26

Capacitance and Dielectrics

CHAPTER OUTLINE

26.1 Definition of Capacitance
26.2 Calculating Capacitance
26.3 Combinations of Capacitors
26.4 Energy Stored in a Charged Capacitor
26.5 Capacitors with Dielectrics
26.6 Electric Dipole in an Electric Field
26.7 An Atomic Description of Dielectrics

ANSWERS TO QUESTIONS

***Q26.1** (a) False. (b) True. In $Q = C\Delta V$, the capacitance is the proportionality constant relating the variables Q and ΔV.

Q26.2 Seventeen combinations:

Individual $C_1,\ C_2,\ C_3$

Parallel $C_1+C_2+C_3,\ C_1+C_2,\ C_1+C_3,\ C_2+C_3$

Series-Parallel $\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}+C_3,\ \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}+C_2,\ \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}+C_1$

$\left(\frac{1}{C_1+C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1+C_3}+\frac{1}{C_2}\right)^{-1},\ \left(\frac{1}{C_2+C_3}+\frac{1}{C_1}\right)^{-1}$

Series $\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1},\ \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}$

***Q26.3** Volume is proportional to radius cubed. Increasing the radius by a factor of $3^{1/3}$ will triple the volume. Capacitance is proportional to radius, so it increases by a factor of $3^{1/3}$. Answer (d).

***Q26.4** Let $C_2 = NC_1$ be the capacitance of the large capacitor and C_1 that of the small one. The equivalent capacitance is

$$C_{eq} = C_{eq} = \frac{1}{1/C_1 + 1/NC_1} = \frac{1}{(N+1)/NC_1} = \frac{N}{N+1}C_1$$

This is slightly less than C_1, answer (d).

***Q26.5** We find the capacitance, voltage, charge, and energy for each capacitor.

(a) $C = 20\ \mu\text{F}$	$\Delta V = 4\ \text{V}$	$Q = C\Delta V = 80\ \mu\text{C}$	$U = (1/2)Q\Delta V = 160\ \mu\text{J}$
(b) $C = 30\ \mu\text{F}$	$\Delta V = Q/C = 3\ \text{V}$	$Q = 90\ \mu\text{C}$	$U = 135\ \mu\text{J}$
(c) $C = Q/\Delta V = 40\ \mu\text{F}$	$\Delta V = 2\ \text{V}$	$Q = 80\ \mu\text{C}$	$U = 80\ \mu\text{J}$
(d) $C = 10\ \mu\text{F}$	$\Delta V = (2U/C)^{1/2} = 5\ \text{V}$	$Q = 50\ \mu\text{C}$	$U = 125\ \mu\text{J}$
(e) $C = 2U/\Delta V^2 = 5\ \mu\text{F}$	$\Delta V = 10\ \text{V}$	$Q = 50\ \mu\text{C}$	$U = 250\ \mu\text{J}$
(f) $C = Q^2/2U = 20\ \mu\text{F}$	$\Delta V = 6\ \text{V}$	$Q = 120\ \mu\text{C}$	$U = 360\ \mu\text{J}$

Then (i) the ranking by capacitance is c > b > a = f > d > e.
(ii) The ranking by voltage ΔV is e > f > d > a > b > c.
(iii) The ranking by charge Q is f > b > a = c > d = e.
(iv) The ranking by energy U is f > e > a > b > d > c.

Q26.6 A capacitor stores energy in the electric field between the plates. This is most easily seen when using a "dissectible" capacitor. If the capacitor is charged, carefully pull it apart into its component pieces. One will find that very little residual charge remains on each plate. When reassembled, the capacitor is suddenly "recharged"—by induction—due to the electric field set up and "stored" in the dielectric. This proves to be an instructive classroom demonstration, especially when you ask a student to reconstruct the capacitor without supplying him/her with any rubber gloves or other insulating material. (Of course, this is *after* they sign a liability waiver.)

***Q26.7** (i) According to $Q = C\Delta V$, the answer is (b).
(ii) From $U = (1/2)C\Delta V^2$, the answer is (a).

***Q26.8** The charge stays constant as C is cut in half, so $U = Q^2/2C$ doubles: answer (b).

***Q26.9** (i) Answer (b). (ii) Answer (c). (iii) Answer (c). (iv) Answer (a). (v) Answer (a).

***Q26.10** (i) Answer (b). (ii) Answer (b). (iii) Answer (b). (iv) Answer (c). (v) Answer (b).

Q26.11 The work you do to pull the plates apart becomes additional electric potential energy stored in the capacitor. The charge is constant and the capacitance decreases but the potential difference increases to drive up the potential energy $\frac{1}{2}Q\Delta V$. The electric field between the plates is constant in strength but fills more volume as you pull the plates apart.

Q26.12 The work done, $W = Q\Delta V$, is the work done by an external agent, like a battery, to move a charge through a potential difference, ΔV. To determine the energy in a charged capacitor, we must add the work done to move bits of charge from one plate to the other. Initially, there is no potential difference between the plates of an uncharged capacitor. As more charge is transferred from one plate to the other, the potential difference increases as shown in the textbook graph of ΔV versus Q, meaning that more work is needed to transfer each additional bit of charge. The total work is the area under the curve on this graph, and thus $W = \frac{1}{2}Q\Delta V$.

***Q26.13** Let C = the capacitance of an individual capacitor, and C_s represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\Delta V_{\text{charge}} = \left(5.00\times10^{-4}\ \text{F}\right)(800\ \text{V}) = 0.400\ \text{C}$$

While being discharged in series, $$\Delta V_{\text{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\ \text{C}}{5.00\times10^{-5}\ \text{F}} = 8.00\ \text{kV}$$

(or 10 times the original voltage). Answer (b).

Q26.14 The potential difference must decrease. Since there is no external power supply, the charge on the capacitor, Q, will remain constant—that is, assuming that the resistance of the meter is sufficiently large. Adding a dielectric *increases* the capacitance, which must therefore *decrease* the potential difference between the plates.

***Q26.15** (i) Answer (a). (ii) Because ΔV is constant, $Q = C\Delta V$ increases, answer (a).
(iii) Answer (c). (iv) Answer (c). (v) $U = (1/2)C\Delta V^2$ increases, answer (a).

Q26.16 Put a material with higher dielectric strength between the plates, or evacuate the space between the plates. At very high voltages, you may want to cool off the plates or choose to make them of a different chemically stable material, because atoms in the plates themselves can ionize, showing *thermionic emission* under high electric fields.

Q26.17 The primary choice would be the dielectric. You would want to choose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 26.1). A convenient choice could be thick plastic or Mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between your sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their next neighbors, connect every other plate together. Figure Q26.17 illustrates this idea.

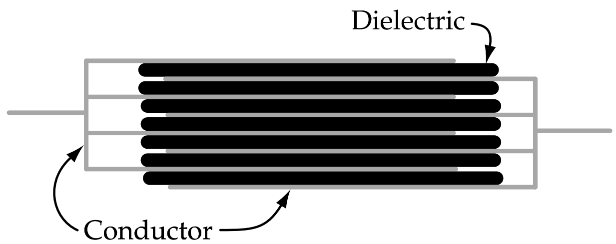


FIG. Q26.17

This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).

SOLUTIONS TO PROBLEMS

Section 26.1 **Definition of Capacitance**

P26.1 (a) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)\left(12.0\text{ V}\right) = 4.80\times10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)\left(1.50\text{ V}\right) = 6.00\times10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

P26.2 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{10.0\times10^{-6}\text{ C}}{10.0\text{ V}} = 1.00\times10^{-6}\text{ F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{100\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ F}} = \boxed{100\text{ V}}$

Section 26.2 **Calculating Capacitance**

P26.3 $E = \dfrac{k_e q}{r^2}$: $\quad q = \dfrac{\left(4.90\times10^{4}\text{ N/C}\right)\left(0.210\text{ m}\right)^2}{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)} = 0.240\ \mu\text{C}$

(a) $\sigma = \dfrac{q}{A} = \dfrac{0.240\times10^{-6}}{4\pi\left(0.120\right)^2} = \boxed{1.33\ \mu\text{C/m}^2}$

(b) $C = 4\pi\in_0 r = 4\pi\left(8.85\times10^{-12}\right)\left(0.120\right) = \boxed{13.3\text{ pF}}$

P26.4 $C=\dfrac{\kappa\in_0 A}{d}=\dfrac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2\right)\left(1.00\times10^3\ \text{m}\right)^2}{\text{N}\cdot\text{m}^2\,(800\ \text{m})}=\boxed{11.1\ \text{nF}}$

The potential between ground and cloud is

$$\Delta V=Ed=\left(3.00\times10^6\ \text{N/C}\right)(800\ \text{m})=2.40\times10^9\ \text{V}$$
$$Q=C(\Delta V)=\left(11.1\times10^{-9}\ \text{C/V}\right)\left(2.40\times10^9\ \text{V}\right)=\boxed{26.6\ \text{C}}$$

P26.5 (a) $\Delta V=Ed$ so $E=\dfrac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}}=\boxed{11.1\ \text{kV/m}}$ toward the negative plate

(b) $E=\dfrac{\sigma}{\in_0}$ so $\sigma=\left(1.11\times10^4\ \text{N/C}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)=\boxed{98.3\ \text{nC/m}^2}$

(c) $C=\dfrac{\in_0 A}{d}=\dfrac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\ \text{cm}^2\right)(1.00\ \text{m}/100\ \text{cm})^2}{1.80\times10^{-3}\ \text{m}}=\boxed{3.74\ \text{pF}}$

(d) $\Delta V=\dfrac{Q}{C}$ so $Q=(20.0\ \text{V})\left(3.74\times10^{-12}\ \text{F}\right)=\boxed{74.7\ \text{pC}}$

P26.6 With $\theta=\pi$, the plates are out of mesh and the overlap area is zero.

With $\theta=0$, the overlap area is that of a semi-circle, $\dfrac{\pi R^2}{2}$. By proportion, the effective area of a single sheet of charge is $\dfrac{(\pi-\theta)R^2}{2}$.

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N-1$ and the total capacitance is

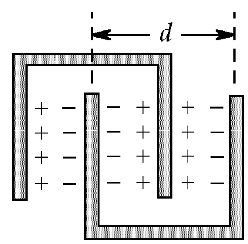


FIG. P26.6

$$C=(2N-1)\frac{\in_0 A_{\text{effective}}}{\text{distance}}=\frac{(2N-1)\in_0(\pi-\theta)R^2/2}{d/2}=\boxed{\frac{(2N-1)\in_0(\pi-\theta)R^2}{d}}.$$

P26.7 $Q=\dfrac{\in_0 A}{d}(\Delta V)$ $\qquad \dfrac{Q}{A}=\sigma=\dfrac{\in_0(\Delta V)}{d}$

$$d=\frac{\in_0(\Delta V)}{\sigma}=\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(30.0\times10^{-9}\ \text{C/cm}^2\right)\left(1.00\times10^4\ \text{cm}^2/\text{m}^2\right)}=\boxed{4.42\ \mu\text{m}}$$

P26.8 $\sum F_y=0$: $\quad T\cos\theta-mg=0$

$\sum F_x=0$: $\quad T\sin\theta-Eq=0$

Dividing, $\quad \tan\theta=\dfrac{Eq}{mg}$

so $\quad E=\dfrac{mg}{q}\tan\theta$

and $\quad \Delta V=Ed=\boxed{\dfrac{mgd\tan\theta}{q}}$

P26.9 (a) $C = \dfrac{\ell}{2k_e \ln(b/a)} = \dfrac{50.0}{2(8.99\times10^9)\ln(7.27/2.58)} = \boxed{2.68\text{ nF}}$

(b) Method 1: $\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$

$$\lambda = \frac{q}{\ell} = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2(8.99\times10^9)(1.62\times10^{-7})\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02\text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10\times10^{-6}}{2.68\times10^{-9}} = \boxed{3.02\text{ kV}}$

***P26.10** The original kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2\times10^{-16}\text{ kg})(2\times10^6\text{ m/s})^2 = 4.00\times10^{-4}\text{ J}$$

The potential difference across the capacitor is $\Delta V = \dfrac{Q}{C} = \dfrac{1\,000\ \mu\text{C}}{10\ \mu\text{F}} = 100\text{ V}$.

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$U = q\Delta V = (-3\times10^{-6}\text{ C})(-100\text{ V}) = 3.00\times10^{-4}\text{ J}$$

Since its original kinetic energy is greater than this, $\boxed{\text{the particle will reach the negative plate}}$.

As the particle moves, the system keeps constant total energy

$$(K+U)_{\text{at +plate}} = (K+U)_{\text{at −plate}}: \qquad 4.00\times10^{-4}\text{ J} + (-3\times10^{-6}\text{ C})(+100\text{ V}) = \frac{1}{2}(2\times10^{-16})v_f^2 + 0$$

$$v_f = \sqrt{\frac{2(1.00\times10^{-4}\text{ J})}{2\times10^{-16}\text{ kg}}} = \boxed{1.00\times10^6\text{ m/s}}$$

P26.11 (a) $C = \dfrac{ab}{k_e(b-a)} = \dfrac{(0.070\,0)(0.140)}{(8.99\times10^9)(0.140-0.070\,0)} = \boxed{15.6\text{ pF}}$

(b) $C = \dfrac{Q}{\Delta V}$ $\qquad \Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{15.6\times10^{-12}\text{ F}} = \boxed{256\text{ kV}}$

Section 26.3 **Combinations of Capacitors**

P26.12 (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00\text{ V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\text{ V}) = \boxed{45.0\ \mu\text{C}}$

and $Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\text{ V}) = \boxed{108\ \mu\text{C}}$

P26.13 (a) In series capacitors add as $$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{5.00\ \mu\text{F}}+\frac{1}{12.0\ \mu\text{F}}$$

and $$C_{eq}=\boxed{3.53\ \mu\text{F}}$$

(c) The charge on the equivalent capacitor is $Q_{eq}=C_{eq}\Delta V=(3.53\ \mu\text{F})(9.00\ \text{V})=31.8\ \mu\text{C}$

Each of the series capacitors has this same charge on it.

So $$Q_1=Q_2=\boxed{31.8\ \mu\text{C}}$$

(b) The potential difference across each is $$\Delta V_1=\frac{Q_1}{C_1}=\frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}}=\boxed{6.35\ \text{V}}$$

and $$\Delta V_2=\frac{Q_2}{C_2}=\frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}}=\boxed{2.65\ \text{V}}$$

P26.14 (a) Capacitors 2 and 3 are in parallel and present equivalent capacitance $6C$. This is in series with capacitor 1, so the battery sees capacitance $\left[\frac{1}{3C}+\frac{1}{6C}\right]^{-1}=\boxed{2C}$.

(b) If they were initially uncharged, C_1 stores the same charge as C_2 and C_3 together. With greater capacitance, C_3 stores more charge than C_2. Then $\boxed{Q_1>Q_3>Q_2}$.

(c) The $(C_2 \parallel C_3)$ equivalent capacitor stores the same charge as C_1. Since it has greater capacitance, $\Delta V=\frac{Q}{C}$ implies that it has smaller potential difference across it than C_1. In parallel with each other, C_2 and C_3 have equal voltages: $\boxed{\Delta V_1>\Delta V_2=\Delta V_3}$.

(d) If C_3 is increased, the overall equivalent capacitance increases. More charge moves through the battery and Q increases. As ΔV_1 increases, ΔV_2 must decrease so Q_2 decreases. Then Q_3 must increase even more: $\boxed{Q_3 \text{ and } Q_1 \text{ increase; } Q_2 \text{ decreases}}$.

P26.15 $C_p=C_1+C_2$ $$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}$$

Substitute $C_2=C_p-C_1$ $$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_p-C_1}=\frac{C_p-C_1+C_1}{C_1(C_p-C_1)}$$

Simplifying, $$C_1^2-C_1C_p+C_pC_s=0$$

$$C_1=\frac{C_p\pm\sqrt{C_p^2-4C_pC_s}}{2}=\frac{1}{2}C_p\pm\sqrt{\frac{1}{4}C_p^2-C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1=\frac{1}{2}C_p+\sqrt{\frac{1}{4}C_p^2-C_pC_s}=\frac{1}{2}(9.00\ \text{pF})+\sqrt{\frac{1}{4}(9.00\ \text{pF})^2-(9.00\ \text{pF})(2.00\ \text{pF})}=\boxed{6.00\ \text{pF}}$$

$$C_2=C_p-C_1=\frac{1}{2}C_p-\sqrt{\frac{1}{4}C_p^2-C_pC_s}=\frac{1}{2}(9.00\ \text{pF})-1.50\ \text{pF}=\boxed{3.00\ \text{pF}}$$

P26.16 $C_p = C_1 + C_2$

and $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Substitute $C_2 = C_p - C_1$: $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1\left(C_p - C_1\right)}$$

Simplifying, $$C_1^2 - C_1C_p + C_pC_s = 0$$

and $$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \boxed{\frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed).

Then, from $$C_2 = C_p - C_1$$

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

P26.17 (a) $$\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}$$

$$C_s = 2.50\ \mu\text{F}$$
$$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$$
$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $Q = C\Delta V = \left(5.96\ \mu\text{F}\right)\left(15.0\ \text{V}\right) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

$$\Delta V = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$$
$$15.0 - 4.47 = 10.53\ \text{V}$$
$Q = C\Delta V = \left(6.00\ \mu\text{F}\right)\left(10.53\ \text{V}\right) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$

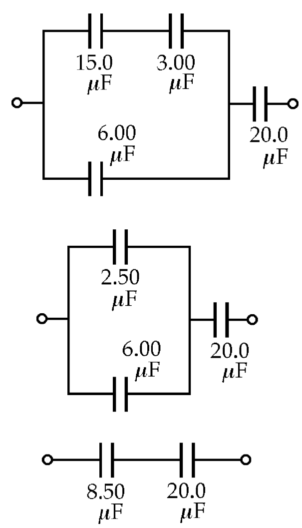


FIG. P26.17

P26.18 (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s}$$
$$C_s = \frac{1}{2.51\times10^{-3}/\mu\text{F}} = \boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$$
$$C_p = \boxed{2.20\ \mu\text{F}}$$

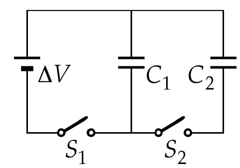


FIG. P26.19

P26.19 $C = \dfrac{Q}{\Delta V}$ so $\quad 6.00 \times 10^{-6} = \dfrac{Q}{20.0}$

and $\quad Q = \boxed{120\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - Q_2$

and $\quad \Delta V = \dfrac{Q}{C}$: $\quad \dfrac{120 - Q_2}{C_1} = \dfrac{Q_2}{C_2}$

or $\quad \dfrac{120 - Q_2}{6.00} = \dfrac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \dfrac{360}{9.00} = \boxed{40.0\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$

P26.20 For C_1 connected by itself, $C_1\Delta V = 30.8\ \mu\text{C}$ where ΔV is the battery voltage: $\Delta V = \dfrac{30.8\ \mu\text{C}}{C_1}$.

For C_1 and C_2 in series:

$$\left(\frac{1}{1/C_1 + 1/C_2}\right)\Delta V = 23.1\ \mu\text{C}$$

substituting, $\dfrac{30.8\ \mu\text{C}}{C_1} = \dfrac{23.1\ \mu\text{C}}{C_1} + \dfrac{23.1\ \mu\text{C}}{C_2} \qquad C_1 = 0.333C_2$

For C_1 and C_3 in series:

$$\left(\frac{1}{1/C_1 + 1/C_3}\right)\Delta V = 25.2\ \mu\text{C}$$

$$\frac{30.8\ \mu\text{C}}{C_1} = \frac{25.2\ \mu\text{C}}{C_1} + \frac{25.2\ \mu\text{C}}{C_3} \qquad C_1 = 0.222C_3$$

For all three:

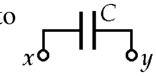
$$Q = \left(\frac{1}{1/C_1 + 1/C_2 + 1/C_3}\right)\Delta V = \frac{C_1\Delta V}{1 + C_1/C_2 + C_1/C_3} = \frac{30.8\ \mu\text{C}}{1 + 0.333 + 0.222} = \boxed{19.8\ \mu\text{C}}$$

This is the charge on each one of the three.

P26.21 $nC = \dfrac{100}{\underbrace{\frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \cdots}_{n \text{ capacitors}}} = \dfrac{100}{n/C}$

$nC = \dfrac{100C}{n}$ so $\quad n^2 = 100 \quad$ and $\quad n = \boxed{10}$

P26.22 According to the suggestion, the combination of capacitors shown is equivalent to



Then
$$\frac{1}{C}=\frac{1}{C_0}+\frac{1}{C+C_0}+\frac{1}{C_0}$$
$$=\frac{C+C_0+C_0+C+C_0}{C_0\left(C+C_0\right)}$$
$$C_0C+C_0^2=2C^2+3C_0C$$
$$2C^2+2C_0C-C_0^2=0$$
$$C=\frac{-2C_0\pm\sqrt{4C_0^2+4\left(2C_0^2\right)}}{4}$$

Only the positive root is physical.

$$\boxed{C=\frac{C_0}{2}\left(\sqrt{3}-1\right)}$$

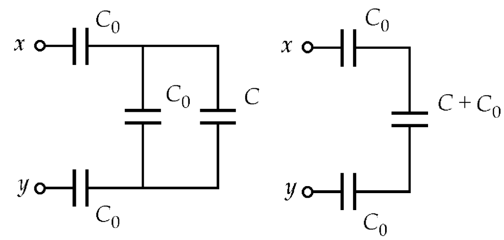


FIG. P26.22

P26.23 $C_s=\left(\frac{1}{5.00}+\frac{1}{10.0}\right)^{-1}=3.33\ \mu\text{F}$

$C_{p1}=2\left(3.33\right)+2.00=8.66\ \mu\text{F}$

$C_{p2}=2\left(10.0\right)=20.0\ \mu\text{F}$

$C_{eq}=\left(\frac{1}{8.66}+\frac{1}{20.0}\right)^{-1}=\boxed{6.04\ \mu\text{F}}$

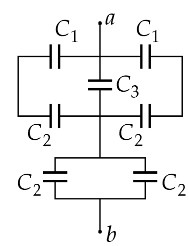


FIG. P26.23

P26.24 $Q_{eq}=C_{eq}\left(\Delta V\right)=\left(6.04\times10^{-6}\text{ F}\right)\left(60.0\text{ V}\right)=3.62\times10^{-4}\text{ C}$

$Q_{p1}=Q_{eq}$, so $\Delta V_{p1}=\frac{Q_{eq}}{C_{p1}}=\frac{3.62\times10^{-4}\text{ C}}{8.66\times10^{-6}\text{ F}}=41.8\text{ V}$

$Q_3=C_3\left(\Delta V_{p1}\right)=\left(2.00\times10^{-6}\text{ F}\right)\left(41.8\text{ V}\right)=\boxed{83.6\ \mu\text{C}}$

P26.25 $C_s=\left(\frac{1}{5.00}+\frac{1}{7.00}\right)^{-1}=2.92\ \mu\text{F}$

$C_p=2.92+4.00+6.00=\boxed{12.9\ \mu\text{F}}$

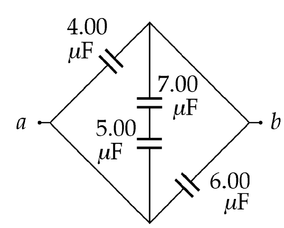


FIG. P26.25

Section 26.4 **Energy Stored in a Charged Capacitor**

P26.26 $U = \frac{1}{2}C\Delta V^2$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(300\text{ J})}{30 \times 10^{-6}\text{ C/V}}} = \boxed{4.47 \times 10^3\text{ V}}$$

P26.27 (a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\text{ V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\text{ V})^2 = \boxed{54.0\ \mu\text{J}}$

P26.28 $U = \frac{1}{2}C(\Delta V)^2$

The circuit diagram is shown at the right.

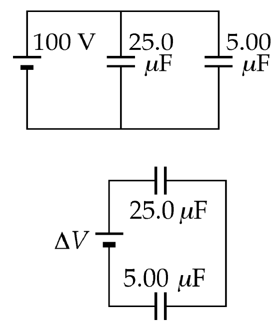


FIG. P26.28

(a) $C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$

$$U = \frac{1}{2}\left(30.0 \times 10^{-6}\right)(100)^2 = \boxed{0.150\text{ J}}$$

(b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$$U = \frac{1}{2}C(\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17 \times 10^{-6}}} = \boxed{268\text{ V}}$$

P26.29 $W = U = \int F dx$

so $F = \frac{dU}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2C}\right) = \frac{d}{dx}\left(\frac{Q^2 x}{2\in_0 A}\right) = \boxed{\frac{Q^2}{2\in_0 A}}$

P26.30 With switch closed, distance $d' = 0.500d$ and capacitance $C' = \frac{\in_0 A}{d'} = \frac{2\in_0 A}{d} = 2C$.

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2\left(2.00 \times 10^{-6}\text{ F}\right)(100\text{ V}) = \boxed{400\ \mu\text{C}}$

(b) The force stretching out one spring is

$$F = \frac{Q^2}{2\in_0 A} = \frac{4C^2(\Delta V)^2}{2\in_0 A} = \frac{2C^2(\Delta V)^2}{(\in_0 A/d)d} = \frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x = \frac{d}{4}$, so

$$k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2} = \frac{8\left(2.00 \times 10^{-6}\text{ F}\right)(100\text{ V})^2}{\left(8.00 \times 10^{-3}\text{ m}\right)^2} = \boxed{2.50\text{ kN/m}}$$

P26.31 (a) $Q = C\Delta V = \left(150\times10^{-12}\text{ F}\right)\left(10\times10^{3}\text{ V}\right) = \boxed{1.50\times10^{-6}\text{ C}}$

(b) $U = \dfrac{1}{2}C\left(\Delta V\right)^2$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2\left(250\times10^{-6}\text{ J}\right)}{150\times10^{-12}\text{ F}}} = \boxed{1.83\times10^{3}\text{ V}}$$

P26.32 (a) $U = \dfrac{1}{2}C\left(\Delta V\right)^2 + \dfrac{1}{2}C\left(\Delta V\right)^2 = \boxed{C\left(\Delta V\right)^2}$

(b) The altered capacitor has capacitance $C' = \dfrac{C}{2}$. The total charge is the same as before:

$$C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V') \qquad \boxed{\Delta V' = \frac{4\Delta V}{3}}$$

(c) $U' = \dfrac{1}{2}C\left(\dfrac{4\Delta V}{3}\right)^2 + \dfrac{1}{2}\dfrac{1}{2}C\left(\dfrac{4\Delta V}{3}\right)^2 = \boxed{4C\dfrac{\left(\Delta V\right)^2}{3}}$

(d) The extra energy comes from work put into the system by the agent pulling the capacitor plates apart.

P26.33 $U = \dfrac{1}{2}C\left(\Delta V\right)^2$ where $C = 4\pi\in_0 R = \dfrac{R}{k_e}$ and $\Delta V = \dfrac{k_eQ}{R} - 0 = \dfrac{k_eQ}{R}$

$$U = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_eQ}{R}\right)^2 = \boxed{\frac{k_eQ^2}{2R}}$$

P26.34 (a) The total energy is $U = U_1 + U_2 = \dfrac{1}{2}\dfrac{q_1^2}{C_1} + \dfrac{1}{2}\dfrac{q_2^2}{C_2} = \dfrac{1}{2}\dfrac{q_1^2}{4\pi\in_0 R_1} + \dfrac{1}{2}\dfrac{\left(Q-q_1\right)^2}{4\pi\in_0 R^2}$.

For a minimum we set $\dfrac{dU}{dq_1} = 0$:

$$\frac{1}{2}\frac{2q_1}{4\pi\in_0 R_1} + \frac{1}{2}\frac{2(Q-q_1)}{4\pi\in_0 R_2}\left(-1\right) = 0$$

$$R_2q_1 = R_1Q - R_1q_1 \qquad \boxed{q_1 = \frac{R_1Q}{R_1+R_2}}$$

4 Then $q_2 = Q - q_1 = \boxed{\dfrac{R_2Q}{R_1+R_2} = q_2}$.

(b) $V_1 = \dfrac{k_eq_1}{R_1} = \dfrac{k_eR_1Q}{R_1\left(R_1+R_2\right)} = \dfrac{k_eQ}{R_1+R_2}$

$$V_2 = \frac{k_eq_2}{R_2} = \frac{k_eR_2Q}{R_2\left(R_1+R_2\right)} = \frac{k_eQ}{R_1+R_2}$$

and $V_1 - V_2 = 0$

P26.35 The energy transferred is $T_{ET} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50.0\text{ C})(1.00\times10^{8}\text{ V}) = 2.50\times10^{9}\text{ J}$

and 1% of this (or $\Delta E_{int} = 2.50\times10^{7}$ J) is absorbed by the tree. If m is the amount of water boiled away,

then $\Delta E_{int} = m(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 30.0^\circ\text{C}) + m(2.26\times10^{6}\text{ J/kg}) = 2.50\times10^{7}\text{ J}$

giving $m = \boxed{9.79\text{ kg}}$

Section 26.5 **Capacitors with Dielectrics**

P26.36 $Q_{max} = C\Delta V_{max}$,

but $\Delta V_{max} = E_{max}d$

Also, $C = \frac{\kappa \in_0 A}{d}$

Thus, $Q_{max} = \frac{\kappa \in_0 A}{d}(E_{max}d) = \kappa \in_0 AE_{max}$

(a) With air between the plates, $\kappa = 1.00$

and $E_{max} = 3.00\times10^{6}\text{ V/m}$

Therefore,

$$Q_{max} = \kappa \in_0 AE_{max} = (8.85\times10^{-12}\text{ F/m})(5.00\times10^{-4}\text{ m}^2)(3.00\times10^{6}\text{ V/m}) = \boxed{13.3\text{ nC}}$$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{max} = 24.0\times10^{6}\text{ V/m}$.

$$Q_{max} = \kappa \in_0 AE_{max} = 2.56(8.85\times10^{-12}\text{ F/m})(5.00\times10^{-4}\text{ m}^2)(24.0\times10^{6}\text{ V/m}) = \boxed{272\text{ nC}}$$

P26.37 (a) $C = \frac{\kappa \in_0 A}{d} = \frac{2.10(8.85\times10^{-12}\text{ F/m})(1.75\times10^{-4}\text{ m}^2)}{4.00\times10^{-5}\text{ m}} = 8.13\times10^{-11}\text{ F} = \boxed{81.3\text{ pF}}$

(b) $\Delta V_{max} = E_{max}d = (60.0\times10^{6}\text{ V/m})(4.00\times10^{-5}\text{ m}) = \boxed{2.40\text{ kV}}$

P26.38 Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them. Suppose the plastic has $\kappa \approx 3$, $E_{max} \sim 10^{7}\text{ V/m}$, and thickness 1 mil $= \frac{2.54\text{ cm}}{1\,000}$. Then,

$$C = \frac{\kappa \in_0 A}{d} \sim \frac{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.4\text{ m}^2)}{2.54\times10^{-5}\text{ m}} \sim \boxed{10^{-6}\text{ F}}$$

$$\Delta V_{max} = E_{max}d \sim (10^{7}\text{ V/m})(2.54\times10^{-5}\text{ m}) \sim \boxed{10^{2}\text{ V}}$$

P26.39 $C = \frac{\kappa \in_0 A}{d}$

or $95.0\times10^{-9} = \frac{3.70(8.85\times10^{-12})(0.070\,0)\ell}{0.025\,0\times10^{-3}}$

$\ell = \boxed{1.04\text{ m}}$

P26.40 Originally, $C = \dfrac{\epsilon_0 A}{d} = \dfrac{Q}{(\Delta V)_i}$

(a) The charge is the same before and after immersion, with value $Q = \dfrac{\epsilon_0 A(\Delta V)_i}{d}$.

$$Q = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0\times10^{-4}\ \text{m}^2\right)(250\ \text{V})}{\left(1.50\times10^{-2}\ \text{m}\right)} = \boxed{369\ \text{pC}}$$

(b) Finally,

$$C_f = \frac{\kappa \epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f} \qquad C_f = \frac{80.0\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0\times10^{-4}\ \text{m}^2\right)}{\left(1.50\times10^{-2}\ \text{m}\right)} = \boxed{118\ \text{pF}}$$

$$(\Delta V)_f = \frac{Qd}{\kappa \epsilon_0 A} = \frac{\epsilon_0 A(\Delta V)_i d}{\kappa \epsilon_0 Ad} = \frac{(\Delta V)_i}{\kappa} = \frac{250\ \text{V}}{80.0} = \boxed{3.12\ \text{V}}$$

(c) Originally, $U_i = \dfrac{1}{2}C(\Delta V)_i^2 = \dfrac{\epsilon_0 A(\Delta V)_i^2}{2d}$

Finally, $U_f = \dfrac{1}{2}C_f(\Delta V)_f^2 = \dfrac{\kappa \epsilon_0 A(\Delta V)_i^2}{2d\kappa^2} = \dfrac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa}$

So, $\Delta U = U_f - U_i = \dfrac{-\epsilon_0 A(\Delta V)_i^2(\kappa-1)}{2d\kappa}$

$$\Delta U = -\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0\times10^{-4}\ \text{m}^2\right)(250\ \text{V})^2(79.0)}{2\left(1.50\times10^{-2}\ \text{m}\right)(80.0)} = \boxed{-45.5\ \text{nJ}}$$

P26.41 The given combination of capacitors is equivalent to the circuit diagram shown to the right.

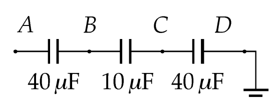


FIG. P26.41

Put charge Q on point A. Then,

$$Q = (40.0\ \mu\text{F})\Delta V_{AB} = (10.0\ \mu\text{F})\Delta V_{BC} = (40.0\ \mu\text{F})\Delta V_{CD}$$

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0$ V. When this occurs,

$$\Delta V_{AB} = \Delta V_{CD} = \frac{1}{4}(\Delta V_{BC}) = 3.75\ \text{V}$$

and $V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75\ \text{V} + 15.0\ \text{V} + 3.75\ \text{V} = \boxed{22.5\ \text{V}}$.

Section 26.6 **Electric Dipole in an Electric Field**

P26.42 (a) The displacement from negative to positive charge is

$$2a = \left(-1.20\hat{\mathbf{i}} + 1.10\hat{\mathbf{j}}\right)\text{ mm} - \left(1.40\hat{\mathbf{i}} - 1.30\hat{\mathbf{j}}\right)\text{ mm} = \left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\times 10^{-3}\text{ m}$$

The electric dipole moment is

$$\vec{\mathbf{p}} = 2\vec{\mathbf{a}}q = \left(3.50\times 10^{-9}\text{ C}\right)\left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\times 10^{-3}\text{ m} = \boxed{\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}}$$

(b) $$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}}\times\vec{\mathbf{E}} = \left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}\right]\times\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times 10^{3}\text{ N/C}\right]$$

$$\vec{\boldsymbol{\tau}} = \left(+44.6\hat{\mathbf{k}} - 65.5\hat{\mathbf{k}}\right)\times 10^{-9}\text{ N}\cdot\text{m} = \boxed{-2.09\times 10^{-8}\text{ N}\cdot\text{m}\hat{\mathbf{k}}}$$

(c) $$U = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}} = -\left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}\right]\cdot\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times 10^{3}\text{ N/C}\right]$$

$$U = \left(71.0 + 41.2\right)\times 10^{-9}\text{ J} = \boxed{112\text{ nJ}}$$

(d) $$\left|\vec{\mathbf{p}}\right| = \sqrt{(9.10)^2 + (8.40)^2}\times 10^{-12}\text{ C}\cdot\text{m} = 12.4\times 10^{-12}\text{ C}\cdot\text{m}$$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{(7.80)^2 + (4.90)^2}\times 10^{3}\text{ N/C} = 9.21\times 10^{3}\text{ N/C}$$

$$U_{\text{max}} = \left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right| = 114\text{ nJ},\qquad U_{\text{min}} = -114\text{ nJ}$$

$$U_{\text{max}} - U_{\text{min}} = \boxed{228\text{ nJ}}$$

P26.43 (a) Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\vec{\mathbf{F}}_{-} = -qE(x)\hat{\mathbf{i}}$. The force on the positive charge is

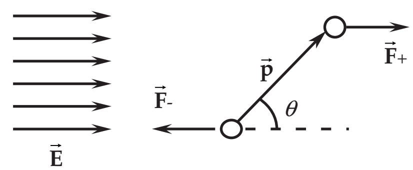


FIG. P26.43(a)

$$\vec{\mathbf{F}}_{+} = +qE(x + 2a\cos\theta)\hat{\mathbf{i}} \approx qE(x)\hat{\mathbf{i}} + q\frac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}}.$$

The force on the dipole is altogether $\vec{\mathbf{F}} = \vec{\mathbf{F}}_{-} + \vec{\mathbf{F}}_{+} = q\dfrac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}} = \boxed{p\dfrac{dE}{dx}\cos\theta\,\hat{\mathbf{i}}}$.

(b) The balloon creates field along the x-axis of $\dfrac{k_e q}{x^2}\hat{\mathbf{i}}$.

Thus, $\dfrac{dE}{dx} = \dfrac{(-2)k_e q}{x^3}$.

At $x = 16.0$ cm, $\dfrac{dE}{dx} = \dfrac{(-2)\left(8.99\times 10^{9}\right)\left(2.00\times 10^{-6}\right)}{(0.160)^3} = \boxed{-8.78\text{ MN/C}\cdot\text{m}}$

$$\vec{\mathbf{F}} = \left(6.30\times 10^{-9}\text{ C}\cdot\text{m}\right)\left(-8.78\times 10^{6}\text{ N/C}\cdot\text{m}\right)\cos 0°\hat{\mathbf{i}} = \boxed{-55.3\hat{\mathbf{i}}\text{ mN}}$$

Section 26.7 An Atomic Description of Dielectrics

P26.44 (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$$EA' + EA' = \frac{Q}{\in A}A', \text{ so } \boxed{E = \frac{Q}{2\in A}} \text{ directed away from the positive sheet.}$$

(b) In the space between the sheets, each creates field $\frac{Q}{2\in A}$ away from the positive and toward the negative sheet. Together, they create a field of

$$\boxed{E = \frac{Q}{\in A}}$$

(c) Assume that the field is in the positive x-direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A}\hat{\mathbf{i}} \cdot \left(-\hat{\mathbf{i}}dx\right) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\in A} = \boxed{\frac{\in A}{d} = \frac{\kappa \in_0 A}{d}}$.

P26.45 $E_{\max}$ occurs at the inner conductor's surface.

$E_{\max} = \frac{2k_e\lambda}{a}$ from an equation derived about this situation in Chapter 24.

$\Delta V = 2k_e\lambda \ln\left(\frac{b}{a}\right)$ from Example 26.1.

$$E_{\max} = \frac{\Delta V}{a\ln(b/a)}$$

$$\Delta V_{\max} = E_{\max} a \ln\left(\frac{b}{a}\right) = \left(18.0\times10^6 \text{ V/m}\right)\left(0.800\times10^{-3}\text{ m}\right)\ln\left(\frac{3.00}{0.800}\right) = \boxed{19.0 \text{ kV}}$$

Additional Problems

P26.46 Imagine the center plate is split along its midplane and pulled apart. We have two capacitors in parallel, supporting the same ΔV and carrying total charge Q. The upper has capacitance $C_1 = \frac{\in_0 A}{d}$ and the lower $C_2 = \frac{\in_0 A}{2d}$. Charge flows from ground onto each of the outside plates so that $Q_1 + Q_2 = Q$ $\qquad \Delta V_1 = \Delta V_2 = \Delta V$

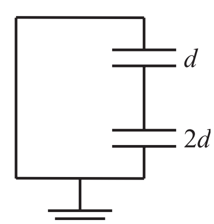


FIG. P26.46

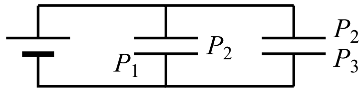
Then $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_1 d}{\in_0 A} = \frac{Q_2 2d}{\in_0 A}$ $\qquad Q_1 = 2Q_2$ $\qquad 2Q_2 + Q_2 = Q$

(a) $Q_2 = \frac{Q}{3}$. $\boxed{\text{On the lower plate the charge is } -\frac{Q}{3}.}$

$Q_1 = \frac{2Q}{3}$. $\boxed{\text{On the upper plate the charge is } -\frac{2Q}{3}.}$

(b) $\Delta V = \frac{Q_1}{C_1} = \boxed{\frac{2Qd}{3\in_0 A}}$

P26.47 (a) Each face of P_2 carries charge, so the three-plate system is equivalent to



Each capacitor by itself has capacitance

$$C = \frac{\kappa \in_0 A}{d} = \frac{1\left(8.85\times10^{-12}\ \text{C}^2\right)7.5\times10^{-4}\ \text{m}^2}{\text{N}\cdot\text{m}^2\quad 1.19\times10^{-3}\ \text{m}} = 5.58\ \text{pF}$$

Then equivalent capacitance $= 5.58 + 5.58 = \boxed{11.2\ \text{pF}}$.

(b) $Q = C\Delta V + C\Delta V = 11.2\times10^{-12}\ \text{F}(12\ \text{V}) = \boxed{134\ \text{pC}}$

(c) Now P_3 has charge on two surfaces and in effect three capacitors are in parallel:

$$C = 3(5.58\ \text{pF}) = \boxed{16.7\ \text{pF}}$$

(d) Only one face of P_4 carries charge: $Q = C\Delta V = 5.58\times10^{-12}\ \text{F}(12\ \text{V}) = \boxed{66.9\ \text{pC}}$

P26.48 From the Example about a cylindrical capacitor,

$$V_b - V_a = -2k_e\lambda \ln\frac{b}{a}$$

$$V_b - 345\ \text{kV} = -2\left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(1.40\times10^{-6}\ \text{C/m}\right)\ln\frac{12\ \text{m}}{0.024\ \text{m}}$$

$$= -2(8.99)\left(1.4\times10^3\ \text{J/C}\right)\ln 500 = -1.564\,3\times10^5\ \text{V}$$

$$V_b = 3.45\times10^5\ \text{V} - 1.56\times10^5\ \text{V} = \boxed{1.89\times10^5\ \text{V}}$$

P26.49 (a) We use the equation $U = Q^2/2C$ to find the potential energy of the capacitor. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are $U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)$ and $U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$.

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$.

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have $W = U_f - U_i = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$.

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i\left(\Delta V_i\right)$, and evaluate: $W = \frac{1}{2}C_i\left(\Delta V_i\right)^2(\kappa - 1) = \frac{1}{2}\left(2.00\times10^{-9}\ \text{F}\right)(100\ \text{V})^2(5.00 - 1.00) = \boxed{4.00\times10^{-5}\ \text{J}}$.

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$.

Substituting $C_f = \frac{C_i}{\kappa}$ and $Q = C_i\left(\Delta V_i\right)$ gives $\Delta V_f = \kappa\Delta V_i = 5.00(100\ \text{V}) = \boxed{500\ \text{V}}$.

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

***P26.50** (a)

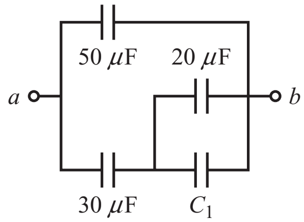


FIG. P26.50a

(b) $\dfrac{1}{1/30+1/(20+C_1)}=20\mu\text{F}$ gives $\dfrac{1}{30}+\dfrac{1}{20+C_1}=\dfrac{1}{20}$ so $\dfrac{1}{20+C_1}=\dfrac{1}{60}$

and $C_1 = \boxed{40.0\ \mu\text{F}}$

(c)

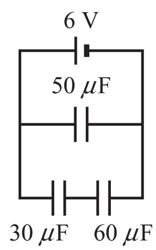


FIG. P26.50c

For the 50 μF, $\Delta V = \boxed{6.00\text{ V}}$ and $Q = C\Delta V = \boxed{300\ \mu\text{C}}$.

For the 30 μF and 60 μF, the equivalent capacitance is 20 μF and $Q = C\Delta V =$ 20 μF 6 V = $\boxed{120\ \mu\text{C}}$.

For 30 μF, $\Delta V = Q/C = 120\ \mu\text{C}/30\ \mu\text{F} = \boxed{4.00\text{ V}}$.

For 20 μF and 40 μF, $\Delta V = 120\ \mu\text{C}/60\ \mu\text{F} = \boxed{2.00\text{ V}}$.

For 20 μF, $Q = C\Delta V = 20\ \mu\text{F}\ 2\text{ V} = \boxed{40.0\ \mu\text{C}}$.

For 40 μF, $Q = C\Delta V = 40\ \mu\text{F}\ 2\text{ V} = \boxed{80.0\ \mu\text{C}}$.

P26.51 $\kappa = 3.00,\ E_{\max} = 2.00\times10^{8}\ \text{V/m} = \dfrac{\Delta V_{\max}}{d}$

For $C = \dfrac{\kappa \in_0 A}{d} = 0.250\times10^{-6}\ \text{F}$

$$A = \frac{Cd}{\kappa \in_0} = \frac{C\Delta V_{\max}}{\kappa \in_0 E_{\max}} = \frac{\left(0.250\times10^{-6}\right)(4\,000)}{3.00\left(8.85\times10^{-12}\right)\left(2.00\times10^{8}\right)} = \boxed{0.188\ \text{m}^2}$$

***P26.52** (a) The partially filled capacitor constitutes two capacitors in series, with separate capacitances

$$\frac{1\in_0 A}{d(1-f)} \quad \text{and} \quad \frac{6.5\in_0 A}{fd} \quad \text{and equivalent capacitance}$$

$$\frac{1}{\dfrac{d(1-f)}{\in_0 A}+\dfrac{fd}{6.5\in_0 A}}=\frac{6.5\in_0 A}{6.5d-6.5df+fd}=\frac{\in_0 A}{d}\frac{6.5}{6.5-5.5f}=\boxed{25.0\ \mu\text{F}(1-0.846f)^{-1}}$$

(b) For $f = 0$, the capacitor is empty so we can expect capacitance $\boxed{25.0\ \mu\text{F}}$, and the expression in part (a) $\boxed{\text{agrees}}$ with this.

(c) For $f = 1$ we expect $6.5(25.0\ \mu\text{F}) = 162\ \mu\text{F}$. The expression in (a) becomes $25.0\ \mu\text{F}(1-0.846)^{-1} = \boxed{162\ \mu\text{F, in agreement}}$.

(d) The charge on the lower plate creates an electric field in the liquid given by

$$E=\frac{Q}{A\kappa\in_0}=\frac{300\ \mu\text{C}}{A\ 6.5\in_0}$$

The charge on the upper plate creates an electric field in the vacuum according to

$$E=\frac{Q}{A\in_0}=\frac{300\ \mu\text{C}}{A\in_0}$$

The change in strength of the field at the upper surface of the liquid is described by

$$\frac{300\ \mu\text{C}}{A\ 6.5\in_0}+\frac{Q_{induced}}{A\in_0}=\frac{300\ \mu\text{C}}{A\in_0}$$

which gives $Q_{induced} = \boxed{254\ \mu\text{C, independent of } f}$. The induced charge will be opposite in sign to the charge on the top capacitor plate and $\boxed{\text{the same in sign as the charge on the lower plate}}$.

P26.53 (a) Put charge Q on the sphere of radius a and $-Q$ on the other sphere. Relative to $V = 0$ at infinity,

the potential at the surface of a is $V_a=\dfrac{k_eQ}{a}-\dfrac{k_eQ}{d}$

and the potential of b is $V_b=\dfrac{-k_eQ}{b}+\dfrac{k_eQ}{d}$.

The difference in potential is $V_a-V_b=\dfrac{k_eQ}{a}+\dfrac{k_eQ}{b}-\dfrac{k_eQ}{d}-\dfrac{k_eQ}{d}$

and $C=\dfrac{Q}{V_a-V_b}=\boxed{\left(\dfrac{4\pi\in_0}{(1/a)+(1/b)-(2/d)}\right)}$

(b) As $d\to\infty$, $\dfrac{1}{d}$ becomes negligible compared to $\dfrac{1}{a}$. Then,

$$C=\frac{4\pi\in_0}{1/a+1/b} \quad \text{and} \quad \frac{1}{C}=\boxed{\frac{1}{4\pi\in_0 a}+\frac{1}{4\pi\in_0 b}}$$

as for two spheres in series.

P26.54 The initial charge on the larger capacitor is

$$Q = C\Delta V = 10\ \mu\text{F}(15\ \text{V}) = 150\ \mu\text{C}$$

An additional charge q is pushed through the 50-V battery, giving the smaller capacitor charge q and the larger charge $150\ \mu\text{C} + q$.

Then $$50\ \text{V} = \frac{q}{5\ \mu\text{F}} + \frac{150\ \mu\text{C} + q}{10\ \mu\text{F}}$$

$$500\ \mu\text{C} = 2q + 150\ \mu\text{C} + q$$

$$q = 117\ \mu\text{C}$$

So across the $5\text{-}\mu\text{F}$ capacitor $$\Delta V = \frac{q}{C} = \frac{117\ \mu\text{C}}{5\ \mu\text{F}} = \boxed{23.3\ \text{V}}$$

Across the $10\text{-}\mu\text{F}$ capacitor $$\Delta V = \frac{150\ \mu\text{C} + 117\ \mu\text{C}}{10\ \mu\text{F}} = \boxed{26.7\ \text{V}}$$

P26.55 Gasoline: $$(126\,000\ \text{Btu/gal})(1\,054\ \text{J/Btu})\left(\frac{1.00\ \text{gal}}{3.786 \times 10^{-3}\ \text{m}^3}\right)\left(\frac{1.00\ \text{m}^3}{670\ \text{kg}}\right) = 5.24 \times 10^7\ \text{J/kg}$$

Battery: $$\frac{(12.0\ \text{J/C})(100\ \text{C/s})(3\,600\ \text{s})}{16.0\ \text{kg}} = 2.70 \times 10^5\ \text{J/kg}$$

Capacitor: $$\frac{\frac{1}{2}(0.100\ \text{F})(12.0\ \text{V})^2}{0.100\ \text{kg}} = 72.0\ \text{J/kg}$$

Gasoline has 194 times the specific energy content of the battery and 727 000 times that of the capacitor.

***P26.56** (a) The portion of the device containing the dielectric has plate area ℓx and capacitance $C_1 = \dfrac{\kappa \in_0 \ell x}{d}$. The unfilled part has area $\ell(\ell - x)$ and capacitance $C_2 = \dfrac{\in_0 \ell(\ell - x)}{d}$. The total capacitance is $C_1 + C_2 = \boxed{\dfrac{\in_0}{d}\left[\ell^2 + \ell x(\kappa - 1)\right]}$.

(b) The stored energy is $U = \dfrac{1}{2}\dfrac{Q^2}{C} = \boxed{\dfrac{Q^2 d}{2\in_0\left(\ell^2 + \ell x(\kappa - 1)\right)}}$.

(c) $\vec{\mathbf{F}} = -\left(\dfrac{dU}{dx}\right)\hat{\mathbf{i}} = \boxed{\dfrac{Q^2 d\,\ell(\kappa - 1)}{2\in_0\left(\ell^2 + \ell x(\kappa - 1)\right)^2}\hat{\mathbf{i}}}$. When $x = 0$, the original value of the force is $\dfrac{Q^2 d(\kappa - 1)}{2\in_0 \ell^3}\hat{\mathbf{i}}$. As the dielectric slides in, the charges on the plates redistribute themselves. The force decreases to its final value $\dfrac{Q^2 d(\kappa - 1)}{2\in_0 \ell^3 \kappa^2}\hat{\mathbf{i}}$.

continued on next page

(d) At $x = \frac{\ell}{2}$, $\vec{\mathbf{F}} = \frac{2Q^2 d(\kappa - 1)}{\epsilon_0 \ell^3 (\kappa+1)^2}\hat{\mathbf{i}}$.

For the constant charge on the capacitor and the initial voltage we have the relationship

$$Q = C_0 \Delta V = \frac{\epsilon_0 \ell^2 \Delta V}{d}$$

Then the force is $\vec{\mathbf{F}} = \frac{2\epsilon_0 \ell (\Delta V)^2 (\kappa-1)}{d(\kappa+1)^2}\hat{\mathbf{i}}$

$$\vec{\mathbf{F}} = \frac{2\left(8.85\times 10^{-12}\ \mathrm{C}^2\right)0.05\ \mathrm{m}(2\ 000\ \mathrm{Nm})^2(4.5-1)}{\mathrm{Nm}^2(0.002\ \mathrm{m})\mathrm{C}^2(4.5+1)^2}\hat{\mathbf{i}} = \boxed{205\mu\mathrm{N}\,\hat{\mathbf{i}}}$$

***P26.57** The portion of the capacitor nearly filled by metal has

capacitance $\frac{\kappa\epsilon_0(\ell x)}{d} \to \infty$

and stored energy $\frac{Q^2}{2C} \to 0$

The unfilled portion has

capacitance $\frac{\epsilon_0 \ell(\ell - x)}{d}$

The charge on this portion is $Q = \frac{(\ell - x)Q_0}{\ell}$

(a) The stored energy is

$$U = \frac{Q^2}{2C} = \frac{\left[(\ell-x)Q_0/\ell\right]^2}{2\epsilon_0 \ell(\ell-x)/d} = \boxed{\frac{Q_0^2(\ell-x)d}{2\epsilon_0 \ell^3}}$$

(b) $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{Q_0^2(\ell-x)d}{2\epsilon_0 \ell^3}\right) = +\frac{Q_0^2 d}{2\epsilon_0 \ell^3}$

$\vec{\mathbf{F}} = \boxed{\frac{Q_0^2 d}{2\epsilon_0 \ell^3}\text{ to the right}}$ (into the capacitor)

(c) Stress $= \frac{F}{\ell d} = \boxed{\frac{Q_0^2}{2\epsilon_0 \ell^4}}$

(d) $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{1}{2}\epsilon_0\left(\frac{Q_0}{\epsilon_0 \ell^2}\right)^2 = \boxed{\frac{Q_0^2}{2\epsilon_0 \ell^4}}$. The answers to parts (c) and (d) are precisely the same.

***P26.58** One capacitor cannot be used by itself—it would burn out. She can use two capacitors in parallel, connected in series to another two capacitors in parallel. One capacitor will be left over. The equivalent capacitance is $\frac{1}{(200\ \mu\mathrm{F})^{-1} + (200\ \mu\mathrm{F})^{-1}} = 100\ \mu\mathrm{F}$. When 90 V is connected across the combination, only $\boxed{45\ \mathrm{V}}$ appears across each individual capacitor.

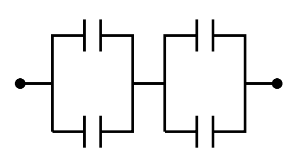


FIG. P26.58

P26.59 Call the unknown capacitance C_u

$$Q = C_u(\Delta V_i) = (C_u + C)(\Delta V_f)$$

$$\boxed{C_u = \frac{C(\Delta V_f)}{(\Delta V_i) - (\Delta V_f)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

***P26.60** Consider a strip of width dx and length W at position x from the front left corner. The capacitance of the lower portion of this strip is $\frac{\kappa_1 \in_0 W\ dx}{t x/L}$. The capacitance of the upper portion is $\frac{\kappa_2 \in_0 W\ dx}{t(1-x/L)}$. The series combination of the two elements has capacitance

$$\frac{1}{\frac{tx}{\kappa_1 \in_0 WL\ dx} + \frac{t(L-x)}{\kappa_2 \in_0 WL\ dx}} = \frac{\kappa_1\kappa_2 \in_0 WL\ dx}{\kappa_2 tx + \kappa_1 tL - \kappa_1 tx}$$

The whole capacitance is a combination of elements in parallel:

$$C = \int_0^L \frac{\kappa_1\kappa_2 \in_0 WL\ dx}{(\kappa_2 - \kappa_1)tx + \kappa_1 tL} = \frac{1}{(\kappa_2 - \kappa_1)t}\int_0^L \frac{\kappa_1\kappa_2 \in_0 WL(\kappa_2 - \kappa_1)tdx}{(\kappa_2 - \kappa_1)tx + \kappa_1 tL}$$

$$= \frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_2 - \kappa_1)t}\ln\left[(\kappa_2 - \kappa_1)tx + \kappa_1 tL\right]_0^L = \frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_2 - \kappa_1)t}\ln\left[\frac{(\kappa_2 - \kappa_1)tL + \kappa_1 tL}{0 + \kappa_1 tL}\right]$$

$$= \frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_2 - \kappa_1)t}\ln\left[\frac{\kappa_2}{\kappa_1}\right] = \frac{\kappa_1\kappa_2 \in_0 WL}{(-1)(\kappa_2 - \kappa_1)t}\ln\left[\left(\frac{\kappa_2}{\kappa_1}\right)^{-1}\right] = \boxed{\frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_1 - \kappa_2)t}\ln\left[\frac{\kappa_1}{\kappa_2}\right]}$$

(b) The capacitor physically has the same capacitance if it is turned upside down, so the answer should be the same with κ_1 and κ_2 interchanged. We have proved that it has this property in the solution to part (a).

(c) Let $\kappa_1 = \kappa_2(1 + x)$. Then $C = \frac{\kappa_2(1+x)\kappa_2 \in_0 WL}{\kappa_2 xt}\ln[1+x]$.

As x approaches zero we have $C = \frac{\kappa(1+0) \in_0 WL}{xt}x = \frac{\kappa \in_0 WL}{t}$ as was to be shown.

P26.61 (a) $C_0 = \frac{\in_0 A}{d} = \frac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}$$

$$U_0 = \frac{C_0(\Delta V_0)^2}{2}$$

$$U = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0(\Delta V_0^2)}{2}$$

and $\frac{U}{U_0} = \kappa$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0 = C_0\Delta V_0$

and $Q = C\Delta V_0 = \kappa C_0 \Delta V_0$

so the charge increases according to $\boxed{\frac{Q}{Q_0} = \kappa}$.

P26.62 Assume a potential difference across a and b, and notice that the potential difference across the $8.00\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:

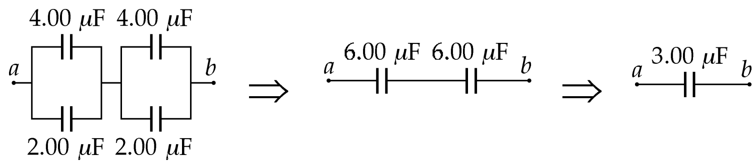


FIG. P26.62

$$C_{ab} = \boxed{3.00\ \mu\text{F}}$$

P26.63 Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1\,500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1\,000\ \mu\text{C} \quad \text{and} \quad \Delta V' = \frac{q'_{\text{total}}}{C_{\text{total}}} = \frac{1\,000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$$

Therefore,

$$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$$

$$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$$

***P26.64** Let charge λ per length be on one wire and $-\lambda$ be on the other. The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}$$

The potential difference between the surfaces of the wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{\lambda}{2\pi \in_0} \int_{D-r}^{r} \frac{dr}{r} = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{D-r}{r}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \in_0} \ln\left(\frac{D-r}{r}\right)$$

With D much larger than r we have nearly $\Delta V = \dfrac{\lambda}{\pi \in_0} \ln\left(\dfrac{D}{r}\right)$

and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda \ell}{(\lambda/\pi \in_0)\ln[D/r]} = \frac{\pi \in_0 \ell}{\ln[D/r]}$$

The capacitance per unit length is $\boxed{\dfrac{C}{\ell} = \dfrac{\pi \in_0}{\ln[D/r]}}$

P26.65 By symmetry, the potential difference across $3C$ is zero, so the circuit reduces to

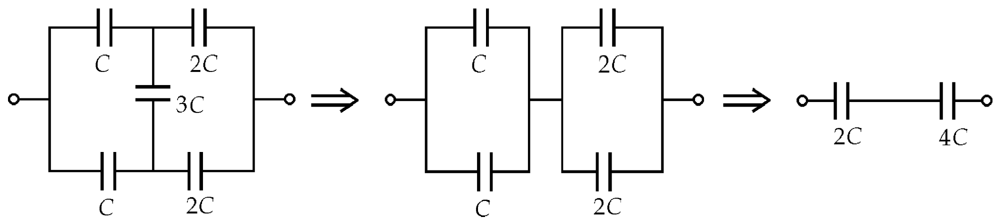


FIG. P26.65

$$C_{eq} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$

P26.66 The condition that we are testing is that the capacitance increases by less than 10%, or,

$$\frac{C'}{C} < 1.10$$

Substituting the expressions for C and C' from Example 26.1, we have

$$\frac{C'}{C} = \frac{\dfrac{\ell}{2k_e \ln(b/1.10a)}}{\dfrac{\ell}{2k_e \ln(b/a)}} = \frac{\ln(b/a)}{\ln(b/1.10a)} < 1.10$$

This becomes

$$\ln\left(\frac{b}{a}\right) < 1.10\ln\left(\frac{b}{1.10a}\right) = 1.10\ln\left(\frac{b}{a}\right) + 1.10\ln\left(\frac{1}{1.10}\right) = 1.10\ln\left(\frac{b}{a}\right) - 1.10\ln(1.10)$$

We can rewrite this as,

$$-0.10\ln\left(\frac{b}{a}\right) < -1.10\ln(1.10)$$

$$\ln\left(\frac{b}{a}\right) > 11.0\ln(1.10) = \ln(1.10)^{11.0}$$

where we have reversed the direction of the inequality because we multiplied the whole expression by –1 to remove the negative signs. Comparing the arguments of the logarithms on both sides of the inequality, we see that

$$\frac{b}{a} > (1.10)^{11.0} = 2.85$$

Thus, if $b > 2.85a$, the increase in capacitance is less than 10% and it is more effective to increase ℓ.

ANSWERS TO EVEN PROBLEMS

P26.2 (a) 1.00 μF (b) 100 V

P26.4 11.1 nF; 26.6 C

P26.6 $\dfrac{(2N-1)\in_0(\pi-\theta)R^2}{d}$

P26.8 $\dfrac{mgd\tan\theta}{q}$

P26.10 Yes; its total energy is sufficient to make the trip; 1.00×10^6 m/s.

P26.12 (a) 17.0 μF (b) 9.00 V (c) 45.0 μC and 108 μC

P26.14 (a) $2C$ (b) $Q_1>Q_3>Q_2$ (c) $\Delta V_1>\Delta V_2=\Delta V_3$ (d) Q_1 and Q_3 increase, Q_2 decreases

P26.16 $\dfrac{C_p}{2}+\sqrt{\dfrac{C_p^2}{4}-C_pC_s}$ and $\dfrac{C_p}{2}-\sqrt{\dfrac{C_p^2}{4}-C_pC_s}$

P26.18 (a) 398 μF in series (b) 2.20 μF in parallel

P26.20 19.8 μC

P26.22 $\left(\sqrt{3}-1\right)\dfrac{C_0}{2}$

P26.24 83.6 μC

P26.26 4.47 kV

P26.28 (a) See the solution. Stored energy = 0.150 J (b) See the solution. Potential difference = 268 V

P26.30 (a) 400 μC (b) 2.50 kN/m

P26.32 (a) $C(\Delta V)^2$ (b) $4\Delta V/3$ (c) $4C(\Delta V)^2/3$ (d) Positive work is done by the agent pulling the plates apart.

P26.34 (a) $q_1=\dfrac{R_1Q}{R_1+R_2}$ and $q_2=\dfrac{R_2Q}{R_1+R_2}$ (b) See the solution.

P26.36 (a) 13.3 nC (b) 272 nC

P26.38 $\sim10^{-6}$ F and $\sim10^2$ V for two 40 cm by 100 cm sheets of aluminum foil sandwiching a thin sheet of plastic.

P26.40 (a) 369 pC (b) 118 pF, 3.12 V (c) –45.5 nJ

P26.42 (a) $\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)$ pC·m (b) -20.9 nN·m$\hat{\mathbf{k}}$ (c) 112 nJ (d) 228 nJ

P26.44 See the solution.

P26.46 (a) $-2Q/3$ on upper plate, $-Q/3$ on lower plate (b) $2Qd/3\in_0A$

P26.48 189 kV

P26.50 (a) See the solution. (b) 40.0 μF (c) 6.00 V across 50 μF with charge 300 μF; 4.00 V across 30 μF with charge 120 μF; 2.00 V across 20 μF with charge 40 μF; 2.00 V across 40 μF with charge 80 μF

P26.52 (a) 25.0 μF $(1 - 0.846\, f\,)^{-1}$ (b) 25.0 μF; the general expression agrees. (c) 162 μF; the general expression agrees. (d) It has the same sign as the lower capacitor plate and its magnitude is 254 μC, independent of f.

P26.54 23.3 V; 26.7 V

P26.56 (a) $\dfrac{\epsilon_0\left[\ell^2 + \ell x(\kappa - 1)\right]}{d}$ (b) $\dfrac{Q^2 d}{2\epsilon_0\left[\ell^2 + \ell x(\kappa - 1)\right]}$ (c) $\dfrac{Q^2 d\ell(\kappa - 1)}{2\epsilon_0\left[\ell^2 + \ell x(\kappa - 1)\right]^2}$ to the right

(d) 205 μN right

P26.58 One capacitor cannot be used by itself—it would burn out. She can use two capacitors in parallel, connected in series to another two capacitors in parallel. One capacitor will be left over. Each of the four capacitors will be exposed to a maximum voltage of 45 V.

P26.60 (a) $\dfrac{\kappa_1\kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2)t}\ln\dfrac{\kappa_1}{\kappa_2}$ (b) The capacitance is the same if κ_1 and κ_2 are interchanged, as it should be.

P26.62 3.00 μF

P26.64 See the solution.

P26.66 See the solution.