39

Relativity

Note: In chapters 39, 40, and 41 we use u to represent the speed of a particle with mass, reserving v for the speeds associated with reference frames, wave functions, and photons.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q39.1 No. The principle of relativity implies that nothing can travel faster than the speed of light in a vacuum, which is 300 Mm/s. The electron would emit light in a conical shock wave of Cerenkov radiation.

***Q39.2** Answer (c). The dimension parallel to the direction of motion is reduced by the factor γ and the other dimensions are unchanged.

***Q39.3** Answer (c). An oblate spheroid. The dimension in the direction of motion would be measured to be scrunched in.

***Q39.4** Answer (e). The relativistic time dilation effect is symmetric between the observers.

Q39.5 Suppose a railroad train is moving past you. One way to measure its length is this: You mark the tracks at the cowcatcher forming the front of the moving engine at 9:00:00 AM, while your assistant marks the tracks at the back of the caboose at the same time. Then you find the distance between the marks on the tracks with a tape measure. You and your assistant must make the marks simultaneously in your frame of reference, for otherwise the motion of the train would make its length different from the distance between marks.

***Q39.6** (i) Answer (c). The Earth observer measures the clock in orbit to run slower.

(ii) Answer (b). They are not synchronized. They both tick at the same rate after return, but a time difference has developed between the two clocks.

Q39.7 (a) Yours does.

(b) His does.

(c) If the velocity of relative motion is constant, both observers have equally valid views.

Q39.8 Get a *Mr. Tompkins* book by George Gamow for a wonderful fictional exploration of this question. Driving home in a hurry, you push on the gas pedal not to increase your speed by very much, but rather to make the blocks get shorter. Big Doppler shifts in wave frequencies make red lights look green as you approach them and make car horns and car radios useless. High-speed transportation is very expensive, requiring huge fuel purchases. And it is dangerous, as a speeding car can knock down a building. Having had breakfast at home, you return hungry for lunch, but you find you have missed dinner. There is a five-day delay in transmission when you watch the Olympics in Australia on live television. It takes ninety-five years for sunlight to reach Earth. We cannot see the Milky Way; the fireball of the Big Bang surrounds us at the distance of Rigel or Deneb.

Q39.9 By a curved line. This can be seen in the middle of Speedo's world-line in Figure 39.11, where he turns around and begins his trip home.

Q39.10 A microwave pulse is reflected from a moving object. The waves that are reflected back are Doppler shifted in frequency according to the speed of the target. The receiver in the radar gun detects the reflected wave and compares its frequency to that of the emitted pulse. Using the frequency shift, the speed can be calculated to high precision. Be forewarned: this technique works if you are either traveling toward or away from your local law enforcement agent!

Q39.11 This system would be seen as a star moving in an elliptical path. Just like the light from a star in a binary star system, the spectrum of light from the star would undergo a series of Doppler shifts depending on the star's speed and direction of motion relative to the observer. The repetition rate of the Doppler shift pattern is the period of the orbit. Information about the orbit size can be calculated from the size of the Doppler shifts.

Q39.12 According to $\vec{\mathbf{p}} = \gamma m\vec{\mathbf{u}}$, doubling the speed u will make the momentum of an object increase by the factor $2\left[\dfrac{c^2 - u^2}{c^2 - 4u^2}\right]^{1/2}$.

***Q39.13** From $E^2 = p^2c^2 + m^2c^4 = (mc^2 + K)^2$ we consider $pc = \sqrt{2Kmc^2 + K^2}$. For a photon, $pc = E$. For particles with mass, the greater the mass the greater the momentum if K is always 1 MeV. The ranking is d > b > c > a.

Q39.14 As the object approaches the speed of light, its kinetic energy grows without limit. It would take an infinite investment of work to accelerate the object to the speed of light.

***Q39.15** (i) Answer (a).

(ii) (c) and (iii) (d). There is no upper limit on the momentum or energy of an electron. As more energy E is fed into the object without limit, its speed approaches the speed of light and its momentum approaches $\dfrac{E}{c}$.

***Q39.16** Answer (b). Quasar light moves at three hundred million meters per second, just like the light from a firefly at rest.

Q39.17 Any physical theory must agree with experimental measurements within some domain. Newtonian mechanics agrees with experiment for objects moving slowly compared to the speed of light. Relativistic mechanics agrees with experiment for objects at all speeds. Thus the two theories must and do agree with each other for ordinary nonrelativistic objects. Both statements given in the question are formally correct, but the first is clumsily phrased. It seems to suggest that relativistic mechanics applies only to fast-moving objects.

Q39.18 The point of intersection moves to the right. To state the problem precisely, let us assume that each of the two cards moves toward the other parallel to the long dimension of the picture, with velocity of magnitude u. The point of intersection moves to the right at speed $\frac{2u}{\tan\phi} = 2u\cot\phi$, where ϕ is the small angle between the cards. As ϕ approaches zero, $\cot\phi$ approaches infinity. Thus the point of intersection can move with a speed faster than c if v is sufficiently large and ϕ sufficiently small. For example, take $u = 500\ \text{m/s}$ and $\phi = 0.000\,19°$. If you are worried about holding the cards steady enough to be sure of the angle, cut the edge of one card along a curve so that the angle will necessarily be sufficiently small at some place along the edge.

Let us assume the spinning flashlight is at the center of a grain elevator, forming a circular screen of radius R. The linear speed of the spot on the screen is given by $v = \omega R$, where ω is the angular speed of rotation of the flashlight. With sufficiently large ω and R, the speed of the spot moving on the screen can exceed c.

Neither of these examples violates the principle of relativity. Both cases are describing a point of intersection: in the first case, the intersection of two cards and in the second case, the intersection of a light beam with a screen. A point of intersection is not made of matter so it has no mass, and hence no energy. A bug momentarily at the intersection point could yelp, take a bite out of one card, or reflect the light. None of these actions would result in communication reaching another bug so soon as the intersection point reaches him. The second bug would have to wait for sound or light to travel across the distance between the first bug and himself, to get the message.

As a child, the author used an Erector set to build a superluminal speed generator using the intersecting-cards method. Can you get a visible dot to run across a computer screen faster than light? Want'a see it again?

Q39.19 Special relativity describes inertial reference frames: that is, reference frames that are not accelerating. General relativity describes all reference frames.

Q39.20 The downstairs clock runs more slowly because it is closer to the Earth and hence in a stronger gravitational field than the upstairs clock.

SOLUTIONS TO PROBLEMS

Section 39.1 **The Principle of Galilean Relativity**

P39.1 The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object $\vec{\mathbf{v}}_1$. The second observer has constant velocity $\vec{\mathbf{v}}_{21}$ relative to the first, and measures the object to have velocity $\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{21}$.

The second observer measures an acceleration of $\vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = \frac{d\vec{\mathbf{v}}_1}{dt}$.

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

P39.2 The laboratory observer notes Newton's second law to hold: $\vec{\mathbf{F}}_1 = m\vec{\mathbf{a}}_1$
(where the subscript 1 denotes the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $\vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_1 - \vec{\mathbf{a}}'$
(where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2$ or $\vec{\mathbf{F}}_1 = m\vec{\mathbf{a}}_2$

(since $\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2 - m\vec{\mathbf{a}}'$, which is *not* Newton's second law.

P39.3 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2\,000\text{ kg})(20.0\text{ m/s}) + (1\,500\text{ kg})(0\text{ m/s}) = 4.00\times10^4\text{ kg}\cdot\text{m/s}$$

$$p_f = (m_1 + m_2)v_f = (2\,000\text{ kg} + 1\,500\text{ kg})v_f$$

Since $p_i = p_f$, $\quad v_f = \dfrac{4.00\times10^4\text{ kg}\cdot\text{m/s}}{2\,000\text{ kg} + 1\,500\text{ kg}} = 11.429\text{ m/s}$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0\text{ m/s} - (+10.0\text{ m/s}) = 10.0\text{ m/s}$$

$$v'_{2i} = v_{2i} - v' = 0\text{ m/s} - (+10.0\text{ m/s}) = -10.0\text{ m/s}$$

$$v'_f = 11.429\text{ m/s} - (+10.0\text{ m/s}) = 1.429\text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2\,000\text{ kg})(10.0\text{ m/s}) + (1\,500\text{ kg})(-10.0\text{ m/s}) = 5\,000\text{ kg}\cdot\text{m/s}$$

and our final momentum has the same value:

$$p'_f = (2\,000\text{ kg} + 1\,500\text{ kg})v'_f = (3\,500\text{ kg})(1.429\text{ m/s}) = 5\,000\text{ kg}\cdot\text{m/s}$$

Section 39.2 **The Michelson-Morley Experiment**

Section 39.3 **Einstein's Principle of Relativity**

Section 39.4 **Consequences of the Special Theory of Relativity**

P39.4 $L = L_p\sqrt{1 - \dfrac{v^2}{c^2}}$ $\qquad v = c\sqrt{1 - \left(\dfrac{L}{L_p}\right)^2}$

Taking $L = \dfrac{L_p}{2}$ where $L_p = 1.00\text{ m}$ gives $v = c\sqrt{1 - \left(\dfrac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \dfrac{1}{4}} = \boxed{0.866c}$

P39.5 $\Delta t = \dfrac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ $\quad$ so $\quad v = c\left[1 - \left(\dfrac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$

For $\Delta t = 2\Delta t_p$ $\qquad v = c\left[1 - \left(\dfrac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \dfrac{1}{4}\right]^{1/2} = \boxed{0.866c}$

P39.6 (a) $\gamma = \dfrac{1}{\sqrt{1 - (v/c)^2}} = \dfrac{1}{\sqrt{1 - (0.500)^2}} = \dfrac{2}{\sqrt{3}}$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma\Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\text{ s}}{75.0}\right) = 0.924\text{ s}$$

Thus, the Earth observer records a pulse rate of $\dfrac{60.0\text{ s/min}}{0.924\text{ s}} = \boxed{64.9/\text{min}}$.

(b) At a relative speed $v = 0.990c$, the relativistic factor γ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the duration of the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

P39.7 $\Delta t = \gamma \Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-v^2/c^2}}$ so $\Delta t_p = \left(\sqrt{1-\dfrac{v^2}{c^2}}\right)\Delta t \cong \left(1-\dfrac{v^2}{2c^2}\right)\Delta t$

and $\Delta t - \Delta t_p = \left(\dfrac{v^2}{2c^2}\right)\Delta t$

If $v = 1\,000 \text{ km/h} = \dfrac{1.00\times 10^6 \text{ m}}{3\,600 \text{ s}} = 277.8 \text{ m/s}$

then $\dfrac{v}{c} = 9.26\times 10^{-7}$

and $(\Delta t - \Delta t_p) = (4.28\times 10^{-13})(3\,600 \text{ s}) = 1.54\times 10^{-9} \text{ s} = \boxed{1.54 \text{ ns}}$

P39.8 For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{4.60 \text{ km}}{0.990c}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60\times 10^3 \text{ m}}{0.990(3.00\times 10^8 \text{ m/s})}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $L = L_p\sqrt{1-\left(\dfrac{v}{c}\right)^2} = \dfrac{L_p}{\gamma} = \dfrac{4.60\times 10^3 \text{ m}}{7.09} = \boxed{649 \text{ m}}$

P39.9 The spaceship is measured by the Earth observer to be length-contracted to

$L = L_p\sqrt{1-\dfrac{v^2}{c^2}}$ or $L^2 = L_p^2\left(1-\dfrac{v^2}{c^2}\right)$

Also, the contracted length is related to the time required to pass overhead by:

$L = vt$ or $L^2 = v^2t^2 = \dfrac{v^2}{c^2}(ct)^2$

Equating these two expressions gives $L_p^2 - L_p^2\dfrac{v^2}{c^2} = (ct)^2\dfrac{v^2}{c^2}$

or $\left[L_p^2 + (ct)^2\right]\dfrac{v^2}{c^2} = L_p^2$

Using the given values: $L_p = 300 \text{ m}$ and $t = 7.50\times 10^{-7} \text{ s}$

this becomes $(1.41\times 10^5 \text{ m}^2)\dfrac{v^2}{c^2} = 9.00\times 10^4 \text{ m}^2$

giving $v = \boxed{0.800c}$

P39.10 (a) The spaceship is measured by Earth observers to be of length L, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad L = v\Delta t$$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Solving for v, $v^2\left(\Delta t^2 + \frac{L_p^2}{c^2}\right) = L_p^2 \qquad \boxed{v = \frac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}}$

(b) The tanks move nonrelativistically, so we have $v = \frac{300\text{ m}}{75\text{ s}} = \boxed{4.00\text{ m/s}}$.

(c) For the data in problem 9,

$$v = \frac{c(300\text{ m})}{\sqrt{(3\times10^8\text{ m/s})^2(0.75\times10^{-6}\text{ s})^2 + (300\text{ m})^2}} = \frac{c(300\text{ m})}{\sqrt{225^2+300^2}\text{ m}} = 0.800c$$

in agreement with problem 9. For the data in part (b),

$$v = \frac{c(300\text{ m})}{\sqrt{(3\times10^8\text{ m/s})^2(75\text{ s})^2 + (300\text{ m})^2}} = \frac{c(300\text{ m})}{\sqrt{(2.25\times10^{10})^2+300^2}\text{ m}}$$

$$= 1.33\times10^{-8}c = 4.00\text{ m/s}$$

in agreement with part (b).

P39.11 We find Cooper's speed: $\frac{GMm}{r^2} = \frac{mv^2}{r}$

Solving, $v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6 + 0.160\times10^6)}\right]^{1/2} = 7.82\text{ km/s}$

Then the time period of one orbit is $T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53\times10^6)}{7.82\times10^3} = 5.25\times10^3\text{ s}$

(a) The time difference for 22 orbits is $\Delta t - \Delta t_p = (\gamma-1)\Delta t_p = \left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right](22T)$

$$\Delta t - \Delta t_p \approx \left(1+\frac{1}{2}\frac{v^2}{c^2}-1\right)(22T) = \frac{1}{2}\left(\frac{7.82\times10^3\text{ m/s}}{3\times10^8\text{ m/s}}\right)^2 22(5.25\times10^3\text{ s}) = \boxed{39.2\ \mu\text{s}}$$

(b) For each one orbit Cooper aged less by $\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

P39.12 $\gamma = \frac{1}{\sqrt{1-(v^2/c^2)}} = 1.01$ so $\boxed{v = 0.140c}$

P39.13 (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$.

(b) His ship is in motion relative to you, so you measure its length contracted to $\boxed{19.0\text{ m}}$.

(c) We have
$$L = L_p\sqrt{1-\frac{v^2}{c^2}}$$
from which
$$\frac{L}{L_p} = \frac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1-\frac{v^2}{c^2}} \text{ and } \boxed{v = 0.312c}$$

P39.14 In the Earth frame, Speedo's trip lasts for a time

$$\Delta t = \frac{\Delta x}{v} = \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 21.05\text{ yr}$$

Speedo's age advances only by the proper time interval

$$\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\text{ yr}\sqrt{1-0.95^2} = 6.574\text{ yr during his trip}$$

Similarly for Goslo,

$$\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0\text{ ly}}{0.750\text{ ly/yr}}\sqrt{1-0.75^2} = 17.64\text{ yr}$$

While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\text{ ly/yr}} - \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 5.614\text{ yr}$$

Then $\boxed{\text{Goslo}}$ ends up older by $17.64\text{ yr} - (6.574\text{ yr} + 5.614\text{ yr}) = \boxed{5.45\text{ yr}}$.

P39.15 The orbital speed of the Earth is as described by $\Sigma F = ma$: $\dfrac{Gm_S m_E}{r^2} = \dfrac{m_E v^2}{r}$

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{1.496\times10^{11}\text{ m}}} = 2.98\times10^4\text{ m/s}$$

The maximum frequency received by the extraterrestrials is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}}$$
$$= 57.005\,66\times10^6\text{ Hz}$$

The minimum frequency received is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-v/c}{1+v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}}$$
$$= 56.994\,34\times10^6\text{ Hz}$$

The difference, which lets them figure out the speed of our planet, is

$$(57.005\,66 - 56.994\,34)\times10^6\text{ Hz} = \boxed{1.13\times10^4\text{ Hz}}$$

P39.16 (a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\dfrac{c+v}{c-v}}$

and, if f is the frequency of the reflected wave, $f = f_c\sqrt{\dfrac{c+v}{c-v}}$

Combining gives $\boxed{f = f_{\text{source}}\dfrac{(c+v)}{(c-v)}}$

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$

which gives $(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v$

The beat frequency is then $f_{\text{beat}} = f - f_{\text{source}} = \dfrac{2f_{\text{source}}v}{c} = \boxed{\dfrac{2v}{\lambda}}$

(c) $$f_{\text{beat}} = \frac{(2)(30.0\ \text{m/s})(10.0\times10^{9}\ \text{Hz})}{3.00\times10^{8}\ \text{m/s}} = \frac{(2)(30.0\ \text{m/s})}{(0.030\ 0\ \text{m})} = 2\ 000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^{8}\ \text{m/s}}{10.0\times10^{9}\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $$v = \frac{f_{\text{beat}}\lambda}{2} \quad \text{so} \quad \Delta v = \frac{\Delta f_{\text{beat}}\lambda}{2} = \frac{(5\ \text{Hz})(0.030\ 0\ \text{m})}{2} = \boxed{0.075\ 0\ \text{m/s} \approx 0.2\ \text{mi/h}}$$

P39.17 (a) When the source moves away from an observer, the observed frequency is

$$f_{\text{obs}} = f_{\text{source}}\left(\frac{c-v_s}{c+v_s}\right)^{1/2} \quad \text{where } v_s = v_{\text{source}}$$

When $v_s << c$, the binomial expansion gives

$$\left(\frac{c-v_s}{c+v_s}\right)^{1/2} = \left[1-\left(\frac{v_s}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_s}{c}\right)\right]^{-1/2} \approx \left(1-\frac{v_s}{2c}\right)\left(1-\frac{v_s}{2c}\right) \approx \left(1-\frac{v_s}{c}\right)$$

So, $$f_{\text{obs}} \approx f_{\text{source}}\left(1-\frac{v_s}{c}\right)$$

The observed wavelength is found from $c = \lambda_{\text{obs}}f_{\text{obs}} = \lambda f_{\text{source}}$:

$$\lambda_{\text{obs}} = \frac{\lambda f_{\text{source}}}{f_{\text{obs}}} \approx \frac{\lambda f_{\text{source}}}{f_{\text{source}}(1-v_s/c)} = \frac{\lambda}{1-v_s/c}$$

$$\Delta\lambda = \lambda_{\text{obs}} - \lambda = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{v_s/c}{1-v_s/c}\right)$$

Since $1-\dfrac{v_s}{c}\approx 1$, $\boxed{\dfrac{\Delta\lambda}{\lambda} \approx \dfrac{v_{\text{source}}}{c}}$

(b) $$v_{\text{source}} = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\ \text{nm}}{397\ \text{nm}}\right) = \boxed{0.050\ 4c}$$

P39.18 For the light as observed

$$f_{\text{obs}} = \frac{c}{\lambda_{\text{obs}}} = \sqrt{\frac{1+v/c}{1-v/c}} f_{\text{source}} = \sqrt{\frac{1+v/c}{1-v/c}} \frac{c}{\lambda_{\text{source}}}$$

$$\sqrt{\frac{1+v/c}{1-v/c}} = \frac{\lambda_{\text{source}}}{\lambda_{\text{obs}}} = \frac{650 \text{ nm}}{520 \text{ nm}} \qquad \frac{1+v/c}{1-v/c} = 1.25^2 = 1.562$$

$$1 + \frac{v}{c} = 1.562 - 1.562\frac{v}{c} \qquad \frac{v}{c} = \frac{0.562}{2.562} = 0.220$$

$$v = \boxed{0.220c} = 6.59 \times 10^7 \text{ m/s}$$

Section 39.5 **The Lorentz Transformation Equations**

P39.19 Let Suzanne be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0,\ t_1 = 0,\ x_2 = 0,\ t_2 = 3\ \mu\text{s}$. Let Mark be fixed in reference frame S′ and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9\ \mu\text{s}$.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$9\ \mu\text{s} = \frac{1}{\sqrt{1-v^2/c^2}}(3\ \mu\text{s} - 0) \qquad \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3}$$

$$\frac{v^2}{c^2} = \frac{8}{9} \qquad \boxed{v = 0.943c}$$

(b) $$x_2' = \gamma(x_2 - vt_2) = 3\left(0 - 0.943c \times 3 \times 10^{-6}\text{ s}\right)\left(\frac{3\times 10^8\text{ m/s}}{c}\right) = \boxed{2.55 \times 10^3 \text{ m}}$$

P39.20 $$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.995^2}} = 10.0$$

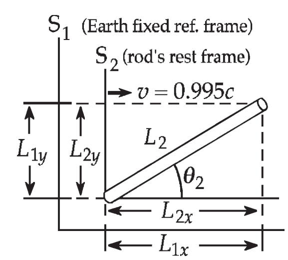

FIG. P39.20

We are also given: $L_1 = 2.00$ m, and $\theta = 30.0°$ (both measured in a reference frame moving relative to the rod).

Thus, $L_{1x} = L_1 \cos\theta_1 = (2.00 \text{ m})(0.867) = 1.73 \text{ m}$

and $L_{1y} = L_1 \sin\theta_1 = (2.00 \text{ m})(0.500) = 1.00 \text{ m}$

L_{2x} is a proper length, related to L_{1x} by $L_{1x} = \dfrac{L_{2x}}{\gamma}$

Therefore, $L_{2x} = 10.0 L_{1x} = 17.3 \text{ m}$

and $L_{2y} = L_{1y} = 1.00 \text{ m}$

(Lengths perpendicular to the motion are unchanged.)

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4 \text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

P39.21 Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. The *S*-frame coordinates of the events we may take as ($x = 0,\ y = 0,\ z = 0,\ t = 0$) and ($x = 100$ m, $y = 0,\ z = 0,\ t = 0$). Then the coordinates in S' are given by the Lorentz transformation. Event A is at ($x' = 0$, $y' = 0,\ z' = 0,\ t' = 0$). The time of event B is

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1-0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\ \text{m})\right) = 1.667\left(-\frac{80\ \text{m}}{3\times10^8\ \text{m/s}}\right) = -4.44\times10^{-7}\ \text{s}$$

The time elapsing before A occurs is $\boxed{444\ \text{ns}}$.

P39.22 (a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$$\Delta x' = \gamma(\Delta x - v\Delta t) \qquad 0 = \gamma\left[2.00\ \text{m} - v\left(8.00\times10^{-9}\ \text{s}\right)\right]$$

$$v = \frac{2.00\ \text{m}}{8.00\times10^{-9}\ \text{s}} = \boxed{2.50\times10^{8}\ \text{m/s}} \qquad \gamma = \frac{1}{\sqrt{1-\left(2.50\times10^8\ \text{m/s}\right)^2/\left(3.00\times10^8\ \text{m/s}\right)^2}} = 1.81$$

(b) Again from the Lorentz transformation,

$$x' = \gamma(x - vt):$$

$$x' = 1.81\left[3.00\ \text{m} - \left(2.50\times10^8\ \text{m/s}\right)\left(1.00\times10^{-9}\ \text{s}\right)\right]$$

$$x' = \boxed{4.97\ \text{m}}$$

(c) $t' = \gamma\left(t - \dfrac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\ \text{s} - \frac{\left(2.50\times10^8\ \text{m/s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2}(3.00\ \text{m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\ \text{s}}$$

Section 39.6 The Lorentz Velocity Transformation Equations

P39.23 $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2} = \dfrac{-0.750c - 0.750c}{1-(-0.750)(0.750)} = -0.960c$

speed = $\boxed{0.960\ \text{c}}$

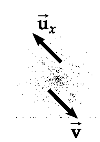

FIG. P39.23

P39.24 u_x = Enterprise velocity

v = Klingon velocity

From Equation 39.16

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.900c - 0.800c}{1-(0.900)(0.800)} = \boxed{0.357c}$$

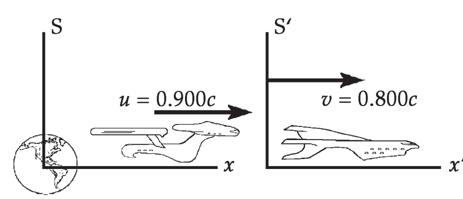

FIG. P39.24

Section 39.7 **Relativistic Linear Momentum**

P39.25 (a) $p = \gamma mu$; for an electron moving at $0.010\,0c$,

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-(0.010\,0)^2}} = 1.000\,05 \approx 1.00$$

Thus, $$p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.010\,0)\left(3.00\times10^{8}\text{ m/s}\right)$$

$$p = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(b) Following the same steps as used in part (a),

we find at $0.500c$, $\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$

P39.26 Using the relativistic form, $$p = \frac{mu}{\sqrt{1-(u/c)^2}} = \gamma mu$$

we find the difference Δ from the classical momentum, mu: $\Delta p = \gamma mu - mu = (\gamma - 1)mu$

(a) The difference is 1.00% when $(\gamma-1)mu = 0.010\,0\gamma mu$: $\gamma = \dfrac{1}{0.990} = \dfrac{1}{\sqrt{1-(u/c)^2}}$

thus $1-\left(\dfrac{u}{c}\right)^2 = (0.990)^2$, and $u = \boxed{0.141c}$

(b) The difference is 10.0% when $(\gamma-1)mu = 0.100\gamma mu$: $\gamma = \dfrac{1}{0.900} = \dfrac{1}{\sqrt{1-(u/c)^2}}$

thus $1-\left(\dfrac{u}{c}\right)^2 = (0.900)^2$ and $u = \boxed{0.436c}$

P39.27 $\dfrac{p-mu}{mu} = \dfrac{\gamma mu - mu}{mu} = \gamma - 1$: $$\gamma - 1 = \frac{1}{\sqrt{1-(u/c)^2}} - 1 \approx 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 - 1 = \frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$\frac{p-mu}{mu} = \frac{1}{2}\left(\frac{90.0\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2 = \boxed{4.50\times10^{-14}}$$

***P39.28** We can express the proportionality of the speeding fine to the excess momentum as $F = d(p - p_\ell)$ where F is the fine, d is a proportionality constant, p is the magnitude of the vehicle's momentum, and p_ℓ is the magnitude of its momentum as it travels at 90 km/h.

(a) METHOD ONE. For low speeds, the classical expression for momentum is accurate. Our equation describes the two cases

$$\$80 = d\left[m(190\ \text{km/h}) - m(90\ \text{km/h})\right] \text{ and } F_a = d\left[m(1\,090\ \text{km/h}) - m(90\ \text{km/h})\right]$$

Dividing gives $\dfrac{F_a}{\$80} = \dfrac{m(1\,000\ \text{km/h})}{m(100\ \text{km/h})} \Rightarrow \boxed{F_a = \$800}$.

METHOD TWO. The relativistic momentum expression is always accurate:

$$\$80 = d\left(\frac{m(190\ \text{km/h})}{\sqrt{1-(190\ \text{km/h})^2/c^2}} - \frac{m(90\ \text{km/h})}{\sqrt{1-(90\ \text{km/h})^2/c^2}}\right)$$

$$F_a = d\left(\frac{m(1\,090\ \text{km/h})}{\sqrt{1-(1\,090\ \text{km/h})^2/c^2}} - \frac{m(190\ \text{km/h})}{\sqrt{1-(190\ \text{km/h})^2/c^2}}\right)$$

To three or even to six digits, the answer is the same: $F_a = \$80(10) = \$800$.

(b) Now the high-speed case must be described relativistically. The speed of light is $3\times10^8\ \text{m/s} = 1.08\times10^9\ \text{km/h}$.

$$\$80 = dm(100\ \text{km/h})$$

$$F_b = d\left(\frac{m(1.000\,000\,09\times10^9\ \text{km/h})}{\sqrt{1-(1.000\,000\,09\times10^9/1.08\times10^9)^2}} - m(90\ \text{km/h})\right)$$

$$F_b = dm(2.65\times10^9\ \text{km/h})$$

again dividing, $\dfrac{F_b}{\$80} = \dfrac{2.65\times10^9}{10^2} \Rightarrow F_b = \boxed{\$2.12\times10^9}$.

P39.29 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$

or $$\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50\times10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$$

or $$\frac{\left(1.67\times10^{-27}\ \text{kg}\right)u_2}{\sqrt{1-\left(u_2/c\right)^2}} = \left(4.960\times10^{-28}\ \text{kg}\right)c$$

Proceeding to solve, we find $$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2 = 1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2} = 1 \text{ and } u_2 = \boxed{0.285c}$$

Section 39.8 **Relativistic Energy**

***P39.30** (a) $K = E - E_R = 5E_R$

$E = 6E_R = 6\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.00\times10^8\ \text{m/s}\right)^2 = 4.92\times10^{-13}\ \text{J} = \boxed{3.07\ \text{MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R$

Thus $\gamma = \dfrac{E}{E_R} = 6 = \dfrac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u = 0.986c}$

P39.31 $$\Sigma W = K_f - K_i = \left(\frac{1}{\sqrt{1-\left(v_f/c\right)^2}} - 1\right)mc^2 - \left(\frac{1}{\sqrt{1-\left(v_i/c\right)^2}}\right)mc^2$$

or $$\Sigma W = \left(\frac{1}{\sqrt{1-\left(v_f/c\right)^2}} - \frac{1}{\sqrt{1-\left(v_i/c\right)^2}}\right)mc^2$$

(a) $$\Sigma W = \left(\frac{1}{\sqrt{1-(0.750)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)\left(1.673\times10^{-27}\ \text{kg}\right)\left(2.998\times10^8\ \text{m/s}\right)^2$$

$$\Sigma W = \boxed{5.37\times10^{-11}\ \text{J}}$$

(b) $$\Sigma W = \left(\frac{1}{\sqrt{1-(0.995)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)\left(1.673\times10^{-27}\ \text{kg}\right)\left(2.998\times10^8\ \text{m/s}\right)^2$$

$$\Sigma W = \boxed{1.33\times10^{-9}\ \text{J}}$$

P39.32 The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$.

For $u = 0.100c$, $\quad K_r = \left(\dfrac{1}{\sqrt{1-0.010\,0}} - 1\right)mc^2 = 0.005\,038mc^2$

The classical equation $K_c = \dfrac{1}{2}mu^2$ gives $\quad K_c = \dfrac{1}{2}m(0.100c)^2 = 0.005\,000mc^2$

different by $\quad \dfrac{0.005\,038 - 0.005\,000}{0.005\,038} = 0.751\%$

For still smaller speeds the agreement will be still better.

P39.33 $E = \gamma mc^2 = 2mc^2$ or $\quad \gamma = 2$

Thus, $\dfrac{u}{c} = \sqrt{1-\left(\dfrac{1}{\gamma}\right)^2} = \dfrac{\sqrt{3}}{2}$ or $\quad u = \dfrac{c\sqrt{3}}{2}$

The momentum is then $\quad p = \gamma mu = 2m\left(\dfrac{c\sqrt{3}}{2}\right) = \left(\dfrac{mc^2}{c}\right)\sqrt{3}$

$$p = \left(\frac{938.3\text{ MeV}}{c}\right)\sqrt{3} = \boxed{1.63\times10^3\ \frac{\text{MeV}}{c}}$$

***P39.34** (a) Using the classical equation, $\quad K = \dfrac{1}{2}mu^2 = \dfrac{1}{2}(78.0\text{ kg})(1.06\times10^5\text{ m/s})^2$

$$= \boxed{4.38\times10^{11}\text{ J}}$$

(b) Using the relativistic equation, $\quad K = \left(\dfrac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2$

$$K = \left[\frac{1}{\sqrt{1-\left(1.06\times10^5/2.998\times10^8\right)^2}} - 1\right](78.0\text{ kg})(2.998\times10^8\text{ m/s})^2 = \boxed{4.38\times10^{11}\text{ J}}$$

(c) When $\dfrac{u}{c} << 1$, the binomial series expansion gives $\quad \left[1-\left(\dfrac{u}{c}\right)^2\right]^{-1/2} \approx 1 + \dfrac{1}{2}\left(\dfrac{u}{c}\right)^2$

Thus, $\quad \left[1-\left(\dfrac{u}{c}\right)^2\right]^{-1/2} - 1 \approx \dfrac{1}{2}\left(\dfrac{u}{c}\right)^2$

and the relativistic expression for kinetic energy becomes $K \approx \dfrac{1}{2}\left(\dfrac{u}{c}\right)^2 mc^2 = \dfrac{1}{2}mu^2$.

That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results. In this situation the two kinetic energy values are experimentally indistinguishable. The fastest-moving macroscopic objects launched by human beings move sufficiently slowly compared to light that relativistic corrections to their energy are negligible.

P39.35 (a) $E_R = mc^2 = (1.67\times10^{-27}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 1.50\times10^{-10}\ \text{J} = \boxed{938\ \text{MeV}}$

(b) $E = \gamma mc^2 = \dfrac{1.50\times10^{-10}\ \text{J}}{\left[1-(0.950c/c)^2\right]^{1/2}} = 4.81\times10^{-10}\ \text{J} = \boxed{3.00\times10^3\ \text{MeV}}$

(c) $K = E - mc^2 = 4.81\times10^{-10}\ \text{J} - 1.50\times10^{-10}\ \text{J} = 3.31\times10^{-10}\ \text{J} = \boxed{2.07\times10^3\ \text{MeV}}$

P39.36 We must conserve both energy and relativistic momentum of the system of fragments. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,

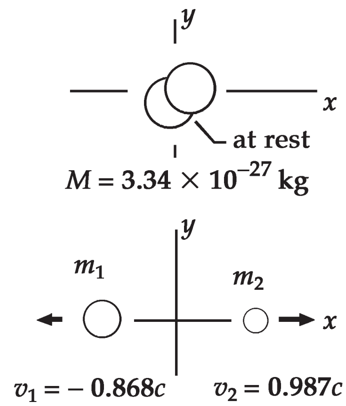

FIG. P39.36

$$\gamma_1 = \frac{1}{\sqrt{1-(0.868)^2}} = 2.01 \text{ and } \gamma_2 = \frac{1}{\sqrt{1-(0.987)^2}} = 6.22$$

Conservation of energy gives $E_1 + E_2 = E_{\text{total}}$

which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01m_1 + 6.22m_2 = 3.34\times10^{-27}\ \text{kg}$

This reduces to: $m_1 + 3.09m_2 = 1.66\times10^{-27}\ \text{kg}$ (1)

Since the final momentum of the system must equal zero, $p_1 = p_2$

gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c)m_1 = (6.22)(0.987c)m_2$

which becomes $m_1 = 3.52m_2$ (2)

Solving (1) and (2) simultaneously, $3.52m_2 + 3.09m_2 = 1.66\times10^{-27}\ \text{kg}$

$m_1 = \boxed{8.84\times10^{-28}\ \text{kg}}$ and $m_2 = \boxed{2.51\times10^{-28}\ \text{kg}}$

P39.37 $E = \gamma mc^2$ $\qquad p = \gamma mu$

$E^2 = (\gamma mc^2)^2 \qquad p^2 = (\gamma mu)^2$

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2 c^2 = \gamma^2\left((mc^2)^2 - (mc)^2u^2\right) = (mc^2)^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1} = (mc^2)^2$$

as was to be demonstrated.

P39.38 (a) $q(\Delta V) = K = (\gamma - 1)m_e c^2$

Thus, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 1 + \dfrac{q(\Delta V)}{m_e c^2} = 1 + \dfrac{25\ 000\ \text{eV}}{511\ 000\ \text{eV}} = 1.0489$

so $1 - (u/c)^2 = 0.9089$ and $\boxed{u = 0.302c}$

(b) $K = (\gamma - 1)m_e c^2 = q(\Delta V) = (1.60\times10^{-19}\ \text{C})(2.50\times10^4\ \text{J/C}) = \boxed{4.00\times10^{-15}\ \text{J}}$

***P39.39** From $K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$ we have $\frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{K+mc^2}{mc^2}$.

$$1 - \frac{u^2}{c^2} = \frac{m^2c^4}{\left(K+mc^2\right)^2}$$

$$\frac{u^2}{c^2} = 1 - \frac{\left(mc^2\right)^2}{\left(K+mc^2\right)^2}$$

$$u = c\left(1 - \left(\frac{mc^2}{K+mc^2}\right)^2\right)^{1/2}$$

(a) Electron: $u = c\left(1 - \left(\frac{0.511}{2.511}\right)^2\right)^{1/2} = \boxed{0.979c}$

(b) Proton: $u = c\left(1 - \left(\frac{938}{940}\right)^2\right)^{1/2} = \boxed{0.065\ 2c}$

(c) $0.979c - 0.065\ 2c = \boxed{0.914c} = 2.74 \times 10^8\ \text{m/s}$

In this case the electron is moving relativistically, but the classical expression $\frac{1}{2}mv^2$ is accurate to two digits for the proton.

(d) Electron: $u = c\left(1 - \left(\frac{0.511}{2\ 000.511}\right)^2\right)^{1/2} = \boxed{0.999\ 999\ 97c}$

Proton: $u = c\left(1 - \left(\frac{938}{2\ 938}\right)^2\right)^{1/2} = \boxed{0.948c}$

Excess speed $= 0.999\ 999\ 97c - 0.948c = \boxed{0.052\ 3c} = 1.57 \times 10^7\ \text{m/s}$

As the kinetic energies of both particles become large, the difference in their speeds approaches zero. By contrast, classically the speed difference would become large without any finite limit.

P39.40 (a) $E = \gamma mc^2 = 20.0$ GeV with $mc^2 = 0.511$ MeV for electrons.

Thus, $\gamma = \frac{20.0 \times 10^9\ \text{eV}}{0.511 \times 10^6\ \text{eV}} = \boxed{3.91 \times 10^4}$.

(b) $\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = 3.91 \times 10^4$ from which $\boxed{u = 0.999\ 999\ 999\ 7c}$

(c) $L = L_p\sqrt{1 - \left(\frac{u}{c}\right)^2} = \frac{L_p}{\gamma} = \frac{3.00 \times 10^3\ \text{m}}{3.91 \times 10^4} = 7.67 \times 10^{-2}\ \text{m} = \boxed{7.67\ \text{cm}}$

P39.41 Conserving total momentum of the decaying particle system, $p_{\text{before decay}} = p_{\text{after decay}} = 0$

$$p_\nu = p_\mu = \gamma m_\mu u = \gamma(207m_e)u$$

Conservation of mass-energy for the system gives $E_\mu + E_\nu = E_\pi$: $\gamma\, m_\mu c^2 + p_\nu c = m_\pi c^2$

$$\gamma(207m_e) + \frac{p_\nu}{c} = 273m_e$$

Substituting from the momentum equation above, $\gamma(207m_e) + \gamma(207m_e)\dfrac{u}{c} = 273m_e$

or $\gamma\left(1+\dfrac{u}{c}\right) = \dfrac{273}{207} = 1.32$: $\dfrac{1+u/c}{1-u/c} = 1.74$ $\dfrac{u}{c} = 0.270$

Then, $K_\mu = (\gamma - 1)m_\mu c^2 = (\gamma - 1)207(m_e c^2)$: $K_\mu = \left(\dfrac{1}{\sqrt{1-(0.270)^2}} - 1\right)207(0.511\text{ MeV})$

$$K_\mu = \boxed{4.08\text{ MeV}}$$

Also, $E_\nu = E_\pi - E_\mu$: $E_\nu = m_\pi c^2 - \gamma\, m_\mu c^2 = (273 - 207\gamma)m_e c^2$

$$E_\nu = \left(273 - \frac{207}{\sqrt{1-(0.270)^2}}\right)(0.511\text{ MeV})$$

$$E_\nu = \boxed{29.6\text{ MeV}}$$

***P39.42** $K = (\gamma - 1)mc^2 = \left(\left(1 - u^2/c^2\right)^{-1/2} - 1\right)mc^2$. We use the series expansion from Appendix B.5:

$$K = mc^2\left[1 + \left(-\tfrac{1}{2}\right)(-u^2/c^2) + \left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\tfrac{1}{2!}(-u^2/c^2)^2 + \cdots - 1\right]$$

$K = \frac{1}{2}mu^2 + \frac{3}{8}m\dfrac{u^4}{c^2} + \cdots$ The actual kinetic energy, given by this relativistic equation, is $\boxed{\text{greater}}$ than the classical $(1/2)mu^2$. The difference, for m = 1 000 kg and u = 25 m/s, is

$$\tfrac{3}{8}(1\,000\text{ kg})\frac{(25\text{ m/s})^4}{(3\times10^8\text{ m/s})^2} = 1.6\times10^{-9}\text{ J}\ \boxed{\sim 10^{-9}\text{ J}}$$

Section 39.9 **Mass and Energy**

***P39.43** $E = 2.86\times10^5$ J leaves the system so the final mass is $\boxed{\text{smaller}}$. The mass-energy relation says that $E = mc^2$. Therefore, $m = \dfrac{E}{c^2} = \dfrac{2.86\times10^5\text{ J}}{\left(3.00\times10^8\text{ m/s}\right)^2} = \boxed{3.18\times10^{-12}\text{ kg}}$

A mass loss of this magnitude, as a fraction of a total of 9.00 g, $\boxed{\text{could not be detected}}$.

P39.44 $\Delta m = \dfrac{E}{c^2} = \dfrac{\mathcal{P}\,\Delta t}{c^2} = \dfrac{0.800\left(1.00\times10^9\text{ J/s}\right)(3.00\text{ yr})\left(3.16\times10^7\text{ s/yr}\right)}{\left(3.00\times10^8\text{ m/s}\right)^2} = \boxed{0.842\text{ kg}}$

P39.45 $\mathcal{P} = \frac{dE}{dt} = \frac{d(mc^2)}{dt} = c^2\frac{dm}{dt} = 3.85\times 10^{26}\ \text{W}$

Thus, $\frac{dm}{dt} = \frac{3.85\times 10^{26}\ \text{J/s}}{(3.00\times 10^{8}\ \text{m/s})^2} = \boxed{4.28\times 10^{9}\ \text{kg/s}}$

P39.46 $2m_e c^2 = 1.02\ \text{MeV}$ $\quad E_\gamma \geq \boxed{1.02\ \text{MeV}}$

Section 39.10 **The General Theory of Relativity**

P39.47 (a) For the satellite $\Sigma F = ma$: $\frac{GM_E m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$

$$GM_E T^2 = 4\pi^2 r^3$$

$$r = \left(\frac{6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2\,(5.98\times 10^{24}\ \text{kg})(43\,080\ \text{s})^2}{\text{kg}^2\,4\pi^2}\right)^{1/3}$$

$$r = \boxed{2.66\times 10^{7}\ \text{m}}$$

(b) $v = \frac{2\pi r}{T} = \frac{2\pi(2.66\times 10^{7}\ \text{m})}{43\,080\ \text{s}} = \boxed{3.87\times 10^{3}\ \text{m/s}}$

(c) The small fractional decrease in frequency received is equal in magnitude to the fractional increase in period of the moving oscillator due to time dilation:

$$\text{fractional change in } f = -(\gamma - 1) = -\left(\frac{1}{\sqrt{1-(3.87\times 10^{3}/3\times 10^{8})^2}} - 1\right)$$

$$= 1 - \left(1 - \frac{1}{2}\left[-\left(\frac{3.87\times 10^{3}}{3\times 10^{8}}\right)^2\right]\right) = \boxed{-8.34\times 10^{-11}}$$

(d) The orbit altitude is large compared to the radius of the Earth, so we must use $U_g = -\frac{GM_E m}{r}$.

$$\Delta U_g = -\frac{6.67\times 10^{-11}\ \text{Nm}^2\,(5.98\times 10^{24}\ \text{kg})m}{\text{kg}^2\ 2.66\times 10^{7}\ \text{m}} + \frac{6.67\times 10^{-11}\ \text{Nm}(5.98\times 10^{24}\ \text{kg})m}{\text{kg}\ 6.37\times 10^{6}\ \text{m}}$$

$$= 4.76\times 10^{7}\ \text{J/kg}\,m$$

$$\frac{\Delta f}{f} = \frac{\Delta U_g}{mc^2} = \frac{4.76\times 10^{7}\ \text{m}^2/\text{s}^2}{(3\times 10^{8}\ \text{m/s})^2} = \boxed{+5.29\times 10^{-10}}$$

(e) $-8.34\times 10^{-11} + 5.29\times 10^{-10} = \boxed{+4.46\times 10^{-10}}$

Additional Problems

P39.48 (a) $d_{\text{earth}} = vt_{\text{earth}} = v\gamma t_{\text{astro}}$ so $(2.00\times10^6 \text{ yr})c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0 \text{ yr}$

$$\sqrt{1-\frac{v^2}{c^2}} = \left(\frac{v}{c}\right)(1.50\times10^{-5}) \qquad 1-\frac{v^2}{c^2} = \frac{v^2(2.25\times10^{-10})}{c^2}$$

$$1 = \frac{v^2}{c^2}(1+2.25\times10^{-10}) \quad \text{so} \quad \frac{v}{c} = (1+2.25\times10^{-10})^{-1/2} = 1-\frac{1}{2}(2.25\times10^{-10})$$

$$\boxed{\frac{v}{c} = 1-1.12\times10^{-10}}$$

(b) $$K = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 = \left(\frac{2.00\times10^6 \text{ yr}}{30 \text{ yr}}-1\right)(1\,000)(1\,000 \text{ kg})(3\times10^8 \text{ m/s})^2$$

$$= \boxed{6.00\times10^{27} \text{ J}}$$

(c) $$6.00\times10^{27} \text{ J} = 6.00\times10^{27} \text{ J}\left(\frac{13¢}{\text{kWh}}\right)\left(\frac{\text{k}}{10^3}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600 \text{ s}}\right) = \boxed{\$2.17\times10^{20}}$$

P39.49 (a) $10^{13} \text{ MeV} = (\gamma-1)m_pc^2$ so $\gamma = 10^{10}$

$v_p \approx c$ $\qquad t' = \dfrac{t}{\gamma} = \dfrac{10^5 \text{ yr}}{10^{10}} = 10^{-5} \text{ yr} \sim \boxed{10^2 \text{ s}}$

(b) $d' = ct' \boxed{\sim10^8 \text{ km}}$

P39.50 (a) When $K_e = K_p$, $\quad m_ec^2(\gamma_e-1) = m_pc^2(\gamma_p-1)$

In this case, $\quad m_ec^2 = 0.511 \text{ MeV}$, $m_pc^2 = 938 \text{ MeV}$

and $\quad \gamma_e = \left[1-(0.750)^2\right]^{-1/2} = 1.511\,9$

Substituting, $$\gamma_p = 1+\frac{m_ec^2(\gamma_e-1)}{m_pc^2} = 1+\frac{(0.511 \text{ MeV})(1.511\,9-1)}{938 \text{ MeV}}$$

$$= 1.000\,279$$

but $$\gamma_p = \frac{1}{\left[1-(u_p/c)^2\right]^{1/2}}$$

Therefore, $\quad u_p = c\sqrt{1-\gamma_p^{-2}} = \boxed{0.023\,6c}$

(b) When $p_e = p_p$ $\quad \gamma_pm_pu_p = \gamma_em_eu_e$ or $\gamma_pu_p = \dfrac{\gamma_em_eu_e}{m_p}$

Thus, $$\gamma_pu_p = \frac{(1.511\,9)(0.511 \text{ MeV}/c^2)(0.750c)}{938 \text{ MeV}/c^2} = 6.177\,2\times10^{-4}c$$

and $$\frac{u_p}{c} = 6.177\,2\times10^{-4}\sqrt{1-\left(\frac{u_p}{c}\right)^2}$$

which yields $\quad u_p = \boxed{6.18\times10^{-4}c} = 185 \text{ km/s}$

***P39.51** (a) We let H represent K/mc^2. Then $H+1=\dfrac{1}{\sqrt{1-u^2/c^2}}$ so $1-u^2/c^2=\dfrac{1}{H^2+2H+1}$

$$\frac{u^2}{c^2}=1-\frac{1}{H^2+2H+1}=\frac{H^2+2H}{H^2+2H+1} \quad \text{and} \quad \boxed{u=c\left(\frac{H^2+2H}{H^2+2H+1}\right)^{1/2}}$$

(b) u goes to 0 as K goes to 0.

(c) u approaches c as K increases without limit.

(d) $a=\dfrac{du}{dt}$

$$=c\frac{1}{2}\left(\frac{H^2+2H}{H^2+2H+1}\right)^{-1/2}\left(\frac{\left[H^2+2H+1\right][2H+2]-\left[H^2+2H\right][2H+2]}{\left[H^2+2H+1\right]^2}\right)\frac{d(K/mc^2)}{dt}$$

$$=a=c\left(\frac{H^2+2H+1}{H^2+2H}\right)^{1/2}\left(\frac{H+1}{[H+1]^4}\right)\frac{\mathcal{P}}{mc^2}=\boxed{\frac{\mathcal{P}}{mcH^{1/2}(H+2)^{1/2}(H+1)^2}}$$

(e) When H is small we have approximately $a=\dfrac{\mathcal{P}}{mcH^{1/2}2^{1/2}}=\dfrac{\mathcal{P}}{(2mK)^{1/2}}$, in agreement with the nonrelativistic case.

(f) When H is large the acceleration approaches $\mathcal{P}/mcH^3=\mathcal{P}m^2c^5/K^3$.

(g) As energy is steadily imparted to particle, the particle's acceleration decreases. It decreases steeply, proportionally to $1/K^3$ at high energy. In this way the particle's speed cannot reach or surpass a certain upper limit, which is the speed of light in vacuum.

P39.52 (a) Since Mary is in the same reference frame, S', as Ted, she measures the ball to have the same speed Ted observes, namely $|u_x'|=\boxed{0.800c}$.

(b) $$\Delta t'=\frac{L_p}{|u_x'|}=\frac{1.80\times10^{12}\text{ m}}{0.800\left(3.00\times10^8\text{ m/s}\right)}=\boxed{7.50\times10^3\text{ s}}$$

(c) $$L=L_p\sqrt{1-\frac{v^2}{c^2}}=\left(1.80\times10^{12}\text{ m}\right)\sqrt{1-\frac{(0.600c)^2}{c^2}}=\boxed{1.44\times10^{12}\text{ m}}$$

Since $v=0.600c$ and $u_x'=-0.800c$, the velocity Jim measures for the ball is

$$u_x=\frac{u_x'+v}{1+u_x'v/c^2}=\frac{(-0.800c)+(0.600c)}{1+(-0.800)(0.600)}=\boxed{-0.385c}$$

(d) Jim measures the ball and Mary to be initially separated by 1.44×10^{12} m. Mary's motion at $0.600c$ and the ball's motion at $0.385c$ nibble into this distance from both ends. The gap closes at the rate $0.600c+0.385c=0.985c$, so the ball and catcher meet after a time

$$\Delta t=\frac{1.44\times10^{12}\text{ m}}{0.985\left(3.00\times10^8\text{ m/s}\right)}=\boxed{4.88\times10^3\text{ s}}$$

P39.53 $$\frac{\Delta mc^2}{mc^2}=\frac{4(938.78\text{ MeV})-3\,728.4\text{ MeV}}{4(938.78\text{ MeV})}\times100\%=\boxed{0.712\%}$$

P39.54 The energy of the first fragment is given by $E_1^2 = p_1^2c^2 + \left(m_1c^2\right)^2 = (1.75\text{ MeV})^2 + (1.00\text{ MeV})^2$

$E_1 = 2.02\text{ MeV}$

For the second, $E_2^2 = (2.00\text{ MeV})^2 + (1.50\text{ MeV})^2 \quad E_2 = 2.50\text{ MeV}$

(a) Energy is conserved, so the unstable object had $E = 4.52$ MeV. Each component of momentum is conserved, so the original object moved

with $p^2 = p_x^2 + p_y^2 = \left(\dfrac{1.75\text{ MeV}}{c}\right)^2 + \left(\dfrac{2.00\text{ MeV}}{c}\right)^2$. Then for

it $(4.52\text{ MeV})^2 = (1.75\text{ MeV})^2 + (2.00\text{ MeV})^2 + \left(mc^2\right)^2 \quad \boxed{m = \dfrac{3.65\text{ MeV}}{c^2}}$

(b) Now $E = \gamma mc^2$ gives $4.52\text{ MeV} = \dfrac{1}{\sqrt{1-v^2/c^2}}3.65\text{ MeV} \quad 1-\dfrac{v^2}{c^2} = 0.654$, $\boxed{v = 0.589c}$

P39.55 Look at the situation from the instructors' viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v+u'}{1+vu'/c^2} = \frac{(-0.280c)-(0.600c)}{1+(-0.280c)(-0.600c)/c^2} = -0.753c \text{ (students relative to clock)}$$

(a) With a proper time interval of $\Delta t_p = 50.0$ min, the time interval measured by the students is:

$$\Delta t = \gamma\,\Delta t_p \qquad \text{with} \qquad \gamma = \frac{1}{\sqrt{1-(0.753c)^2/c^2}} = 1.52$$

Thus, the students measure the exam to last $T = 1.52(50.0\text{ min}) = \boxed{76.0\text{ minutes}}$.

(b) The duration of the exam as measured by observers on Earth is:

$$\Delta t = \gamma\,\Delta t_p \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1-(0.280c)^2/c^2}} \text{ so } T = 1.04(50.0\text{ min}) = \boxed{52.1\text{ minutes}}$$

***P39.56** (a) The speed of light in water is $c/1.33$, so the electron's speed is $1.1c/1.33$. Then

$$\gamma = \frac{1}{\sqrt{1-(1.1/1.33)^2}} = 1.779 \quad \text{and the energy is } \gamma mc^2 = 1.779(0.511\text{ MeV})\boxed{= 0.909\text{ MeV}}$$

(b) $K = 0.909\text{ MeV} - 0.511\text{ MeV} = \boxed{0.398\text{ MeV}}$

(c) $pc = \sqrt{E^2-(mc^2)^2} = \sqrt{\gamma^2-1}\;mc^2 = \sqrt{1.779^2-1}\;0.511\text{ MeV} = 0.752\text{ MeV}$

$p = \boxed{0.752\text{ MeV}/c = 4.01\times10^{-22}\text{ kg}\cdot\text{m/s}}$

(d) $\sin\theta = v/u$ where v is the wave speed, which is the speed of light in water, and u is the source speed. Then $\sin\theta = 1/1.1 = 0.909$ so $\theta = \boxed{65.4°}$

P39.57 (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$.

Then $u_x' = +0.800c$, and $$u_x = \frac{u_x' + v}{1 + (u_x' v)/c^2} = \frac{0.800c + 0.600c}{1 + (0.800)(0.600)} = \boxed{0.946c}$$

(b) $L = \frac{L_p}{\gamma}$: $$L = (0.200 \text{ ly})\sqrt{1 - (0.600)^2} = \boxed{0.160 \text{ ly}}$$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$.

Thus, $$\text{time} = \frac{0.160 \text{ ly}}{0.800c + 0.600c} = \boxed{0.114 \text{ yr}}$$

(d) $K = \left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1\right) mc^2$: $$K = \left(\frac{1}{\sqrt{1 - (0.946)^2}} - 1\right)(4.00 \times 10^5 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$K = \boxed{7.50 \times 10^{22} \text{ J}}$$

***P39.58** (a) From equation 39.18, the speed of light in the laboratory frame is

$$u = \frac{v + c/n}{1 + vc/nc^2} = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)}$$

(b) When v is much less than c we have

$$u = \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 + \frac{v}{nc}\right)^{-1} \approx \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 - \frac{v}{nc}\right) \approx \frac{c}{n}\left(1 + \frac{nv}{c} - \frac{v}{nc}\right) = \frac{c}{n} + v - \frac{v}{n^2}$$

The Galilean velocity transformation equation 4.20 would indeed give $c/n + v$ for the speed of light in the moving water. The third term $-v/n^2$ does represent a relativistic effect that was observed decades before the Michelson-Morley experiment. It is a piece of twentieth-century physics that dropped into the nineteenth century. We could say that light is intrinsically relativistic.

(c) To take the limit as v approaches c we must go back to

$$u = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)} \quad \text{to find } u \text{ approaches } \frac{c\ (1 + nc/c)}{n\ (1 + c/nc)} = \frac{c(1+n)}{n+1} = \boxed{c}$$

P39.59 The observer measures the proper length of the tunnel, 50.0 m, but measures the train contracted to length

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = 100 \text{ m}\sqrt{1 - (0.950)^2} = 31.2 \text{ m}$$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8 \text{ m}}$ so $\boxed{\text{it is completely within the tunnel}}$.

P39.60 If the energy required to remove a mass m from the surface is equal to its rest energy mc^2,

then $\dfrac{GM_s m}{R_g} = mc^2$

and $R_g = \dfrac{GM_s}{c^2} = \dfrac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)^2}$

$R_g = 1.47\times10^{3}\ \text{m} = \boxed{1.47\ \text{km}}$

P39.61 (a) At any speed, the momentum of the particle is given by

$$p = \gamma mu = \frac{mu}{\sqrt{1-(u/c)^2}}$$

Since $F = qE = \dfrac{dp}{dt}$:

$$qE = \frac{d}{dt}\left[mu\left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]$$

$$qE = m\left(1-\frac{u^2}{c^2}\right)^{-1/2}\frac{du}{dt} + \frac{1}{2}mu\left(1-\frac{u^2}{c^2}\right)^{-3/2}\left(\frac{2u}{c^2}\right)\frac{du}{dt}$$

So

$$\frac{qE}{m} = \frac{du}{dt}\left[\frac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$$

and

$$\boxed{a = \frac{du}{dt} = \frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

(b) For u small compared to c, the relativistic expression reduces to the classical $a = \dfrac{qE}{m}$. As u approaches c, the acceleration approaches zero, so that the object can never reach the speed of light.

(c) $\displaystyle\int_0^u \frac{du}{\left(1-u^2/c^2\right)^{3/2}} = \int_{t=0}^{t} \frac{qE}{m}\,dt$ $\qquad\boxed{u = \dfrac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$

$\displaystyle x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}}$ $\qquad\boxed{x = \dfrac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2}-mc\right)}$

P39.62 (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt_S$ is the distance the ship moves toward the mirror in time t_S. Since this observer agrees that the speed of light is c, the time for it to travel distance D is

$$t_S = \frac{D}{c} = \frac{2d - vt_S}{c} \qquad t_S = \boxed{\frac{2d}{c+v}}$$

(b) The observer in the rocket measures a length-contracted initial distance to the mirror of

$$L = d\sqrt{1-\frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as $D = 2(L-y)$ where $y = \frac{vt}{2}$ is the distance the mirror moves toward the ship before the light reflects from it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left[d\sqrt{1-\frac{v^2}{c^2}} - \frac{vt}{2}\right] \text{ so } (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \text{ or } t = \boxed{\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

***P39.63** (a) Conservation of momentum γmu:

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} = \frac{2mu}{3\sqrt{1-u^2/c^2}}$$

Conservation of energy γmc^2:

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

To start solving we can divide: $v_f = \frac{2u}{4} = \frac{u}{2}$. Then

$$\frac{M}{\sqrt{1-u^2/4c^2}} = \frac{4m}{3\sqrt{1-u^2/c^2}} = \frac{M}{(1/2)\sqrt{4-u^2/c^2}}$$

$$\boxed{M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}}}$$

(b) When $u \ll c$, this reduces to $M = \frac{4m}{3}$, in agreement with the classical result, which is the arithmetic sum of the masses of the two colliding particles.

P39.64 Take the two colliding protons as the system

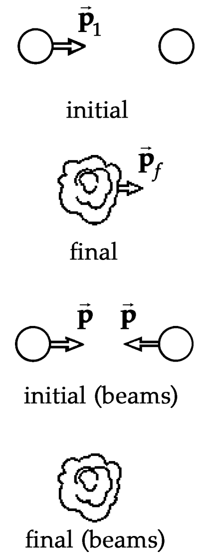

FIG. P39.64

$E_1 = K + mc^2$ $\qquad$ $E_2 = mc^2$

$E_1^2 = p_1^2c^2 + m^2c^4$ $\qquad$ $p_2 = 0$

In the final state, $E_f = K_f + Mc^2$: $\qquad$ $E_f^2 = p_f^2c^2 + M^2c^4$

By energy conservation, $E_1 + E_2 = E_f$, so

$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$

$$p_1^2c^2 + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4$$

$$= p_f^2c^2 + M^2c^4$$

By conservation of momentum, $\qquad$ $p_1 = p_f$

Then

$$M^2c^4 = 2Kmc^2 + 4m^2c^4$$

$$= \frac{4Km^2c^4}{2mc^2} + 4m^2c^4$$

$$Mc^2 = 2mc^2\sqrt{1 + \frac{K}{2mc^2}}$$

By contrast, for colliding beams we have

In the original state, $\qquad$ $E_1 = K + mc^2$

$$E_2 = K + mc^2.$$

In the final state, $\qquad$ $E_f = Mc^2$

$E_1 + E_2 = E_f$: $\qquad$ $K + mc^2 + K + mc^2 = Mc^2$

$$Mc^2 = 2mc^2\left(1 + \frac{K}{2mc^2}\right)$$

P39.65 We choose to write down the answer to part (b) first.

(b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1 - \left(\frac{v}{c}\right)^2} = (6.00\text{ ly})\sqrt{1 - (0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$\frac{3.60\text{ ly}}{1.80c} = 2.00\text{ yr}$$

continued on next page

We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for

$$\frac{3.60\ \text{ly}}{0.200c} = 18.0\ \text{yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

P39.66 Take $m = 1.00$ kg.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50\times10^{16}\ \text{J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}}-1\right)mc^2 = \left(9.00\times10^{16}\ \text{J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}}-1\right)$

$\frac{u}{c}$	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	0.0453×10^{16}
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}

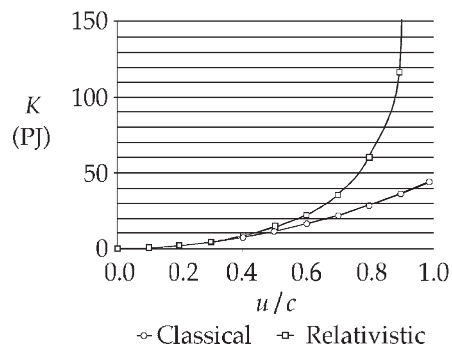

FIG. P39.66

$K_c = 0.990K_r$, when $\frac{1}{2}\left(\frac{u}{c}\right)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}}-1\right]$, yielding $u = \boxed{0.115c}$.

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$

P39.67 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with $mc^2 = 8.60 \times 10^{-9}\text{ J} = 53.8\text{ GeV}$.

Thus, $\left(mc^2 + K\right)^2 = (14.0\text{ keV})^2 + \left(mc^2\right)^2$ or $\left(1 + \dfrac{K}{mc^2}\right)^2 = \left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2 + 1$

So $1 + \dfrac{K}{mc^2} = \sqrt{1 + \left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2} \approx 1 + \dfrac{1}{2}\left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2$ (Binomial Theorem)

and $K \approx \dfrac{(14.0\text{ keV})^2}{2mc^2} = \dfrac{\left(14.0 \times 10^3\text{ eV}\right)^2}{2\left(53.8 \times 10^9\text{ eV}\right)} = \boxed{1.82 \times 10^{-3}\text{ eV}}$

ANSWERS TO EVEN PROBLEMS

P39.2 See the solution.

P39.4 $0.866c$

P39.6 (a) 64.9/min (b) 10.6/min

P39.8 (a) 2.18 μs b) The muon sees the planet surface moving 649 m up toward it.

P39.10 (a) $\dfrac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}$ (b) 4.00 m/s (c) See the solution.

P39.12 $v = 0.140\,c$

P39.14 5.45 yr, Goslo is older

P39.16 (c) 2.00 kHz (d) $\pm$ 0.075 0 m/s $\approx$ 0.2 mi/h

P39.18 $0.220\,c = 6.59 \times 10^7$ m/s

P39.20 (a) 17.4 m (b) 3.30°

P39.22 (a) 2.50×10^8 m/s (b) 4.97 m (c) -1.33×10^{-8} s

P39.24 $0.357c$

P39.26 (a) $0.141c$ (b) $0.436c$

P39.28 (a) \$800 (b) $\$2.12 \times 10^9$

P39.30 (a) 3.07 MeV (b) $0.986c$

P39.32 See the solution.

P39.34 (a) 438 GJ (b) 438 GJ (c) The two kinetic energy values are experimentally indistinguishable, essentially identical. The fastest-moving macroscopic objects launched by human beings move sufficiently slowly compared to light that relativistic corrections to their energy are negligible.

P39.36 8.84×10^{-28} kg and 2.51×10^{-28} kg

P39.38 (a) $0.302c$ (b) 4.00 fJ

P39.40 (a) 3.91×10^4 (b) $u = 0.999\,999\,999\,7\,c$ (c) 7.67 cm

P39.42 larger by $\sim 10^{-9}$ J

P39.44 0.842 kg

P39.46 1.02 MeV

P39.48 (a) $\frac{v}{c} = 1 - 1.12 \times 10^{-10}$ (b) 6.00×10^{27} J (c) $\$2.17 \times 10^{20}$

P39.50 (a) $0.023\,6c$ (b) $6.18 \times 10^{-4}\,c$

P39.52 (a) $0.800\,c$ (b) 7.50 ks (c) 1.44 Tm, $-0.385\,c$ (d) 4.88 ks

P39.54 (a) $\frac{3.65 \text{ MeV}}{c^2}$ (b) $v = 0.589c$

P39.56 (a) 0.909 MeV (b) 0.398 MeV (c) $0.752 \text{ MeV}/c = 4.01 \times 10^{-22}$ kg·m/s (d) 65.4°

P39.58 (c) $u \to c$

P39.60 1.47 km

P39.62 (a) $\frac{2d}{c+v}$ (b) $\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}$

P39.64 See the solution.

P39.66 See the solution, $0.115c$, $0.257c$, $0.786c$.