# 36

# Image Formation

**CHAPTER OUTLINE**



## ANSWERS TO QUESTIONS

**Q36.1** With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving or makeup mirror as an example.

**Q36.2** With a convex spherical mirror, all images of real objects are upright, virtual and smaller than the object. As seen in Question 36.1, you only get a change of orientation when you pass the focal point—but the focal point of a convex mirror is on the non-reflecting side!

***Q36.3** (i) When we flatten a curved mirror we move its center of curvature out to infinity. The focal length is still half the radius of curvature and is infinite. Answer (d).

(ii) The image is actual size and right side up. The magnification is 1. Answer (b).

**Q36.4** The mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=0$ we have $\frac{1}{p}=-\frac{1}{q}$; therefore, $p=-q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M=-\frac{q}{p}=\frac{p}{p}=1$. The image is right side up and actual size.

***Q36.5** (i) Answer (c). (ii) Answer (c). When the object is at the focal point the image can be thought of as a right side up image behind the mirror at infinity, or as an inverted image in front of the mirror.

**Q36.6** In the diagram, only two of the three principal rays have been used to locate images to reduce the amount of visual clutter. The upright shaded arrows are the objects, and the correspondingly numbered inverted arrows are the images. As you can see, object 2 is closer to the focal point than object 1, and image 2 is farther to the left than image 1.

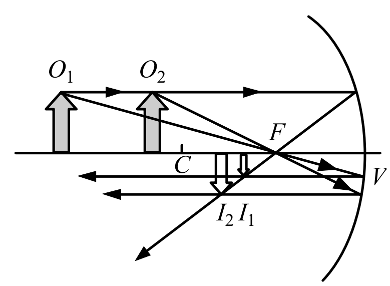


FIG. Q36.6

***Q36.7** (i) (a) positive (b) negative (c) negative (d) negative (e) positive (f) positive (g) positive

(ii) (a) positive (b) positive (c) positive (d) positive (e) negative (f) negative

(iii) (a) positive (b) negative (c) positive (d) negative (e) negative (f) positive

***Q36.8** Answer (c). The angle of refraction for the light coming from fish to person is 60°. The angle of incidence is smaller, so the fish is deeper than it appears.

***Q36.9** The ranking is e > d > g > a > b > f > c. In case e, the object is at infinite distance. In d the object distance is very large but not infinite. In g the object distance is several times the focal length. In a, the object distance is a little larger than the focal length. In b the object distance is very slightly larger than the focal length. In f it is equal to the focal length. In c the object distance is less than the focal length.

**Q36.10** An infinite number. In general, an infinite number of rays leave each point of any object and travel in all directions. Note that the three principal rays that we use for imaging are just a subset of the infinite number of rays. All three principal rays can be drawn in a ray diagram, provided that we extend the plane of the lens as shown in Figure Q36.10.

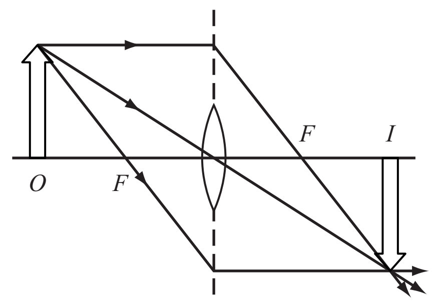


**FIG. Q36.10**

***Q36.11** Answer (d). The entire image is visible, but only at half the intensity. Each point on the object is a source of rays that travel in all directions. Thus, light from all parts of the object goes through all unblocked parts of the lens and forms an image. If you block part of the lens, you are blocking some of the rays, but the remaining ones still come from all parts of the object.

***Q36.12** Answer (e). If the object distance is $2f$ the image distance is also $2f$ and the distance between object and real image is minimal.

***Q36.13** The focal point is defined as the location of the image formed by rays originally parallel to the axis. An object at a large but finite distance will radiate rays nearly but not exactly parallel. Infinite object distance describes the definite limiting case in which these rays become parallel. To measure the focal length of a converging lens, set it up to form an image of the farthest object you can see outside a window. The image distance will be equal to the focal length within one percent or better if the object distance is a hundred times larger or more.

***Q36.14** Use a converging lens as the projection lens in a slide projector. Place the brightly illuminated slide slightly farther than its focal length away from it, so that the lens will produce a real, inverted, enlarged image on the screen.

***Q36.15** Answer (e). The water drop functions as a lens of short focal length, forming a real image of the distant object in space, outside the drop on the side where the light exits the drop. The camera lens is focused on the real image.

**Q36.16** Chromatic aberration arises because a material medium's refractive index can be frequency dependent. A mirror changes the direction of light by reflection, not refraction. Light of all wavelengths follows the same path according to the law of reflection, so no chromatic aberration happens.

**Q36.17** If the converging lens is immersed in a liquid with an index of refraction significantly greater than that of the lens itself, it will make light from a distant source diverge. This is not the case with a converging (concave) mirror, as the law of reflection has nothing to do with the indices of refraction.

**Q36.18** As in the diagram, let the center of curvature $C$ of the fishbowl and the bottom of the fish define the optical axis, intersecting the fishbowl at vertex $V$. A ray from the top of the fish that reaches the bowl surface along a radial line through $C$ has angle of incidence zero and angle of refraction zero. This ray exits from the bowl unchanged in direction. A ray from the top of the fish to $V$ is refracted to bend away from the normal. Its extension back inside the fishbowl determines the location of the image and the characteristics of the image. The image is upright, virtual, and enlarged.

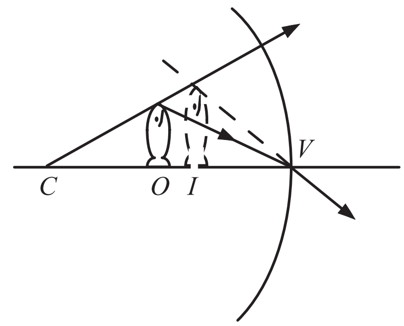


**FIG. Q36.18**

**Q36.19** Because when you look at the ƎƆИA⅃UᗺMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you. Do not jam on your brakes when a MIAMI city bus is right behind you.

**Q36.20** With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.

**Q36.21** Answer (b). The outer surface should be flat so that it will not produce a fuzzy or distorted image for the diver when the mask is used either in air or in water.

**Q36.22** The eyeglasses on the left are diverging lenses that correct for nearsightedness. If you look carefully at the edge of the person's face through the lens, you will see that everything viewed through these glasses is reduced in size. The eyeglasses on the right are converging lenses, which correct for farsightedness. These lenses make everything that is viewed through them look larger.

**Q36.23** The eyeglass wearer's eye is at an object distance from the lens that is quite small—the eye is on the order of $10^{-2}$ meter from the lens. The focal length of an eyeglass lens is several decimeters, positive or negative. Therefore the image distance will be similar in magnitude to the object distance. The onlooker sees a sharp image of the eye behind the lens. Look closely at the left side of Figure Q36.22 and notice that the wearer's eyes seem not only to be smaller, but also positioned a bit behind the plane of his face—namely where they would be if he were not wearing glasses. Similarly, in the right half of Figure Q36.22, his eyes seem to be in front of the plane of his face and magnified. We as observers take this light information coming from the object through the lens and perceive or photograph the image as if it were an object.

**Q36.24** Absolutely. Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold and solid as megajoules of light energy pass through it.

**Q36.25** Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).

**Q36.26** The artist's statements are accurate, perceptive, and eloquent. The image you see is "almost one's whole surroundings," including things behind you and things farther in front of you than the globe is, but nothing eclipsed by the opaque globe or by your head. For example, we cannot see Escher's index and middle fingers or their reflections in the globe.

The point halfway between your eyes is indeed the focus in a figurative sense, but it is not an optical focus. The principal axis will always lie in a line that runs through the center of the sphere and the bridge of your nose. Outside the globe, you are at the center of your observable universe. If you wink at the ball, the center of the looking-glass world hops over to the location of the image of your open eye.

**Q36.27** You have likely seen a Fresnel mirror for sound. The diagram represents first a side view of a band shell. It is a concave mirror for sound, designed to channel sound into a beam toward the audience in front of the band shell. Sections of its surface can be kept at the right orientations as they are pushed around inside a rectangular box to form an auditorium with good diffusion of sound from stage to audience, with a floor plan suggested by the second part of the diagram.

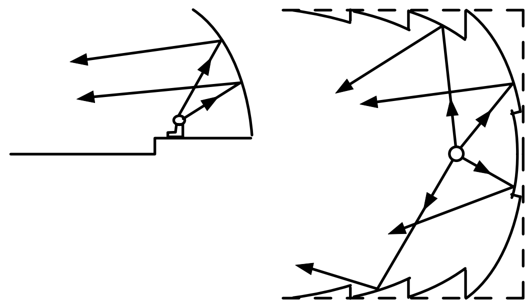

FIG. Q36.27

## SOLUTIONS TO PROBLEMS

### Section 36.1 Images Formed by Flat Mirrors

**P36.1** I stand 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time

$$\frac{0.8\text{ m}}{3\times10^{8}\text{ m/s}}\boxed{\sim10^{-9}\text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

**P36.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is $0.800\text{ m}+5.30\text{ m}=6.10\text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600\text{ m}}=\frac{6.10\text{ m}}{0.800\text{ m}}$$

or $$h'=(0.600\text{ m})\left(\frac{6.10\text{ m}}{0.800\text{ m}}\right)=\boxed{4.58\text{ m}}$$

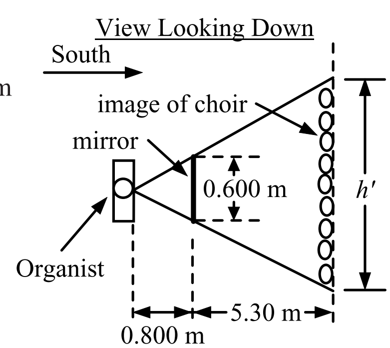


FIG. P36.2

**P36.3** The flatness of the mirror is described

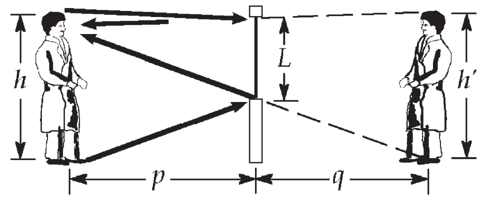


**FIG. P36.3**

by $R = \infty$, $f = \infty$

and $\frac{1}{f} = 0$

By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0$$

or $q = -p$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ inches

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}$$

Thus, the mirror must be $\boxed{\text{at least 35.0 inches high}}$.

**P36.4** (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft}}$ from the person.

**P36.5** (a) The flat mirrors have

$$R \to \infty$$

and $$f \to \infty$$

The upper mirror $\mathbf{M}_1$ produces a virtual, actual sized image $I_1$ according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{\infty} = 0$$

$$q_1 = -p_1$$

with $$M_1 = -\frac{q_1}{p_1} = +1$$

As shown, this image is above the upper mirror. It is the object for mirror $\mathrm{M}_2$, at object distance

$$p_2 = p_1 + h$$

The lower mirror produces a virtual, actual-size, right-side-up image according to

$$\frac{1}{p_2} + \frac{1}{q_2} = 0$$

$$q_2 = -p_2 = -(p_1 + h)$$

with $M_2 = -\dfrac{q_2}{p_2} = +1$ and $M_{\text{overall}} = M_1 M_2 = 1.$

Thus the final image is at distance $\boxed{p_1 + h}$ behind the lower mirror.

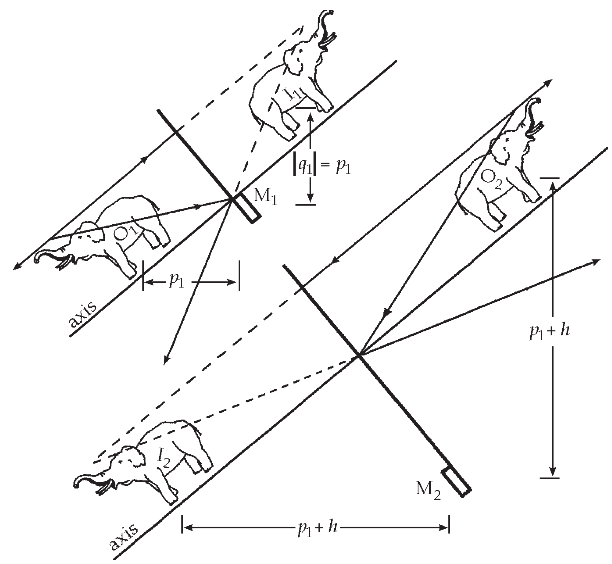


**FIG. P36.5**

(b) It is $\boxed{\text{virtual}}$.

(c) $\boxed{\text{Upright}}$

(d) With magnification $\boxed{+1}$.

(e) It $\boxed{\text{does not appear to be reversed}}$ left and right. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left.

## Section 36.2 **Images Formed by Spherical Mirrors**

**P36.6** For a concave mirror, both $R$ and $f$ are positive.

We also know that $$f = \frac{R}{2} = 10.0 \text{ cm}$$

(a) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{3}{40.0 \text{ cm}}$$

and $$\boxed{q = 13.3 \text{ cm}}$$

$$M = \frac{q}{p} = -\frac{13.3 \text{ cm}}{40.0 \text{ cm}} = \boxed{-0.333}$$

The image is 13.3 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(b) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{1}{20.0 \text{ cm}}$$

and $$\boxed{q = 20.0 \text{ cm}}$$

$$M = \frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = \boxed{-1.00}$$

The image is 20.0 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(c) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = 0$$

Thus, $q = \text{infinity}$

$\boxed{\text{No image is formed}}$. The rays are reflected parallel to each other.

**P36.7** (a) $$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \quad \text{gives} \quad \frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$$

$$\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}} = -0.083\,3 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-12.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{30.0 \text{ cm}} = \boxed{0.400}$$

(b) $$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \quad \text{gives} \quad \frac{1}{60.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$$

$$\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{60.0 \text{ cm}} = -0.066\,6 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-15.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-15.0 \text{ cm})}{60.0 \text{ cm}} = \boxed{0.250}$$

(c) Since $M > 0$, the images are $\boxed{\text{upright}}$.

**P36.8** $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=-\frac{1}{0.275\text{ m}}-\frac{1}{10.0\text{ m}}$ gives $\boxed{q=-0.267\text{ m}}$

Thus, the image is $\boxed{\text{virtual}}$.

$$M=\frac{-q}{p}=-\frac{-0.267}{10.0\text{ m}}=\boxed{0.026\,7}$$

Thus, the image is $\boxed{\text{upright}}$ $(+M)$ and $\boxed{\text{diminished}}$ $(|M|<1)$

**P36.9** (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{90.0\text{ cm}}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=\boxed{-0.500}$

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{20.0\text{ cm}}$

$q=\boxed{-60.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$

(c) The image (a) is real, inverted, and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figures 36.13(a) and 36.13(b) in the text, respectively.

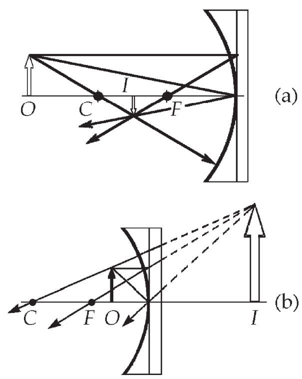

FIG. P36.9

**P36.10** With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f=\frac{R}{2}=1.25\text{ m}$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance $q$ from the back of the niche, where

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\quad\text{so}\quad\frac{1}{2.00\text{ m}}+\frac{1}{q}=\frac{1}{1.25\text{ m}}$$

$$q=\boxed{3.33\text{ m}}$$

***P36.11** (a) Since the object is in front of the mirror, $p>0$. With the image behind the mirror, $q<0$. The mirror equation gives the radius of curvature as $\frac{2}{R}=\frac{1}{p}+\frac{1}{q}=\frac{1}{1.00\text{ cm}}-\frac{1}{10.0\text{ cm}}=\frac{10-1}{10.0\text{ cm}}$,

or $R=2\left(\frac{10.0\text{ cm}}{9}\right)=\boxed{+2.22\text{ cm}}$.

(b) The magnification is $M=-\frac{q}{p}=-\frac{(-10.0\text{ cm})}{1.00\text{ cm}}=\boxed{+10.0}$.

**P36.12** The ball is a convex mirror with $R = -4.25$ cm and $f = \frac{R}{2} = -2.125$ cm. We have

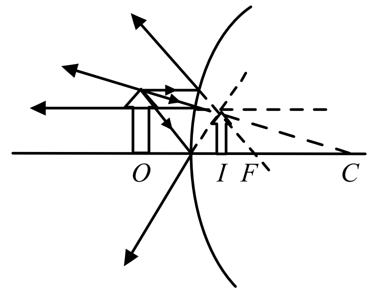


**FIG. P36.12**

$$M = \frac{3}{4} = -\frac{q}{p}$$

$$q = -\frac{3}{4}p$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-(3/4)p} = \frac{1}{-2.125 \text{ cm}}$$

$$\frac{3}{3p} - \frac{4}{3p} = \frac{1}{-2.125 \text{ cm}}$$

$$3p = 2.125 \text{ cm}$$

$\boxed{p = 0.708 \text{ cm}}$ in front of the sphere.

The image is upright, virtual, and diminished.

**P36.13** (a) $M = -4 = -\frac{q}{p}$ $\qquad q = 4p$

$q - p = 0.60 \text{ m} = 4p - p \qquad p = 0.2 \text{ m} \qquad q = 0.8 \text{ m}$

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.2 \text{ m}} + \frac{1}{0.8 \text{ m}} \qquad f = \boxed{160 \text{ mm}}$

(b) $M = +\frac{1}{2} = -\frac{q}{p}$ $\qquad p = -2q$

$|q| + p = 0.20 \text{ m} = -q + p = -q - 2q$

$q = -66.7 \text{ mm} \qquad p = 133 \text{ mm}$

$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{0.133 \text{ m}} + \frac{1}{-0.066\,7 \text{ m}} \qquad R = \boxed{-267 \text{ mm}}$

**P36.14** $M = -\frac{q}{p}$

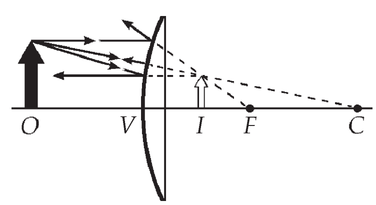


**FIG. P36.14**

$$q = -Mp = -0.013(30 \text{ cm}) = -0.39 \text{ cm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$$

$$\frac{1}{30 \text{ cm}} + \frac{1}{-0.39 \text{ cm}} = \frac{2}{R}$$

$$R = \frac{2}{-2.53 \text{ m}^{-1}} = -0.790 \text{ cm}$$

The cornea is convex, with radius of curvature $\boxed{0.790 \text{ cm}}$.

**P36.15** With

$$M=\frac{h'}{h}=\frac{+4.00\text{ cm}}{10.0\text{ cm}}=+0.400=-\frac{q}{p}$$

$$q=-0.400p$$

the image must be virtual.

(a) It is a $\boxed{\text{convex}}$ mirror that produces a diminished upright virtual image.

(b) We must have

$$p+|q|=42.0\text{ cm}=p-q$$

$$p=42.0\text{ cm}+q$$

$$p=42.0\text{ cm}-0.400p$$

$$p=\frac{42.0\text{ cm}}{1.40}=30.0\text{ cm}$$

The mirror is $\boxed{\text{at the 30.0 cm mark}}$.

(c) $$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{30\text{ cm}}+\frac{1}{-0.4(30\text{ cm})}=\frac{1}{f}=-0.050\ 0/\text{cm} \qquad \boxed{f=-20.0\text{ cm}}$$

The ray diagram looks like Figure 36.13(c) in the text.

**P36.16** Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image ($q=-10.0$ cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R=|R|$, $q=-30.0$ cm)

$$\frac{1}{p}-\frac{1}{30.0}=\frac{2}{|R|}$$

or

$$\frac{2}{|R|}=\frac{30.0\text{ cm}-p}{(30.0\text{ cm})p} \qquad (1)$$

(convex side: $R=-|R|$, $q=-10.0$ cm)

$$\frac{1}{p}-\frac{1}{10.0}=-\frac{2}{|R|}$$

or

$$\frac{2}{|R|}=\frac{p-10.0\text{ cm}}{(10.0\text{ cm})p} \qquad (2)$$

(a) Equating Equations (1) and (2) gives:

$$\frac{30.0\text{ cm}-p}{3.00}=p-10.0\text{ cm}$$

or

$$p=15.0\text{ cm}$$

Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

*continued on next page*

(b) Using the above result ($p = 15.0$ cm) in Equation (1) gives:

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})}$$

or $$\frac{2}{|R|} = \frac{1}{30.0\text{ cm}}$$

and $$|R| = 60.0\text{ cm}$$

The radius of the hubcap is $\boxed{60.0\text{ cm}}$.

**P36.17** (a) $q = (p + 5.00\text{ m})$ and, since the image must be real,

$$M = -\frac{q}{p} = -5 \qquad \text{or} \qquad q = 5p$$

Therefore, $\quad p + 5.00\text{ m} = 5p$

or $\quad p = 1.25\text{ m} \quad$ and $\quad q = 6.25\text{ m}$

From $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{2}{R}, \qquad R = \dfrac{2pq}{p+q} = \dfrac{2(1.25)(6.25)}{1.25 + 6.25}$

$$= \boxed{2.08\text{ m}(\text{concave})}$$

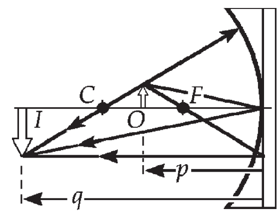


**FIG. P36.17**

(b) From part (a), $p = 1.25$ m; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

**P36.18** (a) The flat mirror produces an image according to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \qquad \frac{1}{24\text{ cm}} + \frac{1}{q} = \frac{1}{\infty} = 0 \qquad q = -24.0\text{ m}$$

The image is 24.0 m behind the mirror, distant from your eyes by

$$1.55\text{ m} + 24.0\text{ m} = \boxed{25.6\text{ m}}$$

(b) The image is the same size as the object, so $\quad \theta = \dfrac{h}{d} = \dfrac{1.50\text{ m}}{25.6\text{ m}} = \boxed{0.058\ 7\text{ rad}}$

(c) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{2}{R} \qquad \dfrac{1}{24\text{ m}} + \dfrac{1}{q} = \dfrac{2}{(-2\text{ m})} \qquad q = \dfrac{1}{-(1/1\text{ m}) - (1/24\text{ m})} = -0.960\text{ m}$

This image is distant from your eyes by $\quad 1.55\text{ m} + 0.960\text{ m} = \boxed{2.51\text{ m}}$

(d) The image size is given by $M = \dfrac{h'}{h} = -\dfrac{q}{p} \qquad h' = -h\dfrac{q}{p} = -1.50\text{ m}\left(\dfrac{-0.960\text{ m}}{24\text{ m}}\right)$

$$= 0.060\ 0\text{ m}.$$

So its angular size at your eye is $\quad \theta' = \dfrac{h'}{d} = \dfrac{0.06\text{ m}}{2.51\text{ m}} = \boxed{0.023\ 9\text{ rad}}$

(e) Your brain assumes that the car is 1.50 m high and calculates its distance as

$$d' = \frac{h}{\theta'} = \frac{1.50\text{ m}}{0.023\ 9} = \boxed{62.8\text{ m}}$$

**P36.19** (a) The image starts from a point whose height above the mirror vertex is given by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R} \qquad \frac{1}{3.00\text{ m}}+\frac{1}{q}=\frac{1}{0.500\text{ m}} \qquad \text{Therefore,} \qquad q=0.600\text{ m}$$

As the ball falls, $p$ decreases and $q$ increases. Ball and image pass when $q_1 = p_1$. When this is true,

$$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500\text{ m}}=\frac{2}{p_1} \qquad \text{or} \qquad p_1=1.00\text{ m}$$

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2 = q_2 = 0$.

(b) The falling ball passes its real image when it has fallen

$$3.00\text{ m}-1.00\text{ m}=2.00\text{ m}=\frac{1}{2}gt^2,\text{ or when } t=\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.639\text{ s}}$$

The ball reaches its virtual image when it has traversed

$$3.00\text{ m}-0=3.00\text{ m}=\frac{1}{2}gt^2,\text{ or at } t=\sqrt{\frac{2(3.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.782\text{ s}}$$

## Section 36.3 Images Formed by Refraction

**P36.20** When $R\to\infty$, the equation describing image formation at a single refracting surface becomes $q=-p\left(\frac{n_2}{n_1}\right)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate.

$$q_{B1}=-\left(\frac{1.33}{1.66}\right)(8.00\text{ cm})=-6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass of 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2}=-\left(\frac{1.00}{1.33}\right)(18.41\text{ cm})=-13.84\text{ cm} \qquad \text{or} \qquad 13.84\text{ cm below the water surface}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3=-\left(\frac{1}{1.33}\right)(12.0\text{ cm})=-9.02\text{ cm} \qquad \text{or} \qquad 9.02\text{ cm below the water surface}$$

Therefore, the apparent thickness of the glass is $\Delta t=13.84\text{ cm}-9.02\text{ cm}=\boxed{4.82\text{ cm}}$.

**P36.21** $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}=0$ and $R\to\infty$

$$q=-\frac{n_2}{n_1}p=-\frac{1}{1.309}(50.0\text{ cm})=-38.2\text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

**P36.22** $\dfrac{n_1}{p}+\dfrac{n_2}{q}=\dfrac{n_2-n_1}{R}$ becomes $\dfrac{1.00}{p}+\dfrac{1.50}{q}=\dfrac{1.50-1.00}{6.00\text{ cm}}=\dfrac{1}{12.0\text{ cm}}$

(a) $\dfrac{1.00}{20.0\text{ cm}}+\dfrac{1.50}{q}=\dfrac{1}{12.0\text{ cm}}$ or $q=\dfrac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/20.0\text{ cm})\right]}=\boxed{45.0\text{ cm}}$

(b) $\dfrac{1.00}{10.0\text{ cm}}+\dfrac{1.50}{q}=\dfrac{1}{12.0\text{ cm}}$ or $q=\dfrac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/10.0\text{ cm})\right]}=\boxed{-90.0\text{ cm}}$

(c) $\dfrac{1.00}{3.0\text{ cm}}+\dfrac{1.50}{q}=\dfrac{1}{12.0\text{ cm}}$ or $q=\dfrac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/3.0\text{ cm})\right]}=\boxed{-6.00\text{ cm}}$

**P36.23** From Equation 36.8 $\dfrac{n_1}{p}+\dfrac{n_2}{q}=\dfrac{n_2-n_1}{R}$

Solve for $q$ to find $q=\dfrac{n_2Rp}{p(n_2-n_1)-n_1R}$

In this case, $n_1=1.50,\ n_2=1.00,\ R=-15.0\text{ cm}$

and $p=10.0\text{ cm}$

So $q=\dfrac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00-1.50)-(1.50)(-15.0\text{ cm})}=-8.57\text{ cm}$

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$

***P36.24** In the right triangle lying between $O$ and the center of the curved surface, $\tan\theta_1=h/p$. In the right triangle lying between $I$ and the center of the surface, $\tan\theta_2=-h'/q$. We need the negative sign because the image height is counted as negative while the angle is not. We substitute into the given $n_1\tan\theta_1=n_2\tan\theta_2$ to obtain $n_1\,h/p=-n_2\,h'/q$. Then the magnification, defined by $M=h'/h$, is given by $M=h'/h=-n_1\,q/n_2\,p$.

***P36.25** (a) The center of curvature is on the object side, so the radius of curvature is negative.

$\dfrac{n_1}{p}+\dfrac{n_2}{q}=\dfrac{n_2-n_1}{R}$ becomes $\dfrac{1.33}{30\text{ cm}}+\dfrac{1.00}{q}=\dfrac{1.00-1.33}{-80\text{ cm}}$ $q=-24.9\text{ cm}$

So $\boxed{\text{the image is inside the tank, 24.9 cm behind the front wall; virtual, right side up, enlarged.}}$

(b) Now we have $\dfrac{1.33}{90\text{ cm}}+\dfrac{1.00}{q}=\dfrac{1.00-1.33}{-80\text{ cm}}$ $q=-93.9\text{ cm}$

So $\boxed{\text{the image is inside the tank, 93.9 cm behind the front wall; virtual, right side up, enlarged.}}$

(c) In case (a) the result of problem 24 gives $M=-\dfrac{n_1q}{n_2p}=-\dfrac{1.33(-24.9)}{1.00(30)}=\boxed{+1.10}$

In case (b) we have $M=-\dfrac{1.33(-93.9)}{1.00(90)}=\boxed{+1.39}$

(d) In case (a) $h'=Mh=1.10(9.00\text{ cm})=\boxed{9.92\text{ cm}}$. In case (b), the farther lobster looms larger:

$h'=Mh=1.30(9.00\text{ cm})=\boxed{12.5\text{ cm}}$

(e) The plastic has uniform thickness, so the surfaces of entry and exit for any particular ray are very nearly parallel. The ray is slightly displaced, but it would not be changed in direction by going through the plastic wall with air on both sides. Only the difference between the air and water is responsible for the refraction of the light.

**P36.26** For a plane surface, $\dfrac{n_1}{p}+\dfrac{n_2}{q}=\dfrac{n_2-n_1}{R}$ becomes $q=-\dfrac{n_2 p}{n_1}$

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right|=\frac{n_2}{n_1}\left|\frac{dp}{dt}\right|$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}}=\left|\frac{dq}{dt}\right|=\frac{1.00}{1.33}(2.00\text{ cm/s})=\boxed{1.50\text{ cm/s}}$$

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## Section 36.4 **Thin Lenses**

**P36.27** (a) $$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(0.440)\left[\frac{1}{12.0\text{ cm}}-\frac{1}{(-18.0\text{ cm})}\right]$$

$$f=\boxed{16.4\text{ cm}}$$

(b) $$\frac{1}{f}=(0.440)\left[\frac{1}{18.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$$

$$f=\boxed{16.4\text{ cm}}$$

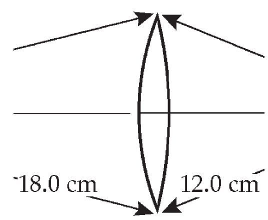



**FIG. P36.27**

**P36.28** Let $R_1$ = outer radius and $R_2$ = inner radius

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{2.00\text{ m}}-\frac{1}{2.50\text{ cm}}\right]=0.050\,0\text{ cm}^{-1}$$

so $f=\boxed{20.0\text{ cm}}$

**P36.29** For a converging lens, $f$ is positive. We use $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{f}$.

(a) $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{40.0\text{ cm}}=\frac{1}{40.0\text{ cm}}\qquad\boxed{q=40.0\text{ cm}}$$

$$M=-\frac{q}{p}=-\frac{40.0}{40.0}=\boxed{-1.00}$$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{20.0\text{ cm}}=0\qquad\boxed{q=\text{infinity}}$$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\text{ cm}}-\frac{1}{10.0\text{ cm}}=-\frac{1}{20.0\text{ cm}}\qquad\boxed{q=-20.0\text{ cm}}$$

$$M=-\frac{q}{p}=-\frac{(-20.0)}{10.0}=\boxed{2.00}$$

The image is $\boxed{\text{upright, virtual}}$ and 20.0 cm in front of the lens.

**P36.30** (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\quad \frac{1}{32.0\text{ cm}}+\frac{1}{8.00\text{ cm}}=\frac{1}{f}$

so $\quad \boxed{f=6.40\text{ cm}}$

(b) $M=-\frac{q}{p}=-\frac{8.00\text{ cm}}{32.0\text{ cm}}=\boxed{-0.250}$

(c) Since $f>0$, the lens is $\boxed{\text{converging}}$.

**P36.31** We are looking at an enlarged, upright, virtual image:

$M=\frac{h'}{h}=2=-\frac{q}{p}$ so $\quad p=-\frac{q}{2}=-\frac{(-2.84\text{ cm})}{2}=+1.42\text{ cm}$

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ gives $\quad \frac{1}{1.42\text{ cm}}+\frac{1}{(-2.84\text{ cm})}=\frac{1}{f}$

$\boxed{f=2.84\text{ cm}}$

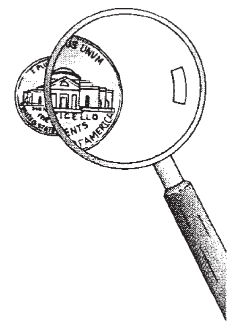


FIG. P36.31

**P36.32** $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\quad p^{-1}+q^{-1}=\text{constant}$

We may differentiate through with respect to $p$: $\quad -1p^{-2}-1q^{-2}\frac{dq}{dp}=0$

$$\frac{dq}{dp}=-\frac{q^2}{p^2}=-M^2$$

**P36.33** (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $\quad \frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{1}{(-32.0\text{ cm})}$

so $\quad q=-\left(\frac{1}{20.0}+\frac{1}{32.0}\right)^{-1}=\boxed{-12.3\text{ cm}}$

The image is 12.3 cm to the left of the lens.

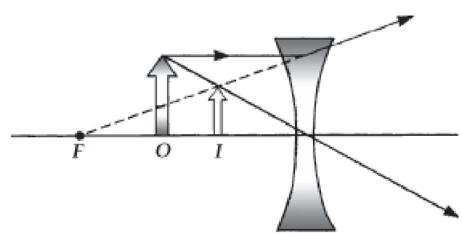



FIG. P36.33

(b) $M=-\frac{q}{p}=-\frac{(-12.3\text{ cm})}{20.0\text{ cm}}=\boxed{0.615}$

(c) See the ray diagram to the right.

**P36.34** The image is inverted: $\quad M=\frac{h'}{h}=\frac{-1.80\text{ m}}{0.024\,0\text{ m}}=-75.0=\frac{-q}{p}$ $\quad q=75.0p$

(a) $q+p=3.00\text{ m}=75.0p+p$ $\quad p=\boxed{39.5\text{ mm}}$

(b) $q=2.96\text{ m}$ $\quad \frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{0.039\,5\text{ m}}+\frac{1}{2.96\text{ m}}$ $\quad f=\boxed{39.0\text{ mm}}$

***P36.35** Comparing $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ with $\frac{1}{p}+\frac{1}{-3.5p}=\frac{1}{7.5\text{ cm}}$ we see $q=-3.5p$ and $f=7.50$ cm for a converging lens.

(a) To solve, we add the fractions:

$$\frac{-3.5+1}{-3.5p}=\frac{1}{7.5\text{ cm}}$$

$$\frac{3.5p}{2.5}=7.5\text{ cm}$$

$$p=\boxed{5.36\text{ cm}}$$

(b) $q=-3.5(5.36\text{ cm})=\boxed{-18.8\text{ cm}}$ $\quad M=-\frac{q}{p}=-\frac{-18.8\text{ cm}}{5.36\text{ cm}}=+3.50$

(c)

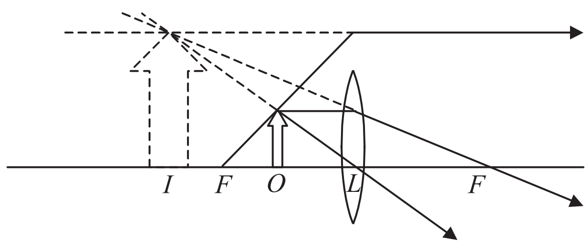



**FIG. P36.35(c)**

The image is $\boxed{\text{enlarged, upright, and virtual.}}$

(d) The lens is being used as a magnifying glass. Statement: A magnifying glass with focal length 7.50 cm is used to form an image of a stamp, enlarged 3.50 times. Find the object distance. Locate and describe the image.

**P36.36** In $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ or $p^{-1}+q^{-1}=\text{constant}$, we differentiate with respect to time:

$$-1\left(p^{-2}\right)\frac{dp}{dt}-1\left(q^{-2}\right)\frac{dq}{dt}=0$$

$$\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}$$

We must find the momentary image location $q$:

$$\frac{1}{20\text{ m}}+\frac{1}{q}=\frac{1}{0.3\text{ m}}$$

$$q=0.305\text{ m}$$

Now $\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20\text{ m})^2}5\text{ m/s}=-0.00116\text{ m/s}=\boxed{1.16\text{ mm/s toward the lens}}$.

**P36.37** (a) The image distance is: $q = d - p$

Thus, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{p}+\frac{1}{d-p}=\frac{1}{f}$

This reduces to a quadratic equation: $p^2 + (-d)p + fd = 0$

which yields: $\boxed{p = \frac{d \pm \sqrt{d^2 - 4fd}}{2} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - fd}}$

Since $f < \frac{d}{4}$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for $p$ gives a larger value for $q$, with a $\boxed{\text{real, enlarged, inverted image}}$.

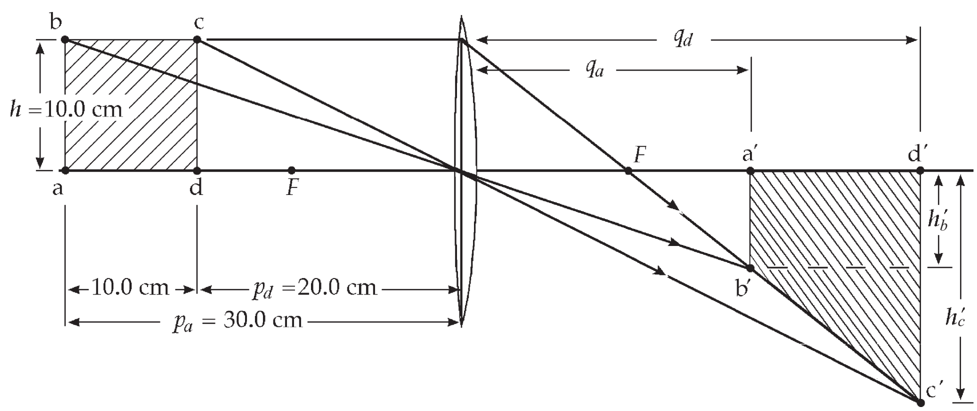
The larger solution for $p$ describes a $\boxed{\text{real, diminished, inverted image}}$.

***P36.38** (a) $\frac{1}{p_a}+\frac{1}{q_a}=\frac{1}{f}$ becomes $\frac{1}{30.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{14.0\text{ cm}}$ or $\boxed{q_a = 26.2\text{ cm}}$

$$h_b' = hM_a = h\left(\frac{-q_a}{p_a}\right) = (10.0\text{ cm})(-0.875) = \boxed{-8.75\text{ cm}}$$

$$\frac{1}{20.0\text{ cm}}+\frac{1}{q_d}=\frac{1}{14.0\text{ cm}} \quad \text{or} \quad \boxed{q_d = 46.7\text{ cm}}$$

$$h_c' = hM_d = (10.0\text{ cm})(-2.33) = \boxed{-23.3\text{ cm}}$$




The square is imaged as a trapezoid.

**FIG. P36.38(b)**

(b) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{p}+\frac{1}{q}=\frac{1}{14\text{ cm}}$ or $1/p = 1/14\text{ cm} - 1/q$

$$|h'| = |hM| = \left|h\left(\frac{-q}{p}\right)\right| = (10.0\text{ cm})q\left(\frac{1}{14\text{ cm}} - \frac{1}{q}\right)$$

(c) The quantity $\int_{q_a}^{q_d} |h'|dq$ adds up the geometrical (positive) areas of thin vertical ribbons comprising the whole area of the image. We have

$$\int_{q_a}^{q_d} |h'|dq = \int_{q_a}^{q_d} (10.0\text{ cm})\left(\frac{q}{14\text{ cm}} - 1\right)dq = (10.0\text{ cm})\left(\frac{q^2}{28\text{ cm}} - q\right)\Bigg|_{26.2\text{ cm}}^{46.7\text{ cm}}$$

$$\text{Area} = (10.0\text{ cm})\left(\frac{46.7^2 - 26.2^2}{28} - 46.7 + 26.2\right)\text{ cm} = \boxed{328\text{ cm}^2}$$

**P36.39** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f}$$

becomes $$\frac{1}{2\,000\text{ mm}}+\frac{1}{q_2}=\frac{1}{65.0\text{ mm}}$$

and $$q_2=(65.0\text{ mm})\left(\frac{2\,000}{2\,000-65.0}\right)$$

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D=q_2-q_1=(65.0\text{ mm})\left(\frac{2\,000}{2\,000-65.0}\right)-65.0\text{ mm}=\boxed{2.18\text{ mm}}$$

## Section 36.5 **Lens Aberrations**

**P36.40** (a) The focal length of the lens is given by

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.53-1.00)\left(\frac{1}{-32.5\text{ cm}}-\frac{1}{42.5\text{ cm}}\right)$$

$f=-34.7$ cm

Note that $R_1$ is negative because the center of curvature of the first surface is on the virtual image side.

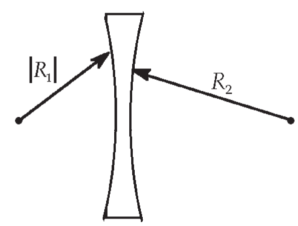
When $p=\infty$

the thin lens equation gives $q=f$

Thus, the violet image of a very distant object is formed

at $\boxed{q=-34.7\text{ cm}}$

The image is $\boxed{\text{virtual, upright and diminshed}}$.



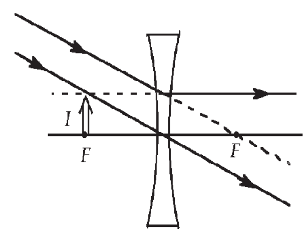




FIG. P36.40

(b) The same ray diagram and image characteristics apply for red light.

Again, $q=f$

and now $$\frac{1}{f}=(1.51-1.00)\left(\frac{1}{-32.5\text{ cm}}-\frac{1}{42.5\text{ cm}}\right)$$

giving $f=\boxed{-36.1\text{ cm}}$

**P36.41** Ray $h_1$ is undeviated at the plane surface and strikes the second surface at angle of incidence given by

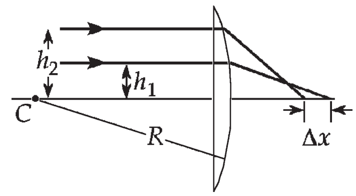



**FIG. P36.41**

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43°$$

Then, $$1.00\sin\theta_2 = 1.60\sin\theta_1 = (1.60)\left(\frac{0.500}{20.0\text{ cm}}\right)$$

so $$\theta_2 = 2.29°$$

The angle this emerging ray makes with the horizontal is $\theta_2 - \theta_1 = 0.860°$

It crosses the axis at a point farther out by $f_1$

where $$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500\text{ cm}}{\tan(0.860°)} = 33.3\text{ cm}$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (0.500\text{ cm})^2} = 0.006\ 25\text{ cm}$$

so ray $h_1$ crosses the axis at this distance from the vertex:

$$x_1 = 33.3\text{ cm} - 0.006\ 25\text{ cm} = 33.3\text{ cm}$$

Now we repeat this calculation for ray $h_2$:

$$\theta = \sin^{-1}\left(\frac{12.0\text{ cm}}{20.0\text{ cm}}\right) = 36.9°$$

$$1.00\sin\theta_2 = 1.60\sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_1 - \theta_2)} = \frac{12.0\text{ cm}}{\tan 36.8°} = 16.0\text{ cm}$$

$$x_2 = (16.0\text{ cm})\left(20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (12.0\text{ cm})^2}\right) = 12.0\text{ cm}$$

Now $$\Delta x = 33.3\text{ cm} - 12.0\text{ cm} = \boxed{21.3\text{ cm}}$$

## Section 36.6 **The Camera**

**P36.42** The same light intensity is received from the subject, and the same light energy on the film is required:

$$IA_1\Delta t_1 = IA_2\Delta t_2$$

$$\frac{\pi d_1^2}{4}\Delta t_1 = \frac{\pi d_2^2}{4}\Delta t_2$$

$$\left(\frac{f}{4}\right)^2\left(\frac{1}{16}\text{ s}\right) = d_2^2\left(\frac{1}{128}\text{ s}\right)$$

$$d_2 = \sqrt{\frac{128}{16}}\frac{f}{4} = \boxed{\frac{f}{1.41}}$$

## Section 36.7 The Eye

**P36.43** $P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} - \frac{1}{0.250 \text{ m}} = -4.00 \text{ diopters} = \boxed{-4.00 \text{ diopters, a diverging lens}}$

**P36.44** For starlight going through Nick's glasses, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$$\frac{1}{\infty} + \frac{1}{(-0.800 \text{ m})} = \frac{1}{f} = -1.25 \text{ diopters}$$

For a nearby object, $\frac{1}{p} + \frac{1}{(-0.180 \text{ m})} = -1.25 \text{ m}^{-1}$, so $p = \boxed{23.2 \text{ cm}}$

## Section 36.8 The Simple Magnifier

## Section 36.9 The Compound Microscope

## Section 36.10 The Telescope

**P36.45** (a) From the thin lens equation: $\frac{1}{p} + \frac{1}{(-25.0 \text{ cm})} = \frac{1}{5.00 \text{ cm}}$ or $p = \boxed{4.17 \text{ cm}}$

(b) $M = -\frac{q}{p} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{5.00 \text{ cm}} = \boxed{6.00}$

**P36.46** Using Equation 36.26, $M \approx -\left(\frac{L}{f_o}\right)\left(\frac{25.0 \text{ cm}}{f_e}\right) = -\left(\frac{23.0 \text{ cm}}{0.400 \text{ cm}}\right)\left(\frac{25.0 \text{ cm}}{2.50 \text{ cm}}\right) = \boxed{-575}$

**P36.47** $f_o = 20.0 \text{ m}$ $\quad f_e = 0.025\,0 \text{ m}$

(a) The angular magnification produced by this telescope is $m = -\frac{f_o}{f_e} = \boxed{-800}$.

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

**P36.48** (a) The mirror-and-lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

gives $q = \frac{1}{1/f - 1/p} = \frac{1}{(p-f)/fp} = \frac{fp}{p-f}$

Then, $M = \frac{h'}{h} = -\frac{q}{p} = -\frac{f}{p-f}$

gives $\boxed{h' = \frac{hf}{f-p}}$

(b) For $p >> f$, $f - p \approx -p$. Then, $h' = \boxed{-\frac{hf}{p}}$

(c) Suppose the telescope observes the space station at the zenith:

$$h' = -\frac{hf}{p} = -\frac{(108.6 \text{ m})(4.00 \text{ m})}{407 \times 10^3 \text{ m}} = \boxed{-1.07 \text{ mm}}$$

**P36.49** Let $I_0$ represent the intensity of the light from the nebula and $\theta_0$ its angular diameter. With the first telescope, the image diameter $h'$ on the film is given by $\theta_o = -\dfrac{h'}{f_o}$ as $h' = -\theta_o(2\,000 \text{ mm})$.

The light power captured by the telescope aperture is $\mathcal{P}_1 = I_0 A_1 = I_0\left[\dfrac{\pi(200 \text{ mm})^2}{4}\right]$, and the light energy focused on the film during the exposure is $E_1 = \mathcal{P}_1 \Delta t_1 = I_0\left[\dfrac{\pi(200 \text{ mm})^2}{4}\right](1.50 \text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is

$\mathcal{P}_2 = I_0 A_2 = I_0\left[\dfrac{\pi(60.0 \text{ mm})^2}{4}\right]$ and the light energy is $E_2 = I_0\left[\dfrac{\pi(60.0 \text{ mm})^2}{4}\right]\Delta t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0 \text{ mm})^2/4\right]\Delta t_2}{\pi\left[\theta_o(900 \text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200 \text{ mm})^2/4\right](1.50 \text{ min})}{\pi\left[\theta_o(2\,000 \text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$\Delta t_2 = \frac{(200 \text{ mm})^2(900 \text{ mm})^2}{(60.0 \text{ mm})^2(2\,000 \text{ mm})^2}(1.50 \text{ min}) = \boxed{3.38 \text{ min}}$$

## Additional Problems

**P36.50** (a) $$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{5 \text{ cm}} - \frac{1}{7.5 \text{ cm}} \qquad \therefore q_1 = 15 \text{ cm}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{15 \text{ cm}}{7.5 \text{ cm}} = -2$$

$$M = M_1 M_2 \qquad \therefore 1 = (-2)M_2$$

$$\therefore M_2 = -\frac{1}{2} = -\frac{q_2}{p_2} \qquad \therefore p_2 = 2q_2$$

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \qquad \therefore \frac{1}{2q_2} + \frac{1}{q_2} = \frac{1}{10 \text{ cm}} \qquad \therefore q_2 = 15 \text{ cm},\ p_2 = 30 \text{ cm}$$

$$p_1 + q_1 + p_2 + q_2 = 7.5 \text{ cm} + 15 \text{ cm} + 30 \text{ cm} + 15 \text{ cm} = \boxed{67.5 \text{ cm}}$$

(b) $$\frac{1}{p_1'} + \frac{1}{q_1'} = \frac{1}{f_1} = \frac{1}{5 \text{ cm}}$$

Solve for $q_1'$ in terms of $p_1'$: $q_1' = \dfrac{5p_1'}{p_1' - 5}$ (1)

$M_1' = -\dfrac{q_1'}{p_1'} = -\dfrac{5}{p_1' - 5}$, using (1).

$$M' = M_1' M_2' \qquad \therefore M_2' = \frac{M'}{M_1'} = -\frac{3}{5}(p_1' - 5) = -\frac{q_2'}{p_2'}$$

$\therefore q_2' = \dfrac{3}{5}p_2'(p_1' - 5)$ (2)

Substitute (2) into the lens equation $\dfrac{1}{p_2'} + \dfrac{1}{q_2'} = \dfrac{1}{f_2} = \dfrac{1}{10 \text{ cm}}$ and obtain $p_2'$ in terms of $p_1'$:

*continued on next page*

$$p_2' = \frac{10(3p_1' - 10)}{3(p_1' - 5)} \tag{3}$$

Substituting (3) in (2), obtain $q_2'$ in terms of $p_1'$:

$$q_2' = 2(3p_1' - 10) \tag{4}$$

Now, $p_1' + q_1' + p_2' + q_2' =$ a constant.

Using (1), (3), and (4), and the value obtained in (a):

$$p_1' + \frac{5p_1'}{p_1' - 5} + \frac{10(3p_1' - 10)}{3(p' - 5)} + 2(3p_1' - 10) = 67.5$$

This reduces to the quadratic equation

$$21p_1'^2 - 322.5p_1' + 1\,212.5 = 0$$

which has solutions $p_1' = 8.784$ cm and 6.573 cm.

Case 1: $p_1' = 8.784$ cm

$\therefore\ p_1' - p_1 = 8.784\text{ cm} - 7.5\text{ cm} = 1.28\text{ cm}$

From (4): $q_2' = 32.7$ cm

$\therefore\ q_2' - q_2 = 32.7\text{ cm} - 15\text{ cm} = 17.7\text{ cm}$

Case 2: $p_1' = 6.573$ cm

$\therefore\ p_1' - p_1 = 6.573\text{ cm} - 7.5\text{ cm} = -0.927\text{ cm}$

From (4): $q_2' = 19.44$ cm

$\therefore\ q_2' = q_2 = 19.44\text{ cm} - 15\text{ cm} = 4.44\text{ cm}$

From these results it is concluded that:

| |
|---|
| The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object. |

**P36.51** Only a diverging lens gives an upright diminished image. The image is virtual and

$d = p - |q| = p + q$: $\quad M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$p = \dfrac{d}{1-M}$: $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{-Mp} = \dfrac{-M+1}{-Mp} = \dfrac{(1-M)^2}{-Md}$

$$f = \frac{-Md}{(1-M)^2} = \frac{-(0.500)(20.0\text{ cm})}{(1-0.500)^2} = \boxed{-40.0\text{ cm}}$$

**P36.52** If $M < 1$, the lens is diverging and the image is virtual. $d = p - |q| = p + q$

$M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$p = \dfrac{d}{1-M}$: $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{(-Mp)} = \dfrac{(-M+1)}{-Mp} = \dfrac{(1-M)^2}{-Md}$ $\quad \boxed{f = \dfrac{-Md}{(1-M)^2}}$

If $M > 1$, the lens is converging and the image is still virtual.

Now $\quad d = -q - p$

We obtain in this case $\boxed{f = \dfrac{Md}{(M-1)^2}}$.

***P36.53** The real image formed by the concave mirror serves as a real object for the convex mirror with $p = 50$ cm and $q = -10$ cm. Therefore,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad \frac{1}{f} = \frac{1}{50\text{ cm}} + \frac{1}{(-10\text{ cm})}$$

gives $f = -12.5$ cm and $R = 2f = \boxed{-25.0\text{ cm}}$.

***P36.54** Start with the first pass through the lens.

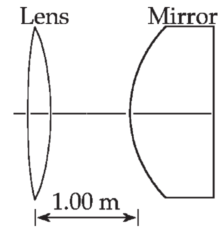



**FIG. P36.54**

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0\text{ cm}} - \frac{1}{100\text{ cm}} \qquad q_1 = 400\text{ cm to right of lens}$$

For the mirror, $p_2 = -300$ cm

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-50.0\text{ cm})} - \frac{1}{(-300\text{ cm})} \qquad q_2 = -60.0\text{ cm}$$

For the second pass through the lens, $p_3 = 160$ cm

$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0\text{ cm}} - \frac{1}{160\text{ cm}} \qquad q_3 = \boxed{160\text{ cm to the left of lens}}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{400\text{ cm}}{100\text{ cm}} = -4.00 \qquad M_2 = -\frac{q_2}{p_2} = -\frac{(-60.0\text{ cm})}{(-300\text{ cm})} = -\frac{1}{5}$$

$$M_3 = -\frac{q_3}{p_3} = -\frac{160\text{ cm}}{160\text{ cm}} = -1 \qquad M = M_1M_2M_3 = \boxed{-0.800}$$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

***P36.55** When the meterstick coordinate of the object is 0, its object distance is $p_i = 32$ cm. When the meterstick coordinate of the object is $x$, its object distance is $p = 32\text{ cm} - x$. The image distance from the lens is given by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{32 - x} + \frac{1}{q} = \frac{1}{26} \qquad \frac{1}{q} = \frac{32 - x - 26}{26(32 - x)} \qquad q = \frac{832 - 26x}{6 - x}$$

The image meterstick coordinate is

$x' = 32 + q = (32(6 - x) + 832 - 26x)/(6 - x) = \boxed{(1\,024\text{ cm} - 58x)\text{ cm}/(6\text{ cm} - x)}$. The image starts at the position $x_i' = 171$ cm and moves in the positive $x$ direction, faster and faster, until it is out at infinity when the object is at the position $x = 6$ cm. At this instant the rays from the top of the object are parallel as they leave the lens. Their intersection point can be described as at $x' = \infty$ to the right or equally well at $x' = -\infty$ on the left. From $x' = -\infty$ the image continues moving to the right, now slowing down. It reaches, for example, –280 cm when the object is at 8 cm, and –55 cm when the object is finally at 12 cm. The image has traveled always to the right, to infinity and beyond.

**P36.56** $\dfrac{1}{q_1} = \dfrac{1}{f_1} - \dfrac{1}{p_1} = \dfrac{1}{10.0\text{ cm}} - \dfrac{1}{12.5\text{ cm}}$

so $q_1 = 50.0\text{ cm}$ (to left of mirror)

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-16.7\text{ cm})} - \frac{1}{(-25.0\text{ cm})} \text{ and } q_2 = -50.3\text{ cm}$$

meaning 50.3 cm to the right of the lens. Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0\text{ cm}}{12.5\text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-50.3\text{ cm})}{(-25.0\text{ cm})} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

**P36.57** A telescope with an eyepiece decreases the diameter of a beam of parallel rays. When light is sent through the same device in the opposite direction, the beam expands. Send the light first through the diverging lens. It will then be diverging from a virtual image found like this:

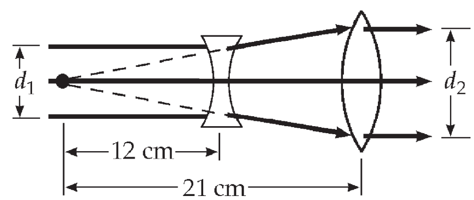



FIG. P36.57

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{\infty} + \frac{1}{q} = \frac{1}{-12\text{ cm}}$$

$$q = -12\text{ cm}$$

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p = 21\text{cm}$. Then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{21\text{ cm}} + \frac{1}{q} = \frac{1}{21\text{ cm}}$$

$$q = \infty$$

The exiting rays will be parallel. The lenses must be 21.0 cm − 12.0 cm = 9.00 cm apart.

By similar triangles, $\dfrac{d_2}{d_1} = \dfrac{21\text{ cm}}{12\text{ cm}} = \boxed{\text{1.75 times}}$

***P36.58** (a) For the lens in air,

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{79\text{ cm}}=(1.55-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For the same lens in water

$$\frac{1}{f'}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f'}=\left(\frac{1.55}{1.33}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

By division,

$$\frac{1/79\text{ cm}}{1/f'}=\frac{0.55}{0.165}=\frac{f'}{79\text{ cm}}$$

$$f'=79\text{ cm}(3.32)=\boxed{263\text{ cm}}$$

(b) The path of a reflected ray does not depend on the refractive index of the medium which the reflecting surface bounds. Therefore the focal length of a mirror does not change when it is put into a different medium: $f'=\frac{R}{2}=f=\boxed{79.0\text{ cm}}$.

**P36.59** A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R=-6.00$ cm.

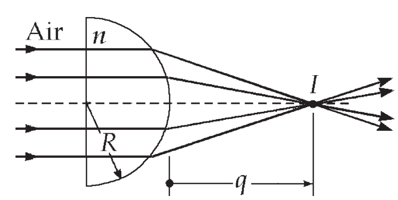



FIG. P36.59

The incident rays are parallel, so $p=\infty$

Then, $$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

becomes $$0+\frac{1}{q}=\frac{1.00-1.56}{-6.00\text{ cm}}$$

and $$\boxed{q=10.7\text{ cm}}$$

**P36.60** (a) $I=\frac{\mathcal{P}}{4\pi r^2}=\frac{4.50\text{ W}}{4\pi\left(1.60\times10^{-2}\text{ m}\right)^2}=\boxed{1.40\text{ kW/m}^2}$

(b) $I=\frac{\mathcal{P}}{4\pi r^2}=\frac{4.50\text{ W}}{4\pi(7.20\text{ m})^2}=\boxed{6.91\text{ mW/m}^2}$

(c) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $$\frac{1}{7.20\text{ m}}+\frac{1}{q}=\frac{1}{0.350\text{ m}}$$

so $$q=0.368\text{ m}$$

and $$M=\frac{h'}{3.20\text{ cm}}=-\frac{q}{p}=-\frac{0.368\text{ m}}{7.20\text{ m}}$$

$$h'=\boxed{0.164\text{ cm}}$$

(d) The lens intercepts power given by $$\mathcal{P}=IA=\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\frac{\pi}{4}(0.150\text{ m})^2\right]$$

and puts it all onto the image where $$I=\frac{\mathcal{P}}{A}=\frac{\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\pi(15.0\text{ cm})^2/4\right]}{\pi(0.164\text{ cm})^2/4}$$

$$I=\boxed{58.1\text{ W/m}^2}$$

**P36.61** From the thin lens equation, $q_1 = \dfrac{f_1 p_1}{p_1 - f_1} = \dfrac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$.

In this case, $p_2 = d - (-4.00\text{ cm})$

Therefore, $d + 4.00\text{ cm} = f_2 = 12.0\text{ cm}$ and $d = \boxed{8.00\text{ cm}}$

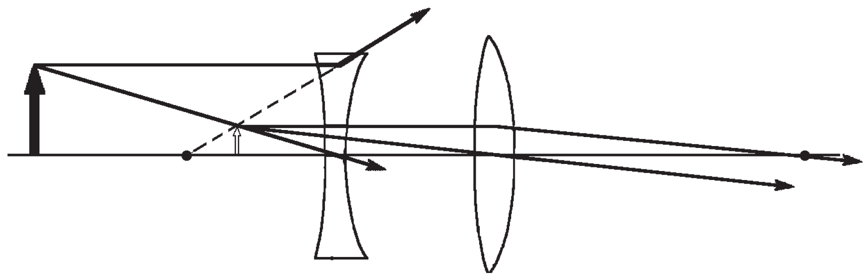


FIG. P36.61

**P36.62** (a) For the light the mirror intercepts,

$$\mathcal{P} = I_0 A = I_0 \pi R_a^2$$

$$350\text{ W} = \left(1\,000\text{ W/m}^2\right)\pi R_a^2$$

and $R_a = \boxed{0.334\text{ m or larger}}$

(b) In $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{2}{R}$

we have $p \to \infty$

so $q = \dfrac{R}{2}$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $h' = -q\left(\dfrac{h}{p}\right) = -\left(\dfrac{R}{2}\right)\left[0.533°\left(\dfrac{\pi\text{ rad}}{180°}\right)\right] = -\left(\dfrac{R}{2}\right)(9.30\text{ m rad})$

where $\dfrac{h}{p}$ is the angle the Sun subtends. The intensity at the image is

then $I = \dfrac{\mathcal{P}}{\pi h'^2/4} = \dfrac{4 I_0 \pi R_a^2}{\pi h'^2} = \dfrac{4 I_0 R_a^2}{(R/2)^2\left(9.30\times 10^{-3}\text{ rad}\right)^2}$

$$120\times 10^3\text{ W/m}^2 = \frac{16\left(1\,000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times 10^{-3}\text{ rad}\right)^2}$$

so $\boxed{\dfrac{R_a}{R} = 0.025\,5\text{ or larger}}$

**P36.63** For the mirror, $f = \frac{R}{2} = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty$.

The mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, then gives $\qquad q = f = \boxed{1.50 \text{ m}}$

Now, in $\qquad M = -\frac{q}{p} = \frac{h'}{h}$

the magnification is nearly zero, but we can be more precise: $\frac{h}{p}$ is the angular diameter of the object. Thus, the image diameter is

$$h' = -\frac{hq}{p} = \left(-0.533°\right)\left(\frac{\pi \text{ rad}}{180°}\right)(1.50 \text{ m}) = -0.014\,0 \text{ m} = \boxed{-1.40 \text{ cm}}$$

**P36.64** (a) The lens makers' equation, $\qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

becomes: $\frac{1}{5.00 \text{ cm}} = (n-1)\left[\frac{1}{9.00 \text{ cm}} - \frac{1}{(-11.0 \text{ cm})}\right]$ giving $n = \boxed{1.99}$.

(b) As the light passes through the lens for the first time, the thin lens equation

$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$ $\qquad$ becomes: $\qquad \frac{1}{8.00 \text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00 \text{ cm}}$

or $\quad q_1 = 13.3$ cm, $\quad$ and $\quad M_1 = -\frac{q_1}{p_1} = -\frac{13.3 \text{ cm}}{8.00 \text{ cm}} = -1.67$

This image becomes the object for the concave mirror with:

$p_m = 20.0 \text{ cm} - q_1 = 20.0 \text{ cm} - 13.3 \text{ cm} = 6.67 \text{ cm}$ $\quad$ and $\quad f = \frac{R}{2} = +4.00$ cm

The mirror equation becomes: $\qquad \frac{1}{6.67 \text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00 \text{ cm}}$

giving $\qquad q_m = 10.0$ cm $\quad$ and $\quad M_2 = -\frac{q_m}{p_m} = -\frac{10.0 \text{ cm}}{6.67 \text{ cm}} = -1.50$

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3 = 20.0 \text{ cm} - q_m = +10.0 \text{ cm}$$

The thin lens equation yields: $\qquad \frac{1}{10.0 \text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00 \text{ cm}}$

or $\qquad q_3 = 10.0$ cm $\quad$ and $\quad M_3 = -\frac{q_3}{p_3} = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00$

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $\qquad M_{\text{total}} = M_1 M_2 M_3 = \boxed{-2.50}$

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

**P36.65** In the original situation, $p_1 + q_1 = 1.50\text{ m}$

In the final situation, $p_2 = p_1 + 0.900\text{ m}$

and $q_2 = q_1 - 0.900\text{ m} = 0.600\text{ m} - p_1$

Our lens equation is $$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}$$

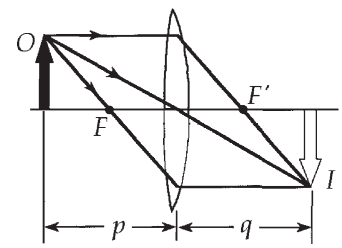



FIG. P36.65

Substituting, we have $$\frac{1}{p_1} + \frac{1}{1.50\text{ m} - p_1} = \frac{1}{p_1 + 0.900} + \frac{1}{0.600 - p_1}$$

Adding the fractions, $$\frac{1.50\text{ m} - p_1 + p_1}{p_1(1.50\text{ m} - p_1)} = \frac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}$$

Simplified, this becomes $$p_1(1.50\text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1)$$

(a) Thus, $p_1 = \frac{0.540}{1.80}\text{ m} = \boxed{0.300\text{ m}}$ $\quad p_2 = p_1 + 0.900 = \boxed{1.20\text{ m}}$

(b) $\frac{1}{f} = \frac{1}{0.300\text{ m}} + \frac{1}{1.50\text{ m} - 0.300\text{ m}}$ and $f = \boxed{0.240\text{ m}}$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$

with $M = -\frac{q_2}{p_2} = \boxed{-0.250}$

**P36.66** The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the $\boxed{\text{image is real, inverted, and actual size}}$.

For the upper mirror:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \qquad \frac{1}{7.50\text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50\text{ cm}} \qquad q_1 = \infty$$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50\text{ cm}} \qquad q_2 = 7.50\text{ cm}$$

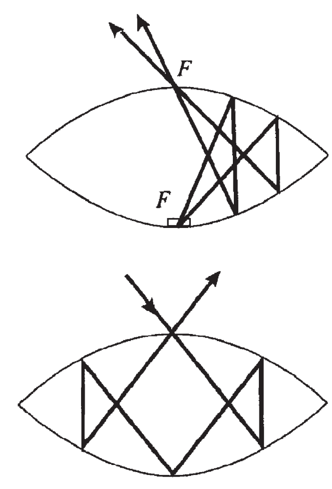



FIG. P36.66

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

**P36.67** (a) For lens one, as shown in the first figure,

$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}$$
$$q_1 = 120\text{ cm}$$
$$M_1 = -\frac{q_1}{p_1} = -\frac{120\text{ cm}}{40.0\text{ cm}} = -3.00$$

This real image $I_1 = O_2$ is a virtual object for the second lens. That is, it is *behind* the lens, as shown in the second figure. The object distance is

$$p_2 = 110\text{ cm} - 120\text{ cm} = -10.0\text{ cm}$$
$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}:$$
$$q_2 = \boxed{20.0\text{ cm}}$$
$$M_2 = -\frac{q_2}{p_2} = -\frac{20.0\text{ cm}}{(-10.0\text{ cm})} = +2.00$$
$$M_{\text{overall}} = M_1 M_2 = \boxed{-6.00}$$

(b) $M_{\text{overall}} < 0$, so final image is $\boxed{\text{inverted}}$.

(c) If lens two is a converging lens (third figure):

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$
$$q_2 = \boxed{6.67\text{ cm}}$$
$$M_2 = -\frac{6.67\text{ cm}}{(-10.0\text{ cm})} = +0.667$$
$$M_{\text{overall}} = M_1 M_2 = \boxed{-2.00}$$

Again, $M_{\text{overall}} < 0$ and the final image is $\boxed{\text{inverted}}$.

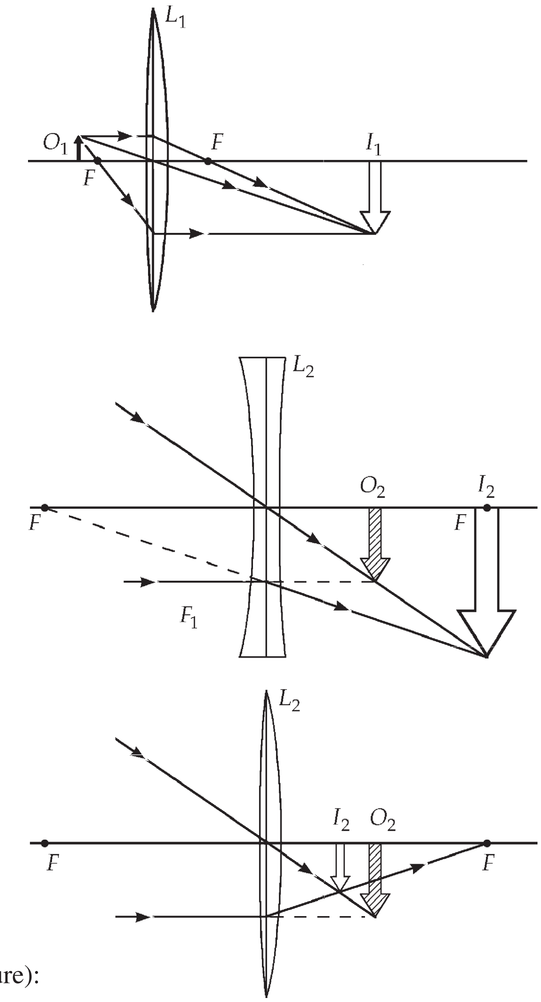



FIG. P36.67

**P36.68** The first lens has focal length described by

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_{11}} - \frac{1}{R_{12}}\right) = (n_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = -\frac{n_1 - 1}{R}$$

For the second lens

$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_{21}} - \frac{1}{R_{22}}\right) = (n_2 - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right) = +\frac{2(n_2 - 1)}{R}$$

Let an object be placed at any distance $p_1$ large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{q_1} = \frac{-n_1 + 1}{R} - \frac{1}{p_1}$$

This virtual $(q_1 < 0)$ image is a real object for the second lens at distance $p_2 = -q_1$. For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{q_2} = \frac{2n_2 - 2}{R} - \frac{1}{p_2} = \frac{2n_2 - 2}{R} + \frac{1}{q_1} = \frac{2n_2 - 2}{R} + \frac{-n_1 + 1}{R} - \frac{1}{p_1} = \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1}$$

Then $\frac{1}{p_1} + \frac{1}{q_2} = \frac{2n_2 - n_1 - 1}{R}$ so the doublet behaves like a single lens with $\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$.

## ANSWERS TO EVEN PROBLEMS

**P36.2** 4.58 m

**P36.4** 10.0 ft, 30.0 ft, 40.0 ft

**P36.6** (a) 13.3 cm, –0.333, real and inverted (b) 20.0 cm, –1.00, real and inverted (c) No image is formed.

**P36.8** at $q = -0.267$ m virtual, upright, and diminished with $M = 0.026\,7$

**P36.10** at 3.33 m from the deepest point of the niche

**P36.12** At 0.708 cm in front of the reflecting surface. Image is virtual, upright, and diminished.

**P36.14** 7.90 mm

**P36.16** (a) 15.0 cm (b) 60.0 cm

**P36.18** (a) 25.6 m (b) 0.058 7 rad (c) 2.51 m (d) 0.023 9 rad (e) 62.8 m from your eyes

**P36.20** 4.82 cm

**P36.22** (a) 45.0 cm (b) –90.0 cm (c) –6.00 cm

**P36.24** See the solution.

**P36.26** $1.50 \text{ cm/s}$

**P36.28** 20.0 cm

**P36.30** (a) 6.40 cm (b) –0.250 (c) converging

**P36.32** See the solution.

**P36.34** (a) 39.0 mm (b) 39.5 mm

**P36.36** $1.16 \text{ mm/s}$ toward the lens

**P36.38** (a) $q_a = 26.2$ cm $q_d = 46.7$ cm $h_b' = -8.75$ cm $h_c' = -23.3$ cm. See the solution. (b) The equation follows from $h'/h = -q/p$ and $1/p + 1/q = 1/f$. (c) The integral stated adds up the areas of ribbons covering the whole image, each with vertical dimension $|h'|$ and horizontal width $dq$. 328 $\text{cm}^2$

**P36.40** (a) at $q = -34.7$ cm virtual, upright, and diminished (b) at $q = -36.1$ cm virtual, upright, and diminished

**P36.42** $\dfrac{f}{1.41}$

**P36.44** 23.2 cm

**P36.46** –575

**P36.48** (a) See the solution. (b) $h' = -\dfrac{hf}{p}$ (c) $-1.07$ mm

**P36.50** (a) 67.5 cm (b) The lenses can be displaced in two ways. The first lens can be displaced 1.28 cm farther away from the object, and the second lens 17.7 cm toward the object. Alternatively, the first lens can be displaced 0.927 cm toward the object and the second lens 4.44 cm toward the object.

**P36.52** if $M < 1$, $f = \dfrac{-Md}{(1-M)^2}$, if $M > 1$, $f = \dfrac{Md}{(M-1)^2}$

**P36.54** 160 cm to the left of the lens, inverted, $M = -0.800$

**P36.56** 25.3 cm to right of mirror, virtual, upright, enlarged 8.05 times

**P36.58** (a) 263 cm (b) 79.0 cm

**P36.60** (a) $1.40 \text{ kW/m}^2$ (b) $6.91 \text{ mW/m}^2$ (c) 0.164 cm (d) $58.1 \text{ W/m}^2$

**P36.62** (a) 0.334 m or larger (b) $\dfrac{R_a}{R} = 0.025\,5$ or larger

**P36.64** (a) 1.99 (b) 10.0 cm to the left of the lens; $-2.50$ (c) inverted

**P36.66** See the solution; real, inverted, and actual size.

**P36.68** See the solution.