42

Atomic Physics

Note: In chapters 39, 40, and 41 we used u to represent the speed of a particle with mass. In this chapter 42 and the remaining chapters we go back to using v for the symbol for speed.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q42.1 If an electron moved like a hockey puck, it could have any arbitrary frequency of revolution around an atomic nucleus. If it behaved like a charge in a radio antenna, it would radiate light with frequency equal to its own frequency of oscillation. Thus, the electron in hydrogen atoms would emit a continuous spectrum, electromagnetic waves of all frequencies smeared together.

***Q42.2** (a) Yes, provided that the energy of the photon is *precisely* enough to put the electron into one of the allowed energy states. Strangely—more precisely non-classically—enough, if the energy of the photon is not sufficient to put the electron into a particular excited energy level, the photon will not interact with the atom at all!

(b) Yes, a photon of any energy greater than 13.6 eV will ionize the atom. Any "extra" energy will go into kinetic energy of the newly liberated electron.

***Q42.3** Answer (a). The 10.5-eV bombarding energy does not match the 10.2-eV excitation energy required to lift the atom from state 1 to state 2. But the atom can be excited into state 2 and the bombarding particle can carry off the excess energy.

***Q42.4** (i) b (ii) g From Equations 42.7, 42.8 and 42.9, we have $-|E| = -\frac{k_e e^2}{2r} = +\frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = K + U_e$.
Then $K = |E|$ and $U_e = -2|E|$.

***Q42.5** In $\Delta E = (13.6 \text{ eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$ for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(a) for $n_i = 2$ and $n_f = 5$, $\Delta E = 2.86$ eV (absorption)

(b) for $n_i = 5$ and $n_f = 3$, $\Delta E = -0.967$ eV (emission)

(c) for $n_i = 7$ and $n_f = 4$, $\Delta E = -0.572$ eV (emission)

(d) for $n_i = 4$ and $n_f = 7$, $\Delta E = 0.572$ eV (absorption)

(i) In order of energy change, the ranking is a > d > c > b

(ii) $E = \frac{hc}{\lambda}$ so the ranking in order of decreasing wavelength of the associated photon is c = d > b > a.

***Q42.6** (a) Yes. (b) No. The greatest frequency is that of the Lyman series limit. (c) Yes. We can imagine arbitrarily low photon energies for transitions between adjacent states with n large.

Q42.7 Bohr modeled the electron as moving in a perfect circle, with zero uncertainty in its radial coordinate. Then its radial velocity is always zero with zero uncertainty. Bohr's theory violates the uncertainty principle by making the uncertainty product $\Delta r \Delta p_r$ be zero, less than the minimum allowable $\frac{\hbar}{2}$.

***Q42.8** (i) d (ii) c and d (iii) b and c. Having n start from 1 and ℓ start from zero, with ℓ always less than n, is a way of reminding ourselves that the minimum kinetic energy of a bound quantum particle is greater than zero and the minimum angular momentum is precisely zero.

Q42.9 Fundamentally, three quantum numbers describe an orbital wave function because we live in three-dimensional space. They arise mathematically from boundary conditions on the wave function, expressed as a product of a function of r, a function of θ, and a function of ϕ.

Q42.10 Bohr's theory pictures the electron as moving in a flat circle like a classical particle described by $\Sigma F = ma$. Schrödinger's theory pictures the electron as a cloud of probability amplitude in the three-dimensional space around the hydrogen nucleus, with its motion described by a wave equation. In the Bohr model, the ground-state angular momentum is $1\hbar$; in the Schrödinger model the ground-state angular momentum is zero. Both models predict that the electron's energy is limited to discrete energy levels, given by $\frac{-13.606 \text{ eV}}{n^2}$ with $n = 1, 2, 3.$

Q42.11 Practically speaking, no. Ions have a net charge and the magnetic force $q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$ would deflect the beam, making it difficult to separate the atoms with different orientations of magnetic moments.

Q42.12 The deflecting force on an atom with a magnetic moment is proportional to the *gradient* of the magnetic field. Thus, atoms with oppositely directed magnetic moments would be deflected in *opposite* directions in an inhomogeneous magnetic field.

Q42.13 If the exclusion principle were not valid, the elements and their chemical behavior would be grossly different because every electron would end up in the lowest energy level of the atom. All matter would be nearly alike in its chemistry and composition, since the shell structures of all elements would be identical. Most materials would have a much higher density. The spectra of atoms and molecules would be very simple, and there would be very little color in the world.

Q42.14 In a neutral helium atom, one electron can be modeled as moving in an electric field created by the nucleus and the other electron. According to Gauss's law, if the electron is above the ground state it moves in the electric field of a net charge of $+2e-1e=+1e$. We say the nuclear charge is *screened* by the inner electron. The electron in a He^{+} ion moves in the field of the unscreened nuclear charge of 2 protons. Then the potential energy function for the electron is about double that of one electron in the neutral atom.

Q42.15 The three elements have similar electronic configurations. Each has filled inner shells plus one electron in an *s* orbital. Their single outer electrons largely determine their chemical interactions with other atoms.

Q42.16 Each of the electrons must have at least one quantum number different from the quantum numbers of each of the other electrons. They can differ (in m_s) by being spin-up or spin-down. They can also differ (in ℓ) in angular momentum and in the general shape of the wave function. Those electrons with $\ell=1$ can differ (in m_ℓ) in orientation of angular momentum—look at Figure Q42.16.

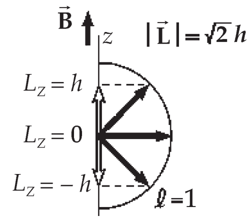


FIG. Q42.16

***Q42.17** The M means that the electron falls down into the M shell. The final principal quantum number is 3. M_α would refer to $4\to3$ and M_β refers to $5\to3$. Answers: (i) e (ii) c

Q42.18 No. Laser light is collimated. The energy generally travels in the same direction. The intensity of a laser beam stays remarkably constant, independent of the distance it has traveled.

Q42.19 Stimulated emission coerces atoms to emit photons along a specific axis, rather than in the random directions of spontaneously emitted photons. The photons that are emitted through stimulation can be made to accumulate over time. The fraction allowed to escape constitutes the intense, collimated, and coherent laser beam. If this process relied solely on spontaneous emission, the emitted photons would not exit the laser tube or crystal in the same direction. Neither would they be coherent with one another.

Q42.20 (a) The terms "I define" and "this part of the universe" seem vague, in contrast to the precision of the rest of the statement. But the statement is true in the sense of being experimentally verifiable. The way to test the orientation of the magnetic moment of an electron is to apply a magnetic field to it. When that is done for any electron, it has precisely a 50% chance of being either spin-up or spin-down. Its spin magnetic moment vector must make one of two allowed angles with the applied magnetic field. They are given by $\cos\theta=\frac{S_z}{S}=\frac{1/2}{\sqrt{3/2}}$ and $\cos\theta=\frac{-1/2}{\sqrt{3/2}}$. You can calculate as many digits of the two angles allowed by "space quantization" as you wish.

(b) This statement may be true. There is no reason to suppose that an ant can comprehend the cosmos, and no reason to suppose that a human can comprehend all of it. Our experience with macroscopic objects does not prepare us to understand quantum particles. On the other hand, what seems strange to us now may be the common knowledge of tomorrow. Looking back at the past 150 years of physics, great strides in understanding the Universe—from the quantum to the galactic scale—have been made. Think of trying to explain the photoelectric effect using Newtonian mechanics. What seems strange sometimes just has an underlying structure that has not yet been described fully. On the other hand still, it has been demonstrated that a "hidden-variable" theory, that would model quantum uncertainty as caused by some determinate but fluctuating quantity, cannot agree with experiment.

***Q42.21** People commonly say that science is difficult to learn. Many other branches of knowledge are constructed by humans. Learning them can require getting your mind to follow the path of someone else's mind. A well-developed science does not mirror human patterns of thought, but the way nature works. Thus we agree with the view stated in the problem. A scientific discovery can be like a garbled communication in the sense that it does not explain itself. It can seem fragmentary and in need of context. It can seem unfriendly in the sense that it has no regard for human desires or ease of human comprehension. Evolution has adapted the human mind for catching rabbits and outwitting bison and leopards, but not necessarily for understanding atoms. Education in science gives the student opportunities for understanding a wide variety of ideas; for evaluating new information; and for recognizing that extraordinary evidence must be adduced for extraordinary claims.

SOLUTIONS TO PROBLEMS

Section 42.1 **Atomic Spectra of Gases**

P42.1 (a) Lyman series $\frac{1}{\lambda} = R\left(1 - \frac{1}{n_i^2}\right)$ $\quad n_i = 2, 3, 4, \ldots$

$$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = \left(1.097 \times 10^7\right)\left(1 - \frac{1}{n_i^2}\right) \qquad \boxed{n_i = 5}$$

(b) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$ $\quad n_i = 4, 5, 6, \ldots$

The shortest wavelength for this series corresponds to $n_i = \infty$ for ionization

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{9} - \frac{1}{n_i^2}\right)$$

For $n_i = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$.

Balmer series: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$ $\quad n_i = 3, 4, 5, \ldots$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{4} - \frac{1}{n_i^2}\right) \qquad \text{with } n_i = \infty \text{ for ionization, } \lambda_{\min} = 365 \text{ nm}$$

Once again the shorter given wavelength $\boxed{\text{cannot be associated with the Balmer series}}$.

***P42.2** (a) The fifth excited state must lie above the second excited state by the photon energy

$$E_{52} = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3\times10^{8}\text{ m/s})}{(520\times10^{-9}\text{ m})} = 3.82\times10^{-19}\text{ J}$$

The sixth excited state exceeds the second in energy by

$$E_{62} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3\times10^{8}\text{ m/s})}{(410\times10^{-9}\text{ m})} = 4.85\times10^{-19}\text{ J}$$

Then the sixth excited state is above the fifth by $(4.85-3.82)10^{-19}\text{ J} = 1.03\times10^{-19}\text{ J}$. In the 6 to 5 transition the atom emits a photon with the infrared wavelength

$$\lambda = \frac{hc}{E_{62}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3\times10^{8}\text{ m/s})}{1.03\times10^{-19}\text{ J}} = \boxed{1.94\times10^{-6}\text{ m}}$$

(b) The same steps solve the symbolic problem.

$$E_{CA} = E_{CB} + E_{BA}$$

$$\frac{hc}{\lambda_{CA}} = \frac{hc}{\lambda_{CB}} + \frac{hc}{\lambda_{BA}}$$

$$\frac{1}{\lambda_{CB}} = \frac{1}{\lambda_{CA}} - \frac{1}{\lambda_{BA}}$$

$$\boxed{\lambda_{CB} = \left(\frac{1}{\lambda_{CA}} - \frac{1}{\lambda_{BA}}\right)^{-1}}$$

Section 42.2 Early Models of the Atom

P42.3 (a) For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1}{4\pi\in_0}\frac{e^2}{r^2 m_e}$$

$$E = -\frac{e^2}{4\pi\in_0 r} + \frac{m_e v^2}{2} = -\frac{e^2}{8\pi\in_0 r}$$

so
$$\frac{dE}{dt} = \frac{e^2}{8\pi\in_0 r^2}\frac{dr}{dt} = \frac{-1}{6\pi\in_0}\frac{e^2a^2}{c^3}$$

$$= \frac{-e^2}{6\pi\in_0 c^3}\left(\frac{e^2}{4\pi\in_0 r^2 m_e}\right)^2$$

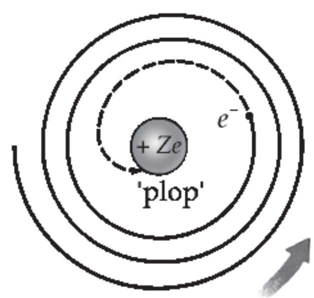


FIG. P42.3

Therefore, $\dfrac{dr}{dt} = -\dfrac{e^4}{12\pi^2\in_0^2 r^2 m_e^2 c^3}$.

(b) $$-\int_{2.00\times10^{-10}\text{ m}}^{0} 12\pi^2\in_0^2 r^2 m_e^2 c^3\,dr = e^4\int_0^T dt \qquad \frac{12\pi^2\in_0^2 m_e^2 c^3}{e^4}\left.\frac{r^3}{3}\right|_0^{2.00\times10^{-10}} = T = \boxed{8.46\times10^{-10}\text{ s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

P42.4 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{Au}}}{r}$$

or
$$r_{\min} = \frac{k_e(2e)(79e)}{E}$$

$$r_{\min} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(4.00\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = \boxed{5.68\times10^{-14}\ \text{m}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{Au}}}{r_{\min}^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(5.68\times10^{-14}\ \text{m}\right)^2} = \boxed{11.3\ \text{N}}$$ away from the nucleus.

Section 42.3 **Bohr's Model of the Hydrogen Atom**

***P42.5** (a) Longest wavelength implies lowest frequency and smallest energy:

the atom falls from $n = 3$ to $n = 2$

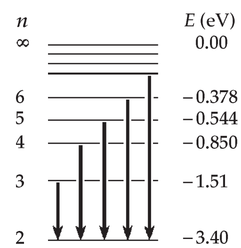
losing energy $-\dfrac{13.6\ \text{eV}}{3^2} + \dfrac{13.6\ \text{eV}}{2^2} = \boxed{1.89\ \text{eV}}$

The photon frequency is $f = \dfrac{\Delta E}{h}$

and its wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\ \times10^{8}\ \text{m/s}\right)}{(1.89\ \text{eV})}\left(\frac{\text{eV}}{1.602\times10^{-19}\ \text{J}}\right)$$

$$\lambda = \boxed{656\ \text{nm}}$$



Balmer Series

FIG. P42.5

This is the red Balmer-alpha line, which gives its characteristic color to the chromosphere of the Sun and to photographs of the Orion nebula.

(b) The biggest energy loss is for an atom to fall from an ionized configuration,

$n = \infty$ to the $n = 2$ state

It loses energy $-\dfrac{13.6\ \text{eV}}{\infty} + \dfrac{13.6\ \text{eV}}{2^2} = \boxed{3.40\ \text{eV}}$

to emit light of wavelength $\lambda = \dfrac{hc}{\Delta E} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(3.40\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{365\ \text{nm}}$

This is the Balmer series limit, in the near ultraviolet.

P42.6 (a) $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$

where $r_1 = (1)^2 a_0 = 0.005\,29\text{ nm} = 5.29 \times 10^{-11}\text{ m}$

$$v_1 = \sqrt{\frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})^2}{(9.11 \times 10^{-31}\text{ kg})(5.29 \times 10^{-11}\text{ m})}} = \boxed{2.19 \times 10^6\text{ m/s}}$$

(b) $K_1 = \dfrac{1}{2} m_e v_1^2 = \dfrac{1}{2}(9.11 \times 10^{-31}\text{ kg})(2.19 \times 10^6\text{ m/s})^2 = 2.18 \times 10^{-18}\text{ J} = \boxed{13.6\text{ eV}}$

(c) $U_1 = -\dfrac{k_e e^2}{r_1} = -\dfrac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})^2}{5.29 \times 10^{-11}\text{ m}} = -4.35 \times 10^{-18}\text{ J} = \boxed{-27.2\text{ eV}}$

P42.7 (a) $r_2^2 = (0.052\,9\text{ nm})(2)^2 = \boxed{0.212\text{ nm}}$

(b) $m_e v_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}} = \sqrt{\dfrac{(9.11 \times 10^{-31}\text{ kg})(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})^2}{0.212 \times 10^{-9}\text{ m}}}$

$m_e v_2 = \boxed{9.95 \times 10^{-25}\text{ kg}\cdot\text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = (9.95 \times 10^{-25}\text{ kg}\cdot\text{m/s})(0.212 \times 10^{-9}\text{ m}) = \boxed{2.11 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $K_2 = \dfrac{1}{2} m_e v_2^2 = \dfrac{(m_e v_2)^2}{2m_e} = \dfrac{(9.95 \times 10^{-25}\text{ kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31}\text{ kg})} = 5.43 \times 10^{-19}\text{ J} = \boxed{3.40\text{ eV}}$

(e) $U_2 = -\dfrac{k_e e^2}{r_2} = -\dfrac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})^2}{0.212 \times 10^{-9}\text{ m}} = -1.09 \times 10^{-18}\text{ J} = \boxed{-6.80\text{ eV}}$

(f) $E_2 = K_2 + U_2 = 3.40\text{ eV} - 6.80\text{ eV} = \boxed{-3.40\text{ eV}}$

***P42.8** (a) The batch of excited atoms must make these six transitions to get back to state one: $2 \to 1$, and also $3 \to 2$ and $3 \to 1$, and also $4 \to 3$ and $4 \to 2$ and $4 \to 1$. Thus, the incoming light must have just enough energy to produce the $1 \to 4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The absorbing atom changes from energy

$$E_i = -\frac{13.6\text{ eV}}{1^2} = -13.6\text{ eV} \text{ to } E_f = -\frac{13.6\text{ eV}}{4^2} = -0.850\text{ eV}$$

so the incoming photons have wavelength $\lambda = c/f = hc/E_{\text{photon}}$

$$\lambda = \frac{hc}{E_f - E_i} = \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^8\text{ m/s})}{-0.850\text{ eV} - (-13.6\text{ eV})}\left(\frac{1.00\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right).$$

$$= 9.74 \times 10^{-8}\text{ m} = \boxed{97.4\text{ nm}}$$

(b) The longest of the six wavelengths corresponds to the lowest photon energy, emitted in the $4 \to 3$ transition. Here $E_i = -\dfrac{13.6\text{ eV}}{4^2} = -0.850\text{ eV}$ and $E_f = -\dfrac{13.6\text{ eV}}{3^2} = -1.51\text{ eV}$,

so $\lambda = \dfrac{hc}{E_f - E_i} = \dfrac{1240\text{ eV}\cdot\text{nm}}{-0.850\text{ eV} - (-1.51\text{ eV})} = \boxed{1.88\ \mu\text{m}}$. This infrared wavelength is part of the Paschen series, since the lower state has $n = 3$.

(c) The shortest wavelength emitted is the same as the wavelength absorbed: 97.4 nm, ultraviolet, Lyman series.

P42.9 (a) The energy levels of a hydrogen-like ion whose charge number is Z are given by

$$E_n = (-13.6\text{ eV})\frac{Z^2}{n^2}$$

Thus for Helium $(Z=2)$, the energy levels are

$$\boxed{E_n = -\frac{54.4\text{ eV}}{n^2} \quad n = 1,\ 2,\ 3,\ \ldots}$$

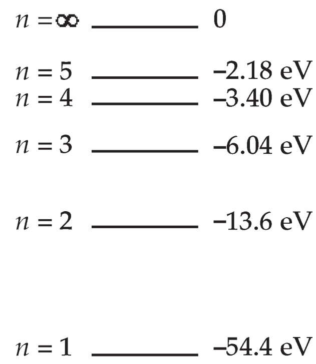


FIG. P42.9

(b) For He^+, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n=1$ to the $n=\infty$ state) is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\text{ eV})(2)^2}{(1)^2} = \boxed{54.4\text{ eV}}$$

***P42.10** (a) The photon has energy 2.28 eV.

And $\dfrac{13.6\text{ eV}}{2^2} = 3.40\text{ eV}$ is required to ionize a hydrogen atom from state $n = 2$. So while the photon cannot ionize a hydrogen atom pre-excited to $n = 2$, it can ionize a hydrogen atom in the $n = \boxed{3}$ state, with energy $-\dfrac{13.6\text{ eV}}{3^2} = -1.51\text{ eV}$.

(b) The electron thus freed can have kinetic energy $K_e = 2.28\text{ eV} - 1.51\text{ eV} = 0.769\text{ eV} = \dfrac{1}{2}m_e v^2$.

Therefore $v = \sqrt{\dfrac{2(0.769)\left(1.60\times10^{-19}\right)\text{ J}}{9.11\times10^{-31}\text{ kg}}} = \boxed{520\text{ km/s}}$.

P42.11 Let r represent the distance between the electron and the positron. The two move in a circle of radius $\dfrac{r}{2}$ around their center of mass with opposite velocities. The total angular momentum of the electron-positron system is quantized according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar$$

where $n = 1,\ 2,\ 3,\ \ldots$

For each particle, $\Sigma F = ma$ expands to $\dfrac{k_e e^2}{r^2} = \dfrac{mv^2}{r/2}$

We can eliminate $v = \dfrac{n\hbar}{mr}$ to find $\dfrac{k_e e^2}{r} = \dfrac{2mn^2\hbar^2}{m^2 r^2}$

So the separation distances are $r = \dfrac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2 = \boxed{\left(1.06\times10^{-10}\text{ m}\right)n^2}$

The orbital radii are $\dfrac{r}{2} = a_0 n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $E = K + U = \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2 - \dfrac{k_e e^2}{r}$

Since $mv^2 = \dfrac{k_e e^2}{2r}$, $E = \dfrac{k_e e^2}{2r} - \dfrac{k_e e^2}{r} = -\dfrac{k_e e^2}{2r} = \dfrac{-k_e e^2}{4a_0 n^2} = \boxed{-\dfrac{6.80\text{ eV}}{n^2}}$

***P42.12** (a) From the Bohr theory we have for the speed of the electron $v = \dfrac{n\hbar}{m_e r}$.

The period of its orbital motion is $T = \dfrac{2\pi r}{v} = \dfrac{2\pi r m_e r}{n\hbar}$. Substituting the orbital radius

$r = \dfrac{n^2\hbar^2}{m_e k_e e^2}$ gives $T = \dfrac{2\pi m_e n^4 \hbar^4}{n\hbar m_e^2 k_e^2 e^4} = \dfrac{2\pi\hbar^3}{m_e k_e^2 e^4} n^3$. Thus we have the periods determined in terms of the ground-state period

$$t_0 = \frac{2\pi\hbar^3}{m_e k_e^2 e^4} = \frac{2\pi(1.055\times10^{-34}\ \text{J}\cdot\text{s})^3}{9.11\times10^{-31}\ \text{kg}(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)^2(1.6\times10^{-19}\ \text{C})^4} = \boxed{153\ \text{as}}$$

(b) In the $n = 2$ state the period is $(153 \times 10^{-18}\ \text{s})2^3 = 1.22 \times 10^{-15}$ s so the number of orbits completed in the excited state is $10 \times 10^{-6}\ \text{s}/1.22 \times 10^{-15}\ \text{s} = \boxed{8.18 \times 10^9\ \text{revolutions}}$.

(c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so we can think of it as a long time.

Section 42.4 **The Quantum Model of the Hydrogen Atom**

P42.13 The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

All the factors in the given equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

For hydrogen, $\mu = \dfrac{m_p m_e}{m_p + m_e} \approx m_e$ The photon energy is $\Delta E = E_3 - E_2$

Its wavelength is $\lambda = 656.3$ nm where $\lambda = \dfrac{c}{f} = \dfrac{hc}{\Delta E}$

(a) For positronium, $\mu = \dfrac{m_e m_e}{m_e + m_e} = \dfrac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium." The photon energy is inversely proportional to its wavelength, so for positronium,

$$\lambda_{32} = 2(656.3\ \text{nm}) = \boxed{1.31\ \mu\text{m}}\ \text{(in the infrared region)}$$

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$

so the transition energy is $2^2 = 4$ times larger than hydrogen

Then, $$\lambda_{32} = \left(\frac{656}{4}\right)\text{nm} = \boxed{164\ \text{nm}}\ \text{(in the ultraviolet region)}$$

***P42.14** (a) For a particular transition from n_i to n_f, $\Delta E_{\text{H}} = -\frac{\mu_{\text{H}} k_e^{\,2} e^4}{2\hbar^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = \frac{hc}{\lambda_{\text{H}}}$ and

$$\Delta E_{\text{D}} = -\frac{\mu_{\text{D}} k_e^{\,2} e^4}{2\hbar^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = \frac{hc}{\lambda_{\text{D}}} \quad \text{where } \mu_{\text{H}} = \frac{m_e m_p}{m_e + m_p} \text{ and } \mu_{\text{D}} = \frac{m_e m_{\text{D}}}{m_e + m_{\text{D}}}.$$

By division, $\frac{\Delta E_{\text{H}}}{\Delta E_{\text{D}}} = \frac{\mu_{\text{H}}}{\mu_{\text{D}}} = \frac{\lambda_{\text{D}}}{\lambda_{\text{H}}}$ or $\lambda_{\text{D}} = \left(\frac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}$. Then, $\boxed{\lambda_{\text{H}} - \lambda_{\text{D}} = \left(1 - \frac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}}$.

(b) $$\frac{\mu_{\text{H}}}{\mu_{\text{D}}} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_{\text{D}}}{m_e m_{\text{D}}}\right) = \frac{(1.007\ 276\text{ u})(0.000\ 549\text{ u} + 2.013\ 553\text{ u})}{(0.000\ 549\text{ u} + 1.007\ 276\text{ u})(2.013\ 553\text{ u})} = 0.999\ 728$$

$$\lambda_{\text{H}} - \lambda_{\text{D}} = (1 - 0.999\ 728)(656.3\text{ nm}) = \boxed{0.179\text{ nm}}$$

***P42.15** (a) $\Delta x \Delta p \ge \frac{\hbar}{2}$ so if $\Delta x = r$, $\Delta p \ge \boxed{\frac{\hbar}{2r}}$

(b) Arbitrarily choosing $\Delta p \approx \frac{\hbar}{r}$, we find $K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \boxed{\frac{\hbar^2}{2m_e r^2}}$

$U = \frac{-k_e e^2}{r}$, so $E = K + U \approx \boxed{\frac{\hbar^2}{2m_e r^2} - \frac{k_e e^2}{r}}$

(c) To minimize E,

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{k_e e^2}{r^2} = 0 \to r = \boxed{\frac{\hbar^2}{m_e k_e e^2} = a_0} \quad \text{(the Bohr radius)}$$

Then, $E = \frac{\hbar^2}{2m_e}\left(\frac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2\left(\frac{m_e k_e e^2}{\hbar^2}\right) = -\frac{m_e k_e^2 e^4}{2\hbar^2} = \boxed{-13.6\text{ eV}}$. With our particular choice for the momentum uncertainty as double its minimum possible value, we find precisely the Bohr results for the orbital radius and for the ground-state energy.

Section 42.5 The Wave Functions of Hydrogen

P42.16 $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ is the ground state hydrogen wave function.

$P_{1s}(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}$ is the ground state radial probability distribution function.

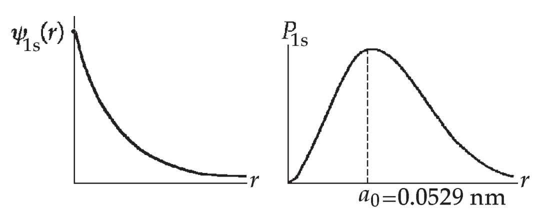


FIG. P42.16

P42.17 (a) $\int |\psi|^2 \, dV = 4\pi \int_0^\infty |\psi|^2 \, r^2 dr = 4\pi \left(\frac{1}{\pi a_0^3}\right) \int_0^\infty r^2 e^{-2r/a_0} dr$

Using integral tables, $\int |\psi|^2 \, dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_0^\infty = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$

so the wave function as given is normalized.

(b) $P_{a_0/2 \to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 \, r^2 dr = 4\pi \left(\frac{1}{\pi a_0^3}\right) \int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0} dr$

Again, using integral tables,

$$P_{a_0/2 \to 3a_0/2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2} = -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right) - e^{-1}\left(\frac{5a_0^2}{4}\right)\right] = \boxed{0.497}$$

P42.18 $\psi = \frac{1}{\sqrt{3}} \frac{1}{(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$

so $P_r = 4\pi r^2 \left|\psi^2\right| = 4\pi r^2 \frac{r^2}{24a_0^5} e^{-r/a_0}$

Set $\frac{dP}{dr} = \frac{4\pi}{24a_0^5}\left[4r^3 e^{-r/a_0} + r^4\left(-\frac{1}{a_0}\right)e^{-r/a_0}\right] = 0$

Solving for r, this is a maximum at $\boxed{r = 4a_0}$.

P42.19 $\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ $\qquad \frac{2}{r}\frac{d\psi}{dr} = \frac{-2}{r\sqrt{\pi a_0^5}} e^{-r/a_0} = -\frac{2}{ra_0}\psi$

$$\frac{d^2\psi}{dr^2} = \frac{1}{\sqrt{\pi a_0^7}} e^{-r/a_0} = \frac{1}{a_0^2}\psi$$

Substitution into the Schrödinger equation to test the validity of the solution yields

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi \in_0 r}\psi = E\psi$$

But $a_0 = \frac{\hbar^2 (4\pi \in_0)}{m_e e^2}$

so $-\frac{e^2}{8\pi \in_0 a_0} = E$ or $\boxed{E = -\frac{k_e e^2}{2a_0}}$

This is true, so the Schrödinger equation is satisfied.

P42.20 The hydrogen ground-state radial probability density is

$$P(r) = 4\pi r^2 |\psi_{1s}|^2 = \frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right)$$

The number of observations at $2a_0$ is, by proportion

$$N = 1\,000\frac{P(2a_0)}{P(a_0/2)} = 1\,000\frac{(2a_0)^2}{(a_0/2)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}} = 1\,000(16)e^{-3} = \boxed{797 \text{ times}}$$

Section 42.6 **Physical Interpretation of the Quantum Numbers**

Note: Problems 31 and 36 in Chapter 29 and Problem 62 in Chapter 30 can be assigned with this section.

P42.21 (a) In the 3d subshell, $n = 3$ and $\ell = 2$,

we have										
n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	–1	–1	–2	–2
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(A total of 10 states)

(b) In the 3p subshell, $n = 3$ and $\ell = 1$,

we have						
n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	0	0	–1	–1
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(A total of 6 states)

P42.22 (a) For the d state, $\ell = 2$, $L = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J}\cdot\text{s}$

(b) For the f state, $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.65 \times 10^{-34}\ \text{J}\cdot\text{s}$

***P42.23** (a) The problem: Find the orbital quantum number of a hydrogen atom in a state in which it has orbital angular momentum $4.714 \times 10^{-34}\ \text{J}\cdot\text{s}$.

(b) The solution: $L = \sqrt{\ell(\ell+1)}\hbar$ $\qquad 4.714 \times 10^{-34} = \sqrt{\ell(\ell+1)}\left(\dfrac{6.626 \times 10^{-34}}{2\pi}\right)$

$$\ell(\ell+1) = \frac{\left(4.714 \times 10^{-34}\right)^2 (2\pi)^2}{\left(6.626 \times 10^{-34}\right)^2} = 1.998 \times 10^1 \approx 20 = 4(4+1)$$

so the orbital quantum number is $\boxed{\ell = 4}$.

P42.24 The 5th excited state has $n = 6$, energy $\dfrac{-13.6\ \text{eV}}{36} = -0.378\ \text{eV}$.

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{\left(1\,090 \times 10^{-9}\ \text{m}\right)\left(1.60 \times 10^{-19}\ \text{J/eV}\right)} = 1.14\ \text{eV}$

to end up with energy $-0.378\ \text{eV} - 1.14\ \text{eV} = -1.52\ \text{eV}$

which is the energy in state 3: $-\dfrac{13.6\ \text{eV}}{3^3} = -1.51\ \text{eV}$

While $n = 3$, ℓ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar}$.

P42.25 (a) $n=1$: For $n=1,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$

n	ℓ	m_ℓ	m_s
1	0	0	–1/2
1	0	0	+1/2

Yields 2 sets; $2n^2=2(1)^2=\boxed{2}$

(b) $n=2$: For $n=2$,

we have

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	–1	±1/2
2	1	0	±1/2
2	1	1	±1/2

Yields 8 sets; $2n^2=2(2)^2=\boxed{8}$

Note that the number is twice the number of m_ℓ values. Also, for each ℓ there are $(2\ell+1)$ different m_ℓ values. Finally, ℓ can take on values ranging from 0 to $n-1$.

So the general expression is number $=\sum_{0}^{n-1}2(2\ell+1)$

The series is an arithmetic progression: $2+6+10+14\ldots$

the sum of which is number $=\frac{n}{2}[2a+(n-1)d]$

where $a=2,\ d=4$: number $=\frac{n}{2}[4+(n-1)4]=2n^2$

(c) $n=3$: $2(1)+2(3)+2(5)=2+6+10=18$ $\quad 2n^2=2(3)^2=\boxed{18}$

(d) $n=4$: $2(1)+2(3)+2(5)+2(7)=32$ $\quad 2n^2=2(4)^2=\boxed{32}$

(e) $n=5$: $32+2(9)=32+18=50$ $\quad 2n^2=2(5)^2=\boxed{50}$

P42.26 For a $3d$ state, $n=3$ and $\ell=2$

Therefore, $L=\sqrt{\ell(\ell+1)}\hbar=\boxed{\sqrt{6}\hbar}=2.58\times10^{-34}\ \text{J}\cdot\text{s}$

m_ℓ can have the values –2, –1, 0, 1, and 2

so $\boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

Using the relation $\cos\theta=\frac{L_z}{L}$

we find the possible values of θ $\boxed{145°,\ 114°,\ 90.0°,\ 65.9°,\text{ and } 35.3°}$

***P42.27** (a) Density of a proton: $\rho = \dfrac{m}{V} = \dfrac{1.67\times10^{-27}\text{ kg}}{(4/3)\pi\left(1.00\times10^{-15}\text{ m}\right)^3} = \boxed{3.99\times10^{17}\text{ kg/m}^3}$

(b) Size of model electron: $r = \left(\dfrac{3m}{4\pi\rho}\right)^{1/3} = \left(\dfrac{3\left(9.11\times10^{-31}\text{ kg}\right)}{4\pi\left(3.99\times10^{17}\text{ kg/m}^3\right)}\right)^{1/3} = \boxed{8.17\times10^{-17}\text{ m}}$

(c) Moment of inertia: $I = \dfrac{2}{5}mr^2 = \dfrac{2}{5}\left(9.11\times10^{-31}\text{ kg}\right)\left(8.17\times10^{-17}\text{ m}\right)^2$

$$= 2.43\times10^{-63}\text{ kg}\cdot\text{m}^2$$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore, $v = \dfrac{\hbar r}{2I} = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(8.17\times10^{-17}\text{ m}\right)}{2\pi\left(2\times2.43\times10^{-63}\text{ kg}\cdot\text{m}^2\right)} = \boxed{1.77\times10^{12}\text{ m/s}}$

(d) This is $\boxed{5.91\times10^3\text{ times larger}}$ than the speed of light. So the spinning-solid-ball model of an electron with spin angular momentum is absurd.

P42.28 In the N shell, $n = 4$. For $n = 4$, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z = m_\ell\hbar$, the maximum value for L_z is $L_z = \boxed{3\hbar}$.

P42.29 The $3d$ subshell has $\ell = 2$, and $n = 3$. Also, we have $s = 1$.

Therefore, we can have $\boxed{n = 3,\ \ell = 2;\ m_\ell = -2,\ -1,\ 0,\ 1,\ 2;\ s = 1;\text{ and } m_s = -1,\ 0,\ 1}$

leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

Section 42.7 **The Exclusion Principle and the Periodic Table**

P42.30 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the $1s$ electrons, $n=1,\ \ell=0,\ m_\ell=0,$ $m_s=+\frac{1}{2}$ and $-\frac{1}{2}$

For the two $2s$ electrons, $n=2,\ \ell=0,\ m_\ell=0,$ $m_s=+\frac{1}{2}$ and $-\frac{1}{2}$

For the four $2p$ electrons, $n=2;\ \ell=1;\ m_\ell=-1$, 0, or 1; and $m_s=+\frac{1}{2}$ or $-\frac{1}{2}$

So one possible set of quantum numbers is

n	1	1	2	2	2	2	2	2
ℓ	0	0	0	0	1	1	1	1
m_ℓ	0	0	0	0	–1	0	1	–1
m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

P42.31 The $\boxed{4s\text{ subshell fills first}}$, for potassium and calcium, before the $3d$ subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\mathrm{Ar}]3d^4 4s^2$ would have lower energy than $[\mathrm{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\mathrm{Ar}]3d^5 4s^1$ is the ground state for chromium.

P42.32 Electronic configuration: Sodium to Argon

$[1s^2 2s^2 2p^6]$	$+3s^1$	$\to$	Na^{11}
	$+3s^2$	$\to$	Mg^{12}
	$+3s^2 3p^1$	$\to$	Al^{13}
	$+3s^2 3p^2$	$\to$	Si^{14}
	$+3s^2 3p^3$	$\to$	P^{15}
	$+3s^2 3p^4$	$\to$	S^{16}
	$+3s^2 3p^5$	$\to$	Cl^{17}
	$+3s^2 3p^6$	$\to$	Ar^{18}
$[1s^2 2s^2 2p^6 3s^2 3p^6]4s^1$		$\to$	K^{19}

P42.33 In the table of electronic configurations in the text, or on a periodic table, we look for the element whose last electron is in a $3p$ state and which has three electrons outside a closed shell. Its electron configuration then ends in $3s^2 3p^1$. The element is $\boxed{\text{aluminum}}$.

P42.34 (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$. The possible states are listed here in columns giving the other quantum numbers:

electron	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
one	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
two	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
electron	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
one	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
two	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are thirty allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

P42.35 (a)

$n+\ell$	1	2	3	4	5	6	7
subshell	$1s$	$2s$	$2p$, $3s$	$3p$, $4s$	$3d$, $4p$, $5s$	$4d$, $5p$, $6s$	$4f$, $5d$, $6p$, $7s$

(b) $Z = 15$:
Filled subshells: $1s$, $2s$, $2p$, $3s$ (12 electrons)
Valence subshell: 3 electrons in $3p$ subshell
Prediction: Valence $= -3$ or $+5$
Element is phosphorus, Valence $= -3$ or $+5$ (Prediction correct)

$Z = 47$:
Filled subshells: $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$ (38 electrons)
Outer subshell: 9 electrons in $4d$ subshell
Prediction: Valence $= -1$
Element is silver, (Prediction fails) Valence is $+1$

$Z = 86$:
Filled subshells: $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$, $4d$, $5p$, $6s$, $4f$, $5d$, $6p$ (86 electrons)
Prediction Outer subshell is full: inert gas
Element is radon, inert (Prediction correct)

P42.36 Listing subshells in the order of filling, we have for element 110,

$$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^{14} 6d^8$$

In order of increasing principal quantum number, this is

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2}$$

P42.37 In the ground state of sodium, the outermost electron is in an s state. This state is spherically symmetric, so it generates no magnetic field by orbital motion, and has the same energy no matter whether the electron is spin-up or spin-down. The energies of the states $3p\uparrow$ and $3p\downarrow$ above $3s$ are $hf_1 = \dfrac{hc}{\lambda}$ and $hf_2 = \dfrac{hc}{\lambda_2}$.

The energy difference is

$$2\mu_B B = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

so

$$B = \frac{hc}{2\mu_B}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{2\left(9.27\times10^{-24}\ \text{J/T}\right)}\left(\frac{1}{588.995\times10^{-9}\ \text{m}} - \frac{1}{589.592\times10^{-9}\ \text{m}}\right)$$

$$B = \boxed{18.4\ \text{T}}$$

Section 42.8 More on Atomic Spectra: Visible and X-ray

P42.38 (a) $\boxed{n = 3,\ \ell = 0,\ m_\ell = 0}$

$\boxed{n = 3,\ \ell = 1,\ m_\ell = -1,\ 0,\ 1}$

For $\boxed{n = 3,\ \ell = 2,\ m_\ell = -2,\ -1,\ 0,\ 1,\ 2}$

(b) ψ_{300} corresponds to $E_{300} = -\dfrac{Z^2E_0}{n^2} = -\dfrac{2^2(13.6)}{3^2} = \boxed{-6.05\ \text{eV}}$.

ψ_{31-1}, ψ_{310}, ψ_{311} have the same energy since n is the same.

ψ_{32-2}, ψ_{32-1}, ψ_{320}, ψ_{321}, ψ_{322} have the same energy since n is the same.

All states are degenerate.

***P42.39** For the 3p state, $E_n = \dfrac{-13.6\text{ eV } Z_{eff}^2}{n^2}$ becomes $-3.0\text{ eV} = \dfrac{-13.6\text{ eV } Z_{eff}^2}{3^2}$ so $Z_{eff} = \boxed{1.4}$

For the 3d state $-1.5\text{ eV} = \dfrac{-13.6\text{ eV } Z_{eff}^2}{3^2}$ so $Z_{eff} = \boxed{1.0}$

When the outermost electron in sodium is promoted from the 3s state into a 3p state, its wave function still overlaps somewhat with the ten electrons below it. It therefore sees the $+11e$ nuclear charge not fully screened, and on the average moves in an electric field like that created by a particle with charge $+11e - 9.6e = 1.4e$. When this valence electron is lifted farther to a 3p state, it is essentially entirely outside the cloud of ten electrons below it, and moves in the field of a net charge $+11e - 10e = 1e$.

***P42.40** (a) Picture all of the energy of an electron after its acceleration going into producing a single photon. Then we have $E = \dfrac{hc}{\lambda} = e\Delta V$ and

$$\Delta V = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s } 2.998\times10^{8}\text{ m/s}}{\lambda \qquad 1.602\times10^{-19}\text{ J/eV}} = \boxed{\frac{1240\text{ V}\cdot\text{nm}}{\lambda}}$$

(b) The potential difference is inversely proportional to the wavelength.

(c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure.

(d) Yes, but it might be unlikely for a very high-energy electron to stop in a single interaction to produce a high-energy gamma ray; and it might be difficult to observe the very low-intensity radio waves produced as bremsstrahlung by low-energy electrons. The potential difference goes to infinity as the wavelength goes to zero. The potential difference goes to zero as the wavelength goes to infinity.

P42.41 Following Example 42.5 $\qquad E_\gamma = \dfrac{3}{4}(42-1)^2(13.6\text{ eV}) = 1.71\times10^4\text{ eV} = 2.74\times10^{-15}\text{ J}$

$$f = 4.14\times10^{18}\text{ Hz}$$

and $\qquad \lambda = \boxed{0.072\,5\text{ nm}}$

P42.42 $E = \dfrac{hc}{\lambda} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{\lambda} = \dfrac{1.240\text{ keV}\cdot\text{nm}}{\lambda}$

For $\lambda_1 = 0.018\,5\text{ nm}$, $\qquad E = 67.11\text{ keV}$

$\lambda_2 = 0.020\,9\text{ nm}$, $\qquad E = 59.4\text{ keV}$

$\lambda_3 = 0.021\,5\text{ nm}$, $\qquad E = 57.7\text{ keV}$

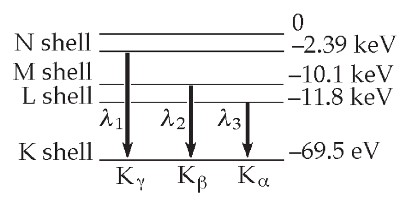


FIG. P42.42

The ionization energy for the K shell is 69.5 keV, so the ionization energies for the other shells are:

$\boxed{\text{L shell} = 11.8\text{ keV}}$ $\qquad \boxed{\text{M shell} = 10.1\text{ keV}}$ $\qquad \boxed{\text{N shell} = 2.39\text{ keV}}$

P42.43 The K_β x-rays are emitted when there is a vacancy in the $(n=1)$ K shell and an electron from the $(n=3)$ M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda}=-\frac{13.6(Z-9)^2}{3^2}\text{ eV}+\frac{13.6(Z-1)^2}{1^2}\text{ eV}$$

$$\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.152\times10^{-9}\text{ m}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}=(13.6\text{ eV})\left(-\frac{Z^2}{9}+\frac{18Z}{9}-\frac{81}{9}+Z^2-2Z+1\right)$$

$$8.17\times10^{3}\text{ eV}=(13.6\text{ eV})\left(\frac{8Z^2}{9}-8\right)$$

so $\quad 601=\dfrac{8Z^2}{9}-8$

and $\quad Z=26 \qquad \boxed{\text{Iron}}$

Section 42.9 **Spontaneous and Stimulated Transitions**

Section 42.10 **Lasers**

P42.44 The photon energy is $\quad E_4-E_3=(20.66-18.70)\text{ eV}=1.96\text{ eV}=\dfrac{hc}{\lambda}$

$$\lambda=\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{1.96\left(1.60\times10^{-19}\text{ J}\right)}=\boxed{633\text{ nm}}$$

P42.45 $f=\dfrac{E}{h}=\dfrac{0.117\text{ eV}}{6.630\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ C}}{e}\right)\left(\dfrac{1\text{ J}}{1\text{ V}\cdot\text{C}}\right)=\boxed{2.82\times10^{13}\text{ s}^{-1}}$

$$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{2.82\times10^{13}\text{ s}^{-1}}=\boxed{10.6\ \mu\text{m}},\ \boxed{\text{infrared}}$$

P42.46 (a) $I=\dfrac{\left(3.00\times10^{-3}\text{ J}\right)}{\left(1.00\times10^{-9}\text{ s}\right)\left[\pi\left(15.0\times10^{-6}\text{ m}\right)^2\right]}=\boxed{4.24\times10^{15}\text{ W/m}^2}$

(b) $\left(3.00\times10^{-3}\text{ J}\right)\dfrac{\left(0.600\times10^{-9}\text{ m}\right)^2}{\left(30.0\times10^{-6}\text{ m}\right)^2}=\boxed{1.20\times10^{-12}\text{ J}}=7.50\text{ MeV}$

P42.47 $E=\mathcal{P}\,\Delta t=\left(1.00\times10^{6}\text{ W}\right)\left(1.00\times10^{-8}\text{ s}\right)=0.010\,0\text{ J}$

$$E_\gamma=hf=\frac{hc}{\lambda}=\frac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)}{694.3\times10^{-9}}\text{ J}=2.86\times10^{-19}\text{ J}$$

$$N=\frac{E}{E_\gamma}=\frac{0.010\,0}{2.86\times10^{-19}}=\boxed{3.49\times10^{16}\text{ photons}}$$

P42.48 (a) $$\frac{N_3}{N_2}=\frac{N_g e^{-E_3/(k_B\cdot 300\text{ K})}}{N_g e^{-E_2/(k_B\cdot 300\text{ K})}}=e^{-(E_3-E_2)/(k_B\cdot 300\text{ K})}=e^{-hc/\lambda(k_B\cdot 300\text{ K})}$$

where λ is the wavelength of light radiated in the $3\to 2$ transition.

$$\frac{N_3}{N_2}=e^{-(6.63\times10^{-34}\text{ J}\cdot\text{s})(3\times10^{8}\text{ m/s})/(632.8\times10^{-9}\text{ m})(1.38\times10^{-23}\text{ J/K})(300\text{ K})}$$

$$\frac{N_3}{N_2}=e^{-75.9}=\boxed{1.07\times10^{-33}}$$

(b) $$\frac{N_u}{N_\ell}=e^{-(E_u-E_\ell)/k_BT}$$

where the subscript u refers to an upper energy state and the subscript ℓ to a lower energy state.

Since $E_u-E_\ell=E_{\text{photon}}=\dfrac{hc}{\lambda}$ $\qquad \dfrac{N_u}{N_\ell}=e^{-hc/\lambda k_BT}$

Thus, we require $\qquad 1.02=e^{-hc/\lambda k_BT}$

or $$\ln(1.02)=-\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3\times10^{8}\text{ m/s})}{(632.8\times10^{-9}\text{ m})(1.38\times10^{-23}\text{ J/K})T}$$

$$T=-\frac{2.28\times10^{4}}{\ln(1.02)}=\boxed{-1.15\times10^{6}\text{ K}}$$

A negative-temperature state is not achieved by cooling the system below 0 K, but by heating it above $T=\infty$, for as $T\to\infty$ the populations of upper and lower states approach equality.

(c) Because $E_u-E_\ell>0$, and in any real equilibrium state $T>0$,

$e^{-(E_u-E_\ell)/k_BT}<1$ $\qquad$ and $\qquad N_u<N_\ell$

Thus, a population inversion cannot happen in thermal equilibrium.

P42.49 (a) The light in the cavity is incident perpendicularly on the mirrors, although the diagram shows a large angle of incidence for clarity. We ignore the variation of the index of refraction with wavelength. To minimize reflection at a vacuum wavelength of 632.8 nm, the net phase difference between rays (1) and (2) should be 180°. There is automatically a 180° shift in one of the two rays upon reflection, so the extra distance traveled by ray (2) should be one whole wavelength:

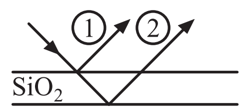


FIG. P42.49

$$2t=\frac{\lambda}{n}$$

$$t=\frac{\lambda}{2n}=\frac{632.8\text{ nm}}{2(1.458)}=\boxed{217\text{ nm}}$$

(b) The total phase difference should be 360°, including contributions of 180° by reflection and 180° by extra distance traveled

$$2t=\frac{\lambda}{2n}$$

$$t=\frac{\lambda}{4n}=\frac{543\text{ nm}}{4(1.458)}=\boxed{93.1\text{ nm}}$$

Additional Problems

P42.50 (a) Using the same procedure that was used in the Bohr model of the hydrogen atom, we apply Newton's second law to the Earth. We simply replace the Coulomb force by the gravitational force exerted by the Sun on the Earth and find

$$G\frac{M_S M_E}{r^2} = M_E \frac{v^2}{r} \tag{1}$$

where v is the orbital speed of the Earth. Next, we apply the postulate that angular momentum of the Earth is quantized in multiples of $\hbar$:

$$M_E v r = n\hbar \qquad (n = 1,\ 2,\ 3,\ \ldots)$$

Solving for v gives

$$v = \frac{n\hbar}{M_E r} \tag{2}$$

Substituting (2) into (1), we find

$$r = \frac{n^2\hbar^2}{GM_S M_E^2} \tag{3}$$

(b) Solving (3) for n gives

$$n = \sqrt{GM_S r}\,\frac{M_E}{\hbar} \tag{4}$$

Taking $M_S = 1.99 \times 10^{30}$ kg, and $M_E = 5.98 \times 10^{24}$ kg, $r = 1.496 \times 10^{11}$ m, $G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$, and $\hbar = 1.055 \times 10^{-34}$ Js, we find

$$n = \boxed{2.53 \times 10^{74}}$$

(c) We can use (3) to determine the radii for the orbits corresponding to the quantum numbers n and $n+1$:

$$r_n = \frac{n^2\hbar^2}{GM_S M_E^2} \qquad \text{and} \qquad r_{n+1} = \frac{(n+1)^2\hbar^2}{GM_S M_E^2}$$

Hence, the separation between these two orbits is

$$\Delta r = \frac{\hbar^2}{GM_S M_E^2}\left[(n+1)^2 - n^2\right] = \frac{\hbar^2}{GM_S M_E^2}(2n+1)$$

Since n is very large, we can neglect the number 1 in the parentheses and express the separation as

$$\Delta r \approx \frac{\hbar^2}{GM_S M_E^2}(2n) = \boxed{1.18 \times 10^{-63}\ \mathrm{m}}$$

This number is *much smaller* than the radius of an atomic nucleus $(\sim 10^{-15}\ \mathrm{m})$, so the distance between quantized orbits of the Earth is too small to observe.

P42.51 (a) $\Delta E = \dfrac{e\hbar B}{m_e} = \dfrac{1.60\times10^{-19}\text{ C}\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)(5.26\text{ T})}{2\pi\left(9.11\times10^{-31}\text{ kg}\right)}\left(\dfrac{\text{N}\cdot\text{s}}{\text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = 9.75\times10^{-23}\text{ J}$

$= \boxed{609\ \mu\text{eV}}$

(b) $k_BT = \left(1.38\times10^{-23}\text{ J/K}\right)\left(80\times10^{-3}\text{ K}\right) = 1.10\times10^{-24}\text{ J} = \boxed{6.9\ \mu\text{eV}}$

(c) $f = \dfrac{\Delta E}{h} = \dfrac{9.75\times10^{-23}\text{ J}}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{1.47\times10^{11}\text{ Hz}}$

$\lambda = \dfrac{c}{f} = \dfrac{3\times10^{8}\text{ m/s}}{1.47\times10^{11}\text{ Hz}} = \boxed{2.04\times10^{-3}\text{ m}}$

P42.52 (a) Probability $= \int_r^\infty P_{1s}(r')\,dr' = \dfrac{4}{a_0^3}\int_r^\infty r'^2 e^{-2r'/a_0}\,dr' = \left[-\left(\dfrac{2r'^2}{a_0^2}+\dfrac{2r'}{a_0}+1\right)e^{-2r'/a_0}\right]_r^\infty$

using integration by parts, we find

$$= \boxed{\left(\frac{2r^2}{a_0^2}+\frac{2r}{a_0}+1\right)e^{-2r/a_0}}$$

(b)

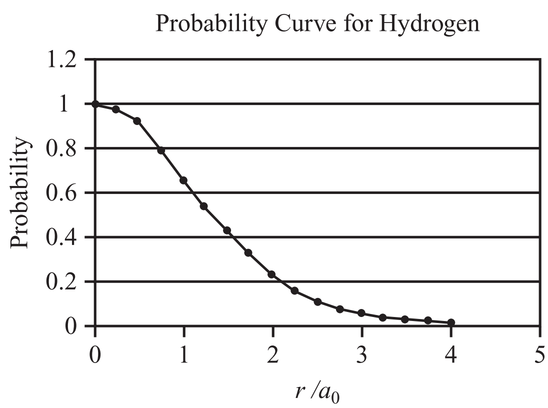


FIG. P42.52

(c) The probability of finding the electron inside or outside the sphere of radius r is $\dfrac{1}{2}$.

$\therefore \left(\dfrac{2r^2}{a_0^2}+\dfrac{2r}{a_0}+1\right)e^{-2r/a_0} = \dfrac{1}{2}$ or $z^2+2z+2=e^z$ where $z=\dfrac{2r}{a_0}$

One can home in on a solution to this transcendental equation for r on a calculator, the result being $r = \boxed{1.34a_0}$ to three digits.

P42.53 $hf = \Delta E = \dfrac{4\pi^2 m_e k_e^2 e^4}{2h^2}\left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2}\right)$ $\qquad f = \dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\dfrac{2n-1}{(n-1)^2 n^2}\right)$

As n approaches infinity, we have f approaching $\dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

The classical frequency is $\qquad f = \dfrac{v}{2\pi r} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e}}\dfrac{1}{r^{3/2}}$

where $\qquad r = \dfrac{n^2 h^2}{4\pi m_e k_e e^2}$

Using this equation to eliminate r from the expression for f, we find $f = \dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\dfrac{2}{n^3}$ in agreement with the Bohr result for large n.

P42.54 (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $\qquad E = E_2 - E_1 = \dfrac{(-13.6\text{ eV})}{4} - \dfrac{(-13.6\text{ eV})}{1} = 10.2\text{ eV} = \boxed{1.63\times 10^{-18}\text{ J}}$

(b) $E = \dfrac{3}{2}k_B T$ or $\qquad T = \dfrac{2E}{3k_B} = \dfrac{2(1.63\times 10^{-18}\text{ J})}{3(1.38\times 10^{-23}\text{ J/K})} = \boxed{7.88\times 10^4\text{ K}}$

P42.55 (a) The energy of the ground state is: $\qquad E_1 = -\dfrac{hc}{\lambda_{\text{series limit}}} = -\dfrac{1\,240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}}$

$= \boxed{-8.16\text{ eV}}$

From the wavelength of the Lyman α line: $\qquad E_2 - E_1 = \dfrac{hc}{\lambda} = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}} = 6.12\text{ eV}$

$E_2 = E_1 + 6.12\text{ eV} = \boxed{-2.04\text{ eV}}$

The wavelength of the Lyman β line gives: $\qquad E_3 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}} = 7.26\text{ eV}$

so $\qquad E_3 = \boxed{-0.902\text{ eV}}$

Next, using the Lyman γ line gives: $\qquad E_4 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}} = 7.65\text{ eV}$

and $\qquad E_4 = \boxed{-0.508\text{ eV}}$

From the Lyman δ line, $\qquad E_5 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}} = 7.83\text{ eV}$

so $\qquad E_5 = \boxed{-0.325\text{ eV}}$

continued on next page

(b) For the Balmer series, $\frac{hc}{\lambda} = E_i - E_2$, or $\lambda = \frac{1\,240 \text{ nm}\cdot\text{eV}}{E_i - E_2}$

For the α line, $E_i = E_3$ and so

$$\lambda_a = \frac{1\,240 \text{ nm}\cdot\text{eV}}{(-0.902 \text{ eV}) - (-2.04 \text{ eV})} = \boxed{1\,090 \text{ nm}}$$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811 \text{ nm}}$, $\boxed{724 \text{ nm}}$, and $\boxed{609 \text{ nm}}$.

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

$0.600(202.6 \text{ nm}) = \boxed{122 \text{ nm}}$, $0.600(170.9 \text{ nm}) = \boxed{103 \text{ nm}}$,

$0.600(162.1 \text{ nm}) = \boxed{97.3 \text{ nm}}$, $0.600(158.3 \text{ nm}) = \boxed{95.0 \text{ nm}}$,

and $0.600(152.0 \text{ nm}) = \boxed{91.2 \text{ nm}}$

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f} = \frac{\lambda}{\lambda'} = \sqrt{\frac{1-(v/c)}{1+(v/c)}} = 0.600 \quad \text{yielding} \quad \boxed{v = 0.471c}$$

***P42.56** (a) The energy emitted by the atom is $\Delta E = E_4 - E_2 = -13.6 \text{ eV}\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 2.55 \text{ eV}$. The wavelength of the photon produced is then

$$\lambda = \frac{hc}{E_\gamma} = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{(2.55 \text{ eV})(1.60\times10^{-19} \text{ J/eV})} = 4.87\times10^{-7} \text{ m} = \boxed{487 \text{ nm}}$$

(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus, $p = m_{\text{atom}}v = \frac{h}{\lambda}$ or

$$v = \frac{h}{m_{\text{atom}}\lambda} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(1.67\times10^{-27} \text{ kg})(4.87\times10^{-7} \text{ m})} = \boxed{0.814 \text{ m/s}}$$

P42.57 The wave function for the 2s state is given by Equation 42.26:

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2 - \frac{r}{a_0}\right]e^{-r/2a_0}.$$

(a) Taking $r = a_0 = 0.529\times10^{-10} \text{ m}$

we find $\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{0.529\times10^{-10} \text{ m}}\right)^{3/2}[2-1]e^{-1/2} = \boxed{1.57\times10^{14} \text{ m}^{-3/2}}$

(b) $|\psi_{2s}(a_0)|^2 = (1.57\times10^{14} \text{ m}^{-3/2})^2 = \boxed{2.47\times10^{28} \text{ m}^{-3}}$

(c) Using Equation 42.24 and the results to (b) gives $P_{2s}(a_0) = 4\pi a_0^2|\psi_{2s}(a_0)|^2 = \boxed{8.69\times10^8 \text{ m}^{-1}}$.

***P42.58** The average squared separation distance is

$$\left\langle r^2 \right\rangle = \int_{\text{all space}} \psi_{1s}^* r^2 \psi_{1s} dV = \int_{r=0}^{\infty} \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} r^2 \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} 4\pi r^2 dr = \frac{4\pi}{\pi a_0^3} \int_0^{\infty} r^4 e^{-2r/a_0} dr$$

We use $\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ from Table B.6.

$$\left\langle r^2 \right\rangle = \frac{4}{a_0^3} \frac{4!}{(2/\ a_0)^5} = \frac{a_0^2 96}{32} = 3a_0^2$$

$$\Delta r = \sqrt{\left\langle r^2 \right\rangle - \langle r \rangle^2} = \left(3a_0^2 - \left(\frac{3a_0}{2} \right)^2 \right)^{1/2} = \left(3a_0^2 - \frac{9a_0^2}{4} \right)^{1/2} = \boxed{\left(\frac{3}{4} \right)^{1/2} a_0}$$

P42.59 (a) $(3.00 \times 10^8 \text{ m/s})(14.0 \times 10^{-12} \text{ s}) = \boxed{4.20 \text{ mm}}$

(b) $E = \frac{hc}{\lambda} = 2.86 \times 10^{-19} \text{ J}$

$$N = \frac{3.00 \text{ J}}{2.86 \times 10^{-19} \text{ J}} = \boxed{1.05 \times 10^{19} \text{ photons}}$$

(c) $V = (4.20 \text{ mm})\left[\pi (3.00 \text{ mm})^2\right] = 119 \text{ mm}^3$

$$n = \frac{1.05 \times 10^{19}}{119} = \boxed{8.82 \times 10^{16} \text{ mm}^{-3}}$$

P42.60 (a) The length of the pulse is $\Delta L = \boxed{c\Delta t}$.

(b) The energy of each photon is $E_\gamma = \frac{hc}{\lambda}$ so $N = \frac{E}{E_\gamma} = \boxed{\frac{E\lambda}{hc}}$

(c) $V = \Delta L \pi \frac{d^2}{4}$ $\qquad n = \frac{N}{V} = \boxed{\left(\frac{4}{c\Delta t \pi d^2} \right) \left(\frac{E\lambda}{hc} \right)}$

P42.61 The fermions are described by the exclusion principle. Two of them, one spin-up and one spin-down, will be in the ground energy level, with

$$d_{\text{NN}} = L = \frac{1}{2}\lambda,\ \lambda = 2L = \frac{h}{p}\text{, and } p = \frac{h}{2L} \qquad K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{8mL^2}$$

The third must be in the next higher level, with

$$d_{\text{NN}} = \frac{L}{2} = \frac{\lambda}{2},\ \lambda = L\text{, and } p = \frac{h}{L} \qquad K = \frac{p^2}{2m} = \frac{h^2}{2mL^2}$$

The total energy is then $\quad \frac{h^2}{8mL^2} + \frac{h^2}{8mL^2} + \frac{h^2}{2mL^2} = \boxed{\frac{3h^2}{4mL^2}}$

P42.62 $\Delta z = \frac{at^2}{2} = \frac{1}{2}\left(\frac{F_z}{m_0}\right)t^2 = \frac{\mu_z\left(dB_z/dz\right)}{2m_0}\left(\frac{\Delta x}{v}\right)^2$ and $\mu_z = \frac{e\hbar}{2m_e}$

$$\frac{dB_z}{dz} = \frac{2m_0(\Delta z)v^2(2m_e)}{\Delta x^2 e\hbar} = \frac{2(108)\left(1.66\times10^{-27}\text{ kg}\right)\left(10^{-3}\text{ m}\right)\left(10^4\text{ m}^2/\text{s}^2\right)\left(2\times9.11\times10^{-31}\text{ kg}\right)}{\left(1.00\text{ m}^2\right)\left(1.60\times10^{-19}\text{ C}\right)\left(1.05\times10^{-34}\text{ J}\cdot\text{s}\right)}$$

$$\frac{dB_z}{dz} = \boxed{0.389\text{ T/m}}$$

P42.63 We use $\psi_{2s}(r) = \frac{1}{4}\left(2\pi a_0^3\right)^{-1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$

By Equation 42.24, $P(r) = 4\pi r^2\psi^2 = \frac{1}{8}\left(\frac{r^2}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)^2 e^{-r/a_0}$

(a) $$\frac{dP(r)}{dr} = \frac{1}{8}\left[\frac{2r}{a_0^3}\left(2-\frac{r}{a_0}\right)^2 - \frac{2r^2}{a_0^3}\left(\frac{1}{a_0}\right)\left(2-\frac{r}{a_0}\right) - \frac{r^2}{a_0^3}\left(2-\frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0} = 0$$

or $$\frac{1}{8}\left(\frac{r}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)\left[2\left(2-\frac{r}{a_0}\right) - \frac{2r}{a_0} - \frac{r}{a_0}\left(2-\frac{r}{a_0}\right)\right]e^{-r/a_0} = 0$$

The roots of $\frac{dP}{dr} = 0$ at $r = 0$, $r = 2a_0$, and $r = \infty$ are minima with $P(r) = 0$.

Therefore we require $[.....] = 4 - \left(\frac{6r}{a_0}\right) + \left(\frac{r}{a_0}\right)^2 = 0$

with solutions $r = \left(3\pm\sqrt{5}\right)a_0$

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = \left(3-\sqrt{5}\right)a_0 = 0.763\,9a_0$, $P(r) = \frac{0.051\,9}{a_0}$

When $r = \left(3+\sqrt{5}\right)a_0 = 5.236a_0$, $P(r) = \frac{0.191}{a_0}$

Therefore, the most probable value of r is $\left(3+\sqrt{5}\right)a_0 = \boxed{5.236a_0}$

(b) $$\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}\left(\frac{r^2}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)^2 e^{-r/a_0}dr$$

Let $u = \frac{r}{a_0}$, $dr = a_0 du$,

$$\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}u^2\left(4-4u+u^2\right)e^{-u}dr = \int_0^\infty \frac{1}{8}\left(u^4-4u^3+4u^2\right)e^{-u}du$$

$$= -\frac{1}{8}\left(u^4+4u^2+8u+8\right)e^{-u}\Big|_0^\infty = 1$$

$\boxed{\text{This is as required for normalization.}}$

P42.64 (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\hat{\mathbf{i}}+\frac{h}{\lambda}\left(-\hat{\mathbf{i}}\right)=mv_f\hat{\mathbf{i}} \qquad \text{so} \qquad v_f-v_i=-\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\ \text{s}=\Delta t$. Then,

$$a=\frac{v_f-v_i}{\Delta t}=-\frac{h}{m\lambda\Delta t}\sim-\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(10^{-25}\ \text{kg}\right)\left(500\times10^{-9}\ \text{m}\right)\left(10^{-8}\ \text{s}\right)}\sim\boxed{-10^{6}\ \text{m/s}^2}$$

(b) With constant average acceleration,

$$v_f^2=v_i^2+2a\Delta x \qquad 0\sim\left(10^3\ \text{m/s}\right)^2+2\left(-10^6\ \text{m/s}^2\right)\Delta x$$

so
$$\Delta x\sim\frac{\left(10^3\ \text{m/s}\right)^2}{10^6\ \text{m/s}^2}\boxed{\sim 1\ \text{m}}$$

P42.65 With one vacancy in the K shell, excess energy

$$\Delta E\approx-(Z-1)^2(13.6\ \text{eV})\left(\frac{1}{2^2}-\frac{1}{1^2}\right)=5.40\ \text{keV}$$

We suppose the outermost 4*s* electron is shielded by 22 electrons inside its orbit:

$$E_{\text{ionization}}\approx\frac{2^2(13.6\ \text{eV})}{4^2}=3.40\ \text{eV}$$

As evidence that this is of the right order of magnitude, note that the experimental ionization energy is 6.76 eV.

$$K=\Delta E-E_{\text{ionization}}\approx\boxed{5.39\ \text{keV}}$$

P42.66 $E=\dfrac{hc}{\lambda}=\dfrac{1\,240\ \text{eV}\cdot\text{nm}}{\lambda}=\Delta E$

$\lambda_1=310\ \text{nm}$, so $\Delta E_1=4.00\ \text{eV}$

$\lambda_2=400\ \text{nm}$, $\Delta E_2=3.10\ \text{eV}$

$\lambda_3=1\,378\ \text{nm}$, $\Delta E_3=0.900\ \text{eV}$

and the ionization energy $=4.10\ \text{eV}$.

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.

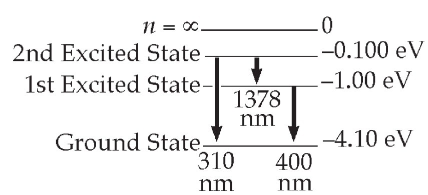


FIG. P42.66

P42.67 $P=\displaystyle\int_{2.50a_0}^{\infty}\frac{4r^2}{a_0^3}e^{-2r/a_0}dr=\frac{1}{2}\int_{5.00}^{\infty}z^2e^{-z}dz$ where $z\equiv\dfrac{2r}{a_0}$

$$P=-\frac{1}{2}\left(z^2+2z+2\right)e^{-z}\Big|_{5.00}^{\infty}=-\frac{1}{2}[0]+\frac{1}{2}(25.0+10.0+2.00)e^{-5}=\left(\frac{37}{2}\right)(0.006\,74)=\boxed{0.125}$$

P42.68 (a) One molecule's share of volume

Al: $$V = \frac{\text{mass per molecule}}{\text{density}} = \left(\frac{27.0\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}}\right)\left(\frac{1.00\times10^{-6}\ \text{m}^3}{2.70\ \text{g}}\right)$$
$$= 1.66\times10^{-29}\ \text{m}^3$$

$$\sqrt[3]{V} = \boxed{2.55\times10^{-10}\ \text{m}\sim10^{-1}\ \text{nm}}$$

U: $$V = \left(\frac{238\ \text{g}}{6.02\times10^{23}\ \text{molecules}}\right)\left(\frac{1.00\times10^{-6}\ \text{m}^3}{18.9\ \text{g}}\right) = 2.09\times10^{-29}\ \text{m}^3$$

$$\sqrt[3]{V} = \boxed{2.76\times10^{-10}\ \text{m}\sim10^{-1}\ \text{nm}}$$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge $+Ze-(Z-1)e=+e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is $\frac{a_0}{Z}$.

P42.69 $\Delta E = 2\mu_B B = hf$

so $2\left(9.27\times10^{-24}\ \text{J/T}\right)(0.350\ \text{T}) = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)f$

and $f = \boxed{9.79\times10^{9}\ \text{Hz}}$

ANSWERS TO EVEN PROBLEMS

P42.2 (a) 1.94 μm (b) $\lambda_{CB} = \dfrac{1}{1/\lambda_{CA} - 1/\lambda_{BA}}$

P42.4 (a) 56.8 fm (b) 11.3 N away from the nucleus

P42.6 (a) 2.19 Mm/s (b) 13.6 eV (c) −27.2 eV

P42.8 (a) The atoms must be excited to energy level $n = 4$, to emit six different photon energies in the downward transitions $4\to3$, $4\to2$, $4\to1$, $3\to2$, $3\to1$, and $2\to1$. The photon energy absorbed in the $1\to4$ transition is 12.8 eV, making the wavelength 97.4 nm. (b) 1.88 μm, infrared, Paschen (c) 97.4 nm, ultraviolet, Lyman

P42.10 (a) 3 (b) 520 km/s

P42.12 (a) 153 as (b) 8.18×10^{9} revolutions (c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so you can think of it as a long time.

P42.14 (a) See the solution. (b) 0.179 nm

P42.16 See the solution.

P42.18 $4a_0$

P42.20 797 times

P42.22 (a) $\sqrt{6}\hbar = 2.58\times10^{-34}\ \text{J}\cdot\text{s}$ (b) $\sqrt{12}\hbar = 3.65\times10^{-34}\ \text{J}\cdot\text{s}$

P42.24 $\sqrt{6}\hbar$

P42.26 $\sqrt{6}\hbar;\ -2\hbar,\ -\hbar,\ 0,\ \hbar,\ 2\hbar$; 145°, 114°, 90.0°, 65.9°, 35.3°

P42.28 $3\hbar$

P42.30 (a) $1s^2 2s^2 2p^4$

(b)

n	1	1	2	2	2	2	2	2
ℓ	0	0	0	0	1	1	1	1
m_ℓ	0	0	0	0	1	1	0	−1
m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

P42.32 See the solution.

P42.34 (a) See the solution. (b) 36 states instead of 30

P42.36 $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10}\ 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2$

P42.38 (a) $\ell = 0$ with $m_\ell = 0$; $\ell = 1$ with $m_\ell = 1$, 0, or 1; and $\ell = 2$ with $m_\ell = -2, -1, 0, 1, 2$ (b) –6.05 eV

P42.40 (a) $\Delta V = 1240\ \text{V}\cdot\text{nm}/\lambda$. (b) The potential difference is inversely proportional to the wavelength. (c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure. (d) Yes, but it might be unlikely for a very high-energy electron to stop in a single interaction to produce a high-energy gamma ray; and it might be difficult to observe the very low-intensity radio waves produced as bremsstrahlung by low-energy electrons. The potential difference goes to infinity as the wavelength goes to zero. The potential difference goes to zero as the wavelength goes to infinity.

P42.42 L shell 11.8 keV, M shell 10.1 keV, N shell 2.39 keV. See the solution.

P42.44 See the solution.

P42.46 (a) $4.24\ \text{PW/m}^2$ (b) $1.20\ \text{pJ} = 7.50\ \text{MeV}$

P42.48 (a) 1.07×10^{-33} (b) $-1.15\times10^{6}\ \text{K}$ (c) Negative temperatures do not describe systems in thermal equilibrium.

P42.50 (a) See the solution. (b) 2.53×10^{74} (c) $1.18\times10^{-63}\ \text{m}$, unobservably small

P42.52 (a) Probability $= \left(\dfrac{2r^2}{a_0^2} + \dfrac{2r}{a_0} + 1\right)e^{-2r/a_0}$ (b) See the solution. (c) $1.34a_0$

P42.54 (a) $10.2\ \text{eV} = 1.63\ \text{aJ}$ (b) $7.88\times10^{4}\ \text{K}$

P42.56 (a) 487 nm (b) 0.814 m/s

P42.58 $\sqrt{3/4}\ a_0 = 0.866\ a_0$

P42.60 (a) $c\Delta t$ (b) $\dfrac{E\lambda}{hc}$ (c) $\dfrac{4E\lambda}{\Delta t\pi d^2 hc^2}$

P42.62 0.389 T/m

P42.64 (a) $\sim -10^6$ m/s^2 (b) ~1 m

P42.66 Energy levels at 0, –0.100 eV, –1.00 eV, and –4.10 eV

P42.68 (a) diameter $\sim 10^{-1}$ nm for both (b) A K-shell electron moves in an orbit with size on the order of $\dfrac{a_0}{Z}$.