11

Angular Momentum

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q11.1 No to both questions. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis, so the value of the torque depends on the location of the axis.

***Q11.2** (i) Down–cross–left is away from you: $-\hat{\mathbf{j}}\times\left(-\hat{\mathbf{i}}\right)=-\hat{\mathbf{k}}$ answer (f), as in the first picture.

(ii) Left–cross–down is toward you: $-\hat{\mathbf{i}}\times\left(-\hat{\mathbf{j}}\right)=\hat{\mathbf{k}}$ answer (e), as in the second picture.

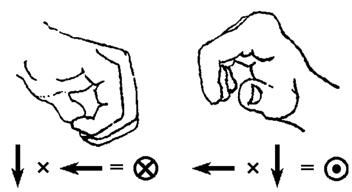

FIG. Q11.2

***Q11.3** (3 m down) $\times$ (2 N toward you) = 6 N $\cdot$ m left. The answers are (i) a (ii) a (iii) f

***Q11.4** The unit vectors have magnitude 1, so the magnitude of each cross product is $|1 \cdot 1 \cdot \sin\theta|$ where θ is the angle between the factors. Thus for (a) the magnitude of the cross product is $\sin 0° = 0$. For (b), $|\sin 135°| = 0.707$ (c) $\sin 90° = 1$ (d) $\sin 45° = 0.707$ (e) $\sin 90° = 1$. The assembled answer is c = e > b = d > a = 0.

Q11.5 Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.

Q11.6 No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

***Q11.7** (a) Yes. Rotational kinetic energy is one contribution to a system's total energy.

(b) No. Pulling down on one side of a steering wheel and pushing up equally hard on the other side causes a total torque on the wheel with zero total force.

(c) No. A top spinning with its center of mass on a fixed axis has angular momentum with no momentum. A car driving straight toward a light pole has momentum but no angular momentum about the axis of the pole.

Q11.8 The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.

***Q11.9** Her angular momentum stays constant as I is cut in half and ω doubles. Then $(1/2)I\omega^2$ doubles. Answer (b).

Q11.10 Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

As the source reel radius R shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed v constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2. Thus, on the whole the moment of inertia is proportional to R^4. The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to R^4, will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

***Q11.11** The angular momentum of the mouse-turntable system is initially zero, with both at rest. The frictionless axle isolates the mouse-turntable system from outside torques, so its angular momentum must stay constant with the value zero.

(i) The mouse makes some progress north, or counterclockwise. Answer (a).

(ii) The turntable will rotate clockwise. The turntable rotates in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero. Answer (b).

(iii) No, mechanical energy changes as the mouse converts some chemical into mechanical energy, positive for the motions of both the mouse and the turntable.

(iv) No, momentum is not conserved. The turntable has zero momentum while the mouse has a bit of northward momentum. The sheave around the turntable axis exerts a force northward to feed in this momentum.

(v) Yes, angular momentum is constant with the value zero.

***Q11.12** (i) The angular momentum is constant. The moment of inertia decreases, so the angular speed must increase. Answer (a).

(ii) No, mechanical energy increases. The ponies must do work to push themselves inward.

(iii) Yes, momentum stays constant with the value zero.

(iv) Yes, angular momentum is constant with a nonzero value.

***Q11.13** Angular momentum is conserved according to the equation $I_1\omega_0 + 0 = (I_1 + I_2)\omega_f$. Solving for ω_f gives answer (c).

Q11.14 Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

Q11.15 Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases. We would not have more hours in a day, but more nanoseconds.

Q11.16 The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will want to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

SOLUTIONS TO PROBLEMS

Section 11.1 The Vector Product and Torque

P11.1 $$\vec{\mathbf{M}}\times\vec{\mathbf{N}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = \hat{\mathbf{i}}(-6-1) + \hat{\mathbf{j}}(-2+18) + \hat{\mathbf{k}}(-6-4) = \boxed{-7.00\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}} - 10.0\hat{\mathbf{k}}}$$

P11.2 (a) $$\text{area} = \left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0° - 15.0°) = \boxed{740\text{ cm}^2}$$

(b) $$\vec{\mathbf{A}}+\vec{\mathbf{B}} = \left[(42.0\text{ cm})\cos 15.0° + (23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}$$
$$+\left[(42.0\text{ cm})\sin 15.0° + (23.0\text{ cm})\sin 65.0°\right]\hat{\mathbf{j}}$$
$$\vec{\mathbf{A}}+\vec{\mathbf{B}} = (50.3\text{ cm})\hat{\mathbf{i}} + (31.7\text{ cm})\hat{\mathbf{j}}$$
$$\text{length} = \left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$$

P11.3 (a) $$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \boxed{-17.0\hat{\mathbf{k}}}$$

(b) $$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|\sin\theta$$
$$17 = 5\sqrt{13}\sin\theta$$
$$\theta = \sin^{-1}\left(\frac{17}{5\sqrt{13}}\right) = \boxed{70.6°}$$

P11.4 $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = -3.00(6.00)+7.00(-10.0)+(-4.00)(9.00) = -124$

$$AB = \sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2} = 127$$

(a) $\cos^{-1}\left(\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}(-0.979) = \boxed{168°}$

(b) $\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3.00 & 7.00 & -4.00 \\ 6.00 & -10.0 & 9.00 \end{vmatrix} = 23.0\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-12.0\hat{\mathbf{k}}$

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \sqrt{(23.0)^2+(3.00)^2+(-12.0)^2} = 26.1$$

$$\sin^{-1}\left(\frac{\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|}{AB}\right) = \sin^{-1}(0.206) = \boxed{11.9°} \text{ or } 168°$$

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

P11.5 $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}$

$= 0.450\text{ m}(0.785\text{ N})\sin(90°-14°)\,\text{up}\times\text{east}$

$= \boxed{0.343\text{ N}\cdot\text{m} \quad \text{horizontally north}}$

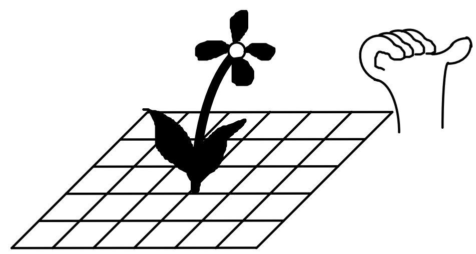

FIG. P11.5

P11.6 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}) = 0$?

We have $8-9-4 = -5 \neq 0$ so the answer is

$\boxed{\text{No. The cross product could not work out that way.}}$

P11.7 $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}} \quad\Rightarrow\quad AB\sin\theta = AB\cos\theta \quad\Rightarrow\quad \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

***P11.8** (a) $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 6 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(8-18) = \boxed{(-10.0\ \text{N}\cdot\text{m})\hat{\mathbf{k}}}$

(b) Yes. The line of action of the force is the dashed line in the diagram. The point or axis must be on the other side of the line of action, and half as far from this line along which the force acts. Then the lever arm of the force about this new axis will be half as large and the force will produce counterclockwise instead of clockwise torque. There are infinitely many such points, along the dotted line in the diagram. But the locus of these points intersects the y axis in only one point, which we now determine.

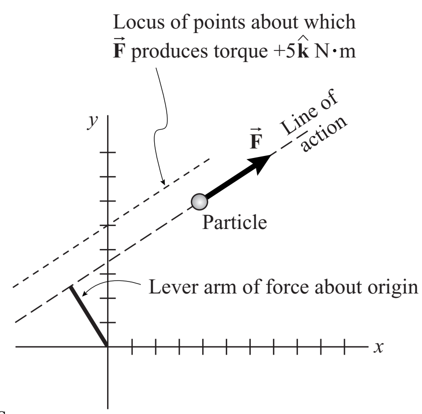

FIG. P11.8(b)

Let (0, y) represent the coordinates of the special axis of rotation located on the y axis of coordinates. Then the displacement from this point to the particle feeling the force is $\vec{\mathbf{r}}_{new} = 4\hat{\mathbf{i}} + (6-y)\hat{\mathbf{j}}$ meters. The torque of the force about this new axis is

$$\vec{\boldsymbol{\tau}}_{new} = \vec{\mathbf{r}}_{new} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 6-y & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(8-18+3y) = (+5\ \text{N}\cdot\text{m})\hat{\mathbf{k}}$$

Then we need only $-10 + 3y = 5$ $\quad y = 5$ m. The position vector of the new axis is $\boxed{5.00\hat{\mathbf{j}}\ \text{m}}$.

P11.9 $\boxed{\left|\vec{\mathbf{F}}_3\right| = \left|\vec{\mathbf{F}}_1\right| + \left|\vec{\mathbf{F}}_2\right|}$

The torque produced by $\vec{\mathbf{F}}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\vec{\mathbf{F}}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

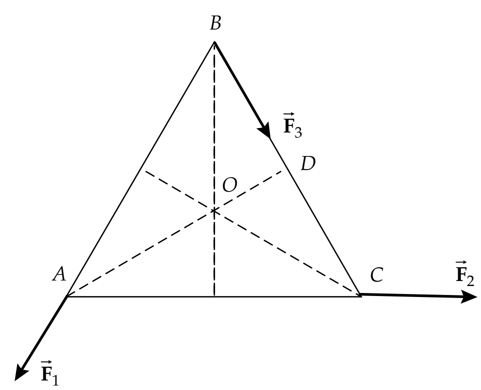

FIG. P11.9

P11.10 $\left|\hat{\mathbf{i}} \times \hat{\mathbf{i}}\right| = 1 \cdot 1 \cdot \sin 0° = 0$

$\hat{\mathbf{j}} \times \hat{\mathbf{j}}$ and $\hat{\mathbf{k}} \times \hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$\left|\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right| = 1 \cdot 1 \cdot \sin 90° = 1$

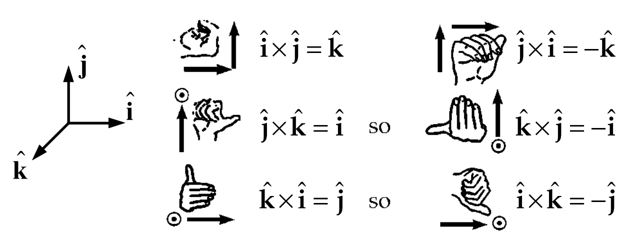

FIG. P11.10

Section 11.2 Angular Momentum

P11.11 $L = \sum m_i v_i r_i$

$$= (4.00\ \text{kg})(5.00\ \text{m/s})(0.500\ \text{m}) + (3.00\ \text{kg})(5.00\ \text{m/s})(0.500\ \text{m})$$

$L = 17.5\ \text{kg}\cdot\text{m}^2/\text{s}$, and

$$\boxed{\vec{\mathbf{L}} = \left(17.5\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

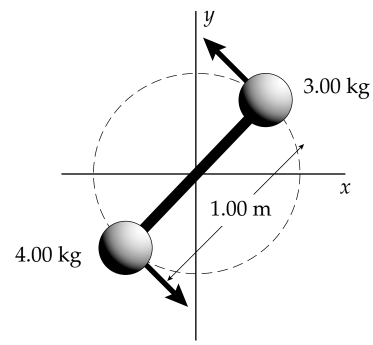

FIG. P11.11

P11.12 $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$

$$\vec{\mathbf{L}} = \left(1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right)\ \text{m} \times (1.50\ \text{kg})\left(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\vec{\mathbf{L}} = \left(-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m}^2/\text{s} = \boxed{\left(-22.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.13 $\vec{\mathbf{r}} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\ \text{m}\right)$ $\qquad \vec{\mathbf{v}} = \dfrac{d\vec{\mathbf{r}}}{dt} = 5.00\hat{\mathbf{j}}\ \text{m/s}$

so

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = 2.00\ \text{kg}\left(5.00\hat{\mathbf{j}}\ \text{m/s}\right) = 10.0\hat{\mathbf{j}}\ \text{kg}\cdot\text{m/s}$$

and

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.14 $\sum F_x = ma_x \qquad T\sin\theta = \dfrac{mv^2}{r}$

$\sum F_y = ma_y \qquad T\cos\theta = mg$

So

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg} \qquad v = \sqrt{rg\frac{\sin\theta}{\cos\theta}}$$

$$L = rmv\sin 90.0°$$

$$L = rm\sqrt{rg\frac{\sin\theta}{\cos\theta}}$$

$$L = \sqrt{m^2 g r^3 \frac{\sin\theta}{\cos\theta}}$$

$r = \ell\sin\theta$, so

$$L = \boxed{\sqrt{m^2 g \ell^3 \frac{\sin^4\theta}{\cos\theta}}}$$

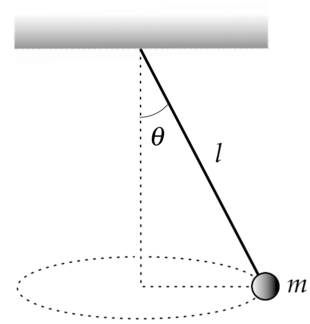

FIG. P11.14

P11.15 The angular displacement of the particle around the circle is $\theta = \omega t = \frac{vt}{R}$.

The vector from the center of the circle to the mass is then $R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$.

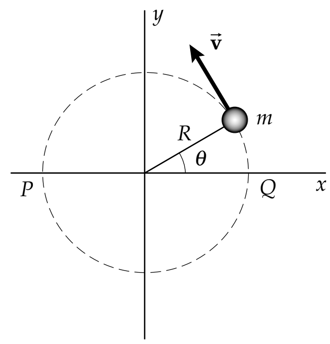

FIG. P11.15

The vector from point P to the mass is

$$\vec{\mathbf{r}} = R\hat{\mathbf{i}} + R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}} = R\left[\left(1+\cos\left(\frac{vt}{R}\right)\right)\hat{\mathbf{i}} + \sin\left(\frac{vt}{R}\right)\hat{\mathbf{j}}\right]$$

The velocity is

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = -v\sin\left(\frac{vt}{R}\right)\hat{\mathbf{i}} + v\cos\left(\frac{vt}{R}\right)\hat{\mathbf{j}}$$

So

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = mvR\left[(1+\cos\omega t)\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right] \times \left[-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{L}} = \boxed{mvR\hat{\mathbf{k}}\left[\cos\left(\frac{vt}{R}\right)+1\right]}$$

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$|\vec{\tau}| = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = 8.00\times10^{-2}\ \text{m}(4.00\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{3.14\ \text{N}\cdot\text{m}}$$

(b) $\left|\vec{\mathbf{L}}\right| = \left|\vec{\mathbf{r}} \times m\vec{\mathbf{v}}\right| + I\omega$ $\qquad \left|\vec{\mathbf{L}}\right| = Rmv + \frac{1}{2}MR^2\left(\frac{v}{R}\right) = R\left(m + \frac{M}{2}\right)v = \boxed{(0.400\ \text{kg}\cdot\text{m})v}$

(c) $\tau = \frac{dL}{dt} = (0.400\ \text{kg}\cdot\text{m})a$ $\qquad a = \frac{3.14\ \text{N}\cdot\text{m}}{0.400\ \text{kg}\cdot\text{m}} = \boxed{7.85\ \text{m/s}^2}$

P11.17 (a) $\boxed{\text{zero}}$

(b) At the highest point of the trajectory,

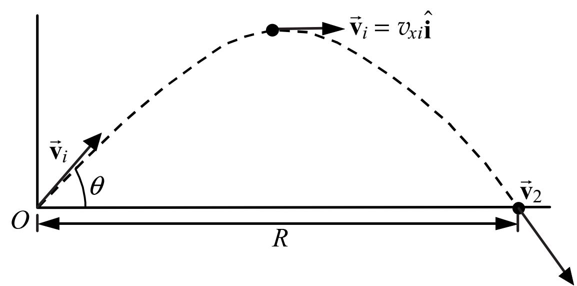

FIG. P11.17

$$x = \frac{1}{2}R = \frac{v_i^2\sin 2\theta}{2g} \quad \text{and}$$

$$y = h_{\max} = \frac{(v_i\sin\theta)^2}{2g}$$

$$\vec{\mathbf{L}}_1 = \vec{\mathbf{r}}_1 \times m\vec{\mathbf{v}}_1$$

$$= \left[\frac{v_i^2\sin 2\theta}{2g}\hat{\mathbf{i}} + \frac{(v_i\sin\theta)^2}{2g}\hat{\mathbf{j}}\right] \times mv_{xi}\hat{\mathbf{i}}$$

$$= \boxed{\frac{-m(v_i\sin\theta)^2 v_i\cos\theta}{2g}\hat{\mathbf{k}}}$$

continued on next page

(c) $\vec{\mathbf{L}}_2 = R\hat{\mathbf{i}} \times m\vec{\mathbf{v}}_2$, where $R = \dfrac{v_i^2 \sin 2\theta}{g}$

$$= mR\hat{\mathbf{i}} \times \left(v_i \cos\theta\,\hat{\mathbf{i}} - v_i \sin\theta\,\hat{\mathbf{j}}\right)$$

$$= -mRv_i \sin\theta\,\hat{\mathbf{k}} = \boxed{\frac{-mv_i^3 \sin 2\theta \sin\theta}{g}\hat{\mathbf{k}}}$$

(d) $\boxed{\text{The downward force of gravity exerts a torque in the } -z \text{ direction.}}$

P11.18 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

(a) $\vec{\mathbf{r}} = (4.30\text{ km})\hat{\mathbf{k}} = \left(4.30 \times 10^3\text{ m}\right)\hat{\mathbf{k}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = 12\,000\text{ kg}\left(-175\hat{\mathbf{i}}\text{ m/s}\right) = -2.10 \times 10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \left(4.30 \times 10^3\hat{\mathbf{k}}\text{ m}\right) \times \left(-2.10 \times 10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}\right) = \boxed{\left(-9.03 \times 10^9\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}}$$

(b) $\boxed{\text{No.}}$ $L = |\vec{\mathbf{r}}||\vec{\mathbf{p}}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L = \textit{constant}$ as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero.}}$ The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L = mvr\sin 180° = 0$.

***P11.19** (a) The vector from P to the falling ball is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{r}} = \left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$

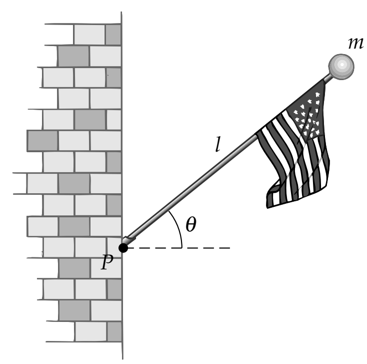

FIG. P11.19

The velocity of the ball is

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = 0 - gt\hat{\mathbf{j}}$$

So

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = m\left[\left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right] \times \left(-gt\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{L}} = \boxed{-m\ell gt\cos\theta\,\hat{\mathbf{k}}}$$

(b) $\boxed{\text{The Earth exerts a gravitational torque on the ball.}}$

(c) Differentiating with respect to time, we have $\boxed{-mg\ell\,\cos\theta\,\hat{\mathbf{k}}}$ for the rate of change of angular momentum, which is also the torque due to the gravitational force on the ball.

***P11.20** (a) $\int_0^{\vec{\mathbf{r}}} d\vec{\mathbf{r}} = \int_0^t \vec{\mathbf{v}}dt = \vec{\mathbf{r}} - 0 = \int_0^t (6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}})dt = \vec{\mathbf{r}} = \left(6t^3/3\right)\hat{\mathbf{i}} + \left(2t^2/2\right)\hat{\mathbf{j}}$

$= \boxed{2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}}$ meters, where t is in seconds

(b) The particle starts from rest at the origin, starts moving in the y direction, and gains speed faster and faster while turning to move more and more nearly parallel to the x axis.

(c) $\vec{\mathbf{a}} = (d/dt)(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = \boxed{(12t\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{m/s}^2}$

(d) $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = (5\ \text{kg})(12t\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{m/s}^2 = \boxed{(60t\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\ \text{N}}$

(e) $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = (2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}) \times (60t\hat{\mathbf{i}} + 10\hat{\mathbf{j}}) = 20t^3\hat{\mathbf{k}} - 60t^3\hat{\mathbf{k}} = \boxed{-40t^3\hat{\mathbf{k}}\ \text{N}\cdot\text{m}}$

(f) $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}} = (5\ \text{kg})(2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}) \times (6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = 5(4t^4\hat{\mathbf{k}} - 6t^4\hat{\mathbf{k}}) = \boxed{-10t^4\hat{\mathbf{k}}\ \text{kg}\cdot\text{m}^2/\text{s}}$

(g) $K = \frac{1}{2}m\vec{\mathbf{v}}\cdot\vec{\mathbf{v}} = \frac{1}{2}(5\ \text{kg})(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}})\cdot(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = (2.5)(36t^4 + 4t^2) = \boxed{(90t^4 + 10t^2)\ \text{J}}$

(h) $\mathcal{P} = (d/dt)(90t^4 + 10t^2)\ \text{J}\ \boxed{= (360t^3 + 20t)\ \text{W}}$, all where t is in seconds.

Section 11.3 **Angular Momentum of a Rotating Rigid Object**

P11.21 $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{L^2}{2I}$

P11.22 The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0\ \text{kg})(0.500\ \text{m})^2 = 1.50\ \text{kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \left(1.50\ \text{kg}\cdot\text{m}^2\right)(3.00\ \text{rad/s}) = 4.50\ \text{kg}\cdot\text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus,

$$\boxed{\vec{\mathbf{L}} = \left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.23 (a) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(3.00\ \text{kg})(0.200\ \text{m})^2(6.00\ \text{rad/s}) = \boxed{0.360\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$

$= \frac{3}{4}(3.00\ \text{kg})(0.200\ \text{m})^2(6.00\ \text{rad/s}) = \boxed{0.540\ \text{kg}\cdot\text{m}^2/\text{s}}$

***P11.24** (a) $I = (2/5)MR^2 = (2/5)(5.98 \times 10^{24}\text{ kg})(6.37 \times 10^6\text{ m})^2 = 9.71 \times 10^{37}\text{ kg}\cdot\text{m}^2$

$\omega = 1\text{ rev}/24\text{ h} = 2\pi\text{ rad}/86400\text{ s} = 7.27 \times 10^{-5}/\text{s}$

$L = I\omega = (9.71 \times 10^{37}\text{ kg}\cdot\text{m}^2)(7.27 \times 10^{-5}/\text{s}) = \boxed{7.06\times10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}}$

The earth turns toward the east, counterclockwise as seen from above north, so the vector angular momentum points north along the earth's axis, $\boxed{\text{toward the north celestial pole}}$ or nearly toward the star Polaris.

(b) $I = MR^2 = (5.98 \times 10^{24}\text{ kg})(1.496 \times 10^{11}\text{ m})^2 = 1.34 \times 10^{47}\text{ kg}\cdot\text{m}^2$

$\omega = 1\text{ rev}/365.25\text{ d} = 2\pi\text{ rad}/(365.25 \times 86400\text{ s}) = 1.99 \times 10^{-7}/\text{s}$

$L = I\omega = (1.34 \times 10^{47}\text{ kg}\cdot\text{m}^2)(1.99 \times 10^{-7}/\text{s}) = \boxed{2.66\times10^{40}\ \text{kg}\cdot\text{m}^2/\text{s}}$

The earth plods around the Sun, counterclockwise as seen from above north, so the vector angular momentum points north perpendicular to the plane of the ecliptic, $\boxed{\text{toward the north ecliptic pole}}$ or 23.5° away from Polaris, toward the center of the circle that the north celestial pole moves in as the equinoxes precess. The north ecliptic pole is in the constellation Draco.

(c) The earth is so far from the Sun that the $\boxed{\text{orbital angular momentum is much larger,}}$ $\boxed{\text{by } 3.78 \times 10^6 \text{ times.}}$

P11.25 (a) $I = \frac{1}{12}m_1L^2 + m_2(0.500)^2 = \frac{1}{12}(0.100)(1.00)^2 + 0.400(0.500)^2 = 0.108\,3\text{ kg}\cdot\text{m}^2$

$L = I\omega = 0.108\,3(4.00) = \boxed{0.433\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $I = \frac{1}{3}m_1L^2 + m_2R^2 = \frac{1}{3}(0.100)(1.00)^2 + 0.400(1.00)^2 = 0.433$

$L = I\omega = 0.433(4.00) = \boxed{1.73\ \text{kg}\cdot\text{m}^2/\text{s}}$

P11.26 The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with

$$I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$$

and

$$I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$$

In addition,

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\ \text{rad/s}$$

while

$$\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\ \text{rad/s}$$

Thus,

$$L = 146\text{ kg}\cdot\text{m}^2\left(1.45\times10^{-4}\ \text{rad/s}\right) + 675\text{ kg}\cdot\text{m}^2\left(1.75\times10^{-3}\ \text{rad/s}\right)$$

or $\boxed{L = 1.20\ \text{kg}\cdot\text{m}^2/\text{s}}$ The hands turn clockwise, so their vector angular momentum is $\boxed{\text{perpendicularly into the clock face}}$.

P11.27 We require $a_c = g = \frac{v^2}{r} = \omega^2 r$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.80\ \text{m/s}^2)}{100\ \text{m}}} = 0.313\ \text{rad/s}$$
$$I = Mr^2 = 5\times10^4\ \text{kg}(100\ \text{m})^2 = 5\times10^8\ \text{kg}\cdot\text{m}^2$$

(a) $L = I\omega = 5\times10^8\ \text{kg}\cdot\text{m}^2\ 0.313/\text{s} = \boxed{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $\sum\tau = I\alpha = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$\sum\tau\Delta t = I\omega_f - I\omega_i = L_f - L_i$

This is the angular impulse-angular momentum theorem.

(c) $\Delta t = \frac{L_f - 0}{\sum\tau} = \frac{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}{2(125\ \text{N})(100\ \text{m})} = \boxed{6.26\times10^3\ \text{s}} = 1.74\ \text{h}$

P11.28 $\sum F_x = ma_x$: $\quad +f_s = ma_x$

We must use the center of mass as the axis in

$\sum\tau = I\alpha$: $\quad F_g(0) - n(77.5\ \text{cm}) + f_s(88\ \text{cm}) = 0$

$\sum F_y = ma_y$: $\quad +n - F_g = 0$

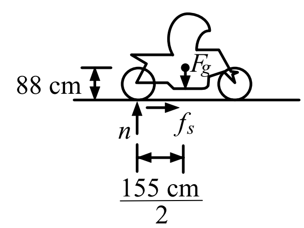

FIG. P11.28

We combine the equations by substitution:

$$-mg(77.5\ \text{cm}) + ma_x(88\ \text{cm}) = 0$$
$$a_x = \frac{(9.80\ \text{m/s}^2)77.5\ \text{cm}}{88\ \text{cm}} = \boxed{8.63\ \text{m/s}^2}$$

Section 11.4 **Conservation of Angular Momentum**

P11.29 (a) From conservation of angular momentum for the system of two cylinders:

$(I_1 + I_2)\omega_f = I_1\omega_i$ or $\omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2}\ \text{which is less than 1}}$$

***P11.30** (a) We choose to solve by conservation of angular momentum, because it would be true even if the rod had considerable mass:

$$I\omega_{initial} = I\omega_{final} \qquad mR^2(v/R)_i = (mR^2 + m_pR^2)(v/R)_f$$

$(2.4\text{ kg})(1.5\text{ m})(5\text{ m/s}) = (2.4 + 1.3)\text{kg}(1.5\text{ m})\, v_f \quad v_f = 3.24\text{ m/s} = 2\pi(1.5\text{ m})/T \quad T = \boxed{2.91\text{ s}}$

(b) Angular momentum of the puck-putty system is conserved because the pivot exerts no torque.

(c) If the putty-puck collision lasts so short a time that the puck slides through a negligibly small arc of the circle, then momentum is also conserved. But the pivot pin is always pulling on the rod to change the direction of the momentum.

(d) No. Some mechanical energy is converted into internal energy. The collision is perfectly inelastic.

***P11.31** (a) We solve by using conservation of angular momentum for the turntable-clay system, which is isolated from outside torques:

$$I\omega_{initial} = I\omega_{final} \qquad (1/2)mR^2(\omega)_i = [(1/2)mR^2 + m_cR^2)]\omega_f$$

$(1/2)(30\text{ kg})(1.9\text{ m})^2(4\pi\text{ /s}) = [(1/2)(30\text{ kg})(1.9\text{ m})^2 + (2.25\text{ kg})(1.8\text{ m})^2]\,\omega_f$

$(54.15)(4\pi) = (61.44)\omega_f \quad \omega_f = \boxed{11.1\text{ rad/s counterclockwise}}$

(b) $\boxed{\text{No.}}$ The "angular collision" is completely inelastic, so some mechanical energy is degraded into internal energy. The initial energy is $(1/2)I\omega_i^2 = (1/2)(54.15)(4\pi)^2 = 4276$ J. The final mechanical energy is $(1/2)(61.44)(11.1)^2 = 3768$ J. Thus $\boxed{\text{507 J of extra internal energy appears}}$.

(c) $\boxed{\text{No.}}$ The turntable bearing must exert an impulsive force toward the north.The original horizontal momentum is zero. As soon as the clay has stopped skidding on the turntable, the final momentum is $2.25\text{ kg }(1.8\text{ m})(11.1\text{ /s}) = \boxed{44.9\text{ kg}\cdot\text{m/s north. This is the amount of impulse injected by the bearing.}}$ The bearing thereafter keeps changing the system momentum to change the direction of the motion of the clay.

P11.32 (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2\left(mr^2\right) + 3.00\text{ kg}\cdot\text{m}^2$$

Before:

$$r = 1.00\text{ m}$$

Thus,

$$I_i = 2(3.00\text{ kg})(1.00\text{ m})^2 + 3.00\text{ kg}\cdot\text{m}^2 = 9.00\text{ kg}\cdot\text{m}^2$$

After:

$$r = 0.300\text{ m}$$

Thus,

$$I_f = 2(3.00\text{ kg})(0.300\text{ m})^2 + 3.00\text{ kg}\cdot\text{m}^2 = 3.54\text{ kg}\cdot\text{m}^2$$

We now use conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.00}{3.54}\right)(0.750\text{ rad/s}) = \boxed{1.91\text{ rad/s}}$$

continued on next page

(b) $K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2}\left(9.00 \text{ kg}\cdot\text{m}^2\right)\left(0.750 \text{ rad/s}\right)^2 = \boxed{2.53 \text{ J}}$

$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}\left(3.54 \text{ kg}\cdot\text{m}^2\right)\left(1.91 \text{ rad/s}\right)^2 = \boxed{6.44 \text{ J}}$

P11.33 $I_i\omega_i = I_f\omega_f$: $\left(250 \text{ kg}\cdot\text{m}^2\right)\left(10.0 \text{ rev/min}\right) = \left[250 \text{ kg}\cdot\text{m}^2 + 25.0 \text{ kg}\left(2.00 \text{ m}\right)^2\right]\omega_2$

$\omega_2 = \boxed{7.14 \text{ rev/min}}$

***P11.34** (a) Let M = mass of rod and m = mass of each bead. From $I_i\omega_i = I_f\omega_f$ between the moment of release and the moment the beads slide off, we have

$$\left[\frac{1}{12}M\ell^2 + 2mr_1^2\right]\omega_i = \left[\frac{1}{12}M\ell^2 + 2mr_2^2\right]\omega_f$$

When $M = 0.3$ kg, $\ell = 0.500$ m, $r_1 = 0.100$ m, $r_2 = 0.250$ m, $\omega_i = 36$/s, we find

$[0.00625 + 0.02\ m]36 = [0.00625 + 0.125\ m]\ \omega_f$

$\boxed{\omega_f = (36/\text{s})(1 + 3.2\ m)/(1 + 20\ m)}$

(b) The denominator of this fraction always exceeds the numerator, so

ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of $(36 \times 3.2/20) = 5.76$ rad/s as $m \to \infty$.

As a bonus, we find the work that the bar does on the beads as a function of m. Consider the beads alone. Their kinetic energy increases because of work done on them by the bar.

initial kinetic energy + work = final kinetic energy

$(1/2)(2mr_1^{\,2})(\omega_i)^2 + W_b = (1/2)(2mr_2^{\,2})(\omega_f)^2$

$m\ (0.1)^2(36)^2 + W_b = m\ (0.25)^2[(36/\text{s})(1 + 3.2\ m)/(1 + 20\ m)]^2$

$W_b = m[81(1 + 3.2m)^2 - 12.96(1 + 20m)^2]/\ (1 + 20\ m)^2$
$\quad = (68.04\ m)(1 - 64\ m^2)/(1 + 20\ m)^2$ joules

W_b increases from 0 for $m = 0$ toward a maximum value of about 0.8 J at about $m = 0.035$ kg, and then decreases and goes negative, diverging to $-\infty$ as $m \to \infty$.

***P11.35** (a) Mechanical energy is not conserved; some chemical energy is converted into mechanical energy. Momentum is not conserved. The turntable bearing exerts an external northward force on the axle. Angular momentum is conserved. The bearing isolates the system from outside torques. The table turns opposite to the way the woman walks, so its angular momentum cancels that of the woman.

continued on next page

(b) From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$

so,

$$L_f = I_{\text{woman}}\omega_{\text{woman}} + I_{\text{table}}\omega_{\text{table}} = 0$$

and

$$\omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}}\right)\omega_{\text{woman}} = \left(-\frac{m_{\text{woman}}r^2}{I_{\text{table}}}\right)\left(\frac{v_{\text{woman}}}{r}\right) = -\frac{m_{\text{woman}}rv_{\text{woman}}}{I_{\text{table}}}$$

$$\omega_{\text{table}} = -\frac{60.0\text{ kg}(2.00\text{ m})(1.50\text{ m/s})}{500\text{ kg}\cdot\text{m}^2} = -0.360\text{ rad/s}$$

or

$$\omega_{\text{table}} = \boxed{0.360\text{ rad/s (counterclockwise)}}$$

(c) chemical energy converted into mechanical $= \Delta K = K_f - 0 = \frac{1}{2}m_{\text{woman}}v_{\text{woman}}^2 + \frac{1}{2}I\omega_{\text{table}}^2$

$$\Delta K = \frac{1}{2}(60\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(500\text{ kg}\cdot\text{m}^2)(0.360\text{ rad/s})^2 = \boxed{99.9\text{ J}}$$

P11.36 When they touch, the center of mass is distant from the center of the larger puck by

$$y_{\text{CM}} = \frac{0 + 80.0\text{ g}(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a) $L = r_1m_1v_1 + r_2m_2v_2 = 0 + (6.00\times10^{-2}\text{ m})(80.0\times10^{-3}\text{ kg})(1.50\text{ m/s})$

$= \boxed{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1r_1^2 + m_1d_1^2\right) + \left(\frac{1}{2}m_2r_2^2 + m_2d_2^2\right)$$

$$I = \frac{1}{2}(0.120\text{ kg})(6.00\times10^{-2}\text{ m})^2 + (0.120\text{ kg})(4.00\times10^{-2})^2$$

$$+\frac{1}{2}(80.0\times10^{-3}\text{ kg})(4.00\times10^{-2}\text{ m})^2 + (80.0\times10^{-3}\text{ kg})(6.00\times10^{-2}\text{ m})^2$$

$$I = 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = \boxed{9.47\text{ rad/s}}$$

P11.37 (a) $L_i = mv\ell \qquad \sum \tau_{\text{ext}} = 0,$

so

$$L_f = L_i = \boxed{mv\ell}$$

$$L_f = (m+M)v_f\ell$$

$$v_f = \left(\frac{m}{m+M}\right)v$$

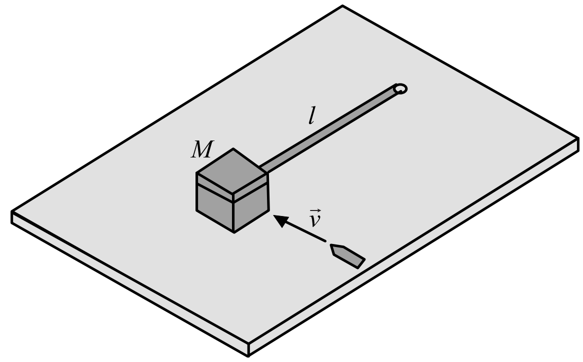

FIG. P11.37

(b) $K_i = \frac{1}{2}mv^2$

$$K_f = \frac{1}{2}(M+m)v_f^2$$

$$v_f = \left(\frac{m}{M+m}\right)v \Rightarrow \text{ velocity of the bullet and block}$$

$$\text{Fraction of } K \text{ lost } = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m^2v^2/(M+m)}{\frac{1}{2}mv^2} = \boxed{\frac{M}{M+m}}$$

P11.38 For one of the crew,

$$\sum F_r = ma_r: \qquad n = \frac{mv^2}{r} = m\omega_i^2 r$$

We require $\qquad n = mg$, so $\omega_i = \sqrt{\frac{g}{r}}$

Now, $\qquad I_i\omega_i = I_f\omega_f$

$$\left[5.00\times10^8 \text{ kg}\cdot\text{m}^2 + 150\times65.0 \text{ kg}\times(100 \text{ m})^2\right]\sqrt{\frac{g}{r}}$$

$$= \left[5.00\times10^8 \text{ kg}\cdot\text{m}^2 + 50\times65.0 \text{ kg}(100 \text{ m})^2\right]\omega_f$$

$$\left(\frac{5.98\times10^8}{5.32\times10^8}\right)\sqrt{\frac{g}{r}} = \omega_f = 1.12\sqrt{\frac{g}{r}}$$

Now,

$$|a_r| = \omega_f^2 r = 1.26g = \boxed{12.3 \text{ m/s}^2}$$

***P11.39** (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

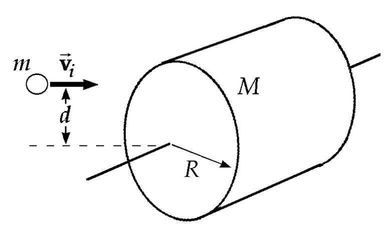

FIG. P11.39

$$L_f = L_i: \qquad I\omega = mv_id$$

or

$$\left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_id$$

Thus,

$$\omega = \boxed{\frac{2mv_id}{(M+2m)R^2}}$$

continued on next page

(b) $\boxed{\text{No; some mechanical energy changes into internal energy.}}$

(c) $\boxed{\text{Momentum is not conserved. The axle exerts a backward force on the cylinder.}}$

P11.40 (a) Let ω be the angular speed of the signboard when it is vertical.

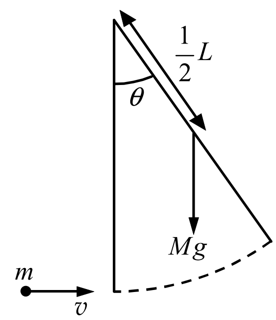

FIG. P11.40

$$\frac{1}{2}I\omega^2 = Mgh$$

$$\therefore \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = Mg\frac{1}{2}L(1-\cos\theta)$$

$$\therefore \omega = \sqrt{\frac{3g(1-\cos\theta)}{L}}$$

$$= \sqrt{\frac{3\left(9.80\ \text{m/s}^2\right)(1-\cos 25.0°)}{0.50\ \text{m}}}$$

$$= \boxed{2.35\ \text{rad/s}}$$

(b) $I_i\omega_i - mvL = I_f\omega_f$ represents angular momentum conservation for the sign-snowball system.

In more detail,

$$\left(\frac{1}{3}ML^2 + mL^2\right)\omega_f = \frac{1}{3}ML^2\omega_i - mvL$$

Solving,

$$\omega_f = \frac{\frac{1}{3}ML\omega_i - mv}{\left(\frac{1}{3}M + m\right)L}$$

$$= \frac{\frac{1}{3}(2.40\ \text{kg})(0.5\ \text{m})(2.347\ \text{rad/s}) - (0.4\ \text{kg})(1.6\ \text{m/s})}{\left[\frac{1}{3}(2.40\ \text{kg}) + 0.4\ \text{kg}\right](0.5\ \text{m})} = \boxed{0.498\ \text{rad/s}}$$

(c) Let h_{CM} = distance of center of mass from the axis of rotation.

$$h_{\text{CM}} = \frac{(2.40\ \text{kg})(0.25\ \text{m}) + (0.4\ \text{kg})(0.50\ \text{m})}{2.40\ \text{kg} + 0.4\ \text{kg}} = 0.285\,7\ \text{m}$$

Apply conservation of mechanical energy:

$$(M+m)gh_{\text{CM}}(1-\cos\theta) = \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2$$

$$\therefore \theta = \cos^{-1}\left[1 - \frac{\left(\frac{1}{3}M + m\right)L^2\omega^2}{2(M+m)gh_{\text{CM}}}\right]$$

$$= \cos^{-1}\left\{1 - \frac{\left[\frac{1}{3}(2.40\ \text{kg}) + 0.4\ \text{kg}\right](0.50\ \text{m})^2(0.498\ \text{rad/s})^2}{2(2.40\ \text{kg} + 0.4\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.285\,7\ \text{m})}\right\}$$

$$= \boxed{5.58°}$$

P11.41 $\boxed{\text{The meteor will slow the rotation of the Earth by the largest amount if its line of motion passes farthest from the Earth's axis. The meteor should be headed west and strike a point on the equator tangentially.}}$

Let the z axis coincide with the axis of the Earth with $+z$ pointing northward. Then, conserving angular momentum about this axis,

$$\sum \vec{\mathbf{L}}_f = \sum \vec{\mathbf{L}}_i \Rightarrow I\omega_f = I\omega_i + m\vec{\mathbf{v}} \times \vec{\mathbf{r}}$$

or

$$\frac{2}{5}MR^2\omega_f\hat{\mathbf{k}} = \frac{2}{5}MR^2\omega_i\hat{\mathbf{k}} - mvR\hat{\mathbf{k}}$$

Thus,

$$\omega_i - \omega_f = \frac{mvR}{\frac{2}{5}MR^2} = \frac{5mv}{2MR}$$

or

$$\omega_i - \omega_f = \frac{5\left(3.00\times10^{13}\text{ kg}\right)\left(30.0\times10^3\text{ m/s}\right)}{2\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^6\text{ m}\right)} = 5.91\times10^{-14}\text{ rad/s}$$

$$\boxed{\left|\Delta\omega_{\max}\right| \sim 10^{-13}\text{ rad/s}}$$

Section 11.5 **The Motion of Gyroscopes and Tops**

P11.42 Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \qquad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$-20\text{ kg}\cdot\text{m}^2\left(-100\text{ rad/s}\right) = 5\times10^5\text{ kg}\cdot\text{m}^2\left(\frac{30°}{t}\right)\left(\frac{\pi\text{ rad}}{180°}\right)$$

$$t = \frac{2.62\times10^5\text{ s}}{2\,000} = \boxed{131\text{ s}}$$

P11.43 $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^6\text{ m}\right)^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$

$$L = I\omega = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2\left(\frac{2\pi\text{ rad}}{86\,400\text{ s}}\right) = 7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\tau = L\omega_p = \left(7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}\right)\left(\frac{2\pi\text{ rad}}{2.58\times10^4\text{ yr}}\right)\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right) = \boxed{5.45\times10^{22}\text{ N}\cdot\text{m}}$$

Additional Problems

P11.44 First, we define the following symbols:

I_P = moment of inertia due to mass of people on the equator

I_E = moment of inertia of the Earth alone (without people)

ω = angular velocity of the Earth (due to rotation on its axis)

$T = \dfrac{2\pi}{\omega}$ = rotational period of the Earth (length of the day)

R = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P\omega_i + I_E\omega_i = (I_P + I_E)\omega_i$$

When the Earth has angular speed ω, the tangential speed of a point on the equator is $v_t = R\omega$. Thus, when the people run eastward along the equator at speed v relative to the surface of the Earth, their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is $\omega_P = \dfrac{v_p}{R} = \omega + \dfrac{v}{R}$.

The angular momentum of the system after the people begin to run is

$$L_f = I_P\omega_p + I_E\omega = I_P\left(\omega + \frac{v}{R}\right) + I_E\omega = (I_P + I_E)\omega + \frac{I_P v}{R}$$

Since no external torques have acted on the system, angular momentum is conserved $(L_f = L_i)$, giving $(I_P + I_E)\omega + \dfrac{I_P v}{R} = (I_P + I_E)\omega_i$. Thus, the final angular velocity of the Earth is

$\omega = \omega_i - \dfrac{I_P v}{(I_P + I_E)R} = \omega_i(1 - x) =$, where $x \equiv \dfrac{I_P v}{(I_P + I_E)R\omega_i}$

The new length of the day is $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\omega_i(1-x)} = \dfrac{T_i}{1-x} \approx T_i(1+x)$, so the increase in the length of the day is $\Delta T = T - T_i \approx T_i x = T_i\left[\dfrac{I_P v}{(I_P + I_E)R\omega_i}\right]$. Since $\omega_i = \dfrac{2\pi}{T_i}$, this may be written as

$$\Delta T \approx \frac{T_i^2 I_P v}{2\pi(I_P + I_E)R}$$

To obtain a numeric answer, we compute

$$I_P = m_p R^2 = \left[(7\times10^9)(70\text{ kg})\right](6.37\times10^6\text{ m})^2 = 1.99\times10^{25}\text{ kg}\cdot\text{m}^2$$

and

$$I_E = \frac{2}{5}m_E R^2 = \frac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

Thus,

$$\Delta T \approx \frac{(8.64\times10^4\text{ s})^2(1.99\times10^{25}\text{ kg}\cdot\text{m}^2)(2.5\text{ m/s})}{2\pi\left[(1.99\times10^{25} + 9.71\times10^{37})\text{ kg}\cdot\text{m}^2\right](6.37\times10^6\text{ m})} = \boxed{9.55\times10^{-11}\text{ s}}$$

***P11.45** (a) Momentum is conserved for the system of two men:
(162 kg)(+ 8 m/s) + (81 kg)(−11 m/s) = (243 kg) $\vec{\mathbf{v}}_f$ $\qquad \vec{\mathbf{v}}_f = \boxed{1.67\ \hat{\mathbf{i}}\ \text{m/s}}$

(b) original mechanical energy = (1/2)(162 kg)(+8 m/s)^2 + (1/2)(81 kg)(−11 m/s)^2 = 10 084 J
final mechanical energy = (1/2)(243 kg)(1.67 m/s)^2 = 338 J

Thus the fraction remaining is 338/10 084 = $\boxed{0.033\ 5}$ = 3.35%

(c) The calculation in part (a) still applies: $\vec{\mathbf{v}}_f = \boxed{1.67\ \hat{\mathbf{i}}\ \text{m/s}}$

(d) With half the mass of Perry, Flutie is distant from the center of mass by (2/3)(1.2 m) = 0.8 m. His angular speed relative to the center of mass just before they link arms is $\omega = v/r$ = (11 + 1.67)(m/s)/0.8 m = 15.8 rad/s. That of Perry is necessarily the same (8 − 1.67)/0.4 m = 15.8 rad/s.

In their linking of arms, angular momentum is conserved. Their total moment of inertia stays constant, so their angular speed also stays constant at $\boxed{15.8\ \text{rad/s}}$.

(e) Only the men's direction of motion is changed by their linking arms. Each keeps constant speed relative to the center of mass and the center of mass keeps constant speed, so all of the kinetic energy is still present. The fraction remaining mechanical is $\boxed{1.00 = 100\%}$.

We can compute this explicitly: the final total kinetic energy is
(1/2)(243 kg)(1.67 m/s)^2 + (1/2)[(81 kg)(0.8 m)^2 + (162 kg)(0.4 m)^2]((15.8 /s)^2 =
338 J + 6498 J + 3249 J = 10 084 J, the same as the original kinetic energy.

P11.46 (a) $\left(K+U_s\right)_A = \left(K+U_s\right)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gy_A} = \sqrt{2\left(9.8\ \text{m/s}^2\right)6.30\ \text{m}} = \boxed{11.1\ \text{m/s}}$$

(b) $L = mvr = 76\ \text{kg}\ 11.1\ \text{m/s}\ 6.3\ \text{m} = \boxed{5.32\times 10^3\ \text{kg}\cdot\text{m}^2/\text{s}}$ toward you along the axis of the channel.

(c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum.

(d) $L = mvr \qquad v = \dfrac{5.32\times 10^3\ \text{kg}\cdot\text{m}^2/\text{s}}{76\ \text{kg}\ 5.85\ \text{m}} = \boxed{12.0\ \text{m/s}}$

(e) $\left(K+U_g\right)_B + U_{chemical,B} = \left(K+U_g\right)_C$

$$\frac{1}{2}76\ \text{kg}\left(11.1\ \text{m/s}\right)^2 + 0 + U_{chem} = \frac{1}{2}76\ \text{kg}\left(12.0\ \text{m/s}\right)^2 + 76\ \text{kg}\ 9.8\ \text{m/s}^2\ 0.45\ \text{m}$$

$$U_{chem} = 5.44\ \text{kJ} - 4.69\ \text{kJ} + 335\ \text{J} = \boxed{1.08\ \text{kJ}}$$

(f) $\left(K+U_g\right)_C = \left(K+U_g\right)_D$

$$\frac{1}{2}76\ \text{kg}\left(12.0\ \text{m/s}\right)^2 + 0 = \frac{1}{2}76\ \text{kg}v_D^2 + 76\ \text{kg}\ 9.8\ \text{m/s}^2\ 5.85\ \text{m}$$

$$v_D = \boxed{5.34\ \text{m/s}}$$

continued on next page

(g) Let point E be the apex of his flight:

$$\left(K+U_g\right)_D=\left(K+U_g\right)_E$$

$$\frac{1}{2}76\text{ kg}(5.34\text{ m/s})^2+0=0+76\text{ kg}\left(9.8\text{ m/s}^2\right)\left(y_E-y_D\right)$$

$$\left(y_E-y_D\right)=\boxed{1.46\text{ m}}$$

(h) For the motion between takeoff and touchdown

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$

$$-2.34\text{ m}=0+5.34\text{ m/s}\,t-4.9\text{ m/s}^2\,t^2$$

$$t=\frac{-5.34\pm\sqrt{5.34^2+4(4.9)(2.34)}}{-9.8}=\boxed{1.43\text{ s}}$$

(i) This solution is more accurate. In Chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

P11.47 (a) $I=\sum m_ir_i^2$

$$=m\left(\frac{4d}{3}\right)^2+m\left(\frac{d}{3}\right)^2+m\left(\frac{2d}{3}\right)^2$$

$$=\boxed{7m\frac{d^2}{3}}$$

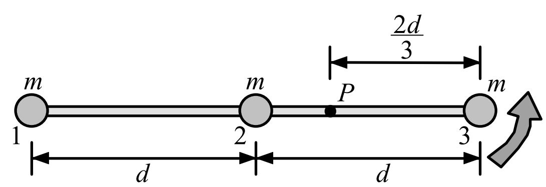

FIG. P11.47

(b) Think of the whole weight, $3mg$, acting at the center of gravity.

$$\vec{\boldsymbol{\tau}}=\vec{\mathbf{r}}\times\vec{\mathbf{F}}=\left(\frac{d}{3}\right)\left(-\hat{\mathbf{i}}\right)\times3mg\left(-\hat{\mathbf{j}}\right)=\boxed{(mgd)\hat{\mathbf{k}}}$$

(c) $$\alpha=\frac{\tau}{I}=\frac{3mgd}{7md^2}=\boxed{\frac{3g}{7d}\text{ counterclockwise}}$$

(d) $$a=\alpha r=\left(\frac{3g}{7d}\right)\left(\frac{2d}{3}\right)=\boxed{\frac{2g}{7}\text{ up}}$$

The angular acceleration is not constant, but energy is.

$$(K+U)_i+\Delta E=(K+U)_f$$

$$0+(3m)g\left(\frac{d}{3}\right)+0=\frac{1}{2}I\omega_f^2+0$$

(e) maximum kinetic energy $=\boxed{mgd}$

(f) $$\omega_f=\boxed{\sqrt{\frac{6g}{7d}}}$$

(g) $$L_f=I\omega_f=\frac{7md^2}{3}\sqrt{\frac{6g}{7d}}=\boxed{\left(\frac{14g}{3}\right)^{1/2}md^{3/2}}$$

(h) $$v_f=\omega_fr=\sqrt{\frac{6g}{7d}}\frac{d}{3}=\boxed{\sqrt{\frac{2gd}{21}}}$$

P11.48 (a) $\sum \tau = MgR - MgR = \boxed{0}$

(b) $\sum \tau = \frac{dL}{dt}$, and since $\sum \tau = 0$, $L = \text{constant}$.

Since the total angular momentum of the system is zero, the $\boxed{\text{monkey and bananas move upward with the same speed}}$ at any instant, and $\boxed{\text{he will not reach the bananas}}$ (until they get tangled in the pulley). To state the evidence differently, the tension in the rope is the same on both sides. Newton's second law applied to the monkey and bananas give the same acceleration upwards.

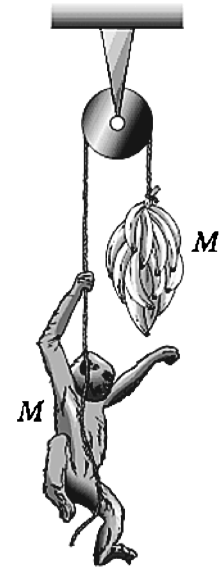

FIG. P11.48

P11.49 Using conservation of angular momentum, we have

$$L_{\text{aphelion}} = L_{\text{perihelion}} \text{ or } \left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p$$

Thus,

$$\left(mr_a^2\right)\frac{v_a}{r_a} = \left(mr_p^2\right)\frac{v_p}{r_p} \text{ giving}$$

$$r_a v_a = r_p v_p \text{ or } v_a = \frac{r_p}{r_a} v_p = \frac{0.590 \text{ AU}}{35.0 \text{ AU}}(54.0 \text{ km/s}) = \boxed{0.910 \text{ km/s}}$$

P11.50 (a) Angular momentum is conserved:

$$\frac{mv_i d}{2} = \left(\frac{1}{12}Md^2 + m\left(\frac{d}{2}\right)^2\right)\omega$$

$$\omega = \boxed{\frac{6mv_i}{Md + 3md}}$$

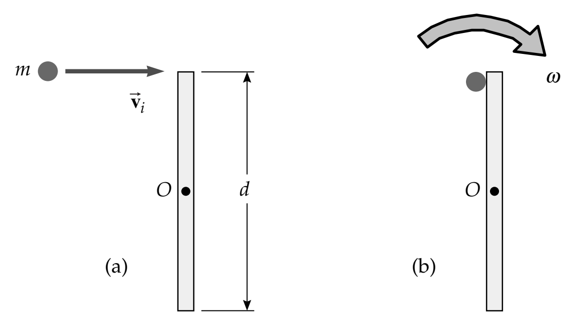

FIG. P11.50

(b) The original energy is $\frac{1}{2}mv_i^2$.

The final energy is

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{12}Md^2 + \frac{md^2}{4}\right)\frac{36m^2v_i^2}{(Md+3md)^2} = \frac{3m^2v_i^2 d}{2(Md+3md)}$$

The loss of energy is

$$\frac{1}{2}mv_i^2 - \frac{3m^2v_i^2 d}{2(Md+3md)} = \frac{mMv_i^2 d}{2(Md+3md)}$$

and the fractional loss of energy is

$$\frac{mMv_i^2 d2}{2(Md+3md)mv_i^2} = \boxed{\frac{M}{M+3m}}$$

P11.51 (a) $\tau = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = \left|\vec{\mathbf{r}}\right|\left|\vec{\mathbf{F}}\right| \sin 180° = 0$

Angular momentum is conserved.

$$L_f = L_i$$
$$mrv = mr_i v_i$$
$$v = \boxed{\frac{r_i v_i}{r}}$$

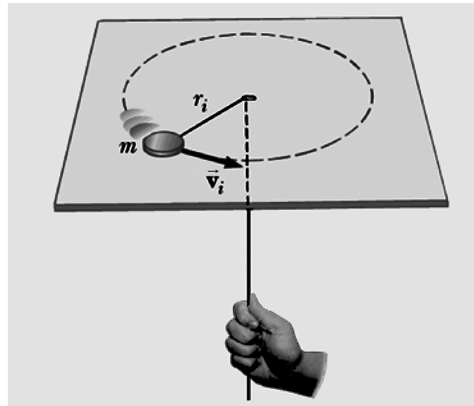

FIG. P11.51

(b) $T = \dfrac{mv^2}{r} = \boxed{\dfrac{m(r_i v_i)^2}{r^3}}$

(c) The work is done by the centripetal force in the *negative-r*, inward direction.

METHOD 1:

$$W = \int F \cdot d\ell = -\int T dr' = -\int_{r_i}^{r} \frac{m(r_i v_i)^2}{(r')^3} dr' = \left.\frac{m(r_i v_i)^2}{2(r')^2}\right|_{r_i}^{r}$$

$$= \frac{m(r_i v_i)^2}{2}\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

METHOD 2:

$$W = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_i^2 = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

(d) Using the data given, we find

$$v = \boxed{4.50 \text{ m/s}} \qquad T = \boxed{10.1 \text{ N}} \qquad W = \boxed{0.450 \text{ J}}$$

***P11.52** (a) The equation simplifies to

$$(1.75 \text{ kg} \cdot \text{m}^2/\text{s} - 0.181 \text{ kg} \cdot \text{m}^2/\text{s})\,\hat{\mathbf{j}} = (0.745 \text{ kg} \cdot \text{m}^2)\vec{\omega} \qquad \vec{\omega} = \boxed{2.11\hat{\mathbf{j}} \text{ rad/s}}$$

(b) We take the x axis east, the y axis up, and the z axis south.

The child has moment of inertia $0.730 \text{ kg} \cdot \text{m}^2$ about the axis of the stool and is originally turning counterclockwise at 2.40 rad/s. At a point 0.350 m to the east of the axis, he catches a 0.120 kg ball moving toward the south at 4.30 m/s. He continues to hold the ball in his outstretched arm. Find his final angular velocity.

(c) Yes, with the left-hand side representing the final situation and the right-hand side representing the original situation, the equation describes the throwing process.

P11.53 (a) $L_i = m_1 v_{1i} r_{1i} + m_2 v_{2i} r_{2i} = 2mv\left(\frac{d}{2}\right)$

$L_i = 2(75.0\ \text{kg})(5.00\ \text{m/s})(5.00\ \text{m})$

$L_i = \boxed{3\,750\ \text{kg}\cdot\text{m}^2/\text{s}}$

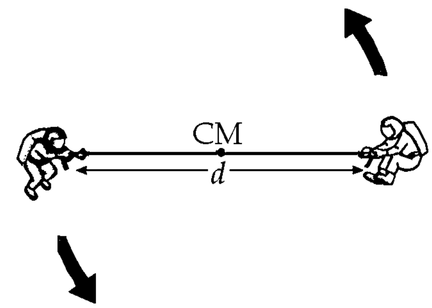

FIG. P11.53

(b) $K_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$

$K_i = 2\left(\frac{1}{2}\right)(75.0\ \text{kg})(5.00\ \text{m/s})^2 = \boxed{1.88\ \text{kJ}}$

(c) Angular momentum is conserved: $L_f = L_i = \boxed{3\,750\ \text{kg}\cdot\text{m}^2/\text{s}}$

(d) $v_f = \frac{L_f}{2(mr_f)} = \frac{3\,750\ \text{kg}\cdot\text{m}^2/\text{s}}{2(75.0\ \text{kg})(2.50\ \text{m})} = \boxed{10.0\ \text{m/s}}$

(e) $K_f = 2\left(\frac{1}{2}\right)(75.0\ \text{kg})(10.0\ \text{m/s})^2 = \boxed{7.50\ \text{kJ}}$

(f) $W = K_f - K_i = \boxed{5.62\ \text{kJ}}$

P11.54 (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

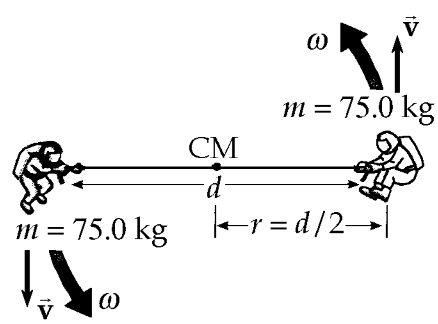

FIG. P11.54

(b) $K = 2\left(\frac{1}{2}Mv^2\right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \frac{L_f}{2Mr_f} = \frac{Mvd}{2M\left(\frac{d}{4}\right)} = \boxed{2v}$

(e) $K_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W = K_f - K_i = \boxed{3Mv^2}$

***P11.55** (a) At the moment of release, two stones are moving with speed v_0. The total momentum has magnitude $\boxed{2mv_0}$. It keeps this same horizontal component of momentum as it flies away.

(b) The center of mass speed relative to the hunter is $2mv_0/3m = \boxed{2v_0/3}$, before the hunter lets go and, as far as horizontal motion is concerned, afterward.

(c) The one ball just being released is at distance $4\ell/3$ from the center of mass and is moving at speed $2v_0/3$ relative to the center of mass. Its angular speed is $\omega = v/r = (2v_0/3)/(4\ell/3) = v_0/2\ell$.

The other two balls are at distance $2\ell/3$ from the center of mass and moving relative to it at speed $v_0/3$. Their angular speed is necessarily the same $\omega = v/r = (v_0/3)/(2\ell/3) = v_0/2\ell$.

The total angular momentum around the center of mass is
$\Sigma mvr = m(2v_0/3)(4\ell/3) + 2m(v_0/3)(2\ell/3) = \boxed{4m\ell v_0/3}$. The angular momentum remains constant with this value as the bola flies away.

(d) As computed in part (c), the angular speed at the moment of release is $v_0/2\ell$. As it moves through the air, the bola keeps constant angular momentum, but its moment of inertia changes to $3m\ell^2$. Then the new angular speed is given by $L = I\omega \quad 4m\ell v_0/3 = 3m\ell^2\omega \quad \omega = \boxed{4v_0/9\ell}$
The angular speed drops as the moment of inertia increases.

(e) At the moment of release, $K = (1/2)m(0)^2 + (1/2)(2m)v_0^2 = \boxed{mv_0^2}$

(f) As it flies off in its horizontal motion it has kinetic energy
$(1/2)(3m)(v_{CM})^2 + (1/2)I\omega^2 = (1/2)(3m)(2v_0/3)^2 + (1/2)(3m\ell^2)(4v_0/9\ell)^2 = \boxed{(26/27)mv_0^2}$

(g) No horizontal forces act on the bola from outside after release, so the horizontal momentum stays constant. Its center of mass moves steadily with the horizontal velocity it had at release. No torques about its axis of rotation act on the bola, so its spin angular momentum stays constant. Internal forces cannot affect momentum conservation and angular momentum conservation, but they can affect mechanical energy. Energy $mv_0^2/27$ changes from mechanical energy into internal energy as the bola takes its stable configuration.

P11.56 For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation AB. To do this, the CM must be raised a distance of $a\left(\sqrt{2}-1\right)$.

For conservation of energy as the cube turns,

$$Mga\left(\sqrt{2}-1\right)=\frac{1}{2}I_{\text{cube}}\omega^2$$

From conservation of angular momentum,

$$\frac{4a}{3}mv=\left(\frac{8Ma^2}{3}\right)\omega$$

$$\omega=\frac{mv}{2Ma}$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\frac{m^2v^2}{4M^2a^2}=Mga\left(\sqrt{2}-1\right)$$

$$v=\boxed{\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}}$$

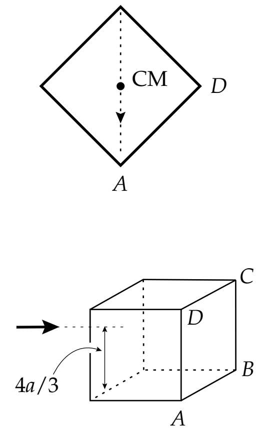

FIG. P11.56

***P11.57** The moment of inertia of the rest of the Earth is

$$I=\frac{2}{5}MR^2=\frac{2}{5}5.98\times10^{24}\text{ kg}\left(6.37\times10^6\text{ m}\right)^2=9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

For the original ice disks,

$$I=\frac{1}{2}Mr^2=\frac{1}{2}2.30\times10^{19}\text{ kg}\left(6\times10^5\text{ m}\right)^2=4.14\times10^{30}\text{ kg}\cdot\text{m}^2$$

For the final thin shell of water,

$$I=\frac{2}{3}Mr^2=\frac{2}{3}2.30\times10^{19}\text{ kg}\left(6.37\times10^6\text{ m}\right)^2=6.22\times10^{32}\text{ kg}\cdot\text{m}^2$$

Conservation of angular momentum for the spinning planet is expressed by $I_i\omega_i=I_f\omega_f$

$$\left(4.14\times10^{30}+9.71\times10^{37}\right)\frac{2\pi}{86\,400\text{ s}}=\left(6.22\times10^{32}+9.71\times10^{37}\right)\frac{2\pi}{\left(86\,400\text{ s}+\delta\right)}$$

$$\left(1+\frac{\delta}{86\,400\text{ s}}\right)\left(1+\frac{4.14\times10^{30}}{9.71\times10^{37}}\right)=\left(1+\frac{6.22\times10^{32}}{9.71\times10^{37}}\right)$$

$$\frac{\delta}{86\,400\text{ s}}=\frac{6.22\times10^{32}}{9.71\times10^{37}}-\frac{4.14\times10^{30}}{9.71\times10^{37}}$$

$$\boxed{\delta=0.550\text{ s}}$$

$\boxed{\text{It is a measurable change, but not significant for everyday life.}}$

P11.58 (a) The net torque is zero at the point of contact, so the angular momentum before and after the collision must be equal.

$$\left(\frac{1}{2}MR^2\right)\omega_i = \left(\frac{1}{2}MR^2\right)\omega + \left(MR^2\right)\omega \qquad \omega = \boxed{\frac{\omega_i}{3}}$$

(b) $$\frac{\Delta E}{E} = \frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\omega_i/3\right)^2 + \frac{1}{2}M\left(R\omega_i/3\right)^2 - \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2} = \boxed{-\frac{2}{3}}$$

P11.59 (a) $$\Delta t = \frac{\Delta p}{f} = \frac{Mv}{\mu Mg} = \frac{MR\omega}{\mu Mg} = \boxed{\frac{R\omega_i}{3\mu g}}$$

(b) $$W = \Delta K_{translational} = \frac{1}{2}Mv^2 - 0 = \frac{1}{2}M(\omega R)^2 = \frac{1}{2}M\left(\frac{\omega_i}{3}R\right)^2 = \frac{1}{18}MR^2\omega_i^2$$

(See Problem 11.58)

$$\mu Mgx = \frac{1}{18}MR^2\omega_i^2 \qquad \boxed{x = \frac{R^2\omega_i^2}{18\mu g}}$$

P11.60 Angular momentum is conserved during the inelastic collision.

$$Mva = I\omega$$

$$\omega = \frac{Mva}{I} = \frac{3v}{8a}$$

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{max} = a\sqrt{2}$. Using conservation of energy:

$$\frac{1}{2}I\omega^2 = Mg\left(a\sqrt{2} - a\right)$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2 = Mg\left(a\sqrt{2} - a\right)$$

$$v^2 = \frac{16}{3}ga\left(\sqrt{2} - 1\right)$$

$$v = \boxed{4\left[\frac{ga}{3}\left(\sqrt{2} - 1\right)\right]^{1/2}}$$

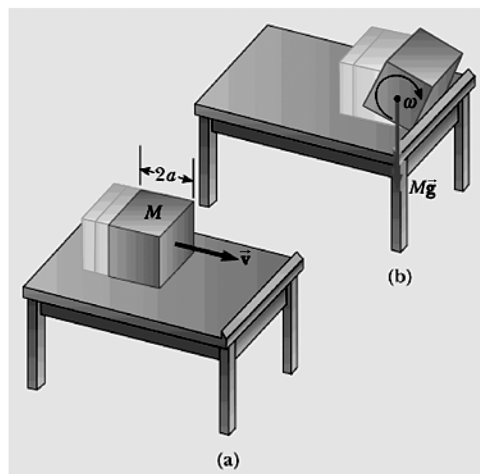

FIG. P11.60

ANSWERS TO EVEN PROBLEMS

P11.2 (a) 740 cm^2 (b) 59.5 cm

P11.4 (a) 168° (b) 11.9° principal value (c) Only the first is unambiguous.

P11.6 No. The cross product must be perpendicular to each factor.

P11.8 (a) $-10.0\ \hat{\mathbf{k}}$ N · m (b) yes; yes, infinitely many; yes; no, only one. $\vec{\mathbf{r}} = 5.00\hat{\mathbf{j}}$ m

P11.10 see the solution

P11.12 $\left(-22.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$

P11.14 see the solution

P11.16 (a) $3.14\ \text{N}\cdot\text{m}$ (b) $(0.400\ \text{kg}\cdot\text{m})v$ (c) $7.85\ \text{m/s}^2$

P11.18 (a) $\left(+9.03\times10^{9}\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ south (b) no (c) 0

P11.20 (a) $\vec{\mathbf{r}} = (2t^3\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}})$ m, where t is in s. (b) The particle starts from rest at the origin, starts moving in the y direction, and gains speed faster and faster while turning to move more and more nearly parallel to the x axis. (c) $\vec{\mathbf{a}} = (12t\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})$ m/s² (d) $\vec{\mathbf{F}} = (60t\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}})$ N
(e) $\vec{\tau} = (-40t^3\hat{\mathbf{k}})$ N · m (f) $\vec{\mathbf{L}} = -10t^4\,\hat{\mathbf{k}}$ kg · m²/s (g) $K = (90t^4 + 10t^2)$ J (h) $\mathcal{P} = (360t^3 + 20t)$ W, all where t is in s.

P11.22 $\left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ up

P11.24 (a) 7.06×10^{33} kg · m²/s toward Polaris (b) 2.66×10^{40} kg · m²/s toward Draco (c) The orbital angular momentum is much larger, by 3.78×10^{6} times.

P11.26 $1.20\ \text{kg}\cdot\text{m}^2/\text{s}$ perpendicularly into the clock face

P11.28 $8.63\ \text{m/s}^2$

P11.30 (a) 2.91 s (b) yes (c) Yes, but the pivot pin is always pulling on the rod to change the direction of the momentum. (d) No. Some mechanical energy is converted into internal energy.

P11.32 (a) $1.91\ \text{rad/s}$ (b) 2.53 J; 6.44 J

P11.34 (a) $\omega_f = (36\ \text{rad/s})(1 + 3.2\,m)/(1 + 20\,m)$ (b) ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of 5.76 rad/s as $m \to \infty$.

P11.36 (a) $7.20\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}$ (b) $9.47\ \text{rad/s}$

P11.38 $12.3\ \text{m/s}^2$

P11.40 (a) $2.35\ \text{rad/s}$ (b) $0.498\ \text{rad/s}$ (c) 5.58°

P11.42 131 s

P11.44 $9.55\times10^{-11}\ \text{s}$

P11.46 (a) $11.1\ \text{m/s}$ (b) $5.32\times10^{3}\ \text{kg}\cdot\text{m}^2/\text{s}$ (c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum. (d) $12.0\ \text{m/s}$ (e) 1.08 kJ (f) $5.34\ \text{m/s}$ (g) 1.46 m (h) 1.43 s (i) This solution is more accurate. In Chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

P11.48 (a) 0 (b) The total angular momentum is constant, zero if the system is initially at rest. The monkey and the bananas move upward with the same speed. He will not reach the bananas.

P11.50 (a) $\dfrac{6mv_i}{Md+3md}$ (b) $\dfrac{M}{M+3m}$

P11.52 (a) 2.11p $\hat{\mathbf{j}}$ rad/s (b) The child has moment of inertia $0.730\ \text{kg}\cdot\text{m}^2$ about the axis of the stool, and is originally turning counterclockwise at 2.40 rad/s. At a point 0.350 m to the east of the axis, he catches a 0.120 kg ball moving toward the south at 4.30 m/s. In his outstretched arm he continues to hold the ball. Find his final angular velocity. (c) Yes, with the left-hand side representing the final situation and the right-hand side representing the original situation, the equation describes the throwing process.

P11.54 (a) Mvd (b) Mv^2 (c) Mvd (d) $2v$ (e) $4Mv^2$ (f) $3Mv^2$

P11.56 $\dfrac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}$

P11.58 (a) $\dfrac{\omega_i}{3}$ (b) $\dfrac{\Delta E}{E}=-\dfrac{2}{3}$

P11.60 $4\left[\dfrac{ga}{3}\left(\sqrt{2}-1\right)\right]^{1/2}$