15

Oscillatory Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q15.1 Neither are examples of simple harmonic motion, although they are both periodic motion. In neither case is the acceleration proportional to the position. Neither motion is so smooth as SHM. The ball's acceleration is very large when it is in contact with the floor, and the student's when the dismissal bell rings.

***Q15.2** (i) Answer (c). At 120 cm we have the midpoint between the turning points, so it is the equilibrium position and the point of maximum speed.

(ii) Answer (a). In simple harmonic motion the acceleration is a maximum when the excursion from equilibrium is a maximum.

(iii) Answer (a), by the same logic as in part (ii).

(iv) Answer (c), by the same logic as in part (i).

(v) Answer (c), by the same logic as in part (i).

(vi) Answer (e). The total energy is a constant.

Q15.3 You can take $\phi = \pi$, or equally well, $\phi = -\pi$. At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.

***Q15.4** The amplitude does not affect the period in simple harmonic motion; neither do constant forces that offset the equilibrium position. Thus a, b, e, and f all have equal periods. The period is proportional to the square root of mass divided by spring constant. So c, with larger mass, has larger period than a. And d with greater stiffness has smaller period. In situation g the motion is not quite simple harmonic, but has slightly smaller angular frequency and so slightly longer period. Thus the ranking is c > g > a = b = e = f > d.

***Q15.5** (a) Yes. In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.

(b) Yes. Velocity and acceleration are in the same direction half the time.

(c) No. Acceleration is always opposite to the position vector, and never in the same direction.

***Q15.6** Answer (e). We assume that the coils of the spring do not hit one another. The frequency will be higher than f by the factor $\sqrt{2}$. When the spring with two blocks is set into oscillation in space, the coil in the center of the spring does not move. We can imagine clamping the center coil in place without affecting the motion. We can effectively duplicate the motion of each individual block in space by hanging a single block on a half-spring here on Earth. The half-spring with its center coil clamped—or its other half cut off—has twice the spring constant as the original uncut spring, because an applied force of the same size would produce only one-half the extension distance. Thus the oscillation frequency in space is $\left(\frac{1}{2\pi}\right)\left(\frac{2k}{m}\right)^{1/2} = \sqrt{2}f$. The absence of a force required to support the vibrating system in orbital free fall has no effect on the frequency of its vibration.

***Q15.7** Answer (c). The equilibrium position is 15 cm below the starting point. The motion is symmetric about the equilibrium position, so the two turning points are 30 cm apart.

Q15.8 Since the acceleration is not constant in simple harmonic motion, none of the equations in Table 2.2 are valid.

Equation	*Information given by equation*
$x(t) = A\cos(\omega t + \phi)$	position as a function of time
$v(t) = -\omega A\sin(\omega t + \phi)$	velocity as a function of time
$v(x) = \pm\omega(A^2 - x^2)^{1/2}$	velocity as a function of position
$a(t) = -\omega^2 A\cos(\omega t + \phi)$	acceleration as a function of time
$a(t) = -\omega^2 x(t)$	acceleration as a function of position

The angular frequency ω appears in every equation. It is a good idea to figure out the value of angular frequency early in the solution to a problem about vibration, and to store it in calculator memory.

***Q15.9** (i) Answer (e). We have $T_i = \sqrt{\frac{L_i}{g}}$ and $T_f = \sqrt{\frac{L_f}{g}} = \sqrt{\frac{4L_i}{g}} = 2T_i$. The period gets larger by 2 times, to become 5 s.

(ii) Answer (c). Changing the mass has no effect on the period of a simple pendulum.

***Q15.10** (i) Answer (b). The upward acceleration has the same effect as an increased gravitational field.

(ii) Answer (a). The restoring force is smaller for the same displacement.

(iii) Answer (c).

Q15.11 (a) No force is exerted on the particle. The particle moves with constant velocity.

(b) The particle feels a constant force toward the left. It moves with constant acceleration toward the left. If its initial push is toward the right, it will slow down, turn around, and speed up in motion toward the left. If its initial push is toward the left, it will just speed up.

(c) A constant force towards the right acts on the particle to produce constant acceleration toward the right.

(d) The particle moves in simple harmonic motion about the lowest point of the potential energy curve.

Q15.12 The motion will be periodic—that is, it will repeat. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as θ increases farther.

***Q15.13** The mechanical energy of a damped oscillator changes back and forth between kinetic and potential while it gradually and permanently changes into internal energy.

***Q15.14** The oscillation of an atom in a crystal at constant temperature is not damped but keeps constant amplitude forever.

Q15.15 No. If the resistive force is greater than the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

Q15.16 Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.

Q15.17 Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight. Thus the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ is greater for you bouncing on the center of the board.

Q15.18 An imperceptibly slight breeze may be blowing past the leaves in tiny puffs. As a leaf twists in the wind, the fibers in its stem provide a restoring torque. If the frequency of the breeze matches the natural frequency of vibration of one particular leaf as a torsional pendulum, that leaf can be driven into a large-amplitude resonance vibration. Note that it is not the *size* of the driving force that sets the leaf into resonance, but the *frequency* of the driving force. If the frequency changes, another leaf will be set into resonant oscillation.

Q15.19 We assume the diameter of the bob is not very small compared to the length of the cord supporting it. As the water leaks out, the center of mass of the bob moves down, increasing the effective length of the pendulum and slightly lowering its frequency. As the last drops of water dribble out, the center of mass of the bob hops back up to the center of the sphere, and the pendulum frequency quickly increases to its original value.

SOLUTIONS TO PROBLEMS

Section 15.1 **Motion of an Object Attached to a Spring**

P15.1 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$.

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.904 \text{ s}$.

This equals one-half the period, so $T = 2(0.904 \text{ s}) = \boxed{1.81 \text{ s}}$.

(c) The motion is not simple harmonic. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

Section 15.2 **The Particle in Simple Harmonic Motion**

P15.2 (a) $x = (5.00 \text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00 \text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33 \text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0 \text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00 \text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -(20.0 \text{ cm/s}^2)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3 \text{ cm/s}^2}$

(d) $A = \boxed{5.00 \text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14 \text{ s}}$

P15.3 $x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $T = \frac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m})\cos(1.75\pi) = \boxed{2.83 \text{ m}}$

P15.4 (a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{0.45 \text{ kg } 9.8 \text{ m/s}^2}{0.35 \text{ m}} = 12.6 \text{ N/m}$$

we take the x-axis pointing downward, so $\phi = 0$

$$x = A\cos\omega t = 18.0 \text{ cm}\cos\sqrt{\frac{12.6 \text{ kg}}{0.45 \text{ kg}\cdot\text{s}^2}}84.4 \text{ s} = 18.0 \text{ cm}\cos 446.6 \text{ rad} = \boxed{15.8 \text{ cm}}$$

We choose to solve the parts in a different order.

(d) Now $446.6 \text{ rad} = 71\times 2\pi + 0.497 \text{ rad}$. In each cycle the object moves $4(18) = 72$ cm, so it has moved $71(72 \text{ cm}) + (18 - 15.8) \text{ cm} = \boxed{51.1 \text{ m}}$.

(b) By the same steps, $k = \frac{0.44 \text{ kg } 9.8 \text{ m/s}^2}{0.355 \text{ m}} = 12.1 \text{ N/m}$

$$x = A\cos\sqrt{\frac{k}{m}}t = 18.0 \text{ cm}\cos\sqrt{\frac{12.1}{0.44}}84.4 = 18.0 \text{ cm } \cos 443.5 \text{ rad} = \boxed{-15.9 \text{ cm}}$$

(e) $443.5 \text{ rad} = 70(2\pi) + 3.62 \text{ rad}$

$$\text{Distance moved} = 70(72 \text{ cm}) + 18 + 15.9 \text{ cm} = \boxed{50.7 \text{ m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

P15.5 (a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$

and $\qquad v = v_i \cos\omega t$

Since $f = 1.50$ Hz, $\qquad \omega = 2\pi f = 3.00\pi$

Also, $A = 2.00$ cm, so that $\qquad \boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi\ \text{cm/s} = \boxed{18.8\ \text{cm/s}}$

The particle has this speed at $t = 0$ and next at $\qquad t = \dfrac{T}{2} = \boxed{\dfrac{1}{3}\ \text{s}}$

(c) $a_{\max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2\ \text{cm/s}^2 = \boxed{178\ \text{cm/s}^2}$

This positive value of acceleration first occurs at $\qquad t = \dfrac{3}{4}T = \boxed{0.500\ \text{s}}$

(d) Since $T = \dfrac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in $1.00\ \text{s}\left(= \dfrac{3}{2}T\right)$, the particle will travel $\qquad 8.00\ \text{cm} + 4.00\ \text{cm} = \boxed{12.0\ \text{cm}}$.

P15.6 (a) $T = \dfrac{12.0\ \text{s}}{5} = \boxed{2.40\ \text{s}}$

(b) $f = \dfrac{1}{T} = \dfrac{1}{2.40} = \boxed{0.417\ \text{Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\ \text{rad/s}}$

P15.7 $\qquad f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} \qquad$ or $\qquad T = \dfrac{1}{f} = 2\pi\sqrt{\dfrac{m}{k}}$

Solving for k, $\qquad k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(7.00\ \text{kg})}{(2.60\ \text{s})^2} = \boxed{40.9\ \text{N/m}}$

***P15.8** (a) For constant acceleration position is given as a function of time by

$$x = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$= 0.27\ \text{m} + (0.14\ \text{m/s})(4.5\ \text{s}) + \frac{1}{2}(-0.32\ \text{m/s}^2)(4.5\ \text{s})^2$$

$$= \boxed{-2.34\ \text{m}}$$

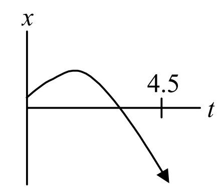


FIG. P15.8(a)

(b) $v_x = v_{xi} + a_x t = 0.14\ \text{m/s} - (0.32\ \text{m/s}^2)(4.5\ \text{s}) = \boxed{-1.30\ \text{m/s}}$

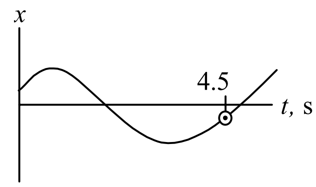
(c) For simple harmonic motion we have instead $x = A\cos(\omega t + \phi)$ and $v = -A\omega\sin(\omega t + \phi)$ where $a = -\omega^2 x$, so that $-0.32\ \text{m/s}^2 = -\omega^2(0.27\ \text{m})$, $\omega = 1.09\ \text{rad/s}$. At $t = 0$, $0.27\ \text{m} = A\cos\phi$ and $0.14\ \text{m/s} = -A(1.09/\text{s})\sin\phi$. Dividing gives $\dfrac{0.14\ \text{m/s}}{0.27\ \text{m}} = -(1.09/\text{s})\tan\phi$, $\tan\phi = -0.476$, $\phi = -25.5°$. Still at $t = 0$, $0.27\ \text{m} = A\cos(-25.5°)$, $A = 0.299\ \text{m}$. Now at $t = 4.5$ s,

$$x = (0.299\ \text{m})\cos\left[(1.09\ \text{rad/s})(4.5\ \text{s}) - 25.5°\right] = (0.299\ \text{m})\cos(4.90\ \text{rad} - 25.5°)$$

$$= (0.299\ \text{m})\cos 255° = \boxed{-0.076\,3\ \text{m}}$$

continued on next page

(d) $v = -(0.299\text{ m})(1.09/\text{s})\sin 255° = \boxed{+0.315\text{ m/s}}$



P15.9 $x = A\cos\omega t \qquad A = 0.05\text{ m} \qquad v = -A\omega\sin\omega t \qquad a = -A\omega^2\cos\omega t$

If $f = 3\,600\text{ rev/min} = 60\text{ Hz}$, then $\omega = 120\pi\text{ s}^{-1}$

$v_{\max} = 0.05(120\pi)\text{ m/s} = \boxed{18.8\text{ m/s}} \qquad a_{\max} = 0.05(120\pi)^2\text{ m/s}^2 = \boxed{7.11\text{ km/s}^2}$

P15.10 $m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm. At $t = 0$, $x = -3.00$ cm

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25.0}{1.00}} = 5.00\text{ rad/s}$

so that, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{\max} = A\omega = 3.00\times10^{-2}\text{ m}(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$

$a_{\max} = A\omega^2 = 3.00\times10^{-2}\text{ m}(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$

or $\boxed{x = -3.00\cos(5.00t)\text{ cm}}$

$v = \dfrac{dx}{dt} = \boxed{15.0\sin(5.00t)\text{ cm/s}}$

$a = \dfrac{dv}{dt} = \boxed{75.0\cos(5.00t)\text{ cm/s}^2}$

P15.11 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{8.00\text{ N/m}}{0.500\text{ kg}}} = 4.00\text{ s}^{-1}$ so position is given by $x = 10.0\sin(4.00t)$ cm

From this we find that $v = 40.0\cos(4.00t)$ cm/s $\quad v_{\max} = \boxed{40.0\text{ cm/s}}$

$a = -160\sin(4.00t)\text{ cm/s}^2 \quad a_{\max} = \boxed{160\text{ cm/s}^2}$

(b) $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$ and when $x = 6.00$ cm, $t = 0.161$ s.

We find $v = 40.0\cos[4.00(0.161)] = \boxed{32.0\text{ cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\text{ cm/s}^2}$

(c) Using $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00$ cm, $t = 0.232$ s

Therefore, $\Delta t = \boxed{0.232\text{ s}}$

***P15.12** We assume that the mass of the spring is negligible and that we are on Earth. Let m represent the mass of the object. Its hanging at rest is described by

$$\Sigma F_y = 0 \qquad -F_g + kx = 0 \qquad mg = k(0.183\text{ m}) \qquad m/k = (0.183\text{ m})/(9.8\text{ N/kg})$$

The object's bouncing is described by $T = 2\pi(m/k)^{1/2} = 2\pi[(0.183\text{ m})/(9.8\text{ m/s}^2)]^{1/2} = \boxed{0.859\text{ s}}$

We do have enough information to find the period. Whether the object has small or large mass, the ratio m/k must be equal to 0.183 m/(9.80 m/s^2). The period is 0.859 s.

P15.13 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

and $v_{\max} = \omega A = (6.28/\text{s})(0.100\text{ m}) = \boxed{0.628\text{ m/s}}$.

Section 15.3 **Energy of the Simple Harmonic Oscillator**

P15.14 $m = 200\text{ g},\ T = 0.250\text{ s}, E = 2.00\text{ J}; \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.250} = 25.1\text{ rad/s}$

(a) $k = m\omega^2 = 0.200\text{ kg}(25.1\text{ rad/s})^2 = \boxed{126\text{ N/m}}$

(b) $E = \dfrac{kA^2}{2} \Rightarrow A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(2.00)}{126}} = \boxed{0.178\text{ m}}$

P15.15 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2: \qquad v = x\sqrt{\frac{k}{m}} = \left(3.16\times10^{-2}\text{ m}\right)\sqrt{\frac{5.00\times10^6}{10^3}} = \boxed{2.23\text{ m/s}}$$

P15.16 (a) $E = \dfrac{kA^2}{2} = \dfrac{250\text{ N/m}\left(3.50\times10^{-2}\text{ m}\right)^2}{2} = \boxed{0.153\text{ J}}$

(b) $v_{\max} = A\omega \qquad \text{where} \qquad \omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250}{0.500}} = 22.4\text{ s}^{-1} \qquad v_{\max} = \boxed{0.784\text{ m/s}}$

(c) $a_{\max} = A\omega^2 = 3.50\times10^{-2}\text{ m}\left(22.4\text{ s}^{-1}\right)^2 = \boxed{17.5\text{ m/s}^2}$

P15.17 (a) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(35.0\text{ N/m})\left(4.00\times10^{-2}\text{ m}\right)^2 = \boxed{28.0\text{ mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2}$

$$|v| = \sqrt{\frac{35.0}{50.0\times10^{-3}}}\sqrt{\left(4.00\times10^{-2}\right)^2 - \left(1.00\times10^{-2}\right)^2} = \boxed{1.02\text{ m/s}}$$

(c) $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2 = \dfrac{1}{2}(35.0)\left[\left(4.00\times10^{-2}\right)^2 - \left(3.00\times10^{-2}\right)^2\right] = \boxed{12.2\text{ mJ}}$

(d) $\dfrac{1}{2}kx^2 = E - \dfrac{1}{2}mv^2 = \boxed{15.8\text{ mJ}}$

P15.18 (a) $k = \frac{|F|}{x} = \frac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0}\text{ rad/s}$ so $f = \frac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$

(c) $v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\text{ m/s}}$ at $x = 0$

(d) $a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2}$ at $x = \pm A$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$

(f) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\frac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$

(g) $|a| = \omega^2 x = 50.0\left(\frac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$

P15.19 Model the oscillator as a block-spring system.

From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$v_{\max} = \omega A$ and $v = \frac{\omega A}{2}$ so $\left(\frac{\omega A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$

From this we find $x^2 = \frac{3}{4}A^2$ and $x = \frac{\sqrt{3}}{2}A = \boxed{\pm 2.60\text{ cm}}$ where $A = 3.00\text{ cm}$

P15.20 (a) $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$-11\text{ m} = 0 + 0 + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$

$$t = \sqrt{\frac{22\text{ m}\cdot\text{s}^2}{9.8\text{ m}}} = \boxed{1.50\text{ s}}$$

(b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$

$$0 + mgy + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$65\text{ kg }9.8\text{ m/s}^2\ 36\text{ m} = \frac{1}{2}k(25\text{ m})^2$$

$$k = \boxed{73.4\text{ N/m}}$$

(c) The spring extension at equilibrium is $x = \frac{F}{k} = \frac{65\text{ kg }9.8\text{ m/s}^2}{73.4\text{ N/m}} = 8.68\text{ m}$, so this point is $11 + 8.68\text{ m} = \boxed{19.7\text{ m below the bridge}}$ and the amplitude of her oscillation is $36 - 19.7 = 16.3\text{ m}$.

(d) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{73.4\text{ N/m}}{65\text{ kg}}} = \boxed{1.06\text{ rad/s}}$

continued on next page

(e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher, where $x = -8.68$ m:

In $x = A\cos\omega t$ $\qquad$ $16.3\text{ m} = 16.3\text{ m}\cos 0$

$-8.68\text{ m} = 16.3\text{ m}\cos\left(1.06\ \dfrac{t}{\text{s}}\right)$ $\qquad$ $1.06\dfrac{t}{\text{s}} = -122° = -2.13\text{ rad}$

$t = -2.01$ s

Then $\boxed{+2.01\text{ s}}$ is the time over which the spring stretches.

(f) total time $= 1.50\text{ s} + 2.01\text{ s} = \boxed{3.50\text{ s}}$

P15.21 The potential energy is

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t)$$

The rate of change of potential energy is

$$\frac{dU_s}{dt} = \frac{1}{2}kA^2 2\cos(\omega t)\left[-\omega\sin(\omega t)\right] = -\frac{1}{2}kA^2\omega\sin 2\omega t$$

(a) This rate of change is maximal and negative at

$$2\omega t = \frac{\pi}{2},\ 2\omega t = 2\pi + \frac{\pi}{2},\text{ or in general, } 2\omega t = 2n\pi + \frac{\pi}{2}\text{ for integer } n$$

Then,

$$t = \frac{\pi}{4\omega}(4n+1) = \frac{\pi(4n+1)}{4\left(3.60\text{ s}^{-1}\right)}$$

For $n = 0$, this gives $t = \boxed{0.218\text{ s}}$ while $n = 1$ gives $t = \boxed{1.09\text{ s}}$.

All other values of n yield times outside the specified range.

(b) $\left|\dfrac{dU_s}{dt}\right|_{\max} = \dfrac{1}{2}kA^2\omega = \dfrac{1}{2}(3.24\ \text{N/m})\left(5.00\times10^{-2}\ \text{m}\right)^2\left(3.60\ \text{s}^{-1}\right) = \boxed{14.6\text{ mW}}$

Section 15.4 **Comparing Simple Harmonic Motion with Uniform Circular Motion**

P15.22 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is

$$x = A\cos\theta = A\cos\omega t$$

where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.

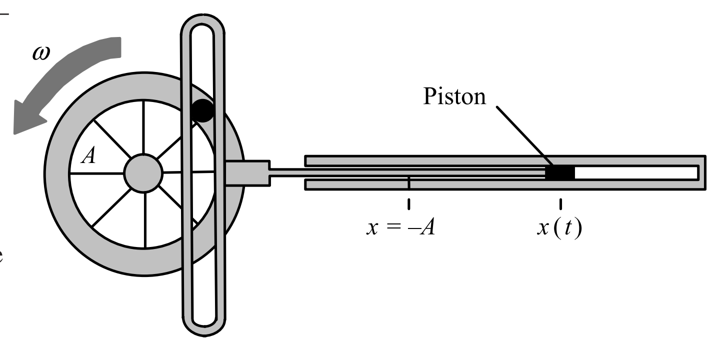


FIG. P15.22

P15.23 (a) $\boxed{\text{The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the bump projected in a plane perpendicular to the tire.}}$

(b) Since the car is moving with speed $v = 3.00\ \text{m/s}$, and its radius is 0.300 m, we have

$$\omega = \frac{3.00\ \text{m/s}}{0.300\ \text{m}} = 10.0\ \text{rad/s}$$

Therefore, the period of the motion is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0\ \text{rad/s})} = \boxed{0.628\ \text{s}}$$

Section 15.5 **The Pendulum**

P15.24 The period in Tokyo is $T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$

We know $T_T = T_C = 2.00\ \text{s}$

For which, we see $\frac{L_T}{g_T} = \frac{L_C}{g_C}$

or $\frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$

P15.25 Using the simple harmonic motion model:

$$A = r\theta = 1\ \text{m}\ 15°\frac{\pi}{180°} = 0.262\ \text{m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\ \text{m/s}^2}{1\ \text{m}}} = 3.13\ \text{rad/s}$$

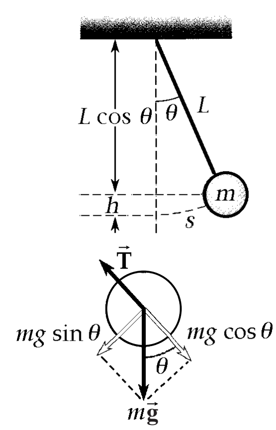


FIG. P15.25

(a) $v_{\max} = A\omega = 0.262\ \text{m}\ 3.13/\text{s} = \boxed{0.820\ \text{m/s}}$

(b) $a_{\max} = A\omega^2 = 0.262\ \text{m}(3.13/\text{s})^2 = 2.57\ \text{m/s}^2$

$a_{\tan} = r\alpha \qquad \alpha = \frac{a_{\tan}}{r} = \frac{2.57\ \text{m/s}^2}{1\ \text{m}} = \boxed{2.57\ \text{rad/s}^2}$

(c) $F = ma = 0.25\ \text{kg}\ 2.57\ \text{m/s}^2 = \boxed{0.641\ \text{N}}$

More precisely,

(a) $mgh = \frac{1}{2}mv^2$ and $h = L(1-\cos\theta)$

$\therefore v_{\max} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817\ \text{m/s}}$

continued on next page

(b) $I\alpha = mgL\sin\theta$

$$\alpha_{\max} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54\ \text{rad/s}^2}$$

(c) $F_{\max} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634\ \text{N}}$

> The answers agree to two digits. The answers computed from conservation of energy and from Newton's second law are more precisely correct. With this amplitude the motion of the pendulum is approximately simple harmonic.

***P15.26** Note that the angular amplitude 0.032 rad = 1.83 degree is small, as required for the SHM model of a pendulum.

$$\omega = \frac{2\pi}{T}: \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42\ \text{s}}$$

$$\omega = \sqrt{\frac{g}{L}}: \qquad L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499\ \text{m}}$$

P15.27 Referring to the sketch we have

$$F = -mg\sin\theta \qquad \text{and} \qquad \tan\theta = \frac{x}{R}$$

For small displacements, $\tan\theta \approx \sin\theta$

and $F = -\frac{mg}{R}x = -kx$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

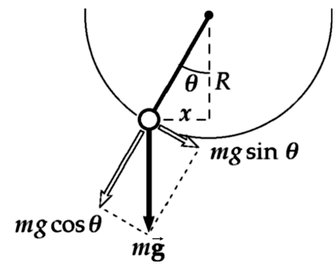


FIG. P15.27

Comparing to $F = -m\omega^2 x$ shows $\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{R}}}$.

P15.28 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field of $(9.80 + 5.00)\ \text{m/s}^2$. Thus the period is

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{14.8\ \text{m/s}^2}}$$

$$T = \boxed{3.65\ \text{s}}$$

(b) $T = 2\pi\sqrt{\dfrac{5.00\ \text{m}}{(9.80\ \text{m/s}^2 - 5.00\ \text{m/s}^2)}} = \boxed{6.41\ \text{s}}$

(c) $g_{eff} = \sqrt{(9.80\ \text{m/s}^2)^2 + (5.00\ \text{m/s}^2)^2} = 11.0\ \text{m/s}^2$

$$T = 2\pi = \sqrt{\frac{5.00\ \text{m}}{11.0\ \text{m/s}^2}} = \boxed{4.24\ \text{s}}$$

P15.29 $f = 0.450$ Hz, $d = 0.350$ m, and $m = 2.20$ kg

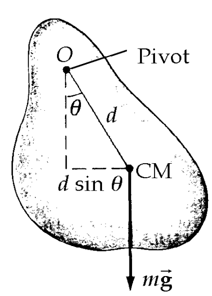


FIG. P15.29

$$T = \frac{1}{f};$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}; \qquad T^2 = \frac{4\pi^2 I}{mgd}$$

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2(0.450\ \text{s}^{-1})^2} = \boxed{0.944\ \text{kg}\cdot\text{m}^2}$$

P15.30 (a) From $T = \dfrac{\text{total measured time}}{50}$

the measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422

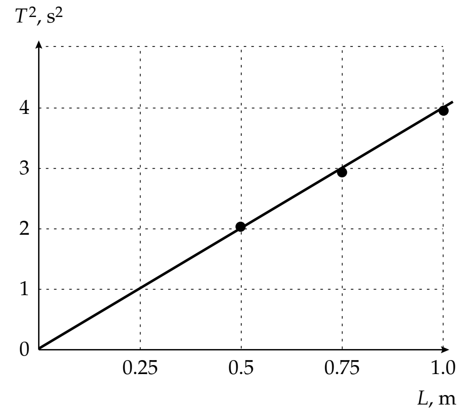


FIG. P15.30

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
g (m/s^2)	9.91	9.87	9.76

Thus, $g_{\text{ave}} = 9.85\,\text{m/s}^2$ $\boxed{\text{This agrees with the accepted value of } g = 9.80\ \text{m/s}^2 \text{ within } 0.5\%.}$

(c) From $T^2 = \left(\dfrac{4\pi^2}{g}\right)L$, the slope of T^2 versus L graph $= \dfrac{4\pi^2}{g} = 4.01\ \text{s}^2/\text{m}$.

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = \boxed{9.85\ \text{m/s}^2}$. This is the same as the value in (b).

P15.31 (a) The parallel axis theorem says directly $I = I_{\text{CM}} + md^2$

so $T = 2\pi\sqrt{\dfrac{I}{mgd}} = \boxed{2\pi\sqrt{\dfrac{\left(I_{\text{CM}} + md^2\right)}{mgd}}}$

(b) When d is very large $T \to 2\pi\sqrt{\dfrac{d}{g}}$ gets large.

When d is very small $T \to 2\pi\sqrt{\dfrac{I_{\text{CM}}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{-1/2}$$

$$= 2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}\left(-\frac{1}{2}\right)(mgd)^{-3/2}mg + 2\pi(mgd)^{-1/2}\left(\frac{1}{2}\right)\left(I_{\text{CM}} + md^2\right)^{-1/2}2md$$

$$= \frac{-\pi\left(I_{\text{CM}} + md^2\right)mg}{\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{3/2}} + \frac{2\pi\, md\, mgd}{\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{3/2}} = 0$$

This requires

$$-I_{\text{CM}} - md^2 + 2md^2 = 0$$

or $\boxed{I_{\text{CM}} = md^2}$.

P15.32 (a) The parallel-axis theorem:

$$I = I_{CM} + Md^2 = \frac{1}{12}ML^2 + Md^2$$

$$= \frac{1}{12}M(1.00\ \text{m})^2 + M(1.00\ \text{m})^2 = M\left(\frac{13}{12}\ \text{m}^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(13\ \text{m}^2)}{12Mg(1.00\ \text{m})}}$$

$$= 2\pi\sqrt{\frac{13\ \text{m}}{12(9.80\ \text{m/s}^2)}} = \boxed{2.09\ \text{s}}$$

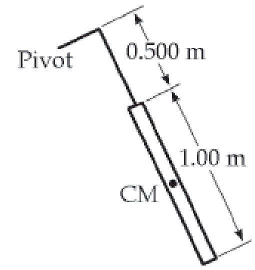


FIG. P15.32

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00\ \text{m}}{9.80\ \text{m/s}^2}} = 2.01\ \text{s} \qquad \text{difference} = \frac{2.09\ \text{s} - 2.01\ \text{s}}{2.01\ \text{s}} = \boxed{4.08\%}$$

P15.33 $T = 0.250\ \text{s},\ I = mr^2 = (20.0\times10^{-3}\ \text{kg})(5.00\times10^{-3}\ \text{m})^2$

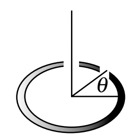


FIG. P15.33

(a) $I = \boxed{5.00\times10^{-7}\ \text{kg}\cdot\text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\ \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = (5.00\times10^{-7})\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$$

Section 15.6 **Damped Oscillations**

P15.34 The total energy is $E = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

Taking the time derivative, $\dfrac{dE}{dt} = mv\dfrac{d^2x}{dt^2} + kxv$

Use Equation 15.31: $\dfrac{md^2x}{dt^2} = -kx - bv$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

Thus, $\boxed{\dfrac{dE}{dt} = -bv^2 < 0}$

We have proved that the mechanical energy of a damped oscillator is always decreasing.

P15.35 $\theta_i = 15.0°$ $\qquad \theta(t = 1\,000) = 5.50°$

$x = Ae^{-bt/2m}$ $\qquad \dfrac{x_{1\,000}}{x_i} = \dfrac{Ae^{-bt/2m}}{A} = \dfrac{5.50}{15.0} = e^{-b(1\,000)/2m}$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b(1\,000)}{2m}$$

$$\therefore \frac{b}{2m} = \boxed{1.00\times10^{-3}\ \text{s}^{-1}}$$

P15.36 To show that $x = Ae^{-bt/2m}\cos(\omega t + \phi)$

is a solution of $-kx - b\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$ (1)

where $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$ (2)

We take $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ and compute (3)

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi) \quad (4)$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right]$$
$$-\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\omega^2\cos(\omega t + \phi)\right] \quad (5)$$

We substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t + \phi) + \frac{b^2}{2m}Ae^{-bt/2m}\cos(\omega t + \phi) + b\omega Ae^{-bt/2m}\sin(\omega t + \phi)$$
$$= -\frac{b}{2}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right]$$
$$+\frac{b}{2}Ae^{-bt/2m}\omega\sin(\omega t + \phi) - m\omega^2Ae^{-bt/2m}\cos(\omega t + \phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t + \phi)$ and $Ae^{-bt/2m}\sin(\omega t + \phi)$:

cosine-term: $-k + \dfrac{b^2}{2m} = -\dfrac{b}{2}\left(-\dfrac{b}{2m}\right) - m\omega^2 = \dfrac{b^2}{4m} - m\left(\dfrac{k}{m} - \dfrac{b^2}{4m^2}\right) = -k + \dfrac{b^2}{2m}$

sine-term: $b\omega = +\dfrac{b}{2}(\omega) + \dfrac{b}{2}(\omega) = b\omega$

Since the coefficients are equal, $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ is a solution of the equation.

P15.37 The frequency if undamped would be $\omega_0 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2.05\times10^4\ \text{N/m}}{10.6\ \text{kg}}} = 44.0/\text{s}$.

(a) With damping

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \sqrt{\left(44\ \frac{1}{\text{s}}\right)^2 - \left(\frac{3\ \text{kg}}{\text{s}\ 2\ 10.6\ \text{kg}}\right)}$$
$$= \sqrt{1\,933.96 - 0.02} = 44.0\ \frac{1}{\text{s}}$$
$$f = \frac{\omega}{2\pi} = \frac{44.0}{2\pi\ \text{s}} = \boxed{7.00\ \text{Hz}}$$

(b) In $x = A_0e^{-bt/2m}\cos(\omega t + \phi)$ over one cycle, a time $T = \dfrac{2\pi}{\omega}$, the amplitude changes from A_0 to $A_0e^{-b2\pi/2m\omega}$ for a fractional decrease of

$$\frac{A_0 - A_0e^{-\pi b/m\omega}}{A_0} = 1 - e^{-\pi 3/(10.6\cdot 44.0)} = 1 - e^{-0.020\,2} = 1 - 0.979\,98 = 0.020\,0 = \boxed{2.00\%}$$

continued on next page

(c) The energy is proportional to the square of the amplitude, so its fractional rate of decrease is twice as fast:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kA_0^2 e^{-2bt/2m} = E_0 e^{-bt/m}$$

We specify

$$0.05E_0 = E_0 e^{-3t/10.6}$$
$$0.05 = e^{-3t/10.6}$$
$$e^{+3t/10.6} = 20$$
$$\frac{3t}{10.6} = \ln 20 = 3.00$$
$$t = \boxed{10.6\text{ s}}$$

Section 15.7 **Forced Oscillations**

P15.38 (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30\times10^3\text{ N/m}}{12.5\text{ kg}}} = \boxed{2.95\text{ Hz}}$$

(b) From $x = A\cos\omega t$, $v = \frac{dx}{dt} = -A\omega\sin\omega t$, and $a = \frac{dv}{dt} = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \qquad \text{or} \qquad A = \frac{g}{\omega^2} = \frac{g}{k/m} = \frac{gm}{k} \qquad A = \frac{(9.80\text{ m/s}^2)(12.5\text{ kg})}{4.30\times10^3\text{ N/m}} = \boxed{2.85\text{ cm}}$$

P15.39 $F = 3.00\sin(2\pi t)$ N and $k = 20.0$ N/m

(a) $\omega = \frac{2\pi}{T} = 2\pi$ rad/s so $T = \boxed{1.00\text{ s}}$

(b) In this case, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0}{2.00}} = 3.16$ rad/s

The equation for the amplitude of a driven oscillator,

with $b = 0$, gives $A = \left(\frac{F_0}{m}\right)\left(\omega^2 - \omega_0^2\right)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$

Thus, $A = 0.050\,9\text{ m} = \boxed{5.09\text{ cm}}$

P15.40 $F_0 \sin\omega t - kx = m\dfrac{d^2x}{dt^2} \qquad \omega_0 = \sqrt{\dfrac{k}{m}}$ (1)

$x = A\cos(\omega t + \phi)$ (2)

$\dfrac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ (3)

$\dfrac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1): $F_0 \sin\omega t - kA\cos(\omega t + \phi) = m(-A\omega^2)\cos(\omega t + \phi)$

Solve for the amplitude: $(kA - mA\omega^2)\cos(\omega t + \phi) = F_0\sin\omega t = F_0\cos(\omega t - 90°)$

These will be equal, provided only that ϕ must be $-90°$ and $kA - mA\omega^2 = F_0$

Thus,

$$A = \frac{F_0/m}{(k/m) - \omega^2}$$

P15.41 From the equation for the amplitude of a driven oscillator with no damping,

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \frac{F_0/m}{\omega^2 - \omega_0^2}$$

$$\omega = 2\pi f = (20.0\pi \text{ s}^{-1}) \qquad \omega_0^2 = \frac{k}{m} = \frac{200}{(40.0/9.80)} = 49.0 \text{ s}^{-2}$$

$$F_0 = mA(\omega^2 - \omega_0^2)$$

$$F_0 = \left(\frac{40.0}{9.80}\right)(2.00 \times 10^{-2})(3\,950 - 49.0) = \boxed{318 \text{ N}}$$

P15.42 $A = \dfrac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$

With $b = 0$, $A = \dfrac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \dfrac{F_{\text{ext}}/m}{\pm(\omega^2 - \omega_0^2)} = \pm\dfrac{F_{\text{ext}}/m}{\omega^2 - \omega_0^2}$

Thus, $\omega^2 = \omega_0^2 \pm \dfrac{F_{\text{ext}}/m}{A} = \dfrac{k}{m} \pm \dfrac{F_{\text{ext}}}{mA} = \dfrac{6.30 \text{ N/m}}{0.150 \text{ kg}} \pm \dfrac{1.70 \text{ N}}{(0.150 \text{ kg})(0.440 \text{ m})}$

This yields $\omega = 8.23$ rad/s or $\omega = 4.03$ rad/s

Then, $f = \dfrac{\omega}{2\pi}$ gives either $f = \boxed{1.31 \text{ Hz}}$ or $f = \boxed{0.641 \text{ Hz}}$

P15.43 The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80 \text{ m/s}^2}{0.082\,1 \text{ m}}} = \boxed{1.74 \text{ Hz}}$$

Additional Problems

***P15.44** (a) Consider the first process of spring compression. It continues as long as glider 1 is moving faster than glider 2. The spring instantaneously has maximum compression when both gliders are moving with the same speed v_a.

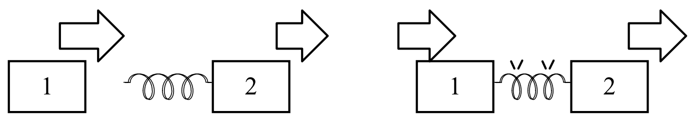


FIG. P15.44(a)

Momentum conservation:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(0.24\text{ kg})(0.74\text{ m/s}) + (0.36\text{ kg})(0.12\text{ m/s}) = (0.24\text{ kg})v_a + (0.36\text{ kg})v_a$$

$$\frac{0.220\,8\text{ kg}\cdot\text{m/s}}{0.60\text{ kg}} = v_a \qquad \vec{\mathbf{v}}_a = \boxed{0.368\text{ m/s}\,\hat{\mathbf{i}}}$$

(b) Energy conservation:

$$\left(K_1 + K_2 + U_s\right)_i = \left(K_1 + K_2 + U_s\right)_f$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}\left(m_1 + m_2\right)v_a^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}(0.24\text{ kg})(0.74\text{ m/s})^2 + \frac{1}{2}(0.36\text{ kg})(0.12\text{ m/s})^2$$

$$= \frac{1}{2}(0.60\text{ kg})(0.368\text{ m/s})^2 + \frac{1}{2}(45\text{ N/m})x^2$$

$$0.068\,3\text{ J} = 0.040\,6\text{ J} + \frac{1}{2}(45\text{ N/m})x^2$$

$$x = \left(\frac{2(0.027\,7\text{ J})}{45\text{ N/m}}\right)^{1/2} = \boxed{0.035\,1\text{ m}}$$

(c) Conservation of momentum guarantees that the center of mass moves with constant velocity. Imagine viewing the gliders from a reference frame moving with the center of mass. We see the two gliders approach each other with momenta in opposite directions of equal magnitude. Upon colliding they compress the ideal spring and then together bounce, extending and compressing it cyclically.

(d) $$\frac{1}{2}m_{\text{tot}}v_{\text{CM}}^2 = \frac{1}{2}(0.60\text{ kg})(0.368\text{ m/s})^2 = \boxed{0.040\,6\text{ J}}$$

(e) $$\frac{1}{2}kA^2 = \frac{1}{2}(45\text{ N/m})(0.035\,1\text{ m})^2 = \boxed{0.027\,7\text{ J}}$$

***P15.45** From $a = -\omega^2 x$, the maximum acceleration is given by $a_{max} = \omega^2 A$. Then $108\text{ cm/s}^2 = \omega^2(12\text{ cm})$ $\omega = 3.00/\text{s}$.

(a) $T = 1/f = 2\pi/\omega = 2\pi/(3/\text{s}) = \boxed{2.09\text{ s}}$

(b) $f = \omega/2\pi = (3/\text{s})/2\omega = \boxed{0.477\text{ Hz}}$

(c) $v_{max} = \omega A = (3/\text{s})(12\text{ cm}) = \boxed{36.0\text{ cm/s}}$

(d) $E = (1/2)\, m\, v_{max}^{\ 2} = (1/2)\, m\, (0.36\text{ m/s})^2 = \boxed{(0.0648\text{ m}^2/\text{s}^2)m}$

(e) $\omega^2 = k/m \qquad k = \omega^2 m = (3/\text{s})^2 m = \boxed{(9.00/\text{s}^2)m}$

(f) Period, frequency, and maximum speed are all independent of mass in this situation. The energy and the force constant are directly proportional to mass.

***P15.46** (a) From $a = -\omega^2 x$, the maximum acceleration is given by $a_{max} = \omega^2 A$. As A increases, the maximum acceleration increases. When it becomes greater than the acceleration due to gravity, the rock will no longer stay in contact with the vibrating ground, but lag behind as the ground moves down with greater acceleration. We have then

$A = g/\omega^2 = g/(2\pi f)^2 = g/4\pi^2 f^2 = (9.8\text{ m/s}^2)/4\pi^2(2.4/\text{s})^2 = \boxed{4.31\text{ cm}}$

(b) When the rock is on the point of lifting off, the surrounding water is also barely in free fall. No pressure gradient exists in the water, so no buoyant force acts on the rock.

P15.47 Let F represent the tension in the rod.

(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point P. Thus, the tension in the rod at point P is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}$$

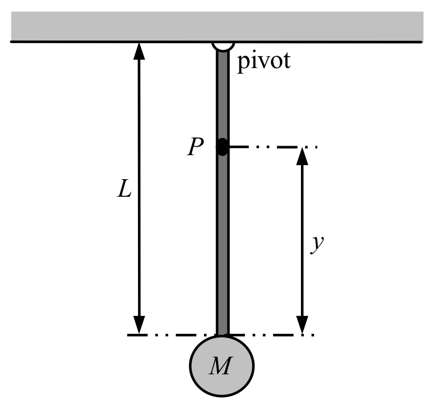


FIG. P15.47

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$ where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M(L/2) + ML}{M + M} = \frac{3L}{4} \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{(4/3)ML^2}{(2M)g(3L/4)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}$$

For $L = 2.00\text{ m}$, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{2.68\text{ s}}$.

P15.48 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\ \text{N/m})(0.200\ \text{m})^2 = 2.00\ \text{J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(16.0\ \text{kg})v^2 = (8.00\ \text{kg})v^2$$

Therefore,

$$(8.00\ \text{kg})v^2 = 2.00\ \text{J, and } v = \boxed{0.500\ \text{m/s}}$$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1-spring system at equilibrium is:

$$\frac{1}{2}m_1 v^2 = \frac{1}{2}(9.00\ \text{kg})(0.500\ \text{m/s})^2 = 1.125\ \text{J}$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system.

Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125 \qquad \text{or} \qquad A' = 0.150\ \text{m}$$

The period of the m_1-spring system is $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\ \text{s}$

and it takes $\frac{1}{4}T = 0.471\ \text{s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500\ \text{m/s}(0.471\ \text{s}) - 0.150\ \text{m} = 0.085\,6 = \boxed{8.56\ \text{cm}}$$

P15.49 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block *B* is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2 Af^2\right)$$

$$A = \frac{\mu_s g}{4\pi^2 f^2} = \frac{0.6(980\ \text{cm/s}^2)}{4\pi^2(1.5/\text{s})^2} = \boxed{6.62\ \text{cm}}$$

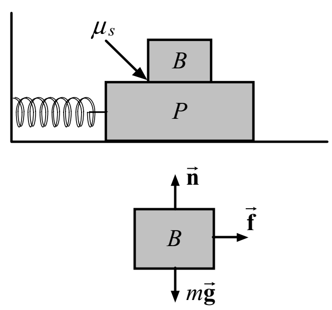


FIG. P15.49

P15.50 Refer to the diagram in the previous problem. The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block *B* is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2 Af^2\right) \qquad \text{or} \qquad A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}$$

P15.51 Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as D and to the diatomic molecule of hydrogen-1 as H.

$$M_D = 2M_H \qquad \frac{\omega_D}{\omega_H} = \frac{\sqrt{k/M_D}}{\sqrt{k/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}} \qquad f_D = \frac{f_H}{\sqrt{2}} = \boxed{0.919 \times 10^{14}\ \text{Hz}}$$

***P15.52** (a) A time interval. If the interaction occupied no time, each ball would move with infinite acceleration. The force exerted by each ball on the other would be infinite, and that cannot happen.

(b) $k = |F|/|x| = 16\ 000\ \text{N}/0.000\ 2\ \text{m} = \boxed{80\ \text{MN/m}}$

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1.

Then its mass is $\rho V = \rho\,(4/3)\pi r^3 = (4\pi/3)(7860\ \text{kg/m}^3)(0.0254\ \text{m}/2)^3 = 0.0674\ \text{kg}$

and $K = (1/2)\,mv^2 = (1/2)\,(0.0674\ \text{kg})(5\ \text{m/s})^2 = \boxed{0.843\ \text{J}}$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$$0.843\ \text{J} = (1/2)\,(8 \times 10^7\ \text{N/m})x^2 \qquad x = \boxed{0.145\ \text{mm}}$$

(e) The half-cycle is from the equilibrium position of the model spring to maximum compression and back to equilibrium again. The time is one-half the period,

$$(1/2)T = (1/2)2\pi(m/k)^{1/2} = \pi(0.0674\ \text{kg}/80 \times 10^6\ \text{kg/s}^2)^{1/2} = \boxed{9.12 \times 10^{-5}\,\text{s}}$$

P15.53 (a)

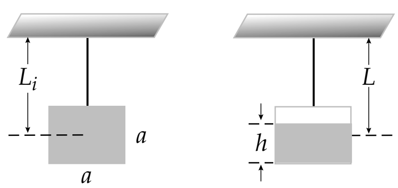


FIG. P15.53(a)

(b)
$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \frac{dT}{dt} = \frac{\pi}{\sqrt{g}}\frac{1}{\sqrt{L}}\frac{dL}{dt} \tag{1}$$

We need to find $L(t)$ and $\frac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2} \qquad \text{and} \qquad \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}$$

But $\frac{dM}{dt} = \rho\frac{dV}{dt} = -\rho A\frac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt} \qquad \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \tag{2}$$

Also,

$$\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right)t = L - L_i \tag{3}$$

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \boxed{\frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \left(t/2\rho a^2\right)\left(dM/dt\right)}}}$$

continued on next page

(c) Substitute Equation (3) into the equation for the period.

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

Or one can obtain T by integrating (b):

$$\int_T^{T_i} dT = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \left(1/2\rho a^2\right)\left(dM/dt\right)t}}$$

$$T - T_i = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\left[\frac{2}{\left(1/2\rho a^2\right)\left(dM/dt\right)}\right]\left[\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t} - \sqrt{L_i}\right]$$

But $T_i = 2\pi\sqrt{\frac{L_i}{g}}$, so $T = \frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}$

P15.54 $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

(a) $k = \omega^2 m = \boxed{\frac{4\pi^2 m}{T^2}}$ (b) $m' = \frac{k\left(T'\right)^2}{4\pi^2} = \boxed{m\left(\frac{T'}{T}\right)^2}$

P15.55 We draw a free-body diagram of the pendulum. The force **H** exerted by the hinge causes no torque about the axis of rotation.

$\tau = I\alpha$ and $\frac{d^2\theta}{dt^2} = -\alpha$

$$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$$

For small amplitude vibrations, use the approximations: $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$.

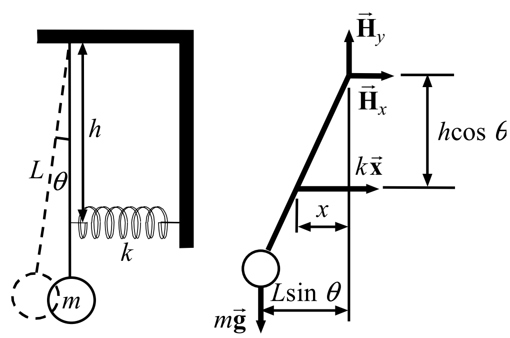


FIG. P15.55

Therefore,

$$\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta \qquad \omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$$

P15.56 (a) In $x = A\cos(\omega t + \phi)$, $v = -\omega A \sin(\omega t + \phi)$

we have at $t = 0$ $v = -\omega A \sin\phi = -v_{max}$

This requires $\phi = 90°$, so $x = A\cos(\omega t + 90°)$

And this is equivalent to $x = -A\sin\omega t$

Numerically we have $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\text{ N/m}}{0.5\text{ kg}}} = 10\text{ s}^{-1}$

and $v_{max} = \omega A$ $20\text{ m/s} = (10\text{ s}^{-1})A$ $A = 2\text{ m}$

So $\boxed{x = (-2\text{ m})\sin\left[(10\text{ s}^{-1})t\right]}$

(b) In $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, $\frac{1}{2}kx^2 = 3\left(\frac{1}{2}mv^2\right)$

implies $\frac{1}{3}\frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ $\frac{4}{3}x^2 = A^2$

$$x = \pm\sqrt{\frac{3}{4}}A = \pm 0.866A = \boxed{\pm 1.73\text{ m}}$$

(c) $\omega = \sqrt{\frac{g}{L}}$ $L = \frac{g}{\omega^2} = \frac{9.8\text{ m/s}^2}{(10\text{ s}^{-1})^2} = \boxed{0.098\,0\text{ m}}$

(d) In $x = (-2\text{ m})\sin\left[(10\text{ s}^{-1})t\right]$

the particle is at $x = 0$ at $t = 0$, at $10t = \pi$ s, and so on.

The particle is at $x = 1\text{ m}$

when $-\frac{1}{2} = \sin\left[(10\text{ s}^{-1})t\right]$

with solutions $(10\text{ s}^{-1})t = -\frac{\pi}{6}$

$(10\text{ s}^{-1})t = \pi + \frac{\pi}{6}$, and so on.

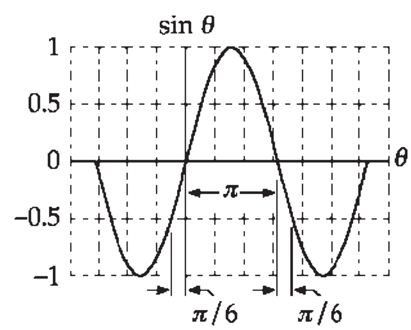


FIG. P15.56(d)

The minimum time for the motion is Δt in $10\Delta t = \left(\frac{\pi}{6}\right)\text{s}$

$$\Delta t = \left(\frac{\pi}{60}\right)\text{s} = \boxed{0.052\,4\text{ s}}$$

P15.57 (a) At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0 L$$

where x_0 is the equilibrium compression.

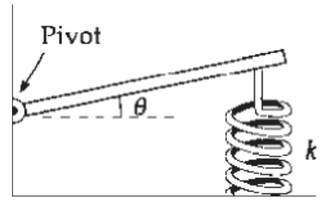


FIG. P15.57

After displacement by a small angle,

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k\left(x_0 - L\theta\right)L = -k\theta L^2$$

But,

$$\sum \tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

So

$$\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$$

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \frac{3k}{m}}$.

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\ \text{N/m})}{5.00\ \text{kg}}} = \boxed{1.23\ \text{Hz}}$

P15.58 As it passes through equilibrium, the 4-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{100\ \text{N/m}}{4\ \text{kg}}}2\ \text{m} = 10.0\ \text{m/s}$$

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

$$4\ \text{kg}(10\ \text{m/s}) + (6\ \text{kg})0 = (10\ \text{kg})v_{\max}$$
$$v_{\max} = 4.00\ \text{m/s}$$

(a) The new amplitude is given by $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$

$$10\ \text{kg}(4\ \text{m/s})^2 = (100\ \text{N/m})A^2$$
$$A = 1.26\ \text{m}$$

Thus the amplitude has $\boxed{\text{decreased by}}$ $2.00\ \text{m} - 1.26\ \text{m} = \boxed{0.735\ \text{m}}$

(b) The old period was $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\ \text{kg}}{100\ \text{N/m}}} = 1.26\ \text{s}$

The new period is $T = 2\pi\sqrt{\frac{10}{100}\ \text{s}^2} = 1.99\ \text{s}$

The period has $\boxed{\text{increased by}}$ $1.99\ \text{m} - 1.26\ \text{m} = \boxed{0.730\ \text{s}}$

continued on next page

(c) The old energy was $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}(4\text{ kg})(10\text{ m/s})^2 = 200\text{ J}$

The new mechanical energy is $\frac{1}{2}(10\text{ kg})(4\text{ m/s})^2 = 80\text{ J}$

The energy has $\boxed{\text{decreased by 120 J}}$.

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.

P15.59 (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \boxed{3.00\text{ s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74)(2.06)^2 = \boxed{14.3\text{ J}}$

(c) At maximum angular displacement $mgh = \frac{1}{2}mv^2$ $h = \frac{v^2}{2g} = 0.217\text{ m}$

$h = L - L\cos\theta = L(1-\cos\theta)$ $\cos\theta = 1 - \frac{h}{L}$ $\boxed{\theta = 25.5°}$

P15.60 One can write the following equations of motion:

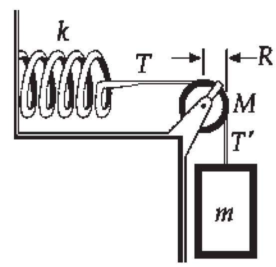


FIG. P15.60

$T - kx = 0$ (describes the spring)

$mg - T' = ma = m\frac{d^2x}{dt^2}$ (for the hanging object)

$R(T' - T) = I\frac{d^2\theta}{dt^2} = \frac{I}{R}\frac{d^2x}{dt^2}$ (for the pulley)

with $I = \frac{1}{2}MR^2$

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg$$

The solution is $x(t) = A\sin\omega t + \frac{mg}{k}$ (where $\frac{mg}{k}$ arises because of the extension of the spring due to the weight of the hanging object), with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100\text{ N/m}}{0.200\text{ kg} + \frac{1}{2}M}}$$

(a) For $M = 0$ $f = \boxed{3.56\text{ Hz}}$

(b) For $M = 0.250\text{ kg}$ $f = \boxed{2.79\text{ Hz}}$

(c) For $M = 0.750\text{ kg}$ $f = \boxed{2.10\text{ Hz}}$

P15.61 Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then,

$$k = \frac{F}{x} = \frac{980\text{ N}}{2.00\times 10^{-2}\text{ m}} = 4.90\times 10^{4}\text{ N/m}$$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90\times 10^{4}\text{ N/m}}{500\text{ kg}}} = 1.58\text{ Hz}$$

If he encounters washboard bumps at the same frequency as the free vibration, resonance will make the motorcycle bounce a lot. It may bounce so much as to interfere with the rider's control of the machine.

Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0\text{ m/s}}{1.58\text{ s}^{-1}} = 12.7\text{ m}\ \boxed{\sim 10^{1}\text{ m}}$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacing of bumps will excite all of these other resonances.

P15.62 (a) For each segment of the spring

$$dK = \frac{1}{2}(dm)v_x^2$$

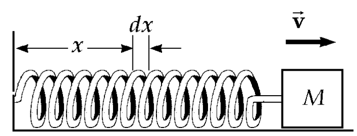


FIG. P15.62

Also,

$$v_x = \frac{x}{\ell}v \qquad \text{and} \qquad dm = \frac{m}{\ell}dx$$

Therefore, the total kinetic energy of the block-spring system is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^{\ell}\left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}$$

(b) $\omega = \sqrt{\dfrac{k}{m_{eff}}}$ and $\dfrac{1}{2}m_{eff}v^2 = \dfrac{1}{2}\left(M + \dfrac{m}{3}\right)v^2$

Therefore,

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + m/3}{k}}}$$

P15.63 (a) $\sum \vec{\mathbf{F}} = -2T\sin\theta\,\hat{\mathbf{j}}$ where $\theta = \tan^{-1}\left(\dfrac{y}{L}\right)$

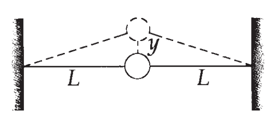


FIG. P15.63

Therefore, for a small displacement

$$\sin\theta \approx \tan\theta = \frac{y}{L} \qquad \text{and} \qquad \boxed{\sum \vec{\mathbf{F}} = \frac{-2Ty}{L}\hat{\mathbf{j}}}$$

(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum \vec{\mathbf{F}} = -k\vec{\mathbf{x}} \qquad \text{becomes here} \qquad \sum \vec{\mathbf{F}} = -\frac{2T}{L}\vec{\mathbf{y}}$$

Therefore, the effective spring constant is $\dfrac{2T}{L}$ and $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2T}{mL}}}$.

P15.64 (a) Assuming a Hooke's Law type spring,

$$F = Mg = kx$$

and empirically

$$Mg = 1.74x - 0.113$$

so

$$k = \boxed{1.74\ \text{N/m} \pm 6\%}$$

M, kg	x, m	Mg, N
0.020 0	0.17	0.196
0.040 0	0.293	0.392
0.050 0	0.353	0.49
0.060 0	0.413	0.588
0.070 0	0.471	0.686
0.080 0	0.493	0.784

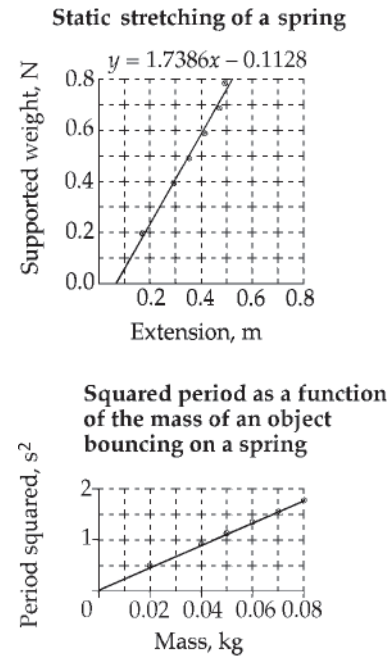


FIG. P15.64

(b) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7M + 0.058\,9$$

so

$$k = \frac{4\pi^2}{21.7} = \boxed{1.82\ \text{N/m} \pm 3\%}$$

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.020 0	0.494
9.62	0.962	0.040 0	0.925
10.67	1.067	0.050 0	1.138
11.67	1.167	0.060 0	1.362
12.52	1.252	0.070 0	1.568
13.41	1.341	0.080 0	1.798

The k values $1.74\ \text{N/m} \pm 6\%$

and $1.82\ \text{N/m} \pm 3\%$ differ by 4% so $\boxed{\text{they agree}}$.

(c) Utilizing the axis-crossing point, $m_s = 3\left(\frac{0.058\,9}{21.7}\right)\text{kg} = \boxed{8\ \text{grams} \pm 12\%}$

$\boxed{\text{in agreement}}$ with 7.4 grams.

P15.65 (a) $\Delta K + \Delta U = 0$

Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$

where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2} I\omega^2$, but

$$h = R - R\cos\theta = R(1-\cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

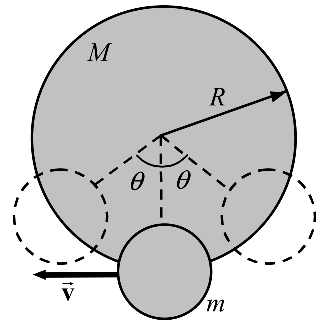


FIG. P15.65

Substituting we find

$$mgR(1-\cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1-\cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right]v^2$$

and $v^2 = 4gR\dfrac{(1-\cos\theta)}{\left(M/m + r^2/R^2 + 2\right)}$

so $\boxed{v = 2\sqrt{\dfrac{Rg(1-\cos\theta)}{M/m + r^2/R^2 + 2}}}$

(b) $T = 2\pi\sqrt{\dfrac{I}{m_T g d_{\text{CM}}}}$

$m_T = m + M \qquad d_{\text{CM}} = \dfrac{mR + M(0)}{m + M}$

$$T = \boxed{2\pi\sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}}$$

P15.66 (a) We require $Ae^{-bt/2m} = \dfrac{A}{2} \qquad e^{+bt/2m} = 2$

or $\dfrac{bt}{2m} = \ln 2$ or $\dfrac{0.100\ \text{kg/s}}{2(0.375\ \text{kg})}t = 0.693 \qquad \therefore t = \boxed{5.20\ \text{s}}$

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where

$\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/m}$. We require $\frac{1}{2}kA^2e^{-bt/m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2 \qquad \therefore t = \dfrac{m\ln 2}{b} = \dfrac{0.375\ \text{kg}(0.693)}{0.100\ \text{kg/s}} = \boxed{2.60\ \text{s}}$

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

$$\frac{dE/dt}{E} = \frac{(d/dt)\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)(dA/dt)}{\frac{1}{2}kA^2} = 2\frac{dA/dt}{A}$$

two times faster than the fractional rate of change in amplitude.

P15.67 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.

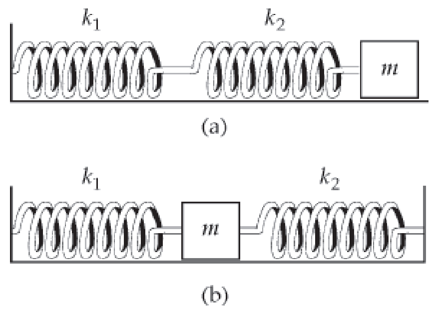


FIG. P15.67

By Newton's third law, we expect

$$k_1x_1 = k_2x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

we find $\quad x_1 = \left[\dfrac{k_2}{k_1+k_2}\right]x$

The force on either spring is given by $\quad F_1 = \left[\dfrac{k_1k_2}{k_1+k_2}\right]x = ma$

where a is the acceleration of the mass m.

This is in the form $\quad F = k_{eff}x = ma$

and $\quad T = 2\pi\sqrt{\dfrac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1k_2}}}$

(b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1+k_2)x \qquad \text{and} \qquad k_{eff} = k_1 + k_2$$

so that $\quad T = \boxed{2\pi\sqrt{\dfrac{m}{(k_1+k_2)}}}$

P15.68 Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\sum F_y = 0 \Rightarrow -Mg + \rho\pi r^2\ell g = 0$$

Now with any excursion x from equilibrium

$$-Mg + \rho\pi r^2(\ell - x)g = Ma$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 gx = Ma$$

$$a = -\left(\frac{\rho\pi r^2 g}{M}\right)x = -\omega^2 x$$

$\boxed{\text{The opposite direction and direct proportionality of } a \text{ to } x \text{ imply SHM}}$ with angular frequency

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}}$$

The acceleration $a = -\rho\pi r^2 gx/M$ is a negative constant times the displacement from equilibrium.

$$T = \frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}$$

P15.69 (a) Newton's law of universal gravitation is $F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2}\left(\frac{4}{3}\pi r^3\right)\rho$

Thus, $F = -\left(\frac{4}{3}\pi\rho Gm\right)r$

Which is of Hooke's law form with $k = \frac{4}{3}\pi\rho Gm$

(b) The sack of mail moves without friction according to $-\left(\frac{4}{3}\right)\pi\rho Gmr = ma$

$$a = -\left(\frac{4}{3}\right)\pi\rho Gr = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$\omega = \sqrt{\frac{4\pi\rho G}{3}}$ and period $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$

The time for a one-way trip through the earth is $\frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$

We have also $g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3\rho}{3R_e^2} = \frac{4}{3}\pi\rho GR_e$

so

$$\frac{4\rho G}{3} = \frac{g}{(\pi R_e)}$$

and

$$\frac{T}{2} = \pi\sqrt{\frac{R_e}{g}} = \pi\sqrt{\frac{6.37\times 10^6\ \text{m}}{9.8\ \text{m/s}^2}} = 2.53\times 10^3\ \text{s} = \boxed{42.2\ \text{min}}$$

P15.70 (a) The block moves with the board in what we take as the positive *x* direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction

$\mu_s n = \mu_s mg$. This occurs at $x = \frac{\mu_s mg}{k}$

(b) Since v is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

$$-kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) = -kx_{rel}$$

where x_{rel} is the excursion of the block away from the point $\frac{\mu_k mg}{k}$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\frac{\mu_k mg}{k}$.

(d) The amplitude of its motion is its original displacement, $A = \frac{\mu_s mg}{k} - \frac{\mu_k mg}{k}$. It first comes to rest at spring extension $\frac{\mu_k mg}{k} - A = \frac{(2\mu_k - \mu_s)mg}{k}$. Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

continued on next page

(c) The graph of the motion looks like this:

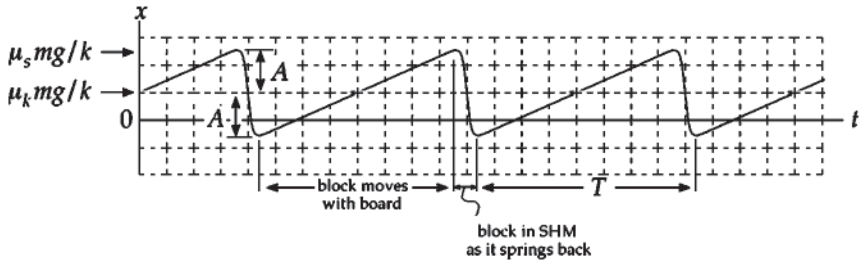


FIG. P15.70(c)

(e) The time during each cycle when the block is moving with the board is $\frac{2A}{v} = \frac{2(\mu_s - \mu_k)mg}{kv}$.
The time for which the block is springing back is one half a cycle of simple harmonic motion, $\frac{1}{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \pi\sqrt{\frac{m}{k}}$. We ignore the times at the end points of the motion when the speed of the block changes from v to 0 and from 0 to v. Since v is small compared to $\frac{2A}{\pi\sqrt{m/k}}$, these times are negligible. Then the period is

$$T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$$

(f) $$T = \frac{2(0.4 - 0.25)(0.3\text{ kg})(9.8\text{ m/s}^2)}{(0.024\text{ m/s})(12\text{ N/m})} + \pi\sqrt{\frac{0.3\text{ kg}}{12\text{ N/m}}} = 3.06\text{ s} + 0.497\text{ s} = 3.56\text{ s}$$

Then

$$f = \frac{1}{T} = \boxed{0.281\text{ Hz}}$$

(g) $T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$ increases as m increases, so the frequency $\boxed{\text{decreases}}$.

(h) As k increases, T decreases and f $\boxed{\text{increases}}$.

(i) As v increases, T decreases and f $\boxed{\text{increases}}$.

(j) As $(\mu_s - \mu_k)$ increases, T increases and f $\boxed{\text{decreases}}$.

ANSWERS TO EVEN PROBLEMS

P15.2 (a) 4.33 cm (b) -5.00 cm/s (c) -17.3 cm/s^2 (d) 3.14 s; 5.00 cm

P15.4 (a) 15.8 cm (b) –15.9 cm (c) The patterns of oscillation diverge from each other, starting out in phase but becoming completely out of phase. To calculate the future we would need exact knowledge of the present, an impossibility. (d) 51.1 m (e) 50.7 m

P15.6 (a) 2.40 s (b) 0.417 Hz (c) 2.62 rad/s

P15.8 (a) -2.34 m (b) -1.30 m/s (c) –0.076 3 m (d) 0.315 m/s

P15.10 (a) 1.26 s (b) 0.150 m/s; 0.750 m/s^2 (c) $x = -3 \text{ cm} \cos 5t$; $v = \left(\frac{15 \text{ cm}}{\text{s}}\right)\sin 5t$; $a = \left(\frac{75 \text{ cm}}{\text{s}^2}\right)\cos 5t$

P15.12 Yes. Whether the object has small or large mass, the ratio m/k must be equal to 0.183 m/(9.80 m/s^2). The period is 0.859 s.

P15.14 (a) 126 N/m (b) 0.178 m

P15.16 (a) 0.153 J (b) 0.784 m/s (c) 17.5 m/s^2

P15.18 (a) 100 N/m (b) 1.13 Hz (c) 1.41 m/s at $x = 0$ (d) 10.0 m/s^2 at $x = \pm A$ (e) 2.00 J (f) 1.33 m/s (g) 3.33 m/s^2

P15.20 (a) 1.50 s (b) 73.4 N/m (c) 19.7 m below the bridge (d) 1.06 rad/s (e) 2.01 s (f) 3.50 s

P15.22 The position of the piston is given by $x = A\cos \omega t$.

P15.24 $\frac{g_C}{g_T} = 1.001\,5$

P15.26 1.42 s; 0.499 m

P15.28 (a) 3.65 s (b) 6.41 s (c) 4.24 s

P15.30 (a) see the solution (b), (c) 9.85 m/s^2, agreeing with the accepted value within 0.5%

P15.32 (a) 2.09 s (b) 4.08%

P15.34 see the solution

P15.36 see the solution

P15.38 (a) 2.95 Hz (b) 2.85 cm

P15.40 see the solution

P15.42 either 1.31 Hz or 0.641 Hz

P15.44 (a) 0.368 $\hat{\mathbf{i}}$ m/s (b) 3.51 cm (c) Conservation of momentum for the glider-spring-glider system requires that the center of mass move with constant velocity. Conservation of mechanical energy for the system implies that in the center-of-mass reference frame the gliders both oscillate after they couple together. (d) 40.6 mJ (e) 27.7 mJ

P15.46 (a) 4.31 cm (b) When the rock is on the point of lifting off, the surrounding water is also barely in free fall. No pressure gradient exists in the water, so no buoyant force acts on the rock.

P15.48 (a) 0.500 m/s (b) 8.56 cm

P15.50 $A = \frac{\mu_s g}{4\pi^2 f^2}$

P15.52 (a) A time interval. If the interaction occupied no time, the force exerted by each ball on the other would be infinite, and that cannot happen. (b) 80.0 MN/m (c) 0.843 J. (d) 0.145 mm (e) 9.12×10^{-5} s

P15.54 (a) $k = \dfrac{4\pi^2 m}{T^2}$ (b) $m' = m\left(\dfrac{T'}{T}\right)^2$

P15.56 (a) $x = -(2\text{ m})\sin(10\,t)$ (b) at $x = \pm 1.73$ m (c) 98.0 mm (d) 52.4 ms

P15.58 (a) The amplitude is reduced by 0.735 m (b) The period increases by 0.730 s (c) The energy decreases by 120 J (d) Mechanical energy is converted into internal energy in the perfectly inelastic collision.

P15.60 (a) 3.56 Hz (b) 2.79 Hz (c) 2.10 Hz

P15.62 (a) $\dfrac{1}{2}\left(M + \dfrac{m}{3}\right)v^2$ (b) $T = 2\pi\sqrt{\dfrac{M + m/3}{k}}$

P15.64 see the solution (a) $k = 1.74\ \text{N/m} \pm 6\%$ (b) $1.82\ \text{N/m} \pm 3\%$; they agree (c) $8\ \text{g} \pm 12\%$; it agrees

P15.66 (a) 5.20 s (b) 2.60 s (c) see the solution

P15.68 The acceleration $a = -\rho\pi r^2 gx/M$ is a negative constant times the displacement from equilibrium.

$$T = \frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}$$

P15.70 see the solution (f) 0.281 Hz (g) decreases (h) increases (i) increases (j) decreases