40

Introduction to Quantum Physics

Note: In chapters 39, 40, and 41 we use u to represent the speed of a particle with mass, reserving v for the speeds associated with reference frames, wave functions, and photons.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q40.1 The first flaw is that the Rayleigh–Jeans law predicts that the intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases. This is known as the *ultraviolet catastrophe*. The second flaw is the prediction of much more power output from a blackbody than is shown experimentally. The intensity of radiation from the blackbody is given by the area under the red $I(\lambda, T)$ *vs.* λ curve in Figure 40.5 in the text, not by the area under the blue curve.

Planck's Law dealt with both of these issues and brought the theory into agreement with the experimental data by adding an exponential term to the denominator that depends on $\frac{1}{\lambda}$. This both keeps the predicted intensity from approaching infinity as the wavelength decreases and keeps the area under the curve finite.

Q40.2 Our eyes are not able to detect all frequencies of electromagnetic waves. For example, all objects that are above 0 K in temperature emit electromagnetic radiation in the infrared region. This describes *everything* in a dark room. We are only able to see objects that emit or reflect electromagnetic radiation in the visible portion of the spectrum.

***Q40.3** (i) The power input to the filament has increased by $8 \times 2 = 16$ times. The filament radiates this greater power according to Stefan's law, so its absolute temperature is higher by the fourth root of 16. It is two times higher. Answer (d).

(ii) Wien's displacement law then says that the wavelength emitted most strongly is half as large: answer (f).

Q40.4 No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

Q40.5 Comparing Equation 40.9 with the slope-intercept form of the equation for a straight line, $y = mx + b$, we see that the slope in Figure 40.11 in the text is Planck's constant h and that the y intercept is $-\phi$, the negative of the work function. If a different metal were used, the slope would remain the same but the work function would be different. Thus, data for different metals appear as parallel lines on the graph.

Q40.6 Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur.

Q40.7 The stopping voltage measures the kinetic energy of the most energetic photoelectrons. Each of them has gotten its energy from a single photon. According to Planck's $E = hf$, the photon energy depends on the frequency of the light. The intensity controls only the number of photons reaching a unit area in a unit time.

Q40.8 Ultraviolet light has shorter wavelength and higher photon energy than visible light.

***Q40.9** Answer (c). UV light has the highest frequency of the three, and hence each photon delivers more energy to a skin cell. This explains why you can become sunburned on a cloudy day: clouds block visible light and infrared, but not much ultraviolet. You usually do not become sunburned through window glass, even though you can see the visible light from the Sun coming through the window, because the glass absorbs much of the ultraviolet and reemits it as infrared.

Q40.10 The Compton effect describes the *scattering* of photons from electrons, while the photoelectric effect predicts the ejection of electrons due to the *absorption* of photons by a material.

***Q40.11** Answer (a). The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

Q40.12 A few photons would only give a few dots of exposure, apparently randomly scattered.

***Q40.13** (i) a and c. Some people would say that electrons and protons possess mass and photons do not.

(ii) a and c (iii) a, b, and c (iv) a, b, and c (v) b (vi) a, b, and c

Q40.14 Light has both classical-wave and classical-particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, *hf*. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle." It is customary to call a photon a *quantum particle*, different from a classical particle.

Q40.15 An electron has both classical-wave and classical-particle characteristics. In single- and double-slit diffraction and interference experiments, electrons behave like classical waves. An electron has mass and charge. It carries kinetic energy and momentum in parcels of definite size, as classical particles do. At the same time it has a particular wavelength and frequency. Since an electron displays characteristics of both classical waves and classical particles, it is neither a classical wave nor a classical particle. It is customary to call it a *quantum particle*, but another invented term, such as "wavicle," could serve equally well.

Q40.16 The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

***Q40.17** Answer (a). If we set $\frac{p^2}{2m} = q\Delta V$, which is the same for both particles, then we see that the electron has the smaller momentum and therefore the longer wavelength $\left(\lambda = \frac{h}{p}\right)$.

Q40.18 Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength and does not exhibit quantum behavior.

***Q40.19** The wavelength is described by $\lambda = h/p$ in all cases, so e, f, and g are all the same. For the photons the momentum is given by $p = E/c$, so a is also the same, and d has wavelength ten times larger. For the particles with mass, $pc = (E^2 - m^2c^4)^{1/2} = ([K + mc^2]^2 - m^2c^4)^{1/2} = (K^2 + 2Kmc^2)^{1/2}$. Thus a particle with larger mass has more momentum for the same kinetic energy, and a shorter wavelength. The ranking is then d > a = e = f = g > b > c.

Q40.20 The *intensity* of electron waves in some small region of space determines the *probability* that an electron will be found in that region.

Q40.21 The wavelength of violet light is on the order of $\frac{1}{2}$ μm, while the de Broglie wavelength of an electron can be 4 orders of magnitude smaller. Would your collar size be measured more precisely with an unruled meter stick or with one engraved with divisions down to $\frac{1}{10}$ mm?

***Q40.22** Answer (c). The proton has 1836 time more momentum, thus more momentum uncertainty, and thus possibly less position uncertainty.

Q40.23 The spacing between repeating structures on the surface of the feathers or scales is on the order of 1/2 the wavelength of light. An optical microscope would not have the resolution to see such fine detail, while an electron microscope can. The electrons can have much shorter wavelength.

Q40.24 (a) The slot is blacker than any black material or pigment. Any radiation going in through the hole will be absorbed by the walls or the contents of the box, perhaps after several reflections. Essentially none of that energy will come out through the hole again. Figure 40.1 in the text shows this effect if you imagine the beam getting weaker at each reflection.

(b) The open slots between the glowing tubes are brightest. When you look into a slot, you receive direct radiation emitted by the wall on the far side of a cavity enclosed by the fixture; and you also receive radiation that was emitted by other sections of the cavity wall and has bounced around a few or many times before escaping through the slot. In Figure 40.1 in the text, reverse all of the arrowheads and imagine the beam getting stronger at each reflection. Then the figure shows the extra efficiency of a cavity radiator. Here is the conclusion of Kirchhoff's thermodynamic argument: … energy radiated. A poor reflector—a good absorber—avoids rising in temperature by being an efficient emitter. Its emissivity is equal to its absorptivity: $e = a$. The slot in the box in part (a) of the question is a blackbody with reflectivity zero and absorptivity 1, so it must also be the most efficient possible radiator, to avoid rising in temperature above its surroundings in thermal equilibrium. Its emissivity in Stefan's law is $100\% = 1$, higher than perhaps 0.9 for black paper, 0.1 for light-colored paint, or 0.04 for shiny metal. Only in this way can the material objects underneath these different surfaces maintain equal temperatures after they come to thermal equilibrium and continue to exchange energy by electromagnetic radiation. By considering one blackbody facing another, Kirchhoff proved logically that the material forming the walls of the cavity made no difference to the radiation. By thinking about inserting color filters between two cavity radiators, he proved that the spectral distribution of blackbody radiation must be a universal function of wavelength, the same for all materials and depending only on the temperature. Blackbody radiation is a fundamental connection between the matter and the energy that physicists had previously studied separately.

SOLUTIONS TO PROBLEMS

Section 40.1 Blackbody Radiation and Planck's Hypothesis

P40.1 $$T = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} = \boxed{5.18\times10^{3}\ \text{K}}$$

***P40.2** (a) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2900\ \text{K}} = \boxed{999\ \text{nm}}$$

(b) The wavelength emitted most strongly is infrared (greater than 700 nm), and much more energy is radiated at wavelengths longer than $\lambda_{\max}$ than at shorter wavelengths.

***P40.3** The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.289\ 8\times10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{0.289\ 8\times10^{-2}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} \approx \boxed{5\ 200\ \text{K}}$$

Clearly, a firefly is not at this temperature, so $\boxed{\text{this is not blackbody radiation}}$.

P40.4 (a) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} \sim \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^{4}\ \text{K}} \boxed{\sim10^{-7}\ \text{m}} \quad \boxed{\text{ultraviolet}}$$

(b) $$\lambda_{\max} \sim \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^{7}\ \text{K}} \boxed{\sim10^{-10}\ \text{m}} \quad \boxed{\gamma\text{-ray}}$$

P40.5 Planck's radiation law gives intensity-per-wavelength. Taking E to be the photon energy and n to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\mathcal{P} = \frac{2\pi hc^2(\lambda_2-\lambda_1)\pi(d/2)^2}{\left[(\lambda_1+\lambda_2)/2\right]^5\left(e^{2hc/\left[(\lambda_1+\lambda_2)k_BT\right]}-1\right)} = En = nhf \qquad \text{where} \qquad E = hf = \frac{2hc}{\lambda_1+\lambda_2}$$

$$n = \frac{\mathcal{P}}{E} = \frac{8\pi^2cd^2(\lambda_2-\lambda_1)}{(\lambda_1+\lambda_2)^4\left(e^{2hc/\left[(\lambda_1+\lambda_2)k_BT\right]}-1\right)}$$

$$= \frac{8\pi^2(3.00\times10^{8}\ \text{m/s})(5.00\times10^{-5}\ \text{m})^2(1.00\times10^{-9}\ \text{m})}{(1\,001\times10^{-9}\ \text{m})^4\left(e^{\left[2(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})\right]/\left[(1\,001\times10^{-9}\ \text{m})(1.38\times10^{-23}\ \text{J/K})(7.50\times10^{3}\ \text{K})\right]}-1\right)}$$

$$n = \frac{5.90\times10^{16}/\text{s}}{(e^{3.84}-1)} = \boxed{1.30\times10^{15}/\text{s}}$$

P40.6 (a) $\mathcal{P} = eA\sigma T^4 = 1(20.0\times10^{-4}\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(5\,000\ \text{K})^4 = \boxed{7.09\times10^4\ \text{W}}$

(b) $\lambda_{\max}T = \lambda_{\max}(5\,000\ \text{K}) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \Rightarrow \lambda_{\max} = \boxed{580\ \text{nm}}$

(c) We compute: $\dfrac{hc}{k_B T} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{(1.38\times10^{-23}\ \text{J/K})(5\,000\ \text{K})} = 2.88\times10^{-6}\ \text{m}$

The power per wavelength interval is $\mathcal{P}(\lambda) = AI(\lambda) = \dfrac{2\pi hc^2 A}{\lambda^5\left[\exp(hc/\lambda k_B T)-1\right]}$, and

$$2\pi hc^2 A = 2\pi(6.626\times10^{-34})(3.00\times10^8)^2(20.0\times10^{-4}) = 7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}$$

$$\mathcal{P}(580\ \text{nm}) = \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{(580\times10^{-9}\ \text{m})^5\left[\exp(2.88\ \mu\text{m}/0.580\ \mu\text{m})-1\right]} = \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1}$$

$$= \boxed{7.99\times10^{10}\ \text{W/m}}$$

(d)–(i) The other values are computed similarly:

	λ	$\dfrac{hc}{\lambda k_B T}$	$e^{hc/\lambda k_B T}-1$	$\dfrac{2\pi hc^2 A}{\lambda^5}$	$\mathcal{P}(\lambda)$, W/m
(d)	1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
(e)	5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
(f)	400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
(c)	580 nm	4.97	143.5	1.15×10^{13}	7.99×10^{10}
(g)	700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
(h)	1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
(i)	10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $\mathcal{P}(\lambda)$ versus λ curve, between 400 nm and 700 nm, as two trapezoids:

$$\mathcal{P} = \frac{\left[(5.44+7.99)\times10^{10}\ \text{W/m}\right]\left[(580-400)\times10^{-9}\ \text{m}\right]}{2} + \frac{\left[(7.99+7.38)\times10^{10}\ \text{W/m}\right]\left[(700-580)\times10^{-9}\ \text{m}\right]}{2}$$

$\mathcal{P} = 2.13\times10^4\ \text{W}$ so the power radiated as visible light is $\boxed{\text{approximately 20 kW}}$.

P40.7 (a) $\mathcal{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathcal{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.85\times10^{26}\ \text{W}}{1\left[4\pi(6.96\times10^8\ \text{m})^2\right](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{5.78\times10^3\ \text{K}}$$

(b) $\lambda_{\max} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.78\times10^3\ \text{K}} = 5.01\times10^{-7}\ \text{m} = \boxed{501\ \text{nm}}$

P40.8 Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(500\times10^{-9}\ \text{m})} = 3.98\times10^{-19}\ \text{J}$$

The energy entering the eye each second

$$E = \mathcal{P}\Delta t = IA\Delta t = (4.00\times10^{-11}\ \text{W/m}^2)\left[\frac{\pi}{4}(8.50\times10^{-3}\ \text{m})^2\right](1.00\ \text{s}) = 2.27\times10^{-15}\ \text{J}$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \text{J}}{3.98\times10^{-19}\ \text{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}$$

P40.9 (a) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(620\times10^{12}\ \text{s}^{-1})\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.10\times10^{9}\ \text{s}^{-1})\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$

(c) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(46.0\times10^{6}\ \text{s}^{-1})\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$

(d) $\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

P40.10 We take $\theta = 0.030\,0$ radians. Then the pendulum's total energy is

$$E = mgh = mg(L - L\cos\theta)$$

$$E = (1.00\ \text{kg})(9.80\ \text{m/s}^2)(1.00 - 0.999\,5) = 4.41\times10^{-3}\ \text{J}$$

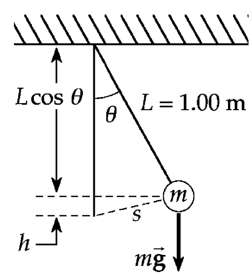

FIG. P40.10

The frequency of oscillation is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 0.498\ \text{Hz}$

The energy is quantized, $E = nhf$

Therefore, $n = \frac{E}{hf} = \frac{4.41\times10^{-3}\ \text{J}}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(0.498\ \text{s}^{-1})}$

$$= \boxed{1.34\times10^{31}}$$

P40.11 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^{6}) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $\frac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photon}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$

***P40.12** (a) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (0.02\ \text{m})^3 = 3.35 \times 10^{-5}\ \text{m}^3$

$$m = \rho V = 7.86 \times 10^3\ \text{kg/m}^3\ \ 3.35 \times 10^{-5}\ \text{m}^3 = \boxed{0.263\ \text{kg}}$$

(b) $A = 4\pi r^2 = 4\pi (0.02\ \text{m})^2 = 5.03 \times 10^{-3}\ \text{m}^2$

$$\mathcal{P} = \sigma A e T^4 = \left(5.67 \times 10^{-8}\ \text{W/m}^2\ \text{K}^4\right)\ 5.03 \times 10^{-3}\ \text{m}^2\ \ 0.86 (293\ \text{K})^4 = \boxed{1.81\ \text{W}}$$

(c) It emits but does not absorb radiation, so its temperature must drop according to

$$Q = mc\Delta T = mc\left(T_f - T_i\right) \qquad \frac{dQ}{dt} = mc\frac{dT_f}{dt}$$

$$\frac{dT_f}{dt} = \frac{dQ/dt}{mc} = \frac{-\mathcal{P}}{mc} = \frac{-1.81\ \text{J/s}}{0.263\ \text{kg}\ 448\ \text{J/kg}\cdot\text{C}°} = \boxed{-0.015\ 3°\text{C/s}} = -0.919\,°\text{C/min}$$

(d) $\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m}\cdot\text{K}$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{293\ \text{K}} = \boxed{9.89 \times 10^{-6}\ \text{m}}\ \ \text{infrared}$$

(e) $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\ 3 \times 10^8\ \text{m/s}}{9.89 \times 10^{-6}\ \text{m}} = \boxed{2.01 \times 10^{-20}\ \text{J}}$

(f) The energy output each second is carried by photons according to

$$\mathcal{P} = \left(\frac{N}{\Delta t}\right)E$$

$$\frac{N}{\Delta t} = \frac{\mathcal{P}}{E} = \frac{1.81\ \text{J/s}}{2.01 \times 10^{-20}\ \text{J/photon}} = \boxed{8.98 \times 10^{19}\ \text{photon/s}}$$

Matter is coupled to radiation quite strongly, in terms of photon numbers.

P40.13 Planck's radiation law is $I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$

Using the series expansion $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

Planck's law reduces to $I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left[\left(1 + hc/\lambda k_B T + \cdots\right) - 1\right]} \approx \frac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_B T\right)} = \frac{2\pi c k_B T}{\lambda^4}$

which is the Rayleigh-Jeans law, for very long wavelengths.

Section 40.2 **The Photoelectric Effect**

P40.14 (a) $$\lambda_c = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(4.20\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{296\text{ nm}}$$

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^{8}\text{ m/s}}{296\times10^{-9}\text{ m}} = \boxed{1.01\times10^{15}\text{ Hz}}$$

(b) $\dfrac{hc}{\lambda} = \phi + e\Delta V_S$: $$\frac{(6.626\times10^{-34})(3.00\times10^{8})}{180\times10^{-9}} = (4.20\text{ eV})(1.60\times10^{-19}\text{ J/eV}) + (1.60\times10^{-19})\Delta V_S$$

Therefore, $\boxed{\Delta V_S = 2.71\text{ V}}$

P40.15 (a) $$e\Delta V_S = \frac{hc}{\lambda} - \phi \to \phi = \frac{1\,240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$$

(b) $$e\Delta V_S = \frac{hc}{\lambda} - \phi = \frac{1\,240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \to \boxed{\Delta V_S = 0.216\text{ V}}$$

P40.16 $$K_{\max} = \frac{1}{2}mu_{\max}^2 = \frac{1}{2}(9.11\times10^{-31})(4.60\times10^{5})^2 = 9.64\times10^{-20}\text{ J} = 0.602\text{ eV}$$

(a) $$\phi = E - K_{\max} = \frac{1\,240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ nm} = \boxed{1.38\text{ eV}}$$

(b) $$f_c = \frac{\phi}{h} = \frac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right) = \boxed{3.34\times10^{14}\text{ Hz}}$$

***P40.17** (a) We could find the energy of a photon with wavelength 400 nm and check whether it exceeds the work function. But we can instead use $\lambda_c = \dfrac{hc}{\phi}$ to find the threshold wavelength for each sample:

Li: $$\lambda_c = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(2.30\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 540\text{ nm}$$

Be: $$\lambda_c = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(3.90\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 319\text{ nm}$$

Hg: $$\lambda_c = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(4.50\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 276\text{ nm}$$

We must have $\lambda < \lambda_c$ for photocurrent to exist.

$\boxed{\text{Thus, only lithium will exhibit the photoelectric effect.}}$

(b) For lithium, $\dfrac{hc}{\lambda} = \phi + K_{\max}$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{400\times10^{-9}\text{ m}} = (2.30\text{ eV})(1.60\times10^{-19}) + K_{\max}$$

$$K_{\max} = 1.29\times10^{-19}\text{ J} = \boxed{0.806\text{ eV}}$$

P40.18 The energy needed is $E = 1.00\text{ eV} = 1.60\times10^{-19}\text{ J}$

The energy absorbed in time interval Δt is $E = \mathcal{P}\Delta t = IA\Delta t$

so $$\Delta t = \frac{E}{IA} = \frac{1.60\times10^{-19}\text{ J}}{\left(500\text{ J/s}\cdot\text{m}^2\right)\left[\pi\left(2.82\times10^{-15}\text{ m}\right)^2\right]} = 1.28\times10^{7}\text{ s} = \boxed{148\text{ days}}$$

The success of quantum mechanics contrasts with the gross failure of the classical theory of the photoelectric effect.

P40.19 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$,

or $$K_{\max} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{200\times10^{-9}\text{ m}}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 4.70\text{ eV} = 1.51\text{ eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad \text{or} \quad Q = \frac{rV}{k_e} = \frac{\left(5.00\times10^{-2}\text{ m}\right)\left(1.51\text{ N}\cdot\text{m/C}\right)}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41\times10^{-12}\text{ C}}$$

P40.20 (a) By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) If $v = 0.280c$, $$f' = f\sqrt{\frac{1+v/c}{1-v/c}} = \left(7.00\times10^{14}\right)\sqrt{\frac{1.28}{0.720}} = 9.33\times10^{14}\text{ Hz}$$

Therefore, $$\phi = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(9.33\times10^{14}\text{ Hz}\right) = 6.18\times10^{-19}\text{ J} = \boxed{3.87\text{ eV}}$$

(c) At $v = 0.900c$, $f = 3.05\times10^{15}\text{ Hz}$

and $$K_{\max} = hf - \phi = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.05\times10^{15}\text{ Hz}\right)\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 3.87\text{ eV}$$

$$= \boxed{8.78\text{ eV}}$$

Section 40.3 **The Compton Effect**

P40.21 $$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{700\times10^{-9}\text{ m}} = 2.84\times10^{-19}\text{ J} = \boxed{1.78\text{ eV}}$$

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$$

***P40.22** (a) and (b) From $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$ we calculate the wavelength of the scattered photon. For example, at $\theta = 30°$ we have

$$\lambda' + \Delta\lambda = 120\times10^{-12} + \frac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 30.0°\ ) = \boxed{120.3\times10^{-12}\ \text{m}}$$

The electron carries off the energy the photon loses:

$$K_e = \frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}\ 3\times10^{8}\ \text{m}}{(1.6\times10^{-19}\ \text{J/eV})\ \text{s}\ 10^{-12}\ \text{m}}\left(\frac{1}{120} - \frac{1}{120.3}\right) = 27.9\ \text{eV}$$

The other entries are computed similarly.

θ, degrees	0	30	60	90	120	150	180
λ', pm	120.0	120.3	121.2	122.4	123.6	124.5	124.8
K_e, eV	0	27.9	104	205	305	376	402

(c) 180°. We could answer like this: The photon imparts the greatest momentum to the originally stationary electron in a head-on collision. Here the photon recoils straight back and the electron has maximum kinetic energy.

P40.23 With $K_e = E'$, $K_e = E_0 - E'$ gives $\quad E' = E_0 - E'$

$$E' = \frac{E_0}{2} \text{ and } \lambda' = \frac{hc}{E'} \qquad \lambda' = \frac{hc}{E_0/2} = 2\frac{hc}{E_0} = 2\lambda_0$$

$$\lambda' = \lambda_0 + \lambda_C(1-\cos\theta) \qquad 2\lambda_0 = \lambda_0 + \lambda_C(1-\cos\theta)$$

$$1-\cos\theta = \frac{\lambda_0}{\lambda_C} = \frac{0.001\,60}{0.002\,43} \qquad \theta = \boxed{70.0°}$$

P40.24 (a) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$: $\Delta\lambda = \dfrac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 37.0°\ ) = \boxed{4.88\times10^{-13}\ \text{m}}$

(b) $E_0 = \dfrac{hc}{\lambda_0}$: $\qquad (300\times10^{3}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) = \dfrac{(6.626\times10^{-34})(3.00\times10^{8}\ \text{m/s})}{\lambda_0}$

$$\lambda_0 = 4.14\times10^{-12}\ \text{m}$$

and $\qquad \lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\ \text{m}$

$$E' = \frac{hc}{\lambda'} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{4.63\times10^{-12}\ \text{m}} = 4.30\times10^{-14}\ \text{J} = \boxed{268\ \text{keV}}$$

(c) $K_e = E_0 - E' = 300\ \text{keV} - 268.5\ \text{keV} = \boxed{31.5\ \text{keV}}$

P40.25 (a) Conservation of momentum in the x direction gives: $p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$

or since $\theta = \phi$, $\dfrac{h}{\lambda_0} = \left(p_e + \dfrac{h}{\lambda'}\right)\cos\theta$ [1]

Conservation of momentum in the y direction gives: $0 = p'_\gamma \sin\theta - p_e \sin\theta$

which (neglecting the trivial solution $\theta = 0$) gives: $p_e = p'_\gamma = \dfrac{h}{\lambda'}$ [2]

Substituting [2] into [1] gives: $\dfrac{h}{\lambda_0} = \dfrac{2h}{\lambda'}\cos\theta$, or $\lambda' = 2\lambda_0 \cos\theta$ [3]

Then the Compton equation is $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1-\cos\theta)$

giving $2\lambda_0 \cos\theta - \lambda_0 = \dfrac{h}{m_e c}(1-\cos\theta)$

or $2\cos\theta - 1 = \dfrac{hc}{\lambda_0}\dfrac{1}{m_e c^2}(1-\cos\theta)$

Since $E_\gamma = \dfrac{hc}{\lambda_0}$, this may be written as: $2\cos\theta - 1 = \left(\dfrac{E_\gamma}{m_e c^2}\right)(1-\cos\theta)$

which reduces to: $\left(2 + \dfrac{E_\gamma}{m_e c^2}\right)\cos\theta = 1 + \dfrac{E_\gamma}{m_e c^2}$

or $\cos\theta = \dfrac{m_e c^2 + E_\gamma}{2m_e c^2 + E_\gamma} = \dfrac{0.511\text{ MeV} + 0.880\text{ MeV}}{1.02\text{ MeV} + 0.880\text{ MeV}} = 0.732$ so that $\boxed{\theta = \phi = 43.0°}$

(b) Using Equation (3): $E'_\gamma = \dfrac{hc}{\lambda'} = \dfrac{hc}{\lambda_0(2\cos\theta)} = \dfrac{E_\gamma}{2\cos\theta} = \dfrac{0.880\text{ MeV}}{2\cos 43.0°}$

$= 0.602\text{ MeV} = \boxed{602\text{ keV}}$

Then, $p'_\gamma = \dfrac{E'_\gamma}{c} = \dfrac{0.602\text{ MeV}}{c} = \boxed{3.21\times10^{-22}\ \text{kg}\cdot\text{m/s}}$

(c) From Equation (2), $p_e = p'_\gamma = \dfrac{0.602\text{ MeV}}{c} = \boxed{3.21\times10^{-22}\ \text{kg}\cdot\text{m/s}}$.

From energy conservation:

$K_e = E_\gamma - E'_\gamma = 0.880\text{ MeV} - 0.602\text{ MeV} = 0.278\text{ MeV} = \boxed{278\text{ keV}}$

P40.26 The energy of the incident photon is $E_0 = p_\gamma c = \dfrac{hc}{\lambda_0}$.

(a) Conserving momentum in the x direction gives

$$p_\lambda = p_e \cos\phi + p'_\gamma \cos\theta, \text{ or since } \phi = \theta,\ \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta \qquad [1]$$

Conserving momentum in the y direction (with $\phi = \theta$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta, \text{ or } p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta, \text{ or } \lambda' = \frac{2hc}{E_0}\cos\theta \qquad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1-\cos\theta)$, $\quad \dfrac{2hc}{E_0}\cos\theta - \dfrac{2hc}{E_0} = \dfrac{h}{m_e c}(1-\cos\theta)$

which reduces to $\quad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$

Thus, $\quad \boxed{\phi = \theta = \cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$

(b) From Equation [3], $\quad \lambda' = \dfrac{2hc}{E_0}\cos\theta = \dfrac{2hc}{E_0}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$

Therefore,

$$E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{\left(2hc/E_0\right)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)}$$

$$= \boxed{\frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$$

and

$$p'_\gamma = \frac{E'_\gamma}{c} = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$$

(c) From conservation of energy, $\quad K_e = E_0 - E'_\gamma = E_0 - \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or

$$K_e = \frac{E_0}{2}\left(\frac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\frac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$$

Finally, from Equation (2), $\quad p_e = p'_\gamma = \boxed{\dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$.

***P40.27** (a) $K = \frac{1}{2}m_e u^2$: $\quad K = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(1.40\times10^{6}\text{ m/s}\right)^2 = 8.93\times10^{-19}\text{ J} = 5.58\text{ eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1\,240\text{ eV}\cdot\text{nm}}{0.800\text{ nm}} = 1\,550\text{ eV}$$

$E' = E_0 - K$ and $\quad \lambda' = \frac{hc}{E'} = \frac{1\,240\text{ eV}\cdot\text{nm}}{1\,550\text{ eV} - 5.58\text{ eV}} = 0.803\text{ nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.002\,89\text{ nm} = \boxed{2.89\text{ pm}}$$

(b) $\Delta\lambda = \lambda_C\left(1-\cos\theta\right)$: $\quad \cos\theta = 1 - \frac{\Delta\lambda}{\lambda_C} = 1 - \frac{0.002\,89\text{ nm}}{0.002\,43\text{ nm}} = -0.189$

so $\quad \boxed{\theta = 101°}$

P40.28 The electron's kinetic energy is

$$K = \frac{1}{2}mu^2 = \frac{1}{2}9.11\times10^{-31}\text{ kg}\left(2.18\times10^{6}\text{ m/s}\right)^2 = 2.16\times10^{-18}\text{ J}$$

This is the energy lost by the photon, $hf_0 - hf'$

$\frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = 2.16\times10^{-18}\text{ J}$. We also have

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta) = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}\cdot\text{s}}{9.11\times10^{-31}\text{ kg}\left(3\times10^{8}\text{ m}\right)}(1-\cos 17.4°)$$

$$\lambda' = \lambda_0 + 1.11\times10^{-13}\text{ m}$$

(a) Combining the equations by substitution,

$$\frac{1}{\lambda_0} - \frac{1}{\lambda_0 + 0.111\text{ pm}} = \frac{2.16\times10^{-18}\text{ J}\cdot\text{s}}{6.63\times10^{-34}\text{ J}\cdot\text{s}\left(3\times10^{8}\text{ m}\right)} = 1.09\times10^{7}/\text{m}$$

$$\frac{\lambda_0 + 0.111\text{ pm} - \lambda_0}{\lambda_0^2 + \lambda_0(0.111\text{ pm})} = 1.09\times10^{7}/\text{m}$$

$$0.111\text{ pm} = \left(1.09\times10^{7}/\text{m}\right)\lambda_0^2 + 1.21\times10^{-6}\lambda_0$$

$$1.09\times10^{7}\lambda_0^2 + 1.21\times10^{-6}\text{ m}\lambda_0 - 1.11\times10^{-13}\text{ m}^2 = 0$$

$$\lambda_0 = \frac{-1.21\times10^{-6}\text{ m} \pm \sqrt{\left(1.21\times10^{-6}\text{ m}\right)^2 - 4\left(1.09\times10^{7}\right)\left(-1.11\times10^{-13}\text{ m}^2\right)}}{2\left(1.09\times10^{7}\right)}$$

only the positive answer is physical: $\lambda_0 = \boxed{1.01\times10^{-10}\text{ m}}$.

(b) Then $\lambda' = 1.01\times10^{-10}\text{ m} + 1.11\times10^{-13}\text{ m} = 1.01\times10^{-10}\text{ m}$.
Conservation of momentum in the transverse direction:

$$0 = \frac{h}{\lambda'}\sin\theta - \gamma m_e u \sin\phi$$

$$\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{1.01\times10^{-10}\text{ m}}\sin 17.4° = \frac{9.11\times10^{-31}\text{ kg}\left(2.18\times10^{6}\text{ m/s}\right)\sin\phi}{\sqrt{1-\left(2.18\times10^{6}/3\times10^{8}\right)^2}}$$

$$1.96\times10^{-24} = 1.99\times10^{-24}\sin\phi \qquad \phi = \boxed{81.1°}$$

***P40.29** It is, because Compton's equation and the conservation of vector momentum give three independent equations in the unknowns λ', λ_0, and u. They are

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos 90°) \qquad \text{so } \lambda' = \lambda_0 + \frac{h}{m_e c}$$

$$\frac{h}{\lambda_0} = \gamma m_e u \cos 20°$$

$$\frac{h}{\lambda'} = \gamma m_e u \sin 20°$$

Dividing the latter two equations gives $\frac{\lambda_0}{\lambda'} = \tan 20°$ Then substituting, $\lambda' = \lambda' \tan 20° + \frac{h}{m_e c}$

So $\lambda' = 2.43 \times 10^{-12}$ m/(1 – tan 20°) = $\boxed{3.82 \text{ pm}}$

P40.30 $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

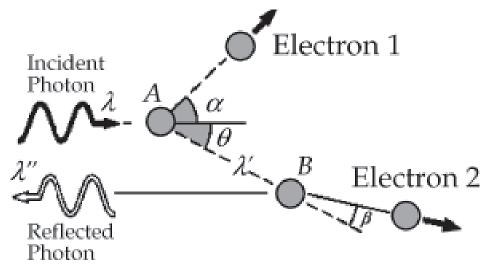

FIG. P40.30

$$\lambda'' - \lambda' = \frac{h}{m_e c}\left[1-\cos(\pi-\theta)\right]$$

$$\lambda'' - \lambda = \frac{h}{m_e c} - \frac{h}{m_e c}\cos(\pi-\theta) + \frac{h}{m_e c} - \frac{h}{m_e c}\cos\theta$$

Now $\cos(\pi-\theta) = -\cos\theta$, so

$$\lambda'' - \lambda = 2\frac{h}{m_e c} = \boxed{0.004\,86 \text{ nm}}$$

P40.31 Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then $\Delta\lambda = (1-\cos 180°)\left(\frac{h}{mc}\right) = \frac{2h}{mc}$ where m is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}$$

Further, $\lambda_0 = \frac{hc}{E_0}$, so $\frac{E_0 - E'}{E_0} = \frac{2h/mc}{hc/E_0 + 2h/mc} = \frac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{0.511 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.667}$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{938 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.001\,09}$$

Section 40.4 **Photons and Electromagnetic Waves**

***P40.32** With photon energy $10.0\text{ eV} = hf$ a photon would have

$$f = \frac{10.0\left(1.6\times10^{-19}\text{ J}\right)}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = 2.41\times10^{15}\text{ Hz and } \lambda = c/f = (3\times10^{8}\text{ m/s})/(2.41\times10^{15}/\text{s}) = 124\text{ nm}$$

$\boxed{\text{To have photon energy 10 eV or greater, according to this definition, ionizing radiation is the ultraviolet light, x-rays, and } \gamma \text{ rays with wavelength shorter than 124 nm; that is, with frequency higher than } 2.41\times10^{15}\text{ Hz.}}$

P40.33 The photon energy is $E = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}\left(3\times10^{8}\text{ m/s}\right)}{633\times10^{-9}\text{ m}} = 3.14\times10^{-19}\text{ J}$. The power carried by the beam is $\left(2\times10^{18}\text{ photons/s}\right)\left(3.14\times10^{-19}\text{ J/photon}\right) = 0.628\text{ W}$. Its intensity is the average Poynting vector $I = S_{\text{av}} = \dfrac{\mathcal{P}}{\pi r^2} = \dfrac{0.628\text{ W}(4)}{\pi\left(1.75\times10^{-3}\text{ m}\right)^2} = 2.61\times10^{5}\text{ W/m}^2$.

(a) $S_{\text{av}} = \dfrac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}}\sin 90° = \dfrac{1}{\mu_0}\dfrac{E_{\text{max}}}{\sqrt{2}}\dfrac{B_{\text{max}}}{\sqrt{2}}$. Also $E_{\text{max}} = B_{\text{max}}c$. So $S_{\text{av}} = \dfrac{E_{\text{max}}^2}{2\mu_0 c}$.

$$E_{\text{max}} = \left(2\mu_0 c S_{\text{av}}\right)^{1/2} = \left(2\left(4\pi\times10^{-7}\text{ Tm/A}\right)\left(3\times10^{8}\text{ m/s}\right)\left(2.61\times10^{5}\text{ W/m}^2\right)\right)^{1/2}$$
$$= \boxed{1.40\times10^{4}\text{ N/C}}$$
$$B_{\text{max}} = \frac{1.40\times10^{4}\text{ N/C}}{3\times10^{8}\text{ m/s}} = \boxed{4.68\times10^{-5}\text{ T}}$$

(b) Each photon carries momentum $\dfrac{E}{c}$. The beam transports momentum at the rate $\dfrac{\mathcal{P}}{c}$.

It imparts momentum to a perfectly reflecting surface at the rate

$$\frac{2\mathcal{P}}{c} = \text{force} = \frac{2(0.628\text{ W})}{3\times10^{8}\text{ m/s}} = \boxed{4.19\times10^{-9}\text{ N}}$$

(c) The block of ice absorbs energy $mL = \mathcal{P}\Delta t$ melting

$$m = \frac{\mathcal{P}\Delta t}{L} = \frac{0.628\text{ W}(1.5\times3\,600\text{ s})}{3.33\times10^{5}\text{ J/kg}} = \boxed{1.02\times10^{-2}\text{ kg}}$$

Section 40.5 The Wave Properties of Particles

P40.34 $\lambda = \dfrac{h}{p} = \dfrac{h}{mu} = \dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(1.00 \times 10^{6}\ \text{m/s}\right)} = \boxed{3.97 \times 10^{-13}\ \text{m}}$

P40.35 (a) $\dfrac{p^2}{2m} = (50.0)\left(1.60 \times 10^{-19}\ \text{J}\right)$

$p = 3.81 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{0.174\ \text{nm}}$

(b) $\dfrac{p^2}{2m} = \left(50.0 \times 10^{3}\right)\left(1.60 \times 10^{-19}\ \text{J}\right)$

$p = 1.20 \times 10^{-22}\ \text{kg}\cdot\text{m/s}$

$\lambda = \dfrac{h}{p} = 5.49 \times 10^{-12}\ \text{m}$

The relativistic answer is slightly more precise:

$$\lambda = \frac{h}{p} = \frac{hc}{\left[\left(mc^2 + K\right)^2 - m^2c^4\right]^{1/2}} = \boxed{5.37 \times 10^{-12}\ \text{m}}$$

P40.36 (a) Electron: $\lambda = \dfrac{h}{p}$ and $K = \dfrac{1}{2}m_e u^2 = \dfrac{m_e^2 u^2}{2m_e} = \dfrac{p^2}{2m_e}$ so $p = \sqrt{2m_e K}$

and $\lambda = \dfrac{h}{\sqrt{2m_e K}} = \dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11 \times 10^{-31}\ \text{kg}\right)(3.00)\left(1.60 \times 10^{-19}\ \text{J}\right)}}$

$\lambda = 7.09 \times 10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}$

(b) Photon: $\lambda = \dfrac{c}{f}$ and $E = hf$ so $f = \dfrac{E}{h}$

and $\lambda = \dfrac{hc}{E} = \dfrac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{3\left(1.60 \times 10^{-19}\ \text{J}\right)} = 4.14 \times 10^{-7}\ \text{m} = \boxed{414\ \text{nm}}$

***P40.37** (a) $\lambda \sim 10^{-14}\ \text{m}$ or less. $p = \dfrac{h}{\lambda} \sim \dfrac{6.6 \times 10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 10^{-19}\ \text{kg}\cdot\text{m/s}$ or more

The energy of the electron is $E = \sqrt{p^2c^2 + m_e^2c^4} \sim \sqrt{\left(10^{-19}\right)^2\left(3 \times 10^{8}\right)^2 + \left(9 \times 10^{-31}\right)^2\left(3 \times 10^{8}\right)^4}$

or $E \sim 10^{-11}\ \text{J} \sim 10^{8}\ \text{eV}$ or more

so that $K = E - m_e c^2 \sim 10^{8}\ \text{eV} - \left(0.5 \times 10^{6}\ \text{eV}\right)\boxed{\sim 10^{8}\ \text{eV}}$ or more

(b) If the nucleus contains ten protons, the electric potential energy of the electron-nucleus system would be

$$U_e = \frac{k_e q_1 q_2}{r} \sim \frac{\left(9 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10 \times 10^{-19}\ \text{C}\right)(-e)}{5 \times 10^{-15}\ \text{m}}\boxed{\sim -10^{6}\ \text{eV}}$$

With its $K + U_e >> 0$ $\boxed{\text{the electron would immediately escape the nucleus}}$.

P40.38 From the condition for Bragg reflection,

$$m\lambda = 2d\sin\theta = 2d\cos\left(\frac{\phi}{2}\right)$$

But $d = a\sin\left(\frac{\phi}{2}\right)$

where a is the lattice spacing.

Thus, with $m = 1$, $\lambda = 2a\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) = a\sin\phi$

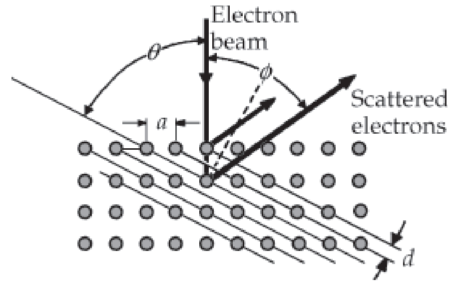

FIG. P40.38

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} \qquad \lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(54.0\times1.60\times10^{-19}\ \text{J}\right)}} = 1.67\times10^{-10}\ \text{m}$$

Therefore, the lattice spacing is $a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\ \text{m}}{\sin 50.0°} = 2.18\times10^{-10} = \boxed{0.218\ \text{nm}}$.

P40.39 (a) $E^2 = p^2c^2 + m^2c^4$

with $E = hf$, $p = \frac{h}{\lambda}$ and $mc = \frac{h}{\lambda_C}$

so $h^2f^2 = \frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda_C^2}$ and $\left(\frac{f}{c}\right)^2 = \frac{1}{\lambda^2} + \frac{1}{\lambda_C^2}$ (Eq. 1)

(b) For a photon $\frac{f}{c} = \frac{1}{\lambda}$

The third term $\frac{1}{\lambda_C}$ in Equation 1 for electrons and other massive particles shows that $\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$.

***P40.40** (a) For the massive particle, $K = (\gamma - 1)mc^2$ and $\lambda_m = \frac{h}{p} = \frac{h}{\gamma mu}$. For the photon (which we represent as γ), $E = K$ and $\lambda_\gamma = \frac{c}{f} = \frac{ch}{E} = \frac{ch}{K} = \frac{ch}{(\gamma-1)mc^2}$. Then the ratio is

$$\frac{\lambda_\gamma}{\lambda_m} = \frac{ch\gamma mu}{(\gamma-1)mc^2h} = \boxed{\frac{\gamma}{\gamma-1}\frac{u}{c} \quad \text{where } \gamma = \frac{1}{\sqrt{1-u^2/c^2}}}$$. The ratio can be written

$$\frac{u}{c\left(1-\sqrt{1-u^2/c^2}\right)}$$

(b) $$\frac{\lambda_\gamma}{\lambda_m} = \frac{1(0.9)}{\sqrt{1-0.9^2}\left[\left(1/\sqrt{1-0.9^2}\right)-1\right]} = \boxed{1.60}$$

(c) The ratio for a particular particle speed does not depend on the particle mass: There would be no change.

(d) $$\frac{\lambda_\gamma}{\lambda_m} = \frac{1(0.001)}{\sqrt{1-(0.001)^2}\left[\left(1/\sqrt{1-(0.001)^2}\right)-1\right]} = \boxed{2.00\times10^3}$$

(e) As $\frac{u}{c} \to 1$, $\gamma \to \infty$ and $\gamma - 1$ becomes nearly equal to γ. Then $\frac{\lambda_\gamma}{\lambda_m} \to \frac{\gamma}{\gamma}1 = \boxed{1}$.

(f) As $\frac{u}{c} \to 0$, $\left(1-\frac{u^2}{c^2}\right)^{-1/2} - 1 \approx 1 - \left(-\frac{1}{2}\right)\frac{u^2}{c^2} - 1 = \frac{1}{2}\frac{u^2}{c^2}$ and $\frac{\lambda_\gamma}{\lambda_m} \to 1\frac{u/c}{(1/2)\left(u^2/c^2\right)} = \frac{2c}{u} \to \boxed{\infty}$.

P40.41 $\lambda = \dfrac{h}{p}$ $\qquad p = \dfrac{h}{\lambda} = \dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{1.00 \times 10^{-11}\ \text{m}} = 6.63 \times 10^{-23}\ \text{kg}\cdot\text{m/s}$

(a) electrons: $\quad K_e = \dfrac{p^2}{2m_e} = \dfrac{\left(6.63 \times 10^{-23}\right)^2}{2\left(9.11 \times 10^{-31}\right)}\ \text{J} = 15.1\ \text{keV}$

The relativistic answer is more precisely correct:

$$K_e = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2 = \boxed{14.9\ \text{keV}}$$

(b) photons: $\quad E_\gamma = pc = \left(6.63 \times 10^{-23}\right)\left(3.00 \times 10^8\right) = \boxed{124\ \text{keV}}$

***P40.42** (a) The wavelength of the student is $\lambda = \dfrac{h}{p} = \dfrac{h}{mu}$. If w is the width of the diffracting aperture,

then we need $\quad w \le 10.0\lambda = 10.0\left(\dfrac{h}{mu}\right)$

so that $\quad u \le 10.0\dfrac{h}{mw} = 10.0\left(\dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{(80.0\ \text{kg})(0.750\ \text{m})}\right) = \boxed{1.10 \times 10^{-34}\ \text{m/s}}$

(b) Using $\Delta t = \dfrac{d}{u}$ we get: $\quad \Delta t \ge \dfrac{0.150\ \text{m}}{1.10 \times 10^{-34}\ \text{m/s}} = \boxed{1.36 \times 10^{33}\ \text{s}}$, which is more than 10^{15} times the age of the Universe.

(c) $\boxed{\text{He should not worry.}}$ The student cannot be diffracted to a measurable extent. Even if he tries to stand still, molecular bombardment will give him a sufficiently high speed to make his wavelength immeasurably small.

***P40.43** (a) $E = \gamma mc^2 \qquad \gamma = E/mc^2 = 20\,000\ \text{MeV}/0.511\ \text{MeV} = \boxed{3.91 \times 10^4}$

(b) $pc = [E^2 - (mc^2)^2]^{1/2} = [(20\,000\ \text{MeV})^2 - (0.511\ \text{MeV})^2]^{1/2} = 20.0\ \text{GeV}$

$p = \boxed{20.0\ \text{GeV}/c = 1.07 \times 10^{-17}\ \text{kg}\cdot\text{m/s}}$

(c) $\lambda = h/p = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s}/1.07 \times 10^{-17}\ \text{kg}\cdot\text{m/s} = \boxed{6.21 \times 10^{-17}\ \text{m}}$, small compared to the size of the nucleus. The scattering of the electrons can give information about the particles forming the nucleus.

Section 40.6 **The Quantum Particle**

P40.44 $E = K = \dfrac{1}{2}mu^2 = hf$ and $\lambda = \dfrac{h}{mu}$

$$v_{\text{phase}} = f\lambda = \frac{mu^2}{2h}\frac{h}{mu} = \boxed{\frac{u}{2} = v_{\text{phase}}}$$

This is different from the speed u at which the particle transports mass, energy, and momentum.

P40.45 As a bonus, we begin by proving that the phase speed $v_p = \dfrac{\omega}{k}$ is not the speed of the particle.

$$v_p = \frac{\omega}{k} = \frac{\sqrt{p^2c^2 + m^2c^4}\,\hbar}{\hbar\gamma mu} = \frac{\sqrt{\gamma^2m^2u^2c^2 + m^2c^4}}{\sqrt{\gamma^2m^2u^2}} = c\sqrt{1 + \frac{c^2}{\gamma^2u^2}} = c\sqrt{1 + \frac{c^2}{u^2}\left(1 - \frac{u^2}{c^2}\right)}$$

$$= c\sqrt{1 + \frac{c^2}{u^2} - 1} = \frac{c^2}{u}$$

In fact, the phase speed is larger than the speed of light. A point of constant phase in the wave function carries no mass, no energy, and no information.

Now for the group speed:

$$v_g = \frac{d\omega}{dk} = \frac{d\hbar\omega}{d\hbar k} = \frac{dE}{dp} = \frac{d}{dp}\sqrt{m^2c^4 + p^2c^2}$$

$$v_g = \frac{1}{2}\left(m^2c^4 + p^2c^2\right)^{-1/2}\left(0 + 2pc^2\right) = \sqrt{\frac{p^2c^4}{p^2c^2 + m^2c^4}}$$

$$v_g = c\sqrt{\frac{\gamma^2m^2u^2}{\gamma^2m^2u^2 + m^2c^2}} = c\sqrt{\frac{u^2/\left(1 - u^2/c^2\right)}{u^2/\left(1 - u^2/c^2\right) + c^2}} = c\sqrt{\frac{u^2/\left(1 - u^2/c^2\right)}{\left(u^2 + c^2 - u^2\right)/\left(1 - u^2/c^2\right)}} = u$$

It is this speed at which mass, energy, and momentum are transported.

Section 40.7 **The Double-Slit Experiment Revisited**

P40.46 Consider the first bright band away from the center:

$d\sin\theta = m\lambda$ $\qquad$ $\left(6.00\times10^{-8}\ \text{m}\right)\sin\left(\tan^{-1}\left[\dfrac{0.400}{200}\right]\right) = (1)\,\lambda = 1.20\times10^{-10}\ \text{m}$

$\lambda = \dfrac{h}{m_eu}$ so $\qquad$ $m_eu = \dfrac{h}{\lambda}$

and $\qquad$ $K = \dfrac{1}{2}m_eu^2 = \dfrac{m_e^2u^2}{2m_e} = \dfrac{h^2}{2m_e\lambda^2} = e\Delta V$

$\Delta V = \dfrac{h^2}{2em_e\lambda^2}$ $\qquad$ $\Delta V = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(1.60\times10^{-19}\ \text{C}\right)\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.20\times10^{-10}\ \text{m}\right)^2} = \boxed{105\ \text{V}}$

P40.47 (a) $\lambda = \dfrac{h}{mu} = \dfrac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(0.400\ \text{m/s}\right)} = \boxed{9.92 \times 10^{-7}\ \text{m}}$

(b) For destructive interference in a multiple-slit experiment, $d \sin\theta = \left(m + \dfrac{1}{2}\right)\lambda$, with $m = 0$ for the first minimum. Then

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 0.028\,4^\circ$$

$$\frac{y}{L} = \tan\theta \qquad y = L\tan\theta = (10.0\ \text{m})\left(\tan 0.028\,4^\circ\right) = \boxed{4.96\ \text{mm}}$$

(c) We cannot say the neutron passed through one slit. If its detection forms part of an interference pattern, we can only say it passed through the array of slits. If we test to see which slit a particular neutron passes through, it will not form part of the interference pattern.

P40.48 We find the speed of each electron from energy conservation in the firing process:

$$0 = K_f + U_f = \frac{1}{2}mu^2 - \text{eV}$$

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19}\ \text{C}(45\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}} = 3.98 \times 10^6\ \text{m/s}$$

The time of flight is $\Delta t = \dfrac{\Delta x}{u} = \dfrac{0.28\ \text{m}}{3.98 \times 10^6\ \text{m/s}} = 7.04 \times 10^{-8}\ \text{s}$. The current when electrons are 28 cm apart is $I = \dfrac{q}{t} = \dfrac{e}{\Delta t} = \dfrac{1.6 \times 10^{-19}\ \text{C}}{7.04 \times 10^{-8}\ \text{s}} = \boxed{2.27 \times 10^{-12}\ \text{A}}$.

Section 40.8 **The Uncertainty Principle**

P40.49 For the electron, $\Delta p = m_e \Delta u = \left(9.11 \times 10^{-31}\ \text{kg}\right)(500\ \text{m/s})\left(1.00 \times 10^{-4}\right) = 4.56 \times 10^{-32}\ \text{kg} \cdot \text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{4\pi\left(4.56 \times 10^{-32}\ \text{kg} \cdot \text{m/s}\right)} = \boxed{1.16\ \text{mm}}$$

For the bullet, $\Delta p = m\Delta u = (0.020\,0\ \text{kg})(500\ \text{m/s})\left(1.00 \times 10^{-4}\right) = 1.00 \times 10^{-3}\ \text{kg} \cdot \text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \boxed{5.28 \times 10^{-32}\ \text{m}}$$

P40.50 (a) $\Delta p \Delta x = m \Delta u \Delta x \geq \dfrac{\hbar}{2}$ so $\Delta u \geq \dfrac{h}{4\pi\, m\Delta x} = \dfrac{2\pi\ \text{J} \cdot \text{s}}{4\pi(2.00\ \text{kg})(1.00\ \text{m})} = \boxed{0.250\ \text{m/s}}$

(b) The duck might move by $(0.25\ \text{m/s})(5\ \text{s}) = 1.25$ m. With original position uncertainty of 1.00 m, we can think of Δx growing to $1.00\ \text{m} + 1.25\ \text{m} = \boxed{2.25\ \text{m}}$.

P40.51 $\frac{\Delta y}{x} = \frac{\Delta p_y}{p_x}$ and $d\Delta p_y \geq \frac{h}{4\pi}$

Eliminate Δp_y and solve for x.

$$x = 4\pi\, p_x(\Delta y)\frac{d}{h}: \qquad x = 4\pi\left(1.00\times10^{-3}\text{ kg}\right)(100\text{ m/s})\left(1.00\times10^{-2}\text{ m}\right)\frac{\left(2.00\times10^{-3}\text{ m}\right)}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)}$$

The answer, $x = \boxed{3.79\times10^{28}\text{ m}}$, is 190 times greater than the diameter of the observable Universe.

P40.52 With $\Delta x = 2\times10^{-15}$ m, the uncertainty principle requires $\Delta p_x \geq \frac{\hbar}{2\Delta x} = 2.6\times10^{-20}\text{ kg}\cdot\text{m/s}$.

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} \approx 3\times10^{-20}\text{ kg}\cdot\text{m/s}$. For an electron, the non-relativistic approximation $p = m_e u$ would predict $u \approx 3\times10^{10}$ m/s, while u cannot be greater than c.

Thus, a better solution would be $E = \left[\left(m_e c^2\right)^2 + (pc)^2\right]^{1/2} \approx 56\text{ MeV} = \gamma m_e c^2$

$$\gamma \approx 110 = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{so} \qquad u \approx 0.999\,96c$$

For a proton, $u = \frac{p}{m}$ gives $u = 1.8\times10^7$ m/s, less than one-tenth the speed of light.

P40.53 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta u_x = \frac{\hbar}{2\Delta x_i}$. It falls to the floor in a travel time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{\frac{2H}{g}}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta u_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}$$

where $A = \frac{\hbar}{2m}\sqrt{\frac{2H}{g}}$

To minimize Δx_f, we require $\frac{d(\Delta x_f)}{d(\Delta x_i)} = 0$ or $1 - \frac{A}{\Delta x_i^2} = 0$

so $\Delta x_i = \sqrt{A}$

The minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left.\left(\Delta x_i + \frac{A}{\Delta x_i}\right)\right|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.054\,6\times10^{-34}\text{ J}\cdot\text{s}\right)}{5.00\times10^{-4}\text{ kg}}\right]^{1/2}\left[\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}\right]^{1/4} = \boxed{5.19\times10^{-16}\text{ m}}$$

Additional Problems

***P40.54** The condition on electric power delivered to the filament is

$$\mathcal{P} = I\,\Delta V = \frac{(\Delta V)^2}{R} = \frac{(\Delta V)^2 A}{\rho\,\ell} = \frac{(\Delta V)^2 \pi r^2}{\rho\,\ell} \text{ so } r = \left(\frac{\mathcal{P}\,\rho\,\ell}{\pi(\Delta V)^2}\right)^{1/2}. \text{ Here } \mathcal{P} = 75 \text{ W},$$

$\rho = 7.13\times10^{-7}\,\Omega\cdot\text{m}$, and $\Delta V = 120$ V. As the filament radiates in steady state, it must emit all of this power through its lateral surface area $\mathcal{P} = \sigma\,eAT^4 = \sigma\,e2\pi r\ell T^4$. We combine the conditions by substitution:

$$\mathcal{P} = \sigma\,e2\pi\left(\frac{\mathcal{P}\,\rho\,\ell}{\pi(\Delta V)^2}\right)^{1/2}\ell T^4$$

$$(\Delta V)\mathcal{P}^{1/2} = \sigma\,e2\pi^{1/2}\rho^{1/2}\ell^{3/2}T^4$$

$$\ell = \left(\frac{(\Delta V)\mathcal{P}^{1/2}}{\sigma\,e2\pi^{1/2}\rho^{1/2}T^4}\right)^{2/3}$$

$$= \left(\frac{120\text{ V}(75\text{ W})^{1/2}\ \text{m}^2\text{K}^4}{5.67\times10^{-8}\ \text{W}\ 0.450(2)\pi^{1/2}\left(7.13\times10^{-7}\,\Omega\cdot\text{m}\right)^{1/2}(2\,900\text{ K})^4}\right)^{2/3}$$

$$= \left(0.192\text{ m}^{3/2}\right)^{2/3} = \boxed{0.333\text{ m} = \ell}$$

and $$r = \left(\frac{\mathcal{P}\,\rho\,\ell}{\pi(\Delta V)^2}\right)^{1/2} = \left(\frac{75\text{ W }7.13\times10^{-7}\,\Omega\cdot\text{m }0.333\text{ m}}{\pi(120\text{ V})^2}\right)^{1/2} = \boxed{r = 1.98\times10^{-5}\text{ m}}$$

P40.55 We want an Einstein plot of $K_{\max}$ versus f

λ, nm	f, 10^{14} Hz	$K_{\max}$, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63

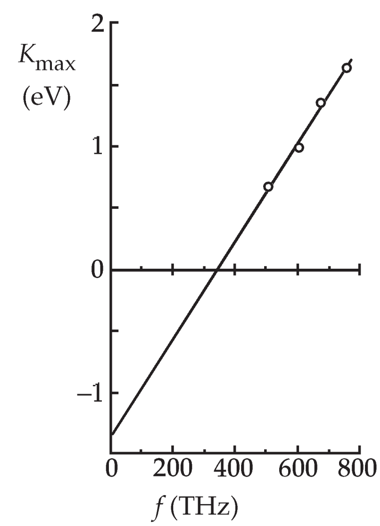

FIG. P40.55

(a) slope $= \dfrac{0.402\text{ eV}}{10^{14}\text{ Hz}} \pm 8\%$

(b) $e\Delta V_S = hf - \phi$

$$h = (0.402)\left(\frac{1.60\times10^{-19}\ \text{J}\cdot\text{s}}{10^{14}}\right) = \boxed{6.4\times10^{-34}\ \text{J}\cdot\text{s}\pm8\%}$$

(c) $K_{\max} = 0$

at $f \approx 344\times10^{12}$ Hz

$\phi = hf = 2.32\times10^{-19}\ \text{J} = \boxed{1.4\text{ eV}}$

P40.56 $\Delta V_S = \left(\frac{h}{e}\right) f - \frac{\phi}{e}$

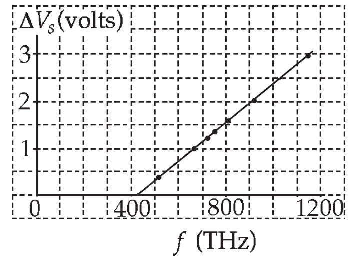

FIG. P40.56

From two points on the graph $\quad 0 = \left(\frac{h}{e}\right)\left(4.1\times10^{14}\text{ Hz}\right) - \frac{\phi}{e}$

and $\quad 3.3\text{ V} = \left(\frac{h}{e}\right)\left(12\times10^{14}\text{ Hz}\right) - \frac{\phi}{e}$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\text{ eV}}$

(b) $\frac{h}{e} = \boxed{4.2\times10^{-15}\text{ V}\cdot\text{s}}$

(c) At the cutoff wavelength $\frac{hc}{\lambda_c} = \phi = \left(\frac{h}{e}\right)\frac{ec}{\lambda_c}$

$$\lambda_c = \left(4.2\times10^{-15}\text{ V}\cdot\text{s}\right)\left(1.6\times10^{-19}\text{ C}\right)\frac{\left(3\times10^{8}\text{ m/s}\right)}{(1.7\text{ eV})\left(1.6\times10^{-19}\text{ J/eV}\right)} = \boxed{730\text{ nm}}$$

P40.57 From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

$$K_{\max} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $\quad K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$

Thus, the work function is $\quad \phi = \frac{hc}{\lambda} - K_{\max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$

***P40.58** (a) We find the energy of one photon:

$$3.44\text{ eV}\left(\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}\right) + \tfrac{1}{2}9.11\times10^{-31}\text{ kg}\left(420\times10^{3}\text{ m/s}\right)^2 = 5.50\times10^{-19}\text{ J} + 0.804\times10^{-19}\text{ J}$$

$$= 6.31\times10^{-19}\text{ J}$$

The number intensity of photon bombardment is

$$\frac{550\text{ J/s}\cdot\text{m}^2}{6.31\times10^{-19}\text{ J}}\left(\frac{1\text{ m}^2}{10^4\text{ cm}^2}\right) = \boxed{8.72\times10^{16}\text{ 1/s}\cdot\text{cm}^2}$$

(b) The density of the current the imagined electrons comprise is

$$8.72\times10^{16}\frac{1}{\text{s}\cdot\text{cm}^2}1.6\times10^{-19}\text{ C} = \boxed{14.0\text{ mA/cm}^2}$$

(c) Many photons are likely reflected or give their energy to the metal as internal energy, so the actual current is probably a small fraction of 14.0 mA.

P40.59 Isolate the terms involving ϕ in Equations 40.13 and 40.14. Square and add to eliminate ϕ.

$$h^2\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]=\gamma^2 m_e^2 u^2$$

Solve for $\frac{u^2}{c^2}=\frac{b}{\left(b+c^2\right)}$ where we define $b=\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$

Substitute into Eq. 40.12: $$1+\left(\frac{h}{m_e c}\right)\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]=\gamma=\left(1-\frac{b}{b+c^2}\right)^{-1/2}=\sqrt{\frac{c^2+b}{c^2}}$$

Square each side: $$c^2+\frac{2hc}{m_e}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]+\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]^2$$
$$=c^2+\left(\frac{h^2}{m_e^2}\right)\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

From this we get Eq. 40.11: $$\lambda'-\lambda_0=\left(\frac{h}{m_e c}\right)[1-\cos\theta]$$

P40.60 We show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved.

Energy: $$\frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+K_e=m_e c^2\left(\gamma-1\right)\quad\text{if}\quad\frac{hc}{\lambda'}=0 \tag{1}$$

Momentum: $$\frac{h}{\lambda_0}=\frac{h}{\lambda'}+\gamma\, m_e u=\gamma\, m_e u\quad\text{if}\quad\lambda'=\infty \tag{2}$$

From (1), $$\gamma=\frac{h}{\lambda_0 m_e c}+1 \tag{3}$$

$$u=c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2} \tag{4}$$

Substitute (3) and (4) into (2) and show the inconsistency:

$$\frac{h}{\lambda_0}=\left(1+\frac{h}{\lambda_0 m_e c}\right)m_e c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2}=\frac{\lambda_0 m_e c+h}{\lambda_0}\sqrt{\frac{h\left(h+2\lambda_0 m_e c\right)}{\left(h+\lambda_0 m_e c\right)^2}}=\frac{h}{\lambda_0}\sqrt{\frac{h+2\lambda_0 m_e c}{h}}$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

P40.61 (a) Starting with Planck's law,
$$I(\lambda,\ T)=\frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T}-1\right]}$$

the total power radiated per unit area
$$\int_0^\infty I(\lambda,\ T)\,d\lambda=\int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T}-1\right]}\,d\lambda$$

Change variables by letting
$$x=\frac{hc}{\lambda k_B T}$$

and
$$dx=-\frac{hc\,d\lambda}{k_B T\lambda^2}$$

Note that as λ varies from $0\to\infty$, x varies from $\infty\to 0$.

Then
$$\int_0^\infty I(\lambda,\ T)\,d\lambda=-\frac{2\pi k_B^4T^4}{h^3c^2}\int_\infty^0\frac{x^3}{\left(e^x-1\right)}\,dx=\frac{2\pi k_B^4T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right)$$

Therefore,
$$\boxed{\int_0^\infty I(\lambda,\ T)\,d\lambda=\left(\frac{2\pi^5k_B^4}{15h^3c^2}\right)T^4=\sigma T^4}$$

(b) From part (a),
$$\sigma=\frac{2\pi^5k_B^4}{15h^3c^2}=\frac{2\pi^5\left(1.38\times10^{-23}\ \text{J/K}\right)^4}{15\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3\left(3.00\times10^8\ \text{m/s}\right)^2}$$

$$\sigma=\boxed{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}$$

P40.62 Planck's law states $I(\lambda,\ T)=\dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T}-1\right]}=2\pi hc^2\lambda^{-5}\left[e^{hc/\lambda k_B T}-1\right]^{-1}$.

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda}=2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_B T}-1\right]^{-1}-\lambda^{-5}\left[e^{hc/\lambda k_B T}-1\right]^{-2}e^{hc/\lambda k_B T}\left(-\frac{hc}{\lambda^2k_B T}\right)\right\}=0$$

$$\frac{dI}{d\lambda}=\frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_B T}-1\right]}\left\{-5+\frac{hc}{\lambda k_B T}\frac{e^{hc/\lambda k_B T}}{\left[e^{hc/\lambda k_B T}-1\right]}\right\}=0$$

Letting $x=\dfrac{hc}{\lambda k_B T}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x-1}=5$.

We zero in on the solution to this transcendental equation by iterations as shown in the table below.

continued on next page

x	$xe^x/(e^x - 1)$
4.000 00	4.074 629 4
4.500 00	4.550 552 1
5.000 00	5.033 918 3
4.900 00	4.936 762 0
4.950 00	4.985 313 0
4.975 00	5.009 609 0
4.963 00	4.997 945 2
4.969 00	5.003 776 7
4.966 00	5.000 860 9

x	$xe^x/(e^x - 1)$
4.964 50	4.999 403 0
4.965 50	5.000 374 9
4.965 00	4.999 889 0
4.965 25	5.000 132 0
4.965 13	5.000 015 3
4.965 07	4.999 957 0
4.965 10	4.999 986 2
4.965 115	5.000 000 8

The solution is found to be

$$x = \frac{hc}{\lambda_{\max} k_B T} = 4.965\,115 \text{ and } \qquad \lambda_{\max} T = \frac{hc}{4.965\,115 k_B}$$

Thus, $\lambda_{\max} T = \dfrac{(6.626\,075 \times 10^{-34}\ \text{J} \cdot \text{s})(2.997\,925 \times 10^{8}\ \text{m/s})}{4.965\,115(1.380\,658 \times 10^{-23}\ \text{J/K})} = \boxed{2.897\,755 \times 10^{-3}\ \text{m} \cdot \text{K}}$.

This result agrees with Wien's experimental value of $\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$ for this constant.

P40.63 $p = mu = \sqrt{2mE} = \sqrt{2(1.67 \times 10^{-27}\ \text{kg})(0.040\,0\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})}$

$$\lambda = \frac{h}{mu} = 1.43 \times 10^{-10}\ \text{m} = \boxed{0.143\ \text{nm}}$$

This is of the same order of magnitude as the spacing between atoms in a crystal, so diffraction should appear. A diffraction pattern with maxima and minima at the same angles can be produced with x-rays, with neutrons, and with electrons of much higher kinetic energy, by using incident quantum particles with the same wavelength.

P40.64 (a) $mgy_i = \dfrac{1}{2} m u_f^2$

$$u_f = \sqrt{2gy_i} = \sqrt{2(9.80\ \text{m/s}^2)(50.0\ \text{m})} = 31.3\ \text{m/s}$$

$$\lambda = \frac{h}{mu} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(75.0\ \text{kg})(31.3\ \text{m/s})} = \boxed{2.82 \times 10^{-37}\ \text{m}} \quad \text{(not observable)}$$

(b) $\Delta E \Delta t \geq \dfrac{\hbar}{2}$

$$\text{so } \Delta E \geq \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{4\pi(5.00 \times 10^{-3}\ \text{s})} = \boxed{1.06 \times 10^{-32}\ \text{J}}$$

(c) $\dfrac{\Delta E}{E} = \dfrac{1.06 \times 10^{-32}\ \text{J}}{(75.0\ \text{kg})(9.80\ \text{m/s}^2)(50.0\ \text{m})} = \boxed{2.87 \times 10^{-35}\%}$

P40.65 $\lambda_C = \dfrac{h}{m_e c}$ and $\lambda = \dfrac{h}{p}$: $\qquad \dfrac{\lambda_C}{\lambda} = \dfrac{h/m_e c}{h/p} = \dfrac{p}{m_e c}$

$E^2 = c^2p^2 + (m_e c^2)^2$: $\qquad p = \sqrt{\dfrac{E^2}{c^2} - (m_e c)^2}$

$$\frac{\lambda_C}{\lambda} = \frac{1}{m_e c}\sqrt{\frac{E^2}{c^2} - (m_e c)^2} = \sqrt{\frac{1}{(m_e c)^2}\left[\frac{E^2}{c^2} - (m_e c)^2\right]} = \sqrt{\left(\frac{E}{m_e c^2}\right)^2 - 1}$$

P40.66 $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

P40.67 From the uncertainty principle $\qquad \Delta E \Delta t \geq \dfrac{\hbar}{2}$

or $\qquad \Delta(mc^2)\Delta t = \dfrac{\hbar}{2}$

Therefore, $\qquad \dfrac{\Delta m}{m} = \dfrac{h}{4\pi c^2(\Delta t)m} = \dfrac{h}{4\pi(\Delta t)E_R}$

$$\frac{\Delta m}{m} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(8.70\times10^{-17}\ \text{s}\right)(135\ \text{MeV})}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right)$$

$$= \boxed{2.81\times10^{-8}}$$

P40.68 Let u' represent the final speed of the electron and let $\gamma' = \left(1 - \frac{u'^2}{c^2}\right)^{-1/2}$. We must eliminate β and u' from the three conservation equations:

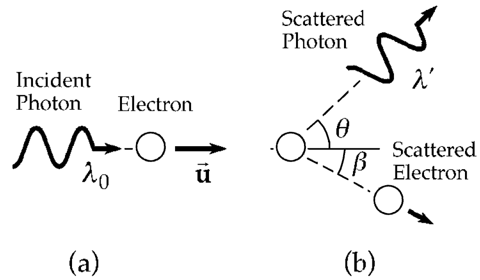

FIG. P40.68

$$\frac{hc}{\lambda_0} + \gamma m_e c^2 = \frac{hc}{\lambda'} + \gamma' m_e c^2 \qquad [1]$$

$$\frac{h}{\lambda_0} + \gamma m_e u - \frac{h}{\lambda'}\cos\theta = \gamma' m_e u' \cos\beta \qquad [2]$$

$$\frac{h}{\lambda'}\sin\theta = \gamma' m_e u' \sin\beta \qquad [3]$$

Square Equations [2] and [3] and add:

$$\frac{h^2}{\lambda_0^2} + \gamma^2 m_e^2 u^2 + \frac{h^2}{\lambda'^2} + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h^2\cos\theta}{\lambda_0\lambda'} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} = \gamma'^2 m_e^2 u'^2$$

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 u^2 + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} - \frac{2h^2\cos\theta}{\lambda_0\lambda'} = \frac{m_e^2 u'^2}{1 - u'^2/c^2}$$

Call the left-hand side b. Then $b - \frac{bu'^2}{c^2} = m_e^2 u'^2$ and $u'^2 = \frac{b}{m_e^2 + b/c^2} = \frac{c^2 b}{m_e^2 c^2 + b}$.

Now square Equation [1] and substitute to eliminate γ':

$$\frac{h^2}{\lambda_0^2} + \gamma^2 m_e^2 c^2 + \frac{h^2}{\lambda'^2} + \frac{2h\gamma\, m_e c}{\lambda_0} - \frac{2h^2}{\lambda_0\lambda'} - \frac{2h\gamma\, m_e c}{\lambda'} = \frac{m_e^2 c^2}{1 - u'^2/c^2} = m_e^2 c^2 + b$$

So we have $\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 c^2 + \frac{2h\gamma\, m_e c}{\lambda_0} - \frac{2h\gamma\, m_e c}{\lambda'} - \frac{2h^2}{\lambda_0\lambda'}$

$$= m_e c^2 + \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 u^2 + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} - \frac{2h^2\cos\theta}{\lambda_0\lambda'}$$

Multiply through by $\frac{\lambda_0\lambda'}{m_e^2 c^2}$

$$\lambda_0\lambda'\gamma^2 + \frac{2h\lambda'\gamma}{m_e c} - \frac{2h\lambda_0\gamma}{m_e c} - \frac{2h^2}{m_e^2 c^2} = \lambda_0\lambda' + \frac{\lambda_0\lambda'\gamma^2 u^2}{m_e c^2} + \frac{2h\lambda' u\gamma}{m_e c^2} - \frac{2h\gamma\lambda_0 u\cos\theta}{m_e c^2} - \frac{2h^2\cos\theta}{m_e^2 c^2}$$

$$\lambda_0\lambda'\left(\gamma^2 - 1 - \frac{\gamma^2 u^2}{c^2}\right) + \frac{2h\gamma\lambda'}{m_e c}\left(1 - \frac{u}{c}\right) = \frac{2h\gamma\lambda_0}{m_e c}\left(1 - \frac{u\cos\theta}{c}\right) + \frac{2h^2}{m_e^2 c^2}(1 - \cos\theta)$$

The first term is zero. Then $\lambda' = \lambda_0\left(\frac{1 - (u\cos\theta)/c}{1 - u/c}\right) + \frac{h\gamma^{-1}}{m_e c}\left(\frac{1}{1 - u/c}\right)(1 - \cos\theta)$

Since $\gamma^{-1} = \sqrt{1 - \left(\frac{u}{c}\right)^2} = \sqrt{\left(1 - \frac{u}{c}\right)\left(1 + \frac{u}{c}\right)}$

this result may be written as $\boxed{\lambda' = \lambda_0\left(\frac{1 - (u\cos\theta)/c}{1 - u/c}\right) + \frac{h}{m_e c}\sqrt{\frac{1 + u/c}{1 - u/c}}(1 - \cos\theta)}$

It shows a specific combination of what looks like a Doppler shift and a Compton shift. This problem is about the same as the first problem in Albert Messiah's graduate text on quantum mechanics.

ANSWERS TO EVEN PROBLEMS

P40.2 (a) 999 nm (b) The wavelength emitted most strongly is infrared, and much more energy is radiated at wavelengths longer than $\lambda_{\max}$ than at shorter wavelengths.

P40.4 (a) $\sim 10^{-7}$ m ultraviolet (b) $\sim 10^{-10}$ m gamma ray

P40.6 (a) 70.9 kW (b) 580 nm (c) 7.99×10^{10} W/m (d) 9.42×10^{-1226} W/m
(e) 1.00×10^{-227} W/m (f) 5.44×10^{10} W/m (g) 7.38×10^{10} W/m (h) 0.260 W/m
(i) 2.60×10^{-9} W/m (j) 20 kW

P40.8 5.71×10^{3} photons

P40.10 1.34×10^{31}

P40.12 (a) 0.263 kg (b) 1.81 W (c) –0.015 3°C/s = –0.919°C/min (d) 9.89 μm (e) 2.01×10^{-20} J
(f) 8.98×10^{19} photon/s

P40.14 (a) 296 nm, 1.01 PHz (b) 2.71 V

P40.16 (a) 1.38 eV (b) 334 THz

P40.18 148 d, the classical theory is a gross failure

P40.20 (a) The incident photons are Doppler shifted to higher frequencies, and hence, higher energy.
(b) 3.87 eV (c) 8.78 eV

P40.22 (a) and (b)

θ, degrees	0	30	60	90	120	150	180
λ', pm	120.0	120.3	121.2	122.4	123.6	124.5	124.8
K_e, eV	0	27.9	104	205	305	376	402

(c) 180°. We could answer like this: The photon imparts the greatest momentum to the originally stationary electron in a head-on collision. Here the photon recoils straight back and the electron has maximum kinetic energy.

P40.24 (a) 488 fm (b) 268 keV (c) 31.5 keV

P40.26 (a) $\cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$ (b) $E'_\gamma = \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$, $p'_\gamma = \dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

(c) $K_e = \dfrac{E_0^2}{2\left(m_e c^2 + E_0\right)}$, $p_e = \dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

P40.28 (a) 0.101 nm (b) 81.1°

P40.30 0.004 86 nm

P40.32 To have photon energy 10 eV or greater, according to this definition, ionizing radiation is the ultraviolet light, x-rays, and γ rays with wavelength shorter than 124 nm; that is, with frequency higher than 2.41×10^{15} Hz.

P40.34 397 fm

P40.36 (a) 0.709 nm (b) 414 nm

P40.38 0.218 nm

P40.40 (a) $\frac{\gamma}{\gamma-1}\frac{u}{c}$ where $\gamma=\frac{1}{\sqrt{1-u^2/c^2}}$ (b) 1.60 (c) no change (d) 2.00×10^3 (e) 1 (f) ∞

P40.42 (a) 1.10×10^{-34} m/s (b) 1.36×10^{33} s is more than 10^{15} times the age of the Universe. (c) The student cannot be diffracted to a measurable extent. Even if he tries to stand still, molecular bombardment will give him a sufficiently high speed to make his wavelength immeasurably small.

P40.44 $v_{\text{phase}} = u/2$

P40.46 105 V

P40.48 2.27 pA

P40.50 (a) 0.250 m/s (b) 2.25 m

P40.52 The electron energy must be ~100 mc^2 or larger. The proton energy can be as small as 1.001 mc^2, which is within the range well described classically.

P40.54 length 0.333 m, radius 19.8 μm

P40.56 (a) 1.7 eV (b) 4.2×10^{-15} V·s (c) 730 nm

P40.58 (a) 8.72×10^{16}/s (b) 14.0 mA (c) Many photons are likely reflected or give their energy to the metal as internal energy, so the actual current is probably a small fraction of 14.0 mA.

P40.60 See the solution.

P40.62 See the solution.

P40.64 (a) 2.82×10^{-37} m (b) 1.06×10^{-32} J (c) 2.87×10^{-35}%

P40.66 See the solution.

P40.68 See the solution.