44

Nuclear Structure

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q44.1 Because of electrostatic repulsion between the positively-charged nucleus and the $+2e$ alpha particle. To drive the α-particle into the nucleus would require extremely high kinetic energy.

***Q44.2** (a) X has a mass number less by 2 than the others. The ranking is W = Y = Z > X.

(b) Y has a greater atomic number, because a neutron in the parent nucleus has turned into a proton. X has an atomic number less by two than W, so the ranking is Y > W = Z > X.

(c) Y has one fewer neutron compared to the parent nucleus W, and X has two fewer neutrons than W. The ranking is W = Z > Y > X.

Q44.3 The nuclear force favors the formation of neutron-proton pairs, so a stable nucleus cannot be too far away from having equal numbers of protons and neutrons. This effect sets the upper boundary of the zone of stability on the neutron-proton diagram. All of the protons repel one another electrically, so a stable nucleus cannot have too many protons. This effect sets the lower boundary of the zone of stability.

Q44.4 Nuclei with more nucleons than bismuth-209 are unstable because the electrical repulsion forces among all of the protons is stronger than the nuclear attractive force between nucleons.

Q44.5 Nucleus *Y* will be more unstable. The nucleus with the higher binding energy requires more energy to be disassembled into its constituent parts.

Q44.6 Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons. The neutrons participate in the net attractive effect of the nuclear force, but feel no Coulomb repulsion.

***Q44.7** (i) Answer (a). The liquid drop model gives a simpler account of a nuclear fission reaction, including the energy released and the probable fission product nuclei.

(ii) Answer (b). The shell model predicts magnetic moments by necessarily describing the spin and orbital angular momentum states of the nucleons.

(iii) Answer (b). Again, the shell model wins when it comes to predicting the spectrum of an excited nucleus, as the quantum model allows only quantized energy states, and thus only specific transitions.

Q44.8 The statement is false. Both patterns show monotonic decrease over time, but with very different shapes. For radioactive decay, maximum activity occurs at time zero. Cohorts of people now living will be dying most rapidly perhaps forty years from now. Everyone now living will be dead within less than two centuries, while the mathematical model of radioactive decay tails off exponentially forever. A radioactive nucleus never gets old. It has constant probability of decay however long it has existed.

***Q44.9** (i) Answer (b). Since the samples are of the same radioactive isotope, their half-lives are the same.

(ii) Answer (b). When prepared, sample G has twice the activity (number of radioactive decays per second) of sample H. After 5 half-lives, the activity of sample G is decreased by a factor of 2^5, and after 5 half-lives the activity of sample H is decreased by a factor of 2^5. So after 5 half-lives, the ratio of activities is still 2:1.

Q44.10 After one half-life, one half the radioactive atoms have decayed. After the second half-life, one half of the remaining atoms have decayed. Therefore $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ of the original radioactive atoms have decayed after two half-lives.

Q44.11 Long-lived progenitors at the top of each of the three natural radioactive series are the sources of our radium. As an example, thorium-232 with a half-life of 14 Gyr produces radium-228 and radium-224 at stages in its series of decays, shown in Figure 44.17.

***Q44.12** Answer (d). A free neutron decays into a proton plus an electron and an antineutrino. This implies that a proton is more stable than a neutron, and in particular the proton has lower mass. Therefore a proton cannot decay into a neutron plus a positron and a neutrino. This reaction satisfies every conservation law except for energy conservation.

***Q44.13** The alpha particle and the daughter nucleus carry equal amounts of momentum in opposite directions. Since kinetic energy can be written as $\frac{p^2}{2m}$, the small-mass alpha particle has much more of the decay energy than the recoiling nucleus.

Q44.14 Yes. The daughter nucleus can be left in its ground state or sometimes in one of a set of excited states. If the energy carried by the alpha particle is mysteriously low, the daughter nucleus can quickly emit the missing energy in a gamma ray.

***Q44.15** Answer (d). The reaction energy is the amount of energy released as a result of a nuclear reaction. Equation 44.28 in the text implies that the reaction energy is (initial mass − final mass) c^2. The Q-value is taken as positive for an exothermic reaction.

***Q44.16** The samples would have started with more carbon-14 than we first thought. We would increase our estimates of their ages.

Q44.17 I_z may have 6 values for $I=\frac{5}{2}$, namely $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, and $-\frac{5}{2}$. Seven I_z values are possible for $I=3$.

***Q44.18** Answer (b). The frequency increases linearly with the magnetic field strength.

Q44.19 The decay of a radioactive nucleus at one particular moment instead of at another instant cannot be predicted and has no cause. Natural events are not just like a perfect clockworks. In history, the idea of a determinate mechanical Universe arose temporarily from an unwarranted wild extrapolation of Isaac Newton's account of planetary motion. Before Newton's time [really you can blame Pierre Simon de Laplace] and again now, no one thought of natural events as just like a perfect row of falling dominos. We can and do use the word "cause" more loosely to describe antecedent enabling events. One gear turning another is intelligible. So is the process of a hot dog getting toasted over a campfire, even though random molecular motion is at the essence of that process. In summary, we say that the future is not determinate. All natural events have causes in the ordinary sense of the word, but not necessarily in the contrived sense of a cause operating infallibly and predictably in a way that can be calculated. We have better reason now than ever before to think of the Universe as intelligible. First describing natural events, and second determining their causes form the basis of science, including physics but also scientific medicine and scientific bread-baking. The evidence alone of the past hundred years of discoveries in physics, finding causes of natural events from the photoelectric effect to x-rays and jets emitted by black holes, suggests that human intelligence is a good tool for figuring out how things go. Even without organized science, humans have always been searching for the causes of natural events, with explanations ranging from "the will of the gods" to Schrödinger's equation. We depend on the principle that things are intelligible as we make significant strides towards understanding the Universe. To hope that our search is not futile is the best part of human nature.

SOLUTIONS TO PROBLEMS

Section 44.1 **Some Properties of Nuclei**

P44.1 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim10^{28}\text{ protons}}$$

and $\boxed{\sim10^{28}\text{ neutrons}}$

The electron number is precisely equal to the proton number, $\boxed{\sim10^{28}\text{ electrons}}$.

P44.2 $\frac{1}{2}mv^2 = q\Delta V$ and $\frac{mv^2}{r} = qvB$

$2m\Delta V = qr^2B^2$: $r = \sqrt{\frac{2m\Delta V}{qB^2}} = \sqrt{\frac{2(1\,000\text{ V})}{(1.60\times10^{-19}\text{ C})(0.200\text{ T})^2}}\sqrt{m}$

$$r = \left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{m}$$

(a) For ^{12}C, $m = 12$ u and $r = \left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{12\left(1.66\times10^{-27}\text{ kg}\right)}$

$$r = 0.078\,9\text{ m} = \boxed{7.89\text{ cm}}$$

For ^{13}C: $r = \left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{13\left(1.66\times10^{-27}\text{ kg}\right)}$

$$r = 0.082\,1\text{ m} = \boxed{8.21\text{ cm}}$$

(b) With $r_1 = \sqrt{\frac{2m_1\Delta V}{qB^2}}$ and $r_2 = \sqrt{\frac{2m_2\Delta V}{qB^2}}$

the ratio gives $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}}$

$$\frac{r_1}{r_2} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961$$

and $\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961$

so they do agree.

***P44.3** (a) Let V represent the volume of the tank. The number of moles present is

$$n = \frac{PV}{RT} = \frac{1.013\times10^5\text{ N }V\text{ mol}\cdot\text{K}}{8.314\text{ N}\cdot\text{m m}^2\text{ 273 K}} = 44.6\ V\text{ mol/m}^3$$

Then the number of molecules is $6.023\times10^{23}\times44.6\ V/\text{m}^3$.

The volume of one molecule is $2\frac{4}{3}\pi r^3 = \frac{8\pi}{3}\left(\frac{10^{-10}\text{ m}}{2}\right)^3 = 1.047\times10^{-30}\text{ m}^3$.

The volume of all the molecules is $2.69\times10^{25}\ V\,(1.047\times10^{-30}) = 2.81\times10^{-5}\ V$.
So the fraction of the volume occupied by the hydrogen molecules is $\boxed{2.81\times10^{-5}}$.
An atom is precisely one half of a molecule.

(b) $$\frac{\textit{nuclear volume}}{\textit{atomic volume}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(d/2)^3} = \left(\frac{1.20\times10^{-15}\text{ m}}{0.5\times10^{-10}\text{ m}}\right)^3 = \boxed{1.38\times10^{-14}}$$

In linear dimension, the nucleus is small inside the atom in the way a fat strawberry is small inside the width of the Grand Canyon. In terms of volume, the nucleus is *really* small.

P44.4 $E_\alpha = 7.70 \text{ MeV}$

(a) $d_{\min} = \dfrac{4k_e Ze^2}{mv^2} = \dfrac{2k_e Ze^2}{E_\alpha} = \dfrac{2\left(8.99\times10^9\right)(79)\left(1.60\times10^{-19}\right)^2}{7.70\left(1.60\times10^{-13}\right)} = 29.5\times10^{-15} \text{ m} = \boxed{29.5 \text{ fm}}$

(b) The de Broglie wavelength of the α is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626\times10^{-34}}{\sqrt{2\left(6.64\times10^{-27}\right)7.70\left(1.60\times10^{-13}\right)}} = 5.18\times10^{-15} \text{ m} = \boxed{5.18 \text{ fm}}$$

(c) Since $\boxed{\lambda \text{ is much less than the distance of closest approach}}$, the α may be considered a particle.

P44.5 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_e qQ}{r_{\min}}$$

$$r_{\min} = \frac{k_e qQ}{K_i} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19} \text{ C}\right)^2}{(0.500 \text{ MeV})\left(1.60\times10^{-13} \text{ J/MeV}\right)} = \boxed{4.55\times10^{-13} \text{ m}}$$

(b) Since $K_i = \dfrac{1}{2}m_\alpha v_i^2 = \dfrac{k_e qQ}{r_{\min}}$,

$$v_i = \sqrt{\frac{2k_e qQ}{m_\alpha r_{\min}}} = \sqrt{\frac{2\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19} \text{ C}\right)^2}{(4.00 \text{ u})\left(1.66\times10^{-27} \text{ kg/u}\right)\left(3.00\times10^{-13} \text{ m}\right)}} = \boxed{6.04\times10^6 \text{ m/s}}$$

P44.6 (a) $r = r_0 A^{1/3} = \left(1.20\times10^{-15} \text{ m}\right)(4)^{1/3} = \boxed{1.90\times10^{-15} \text{ m}}$

(b) $r = r_0 A^{1/3} = \left(1.20\times10^{-15} \text{ m}\right)(238)^{1/3} = \boxed{7.44\times10^{-15} \text{ m}}$

P44.7 The number of nucleons in a star of two solar masses is

$$A = \frac{2\left(1.99\times10^{30} \text{ kg}\right)}{1.67\times10^{-27} \text{ kg/nucleon}} = 2.38\times10^{57} \text{ nucleons}$$

Therefore $r = r_0 A^{1/3} = \left(1.20\times10^{-15} \text{ m}\right)\left(2.38\times10^{57}\right)^{1/3} = \boxed{16.0 \text{ km}}$

P44.8 $V = \dfrac{4}{3}\pi r^4 = \dfrac{4}{3}\pi(0.021\,5 \text{ m})^3 = 4.16\times10^{-5} \text{ m}^3$

We take the nuclear density from Example 44.2

$$m = \rho V = \left(2.3\times10^{17} \text{ kg/m}^3\right)\left(4.16\times10^{-5} \text{ m}^3\right) = 9.57\times10^{12} \text{ kg}$$

and $F = G\dfrac{m_1 m_2}{r^2} = \left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\dfrac{\left(9.57\times10^{12} \text{ kg}\right)^2}{(1.00 \text{ m})^2}$

$F = \boxed{6.11\times10^{15} \text{ N}}$ toward the other ball.

Section 44.2 **Nuclear Binding Energy**

P44.9 Using atomic masses as given in the table in the text,

(a) For ${}^{2}_{1}\text{H}$: $\dfrac{-2.014\,102+1(1.008\,665)+1(1.007\,825)}{2}$

$$\frac{E_b}{A}=(0.001\,194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right)=\boxed{1.11\text{ MeV/nucleon}}$$

(b) For ${}^{4}_{2}\text{He}$: $\dfrac{2(1.008\,665)+2(1.007\,825)-4.002\,603}{4}$

$$\frac{E_b}{A}=0.007\,59\text{ u}c^2=\boxed{7.07\text{ MeV/nucleon}}$$

(c) For ${}^{56}_{26}\text{Fe}$: $30(1.008\,665)+26(1.007\,825)-55.934\,942=0.528\text{ u}$

$$\frac{E_b}{A}=\frac{0.528}{56}=0.009\,44\text{ u}c^2=\boxed{8.79\text{ MeV/nucleon}}$$

(d) For ${}^{238}_{92}\text{U}$: $146(1.008\,665)+92(1.007\,825)-238.050\,783=1.934\,2\text{ u}$

$$\frac{E_b}{A}=\frac{1.934\,2}{238}=0.008\,13\text{ u}c^2=\boxed{7.57\text{ MeV/nucleon}}$$

***P44.10** $\Delta M=Zm_{\text{H}}+Nm_{\text{n}}-M$ $\qquad \dfrac{E_b}{A}=\dfrac{\Delta M\,(931.5)}{A}$

Nuclei	Z	N	M in u	ΔM in u	$\frac{E_b}{A}$ in MeV
^{55}Mn	25	30	54.938 050	0.517 5	8.765
^{56}Fe	26	30	55.934 942	0.528 46	8.790
^{59}Co	27	32	58.933 200	0.555 35	8.768

$\therefore$ ^{56}Fe has a greater $\dfrac{E_b}{A}$ than its neighbors. This tells us finer detail than is shown in Figure 44.5.

P44.11 (a) The neutron-to-proton ratio $\dfrac{A-Z}{Z}$ is greatest for $\boxed{{}^{139}_{55}\text{Cs}}$ and is equal to 1.53.

(b) $\boxed{{}^{139}\text{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ^{139}Cs with a mass of 138.913 u. We locate the nuclei carefully on Figure 44.4, the neutron–proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

P44.12 Use Equation 44.2.

Then for ${}^{23}_{11}\text{Na}$, $\dfrac{E_b}{A}=8.11\text{ MeV/nucleon}$

and for ${}^{23}_{12}\text{Mg}$, $\dfrac{E_b}{A}=7.90\text{ MeV/nucleon}$

The binding energy per nucleon is greater for ${}^{23}_{11}\text{Na}$ by $\boxed{0.210\text{ MeV}}$. There is less proton repulsion in Na^{23}. It is the more stable nucleus.

P44.13 The binding energy of a nucleus is $E_b(\text{MeV}) = \left[ZM(\text{H}) + Nm_n - M\left({}^A_Z\text{X}\right)\right](931.494\ \text{MeV/u})$

For ${}^{15}_{8}\text{O}$: $E_b = [8(1.007\,825\ \text{u}) + 7(1.008\,665\ \text{u}) - 15.003\,065\ \text{u}](931.494\ \text{MeV/u}) = 111.96\ \text{MeV}$

For ${}^{15}_{7}\text{N}$: $E_b = [7(1.007\,825\ \text{u}) + 8(1.008\,665\ \text{u}) - 15.000\,109\ \text{u}](931.494\ \text{MeV/u}) = 115.49\ \text{MeV}$

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$.

P44.14 (a) The radius of the ${}^{40}\text{Ca}$ nucleus is: $R = r_0 A^{1/3} = (1.20 \times 10^{-15}\ \text{m})(40)^{1/3} = 4.10 \times 10^{-15}\ \text{m}$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_e Q^2}{5R} = \frac{3(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[20(1.60 \times 10^{-19}\ \text{C})\right]^2}{5(4.10 \times 10^{-15}\ \text{m})} = 1.35 \times 10^{-11}\ \text{J} = \boxed{84.1\ \text{MeV}}$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ is

$$E_b = [20(1.007\,825\ \text{u}) + 20(1.008\,665\ \text{u}) - 39.962\,591\ \text{u}](931.5\ \text{MeV/u}) = \boxed{342\ \text{MeV}}$$

(c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

P44.15 Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is S_n, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + S_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass}(\text{n})$$

$$S_n = (41.958\,618 + 1.008\,665 - 42.958\,767)\ \text{u} = (0.008\,516\ \text{u})(931.5\ \text{MeV/u}) = \boxed{7.93\ \text{MeV}}$$

Section 44.3 **Nuclear Models**

P44.16 (a) The first term overstates the importance of volume and the second term *subtracts* this overstatement.

(b) For spherical volume $\dfrac{(4/3)\pi R^3}{4\pi R^2} = \boxed{\dfrac{R}{3}}$. For cubical volume $\dfrac{R^3}{6R^2} = \boxed{\dfrac{R}{6}}$.

The maximum binding energy or lowest state of energy is achieved by building "nearly" spherical nuclei.

P44.17 $\Delta E_b = E_{bf} - E_{bi}$

For $A = 200$, $\dfrac{E_b}{A} = 7.8\ \text{MeV}$

so $E_{bi} = 200(7.8\ \text{MeV}) = 1\,560\ \text{MeV}$

For $A \approx 100$, $\dfrac{E_b}{A} = 8.7\ \text{MeV}$

so $E_{bf} = 2(100)(8.7\ \text{MeV})$
$= 1\,740\ \text{MeV}$

$\Delta E_b = E_{bf} - E_{bi}$: $E_b = 1\,740\ \text{MeV} - 1\,560\ \text{MeV}$
$\approx \boxed{200\ \text{MeV}}$

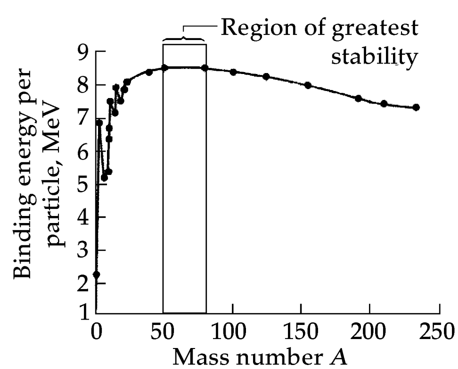

FIG. P44.17

P44.18 (a) "Volume" term: $E_1 = C_1 A = (15.7\text{ MeV})(56) = 879\text{ MeV}$

"Surface" term: $E_2 = -C_2 A^{2/3} = -(17.8\text{ MeV})(56)^{2/3} = -260\text{ MeV}$

"Coulomb" term: $E_3 = -C_3 \dfrac{Z(Z-1)}{A^{1/3}} = -(0.71\text{ MeV})\dfrac{(26)(25)}{(56)^{1/3}} = -121\text{ MeV}$

"Asymmetry" term: $E_4 = C_4 \dfrac{(A-2Z)^2}{A} = -(23.6\text{ MeV})\dfrac{(56-52)^2}{56} = -6.74\text{ MeV}$

$\boxed{E_b = 491\text{ MeV}}$

(b) $\dfrac{E_1}{E_b} = 179\%$; $\dfrac{E_2}{E_b} = -53.0\%$; $\dfrac{E_3}{E_b} = -24.6\%$; $\dfrac{E_4}{E_b} = -1.37\%$

Section 44.4 **Radioactivity**

P44.19 $\dfrac{dN}{dt} = -\lambda N$

so $\lambda = \dfrac{1}{N}\left(-\dfrac{dN}{dt}\right) = \left(1.00\times10^{-15}\right)\left(6.00\times10^{11}\right) = 6.00\times10^{-4}\text{ s}^{-1}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = \boxed{1.16\times10^{3}\text{ s}}\,(= 19.3\text{ min})$

P44.20 $R = R_0 e^{-\lambda t} = (6.40\text{ mCi})e^{-(\ln 2/8.04\text{ d})(40.2\text{ d})} = (6.40\text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40\text{ mCi})\left(\dfrac{1}{2^5}\right) = \boxed{0.200\text{ mCi}}$

P44.21 (a) From $R = R_0 e^{-\lambda t}$,

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58\times10^{-2}\text{ h}^{-1} = \boxed{1.55\times10^{-5}\text{ s}^{-1}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$$

(b) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{10.0\times10^{-3}\text{ Ci}}{1.55\times10^{-5}/\text{s}}\left(\dfrac{3.70\times10^{10}/\text{s}}{1\text{ Ci}}\right) = \boxed{2.39\times10^{13}\text{ atoms}}$

(c) $R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp\left(-5.58\times10^{-2}\times30.0\right) = \boxed{1.88\text{ mCi}}$

***P44.22** From the law of radioactive decay $N = N_0\, e^{-\lambda t}$ the decay rate is originally $R_0 = -dN/dt$ and decreases according to $R = -dN/dt = +N_0\, \lambda\, e^{-\lambda t}$ $\quad R = R_0\, e^{-\lambda t}$

Then algebra to isolate the decay constant gives $R_0/R = e^{\lambda t}$ $\quad \ln(R_0/R) = \lambda t$ $\quad \lambda = \dfrac{1}{t}\ln\left(\dfrac{R_0}{R}\right)$

Now $\lambda = \dfrac{\ln 2}{T_{1/2}}$ gives $\dfrac{\ln 2}{T_{1/2}} = \dfrac{1}{t}\ln\left(\dfrac{R_0}{R}\right)$ $\qquad T_{1/2} = \dfrac{(\ln 2)t}{\ln(R_0/R)}$

where $t = \Delta t$ is the time interval during which the activity decreases from R_0 to R.

P44.23 The number of nuclei that decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find λ: $\qquad \lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{64.8\text{ h}} = 0.010\,7\text{ h}^{-1} = 2.97\times10^{-6}\text{ s}^{-1}$

and $\qquad N_0 = \dfrac{R_0}{\lambda} = \dfrac{(40.0\ \mu\text{Ci})\left(3.70\times10^4\ \text{s}^{-1}/\mu\text{Ci}\right)}{2.97\times10^{-6}\ \text{s}^{-1}} = 4.98\times10^{11}$ nuclei

Substituting these values, $\qquad N_1 - N_2 = \left(4.98\times10^{11}\right)\left[e^{-\left(0.010\,7\text{ h}^{-1}\right)(10.0\text{ h})} - e^{-\left(0.010\,7\text{ h}^{-1}\right)(12.0\text{ h})}\right]$

Hence, the number of nuclei decaying during the interval is $N_1 - N_2 = \boxed{9.47\times10^9\ \text{nuclei}}$.

P44.24 The number of nuclei that decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find λ: $\qquad \lambda = \dfrac{\ln 2}{T_{1/2}}$

so $\qquad e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$

and $\qquad N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$

Substituting in these values $\qquad N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$

***P44.25** The number remaining after time $\dfrac{T_{1/2}}{2} = \dfrac{\ln 2}{2\lambda}$

is $\qquad N = N_0 e^{-\lambda t} = N_0\, e^{-\lambda \ln 2/2\lambda} = N_0\left(e^{-\ln 2}\right)^{1/2} = N_0\left(\dfrac{1}{2}\right)^{1/2} = 0.7071\, N_0$

The number decaying in this first half of the first half-life is $N_0 - 0.7071\, N_0 = 0.2929\, N_0$.
The number remaining after time $T_{1/2}$ is $0.500\, N_0$, so the number decaying in the second half of the first half-life is $0.7071\, N_0 - 0.500\, N_0 = 0.2071\, N_0$.
The ratio required is then $\quad 0.2929\, N_0/0.2071\, N_0 = \boxed{1.41}$

P44.26 (a) $\frac{dN_2}{dt}$ = rate of change of N_2

= rate of production of N_2 − rate of decay of N_2

= rate of decay of N_1 − rate of decay of N_2

$= \lambda_1 N_1 - \lambda_2 N_2$

(b) From the trial solution

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

$$\therefore \frac{dN_2}{dt} = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t}\right) \qquad (1)$$

$$\therefore \frac{dN_2}{dt} + \lambda_2 N_2 = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}\right)$$

$$= \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(\lambda_1 - \lambda_2\right)e^{-\lambda_1 t}$$

$$= \lambda_1 N_1$$

So $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$ as required.

(c) The functions to be plotted are

$$N_1(t) = 1\,000 e^{-\left(0.223\,6\ \text{min}^{-1}\right)t}$$

$$N_2(t) = 1\,130.8\left[e^{-\left(0.223\,6\ \text{min}^{-1}\right)t} - e^{-\left(0.025\,9\ \text{min}^{-1}\right)t}\right]$$

From the graph: $t_m \approx \boxed{10.9\ \text{min}}$

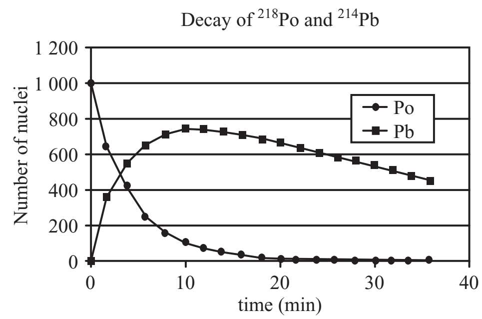

FIG. P44.26(c)

(d) From (1), $\frac{dN_2}{dt} = 0$ if $\lambda_2 e^{-\lambda_2 t} = \lambda_1 e^{-\lambda_1 t}$. $\therefore e^{(\lambda_1 - \lambda_2)t} = \frac{\lambda_1}{\lambda_2}$. Thus, $t = \boxed{t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}}$.

With $\lambda_1 = 0.223\,6\ \text{min}^{-1}$, $\lambda_2 = 0.025\,9\ \text{min}^{-1}$, this formula gives $t_m = \boxed{10.9\ \text{min}}$, in agreement with the result of part (c).

P44.27 We have all this information: $N_x(0) = 2.50 N_y(0)$

$$N_x(3\text{d}) = 4.20 N_y(3\text{d})$$

$$N_x(0)e^{-\lambda_x 3\text{d}} = 4.20 N_y(0)e^{-\lambda_y 3\text{d}} = 4.20\frac{N_x(0)}{2.50}e^{-\lambda_y 3\text{d}}$$

$$e^{3\text{d}\lambda_x} = \frac{2.5}{4.2}e^{3\text{d}\lambda_y}$$

$$3\text{d}\lambda_x = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\lambda_y$$

$$3\text{d}\frac{0.693}{T_{1/2\,x}} = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\frac{0.693}{1.60\ \text{d}} = 0.781$$

$$T_{1/2\,x} = \boxed{2.66\ \text{d}}$$

Section 44.5 **The Decay Processes**

P44.28 (a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z = 0$ and $A = 0$. Keeping the total values of Z and A for the system conserved then requires $Z = 28$ and $A = 65$ for X. With this atomic number it must be nickel, and the nucleus must be in an exited state, so it is ${}^{65}_{28}\text{Ni}^*$.

(b) $\alpha = {}^4_2\text{He}$ has $Z = 2$ and $A = 4$
so for X we require $Z = 84 - 2 = 82$
for Pb and $A = 215 - 4 = 211$, $X = {}^{211}_{82}\text{Pb}$

(c) A positron $e^+ = {}^0_1e$ has charge the same as a nucleus with $Z = 1$. A neutrino ${}^0_0\nu$ has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z = 26 + 1 = 27$. It is Co. And $A = 55 + 0 = 55$. It is ${}^{55}_{27}\text{Co}$.

Similar reasoning about balancing the sums of Z and A across the reaction reveals:

(d) ${}^{0}_{-1}e$

(e) ${}^1_1\text{H}$ (or p). Note that this process is a nuclear reaction, rather than radioactive decay. We can solve it from the same principles, which are fundamentally conservation of charge and conservation of baryon number.

P44.29 $Q = (M_{\text{U-238}} - M_{\text{Th-234}} - M_{\text{He-4}})(931.5\ \text{MeV/u})$

$$Q = (238.050\,783 - 234.043\,596 - 4.002\,603)\ \text{u}(931.5\ \text{MeV/u}) = \boxed{4.27\ \text{MeV}}$$

P44.30 $N_C = \left(\dfrac{0.021\,0\ \text{g}}{12.0\ \text{g/mol}}\right)(6.02\times10^{23}\ \text{molecules/mol})$

$(N_C = 1.05\times10^{21}\ \text{carbon atoms})$ of which 1 in 7.70×10^{11} is a ^{14}C atom

$$(N_0)_{\text{C-14}} = 1.37\times10^{9}, \qquad \lambda_{\text{C-14}} = \frac{\ln 2}{5\,730\ \text{yr}} = 1.21\times10^{-4}\ \text{yr}^{-1} = 3.83\times10^{-12}\ \text{s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$,
$$R_0 = \lambda N_0 = (3.83\times10^{-12}\ \text{s}^{-1})(1.37\times10^{9})\left[\frac{7(86\,400\ \text{s})}{1\ \text{week}}\right]$$
$$= 3.17\times10^{3}\ \text{decays/week}$$

At time t,
$$R = \frac{837}{0.88} = 951\ \text{decays/week}$$

Taking logarithms,
$$\ln\frac{R}{R_0} = -\lambda t \quad \text{so} \quad t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

$$t = \frac{-1}{1.21\times10^{-4}\ \text{yr}^{-1}}\ln\left(\frac{951}{3.17\times10^{3}}\right) = \boxed{9.96\times10^{3}\ \text{yr}}$$

***P44.31** (a) The decay constant is $\lambda = \ln 2/10 \text{ h} = 0.0693/\text{h}$.
The number of parent nuclei is given by $10^6\, e^{-0.0693\,t}$ where t is in hours.
The number of daughter nuclei is equal to the number of missing parent nuclei,
$N_d = 10^6 - 10^6\, e^{-0.0693t}$ $\boxed{N_d = 10^6(1 - e^{-0.0693t}) \text{ where } t \text{ is in hours}}$.

(b) The number of daughter nuclei starts from zero at $t = 0$. The number of stable product nuclei always increases with time and asymptotically approaches 1.00×10^6 as t increases without limit.

Its rate of increase is $\dfrac{dN_d}{dt} = 10^6(0 + 0.0693\; e^{-0.0693t}) = 6.93 \times 10^4\, \dfrac{1}{\text{h}}\, e^{-0.0693t}$.

The number of daughter nuclei first increases most rapidly, at 6.93×10^4/h, and then more and more slowly. Its rate of change approaches zero in the far future.

P44.32 ${}^3_1\text{H nucleus} \rightarrow {}^3_2\text{He nucleus} + e^- + \bar{\nu}$

becomes ${}^3_1\text{H nucleus} + e^- \rightarrow {}^3_2\text{He nucleus} + 2e^- + \bar{\nu}$

Ignoring the slight difference in ionization energies,

we have ${}^3_1\text{H atom} \rightarrow {}^3_2\text{He atom} + \bar{\nu}$

$$3.016\,049 \text{ u} = 3.016\,029 \text{ u} + 0 + \frac{Q}{c^2}$$

$$Q = (3.016\,049 \text{ u} - 3.016\,029 \text{ u})(931.5 \text{ MeV/u}) = 0.018\,6 \text{ MeV} = \boxed{18.6 \text{ keV}}$$

P44.33 (a) $\boxed{e^- + p \rightarrow n + \nu}$

(b) For nuclei, ${}^{15}\text{O} + e^- \rightarrow {}^{15}\text{N} + \nu$
Add seven electrons to both sides to obtain $\boxed{{}^{15}_{8}\text{O atom} \rightarrow {}^{15}_{7}\text{N atom} + \nu}$

(c) From the table of isotopic masses in the chapter text, $m({}^{15}\text{O}) = m({}^{15}\text{N}) + \dfrac{Q}{c^2}$

$$\Delta m = 15.003\,065 \text{ u} - 15.000\,109 \text{ u} = 0.002\,956 \text{ u}$$

$$Q = (931.5 \text{ MeV/u})(0.002\,956 \text{ u}) = \boxed{2.75 \text{ MeV}}$$

P44.34 (a) For e^+ decay,

$$Q = (M_X - M_Y - 2m_e)c^2 = [39.962\,591 \text{ u} - 39.963\,999 \text{ u} - 2(0.000\,549 \text{ u})](931.5 \text{ MeV/u})$$

$Q = -2.33$ MeV

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For alpha decay,

$$Q = (M_X - M_\alpha - M_Y)c^2 = [97.905\,287 \text{ u} - 4.002\,603 \text{ u} - 93.905\,088 \text{ u}](931.5 \text{ MeV/u})$$

$Q = -2.24$ MeV

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For alpha decay,

$$Q = (M_X - M_\alpha - M_Y)c^2 = [143.910\,083 \text{ u} - 4.002\,603 \text{ u} - 139.905\,434 \text{ u}](931.5 \text{ MeV/u})$$

$Q = 1.91$ MeV

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

Section 44.6 **Natural Radioactivity**

P44.35

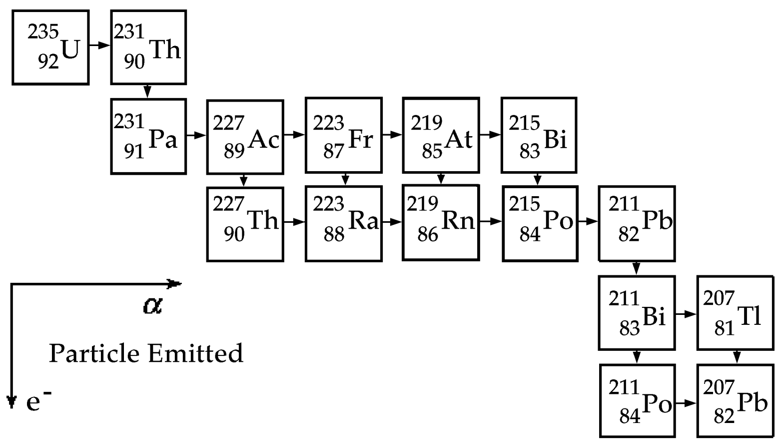

FIG. P44.35

P44.36 (a) Let N be the number of ${}^{238}\text{U}$ nuclei and N' be ${}^{206}\text{Pb}$ nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$.

Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = \frac{\ln 2}{T_{1/2}}$

Thus, $t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$

If $\frac{N}{N'} = 1.164$ for the ${}^{238}\text{U} \to {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$.

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the ${}^{235}\text{U} \to {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938 \text{ and } \boxed{\frac{N}{N'} = 0.0199}$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the ${}^{232}\text{Th} \to {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.1966 \text{ and } \boxed{\frac{N}{N'} = 4.60}$$

P44.37 (a) $4.00 \text{ pCi/L} = \left(\dfrac{4.00\times10^{-12}\text{ Ci}}{1\text{ L}}\right)\left(\dfrac{3.70\times10^{10}\text{ Bq}}{1\text{ Ci}}\right)\left(\dfrac{1.00\times10^{3}\text{ L}}{1\text{ m}^3}\right) = \boxed{148\text{ Bq/m}^3}$

(b) $N = \dfrac{R}{\lambda} = R\left(\dfrac{T_{1/2}}{\ln 2}\right) = \left(148\text{ Bq/m}^3\right)\left(\dfrac{3.82\text{ d}}{\ln 2}\right)\left(\dfrac{86\,400\text{ s}}{1\text{ d}}\right) = \boxed{7.05\times10^{7}\text{ atoms/m}^3}$

(c) $\text{mass } = \left(7.05\times10^{7}\text{ atoms/m}^3\right)\left(\dfrac{1\text{ mol}}{6.02\times10^{23}\text{ atoms}}\right)\left(\dfrac{222\text{ g}}{1\text{ mol}}\right) = 2.60\times10^{-14}\text{ g/m}^3$

Since air has a density of 1.20 kg/m^3, the fraction consisting of radon is

$\text{fraction } = \dfrac{2.60\times10^{-14}\text{ g/m}^3}{1\,200\text{ g/m}^3} = \boxed{2.17\times10^{-17}}$

.

P44.38 Number remaining: $N = N_0 e^{-(\ln 2)t/T_{1/2}}$

Fraction remaining: $\dfrac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$

(a) With $T_{1/2} = 3.82\text{ d}$ and $t = 7.00\text{ d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$

(b) When $t = 1.00\text{ yr} = 365.25\text{ d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

Section 44.7 **Nuclear Reactions**

P44.39 (a) For X, $A = 24+1-4 = 21$
and $Z = 12+0-2 = 10$, so X is $\boxed{^{21}_{10}\text{Ne}}$

(b) $A = 235+1-90-2 = 144$
and $Z = 92+0-38-0 = 54$, so X is $\boxed{^{144}_{54}\text{Xe}}$

(c) $A = 2-2 = 0$
and $Z = 2-1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{\text{X} = {}^{0}_{1}\text{e}^{+}}$ and $\boxed{\text{X}' = {}^{0}_{0}\nu}$

P44.40 (a) Add two electrons to both sides of the reaction to have it in neutral-atom terms:

$$4\,{}^{1}_{1}\text{H atom} \rightarrow {}^{4}_{2}\text{He atom} + Q \qquad Q = \Delta mc^2 = \left[4M_{{}^{1}_{1}\text{H}} - M_{{}^{4}_{2}\text{He}}\right]c^2$$

$$Q = \left[4(1.007\,825\text{ u}) - 4.002\,603\text{ u}\right](931.5\text{ MeV/u})\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{4.28\times10^{-12}\text{ J}}$$

(b) $$N = \frac{1.99\times10^{30}\text{ kg}}{1.67\times10^{-27}\text{ kg/atom}} = \boxed{1.19\times10^{57}\text{ atoms}} = 1.19\times10^{57}\text{ protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$\left(1.19\times10^{57}\text{ protons}\right)\left(\frac{4.28\times10^{-12}\text{ J}}{4\text{ protons}}\right) = 1.27\times10^{45}\text{ J}$$

$$\mathcal{P} = \frac{E}{\Delta t} \qquad \text{so} \qquad \Delta t = \frac{E}{\mathcal{P}} = \frac{1.27\times10^{45}\text{ J}}{3.85\times10^{26}\text{ W}} = 3.31\times10^{18}\text{ s} = \boxed{105\text{ billion years}}$$

P44.41 (a) $$\boxed{{}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{79}\text{Au}^* \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}}$$

(b) Consider adding 79 electrons:

$${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$$

$$Q = \left[M_{{}^{197}\text{Au}} + m_n - M_{{}^{198}\text{Hg}}\right]c^2$$

$$Q = \left[196.966\,552 + 1.008\,665 - 197.966\,752\right]\text{ u}(931.5\text{ MeV/u}) = \boxed{7.89\text{ MeV}}$$

P44.42 Neglect recoil of product nucleus (i.e., do not require momentum conservation for the system of colliding particles). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find Q:

$$Q = \left[(M_{\text{H}} + M_{\text{Al}}) - (M_{\text{Si}} + m_n)\right]c^2$$

$$Q = \left[(1.007\,825 + 26.981\,539) - (26.986\,705 + 1.008\,665)\right]\text{ u}(931.5\text{ MeV/u}) = -5.59\text{ MeV}$$

Thus, $K_{\text{emerging}} = 6.61\text{ MeV} - 5.59\text{ MeV} = \boxed{1.02\text{ MeV}}$.

P44.43 $${}^{9}_{4}\text{Be} + 1.665\text{ MeV} \rightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}\text{n},\text{ so } M_{{}^{8}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} - \frac{Q}{c^2} - m_n$$

$$M_{{}^{8}_{4}\text{Be}} = 9.012\,182\text{ u} - \frac{(-1.665\text{ MeV})}{931.5\text{ MeV/u}} - 1.008\,665\text{ u} = \boxed{8.005\,3\text{ u}}$$

$${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} \rightarrow {}^{10}_{4}\text{Be} + 6.812\text{ MeV},\text{ so } M_{{}^{10}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} + m_n - \frac{Q}{c^2}$$

$$M_{{}^{10}_{4}\text{Be}} = 9.012\,182\text{ u} + 1.008\,665\text{ u} - \frac{6.812\text{ MeV}}{931.5\text{ MeV/u}} = \boxed{10.013\,5\text{ u}}$$

P44.44 (a) $${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{13}_{6}\text{C} + {}^{1}_{1}\text{H}$$

The product nucleus is $\boxed{{}^{13}_{6}\text{C}}$.

(b) $${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \rightarrow {}^{10}_{5}\text{B} + {}^{4}_{2}\text{He}$$

The product nucleus is $\boxed{{}^{10}_{5}\text{B}}$.

Section 44.8 **Nuclear Magnetic Resonance and Magnetic Resonance Imagining**

P44.45 (a) $$f_n = \frac{|2\mu B|}{h} = \frac{2(1.913\,5)(5.05\times10^{-27}\text{ J/T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{29.2\text{ MHz}}$$

(b) $$f_p = \frac{2(2.792\,8)(5.05\times10^{-27}\text{ J/T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{42.6\text{ MHz}}$$

(c) In the Earth's magnetic field,

$$f_p = \frac{2(2.792\,8)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\text{ kHz}}$$

P44.46

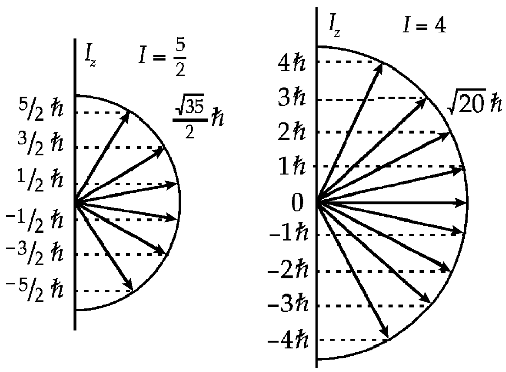

FIG. P44.46

Additional Problems

P44.47 (a) $Q = \left[M_{^9\text{Be}} + M_{^4\text{He}} - M_{^{12}\text{C}} - m_n\right]c^2$

$Q = [9.012\,182\text{ u} + 4.002\,603\text{ u} - 12.000\,000\text{ u} - 1.008\,665\text{ u}](931.5\text{ MeV/u}) = \boxed{5.70\text{ MeV}}$

(b) $Q = \left[2M_{^2\text{H}} - M_{^3\text{He}} - m_n\right]$

$Q = \left[2(2.014\,102) - 3.016\,029 - 1.008\,665\right]\text{u}(931.5\text{ MeV/u}) = \boxed{3.27\text{ MeV (exothermic)}}$

P44.48 (a) With m_n and v_n as the mass and speed of the neutrons, Equation 9.24 of the text becomes, after making appropriate notational changes, for the two collisions $v_1 = \left(\frac{2m_n}{m_n + m_1}\right)v_n$, and

$$v_2 = \left(\frac{2m_n}{m_n + m_2}\right)v_n$$

$$\therefore (m_n + m_2)v_2 = (m_n + m_1)v_1 = 2m_n v_n$$

$$\therefore m_n(v_2 - v_1) = m_1 v_1 - m_2 v_2$$

$$\therefore m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b) $$m_n = \frac{(1\text{ u})(3.30\times10^7\text{ m/s}) - (14\text{ u})(4.70\times10^6\text{ m/s})}{4.70\times10^6\text{ m/s} - 3.30\times10^7\text{ m/s}} = \boxed{1.16\text{ u}}$$

P44.49 (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles that have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation for the projectile-target system:

$$M_a v_a = (M_a + M_X) v_c$$

The initial energy is: $E_i = \frac{1}{2} M_a v_a^2$

The final kinetic energy is:

$$E_f = \frac{1}{2}(M_a + M_X) v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{M_a + M_X}\right] E_i$$

From this, we see that E_f is always less than E_i and the change in energy, $E_f - E_i$, is given by

$$E_f - E_i = \left[\frac{M_a}{M_a + M_X} - 1\right] E_i = -\left[\frac{M_X}{M_a + M_X}\right] E_i$$

This loss of kinetic energy in the isolated system corresponds to an increase in mass-energy during the reaction. Thus, the absolute value of this kinetic energy change is equal to $-Q$ (remember that Q is negative in an endothermic reaction). The initial kinetic energy E_i is the threshold energy E_{th}.

Therefore, $$-Q = \left[\frac{M_X}{M_a + M_X}\right] E_{th}$$

or $$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}$$

(b) First, calculate the Q-value for the reaction: $Q = \left[M_{\text{N-14}} + M_{\text{He-4}} - M_{\text{O-17}} - M_{\text{H-1}}\right] c^2$

$$Q = [14.003\,074 + 4.002\,603 - 16.999\,132 - 1.007\,825]\,\text{u}(931.5 \text{ MeV/u}) = -1.19 \text{ MeV}$$

Then, $E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19 \text{ MeV})\left[1 + \frac{4.002\,603}{14.003\,074}\right] = \boxed{1.53 \text{ MeV}}$.

***P44.50** (a) The system of a separated proton and electron puts out energy 13.606 eV to become a hydrogen atom in its ground state. This decrease in its rest energy appears also as a decrease in mass: the mass is $\boxed{\text{smaller}}$.

(b) $$|\Delta m| = \frac{|E|}{c^2} = \frac{13.6 \text{ eV}}{(3 \times 10^8 \text{ m/s})^2}\left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 2.42 \times 10^{-35} \text{ kg}$$

$$= 2.42 \times 10^{-35} \text{ kg}\left(\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}}\right) = \boxed{1.46 \times 10^{-8} \text{ u}}$$

(c) $$\frac{1.46 \times 10^{-8} \text{ u}}{1.007\,825 \text{ u}} = 1.45 \times 10^{-8} = \boxed{1.45 \times 10^{-6}\%}$$

(d) The textbook table lists 1.007 825 u as the atomic mass of hydrogen. This correction of 0.000 000 01 u is on the order of 100 times $\boxed{\text{too small}}$ to affect the values listed.

***P44.51** The problem statement can be: Find the reaction energy (Q value) of the reaction ${}^{10}_{5}B + {}^{4}_{2}He \rightarrow {}^{13}_{6}C + {}^{1}_{1}H$.
Solving gives $Q = (14.015\,54 - 14.011\,18)$ u $c^2(931.5$ MeV/u $c^2) = 4.06$ MeV for the energy released by the reaction as it is converted from rest energy into other forms.

P44.52 (a) $$N_0 = \frac{\text{mass}}{\text{mass per atom}} = \frac{1.00 \text{ kg}}{(239.05 \text{ u})(1.66\times10^{-27} \text{ kg/u})} = \boxed{2.52\times10^{24}}$$

(b) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(2.412\times10^4 \text{ yr})(3.156\times10^7 \text{ s/yr})} = 9.106\times10^{-13} \text{ s}^{-1}$$
$$R_0 = \lambda N_0 = (9.106\times10^{-13} \text{ s}^{-1})(2.52\times10^{24}) = \boxed{2.29\times10^{12} \text{ Bq}}$$

(c) $$R = R_0 e^{-\lambda t}, \text{ so } t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right) = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$$
$$t = \frac{1}{9.106\times10^{-13} \text{ s}^{-1}}\ln\left(\frac{2.29\times10^{12} \text{ Bq}}{0.100 \text{ Bq}}\right) = 3.38\times10^{13} \text{ s}\left(\frac{1 \text{ yr}}{3.156\times10^7 \text{ s}}\right) = \boxed{1.07\times10^6 \text{ yr}}$$

P44.53 (a) ${}^{57}_{27}Co \rightarrow {}^{57}_{26}Fe + {}^{0}_{+1}e + {}^{0}_{0}\nu$

The Q-value for this positron emission is $Q = \left[M_{{}^{57}Co} - M_{{}^{57}Fe} - 2m_e\right]c^2$.

$Q = \left[56.936\,296 - 56.935\,399 - 2(0.000\,549)\right]\text{u}(931.5 \text{ MeV/u}) = -0.187 \text{ MeV}$

Since $Q < 0$, this reaction $\boxed{\text{cannot spontaneously occur}}$.

(b) ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + {}^{0}_{0}\overline{\nu}$

The Q-value for this e$^-$ decay is $Q = \left[M_{{}^{14}C} - M_{{}^{14}N}\right]c^2$.

$Q = [14.003\,242 - 14.003\,074]\text{u}(931.5 \text{ MeV/u}) = 0.156 \text{ MeV} = 156 \text{ keV}$

Since $Q > 0$, the decay $\boxed{\text{can spontaneously occur}}$.

(c) The energy released in the reaction of (b) is shared by the electron and neutrino. Thus, $\boxed{K_e \text{ can range from zero to 156 keV}}$.

P44.54 (a) $r = r_0 A^{1/3} = 1.20\times10^{-15} A^{1/3}$ m

When $A = 12$, $r = \boxed{2.75\times10^{-15} \text{ m}}$

(b) $$F = \frac{k_e(Z-1)e^2}{r^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(Z-1)(1.60\times10^{-19} \text{ C})^2}{r^2}$$

When $Z = 6$ and $r = 2.75\times10^{-15}$ m, $F = \boxed{152 \text{ N}}$

(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{k_e(Z-1)e^2}{r} = \frac{(8.99\times10^9)(Z-1)(1.6\times10^{-19})^2}{r}$$

When $Z = 6$ and $r = 2.75\times10^{-15}$ m, $U = 4.19\times10^{-13} \text{ J} = \boxed{2.62 \text{ MeV}}$

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44\times10^{-15} \text{ m}}$ $F = \boxed{379 \text{ N}}$

and $U = 2.82\times10^{-12} \text{ J} = \boxed{17.6 \text{ MeV}}$

P44.55 (a) Because the reaction $p \to n + e^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$

$$m_p = 1.007\,276 \text{ u} \qquad m_n = 1.008\,665 \text{ u} \qquad m_{e^+} = 5.49 \times 10^{-4} \text{ u}$$

Note that $m_n + m_{e^+} > m_p$

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the parent nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei ${}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + e^+ + \nu$ to obtain the reaction for neutral atoms ${}^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + e^+ + e^- + \nu$

$$Q = c^2\left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{e^+} - m_{e^-} - m_\nu\right]$$

$$Q = (931.5 \text{ MeV/u})\left[13.005\,739 - 13.003\,355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5 \text{ MeV/u})\left(1.286 \times 10^{-3} \text{ u}\right) = \boxed{1.20 \text{ MeV}}$$

P44.56 (a) A least-square fit to the graph yields:

$$\lambda = -\text{slope} = -\left(-0.250 \text{ h}^{-1}\right) = 0.250 \text{ h}^{-1}$$

and

$$\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30$$

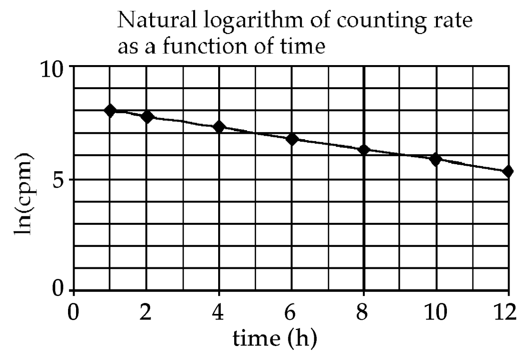

FIG. P44.56

(b) $\lambda = 0.250 \text{ h}^{-1}\left(\dfrac{1 \text{ h}}{60.0 \text{ min}}\right) = \boxed{4.17 \times 10^{-3} \text{ min}^{-1}}$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17 \times 10^{-3} \text{ min}^{-1}}$$

$$= 166 \text{ min} = \boxed{2.77 \text{ h}}$$

(c) From (a), intercept $= \ln(\text{cpm})_0 = 8.30$

Thus, $(\text{cpm})_0 = e^{8.30}$ counts/min $= \boxed{4.02 \times 10^3 \text{ counts/min}}$

(d) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{1}{\lambda}\dfrac{(\text{cpm})_0}{\text{Eff}} = \dfrac{4.02 \times 10^3 \text{ counts/min}}{\left(4.17 \times 10^{-3} \text{ min}^{-1}\right)(0.100)} = \boxed{9.65 \times 10^6 \text{ atoms}}$

P44.57 (a) One liter of milk contains this many ${}^{40}\text{K}$ nuclei:

$$N = (2.00 \text{ g})\left(\frac{6.02 \times 10^{23} \text{ nuclei/mol}}{39.1 \text{ g/mol}}\right)\left(\frac{0.011\,7}{100}\right) = 3.60 \times 10^{18} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28 \times 10^9 \text{ yr}}\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = 1.72 \times 10^{-17} \text{ s}^{-1}$$

$$R = \lambda N = \left(1.72 \times 10^{-17} \text{ s}^{-1}\right)\left(3.60 \times 10^{18}\right) = \boxed{61.8 \text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\lambda = \dfrac{\ln 2}{8.04 \text{ d}}$

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04 \text{ d}}{\ln 2}\ln\left(\frac{2\,000}{61.8}\right) = \boxed{40.3 \text{ d}}$$

P44.58 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy for the nucleus-photon system gives

$$\Delta E = hf + E_r \tag{1}$$

Where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \tag{2}$$

Since system momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \tag{3}$$

Hence, E_r can be expresses as $E_r = \dfrac{(hf)^2}{2Mc^2}$

When $hf << Mc^2$

we can make the approximation that $hf \approx \Delta E$

so $E_r \approx \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\Delta E = 0.014\,4$ MeV

and $Mc^2 = (57 \text{ u})(931.5 \text{ MeV/u}) = 5.31 \times 10^4 \text{ MeV}$

Therefore, $E_r = \dfrac{(1.44 \times 10^{-2} \text{ MeV})^2}{2(5.31 \times 10^4 \text{ MeV})} = \boxed{1.94 \times 10^{-3} \text{ eV}}$

P44.59 We have $N_{235} = N_{0,235} e^{-\lambda_{235} t}$

and $N_{238} = N_{0,238} e^{-\lambda_{238} t}$

$$\frac{N_{235}}{N_{238}} = 0.007\,25 = e^{\left(-(\ln 2)t/T_{h,235} + (\ln 2)t/T_{h,\,238}\right)}$$

Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704 \times 10^9 \text{ yr}} + \frac{\ln 2}{4.47 \times 10^9 \text{ yr}}\right)t$$

or

$$-4.93 = \left(-\frac{1}{0.704 \times 10^9 \text{ yr}} + \frac{1}{4.47 \times 10^9 \text{ yr}}\right)(\ln 2)t$$

$$t = \frac{-4.93}{(-1.20 \times 10^{-9} \text{ yr}^{-1})\ln 2} = \boxed{5.94 \times 10^9 \text{ yr}}$$

P44.60 (a) For cobalt-56,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{77.1 \text{ d}}\left(\frac{365.25 \text{ d}}{1 \text{ yr}}\right) = 3.28 \text{ yr}^{-1}$$

The elapsed time from July 1054 to July 2007 is 953 yr.

$$R = R_0 e^{-\lambda t}$$

implies $$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28 \text{ yr}^{-1}\right)(953 \text{ yr})} = e^{-3\,129} = e^{-(\ln 10)1\,359} = \boxed{\sim 10^{-1\,359}}$$

(b) For carbon-14,

$$\lambda = \frac{\ln 2}{5\,730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}$$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times 10^{-4} \text{ yr}^{-1}\right)(953 \text{ yr})} = e^{-0.115} = \boxed{0.891}$$

P44.61 $E = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$

$\mu = 2.792\,8\mu_n$ and $\mu_n = 5.05 \times 10^{-27}$ J/T

$$\Delta E = 2\mu B = 2(2.792\,8)\left(5.05 \times 10^{-27} \text{ J/T}\right)(12.5 \text{ T}) = 3.53 \times 10^{-25} \text{ J} = \boxed{2.20 \times 10^{-6} \text{ eV}}$$

P44.62 $R = R_0 \exp(-\lambda t)$ lets us write $\ln R = \ln R_0 - \lambda t$ which is the equation of a straight line with $|\text{slope}| = \lambda$. The logarithmic plot shown in Figure P44.62 is fitted by $\ln R = 8.44 - 0.262t$. If t is measured in minutes, then decay constant λ is 0.262 per minute. The half–life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262/\text{min}} = \boxed{2.64 \text{ min}}.$$

The reported half–life of ^{137}Ba is 2.55 min. The difference reflects experimental uncertainties.

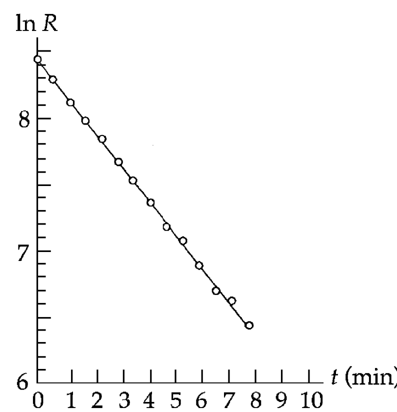

FIG. P44.62

P44.63 $K = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.040\,0 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}} = 2.77 \times 10^3 \text{ m/s}$

The time for the trip is $t = \frac{x}{v} = \frac{1.00 \times 10^4 \text{ m}}{2.77 \times 10^3 \text{ m/s}} = 3.61 \text{ s}$.

The number of neutrons finishing the trip is given by $N = N_0 e^{-\lambda t}$.

The fraction decaying is $1 - \frac{N}{N_0} = 1 - e^{-(\ln 2)t/T_{1/2}} = 1 - e^{-(\ln 2)(3.61 \text{ s}/624 \text{ s})} = 0.004\,00 = \boxed{0.400\%}$.

P44.64 For an electric charge density $\rho = \frac{Ze}{(4/3)\pi R^3}$.

Using Gauss's Law inside the sphere,

$$E \cdot 4\pi r^2 = \frac{(4/3)\pi r^3}{\epsilon_0} \frac{Ze}{(4/3)\pi R^3}: \qquad E = \frac{1}{4\pi \epsilon_0} \frac{Zer}{R^3} \qquad (r \le R)$$

$$E = \frac{1}{4\pi \epsilon_0} \frac{Ze}{r^2} \qquad (r \ge R)$$

We now find the electrostatic energy: $U = \int_{r=0}^{\infty} \frac{1}{2} \epsilon_0 E^2 4\pi r^2 dr$

$$U = \frac{1}{2}\epsilon_0 \int_0^R \left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{Z^2 e^2 r^2}{R^6} 4\pi r^2 dr + \frac{1}{2}\epsilon_0 \int_R^{\infty} \left(\frac{1}{4\pi \epsilon_0}\right)^2 \frac{Z^2 e^2}{r^4} 4\pi r^2 dr = \frac{Z^2 e^2}{8\pi \epsilon_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right]$$

$$= \boxed{\frac{3}{20} \frac{Z^2 e^2}{\pi \epsilon_0 R}}$$

P44.65 (a) If we assume all the ^{87}Sr came from ^{87}Rb,

then $N = N_0 e^{-\lambda t}$

yields $t = \frac{-1}{\lambda} \ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2} \ln\left(\frac{N_0}{N}\right)$

where $N = N_{\text{Rb-87}}$

and $N_0 = N_{\text{Sr-87}} + N_{\text{Rb-87}}$

$$t = \frac{(4.75 \times 10^{10} \text{ yr})}{\ln 2} \ln\left(\frac{1.82 \times 10^{10} + 1.07 \times 10^9}{1.82 \times 10^{10}}\right) = \boxed{3.91 \times 10^9 \text{ yr}}$$

(b) It could be $\boxed{\text{no older}}$. The rock could be younger if some ^{87}Sr were originally present. We must make some assumption about the original quantity of radioactive material. In part (a) we assumed that the rock originally contained no strontium.

P44.66 (a) For the electron capture, $^{93}_{43}\text{Tc} + ^{0}_{-1}\text{e} \to ^{93}_{42}\text{Mo} + \gamma$

The disintegration energy is $Q = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}}\right]c^2$.

$$Q = [92.910\,2 - 92.906\,8]\text{u}(931.5\ \text{MeV/u}) = 3.17\ \text{MeV} > 2.44\ \text{MeV}$$

Electron capture is allowed $\boxed{\text{to all specified excited states}}$ in $^{93}_{42}\text{Mo}$.

For positron emission, $^{93}_{43}\text{Tc} \to ^{93}_{42}\text{Mo} + ^{0}_{+1}\text{e} + \gamma$

The disintegration energy is $Q' = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}} - 2m_e\right]c^2$.

$$Q' = [92.910\,2 - 92.906\,8 - 2(0.000\,549)]\text{u}(931.5\ \text{MeV/u}) = 2.14\ \text{MeV}$$

Positron emission can reach $\boxed{\text{the 1.35, 1.48, and 2.03 MeV states}}$ but there is insufficient energy to reach the 2.44 MeV state.

(b) The daughter nucleus in both forms of decay is $\boxed{^{93}_{42}\text{Mo}}$.

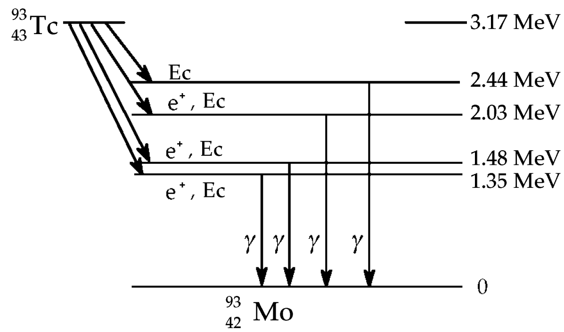

FIG. P44.66

ANSWERS TO EVEN PROBLEMS

P44.2 (a) 7.89 cm and 8.21 cm (b) See the solution.

P44.4 (a) 29.5 fm (b) 5.18 fm (c) See the solution.

P44.6 (a) 1.90 fm (b) 7.44 fm

P44.8 6.11 PN toward the other ball

P44.10 They agree with the figure.

P44.12 0.210 MeV greater for ^{23}Na because it has less proton repulsion

P44.14 (a) 84.1 MeV (b) 342 MeV (c) The nuclear force of attraction dominates over electrical repulsion.

P44.16 (a) See the solution. (b) $\frac{R}{3}$ and $\frac{R}{6}$; see the solution.

P44.18 (a) 491 MeV (b) 179%, –53.0%, –24.6%, –1.37%

P44.20 0.200 mCi

P44.22 See the solution.

P44.24 $\frac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)$

P44.26 (a) See the solution. (b) See the solution. (c) See the solution; 10.9 min.
(d) $t_m = \dfrac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$; yes

P44.28 (a) ${}^{65}_{28}\mathrm{Ni}^*$ (b) ${}^{211}_{82}\mathrm{Pb}$ (c) ${}^{55}_{27}\mathrm{Co}$ (d) ${}^{0}_{-1}\mathrm{e}$ (e) ${}^{1}_{1}\mathrm{H}$

P44.30 9.96×10^3 yr

P44.32 ${}^{3}_{1}\mathrm{H}$ atom $\rightarrow$ ${}^{3}_{2}\mathrm{He}$ atom $+ \bar{\nu}$; 18.6 keV

P44.34 (a) cannot occur (b) cannot occur (c) can occur

P44.36 (a) 4.00 Gyr (b) 0.019 9 and 4.60

P44.38 (a) 0.281 (b) 1.65×10^{-29} (c) See the solution.

P44.40 (a) 4.28 pJ (b) 1.19×10^{57} atoms (c) 105 Gyr

P44.42 1.02 MeV

P44.44 (a) ${}^{13}_{6}\mathrm{C}$ (b) ${}^{10}_{5}\mathrm{B}$

P44.46 See the solution.

P44.48 (a) See the solution. (b) 1.16 u

P44.50 (a) smaller (b) 1.46×10^{-8} u (c) 1.45×10^{-6} % (d) no

P44.52 (a) 2.52×10^{24} (b) 2.29 TBq (c) 1.07 Myr

P44.54 (a) 2.75 fm (b) 152 N (c) 2.62 MeV (d) 7.44 fm, 379 N, 17.6 MeV

P44.56 (a) See the solution. (b) 4.17×10^{-3} min^{-1}; 2.77 h (c) 4.02×10^3 counts/min
(d) 9.65×10^6 atoms

P44.58 (a) See the solution. (b) 1.94 meV

P44.60 (a) $\sim 10^{-1\,359}$ (b) 0.891

P44.62 2.64 min

P44.64 See the solution.

P44.66 (a) electron capture to all; positron emission to the 1.35 MeV, 1.48 MeV, and 2.03 MeV states
(b) ${}^{93}_{42}\mathrm{Mo}$; see the solution.