\text{ mm}$. The ranking is b = d > a > c.

***Q37.4** Yes. A single beam of laser light going into the slits divides up into several fuzzy-edged beams diverging from the point halfway between the slits.

***Q37.5** Answer (c). Underwater, the wavelength of the light decreases according to $\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}}$. Since the angles between positions of light and dark bands are proportional to λ, the underwater fringe separations decrease.

Q37.6 Every color produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With white light, the central maximum is white. The first side maximum is a full spectrum with violet on the inner edge and red on the outer edge on each side. Each side maximum farther out is in principle a full spectrum, but they overlap one another and are hard to distinguish. Using monochromatic light can eliminate this problem.

***Q37.7** With two fine slits separated by a distance d slightly less than λ, the equation $d \sin\theta = 0$ has the usual solution $\theta = 0$. But $d \sin\theta = 1\lambda$ has no solution. There is no first side maximum. $d \sin\theta = (1/2)\lambda$ has a solution. Minima flank the central maximum on each side. Answer (b).

Q37.8 As water evaporates from the "soap" bubble, the thickness of the bubble wall approaches zero. Since light reflecting from the front of the water surface is phase-shifted 180° and light reflecting from the back of the soap film is phase-shifted 0°, the reflected light meets the conditions for a minimum. Thus the soap film appears black, as in the textbook illustration accompanying this question.

***Q37.9** Answer (b). If the thickness of the oil film were smaller than half of the wavelengths of visible light, no colors would appear. If the thickness of the oil film were much larger, the colors would overlap to mix to white or gray.

***Q37.10** (i) Answer (b). If the oil film is brightest where it is thinnest, then $n_{air} < n_{oil} < n_{glass}$. With this condition, light reflecting from both the top and the bottom surface of the oil film will undergo phase reversal. Then these two beams will be in phase with each other where the film is very thin. This is the condition for constructive interference as the thickness of the oil film decreases toward zero. If the oil film is dark where it is thinnest, then $n_{air} < n_{oil} > n_{glass}$. In this case, reflecting light undergoes phase reversal upon reflection from the front surface but no phase reversal upon reflection from the back surface. The two reflected beams are 180° out of phase and interfere destructively as the oil film thickness goes to zero.

(ii) Yes. It should be lower in index than both kinds of glass.

(iii) Yes. It should be higher in refractive index than both kinds of glass.

(iv) No.

Q37.11 If R is large, light reflecting from the lower surface of the lens can interfere with light reflecting from the upper surface of the flat. The latter undergoes phase reversal on reflection while the former does not. Where there is negligible distance between the surfaces, at the center of the pattern you will see a dark spot because of the destructive interference associated with the 180° phase shift. Colored rings surround the dark spot. If the lens is a perfect sphere the rings are perfect circles. Distorted rings reveal bumps or hollows on the fine scale of the wavelength of visible light.

Q37.12 A camera lens will have more than one element, to correct (at least) for chromatic aberration. It will have several surfaces, each of which would reflect some fraction of the incident light. To maximize light throughput the surfaces need antireflective coatings. The coating thickness is chosen to produce destructive interference for reflected light of some wavelength.

***Q37.13** (i) Answer (c). The distance between nodes is one-half the wavelength.

(ii) Answer (d). Moving one mirror by 125 nm lengthens the path of light reflecting from it by 250 nm. Since this is half a wavelength, the action reverses constructive into destructive interference.

(iii) Answer (e). The wavelength of the light in the film is 500 nm/2 = 250 nm. If the film is made 62.5 nm thicker, the light reflecting inside the film has a path length 125 nm greater. This is half a wavelength, to reverse constructive into destructive interference.

***Q37.14** Answer (a). If the mirrors do not move the character of the interference stays the same. The light does not get tired before entering the interferometer or undergo any change on the way from the source to the half-silvered mirror.

SOLUTIONS TO PROBLEMS

Section 37.1 **Conditions for Interference**

Section 37.2 **Young's Double-Slit Experiment**

Section 37.3 **Light Waves in Interference**

P37.1 $\Delta y_{\text{bright}} = \dfrac{\lambda L}{d} = \dfrac{(632.8\times 10^{-9})(5.00)}{2.00\times 10^{-4}}\text{ m} = \boxed{1.58\text{ cm}}$

P37.2 $y_{\text{bright}} = \dfrac{\lambda L}{d} m$

For $m = 1$, $\lambda = \dfrac{yd}{L} = \dfrac{(3.40\times 10^{-3}\text{ m})(5.00\times 10^{-4}\text{ m})}{3.30\text{ m}} = \boxed{515\text{ nm}}$

P37.3

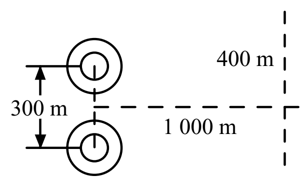

FIG. P37.3

Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. We treat the interference as a Fraunhofer pattern.

(a) At the $m = 2$ maximum, $\tan\theta = \dfrac{400\text{ m}}{1\,000\text{ m}} = 0.400$

$$\theta = 21.8°$$

So $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(300\text{ m})\sin 21.8°}{2} = \boxed{55.7\text{ m}}$

(b) The next minimum encountered is the $m = 2$ minimum, and at that point,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

which becomes $d\sin\theta = \dfrac{5}{2}\lambda$

or $\sin\theta = \dfrac{5}{2}\dfrac{\lambda}{d} = \dfrac{5}{2}\left(\dfrac{55.7\text{ m}}{300\text{ m}}\right) = 0.464$

and $\theta = 27.7°$

so $y = (1\,000\text{ m})\tan 27.7° = 524\text{ m}$

Therefore, the car must travel an additional $\boxed{124\text{ m}}$.

If we considered Fresnel interference, we would more precisely find

(a) $\lambda = \dfrac{1}{2}\left(\sqrt{550^2 + 1\,000^2} - \sqrt{250^2 + 1\,000^2}\right) = 55.2\text{ m}$ and (b) 123 m

P37.4 $\lambda = \dfrac{v}{f} = \dfrac{354\ \text{m/s}}{2\,000\ \text{s}^{-1}} = 0.177\ \text{m}$

(a) $d\sin\theta = m\lambda$ so $(0.300\ \text{m})\sin\theta = 1(0.177\ \text{m})$ and $\theta = \boxed{36.2°}$

(b) $d\sin\theta = m\lambda$ so $d\sin 36.2° = 1(0.030\,0\ \text{m})$ and $d = \boxed{5.08\ \text{cm}}$

(c) $(1.00\times10^{-6}\ \text{m})\sin 36.2° = (1)\lambda$ so $\lambda = 590\ \text{nm}$

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{5.90\times10^{-7}\ \text{m}} = \boxed{508\ \text{THz}}$$

P37.5 In the equation $d\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda$

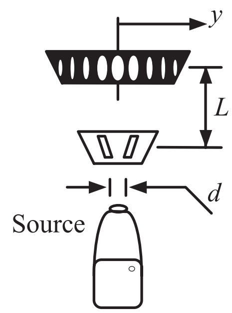

FIG. P37.5

The first minimum is described by $m = 0$

and the tenth by $m = 9$: $\sin\theta = \dfrac{\lambda}{d}\left(9 + \dfrac{1}{2}\right)$

Also, $\tan\theta = \dfrac{y}{L}$

but for small θ, $\sin\theta \approx \tan\theta$

Thus, $d = \dfrac{9.5\lambda}{\sin\theta} = \dfrac{9.5\lambda L}{y}$

$$d = \frac{9.5(5\,890\times10^{-10}\ \text{m})(2.00\ \text{m})}{7.26\times10^{-3}\ \text{m}} = 1.54\times10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}}$$

***P37.6** Problem statement: A single oscillator makes the two speakers of a boom box, 35.0 cm apart, vibrate in phase at 1.62 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, will a distant observer hear maximum sound intensity? Minimum sound intensity? The ambient temperature is 20°C.

We solve the first equation for λ, substitute into the others, and solve for each angle to find this answer: The wavelength of the sound is 21.2 cm. Interference maxima occur at angles of 0° and 37.2° to the left and right. Minima occur at angles of 17.6° and 65.1°. No second-order or higher-order maximum exists. No angle exists, smaller or larger than 90°, for which $\sin\theta_{2\ \text{loud}} = 1.21$. No location exists in the Universe that is two wavelengths farther from one speaker than from the other.

P37.7 (a) For the bright fringe,

$$y_{\text{bright}} = \frac{m\lambda L}{d} \text{ where } m = 1$$

$$y = \frac{(546.1\times10^{-9}\ \text{m})(1.20\ \text{m})}{0.250\times10^{-3}\ \text{m}} = 2.62\times10^{-3}\ \text{m} = \boxed{2.62\ \text{mm}}$$

(b) For the dark bands, $y_{\text{dark}} = \dfrac{\lambda L}{d}\left(m + \dfrac{1}{2}\right);\ m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1 + \frac{1}{2}\right) - \left(0 + \frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1)$$

$$= \frac{(546.1\times10^{-9}\ \text{m})(1.20\ \text{m})}{0.250\times10^{-3}\ \text{m}}$$

$\Delta y = \boxed{2.62\ \text{mm}}$

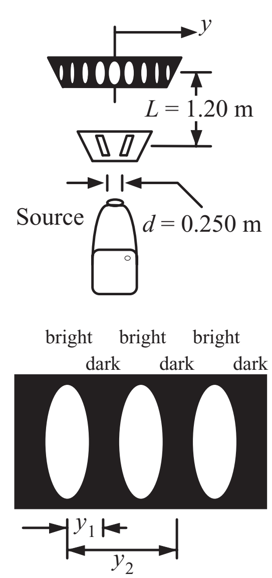

FIG. P37.7

P37.8 Location of A = central maximum,

Location of B = first minimum.

So, $\Delta y = [y_{min} - y_{max}] = \dfrac{\lambda L}{d}\left(0 + \dfrac{1}{2}\right) - 0 = \dfrac{1}{2}\dfrac{\lambda L}{d} = 20.0 \text{ m}$

Thus, $d = \dfrac{\lambda L}{2(20.0 \text{ m})} = \dfrac{(3.00 \text{ m})(150 \text{ m})}{40.0 \text{ m}} = \boxed{11.3 \text{ m}}$

P37.9 Taking $m = 0$ and $y = 0.200$ mm in Equations 37.3 and 37.4 gives

$L \approx \dfrac{2dy}{\lambda} = \dfrac{2(0.400 \times 10^{-3} \text{ m})(0.200 \times 10^{-3} \text{ m})}{442 \times 10^{-9} \text{ m}} = 0.362 \text{ m}$

$L \approx \boxed{36.2 \text{ cm}}$

Geometric optics or a particle theory of light would incorrectly predict bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

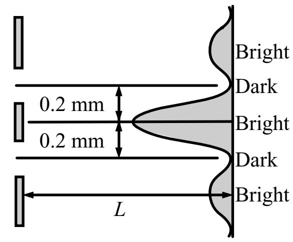

FIG. P37.9

P37.10 At 30.0°, $d \sin\theta = m\lambda$

$(3.20 \times 10^{-4} \text{ m})\sin 30.0° = m(500 \times 10^{-9} \text{ m})$ so $m = 320$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{\text{641 maxima}}$.

P37.11 Observe that the pilot must not only home in on the airport, but must be headed in the right direction when she arrives at the end of the runway.

(a) $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^8 \text{ m/s}}{30 \times 10^6 \text{ s}^{-1}} = \boxed{10.0 \text{ m}}$

(b) The first side maximum is at an angle given by $d \sin\theta = (1)\lambda$.

$(40 \text{ m})\sin\theta = 10 \text{ m}$ $\theta = 14.5°$ $\tan\theta = \dfrac{y}{L}$

$y = L\tan\theta = (2\,000 \text{ m})\tan 14.5° = \boxed{516 \text{ m}}$

(c) The signal of 10-m wavelength in parts (a) and (b) would show maxima at 0°, 14.5°, 30.0°, 48.6°, and 90°. A signal of wavelength 11.23 m would show maxima at 0°, 16.3°, 34.2°, and 57.3°. The only value in common is 0°. If λ_1 and λ_2 were related by a ratio of small integers (a just musical consonance!) in $\dfrac{\lambda_1}{\lambda_2} = \dfrac{n_1}{n_2}$, then the equations $d\sin\theta = n_2\lambda_1$ and $d\sin\theta = n_1\lambda_2$ would both be satisfied for the same nonzero angle. The pilot could come flying in with that inappropriate bearing, and run off the runway immediately after touchdown.

P37.12 In $d\sin\theta = m\lambda$ $d\dfrac{y}{L} = m\lambda$ $y = \dfrac{m\lambda L}{d}$

$\dfrac{dy}{dt} = \dfrac{m\lambda}{d}\dfrac{dL}{dt} = \dfrac{1(633 \times 10^{-9} \text{ m})}{(0.3 \times 10^{-3} \text{ m})} 3 \text{ m/s} = \boxed{6.33 \text{ mm/s}}$

P37.13 $\phi = \frac{2\pi}{\lambda} d\sin\theta = \frac{2\pi}{\lambda} d\left(\frac{y}{L}\right)$

(a) $\phi = \frac{2\pi}{\left(5.00\times10^{-7}\text{ m}\right)}\left(1.20\times10^{-4}\text{ m}\right)\sin(0.500°) = \boxed{13.2\text{ rad}}$

(b) $\phi = \frac{2\pi}{\left(5.00\times10^{-7}\text{ m}\right)}\left(1.20\times10^{-4}\text{ m}\right)\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = \boxed{6.28\text{ rad}}$

(c) If $\phi = 0.333\text{ rad} = \frac{2\pi d\sin\theta}{\lambda}$ $\quad \theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{\left(5.00\times10^{-7}\text{ m}\right)(0.333\text{ rad})}{2\pi\left(1.20\times10^{-4}\text{ m}\right)}\right]$

$\theta = \boxed{1.27\times10^{-2}\text{ deg}}$

(d) If $d\sin\theta = \frac{\lambda}{4}$ $\quad \theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{5\times10^{-7}\text{ m}}{4\left(1.20\times10^{-4}\text{ m}\right)}\right]$

$\theta = \boxed{5.97\times10^{-2}\text{ deg}}$

P37.14 The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$
For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$\boxed{d\left(\sin\theta_1 - \sin\theta_2\right) = m\lambda}$

***P37.15** (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$$\delta = \frac{yd}{L} = \frac{\left(1.80\times10^{-2}\text{ m}\right)\left(1.50\times10^{-4}\text{ m}\right)}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$

(b) $\frac{\delta}{\lambda} = \frac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ because the path difference is an integer multiple of the wavelength.

Section 37.4 **Intensity Distribution of the Double-Slit Interference Pattern**

P37.16 (a) $\frac{I}{I_{\max}} = \cos^2\left(\frac{\phi}{2}\right)$ (Equation 37.12)

Therefore, $\quad \phi = 2\cos^{-1}\sqrt{\frac{I}{I_{\max}}} = 2\cos^{-1}\sqrt{0.640} = \boxed{1.29\text{ rad}}$

(b) $\delta = \frac{\lambda\phi}{2\pi} = \frac{(486.1\text{ nm})(1.29\text{ rad})}{2\pi} = \boxed{99.6\text{ nm}}$

P37.17 $I_{av} = I_{\max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$

For small θ, $\quad \sin\theta = \dfrac{y}{L}$

and $\quad I_{av} = 0.750 I_{\max}$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\sqrt{\frac{I_{av}}{I_{\max}}}$$

$$y = \frac{\left(6.00\times10^{-7}\right)(1.20\text{ m})}{\pi\left(2.50\times10^{-3}\text{ m}\right)}\cos^{-1}\sqrt{\frac{0.750 I_{\max}}{I_{\max}}} = \boxed{48.0\ \mu\text{m}}$$

P37.18 $I = I_{\max}\cos^2\left(\dfrac{\pi yd}{\lambda L}\right)$

$$\frac{I}{I_{\max}} = \cos^2\left[\frac{\pi\left(6.00\times10^{-3}\text{ m}\right)\left(1.80\times10^{-4}\text{ m}\right)}{\left(656.3\times10^{-9}\text{ m}\right)(0.800\text{ m})}\right] = \boxed{0.968}$$

***P37.19** We use trigonometric identities to write

$6\sin(100\pi t) + 8\sin(100\pi t + \pi/2) = 6\sin(100\pi t) + 8\sin(100\pi t)\cos(\pi/2) + 8\cos(100\pi t)\sin(\pi/2)$

$E_1 + E_2 = 6\sin(100\pi t) + 8\cos(100\pi t)$

and $E_R\sin(100\pi t + \phi) = E_R\sin(100\pi t)\cos\phi + E_R\cos(100\pi t)\sin\phi$

The equation $E_1 + E_2 = E_R\sin(100\pi t + \phi)$ is satisfied if we require just

$6 = E_R\cos\phi$ and $8 = E_R\sin\phi$

or $6^2 + 8^2 = E_R^2(\cos^2\phi + \sin^2\phi)$ $\qquad \boxed{E_R = 10}$

and $\tan\phi = \sin\phi/\cos\phi = 8/6 = 1.33$ $\qquad \boxed{\phi = 53.1°}$

***P37.20** In $I_{av} = I_{\max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$ for angles between $-0.3°$ and $+0.3°$ we take $\sin\theta = \theta$ to find

$I = I_{\max}\cos^2\left(\dfrac{\pi\, 250\ \mu\text{m}\ \theta}{0.5461\ \mu\text{m}}\right) \qquad I/I_{\max} = \cos^2(1438\ \theta)$

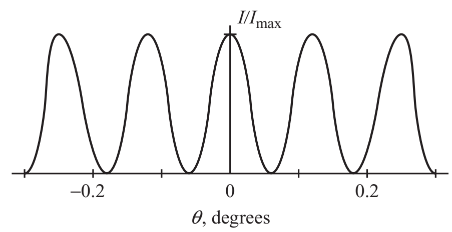

FIG. P37.20

This equation is correct assuming θ is in radians; but we can then equally well substitute in values for θ in degrees and interpret the argument of the cosine function as a number of degrees. We get the same answers for θ negative and for θ positive. We evaluate

θ, degrees	–0.3	–0.25	–0.2	–0.15	–0.1	–0.05	0	0.05	0.1	0.15	0.2	0.25	0.3
$I/I_{\max}$	0.101	1.00	0.092	0.659	0.652	0.096	1	0.096	0.652	0.659	0.092	1.00	0.101

The cosine-squared function has maximum values of 1 at $\theta = 0$, at $1438\ \theta = 180°$ with $\theta = 0.125°$, and at $1438\ \theta = 360°$ with $\theta = 0.250°$. It has minimum values of zero halfway between the maximum values. The graph then has the appearance shown.

P37.21 (a) From Equation 37.9,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2+D^2}}$$

$$\phi \approx \frac{2\pi yd}{\lambda D} = \frac{2\pi\left(0.850\times10^{-3}\text{ m}\right)\left(2.50\times10^{-3}\text{ m}\right)}{\left(600\times10^{-9}\text{ m}\right)(2.80\text{ m})} = \boxed{7.95\text{ rad}}$$

(b) $$\frac{I}{I_{\max}} = \frac{\cos^2\left[(\pi d/\lambda)\sin\theta\right]}{\cos^2\left[(\pi d/\lambda)\sin\theta_{\max}\right]} = \frac{\cos^2(\phi/2)}{\cos^2 m\pi}$$

$$\frac{I}{I_{\max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

***P37.22** (a) The resultant amplitude is

$$E_r = E_0\sin\omega t + E_0\sin(\omega t+\phi) + E_0\sin(\omega t+2\phi),\text{ where }\phi = \frac{2\pi}{\lambda}d\sin\theta$$

$$E_r = E_0\left(\sin\omega t + \sin\omega t\cos\phi + \cos\omega t\sin\phi + \sin\omega t\cos 2\phi + \cos\omega t\sin 2\phi\right)$$

$$E_r = E_0(\sin\omega t)\left(1+\cos\phi+2\cos^2\phi-1\right) + E_0(\cos\omega t)(\sin\phi + 2\sin\phi\cos\phi)$$

$$E_r = E_0(1+2\cos\phi)(\sin\omega t\cos\phi + \cos\omega t\sin\phi) = E_0(1+2\cos\phi)\sin(\omega t+\phi)$$

Then the intensity is $\quad I \propto E_r^2 = E_0^2(1+2\cos\phi)^2\left(\frac{1}{2}\right)$

where the time average of $\sin^2(\omega t+\phi)$ is $\frac{1}{2}$.

From one slit alone we would get intensity $I_{\max} \propto E_0^2\left(\frac{1}{2}\right)$ so

$$\boxed{I = I_{\max}\left[1+2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2}$$

(b) Look at the $N = 3$ graph in the textbook Figure 37.8. Minimum intensity is zero, attained where $\cos\phi = -\frac{1}{2}$. One relative maximum occurs at $\cos\phi = -1.00$, where

$I = I_{\max}[1-2]^2 = I_{\max}$

The larger local maximum happens where $\cos\phi = +1.00$, giving $I = I_{\max}[1+2]^2 = 9.00 I_{\max}$.

The ratio of intensities at primary versus secondary maxima is $\boxed{9.00}$.

Section 37.5 **Change of Phase Due to Reflection**

Section 37.6 **Interference in Thin Films**

P37.23 (a) The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

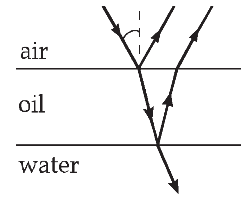

FIG. P37.23

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m+(1/2)} = \frac{2(1.45)(280\text{ nm})}{m+(1/2)}$$

Substituting for m gives: $m = 0$, $\lambda_0 = 1\,620$ nm (infrared)

$m = 1$, $\lambda_1 = 541$ nm (green)

$m = 2$, $\lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda$$

or $$\lambda_m = \frac{2nt}{m} = \frac{812\text{ nm}}{m}$$

Substituting for m gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet)

Of these, the only wavelength visible to the human eye (and hence the dominant wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

P37.24 Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\lambda_n = \dfrac{\lambda}{n}$ is the wavelength in the material.

Then $$2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$$

$$\lambda = 4nt = 4(1.33)(115\text{ nm}) = \boxed{612\text{ nm}}$$

P37.25 Since $1 < 1.25 < 1.33$, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require $2t = \dfrac{m\lambda_{\text{cons}}}{n}$

and for destructive interference, $2t = \dfrac{[m + (1/2)]\lambda_{\text{des}}}{n}$

Then $\dfrac{\lambda_{\text{cons}}}{\lambda_{\text{dest}}} = 1 + \dfrac{1}{2m} = \dfrac{640\text{ nm}}{512\text{ nm}} = 1.25$ and $m = 2$

Therefore, $t = \dfrac{2(640\text{ nm})}{2(1.25)} = \boxed{512\text{ nm}}$

P37.26 Treating the anti-reflectance coating like a camera-lens coating,

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

Let $m = 0$: $t = \dfrac{\lambda}{4n} = \dfrac{3.00\text{ cm}}{4(1.50)} = \boxed{0.500\text{ cm}}$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar to 1.50 cm. Then the coating would exhibit maximum reflection!

P37.27 $2nt = \left(m + \dfrac{1}{2}\right)\lambda$ so $t = \left(m + \dfrac{1}{2}\right)\dfrac{\lambda}{2n}$

Minimum $t = \left(\dfrac{1}{2}\right)\dfrac{(500\text{ nm})}{2(1.30)} = \boxed{96.2\text{ nm}}$

P37.28 Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is $2nt = m\lambda$, or $\lambda = \dfrac{2nt}{m}$. The film thickness is $t = 1.00 \times 10^{-5}\text{ cm} = 1.00 \times 10^{-7}\text{ m} = 100\text{ nm}$. Therefore, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2(1.38)(100\text{ nm})}{m} = \frac{276\text{ nm}}{m} \text{ where } m = 1, 2, 3, \ldots$$

or $\lambda_1 = 276\text{ nm}$, $\lambda_2 = 138\text{ nm}$, ... All reflection maxima are in the ultraviolet and beyond.

$\boxed{\text{No visible wavelengths are intensified.}}$

P37.29 (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \dfrac{\lambda}{n}$.

$$t = \frac{\lambda}{2n} = \frac{656.3\text{ nm}}{2(1.378)} = \boxed{238\text{ nm}}$$

(b) The filter will undergo thermal expansion. As t increases in $2nt = \lambda$, so does $\boxed{\lambda\text{ increase}}$.

(c) Destructive interference for reflected light happens also for λ in $2nt = 2\lambda$,

or $\lambda = 1.378(238\text{ nm}) = \boxed{328\text{ nm}}$ (near ultraviolet)

P37.30 If the path length difference $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580 \text{ nm}}{2} = \boxed{290 \text{ nm}}$$

P37.31 For destructive interference in the air,

$$2t = m\lambda$$

For 30 dark fringes, including the one where the plates meet,

$$t = \frac{29(600 \text{ nm})}{2} = 8.70 \times 10^{-6} \text{ m}$$

Therefore, the *radius* of the wire is

$$r = \frac{t}{2} = \frac{8.70\ \mu\text{m}}{2} = \boxed{4.35\ \mu\text{m}}$$

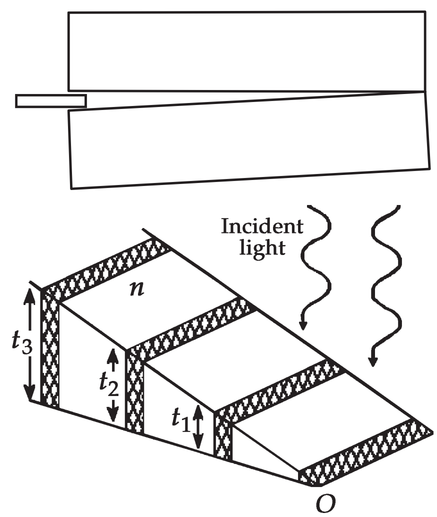

FIG. P37.31

P37.32 The condition for bright fringes is

$$2t + \frac{\lambda}{2n} = m\frac{\lambda}{n} \qquad m = 1,\ 2,\ 3,\ \ldots$$

From the sketch, observe that

$$t = R(1 - \cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$$

The condition for a bright fringe becomes $\quad \dfrac{r^2}{R} = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda}{n}$

Thus, for fixed m and λ, $\quad nr^2 = \text{constant}$

Therefore, $n_{\text{liquid}} r_f^2 = n_{\text{air}} r_i^2$ and $\quad n_{\text{liquid}} = (1.00)\dfrac{(1.50 \text{ cm})^2}{(1.31 \text{ cm})^2} = \boxed{1.31}$

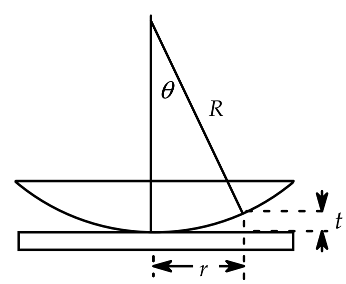

FIG. P37.32

P37.33 For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface, the condition for destructive interference is

$$2n_{\text{air}} t = m\lambda \qquad m = 0,\ 1,\ 2,\ \ldots$$

The least common multiple of these two wavelengths is 1 200 nm, so we get no reflected light at $2(1.00)t = 3(400 \text{ nm}) = 2(600 \text{ nm}) = 1\,200$ nm, so $t = 600$ nm at this second dark fringe.

By similar triangles, $\quad \dfrac{600 \text{ nm}}{x} = \dfrac{0.050\,0 \text{ mm}}{10.0 \text{ cm}}$

or the distance from the contact point is $\quad x = \left(600 \times 10^{-9} \text{ m}\right)\left(\dfrac{0.100 \text{ m}}{5.00 \times 10^{-5} \text{ m}}\right) = \boxed{1.20 \text{ mm}}$

Section 37.7 The Michelson Interferometer

P37.34 Distance $= 2\left(3.82\times10^{-4}\text{ m}\right) = 1\,700\lambda$ $\qquad \lambda = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$

The light is $\boxed{\text{blue}}$.

P37.35 When the mirror on one arm is displaced by $\Delta\ell$, the path difference changes by $2\Delta\ell$. A shift resulting in the reversal between dark and bright fringes requires a path length change of one-half wavelength. Therefore, $2\Delta\ell = \dfrac{m\lambda}{2}$, where in this case, $m = 250$.

$$\Delta\ell = m\frac{\lambda}{4} = \frac{(250)\left(6.328\times10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

P37.36 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \dfrac{2L}{\lambda}$. It changes to $m_2 = \dfrac{2L}{\lambda/n} = \dfrac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \dfrac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1+\frac{N\lambda}{2L}}$$

Additional Problems

***P37.37** The same light source will radiate into the syrup light with wavelength 560 nm/1.38 = 406 nm. The first side bright fringe is separated from the central bright fringe by distance y described by

$d\sin\theta = 1\lambda \quad dy/L \approx \lambda \qquad y = \lambda L/d = 406\times10^{-9}\text{m}(1.20\text{ m})/(30\times10^{-6}\text{m}) = \boxed{1.62\text{ cm}}$

P37.38 (a) Where fringes of the two colors coincide we have $d\sin\theta = m\lambda = m'\lambda'$, requiring $\dfrac{\lambda}{\lambda'} = \dfrac{m'}{m}$.

(b) $\lambda = 430$ nm, $\lambda' = 510$ nm

$\therefore \dfrac{m'}{m} = \dfrac{430\text{ nm}}{510\text{ nm}} = \dfrac{43}{51}$, which cannot be reduced any further. Then $m = 51$, $m' = 43$.

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(51)\left(430\times10^{-9}\text{ m}\right)}{0.025\times10^{-3}\text{ m}}\right] = 61.3^\circ$$

$$y_m = L\tan\theta_m = (1.5\text{ m})\tan 61.3^\circ = \boxed{2.74\text{ m}}$$

P37.39 The wavelength is $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{60.0\times10^6\text{ s}^{-1}} = 5.00\text{ m}$.

Along the line AB the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from B, when it arrives at A, will always be in phase with transmitter B. Since B is 180° out of phase with A, the two signals always interfere destructively at the position of A.

The first antinode (point of constructive interference) is located at distance

$\dfrac{\lambda}{4} = \dfrac{5.00\text{ m}}{4} = \boxed{1.25\text{ m}}$ from the node at A.

P37.40 Along the line of length d joining the source, two identical waves moving in opposite directions add to give a standing wave.

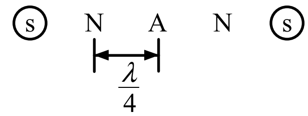

FIG. P37.40

An antinode is halfway between the sources. If $\frac{d}{2} > \frac{\lambda}{2}$, there is space for two more antinodes for a total of three. If $\frac{d}{2} > \lambda$, there will be at least five antinodes, and so on. To repeat, if $\frac{d}{\lambda} > 0$, the number of antinodes is 1 or more. If $\frac{d}{\lambda} > 1$, the number of antinodes is 3 or more. If $\frac{d}{\lambda} > 2$, the number of antinodes is 5 or more. In general,

$$\boxed{\text{the number of antinodes is 1 plus 2 times the greatest integer less than or equal to } \frac{d}{\lambda}.}$$

If $\frac{d}{2} < \frac{\lambda}{4}$, there will be no nodes. If $\frac{d}{2} > \frac{\lambda}{4}$, there will be space for at least two nodes, as shown in the picture. If $\frac{d}{2} > \frac{3\lambda}{4}$, there will be at least four nodes. If $\frac{d}{2} > \frac{5\lambda}{4}$, six or more nodes will fit in, and so on. To repeat, if $2d < \lambda$, the number of nodes is 0. If $2d > \lambda$, the number of nodes is 2 or more. If $2d > 3\lambda$, the number of nodes is 4 or more. If $2d > 5\lambda$, the number of nodes is 6 or more. Again, if $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 1$, the number of nodes is at least 2. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 2$, the number of nodes is at least 4. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 3$, the number of nodes is at least 6. In general,

$$\boxed{\text{the number of nodes is 2 times the greatest nonzero integer less than } \left(\frac{d}{\lambda} + \frac{1}{2}\right).}$$

Next, we enumerate the zones of constructive interference. They are described by $d\sin\theta = m\lambda$, $m = 0, 1, 2, \ldots$ with θ counted as positive both left and right of the maximum at $\theta = 0$ in the center. The number of side maxima on each side is the greatest integer satisfying $\sin\theta \le 1$, $d1 \ge m\lambda$, $m \le \frac{d}{\lambda}$. So the total

$$\boxed{\text{number of bright fringes is one plus 2 times the greatest integer less than or equal to } \frac{d}{\lambda}.}$$

It is equal to the number of antinodes on the line joining the sources.

The interference minima are to the left and right at angles described by $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$. With $\sin\theta < 1$, $d1 > \left(m_{\text{max}} + \frac{1}{2}\right)\lambda$, $m_{\text{max}} < \frac{d}{\lambda} - \frac{1}{2}$ or $m_{\text{max}} + 1 < \frac{d}{\lambda} + \frac{1}{2}$. Let $n = 1, 2, 3, \ldots$. Then the number of side minima is the greatest integer n less than $\frac{d}{\lambda} + \frac{1}{2}$. Counting both left and right,

$$\boxed{\text{the number of dark fringes is two times the greatest positive integer less than } \left(\frac{d}{\lambda} + \frac{1}{2}\right).}$$

It is equal to the number of nodes in the standing wave between the sources.

P37.41 My middle finger has width $d = 2$ cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$$d\sin\theta = m\lambda$$

$$\theta_0 = 0$$

$$\left(2\times10^{-2}\text{ m}\right)\sin\theta_1 = 1\left(6\times10^{-7}\text{ m}\right)$$

Thus, $\theta_1 = 2\times10^{-3}$ degree

and $\theta_1 - \theta_0 \boxed{\sim10^{-3}\text{ degree}}$

(b) Choose $\theta_1 = 20°$

$$\left(2\times10^{-2}\text{ m}\right)\sin 20° = (1)\lambda$$

$$\lambda = 7\text{ mm}$$

Millimeter waves are $\boxed{\text{microwaves}}$.

$f = \dfrac{c}{\lambda}$: $\quad f = \dfrac{3\times10^{8}\text{ m/s}}{7\times10^{-3}\text{ m}}\boxed{\sim10^{11}\text{ Hz}}$

***P37.42** Constructive interference occurs where $m = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots$, for

$$\left(\frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right) = 2\pi m \qquad \frac{2\pi(x_1 - x_2)}{\lambda} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

$$\frac{(x_1 - x_2)}{650\text{ nm}} + \frac{1}{12} - \frac{1}{16} = m \boxed{x_1 - x_2 = \left(m - \frac{1}{48}\right)650\text{ nm with } m = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots}$$

***P37.43** A bright line for the green light requires $dy/L = m_1\lambda_1$. A blue interference maximum requires $dy/L = m_2\lambda_2$ for integers m_1 and m_2. Then m_1 540 nm $= m_2$ 450 nm. The smallest integers satisfying the equation are $m_1 = 5$ and $m_2 = 6$. Then for both

$dy/L = 2\,700$ nm $\qquad y = (1.4\text{ m})\ 2.7\ \mu m/\ 150\mu m = \boxed{2.52\text{ cm}}$

P37.44 If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic t,

$\dfrac{t}{\lambda} + \dfrac{1}{2} = \dfrac{t}{\lambda/n} = \dfrac{nt}{\lambda}$ or $t = \boxed{\dfrac{\lambda}{2(n-1)}}$ where n is the index of refraction for the plastic.

P37.45 No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\frac{\lambda}{2}$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is

then $$\delta = 2nt + \frac{\lambda}{2}$$

For constructive interference, $\delta = m\lambda$

or $$2(1.00)t + \frac{\lambda}{2} = m\lambda$$

Thus, the film thickness for the mth order bright fringe is

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

and the thickness for the $m-1$ bright fringe is:

$$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

Therefore, the change in thickness required to go from one bright fringe to the next is

$$\Delta t = t_m - t_{m-1} = \frac{\lambda}{2}$$

To go through 200 bright fringes, the change in thickness of the air film must be:

$$200\left(\frac{\lambda}{2}\right) = 100\lambda$$

Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00 \times 10^{-7}\text{ m}\right) = 5.00 \times 10^{-5}\text{ m}$$

From $$\Delta L = L_i \alpha\, \Delta T$$

we have: $$\alpha = \frac{\Delta L}{L_i \Delta T} = \frac{5.00 \times 10^{-5}\text{ m}}{(0.100\text{ m})(25.0\,°\text{C})} = \boxed{20.0 \times 10^{-6}\,°\text{C}^{-1}}$$

P37.46 Since $1 < 1.25 < 1.34$, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then $2t$, which must be equal to

$$m\lambda_n = \frac{m\lambda}{n}$$

for maximum reflection, with $m = 1$ for the given first-order condition and $n = 1.25$. So

$$t = \frac{m\lambda}{2n} = \frac{1(500\text{ nm})}{2(1.25)} = 200\text{ nm}$$

The volume we assume to be constant: $1.00\text{ m}^3 = (200\text{ nm})A$

$$A = \frac{1.00\text{ m}^3}{200\left(10^{-9}\text{ m}\right)} = 5.00 \times 10^6\text{ m}^2 = \boxed{5.00\text{ km}^2}$$

P37.47 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P.

Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad (1)$$

It is equally far from P to R as from P to R', the mirror image of the telescope.

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at R'.

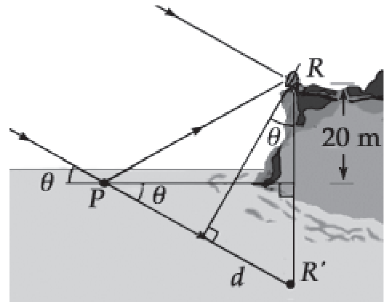

FIG. P37.47

So the path difference is $d = 2(20.0\text{ m})\sin\theta = (40.0\text{ m})\sin\theta$

The wavelength is $\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{60.0\times10^{6}\text{ Hz}} = 5.00\text{ m}$

Substituting for d and λ in Equation (1), $(40.0\text{ m})\sin\theta = \frac{5.00\text{ m}}{2}$

Solving for the angle θ, $\sin\theta = \frac{5.00\text{ m}}{80.0\text{ m}}$ and $\boxed{\theta = 3.58°}$.

P37.48 For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\frac{\pi}{2}$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Modifying Equation 37.7,

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1(5.00\times10^{-7}\text{ m})(100\text{ m})}{(2.00\times10^{-2}\text{ m})} = \boxed{2.50\text{ mm}}$$

P37.49 $2\sqrt{(15.0\text{ km})^2 + h^2} = 30.175\text{ km}$

$(15.0\text{ km})^2 + h^2 = 227.63$

$h = \boxed{1.62\text{ km}}$

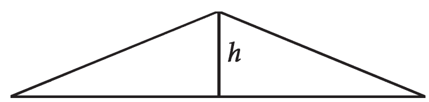

FIG. P37.49

P37.50 For dark fringes, $2nt = m\lambda$

and at the edge of the wedge, $t = \frac{84(500\text{ nm})}{2}$

When submerged in water, $2nt = m\lambda$

$$m = \frac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$$

so $m + 1 = \boxed{113\text{ dark fringes}}$

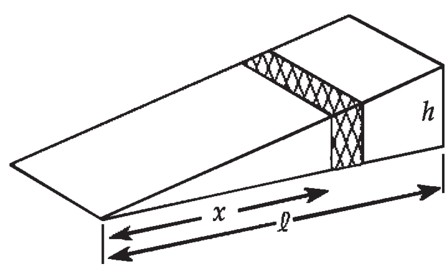

FIG. P37.50

P37.51 From Equation 37.14, $$\frac{I}{I_{\max}} = \cos^2\left(\frac{\pi yd}{\lambda L}\right)$$

Let λ_2 equal the wavelength for which $$\frac{I}{I_{\max}} \to \frac{I_2}{I_{\max}} = 0.640$$

Then $$\lambda_2 = \frac{\pi yd/L}{\cos^{-1}\left(I_2/I_{\max}\right)^{1/2}}$$

But $$\frac{\pi yd}{L} = \lambda_1 \cos^{-1}\left(\frac{I_1}{I_{\max}}\right)^{1/2} = (600\text{ nm})\cos^{-1}(0.900) = 271\text{ nm}$$

Substituting this value into the expression for λ_2, $$\lambda_2 = \frac{271\text{ nm}}{\cos^{-1}\left(0.640^{1/2}\right)} = \boxed{421\text{ nm}}$$

Note that in this problem, $\cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2}$ must be expressed in radians.

P37.52 At entrance, $1.00\sin 30.0° = 1.38\sin\theta_2$ $\qquad \theta_2 = 21.2°$
Call t the unknown thickness. Then

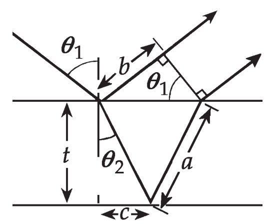

FIG. P37.52

$\cos 21.2° = \dfrac{t}{a}$ $\qquad a = \dfrac{t}{\cos 21.2°}$

$\tan 21.2° = \dfrac{c}{t}$ $\qquad c = t\tan 21.2°$

$\sin\theta_1 = \dfrac{b}{2c}$ $\qquad b = 2t\tan 21.2°\sin 30.0°$

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor n accounts for the shorter wavelength in the film. For constructive interference, we require

$$2an - b - \frac{\lambda}{2} = m\lambda$$

The minimum thickness will be given by $$2an - b - \frac{\lambda}{2} = 0$$

$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t\,(\tan 21.2°)\sin 30.0°$$

$$\frac{590\text{ nm}}{2} = \left(\frac{2\times 1.38}{\cos 21.2°} - 2\tan 21.2°\ \sin 30.0°\right)t = 2.57t \qquad t = \boxed{115\text{ nm}}$$

P37.53 The shift between the two reflected waves is $\delta = 2na - b - \dfrac{\lambda}{2}$ where a and b are as shown in the ray diagram, n is the index of refraction, and the term $\dfrac{\lambda}{2}$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where m has integer values. This condition becomes

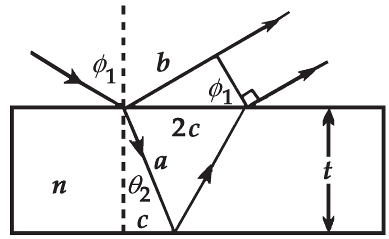

FIG. P37.53

$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

From the figure's geometry, $a = \dfrac{t}{\cos\theta_2}$

$$c = a\sin\theta_2 = \frac{t\sin\theta_2}{\cos\theta_2}$$

$$b = 2c\sin\phi_1 = \frac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$$

Also, from Snell's law, $\sin\phi_1 = n\sin\theta_2$

Thus, $b = \dfrac{2nt\sin^2\theta_2}{\cos\theta_2}$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda$$

or $\boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}$

***P37.54** (a) For a linear function taking the value 1.90 at $y = 0$ and 1.33 at $y = 20$ cm we write $n(y) = 1.90 + (1.33 - 1.90)y/20\text{ cm}$ or $\boxed{n(y) = 1.90 - 0.0285\ y/cm}$

(b) $$\int_0^{20\text{ cm}} n(y)dy = \int_0^{20\text{ cm}} [1.90 - 0.0285y / \text{cm}]dy = 1.90y - \frac{0.0285y^2}{2}\Bigg|_0^{20\text{ cm}}$$

$$= 38.0\text{ cm} - 5.7\text{ cm}$$

$$= \boxed{32.3\text{ cm}}$$

(c) The beam will continuously curve downward.

P37.55 (a) Minimum: $2nt = m\lambda_2$ for $m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ for $m' = 0, 1, 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$

so $m' = m - 1$

Then $2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$2m\lambda_2 = 2m\lambda_1 - \lambda_1$

so $\boxed{m = \dfrac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$

(b) $m = \dfrac{500}{2(500 - 370)} = 1.92 \to 2$ (wavelengths measured to ± 5 nm)

Minimum: $2nt = m\lambda_2$

$2(1.40)t = 2(370 \text{ nm})$ $t = 264$ nm

Maximum: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$

$2(1.40)t = 1.5(500 \text{ nm})$ $t = 268$ nm

Film thickness = $\boxed{266 \text{ nm}}$

P37.56 From the sketch, observe that

$$x = \sqrt{h^2 + \left(\frac{d}{2}\right)^2} = \frac{\sqrt{4h^2 + d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \frac{\lambda}{2}$.

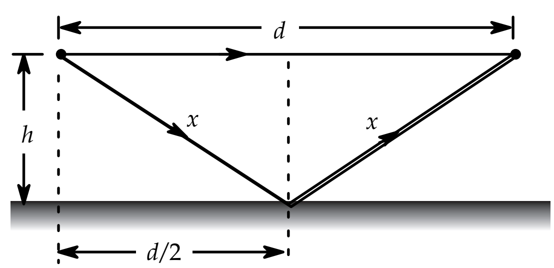

FIG. P37.56

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where $m = 0, 1, 2, 3, \ldots$

Thus, $2x - d = \left(m + \frac{1}{2}\right)\lambda$ or $\lambda = \dfrac{4x - 2d}{2m + 1}$

For the longest wavelength, $m = 0$, giving $\lambda = 4x - 2d = \boxed{2\sqrt{4h^2 + d^2} - 2d}$

(b) For destructive interference, $\delta = \left(m - \frac{1}{2}\right)\lambda$ where $m = 0, 1, 2, 3, \ldots$

Thus, $2x - d = m\lambda$ or $\lambda = \dfrac{2x - d}{m}$

For the longest wavelength, $m = 1$ giving $\lambda = 2x - d = \boxed{\sqrt{4h^2 + d^2} - d}$

P37.57 Call t the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference ϕ is

$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)$$

The corresponding difference in **path length** Δr is

$$\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)\left(\frac{\lambda_a}{2\pi}\right) = t(n-1)$$

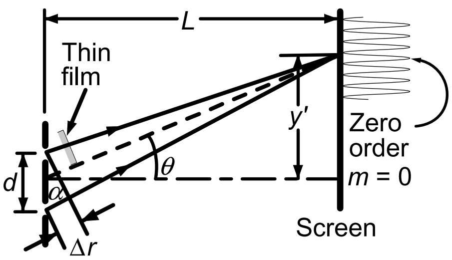

FIG. P37.57

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as $\tan\theta = \dfrac{\Delta r}{d} = \dfrac{y'}{L}$

Eliminating Δr by substitution, $\dfrac{y'}{L} = \dfrac{t(n-1)}{d}$ gives $\boxed{y' = \dfrac{t(n-1)L}{d}}$

P37.58 Bright fringes occur when $2t = \dfrac{\lambda}{n}\left(m+\dfrac{1}{2}\right)$

and dark fringes occur when $2t = \left(\dfrac{\lambda}{n}\right)m$

The thickness of the film at x is $t = \left(\dfrac{h}{\ell}\right)x$

Therefore, $\boxed{x_{\text{bright}} = \dfrac{\lambda\ell}{2hn}\left(m+\dfrac{1}{2}\right)}$ and $\boxed{x_{\text{dark}} = \dfrac{\lambda\ell m}{2hn}}$.

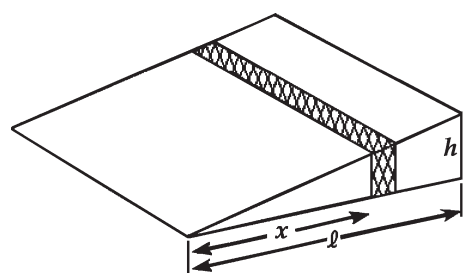

FIG. P37.58

P37.59 (a) Constructive interference in the reflected light requires $2t = \left(m+\dfrac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \frac{54.5\left(650\times10^{-9}\text{ m}\right)}{2} = 17.7\ \mu\text{m}$$

Now from the geometry in textbook Figure 37.12, we can find the distance t from the curved surface down to the flat plate by considering distances down from the center of curvature:

$$\sqrt{R^2 - r^2} = R - t \text{ or } R^2 - r^2 = R^2 - 2Rt + t^2$$

$$R = \frac{r^2+t^2}{2t} = \frac{\left(5.00\times10^{-2}\text{ m}\right)^2 + \left(1.77\times10^{-5}\text{ m}\right)^2}{2\left(1.77\times10^{-5}\text{ m}\right)} = \boxed{70.6\text{ m}}$$

(b) $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_2} - \dfrac{1}{R_2}\right) = 0.520\left(\dfrac{1}{\infty} - \dfrac{1}{-70.6\text{ m}}\right)$ so $f = \boxed{136\text{ m}}$

P37.60 The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness t is $\delta = 2tn_{\text{film}} + \dfrac{\lambda}{2}$, with the factor of $\dfrac{\lambda}{2}$ being due to a phase reversal at *one* of the surfaces.

For the dark rings (destructive interference), the total shift should be $\delta = \left(m+\dfrac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$.

This requires that $t = \dfrac{m\lambda}{2n_{\text{film}}}$.

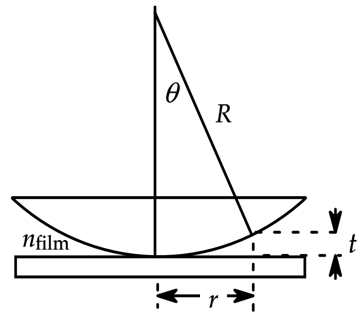

FIG. P37.60

To find t in terms of r and R, $\quad R^2 = r^2 + (R-t)^2 \quad$ so $\quad r^2 = 2Rt + t^2$

Since t is much smaller than R, $\quad t^2 << 2Rt \quad$ and $\quad r^2 \approx 2Rt = 2R\left(\dfrac{m\lambda}{2n_{\text{film}}}\right)$

Thus, where m is an integer, $\quad \boxed{r \approx \sqrt{\dfrac{m\lambda R}{n_{\text{film}}}}}$

P37.61 Light reflecting from the upper interface of the air layer suffers no phase change, while light reflecting from the lower interface is reversed 180°. Then there is indeed a dark fringe at the outer circumference of the lens, and a dark fringe wherever the air thickness t satisfies $2t = m\lambda$, $m = 0, 1, 2, \ldots$.

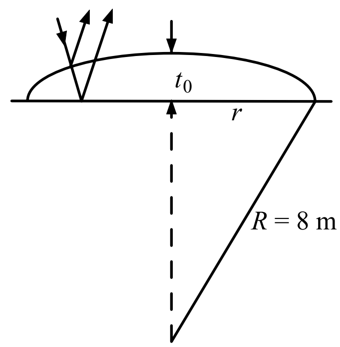

FIG. P37.61

(a) At the central dark spot $m = 50$ and

$$t_0 = \frac{50\lambda}{2} = 25\left(589\times10^{-9}\text{ m}\right) = \boxed{1.47\times10^{-5}\text{ m}}$$

(b) In the right triangle,

$$(8\text{ m})^2 = r^2 + \left(8\text{ m} - 1.47\times10^{-5}\text{ m}\right)^2 = r^2 + (8\text{ m})^2$$
$$-2(8\text{ m})\left(1.47\times10^{-5}\text{ m}\right) + 2\times10^{-10}\text{ m}^2$$

The last term is negligible. $r = \sqrt{2(8\text{ m})\left(1.47\times10^{-5}\text{ m}\right)} = \boxed{1.53\times10^{-2}\text{ m}}$

(c) $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (1.50-1)\left(\dfrac{1}{\infty} - \dfrac{1}{8.00\text{ m}}\right)$

$\boxed{f = -16.0\text{ m}}$

P37.62 For bright rings the gap t between surfaces is given by $2t = \left(m + \frac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the hundredth has $m = 99$.

So, $t = \frac{1}{2}(99.5)\left(500 \times 10^{-9}\text{ m}\right) = 24.9\ \mu\text{m}$.

Call r_b the ring radius. From the geometry of the figure at the right,

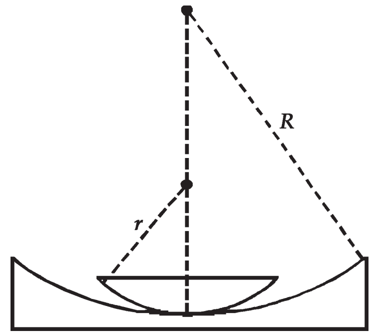

FIG. P37.62

$$t = r - \sqrt{r^2 - r_b^2} - \left(R - \sqrt{R^2 - r_b^2}\right)$$

Since $r_b << r$, we can expand in series:

$$t = r - r\left(1 - \frac{1}{2}\frac{r_b^2}{r^2}\right) - R + R\left(1 - \frac{1}{2}\frac{r_b^2}{R^2}\right) = \frac{1}{2}\frac{r_b^2}{r} - \frac{1}{2}\frac{r_b^2}{R}$$

$$r_b = \left[\frac{2t}{1/r - 1/R}\right]^{1/2} = \left[\frac{2\left(24.9 \times 10^{-6}\text{ m}\right)}{1/4.00\text{ m} - 1/12.0\text{ m}}\right]^{1/2} = \boxed{1.73\text{ cm}}$$

P37.63 (a) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$.

Hence, the first bright band ($m = 0$) corresponds to $nt = \frac{\lambda}{4}$.

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$

we have $x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right) = \boxed{4.86\text{ cm}}$

(b) $t_1 = \frac{\lambda_1}{4n} = \frac{420\text{ nm}}{4(1.33)} = \boxed{78.9\text{ nm}}$

$t_2 = \frac{\lambda_2}{4n} = \frac{680\text{ nm}}{4(1.33)} = \boxed{128\text{ nm}}$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9\text{ nm}}{3.00\text{ cm}} = \boxed{2.63 \times 10^{-6}\text{ rad}}$

P37.64 Depth = one-quarter of the wavelength in plastic.

$$t = \frac{\lambda}{4n} = \frac{780\text{ nm}}{4(1.50)} = \boxed{130\text{ nm}}$$

P37.65 $2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ bright

$2h\left(\frac{\Delta y}{2L}\right) = \frac{1}{2}\lambda$ so $h = \frac{L\lambda}{2\Delta y} = \frac{(2.00\text{ m})\left(606 \times 10^{-9}\text{ m}\right)}{2\left(1.2 \times 10^{-3}\text{ m}\right)} = \boxed{0.505\text{ mm}}$

P37.66 Superposing the two vectors, $E_R = \left|\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right|$

$$E_R = \left|\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right| = \sqrt{\left(E_0 + \frac{E_0}{3}\cos\phi\right)^2 + \left(\frac{E_0}{3}\sin\phi\right)^2} = \sqrt{E_0^2 + \frac{2}{3}E_0^2\cos\phi + \frac{E_0^2}{9}\cos^2\phi + \frac{E_0^2}{9}\sin^2\phi}$$

$$E_R = \sqrt{\frac{10}{9}E_0^2 + \frac{2}{3}E_0^2\cos\phi}$$

Since intensity is proportional to the square of the amplitude,

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\cos\phi$$

Using the trigonometric identity $\cos\phi = 2\cos^2\frac{\phi}{2} - 1$, this becomes

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\left(2\cos^2\frac{\phi}{2} - 1\right) = \frac{4}{9}I_{\max} + \frac{4}{3}I_{\max}\cos^2\frac{\phi}{2}$$

or $\boxed{I = \frac{4}{9}I_{\max}\left(1 + 3\cos^2\frac{\phi}{2}\right)}$

P37.67 Represent the light radiated from each slit to point P as a phasor. The two have equal amplitudes E. Since intensity is proportional to amplitude squared, they add to amplitude $\sqrt{3}E$.

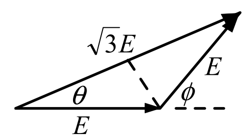

FIG. P37.67

Then $\cos\theta = \frac{\sqrt{3}E/2}{E}$, $\theta = 30°$. Next, the obtuse angle between the two phasors is $180 - 30 - 30 = 120°$, and $\phi = 180 - 120° = 60°$. The phase difference between the two phasors is caused by the path difference $\delta = \overline{SS_2} - \overline{SS_1}$ according to $\frac{\delta}{\lambda} = \frac{\phi}{360°}$, $\delta = \lambda\frac{60°}{360°} = \frac{\lambda}{6}$. Then

$$\sqrt{L^2 + d^2} - L = \frac{\lambda}{6}$$

$$L^2 + d^2 = L^2 + \frac{2L\lambda}{6} + \frac{\lambda^2}{36}$$

The last term is negligible, so

$$d = \left(\frac{2L\lambda}{6}\right)^{1/2} = \sqrt{\frac{2(1.2\text{ m})620\times10^{-9}\text{ m}}{6}} = \boxed{0.498\text{ mm}}$$

ANSWERS TO EVEN PROBLEMS

P37.2 515 nm

P37.4 (a) 36.2° (b) 5.08 cm (c) 508 THz

P37.6 Question: A single oscillator makes the two speakers of a boom box, 35.0 cm apart, vibrate in phase at 1.62 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, will a distant observer hear maximum sound intensity? Minimum sound intensity? The ambient temperature is 20°C. Answer: The wavelength of the sound is 21.2 cm. Interference maxima occur at angles of 0° and 37.2° to the left and right. Minima occur at angles of 17.6° and 65.1°. No second-order or higher-order maximum exists. No angle exists, smaller or larger than 90°, for which $\sin\theta_{2\text{ loud}} = 1.21$. No location exists in the Universe that is two wavelengths farther from one speaker than from the other.

P37.8 11.3 m

P37.10 641

P37.12 6.33 mm/s

P37.14 See the solution.

P37.16 (a) 1.29 rad (b) 99.6 nm

P37.18 0.968

P37.20 See the solution.

P37.22 (a) See the solution. (b) The cosine function takes on the extreme value −1 to describe the secondary maxima. The cosine function takes on the extreme value +1 to describe the primary maxima. The ratio is 9.00.

P37.24 612 nm

P37.26 0.500 cm

P37.28 No reflection maxima in the visible spectrum

P37.30 290 nm

P37.32 1.31

P37.34 449 nm; blue

P37.36 $1 + N\lambda/2L$

P37.38 (a) See the solution. (b) 2.74 m

P37.40 number of antinodes = number of constructive interference zones

$$= 1 \text{ plus 2 times the greatest positive integer} \le \frac{d}{\lambda}$$

number of nodes = number of destructive interference zones

$$= 2 \text{ times the greatest positive integer} < \left(\frac{d}{\lambda} + \frac{1}{2}\right)$$

P37.42 $x_1 - x_2 = \left(m - \frac{1}{48}\right) 650 \text{ nm}$ with $m = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots$

P37.44 $\dfrac{\lambda}{2(n-1)}$

P37.46 5.00 km^2

P37.48 2.50 mm

P37.50 113

P37.52 115 nm

P37.54 (a) $n(y) = 1.90 - 0.0285\ y/\text{cm}$ (b) 32.3 cm (c) The beam will continuously curve downward.

P37.56 (a) $2(4h^2 + d^2)^{1/2} - 2d$ (b) $(4h^2 + d^2)^{1/2} - d$

P37.58 See the solution.

P37.60 See the solution.

P37.62 1.73 cm

P37.64 130 nm

P37.66 See the solution.