29

Magnetic Fields

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q29.1** (a) Yes, as described by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$.
(b) No, as described by $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
(c) Yes. (d) Yes. (e) No. The wire is uncharged.
(f) Yes. (g) Yes. (h) Yes.

***Q29.2** (i) (b). (ii) (a). Electron A has a smaller radius of curvature, as described by $qvB = mv^2/r$.

***Q29.3** (i) (c) (ii) (c) (iii) (c) (iv) (b) (v) (d) (vi) (b) (vii) (b) (viii) (b)

***Q29.4** We consider the quantity $|\,qvB \sin\theta\,|$, in units of e (m/s)(T). For (a) it is $1 \times 10^6\ 10^{-3}\ 1 = 10^3$. For (b) it is $1 \times 10^6\ 10^{-3}\ 0 = 0$. For (c) $2 \times 10^6\ 10^{-3}\ 1 = 2\,000$. For (d) $1 \times 10^6\ 2 \times 10^{-3}\ 1 = 2\,000$. For (e) $1 \times 10^6\ 10^{-3}\ 1 = 10^3$. For (f) $1 \times 10^6\ 10^{-3}\ 0.707 = 707$. The ranking is then $c = d > a = e > f > b$.

***Q29.5** $\hat{\mathbf{i}} \times (-\hat{\mathbf{k}}) = \hat{\mathbf{j}}$. Answer (c).

***Q29.6** Answer (c). It is not necessarily zero. If the magnetic field is parallel or antiparallel to the velocity of the charged particle, then the particle will experience no magnetic force.

Q29.7 If they are projected in the same direction into the same magnetic field, the charges are of opposite sign.

Q29.8 Send the particle through the uniform field and look at its path. If the path of the particle is parabolic, then the field must be electric, as the electric field exerts a constant force on a charged particle, independent of its velocity. If you shoot a proton through an electric field, it will feel a constant force in the same direction as the electric field—it's similar to throwing a ball through a gravitational field.
If the path of the particle is helical or circular, then the field is magnetic.
If the path of the particle is straight, then observe the speed of the particle. If the particle accelerates, then the field is electric, as a constant force on a proton with or against its motion will make its speed change. If the speed remains constant, then the field is magnetic.

***Q29.9** Answer (d). The electrons will feel a constant electric force and a magnetic force that will change in direction and in magnitude as their speed changes.

Q29.10 Yes. If the magnetic field is perpendicular to the plane of the loop, then it exerts no torque on the loop.

Q29.11 If you can hook a spring balance to the particle and measure the force on it in a known electric field, then $q = \frac{F}{E}$ will tell you its charge. You cannot hook a spring balance to an electron. Measuring the acceleration of small particles by observing their deflection in known electric and magnetic fields can tell you the charge-to-mass ratio, but not separately the charge or mass. Both an acceleration produced by an electric field and an acceleration caused by a magnetic field depend on the properties of the particle only by being proportional to the ratio $\frac{q}{m}$.

Q29.12 If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation—the torque is zero if the field is along the axis of the loop.

Q29.13 The Earth's magnetic field exerts force on a charged incoming cosmic ray, tending to make it spiral around a magnetic field line. If the particle energy is low enough, the spiral will be tight enough that the particle will first hit some matter as it follows a field line down into the atmosphere or to the surface at a high geographic latitude.

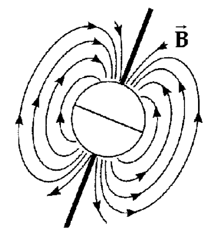


FIG. Q29.13

Q29.14 No. Changing the velocity of a particle requires an accelerating force. The magnetic force is proportional to the speed of the particle. If the particle is not moving, there can be no magnetic force on it.

SOLUTIONS TO PROBLEMS

Section 29.1 Magnetic Fields and Forces

P29.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page

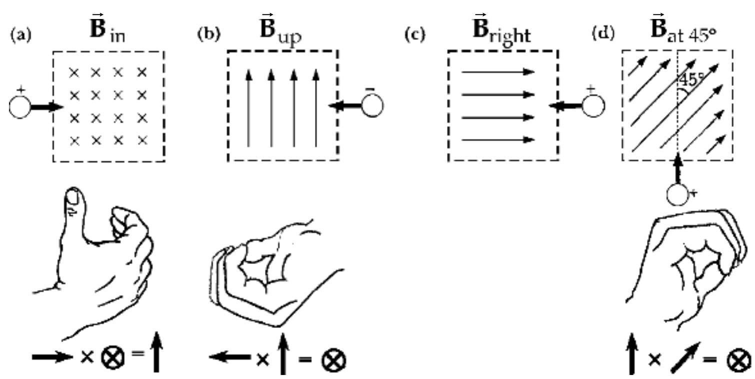


FIG. P29.1

P29.2 At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is opposite in direction to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Figures are drawn looking down.

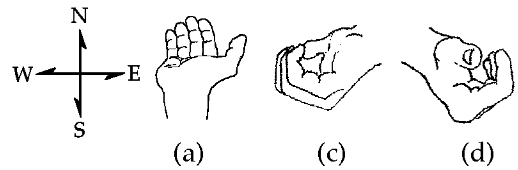


FIG. P29.2

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$.

(b) North × North $= \sin 0° = 0$: $\boxed{\text{Zero deflection}}$.

(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$.

(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$.

P29.3 $F = ma = \left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{13}\text{ m/s}^2\right) = 3.34\times10^{-14}\text{ N} = qvB\sin 90°$

$$B = \frac{F}{qv} = \frac{3.34\times10^{-14}\text{ N}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{7}\text{ m/s}\right)} = \boxed{2.09\times10^{-2}\text{ T}}$$

The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

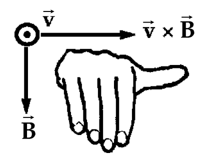


FIG. P29.3

P29.4 (a) $F_B = qvB\sin\theta = \left(1.60\times10^{-19}\text{ C}\right)\left(3.00\times10^{6}\text{ m/s}\right)\left(3.00\times10^{-1}\text{ T}\right)\sin 37.0°$

$F_B = \boxed{8.67\times10^{-14}\text{ N}}$

(b) $a = \dfrac{F}{m} = \dfrac{8.67\times10^{-14}\text{ N}}{1.67\times10^{-27}\text{ kg}} = \boxed{5.19\times10^{13}\text{ m/s}^2}$

P29.5 $F_B = qvB\sin\theta$ so $8.20\times10^{-13}\text{ N} = \left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{6}\text{ m/s}\right)(1.70\text{ T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9° \text{ or } 131°}$

P29.6 First find the speed of the electron.

$$\Delta K = \frac{1}{2}mv^2 = e\Delta V = \Delta U: \qquad v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\text{ C}\right)\left(2\,400\text{ J/C}\right)}{\left(9.11\times10^{-31}\text{ kg}\right)}} = 2.90\times10^{7}\text{ m/s}$$

(a) $F_{B,\max} = qvB = \left(1.60\times10^{-19}\text{ C}\right)\left(2.90\times10^{7}\text{ m/s}\right)(1.70\text{ T}) = \boxed{7.90\times10^{-12}\text{ N}}$

(b) $F_{B,\min} = \boxed{0}$ occurs when $\vec{\mathbf{v}}$ is either parallel to or anti-parallel to $\vec{\mathbf{B}}$.

P29.7 $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$$\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12-2)\hat{\mathbf{i}} + (1+6)\hat{\mathbf{j}} + (4+4)\hat{\mathbf{k}} = 10\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \sqrt{10^2 + 7^2 + 8^2} = 14.6\text{ T}\cdot\text{m/s}$$

$$\left|\vec{\mathbf{F}}_B\right| = q\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \left(1.60\times10^{-19}\text{ C}\right)(14.6\text{ T}\cdot\text{m/s}) = \boxed{2.34\times10^{-18}\text{ N}}$$

P29.8 $q\vec{\mathbf{E}}=\left(-1.60\times10^{-19}\,\mathrm{C}\right)(20.0\ \ \mathrm{N/C})\hat{\mathbf{k}}=\left(-3.20\times10^{-18}\,\mathrm{N}\right)\hat{\mathbf{k}}$

$$\sum\vec{\mathbf{F}}=q\vec{\mathbf{E}}+q\vec{\mathbf{v}}\times\vec{\mathbf{B}}=m\vec{\mathbf{a}}$$

$$\left(-3.20\times10^{-18}\,\mathrm{N}\right)\hat{\mathbf{k}}-1.60\times10^{-19}\,\mathrm{C}\left(1.20\times10^{4}\ \mathrm{m/s}\,\hat{\mathbf{i}}\right)\times\vec{\mathbf{B}}=\left(9.11\times10^{-31}\right)\left(2.00\times10^{12}\ \mathrm{m/s^2}\right)\hat{\mathbf{k}}$$

$$-\left(3.20\times10^{-18}\,\mathrm{N}\right)\hat{\mathbf{k}}-\left(1.92\times10^{-15}\,\mathrm{C\cdot m/s}\right)\hat{\mathbf{i}}\times\vec{\mathbf{B}}=\left(1.82\times10^{-18}\,\mathrm{N}\right)\hat{\mathbf{k}}$$

$$\left(1.92\times10^{-15}\,\mathrm{C\cdot m/s}\right)\hat{\mathbf{i}}\times\vec{\mathbf{B}}=-\left(5.02\times10^{-18}\,\mathrm{N}\right)\hat{\mathbf{k}}$$

The magnetic field $\boxed{\text{may have any } x\text{-component}}$. $B_z=\boxed{0}$ and $B_y=\boxed{-2.62\ \mathrm{mT}}$.

Section 29.2 **Motion of a Charged Particle in a Uniform Magnetic Field**

P29.9 (a) $B=50.0\times10^{-6}\ \mathrm{T}$; $v=6.20\times10^{6}\ \mathrm{m/s}$

Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

$$F_B=qvB\sin\theta$$

$$F_B=\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(6.20\times10^{6}\ \mathrm{m/s}\right)\left(50.0\times10^{-6}\,\mathrm{T}\right)\sin 90.0^\circ$$

$$=\boxed{4.96\times10^{-17}\,\mathrm{N}}$$

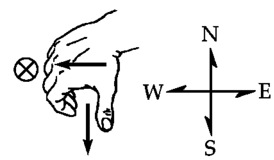


FIG. P29.9

(b) $F=\dfrac{mv^2}{r}$ so

$$r=\frac{mv^2}{F}=\frac{\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(6.20\times10^{6}\ \mathrm{m/s}\right)^2}{4.96\times10^{-17}\,\mathrm{N}}=\boxed{1.29\ \mathrm{km}}$$

***P29.10** (a) The horizontal velocity component of the electrons is given by $(1/2)mv_x^2=|q|V$.

$$v_x=\sqrt{\frac{2|q|V}{m}}=\sqrt{\frac{2(1.6\times10^{-19}\,\mathrm{C})\ 2\,500\ \mathrm{J/C}}{9.11\times10^{-31}\,\mathrm{kg}}}=2.96\times10^{7}\,\mathrm{m/s}$$

Its time of flight is $t=x/v_x=0.35\ \mathrm{m}/2.96\times10^7\ \mathrm{m/s}$
$=1.18\times10^{-8}\ \mathrm{s}$.

Its vertical deflection is $y=(1/2)\,gt^2=(1/2)\ 9.8\ \mathrm{m/s^2}\ (1.18\times10^{-8}\ \mathrm{s})^2=\boxed{6.84\times10^{-16}\ \mathrm{m\ down}}$, which is unobservably small.

(b) The magnetic force is in the direction –north × down = –west = east. The beam is deflected into a circular path with radius

$$r=\frac{mv}{|q|B}=\frac{9.11\times10^{-31}\,\mathrm{kg}\ 2.96\times10^{7}\mathrm{m/s}}{1.6\times10^{-19}\,\mathrm{C}\ 20\times10^{-6}\mathrm{N\cdot s/C\cdot m}}=8.44\ \mathrm{m}.$$

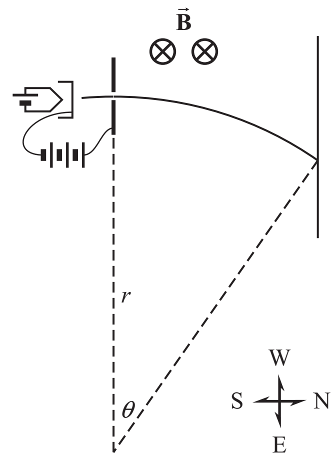


FIG. P29.10

Their path to the screen subtends at the center of curvature an angle given by $\sin\theta=0.35\ \mathrm{m}/8.44\ \mathrm{m}=2.38^\circ$. Their deflection is $8.44\ \mathrm{m}(1-\cos 2.38^\circ)=\boxed{7.26\ \mathrm{mm\ east}}$. It does not move as a projectile, but its northward velocity component stays nearly constant, changing from 2.96×10^7 m/s cos 0° to 2.96×10^7 m/s cos 2.38°. That is, it is constant within 0.09%. It is a good approximation to think of it as moving on a parabola as it really moves on a circle.

P29.11 $q(\Delta V)=\frac{1}{2}mv^2$ or $v=\sqrt{\frac{2q(\Delta V)}{m}}$

Also, $qvB=\frac{mv^2}{r}$ so $r=\frac{mv}{qB}=\frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}}=\sqrt{\frac{2m(\Delta V)}{qB^2}}$

Therefore, $r_p^2=\frac{2m_p(\Delta V)}{eB^2}$

$$r_d^2=\frac{2m_d(\Delta V)}{q_dB^2}=\frac{2(2m_p)(\Delta V)}{eB^2}=2\left(\frac{2m_p(\Delta V)}{eB^2}\right)=2r_p^2$$

and $r_\alpha^2=\frac{2m_\alpha(\Delta V)}{q_\alpha B^2}=\frac{2(4m_p)(\Delta V)}{(2e)B^2}=2\left(\frac{2m_p(\Delta V)}{eB^2}\right)=2r_p^2$

The conclusion is: $\boxed{r_\alpha=r_d=\sqrt{2}r_p}$

P29.12 For each electron, $|q|vB\sin 90.0°=\frac{mv^2}{r}$ and $v=\frac{eBr}{m}$.

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K=\frac{1}{2}mv_{1i}^2+0=\frac{1}{2}mv_{1f}^2+\frac{1}{2}mv_{2f}^2$$

$$K=\frac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right)+\frac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right)=\frac{e^2B^2}{2m}\left(R_1^2+R_2^2\right)$$

$$K=\frac{e\left(1.60\times10^{-19}\text{ C}\right)\left(0.044\,0\text{ N}\cdot\text{s/C}\cdot\text{m}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)}\left[\left(0.010\,0\text{ m}\right)^2+\left(0.024\,0\text{ m}\right)^2\right]=\boxed{115\text{ keV}}$$

P29.13 (a) We begin with $qvB=\frac{mv^2}{R}$

or $qRB=mv$

But $L=mvR=qR^2B$

Therefore, $R=\sqrt{\frac{L}{qB}}=\sqrt{\frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{-3}\text{ T}\right)}}=0.050\,0\text{ m}=\boxed{5.00\text{ cm}}$

(b) Thus, $v=\frac{L}{mR}=\frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)\left(0.050\,0\text{ m}\right)}=\boxed{8.78\times10^6\text{ m/s}}$

P29.14 $\frac{1}{2}mv^2=q(\Delta V)$ so $v=\sqrt{\frac{2q(\Delta V)}{m}}$

$r=\frac{mv}{qB}$ so $r=\frac{m\sqrt{2q(\Delta V)/m}}{qB}$

$r^2=\frac{m}{q}\cdot\frac{2(\Delta V)}{B^2}$ and $(r')^2=\frac{m'}{q'}\cdot\frac{2(\Delta V)}{B^2}$

$m=\frac{qB^2r^2}{2(\Delta V)}$ and $(m')=\frac{(q')B^2(r')^2}{2(\Delta V)}$ so $\frac{m'}{m}=\frac{q'}{q}\cdot\frac{(r')^2}{r^2}=\left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^2=\boxed{8}$

P29.15 $E = \frac{1}{2}mv^2 = e\Delta V$

and $evB\sin 90° = \frac{mv^2}{R}$

$$B = \frac{mv}{eR} = \frac{m}{eR}\sqrt{\frac{2e\Delta V}{m}} = \frac{1}{R}\sqrt{\frac{2m\Delta V}{e}}$$

$$B = \frac{1}{5.80\times10^{10}\text{ m}}\sqrt{\frac{2\left(1.67\times10^{-27}\text{ kg}\right)\left(10.0\times10^{6}\text{ V}\right)}{1.60\times10^{-19}\text{ C}}} = \boxed{7.88\times10^{-12}\text{ T}}$$

P29.16 (a) The boundary between a region of strong magnetic field and a region of zero field cannot be perfectly sharp, but we ignore the thickness of the transition zone. In the field the electron moves on an arc of a circle:

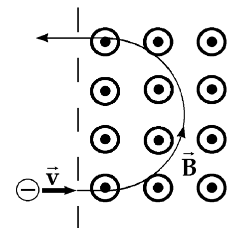


FIG. P29.16(a)

$\sum F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$\frac{v}{r} = \omega = \frac{|q|B}{m} = \frac{\left(1.60\times10^{-19}\text{C}\right)\left(10^{-3}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{\left(9.11\times10^{-31}\text{kg}\right)} = 1.76\times10^{8}\text{ rad/s}$$

The time for one half revolution is,

from $\Delta\theta = \omega\Delta t$

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\pi\text{ rad}}{1.76\times10^{8}\text{ rad/s}} = \boxed{1.79\times10^{-8}\text{ s}}$$

(b) The maximum depth of penetration is the radius of the path.

Then $v = \omega r = \left(1.76\times10^{8}\text{s}^{-1}\right)(0.02\text{ m}) = 3.51\times10^{6}\text{ m/s}$

and

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{kg}\right)\left(3.51\times10^{6}\text{ m/s}\right)^2 = 5.62\times10^{-18}\text{ J} = \frac{5.62\times10^{-18}\text{ J}\cdot\text{e}}{1.60\times10^{-19}\text{ C}}$$

$$= \boxed{35.1\text{ eV}}$$

Section 29.3 **Applications Involving Charged Particles Moving in a Magnetic Field**

P29.17 $F_B = F_e$

so $qvB = qE$

where $v = \sqrt{\frac{2K}{m}}$ and K is kinetic energy of the electron.

$$E = vB = \sqrt{\frac{2K}{m}}B = \sqrt{\frac{2(750)\left(1.60\times10^{-19}\right)}{9.11\times10^{-31}}}(0.015\,0) = \boxed{244\text{ kV/m}}$$

P29.18 $K = \frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\frac{2q(\Delta V)}{m}}$

$$\left|\vec{\mathbf{F}}_B\right| = \left|q\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \frac{mv^2}{r} \qquad r = \frac{mv}{qB} = \frac{m}{q}\sqrt{\frac{2q(\Delta V)/m}{B}} = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$$

(a) $r_{238} = \sqrt{\frac{2\left(238 \times 1.66 \times 10^{-27}\right)2\,000}{1.60 \times 10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28 \times 10^{-2}\text{ m} = \boxed{8.28\text{ cm}}$

(b) $r_{235} = \boxed{8.23\text{ cm}}$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.006\,4$$

The ratios of the orbit radius for different ions are independent of ΔV and B.

P29.19 In the velocity selector: $v = \frac{E}{B} = \frac{2\,500\text{ V/m}}{0.035\,0\text{ T}} = 7.14 \times 10^4\text{ m/s}$

In the deflection chamber: $r = \frac{mv}{qB} = \frac{\left(2.18 \times 10^{-26}\text{ kg}\right)\left(7.14 \times 10^4\text{ m/s}\right)}{\left(1.60 \times 10^{-19}\text{ C}\right)\left(0.035\,0\text{ T}\right)} = \boxed{0.278\text{ m}}$

P29.20 Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by

$\sum F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$|q|B = m\frac{v}{r} = m\omega$$

(a) $\omega = \frac{|q|B}{m} = \frac{\left(1.60 \times 10^{-19}\text{C}\right)\left(0.8\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{\left(1.67 \times 10^{-27}\text{kg}\right)}\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = \boxed{7.66 \times 10^7\text{ rad/s}}$

(b) $v = \omega r = \left(7.66 \times 10^7\text{ rad/s}\right)(0.350\text{ m})\left(\frac{1}{1\text{ rad}}\right) = \boxed{2.68 \times 10^7\text{ m/s}}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67 \times 10^{-27}\text{kg}\right)\left(2.68 \times 10^7\text{ m/s}\right)^2\left(\frac{1\text{ eV}}{1.6 \times 10^{-19}\text{J}}\right) = \boxed{3.76 \times 10^6\text{ eV}}$

(d) The proton gains 600 eV twice during each revolution, so the number of revolutions is

$$\frac{3.76 \times 10^6\text{ eV}}{2(600\text{ eV})} = \boxed{3.13 \times 10^3\text{ revolutions}}$$

(e) $\theta = \omega t \qquad t = \frac{\theta}{\omega} = \frac{3.13 \times 10^3\text{ rev}}{7.66 \times 10^7\text{ rad/s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{2.57 \times 10^{-4}\text{ s}}$

P29.21 (a) $F_B = qvB = \frac{mv^2}{R}$

$$\omega = \frac{v}{R} = \frac{qBR}{mR} = \frac{qB}{m} = \frac{\left(1.60 \times 10^{-19}\text{C}\right)(0.450\text{ T})}{1.67 \times 10^{-27}\text{kg}} = \boxed{4.31 \times 10^7\text{ rad/s}}$$

(b) $v = \frac{qBR}{m} = \frac{\left(1.60 \times 10^{-19}\text{C}\right)(0.450\text{ T})(1.20\text{ m})}{1.67 \times 10^{-27}\text{kg}} = \boxed{5.17 \times 10^7\text{ m/s}}$

***P29.22** (a) The path radius is $r = mv/qB$, which we can put in terms of energy E by $(1/2)mv^2 = E$.

$v = (2E/m)^{1/2}$ so $r = (m/qB)\,(2E/m)^{1/2} = (2m)^{1/2}(qB)^{-1}E^{1/2}$

Then $dr/dt = = (2m)^{1/2}(qB)^{-1}(1/2)\,E^{-1/2}\,dE/dt = \dfrac{\sqrt{2m}}{qB2}\dfrac{\sqrt{2m}}{qBr}\dfrac{q^2B\Delta V}{\pi m} = \dfrac{1}{r}\dfrac{\Delta V}{\pi B}$

(b) The dashed red line should spiral around many times, with its turns relatively far apart on the inside and closer together on the outside.

(c) $$\frac{dr}{dt} = \frac{1}{r}\frac{\Delta V}{\pi B} = \frac{600\text{ V}}{0.35\text{ m }\pi\ 0.8\text{ N s}}\frac{\text{C m N m}}{\text{V C}} = \boxed{682\text{ m/s}}$$

(d) $$\Delta r = \frac{dr}{dt}T = \frac{1}{r}\frac{\Delta V}{\pi B}\frac{2\pi m}{qB} = \frac{2\Delta V m}{rqB^2} = \frac{2\times 600\text{ V }1.67\times 10^{-27}\text{kg}}{0.35\text{ m }1.6\times 10^{-19}\text{C }0.8^2}\frac{\text{C}^2\ \text{m}^2\ \text{N m}}{\text{N}^2\ \text{s}^2\ \text{V C}} = \boxed{55.9\ \mu\text{m}}$$

P29.23 $\theta = \tan^{-1}\left(\dfrac{25.0}{10.0}\right) = 68.2°$ and $R = \dfrac{1.00\text{ cm}}{\sin 68.2°} = 1.08\text{ cm}$

Ignoring relativistic correction, the kinetic energy of the electrons is

$\dfrac{1}{2}mv^2 = q\Delta V$ so $v = \sqrt{\dfrac{2q\Delta V}{m}} = 1.33\times 10^8\text{ m/s}$

From Newton's second law, $\dfrac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11\times 10^{-31}\text{ kg})(1.33\times 10^8\text{ m/s})}{(1.60\times 10^{-19}\text{ C})(1.08\times 10^{-2}\text{ m})} = \boxed{70.1\text{ mT}}$$

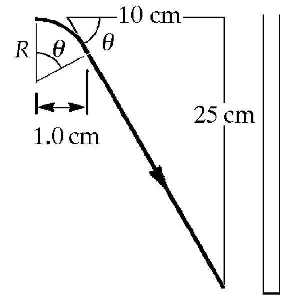


FIG. P29.23

***P29.24** (a) Yes: The constituent of the beam is present in all kinds of atoms.

(b) Yes: Everything in the beam has a single charge-to-mass ratio.

(c) In a charged macroscopic object most of the atoms are uncharged. A molecule never has all of its atoms ionized. Any atom other than hydrogen contains neutrons and so has more mass per charge if it is ionized than hydrogen does. The greatest charge-to-mass ratio Thomson could expect was then for ionized hydrogen, 1.6×10^{-19} C/1.67×10^{-27} kg, smaller than the value e/m he measured, 1.6×10^{-19} C/9.11×10^{-31} kg, by 1 836 times. The particles in his beam could not be whole atoms, but rather must be much smaller in mass.

(d) With kinetic energy 100 eV, an electron has speed given by $(1/2)mv^2 = 100$ eV

$v = \sqrt{\dfrac{200\cdot 1.6\times 10^{-19}\text{C 1 J/C}}{9.11\times 10^{-31}\text{kg}}} = 5.93\times 10^6\text{ m/s}$. The time to travel 40 cm is

$0.4\text{ m}/(5.93\times 10^6\text{ m/s}) = 6.75\times 10^{-8}$ s. If it is fired horizontally it will fall vertically by $(1/2)gt^2 = (1/2)(9.8\text{ m/s}^2)(6.75\times 10^{-8}\text{ s})^2 = 2.23\times 10^{-14}$ m, an immeasurably small amount. An electron with higher energy falls by a smaller amount.

Section 29.4 **Magnetic Force Acting on a Current-Carrying Conductor**

P29.25 $F_B = ILB\sin\theta$ with $F_B = F_g = mg$

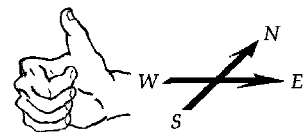


FIG. P29.25

$mg = ILB\sin\theta$ so $\frac{m}{L}g = IB\sin\theta$

$I = 2.00\text{ A}$ and $\frac{m}{L} = (0.500\text{ g/cm})\left(\frac{100\text{ cm/m}}{1\,000\text{ g/kg}}\right) = 5.00\times10^{-2}\text{ kg/m}$

Thus $\left(5.00\times10^{-2}\right)(9.80) = (2.00)B\sin 90.0°$

$B = \boxed{0.245\text{ Tesla}}$ with the direction given by right-hand rule: $\boxed{\text{eastward}}$.

P29.26 $\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = (2.40\text{ A})(0.750\text{ m})\hat{\mathbf{i}}\times(1.60\text{ T})\hat{\mathbf{k}} = \boxed{\left(-2.88\hat{\mathbf{j}}\right)\text{ N}}$

P29.27 (a) $F_B = ILB\sin\theta = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 60.0° = \boxed{4.73\text{ N}}$

(b) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 90.0° = \boxed{5.46\text{ N}}$

(c) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 120° = \boxed{4.73\text{ N}}$

***P29.28** The magnetic force should counterbalance the gravitational force on each section of wire:

$I\ell B\sin 90° = mg$ $\quad I = \frac{m}{\ell}\frac{g}{B} = 2.4\times10^{-3}\frac{\text{kg}}{\text{m}}\frac{9.8\text{ m/s}^2\text{C m}}{28\times10^{-6}\text{N s}} = \boxed{840\text{ A}}$

The current should be $\boxed{\text{east}}$ so that the magnetic force will be east $\times$ north = up.

P29.29 The rod feels force $\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{d}}\times\vec{\mathbf{B}}\right) = Id\left(\hat{\mathbf{k}}\right)\times B\left(-\hat{\mathbf{j}}\right) = IdB\left(\hat{\mathbf{i}}\right)$.

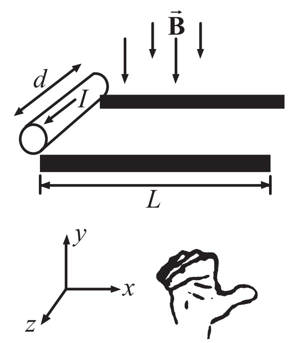


FIG. P29.29

The work-energy theorem is $\left(K_{\text{trasn}} + K_{\text{rot}}\right)_i + \Delta E = \left(K_{\text{trans}} + K_{\text{rot}}\right)_f$

$$0 + 0 + F_s\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$IdBL\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \text{ and } IdBL = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{4IdBL}{3m}} = \sqrt{\frac{4(48.0\text{ A})(0.120\text{ m})(0.240\text{ T})(0.450\text{ m})}{3(0.720\text{ kg})}} = \boxed{1.07\text{ m/s}}$$

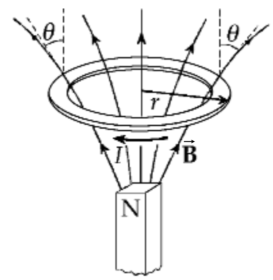
P29.30 The rod feels force $\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{d}} \times \vec{\mathbf{B}}\right) = Id\left(\hat{\mathbf{k}}\right) \times B\left(-\hat{\mathbf{j}}\right) = IdB\left(\hat{\mathbf{i}}\right)$.

The work-energy theorem is $\left(K_{\text{trans}} + K_{\text{rot}}\right)_i + \Delta E = \left(K_{\text{trans}} + K_{\text{rot}}\right)_f$

$$0 + 0 + Fs\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$IdBL\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \text{ and } v = \boxed{\sqrt{\frac{4IdBL}{3m}}}$$

P29.31 The magnetic force on each bit of ring is $Id\vec{\mathbf{s}} \times \vec{\mathbf{B}} = IdsB$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $IdsB\sin\theta$ all add to $\boxed{I2\pi rB\sin\theta \text{ up}}$.



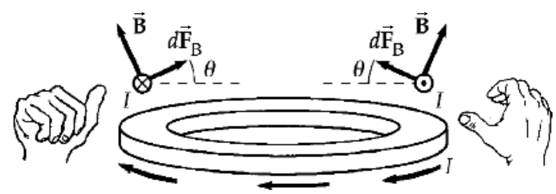


FIG. P29.31

***P29.32** (a) For each segment, $I = 5.00$ A and $\vec{\mathbf{B}} = 0.020\,0$ N/A·m $\hat{\mathbf{j}}$.

Segment	$\vec{\ell}$	$\vec{\mathbf{F}}_B = I\left(\vec{\ell} \times \vec{\mathbf{B}}\right)$
ab	-0.400 m $\hat{\mathbf{j}}$	$\boxed{0}$
bc	0.400 m $\hat{\mathbf{k}}$	$\boxed{(40.0 \text{ mN})\left(-\hat{\mathbf{i}}\right)}$
cd	-0.400 m $\hat{\mathbf{i}} + 0.400$ m $\hat{\mathbf{j}}$	$\boxed{(40.0 \text{ mN})\left(-\hat{\mathbf{k}}\right)}$
da	0.400 m $\hat{\mathbf{i}} - 0.400$ m $\hat{\mathbf{k}}$	$\boxed{(40.0 \text{ mN})\left(\hat{\mathbf{k}} + \hat{\mathbf{i}}\right)}$

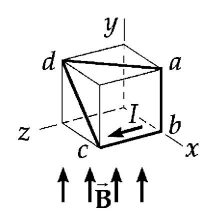


FIG. P29.32

(b) The forces on the four segments must add to zero, so the force on the fourth segment must be the negative of the resultant of the forces on the other three.

P29.33 Take the x-axis east, the y-axis up, and the z-axis south. The field is

$$\vec{\mathbf{B}} = (52.0\ \mu\text{T})\cos 60.0°\left(-\hat{\mathbf{k}}\right) + (52.0\ \mu\text{T})\sin 60.0°\left(-\hat{\mathbf{j}}\right)$$

The current then has equivalent length: $\vec{\mathbf{L}}' = 1.40\ \text{m}\left(-\hat{\mathbf{k}}\right) + 0.850\ \text{m}\left(\hat{\mathbf{j}}\right)$

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}}' \times \vec{\mathbf{B}} = (0.035\,0\ \text{A})\left(0.850\hat{\mathbf{j}} - 1.40\hat{\mathbf{k}}\right)\ \text{m} \times \left(-45.0\hat{\mathbf{j}} - 26.0\hat{\mathbf{k}}\right)10^{-6}\,\text{T}$$

$$\vec{\mathbf{F}}_B = 3.50\times10^{-8}\ \text{N}\left(-22.1\hat{\mathbf{i}} - 63.0\hat{\mathbf{i}}\right) = 2.98\times10^{-6}\ \text{N}\left(-\hat{\mathbf{i}}\right) = \boxed{2.98\ \mu\text{N west}}$$

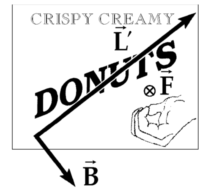


FIG. P29.33

Section 29.5 **Torque on a Current Loop in a Uniform Magnetic Field**

P29.34 (a) $2\pi r = 2.00\ \text{m}$ so $r = 0.318\ \text{m}$

$$\mu = IA = \left(17.0\times10^{-3}\,\text{A}\right)\left[\pi(0.318)^2\ \text{m}^2\right] = \boxed{5.41\ \text{mA}\cdot\text{m}^2}$$

(b) $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ so $\tau = \left(5.41\times10^{-3}\,\text{A}\cdot\text{m}^2\right)(0.800\ \text{T}) = \boxed{4.33\ \text{mN}\cdot\text{m}}$

P29.35 $\tau = NBAI\sin\phi$

$\tau = 100(0.800\ \text{T})\left(0.400\times0.300\ \text{m}^2\right)(1.20\ \text{A})\sin 60°$

$\tau = \boxed{9.98\ \text{N}\cdot\text{m}}$

Note that ϕ is the angle between the magnetic moment and the $\vec{\mathbf{B}}$ field. The loop will rotate so as to align the magnetic moment with the $\vec{\mathbf{B}}$ field. Looking down along the y-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.

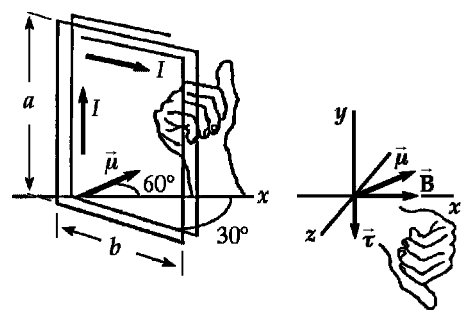


FIG. P29.35

P29.36 Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B\sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ. According to the mechanical equations $\Delta U = -\int dW$ and $dW = \tau\, d\theta$, the potential energy of the dipole-field system is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B\sin\theta\, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \qquad \text{or} \qquad \boxed{U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}}.$$

P29.37 (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at 48.0° below the horizontal}}$

where its energy is $U_{\text{min}} = -\mu B\cos 0° = -\left(9.70\times10^{-3}\,\text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = -5.34\times10^{-7}\ \text{J}$.

It has maximum energy when pointing in the opposite direction,

$\boxed{\text{south at 48.0° above the horizontal}}$

where its energy is $U_{\text{max}} = -\mu B\cos 180° = +\left(9.70\times10^{-3}\,\text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\,\text{T}\right) = +5.34\times10^{-7}\,\text{J}$.

(b) $U_{\text{min}} + W = U_{\text{max}}$: $\quad W = U_{\text{max}} - U_{\text{min}} = +5.34\times10^{-7}\ \text{J} - \left(-5.34\times10^{-7}\ \text{J}\right) = \boxed{1.07\ \mu\text{J}}$

P29.38 (a) $\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$, so $\tau = \left|\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}\right| = \mu B \sin\theta = NIAB\sin\theta$

$$\tau_{\max} = NIAB\sin 90.0° = 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) = \boxed{118\ \mu\text{N}\cdot\text{m}}$$

(b) $U = -\vec{\mu}\cdot\vec{\mathbf{B}}$, so $-\mu B \le U \le +\mu B$

Since $\mu B = (NIA)B = 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) = 118\ \mu\text{J}$,

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$.

***P29.39** For a single-turn circle, $r = 1.5\text{ m}/2\pi$. The magnetic moment is

$\mu = NIA = 1\,(30\times10^{-3}\text{ A})\ \pi(1.5\text{ m}/2\pi)^2 = 5.37\times10^{-3}\text{ A}\cdot\text{m}^2$

For a single-turn square, $\ell = 1.5\text{ m}/4$. The magnetic moment is

$\mu = NIA = 1\,(30\times10^{-3}\text{ A})\,(1.5\text{ m}/4)^2 = 4.22\times10^{-3}\text{ A}\cdot\text{m}^2$

For a single-turn triangle, $\ell = 1.5\text{ m}/3 = 0.5\text{ m}$. The magnetic moment is

$\mu = NIA = 1\,(30\times10^{-3}\text{ A})\,(1/2)\,0.5\text{ m}\,(0.5^2 - 0.25^2)^{1/2}\text{ m} = 3.25\times10^{-3}\text{ A}\cdot\text{m}^2$

For a flat compact circular coil with N turns, $r = 1.5\text{ m}/2\pi N$. The magnetic moment is

$\mu = NIA = N\,(30\times10^{-3}\text{ A})\ \pi(1.5\text{ m}/2\pi N)^2 = 5.37\times10^{-3}\text{ A}\cdot\text{m}^2/N$

> The magnetic moment cannot go to infinity. Its maximum value is 5.37 mA·m² for a single-turn circle. Smaller by 21% and by 40% are the magnetic moments for the single-turn square and triangle. Circular coils with several turns have magnetic moments inversely proportional to the number of turns, approaching zero as the number of turns goes to infinity.

P29.40 (a) $|\vec{\boldsymbol{\tau}}| = \left|\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}\right| = NIAB\sin\theta$

$$\tau_{\max} = 80(10^{-2}\text{ A})(0.025\text{ m}\cdot0.04\text{ m})(0.8\text{ N/A}\cdot\text{m})\sin 90° = \boxed{6.40\times10^{-4}\text{ N}\cdot\text{m}}$$

(b) $$\mathcal{P}_{\max} = \tau_{\max}\omega = 6.40\times10^{-4}\text{ N}\cdot\text{m}(3\,600\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{0.241\text{ W}}$$

(c) In one half revolution the work is

$$W = U_{\max} - U_{\min} = -\mu B\cos 180° - (-\mu B\cos 0°) = 2\mu B$$
$$= 2NIAB = 2(6.40\times10^{-4}\text{ N}\cdot\text{m}) = 1.28\times10^{-3}\text{ J}$$

In one full revolution, $W = 2(1.28\times10^{-3}\text{ J}) = \boxed{2.56\times10^{-3}\text{ J}}$.

(d) $$\mathcal{P}_{\text{avg}} = \frac{W}{\Delta t} = \frac{2.56\times10^{-3}\text{ J}}{(1/60)\text{ s}} = \boxed{0.154\text{ W}}$$

The peak power in (b) is greater by the factor $\frac{\pi}{2}$.

Section 29.6 **The Hall Effect**

P29.41 $B = \dfrac{nqt(\Delta V_H)}{I} = \dfrac{(8.46\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})(5.00\times10^{-3}\text{ m})(5.10\times10^{-12}\text{ V})}{8.00\text{ A}}$

$B = 4.31\times10^{-5}\text{ T} = \boxed{43.1\,\mu\text{T}}$

P29.42 (a) $\Delta V_H = \dfrac{IB}{nqt}$ so $\dfrac{nqt}{I} = \dfrac{B}{\Delta V_H} = \dfrac{0.080\,0\text{ T}}{0.700\times10^{-6}\text{ V}} = 1.14\times10^{5}\text{ T/V}$

Then, the unknown field is $B = \left(\dfrac{nqt}{I}\right)(\Delta V_H)$

$B = (1.14\times10^{5}\text{ T/V})(0.330\times10^{-6}\text{ V}) = 0.037\,7\text{ T} = \boxed{37.7\text{ mT}}$

(b) $\dfrac{nqt}{I} = 1.14\times10^{5}\text{ T/V}$ so $n = (1.14\times10^{5}\text{ T/V})\dfrac{I}{qt}$

$n = (1.14\times10^{5}\text{ T/V})\dfrac{0.120\text{ A}}{(1.60\times10^{-19}\text{ C})(2.00\times10^{-3}\text{ m})} = \boxed{4.29\times10^{25}\text{ m}^{-3}}$

Additional Problems

***P29.43** (a) Define vector $\vec{\mathbf{h}}$ to have the downward direction of the current, and vector $\vec{\mathbf{L}}$ to be along the pipe into the page as shown. The $\boxed{\text{electric current experiences a magnetic force}}$.

$I(\vec{\mathbf{h}}\times\vec{\mathbf{B}})$ in the direction of $\vec{\mathbf{L}}$

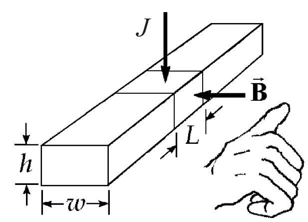


FIG. P29.43

(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length *L*, electrons drift upward to constitute downward electric current $J\times(\text{area}) = J\,Lw$.

The current then feels a magnetic force $I\left|\vec{\mathbf{h}}\times\vec{\mathbf{B}}\right| = JLwhB\sin 90°$.

This force along the pipe axis will make the fluid move, exerting pressure

$\dfrac{F}{\text{area}} = \dfrac{JLwhB}{hw} = \boxed{JLB}$

(c) Charge moves within the fluid inside the length *L*, but charge does not accumulate: the fluid is not charged after it leaves the pump. It is not current-carrying and it is not magnetized.

P29.44 (a) At the moment shown in Figure 29.10, the particle must be moving upward in order for the magnetic force on it to be  into the page, toward the center of this turn of its spiral path. Throughout its motion it circulates clockwise.

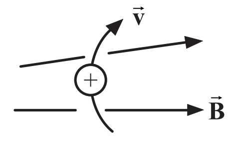


FIG. P29.44(a)

(b) After the particle has passed the middle of the bottle and moves into the region of increasing magnetic field, the magnetic force on it has a component to the left (as well as a radially inward component) as shown. This force in the $-x$ direction slows and reverses the particle's motion along the axis.

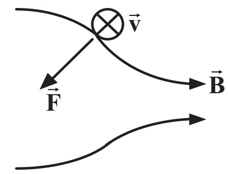


FIG. P29.44(b)

(c) The magnetic force is perpendicular to the velocity and does no work on the particle. The particle keeps constant kinetic energy. As its axial velocity component decreases, its tangential velocity component increases.

(d) The orbiting particle constitutes a loop of current in the yz plane and therefore a magnetic dipole moment $IA = \frac{q}{T}A$ in the $-x$ direction. It is like a little bar magnet with its N pole on the left.

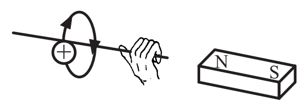


FIG. P29.44(d)

(e) Problem 31 showed that a nonuniform magnetic field exerts a net force on a magnetic dipole. When the dipole is aligned opposite to the external field, the force pushes it out of the region of stronger field. Here it is to the left, a force of repulsion of one magnetic south pole on another south pole.

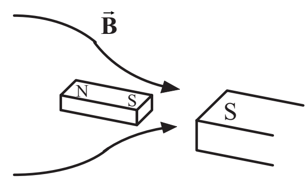


FIG. P29.44(e)

***P29.45** The particle moves in an arc of a circle with radius

$r = \frac{mv}{qB} = \frac{1.67\times10^{-27}\ \text{kg}\ \ 3\times10^{7}\ \text{m/s}\ \ \text{C m}}{1.6\times10^{-19}\ \text{C}\ \ 25\times10^{-6}\ \text{N s}} = \boxed{12.5\ \text{km}}$. It will not hit the Earth, but perform a hairpin turn and go back parallel to its original direction.

P29.46

$$\sum F_y = 0: \qquad +n - mg = 0$$

$$\sum F_x = 0: \qquad -\mu_k n + IBd\sin 90.0^\circ = 0$$

$$\text{B} = \frac{\mu_k mg}{Id} = \frac{0.100(0.200\ \text{kg})(9.80\ \text{ms}^2)}{(10.0\ \text{A})(0.500\ \text{m})} = \boxed{39.2\ \text{mT}}$$

P29.47 The magnetic force on each proton, $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = qvB\sin 90°$ downward perpendicular to velocity, causes centripetal acceleration, guiding it into a circular path of radius r, with

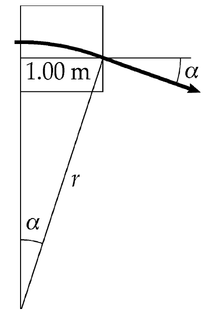


FIG. P29.47

$$qvB = \frac{mv^2}{r}$$

and $$r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed:

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.00\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{1.67\times10^{-27}\,\text{kg}}} = 3.10\times10^{7}\ \text{m/s}$$

Now, $$r = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.10\times10^{7}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.050\,0\ \text{N}\cdot\text{s/C}\cdot\text{m}\right)} = 6.46\ \text{m}$$

(b) We can most conveniently do part (b) first. From the figure observe that

$$\sin\alpha = \frac{1.00\ \text{m}}{r} = \frac{1\ \text{m}}{6.46\ \text{m}}$$

$$\boxed{\alpha = 8.90°}$$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$$-\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.10\times10^{7}\ \text{m/s}\right)\sin 8.90° = \boxed{-8.00\times10^{-21}\ \text{kg}\cdot\text{m/s}}$$

***P29.48** (a) If $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$, $\quad \vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = e\left(v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right) = 0 + ev_iB_y\hat{\mathbf{k}} - ev_iB_z\hat{\mathbf{j}}.$

Since the force actually experienced is $\vec{\mathbf{F}}_B = F_i\hat{\mathbf{j}}$, observe that

$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y = 0}$, and $\boxed{B_z = -\dfrac{F_i}{ev_i}}$.

(b) If $\vec{\mathbf{v}} = -v_i\hat{\mathbf{i}}$, then $\quad \vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = e\left(-v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \dfrac{F_i}{ev_i}\hat{\mathbf{k}}\right) = \boxed{-F_i\hat{\mathbf{j}}}$

(c) If $q = -e$ and $\vec{\mathbf{v}} = -v_i\hat{\mathbf{i}}$, then $\quad \vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -e\left(-v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \dfrac{F_i}{ev_i}\hat{\mathbf{k}}\right) = \boxed{+F_i\hat{\mathbf{j}}}$

Reversing either the velocity or the sign of the charge reverses the force.

P29.49 (a) The net force is the Lorentz force given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$$

$$\vec{\mathbf{F}} = \left(3.20\times10^{-19}\right)\left[\left(4\hat{\mathbf{i}} - 1\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right) + \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 1\hat{\mathbf{k}}\right)\times\left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\right]\ \text{N}$$

Carrying out the indicated operations, we find:

$$\vec{\mathbf{F}} = \boxed{\left(3.52\hat{\mathbf{i}} - 1.60\hat{\mathbf{j}}\right)\times10^{-18}\ \text{N}}.$$

(b) $$\theta = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$$

***P29.50** (a) The field should be in the $+z$ direction, perpendicular to the final as well as to the initial velocity, and with $\hat{\mathbf{i}}\times\hat{\mathbf{k}}=-\hat{\mathbf{j}}$ as the direction of the initial force.

(b) $r=\dfrac{mv}{qB}=\dfrac{\left(1.67\times10^{-27}\text{ kg}\right)\left(20\times10^{6}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.3\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}=\boxed{0.696\text{ m}}$

(c) The path is a quarter circle, of length $(\pi/2)0.696\text{ m}=\boxed{1.09\text{ m}}$

(d) $\Delta t=1.09\text{ m}/20\times10^{6}\text{ m/s}=\boxed{54.7\text{ ns}}$

P29.51 A key to solving this problem is that reducing the normal force will reduce the friction force: $F_B=BIL$ or $B=\dfrac{F_B}{IL}$.

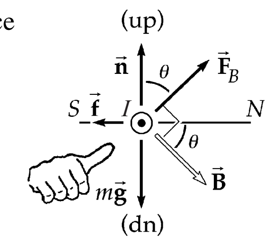


FIG. P29.51

When the wire is just able to move, $\sum F_y=n+F_B\cos\theta-mg=0$

so $n=mg-F_B\cos\theta$

and $f=\mu\left(mg-F_B\cos\theta\right)$

Also, $\sum F_x=F_B\sin\theta-f=0$

so $F_B\sin\theta=f$: $F_B\sin\theta=\mu\left(mg-F_B\cos\theta\right)$ and $F_B=\dfrac{\mu mg}{\sin\theta+\mu\cos\theta}$

We minimize B by minimizing F_B: $\dfrac{dF_B}{d\theta}=(\mu mg)\dfrac{\cos\theta-\mu\sin\theta}{\left(\sin\theta+\mu\cos\theta\right)^2}=0\Rightarrow\mu\sin\theta=\cos\theta$

Thus, $\theta=\tan^{-1}\left(\dfrac{1}{\mu}\right)=\tan^{-1}(5.00)=78.7°$ for the smallest field, and

$$B=\frac{F_B}{IL}=\left(\frac{\mu g}{I}\right)\frac{(m/L)}{\sin\theta+\mu\cos\theta}$$

$$B_{\min}=\left[\frac{(0.200)\left(9.80\text{ m/s}^2\right)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin 78.7°+(0.200)\cos 78.7°}=0.128\text{ T}$$

$$\boxed{B_{\min}=0.128\text{ T pointing north at an angle of }78.7°\text{ below the horizontal}}$$

P29.52 Let v_i represent the original speed of the alpha particle. Let v_α and v_p represent the particles' speeds after the collision. We have conservation of momentum $4m_pv_i=4m_pv_\alpha+m_pv_p$ and the relative velocity equation $v_i-0=v_p-v_\alpha$. Eliminating v_i,

$4v_p-4v_\alpha=4v_\alpha+v_p$ $\qquad 3v_p=8v_\alpha$ $\qquad v_\alpha=\dfrac{3}{8}v_p$

For the proton's motion in the magnetic field,

$\sum F=ma$ $\qquad ev_pB\sin 90°=\dfrac{m_pv_p^2}{R}$ $\qquad \dfrac{eBR}{m_p}=v_p$

For the alpha particle,

$2ev_\alpha B\sin 90°=\dfrac{4m_pv_\alpha^2}{r_\alpha}$ $\qquad r_\alpha=\dfrac{2m_pv_\alpha}{eB}$ $\qquad r_\alpha=\dfrac{2m_p}{eB}\dfrac{3}{8}v_p=\dfrac{2m_p}{eB}\dfrac{3}{8}\dfrac{eBR}{m_p}=\boxed{\dfrac{3}{4}R}$

P29.53 Let Δx_1 be the elongation due to the weight of the wire and let Δx_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2k\Delta x_2$ where k is the force constant of the spring and can be determined from $k = \dfrac{mg}{2\Delta x_1}$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find

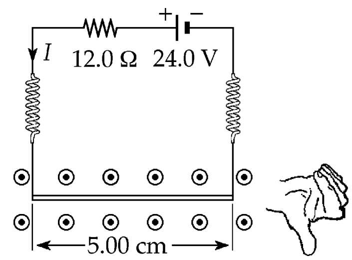


FIG. P29.53

$$F_{\text{magnetic}} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\Delta x_2}{\Delta x_1}; \quad \text{but } \left|\vec{\mathbf{F}}_B\right| = I\left|\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right| = ILB$$

Therefore, where $I = \dfrac{24.0\text{ V}}{12.0\ \Omega} = 2.00\text{ A}$, $B = \dfrac{mg\Delta x_2}{IL\Delta x_1} = \dfrac{(0.100)(9.80)\left(3.00 \times 10^{-3}\right)}{(2.00)(0.050\,0)\left(5.00 \times 10^{-3}\right)} = \boxed{0.588\text{ T}}$.

P29.54 Suppose the input power is

$120\text{ W} = (120\text{ V})I$: $\boxed{I \sim 1\text{ A} = 10^0\text{ A}}$

Suppose $\omega = 2\,000\text{ rev/min}\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) \sim 200\text{ rad/s}$

and the output power is $20\text{ W} = \tau\omega = \tau(200\text{ rad/s})$ $\boxed{\tau \sim 10^{-1}\text{ N}\cdot\text{m}}$

Suppose the area is about $(3\text{ cm}) \times (4\text{ cm})$, or $\boxed{A \sim 10^{-3}\text{ m}^2}$

Suppose that the field is $\boxed{B \sim 10^{-1}\text{ T}}$

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$$0.1\text{ N}\cdot\text{m} \sim N(1\text{ C/s})\left(10^{-3}\text{ m}^2\right)\left(10^{-1}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)$$

giving $\boxed{N \sim 10^3}$

P29.55 The sphere is in translational equilibrium; thus

$$f_s - Mg\sin\theta = 0 \qquad (1)$$

The sphere is in rotational equilibrium. If torques are taken about the center of the sphere, the magnetic field produces a clockwise torque of magnitude $\mu B\sin\theta$, and the frictional force a counterclockwise torque of magnitude f_sR, where R is the radius of the sphere. Thus:

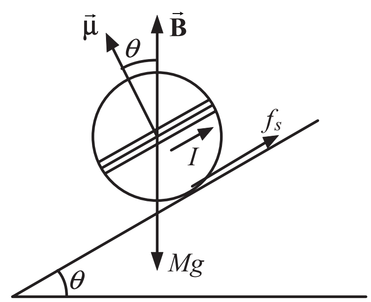


FIG. P29.55

$$f_sR - \mu B\sin\theta = 0 \qquad (2)$$

From (1): $f_s = Mg\sin\theta$. Substituting this in (2) and canceling out $\sin\theta$, one obtains

$$\mu B = MgR. \qquad (3)$$

Now $\mu = NI\pi R^2$. Thus (3) gives $I = \dfrac{Mg}{\pi NBR} = \dfrac{(0.08\text{ kg})\left(9.80\text{ m/s}^2\right)}{\pi(5)(0.350\text{ T})(0.2\text{ m})} = \boxed{0.713\text{ A}}$.

The current must be counterclockwise as seen from above.

P29.56 Call the length of the rod L and the tension in each wire alone $\frac{T}{2}$. Then, at equilibrium:

$$\sum F_x = T\sin\theta - ILB\sin 90.0° = 0 \quad \text{or} \quad T\sin\theta = ILB$$

$$\sum F_y = T\cos\theta - mg = 0, \quad \text{or} \quad T\cos\theta = mg$$

$$\tan\theta = \frac{ILB}{mg} = \frac{IB}{(m/L)g} \quad \text{or} \quad B = \frac{(m/L)g}{I}\tan\theta = \boxed{\frac{\lambda g}{I}\tan\theta}$$

P29.57 $\sum F = ma$ or $qvB\sin 90.0° = \frac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\frac{v}{r} = \omega = \frac{qB}{m} = 2\pi f$ and

$$\Delta\omega = \omega_{12} - \omega_{14} = qB\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{(1.60\times10^{-19}\,\text{C})(2.40\ \text{T})}{(1.66\times10^{-27}\ \text{kg/u})}\left(\frac{1}{12.0\ \text{u}} - \frac{1}{14.0\ \text{u}}\right)$$

$$\Delta\omega = 2.75\times10^{6}\,\text{s}^{-1} = \boxed{2.75\ \text{Mrad/s}}$$

P29.58 Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.

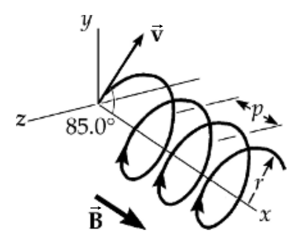


FIG. P29.58

(a) The pitch of trajectory is the distance moved along x by the positron during each period, T (determined by the cyclotron frequency):

$$p = v_x T = (v\cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00\times10^{6})(\cos 85.0°)(2\pi)(9.11\times10^{-31})}{0.150(1.60\times10^{-19})} = \boxed{1.04\times10^{-4}\,\text{m}}$$

(b) The equation about circular motion in a magnetic field still applies to the radius of the spiral:

$$r = \frac{mv_\perp}{Bq} = \frac{mv\sin 85.0°}{Bq}$$

$$r = \frac{(9.11\times10^{-31})(5.00\times10^{6})(\sin 85.0°)}{(0.150)(1.60\times10^{-19})} = \boxed{1.89\times10^{-4}\,\text{m}}$$

P29.59 $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \frac{\Delta q}{\Delta t} = \frac{q}{T}$

and the period of the motion is $T = \frac{2\pi R}{v}$

The electron's speed in its orbit is found by requiring $\frac{k_e q^2}{R^2} = \frac{mv^2}{R}$ or $v = q\sqrt{\frac{k_e}{mR}}$

Substituting this expression for v into the equation for T, we find $T = 2\pi\sqrt{\frac{mR^3}{q^2 k_e}}$

$$T = 2\pi\sqrt{\frac{(9.11\times10^{-31})(5.29\times10^{-11})^3}{(1.60\times10^{-19})^2(8.99\times10^{9})}} = 1.52\times10^{-16}\ \text{s}$$

Therefore, $|\tau| = \left(\frac{q}{T}\right)AB = \frac{1.60\times10^{-19}}{1.52\times10^{-16}}\pi(5.29\times10^{-11})^2(0.400) = \boxed{3.70\times10^{-24}\,\text{N}\cdot\text{m}}$.

P29.60 (a) $K = \frac{1}{2}mv^2 = 6.00 \text{ MeV} = (6.00 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})$

$K = 9.60 \times 10^{-13} \text{ J}$

$$v = \sqrt{\frac{2(9.60 \times 10^{-13} \text{ J})}{1.67 \times 10^{-27} \text{ kg}}} = 3.39 \times 10^7 \text{ m/s}$$

$F_B = qvB = \frac{mv^2}{R}$ so

$$R = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg})(3.39 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(1.00 \text{ T})} = 0.354 \text{ m}$$

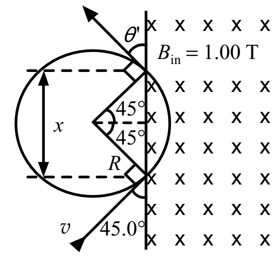


FIG. P29.60

Then, from the diagram, $x = 2R \sin 45.0° = 2(0.354 \text{ m}) \sin 45.0° = \boxed{0.501 \text{ m}}$.

(b) From the diagram, observe that $\theta' = \boxed{45.0°}$.

P29.61 (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force, so

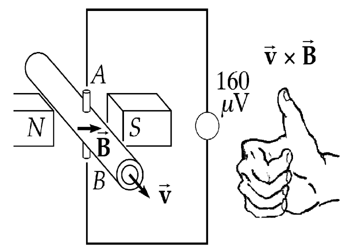


FIG. P29.61

$$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{(160 \times 10^{-6} \text{ V})}{(0.040\,0 \text{ T})(3.00 \times 10^{-3} \text{ m})} = \boxed{1.33 \text{ m/s}}$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

P29.62 (a) See graph to the right. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\Delta V_H = (1.00 \times 10^{-4} \text{ V/T})B}$$

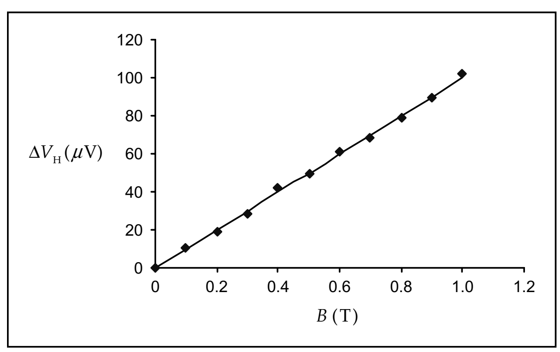


FIG. P29.62

(b) Comparing the equation of the line which fits the data best to

$$\Delta V_H = \left(\frac{1}{nqt}\right)B$$

continued on next page

observe that: $\dfrac{I}{nqt}=1.00\times10^{-4}\ \text{V/T}$, or $t=\dfrac{I}{nq\left(1.00\times10^{-4}\ \text{V/T}\right)}$.

Then, if $I=0.200\ \text{A}$, $q=1.60\times10^{-19}\ \text{C}$, and $n=1.00\times10^{26}\,\text{m}^{-3}$, the thickness of the sample is

$$t=\frac{0.200\ \text{A}}{\left(1.00\times10^{26}\text{m}^{-3}\right)\left(1.60\times10^{-19}\text{C}\right)\left(1.00\times10^{-4}\ \text{V/T}\right)}=1.25\times10^{-4}\,\text{m}=\boxed{0.125\ \text{mm}}$$

P29.63 When in the field, the particles follow a circular path according to $qvB=\dfrac{mv^2}{r}$, so the radius of the path is $r=\dfrac{mv}{qB}$.

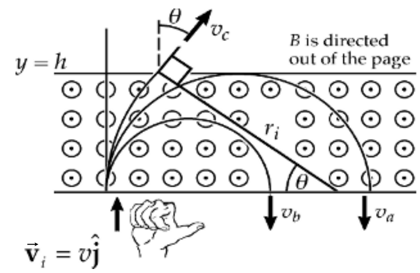


FIG. P29.63

(a) When $r=h=\dfrac{mv}{qB}$, that is, when $\boxed{v=\dfrac{qBh}{m}}$, the particle will cross the band of field. It will move in a full semicircle of radius h, leaving the field at $(2h,\ 0,\ 0)$ with velocity $\boxed{\vec{\mathbf{v}}_f=-v\hat{\mathbf{j}}}$.

(b) When $v<\dfrac{qBh}{m}$, the particle will move in a smaller $\boxed{\text{semicircle}}$ of radius $r=\dfrac{mv}{qB}<h$. It will leave the field at $(2r,\ 0,\ 0)$ with velocity $\boxed{\vec{\mathbf{v}}_f=-v\hat{\mathbf{j}}}$.

(c) When $v>\dfrac{qBh}{m}$, the particle moves in a $\boxed{\text{circular arc}}$ of radius $r=\dfrac{mv}{qB}>h$, centered at $(r,\ 0,\ 0)$. The arc subtends an angle given by $\theta=\sin^{-1}\left(\dfrac{h}{r}\right)$. It will leave the field at the point with coordinates $\left[r(1-\cos\theta),\ h,\ 0\right]$ with velocity $\boxed{\vec{\mathbf{v}}_f=v\sin\theta\,\hat{\mathbf{i}}+v\cos\theta\,\hat{\mathbf{j}}}$.

P29.64 (a) $I=\dfrac{ev}{2\pi r}$ $\qquad$ $\mu=IA=\left(\dfrac{ev}{2\pi r}\right)\pi r^2=\boxed{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

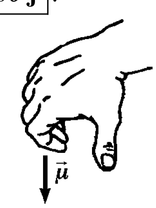


FIG. P29.64

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

***P29.65** (a) The torque on the dipole $\vec{\tau}=\vec{\mu}\times\vec{\mathbf{B}}$ has magnitude $\mu B\sin\theta\approx\mu B\theta$, proportional to the angular displacement if the angle is small. It is a restoring torque, tending to turn the dipole toward its equilibrium orientation. Then the statement that its motion is simple harmonic is true for small angular displacements.

(b) $\tau=I\alpha$ becomes $\quad -\mu B\theta=I\,d^2\theta/dt^2 \qquad d^2\theta/dt^2=-(\mu B/I)\theta=-\omega^2\theta$

where $\omega=(\mu B/I)^{1/2}$ is the angular frequency and $f=\omega/2\pi=\dfrac{1}{2\pi}\sqrt{\dfrac{\mu B}{I}}$ is the frequency in hertz.

(c) The equilibrium orientation of the needle shows the direction of the field. In a stronger field, the frequency is higher. The frequency is easy to measure precisely over a wide range of values. The equation in part (c) gives
$0.680\ \text{Hz}=(1/2\pi)(\mu/I)^{1/2}(39.2\ \mu\text{T})^{1/2}$ and $4.90\ \text{Hz}=(1/2\pi)(\mu/I)^{1/2}(B_2)^{1/2}$. We square and divide to find $B_2/39.2\ \mu\text{T}=(4.90\ \text{Hz}/0.680\ \text{Hz})^2$ so $B_2=51.9(39.2\ \mu\text{T})=\boxed{2.04\ \text{mT}}$.

ANSWERS TO EVEN PROBLEMS

P29.2 (a) west (b) no deflection (c) up (d) down

P29.4 (a) 86.7 fN (b) 51.9 $\mathrm{Tm/s^2}$

P29.6 (a) 7.90 pN (b) 0

P29.8 $B_y = -2.62$ mT. $B_z = 0$. B_x may have any value.

P29.10 (a) 6.84×10^{-16} m down (b) 7.24 mm east. The beam moves on an arc of a circle rather than on a parabola, but its northward velocity component stays constant within 0.09%, so it is a good approximation to treat it as constant.

P29.12 115 keV

P29.14 $\dfrac{m'}{m} = 8$

P29.16 (a) 17.9 ns (b) 35.1 eV

P29.18 (a) 8.28 cm (b) 8.23 cm; ratio is independent of both ΔV and B

P29.20 (a) 7.66×10^7 rad/s (b) 26.8 Mm/s (c) 3.76 MeV (d) 3.13×10^3 rev (e) 257 μs

P29.22 (b) The dashed red line should spiral around many times, with its turns relatively far apart on the inside and closer together on the outside. (c) 682 m/s (d) 55.9 μm

P29.24 (a) Yes: The constituent of the beam is present in all kinds of atoms. (b) Yes: Everything in the beam has a single charge-to-mass ratio. (c) Thomson pointed out that ionized hydrogen had the largest charge-to-mass ratio previously known, and that the particles in his beam had a charge-to-mass ratio about 2 000 times larger. The particles in his beam could not be whole atoms, but rather must be much smaller in mass. (d) No. The particles move with speed on the order of ten million meters per second, to fall by an immeasurably small amount over a distance of less than a meter.

P29.26 $\left(-2.88\hat{\mathbf{j}}\right)$ N

P29.28 840 A east

P29.30 $\left(\dfrac{4IdBL}{3m}\right)^{1/2}$

P29.32 (a) $\vec{\mathbf{F}}_{ab} = 0$; $\vec{\mathbf{F}}_{bc} = 40.0\ \mathrm{mN}\left(-\hat{\mathbf{i}}\right)$; $\vec{\mathbf{F}}_{cd} = 40.0\ \mathrm{mN}\left(-\hat{\mathbf{k}}\right)$; $\vec{\mathbf{F}}_{da} = (40.0\ \mathrm{mN})\left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right)$ (b) The forces on the four segments must add to zero, so the force on the fourth segment must be the negative of the resultant of the forces on the other three.

P29.34 (a) $5.41\ \mathrm{mA \cdot m^2}$ (b) $4.33\ \mathrm{mN \cdot m}$

P29.36 See the solution.

P29.38 (a) 118 μN · m (b) $-118\ \mu\mathrm{J} \le U \le 118\ \mu\mathrm{J}$

P29.40 (a) 640 $\mu\mathrm{N \cdot m}$ (b) 241 mW (c) 2.56 mJ (d) 154 mW

P29.42 (a) 37.7 mT (b) $4.29 \times 10^{25}/\text{m}^3$

P29.44 See the solution.

P29.46 39.2 mT

P29.48 (a) B_x is indeterminate. $B_y = 0$. $B_z = \dfrac{-F_i}{ev_i}$ (b) $-F_i\hat{\mathbf{j}}$ (c) $+F_i\hat{\mathbf{j}}$

P29.50 (a) the $+z$ direction (b) 0.696 m (c) 1.09 m (d) 54.7 ns

P29.52 $\dfrac{3R}{4}$

P29.54 $B \sim 10^{-1}$ T; $\tau \sim 10^{-1}$ N·m; $I \sim 1$ A; $A \sim 10^{-3}$ m^2; $N \sim 10^3$

P29.56 $\dfrac{\lambda g \tan\theta}{I}$

P29.58 (a) 0.104 mm; (b) 0.189 mm

P29.60 (a) 0.501 m (b) 45.0°

P29.62 (a) See the solution. Empirically, $\Delta V_{\text{H}} = (100\ \mu\text{V/T})B$ (b) 0.125 mm

P28.64 (a) 9.27×10^{-24} A · m^2 (b) down