3

Vectors

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q3.1 Only force and velocity are vectors. None of the other quantities requires a direction to be described. The answers are (a) yes (b) no (c) no (d) no (e) no (f) yes (g) no.

Q3.2 The book's displacement is zero, as it ends up at the point from which it started. The distance traveled is 6.0 meters.

***Q3.3** The vector $-2\vec{\mathbf{D}}_1$ will be twice as long as $\vec{\mathbf{D}}_1$ and in the opposite direction, namely northeast. Adding $\vec{\mathbf{D}}_2$, which is about equally long and southwest, we get a sum that is still longer and due east, choice (a).

***Q3.4** The magnitudes of the vectors being added are constant, and we are considering the magnitude only—not the direction—of the resultant. So we need look only at the angle between the vectors being added in each case. The smaller this angle, the larger the resultant magnitude. Thus the ranking is c = e > a > d > b.

***Q3.5** (a) leftward: negative. (b) upward: positive (c) rightward: positive (d) downward: negative (e) Depending on the signs and angles of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, the sum could be in any quadrant. (f) Now $-\vec{\mathbf{A}}$ will be in the fourth quadrant, so $-\vec{\mathbf{A}}+\vec{\mathbf{B}}$ will be in the fourth quadrant.

***Q3.6** (i) The magnitude is $\sqrt{10^2+10^2}$ m/s, answer (f). (ii) Having no y component means answer (a).

***Q3.7** The vertical component is opposite the 30° angle, so sin 30° = (vertical component)/50 m and the answer is (h).

***Q3.8** Take the difference of the coordinates of the ends of the vector. Final first means head end first. (i) $-4-2=-6$ cm, answer (j) (ii) $1-(-2)=3$ cm, answer (c)

Q3.9 (i) If the direction-angle of $\vec{\mathbf{A}}$ is between 180 degrees and 270 degrees, its components are both negative: answer (c). If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs: answer (b) or (d).

Q3.10 Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.

Q3.11 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

Q3.12 Addition of a vector to a scalar is not defined. Think of numbers of apples and of clouds.

SOLUTIONS TO PROBLEMS

Section 3.1 Coordinate Systems

P3.1 $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

P3.2 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2}\text{ m} = \boxed{4.55\text{ m}}$

P3.3 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$; $\vec{\mathbf{r}} = \boxed{2.24\text{ m},\ 26.6°}$

P3.4 We have $2.00 = r\cos 30.0°$

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

and $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

P3.5 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}.$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$ This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$ This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

P3.6 $-\vec{\mathbf{R}} = \boxed{310\text{ km at }57°\text{ S of W}}$

(Scale: 1 unit = 20 km)

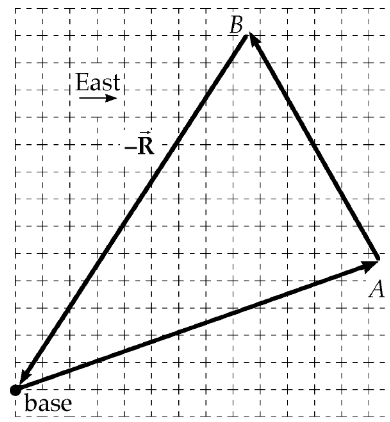


FIG. P3.6

P3.7 $\tan 35.0° = \dfrac{x}{100\text{ m}}$

$$x = (100\text{ m})\tan 35.0° = \boxed{70.0\text{ m}}$$

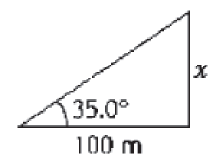


FIG. P3.7

P3.8 Find the resultant $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$. The resultant force vector $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ is of magnitude $\boxed{9.5\text{ N}}$ and at an angle of $\boxed{57°\text{ above the }x\text{ axis}}$.

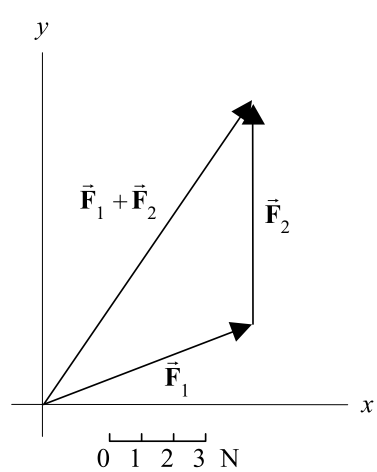


FIG. P3.8

P3.9 (a) $\left|\vec{\mathbf{d}}\right| = \left|-10.0\hat{\mathbf{i}}\right| = \boxed{10.0 \text{ m}}$ since the displacement is in a straight line from point A to point B.

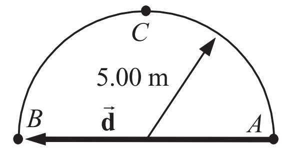


FIG. P3.9

(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (ACB).

$$s = \frac{1}{2}(2\pi r) = 5\pi = \boxed{15.7 \text{ m}}$$

(c) If the circle is complete, $\vec{\mathbf{d}}$ begins and ends at point **A**. Hence, $\left|\vec{\mathbf{d}}\right| = \boxed{0}$.

P3.10 (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, and west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as

$\sim 10^5(0.03 \text{ m}) + 10^2(1 \text{ m})\ \boxed{\sim 10^3 \text{ m upward}}$.

P3.11 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 5.2$ m at 60°

(b) $\vec{\mathbf{A}} - \vec{\mathbf{B}} = 3.0$ m at 330°

(c) $\vec{\mathbf{B}} - \vec{\mathbf{A}} = 3.0$ m at 150°

(d) $\vec{\mathbf{A}} - 2\mathbf{B} = 5.2$ m at 300°

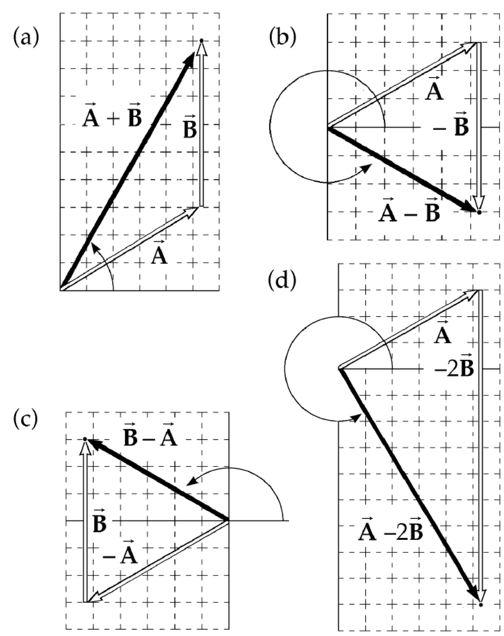


FIG. P3.11

P3.12 The three diagrams shown below represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$. We observe that $\vec{\mathbf{R}}_1 = \vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_3$, illustrating that

$\boxed{\text{the sum of a set of vectors is not affected by the order in which the vectors are added}}$.

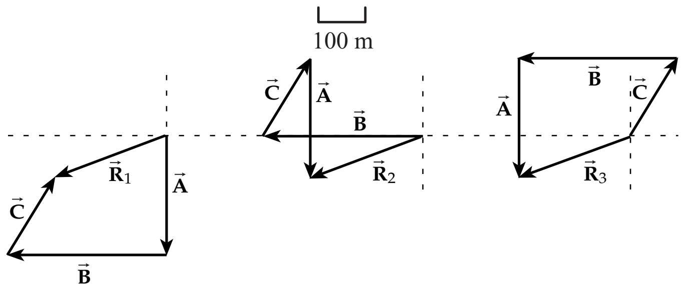


FIG. P3.12

P3.13 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be

$\boxed{d = 420 \text{ ft and } \theta = -3°}$.

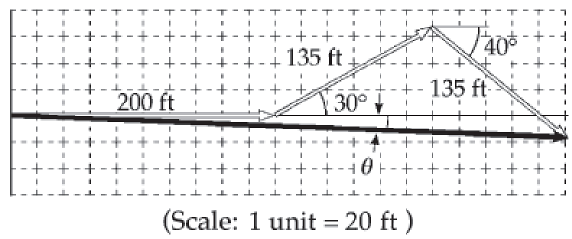


FIG. P3.13

Section 3.4 **Components of a Vector and Unit Vectors**

***P3.14** We assume the floor is level. Take the x axis in the direction of the first displacement. $\boxed{\text{If both of the 90° turns are to the right or both to the left}}$, the displacements add like

$$40.0 \text{ m } \hat{\mathbf{i}} + 15.0 \text{ m } \hat{\mathbf{j}} - 20.0 \text{ m } \hat{\mathbf{i}} = \left(20.0\ \hat{\mathbf{i}} + 15.0\ \hat{\mathbf{j}}\right) \text{m}$$

to give (a) displacement magnitude $(20^2 + 15^2)^{1/2}$ m = $\boxed{25.0 \text{ m}}$

at (b) $\tan^{-1}(15/20) = \boxed{36.9°}$.

$\boxed{\text{If one turn is right and the other is left}}$, the displacements add like

$$40.0 \text{ m } \hat{\mathbf{i}} + 15.0 \text{ m } \hat{\mathbf{j}} + 20.0 \text{ m } \hat{\mathbf{i}} = \left(60.0\ \hat{\mathbf{i}} + 15.0\ \hat{\mathbf{j}}\right) \text{m}$$

to give (a) displacement magnitude $(60^2 + 15^2)^{1/2}$ m = $\boxed{61.8 \text{ m}}$

at (b) $\tan^{-1}(15/60) = \boxed{14.0°.\ \text{Just two answers are possible.}}$

P3.15 $A_x = -25.0$

$A_y = 40.0$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

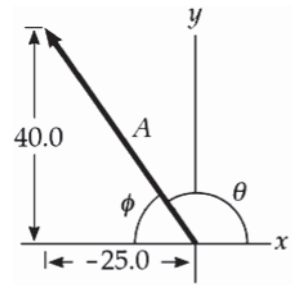


FIG. P3.15

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

P3.16 The person would have to walk $3.10\sin(25.0°) = \boxed{1.31 \text{ km north}}$, and

$$3.10\cos(25.0°) = \boxed{2.81 \text{ km east}}.$$

***P3.17** Let v represent the speed of the camper. The northward component of its velocity is v cos 8.5°. To avoid crowding the minivan we require $v \cos 8.5° \geq 28$ m/s.

$\boxed{\text{We can satisfy this requirement simply by taking } v \geq (28 \text{ m/s})/\cos 8.5° = 28.3 \text{ m/s.}}$

P3.18 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks}$$

and the angle the resultant makes with the x axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0 \text{ blocks}}$.

P3.19 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x, y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x, y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x, y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

P3.20 $x = d\cos\theta = (50.0 \text{ m})\cos(120) = -25.0$ m

$y = d\sin\theta = (50.0 \text{ m})\sin(120) = 43.3$ m

$\vec{\mathbf{d}} = \boxed{(-25.0 \text{ m})\hat{\mathbf{i}} + (43.3 \text{ m})\hat{\mathbf{j}}}$

P3.21 Let $+x$ be East and $+y$ be North.

$$\sum x = 250 + 125\cos 30° = 358 \text{ m}$$
$$\sum y = 75 + 125\sin 30° - 150 = -12.5 \text{ m}$$
$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358 \text{ m}$$
$$\tan\theta = \frac{\left(\sum y\right)}{\left(\sum x\right)} = -\frac{12.5}{358} = -0.0349$$
$$\theta = -2.00°$$
$$\boxed{\vec{\mathbf{d}} = 358 \text{ m at } 2.00° \text{ S of E}}$$

P3.22 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}} = 730\cos 5.00° - 560\sin 21.0° = 527 \text{ miles}$$
$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}} = 730\sin 5.00° + 560\cos 21.0° = 586 \text{ miles}$$

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$.

$$\text{Then } \tan\theta = \frac{d_{\text{DC north}}}{d_{\text{DC east}}} = 1.11 \text{ and } \theta = 48.0°.$$

Thus, Chicago is $\boxed{\text{788 miles at 48.0° northeast of Dallas}}$.

P3.23 We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$
$$A_y = 150\sin 120° = 130 \text{ cm}$$
$$R_x = 140\cos 35.0° = 115 \text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

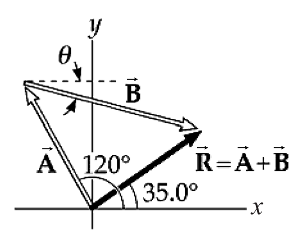


FIG. P3.23

Therefore,

$$\vec{\mathbf{B}} = [115 - (-75)]\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right) \text{ cm}$$
$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}.$$

P3.24 (a) See figure to the right.

(b) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$$\vec{\mathbf{C}} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$$

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$$

$$\vec{\mathbf{D}} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$$

$$\vec{\mathbf{D}} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$$

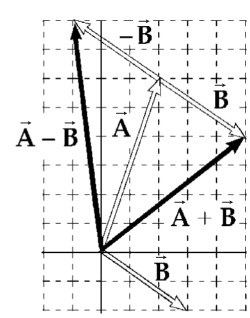


FIG. P3.24

P3.25 (a) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right)=\left(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}\right)+\left(-\hat{\mathbf{i}}-4\hat{\mathbf{j}}\right)=\boxed{2\hat{\mathbf{i}}-6\hat{\mathbf{j}}}$

(b) $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)=\left(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}\right)-\left(-\hat{\mathbf{i}}-4\hat{\mathbf{j}}\right)=\boxed{4\hat{\mathbf{i}}+2\hat{\mathbf{j}}}$

(c) $\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right|=\sqrt{2^2+6^2}=\boxed{6.32}$

(d) $\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right|=\sqrt{4^2+2^2}=\boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|}=\tan^{-1}\left(-\frac{6}{2}\right)=-71.6°=\boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|}=\tan^{-1}\left(\frac{2}{4}\right)=\boxed{26.6°}$

***P3.26** We take the x axis along the slope uphill. Students, get used to this choice! The y axis is perpendicular to the slope, at 35° to the vertical. Then the displacement of the snow makes an angle of $90° - 35° - 20° = 35°$ with the x axis.

(a) Its component parallel to the surface is 5 m cos 35° = $\boxed{\text{4.10 m toward the top of the hill}}$.

(b) Its component perpendicular to the surface is 5 m sin 35° = $\boxed{\text{2.87 m}}$.

P3.27 $\vec{\mathbf{d}}_1=\left(-3.50\hat{\mathbf{j}}\right)$ m

$\vec{\mathbf{d}}_2=8.20\cos 45.0°\hat{\mathbf{i}}+8.20\sin 45.0°\hat{\mathbf{j}}=\left(5.80\hat{\mathbf{i}}+5.80\hat{\mathbf{j}}\right)$ m

$\vec{\mathbf{d}}_3=\left(-15.0\hat{\mathbf{i}}\right)$ m

$\vec{\mathbf{R}}=\vec{\mathbf{d}}_1+\vec{\mathbf{d}}_2+\vec{\mathbf{d}}_3=(-15.0+5.80)\hat{\mathbf{i}}+(5.80-3.50)\hat{\mathbf{j}}=\boxed{\left(-9.20\hat{\mathbf{i}}+2.30\hat{\mathbf{j}}\right)\text{ m}}$

(or 9.20 m west and 2.30 m north)
The magnitude of the resultant displacement is

$$\left|\vec{\mathbf{R}}\right|=\sqrt{R_x^2+R_y^2}=\sqrt{(-9.20)^2+(2.30)^2}=\boxed{9.48\text{ m}}.$$

The direction is $\theta=\arctan\left(\frac{2.30}{-9.20}\right)=\boxed{166°}$.

P3.28 Refer to the sketch

$$\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=-10.0\hat{\mathbf{i}}-15.0\hat{\mathbf{j}}+50.0\hat{\mathbf{i}}$$
$$=40.0\hat{\mathbf{i}}-15.0\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{R}}\right|=\left[(40.0)^2+(-15.0)^2\right]^{1/2}=\boxed{42.7\text{ yards}}$$

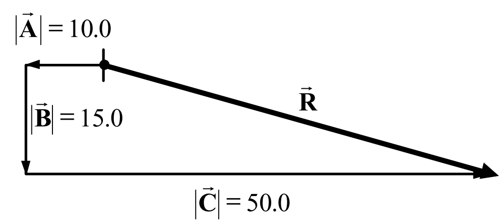


FIG. P3.28

P3.29

East x	North y
0 m	4.00 m
1.41	1.41
–0.500	–0.866
+0.914	4.55

$\left|\vec{\mathbf{R}}\right|=\sqrt{|x|^2+|y|^2}$ at $\tan^{-1}(y/x)=\boxed{4.64\text{ m at }78.6°\text{ N of E}}$

P3.30 $\vec{\mathbf{A}} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$: $\quad 3\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$

$$\vec{\mathbf{C}} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or $\quad C_x = \boxed{7.30 \text{ cm}};\ C_y = \boxed{-7.20 \text{ cm}}$

P3.31 (a) $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

$$\vec{\mathbf{F}} = 120\cos(60.0°)\,\hat{\mathbf{i}} + 120\sin(60.0°)\,\hat{\mathbf{j}} - 80.0\cos(75.0°)\,\hat{\mathbf{i}} + 80.0\sin(75.0°)\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{39.3^2 + 181^2} = \boxed{185 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) $\vec{\mathbf{F}}_3 = -\vec{\mathbf{F}} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$

P3.32 $\vec{\mathbf{A}} = 3.00 \text{ m},\ \theta_A = 30.0°$ $\qquad \vec{\mathbf{B}} = 3.00 \text{ m},\ \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60 \text{ m}$ $\qquad A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50 \text{ m}$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) \text{ m}$$

$B_x = 0,\ B_y = 3.00 \text{ m}$ $\qquad$ so $\qquad \vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} \text{ m}$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right) \text{ m}}$$

P3.33 $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$

$$\left|\vec{\mathbf{B}}\right| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$ is the angle with the x axis

$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$ is the angle with the y axis

$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$ is the angle with the z axis

P3.34 (a) $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{2^2 + 2^2} = \boxed{2.83 \text{ m at } \theta = 315°}$$

(b) $\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}} = -6\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{6^2 + 12^2} = \boxed{13.4 \text{ m at } \theta = 117°}$$

P3.35 (a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right) \text{ m}}$

$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2} \text{ m} = \boxed{5.92 \text{ m}}$$

(b) $\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right) \text{ m}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2} \text{ m} = \boxed{19.0 \text{ m}}$$

P3.36 Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\vec{\mathbf{d}} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right) \text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right) \text{ cm} = \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right) \text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2} \text{ cm} = \boxed{10.4 \text{ cm}}$.

(b) Its angle with the y axis follows from $\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

P3.37 (a) $\vec{\mathbf{A}} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{A}}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\vec{\mathbf{C}} = -3\vec{\mathbf{A}} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.38 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 8\ 040 \text{ m/30 s} = 268 \text{ m/s}$. The position vector as a function of time is

$$\vec{\mathbf{P}} = (268 \text{ m/s})t\hat{\mathbf{i}} + \left(7.60 \times 10^3 \text{ m}\right)\hat{\mathbf{j}}.$$

At $t = 45.0$ s, $\vec{\mathbf{P}} = \left[1.21 \times 10^4\,\hat{\mathbf{i}} + 7.60 \times 10^3\,\hat{\mathbf{j}}\right]$ m. The magnitude is

$$\vec{\mathbf{P}} = \sqrt{\left(1.21 \times 10^4\right)^2 + \left(7.60 \times 10^3\right)^2} \text{ m} = \boxed{1.43 \times 10^4 \text{ m}}$$

and the direction is

$$\theta = \arctan\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2° \text{ above the horizontal}}.$$

P3.39 The position vector from radar station to ship is

$$\vec{\mathbf{S}} = \left(17.3 \sin 136°\hat{\mathbf{i}} + 17.3 \cos 136°\hat{\mathbf{j}}\right) \text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right) \text{ km}.$$

From station to plane, the position vector is

$$\vec{\mathbf{P}} = \left(19.6 \sin 153°\hat{\mathbf{i}} + 19.6 \cos 153°\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{ km},$$

or

$$\vec{\mathbf{P}} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{ km}.$$

(a) To fly to the ship, the plane must undergo displacement

$$\vec{\mathbf{D}} = \vec{\mathbf{S}} - \vec{\mathbf{P}} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right) \text{ km}}.$$

(b) The distance the plane must travel is

$$D = \left|\vec{\mathbf{D}}\right| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2} \text{ km} = \boxed{6.31 \text{ km}}.$$

P3.40 (a) $\vec{\mathbf{E}} = (17.0 \text{ cm})\cos 27.0°\hat{\mathbf{i}} + (17.0 \text{ cm})\sin 27.0°\hat{\mathbf{j}}$

$\vec{\mathbf{E}} = \boxed{(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}) \text{ cm}}$

(b) $\vec{\mathbf{F}} = -(17.0 \text{ cm})\sin 27.0°\hat{\mathbf{i}} + (17.0 \text{ cm})\cos 27.0°\hat{\mathbf{j}}$

$\vec{\mathbf{F}} = \boxed{(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}) \text{ cm}}$ Note that we do not need to explicitly identify the angle with the positive x axis.

(c) $\vec{\mathbf{G}} = +(17.0 \text{ cm})\sin 27.0°\hat{\mathbf{i}} + (17.0 \text{ cm})\cos 27.0°\hat{\mathbf{j}}$

$\vec{\mathbf{G}} = \boxed{(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}) \text{ cm}}$

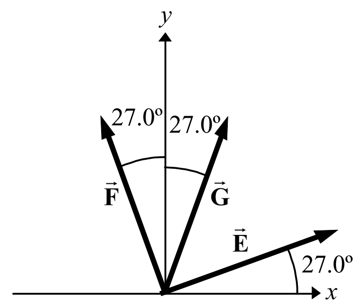


FIG. P3.40

P3.41 $A_x = -3.00,\ A_y = 2.00$

(a) $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $|\vec{\mathbf{A}}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$\tan\theta = \dfrac{A_y}{A_x} = \dfrac{2.00}{(-3.00)} = -0.667,\ \tan^{-1}(-0.667) = -33.7°$

θ is in the 2nd quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

(c) $R_x = 0,\ R_y = -4.00,\ \vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ thus $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00,\ B_y = R_y - A_y = -4.00 - 2.00 = -6.00$.

Therefore, $\vec{\mathbf{B}} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.42 The hurricane's first displacement is $\left(\dfrac{41.0 \text{ km}}{\text{h}}\right)(3.00 \text{ h})$ at 60.0° N of W, and its second displacement is $\left(\dfrac{25.0 \text{ km}}{\text{h}}\right)(1.50 \text{ h})$ due North. With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0°\right)(3.00 \text{ h})(-\hat{\mathbf{i}}) + \left(41.0\frac{\text{km}}{\text{h}}\sin 60.0°\right)(3.00 \text{ h})\hat{\mathbf{j}} + \left(25.0\frac{\text{km}}{\text{h}}\right)(1.50 \text{ h})\hat{\mathbf{j}}$$

$$= 61.5 \text{ km}(-\hat{\mathbf{i}}) + 144 \text{ km}\,\hat{\mathbf{j}}$$

with magnitude $\sqrt{(61.5 \text{ km})^2 + (144 \text{ km})^2} = \boxed{157 \text{ km}}$.

P3.43 (a) $R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$

$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$

$\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b) $|\vec{\mathbf{R}}| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\dfrac{27.1}{49.5}\right) = \boxed{28.7°}$

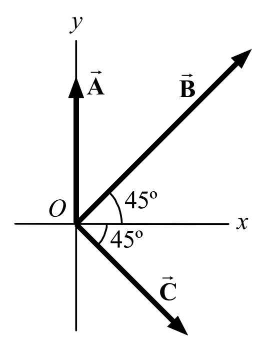


FIG. P3.43

***P3.44** (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0.$$

We solve simultaneously by substituting $a = 1.33\,b - 4.33$ to find $-8(1.33\,b - 4.33) + 3\,b + 19 = 0$

or $7.67b = 53.67$ so $b = 7.00$ and $a = 1.33(7) - 4.33$.

Thus

$$\boxed{a = 5.00,\ b = 7.00}.$$

Therefore,

$$5.00\vec{\mathbf{A}} + 7.00\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0.$$

(b) In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation.

***P3.45** The displacement from the start to the finish is $16\,\hat{\mathbf{i}} + 12\,\hat{\mathbf{j}} - (5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}) = (11\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}})$ meters.
The displacement from the starting point to A is $f(11\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}})$ meters.

(a) The position vector of point A is $5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + f(11\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}}) = \boxed{(5 + 11f)\,\hat{\mathbf{i}} + (3 + 9f)\,\hat{\mathbf{j}}\ \text{meters}}$.

(b) For $f = 0$ we have the position vector $(5 + 0)\,\hat{\mathbf{i}} + (3 + 0)\,\hat{\mathbf{j}}$ meters.

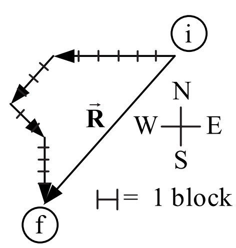
This is reasonable because it is the location of the starting point, $5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ meters.

(c) For $f = 1 = 100\%$, we have position vector $(5 + 11)\,\hat{\mathbf{i}} + (3 + 9)\,\hat{\mathbf{j}}$ meters $= \boxed{16\,\hat{\mathbf{i}} + 12\,\hat{\mathbf{j}}\ \text{meters}}$.

This is reasonable because we have completed the trip and this is the position vector of the endpoint.

***P3.46** We note that $-\hat{\mathbf{i}}$ = west and $-\hat{\mathbf{j}}$ = south. The given mathematical representation of the trip can be written as 6.3 b west + 4 b at 40° south of west + 3 b at 50° south of east + 5 b south.

(a)



(b) The total odometer distance is the sum of the magnitudes of the four displacements:

$$6.3\text{ b} + 4\text{ b} + 3\text{ b} + 5\text{ b} = \boxed{18.3\text{ b}}.$$

(c)

$$\begin{aligned}
\vec{\mathbf{R}} &= (-6.3 - 3.06 + 1.93)\text{ b }\hat{\mathbf{i}} + (-2.57 - 2.30 - 5)\text{ b }\hat{\mathbf{j}} \\
&= -7.44\text{ b }\hat{\mathbf{i}} - 9.87\text{ b }\hat{\mathbf{j}} \\
&= \sqrt{(7.44\text{ b})^2 + (9.87\text{ b})^2}\ \text{at}\ \tan^{-1}\frac{9.87}{7.44}\ \text{south of west} \\
&= 12.4\text{ b at }53.0°\text{ south of west} \\
&= \boxed{12.4\text{ b at }233°\text{ counterclockwise from east}}
\end{aligned}$$

Additional Problems

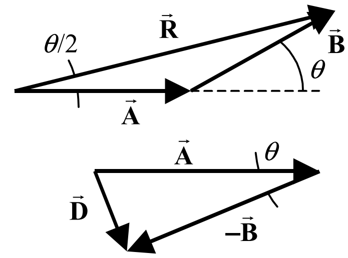


FIG. P3.47

P3.47 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$.

The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$.

This can be seen from applying the law of cosines to the isosceles triangle and using the fact that $B = A$.

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\dfrac{\theta}{2}\right) = 200A\sin\left(\dfrac{\theta}{2}\right)$. This gives $\tan\left(\dfrac{\theta}{2}\right) = 0.010$ and $\boxed{\theta = 1.15°}$.

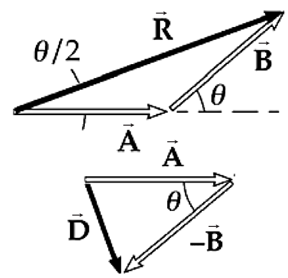


FIG. P3.48

P3.48 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$.

The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$. This can be seen by applying the law of cosines to the isosceles triangle and using the fact that $B = A$.

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = nD$ or $\cos\left(\dfrac{\theta}{2}\right) = n\sin\left(\dfrac{\theta}{2}\right)$ giving $\boxed{\theta = 2\tan^{-1}\left(\dfrac{1}{n}\right)}$.

The larger R is to be compared to D, the smaller the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ becomes.

P3.49 The position vector from the ground under the controller of the first airplane is

$$\begin{aligned}\vec{\mathbf{r}}_1 &= (19.2\text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2\text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8\text{ km})\hat{\mathbf{k}}\\ &= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right)\text{ km.}\end{aligned}$$

The second is at

$$\begin{aligned}\vec{\mathbf{r}}_2 &= (17.6\text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6\text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1\text{ km})\hat{\mathbf{k}}\\ &= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right)\text{ km.}\end{aligned}$$

Now the displacement from the first plane to the second is

$$\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right)\text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29\text{ km}}.$$

P3.50 Take the x axis along the tail section of the snake. The displacement from tail to head is

$$240\text{ m}\hat{\mathbf{i}}+(420-240)\text{ m}\cos(180°-105°)\hat{\mathbf{i}}-180\text{ m}\sin 75°\hat{\mathbf{j}}=287\text{ m}\hat{\mathbf{i}}-174\text{ m}\hat{\mathbf{j}}.$$

Its magnitude is $\sqrt{(287)^2+(174)^2}\text{ m}=335\text{ m}$. From $v=\dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: }\Delta t=\frac{\text{distance}}{v}=\frac{335\text{ m(h)(1 km)(3 600 s)}}{(12\text{ km})(1\,000\text{ m})(1\text{ h})}=101\text{ s}$$

$$\text{Olaf: }\Delta t=\frac{420\text{ m}\cdot\text{s}}{3.33\text{ m}}=126\text{ s}.$$

Inge wins by $126-101=\boxed{25.4\text{ s}}$.

P3.51 Let A represent the distance from island 2 to island 3. The displacement is $\vec{\mathbf{A}}=A$ at 159°. Represent the displacement from 3 to 1 as $\vec{\mathbf{B}}=B$ at 298°. We have 4.76 km at $37°+\vec{\mathbf{A}}+\vec{\mathbf{B}}=0$.

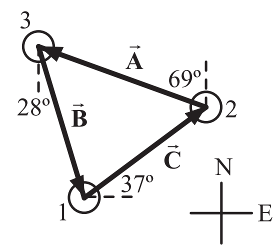


FIG. P3.51

For x components

$$(4.76\text{ km})\cos 37°+A\cos 159°+B\cos 298°=0$$
$$3.80\text{ km}-0.934A+0.469B=0$$
$$B=-8.10\text{ km}+1.99A$$

For y components

$$(4.76\text{ km})\sin 37°+A\sin 159°+B\sin 298°=0$$
$$2.86\text{ km}+0.358A-0.883B=0$$

(a) We solve by eliminating B by substitution:

$$2.86\text{ km}+0.358A-0.883(-8.10\text{ km}+1.99A)=0$$
$$2.86\text{ km}+0.358A+7.15\text{ km}-1.76A=0$$
$$10.0\text{ km}=1.40A$$
$$A=\boxed{7.17\text{ km}}$$

(b) $B=-8.10\text{ km}+1.99(7.17\text{ km})=\boxed{6.15\text{ km}}$

P3.52 (a) $R_x=\boxed{2.00}$, $R_y=\boxed{1.00}$, $R_z=\boxed{3.00}$

(b) $\left|\vec{\mathbf{R}}\right|=\sqrt{R_x^2+R_y^2+R_z^2}=\sqrt{4.00+1.00+9.00}=\sqrt{14.0}=\boxed{3.74}$

(c) $\cos\theta_x=\dfrac{R_x}{\left|\vec{\mathbf{R}}\right|}\Rightarrow\theta_x=\cos^{-1}\left(\dfrac{R_x}{\left|\vec{\mathbf{R}}\right|}\right)=\boxed{57.7°\text{ from }+x}$

$\cos\theta_y=\dfrac{R_y}{\left|\vec{\mathbf{R}}\right|}\Rightarrow\theta_y=\cos^{-1}\left(\dfrac{R_y}{\left|\vec{\mathbf{R}}\right|}\right)=\boxed{74.5°\text{ from }+y}$

$\cos\theta_z=\dfrac{R_z}{\left|\vec{\mathbf{R}}\right|}\Rightarrow\theta_z=\cos^{-1}\left(\dfrac{R_z}{\left|\vec{\mathbf{R}}\right|}\right)=\boxed{36.7°\text{ from }+z}$

P3.53 $\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\hat{\mathbf{i}} + (100\sin 30.0°)\hat{\mathbf{j}}$

$\vec{\mathbf{v}} = \left(387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ mi/h

$|\vec{\mathbf{v}}| = \boxed{390 \text{ mi/h at } 7.37° \text{ N of E}}$

***P3.54** (a) $\vec{\mathbf{A}} = -60 \text{ cm }\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = \left(80\ \cos\theta\ \hat{\mathbf{i}} + 80\ \sin\theta\ \hat{\mathbf{j}}\right)$ cm

so $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 80\cos\theta\ \hat{\mathbf{i}} + (80\sin\theta - 60)\ \hat{\mathbf{j}}$ centimeters and

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80\cos\theta)^2 + (80\sin\theta - 60)^2\right]^{1/2} \text{ cm} = \left[80^2\cos^2\theta + 80^2\sin^2\theta - 2(80)(60)\cos\theta + 60^2\right]^{1/2} \text{ cm}$$

Now $\sin^2\theta + \cos^2\theta = 1$ for all θ, so we have

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[80^2 + 60^2 - 2(80)(60)\cos\theta\right]^{1/2} \text{ cm} = \boxed{[10\,000 - 9\,600\cos\theta]^{1/2} \text{ cm}}$$

(b) For $\boxed{\theta = 270°}$, $\cos\theta = -1$ and the expression takes on its maximum value,
[10 000 + 9 600] 1/2 cm = $\boxed{140 \text{ cm}}$.

(c) For $\boxed{\theta = 90°}$, $\cos\theta = +1$ and the expression takes on its minimum value, [10 000 − 9 600] 1/2 cm = $\boxed{20.0 \text{ cm}}$.

(d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

***P3.55** $$\Delta\vec{\mathbf{r}} = \int_0^{0.380\text{s}}\left(1.2\ \hat{\mathbf{i}}\text{ m/s} - 9.8t\ \hat{\mathbf{j}}\text{ m/s}^2\right)dt = 1.2t\ \hat{\mathbf{i}}\text{ m/s}\Big|_0^{0.38\text{s}} - 9.8\hat{\mathbf{j}}\text{ m/s}^2\ \frac{t^2}{2}\Bigg|_0^{0.38\text{s}}$$

$$= \left(1.2\ \hat{\mathbf{i}}\text{ m/s}\right)(0.38\text{ s} - 0) - 9.8\hat{\mathbf{j}}\text{ m/s}^2\left(\frac{(0.38\text{ s})^2 - 0}{2}\right) = \boxed{0.456\hat{\mathbf{i}}\text{ m} - 0.708\hat{\mathbf{j}}\text{ m}}$$

P3.56 Choose the $+x$ axis in the direction of the first force, and the y axis at 90° counterclockwise from the x axis. Then each force will have only one nonzero component.

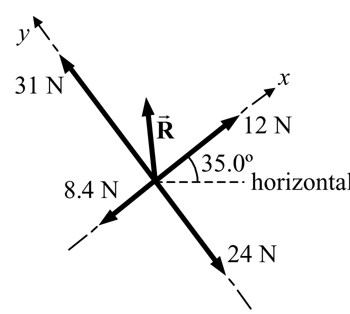


FIG. P3.56

The total force, in newtons, is then

$$12.0\hat{\mathbf{i}} + 31.0\hat{\mathbf{j}} - 8.40\hat{\mathbf{i}} - 24.0\hat{\mathbf{j}} = \boxed{\left(3.60\hat{\mathbf{i}}\right) + \left(7.00\hat{\mathbf{j}}\right)\text{ N}}.$$

The magnitude of the total force is

$$\sqrt{(3.60)^2 + (7.00)^2}\text{ N} = \boxed{7.87\text{ N}}$$

and the angle it makes with our $+x$ axis is given by $\tan\theta = \dfrac{(7.00)}{(3.60)}$, $\theta = 62.8°$.

Thus, its angle counterclockwise from the horizontal is $35.0° + 62.8° = \boxed{97.8°}$.

P3.57 $\vec{\mathbf{d}}_1 = 100\hat{\mathbf{i}}$

$\vec{\mathbf{d}}_2 = -300\hat{\mathbf{j}}$

$\vec{\mathbf{d}}_3 = -150\cos(30.0°)\hat{\mathbf{i}} - 150\sin(30.0°)\hat{\mathbf{j}} = -130\hat{\mathbf{i}} - 75.0\hat{\mathbf{j}}$

$\vec{\mathbf{d}}_4 = -200\cos(60.0°)\hat{\mathbf{i}} + 200\sin(60.0°)\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 173\hat{\mathbf{j}}$

$$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{ m}}$$

$$\left|\vec{\mathbf{R}}\right| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240\text{ m}}$$

$$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2°$$

$$\theta = 180 + \phi = \boxed{237°}$$

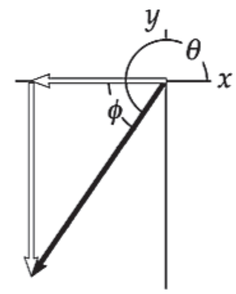


FIG. P3.57

P3.58 $$\frac{d\vec{\mathbf{r}}}{dt} = \frac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{j}}\right)}{dt} = 0 + 0 - 2\hat{\mathbf{j}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{j}}}$$

The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 1\hat{\mathbf{j}}$, and so on. The object is moving straight downward at 2 m/s, so

$$\frac{d\vec{\mathbf{r}}}{dt} \text{ represents } \boxed{\text{its velocity vector}}.$$

P3.59 (a) You start at point A: $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}.$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ to move to

$$\vec{\mathbf{r}}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}.$$

Now the displacement from your current position to C is

$$\vec{\mathbf{r}}_C - \vec{\mathbf{r}}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}.$$

You cover one-third, moving to

$$\vec{\mathbf{r}}_3 = \vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}.$$

The displacement from where you are to D is

$$\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}.$$

You traverse one-quarter of it, moving to

$$\vec{\mathbf{r}}_4 = \vec{\mathbf{r}}_3 + \frac{1}{4}\left(\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right) = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}.$$

The displacement from your new location to **E** is

$$\vec{\mathbf{r}}_E - \vec{\mathbf{r}}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\vec{\mathbf{r}}_4 + \Delta\vec{\mathbf{r}}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}.$$

The treasure is at $\boxed{(10.0\text{ m}, 16.0\text{ m})}$.

continued on next page

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\vec{\mathbf{r}}_A + \frac{1}{2}\left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left(\frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B}{2}\right)$$

then to $\dfrac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)}{2} + \dfrac{\vec{\mathbf{r}}_C - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)/2}{3} = \dfrac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C}{3}$

then to $\dfrac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)}{3} + \dfrac{\vec{\mathbf{r}}_D - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)/3}{4} = \dfrac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D}{4}$

and last to $\dfrac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)}{4} + \dfrac{\vec{\mathbf{r}}_E - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)/4}{5} = \dfrac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D + \vec{\mathbf{r}}_E}{5}$.

This center of mass of the tree distribution is the same location whatever order we take the trees in.

P3.60 (a) Let T represent the force exerted by each child. The x component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0$$

The y component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0.$$

Thus,

$$\sum \vec{\mathbf{F}} = 0$$

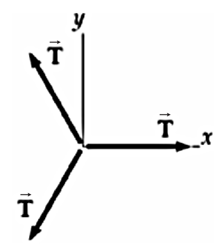


FIG. P3.60

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the whole set of forces acting on the tire turns clockwise by that angle so the total force must turn clockwise by that angle, $\dfrac{360°}{N}$. Because each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. But the force cannot have two different directions. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

P3.61 Since

$$\vec{\mathbf{A}}+\vec{\mathbf{B}}=6.00\hat{\mathbf{j}},$$

we have

$$\left(A_x+B_x\right)\hat{\mathbf{i}}+\left(A_y+B_y\right)\hat{\mathbf{j}}=0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}$$

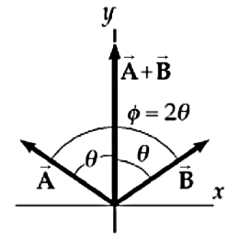


FIG. P3.61

giving

$$A_x+B_x=0 \text{ or } A_x=-B_x \qquad [1]$$

and

$$A_y+B_y=6.00. \qquad [2]$$

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2+A_y^2=B_x^2+B_y^2=5.00^2.$$

From $A_x=-B_x$, it is seen that

$$A_x^2=B_x^2.$$

Therefore, $A_x^2+A_y^2=B_x^2+B_y^2$ gives

$$A_y^2=B_y^2.$$

Then, $A_y=B_y$ and Equation [2] gives

$$A_y=B_y=3.00.$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

$$\cos\theta=\frac{A_y}{A}=\frac{B_y}{B}=\frac{3.00}{5.00}=0.600 \text{ and } \theta=53.1°.$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then $\boxed{\phi=2\theta=106°}$.

P3.62 (a) From the picture, $\vec{\mathbf{R}}_1=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}$ and $\left|\vec{\mathbf{R}}_1\right|=\sqrt{a^2+b^2}$.

(b) $\vec{\mathbf{R}}_2=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$; its magnitude is

$$\sqrt{\left|\vec{\mathbf{R}}_1\right|^2+c^2}=\sqrt{a^2+b^2+c^2}.$$

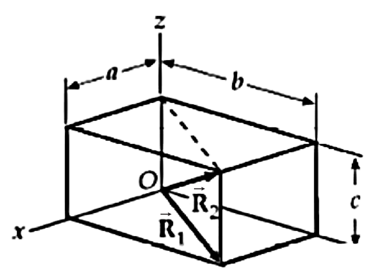


FIG. P3.62

ANSWERS TO EVEN PROBLEMS

P3.2 (a) $(2.17 \text{ m}, 1.25 \text{ m})$; $(-1.90 \text{ m}, 3.29 \text{ m})$ (b) 4.55 m

P3.4 $y=1.15$; $r=2.31$

P3.6 310 km at 57° S of W

P3.8 9.5 N at 57°

P3.10 (a) $\sim 10^5$ m vertically upward (b) $\sim 10^3$ m vertically upward

P3.12 See the solution; the sum of a set of vectors is not affected by the order in which the vectors are added.

P3.14 We assume that the shopping cart stays on the level floor. There are two possibilities. If both of the turns are right or both left, the net displacement is (a) 25.0 m (b) at 36.9°. If one turn is right and one is left, we have (a) 61.8 m (b) at 14.0°.

P3.16 1.31 km north; 2.81 km east

P3.18 (a) 5.00 blocks at 53.1° N of E (b) 13.0 blocks

P3.20 $-25.0\text{ m}\,\hat{\mathbf{i}} + 43.3\text{ m}\,\hat{\mathbf{j}}$

P3.22 788 mi at 48.0° north of east

P3.24 (a) see the solution (b) $5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$, 6.40 at 38.7°, $-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}$, 8.06 at 97.2°

P3.26 (a) 4.10 m toward the top of the hill (b) 2.87 m

P3.28 42.7 yards

P3.30 $\vec{\mathbf{C}} = 7.30\text{ cm}\,\hat{\mathbf{i}} - 7.20\text{ cm}\,\hat{\mathbf{j}}$

P3.32 $\vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.60\,\hat{\mathbf{i}} + 4.50\,\hat{\mathbf{j}})\text{m}$

P3.34 (a) 2.83 m at $\theta = 315°$ (b) 13.4 m at $\theta = 117°$

P3.36 (a) 10.4 cm; (b) 35.5°

P3.38 1.43×10^4 m at 32.2° above the horizontal

P3.40 (a) $\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right)$ cm (b) $\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm (c) $\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm

P3.42 157 km

P3.44 (a) $a = 5.00$ and $b = 7.00$ (b) For vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation.

P3.46 (a) see the solution (b) 18.3 b (c) 12.4 b at 233° counterclockwise from east

P3.48 $2\tan^{-1}\left(\frac{1}{n}\right)$

P3.50 25.4 s

P3.52 (a) 2.00, 1.00, 3.00 (b) 3.74 (c) $\theta_x = 57.7°$, $\theta_y = 74.5°$, $\theta_z = 36.7°$

P3.54 (a) $(10\,000 - 9\,600\cos\theta)^{1/2}$ cm (b) 270° ; 140 cm (c) 90° ; 20.0 cm (d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

P3.56 We choose the *x* axis to the right at 35° above the horizontal and the *y* axis at 90° counterclockwise from the *x* axis. Then each vector has only a single nonzero component. The resultant is 7.87 N at 97.8° counterclockwise from a horizontal line to the right.

P3.58 $(-2.00\text{ m/s})\hat{\mathbf{j}}$; its velocity vector

P3.60 (a) zero (b) see the solution

P3.62 (a) $\vec{\mathbf{R}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$; $\left|\vec{\mathbf{R}}_1\right| = \sqrt{a^2 + b^2}$ (b) $\vec{\mathbf{R}}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; $\left|\vec{\mathbf{R}}_2\right| = \sqrt{a^2 + b^2 + c^2}$