30

Sources of the Magnetic Field

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q30.1** Answers (b) and (c).

***Q30.2** (i) Magnetic field lines line in horizontal planes and go around the wire clockwise as seen from above. East of the wire the field points horizontally south. Answer (b). (ii) The same. Answer (b).

***Q30.3** (i) Answer (f). (ii) Answer (e).

Q30.4 The magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction down × into the paper = to the right, away from wire 1. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force up × into the paper = left, away from wire 2.

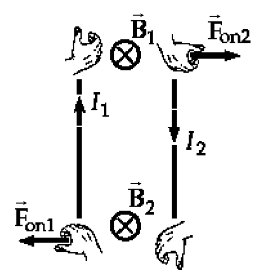


FIG. Q30.4

***Q30.5** Newton's third law describes the relationship. Answer (c).

***Q30.6** (a) No (b) Yes, if all are alike in sign. (c) Yes, if all carry current in the same direction. (d) no

Q30.7 Ampère's law is valid for all closed paths surrounding a conductor, but not always convenient. There are many paths along which the integral is cumbersome to calculate, although not impossible. Consider a circular path around but *not* coaxial with a long, straight current-carrying wire.

Q30.8 The Biot-Savart law considers the contribution of each element of current in a conductor to determine the magnetic field, while for Ampère's law, one need only know the current passing through a given surface. Given situations of high degrees of symmetry, Ampère's law is more convenient to use, even though both laws are equally valid in all situations.

Q30.9 Apply Ampère's law to the circular path labeled 1 in the picture. Since there is no current inside this path, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor. Therefore the magnetic field outside the tube is nonzero.

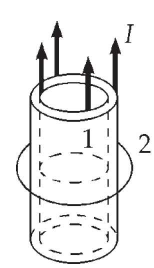


FIG. Q30.9

***Q30.10** (i) Answer (b). (ii) Answer (d), according to $B = \dfrac{\mu_0 NI}{\ell}$. (iii) Answer (b). (iv) Answer (c).

***Q30.11** Answer (a). The adjacent wires carry currents in the same direction.

***Q30.12** Answer (c). The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is maximum when the field is perpendicular to the area of the loop of wire. The flux is zero when there is no component of magnetic field perpendicular to the loop—that is, when the plane of the loop contains the x axis.

***Q30.13** Zero in each case. The fields have no component perpendicular to the area.

***Q30.14** (a) Positive charge for attraction. (b) Larger. The contributions away from + and toward – are in the same direction at the midpoint. (c) Downward (d) Smaller. Clockwise around the left-hand wire and clockwise around the right-hand wire are in opposite directions at the midpoint.

***Q30.15** (a) In units of μ_0(ampere/cm), the field of the straight wire, from $\mu_0 I/2\pi r$, is $3/(2\pi 2) = 0.75/\pi$. As a multiple of the same quantity, $N\mu_0 I/2r$ gives for (b) $10 \times 0.3/2 \times 2 = 0.75$.

(c) $N\mu_0 I/\ell$ gives $1000 \times 0.3/200 = 1.5$ times μ_0(ampere/cm), which is also $1.5 \times 4\pi \times 10^{-7}/0.01\ \text{T} = 0.19\ \text{mT}$.

(d) The field is zero at the center.

(e) 1 mT is larger than 0.19 mT, so it is largest of all.
The ranking is then e > c > b > a > d.

***Q30.16** Yes. Either pole of the magnet creates field that turns atoms inside the iron to align their magnetic moments with the external field. Then the nonuniform field exerts a net force on each atom toward the direction in which the field is getting stronger.

A magnet on a refrigerator door goes through the same steps to exert a strong normal force on the door. Then the magnet is supported by a frictional force.

Q30.17 Magnetic domain alignment within the magnet and then within the first piece of iron creates an external magnetic field. The field of the first piece of iron in turn can align domains in another iron sample. A nonuniform magnetic field exerts a net force of attraction on magnetic dipoles aligned with the field.

Q30.18 The shock misaligns the domains. Heating will also decrease magnetism.

Q30.19 The north magnetic pole is off the coast of Antarctica, near the south geographic pole. Straight up.

***Q30.20** (a) The third magnet from the top repels the second one with a force equal to the weight of the top two. The yellow magnet repels the blue one with a force equal to the weight of the blue one.

(b) The rods (or a pencil) prevents motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. One disk has its north pole on the top side and the adjacent magnets have their north poles on their bottom sides.

(d) If the blue magnet were inverted, it and the yellow one would stick firmly together. The pair would still produce an external field and would float together above the red magnets.

SOLUTIONS TO PROBLEMS

Section 30.1 The Biot-Savart Law

P30.1 $B=\dfrac{\mu_0 I}{2R}=\dfrac{\mu_0 q(v/2\pi R)}{2R}=\boxed{12.5\text{ T}}$

P30.2 $B=\dfrac{\mu_0 I}{2\pi r}=\dfrac{\left(4\pi\times10^{-7}\right)(1.00\text{ A})}{2\pi(1.00\text{ m})}=\boxed{2.00\times10^{-7}\text{ T}}$

P30.3 (a) $B=\dfrac{4\mu_0 I}{4\pi a}\left(\cos\dfrac{\pi}{4}-\cos\dfrac{3\pi}{4}\right)$ where $a=\dfrac{\ell}{2}$

is the distance from any side to the center.

$$B=\frac{4.00\times10^{-6}}{0.200}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)=2\sqrt{2}\times10^{-5}\text{ T}=\boxed{28.3\ \mu\text{T into the paper}}$$

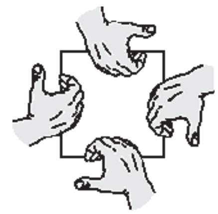


FIG. P30.3

(b) For a single circular turn with $4\ell=2\pi R$,

$$B=\frac{\mu_0 I}{2R}=\frac{\mu_0\pi I}{4\ell}=\frac{\left(4\pi^2\times10^{-7}\right)(10.0)}{4(0.400)}=\boxed{24.7\ \mu\text{T into the paper}}$$

P30.4 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\dfrac{\mu_0 I}{2\pi R}$ and directed into the page) and the field due to the circular loop (having magnitude $\dfrac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is:

$$\boxed{\vec{\mathbf{B}}=\left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}\ \text{(directed into the page)}}.$$

P30.5 For leg 1, $d\vec{\mathbf{s}}\times\hat{\mathbf{r}}=0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B=\frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right)=\boxed{\frac{\mu_0 I}{4\pi x}\text{ into the paper}}$$

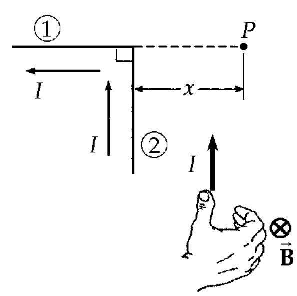


FIG. P30.5

P30.6 Along the axis of a circular loop of radius R,

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{\left(x/R\right)^2 + 1}\right]^{3/2}$$

where $$B_0 \equiv \frac{\mu_0 I}{2R}$$

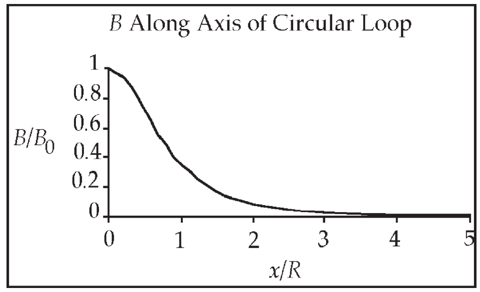


FIG. P30.6

x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

P30.7 Wire 1 creates at the origin magnetic field

$$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{2\pi r} \text{ right hand rule } = \frac{\mu_0 I_1}{2\pi a} \quad = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}$$

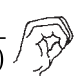
(a) If the total field at the origin is $\frac{2\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$ then the second wire must create field

according to $\vec{\mathbf{B}}_2 = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_2}{2\pi (2a)}$.

Then $I_2 = \boxed{2I_1 \text{ out of the paper}}$.

(b) The other possibility is $\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{2\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right) = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$.

Then $\vec{\mathbf{B}}_2 = \frac{3\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right) = \frac{\mu_0 I_2}{2\pi (2a)}$ $\quad I_2 = \boxed{6I_1 \text{ into the paper}}$



P30.8 Every element of current creates magnetic field in the same direction, into the page, at the center of the arc. The upper straight portion creates one-half of the field that an infinitely long straight wire would create. The curved portion creates one quarter of the field that a circular loop produces at its center. The lower straight segment also creates field $\frac{1}{2}\frac{\mu_0 I}{2\pi r}$.

The total field is

$$\vec{\mathbf{B}} = \left(\frac{1}{2}\frac{\mu_0 I}{2\pi r} + \frac{1}{4}\frac{\mu_0 I}{2r} + \frac{1}{2}\frac{\mu_0 I}{2\pi r}\right) \text{into the page} = \boxed{\frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right) \text{into the plane of the paper}}$$

$$= \left(\frac{0.284\,15\mu_0 I}{r}\right) \text{into the page}$$

P30.9 (a) Above the pair of wires, the field out of the page of the 50 A current will be stronger than the $(-\hat{\mathbf{k}})$ field of the 30 A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y = -|y|$. Here the total field is

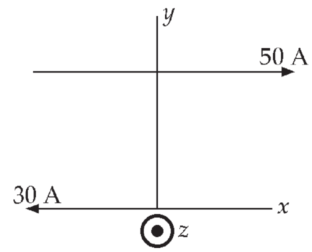


FIG. P30.9

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \quad + \frac{\mu_0 I}{2\pi r} \quad :$$

$$0 = \frac{\mu_0}{2\pi}\left[\frac{50\text{ A}}{(|y| + 0.28\text{ m})}(-\hat{\mathbf{k}}) + \frac{30\text{ A}}{|y|}(\hat{\mathbf{k}})\right]$$

$$50|y| = 30(|y| + 0.28\text{ m})$$

$$50(-y) = 30(0.28\text{ m} - y)$$

$$-20y = 30(0.28\text{ m}) \quad \boxed{\text{at } y = -0.420\text{ m}}$$

(b) At $y = 0.1\text{ m}$ the total field is $\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \quad + \frac{\mu_0 I}{2\pi r} \quad :$

$$\vec{\mathbf{B}} = \frac{4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\left(\frac{50\text{ A}}{(0.28 - 0.10)\text{ m}}(-\hat{\mathbf{k}}) + \frac{30\text{ A}}{0.10\text{ m}}(-\hat{\mathbf{k}})\right) = 1.16 \times 10^{-4}\text{ T}(-\hat{\mathbf{k}})$$

The force on the particle is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = (-2 \times 10^{-6}\text{ C})(150 \times 10^{6}\text{ m/s})(\hat{\mathbf{i}}) \times (1.16 \times 10^{-4}\text{ N}\cdot\text{s/C}\cdot\text{m})(-\hat{\mathbf{k}})$$

$$= \boxed{3.47 \times 10^{-2}\text{ N}(-\hat{\mathbf{j}})}$$

(c) We require $\vec{\mathbf{F}}_e = 3.47 \times 10^{-2}\text{ N}(+\hat{\mathbf{j}}) = q\vec{\mathbf{E}} = (-2 \times 10^{-6}\text{ C})\vec{\mathbf{E}}$

So $\vec{\mathbf{E}} = \boxed{-1.73 \times 10^{4}\,\hat{\mathbf{j}}\text{ N/C}}$

***P30.10** We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\vec{\mathbf{s}}$ is at 0° or 180° to $\hat{\mathbf{r}}$, so $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = 0$. Thus, only the curved section of wire contributes to $\vec{\mathbf{B}}$ at *P*. Hence, $d\vec{\mathbf{s}}$ is tangent to the arc and $\hat{\mathbf{r}}$ is radially inward; so $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |ds| 1 \sin 90° \otimes = |ds| \otimes$. All points along the curve are the same distance $r = 0.600\text{ m}$ from the field point, so

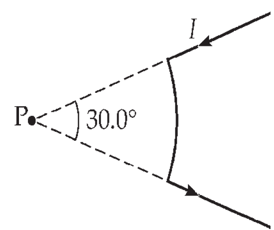


FIG. P30.10

$$B = \int_{\text{all current}} |d\vec{\mathbf{B}}| = \int \frac{\mu_0}{4\pi} \frac{I|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int |ds| = \frac{\mu_0}{4\pi} \frac{I}{r^2} s$$

where *s* is the arc length of the curved wire,

$$s = r\theta = (0.600\text{ m})(30.0°)\left(\frac{2\pi}{360°}\right) = 0.314\text{ m}$$

Then, $B = (10^{-7}\text{ T}\cdot\text{m/A})\frac{(3.00\text{ A})}{(0.600\text{ m})^2}(0.314\text{ m})$ $\qquad B = \boxed{262\text{ nT into the page}}$

***P30.11** Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be $\vec{\mathbf{B}}_1$, $\vec{\mathbf{B}}_2$, and $\vec{\mathbf{B}}_3$, respectively.

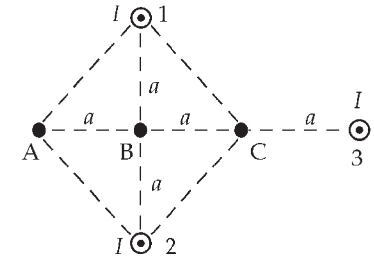


Figure (a)

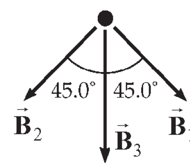


Figure (b)

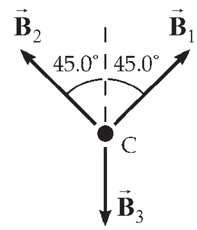


Figure (c)

FIG. P30.11

(a) At point A: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.

The directions of these fields are shown in Figure (b). Observe that the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel while their vertical components both add onto $\vec{\mathbf{B}}_3$.

Therefore, the net field at point A is:

$$B_A = B_1\cos 45.0° + B_2\cos 45.0° + B_3$$

$$= \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$

$$B_A = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi\left(1.00\times10^{-2}\ \text{m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right]$$

$$B_A = \boxed{53.3\ \mu\text{T toward the bottom of the page}}$$

(b) At point B: $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel, leaving

$$B_B = B_3 = \frac{\mu_0 I}{2\pi(2a)}$$

$$B_B = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi(2)\left(1.00\times10^{-2}\ \text{m}\right)}$$

$$= \boxed{20.0\ \mu\text{T toward the bottom of the page}}$$

(c) At point C: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel. The vertical components both oppose $\vec{\mathbf{B}}_3$ giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° - 1\right] = \boxed{0}$$

***P30.12** (a) The upward lightning current creates field in counterclockwise horizontal circles.

$\dfrac{\mu_0 I}{2\pi r}$ righthand rule $= \dfrac{4\pi\times10^{-7}\ \text{T m}\ 20\times10^{3}\ \text{A}}{2\pi\ \text{A}\ 50\ \text{m}}$ north $= 8.00\times10^{-5}$ T north

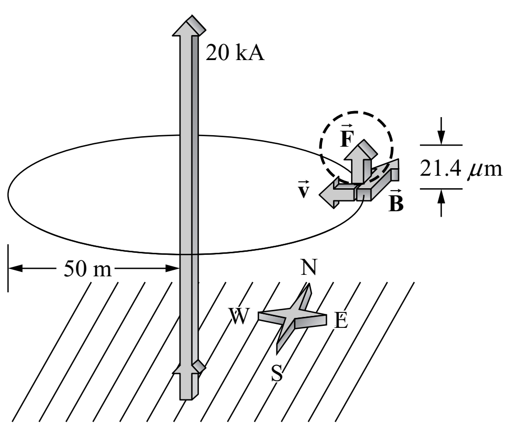


FIG. P30.12

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The force on the electron is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = -1.6 \times 10^{-19}\ \text{C (300 m/s west)} \times 8 \times 10^{-5}\ \text{(N s/C m) north}$$

$$= -3.84 \times 10^{-21}\ \text{N down}\ \sin 90° \ = \boxed{3.84 \times 10^{-21}\ \text{N up}}$$

(b) $r = \dfrac{mv}{qB} = \dfrac{9.11 \times 10^{-31}\ \text{kg 300 m/s}}{1.6 \times 10^{-19}\ \text{C}\ 8 \times 10^{-5}\ \text{N s/C m}} = \boxed{2.14 \times 10^{-5}\ \text{m}}$. This distance is negligible compared to 50 m, so the electron does move in a uniform field.

(c) $\omega = qB/m = 2\pi N/t$

$$N = \frac{qBt}{2\pi m} = \frac{1.6 \times 10^{-19}\ \text{C}\ \left(8 \times 10^{-5}\ \text{N s/C m}\right) 60 \times 10^{-6}\ \text{s}}{2\pi\ 9.11 \times 10^{-31}\ \text{kg}} = \boxed{134\ \text{rev}}$$

***P30.13** (a) We use Equation 30.4 in the chapter text for the field created by a straight wire of limited length. The sines of the angles appearing in that equation are equal to the cosines of the complementary angles shown in our diagram. For the distance a from the wire to the field point we have $\tan 30° = \dfrac{a}{L/2}$, $a = 0.288\,7L$. One wire contributes to the field at P

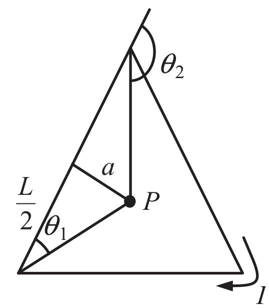


FIG. P30.13(a)

$$B = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_1 - \cos\theta_2\right) = \frac{\mu_0 I}{4\pi(0.288\,7L)}\left(\cos 30° - \cos 150°\right)$$

$$= \frac{\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{1.50\mu_0 I}{\pi L}$$

Each side contributes the same amount of field in the same direction, which is perpendicularly into the paper in the picture. So the total field is

$$3\left(\frac{1.50\mu_0 I}{\pi L}\right) = \boxed{\frac{4.50\mu_0 I}{\pi L}}.$$

(b) As we showed in part (a), one whole side of the triangle creates field at the center $\dfrac{\mu_0 I(1.732)}{4\pi a}$. Now one-half of one nearby side of the triangle will be half as far away from point P_b and have a geometrically similar situation. Then it creates at P_b field $\dfrac{\mu_0 I(1.732)}{4\pi(a/2)} = \dfrac{2\mu_0 I(1.732)}{4\pi a}$. The two half-sides shown crosshatched in the picture create at P_b field $2\left(\dfrac{2\mu_0 I(1.732)}{4\pi a}\right) = \dfrac{4\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \dfrac{6\mu_0 I}{\pi L}$. The rest of the triangle will contribute somewhat more field in the same direction, so we already have a proof that the field at P_b is $\boxed{\text{stronger}}$.

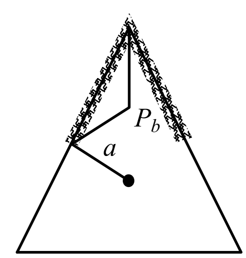


FIG. P30.13(b)

P30.14 Apply the equation from the chapter text for the field created by a straight wire of limited length, three times:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \frac{d}{\sqrt{d^2+a^2}}\right)\text{ toward you} + \frac{\mu_0 I}{4\pi d}\left(\frac{a}{\sqrt{d^2+a^2}} + \frac{a}{\sqrt{d^2+a^2}}\right)\text{ away from you}$$

$$+\frac{\mu_0 I}{4\pi a}\left(\frac{-d}{\sqrt{d^2+a^2}} - \cos 180°\right)\text{ toward you}$$

$$\vec{\mathbf{B}} = \boxed{\frac{\mu_0 I\left(a^2+d^2-d\sqrt{a^2+d^2}\right)}{2\pi ad\sqrt{a^2+d^2}}\text{ away from you}}$$

P30.15 Take the x-direction to the right and the y-direction up in the plane of the paper. Current 1 creates at P a field

$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{\left(2.00\times 10^{-7}\text{ T}\cdot\text{m}\right)(3.00\text{ A})}{\text{A}(0.050\,0\text{ m})}$$

$\vec{\mathbf{B}}_1 = 12.0\ \mu\text{T}$ downward and leftward, at angle 67.4° below the $-x$ axis.
Current 2 contributes

$$B_2 = \frac{\left(2.00\times 10^{-7}\text{ T}\cdot\text{m}\right)(3.00\text{ A})}{\text{A}(0.120\text{ m})}\quad\text{clockwise perpendicular to 12.0 cm}$$

$\vec{\mathbf{B}}_2 = 5.00\ \mu\text{T}$ to the right and down, at angle –22.6°

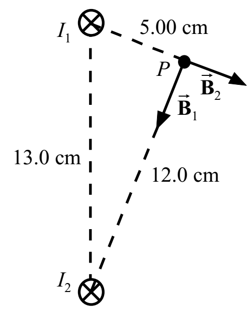


FIG. P30.15

Then, $\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = (12.0\ \mu\text{T})\left(-\hat{\mathbf{i}}\cos 67.4° - \hat{\mathbf{j}}\sin 67.4°\right) + (5.00\ \mu\text{T})\left(\hat{\mathbf{i}}\cos 22.6° - \hat{\mathbf{j}}\sin 22.6°\right)$

$$\vec{\mathbf{B}} = (-11.1\ \mu\text{T})\hat{\mathbf{j}} - (1.92\ \mu\text{T})\hat{\mathbf{j}} = \boxed{(-13.0\ \mu\text{T})\hat{\mathbf{j}}}$$

***P30.16** We apply $B = (\mu_0/2\pi)\,\mu/x^3$ to the center of a face, where $B = 40\,000\ \mu$T and $x = 0.6$ mm, and also to the exterior weak-field point, where $B = 50\ \mu$T and x is the unknown.
Then $Bx^3 = Bx^3$ 40 000 μT (0.6 mm)^3 = 50 μT (d + 0.6 mm)^3

$d = (40\,000/50)^{1/3}$ 0.6 mm – 0.6 mm = $\boxed{4.97\text{ mm}}$

The strong field does not penetrate your painful joint.

Section 30.2 **The Magnetic Force Between Two Parallel Conductors**

P30.17 By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using an equation from the chapter text)

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \frac{\left(4\pi\times 10^{-7}\text{ N/A}^2\right)(5.00\text{ A})(10.0\text{ A})(0.450\text{ m})}{2\pi}\left(\frac{-0.150\text{ m}}{(0.100\text{ m})(0.250\text{ m})}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \left(-2.70\times 10^{-5}\,\hat{\mathbf{i}}\right)\text{ N}$$

or $\vec{\mathbf{F}} = \boxed{2.70\times 10^{-5}\text{ N toward the left}}$

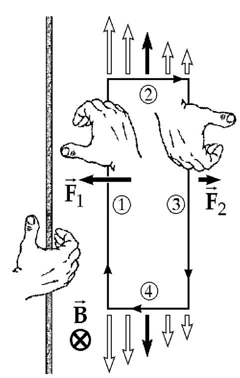


FIG. P30.17

P30.18 Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

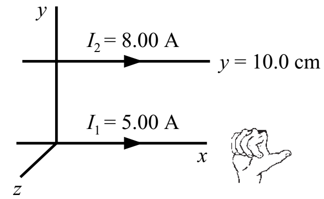


FIG. P30.18(a)

(a) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \boxed{1.00\times10^{-5}\ \mathrm{T\ out\ of\ the\ page}}$$

(b) $$\vec{\mathbf{F}}_B = I_2\vec{\ell}\times\vec{\mathbf{B}} = (8.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}}\times(1.00\times10^{-5}\ \mathrm{T})\hat{\mathbf{k}}\right] = (8.00\times10^{-5}\ \mathrm{N})(-\hat{\mathbf{j}})$$

$$\boxed{\vec{\mathbf{F}}_B = 8.00\times10^{-5}\ \mathrm{N\ toward\ the\ first\ wire}}$$

(c) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}(-\hat{\mathbf{k}}) = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(8.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}(-\hat{\mathbf{k}}) = (1.60\times10^{-5}\ \mathrm{T})(-\hat{\mathbf{k}})$$

$$\vec{\mathbf{B}} = \boxed{1.60\times10^{-5}\ \mathrm{T\ into\ the\ page}}$$

(d) $$\vec{\mathbf{F}}_B = I_1\vec{\ell}\times\vec{\mathbf{B}} = (5.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}}\times(1.60\times10^{-5}\ \mathrm{T})(-\hat{\mathbf{k}})\right] = (8.00\times10^{-5}\ \mathrm{N})(+\hat{\mathbf{j}})$$

$$\vec{\mathbf{F}}_B = \boxed{8.00\times10^{-5}\ \mathrm{N\ towards\ the\ second\ wire}}$$

P30.19 To attract, both currents must be to the right. The attraction is described by

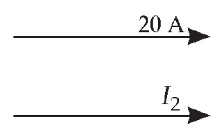


FIG. P30.19

$$F = I_2\ell B\sin 90° = I_2\ell\frac{\mu_0 I}{2\pi r}$$

So $$I_2 = \frac{F}{\ell}\frac{2\pi r}{\mu_0 I_1} = (320\times10^{-6}\ \mathrm{N/m})\left(\frac{2\pi(0.5\ \mathrm{m})}{(4\pi\times10^{-7}\ \mathrm{N\cdot s/C\cdot m})(20\ \mathrm{A})}\right) = 40.0\ \mathrm{A}$$

Let y represent the distance of the zero-field point below the upper wire.

Then $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$$  $$+\frac{\mu_0 I}{2\pi r}$$  $$0 = \frac{\mu_0}{2\pi}\left(\frac{20\ \mathrm{A}}{y}(\mathrm{away}) + \frac{40\ \mathrm{A}}{(0.5\ \mathrm{m}-y)}(\mathrm{toward})\right)$$

$$20(0.5\ \mathrm{m}-y) = 40y \qquad 20(0.5\ \mathrm{m}) = 60y$$

$$y = \boxed{0.167\ \mathrm{m\ below\ the\ upper\ wire}}$$

***P30.20** Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2. We answer part (b) first.

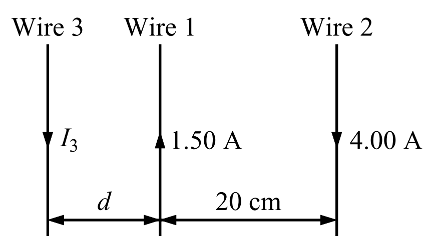


FIG. P30.20

(b) For the equilibrium of wire 3 we have

$$F_{1\ \mathrm{on}\ 3} = F_{2\ \mathrm{on}\ 3} \qquad \frac{\mu_0(1.50\ \mathrm{A})I_3}{2\pi d} = \frac{\mu_0(4\ \mathrm{A})I_3}{2\pi(20\ \mathrm{cm}+d)}$$

$$1.5(20\ \mathrm{cm}+d) = 4d \qquad d = \frac{30\ \mathrm{cm}}{2.5} = \boxed{12.0\ \mathrm{cm\ to\ the\ left\ of\ wire\ 1}}$$

continued on next page

(a) Thus the situation is possible in just one way.

(c) For the equilibrium of wire 1,

$$\frac{\mu_0 I_3 (1.5\text{ A})}{2\pi (12\text{ cm})} = \frac{\mu_0 (4\text{ A})(1.5\text{ A})}{2\pi (20\text{ cm})} \qquad I_3 = \frac{12}{20}4\text{ A} = \boxed{2.40\text{ A down}}$$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

***P30.21** The separation between the wires is

$$a = 2(6.00\text{ cm})\sin 8.00° = 1.67\text{ cm}$$

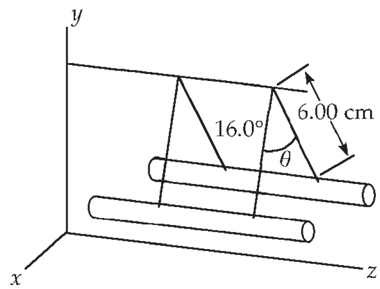


FIG. P30.21

(a) Because the wires repel, the currents are in $\boxed{\text{opposite directions}}$.

(b) Because the magnetic force acts horizontally,

$$\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi amg} = \tan 8.00°$$

$$I^2 = \frac{mg2\pi a}{\ell \mu_0}\tan 8.00° \quad \text{so} \quad I = \boxed{67.8\text{ A}}$$

(c) Smaller. A smaller gravitational force would be pulling down on the wires and so tending to pull the wires together. Then a smaller magnetic force is required to keep the wires apart.

***P30.22** (a) If the conductors are wires, $F = I_2 \ell B \sin 90°$ and $B = \mu_0 I_1/2\pi r$ give $F = I_1 I_2 \mu_0 \ell/2\pi r$

$$r = \frac{I_1 I_2 \mu_0 \ell}{2\pi F} = \frac{(10\text{ A})^2 2\times10^{-7}\text{ T m } 0.5\text{ m}}{1\text{ N} \quad \text{A}} = \boxed{10.0\ \mu\text{m}}$$

(b) For a force of ordinary size in a tabletop mechanics experiments to act on ordinary-to-large size currents, the distance between them must be quite small, but the situation is physically possible. If we tried to use wires with diameter 10 μm on a tabletop, they would feel the force only momentarily after we turn on the current, until they melt. We can use wide, thin sheets of copper, perhaps plated onto glass, and perhaps with water cooling, to have the forces act continuously. A practical electric motor must use coils of wire with many turns.

Section 30.3 **Ampère's Law**

P30.23 Each wire is distant from P by

$$(0.200\text{ m})\cos 45.0° = 0.141\text{ m}$$

Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\text{ A})}{(0.141\text{ m})} = 7.07\ \mu\text{T}$$

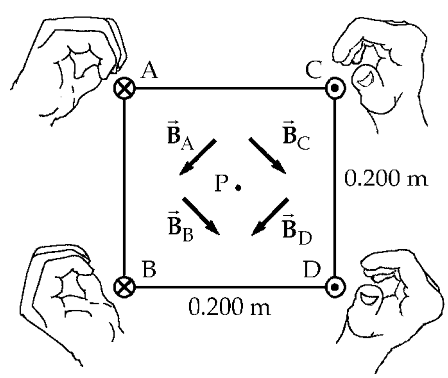


FIG. P30.23

Carrying currents into the page, A produces at P a field of 7.07 μT to the left and down at –135°, while B creates a field to the right and down at –45°. Carrying currents toward you, C produces a field downward and to the right at –45°, while D's contribution is downward and to the left. The total field is then

$$4(7.07\ \mu\text{T})\sin 45.0° = \boxed{20.0\ \mu\text{T}} \text{ toward the bottom of the page.}$$

P30.24 Let the current I be to the right. It creates a field $B=\dfrac{\mu_0 I}{2\pi d}$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force $mg\left(-\hat{\mathbf{j}}\right)+qv\left(-\hat{\mathbf{i}}\right)\times\dfrac{\mu_0 I}{2\pi d}\left(\hat{\mathbf{k}}\right)=0$ at a distance d from the wire

$$d=\frac{qv\mu_0 I}{2\pi mg}=\frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.30\times10^{4}\text{ m/s}\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.20\times10^{-6}\text{ A}\right)}{2\pi\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}=\boxed{5.40\text{ cm}}$$

P30.25 From Ampère's law, the magnetic field at point a is given by $B_a=\dfrac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a=1.00$ A out of the page (the current in the inner conductor), so

$$B_a=\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(1.00\text{ A})}{2\pi\left(1.00\times10^{-3}\text{ m}\right)}=\boxed{200\ \mu\text{T toward top of page}}$$

Similarly at point b: $B_b=\dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b=1.00\text{ A}-3.00\text{ A}=-2.00\text{ A}$, or $I_b=2.00$ A into the page. Therefore,

$$B_b=\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(2.00\text{ A})}{2\pi\left(3.00\times10^{-3}\text{ m}\right)}=\boxed{133\ \mu\text{T toward bottom of page}}$$

P30.26 (a) In $B=\dfrac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\text{ cm}}$.

(b) $\vec{\mathbf{B}}=\dfrac{\mu_0 I}{2\pi r_1}\hat{\mathbf{k}}+\dfrac{\mu_0 I}{2\pi r_2}\left(-\hat{\mathbf{k}}\right)$ so

$$B=\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m}(2.00\text{ A})}{2\pi\text{ A}}\left(\frac{1}{0.398\,5\text{ m}}-\frac{1}{0.401\,5\text{ m}}\right)=\boxed{7.50\text{ nT}}$$

(c) Call r the distance from cord center to field point and $2d=3.00$ mm the distance between conductors.

$$B=\frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d}-\frac{1}{r+d}\right)=\frac{\mu_0 I}{2\pi}\frac{2d}{r^2-d^2}$$

$$7.50\times10^{-10}\text{ T}=\left(2.00\times10^{-7}\text{ T}\cdot\text{m/A}\right)(2.00\text{ A})\frac{\left(3.00\times10^{-3}\text{ m}\right)}{r^2-2.25\times10^{-6}\text{ m}^2}\quad\text{so}\quad r=\boxed{1.26\text{ m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

P30.27 (a) One wire feels force due to the field of the other ninety-nine.

$$B=\frac{\mu_0 I_0 r}{2\pi R^2}=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(99)(2.00\ \mathrm{A})\left(0.200\times10^{-2}\ \mathrm{m}\right)}{2\pi\left(0.500\times10^{-2}\ \mathrm{m}\right)^2}$$

$$=3.17\times10^{-3}\ \mathrm{T}$$

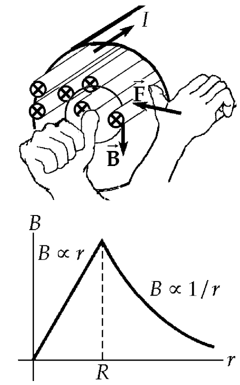


FIG. P30.27

This field points tangent to a circle of radius 0.200 cm and exerts force $\vec{\mathbf{F}}=I\vec{\ell}\times\vec{\mathbf{B}}$ toward the center of the bundle, on the single hundredth wire:

$$\frac{F}{\ell}=IB\sin\theta=(2.00\ \mathrm{A})\left(3.17\times10^{-3}\mathrm{T}\right)\sin90°=6.34\ \ \mathrm{mN/m}$$

$$\frac{F_B}{\ell}=\boxed{6.34\times10^{-3}\ \mathrm{N/m\ \ inward}}$$

(b) $B\propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.

P30.28 (a) $$B_{\text{inner}}=\frac{\mu_0 NI}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(900)\left(14.0\times10^{3}\ \mathrm{A}\right)}{2\pi(0.700\ \mathrm{m})}=\boxed{3.60\ \mathrm{T}}$$

(b) $$B_{\text{outer}}=\frac{\mu_0 NI}{2\pi r}=\frac{\left(2\times10^{-7}\ \mathrm{T\cdot m/A}\right)(900)\left(14.0\times10^{3}\ \mathrm{A}\right)}{1.30\ \mathrm{m}}=\boxed{1.94\ \mathrm{T}}$$

P30.29 We assume the current is vertically upward.

(a) Consider a circle of radius r slightly less than R. It encloses no current so from

$$\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}=\mu_0 I_{\text{inside}}\qquad B(2\pi r)=0$$

we conclude that the magnetic field is $\boxed{\text{zero}}$.

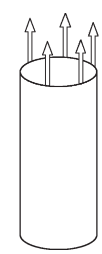


FIG. P30.29(a)

(b) Now let the r be barely larger than R. Ampère's law becomes $B(2\pi R)=\mu_0 I$,

so $$\boxed{B=\frac{\mu_0 I}{2\pi R}}$$

The field's direction is $\boxed{\text{tangent to the wall of the cylinder in a counterclockwise sense}}$.

(c) Consider a strip of the wall of width dx and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s=\dfrac{Idx}{2\pi R}$ up.

The force on it is

$$\vec{\mathbf{F}}=I_s\vec{\ell}\times\vec{\mathbf{B}}=\frac{Idx}{2\pi R}\left(\ell\frac{\mu_0 I}{2\pi R}\right)\widehat{\mathbf{up}}\times\widehat{\mathbf{into\ page}}=\frac{\mu_0 I^2\ell dx}{4\pi^2R^2}\widehat{\mathbf{radially\ inward}}.$$

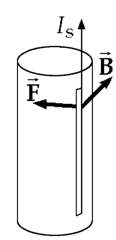


FIG. P30.29(c)

The pressure on the strip and everywhere on the cylinder is

$$P=\frac{F}{A}=\frac{\mu_0 I^2\ell dx}{4\pi^2R^2\ell dx}=\boxed{\frac{\mu_0 I^2}{(2\pi R)^2}\ \text{inward}}$$

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

P30.30 From $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I$, $\quad I = \dfrac{2\pi r B}{\mu_0} = \dfrac{2\pi\left(1.00\times10^{-3}\right)(0.100)}{4\pi\times10^{-7}} = \boxed{500\ \text{A}}$

P30.31 Use Ampère's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$. For current density J, this becomes

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \int J \cdot dA.$$

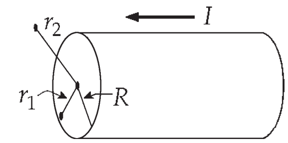


FIG. P30.31

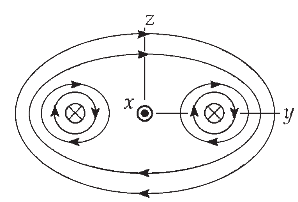
(a) For $r_1 < R$, this gives $\quad B 2\pi r_1 = \mu_0 \int_0^{r_1} (br)(2\pi r dr)$ and

$$B = \boxed{\dfrac{\mu_0 b r_1^2}{3} \quad \left(\text{for } r_1 < R \text{ or inside the cylinder}\right)}$$

(b) When $r_2 > R$, Ampère's law yields $\quad (2\pi r_2) B = \mu_0 \int_0^{R} (br)(2\pi r dr) = \dfrac{2\pi\mu_0 b R^3}{3}$,

or $\quad B = \boxed{\dfrac{\mu_0 b R^3}{3 r_2} \quad \left(\text{for } r_2 > R \text{ or outside the cylinder}\right)}$

***P30.32** (a) See Figure (a) to the right.

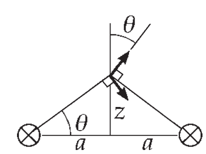


(Currents are into the paper) Figure (a)

(c) We choose to do part (c) before part (b). At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2 + z^2}}$ and is perpendicular to the line from this point to the wire as shown in Figure (b). Combining fields, the vertical components cancel while the horizontal components add, yielding

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}} \sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi\left(a^2+z^2\right)}$$

$$B_y = \frac{4\pi\times10^{-7}\ \text{T m } 8\ \text{A } z}{\pi\left[(0.03\ \text{m})^2 + z^2\right]} \quad \text{so} \quad \vec{\mathbf{B}} = \frac{32\times10^{-7} z\ \text{T}\cdot\text{m}}{9\times10^{-4}\ \text{m}^2 + z^2}\hat{\mathbf{j}}$$



At a distance z above the plane of the conductors Figure (b)

FIG. P30.32

(b) Substituting $z = 0$ gives zero for the field. We can see this from cancellation of the separate fields in either diagram. Taking the limit $z \to \infty$ gives $1/z \to 0$, as we should expect.

(d) The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi\left(a^2+z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2+z^2\right)} = 0 \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{\left(a^2-z^2\right)}{\left(a^2+z^2\right)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a = 3.00\ \text{cm}}$.

(e) The value of the maximum field is $\vec{\mathbf{B}} = \dfrac{32\times10^{-7} 0.03\ \text{m T}\cdot\text{m}}{9\times10^{-4}\,\text{m}^2 + 9\times10^{-4}\,\text{m}^2}\hat{\mathbf{j}} = \boxed{53.3\ \hat{\mathbf{j}}\ \mu\text{T}}$.

***P30.33** $J_s = \dfrac{I}{\ell}$. Each filament of current creates a contribution to the total field that goes counterclockwise around that filament's location. Together, they create field straight up to the right of the sheet and straight down to the left of the sheet.

From Ampère's law applied to the suggested rectangle,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

$B \cdot 2\ell + 0 = \mu_0 J_s \ell$ Therefore the field is uniform in space, with the magnitude $\boxed{B = \dfrac{\mu_0 J_s}{2}}$.

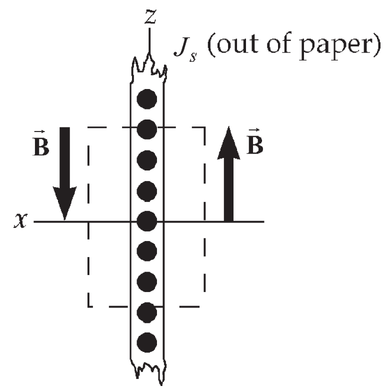


FIG. P30.33

Section 30.4 The Magnetic Field of a Solenoid

***P30.34** In the expression $B = N\mu_0 I/\ell$ for the field within a solenoid with radius much less than 20 cm, all we want to do is increase N.
(a) Make the wire as long and thin as possible without melting when it carries the 5-A current. Then the solenoid can have many turns.
(b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference the wire can form a solenoid with more turns.

P30.35 $B = \mu_0 \dfrac{N}{\ell} I$ so $I = \dfrac{B}{\mu_0 n} = \dfrac{(1.00 \times 10^{-4}\ \text{T})0.400\ \text{m}}{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})1\,000} = \boxed{31.8\ \text{mA}}$

P30.36 Let the axis of the solenoid lie along the y–axis from $y = 0$ to $y = \ell$. We will determine the field at $y = a$. This point will be inside the solenoid if $0 < a < \ell$ and outside if $a < 0$ or $a > \ell$. We think of solenoid as formed of rings, each of thickness dy. Now I is the symbol for the current in each turn of wire and the number of turns per length is $\left(\dfrac{N}{\ell}\right)$. So the number of turns in the ring is $\left(\dfrac{N}{\ell}\right)dy$ and the current in the ring is $I_{\text{ring}} = I\left(\dfrac{N}{\ell}\right)dy$. Now we use the result derived in the chapter text for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where x is the name of the distance from the center of the ring, at location y, to the field point $x = a - y$. Each ring creates field in the same direction, along our y–axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2 + R^2\right)^{3/2}} = \int_0^\ell \frac{\mu_0 I (N/\ell) dy R^2}{2\left((a-y)^2 + R^2\right)^{3/2}} = \frac{\mu_0 I N R^2}{2\ell} \int_0^\ell \frac{dy}{2\left((a-y)^2 + R^2\right)^{3/2}}$$

To perform the integral we change variables to $u = a - y$.

$$B = \frac{\mu_0 I N R^2}{2\ell} \int_a^{a-\ell} \frac{-du}{\left(u^2 + R^2\right)^{3/2}}$$

continued on next page

and then use the table of integrals in the appendix:

(a) $$B = \frac{\mu_0 INR^2}{2\ell} \left.\frac{-u}{R^2\sqrt{u^2+R^2}}\right|_a^{a-\ell} = \boxed{\frac{\mu_0 IN}{2\ell}\left[\frac{a}{\sqrt{a^2+R^2}} - \frac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right]}$$

(b) If ℓ is much larger than R and $a = 0$,

we have $B \cong \dfrac{\mu_0 IN}{2\ell}\left[0 - \dfrac{-\ell}{\sqrt{\ell^2}}\right] = \dfrac{\mu_0 IN}{2\ell}$.

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $a = \ell$ to describe the other end.

P30.37 The field produced by the solenoid in its interior is given by

$$\vec{\mathbf{B}} = \mu_0 nI\left(-\hat{\mathbf{i}}\right) = \left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{30.0}{10^{-2}\ \mathrm{m}}\right)(15.0\ \mathrm{A})\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{B}} = -\left(5.65\times10^{-2}\ \mathrm{T}\right)\hat{\mathbf{i}}$$ 

The force exerted on side AB of the square current loop is

$$\left(\vec{\mathbf{F}}_B\right)_{AB} = I\vec{\mathbf{L}}\times\vec{\mathbf{B}} = (0.200\ \mathrm{A})\left[\left(2.00\times10^{-2}\ \mathrm{m}\right)\hat{\mathbf{j}}\times\left(5.65\times10^{-2}\ \mathrm{T}\right)\left(-\hat{\mathbf{i}}\right)\right]$$

$$\left(\vec{\mathbf{F}}_B\right)_{AB} = \left(2.26\times10^{-4}\ \mathrm{N}\right)\hat{\mathbf{k}}$$ 

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of

$\boxed{226\ \mu\mathrm{N}\ \text{directed away from the center}}$.

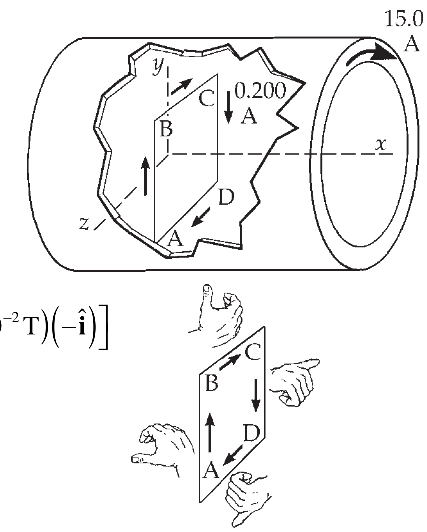


FIG. P30.37

From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by

$$\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}} = (0.200\ \mathrm{A})\left(2.00\times10^{-2}\ \mathrm{m}\right)^2\left(-\hat{\mathbf{i}}\right) = -80.0\ \mu\mathrm{A\cdot m^2}\hat{\mathbf{i}}$$ 

The torque exerted on the loop is then $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} = \left(-80.0\ \mu\mathrm{A\cdot m^2}\hat{\mathbf{i}}\right)\times\left(-5.65\times10^{-2}\ \mathrm{T}\hat{\mathbf{i}}\right) = \boxed{0}$

***P30.38** The number of turns is $N = \dfrac{75\ \mathrm{cm}}{0.1\ \mathrm{cm}} = 750$. We assume that the solenoid is long enough to qualify as a long solenoid. Then the field within it (not close to the ends) is $B = \dfrac{N\mu_0 I}{\ell}$ so

$$I = \frac{B\ell}{N\mu_0} = \frac{\left(8\times10^{-3}\ \mathrm{T}\right)(0.75\ \mathrm{m\cdot A})}{750\left(4\pi\times10^{-7}\ \mathrm{T\cdot m}\right)} = 6.37\ \mathrm{A}$$

The resistance of the wire is

$$R = \frac{\rho\ell_{\text{wire}}}{A} = \frac{\left(1.7\times10^{-8}\ \Omega\cdot\mathrm{m}\right)2\pi(0.05\ \mathrm{m})750}{\pi\left(0.05\times10^{-2}\ \mathrm{m}\right)^2} = 5.10\ \Omega$$

The power delivered is

$$\mathcal{P} = I\Delta V = I^2R = (6.37\ \mathrm{A})^2(5.10\ \Omega) = \boxed{207\ \mathrm{W}}$$

The power required would be smaller if wire were wrapped in several layers.

Section 30.5 **Gauss's Law in Magnetism**

P30.39 (a) $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) \text{ T} \cdot \left(2.50 \times 10^{-2} \text{ m}\right)^2 \hat{\mathbf{i}}$

$\Phi_B = 3.12 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.12 \times 10^{-3} \text{ Wb} = \boxed{3.12 \text{ mWb}}$

(b) $\left(\Phi_B\right)_{\text{total}} = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)

P30.40 (a) $\left(\Phi_B\right)_{\text{flat}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = B\pi R^2 \cos(180 - \theta) = \boxed{-B\pi R^2 \cos\theta}$

(b) The net flux out of the closed surface is zero: $\left(\Phi_B\right)_{\text{flat}} + \left(\Phi_B\right)_{\text{curved}} = 0$.

$\left(\Phi_B\right)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$

P30.41 (a) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA$ where A is the cross-sectional area of the solenoid.

$\Phi_B = \left(\dfrac{\mu_0 NI}{\ell}\right)\left(\pi r^2\right) = \boxed{7.40 \ \mu\text{Wb}}$

(b) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA = \left(\dfrac{\mu_0 NI}{\ell}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right]$

$$\Phi_B = \left[\frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(300)(12.0 \text{ A})}{(0.300 \text{ m})}\right]\pi\left[(8.00)^2 - (4.00)^2\right]\left(10^{-3} \text{ m}\right)^2 = \boxed{2.27 \ \mu\text{Wb}}$$

***P30.42** The field can be uniform in magnitude. Gauss's law for magnetism implies that magnetic field lines never start or stop. If the field is uniform in direction, the lines are parallel and their density stays constant along any one bundle of lines. Thus the magnitude of the field has the same value at all points along a line in the direction of the field. The magnitude of the field could vary over a plane perpendicular to the lines, or it could be constant throughout the volume.

Section 30.6 **Magnetism in Matter**

P30.43 The magnetic moment of one electron is taken as one Bohr magneton μ_B. Let x represent the number of electrons per atom contributing and n the number of atoms per unit volume. Then $nx\mu_B$ is the magnetic moment per volume and the magnetic field (in the absence of any currents in wires) is $B = \mu_0 nx\mu_B = 2.00$ T.

Then $x = \dfrac{B}{\mu_0 \mu_B n} = \dfrac{2.00 \text{ T}}{\left(8.50 \times 10^{28} \text{ m}^{-3}\right)\left(9.27 \times 10^{-24} \text{ N} \cdot \text{m/T}\right)\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)} = \boxed{2.02}$

Section 30.7 The Magnetic Field of the Earth

P30.44 (a) $B_h = B_{\text{coil}} = \dfrac{\mu_0 NI}{2R} = \dfrac{\left(4\pi\times10^{-7}\right)(5.00)(0.600)}{0.300} = \boxed{12.6\ \mu\text{T}}$

(b) $B_h = B\sin\phi \rightarrow B = \dfrac{B_h}{\sin\phi} = \dfrac{12.6\ \mu\text{T}}{\sin 13.0°} = \boxed{56.0\ \mu\text{T}}$

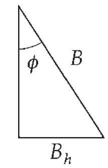


FIG. P30.44

P30.45 (a) Number of unpaired electrons $= \dfrac{8.00\times10^{22}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{8.63\times10^{45}}$.

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\dfrac{1}{2}\left(8.63\times10^{45}\right)$.

(b) $\text{Mass} = \dfrac{\left(4.31\times10^{45}\ \text{atoms}\right)\left(7\,900\ \text{kg/m}^3\right)}{\left(8.50\times10^{28}\ \text{atoms/m}^3\right)} = \boxed{4.01\times10^{20}\ \text{kg}}$

***P30.46** (a) Gravitational field and magnetic field are vectors; atmospheric pressure is a scalar.

(b) At 42° north latitude, 76° west longitude, 260 m above sea level: Gravitational field is 9.803 m/s^2 down. From the Coast and Geodetic Survey, the magnetic field is 54 μT at 12° west of geographic north and 69° below the horizontal. Atmospheric pressure is 98 kPa.

(c) The atmosphere is held on by gravitation, but otherwise the effects are all separate. The magnetic field could be produced by permanent magnetization of a cold iron-nickel deposit within the Earth, so it need not be associated with present-day action of gravitation.

Additional Problems

P30.47 Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current dI in the element dr is

$$dB = \frac{\mu_0 dI}{2\pi r}$$

where $$dI = I\left(\frac{dr}{w}\right)$$

$$\vec{\mathbf{B}} = \int d\vec{\mathbf{B}} = \int_b^{b+w} \frac{\mu_0 I dr}{2\pi wr}\hat{\mathbf{k}} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)\hat{\mathbf{k}}}$$

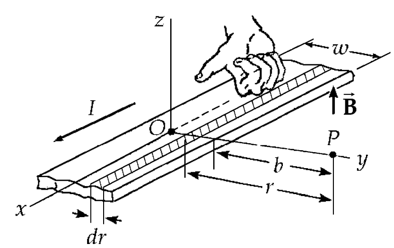


FIG. P30.47

P30.48 $B = \dfrac{\mu_0 IR^2}{2\left(R^2+R^2\right)^{3/2}} = \dfrac{\mu_0 I}{2^{5/2}R}$ $\qquad I = \dfrac{2^{5/2}BR}{\mu_0} = \dfrac{2^{5/2}\left(7.00\times10^{-5}\ \text{T}\right)\left(6.37\times10^6\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}$

so $\boxed{I = 2.01\times10^9\ \text{A}}$ toward the west

P30.49 At a point at distance x from the left end of the bar, current I_2 creates magnetic field $\vec{\mathbf{B}} = \dfrac{\mu_0 I_2}{2\pi\sqrt{h^2+x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta = \dfrac{x}{h}$. This field exerts force on an element of the rod of length dx

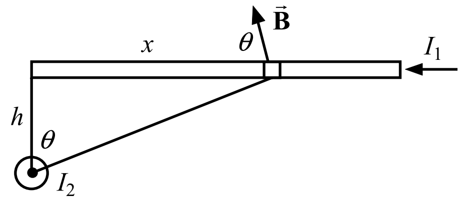


FIG. P30.49

$$d\vec{\mathbf{F}} = I_1\vec{\ell}\times\vec{\mathbf{B}} = I_1\frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2+x^2}}\underset{\text{right hand rule}}{\sin\theta}$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2+x^2}}\frac{x}{\sqrt{h^2+x^2}} \text{ into the page}$$

$$d\vec{\mathbf{F}} = \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2+x^2\right)}\left(-\hat{\mathbf{k}}\right)$$

The whole force is the sum of the forces on all of the elements of the bar:

$$\vec{\mathbf{F}} = \int_{x=0}^{\ell}\frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2+x^2\right)}\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\int_0^{\ell}\frac{2x dx}{h^2+x^2} = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\ln\left(h^2+x^2\right)\Big|_0^{\ell}$$

$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\left[\ln\left(h^2+\ell^2\right)-\ln h^2\right] = \frac{10^{-7}\text{ N}(100\text{ A})(200\text{ A})\left(-\hat{\mathbf{k}}\right)}{\text{A}^2}\ln\left[\frac{(0.5\text{ cm})^2+(10\text{ cm})^2}{(0.5\text{ cm})^2}\right]$$

$$= 2\times10^{-3}\text{ N}\left(-\hat{\mathbf{k}}\right)\ln 401 = \boxed{1.20\times10^{-2}\text{ N}\left(-\hat{\mathbf{k}}\right)}$$

P30.50 Suppose you have two 100-W headlights running from a 12-V battery, with the whole $\dfrac{200\text{ W}}{12\text{ V}} = 17\text{ A}$ current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \approx \mu_0$. Model the current as straight. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\right)17}{2\pi(0.6)}\boxed{\sim10^{-5}\text{ T}}$$

If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim10^{-1}\text{ times as large,}}$ enough to affect the compass noticeably.

P30.51 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a magnitude of

$$B = \frac{\mu_0\omega R^2 q}{4\pi\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0(20.0)(0.100)^2\left(10.0\times10^{-6}\right)}{4\pi\left[(0.050\,0)^2+(0.100)^2\right]^{3/2}} = \boxed{1.43\times10^{-10}\text{ T}}$$

P30.52 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. Therefore, $B = \dfrac{\mu_0\omega R^2 q}{4\pi\left(x^2+R^2\right)^{3/2}}$

when $x = \dfrac{R}{2}$ then $B = \dfrac{\mu_0\omega R^2 q}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\dfrac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}}$

P30.53 (a) Use twice the equation for the field created by a current loop

$$B_x = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

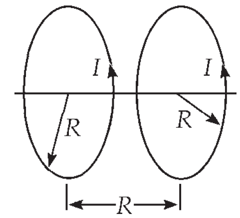


FIG. P30.53

If each coil has N turns, the field is just N times larger.

$$B = B_{x1} + B_{x2} = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left[(R-x)^2 + R^2\right]^{3/2}}\right]$$

$$\boxed{B = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left(2R^2 + x^2 - 2xR\right)^{3/2}}\right]}$$

(b) $$\frac{dB}{dx} = \frac{N\mu_0 I R^2}{2}\left[-\frac{3}{2}(2x)\left(x^2+R^2\right)^{-5/2} - \frac{3}{2}\left(2R^2+x^2-2xR\right)^{-5/2}(2x-2R)\right]$$

Substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{dB}{dx} = 0}$.

$$\frac{d^2B}{dx^2} = \frac{-3N\mu_0 I R^2}{2}\left[\left(x^2+R^2\right)^{-5/2} - 5x^2\left(x^2+R^2\right)^{-7/2} + \left(2R^2+x^2-2xR\right)^{-5/2} - 5(x-R)^2\left(2R^2+x^2-2xR\right)^{-7/2}\right]$$

Again substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{d^2B}{dx^2} = 0}$.

P30.54 "Helmholtz pair" $\rightarrow$ separation distance = radius

$$B = \frac{2\mu_0 I R^2}{2\left[(R/2)^2 + R^2\right]^{3/2}} = \frac{\mu_0 I R^2}{\left[\frac{1}{4}+1\right]^{3/2} R^3} = \frac{\mu_0 I}{1.40R} \text{ for 1 turn}$$

For N turns in each coil, $B = \frac{\mu_0 N I}{1.40R} = \frac{\left(4\pi\times10^{-7}\right)100(10.0)}{1.40(0.500)} = \boxed{1.80\times10^{-3}\text{ T}}$

P30.55 Consider first a solid cylindrical rod of radius R carrying current toward you, uniformly distributed over its cross-sectional area. To find the field at distance r from its center we consider a circular loop of radius r:

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{inside}}$$

$$B2\pi r = \mu_0 \pi r^2 J \qquad B = \frac{\mu_0 J r}{2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 \mathbf{J}}{2}\hat{\mathbf{k}}\times\mathbf{r}$$

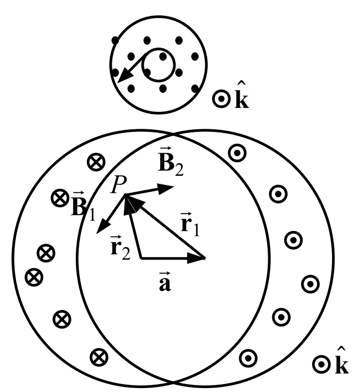


FIG. P30.55

Now the total field at P inside the saddle coils is the field due to a solid rod carrying current toward you, centered at the head of vector $\vec{\mathbf{a}}$, plus the field of a solid rod centered at the tail of vector $\vec{\mathbf{a}}$ carrying current away from you.

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\vec{\mathbf{r}}_1 + \frac{\mu_0 J}{2}\left(-\hat{\mathbf{k}}\right)\times\vec{\mathbf{r}}_2$$

Now note $\vec{\mathbf{a}} + \vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_2$

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\vec{\mathbf{r}}_1 - \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\left(\vec{\mathbf{a}}+\vec{\mathbf{r}}_1\right) = \frac{\mu_0 J}{2}\vec{\mathbf{a}}\times\hat{\mathbf{k}} = \boxed{\frac{\mu_0 J a}{2}\text{ down in the diagram}}$$

P30.56 From Problem 33, the upper sheet creates field

$\vec{\mathbf{B}} = \frac{\mu_0 J_s}{2}\hat{\mathbf{k}}$ above it and $\frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$ below it.

Consider a patch of the sheet of width w parallel to the z axis and length d parallel to the x axis. The charge on it σwd passes a point in time $\frac{d}{v}$, so

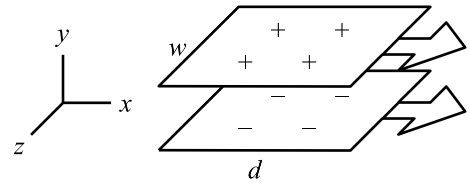


FIG. P30.56

the current it constitutes is $\frac{q}{t} = \frac{\sigma wdv}{d}$ and the linear current density is $J_s = \frac{\sigma wv}{w} = \sigma v$. Then the magnitude of the magnetic field created by the upper sheet is $\frac{1}{2}\mu_0 \sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\frac{1}{2}\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right)$ above it and $\frac{1}{2}\mu_0 \sigma v \hat{\mathbf{k}}$ below it. We choose to write down answer (b) first.

(b) Above both sheets and below both, their equal-magnitude fields add to $\boxed{\text{zero}}$.

(a) Between the plates, their fields add to

$$\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right) = \boxed{\mu_0 \sigma v \text{ away from you horizontally}}.$$

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right)$ will exert a force on the current in the w- by d-section, given by

$$I\vec{\ell}\times\vec{\mathbf{B}} = \sigma wvd\hat{\mathbf{i}} \times \frac{1}{2}\mu_0 \sigma v\left(-\hat{\mathbf{k}}\right) = \frac{1}{2}\mu_0 \sigma^2 v^2 wd\hat{\mathbf{j}}$$

The force per area is $\frac{1}{2}\frac{\mu_0 \sigma^2 v^2 wd}{wd}\hat{\mathbf{j}} = \boxed{\frac{1}{2}\mu_0 \sigma^2 v^2 \text{ up}}$.

(d) The electrical force on our section of the upper plate is $q\vec{\mathbf{E}}_{\text{lower}} = \sigma \ell w \frac{\sigma}{2\epsilon_0}\left(-\hat{\mathbf{j}}\right) = \frac{\ell w \sigma^2}{2\epsilon_0}\left(-\hat{\mathbf{j}}\right)$.

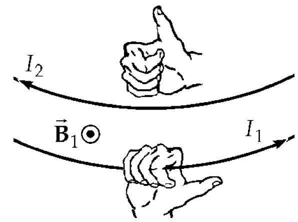
The electrical force per area is $\frac{\ell w \sigma^2}{2\epsilon_0 \ell w}$ down $= \frac{\sigma^2}{2\epsilon_0}$ down. To have $\frac{1}{2}\mu_0 \sigma^2 v^2 = \frac{\sigma^2}{2\epsilon_0}$ we require

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\,(\text{Tm/A})(\text{N/TAm})\,8.85\times10^{-12}\,(\text{C}^2/\text{Nm}^2)(\text{As/C})^2}}$$

$$= \boxed{3.00\times10^8 \text{ m/s}}$$

This is the speed of light, not a possible speed for a metal plate.

***P30.57** Model the two wires as straight parallel wires (!). From the treatment of this situation in the chapter text we have

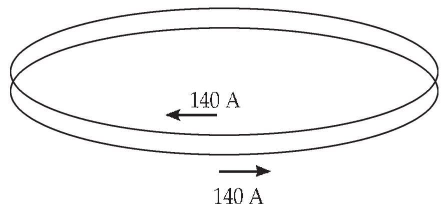


(a) $$F_B = \frac{\mu_0 I^2 \ell}{2\pi a}$$

$$F_B = \frac{\left(4\pi \times 10^{-7}\right)(140)^2 (2\pi)(0.100)}{2\pi\left(1.00 \times 10^{-3}\right)}$$

$$= \boxed{2.46\ \text{N}}\ \text{upward}$$

(b) The magnetic field at the center of the loop or on its axis is much weaker than the magnetic field just outside the wire. The wire has negligible curvature on the scale of 1 mm, so we model the lower loop as a long straight wire to find the field it creates at the location of the upper wire.



(c) $$a_{\text{loop}} = \frac{2.46\ \text{N} - m_{\text{loop}} g}{m_{\text{loop}}} = \boxed{107\ \text{m/s}^2}\ \text{upward}$$

FIG. P30.57

P30.58 (a) In $d\vec{\mathbf{B}} = \dfrac{\mu_0}{4\pi r^2} I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\vec{\mathbf{s}}$ is the direction of $\vec{\mathbf{v}}$, so $d\vec{\mathbf{B}} = \dfrac{\mu_0}{4\pi r^2} nqA(ds)\vec{\mathbf{v}} \times \hat{\mathbf{r}}$. Next, $A(ds)$ is the volume occupied by the moving charge, and $nA(ds) = 1$ for just one charge. Then,

$$\boxed{\vec{\mathbf{B}} = \frac{\mu_0}{4\pi r^2} q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}$$

(b) $$B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{7}\ \text{m/s}\right)}{4\pi\left(1.00 \times 10^{-3}\right)^2} \sin 90.0° = \boxed{3.20 \times 10^{-13}\ \text{T}}$$

(c) $$F_B = q\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{7}\ \text{m/s}\right)\left(3.20 \times 10^{-13}\ \text{T}\right)\sin 90.0°$$

$$F_B = \boxed{1.02 \times 10^{-24}\ \text{N directed away from the first proton}}$$

(d) $$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(1.00 \times 10^{-3}\right)^2}$$

$$F_e = \boxed{2.30 \times 10^{-22}\ \text{N directed away from the first proton}}$$

Both forces act together. The electrical force is stronger by two orders of magnitude. It is productive to think about how it would look to an observer in a reference frame moving along with one proton or the other.

P30.59 (a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(24.0\ \text{A})}{2\pi(0.017\ 5\ \text{m})} = \boxed{2.74 \times 10^{-4}\ \text{T}}$

(b) At point C, conductor AB produces a field $\dfrac{1}{2}(2.74 \times 10^{-4}\ \text{T})(-\hat{\mathbf{j}})$, conductor DE produces a field of $\dfrac{1}{2}(2.74 \times 10^{-4}\ \text{T})(-\hat{\mathbf{j}})$, BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74 \times 10^{-4}\ \text{T}(-\hat{\mathbf{j}})}$.

(c) $\vec{\mathbf{F}}_B = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = (24.0\ \text{A})(0.035\ 0\ \text{m}\hat{\mathbf{k}}) \times \left[5(2.74 \times 10^{-4}\ \text{T})(-\hat{\mathbf{j}})\right] = \boxed{(1.15 \times 10^{-3}\ \text{N})\hat{\mathbf{i}}}$

(d) $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m} = \dfrac{(1.15 \times 10^{-3}\ \text{N})\hat{\mathbf{i}}}{3.0 \times 10^{-3}\ \text{kg}} = \boxed{(0.384\ \text{m/s}^2)\hat{\mathbf{i}}}$

(e) The bar is already so far from AE that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

(f) $v_f^2 = v_i^2 + 2ax = 0 + 2(0.384\ \text{m/s}^2)(1.30\ \text{m})$, so $\vec{\mathbf{v}}_f = \boxed{(0.999\ \text{m/s})\hat{\mathbf{i}}}$

P30.60 Each turn creates field at the center $\dfrac{\mu_0 I}{2R}$. Together they create field

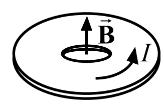


FIG. P30.60

$$\frac{\mu_0 I}{2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{50}}\right) = \frac{4\pi \times 10^{-7}\ \text{Tm}I}{2\ \text{A}}\left(\frac{1}{5.05} + \frac{1}{5.15} + \cdots + \frac{1}{9.95}\right)\frac{1}{10^{-2}\ \text{m}}$$

$$= \mu_0 I(50/\text{m})6.93 = \boxed{347\mu_0 I/\text{m}}$$

P30.61 The central wire creates field $\vec{\mathbf{B}} = \dfrac{\mu_0 I_1}{2\pi R}$ counterclockwise. The curved portions of the loop feels no force since $\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = 0$ there. The straight portions both feel $I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$ forces to the right, amounting to $\vec{\mathbf{F}}_B = I_2 2L \dfrac{\mu_0 I_1}{2\pi R} = \boxed{\dfrac{\mu_0 I_1 I_2 L}{\pi R}\ \text{to the right}}$.

P30.62 (a) From an equation in the chapter text, the magnetic field produced by one loop at the center of the second loop is given by $B = \dfrac{\mu_0 I R^2}{2x^3} = \dfrac{\mu_0 I(\pi R^2)}{2\pi x^3} = \dfrac{\mu_0 \mu}{2\pi x^3}$ where the magnetic moment of either loop is $\mu = I(\pi R^2)$. Therefore,

$$|F_x| = \mu\frac{dB}{dx} = \mu\left(\frac{\mu_0 \mu}{2\pi}\right)\left(\frac{3}{x^4}\right) = \frac{3\mu_0(\pi R^2 I)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}.$$

(b) $|F_x| = \dfrac{3\pi}{2}\dfrac{\mu_0 I^2 R^4}{x^4} = \dfrac{3\pi}{2}\dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})^2(5.00 \times 10^{-3}\ \text{m})^4}{(5.00 \times 10^{-2}\ \text{m})^4} = \boxed{5.92 \times 10^{-8}\ \text{N}}$

P30.63 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $\frac{L}{2}$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz-plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by the integral form of the Biot-Savart law as

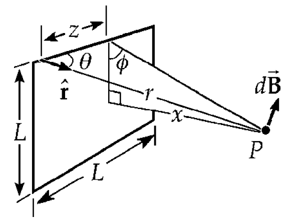


FIG. P30.63

$$\vec{\mathbf{B}}_0 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}}{r^2}$$

From the figure we see that

$$r = \sqrt{x^2 + \left(L^2/4\right) + z^2} \text{ and } \left|d\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}\right| = dz\sin\theta = dz\sqrt{\frac{\left(L^2/4\right)+x^2}{\left(L^2/4\right)+x^2+z^2}}$$

By symmetry all components of the field $\vec{\mathbf{B}}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$$B_{0x} = B_0\cos\phi \quad \text{where} \quad \cos\phi = \frac{L/2}{\sqrt{\left(L^2/4\right)+x^2}}$$

Therefore, $\vec{\mathbf{B}}_{0x} = \frac{\mu_0 I}{4\pi}\int_0^{L/2}\frac{\sin\theta\cos\phi\, dz}{r^2}$ and $B = 8B_{0x}$

Using the expressions given above for $\sin\theta\cos\phi$, and r, we find

$$B = \frac{\mu_0 I L^2}{2\pi\left(x^2+\left(L^2/4\right)\right)\sqrt{x^2+\left(L^2/2\right)}}$$

P30.64 There is no contribution from the straight portion of the wire since $d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = 0$. For the field of the spiral,

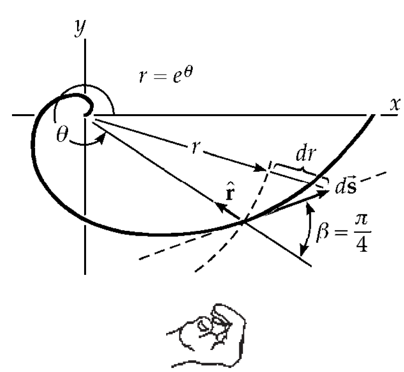


FIG. P30.64

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{(4\pi)}\frac{\left(d\vec{\mathbf{s}}\times\hat{\mathbf{r}}\right)}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{\left|d\vec{\mathbf{s}}\right|\sin\theta\left|\hat{\mathbf{r}}\right|}{r^2} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\left(\sqrt{2}dr\right)\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}dr = -\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\theta}$$

Substitute $r = e^{\theta}$: $B = -\frac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi} = -\frac{\mu_0 I}{4\pi}\left[e^{-2\pi} - e^0\right] =$

$\boxed{\frac{\mu_0 I}{4\pi}\left(1-e^{-2\pi}\right)}$ out of the page.

P30.65 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The contribution to the field from each ring is

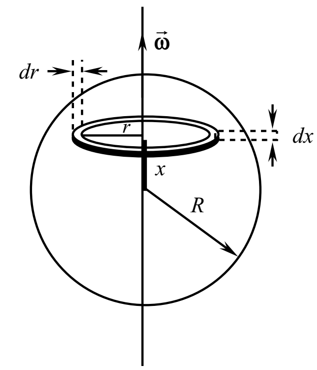


FIG. P30.65

$$dB = \frac{\mu_0 r^2 dI}{2\left(x^2+r^2\right)^{3/2}} \quad \text{where} \quad dI = \frac{dQ}{t} = \frac{\omega dQ}{2\pi}$$

$$dQ = \rho dV = \rho(2\pi r dr)(dx)$$

$$dB = \frac{\mu_0 \rho \omega r^3 dr dx}{2\left(x^2+r^2\right)^{3/2}} \quad \text{where} \quad \rho = \frac{Q}{(4/3)\pi R^3}$$

$$B = \int_{x=-R}^{+R} \int_{r=0}^{\sqrt{R^2-x^2}} \frac{\mu_0 \rho \omega}{2} \frac{r^3 dr dx}{\left(x^2+r^2\right)^{3/2}}$$

Let $v = r^2 + x^2$, $dv = 2r dr$, and $r^2 = v - x^2$.

$$B = \int_{x=-R}^{+R} \int_{v=x^2}^{R^2} \frac{\mu_0 \rho \omega}{2} \frac{\left(v-x^2\right) dv}{2 v^{3/2}} dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[\int_{v=x^2}^{R^2} v^{-1/2} dv - x^2 \int_{v=x^2}^{R^2} v^{-3/2} dv \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2 v^{1/2}\Big|_{x^2}^{R^2} + \left(2x^2\right) v^{-1/2}\Big|_{x^2}^{R^2} \right] dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2\left(R - |x|\right) + 2x^2 \left(\frac{1}{R} - \frac{1}{|x|} \right) \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{-R}^{R} \left[2\frac{x^2}{R} - 4|x| + 2R \right] dx = \frac{2\mu_0 \rho \omega}{4} \int_{0}^{R} \left[2\frac{x^2}{R} - 4x + 2R \right] dx$$

$$B = \frac{2\mu_0 \rho \omega}{4} \left(\frac{2R^3}{3R} - \frac{4R^2}{2} + 2R^2 \right) = \boxed{\frac{\mu_0 \rho \omega R^2}{3}}$$

P30.66 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The current associated with each rotating ring of charge is

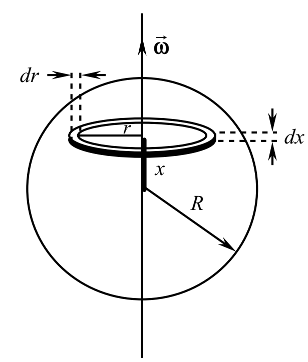


FIG. P30.66

$$dI = \frac{dQ}{t} = \frac{\omega}{2\pi}\left[\rho(2\pi r dr)(dx)\right]$$

The magnetic moment contributed by this ring is

$$d\mu = A(dI) = \pi r^2 \frac{\omega}{2\pi}\left[\rho(2\pi r dr)(dx)\right] = \pi \omega \rho r^3 dr dx$$

$$\mu = \pi \omega \rho \int_{x=-R}^{+R} \left[\int_{r=0}^{\sqrt{R^2-x^2}} r^3 dr \right] dx = \pi \omega \rho \int_{x=-R}^{+R} \frac{\left(\sqrt{R^2-x^2}\right)^4}{4} dx = \pi \omega \rho \int_{x=-R}^{+R} \frac{\left(R^2-x^2\right)^2}{4} dx$$

$$\mu = \frac{\pi \omega \rho}{4} \int_{x=-R}^{+R} \left(R^4 - 2R^2 x^2 + x^4\right) dx = \frac{\pi \omega \rho}{4} \left[R^4 (2R) - 2R^2 \left(\frac{2R^3}{3} \right) + \frac{2R^5}{5} \right]$$

$$\mu = \frac{\pi \omega \rho}{4} R^5 \left(2 - \frac{4}{3} + \frac{2}{5} \right) = \frac{\pi \omega \rho R^5}{4} \left(\frac{16}{15} \right) = \boxed{\frac{4\pi \omega \rho R^5}{15}} \text{ up}$$

P30. 67 Note that the current I exists in the conductor with a current density $J=\frac{I}{A}$, where

$$A=\pi\left[a^2-\frac{a^2}{4}-\frac{a^2}{4}\right]=\frac{\pi a^2}{2}$$

Therefore $J=\frac{2I}{\pi a^2}$.

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $\frac{a}{2}$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

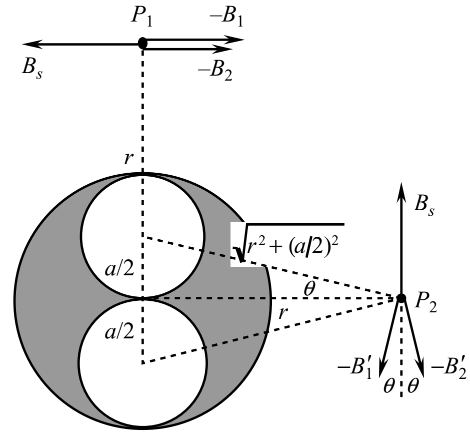


FIG. P30.67

(a) At point P_1, $B_s=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$, $B_1=\frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r-(a/2)\right)}$, and $B_2=\frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r+(a/2)\right)}$.

$$B=B_s-B_1-B_2=\frac{\mu J\pi a^2}{2\pi}\left[\frac{1}{r}-\frac{1}{4\left(r-(a/2)\right)}-\frac{1}{4\left(r+(a/2)\right)}\right]$$

$$B=\frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2-a^2-2r^2}{4r\left(r^2-\left(a^2/4\right)\right)}\right]=\boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2-a^2}{4r^2-a^2}\right]\text{ directed to the left}}$$

(b) At point P_2, $B_s=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$ and $B_1'=B_2'=\frac{\mu_0 J\,\pi(a/2)^2}{2\pi\sqrt{r^2+(a/2)^2}}$.

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B=B_s-B_1'\cos\theta-B_2'\cos\theta=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}-2\left(\frac{\mu_0 J\,\pi a^2/4}{2\pi\sqrt{r^2+\left(a^2/4\right)}}\right)\frac{r}{\sqrt{r^2+\left(a^2/4\right)}}$$

$$B=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1-\frac{r^2}{2\left(r^2+\left(a^2/4\right)\right)}\right]=\frac{\mu_0(2I)}{2\pi r}\left[1-\frac{2r^2}{4r^2+a^2}\right]$$

$$=\boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]\text{ directed toward the top of the page}}$$

ANSWERS TO EVEN PROBLEMS

P30.2 200 nT

P30.4 $\left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}$ into the page

P30.6 See the solution.

P30.8 $\left(\frac{1}{\pi}+\frac{1}{4}\right)\frac{\mu_0 I}{2r}$ into the page

P30.10 262 nT into the page

P30.12 (a) 3.84×10^{-21} N up. See the solution. (b) 21.4 μm. This distance is negligible compared to 50 m, so the electron does move in a uniform field. (c) 134 revolutions.

P30.14 $\frac{\mu_0 I\left(a^2+d^2-d\sqrt{a^2+d^2}\right)}{2\pi ad\sqrt{a^2+d^2}}$ into the page

P30.16 4.97 mm

P30.18 (a) 10.0 μT out of the page (b) 80.0 μN toward wire 1 (c) 16.0 μT into the page

(d) 80.0 μN toward wire 2

P30.20 (a) It is possible in just one way. (b) wire 3 must be 12.0 cm to the left of wire 1, carrying (c) current 2.40 A down

P30.22 (a) 10.0 μm. (b) Yes. If we tried to use wires with diameter 10 μm, they would feel the force only momentarily after we turn on the current, until they melt. We can use wide, thin sheets of copper, perhaps plated onto glass, and perhaps with water cooling, to have the forces act continuously.

P30.24 5.40 cm

P30.26 (a) 400 cm (b) 7.50 nT (c) 1.26 m (d) zero

P30.28 (a) 3.60 T (b) 1.94 T

P30.30 500 A

P30.32 (a) See the solution. (b) zero; zero (c) $\vec{\mathbf{B}} = \frac{32\times10^{-7}\,z\ \mathrm{T\cdot m}}{9\times10^{-4}\ \mathrm{m}^2+z^2}\hat{\mathbf{j}}$ (d) at $d = 3.00$ cm (e) 53.3 $\hat{\mathbf{j}}$ μT

P30.34 (a) Make the wire as long and thin as possible without melting when it carries the 5-A current. Then the solenoid can have many turns. (b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference the wire can form a solenoid with more turns.

P30.36 (a) $\frac{\mu_0 IN}{2\ell}\left[\frac{a}{\sqrt{a^2+R^2}}-\frac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right]$ (b) See the solution.

P30.38 207 W

P30.40 (a) $-B\pi R^2\cos\theta$ (b) $B\pi R^2\cos\theta$

P30.42 The field can be uniform in magnitude. Gauss's law for magnetism implies that magnetic field lines never start or stop. If the field is uniform in direction, the lines are parallel and their density stays constant along any one bundle of lines. Thus the magnitude of the field has the same value at all points along a line in the direction of the field. The magnitude of the field could vary over a plane perpendicular to the lines, or it could be constant throughout the volume.

P30.44 (a) 12.6 μT (b) 56.0 μT

P30.46 (a) Gravitational field and magnetic field are vectors; atmospheric pressure is a scalar. (b) At 42° north latitude, 76° west longitude, 260 m above sea level: Gravitational field is 9.803 m/s^2 down. From the Coast and Geodetic Survey, the magnetic field is 54 μT at 12° west of geographic north and 69° below the horizontal. Atmospheric pressure is 98 kPa. The atmosphere is held on by gravitation, but otherwise the effects are all separate. The magnetic field could be produced by permanent magnetization of a cold iron-nickel deposit within the Earth, so it need not be associated with present-day action of gravitation.

P30.48 2.01 GA west

P30.50 $\sim 10^{-5}\ \text{T}$, enough to affect the compass noticeably

P30.52 $\dfrac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}$

P30.54 1.80 mT

P30.56 (a) $\mu_0 \sigma v$ horizontally away from you (b) 0 (c) $\dfrac{1}{2}\mu_0\sigma^2 v^2$ up (d) $3.00\times 10^8 \text{ m/s}$

P30.58 (a) See the solution. (b) $3.20\times 10^{-13}\ \text{T}$ (c) $1.02\times 10^{-24}\ \text{N}$ away from the first proton (d) $2.30\times 10^{-22}\ \text{N}$ away from the first proton

P30.60 $347\mu_0 I/\text{m}$ perpendicular to the coil

P30.62 (a) See the solution. (b) $59.2\ \text{nN}$

P30.64 $(\mu_0 I/4\pi)(1 - e^{-2\pi})$ out of the plane of the paper

P30.66 $\dfrac{4}{15}\pi\omega\rho R^5$ upward