2

Motion in One Dimension

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

** An asterisk indicates an item new to this edition.*

***Q2.1** Count spaces (intervals), not dots. Count 5, not 6. The first drop falls at time zero and the last drop at 5×5 s = 25 s. The average speed is 600 m/25 s = 24 m/s, answer (b).

Q2.2 The net displacement must be zero. The object could have moved away from its starting point and back again, but it is at its initial position again at the end of the time interval.

Q2.3 Yes. Yes. If the speed of the object varies at all over the interval, the instantaneous velocity will sometimes be greater than the average velocity and will sometimes be less.

***Q2.4** (a) It speeds up and its acceleration is positive. (b) It slows down overall, since final speed 1 m/s is slower than 3 m/s. Its acceleration is positive, meaning to the right. (c) It slows down and its acceleration is negative. (d) It speeds up to final speed 7 m/s. Its acceleration is negative, meaning toward the left or towards increasing-magnitude negative numbers on the track.

Q2.5 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past to give car B greater acceleration just then.

***Q2.6** (c) A graph of velocity versus time slopes down steadily from an original positive (northward) value. The graph cuts through zero and goes through increasing-magnitude negative values, all with the same constant acceleration.

***Q2.7** (i) none. All of the disks are moving. (ii) (b) shows equal spacing, meaning constant nonzero velocity and constant zero acceleration. (iii) (b) This question has the same physical meaning as question (ii). (iv) (c) shows positive acceleration throughout. (v) (a) shows negative (leftward) acceleration in the last three images.

***Q2.8** Tramping hard on the brake at zero speed on a level road, you do not feel pushed around inside the car. The forces of rolling resistance and air resistance have dropped to zero as the car coasted to a stop, so the car's acceleration is zero at this moment and afterward.

Tramping hard on the brake at zero speed on an uphill slope, you feel thrown backward against your seat. Before, during, and after the zero-speed moment, the car is moving with a downhill acceleration if you do not tramp on the brake.

Brian Popp suggested the idea for this question.

***Q2.9** With original velocity zero, displacement is proportional to the square of time in $(1/2)at^2$. Making the time one-third as large makes the displacement one-ninth as large, answer (c).

Q2.10 No. Constant acceleration only. Yes. Zero is a constant.

Q2.11 They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity of magnitude v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

***Q2.12** For the release from rest we have $(4\text{ m/s})^2 = 0^2 + 2\,gh$. For case (i), we have $v_f^2 = (3\text{ m/s})^2 + 2\,gh = (3\text{ m/s})^2 + (4\text{ m/s})^2$. Thus answer (d) is true.

For case (ii) the same steps give the same answer (d).

***Q2.13** (i) Its speed is zero at b and e. Its speed is equal at a and c, and somewhat larger at d. On the bounce it is moving somewhat slower at f than at d, and slower at g than at c. The assembled answer is d > f > a = c > g > b = e.

(ii) The velocity is positive at a, f, and g, zero at b and e, and negative at c and d, with magnitudes as described in part (i). The assembled answer is f > a > g > b = e > c > d.

(iii) The acceleration has a very large positive value at e. At all the other points it is -9.8 m/s^2. The answer is e > a = b = c = d = f = g.

Q2.14 (b) Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point. So your ball must travel a smaller distance to the passing point that the ball your friend throws.

SOLUTIONS TO PROBLEMS

Section 2.1 Position, Velocity, and Speed

P2.1 (a) $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{10\text{ m}}{2\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v_{avg} = \dfrac{5\text{ m}}{4\text{ s}} = \boxed{1.2\text{ m/s}}$

(c) $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$

(d) $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{-5\text{ m} - 5\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$

(e) $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{0 - 0}{8 - 0} = \boxed{0\text{ m/s}}$

P2.2 (a) $v_{avg} = \boxed{2.30\text{ m/s}}$

(b) $v = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\text{ m} - 9.20\text{ m}}{3.00\text{ s}} = \boxed{16.1\text{ m/s}}$

(c) $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\text{ m} - 0\text{ m}}{5.00\text{ s}} = \boxed{11.5\text{ m/s}}$

P2.3 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\ \text{m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\ \text{m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00\ \text{m/s})}$ and $t_2 = \frac{d}{(3.00\ \text{m/s})}$. The average speed is:

$$v_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{d/(5.00\ \text{m/s}) + d/(3.00\ \text{m/s})} = \frac{2d}{(8.00\ \text{m/s})d/(15.0\ \text{m}^2/\text{s}^2)}$$

$$v_{avg} = \frac{2(15.0\ \text{m}^2/\text{s}^2)}{8.00\ \text{m/s}} = \boxed{3.75\ \text{m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

P2.4 $x = 10t^2$: By substitution, for

t(s)	=	2.0	2.1	3.0
x(m)	=	40	44.1	90

(a) $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{50\ \text{m}}{1.0\ \text{s}} = \boxed{50.0\ \text{m/s}}$

(b) $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{4.1\ \text{m}}{0.1\ \text{s}} = \boxed{41.0\ \text{m/s}}$

Section 2.2 **Instantaneous Velocity and Speed**

P2.5 (a) at $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

at $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4 - 1.5)\ \text{s}} = -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

(b) The slope of the tangent line can be found from points C and D. $(t_C = 1.0\ \text{s}, x_C = 9.5\ \text{m})$ and $(t_D = 3.5\ \text{s}, x_D = 0)$,

$$v \approx \boxed{-3.8\ \text{m/s}}.$$

(c) The velocity is zero when x is a minimum. This is at $t \approx \boxed{4\ \text{s}}$.

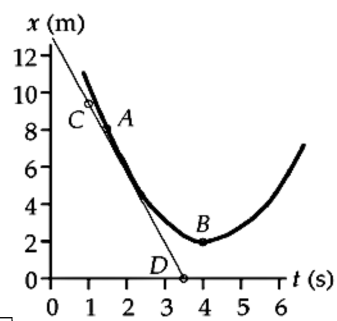


FIG. P2.5

P2.6 (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$.

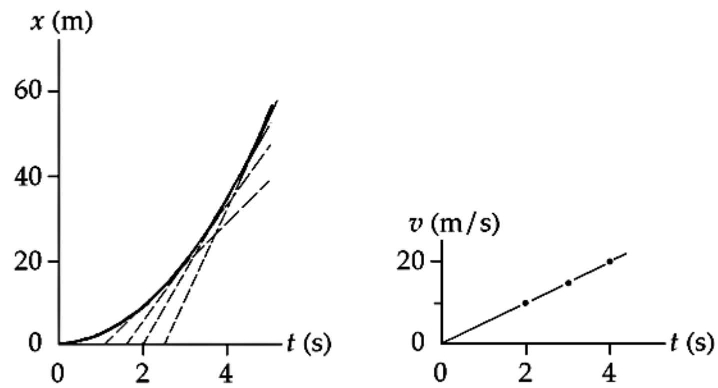
(b) At $t_f = 3.00\ \text{s} + \Delta t$: $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}.$$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0\ \text{m/s} + (3.00\ \text{m/s}^2)\Delta t\right) = \boxed{18.0\ \text{m/s}}.$$

P2.7 (a)



(b) At $t = 5.0$ s, the slope is $v \approx \frac{58 \text{ m}}{2.5 \text{ s}} = \boxed{23 \text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \approx \frac{54 \text{ m}}{3 \text{ s}} = \boxed{18 \text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \approx \frac{49 \text{ m}}{3.4 \text{ s}} = \boxed{14 \text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \approx \frac{36 \text{ m}}{4.0 \text{ s}} = \boxed{9.0 \text{ m/s}}$.

(c) $a_{avg} = \frac{\Delta v}{\Delta t} \approx \frac{23 \text{ m/s}}{5.0 \text{ s}} = \boxed{4.6 \text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$.

P2.8 (a) $v = \frac{(5-0) \text{ m}}{(1-0) \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $v = \frac{(5-10) \text{ m}}{(4-2) \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(c) $v = \frac{(5 \text{ m} - 5 \text{ m})}{(5 \text{ s} - 4 \text{ s})} = \boxed{0}$

(d) $v = \frac{0 - (-5 \text{ m})}{(8 \text{ s} - 7 \text{ s})} = \boxed{+5 \text{ m/s}}$

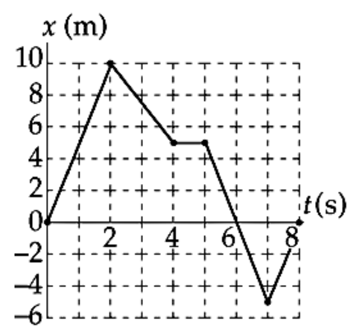


FIG. P2.8

P2.9 Once it resumes the race, the hare will run for a time of

$$t = \frac{x_f - x_i}{v_x} = \frac{1\,000 \text{ m} - 800 \text{ m}}{8 \text{ m/s}} = 25 \text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f - x_i = (0.2 \text{ m/s})(25 \text{ s}) = \boxed{5.00 \text{ m}}.$$

Section 2.3 **Acceleration**

P2.10 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at: \; a = \frac{\Delta v}{\Delta t} = \frac{22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}$$

P2.11 (a) Acceleration is constant over the first ten seconds, so at the end of this interval

$$v_f = v_i + at = 0 + (2.00\ \text{m/s}^2)(10.0\ \text{s}) = \boxed{20.0\ \text{m/s}}.$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0\ \text{m/s} + (-3.00\ \text{m/s}^2)(5.00\ \text{s}) = \boxed{5.00\ \text{m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(2.00\ \text{m/s}^2)(10.0\ \text{s})^2 = 100\ \text{m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + 0 = 200\ \text{m}.$$

And at $t = 20.0$ s,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 200\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + \frac{1}{2}(-3.00\ \text{m/s}^2)(5.00\ \text{s})^2 = \boxed{262\ \text{m}}.$$

P2.12 (a) Acceleration is the slope of the graph of v versus t.

For $0 < t < 5.00$ s, $a = 0$.

For $15.0\ \text{s} < t < 20.0$ s, $a = 0$.

For $5.0\ \text{s} < t < 15.0$ s, $a = \dfrac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60\ \text{m/s}^2$$

We can plot $a(t)$ as shown.

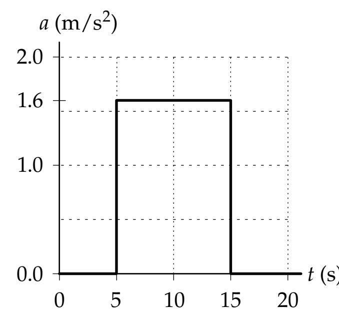


FIG. P2.12

(b) $a = \dfrac{v_f - v_i}{t_f - t_i}$

(i) For $5.00\ \text{s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s,

$t_f = 15.0$ s

$v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60\ \text{m/s}^2}.$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800\ \text{m/s}^2}$$

P2.13 $x = 2.00 + 3.00t - t^2$, so $v = \dfrac{dx}{dt} = 3.00 - 2.00t$, and $a = \dfrac{dv}{dt} = -2.00$

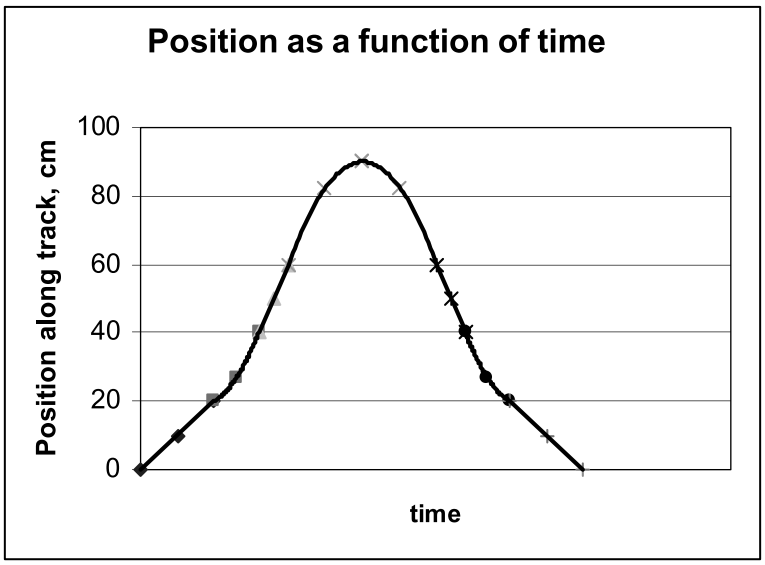
At $t = 3.00$ s:

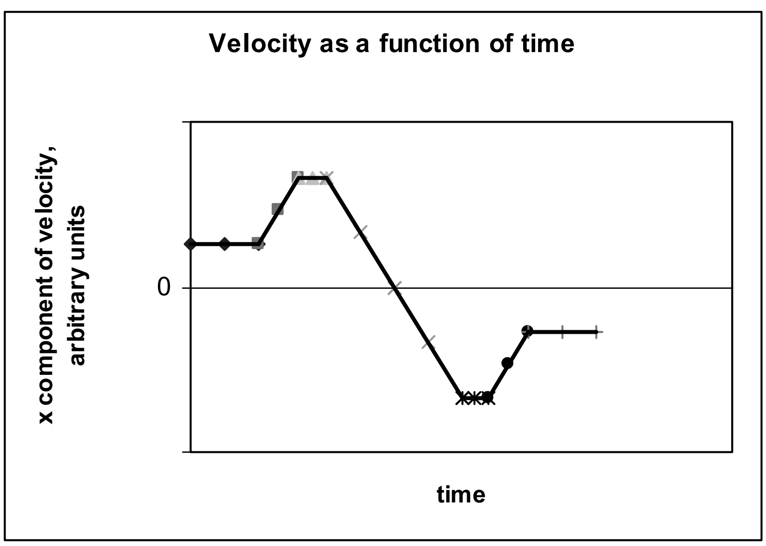
(a) $x = (2.00 + 9.00 - 9.00)\text{ m} = \boxed{2.00\text{ m}}$

(b) $v = (3.00 - 6.00)\text{ m/s} = \boxed{-3.00\text{ m/s}}$

(c) $a = \boxed{-2.00\text{ m/s}^2}$

***P2.14** The acceleration is zero whenever the marble is on a horizontal section. The acceleration has a constant positive value when the marble is rolling on the 20-to-40-cm section and has a constant negative value when it is rolling on the second sloping section. The position graph is a straight sloping line whenever the speed is constant and a section of a parabola when the speed changes.





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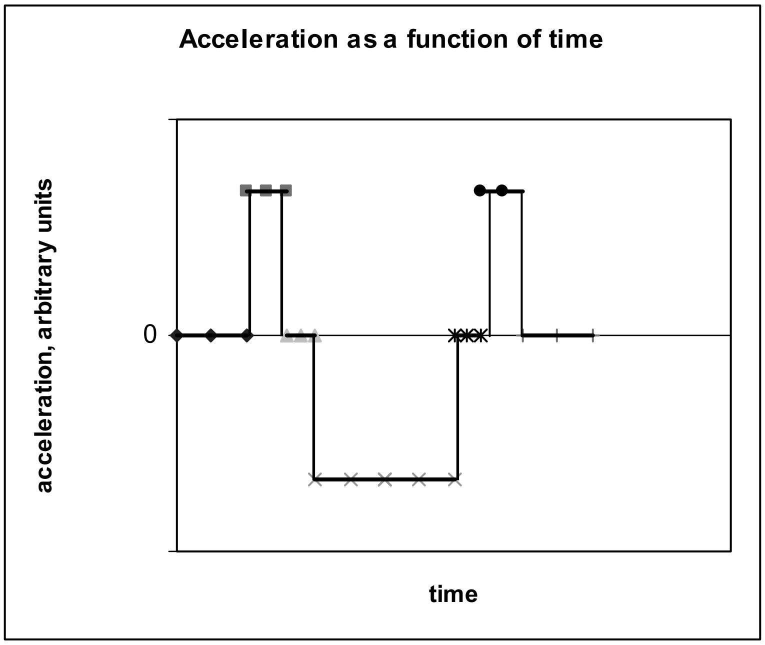


P2.15 (a) At $t = 2.00$ s, $x = \left[3.00(2.00)^2 - 2.00(2.00) + 3.00\right]$ m $= 11.0$ m.

At $t = 3.00$ s, $x = \left[3.00(9.00)^2 - 2.00(3.00) + 3.00\right]$ m $= 24.0$ m

so

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{24.0\text{ m} - 11.0\text{ m}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{13.0\text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00)\text{ m/s}$$

At $t = 2.00$ s, $v = \left[6.00(2.00) - 2.00\right]$ m/s $= \boxed{10.0\text{ m/s}}$.

At $t = 3.00$ s, $v = \left[6.00(3.00) - 2.00\right]$ m/s $= \boxed{16.0\text{ m/s}}$.

(c) $a_{avg} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0\text{ m/s} - 10.0\text{ m/s}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{6.00\text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00t - 2.00) = \boxed{6.00\text{ m/s}^2}$. This includes both $t = 2.00$ s and $t = 3.00$ s.

P2.16 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00\text{ m/s}}{6.00\text{ s}} = \boxed{1.3\text{ m/s}^2}$

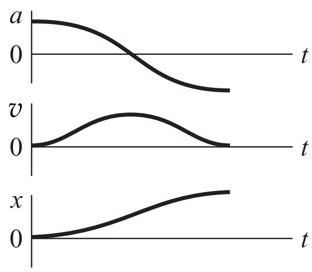
(b) Maximum positive acceleration is at $t = 3$ s, and is the slope of the graph, approximately $(6 - 2)/(4 - 2) = \boxed{2\text{ m/s}^2}$.

(c) $a = 0$ at $\boxed{t = 6\text{ s}}$, and also for $\boxed{t > 10\text{ s}}$.

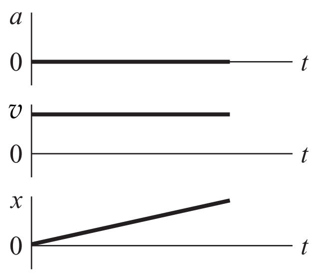
(d) Maximum negative acceleration is at $t = 8$ s, and is the slope of the graph, approximately $\boxed{-1.5\text{ m/s}^2}$.

Section 2.4 **Motion Diagrams**

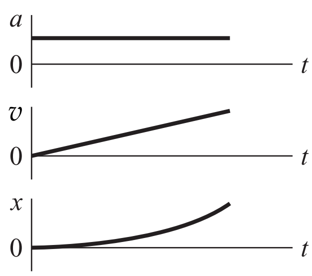
***P2.17** (a) The motion is slow at first, then fast, and then slow again.



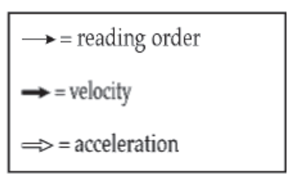
(b) The motion is constant in speed.



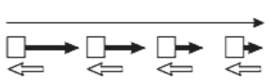
(c) The motion is speeding up, and we suppose the acceleration is constant.

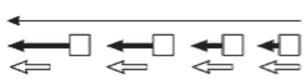


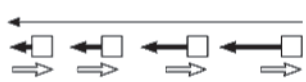
P2.18 (a) 



(b) 

(c) 

(d) 

(e) 

(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

Section 2.5 **One-Dimensional Motion with Constant Acceleration**

***P2.19** (a) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(1\text{ s}) = \boxed{9.00\text{ m/s}}$

(b) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(2\text{ s}) = \boxed{5.00\text{ m/s}}$

(c) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(2.5\text{ s}) = \boxed{3.00\text{ m/s}}$

(d) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(4\text{ s}) = \boxed{-3.00\text{ m/s}}$

(e) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(-1\text{ s}) = \boxed{17.0\text{ m/s}}$

(f) The graph of velocity versus time is a slanting straight line, having the value 13 m/s at 10:05:00 a.m. on the certain date, and sloping down by 4 m/s for every second thereafter.

(g) If we also know the velocity at any one instant, then knowing the value of the constant acceleration tells us the velocity at all other instants.

P2.20 (a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$ becomes $40\text{ m} = \frac{1}{2}(v_i + 2.80\text{ m/s})(8.50\text{ s})$ which yields

$v_i = \boxed{6.61\text{ m/s}}$.

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{2.80\text{ m/s} - 6.61\text{ m/s}}{8.50\text{ s}} = \boxed{-0.448\text{ m/s}^2}$

P2.21 Given $v_i = 12.0\text{ cm/s}$ when $x_i = 3.00\text{ cm}(t=0)$, and at $t = 2.00\text{ s}$, $x_f = -5.00\text{ cm}$,

$$x_f - x_i = v_i t + \frac{1}{2}at^2:\ -5.00 - 3.00 = 12.0(2.00) + \frac{1}{2}a(2.00)^2$$

$$-8.00 = 24.0 + 2a \qquad a = -\frac{32.0}{2} = \boxed{-16.0\text{ cm/s}^2}.$$

P2.22 (a) Total displacement = area under the $(v,\,t)$ curve from $t = 0$ to 50 s.

$$\Delta x = \frac{1}{2}(50\text{ m/s})(15\text{ s}) + (50\text{ m/s})(40-15)\text{ s}$$
$$+\frac{1}{2}(50\text{ m/s})(10\text{ s})$$
$$\Delta x = 1875\text{ m} = \boxed{1.88\text{ km}}$$

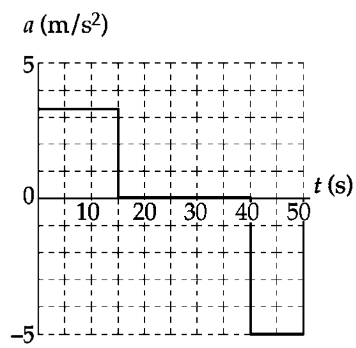


FIG. P2.22

(b) From $t = 10\text{ s}$ to $t = 40\text{ s}$, displacement is

$$\Delta x = \frac{1}{2}(50\text{ m/s} + 33\text{ m/s})(5\text{ s}) + (50\text{ m/s})(25\text{ s}) = \boxed{1.46\text{ km}}$$

(c) $0 \le t \le 15\text{ s}$: $a_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{(50-0)\text{ m/s}}{15\text{ s} - 0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40\text{ s}$: $\boxed{a_2 = 0}$

$40\text{ s} \le t \le 50\text{ s}$: $a_3 = \dfrac{\Delta v}{\Delta t} = \dfrac{(0-50)\text{ m/s}}{50\text{ s} - 40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

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(d) (i) $x_1 = 0 + \frac{1}{2}a_1 t^2 = \frac{1}{2}\left(3.3\ \text{m/s}^2\right)t^2$ or $\boxed{x_1 = \left(1.67\ \text{m/s}^2\right)t^2}$

(ii) $x_2 = \frac{1}{2}(15\ \text{s})[50\ \text{m/s} - 0] + (50\ \text{m/s})(t - 15\ \text{s})$ or $\boxed{x_2 = (50\ \text{m/s})t - 375\ \text{m}}$

(iii) For $40\ \text{s} \le t \le 50\ \text{s}$,

$$x_3 = \begin{pmatrix}\text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40\ \text{s}\end{pmatrix} + \frac{1}{2}a_3(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

or

$$x_3 = 375\ \text{m} + 1\,250\ \text{m} + \frac{1}{2}\left(-5.0\ \text{m/s}^2\right)(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

which reduces to

$$\boxed{x_3 = (250\ \text{m/s})t - \left(2.5\ \text{m/s}^2\right)t^2 - 4\,375\ \text{m}}.$$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1\,875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

P2.23 (a) $v_i = 100\ \text{m/s},\ a = -5.00\ \text{m/s}^2,\ v_f = v_i + at$ so $0 = 100 - 5t,\ v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$ so $0 = (100)^2 - 2(5.00)\left(x_f - 0\right)$. Thus $x_f = 1\,000\ \text{m}$ and $t = \boxed{20.0\ \text{s}}$.

(b) 1 000 m is greater than 800 m. With this acceleration $\boxed{\text{the plane would overshoot the runway: it cannot land}}$.

***P2.24** (a) For the first car the speed as a function of time is $v = v_i + at = -3.5\ \text{cm/s} + 2.4\ \text{cm/s}^2\ t$. For the second car, the speed is + 5.5 cm/s + 0. Setting the two expressions equal gives

$-3.5\ \text{cm/s} + 2.4\ \text{cm/s}^2\ t = 5.5\ \text{cm/s}$ so $t = (9\ \text{cm/s})/(2.4\ \text{cm/s}^2) = \boxed{3.75\ \text{s}}$.

(b) The first car then has speed $-3.5\ \text{cm/s} + (2.4\ \text{cm/s}^2)(3.75\ \text{s}) = \boxed{5.50\ \text{cm/s}}$, and this is the constant speed of the second car also.

(c) For the first car the position as a function of time is $x_i + v_i t + (1/2)at^2 = 15\ \text{cm} - (3.5\ \text{cm/s})t + (0.5)(2.4\ \text{cm/s}^2)\ t^2$.

For the second car, the position is $10\ \text{cm} + (5.5\ \text{cm/s})t + 0$.

At passing, the positions are equal: $15\ \text{cm} - (3.5\ \text{cm/s})t + (1.2\ \text{cm/s}^2)\ t^2 = 10\ \text{cm} + (5.5\ \text{cm/s})t\ (1.2\ \text{cm/s}^2)\ t^2 - (9\ \text{cm/s})\ t + 5\ \text{cm} = 0$.

We solve with the quadratic formula:

$$t = \frac{9 \pm \sqrt{9^2 - 4(1.2)(5)}}{2(1.2)} = \frac{9 + \sqrt{57}}{2.4} \text{ and } \frac{9 - \sqrt{57}}{2.4} = \boxed{6.90\ \text{s and } 0.604\ \text{s}}$$

(d) At 0.604 s, the second and also the first car's position is $10\ \text{cm} + (5.5\ \text{cm/s})0.604\ \text{s} = \boxed{13.3\ \text{cm}}$.

At 6.90 s, both are at position $10\ \text{cm} + (5.5\ \text{cm/s})6.90\ \text{s} = \boxed{47.9\ \text{cm}}$.

continued on next page

(e) The cars are initially moving toward each other, so they soon share the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car. The distance between them will at that moment be staying constant at its maximum value. The distance between the cars will be far from zero, as the accelerating car will be far to the left of the steadily moving car. Thus the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, whizzing past at higher speed than it has ever had before, and giving another answer to (c) that is not an answer to (a). A graph of x versus t for the two cars shows a parabola originally sloping down and then curving upward, intersecting twice with an upward-sloping straight line. The parabola and straight line are running parallel, with equal slopes, at just one point in between their intersections.

P2.25 In the simultaneous equations:

$$\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{array}\right\} \text{ we have } \left\{\begin{array}{c} v_{xf} = v_{xi} - \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) \\ 62.4\ \text{m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\ \text{s}) \end{array}\right\}.$$

So substituting for v_{xi} gives $62.4\ \text{m} = \frac{1}{2}\left[v_{xf} + \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) + v_{xf}\right](4.20\ \text{s})$

$$14.9\ \text{m/s} = v_{xf} + \frac{1}{2}\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}).$$

Thus

$$v_{xf} = \boxed{3.10\ \text{m/s}}.$$

P2.26 Take any two of the standard four equations, such as $\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{array}\right\}$.

Solve one for v_{xi}, and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t.$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}.$$

We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$, this equation represents that displacement is a quadratic function of time.

Our newly derived equation gives us for the situation back in problem 25,

$$62.4\ \text{m} = v_{xf}(4.20\ \text{s}) - \frac{1}{2}\left(-5.60\ \text{m/s}^2\right)(4.20\ \text{s})^2$$

$$v_{xf} = \frac{62.4\ \text{m} - 49.4\ \text{m}}{4.20\ \text{s}} = \boxed{3.10\ \text{m/s}}.$$

P2.27 (a) $a = \dfrac{v_f - v_i}{t} = \dfrac{632(5280/3600)}{1.40} = \boxed{-662 \text{ ft/s}^2} = -202 \text{ m/s}^2$

(b) $x_f = v_i t + \dfrac{1}{2}at^2 = (632)\left(\dfrac{5\,280}{3\,600}\right)(1.40) - \dfrac{1}{2}(662)(1.40)^2 = \boxed{649 \text{ ft}} = \boxed{198 \text{ m}}$

P2.28 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00$ m/s^2. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00 \text{ m/s} - \left(8.00 \text{ m/s}^2\right)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is

$$x = 2.00 \text{ m} + (3.00 \text{ m/s})\left(\frac{3}{8} \text{ s}\right) - \left(4.00 \text{ m/s}^2\right)\left(\frac{3}{8} \text{ s}\right)^2 = \boxed{2.56 \text{ m}}.$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s}$$

and the velocity is $v_f = 3.00 \text{ m/s} - \left(8.00 \text{ m/s}^2\right)\left(\frac{3}{4} \text{ s}\right) = \boxed{-3.00 \text{ m/s}}$.

P2.29 We have $v_i = 2.00 \times 10^4$ m/s, $v_f = 6.00 \times 10^6$ m/s, $x_f - x_i = 1.50 \times 10^{-2}$ m.

(a) $x_f - x_i = \dfrac{1}{2}\left(v_i + v_f\right)t$: $t = \dfrac{2\left(x_f - x_i\right)}{v_i + v_f} = \dfrac{2\left(1.50 \times 10^{-2} \text{ m}\right)}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}} = \boxed{4.98 \times 10^{-9} \text{ s}}$

(b) $v_f^2 = v_i^2 + 2a_x\left(x_f - x_i\right)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left(6.00 \times 10^6 \text{ m/s}\right)^2 - \left(2.00 \times 10^4 \text{ m/s}\right)^2}{2(1.50 \times 10^{-2} \text{ m})} = \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

P2.30 (a) Along the time axis of the graph shown, let $i = 0$ and $f = t_m$. Then $v_{xf} = v_{xi} + a_x t$ gives $v_c = 0 + a_m t_m$

$$\boxed{a_m = \frac{v_c}{t_m}}.$$

(b) The displacement between 0 and t_m is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\frac{v_c}{t_m}t_m^2 = \frac{1}{2}v_c t_m.$$

The displacement between t_m and t_0 is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = v_c\left(t_0 - t_m\right) + 0.$$

The total displacement is

$$\Delta x = \frac{1}{2}v_c t_m + v_c t_0 - v_c t_m = \boxed{v_c\left(t_0 - \frac{1}{2}t_m\right)}.$$

continued on next page

(c) For constant v_c and t_0, Δx is minimized by maximizing t_m to $t_m = t_0$. Then

$$\Delta x_{\min} = v_c\left(t_0 - \frac{1}{2}t_0\right) = \boxed{\frac{v_c t_0}{2}}.$$

(e) This is realized by having the servo motor on all the time.

(d) We maximize Δx by letting t_m approach zero. In the limit $\Delta x = v_c(t_0 - 0) = \boxed{v_c t_0}$.

(e) This cannot be attained because the acceleration must be finite.

P2.31 Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d.$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is $\boxed{\text{equal to } v_d}$ as determined above.

P2.32 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}(-2.00\ \text{m/s}^2)t^2.$$

For the van, $x_{iv} = 155\ \text{m}$, $v_{iv} = 5.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00\ \text{m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$

$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \text{s or } \boxed{11.4\ \text{s}}.$$

The roots are real, not imaginary, so $\boxed{\text{there is a collision}}$. The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155\ \text{m} + (5.00\ \text{m/s})(11.4\ \text{s}) = \boxed{212\ \text{m}}.$$

***P2.33** (a) Starting from rest and accelerating at $a_b = 13.0\ \text{mi/h}\cdot\text{s}$, the bicycle reaches its maximum speed of $v_{b,\max} = 20.0\ \text{mi/h}$ in a time

$$t_{b,1} = \frac{v_{b,\max} - 0}{a_b} = \frac{20.0\ \text{mi/h}}{13.0\ \text{mi/h}\cdot\text{s}} = 1.54\ \text{s}.$$

Since the acceleration a_c of the car is less than that of the bicycle, the car cannot catch the bicycle until some time $t > t_{b,1}$ (that is, until the bicycle is at its maximum speed and coasting). The total displacement of the bicycle at time t is

$$\begin{aligned}\Delta x_b &= \frac{1}{2}a_b t_{b,1}^2 + v_{b,\max}\left(t - t_{b,1}\right)\\ &= \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(13.0\ \frac{\text{mi/h}}{\text{s}}\right)(1.54\ \text{s})^2 + (20.0\ \text{mi/h})(t - 1.54\ \text{s})\right]\\ &= (29.4\ \text{ft/s})t - 22.6\ \text{ft}\end{aligned}$$

The total displacement of the car at this time is

$$\Delta x_c = \frac{1}{2}a_c t^2 = \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(9.00\ \frac{\text{mi/h}}{\text{s}}\right)t^2\right] = (6.62\ \text{ft/s})t^2$$

At the time the car catches the bicycle $\Delta x_c = \Delta x_b$. This gives

$$\left(6.62\ \text{ft/s}^2\right)t^2 = (29.4\ \text{ft/s})t - 22.6\ \text{ft} \qquad \text{or} \qquad t^2 - (4.44\ \text{s})t + 3.42\ \text{s}^2 = 0$$

that has only one physically meaningful solution $t > t_{b,1}$. This solution gives the total time the bicycle leads the car and is $t = \boxed{3.45\ \text{s}}$.

(b) The lead the bicycle has over the car continues to increase as long as the bicycle is moving faster than the car. This means until the car attains a speed of $v_c = v_{b,\max} = 20.0\ \text{mi/h}$. Thus, the elapsed time when the bicycle's lead ceases to increase is

$$t = \frac{v_{b,\max}}{a_c} = \frac{20.0\ \text{mi/h}}{9.00\ \text{mi/h}\cdot\text{s}} = 2.22\ \text{s}$$

At this time, the lead is

$$\left(\Delta x_b - \Delta x_c\right)_{\max} = \left(\Delta x_b - \Delta x_c\right)\Big|_{t=2.22\ \text{s}} = \left[(29.4\ \text{ft/s})(2.22\ \text{s}) - 22.6\ \text{ft}\right] - \left[\left(6.62\ \text{ft/s}^2\right)(2.22\ \text{s})^2\right]$$

or $\left(\Delta x_b - \Delta x_c\right)_{\max} = \boxed{10.0\ \text{ft}}$.

P2.34 As in the algebraic solution to Example 2.9, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and

$$x_{\text{trooper}} = 1.5t^2.$$

They intersect at

$$t = \boxed{31\ \text{s}}.$$

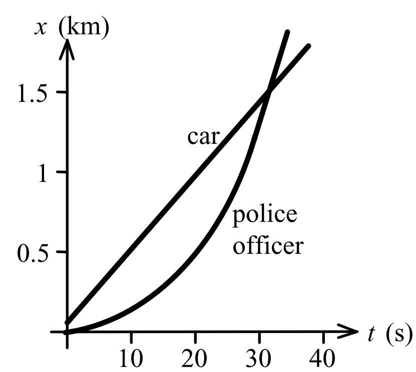

FIG. P2.34

***P2.35** (a) Let a stopwatch start from $t = 0$ as the front end of the glider passes point A. The average speed of the glider over the interval between $t = 0$ and $t = 0.628$ s is 12.4 cm/(0.628 s) = $\boxed{19.7 \text{ cm/s}}$, and this is the instantaneous speed halfway through the time interval, at $t = 0.314$ s.

(b) The average speed of the glider over the time interval between 0.628 + 1.39 = 2.02 s and 0.628 + 1.39 + 0.431 = 2.45 s is 12.4 cm/(0.431 s) = 28.8 cm/s and this is the instantaneous speed at the instant $t = (2.02 + 2.45)/2 = 2.23$ s.

Now we know the velocities at two instants, so the acceleration is found from [(28.8 − 19.7) cm/s]/[(2.23 − 0.314) s] = 9.03/1.92 cm/s^2 = $\boxed{4.70 \text{ cm/s}^2}$.

(c) The time required to pass between A and B is sufficient to find the acceleration, more directly than we could find it from the distance between the points.

Section 2.6 **Freely Falling Objects**

***P2.36** Choose the origin $(y = 0,\ t = 0)$ at the starting point of the cat and take upward as positive. Then $y_i = 0$, $v_i = 0$, and $a = -g = -9.80 \text{ m/s}^2$. The position and the velocity at time t become:

$$y_f - y_i = v_i t + \frac{1}{2}at^2: \qquad y_f = -\frac{1}{2}gt^2 = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)t^2$$

and

$$v_f = v_i + at: \qquad v_f = -gt = -\left(9.80 \text{ m/s}^2\right)t.$$

(a) at $t = 0.1$ s: $y_f = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.1 \text{ s})^2 = \boxed{-0.049\,0 \text{ m}}$

at $t = 0.2$ s: $y_f = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.2 \text{ s})^2 = \boxed{-0.196 \text{ m}}$

at $t = 0.3$ s: $y_f = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.3 \text{ s})^2 = \boxed{-0.441 \text{ m}}$

(b) at $t = 0.1$ s: $v_f = -\left(9.80 \text{ m/s}^2\right)(0.1 \text{ s}) = \boxed{-0.980 \text{ m/s}}$

at $t = 0.2$ s: $v_f = -\left(9.80 \text{ m/s}^2\right)(0.2 \text{ s}) = \boxed{-1.96 \text{ m/s}}$

at $t = 0.3$ s: $v_f = -\left(9.80 \text{ m/s}^2\right)(0.3 \text{ s}) = \boxed{-2.94 \text{ m/s}}$

P2.37 Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80\ \text{m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2 = v_i^2 + 2a(y_f - y_i): \quad 0 = v_i^2 - 2(9.80\ \text{m/s}^2)(1\,609\ \text{m})$$
$$v_i = 178\ \text{m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f - y_i = v_i t + \frac{1}{2}at^2: \ 0 = (178\ \text{m/s})t - \frac{1}{2}(-9.80\ \text{m/s}^2)t^2.$$

The root $t = 0$ describes launch; the other root, $t = 36.2$ s, describes his flight time. His rate of pay is then

$$\text{pay rate } = \frac{\$1.00}{36.2\ \text{s}} = (0.027\,6\ \$/\text{s})(3\,600\ \text{s/h}) = \boxed{\$99.3/\text{h}}.$$

We have assumed that the workman's flight time, "a mile," and "a dollar," were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times.

Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

P2.38 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -(4.90\ \text{m/s}^2)t^2 - (8.00\ \text{m/s})t + 30.0\ \text{m}.$$

Solving for t,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.$$

Using only the positive value for t, we find that $t = \boxed{1.79\ \text{s}}$.

P2.39 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0\ \text{m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68\ \text{m/s}$

$$v_f = \boxed{4.68\ \text{m/s downward}}$$

P2.40 The bill starts from rest $v_i = 0$ and falls with a downward acceleration of $9.80\ \text{m/s}^2$ (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2}gt^2 = 0 - (4.90\ \text{m/s}^2)(0.20\ \text{s})^2 = -0.20\ \text{m}.$$

This distance is about twice the distance between the center of the bill and its top edge $(\approx 8\ \text{cm})$. $\boxed{\text{Thus, David will be unsuccessful.}}$

P2.41 (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00$ s, and $g = 9.80\ \text{m/s}^2$. Therefore,

$$v_i = gt = (9.80\ \text{m/s}^2)(3.00\ \text{s}) = \boxed{29.4\ \text{m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4\ \text{m/s})(3.00\ \text{s}) = \boxed{44.1\ \text{m}}$$

***P2.42** We can solve (a) and (b) at the same time by assuming the rock passes the top of the wall and finding its speed there. If the speed comes out imaginary, the rock will not reach this elevation.

$v_f^2 = v_i^2 + 2a(x_f - x_i) = (7.4\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(3.65\ \text{m} - 1.55\ \text{m}) = 13.6\ \text{m}^2/\text{s}^2$

so $\boxed{\text{the rock does reach the top of the wall with } v_f = 3.69\ \text{m/s}}$.

(c) We find the final speed, just before impact, of the rock thrown down:

$v_f^2 = v_i^2 + 2a(x_f - x_i) = (-7.4\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(1.55\ \text{m} - 3.65\ \text{m}) = 95.9\ \text{m}^2/\text{s}^2$

$v_f = -9.79$ m/s. The change in speed of the rock thrown down is $|9.79 - 7.4| = \boxed{2.39\ \text{m/s}}$

(d) The magnitude of the speed change of the rock thrown up is $|7.4 - 3.69| = 3.71$ m/s. This $\boxed{\text{does not agree}}$ with 2.39 m/s.

The upward-moving rock spends more time in flight, so the planet has more time to change its speed.

P2.43 Time to fall 3.00 m is found from the equation describing position as a function of time, with $v_i = 0$, thus: $3.00\ \text{m} = \frac{1}{2}(9.80\ \text{m/s}^2)t^2$, giving $t = 0.782$ s.

(a) With the horse galloping at $10.0\ \text{m/s}$, the horizontal distance is $vt = \boxed{7.82\ \text{m}}$.

(b) from above $t = \boxed{0.782\ \text{s}}$

P2.44 $y = 3.00t^3$: At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s}\uparrow.$$

If the helicopter releases a small mailbag at this time, the mailbag starts its free fall with velocity 36 m/s upward. The equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for t, (only positive values of t count), $\boxed{t = 7.96\ \text{s}}$.

P2.45 We assume the object starts from rest. Consider the last 30 m of its fall. We find its speed 30 m above the ground:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 30\text{ m} + v_{yi}(1.5\text{ s}) + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(1.5\text{ s})^2$$

$$v_{yi} = \frac{-30\text{ m} + 11.0\text{ m}}{1.5\text{ s}} = -12.6\text{ m/s}.$$

Now consider the portion of its fall above the 30 m point:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$(-12.6\text{ m/s})^2 = 0 + 2\left(-9.8\text{ m/s}^2\right)\Delta y$$

$$\Delta y = \frac{160\text{ m}^2/\text{s}^2}{-19.6\text{ m/s}^2} = -8.16\text{ m}.$$

Its original height was then $30\text{ m} + |-8.16\text{ m}| = \boxed{38.2\text{ m}}$.

Section 2.7 **Kinematic Equations Derived from Calculus**

P2.46 (a) See the graphs at the right.

Choose $x = 0$ at $t = 0$.

At $t = 3\text{ s},\ x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12\text{ m}.$

At $t = 5\text{ s},\ x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28\text{ m}.$

At $t = 7\text{ s},\ x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s}) = 36\text{ m}.$

(b) For $0 < t < 3\text{ s},\ a = \dfrac{8\text{ m/s}}{3\text{ s}} = 2.67\text{ m/s}^2.$

For $3 < t < 5\text{ s},\ a = 0$.

(c) For $5\text{ s} < t < 9\text{ s},\ a = -\dfrac{16\text{ m/s}}{4\text{ s}} = \boxed{-4\text{ m/s}^2}.$

(d) At $t = 6\text{ s},\ x = 28\text{ m} + (6\text{ m/s})(1\text{ s}) = \boxed{34\text{ m}}.$

(e) At $t = 9\text{ s},\ x = 36\text{ m} + \frac{1}{2}(-8\text{ m/s})(2\text{ s}) = \boxed{28\text{ m}}.$

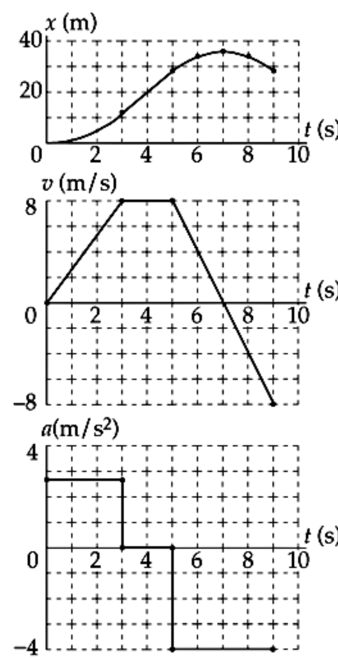

FIG. P2.46

P2.47 (a) $J = \frac{da}{dt} =$ constant

$$da = Jdt$$

$$a = J\int dt = Jt + c_1$$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$. Therefore, $\boxed{a = Jt + a_i}$

$$a = \frac{dv}{dt}$$
$$dv = adt$$
$$v = \int adt = \int (Jt + a_i)dt = \frac{1}{2}Jt^2 + a_i t + c_2$$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_i t + v_i}$

$$v = \frac{dx}{dt}$$
$$dx = vdt$$
$$x = \int vdt = \int \left(\frac{1}{2}Jt^2 + a_i t + v_i\right)dt$$
$$x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + c_3$$
$$x = x_i$$

when $t = 0$, so $c_3 = x_i$. Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + x_i}$.

(b) $a^2 = (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_i t$

$a^2 = a_i^2 + (J^2t^2 + 2Ja_i t)$

$a^2 = a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_i t\right)$

Recall the expression for v: $v = \frac{1}{2}Jt^2 + a_i t + v_i$. So $(v - v_i) = \frac{1}{2}Jt^2 + a_i t$. Therefore,

$$\boxed{a^2 = a_i^2 + 2J(v - v_i)}.$$

P2.48 (a) $a=\frac{dv}{dt}=\frac{d}{dt}\left[-5.00\times10^{7}t^{2}+3.00\times10^{5}t\right]$

$$\boxed{a=-\left(10.0\times10^{7}\ \text{m/s}^{3}\right)t+3.00\times10^{5}\ \text{m/s}^{2}}$$

Take $x_i=0$ at $t=0$. Then $v=\frac{dx}{dt}$

$$x-0=\int_0^t v\,dt=\int_0^t\left(-5.00\times10^{7}t^{2}+3.00\times10^{5}t\right)dt$$

$$x=-5.00\times10^{7}\frac{t^{3}}{3}+3.00\times10^{5}\frac{t^{2}}{2}$$

$$\boxed{x=-\left(1.67\times10^{7}\ \text{m/s}^{3}\right)t^{3}+\left(1.50\times10^{5}\ \text{m/s}^{2}\right)t^{2}}.$$

(b) The bullet escapes when $a=0$, at $-\left(10.0\times10^{7}\ \text{m/s}^{3}\right)t+3.00\times10^{5}\ \text{m/s}^{2}=0$

$$t=\frac{3.00\times10^{5}\ \text{s}}{10.0\times10^{7}}=\boxed{3.00\times10^{-3}\ \text{s}}.$$

(c) New $v=\left(-5.00\times10^{7}\right)\left(3.00\times10^{-3}\right)^{2}+\left(3.00\times10^{5}\right)\left(3.00\times10^{-3}\right)$

$$v=-450\ \text{m/s}+900\ \text{m/s}=\boxed{450\ \text{m/s}}.$$

(d) $x=-\left(1.67\times10^{7}\right)\left(3.00\times10^{-3}\right)^{3}+\left(1.50\times10^{5}\right)\left(3.00\times10^{-3}\right)^{2}$

$x=-0.450\ \text{m}+1.35\ \text{m}=\boxed{0.900\ \text{m}}$

Additional Problems

***P2.49** (a) The velocity is constant between $t_i=0$ and $t=4$ s. Its acceleration is $\boxed{0}$.

(b) $a=(v_9-v_4)/(9\ \text{s}-4\ \text{s})=(18-[-12])\ (\text{m/s})/5\ \text{s}=\boxed{6.0\ \text{m/s}^2}$.

(c) $a=(v_{18}-v_{13})/(18\ \text{s}-13\ \text{s})=(0-18)\ (\text{m/s})/5\ \text{s}=\boxed{-3.6\ \text{m/s}^2}$.

(d) We read from the graph that the speed is zero $\boxed{\text{at } t=6 \text{ s and at } 18 \text{ s}}$.

(e) and (f) The object moves away from $x=0$ into negative coordinates from $t=0$ to $t=6$ s, but then comes back again, crosses the origin and moves farther into positive coordinates until $\boxed{t=18\ \text{s}}$, then attaining its maximum distance, which is the cumulative distance under the graph line:

$(-12\ \text{m/s})(4\ \text{s})+(-6\ \text{m/s})(2\ \text{s})+(9\ \text{m/s})(3\ \text{s})+(18\ \text{m/s})(4\ \text{s})+(9\ \text{m/s})(5\ \text{s})=-60\ \text{m}+144\ \text{m}=\boxed{84\ \text{m}}$.

(g) To gauge the wear on the tires, we consider the total distance rather than the resultant displacement, by counting the contributions computed in part (f) as all positive:

$+60\ \text{m}+144\ \text{m}=\boxed{204\ \text{m}}$.

P2.50 (a) As we see from the graph, from about −50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.

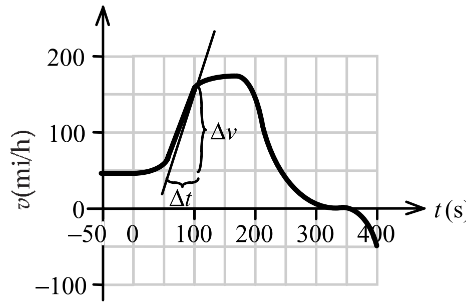

FIG. P2.50(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155-45)\ \text{mi/h}}{(100-50)\ \text{s}} = \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\ \text{m/s}^2.$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

$$\begin{aligned}\Delta x_{0\to 200\ \text{s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\ &\approx (50\ \text{mi/h})(50\ \text{s}) + (50\ \text{mi/h})(50\ \text{s}) \\ &\quad + (160\ \text{mi/h})(100\ \text{s}) \\ &\quad + \frac{1}{2}(50\ \text{s})(100\ \text{mi/h}) \\ &\quad + \frac{1}{2}(100\ \text{s})(170\ \text{mi/h} - 160\ \text{mi/h}) \\ &= 24\,000(\text{mi/h})(\text{s})\end{aligned}$$

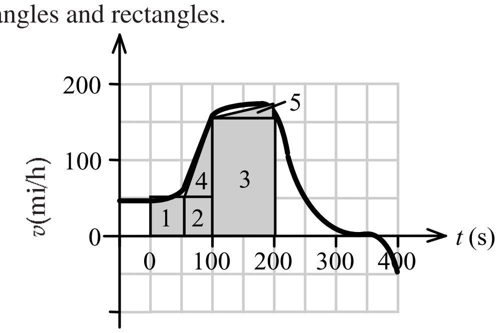

FIG. P2.50(c)

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As $1\ \text{h} = 3\,600\ \text{s}$,

$$\Delta x_{0\to 200\ \text{s}} \approx \left(\frac{24\,000\ \text{mi}}{3\,600\ \text{s}}\right)(\text{s}) = \boxed{6.7\ \text{mi}}.$$

P2.51 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

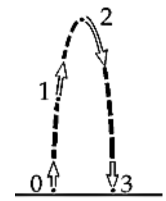

FIG. P2.51

(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120$ m/s

$120 = 80.0 + (4.00)t$ giving $t = 10.0$ s

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735$ m

$0 - 120 = -9.80t$ giving $t = 12.2$ s

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$

$v_f = -184 = (-9.80)t$ giving $t = 18.8$ s

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.52 Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

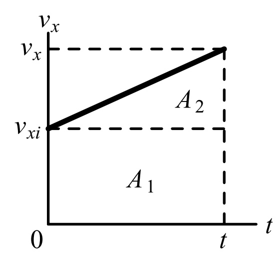

FIG. P2.52

Area A_2 is triangular. Therefore $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$

$$\boxed{A = v_{xi}t + \tfrac{1}{2}a_x t^2}.$$

The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

P2.53 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x = \frac{1}{2}(v + v_i)t_1,\ 100 - x = v(10.2 - t_1),\ \text{and}\ v = v_i + at_1.$$

The first two give

$$100 = \left(10.2 - \frac{1}{2}t_1\right)v = \left(10.2 - \frac{1}{2}t_1\right)at_1$$

$$a = \frac{200}{(20.4 - t_1)t_1}.$$

$$\text{For Maggie: } a = \frac{200}{(18.4)(2.00)} = \boxed{5.43\ \text{m/s}^2}$$

$$\text{For Judy: } a = \frac{200}{(17.4)(3.00)} = \boxed{3.83\ \text{m/s}^2}$$

(b) $v = at_1$

$$\text{Maggie: } v = (5.43)(2.00) = \boxed{10.9\ \text{m/s}}$$

$$\text{Judy: } v = (3.83)(3.00) = \boxed{11.5\ \text{m/s}}$$

(c) At the six-second mark

$$x = \frac{1}{2}at_1^2 + v(6.00 - t_1)$$

$$\text{Maggie: } x = \frac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3\ \text{m}$$

$$\text{Judy: } x = \frac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7\ \text{m}$$

Maggie is ahead by 54.3 m − 51.7 m = $\boxed{2.62\ \text{m}}$. Note that your students may need a reminder that to get the answer in the back of the book they must use calculator memory or a piece of paper to save intermediate results without "rounding off" until the very end.

***P2.54** (a) We first find the distance s_{stop} over which you can stop. The car travels this distance during your reaction time: $\Delta x_1 = v_0(0.6\ \text{s})$. As you brake to a stop, the average speed of the car is $v_0/2$, the interval of time is $(v_f - v_i)/a = -v_0/(-2.40\ \text{m/s}^2) = v_0\ \text{s}^2/2.40\ \text{m}$, and the braking distance is $\Delta x_2 = v_{avg}\ \Delta t = (v_0\ \text{s}^2/2.40\ \text{m})(v_0/2) = v_0^2\ \text{s}^2/4.80\ \text{m}$. The total stopping distance is then $s_{stop} = \Delta x_1 + \Delta x_2 = v_0(0.6\ \text{s}) + v_0^2\ \text{s}^2/4.80\ \text{m}$.

If the car is at this distance from the intersection, it can barely brake to a stop, so it should also be able to get through the intersection at constant speed while the light is yellow, moving a total distance $s_{stop} + 22\ \text{m} = v_0(0.6\ \text{s}) + v_0^2\ \text{s}^2/4.80\ \text{m} + 22\ \text{m}$. This constant-speed motion requires time $\Delta t_y = (s_{stop} + 22\ \text{m})/v_0 = (v_0(0.6\ \text{s}) + v_0^2\ \text{s}^2/4.80\ \text{m} + 22\ \text{m})/v_0 =$ $\boxed{0.6\ \text{s} + v_0\ \text{s}^2/4.80\ \text{m} + 22\ \text{m}/v_0}$.

continued on next page

(b) Substituting, $\Delta t_y = 0.6\text{ s} + (8\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(8\text{ m/s}) = 0.6\text{ s} + 1.67\text{ s} + 2.75\text{ s} = \boxed{5.02\text{ s}}$.

(c) We are asked about higher and higher speeds. For 11 m/s instead of 8 m/s, the time is $0.6\text{ s} + (11\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(11\text{ m/s}) = \boxed{4.89\text{ s}}$ *less* than we had at the lower speed.

(d) Now the time $0.6\text{ s} + (18\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(18\text{ m/s}) = \boxed{5.57\text{ s}}$ begins to increase

(e) $0.6\text{ s} + (25\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(25\text{ m/s}) = \boxed{6.69\text{ s}}$

(f) As v_0 goes to zero, the $22\text{ m}/v_0$ term in the expression for $\boxed{\Delta t_y \text{ becomes large, approaching infinity}}$.

(g) As v_0 grows without limit, the $v_0\text{ s}^2/4.80\text{ m}$ term in the expression for $\boxed{\Delta t_y \text{ becomes large, approaching infinity}}$.

(h) Δt_y decreases steeply from an infinite value at $v_0 = 0$, goes through a rather flat minimum, and then diverges to infinity as v_0 increases without bound. For a very slowly moving car entering the intersection and not allowed to speed up, a very long time is required to get across the intersection. A very fast-moving car requires a very long time to slow down at the constant acceleration we have assumed.

(i) To find the minimum, we set the derivative of Δt_y with respect to v_0 equal to zero:

$$\frac{d}{dv_0}\left(0.6\text{ s} + \frac{v_0\text{ s}^2}{4.8\text{ m}} + 22\text{ m } v_0^{-1}\right) = 0 + \frac{\text{s}^2}{4.8\text{ m}} - 22\text{ m } v_0^{-2} = 0$$

$22\text{ m}/v_0^{\,2} = \text{s}^2/4.8\text{ m} \qquad v_0 = (22\text{m }[4.8\text{ m/s}^2])^{1/2} = \boxed{10.3\text{ m/s}}$

(j) Evaluating again, $\Delta t_y = 0.6\text{ s} + (10.3\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(10.3\text{ m/s}) = \boxed{4.88\text{ s}}$, just a little less than the answer to part (c).

For some students an interesting project might be to measure the yellow-times of traffic lights on local roadways with various speed limits and compare with the minimum

$$\Delta t_{reaction} + (width/2\,|a_{braking}|)^{1/2}$$

implied by the analysis here. But do not let the students string a tape measure across the intersection.

P2.55 $a_1 = 0.100\text{ m/s}^2$ $\qquad a_2 = -0.500\text{ m/s}^2$

$$x = 1\,000\text{ m} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2 \qquad t = t_1 + t_2 \text{ and } v_1 = a_1t_1 = -a_2t_2$$

$$1\,000 = \frac{1}{2}a_1t_1^2 + a_1t_1\left(-\frac{a_1t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2 \qquad 1\,000 = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$$

$$1\,000 = 0.5(0.1)[1 - (0.1/{-0.5})]t_1^{\,2} \quad 20\,000 = 1.20\,t_1^{\,2} \qquad t_1 = \sqrt{\frac{20\,000}{1.20}} = \boxed{129\text{ s}}$$

$$t_2 = \frac{a_1t_1}{-a_2} = \frac{12.9}{0.500} \approx 26\text{ s} \qquad \text{Total time } = t = \boxed{155\text{ s}}$$

***P2.56** (a) From the information in the problem, we model the Ferrari as a particle under constant acceleration. The important "particle" for this part of the problem is the nose of the car. We use the position equation from the particle under constant acceleration model to find the velocity v_0 of the particle as it enters the intersection:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad 28.0\text{ m} = 0 + v_0(3.10\text{ s}) + \tfrac{1}{2}(-2.10\text{ m/s}^2)(3.10\text{ s})^2$$
$$\rightarrow \quad v_0 = 12.3\text{ m/s}$$

Now we use the velocity-position equation in the particle under constant acceleration model to find the displacement of the particle from the first edge of the intersection when the Ferrari stops:

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad x - x_0 = \Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12.3\text{ m/s})^2}{2(-2.10\text{ m/s}^2)} = \boxed{35.9\text{ m}}$$

(b) The time interval during which any part of the Ferrari is in the intersection is that time interval between the instant at which the nose enters the intersection and the instant when the tail leaves the intersection. Thus, the change in position of the nose of the Ferrari is 4.52 m + 28.0 m = 32.52 m. We find the time at which the car is at position $x = 32.52$ m if it is at $x = 0$ and moving at 12.3 m/s at $t = 0$:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad 32.52\text{ m} = 0 + (12.3\text{ m/s})t + \tfrac{1}{2}(-2.10\text{ m/s}^2)t^2$$
$$\rightarrow \quad -1.05t^2 + 12.3t - 32.52 = 0$$

The solutions to this quadratic equation are $t = 4.04$ s and 7.66 s. Our desired solution is the lower of these, so $t = \boxed{4.04\text{ s}}$. (The later time corresponds to the Ferrari stopping and reversing, which it must do if the acceleration truly remains constant, and arriving again at the position $x = 32.52$ m.)

(c) We again define $t = 0$ as the time at which the nose of the Ferrari enters the intersection. Then at time $t = 4.04$ s, the tail of the Ferrari leaves the intersection. Therefore, to find the minimum distance from the intersection for the Corvette, its nose must enter the intersection at $t = 4.04$ s. We calculate this distance from the position equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(5.60\text{ m/s}^2)(4.04\text{ s})^2 = \boxed{45.8\text{ m}}$$

(d) We use the velocity equation:

$$v = v_0 + at = 0 + (5.60\text{ m/s}^2)(4.04\text{ s}) = \boxed{22.6\text{ m/s}}$$

P2.57 (a) $y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2,\ 4.90t^2 + 2.00t - 50.0 = 0$

$$t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$$t = \boxed{3.00\ \text{s}}\ \text{after the first stone is thrown.}$$

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00\ \text{s}$

substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$:

$$v_{i2} = \boxed{15.3\ \text{m/s}}\ \text{downward}$$

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4\ \text{m/s}}$ downward

$v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8\ \text{m/s}}$ downward

P2.58 Let the ball fall freely for 1.50 m after starting from rest. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$v_{xf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(-1.50\ \text{m})$

$$v_{xf} = -5.42\ \text{m/s}$$

If its acceleration were constant its stopping would be described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$0 = (-5.42\ \text{m/s})^2 + 2a_x(-10^{-2}\ \text{m})$$
$$a_x = \frac{-29.4\ \text{m}^2/\text{s}^2}{-2.00\times10^{-2}\ \text{m}} = +1.47\times10^3\ \text{m/s}^2.$$

Upward acceleration of this same order of magnitude will continue for some additional time after the dent is at its maximum depth, to give the ball the speed with which it rebounds from the pavement. The ball's maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \text{m/s}^2}$.

P2.59 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}(3.50\ \text{m/s}^2)(t_k + 1.00)^2 = \frac{1}{2}(4.90\ \text{m/s}^2)(t_k)^2 = x_k$$
$$t_k + 1.00 = 1.183t_k$$
$$t_k = \boxed{5.46\ \text{s}}.$$

(b) $x_k = \frac{1}{2}(4.90\ \text{m/s}^2)(5.46\ \text{s})^2 = \boxed{73.0\ \text{m}}$

(c) $v_k = (4.90\ \text{m/s}^2)(5.46\ \text{s}) = \boxed{26.7\ \text{m/s}}$

$v_s = (3.50\ \text{m/s}^2)(6.46\ \text{s}) = \boxed{22.6\ \text{m/s}}$

P2.60 (a) $d=\frac{1}{2}(9.80)t_1^2$ $\qquad d=336t_2$

$t_1+t_2=2.40$ $\qquad 336t_2=4.90(2.40-t_2)^2$

$4.90t_2^2-359.5t_2+28.22=0$ $\qquad t_2=\frac{359.5\pm\sqrt{359.5^2-4(4.90)(28.22)}}{9.80}$

$t_2=\frac{359.5\pm 358.75}{9.80}=0.076\,5\text{ s}$ so $d=336t_2=\boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d=\frac{1}{2}(9.80)(2.40)^2=28.2\text{ m}$, an error of $\boxed{6.82\%}$.

***P2.61** (a) and (b) We divide each given thinking distance by the corresponding speed to test for the constancy of a proportionality constant. First, 27 ft/25(5280 ft/3600 s) = 0.736 s. Similarly,

constant speed, mi/h	25	35	45	55	65
constant speed, ft/s	36.7	51.3	66	80.7	95.3
thinking distance, ft	27	38	49	60	71
thinking time, s	0.736	0.740	0.742	0.744	0.745

The times can be summarized as 0.742 s ± 0.7%. Their near constancy means that

$\boxed{\text{the car can be modeled as traveling at constant speed, and that the reaction time is 0.742 s}}$.

The proportionality could also be displayed on a graph of thinking distance versus speed, to which a straight line through the origin could be fitted, passing very close to all the points. The line slope is the reaction time.

(c) and (d) According to $v_f^2=v_i^2+2a(x_f-x_i)$ $\qquad 0=v_i^2-2|a|$ (*braking distance*)

$|a|=v_i^2/2$ (*braking distance*), we test for proportionality of speed squared to distance by dividing each squared speed by the given braking distance. The first is (36.7 ft/s)^2/34 ft = 39.5 ft/s^2 = $-2a$. Similarly,

initial speed, ft/s	36.7	51.3	66	80.7	95.3
braking distance, ft	34	67	110	165	231
proportionality constant, ft/s^2	39.5	39.3	39.6	39.4	39.3

The constancy within experimental uncertainty of the last line indicates that the square of initial speed is indeed proportional to the braking distance, so that $\boxed{\text{the braking acceleration is constant}}$. This could be displayed also by graphing initial speed squared versus braking distance. A straight line fits the points convincingly and its slope is $-2a$ = 39.5 ft/s^2, indicating that the braking acceleration is $\boxed{-19.7\text{ ft/s}^2=-6.01\text{ m/s}^2}$.

P2.62

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.62

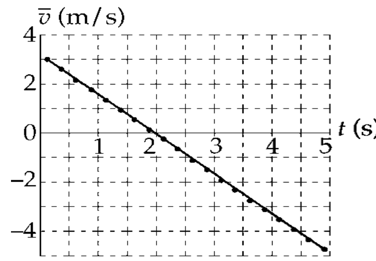

FIG. P2.62

The very convincing fit of a single straight line to the points in the graph of velocity versus time indicates that the rock does fall with constant acceleration. The acceleration is the slope of line:

$$a_{avg} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$$

P2.63 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating through this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt} = -v$.

From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v).$$

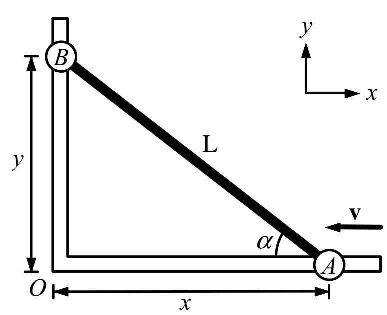

FIG. P2.63

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$. When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$.

ANSWERS TO EVEN PROBLEMS

P2.2 (a) 2.30 m/s (b) 16.1 m/s (c) 11.5 m/s

P2.4 (a) 50.0 m/s (b) 41.0 m/s

P2.6 (a) 27.0 m (b) $27.0\text{ m} + (18.0\text{ m/s})\Delta t + (3.00\text{ m/s}^2)(\Delta t)^2$ (c) 18.0 m/s

P2.8 (a) 5.0 m/s (b) –2.5 m/s (c) 0 (d) 5.0 m/s

P2.10 $1.34 \times 10^4\text{ m/s}^2$

P2.12 (a) see the solution (b) 1.60 m/s^2; 0.800 m/s^2

P2.14 see the solution

P2.16 (a) 1.3 m/s^2 (b) 2.0 m/s^2 at 3 s (c) at $t = 6$ s and for $t > 10$ s (d) -1.5 m/s^2 at 8 s

P2.18 see the solution

P2.20 (a) 6.61 m/s (b) -0.448 m/s^2

P2.22 (a) 1.88 km; (b) 1.46 km (c) see the solution (d) (i) $x_1 = (1.67\text{ m/s}^2)t^2$
(ii) $x_2 = (50\text{ m/s})t - 375\text{ m}$ (iii) $x_3 = (250\text{ m/s})t - (2.5\text{ m/s}^2)t^2 - 4\,375\text{ m}$ (e) 37.5 m/s

P2.24 (a) 3.75 s after release (b) 5.50 cm/s (c) 0.604 s and 6.90 s (d) 13.3 cm and 47.9 cm (e) The cars are initially moving toward each other, so they soon share the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car. The distance between them will at that moment be at its maximum value. The distance between the cars will be far from zero, as the accelerating car will be far to the left of the steadily moving car. Thus the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, whizzing past at higher speed than it has ever had before, and giving another answer to (c) that is not an answer to (a). A graph of x versus t for the two cars shows a parabola originally sloping down and then curving upward, intersecting twice with an upward-sloping straight line. The parabola and straight line are running parallel, with equal slopes, at just one point in between their intersections.

P2.26 $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2$; 3.10 m/s

P2.28 (a) 2.56 m (b) –3.00 m/s

P2.30 (a) v_c/t_m (c) $v_c t_o/2$ (d) $v_c t_o$ (e) The minimum displacement can be attained by having the servo motor on all the time. The maximum displacement cannot be attained because the acceleration must be finite.

P2.32 Yes; 212 m; 11.4 s

P2.34 31.0 s

P2.36 (a) −0.049 0 m, −0.196 m, −0.441 m, with the negative signs all indicating downward
(b) −0.980 m/s, −1.96 m/s, −2.94 m/s

P2.38 1.79 s

P2.40 No; see the solution

P2.42 (a) Yes (b) 3.69 m/s (c) 2.39 m/s (d) +2.39 m/s does not agree with the magnitude of −3.71 m/s. The upward-moving rock spends more time in flight, so its speed change is larger.

P2.44 7.96 s

P2.46 (a) and (b) see the solution (c) $-4\ \mathrm{m/s^2}$ (d) 34 m (e) 28 m

P2.48 (a) $a = -(10.0 \times 10^7\ \mathrm{m/s^3})t + 3.00 \times 10^5\ \mathrm{m/s^2}$; $x = -(1.67 \times 10^7\ \mathrm{m/s^3})t^3 + (1.50 \times 10^5\ \mathrm{m/s^2})t^2$
(b) 3.00×10^{-3} s (c) 450 m/s (d) 0.900 m

P2.50 (a) Acela steadily cruises out of the city center at 45 mi/h. In less than a minute it smoothly speeds up to 150 mi/h; then its speed is nudged up to 170 mi/h. Next it smoothly slows to a very low speed, which it maintains as it rolls into a railroad yard. When it stops, it immediately begins backing up and smoothly speeds up to 50 mi/h in reverse, all in less than seven minutes after it started. (b) 2.2 mi/h/s = 0.98 $\mathrm{m/s^2}$ (c) 6.7 mi.

P2.52 $v_{xi}t + \frac{1}{2}a_x t^2$; displacements agree

P2.54 (a) $\Delta t_y = 0.6\ \mathrm{s} + v_0\ \mathrm{s^2}/4.8\ \mathrm{m} + 22\ \mathrm{m}/v_0$ (b) 5.02 s (c) 4.89 s (d) 5.57 s (e) 6.69 s (f) $\Delta t_y \to \infty$ (g) $\Delta t_y \to \infty$ (h) Δt_y decreases steeply from an infinite value at $v_0 = 0$, goes through a rather flat minimum, and then diverges to infinity as v_0 increases without bound. For a very slowly moving car entering the intersection and not allowed to speed up, a very long time is required to get across the intersection. A very fast-moving car requires a very long time to slow down at the constant acceleration we have assumed. (i) at $v_0 = 10.3$ m/s, (j) $\Delta t_y = 4.88$ s.

P2.56 (a) 35.9 m (b) 4.04 s (c) 45.8 m (d) 22.6 m/s

P2.58 $\sim 10^3\ \mathrm{m/s^2}$

P2.60 (a) 26.4 m (b) 6.82%

P2.62 see the solution; $a_x = -1.63\ \mathrm{m/s^2}$