18

Superposition and Standing Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q18.1 No. Waves with all waveforms interfere. Waves with other wave shapes are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

***Q18.2** (i) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is zero. Answer (d).

(ii) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.1\text{ m}) = 0.2\text{ m}$. Answer (b).

***Q18.3** In the starting situation, the waves interfere constructively. When the sliding section is moved out by 0.1 m, the wave going through it has an extra path length of 0.2 m = $\lambda/4$, to show partial interference. When the slide has come out 0.2 m from the starting configuration, the extra path length is 0.4 m = $\lambda/2$, for destructive interference. Another 0.1 m and we are at $r_2 - r_1 = 3\lambda/4$ for partial interference as before. At last, another equal step of sliding and one wave travels one wavelength farther to interfere constructively. The ranking is then d > a = c > b.

Q18.4 No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.

***Q18.5** Answer (c). The two waves must have slightly different amplitudes at P because of their different distances, so they cannot cancel each other exactly.

Q18.6 Damping, and non–linear effects in the vibration turn the energy of vibration into internal energy.

***Q18.7** The strings have different linear densities and are stretched to different tensions, so they carry string waves with different speeds and vibrate with different fundamental frequencies. They are all equally long, so the string waves have equal wavelengths. They all radiate sound into air, where the sound moves with the same speed for different sound wavelengths. The answer is (b) and (e).

***Q18.8** The fundamental frequency is described by $f_1 = \dfrac{v}{2L}$, where $v = \left(\dfrac{T}{\mu}\right)^{1/2}$

(i) If L is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\dfrac{1}{2}$. Answer (f).

(ii) If μ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\dfrac{1}{\sqrt{2}}$. Answer (e).

(iii) If T is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\sqrt{2}$. Answer (c).

***Q18.9** Answer (d). The energy has not disappeared, but is still carried by the wave pulses. Each particle of the string still has kinetic energy. This is similar to the motion of a simple pendulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the particles of the string do not stop at the equilibrium position of the string when these two waves superimpose.

***Q18.10** The resultant amplitude is greater than either individual amplitude, wherever the two waves are nearly enough in phase that $2A\cos(\phi/2)$ is greater than A. This condition is satisfied whenever the absolute value of the phase difference ϕ between the two waves is less than 120°. Answer (d).

Q18.11 What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two pitches are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.

***Q18.12** The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonies will not be excited because they have a node at the point where the string exhibits its maximum displacement. Answer (c).

***Q18.13** (a) The tuning fork hits the paper repetitively to make a sound like a buzzer, and the paper efficiently moves the surrounding air. The tuning fork will vibrate audibly for a shorter time.

(b) Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner.

(c) The tuning fork in resonance makes the column of air vibrate, especially at the antinode of displacement at the top of the tube. Its area is larger than that of the fork tines, so it radiates louder sound into the environment. The tuning fork will not vibrate for so long.

(d) The tuning fork ordinarily pushes air to the right on one side and simultaneously pushes air to the left a couple of centimeters away, on the far side of its other time. Its net disturbance for sound radiation is small. The slot in the cardboard admits the 'back wave' from the far side of the fork and keeps much of it from interfering destructively with the sound radiated by the tine in front. Thus the sound radiated in front of the screen can become noticeably louder. The fork will vibrate for a shorter time.

All four of these processes exemplify conservation of energy, as the energy of vibration of the fork is transferred faster into energy of vibration of the air. The reduction in the time of audible fork vibration is easy to observe in case (a), but may be challenging to observe in the other cases.

Q18.14 Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large–amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small–amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high–fiber quick–cooking oatmeal into the hot coffee. You do not need a cover on your cup.

***Q18.15** The tape will reduce the frequency of the fork, leaving the string frequency unchanged. If the bit of tape is small, the fork must have started with a frequency 4 Hz below that of the string, to end up with a frequency 5 Hz below that of the string. The string frequency is $262 + 4 = 266$ Hz, answer (d).

Q18.16 Beats. The propellers are rotating at slightly different frequencies.

SOLUTIONS TO PROBLEMS

Section 18.1 **Superposition and Interference**

P18.1 $y = y_1 + y_2 = 3.00\cos(4.00x - 1.60t) + 4.00\sin(5.0x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00,\ t = 1.00$ $\quad y = 3.00\cos(2.40\text{ rad}) + 4.00\sin(+3.00\text{ rad}) = \boxed{-1.65\text{ cm}}$

(b) $x = 1.00,\ t = 0.500$ $\quad y = 3.00\cos(+3.20\text{ rad}) + 4.00\sin(+4.00\text{ rad}) = \boxed{-6.02\text{ cm}}$

(c) $x = 0.500,\ t = 0$ $\quad y = 3.00\cos(+2.00\text{ rad}) + 4.00\sin(+2.50\text{ rad}) = \boxed{1.15\text{ cm}}$

P18.2

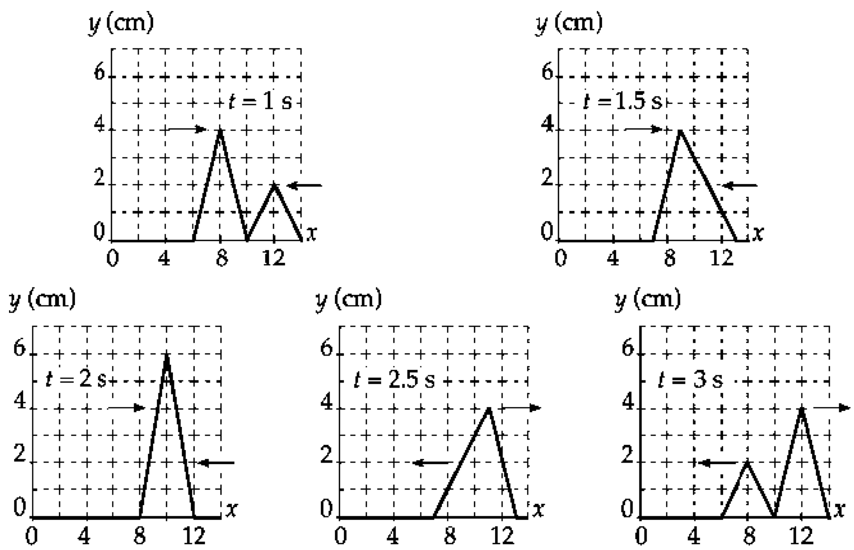

FIG. P18.2

P18.3 (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x\text{ direction}}$

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x\text{ direction}}$

(b) To cancel, $y_1 + y_2 = 0$:
$$\frac{5}{(3x-4t)^2+2} = \frac{+5}{(3x+4t-6)^2+2}$$
$$(3x-4t)^2 = (3x+4t-6)^2$$
$$3x - 4t = \pm(3x+4t-6)$$

from the positive root, $\quad 8t = 6 \quad \boxed{t = 0.750\text{ s}}$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) from the negative root, $\quad 6x = 6 \quad \boxed{x = 1.00\text{ m}}$

(at $x = 1.00$ m, the waves cancel always)

P18.4 Suppose the waves are sinusoidal.

The sum is $(4.00\text{ cm})\sin(kx-\omega t)+(4.00\text{ cm})\sin(kx-\omega t+90.0°)$

$$2(4.00\text{ cm})\sin(kx-\omega t+45.0°)\cos 45.0°$$

So the amplitude is $(8.00\text{ cm})\cos 45.0° = \boxed{5.66\text{ cm}}$.

P18.5 The resultant wave function has the form

$$y=2A_0\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)$$

(a) $A=2A_0\cos\left(\frac{\phi}{2}\right)=2(5.00)\cos\left[\frac{-\pi/4}{2}\right]=\boxed{9.24\text{ m}}$

(b) $f=\frac{\omega}{2\pi}=\frac{1\,200\pi}{2\pi}=\boxed{600\text{ Hz}}$

P18.6 (a) $\Delta x=\sqrt{9.00+4.00}-3.00=\sqrt{13}-3.00=0.606\text{ m}$

The wavelength is $\lambda=\frac{v}{f}=\frac{343\text{ m/s}}{300\text{ Hz}}=1.14\text{ m}$

Thus, $\frac{\Delta x}{\lambda}=\frac{0.606}{1.14}=0.530$ of a wave,

or $\Delta\phi=2\pi(0.530)=\boxed{3.33\text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda}=0.500=f\frac{\Delta x}{v}$

where Δx is a constant in this set up. $f=\frac{v}{2\Delta x}=\frac{343}{2(0.606)}=\boxed{283\text{ Hz}}$

P18.7 We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2+d^2}-L$.

He hears a minimum when this is $\frac{(2n-1)\lambda}{2}$ with $n=1, 2, 3, \ldots$

Then,

$$\sqrt{L^2+d^2}-L=\frac{(n-1/2)v}{f}$$

$$\sqrt{L^2+d^2}=\frac{(n-1/2)v}{f}+L$$

$$L^2+d^2=\frac{(n-1/2)^2v^2}{f^2}+L^2+\frac{2(n-1/2)vL}{f}$$

$$L=\frac{d^2-(n-1/2)^2v^2/f^2}{2(n-1/2)v/f}\quad n=1, 2, 3,\ldots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to d when $L=0$. The number of minima he hears is the greatest integer solution to $d\ge\frac{(n-1/2)v}{f}$

$$n=\text{greatest integer}\le\frac{df}{v}+\frac{1}{2}$$

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(a) $\dfrac{df}{v}+\dfrac{1}{2}=\dfrac{(4.00\text{ m})(200/\text{s})}{330\text{ m/s}}+\dfrac{1}{2}=2.92$

He hears $\boxed{\text{two}}$ minima.

(b) With $n=1$,

$$L=\frac{d^2-(1/2)^2v^2/f^2}{2(1/2)v/f}=\frac{(4.00\text{ m})^2-(330\text{ m/s})^2/4(200/\text{s})^2}{(330\text{ m/s})/200/\text{s}}$$

$$L=\boxed{9.28\text{ m}}$$

With $n=2$,

$$L=\frac{d^2-(3/2)^2v^2/f^2}{2(3/2)v/f}=\boxed{1.99\text{ m}}$$

P18.8 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r=\sqrt{L^2+d^2}-L$.

He hears a minimum when $\Delta r=(2n-1)\left(\dfrac{\lambda}{2}\right)$ with $n=1, 2, 3, \ldots$

Then,

$$\sqrt{L^2+d^2}-L=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$$

$$\sqrt{L^2+d^2}=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)+L$$

$$L^2+d^2=\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2+2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L+L^2$$

$$d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2=2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L=0$.

(a) The number of minima he hears is the greatest integer value for which $L\geq 0$. This is the same as the greatest integer solution to $d\geq\left(n-\dfrac{1}{2}\right)\left(\dfrac{v}{f}\right)$, or

$$\boxed{\text{number of minima heard}=n_{\max}=\text{greatest integer}\leq d\left(\frac{f}{v}\right)+\frac{1}{2}}$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n=\frac{d^2-(n-1/2)^2(v/f)^2}{2(n-1/2)(v/f)}\text{ where }n=1, 2, \ldots, n_{\max}}$$

P18.9 (a) $\phi_1 = (20.0\ \text{rad/cm})(5.00\ \text{cm}) - (32.0\ \text{rad/s})(2.00\ \text{s}) = 36.0\ \text{rad}$

$\phi_1 = (25.0\ \text{rad/cm})(5.00\ \text{cm}) - (40.0\ \text{rad/s})(2.00\ \text{s}) = 45.0\ \text{rad}$

$\Delta\phi = 9.00\ \text{radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = |20.0x - 32.0t - [25.0x - 40.0t]| = |-5.00x + 8.00t|$

At $t = 2.00$ s, the requirement is

$$\Delta\phi = |-5.00x + 8.00(2.00)| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.058\ 4\ \text{cm}}$$

***P18.10** (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344\ \text{m/s}}{21.5\ \text{Hz}} = 16.0\ \text{m}$

Then we note that the path difference equals $9.00\ \text{m} - 1.00\ \text{m} = \frac{1}{2}\lambda$

Point A is one-half wavelength farther from one speaker than from the other. The waves from the two sources interfere destructively, so the receiver records a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then, $$\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$$

Square both sides and simplify to get: $$20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$$

Upon squaring again, this reduces to: $$400x^2 - 10.0\lambda^2 x + \frac{\lambda^4}{16.0} = \lambda^2(x-5.00)^2 + \lambda^2 y^2$$

Substituting $\lambda = 16.0$ m, and reducing, $$9.00x^2 - 16.0y^2 = 144$$

or $$\frac{x^2}{16.0} - \frac{y^2}{9.00} = 1$$

The point should move along the hyperbola $9x^2 - 16y^2 = 144$.

(c) Yes. Far from the origin the equation might as well be $9x^2 - 16y^2 = 0$, so the point can move along the straight line through the origin with slope 0.75 or the straight line through the origin with slope -0.75.

Section 18.2 **Standing Waves**

P18.11 $y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore, $k = \dfrac{2\pi}{\lambda} = 0.400\text{ rad/m}$ $\quad\lambda = \dfrac{2\pi}{0.400\text{ rad/m}} = \boxed{15.7\text{ m}}$

and $\omega = 2\pi f$ so $\quad f = \dfrac{\omega}{2\pi} = \dfrac{200\text{ rad/s}}{2\pi\text{ rad}} = \boxed{31.8\text{ Hz}}$

The speed of waves in the medium is $\quad v = \lambda f = \dfrac{\lambda}{2\pi}2\pi f = \dfrac{\omega}{k} = \dfrac{200\text{ rad/s}}{0.400\text{ rad/m}} = \boxed{500\text{ m/s}}$

P18.12 From $y = 2A_0 \sin kx \cos \omega t$ we find

$$\frac{\partial y}{\partial x} = 2A_0 k \cos kx \cos \omega t \qquad \frac{\partial y}{\partial t} = -2A_0 \omega \sin kx \sin \omega t$$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t \qquad \frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $\quad -2A_0 k^2 \sin kx \cos \omega t = \left(\dfrac{1}{v^2}\right)\left(-2A_0 \omega^2 \sin kx \cos \omega t\right)$

This is satisfied, provided that $v = \dfrac{\omega}{k}$. But this is true, because $v = \lambda f = \dfrac{\lambda}{2\pi}2\pi f = \dfrac{\omega}{k}$

P18.13 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2\left(800\text{ s}^{-1}\right)} = 0.214\text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25\text{ m}}{2} = 0.625$$

Then there is a node at $\quad 0.625 - \dfrac{0.214}{2} = \boxed{0.518\text{ m}}$

a node at $\quad 0.518\text{ m} - 0.214\text{ m} = \boxed{0.303\text{ m}}$

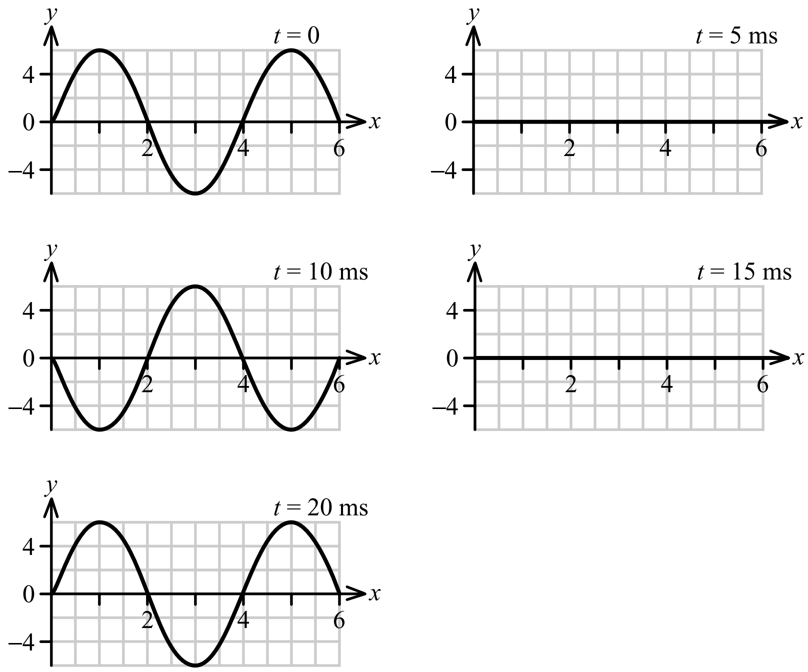
a node at $\quad 0.303\text{ m} - 0.214\text{ m} = \boxed{0.0891\text{ m}}$

a node at $\quad 0.518\text{ m} + 0.214\text{ m} = \boxed{0.732\text{ m}}$

a node at $\quad 0.732\text{ m} + 0.214\text{ m} = \boxed{0.947\text{ m}}$

and a node at $\quad 0.947\text{ m} + 0.214\text{ m} = \boxed{1.16\text{ m}}$ from either speaker.

***P18.14** (a)



(b) $\boxed{\text{In any one picture, the distance from one positive-going zero crossing to the next is } \lambda = 4 \text{ m.}}$

(c) $\boxed{f = 50\text{ Hz. The oscillation at any point starts to repeat after a period of 20 ms, and } f = 1/T.}$

(d) 4 m. By comparison with the wave function $y = (2A \sin kx)\cos \omega t$, we identify $k = \pi/2$, and then compute $\lambda = 2\pi/k$.

(e) 50 Hz. By comparison with the wave function $y = (2A \sin kx)\cos \omega t$, we identify $\omega = 2\pi f = 100\pi$.

P18.15 $y_1 = 3.00\sin\left[\pi(x+0.600t)\right]$ cm; $y_2 = 3.00\sin\left[\pi(x-0.600t)\right]$ cm

$$y = y_1 + y_2 = \left[3.00\sin(\pi x)\cos(0.600\pi t) + 3.00\sin(\pi x)\cos(0.600\pi t)\right] \text{ cm}$$
$$y = (6.00 \text{ cm})\sin(\pi x)\cos(0.600\pi t)$$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.

At $x = 0.250$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get y_{max}:

At $x = 1.50$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

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(d) The antinodes occur when $x = \dfrac{n\lambda}{4} \quad (n = 1, 3, 5, \ldots)$

But $k = \dfrac{2\pi}{\lambda} = \pi$ so $\lambda = 2.00$ cm

and

$$x_1 = \frac{\lambda}{4} = \boxed{0.500\text{ cm}} \text{ as in (b)}$$

$$x_2 = \frac{3\lambda}{4} = \boxed{1.50\text{ cm}} \text{ as in (c)}$$

$$x_3 = \frac{5\lambda}{4} = \boxed{2.50\text{ cm}}$$

***P18.16** (a) The resultant wave is $y = 2A\sin\left(kx + \dfrac{\phi}{2}\right)\cos\left(\omega t - \dfrac{\phi}{2}\right)$

> The oscillation of the $\sin(kx + \phi/2)$ factor means that this wave shows alternating nodes and antinodes. It is a standing wave.

The nodes are located at $kx + \dfrac{\phi}{2} = n\pi$ so $x = \dfrac{n\pi}{k} - \dfrac{\phi}{2k}$

which means that each node is shifted $\dfrac{\phi}{2k}$ to the left by the phase difference between the traveling waves.

> (b) The separation of nodes is $\Delta x = \left[(n+1)\dfrac{\pi}{k} - \dfrac{\phi}{2k}\right] - \left[\dfrac{n\pi}{k} - \dfrac{\phi}{2k}\right]\Delta x = \dfrac{\pi}{k} = \dfrac{\lambda}{2}$
>
> The nodes are still separated by half a wavelength.
>
> (c) As noted in part (a), the nodes are all shifted by the distance $\phi/2k$ to the left.

Section 18.3 **Standing Waves in a String Fixed at Both Ends**

P18.17 $L = 30.0$ m; $\mu = 9.00 \times 10^{-3}$ kg/m; $T = 20.0$ N; $f_1 = \dfrac{v}{2L}$

where $v = \left(\dfrac{T}{\mu}\right)^{1/2} = 47.1$ m/s

so $f_1 = \dfrac{47.1}{60.0} = \boxed{0.786\text{ Hz}}$ $\quad f_2 = 2f_1 = \boxed{1.57\text{ Hz}}$

$f_3 = 3f_1 = \boxed{2.36\text{ Hz}}$ $\quad f_4 = 4f_1 = \boxed{3.14\text{ Hz}}$

P18.18 The tension in the string is $T = (4\text{ kg})(9.8\text{ m/s}^2) = 39.2$ N

Its linear density is $\mu = \dfrac{m}{L} = \dfrac{8 \times 10^{-3}\text{ kg}}{5\text{ m}} = 1.6 \times 10^{-3}$ kg/m

and the wave speed on the string is $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{39.2\text{ N}}{1.6 \times 10^{-3}\text{ kg/m}}} = 156.5$ m/s

In its fundamental mode of vibration, we have $\lambda = 2L = 2(5\text{ m}) = 10$ m

Thus, $f = \dfrac{v}{\lambda} = \dfrac{156.5\text{ m/s}}{10\text{ m}} = \boxed{15.7\text{ Hz}}$

P18.19 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$, and the frequency is $f = \frac{v}{\lambda}$

Thus, $$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$$

and also $$f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$$

Thus, $$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\text{ kg})g}{(16.0\text{ kg})g}} = \frac{5}{4}$$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $$f = \frac{4}{2(2.00\text{ m})}\sqrt{\frac{(25.0\text{ kg})(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}} = \boxed{350\text{ Hz}}$$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$ so $$350\text{ Hz} = \frac{1}{2(2.00\text{ m})}\sqrt{\frac{m(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}}$$

yielding $m = \boxed{400\text{ kg}}$

P18.20 For the whole string vibrating, $d_{NN} = 0.64\text{ m} = \frac{\lambda}{2}$; $\lambda = 1.28\text{ m}$

The speed of a pulse on the string is

$v = f\lambda = 330\frac{1}{s}1.28\text{ m} = 422\text{ m/s}$

(a) When the string is stopped at the fret,

$d_{NN} = \frac{2}{3}0.64\text{ m} = \frac{\lambda}{2}$; $\lambda = 0.853\text{ m}$

$$f = \frac{v}{\lambda} = \frac{422\text{ m/s}}{0.853\text{ m}} = \boxed{495\text{ Hz}}$$

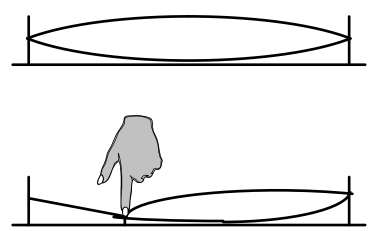

FIG. P18.20(a)

(b) The light touch at a point one third of the way along the string damps out vibration in the two lowest vibration states of the string as a whole. The whole string vibrates in its third resonance possibility: $3d_{NN} = 0.64\text{ m} = 3\frac{\lambda}{2}$

$\lambda = 0.427\text{ m}$

$$f = \frac{v}{\lambda} = \frac{422\text{ m/s}}{0.427\text{ m}} = \boxed{990\text{ Hz}}$$

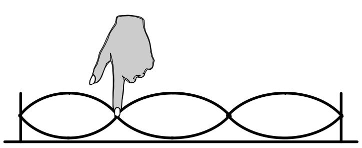

FIG. P18.20(b)

P18.21 $d_{NN} = 0.700\text{ m} = \lambda/2$

$\lambda = 1.40\text{ m}$

$$f\lambda = v = 308\text{ m/s} = \sqrt{\frac{T}{(1.20\times10^{-3})/(0.700)}}$$

(a) $T = \boxed{163\text{ N}}$

(b) With one-third the distance between nodes, the frequency is $f_3 = 3\cdot 220\text{ Hz} = \boxed{660\text{ Hz}}$

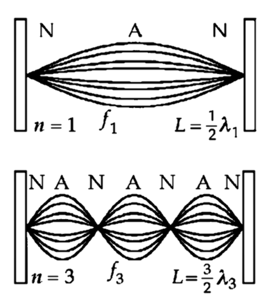

FIG. P18.21

P18.22 $\lambda_G = 2(0.350 \text{ m}) = \dfrac{v}{f_G}$; $\lambda_A = 2L_A = \dfrac{v}{f_A}$

$$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G\left(1 - \frac{f_G}{f_A}\right) = (0.350 \text{ m})\left(1 - \frac{392}{440}\right) = 0.038\,2 \text{ m}$$

Thus, $L_A = L_G - 0.038\,2 \text{ m} = 0.350 \text{ m} - 0.038\,2 \text{ m} = 0.312 \text{ m}$, or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A}\sqrt{\frac{T}{\mu}};\ dL_A = \frac{dT}{4f_A\sqrt{T\mu}};\ \frac{dL_A}{L_A} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600 \text{ cm}}{(35.0 - 3.82) \text{ cm}} = \boxed{3.84\%}$$

P18.23 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos\theta} \text{ or } \lambda = \frac{2L}{\cos\theta}$$

Since the fundamental frequency is f, the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos\theta}$$

Also,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/\overline{AB}}} = \sqrt{\frac{TL}{m\cos\theta}}$$

where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta} = \sqrt{\frac{TL}{m\cos\theta}} \quad \text{or} \quad \frac{4L^2f^2}{\cos^2\theta} = \frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m = \frac{T\cos\theta}{4Lf^2} \qquad [1]$$

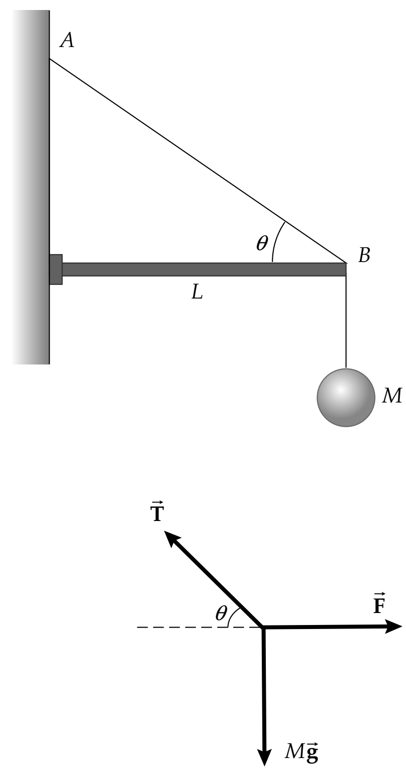

FIG. P18.23

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\sum F_y = T\sin\theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin\theta}$$

Then, from Equation [1], the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2\tan\theta}}$$

P18.24 Let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho V g$. The water exerts a buoyant force $\rho_{\text{water}}\left(\frac{V}{2}\right)g$ on the cylinder, to reduce the tension to

$$T_2 = \rho V g - \rho_{\text{water}}\left(\frac{V}{2}\right)g = \left(\rho - \frac{\rho_{\text{water}}}{2}\right)Vg$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \sqrt{\frac{T_1}{\mu}}\frac{1}{\lambda} \quad \text{to} \quad f_2 = \sqrt{\frac{T_2}{\mu}}\frac{1}{\lambda}$$

where we assume $\lambda = 2L$ is constant.

Then

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{\rho - \rho_{\text{water}}/2}{\rho}} = \sqrt{\frac{8.92 - 1.00/2}{8.92}}$$

$$f_2 = 300\ \text{Hz}\sqrt{\frac{8.42}{8.92}} = \boxed{291\ \text{Hz}}$$

P18.25 Comparing $y = (0.002\ \text{m})\sin((\pi\ \text{rad/m})x)\cos((100\,\pi\ \text{rad/s})t)$

with $y = 2A\sin kx\cos\omega t$

we find $k = \frac{2\pi}{\lambda} = \pi\ \text{m}^{-1}$, $\lambda = 2.00$ m, and $\omega = 2\pi f = 100\pi\ \text{s}^{-1}$: $f = 50.0$ Hz

(a) Then the distance between adjacent nodes is $d_{\text{NN}} = \frac{\lambda}{2} = 1.00$ m

and on the string are $\frac{L}{d_{\text{NN}}} = \frac{3.00\ \text{m}}{1.00\ \text{m}} = \boxed{3\ \text{loops}}$

For the speed we have $v = f\lambda = (50\ \text{s}^{-1})2\ \text{m} = 100\ \text{m/s}$

(b) In the simplest standing wave vibration, $d_{\text{NN}} = 3.00\ \text{m} = \frac{\lambda_b}{2}$, $\lambda_b = 6.00$ m

and

$$f_b = \frac{v_a}{\lambda_b} = \frac{100\ \text{m/s}}{6.00\ \text{m}} = \boxed{16.7\ \text{Hz}}$$

(c) In $v_0 = \sqrt{\frac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100\ \text{m/s}) = 300\ \text{m/s}$$

Then

$$\lambda_c = \frac{v_c}{f_a} = \frac{300\ \text{m/s}}{50\ \text{s}^{-1}} = 6.00\ \text{m} \qquad d_{\text{NN}} = \frac{\lambda_c}{2} = 3.00\ \text{m}$$

and $\boxed{\text{one}}$ loop fits onto the string.

Section 18.4 **Resonance**

P18.26 The wave speed is $v = \sqrt{gd} = \sqrt{(9.80\ \text{m/s}^2)(36.1\ \text{m})} = 18.8\ \text{m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P18.27.

Then, $d_{\text{NA}} = 210 \times 10^3\ \text{m} = \dfrac{\lambda}{4}$

and $\lambda = 840 \times 10^3\ \text{m}$

Therefore, the period is $T = \dfrac{1}{f} = \dfrac{\lambda}{v} = \dfrac{840 \times 10^3\ \text{m}}{18.8\ \text{m/s}} = 4.47 \times 10^4\ \text{s} = \boxed{12\ \text{h}\ 24\ \text{min}}$

The natural frequency of the water sloshing in the bay agrees precisely with that of lunar excitation, so we identify the extra-high tides as amplified by resonance.

P18.27 (a) The wave speed is $v = \dfrac{9.15\ \text{m}}{2.50\ \text{s}} = \boxed{3.66\ \text{m/s}}$

(b) From the figure, there are antinodes at both ends of the pond, so the distance between adjacent antinodes

is $d_{\text{AA}} = \dfrac{\lambda}{2} = 9.15\ \text{m}$

and the wavelength is $\lambda = 18.3\ \text{m}$

The frequency is then $f = \dfrac{v}{\lambda} = \dfrac{3.66\ \text{m/s}}{18.3\ \text{m}} = \boxed{0.200\ \text{Hz}}$

We have assumed the wave speed is the same for all wavelengths.

P18.28 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{\text{NN}} = d_{\text{AA}} = \frac{\lambda}{2} = \frac{20.0\ \text{cm}}{4} = 5.00\ \text{cm}$$

so

$$\lambda = 10.0\ \text{cm}$$

and

$$f = \frac{v}{\lambda} = \frac{900\ \text{m/s}}{0.100\ \text{m}} = 9\ 000\ \text{Hz} = \boxed{9.00\ \text{kHz}}$$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

Section 18.5 **Standing Waves in Air Columns**

P18.29 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \dfrac{v}{4f}$

So,

$$L = \frac{343\ \text{m/s}}{4\left(240\ \text{s}^{-1}\right)} = \boxed{0.357\ \text{m}}$$

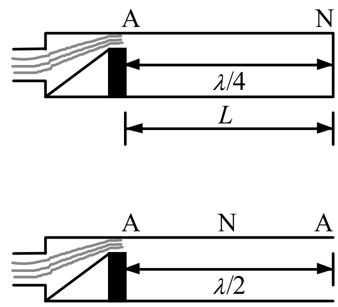

FIG. P18.29

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So,

$$L = \frac{v}{2f} = \frac{343\ \text{m/s}}{2\left(240\ \text{s}^{-1}\right)} = \boxed{0.715\ \text{m}}$$

P18.30 $d_{\text{AA}} = 0.320$ m; $\lambda = 0.640$ m

(a) $f = \dfrac{v}{\lambda} = \boxed{531\ \text{Hz}}$

(b) $\lambda = v/f = 0.085\,0$ m; $d_{\text{AA}} = \boxed{42.5\ \text{mm}}$

P18.31 The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{261.6/\text{s}} = 1.31$ m

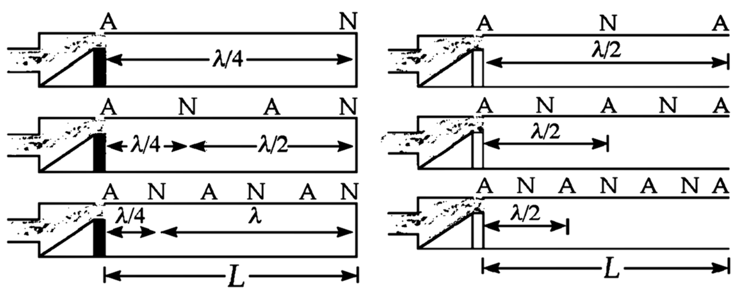

FIG. P18.31

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{\text{A to A}} = \frac{1}{2}\lambda = \boxed{0.656\ \text{m}}$$

A closed pipe has (N-A) for its simplest resonance,
(N-A-N-A) for the second,
and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{\text{N to A}} = \dfrac{5\lambda}{4} = \dfrac{5}{4}(1.31\ \text{m}) = \boxed{1.64\ \text{m}}$

P18.32 The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

with $d_{\text{N to A}} = 3\ \text{cm} = \dfrac{\lambda}{4}$

so $\lambda = 0.12$ m

and $f = \dfrac{v}{\lambda} = \dfrac{343\ \text{m/s}}{0.12\ \text{m}} \approx \boxed{3\ \text{kHz}}$

A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P18.33 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{1}{2}n\lambda, (n = 1, 2, 3, \ldots)$.

i.e.,

$$L = \frac{n\lambda}{2} = \frac{nv}{2f} \qquad \text{and} \qquad f = \frac{nv}{2L}$$

Therefore, with $L = 0.860$ m and $L' = 2.10$ m, the resonant frequencies are $f_n = \boxed{n(206 \text{ Hz})}$ for $L = 0.860$ m for each n from 1 to 9 and $f_n' = \boxed{n(84.5 \text{ Hz})}$ for $L' = 2.10$ m for each n from 2 to 23.

P18.34 The wavelength of sound is $\lambda = \frac{v}{f}$

The distance between water levels at resonance is $d = \frac{v}{2f}$ $\quad \therefore Rt = \pi r^2 d = \frac{\pi r^2 v}{2f}$

and $t = \boxed{\frac{\pi r^2 v}{2Rf}}$

P18.35 For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0 \text{ Hz}}$. Then the pipe length is $d_{NA} = \frac{\lambda}{4} = \frac{v}{4f} = \frac{340 \text{ m/s}}{4(50/\text{s})} = \boxed{1.70 \text{ m}}$.

***P18.36** (a) The open ends of the tunnel are antinodes, so $d_{AA} = 2000$ m/n with $n = 1, 2, 3, \ldots$. Then $\lambda = 2d_{AA} = 4000$ m/n. And $f = v/\lambda = (343 \text{ m/s})/(4000 \text{ m}/n) =$

$\boxed{0.0858\, n \text{ Hz, with } n = 1, 2, 3, \ldots}$

(b) It is a good rule. Any car horn would produce several or many of the closely-spaced resonance frequencies of the air in the tunnel, so it would be greatly amplified. Other drivers might be frightened directly into dangerous behavior, or might blow their horns also.

P18.37 For resonance in a narrow tube open at one end,

$$f = n\frac{v}{4L} \qquad (n = 1, 3, 5, \ldots)$$

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \qquad \text{and} \qquad 384 = \frac{3v}{4(0.683)}$$

In either case, $v = \boxed{350 \text{ m/s}}$.

(b) For the next resonance $n = 5$, and

$$L = \frac{5v}{4f} = \frac{5(350 \text{ m/s})}{4(384 \text{ s}^{-1})} = \boxed{1.14 \text{ m}}$$

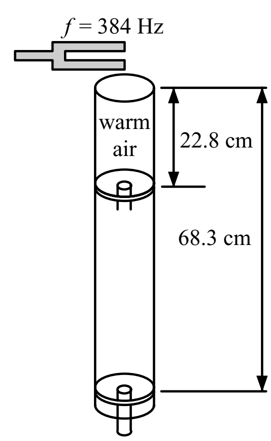

FIG. P18.37

P18.38 The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$: $L_1 = \frac{v}{4f} = \frac{34}{4(512)} = 0.167\text{ m}$.

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to

$$f = \frac{3v}{4L_2} \text{ and } f = \frac{5v}{4L_3}$$

or

$$L_2 = \frac{3v}{4f} = \boxed{0.502\text{ m}} \qquad \text{and} \qquad L_3 = \frac{5v}{4f} = \boxed{0.837\text{ m}}$$

***P18.39** Call L the depth of the well and v the speed of sound.

Then for some integer n $$L = (2n-1)\frac{\lambda_1}{4} = (2n-1)\frac{v}{4f_1} = \frac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})}$$

and for the next resonance $$L = [2(n+1)-1]\frac{\lambda_2}{4} = (2n+1)\frac{v}{4f_2} = \frac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$$

Thus, $$\frac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})} = \frac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$$

and we require an *integer* solution to $$\frac{2n+1}{60.0} = \frac{2n-1}{51.5}$$

The equation gives $n = \frac{111.5}{17} = 6.56$, so the best fitting integer is $n = 7$.

Then the results $$L = \frac{[2(7)-1](343\text{ m/s})}{4(51.5\text{ s}^{-1})} = 21.6\text{ m}$$

and $$L = \frac{[2(7)+1](343\text{ m/s})}{4(60.0\text{ s}^{-1})} = 21.4\text{ m}$$

suggest that we can say

the depth of the well is (21.5 ± 0.1) m. The data suggest 0.6-Hz uncertainty in the frequency measurements, which is only a little more than 1%.

P18.40 (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(880\text{ s}^{-1})} = \boxed{0.195\text{ m}}$$

(b) $v = 331\text{ m/s}\sqrt{1 + \frac{(-5.00)}{273}} = 328\text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328\text{ m/s}}{2(0.195\text{ m})} = \boxed{841\text{ Hz}}$$

The flute is flat by a semitone.

Section 18.6 **Standing Waves in Rod and Membranes**

P18.41 (a) $f = \frac{v}{2L} = \frac{5\,100}{(2)(1.60)} = \boxed{1.59 \text{ kHz}}$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) $f = \frac{v'}{2L} = \frac{3\,560}{(2)(1.60)} = \boxed{1.11 \text{ kHz}}$

P18.42 When the rod is clamped at one-quarter of its length, the vibration pattern reads ANANA and the rod length is $L = 2d_{\text{AA}} = \lambda$.

Therefore,

$$L = \frac{v}{f} = \frac{5\,100 \text{ m/s}}{4\,400 \text{ Hz}} = \boxed{1.16 \text{ m}}$$

Section 18.7 **Beats: Interference in Time**

P18.43 $f \propto v \propto \sqrt{T}$ $\qquad f_{\text{new}} = 110\sqrt{\frac{540}{600}} = 104.4 \text{ Hz}$

$\Delta f = 110/\text{s} - 104.4/\text{s} = \boxed{5.64 \text{ beats/s}}$

P18.44 (a) The string could be tuned to either $\boxed{521 \text{ Hz or } 525 \text{ Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526 \text{ Hz}}$.

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{and} \quad T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523 \text{ Hz}}{526 \text{ Hz}}\right)^2 T_1 = 0.989T_1$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.011\,4 = 1.14\% \text{ lower}$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.

P18.45 For an echo $f' = f\frac{(v + v_s)}{(v - v_s)}$ the beat frequency is $f_b = |f' - f|$.

Solving for f_b, gives $f_b = f\frac{(2v_s)}{(v - v_s)}$ when approaching wall.

(a) $f_b = (256)\frac{2(1.33)}{(343 - 1.33)} = \boxed{1.99 \text{ Hz}}$ beat frequency

(b) When he is moving away from the wall, v_s changes sign. Solving for v_s gives

$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256) - 5} = \boxed{3.38 \text{ m/s}}$$

P18.46 Using the $\boxed{4 \text{ and } 2\frac{2}{3}\text{-foot pipes}}$ produces actual frequencies of 131 Hz and 196 Hz and a combination tone at $(196-131)\,\text{Hz} = 65.4$ Hz, so this pair supplies the so-called missing fundamental. The 4 and 2-foot pipes produce a combination tone $(262-131)\,\text{Hz} = 131$ Hz, so this does not work.

The $\boxed{2\frac{2}{3} \text{ and 2-foot pipes}}$ produce a combination tone at $(262-196)\,\text{Hz} = 65.4$ Hz, so this works.

Also, $\boxed{4,\ 2\frac{2}{3}, \text{ and 2-foot pipes}}$ all playing together produce the 65.4-Hz combination tone.

Section 18.8 **Nonsinusoidal Wave Patterns**

P18.47 We list the frequencies of the harmonics of each note in Hz:

	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1 320.0	1 760.0	2 200.0
C#	554.37	1 108.7	1 663.1	2 217.5	2 771.9
E	659.26	1 318.5	1 977.8	2 637.0	3 296.3

$\boxed{\text{The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C\# is close to the fifth harmonic of A.}}$

P18.48 We evaluate

$$s = 100\sin\theta + 157\sin 2\theta + 62.9\sin 3\theta + 105\sin 4\theta + 51.9\sin 5\theta + 29.5\sin 6\theta + 25.3\sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:

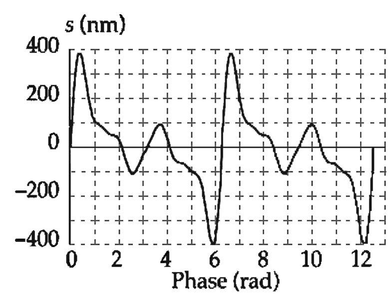

FIG. P18.48

Additional Problems

***P18.49** (a) The yo-yo's downward speed is $dL/dt = 0 + (0.8\text{ m/s}^2)(1.2\text{ s}) = 0.960$ m/s. The instantaneous wavelength of the fundamental string wave is given by $d_{NN} = \lambda/2 = L$ so $\lambda = 2L$ and $d\lambda/dt = 2\,dL/dt = 2(0.96\text{ m/s}) = 1.92$ m/s.

(b) For the second harmonic, the wavelength is equal to the length of the string. Then the rate of change of wavelength is equal to $dL/dt = \boxed{0.960\text{ m/s, half as much as for the first harmonic}}$.

(c) A yo-yo of different mass will hold the string under different tension to make each string wave vibrate with a different frequency, but the geometrical argument given in parts (a) and (b) still applies to the wavelength. $\boxed{\text{The answers are unchanged}}$: $d\lambda_1/dt = 1.92$ m/s and $d\lambda_2/dt = 0.960$ m/s.

***P18.50** (a) Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}$$

With f_1' = frequency of the speaker in front of student and

f_2' = frequency of the speaker behind the student.

$$f_1' = (456\text{ Hz})\frac{(343\text{ m/s} + 1.50\text{ m/s})}{(343\text{ m/s} - 0)} = 458\text{ Hz}$$

$$f_2' = (456\text{ Hz})\frac{(343\text{ m/s} - 1.50\text{ m/s})}{(343\text{ m/s} + 0)} = 454\text{ Hz}$$

Therefore, $f_b = f_1' - f_2' = \boxed{3.99\text{ Hz}}$

(b) The waves broadcast by both speakers have $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{456/\text{s}} = 0.752\text{ m}$. The standing wave between them has $d_{AA} = \dfrac{\lambda}{2} = 0.376\text{ m}$. The student walks from one maximum to the next in time $\Delta t = \dfrac{0.376\text{ m}}{1.50\text{ m/s}} = 0.251\text{ s}$, so the frequency at which she hears maxima is $f = \dfrac{1}{T} = \boxed{3.99\text{ Hz}}$

(c) The answers are identical. The models are equally valid. We may think of the interference of the two waves as interference in space or in time, linked to space by the steady motion of the student.

P18.51 $f = 87.0\text{ Hz}$

speed of sound in air: $v_a = 340\text{ m/s}$

(a) $\lambda_b = \ell$ $\quad v = f\lambda_b = (87.0\text{ s}^{-1})(0.400\text{ m})$

$v = \boxed{34.8\text{ m/s}}$

(b) $\left.\begin{array}{l}\lambda_a = 4L \\ v_a = \lambda_a f\end{array}\right\}$ $\quad L = \dfrac{v_a}{4f} = \dfrac{340\text{ m/s}}{4(87.0\text{ s}^{-1})} = \boxed{0.977\text{ m}}$

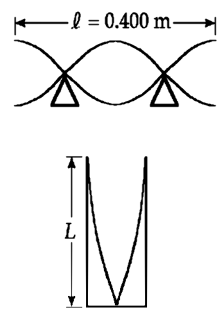

FIG. P18.51

P18.52

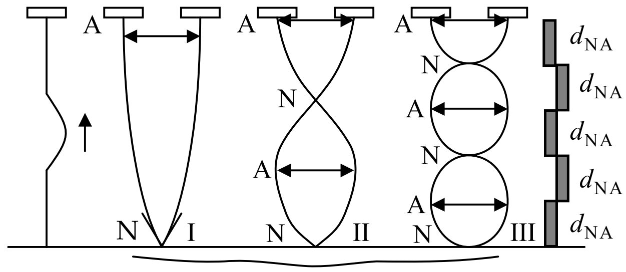

FIG. P18.52

(a) $\mu = \dfrac{5.5\times10^{-3}\ \text{kg}}{0.86\ \text{m}} = 6.40\times10^{-3}\ \ \text{kg/m}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1.30\ \text{kg}\cdot\text{m/s}^2}{6.40\times10^{-3}\ \ \text{kg/m}}} = \boxed{14.3\ \ \text{m/s}}$$

(b) In state I, $d_{NA} = \boxed{0.860\ \text{m}} = \dfrac{\lambda}{4}$

(c) $\lambda = 3.44\ \text{m}$ $\quad f = \dfrac{v}{\lambda} = \dfrac{14.3\ \ \text{m/s}}{3.44\ \text{m}} = \boxed{4.14\ \text{Hz}}$

In state II, $d_{NA} = \dfrac{1}{3}(0.86\ \text{m}) = \boxed{0.287\ \text{m}}$

$$\lambda = 4(0.287\ \text{m}) = 1.15\ \text{m} \qquad f = \frac{v}{\lambda} = \frac{14.3\ \ \text{m/s}}{1.15\ \text{m}} = \boxed{12.4\ \text{Hz}}$$

In state III, $d_{NA} = \dfrac{1}{5}(0.86\ \text{m}) = \boxed{0.172\ \text{m}}$

$$f = \frac{v}{\lambda} = \frac{14.3\ \ \text{m/s}}{4(0.172\ \text{m})} = \boxed{20.7\ \text{Hz}}$$

P18.53 If the train is moving away from station, its frequency is depressed:

$$f' = 180 - 2.00 = 178\ \text{Hz}: \qquad 178 = 180\frac{343}{343-(-v)}$$

Solving for v gives $\quad v = \dfrac{(2.00)(343)}{178}$

Therefore, $\quad v = \boxed{3.85\ \ \text{m/s}\ \text{away from station}}$

If it is moving toward the station, the frequency is enhanced:

$$f' = 180 + 2.00 = 182\ \text{Hz}: \qquad 182 = 180\frac{343}{343-v}$$

Solving for v gives $\quad 4 = \dfrac{(2.00)(343)}{182}$

Therefore, $\quad v = \boxed{3.77\ \ \text{m/s}\ \text{toward the station}}$

P18.54 $v = \sqrt{\dfrac{(48.0)(2.00)}{4.80\times10^{-3}}} = 141\ \ \text{m/s}$

$d_{NN} = 1.00\ \text{m};\ \lambda = 2.00\ \text{m};\ f = \dfrac{v}{\lambda} = 70.7\ \text{Hz}$

$\lambda_a = \dfrac{v_a}{f} = \dfrac{343\ \ \text{m/s}}{70.7\ \text{Hz}} = \boxed{4.85\ \text{m}}$

P18.55 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}=\frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00\times10^{-3}}}=\boxed{59.9\text{ Hz}}$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire. $\mu'=8.00\text{ g/m}$

so $$L'=\frac{1}{2f}\sqrt{\frac{T}{\mu'}}$$

$$L'=\left[\frac{1}{(2)(59.9)}\right]\sqrt{\frac{4.60}{8.00\times10^{-3}}}=\boxed{20.0\text{ cm}}$$ half the length of the thin wire.

***P18.56** The wavelength stays constant at 0.96 m while the wavespeed rises according to $v=(T/\mu)^{1/2}=[(15+2.86t)/0.0016]^{1/2}=[9375+1786t]^{1/2}$ so the frequency rises as $f=v/\lambda=[9375+1786t]^{1/2}/0.96=[10\,173+1938t]^{1/2}$ The number of cycles is $f\,dt$ in each incremental bit of time, or altogether

$$\int_0^{3.5}(10173+1938t)^{1/2}\,dt=\frac{1}{1938}\int_0^{3.5}(10173+1938t)^{1/2}\,1938\,dt$$

$$=\frac{1}{1938}\left.\frac{[10173+1938t]^{3/2}}{3/2}\right|_0^{3.5}=\frac{(16954)^{3/2}-(10173)^{3/2}}{2906}=\boxed{407\text{ cycles}}$$

P18.57 (a) $$f=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

so $$\frac{f'}{f}=\frac{L}{L'}=\frac{L}{2L}=\frac{1}{2}$$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $$\frac{n'}{n}=\sqrt{\frac{T}{T'}}\qquad\text{so}\qquad\frac{T'}{T}=\left(\frac{n}{n'}\right)^2=\left[\frac{n}{n+1}\right]^2$$

The tension must be $$T'=\boxed{\left[\frac{n}{n+1}\right]^2T}$$

(c) $$\frac{f'}{f}=\frac{n'L}{nL'}\sqrt{\frac{T'}{T}}\qquad\text{so}\qquad\frac{T'}{T}=\left(\frac{nf'L'}{n'fL}\right)^2$$

$$\frac{T'}{T}=\left(\frac{3}{2\cdot2}\right)^2\qquad\boxed{\frac{T'}{T}=\frac{9}{16}}$$ to get twice as many antinodes.

P18.58 (a) For the block:

$$\sum F_x = T - Mg\sin 30.0° = 0$$

so $\quad T = Mg\sin 30.0° = \boxed{\frac{1}{2}Mg}$

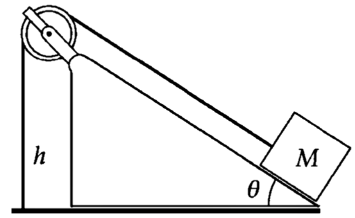

FIG. P18.58

(b) The length of the section of string parallel to the incline is $\frac{h}{\sin 30.0°} = 2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\quad \mu = \boxed{\frac{m}{3h}}$

(d) The speed of waves in the string is $\quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{\text{NN}} = \frac{\lambda}{2} = h$, so the wavelength is $\lambda = 2h$.

The frequency is $\quad f = \frac{v}{\lambda} = \frac{1}{2h}\sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\quad \lambda = \boxed{h}$

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h\sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $\quad f_b = 1.02f - f = \left(2.00\times 10^{-2}\right)f = \boxed{\left(2.00\times 10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}}$

P18.59 We look for a solution of the form

$$5.00\sin(2.00x - 10.0t) + 10.0\cos(2.00x - 10.0t)$$
$$= A\sin(2.00x - 10.0t + \phi)$$
$$= A\sin(2.00x - 10.0t)\cos\phi + A\cos(2.00x - 10.0t)\sin\phi$$

This will be true if both $\quad 5.00 = A\cos\phi$ and $10.0 = A\sin\phi$,

requiring $\quad (5.00)^2 + (10.0)^2 = A^2$

$\quad A = 11.2$ and $\phi = 63.4°$

The resultant wave $\boxed{11.2\sin(2.00x - 10.0t + 63.4°)}$ is sinusoidal.

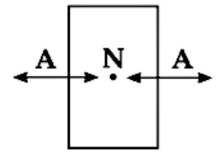

FIG. P18.60

P18.60 $d_{AA} = \dfrac{\lambda}{2} = 7.05 \times 10^{-3}$ m is the distance between antinodes.

Then $\lambda = 14.1 \times 10^{-3}$ m

and $f = \dfrac{v}{\lambda} = \dfrac{3.70 \times 10^3 \text{ m/s}}{14.1 \times 10^{-3} \text{ m}} = \boxed{2.62 \times 10^5 \text{ Hz}}$

The crystal can be tuned to vibrate at 2^{18} Hz, so that binary counters can derive from it a signal at precisely 1 Hz.

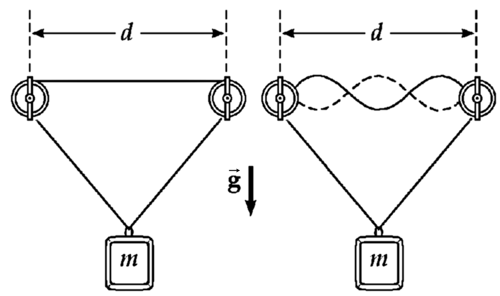

FIG. P18.61

P18.61 (a) Let θ represent the angle each slanted rope makes with the vertical.

In the diagram, observe that:

$$\sin\theta = \frac{1.00 \text{ m}}{1.50 \text{ m}} = \frac{2}{3} \quad \text{or} \quad \theta = 41.8°$$

Considering the mass,

$$\sum F_y = 0: \quad 2T\cos\theta = mg$$

or $T = \dfrac{(12.0 \text{ kg})(9.80 \text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9 \text{ N}}$

(b) The speed of transverse waves in the string is $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{78.9 \text{ N}}{0.001\,00 \text{ kg/m}}} = 281 \text{ m/s}$

For the standing wave pattern shown (3 loops), $d = \dfrac{3}{2}\lambda$

or $\lambda = \dfrac{2(2.00 \text{ m})}{3} = 1.33 \text{ m}$

Thus, the required frequency is $f = \dfrac{v}{\lambda} = \dfrac{281 \text{ m/s}}{1.33 \text{ m}} = \boxed{211 \text{ Hz}}$

ANSWERS TO EVEN PROBLEMS

P18.2 see the solution

P18.4 5.66 cm

P18.6 (a) 3.33 rad (b) 283 Hz

P18.8 (a) The number is the greatest integer $\le d\left(\dfrac{f}{v}\right) + \dfrac{1}{2}$ (b) $L_n = \dfrac{d^2 - (n - 1/2)^2 (v/f)^2}{2(n - 1/2)(v/f)}$ where $n = 1, 2, \ldots, n_{max}$

P18.10 (a) Point A is one half wavelength farther from one speaker than from the other. The waves it receives interfere destructively. (b) Along the hyperbola $9x^2 - 16y^2 = 144$. (c) Yes; along the straight line through the origin with slope 0.75 or the straight line through the origin with slope –0.75.

P18.12 see the solution

P18.14 (a) see the solution (b) 4 m is the distance between crests. (c) 50 Hz. The oscillation at any point starts to repeat after a period of 20 ms, and $f = 1/T$. (d) 4 m. By comparison with equation 18.3, $k = \pi/2$, and $\lambda = 2\pi/k$. (e) 50 Hz. By comparison with equation 18.3, $\omega = 2\pi f = 100\pi$.

P18.16 (a) Yes. The resultant wave contains points of no motion. (b) and (c) The nodes are still separated by $\lambda/2$. They are all shifted by the distance $\phi/2k$ to the left.

P18.18 15.7 Hz

P18.20 (a) 495 Hz (b) 990 Hz

P18.22 31.2 cm from the bridge; 3.84%

P18.24 291 Hz

P18.26 The natural frequency of the water sloshing in the bay agrees precisely with that of lunar excitation, so we identify the extra-high tides as amplified by resonance.

P18.28 9.00 kHz

P18.30 (a) 531 Hz (b) 42.5 mm

P18.32 3 kHz; a small-amplitude external excitation at this frequency can, over times, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P18.34 $\Delta t = \dfrac{\pi r^2 v}{2Rf}$

P18.36 (a) 0.0858 n Hz, with $n = 1, 2, 3, \ldots$ (b) It is a good rule. Any car horn would produce several or many of the closely-spaced resonance frequencies of the air in the tunnel, so it would be greatly amplified. Other drivers might be frightened directly into dangerous behavior, or might blow their horns also.

P18.38 0.502 m; 0.837 m

P18.40 (a) 0.195 m (b) 841 m

P18.42 1.16 m

P18.44 (a) 521 Hz or 525 Hz (b) 526 Hz (c) reduce by 1.14%

P18.46 4-foot and $2\frac{2}{3}$-foot; $2\frac{2}{3}$ and 2-foot; and all three together

P18.48 see the solution

P18.50 (a) 3.99 beats/s (b) 3.99 beats/s (c) The answers are identical. The models are equally valid. We may think of the interference of the two waves as interference in space or in time, linked to space by the steady motion of the student.

P18.52 (a) 14.3 m/s (b) 86.0 cm, 28.7 cm, 17.2 cm (c) 4.14 Hz, 12.4 Hz, 20.7 Hz

P18.54 4.85 m

P18.56 407 cycles

P18.58 (a) $\frac{1}{2}Mg$ (b) $3h$ (c) $\dfrac{m}{3h}$ (d) $\sqrt{\dfrac{3Mgh}{2m}}$ (e) $\sqrt{\dfrac{3Mg}{8mh}}$ (f) $\sqrt{\dfrac{2mh}{3Mg}}$ (g) h (h) $\left(2.00\times10^{-2}\right)\sqrt{\dfrac{3Mg}{8mh}}$

P18.60 262 kHz