31

Faraday's Law

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q31.1 Magnetic flux measures the "flow" of the magnetic field through a given area of a loop—even though the field does not actually flow. By changing the size of the loop, or the orientation of the loop and the field, one can change the magnetic flux through the loop, but the magnetic field will not change.

***Q31.2** The emf is given by the negative of the time derivative of the magnetic flux. We pick out the steepest downward slope at instant F as marking the moment of largest emf. Next comes A. At B and at D the graph line is horizontal so the emf is zero. At E the flux graph slopes gently upward so the emf is slightly negative. At C the emf has its greatest negative value. The answer is then F > A > B = D = 0 > E > C.

***Q31.3** (i) c (ii) b. The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is a maximum when $\vec{\mathbf{B}}$ is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop. The flux is zero when the loop is turned so that the field lies in the plane of its area.

***Q31.4** (i) Answer (c). The magnetic flux through the coil is constant in time, so the induced emf is zero. (ii) Answer (a). Positive test charges in the leading and trailing sides of the square experience a $\vec{\mathbf{F}} = q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$ force that is in direction (to the right) × (perpendicularly into the plane away from you) = toward the top of the square. The charges migrate upward to give positive charge to the top of the square until there is a downward electric field large enough to prevent more charge separation.

Q31.5 By the magnetic force law $\vec{\mathbf{F}} = q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$: the positive charges in the moving bar will feel a magnetic force in direction (right) × (perpendicularly out of the page) = downward toward the bottom end of the bar. These charges will move downward and therefore clockwise in the circuit. If the bar is moving to the left, the positive charge in the bar will flow upward and therefore counterclockwise in the circuit.

***Q31.6** (i) No. A magnetic force acts within the front and back edges of the coil, but produces no current and has no influence on the forward motion of the coil.

(ii) Yes. An induced current exists in the bar, which we can attribute either to an induced emf in the loop or to magnetic force on free charges in the bar. Then a backward magnetic force acts on the current and an external force must counterbalance it to maintain steady motion.

(iii) No. A magnetic force acts within the bar, but produces no current and has no influence on the forward motion of the bar.

Q31.7 As water falls, it gains speed and kinetic energy. It then pushes against turbine blades, transferring its energy to the rotor coils of a large AC generator. The rotor of the generator turns within a strong magnetic field. Because the rotor is spinning, the magnetic flux through its turns changes in time as $\Phi_B = BA\cos\omega t$. Generated in the rotor is an induced emf of $\mathcal{E} = \frac{-Nd\Phi_B}{dt}$. This induced emf is the voltage driving the current in our electric power lines.

Q31.8 Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. In a strong field the plate may fall very slowly.

***Q31.9** Answer (b). The rate of changes of flux of the external magnetic field in the turns of the coil is doubled, to double the maximum induced emf.

Q31.10 The increasing counterclockwise current in the solenoid coil produces an upward magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce clockwise current in the ring. The magnetic field of the solenoid has a radially outward component at each point on the ring. This field component exerts upward force on the current in the ring there. The whole ring feels a total upward force larger than its weight.

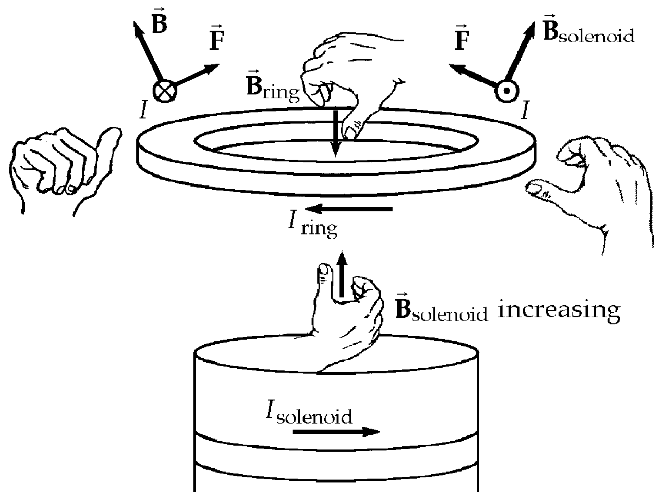


FIG. Q31.10

Q31.11 Oscillating current in the solenoid produces an always-changing magnetic field. Vertical flux through the ring, alternately increasing and decreasing, produces current in it with a direction that is alternately clockwise and counterclockwise. The current through the ring's resistance converts electrically transmitted energy into internal energy at the rate I^2R.

***Q31.12** (i) Answer (a). The south pole of the magnet produces an upward magnetic field that increases as the magnet approaches the loop. The loop opposes change by making its own downward magnetic field; it carries current clockwise, which goes to the left through the resistor.

(ii) Answer (b). The north pole of the magnet produces an upward magnetic field. The loop sees decreasing upward flux as the magnet falls away, and tries to make an upward magnetic field of its own by carrying current counterclockwise, to the right in the resistor.

(iii) Answer (c). The north pole of the magnet creates some downward flux through the section of the coil straight below it, but the south pole creates an equal amount of upward flux. The net flux through the coil due to the magnet is always zero so the induced emf is zero and the current is zero.

***Q31.13** (i) Answer (b). The battery makes counter-clockwise current I_1 in the primary coil, so its magnetic field $\vec{\mathbf{B}}_1$ is to the right and increasing just after the switch is closed. The secondary coil will oppose the change with a leftward field $\vec{\mathbf{B}}_2$, which comes from an induced clockwise current I_2 that goes to the right in the resistor. The upper pair of hands in the diagram represent this effect.

(ii) Answer (d). At steady state the primary magnetic field is unchanging, so no emf is induced in the secondary.

(iii) Answer (a). The primary's field is to the right and decreasing as the switch is opened. The secondary coil opposes this decrease by making its own field to the right, carrying counterclockwise current to the left in the resistor. The lower pair of hands diagrammed represent this chain of events.

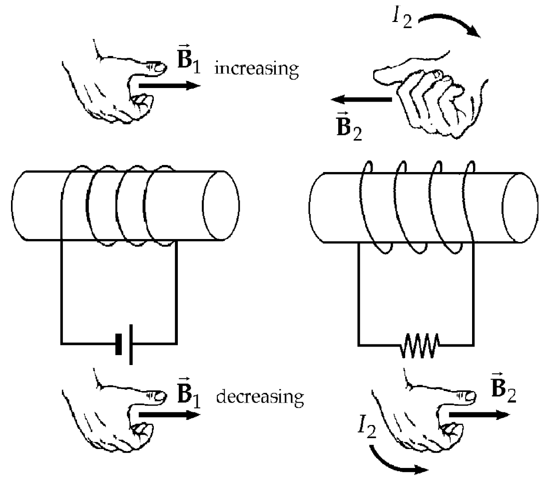


FIG. Q31.13

***Q31.14** A positive electric charge carried around a circular electric field line in the direction of the field gains energy from the field every step of the way. It can be a test charge imagined to exist in vacuum or it can be an actual free charge participating in a current driven by an induced emf. By doing net work on an object carried around a closed path to its starting point, the magnetically-induced electric field exerts by definition a nonconservative force. We can get a larger and larger voltage just by looping a wire around into a coil with more and more turns.

SOLUTIONS TO PROBLEMS

Section 31.1 **Faraday's Law of Induction**

Section 31.3 **Lenz's Law**

***P31.1** (a) $$\mathcal{E} = -N\frac{\Delta BA\cos\theta}{\Delta t} = -1\pi r^2\left(\frac{B_f - B_i}{\Delta t}\right)\cos\theta = -\left[\pi(0.00160\text{ m})^2\right]\left(\frac{1.5\text{ T}-0}{0.120\text{ s}}\right)1$$

$$= -(8.04\times10^{-6}\text{ m}^2)12.5\text{ T/s}$$

$$= \boxed{101\ \mu\text{V tending to produce clockwise current as seen from above}}$$

(b) The rate of change of magnetic field in this case is (–0.5 T – 1.5 T)/0.08 s = 25 T/s. It is twice as large in magnitude and in the opposite sense from the rate of change in case (a), so the emf is also $\boxed{\text{twice as large in magnitude and in the opposite sense}}$.

P31.2 $$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta\left(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}\right)}{\Delta t} = \frac{(2.50\text{ T}-0.500\text{ T})\left(8.00\times10^{-4}\text{ m}^2\right)}{1.00\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{1\text{ N}\cdot\text{m}}\right)$$

$$|\mathcal{E}| = 1.60\text{ mV} \qquad \text{and} \qquad I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60\text{ mV}}{2.00\ \Omega} = \boxed{0.800\text{ mA}}$$

P31.3 $$\mathcal{E} = -N\frac{\Delta BA\cos\theta}{\Delta t} = -NB\pi r^2\left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$$

$$= -25.0\left(50.0\times10^{-6}\text{ T}\right)\left[\pi(0.500\text{ m})^2\right]\left(\frac{\cos 180° - \cos 0°}{0.200\text{ s}}\right)$$

$$\mathcal{E} = \boxed{+9.82\text{ mV}}$$

***P31.4** (a) We evaluate the average emf:

$$\mathcal{E} = -N\frac{\Delta BA\cos\theta}{\Delta t} = -N\pi r^2\left(\frac{B_f - B_i}{\Delta t}\right)\cos\theta = -12\left[\pi(0.0210\text{ m})^2\right]\left(\frac{0-0.11\text{ T}}{0.180\text{ s}}\right)\cos 0°$$
$$= 0.0102\text{ V}$$

The average induced current will then be 0.0102 V/2.3 Ω = 4.42 mA. If the meter has a sufficiently short response time, it will register the current. The average current may even run the meter offscale by a factor of 4.42, so you might wish to slow down the motion of the coil.

(b) Positive. The coil sees decreasing external magnetic flux toward you, so it makes some flux of its own in this direction by carrying counterclockwise current, that enters the red terminal of the ammeter.

P31.5 (a) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = \boxed{\frac{AB_{\max}}{\tau}e^{-t/\tau}}$$

(b) $$\mathcal{E} = \frac{\left(0.160\text{ m}^2\right)(0.350\text{ T})}{2.00\text{ s}}e^{-4.00/2.00} = \boxed{3.79\text{ mV}}$$

(c) At $t = 0$ $\mathcal{E} = \boxed{28.0\text{ mV}}$

***P31.6** $$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right) = -200\left(50.0\times10^{-6}\text{ T}\right)(\cos 62.0°)\left(\frac{39.0\times10^{-4}\text{ m}^2}{1.80\text{ s}}\right) = \boxed{-10.2\ \mu\text{V}}$$

P31.7 Noting unit conversions from $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ and $U = qV$, the induced voltage is

$$\mathcal{E} = -N\frac{d\left(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}\right)}{dt} = -N\left(\frac{0 - B_iA\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})\left(0.200\text{ m}^2\right)\cos 0°}{20.0\times10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right)$$
$$= 3\,200\text{ V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{3\,200\text{ V}}{20.0\ \Omega} = \boxed{160\text{ A}}$$

P31.8 (a) $d\Phi_B = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \dfrac{\mu_0 I}{2\pi x} L dx$: $\Phi_B = \displaystyle\int_h^{h+w} \dfrac{\mu_0 IL}{2\pi} \dfrac{dx}{x} = \boxed{\dfrac{\mu_0 IL}{2\pi} \ln\left(\dfrac{h+w}{h}\right)}$

(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

$$\mathcal{E} = -\frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)(10.0\ \text{A/s})$$
$$= \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle, shown by the first hand in the figure to the right.

As the magnetic flux increases, the rectangle produces its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second hand in the figure).

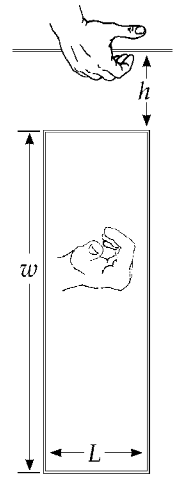


FIG. P31.8

P31.9 $$|\mathcal{E}| = \frac{d(BA)}{dt} = 0.500\mu_0 nA\frac{dI}{dt} = 0.480 \times 10^{-3}\ \text{V}$$

(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \dfrac{4.80 \times 10^{-4}}{3.00 \times 10^{-4}} = \boxed{1.60\ \text{A}}$ counterclockwise

(b) $B_{\text{ring}} = \dfrac{\mu_0 I}{2r_{\text{ring}}} = \boxed{20.1\ \mu\text{T}}$

(c) The coil's field points into the coil, and is increasing,

so $\boxed{B_{\text{ring}} \text{ points away from the center of the coil, or left in the textbook picture}}$.

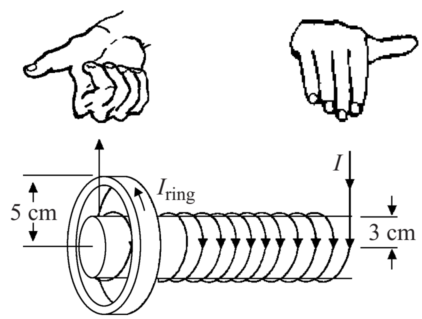


FIG. P31.9

P31.10 $$|\mathcal{E}| = \frac{d(BA)}{dt} = 0.500\mu_0 nA\frac{dI}{dt} = 0.500\mu_0 n\pi r_2^2\frac{\Delta I}{\Delta t}$$

(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 n\pi r_2^2}{2R}\dfrac{\Delta I}{\Delta t}}$ counterclockwise

(b) $B = \dfrac{\mu_0 I}{2r_1} = \boxed{\dfrac{\mu_0^2 n\pi r_2^2}{4r_1 R}\dfrac{\Delta I}{\Delta t}}$

(c) The coil's field points into the coil, and is increasing,

so $\boxed{B_{\text{ring}} \text{ points away from the center of the coil, or left in the textbook picture}}$.

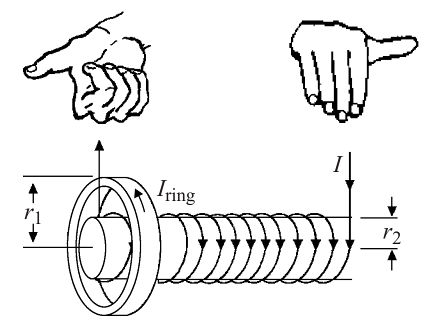


FIG. P31.10

P31.11 $\Phi_B = (\mu_0 n I) A_{\text{solenoid}}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}$$

$$\mathcal{E} = -15.0\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times10^{3}\ \text{m}^{-1}\right)\pi(0.020\,0\ \text{m})^2(600\ \text{A/s})\cos(120t)$$

$$\boxed{\mathcal{E} = -14.2\cos(120t)\ \text{mV}}$$

***P31.12** (a) $\mathcal{E} = -20(130)4\pi\times10^{-7}(0-6[1]/13\times10^{-6})\pi(0.03)^2 1/0.8 = \boxed{5.33\ \text{V}}$

(b) A flat compact circular coil with 130 turns and radius 40.0 cm carries current 3.00 A counterclockwise. The current is smoothly reversed to become 3.00 A clockwise after 13.0 μs. At the center of this primary coil is a secondary coil in the same plane, with 20 turns and radius 3.00 cm. Find the emf induced in the secondary.

P31.13 For a counterclockwise trip around the left-hand loop, with $B = At$

$$\frac{d}{dt}\left[At\left(2a^2\right)\cos 0°\right] - I_1(5R) - I_{PQ}R = 0$$

and for the right-hand loop,

$$\frac{d}{dt}\left[Ata^2\right] + I_{PQ}R - I_2(3R) = 0$$

where $I_{PQ} = I_1 - I_2$ is the upward current in QP.

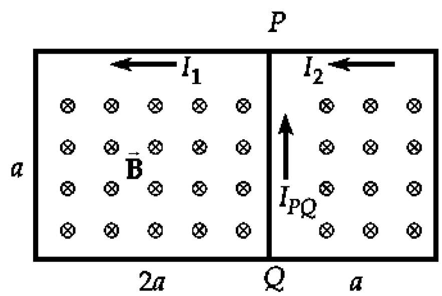


FIG. P31.13

Thus, $2Aa^2 - 5R\left(I_{PQ} + I_2\right) - I_{PQ}R = 0$

and $Aa^2 + I_{PQ}R = I_2(3R)$

$$2Aa^2 - 6RI_{PQ} - \frac{5}{3}\left(Aa^2 + I_{PQ}R\right) = 0$$

$I_{PQ} = \dfrac{Aa^2}{23R}$ upward, and since $R = (0.100\ \Omega/\text{m})(0.650\ \text{m}) = 0.0650\ \Omega$

$$I_{PQ} = \frac{\left(1.00\times10^{-3}\ \text{T/s}\right)(0.650\ \text{m})^2}{23(0.065\,0\ \Omega)} = \boxed{283\ \mu\text{A upward}}.$$

P31.14 $|\mathcal{E}| = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = N\left(\dfrac{dB}{dt}\right)A = N(0.010\,0 + 0.080\,0t)A$

At $t = 5.00$ s, $|\mathcal{E}| = 30.0(0.410\ \text{T/s})\left[\pi(0.040\,0\ \text{m})^2\right] = \boxed{61.8\ \text{mV}}$

P31.15 $B = \mu_0 n I = \mu_0 n(30.0\ \text{A})\left(1 - e^{-1.60t}\right)$

$$\Phi_B = \int B\,dA = \mu_0 n(30.0\ \text{A})\left(1 - e^{-1.60t}\right)\int dA$$

$$\Phi_B = \mu_0 n(30.0\ \text{A})\left(1 - e^{-1.60t}\right)\pi R^2$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n(30.0\ \text{A})\pi R^2(1.60)e^{-1.60t}$$

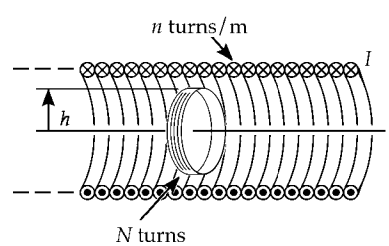


FIG. P31.15

$$\mathcal{E} = -(250)\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(400\ \text{m}^{-1}\right)(30.0\ \text{A})\left[\pi(0.060\,0\ \text{m})^2\right]1.60\ \text{s}^{-1}e^{-1.60t}$$

$$\mathcal{E} = \boxed{(68.2\ \text{mV})e^{-1.60t}\ \text{counterclockwise}}$$

P31.16 (a) Suppose, first, that the central wire is long and straight. The enclosed current of unknown amplitude creates a circular magnetic field around it, with the magnitude of the field given by Ampère's law.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I: \qquad B = \frac{\mu_0 I_{\max} \sin \omega t}{2\pi R}$$

at the location of the Rogowski coil, which we assume is centered on the wire. This field passes perpendicularly through each turn of the toroid, producing flux

$$\vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \frac{\mu_0 I_{\max} A \sin \omega t}{2\pi R}$$

The toroid has $2\pi Rn$ turns. As the magnetic field varies, the emf induced in it is

$$\mathcal{E} = -N \frac{d}{dt} \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = -2\pi Rn \frac{\mu_0 I_{\max} A}{2\pi R} \frac{d}{dt} \sin \omega t = -\mu_0 I_{\max} nA\omega \cos \omega t$$

This is an alternating voltage with amplitude $\mathcal{E}_{\max} = \mu_0 nA\omega I_{\max}$. Measuring the amplitude determines the size $I_{\max}$ of the central current. Our assumptions that the central wire is long and straight and passes perpendicularly through the center of the Rogowski coil are all unnecessary.

(b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

P31.17 $$\mathcal{E} = \frac{d}{dt}\left(NB\ell^2 \cos\theta\right) = \frac{N\ell^2 \Delta B \cos\theta}{\Delta t}$$

$$\ell = \sqrt{\frac{\mathcal{E}\,\Delta t}{N\Delta B \cos\theta}} = \sqrt{\frac{\left(80.0\times10^{-3}\text{ V}\right)(0.400\text{ s})}{(50)\left(600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T}\right)\cos(30.0°)}} = 1.36\text{ m}$$

Length $= 4\ell N = 4(1.36\text{ m})(50) = \boxed{272\text{ m}}$

P31.18 In a toroid, all the flux is confined to the inside of the toroid.

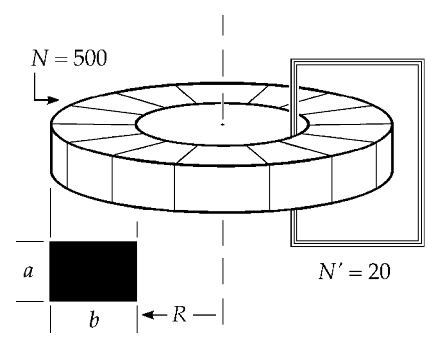


FIG. P31.18

$$B = \frac{\mu_0 NI}{2\pi r} = \frac{500\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B\,dA = \frac{500\mu_0 I_{\max}}{2\pi} \sin\omega t \int \frac{a\,dr}{r}$$

$$\Phi_B = \frac{500\mu_0 I_{\max}}{2\pi} a \sin\omega t \ln\left(\frac{b+R}{R}\right)$$

$$\mathcal{E} = N_2 \frac{d\Phi_B}{dt} = 20\left(\frac{500\mu_0 I_{\max}}{2\pi}\right)\omega a \ln\left(\frac{b+R}{R}\right)\cos\omega t$$

$$\mathcal{E} = \frac{10^4}{2\pi}\left(4\pi\times10^{-7}\text{ N/A}^2\right)(50.0\text{ A})(377\text{ rad/s})(0.020\,0\text{ m})\ln\left(\frac{(3.00+4.00)\text{ cm}}{4.00\text{ cm}}\right)\cos\omega t$$

$$= \boxed{(0.422\text{ V})\cos\omega t}$$

P31.19 The upper loop has area $\pi(0.05\text{ m})^2 = 7.85\times10^{-3}\text{ m}^2$. The induced emf in it is

$$\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -1A\cos 0°\frac{dB}{dt} = -7.85\times10^{-3}\text{ m}^2(2\text{ T/s}) = -1.57\times10^{-2}\text{ V}$$

The minus sign indicates that it tends to produce counterclockwise current, to make its own magnetic field out of the page. Similarly, the induced emf in the lower loop is

$$\mathcal{E} = -NA\cos\theta\frac{dB}{dt} = -\pi(0.09\text{ m})^2\,2\text{ T/s} = -5.09\times10^{-2}\text{ V} = +5.09\times10^{-2}\text{ V to produce}$$

$\boxed{\text{counterclockwise current in the lower loop, which becomes clockwise current in the upper loop}}$.

The net emf for current in this sense around the figure 8 is

$5.09\times10^{-2}\text{ V} - 1.57\times10^{-2}\text{ V} = 3.52\times10^{-2}\text{ V}$.

It pushes current in this sense through series resistance $[2\pi(0.05\text{ m}) + 2\pi(0.09\text{ m})]3\ \Omega/\text{m} = 2.64\ \Omega$.

The current is $I = \frac{\mathcal{E}}{R} = \frac{3.52\times10^{-2}\text{ V}}{2.64\ \Omega} = \boxed{13.3\text{ mA}}$.

Section 31.2 Motional emf

Section 31.3 Lenz's Law

P31.20 (a) For maximum induced emf, with positive charge at the top of the antenna,

$\vec{\mathbf{F}}_+ = q_+(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$, so the auto must move $\boxed{\textit{east}}$.

(b) $\mathcal{E} = B\ell v = (5.00\times10^{-5}\text{ T})(1.20\text{ m})\left(\frac{65.0\times10^3\text{ m}}{3\,600\text{ s}}\right)\cos 65.0° = \boxed{4.58\times10^{-4}\text{ V}}$

***P31.21** (a) $\mathcal{E} = B\ell v = (1.2\times10^{-6}\text{ T})(14.0\text{ m})(70\text{ m/s}) = \boxed{1.18\times10^{-3}\text{ V}}$. A free positive test charge in the wing feels a magnetic force in direction $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ = north × down = west. Then it migrates west to make the left-hand wingtip positive.

(b) No change. The charges in the horizontally-moving wing respond only to the vertical component of the Earth's field.

(c) No. If we tried to connect the wings into a circuit with the light bulb, we would run an extra insulated wire along the wing. With the wing it would form a one-turn coil, in which the emf is zero as the coil moves in a uniform field.

P31.22 $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R} = 2.5(1.2)v/6 = 0.5$

$\boxed{v = 1.00\text{ m/s}}$

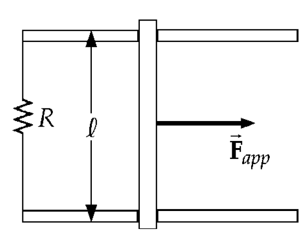


FIG. P31.22

P31.23 (a) $\left|\vec{\mathbf{F}}_B\right| = I\left|\vec{\ell}\times\vec{\mathbf{B}}\right| = I\ell B$

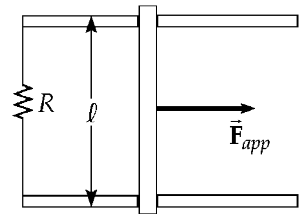


FIG. P31.23

When $\quad I = \dfrac{\mathcal{E}}{R}$

and $\quad \mathcal{E} = B\ell v$

we get $\quad F_B = \dfrac{B\ell v}{R}(\ell B) = \dfrac{B^2\ell^2 v}{R} = \dfrac{(2.50)^2(1.20)^2(2.00)}{6.00}$

$= 3.00\text{ N}.$

The applied force is $\boxed{3.00\text{ N to the right}}$.

(b) $\mathcal{P} = I^2R = \dfrac{B^2\ell^2v^2}{R} = 6.00\text{ W}$ or $\mathcal{P} = Fv = \boxed{6.00\text{ W}}$

***P31.24** $F_B = I\ell B$ and $\mathcal{E} = B\ell v$

$I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$ so $B = \dfrac{IR}{\ell v}$

(a) $F_B = \dfrac{I^2\ell R}{\ell v}$ and $I = \sqrt{\dfrac{F_Bv}{R}} = \boxed{0.500\text{ A}}$

(b) $I^2R = \boxed{2.00\text{ W}}$

(c) For constant force, $\mathcal{P} = \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$.

(d) The powers computed in parts (b) and (c) are mathematically equal. More profoundly, they are physically identical. Each bit of energy delivered to the circuit mechanically immediately goes through being electrically transmitted to the resistor and there becomes additional internal energy. Counting the 2 W twice is like counting your lunch money twice.

P31.25 Observe that the homopolar generator has no commutator and produces a voltage constant in time: DC with no ripple. In time dt, the disk turns by angle $d\theta = \omega dt$. The outer brush slides over distance $rd\theta$. The radial line to the outer brush sweeps over area

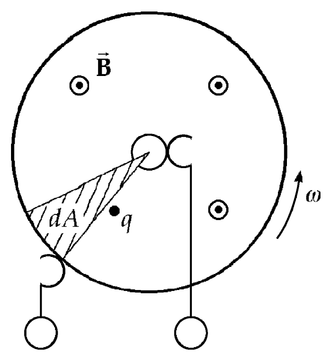


FIG. P31.25

$$dA = \frac{1}{2}rrd\theta = \frac{1}{2}r^2\omega dt$$

The emf generated is $\quad \mathcal{E} = -N\dfrac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$

$$\mathcal{E} = -(1)B\cos 0°\frac{dA}{dt} = -B\left(\frac{1}{2}r^2\omega\right)$$

(We could think of this as following from the result of the example in the chapter text about the helicopter blade.)
The magnitude of the emf is

$$|\mathcal{E}| = B\left(\frac{1}{2}r^2\omega\right) = (0.9\text{ N}\cdot\text{s/C}\cdot\text{m})\left[\frac{1}{2}(0.4\text{ m})^2(3\,200\text{ rev/min})\right]\left(\frac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right)$$

$|\mathcal{E}| = \boxed{24.1\text{ V}}$

A free positive charge q shown, turning with the disk, feels a magnetic force $q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ radially outward. Thus the $\boxed{\text{outer contact is positive}}$.

P31.26 The speed of waves on the wire is

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{267\ \text{N}\cdot\text{m}}{3\times10^{-3}\ \text{kg}}}=298\ \text{m/s}$$

In the simplest standing-wave vibration state,

$$d_{NN}=0.64\ \text{m}=\frac{\lambda}{2}\qquad \lambda=1.28\ \text{m}$$

and $\quad f=\dfrac{v}{\lambda}=\dfrac{298\ \text{m/s}}{1.28\ \text{m}}=233\ \text{Hz}$

(a) The changing flux of magnetic field through the circuit containing the wire will drive current to the left in the wire as it moves up and to the right as it moves down. The emf will have this same frequency of $\boxed{233\ \text{Hz}}$.

(b) The vertical coordinate of the center of the wire is described by
$x=A\cos\omega t=(1.5\ \text{cm})\cos(2\pi 233t/\text{s})$.

Its velocity is $v=\dfrac{dx}{dt}=-(1.5\ \text{cm})(2\pi\,233/\text{s})\sin(2\pi 233\ t/\text{s})$.

Its maximum speed is $1.5\ \text{cm}(2\pi)233/\text{s}=22.0\ \text{m/s}$.

The induced emf is $\mathcal{E}=-B\ell v$, with amplitude

$\mathcal{E}_{\max}=B\ell v_{\max}=4.50\times10^{-3}\ \text{T}(0.02\ \text{m})22\ \text{m/s}=\boxed{1.98\times10^{-3}\ \text{V}}$.

P31.27 $\omega=(2.00\ \text{rev/s})(2\pi\ \text{rad/rev})=4.00\pi\ \text{rad/s}$

$$\mathcal{E}=\frac{1}{2}B\omega\ell^2=\boxed{2.83\ \text{mV}}$$

P31.28 (a) $\vec{\mathbf{B}}_{\text{ext}}=B_{\text{ext}}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_0=B_0\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\vec{\mathbf{B}}_{\text{ext}}=B_{\text{ext}}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\vec{\mathbf{B}}_0=B_0\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed $\boxed{\text{out of the plane}}$ in the textbook picture and to the right in the diagram here.

(c) $\vec{\mathbf{B}}_{\text{ext}}=B_{\text{ext}}\left(-\hat{\mathbf{k}}\right)$ into the paper and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_0=B_0\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$.

(d) By the magnetic force law, $\vec{\mathbf{F}}_B=q\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$. Therefore, a positive charge will move to the top of the bar if $\vec{\mathbf{B}}$ is $\boxed{\text{into the paper}}$.

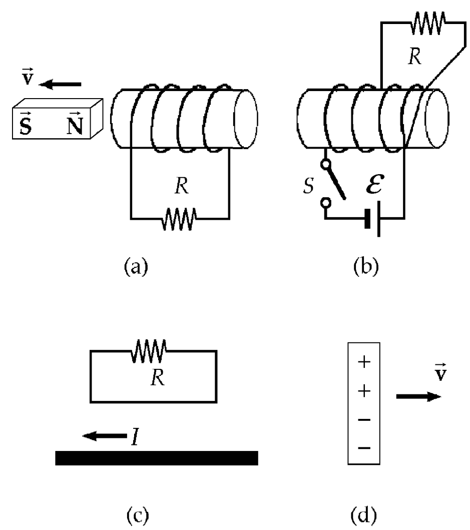


FIG. P31.28

P31.29 (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$

The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv$$

so the current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left}}$$

(b) Once the coil is entirely inside the field,

$\Phi_B = NBA = \text{constant}$,

so $\quad \mathcal{E} = 0,\ I = 0, \quad$ and $\quad F = \boxed{0}$

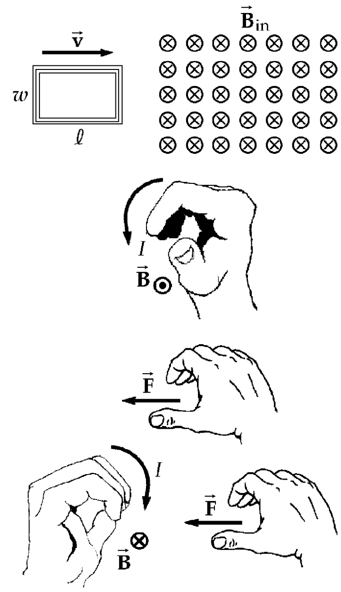


FIG. P31.29

(c) As the coil starts to leave the field, the flux *decreases* at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left again}}$$

P31.30 Look in the direction of *ba*. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counterclockwise current. Therefore, current must flow from b to a through the resistor. Hence, $V_a - V_b$ will be $\boxed{\text{negative}}$.

P31.31 Name the currents as shown in the diagram:

Left loop: $\quad +Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $\quad +Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $\quad I_2 = I_1 + I_3$

Then, $\quad Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

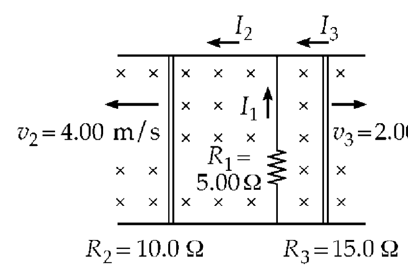


FIG. P31.31

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}$$

So, $\quad Bdv_2 - I_1(R_1 + R_2) - \dfrac{Bdv_3R_2}{R_3} - \dfrac{I_1R_1R_2}{R_3} = 0$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward}$$

$$I_1 = (0.010\,0\text{ T})(0.100\text{ m})\left[\frac{(4.00\text{ m/s})(15.0\ \Omega) - (2.00\text{ m/s})(10.0\ \Omega)}{(5.00\ \Omega)(10.0\ \Omega) + (5.00\ \Omega)(15.0\ \Omega) + (10.0\ \Omega)(15.0\ \Omega)}\right] = \boxed{145\ \mu\text{A}}$$

upward.

Section 31.4 **Induced emf and Electric Fields**

P31.32 (a) $\dfrac{dB}{dt}=6.00t^2-8.00t$

At $t=2.00$ s,

$$|\mathcal{E}|=\frac{d\Phi_B}{dt}$$
$$E=\frac{\pi R^2(dB/dt)}{2\pi r_2}$$
$$=\frac{8.00\pi(0.0250)^2}{2\pi(0.0500)}$$
$$F=qE=\boxed{8.00\times10^{-21}\text{ N}}$$

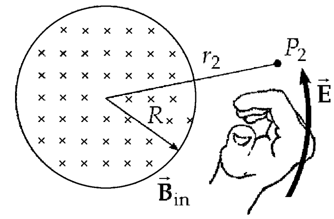


FIG. P31.32

(b) When $6.00t^2-8.00t=0$, $\quad t=\boxed{1.33\text{ s}}$

P31.33 $\dfrac{dB}{dt}=0.060\,0t \qquad |\mathcal{E}|=\dfrac{d\Phi_B}{dt}=\pi r_1^2\dfrac{dB}{dt}=2\pi r_1E$

At $t=3.00$ s,

$$E=\left(\frac{\pi r_1^2}{2\pi r_1}\right)\frac{dB}{dt}=\frac{0.020\,0\text{ m}}{2}\left(0.060\,0\text{ T/s}^2\right)(3.00\text{ s})\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)$$

$$\vec{\mathbf{E}}=\boxed{1.80\times10^{-3}\text{ N/C perpendicular to }r_1\text{ and counterclockwise}}$$

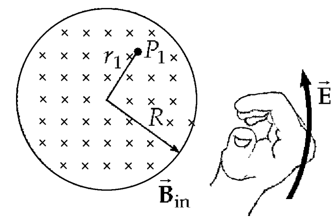


FIG. P31.33

P31.34 (a) $\oint\vec{\mathbf{E}}\cdot d\vec{\ell}=\left|\dfrac{d\Phi_B}{dt}\right|$

$$2\pi rE=\left(\pi r^2\right)\frac{dB}{dt} \qquad \text{so} \qquad E=\boxed{(9.87\text{ mV/m})\cos(100\pi t)}$$

(b) The E field is always opposite to increasing B. Therefore it is $\boxed{\text{clockwise}}$.

Section 31.5 **Generators and Motors**

P31.35 (a) $\mathcal{E}_{\max}=NAB\omega=(1\,000)(0.100)(0.200)(120\pi)=\boxed{7.54\text{ kV}}$

(b) $\mathcal{E}(t)=NBA\omega\sin\omega t=NBA\omega\sin\theta$

$|\mathcal{E}|$ is maximal when $|\sin\theta|=1$

or $\theta=\pm\dfrac{\pi}{2}$

so the $\boxed{\text{plane of coil is parallel to }\vec{\mathbf{B}}}$.

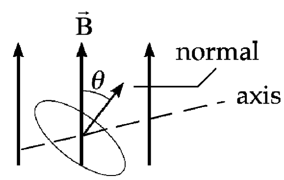


FIG. P31.35

P31.36 For the alternator, $\omega=(3\,000\text{ rev/min})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)=314\text{ rad/s}$

$$\mathcal{E}=-N\frac{d\Phi_B}{dt}=-250\frac{d}{dt}\left[\left(2.50\times10^{-4}\text{ T}\cdot\text{m}^2\right)\cos(314t/\text{s})\right]=+250\left(2.50\times10^{-4}\text{ T}\cdot\text{m}^2\right)(314/\text{s})\sin(314t)$$

(a) $\boxed{\mathcal{E}=(19.6\text{ V})\sin(314t)}$

(b) $\boxed{\mathcal{E}_{\max}=19.6\text{ V}}$

P31.37 $B = \mu_0 nI = \left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(200\ \text{m}^{-1}\right)(15.0\ \text{A}) = 3.77 \times 10^{-3}\ \text{T}$

For the small coil, $\Phi_B = N\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = NBA\cos\omega t = NB\left(\pi r^2\right)\cos\omega t$.

Thus, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = NB\pi r^2 \omega \sin\omega t$

$\mathcal{E} = (30.0)\left(3.77 \times 10^{-3}\ \text{T}\right)\pi(0.080\,0\ \text{m})^2\left(4.00\pi\ \text{s}^{-1}\right)\sin(4.00\pi t) = \boxed{(28.6\ \text{mV})\sin(4.00\pi t)}$

P31.38 As the magnet rotates, the flux through the coil varies sinusoidally in time with $\Phi_B = 0$ at $t = 0$. Choosing the flux as positive when the field passes from left to right through the area of the coil, the flux at any time may be written as $\Phi_B = -\Phi_{\max}\sin\omega t$ so the induced emf is given by

$\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \omega\Phi_{\max}\cos\omega t.$

The current in the coil is then

$I = \dfrac{\mathcal{E}}{R} = \dfrac{\omega\Phi_{\max}}{R}\cos\omega t = \boxed{I_{\max}\cos\omega t}$.

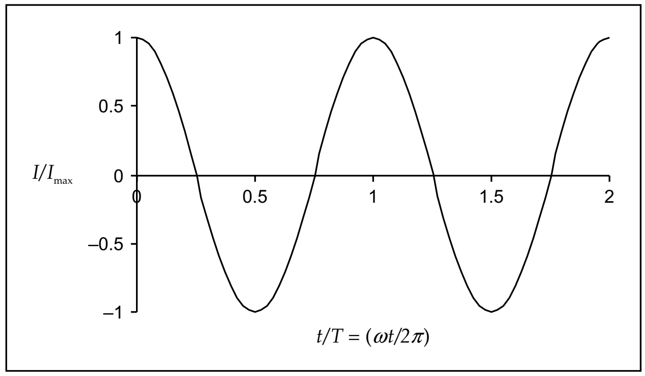
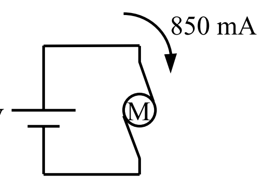
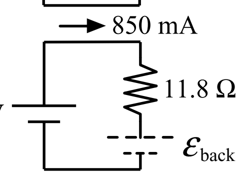


FIG. P31.38



P31.39 120 V To analyze the actual circuit, we model it as



120 V

(a) The loop rule gives $+120\ \text{V} - 0.85\ \text{A}(11.8\ \Omega) - \mathcal{E}_{\text{back}} = 0$ $\qquad \mathcal{E}_{\text{back}} = \boxed{110\ \text{V}}$

(b) The resistor is the device changing electrical work input into internal energy:
$\mathcal{P} = I^2R = (0.85\ \text{A})^2(11.8\ \Omega) = \boxed{8.53\ \text{W}}$

(c) With no motion, the motor does not function as a generator, and $\mathcal{E}_{\text{back}} = 0$. Then
$120\ \text{V} - I_c(11.8\ \Omega) = 0 \qquad I_c = 10.2\ \text{A}$
$\mathcal{P}_c = I_c^2 R = (10.2\ \text{A})^2(11.8\ \Omega) = \boxed{1.22\ \text{kW}}$

P31.40 (a) $\Phi_B = BA\cos\theta = BA\cos\omega t = (0.800\text{ T})(0.010\,0\text{ m}^2)\cos 2\pi(60.0)t$

$$= \boxed{(8.00\text{ mT}\cdot\text{m}^2)\cos(377t)}$$

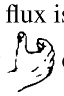
(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \boxed{(3.02\text{ V})\sin(377t)}$

(c) $I = \dfrac{\mathcal{E}}{R} = \boxed{(3.02\text{ A})\sin(377t)}$

(d) $\mathcal{P} = I^2R = \boxed{(9.10\text{ W})\sin^2(377t)}$

(e) $\mathcal{P} = Fv = \tau\omega$ so $\tau = \dfrac{\mathcal{P}}{\omega} = \boxed{(24.1\text{ mN}\cdot\text{m})\sin^2(377t)}$

Section 31.6 **Eddy Currents**

P31.41 The current in the magnet creates an  upward magnetic field, so the N and S poles on the solenoid core are shown correctly. On the rail in front of the brake, the upward flux of $\vec{\mathbf{B}}$ increases as the coil approaches, so a current is induced here to create a downward magnetic field. This is  clockwise current, so the S pole on the rail is shown correctly. On the rail behind the brake, the upward magnetic flux is decreasing. The induced current in the rail will produce upward magnetic field by being  counterclockwise as the picture correctly shows.

P31.42 (a) Consider an annulus of radius r, width dr, height b, and resistivity ρ. Around its circumference, a voltage is induced according to

$$\mathcal{E} = -N\frac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = -1\frac{d}{dt}B_{\text{max}}(\cos\omega t)\pi r^2 = +B_{\text{max}}\pi r^2\omega\sin\omega t$$

The resistance around the loop is $\dfrac{\rho\ell}{A_x} = \dfrac{\rho(2\pi r)}{bdr}$

The eddy current in the ring is $dI = \dfrac{\mathcal{E}}{\text{resistance}} = \dfrac{B_{\text{max}}\pi r^2\omega(\sin\omega t)bdr}{\rho(2\pi r)} = \dfrac{B_{\text{max}}rb\omega dr\sin\omega t}{2\rho}$

The instantaneous power is $d\mathcal{P}_i = \mathcal{E}\,dI = \dfrac{B_{\text{max}}^2\pi r^3 b\omega^2 dr\sin^2\omega t}{2\rho}$

The time average of the function $\sin^2\omega t = \dfrac{1}{2} - \dfrac{1}{2}\cos 2\omega t$ is $\dfrac{1}{2} - 0 = \dfrac{1}{2}$

so the time-averaged power delivered to the annulus is

$$d\mathcal{P} = \frac{B_{\text{max}}^2\pi r^3 b\omega^2 dr}{4\rho}$$

The power delivered to the disk is $\mathcal{P} = \int d\mathcal{P} = \displaystyle\int_0^R \frac{B_{\text{max}}^2\pi b\omega^2}{4\rho}r^3dr$

$$\mathcal{P} = \frac{B_{\text{max}}^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{\pi B_{\text{max}}^2 R^4 b\omega^2}{16\rho}}$$

(b) When B_{max} gets two times larger, B_{max}^2 and $\mathcal{P}$ get $\boxed{4}$ times larger.

(c) When f and $\omega = 2\pi f$ double, ω^2 and $\mathcal{P}$ get $\boxed{4}$ times larger.

(d) When R doubles, R^4 and $\mathcal{P}$ become $2^4 = \boxed{16}$ times larger.

P31.43 (a) At terminal speed,

$$Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_T}{R}\right)wB = \frac{B^2w^2v_T}{R}$$

or $\boxed{v_T = \frac{MgR}{B^2\omega^2}}$

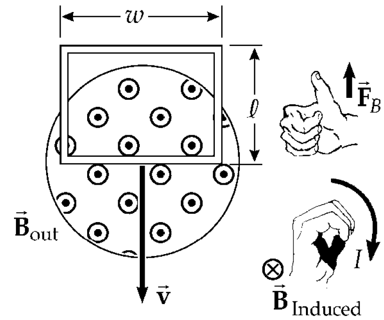


FIG. P31.43

(b) The emf is directly proportional to v_T, but the current is inversely proportional to R. A large R means a small current at a given speed, so the loop must travel faster to get $F_B = mg$.

(c) At a given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small B, the speed must increase to compensate for both the small B and also the current, so $v_T \propto 1/B^2$.

Additional Problems

***P31.44** (a) The circuit encloses increasing flux of magnetic field into the page, so it tries to make its own field out of the page, by carrying $\boxed{\text{counterclockwise}}$ current.

(b) $I = \mathcal{E}/R = B\ell v/R = (0.4\text{ T } 0.8\text{ m } 15\text{ m/s})/48\ \Omega = \boxed{100\text{ mA}}$

(c) The magnetic field exerts a backward magnetic force on the induced current. With the values in (b), this force is $I\ell B = 0.1\text{ A } 0.8\text{ m } 0.4\text{ T} = 0.032\text{ N}$, much less than 0.6 N. The speed of the bar increases until the backward magnetic force exerted on the current in the bar is equal to 0.6 N. The terminal speed is given by $0.6\text{ N} = I\ell B = (B\ell)^2v/R$.

Then $v = 0.6\text{ N } R/(B\ell)^2 = 0.6\text{ N } 48\ \Omega/(0.4\text{ T } 0.8\text{ m})^2 = \boxed{281\text{ m/s}}$.

(d) $Fv = 0.6\text{ N } 281\text{ m/s} = \boxed{169\text{ W}}$

(e) The terminal speed becomes $\boxed{\text{larger}}$. The bar must move faster to generate a larger emf to produce enough current in the larger resistance to feel the 0.6-N magnetic force.

(f) The power delivered to the circuit by the agent moving the bar, and then converted into internal energy by the resistor, is described by $\mathcal{P} = Fv = F^2R/B^2\ell^2$. Thus the power is directly proportional to the resistance and becomes $\boxed{\text{larger}}$ as the bulb heats up.

P31.45 $\mathcal{E} = -N\frac{d}{dt}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0°\left(\frac{dB}{dt}\right)$

$$\mathcal{E} = -(30.0)\left[\pi\left(2.70\times10^{-3}\text{ m}\right)^2\right](1)\frac{d}{dt}\left[50.0\text{ mT} + (3.20\text{ mT})\sin\left(2\pi\left[523t\text{ s}^{-1}\right]\right)\right]$$

$$\mathcal{E} = -(30.0)\left[\pi\left(2.70\times10^{-3}\text{ m}\right)^2\right]\left(3.20\times10^{-3}\text{ T}\right)\left[2\pi\left(523\text{ s}^{-1}\right)\cos\left(2\pi\left[523t\text{ s}^{-1}\right]\right)\right]$$

$$\mathcal{E} = \boxed{-\left(7.22\times10^{-3}\text{ V}\right)\cos\left[2\pi\left(523t\text{ s}^{-1}\right)\right]}$$

P31.46 $\mathcal{E} = -N\dfrac{\Delta}{\Delta t}(BA\cos\theta) = -N\left(\pi r^2\right)\cos°\dfrac{\Delta B}{\Delta t} = -1\left(0.005\ 00\ \text{m}^2\right)(1)\left(\dfrac{1.50\ \text{T} - 5.00\ \text{T}}{20.0\times10^{-3}\ \text{s}}\right) = 0.875\ \text{V}$

(a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.875\ \text{V}}{0.020\ 0\ \Omega} = \boxed{43.8\ \text{A}}$

(b) $\mathcal{P} = \mathcal{E}I = (0.875\ \text{V})(43.8\ \text{A}) = \boxed{38.3\ \text{W}}$

P31.47 (a) Doubling the number of turns has this effect:

$\boxed{\text{Amplitude doubles and period unchanged}}$

(b) Doubling the angular velocity has this effect:

$\boxed{\text{doubles the amplitude and cuts the period in half}}$

(c) Doubling the angular velocity while reducing the number of turns to one half the original value has this effect:

$\boxed{\text{Amplitude unchanged and period is cut in half}}$

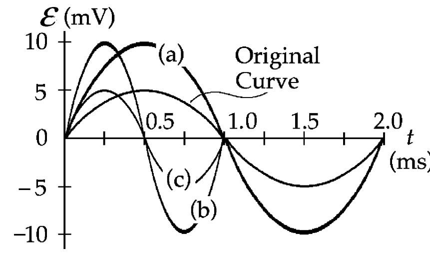


FIG. P31.47

P31.48 In the loop on the left, the induced emf is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.100\ \text{m})^2(100\ \text{T/s}) = \pi\ \text{V}$$

and it attempts to produce a counterclockwise current in this loop.

In the loop on the right, the induced emf is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi(0.150\ \text{m})^2(100\ \text{T/s}) = 2.25\pi\ \text{V}$$

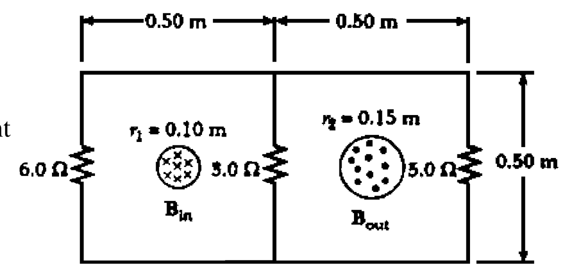


FIG. P31.48

and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00-Ω resistor, I_2 flows down through the 5.00-Ω resistor, and that I_3 flows up through the 3.00-Ω resistor.

From Kirchhoff's junction rule: $I_3 = I_1 + I_2$ (1)

Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$ (2)

Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ (3)

Solving these three equations simultaneously, $I_3 = (\pi - 3I_3)/6 + (2.25\pi - 3I_3)/5$

$I_1 = \boxed{0.062\ 3\ \text{A}}$, $I_2 = \boxed{0.860\ \text{A}}$, and $I_3 = \boxed{0.923\ \text{A}}$.

P31.49 The emf induced between the ends of the moving bar is

$$\mathcal{E} = B\ell v = (2.50\ \text{T})(0.350\ \text{m})(8.00\ \text{m/s}) = 7.00\ \text{V}$$

The left-hand loop contains decreasing flux away from you, so the induced current in it will be clockwise, to produce its own field directed away from you. Let I_1 represent the current flowing upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let I_3 be the upward current in the 5.00-Ω resistor.

(a) Kirchhoff's loop rule then gives: $+7.00\ \text{V} - I_1(2.00\ \Omega) = 0$ $\qquad I_1 = \boxed{3.50\ \text{A}}$

and $+7.00\ \text{V} - I_3(5.00\ \Omega) = 0$ $\qquad I_3 = \boxed{1.40\ \text{A}}$

(b) The total power converted in the resistors of the circuit is

$$\mathcal{P} = \mathcal{E}I_1 + \mathcal{E}I_3 = \mathcal{E}(I_1 + I_3) = (7.00\ \text{V})(3.50\ \text{A} + 1.40\ \text{A}) = \boxed{34.3\ \text{W}}$$

(c) *Method 1*: The current in the sliding conductor is downward with value $I_2 = 3.50\ \text{A} + 1.40\ \text{A} = 4.90\ \text{A}$. The magnetic field exerts a force of $F_m = I\ell B = (4.90\ \text{A})(0.350\ \text{m})(2.50\ \text{T}) = 4.29\ \text{N}$ directed toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\ \text{N}}$ to the left to keep the bar moving.

Method 2: The agent moving the bar must supply the power according to $\mathcal{P} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = Fv\cos 0°$. The force required is then:

$$F = \frac{\mathcal{P}}{v} = \frac{34.3\ \text{W}}{8.00\ \text{m/s}} = \boxed{4.29\ \text{N}}$$

P31.50 $I = \dfrac{\mathcal{E} + \mathcal{E}_{\text{induced}}}{R}$ and $\mathcal{E}_{\text{induced}} = -\dfrac{d}{dt}(BA)$

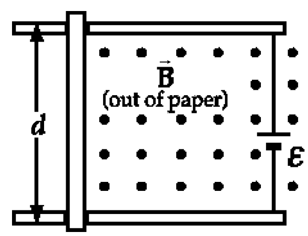


FIG. P31.50

$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}(\mathcal{E} + \mathcal{E}_{\text{induced}})$$

$$\frac{dv}{dt} = \frac{Bd}{mR}(\mathcal{E} - Bvd)$$

To solve the differential equation, let $u = \mathcal{E} - Bvd$

$$\frac{du}{dt} = -Bd\frac{dv}{dt}$$

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u$$

so $$\int_{u_0}^{u}\frac{du}{u} = -\int_0^t \frac{(Bd)^2}{mR}dt$$

Integrating from $t = 0$ to $t = t$, $$\ln\frac{u}{u_0} = -\frac{(Bd)^2}{mR}t$$

or $$\frac{u}{u_0} = e^{-B^2d^2t/mR}$$

Since $v = 0$ when $t = 0$, $u_0 = \mathcal{E}$

and $u = \mathcal{E} - Bvd$

$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR}$$

Therefore, $$\boxed{v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$$

P31.51 Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field 10^{-3} T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in 10^{-1} s. The average induced emf is then

$$\bar{\mathcal{E}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB(\pi r^2)\left(\frac{\cos 180° - \cos 0°}{\Delta t}\right)$$

$$\bar{\mathcal{E}} = -(20)(10^{-3}\text{ T})\pi(0.0150\text{ m})^2\left(\frac{-2}{10^{-1}\text{ s}}\right) \boxed{\sim 10^{-4}\text{ V}}$$

P31.52 (a) $I = \frac{dq}{dt} = \frac{\mathcal{E}}{R}$ where $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ so $\int dq = \frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge passing any point in the circuit will be $|Q| = \frac{N}{R}(\Phi_2 - \Phi_1)$.

(b) $Q = \frac{N}{R}\left[BA\cos 0 - BA\cos\left(\frac{\pi}{2}\right)\right] = \frac{BAN}{R}$

so $B = \frac{RQ}{NA} = \frac{(200\ \Omega)(5.00\times 10^{-4}\text{ C})}{(100)(40.0\times 10^{-4}\text{ m}^2)} = \boxed{0.250\text{ T}}$

P31.53 $I = \frac{\mathcal{E}}{R} = \frac{B}{R}\frac{|A|}{\Delta t}$

so $q = I\Delta t = \frac{(15.0\ \mu\text{T})(0.200\text{ m})^2}{0.500\ \Omega} = \boxed{1.20\ \mu\text{C}}$

P31.54 The enclosed flux is $\Phi_B = BA = B\pi r^2$

The particle moves according to $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$: $qvB\sin 90° = \frac{mv^2}{r}$

$$r = \frac{mv}{qB}$$

Then $\Phi_B = \frac{B\pi m^2v^2}{q^2B^2}$

(a) $v = \sqrt{\frac{\Phi_B q^2 B}{\pi m^2}} = \sqrt{\frac{(15\times 10^{-6}\text{ T}\cdot\text{m}^2)(30\times 10^{-9}\text{ C})^2(0.6\text{ T})}{\pi(2\times 10^{-16}\text{ kg})^2}} = \boxed{2.54\times 10^5\text{ m/s}}$

(b) Energy for the particle-electric field system is conserved in the firing process:

$U_i = K_f$: $q\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2q} = \frac{(2\times 10^{-16}\text{ kg})(2.54\times 10^5\text{ m/s})^2}{2(30\times 10^{-9}\text{ C})} = \boxed{215\text{ V}}$$

P31.55 (a) $\mathcal{E} = B\ell v = 0.360\text{ V}$ $\quad I = \dfrac{\mathcal{E}}{R} = \boxed{0.900\text{ A}}$

(b) $F_B = I\ell B = \boxed{0.108\text{ N}}$

(c) Since the magnetic flux $\vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$ is in effect decreasing, the induced current flow through R is from b to a. $\boxed{\text{Point } b}$ is at higher potential.

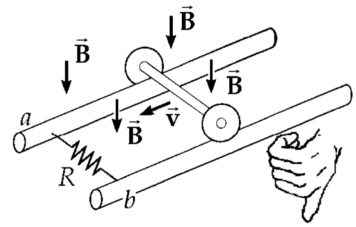


FIG. P31.55

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop to the left of ab. Here counterclockwise current will flow to produce upward magnetic field. The current in R is still from b to a.

P31.56 $\mathcal{E} = B\ell v$ at a distance r from wire

$$|\mathcal{E}| = \left(\frac{\mu_0 I}{2\pi r}\right)\ell v$$

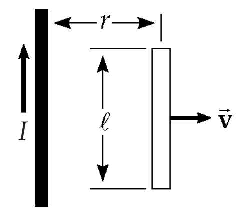


FIG. P31.56

P31.57 $\mathcal{E} = -\dfrac{d}{dt}(NBA) = -1\left(\dfrac{dB}{dt}\right)\pi a^2 = \pi a^2 K$

(a) $Q = C\mathcal{E} = \boxed{C\pi a^2 K}$

(b) $\vec{\mathbf{B}}$ into the paper is decreasing; therefore, current will attempt to counteract this. Positive charge will go to $\boxed{\text{upper plate}}$.

(c) The changing magnetic field through the enclosed area $\boxed{\text{induces an electric field}}$, surrounding the $\vec{\mathbf{B}}$-field, and this pushes on charges in the wire.

***P31.58** (a) We would need to know whether the field is increasing or decreasing.

(b) $\mathcal{P} = \mathcal{E}I = \mathcal{E}^2/R = \left(N\dfrac{dB}{dt}\pi r^2\cos 0°\right)^2 \Big/ R$

$$R = \left(N\frac{dB}{dt}\pi r^2\right)^2 \Big/ \mathcal{P} = \frac{\left[220(0.020\text{ T/s})\pi(0.12\text{ m})^2\right]^2}{160\text{ W}} = \boxed{248\ \mu\Omega}$$

Higher resistance would reduce the power delivered.

P31.59 The flux through the coil is $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta = BA\cos\omega t$. The induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d(\cos\omega t)}{dt} = NBA\omega\sin\omega t$$

(a) $\mathcal{E}_{\max} = NBA\omega = 60.0(1.00\text{ T})\left(0.100 \times 0.200\text{ m}^2\right)(30.0\text{ rad/s}) = \boxed{36.0\text{ V}}$

(b) $\dfrac{d\Phi_B}{dt} = \dfrac{\mathcal{E}}{N}$, thus $\left|\dfrac{d\Phi_B}{dt}\right|_{\max} = \dfrac{\mathcal{E}_{\max}}{N} = \dfrac{36.0\text{ V}}{60.0} = 0.600\text{ V} = \boxed{0.600\text{ Wb/s}}$

(c) At $t = 0.050\,0$ s, $\omega t = 1.50$ rad and

$$\mathcal{E} = \mathcal{E}_{\max}\sin(1.50\text{ rad}) = (36.0\text{ V})\sin(1.50\text{ rad}) = \boxed{35.9\text{ V}}$$

(d) The torque on the coil at any time is in magnitude

$$\tau = \left|\vec{\mu} \times \vec{\mathbf{B}}\right| = \left|NI\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = (NAB)I|\sin\omega t| = \left(\frac{\mathcal{E}_{\max}}{\omega}\right)\left(\frac{\mathcal{E}}{R}\right)|\sin\omega t|$$

When $\mathcal{E} = \mathcal{E}_{\max}$, $\sin\omega t = 1.00$ and $\tau = \dfrac{\mathcal{E}_{\max}^2}{\omega R} = \dfrac{(36.0\text{ V})^2}{(30.0\text{ rad/s})(10.0\ \Omega)} = \boxed{4.32\text{ N}\cdot\text{m}}$.

P31.60 (a) We use $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$, with $N = 1$.

Taking $a = 5.00\times10^{-3}$ m to be the radius of the washer, and $h = 0.500$ m,

$$\Delta\Phi_B = B_2A - B_1A = A(B_2 - B_1) = \pi a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right)$$

$$= \frac{a^2\mu_0 I}{2}\left(\frac{1}{h+a} - \frac{1}{a}\right) = \frac{-\mu_0 ahI}{2(h+a)}$$

The time for the washer to drop a distance h (from rest) is: $\Delta t = \sqrt{\frac{2h}{g}}$.

Therefore, $\mathcal{E} = \frac{\mu_0 ahI}{2(h+a)\Delta t} = \frac{\mu_0 ahI}{2(h+a)}\sqrt{\frac{g}{2h}} = \frac{\mu_0 aI}{2(h+a)}\sqrt{\frac{gh}{2}}$

and $\mathcal{E} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\times10^{-3}\ \text{m})(10.0\ \text{A})}{2(0.500\ \text{m} + 0.005\,00\ \text{m})}\sqrt{\frac{(9.80\ \text{m/s}^2)(0.500\ \text{m})}{2}}$

$= \boxed{97.4\ \text{nV}}$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a $\boxed{\text{clockwise direction}}$.

P31.61 Find an expression for the flux through a rectangular area "swept out" by the bar in time t. The magnetic field at a distance x from wire is

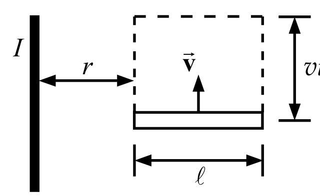


FIG. P31.61

$B = \frac{\mu_0 I}{2\pi x}$ and $\Phi_B = \int B\,dA$. Therefore,

$\Phi_B = \frac{\mu_0 I vt}{2\pi}\int_r^{r+\ell}\frac{dx}{x}$ where vt is the distance the bar has moved in time t.

Then, $|\mathcal{E}| = \frac{d\Phi_B}{dt} = \boxed{\frac{\mu_0 I v}{2\pi}\ln\left(1+\frac{\ell}{r}\right)}$.

P31.62 The magnetic field at a distance x from a long wire is $B = \frac{\mu_0 I}{2\pi x}$. We find an expression for the flux through the loop.

$$d\Phi_B = \frac{\mu_0 I}{2\pi x}(\ell dx) \quad \text{so} \quad \Phi_B = \frac{\mu_0 I\ell}{2\pi}\int_r^{r+w}\frac{dx}{x} = \frac{\mu_0 I\ell}{2\pi}\ln\left(1+\frac{w}{r}\right)$$

Therefore, $\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{\mu_0 I\ell v}{2\pi r}\frac{w}{(r+w)}$ and $I = \frac{\mathcal{E}}{R} = \boxed{\frac{\mu_0 I\ell v}{2\pi Rr}\frac{w}{(r+w)}}$

P31.63 We are given $\Phi_B = \left(6.00t^3 - 18.0t^2\right)\ \text{T}\cdot\text{m}^2$

and $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$

Maximum $\mathcal{E}$ occurs when $\dfrac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$

which gives $t = 1.00\ \text{s}$

Therefore, the maximum current (at $t = 1.00\ \text{s}$) is $I = \dfrac{\mathcal{E}}{R} = \dfrac{(-18.0 + 36.0)\ \text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$

P31.64 For the suspended mass, M: $\sum F = Mg - T = Ma$.

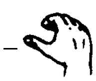
For the sliding bar, m: $\sum F = T - I\ell B = ma$, where $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$

$Mg - \dfrac{B^2\ell^2 v}{R} = (m+M)a$ or $a = \dfrac{dv}{dt} = \dfrac{Mg}{m+M} - \dfrac{B^2\ell^2 v}{R(M+m)}$

$\displaystyle\int_0^v \frac{dv}{(\alpha - \beta v)} = \int_0^t dt$ where $\alpha = \dfrac{Mg}{M+m}$ and $\beta = \dfrac{B^2\ell^2}{R(M+m)}$

Therefore, the velocity varies with time as $v = \dfrac{\alpha}{\beta}\left(1 - e^{-\beta t}\right) = \boxed{\dfrac{MgR}{B^2\ell^2}\left[1 - e^{-B^2\ell^2 t/R(M+m)}\right]}$

P31.65 Suppose the field is vertically down. When an electron is moving away from you the force on it is in the direction given by

$q\vec{\mathbf{v}} \times \vec{\mathbf{B}}_c$ as $-(\text{away}) \times \text{down} = -$  $= -\text{left} = \text{right}$.

Therefore, the electrons circulate clockwise.

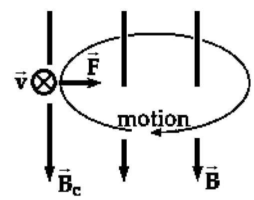


FIG. P31.65

(a) As the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed  counterclockwise, carried by negative electrons moving clockwise. Therefore the original electron motion speeds up.

(b) At the circumference, we have $\sum F_c = ma_c$: $|q|vB_c \sin 90° = \dfrac{mv^2}{r}$

$mv = |q|rB_c$

The increasing magnetic field $\vec{\mathbf{B}}_{\text{av}}$ in the area enclosed by the orbit produces a tangential electric field according to

$\left|\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}\right| = \left|-\dfrac{d}{dt}\vec{\mathbf{B}}_{av}\cdot\vec{\mathbf{A}}\right| \qquad E(2\pi r) = \pi r^2 \dfrac{dB_{av}}{dt} \qquad E = \dfrac{r}{2}\dfrac{dB_{av}}{dt}$

An electron feels a tangential force according to $\sum F_t = ma_t$: $|q|E = m\dfrac{dv}{dt}$

Then $|q|\dfrac{r}{2}\dfrac{dB_{av}}{dt} = m\dfrac{dv}{dt}$ $\qquad |q|\dfrac{r}{2}B_{av} = mv = |q|rB_c$

and $B_{av} = 2B_c$

***P31.66** (a) The induced emf is $\mathcal{E} = B\ell v$ where $B = \frac{\mu_0 I}{2\pi y}$, $v_f = v_i + gt = (9.80\ \text{m/s}^2)t$, and

$$y_f = y_i - \frac{1}{2}gt^2 = 0.800\ \text{m} - (4.90\ \text{m/s}^2)t^2$$

$$\mathcal{E} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(200\ \text{A})}{2\pi\left[0.800\ \text{m} - (4.90\ \text{m/s}^2)t^2\right]}(0.300\ \text{m})(9.80\ \text{m/s}^2)t = \boxed{\frac{(1.18 \times 10^{-4})t}{\left[0.800 - 4.90t^2\right]}\ \text{V}}$$

(b) The emf is zero at $t = 0$.

(c) The emf diverges to infinity at 0.404 s.

(d) At $t = 0.300$ s, $\mathcal{E} = \frac{(1.18 \times 10^{-4})(0.300)}{\left[0.800 - 4.90(0.300)^2\right]}\ \text{V} = \boxed{98.3\ \mu\text{V}}$.

P31.67 The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \frac{\mu_0 I}{2\pi r}$. Thus, the flux linkage is

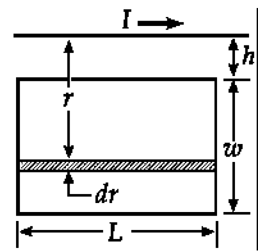


FIG. P31.67

$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{\max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t + \phi)$$

Finally, the induced emf is

$$\mathcal{E} = -\frac{\mu_0 NI_{\max}L\omega}{2\pi}\ln\left(1 + \frac{w}{h}\right)\cos(\omega t + \phi)$$

$$\mathcal{E} = -\frac{(4\pi \times 10^{-7})(100)(50.0)(0.200\ \text{m})(200\pi\ \text{s}^{-1})}{2\pi}\ln\left(1 + \frac{5.00\ \text{cm}}{5.00\ \text{cm}}\right)\cos(\omega t + \phi)$$

$$\mathcal{E} = \boxed{-(87.1\ \text{mV})\cos(200\pi t + \phi)}$$

The term $\sin(\omega t + \phi)$ in the expression for the current in the straight wire does not change appreciably when ωt changes by 0.10 rad or less. Thus, the current does not change appreciably during a time interval

$$\Delta t < \frac{0.10}{(200\pi\ \text{s}^{-1})} = 1.6 \times 10^{-4}\ \text{s}$$

We define a critical length $c\Delta t = (3.00 \times 10^8\ \text{m/s})(1.6 \times 10^{-4}\ \text{s}) = 4.8 \times 10^4\ \text{m}$ equal to the distance to which field changes could be propagated during an interval of 1.6×10^{-4} s. This length is so much larger than any dimension of the coil or its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the angular frequency ω were much larger, say, $200\pi \times 10^5\ \text{s}^{-1}$, the corresponding critical length would be only 48 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a general rule we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \frac{\omega}{2\pi}$, that are less than about 10^6 Hz.

ANSWERS TO EVEN PROBLEMS

P31.2 0.800 mA

P31.4 (a) If the meter has a sufficiently short response time, it will register the current. The average current may even run the meter offscale by a factor of 4.42, so you might wish to slow down the motion of the coil. (b) Positive. The coil sees decreasing external magnetic flux toward you, so it makes some flux of its own in this direction by carrying counterclockwise current, that enters the red terminal of the ammeter.

P31.6 $-10.2\ \mu\text{V}$

P31.8 (a) $(\mu_0 IL/2\pi)\ln(1 + w/h)$ (b) $-4.80\ \mu\text{V}$; current is counterclockwise

P31.10 (a) $\dfrac{\mu_0 n\pi r_2^2}{2R}\dfrac{\Delta I}{\Delta t}$ counterclockwise (b) $\dfrac{\mu_0^2 n\pi r_2^2}{4r_1 R}\dfrac{\Delta I}{\Delta t}$ (c) left

P31.12 (a) 5.33 V (b) A flat compact circular coil with 130 turns and radius 40.0 cm carries current 3.00 A counterclockwise. The current is smoothly reversed to become 3.00 A clockwise after 13.0 μs. At the center of this primary coil is a secondary coil in the same plane, with 20 turns and radius 3.00 cm. Find the emf induced in the secondary.

P31.14 61.8 mV

P31.16 (a) See the solution. (b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

P31.18 $(0.422\text{ V})\cos\omega t$

P31.20 (a) eastward (b) $458\ \mu\text{V}$

P31.22 1.00 m/s

P31.24 (a) 500 mA (b) 2.00 W (c) 2.00 W (d) They are physically identical.

P31.26 (a) 233 Hz (b) 1.98 mV

P31.28 (a) to the right (b) out of the plane of the paper (c) to the right (d) into the paper

P31.30 Negative; see the solution.

P31.32 (a) 8.00×10^{-21} N downward perpendicular to r_1 (b) 1.33 s

P31.34 (a) $(9.87\text{ mV/m})\cos(100\pi t)$ (b) clockwise

P31.36 (a) $(19.6\text{ V})\sin(314t)$ (b) 19.6 V

P31.38 See the solution.

P31.40 (a) 8.00 Wb cos($377t$) (b) 3.02 V sin($377t$) (c) 3.02 A sin($377t$) (d) 9.10 W$\sin^2(377t)$ (e) 24.1 mN·m $\sin^2(377t)$

P31.42 (a) $\dfrac{\pi B_{\max}^2 R^4 b\omega^2}{16\rho}$ (b) 4 times larger (c) 4 times larger (d) 16 times larger

P31.44 (a) counterclockwise (b) 100 mA (c) The speed of the bar increases until the backward magnetic force exerted on the current in the bar is equal to 0.6 N. The terminal speed is 281 m/s (d) 169 W (e) The terminal speed becomes larger. The bar must move faster to generate a larger emf to produce enough current in the larger resistance to feel the 0.6-N magnetic force. (f) The power delivered to the circuit by the agent moving the bar, and then converted into internal energy by the resistor, is described by $\mathcal{P} = Fv = F^2R/B^2\ell^2$. Thus the power is directly proportional to the resistance.

P31.46 (a) 43.8 A (b) 38.3 W

P31.48 62.3 mA down through 6.00 Ω, 860 mA down through 5.00 Ω, 923 mA up through 3.00 Ω

P31.50 See the solution.

P31.52 (a) See the solution. (b) 0.250 T

P31.54 (a) 254 km/s (b) 215 V

P31.56 See the solution.

P31.58 (a) We would need to know whether the field is increasing or decreasing. (b) 248 $\mu\Omega$. Higher resistance would reduce the power delivered.

P31.60 (a) 97.4 nV (b) clockwise

P31.62 $\dfrac{\mu_0 I\ell v}{2\pi Rr}\dfrac{w}{(r+w)}$

P31.64 $\dfrac{MgR}{B^2\ell^2}\left[1-e^{-B^2\ell^2 t/R(M+m)}\right]$

P31.66 (a) $118t$ μV/($0.8 - 4.9t^2$) where t is in seconds. (b) The emf is zero at $t = 0$. (c) The emf diverges to infinity at 0.404. (d) 98.3 μV