41

Quantum Mechanics

Note: In chapters 39, 40, and 41 we use u to represent the speed of a particle with mass, reserving v for the speeds associated with reference frames, wave functions, and photons.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q41.1 A particle's wave function represents its state, containing all the information there is about its location and motion. The squared absolute value of its wave function tells where we would classically think of the particle as spending most its time. $|\Psi|^2$ is the probability distribution function for the position of the particle.

***Q41.2** For the squared wave function to be the probability per length of finding the particle, we require

$$|\psi|^2 = \frac{0.48}{7\text{ nm} - 4\text{ nm}} = \frac{0.16}{\text{nm}} \quad \text{and} \quad \psi = 0.4/\sqrt{\text{nm}}$$

(i) Answer (e). (ii) Answer (e).

***Q41.3** **(i)** For a photon a and b are true, c false, d, e, f, and g true, h false, i and j true.

(ii) For an electron a is true, b false, c, d, e, f true, g false, h, i and j true.

Note that statements a, d, e, f, i, and j are true for both.

***Q41.4** We consider the quantity $h^2n^2/8mL^2$.

In (a) it is $h^2 1/8m_1(3\text{ nm})^2 = h^2/72\, m_1\text{ nm}^2$.
In (b) it is $h^2 4/8m_1(3\text{ nm})^2 = h^2/18\, m_1\text{ nm}^2$.
In (c) it is $h^2 1/16m_1(3\text{ nm})^2 = h^2/144\, m_1\text{ nm}^2$.
In (d) it is $h^2 1/8m_1(6\text{ nm})^2 = h^2/288\, m_1\text{ nm}^2$.
In (e) it is $0^2 1/8m_1(3\text{ nm})^2 = 0$.
The ranking is then b > a > c > d > e.

Q41.5 The motion of the quantum particle does not consist of moving through successive points. The particle has no definite position. It can sometimes be found on one side of a node and sometimes on the other side, but never at the node itself. There is no contradiction here, for the quantum particle is moving as a wave. It is not a classical particle. In particular, the particle does not speed up to infinite speed to cross the node.

Q41.6 Consider a particle bound to a restricted region of space. If its minimum energy were zero, then the particle could have zero momentum and zero uncertainty in its momentum. At the same time, the uncertainty in its position would not be infinite, but equal to the width of the region. In such a case, the uncertainty product $\Delta x \Delta p_x$ would be zero, violating the uncertainty principle. This contradiction proves that the minimum energy of the particle is not zero.

***Q41.7** Compare Figures 41.4 and 41.7 in the text. In the square well with infinitely high walls, the particle's simplest wave function has strict nodes separated by the length L of the well. The particle's wavelength is $2L$, its momentum $\frac{h}{2L}$, and its energy $\frac{p^2}{2m} = \frac{h^2}{8mL^2}$. Now in the well with walls of only finite height, the wave function has nonzero amplitude at the walls. In this finite-depth well …

(i) The particle's wavelength is longer, answer (a).

(ii) The particle's momentum in its ground state is smaller, answer (b).

(iii) The particle has less energy, answer (b).

Q41.8 As Newton's laws are the rules which a particle of large mass follows in its motion, so the Schrödinger equation describes the motion of a quantum particle, a particle of small or large mass. In particular, the states of atomic electrons are confined-wave states with wave functions that are solutions to the Schrödinger equation.

***Q41.9** Answer (b). The reflected amplitude decreases as U decreases. The amplitude of the reflected wave is proportional to the reflection coefficient, R, which is $1-T$, where T is the transmission coefficient as given in equation 41.22. As U decreases, C decreases as predicted by equation 41.23, T increases, and R decreases.

***Q41.10** Answer (a). Because of the exponential tailing of the wave function within the barrier, the tunneling current is more sensitive to the width of the barrier than to its height.

Q41.11 Consider the Heisenberg uncertainty principle. It implies that electrons initially moving at the same speed and accelerated by an electric field through the same distance *need not* all have the same measured speed after being accelerated. Perhaps the philosopher could have said "it is necessary for the very existence of science that the same conditions always produce the same results within the uncertainty of the measurements."

Q41.12 In quantum mechanics, particles are treated as wave functions, not classical particles. In classical mechanics, the kinetic energy is never negative. That implies that $E \geq U$. Treating the particle as a wave, the Schrödinger equation predicts that there is a nonzero probability that a particle can tunnel through a barrier—a region in which $E < U$.

***Q41.13** Answer (c). Other points see a wider potential-energy barrier and carry much less tunneling current.

SOLUTIONS TO PROBLEMS

Section 41.1 An Interpretation of Quantum Mechanics

P41.1 (a) $\psi(x) = Ae^{i(5.00\times10^{10}x)} = A\cos\left(5\times10^{10}x\right) + Ai\sin\left(5\times10^{10}x\right) = A\cos(kx) + Ai\sin(kx)$ goes through a full cycle when x changes by λ and when kx changes by 2π. Then $k\lambda = 2\pi$ where $k = 5.00\times10^{10}\ \text{m}^{-1} = \dfrac{2\pi}{\lambda}$. Then $\lambda = \dfrac{2\pi\ \text{m}}{\left(5.00\times10^{10}\right)} = \boxed{1.26\times10^{-10}\ \text{m}}$.

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.26\times10^{-10}\ \text{m}} = \boxed{5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(c) $m_e = 9.11\times10^{-31}$ kg

$$K = \frac{m_e^2u^2}{2m_e} = \frac{p^2}{2m} = \frac{\left(5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)^2}{\left(2\times9.11\times10^{-31}\ \text{kg}\right)} = 1.52\times10^{-17}\ \text{J} = \frac{1.52\times10^{-17}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = \boxed{95.5\ \text{eV}}$$

P41.2 Probability $$P = \int_{-a}^{a}|\psi(x)|^2 = \int_{-a}^{a}\frac{a}{\pi\left(x^2+a^2\right)}dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\Bigg|_{-a}^{a}$$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{\frac{1}{2}}$$

Section 41.2 The Quantum Particle under Boundary Conditions

P41.3 $E_1 = 2.00\ \text{eV} = 3.20\times10^{-19}\ \text{J}$

For the ground state, $$E_1 = \frac{h^2}{8m_eL^2}$$

(a) $$L = \frac{h}{\sqrt{8m_eE_1}} = 4.34\times10^{-10}\ \text{m} = \boxed{0.434\ \text{nm}}$$

(b) $$\Delta E = E_2 - E_1 = 4\left(\frac{h^2}{8m_eL^2}\right) - \left(\frac{h^2}{8m_eL^2}\right) = \boxed{6.00\ \text{eV}}$$

P41.4 For an electron wave to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.

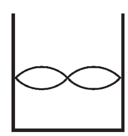

FIG. P41.4

$$\frac{n\lambda}{2} = 1.00\times10^{-9}\ \text{m} \quad \text{so} \quad \lambda = \frac{2.00\times10^{-9}}{n} = \frac{h}{p}$$

(a) Since $$K = \frac{p^2}{2m_e} = \frac{\left(h^2/\lambda^2\right)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{\left(2\times10^{-9}\right)^2} = \left(0.377n^2\right)\ \text{eV}$$

For $K \approx 6$ eV $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03\ \text{eV}}$

P41.5 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:

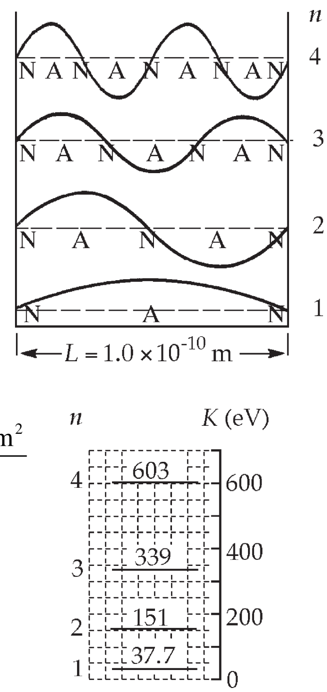

FIG. P41.5

Since $\quad d_{\text{N to N}} = \dfrac{\lambda}{2}$ and $\lambda = \dfrac{h}{p}$

$$p = \frac{h}{\lambda} = \frac{h}{2d}$$

Next, $\quad K = \dfrac{p^2}{2m_e} = \dfrac{h^2}{8m_e d^2} = \dfrac{1}{d^2}\left[\dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\right]$

Evaluating, $\quad K = \dfrac{6.02\times10^{-38}\ \text{J}\cdot\text{m}^2}{d^2} \qquad K = \dfrac{3.77\times10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$

In state 1, $\quad d = 1.00\times10^{-10}\ \text{m} \qquad K_1 = 37.7\ \text{eV}$

In state 2, $\quad d = 5.00\times10^{-11}\ \text{m} \qquad K_2 = 151\ \text{eV}$

In state 3, $\quad d = 3.33\times10^{-11}\ \text{m} \qquad K_3 = 339\ \text{eV}$

In state 4, $\quad d = 2.50\times10^{-11}\ \text{m} \qquad K_4 = 603\ \text{eV}$

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(113\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 11.0\ \text{nm}$$

The wavelengths of the other spectral lines we find similarly:

Transition	$4\rightarrow3$	$4\rightarrow2$	$4\rightarrow1$	$3\rightarrow2$	$3\rightarrow1$	$2\rightarrow1$
$E(\text{eV})$	264	452	565	188	302	113
$\lambda(\text{nm})$	4.71	2.75	2.20	6.60	4.12	11.0

***P41.6** For the bead's energy we have both $(1/2)mu^2$ and $h^2n^2/8mL^2$. Then

$$n = \sqrt{\frac{1}{2}mu^2\frac{8mL^2}{h^2}} = \frac{2muL}{h} \quad \text{note that this expression can be thought of as } \frac{2L}{\lambda} = \frac{L}{d_{NN}}$$

Evaluating, $\quad n = \dfrac{2(0.005\ \text{kg})(10^{-10}\ \text{m})\ 0.2\ \text{m}}{3.156\times10^{7}\ \text{s}\ (6.626\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{9.56\times10^{12}}$

P41.7 $\quad \Delta E = \dfrac{hc}{\lambda} = \left(\dfrac{h^2}{8m_eL^2}\right)\left[2^2 - 1^2\right] = \dfrac{3h^2}{8m_eL^2}$

$$L = \sqrt{\frac{3h\lambda}{8m_ec}} = 7.93\times10^{-10}\ \text{m} = \boxed{0.793\ \text{nm}}$$

P41.8 $$\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_eL^2}\right)\left[2^2 - 1^2\right] = \frac{3h^2}{8m_eL^2}$$

so $$L = \boxed{\sqrt{\frac{3h\lambda}{8m_ec}}}$$

P41.9 The confined proton can be described in the same way as a standing wave on a string. At level 1, the node-to-node distance of the standing wave is 1.00×10^{-14} m, so the wavelength is twice this distance:

$$\frac{h}{p} = 2.00 \times 10^{-14} \text{ m}$$

The proton's kinetic energy is

$$K = \frac{1}{2}mu^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{2\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^{-14} \text{ m}\right)^2}$$

$$= \frac{3.29 \times 10^{-13} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 2.05 \text{ MeV}$$

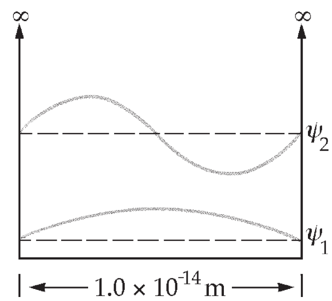

FIG. P41.9

In the first excited state, level 2, the node-to-node distance is half as long as in state 1. The momentum is two times larger and the energy is four times larger: $K = 8.22$ MeV.

The proton has mass, has charge, moves slowly compared to light in a standing wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$$2.05 \text{ MeV} - 8.22 \text{ MeV} = -6.16 \text{ MeV}$$

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $\boxed{+6.16 \text{ MeV}}$.

Its frequency is $$f = \frac{E}{h} = \frac{\left(6.16 \times 10^6 \text{ eV}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)}{6.626 \times 10^{-34} \text{ J}\cdot\text{s}} = 1.49 \times 10^{21} \text{ Hz}$$

And its wavelength is $$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.49 \times 10^{21} \text{ s}^{-1}} = \boxed{2.02 \times 10^{-13} \text{ m}}$$

This is a $\boxed{\text{gamma ray}}$, according to the electromagnetic spectrum chart in Chapter 34.

P41.10 The ground state energy of a particle (mass m) in a 1-dimensional box of width L is $E_1 = \frac{h^2}{8mL^2}$.

(a) For a proton $\left(m = 1.67 \times 10^{-27} \text{ kg}\right)$ in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^{-10} \text{ m}\right)^2} = 8.22 \times 10^{-22} \text{ J} = \boxed{5.13 \times 10^{-3} \text{ eV}}$$

(b) For an electron $\left(m = 9.11 \times 10^{-31} \text{ kg}\right)$ in the same size box:

$$E_1 = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(9.11 \times 10^{-31} \text{ kg}\right)\left(2.00 \times 10^{-10} \text{ m}\right)^2} = 1.51 \times 10^{-18} \text{ J} = \boxed{9.41 \text{ eV}}$$

(c) The electron has a much higher energy because it is much less massive.

***P41.11** $E_n = \left(\dfrac{h^2}{8mL^2}\right)n^2$

$$E_1 = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{-14}\ \text{m}\right)^2} = 8.22\times10^{-14}\ \text{J}$$

$$E_1 = \boxed{0.513\ \text{MeV}} \qquad E_2 = 4E_1 = \boxed{2.05\ \text{MeV}} \qquad E_3 = 9E_1 = \boxed{4.62\ \text{MeV}}$$

Yes; the energy differences are ~1 MeV, which is a typical energy for a γ-ray photon as radiated by an atomic nucleus in an excited state.

P41.12 (a) The energies of the confined electron are $E_n = \dfrac{h^2}{8m_eL^2}n^2$. Its energy gain in the quantum jump from state 1 to state 4 is $\dfrac{h^2}{8m_eL^2}\left(4^2-1^2\right)$ and this is the photon energy: $\dfrac{h^2 15}{8m_eL^2} = hf = \dfrac{hc}{\lambda}$. Then $8m_ecL^2 = 15h\lambda$ and $\boxed{L = \left(\dfrac{15h\lambda}{8m_ec}\right)^{1/2}}$.

(b) Let λ' represent the wavelength of the photon emitted: $\dfrac{hc}{\lambda'} = \dfrac{h^2}{8m_eL^2}4^2 - \dfrac{h^2}{8m_eL^2}2^2 = \dfrac{12h^2}{8m_eL^2}$.

Then $\dfrac{hc}{\lambda}\dfrac{\lambda'}{hc} = \dfrac{h^2 15\left(8m_eL^2\right)}{8m_eL^2 12h^2} = \dfrac{5}{4}$ and $\boxed{\lambda' = 1.25\lambda}$.

***P41.13** (a) From $\Delta x\Delta p \ge \hbar/2$ with $\Delta x = L$, the uncertainty in momentum must be at least $\boxed{\Delta p \approx \hbar/2L}$.

(b) Its energy is all kinetic, $E = p^2/2m = (\Delta p)^2/2m \approx \boxed{\hbar^2/8mL^2} = h^2/(4\pi)^2 8mL^2$.
Compared to the actual $h^2/8mL^2$, this estimate is too low by $4\pi^2 \approx 40$ times. The actual wave function does not have the particular (Gaussian) shape of a minimum-uncertainty wave function. The result correctly displays the pattern of dependence of the energy on the mass and on the length of the well.

P41.14 (a) $$\langle x\rangle = \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left(\frac{1}{2}-\frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$$

$$\langle x\rangle = \frac{1}{L}\frac{x^2}{2}\Bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{\frac{L}{2}}$$

(b) $$\text{Probability} = \int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability} = 0.020 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times10^{-5}}$$

(c) $$\text{Probability}\ \left[\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$$

(d) In the $n = 2$ graph in the text's Figure 41.4(b), it is more probable to find the particle either near $x = \dfrac{L}{4}$ or $x = \dfrac{3L}{4}$ than at the center, where the probability density is zero. Nevertheless, the symmetry of the distribution means that the average position is $\dfrac{L}{2}$.

P41.15 Normalization requires

$$\int_{\text{all space}} |\psi|^2\,dx = 1 \qquad \text{or} \qquad \int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\left(\frac{L}{2}\right) = 1 \qquad \text{or} \qquad \boxed{A = \sqrt{\frac{2}{L}}}$$

***P41.16** (a) The probability is $\int_0^{L/3} |\psi_1|^2\,dx = \frac{2}{L}\int_0^{L/3}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{1}{L}\int_0^{L/3}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right]dx$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{L/3} = \frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right) = \frac{1}{3} - \frac{0.866}{2\pi} = \boxed{0.196}$$

(b) Classically, the particle moves back and forth steadily, spending equal time intervals in each third of the line. Then the classical probability is $\boxed{\text{0.333, significantly larger}}$.

(c) The probability is $\int_0^{L/3} |\psi_{99}|^2\,dx = \frac{2}{L}\int_0^{L/3}\sin^2\left(\frac{99\pi x}{L}\right)dx = \frac{1}{L}\int_0^{L/3}\left[1-\cos\left(\frac{198\pi x}{L}\right)\right]dx$

$$= \frac{1}{L}\left[x - \frac{L}{198\pi}\sin\left(\frac{198\pi x}{L}\right)\right]_0^{L/3} = \frac{1}{3} - \frac{1}{198\pi}\sin(66\pi) = \frac{1}{3} - 0 = \boxed{0.333}$$

$\boxed{\text{in agreement with the classical model}}$.

***P41.17** In $0 \le x \le L$, the argument $\frac{2\pi x}{L}$ of the sine function ranges from 0 to 2π. The probability density $\left(\frac{2}{L}\right)\sin^2\left(\frac{2\pi x}{L}\right)$ reaches maxima at $\sin\theta = 1$ and $\sin\theta = -1$. These points are at

$$\frac{2\pi x}{L} = \frac{\pi}{2} \quad \text{and} \quad \frac{2\pi x}{L} = \frac{3\pi}{2}.$$

Therefore the most probable positions of the particle are $\boxed{\text{at } x = \frac{L}{4} \text{ and } x = \frac{3L}{4}}$.

***P41.18** (a) Probability $= \int_0^{\ell} |\psi_1|^2\, dx = \frac{2}{L}\int_0^{\ell} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{1}{L}\int_0^{\ell}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right] dx$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{\ell} = \boxed{\frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi \ell}{L}\right)}$$

(b)

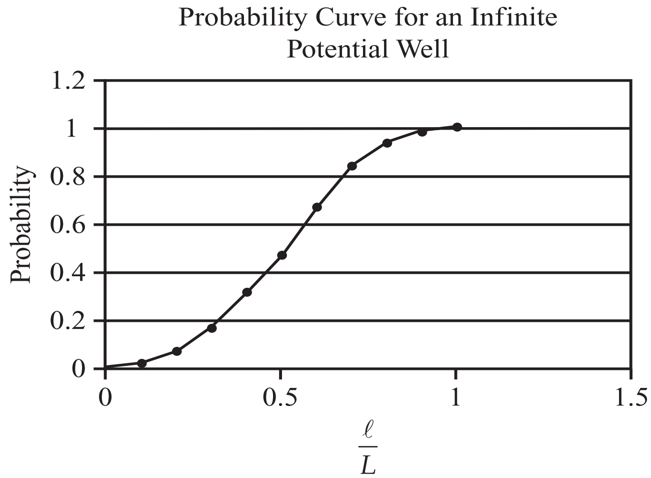

FIG. P41.18(b)

(c) The wave function is zero for $x < 0$ and for $x > L$. The probability at $\ell = 0$ must be zero because the particle is never found at $x < 0$ or exactly at $x = 0$. The probability at $\ell = L$ must be 1 for normalization: the particle is always found somewhere at $x < L$.

(d) The probability of finding the particle between $x = 0$ and $x = \ell$ is $\frac{2}{3}$, and between $x = \ell$ and $x = L$ is $\frac{1}{3}$.

Thus, $\int_0^{\ell} |\psi_1|^2\, dx = \frac{2}{3}$

$$\therefore \frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi \ell}{L}\right) = \frac{2}{3}, \qquad \text{or} \qquad u - \frac{1}{2\pi}\sin 2\pi u = \frac{2}{3}$$

This equation for $\frac{\ell}{L}$ can be solved by homing in on the solution with a calculator, the result being $\frac{\ell}{L} = 0.585$, or $\ell = \boxed{0.585L}$ to three digits.

P41.19 (a) The probability is $P = \int_0^{L/3} |\psi|^2\,dx = \int_0^{L/3} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/3}\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$

$$P = \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_0^{L/3} = \left(\frac{1}{3} - \frac{1}{2\pi}\sin\frac{2\pi}{3}\right) = \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}$$

(b) The probability density is symmetric about $x = \frac{L}{2}$. Thus, the probability of finding the particle between $x = \frac{2L}{3}$ and $x = L$ is the same 0.196. Therefore, the probability of finding it in the range $\frac{L}{3} \le x \le \frac{2L}{3}$ is $P = 1.00 - 2(0.196) = \boxed{0.609}$.

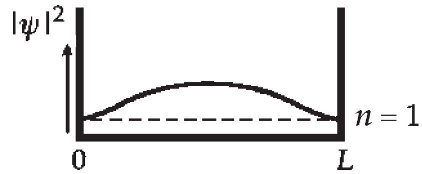

FIG. P41.19(b)

(c) Classically, the electron moves back and forth with constant speed between the walls, and the probability of finding the electron is the same for all points between the walls. Thus, the classical probability of finding the electron in any range equal to one-third of the available space is $P_{\text{classical}} = \frac{1}{3}$. $\boxed{\text{The result of part (a) is significantly smaller}}$, because of the curvature of the graph of the probability density.

Section 41.3 **The Schrödinger Equation**

P41.20 $\psi(x) = A\cos kx + B\sin kx$ $\qquad \frac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$

$\frac{\partial^2\psi}{\partial x^2} = -k^2 A\cos kx - k^2 B\sin kx$ $\qquad -\frac{2m}{\hbar}(E-U)\psi = -\frac{2mE}{\hbar^2}(A\cos kx + B\sin kx)$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2\psi}{\partial x^2} = \left(-\frac{2m}{\hbar^2}\right)(E-U)\psi \quad \text{or} \quad -k^2(A\cos kx + B\sin kx) = \left(-\frac{2mE}{\hbar^2}\right)(A\cos kx + B\sin kx)$$

This is true as an identity (functional equality) for all x if $\boxed{E = \frac{\hbar^2 k^2}{2m}}$.

P41.21 We have $\psi = Ae^{i(kx-\omega t)}$ so $\frac{\partial\psi}{\partial x} = ik\psi$ and $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$.

We test by substituting into Schrödinger's equation: $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = -\frac{2m}{\hbar^2}(E-U)\psi$.

Since $k^2 = \frac{(2\pi)^2}{\lambda^2} = \frac{(2\pi p)^2}{h^2} = \frac{p^2}{\hbar^2}$ and $E - U = \frac{p^2}{2m}$

$\boxed{\text{Thus this equation balances.}}$

P41.22 (a) Setting the total energy E equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$U(x) = \left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\psi(x) = Axe^{-x^2/L^2}$

Then $\frac{d^2\psi}{dx^2} = \left(4Ax^3 - 6AxL^2\right)\frac{e^{-x^2/L^2}}{L^4}$

or $\frac{d^2\psi}{dx^2} = \frac{\left(4x^2 - 6L^2\right)}{L^4}\psi(x)$

and $\boxed{U(x) = \frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2} - 6\right)}$

(b) $\boxed{\text{See the figure to the right.}}$

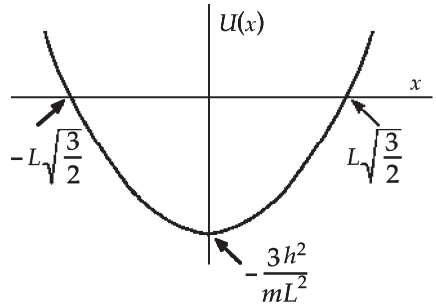

FIG. P41.22(b)

P41.23 Problem 41 in Chapter 16 helps students to understand how to draw conclusions from an identity.

(a) $\psi(x) = A\left(1 - \frac{x^2}{L^2}\right)$ $\frac{d\psi}{dx} = -\frac{2Ax}{L^2}$ $\frac{d^2\psi}{dx^2} = -\frac{2A}{L^2}$

Schrödinger's equation $\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E - U)\psi$

becomes $-\frac{2A}{L^2} = -\frac{2m}{\hbar^2}EA\left(1 - \frac{x^2}{L^2}\right) + \frac{2m}{\hbar^2}\frac{\left(-\hbar^2x^2\right)A\left(1 - x^2/L^2\right)}{mL^2\left(L^2 - x^2\right)}$

$$-\frac{1}{L^2} = -\frac{mE}{\hbar^2} + \frac{mEx^2}{\hbar^2L^2} - \frac{x^2}{L^4}$$

This will be true for all x if both $\frac{1}{L^2} = \frac{mE}{\hbar^2}$

and $\frac{mE}{\hbar^2L^2} - \frac{1}{L^4} = 0$

both these conditions are satisfied for a particle of energy $\boxed{E = \frac{\hbar^2}{L^2m}}$

(b) For normalization, $1 = \int_{-L}^{L} A^2\left(1 - \frac{x^2}{L^2}\right)^2 dx = A^2\int_{-L}^{L}\left(1 - \frac{2x^2}{L^2} + \frac{x^4}{L^4}\right)dx$

$$1 = A^2\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^4}\right]_{-L}^{L} = A^2\left[L - \frac{2}{3}L + \frac{L}{5} + L - \frac{2}{3}L + \frac{L}{5}\right] = A^2\left(\frac{16L}{15}\right) \qquad \boxed{A = \sqrt{\frac{15}{16L}}}$$

(c) $P = \int_{-L/3}^{L/3}\psi^2 dx = \frac{15}{16L}\int_{-L/3}^{L/3}\left(1 - \frac{2x^2}{L^2} + \frac{x^4}{L^4}\right)dx = \frac{15}{16L}\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^5}\right]_{-L/3}^{L/3}$

$$= \frac{30}{16L}\left[\frac{L}{3} - \frac{2L}{81} + \frac{L}{1\,215}\right]$$

$$P = \frac{47}{81} = \boxed{0.580}$$

P41.24 (a) $\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$; $\qquad P_1(x) = |\psi_1(x)|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$

$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$; $\qquad P_2(x) = |\psi_2(x)|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$

$\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$; $\qquad P_3(x) = |\psi_3(x)|^2 = \frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$

(b)

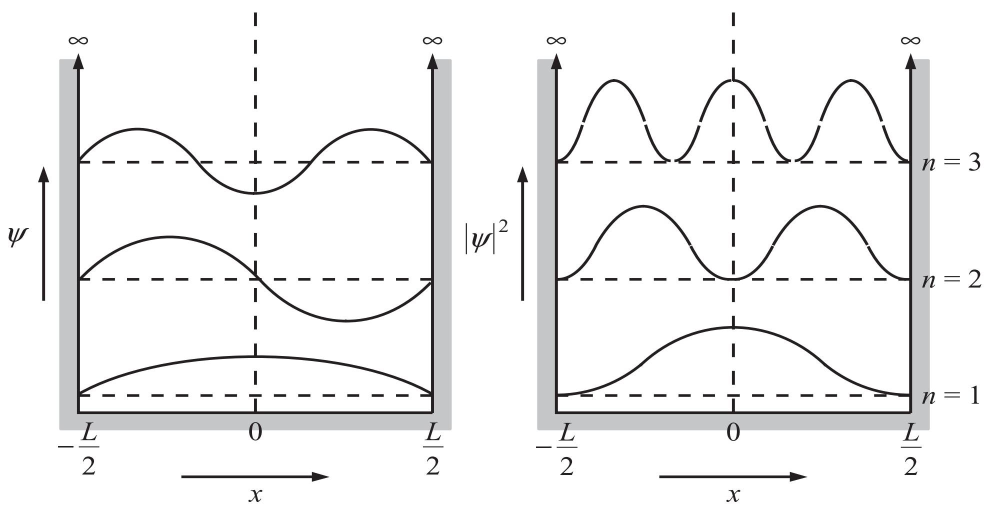

FIG. P41.24(b)

P41.25 (a) With $\psi(x) = A\sin(kx)$

$$\frac{d}{dx}A\sin kx = Ak\cos kx \qquad \text{and} \qquad \frac{d^2}{dx^2}\psi = -Ak^2\sin kx$$

Then $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = +\frac{\hbar^2 k^2}{2m}A\sin kx = \frac{h^2(4\pi^2)}{4\pi^2(\lambda^2)(2m)}\psi = \frac{p^2}{2m}\psi = \frac{m^2u^2}{2m}\psi = \frac{1}{2}mu^2\psi = K\psi$$

(b) With $\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin kx$, the proof given in part (a) applies again.

Section 41.4 **A Particle in a Well of Finite Height**

P41.26

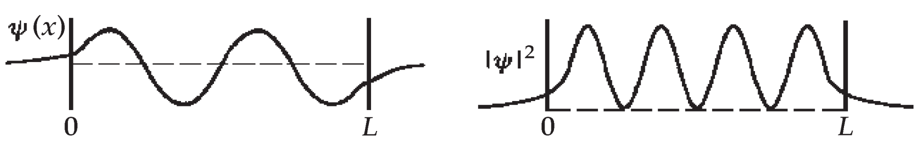

FIG. P41.26

P41.27 (a) $\boxed{\text{See figure to the right.}}$

(b) The wavelength of the transmitted wave traveling to the left is the same as the original wavelength, which equals $\boxed{2L}$.

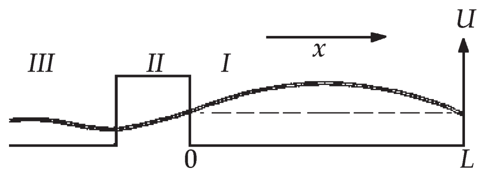

FIG. P41.27(a)

Section 41.5 Tunneling Through a Potential Energy Barrier

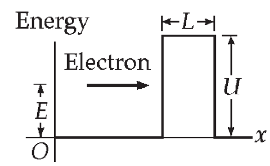

FIG. P41.28

P41.28

$$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$

$$= 3.62\times10^{9}\ \text{m}^{-1}$$

$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$$T = \boxed{1.03\times10^{-3}}$$

P41.29 From problem 28, $C = 3.62\times10^{9}\ \text{m}^{-1}$

We require $10^{-6} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L\right]$.

Taking logarithms, $\quad -13.816 = -2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L$

New $L = 1.91\ \text{nm}$

$$\Delta L = 1.91\ \text{nm} - 0.950\ \text{nm} = \boxed{0.959\ \text{nm}}$$

***P41.30** $T = e^{-2CL}$ where $C = \dfrac{\sqrt{2m(U-E)}}{\hbar}$

(a) $$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(0.01\times1.6\times10^{-19}\right)}}{1.055\times10^{-34}}\left(10^{-10}\right) = 0.102 \qquad T = e^{-0.102} = \boxed{0.903}$$

(b) $$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(1.6\times10^{-19}\right)}}{1.055\times10^{-34}}\left(10^{-10}\right) = 1.02 \qquad T = e^{-1.02} = \boxed{0.359}$$

(c) $$2CL = \frac{2\sqrt{2\left(6.65\times10^{-27}\right)\left(10^{6}\times1.6\times10^{-19}\right)}}{1.055\times10^{-34}}\left(10^{-15}\right) = 0.875 \qquad T = e^{-0.875} = \boxed{0.417}$$

(d) $$2CL = \frac{2\sqrt{2(8)(1)}}{1.055\times10^{-34}}(0.02) = 1.52\times10^{33} \qquad T = e^{-1.52\times10^{33}} = e^{(\ln10)(-1.52\times10^{33}/\ln10)} = \boxed{10^{-6.59\times10^{32}}}$$

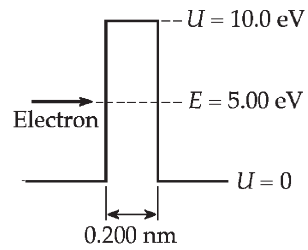

FIG. P41.31

P41.31 $T = e^{-2CL}$ where $C = \dfrac{\sqrt{2m(U-E)}}{\hbar}$

$$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right) = 4.58$$

(a) $T = e^{-4.58} = \boxed{0.010\,3}$, a 1% chance of transmission.

(b) $R = 1 - T = \boxed{0.990}$, a 99% chance of reflection.

P41.32 The original tunneling probability is $T = e^{-2CL}$ where

$$C = \frac{\left(2m(U-E)\right)^{1/2}}{\hbar} = \frac{2\pi\left(2\times9.11\times10^{-31}\text{ kg}(20-12)1.6\times10^{-19}\text{ J}\right)^{1/2}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.448\,1\times10^{10}\text{ m}^{-1}$$

The photon energy is $hf = \dfrac{hc}{\lambda} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{546\text{ nm}} = 2.27$ eV, to make the electron's new kinetic energy $12 + 2.27 = 14.27$ eV and its decay coefficient inside the barrier

$$C' = \frac{2\pi\left(2\times9.11\times10^{-31}\text{ kg}(20-14.27)1.6\times10^{-19}\text{ J}\right)^{1/2}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.225\,5\times10^{10}\text{ m}^{-1}$$

Now the factor of increase in transmission probability is

$$\frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{2\times10^{-9}\text{ m}\times0.223\times10^{10}\text{ m}^{-1}} = e^{4.45} = \boxed{85.9}$$

Section 41.6 **Applications of Tunneling**

P41.33 With the wave function proportional to e^{-CL}, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to e^{-2CL}.

Then, $$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})} = \frac{e^{-2(10.0/\text{nm})(0.500\text{ nm})}}{e^{-2(10.0/\text{nm})(0.515\text{ nm})}} = e^{20.0(0.015)} = \boxed{1.35}$$

P41.34 With transmission coefficient e^{-2CL}, the fractional change in transmission is

$$\frac{e^{-2(10.0/\text{nm})L} - e^{-2(10.0/\text{nm})(L+0.002\,00\text{ nm})}}{e^{-2(10.0/\text{nm})L}} = 1 - e^{-20.0(0.002\,00)} = 0.0392 = \boxed{3.92\%}$$

Section 41.7 **The Simple Harmonic Oscillator**

P41.35 $\psi = Be^{-(m\omega/2\hbar)x^2}$ so $\dfrac{d\psi}{dx} = -\left(\dfrac{m\omega}{\hbar}\right)x\psi$ and $\dfrac{d^2\psi}{dx^2} = \left(\dfrac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\dfrac{m\omega}{\hbar}\right)\psi$

Substituting into the Schrödinger equation gives

$$\left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi = -\left(\frac{2mE}{\hbar^2}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi$$

which is satisfied provided that $E = \dfrac{\hbar\omega}{2}$.

P41.36 Problem 41 in Chapter 16 helps students to understand how to draw conclusions from an identity.

$\psi = Axe^{-bx^2}$ so $\dfrac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2Ae^{-bx^2}$

and $\dfrac{d^2\psi}{dx^2} = -2bxAe^{-bx^2} - 4bxAe^{-bx^2} + 4b^2x^3e^{-bx^2} = -6b\psi + 4b^2x^2\psi$

Substituting into the Schrödinger equation, $-6b\psi + 4b^2x^2\psi = -\left(\dfrac{2mE}{\hbar}\right)\psi + \left(\dfrac{m\omega}{\hbar}\right)^2 x^2\psi$

For this to be true as an identity, it must be true for all values of x.

So we must have both $-6b = -\dfrac{2mE}{\hbar^2}$ and $4b^2 = \left(\dfrac{m\omega}{\hbar}\right)^2$

(a) Therefore $\boxed{b = \dfrac{m\omega}{2\hbar}}$

(b) and $E = \dfrac{3b\hbar^2}{m} = \boxed{\dfrac{3}{2}\hbar\omega}$

(c) The wave function is that of the $\boxed{\text{first excited state}}$.

P41.37 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator:

$$\frac{hc}{\lambda} = \hbar\omega = \hbar\sqrt{\frac{k}{m}} \text{ so } \lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times10^8 \text{ m/s}\right)\left(\frac{9.11\times10^{-31} \text{ kg}}{8.99 \text{ N/m}}\right)^{1/2} = \boxed{600 \text{ nm}}$$

P41.38 (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all } x} |\psi|^2\, dx = 1$

becomes $1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2}\, dx = 2B^2 \int_0^{\infty} e^{-(m\omega/\hbar)x^2}\, dx = 2B^2 \frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}}$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $1 = B^2 \sqrt{\frac{\pi\hbar}{m\omega}}$ and $\boxed{B = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}$.

(b) For small δ, the probability of finding the particle in the range $-\frac{\delta}{2} < x < \frac{\delta}{2}$ is

$$\int_{-\delta/2}^{\delta/2} |\psi|^2\, dx = \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

P41.39 (a) For the center of mass to be fixed, $m_1u_1 + m_2u_2 = 0$. Then

$$u = |u_1| + |u_2| = |u_1| + \frac{m_1}{m_2}|u_1| = \frac{m_2 + m_1}{m_2}|u_1| \quad \text{and} \quad |u_1| = \frac{m_2 u}{m_1 + m_2}$$

Similarly, $u = \frac{m_2}{m_1}|u_2| + |u_2|$

and $|u_2| = \frac{m_1 u}{m_1 + m_2}$. Then

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 + \frac{1}{2}kx^2 = \frac{1}{2}\frac{m_1m_2^2u^2}{(m_1+m_2)^2} + \frac{1}{2}\frac{m_2m_1^2u^2}{(m_1+m_2)^2} + \frac{1}{2}kx^2$$

$$= \frac{1}{2}\frac{m_1m_2(m_1+m_2)}{(m_1+m_2)^2}u^2 + \frac{1}{2}kx^2 = \frac{1}{2}\mu u^2 + \frac{1}{2}kx^2$$

(b) $\frac{d}{dx}\left(\frac{1}{2}\mu u^2 + \frac{1}{2}kx^2\right) = 0$ because energy is constant

$$0 = \frac{1}{2}\mu\, 2u\frac{du}{dx} + \frac{1}{2}k2x = \mu\frac{dx}{dt}\frac{du}{dx} + kx = \mu\frac{du}{dt} + kx$$

Then $\mu a = -kx$, $a = -\frac{kx}{\mu}$. This is the condition for simple harmonic motion, that the acceleration of the equivalent particle be a negative constant times the excursion from equilibrium. By identification with $a = -\omega^2 x$, $\omega = \sqrt{\frac{k}{\mu}} = 2\pi f$ and $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}}$.

P41.40 (a) With $\langle x \rangle = 0$ and $\langle p_x \rangle = 0$, the average value of x^2 is $(\Delta x)^2$ and the average value of p_x^2 is $(\Delta p_x)^2$. Then $\Delta x \ge \dfrac{\hbar}{2\Delta p_x}$ requires

$$E \ge \frac{p_x^2}{2m} + \frac{k}{2}\frac{\hbar^2}{4p_x^2} = \boxed{\frac{p_x^2}{2m} + \frac{k\hbar^2}{8p_x^2}}$$

(b) To minimize this as a function of p_x^2, we require $\dfrac{dE}{dp_x^2} = 0 = \dfrac{1}{2m} + \dfrac{k\hbar^2}{8}(-1)\dfrac{1}{p_x^4}$

Then $\dfrac{k\hbar^2}{8p_x^4} = \dfrac{1}{2m}$ so $p_x^2 = \left(\dfrac{2mk\hbar^2}{8}\right)^{1/2} = \dfrac{\hbar\sqrt{mk}}{2}$

and $E \ge \dfrac{\hbar\sqrt{mk}}{2(2m)} + \dfrac{k\hbar^2 2}{8\hbar\sqrt{mk}} = \dfrac{\hbar}{4}\sqrt{\dfrac{k}{m}} + \dfrac{\hbar}{4}\sqrt{\dfrac{k}{m}}$

$$E_{\min} = \frac{\hbar}{2}\sqrt{\frac{k}{m}} = \boxed{\frac{\hbar\omega}{2}}$$

Additional Problems

P41.41 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m}) = 0.023\,5\text{ J}$$

and $E = K = \dfrac{1}{2}mu^2 = \dfrac{1}{2}(0.02\text{ kg})(0.8\text{ m/s})^2 = 0.006\,4\text{ J}$

Then $C = \dfrac{\sqrt{2m(U-E)}}{\hbar} = \dfrac{\sqrt{2(0.02\text{ kg})(0.017\,1\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 2.5\times10^{32}\text{ m}^{-1}$

and the transmission coefficient is

$$e^{-2CL} = e^{-2(2.5\times10^{32})(2\times10^{-3})} = e^{-10\times10^{29}} = e^{-2.30(4.3\times10^{29})} = 10^{-4.3\times10^{29}} = \boxed{\sim 10^{-10^{30}}}$$

P41.42 (a) $\lambda = 2L = \boxed{2.00\times10^{-10}\text{ m}}$

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2.00\times10^{-10}\text{ m}} = \boxed{3.31\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(c) $E = \dfrac{p^2}{2m} = \boxed{0.172\text{ eV}}$

P41.43 (a) $\boxed{\text{See the figure.}}$ (b) $\boxed{\text{See the figure.}}$

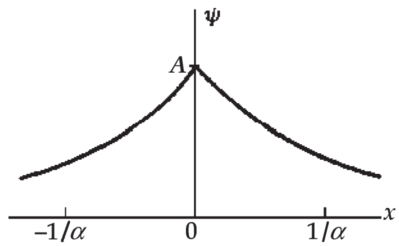

FIG. P41.43(a)

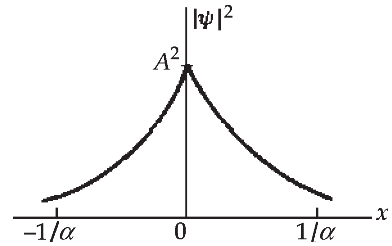

FIG. P41.43(b)

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$.

(d) Since ψ is symmetric,

$$\int_{-\infty}^{\infty} |\psi|^2\, dx = 2\int_0^{\infty} |\psi|^2\, dx = 1$$

or $$2A^2 \int_0^{\infty} e^{-2\alpha x} dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1$$

This gives $\boxed{A = \sqrt{\alpha}}$.

(e) $$P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{a}\right)^2 \int_{x=0}^{1/2\alpha} e^{-2\alpha x} dx = \left(\frac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha} - 1\right) = \left(1 - e^{-1}\right) = \boxed{0.632}$$

***P41.44** If we had $n = 0$ for a quantum particle in a box, its momentum would be zero. The uncertainty in its momentum would be zero. The uncertainty in its position would not be infinite, but just equal to the width of the box. Then the uncertainty product would be zero, to violate the uncertainty principle. The contradiction shows that the quantum number cannot be zero. In its ground state the particle has some nonzero zero-point energy.

***P41.45** (a) With ground state energy 0.3 eV, the energy in the $n = 2$ state is $2^2 \times 0.3$ eV = 1.2 eV. The energy in state 3 is 9×0.3 eV = 2.7 eV. The energy in state 4 is 16×0.3 eV = 4.8 eV. For the transition from the $n = 3$ level to the $n = 1$ level, the electron loses energy (2.7 – 0.3) eV = 2.4 eV. The photon carries off this energy and has wavelength hc/E = 1240 eV·nm/2.4 eV = $\boxed{517\text{ nm}}$.

(b) For the transition from level 2 to level 1, the photon energy is 0.9 eV and the photon wavelength is $\lambda = hc/E$ = 1240 eV · nm/0.9 eV = $\boxed{1.38\ \mu\text{m}}$. This photon, with wavelength greater than 700 nm, is $\boxed{\text{infrared}}$.

For level 4 to 1, E = 4.5 eV and λ = $\boxed{\text{276 nm ultraviolet}}$.

For 3 to 2, E = 1.5 eV and $\boxed{\lambda = \text{827 nm infrared}}$.

For 4 to 2, E = 3.6 eV and $\boxed{\lambda = \text{344 nm near ultraviolet}}$.

For 4 to 3, E = 2.1 eV and $\boxed{\lambda = \text{590 nm yellow-orange visible}}$.

P41.46 (a) Use Schrödinger's equation

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$

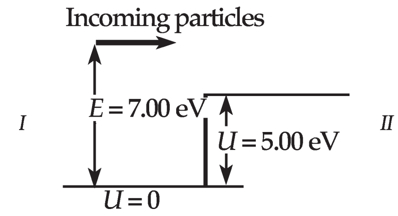

FIG. P41.46(a)

with solutions

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \qquad \text{[region } I\text{]}$$

$$\psi_2 = Ce^{ik_2x} \qquad \text{[region } II\text{]}$$

Where $k_1 = \dfrac{\sqrt{2mE}}{\hbar}$

and $k_2 = \dfrac{\sqrt{2m(E-U)}}{\hbar}$

Then, matching functions and derivatives at $x = 0$

$(\psi_1)_0 = (\psi_2)_0$ gives $A + B = C$

and $\left(\dfrac{d\psi_1}{dx}\right)_0 = \left(\dfrac{d\psi_2}{dx}\right)_0$ gives $k_1(A-B) = k_2C$

Then $B = \dfrac{1-k_2/k_1}{1+k_2/k_1}A$

and $C = \dfrac{2}{1+k_2/k_1}A$

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \dfrac{B^2}{A^2} = \dfrac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2}$

$$= \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$$

(b) With $E = 7.00 \text{ eV}$

and $U = 5.00 \text{ eV}$

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00}{7.00}} = 0.535$$

The reflection probability is $R = \dfrac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.092\ 0}$

The probability of transmission is $T = 1 - R = \boxed{0.908}$

P41.47 $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{(1 - k_2/k_1)^2}{(1 + k_2/k_1)^2}$

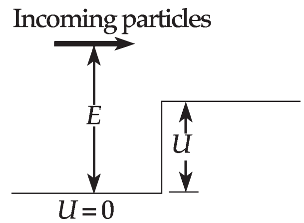

FIG. P41.47

$$\frac{\hbar^2 k^2}{2m} = E - U \text{ for constant } U$$

$$\frac{\hbar^2 k_1^2}{2m} = E \text{ since } U = 0 \qquad (1)$$

$$\frac{\hbar^2 k_2^2}{2m} = E - U \qquad (2)$$

Dividing (2) by (1), $\frac{k_2^2}{k_1^2} = 1 - \frac{U}{E} = 1 - \frac{1}{2} = \frac{1}{2}$ so $\frac{k_2}{k_1} = \frac{1}{\sqrt{2}}$

and therefore, $R = \frac{(1 - 1/\sqrt{2})^2}{(1 + 1/\sqrt{2})^2} = \frac{(\sqrt{2} - 1)^2}{(\sqrt{2} + 1)^2} = \boxed{0.029\,4}$

P41.48 (a) The wave functions and probability densities are the same as those shown in the two lower curves in Figure 41.4 of the textbook.

(b) $$P_1 = \int_{0.150\text{ nm}}^{0.350\text{ nm}} |\psi_1|^2\, dx = \left(\frac{2}{1.00\text{ nm}}\right)\int_{0.150}^{0.350} \sin^2\left(\frac{\pi x}{1.00\text{ nm}}\right)dx$$

$$= (2.00/\text{nm})\left[\frac{x}{2} - \frac{1.00\text{ nm}}{4\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$$

In the above result we used $\int \sin^2 ax dx = \left(\frac{x}{2}\right) - \left(\frac{1}{4a}\right)\sin(2ax)$.

Therefore, $P_1 = (1.00/\text{nm})\left[x - \frac{1.00\text{ nm}}{2\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$

$$P_1 = (1.00/\text{nm})\left\{0.350\text{ nm} - 0.150\text{ nm} - \frac{1.00\text{ nm}}{2\pi}[\sin(0.700\pi) - \sin(0.300\pi)]\right\} = \boxed{0.200}$$

(c) $$P_2 = \frac{2}{1.00}\int_{0.150}^{0.350} \sin^2\left(\frac{2\pi x}{1.00}\right)dx = 2.00\left[\frac{x}{2} - \frac{1.00}{8\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$P_2 = 1.00\left[x - \frac{1.00}{4\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$= 1.00\left\{(0.350 - 0.150) - \frac{1.00}{4\pi}[\sin(1.40\pi) - \sin(0.600\pi)]\right\}$$

$$= \boxed{0.351}$$

(d) Using $E_n = \frac{n^2 h^2}{8mL^2}$, we find that $E_1 = \boxed{0.377\text{ eV}}$ and $E_2 = \boxed{1.51\text{ eV}}$.

P41.49 (a) $f = \dfrac{E}{h} = \dfrac{(1.80\ \text{eV})}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}\left(\dfrac{1.60\times10^{-19}\ \text{J}}{1.00\ \text{eV}}\right) = \boxed{4.34\times10^{14}\ \text{Hz}}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{4.34\times10^{14}\ \text{Hz}} = 6.91\times10^{-7}\ \text{m} = \boxed{691\ \text{nm}}$

(c) $\Delta E\Delta t \ge \dfrac{\hbar}{2}$ so $\Delta E \ge \dfrac{\hbar}{2\Delta t} = \dfrac{h}{4\pi(\Delta t)} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi(2.00\times10^{-6}\ \text{s})} = 2.64\times10^{-29}\ \text{J} = \boxed{1.65\times10^{-10}\ \text{eV}}$

P41.50 (a) Taking $L_x = L_y = L$, we see that the expression for E becomes

$$E = \frac{h^2}{8m_eL^2}\left(n_x^2 + n_y^2\right)$$

For a normalizable wave function describing a particle, neither n_x nor n_y can be zero. The ground state, corresponding to $n_x = n_y = 1$, has an energy of

$$E_{1,1} = \frac{h^2}{8m_eL^2}\left(1^2 + 1^2\right) = \boxed{\frac{h^2}{4m_eL^2}}$$

The first excited state, corresponding to either $n_x = 2,\ n_y = 1$ or $n_x = 1,\ n_y = 2$, has an energy

$$E_{2,1} = E_{1,2} = \frac{h^2}{8m_eL^2}\left(2^2 + 1^2\right) = \boxed{\frac{5h^2}{8m_eL^2}}$$

The second excited state, corresponding to $n_x = 2,\ n_y = 2$, has an energy of

$$E_{2,2} = \frac{h^2}{8m_eL^2}\left(2^2 + 2^2\right) = \boxed{\frac{h^2}{m_eL^2}}$$

Finally, the third excited state, corresponding to either $n_x = 1,\ n_y = 3$ or $n_x = 3,\ n_x = 1$, has an energy

$$E_{1,3} = E_{3,1} = \frac{h^2}{8m_eL^2}\left(1^2 + 3^2\right) = \boxed{\frac{5h^2}{4m_eL^2}}$$

(b) The energy difference between the second excited state and the ground state is given by

$$\Delta E = E_{2,2} - E_{1,1} = \frac{h^2}{m_eL^2} - \frac{h^2}{4m_eL^2}$$

$$= \boxed{\frac{3h^2}{4m_eL^2}}$$

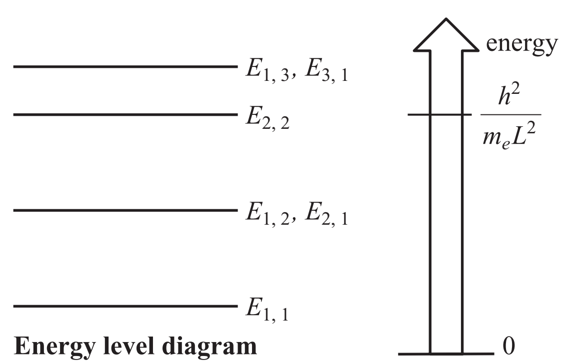

FIG. P41.50(b)

P41.51 $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 \, dx$

For a one-dimensional box of width L, $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$.

Thus, $\langle x^2 \rangle = \frac{2}{L} \int_0^L x^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = \boxed{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}}$ (from integral tables).

P41.52 (a) $\int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$ becomes

$$A^2 \int_{-L/4}^{L/4} \cos^2\left(\frac{2\pi x}{L}\right) dx = A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi x}{L} + \frac{1}{4}\sin\left(\frac{4\pi x}{L}\right)\right]_{-L/4}^{L/4} = A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi}{2}\right] = 1$$

or $A^2 = \frac{4}{L}$ and $\boxed{A = \frac{2}{\sqrt{L}}}$.

(b) The probability of finding the particle between 0 and $\frac{L}{8}$ is

$$\int_0^{L/8} |\psi|^2 \, dx = A^2 \int_0^{L/8} \cos^2\left(\frac{2\pi x}{L}\right) dx = \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}$$

P41.53 For a particle with wave function

$\psi(x) = \sqrt{\frac{2}{a}} e^{-x/a}$ for $x > 0$

and 0 for $x < 0$

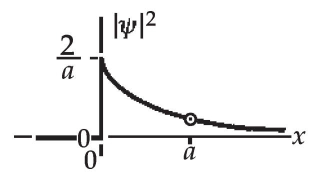

FIG. P41.53

(a) $|\psi(x)|^2 = 0, \; x < 0$ and $|\psi^2(x)| = \frac{2}{a} e^{-2x/a}, \; x > 0$ as shown

(b) $\text{Prob}(x < 0) = \int_{-\infty}^{0} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} (0) \, dx = \boxed{0}$

(c) Normalization $\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} |\psi|^2 \, dx + \int_0^{\infty} |\psi|^2 \, dx = 1$

$$\int_{-\infty}^{0} 0 dx + \int_0^{\infty} \left(\frac{2}{a}\right) e^{-2x/a} dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0 < x < a) = \int_0^a |\psi|^2 \, dx = \int_0^a \left(\frac{2}{a}\right) e^{-2x/a} dx$$

$$= -e^{-2x/a}\Big|_0^a = 1 - e^{-2} = \boxed{0.865}$$

P41.54 (a) The requirement that $\frac{n\lambda}{2} = L$ so $p = \frac{h}{\lambda} = \frac{nh}{2L}$ is still valid.

$$E = \sqrt{(pc)^2 + (mc^2)^2} \Rightarrow E_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2}$$

$$K_n = E_n - mc^2 = \boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2} - mc^2}$$

(b) Taking $L = 1.00 \times 10^{-12}$ m, $m = 9.11 \times 10^{-31}$ kg, and $n = 1$, we find $K_1 = \boxed{4.69 \times 10^{-14}\ \text{J}}$.

Nonrelativistic, $E_1 = \frac{h^2}{8mL^2} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11 \times 10^{-31}\ \text{kg})(1.00 \times 10^{-12}\ \text{m})^2} = 6.02 \times 10^{-14}$ J.

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

P41.55 (a) $U = \frac{e^2}{4\pi \in_0 d}\left[-1 + \frac{1}{2} - \frac{1}{3} + \left(-1 + \frac{1}{2}\right) + (-1)\right] = \frac{(-7/3)e^2}{4\pi \in_0 d} = \boxed{-\frac{7k_e e^2}{3d}}$

(b) From Equation 41.14, $K = 2E_1 = \frac{2h^2}{8m_e(9d^2)} = \boxed{\frac{h^2}{36m_e d^2}}$.

(c) $E = U + K$ and $\frac{dE}{dd} = 0$ for a minimum: $\frac{7k_e e^2}{3d^2} - \frac{h^2}{18m_e d^3} = 0$

$$d = \frac{3h^2}{(7)(18k_e e^2 m_e)} = \frac{h^2}{42m_e k_e e^2} = \frac{(6.626 \times 10^{-34})^2}{(42)(9.11 \times 10^{-31})(8.99 \times 10^9)(1.60 \times 10^{-19}\ \text{C})^2}$$

$$= \boxed{0.049\,9\ \text{nm}}$$

(d) Since the lithium spacing is a, where $Na^3 = V$, and the density is $\frac{Nm}{V}$, where m is the mass of one atom, we get:

$$a = \left(\frac{Vm}{Nm}\right)^{1/3} = \left(\frac{m}{\text{density}}\right)^{1/3} = \left(\frac{1.66 \times 10^{-27}\ \text{kg} \times 7}{530\ \text{kg}}\right)^{1/3} \text{m} = 2.80 \times 10^{-10}\ \text{m} = \boxed{0.280\ \text{nm}}$$

The lithium interatomic spacing of 280 pm is 5.62 times larger than the answer to (c). Thus it is of the same order of magnitude as the interatomic spacing $2d$ here.

P41.56 (a) $\psi = Bxe^{-(m\omega/2\hbar)x^2}$

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(-\frac{m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2 x^3e^{-(m\omega/2\hbar)x^2}$$

Substituting into the Schrödinger equation, we have

$$-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2 x^3e^{-(m\omega/2\hbar)x^2} = -\frac{2mE}{\hbar^2}Bxe^{-(m\omega/2\hbar)x^2} + \left(\frac{m\omega}{\hbar}\right)^2 x^2Bxe^{-(m\omega/2\hbar)x^2}$$

This is true if $-3\omega = -\frac{2E}{\hbar}$; it is true if $\boxed{E = \frac{3\hbar\omega}{2}}$.

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) ψ is maximized if $\frac{d\psi}{dx} = 0 = 1 - x^2\left(\frac{m\omega}{\hbar}\right)$, which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$.

(d) We require $\int_{-\infty}^{\infty}|\psi|^2\,dx = 1$:

$$1 = \int_{-\infty}^{\infty} B^2x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\int x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\,\frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = \frac{B^2}{2}\frac{\pi^{1/2}\hbar^{3/2}}{(m\omega)^{3/2}}$$

Then $B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$.

(e) At $x = 2\sqrt{\frac{\hbar}{m\omega}}$, the potential energy is $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2\left(\frac{4\hbar}{m\omega}\right) = 2\hbar\omega$. This is larger than the total energy $\frac{3\hbar\omega}{2}$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) Probability $= |\psi|^2\,dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2\delta = \delta B^2x^2e^{-(m\omega/\hbar)x^2}$

Probability $= \delta\frac{2}{\pi^{1/2}}\left(\frac{m\omega}{\hbar}\right)^{3/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}}$

P41.57 (a) $\int_0^L |\psi|^2\,dx = 1$: $\qquad A^2\int_0^L \left[\sin^2\left(\frac{\pi x}{L}\right) + 16\sin^2\left(\frac{2\pi x}{L}\right) + 8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right]dx = 1$

$$A^2\left[\left(\frac{L}{2}\right) + 16\left(\frac{L}{2}\right) + 8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right] = 1$$

$$A^2\left[\frac{17L}{2} + 16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right] = A^2\left[\frac{17L}{2} + \frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Bigg|_{x=0}^{x=L}\right] = 1$$

$A^2 = \frac{2}{17L}$, so the normalization constant is $\boxed{A = \sqrt{\frac{2}{17L}}}$.

(b) $\int_{-a}^{a} |\psi|^2\,dx = 1$: $\qquad \int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right) + |B|^2\sin^2\left(\frac{\pi x}{a}\right) + 2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx = 1$

The first two terms are $|A|^2 a$ and $|B|^2 a$. The third term is:

$$2|A||B|\int_{-a}^{a}\cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx = 4|A||B|\int_{-a}^{a}\cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx$$

$$= \frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Bigg|_{-a}^{a} = 0$$

so that $a\left(|A|^2 + |B|^2\right) = 1$, giving $\boxed{|A|^2 + |B|^2 = \frac{1}{a}}$.

P41.58 (a) $\langle x\rangle_0 = \int_{-\infty}^{\infty} x\left(\frac{a}{\pi}\right)^{1/2} e^{-ax^2}dx = \boxed{0}$, since the integrand is an odd function of x.

(b) $\langle x\rangle_1 = \int_{-\infty}^{\infty} x\left(\frac{4a^3}{\pi}\right)^{1/2} x^2 e^{-ax^2}dx = \boxed{0}$, since the integrand is an odd function of x.

(c) $\langle x\rangle_{01} = \int_{-\infty}^{\infty} x\frac{1}{2}\left(\psi_0 + \psi_1\right)^2 dx = \frac{1}{2}\langle x\rangle_0 + \frac{1}{2}\langle x\rangle_1 + \int_{-\infty}^{\infty} x\psi_0(x)\psi_1(x)\,dx$

The first two terms are zero, from (a) and (b). Thus:

$$\langle x\rangle_{01} = \int_{-\infty}^{\infty} x\left(\frac{a}{\pi}\right)^{1/4} e^{-ax^2/2}\left(\frac{4a^3}{\pi}\right)^{1/4} xe^{-ax^2/2}dx = 2\left(\frac{2a^2}{\pi}\right)^{1/2}\int_0^{\infty} x^2 e^{-ax^2}dx$$

$$= 2\left(\frac{2a^2}{\pi}\right)^{1/2}\frac{1}{4}\left(\frac{\pi}{a^3}\right)^{1/2}, \text{ from Table B.6}$$

$$= \boxed{\frac{1}{\sqrt{2a}}}$$

P41.59 With one slit open $P_1 = |\psi_1|^2$ or $P_2 = |\psi_2|^2$

With both slits open, $P = |\psi_1 + \psi_2|^2$

At a maximum, the wave functions are in phase $P_{\max} = \left(|\psi_1| + |\psi_2|\right)^2$

At a minimum, the wave functions are out of phase $P_{\min} = \left(|\psi_1| - |\psi_2|\right)^2$

Now $\dfrac{P_1}{P_2} = \dfrac{|\psi_1|^2}{|\psi_2|^2} = 25.0$, so $\dfrac{|\psi_1|}{|\psi_2|} = 5.00$

and $$\frac{P_{\max}}{P_{\min}} = \frac{\left(|\psi_1| + |\psi_2|\right)^2}{\left(|\psi_1| - |\psi_2|\right)^2} = \frac{\left(5.00|\psi_2| + |\psi_2|\right)^2}{\left(5.00|\psi_2| - |\psi_2|\right)^2} = \frac{(6.00)^2}{(4.00)^2} = \frac{36.0}{16.0} = \boxed{2.25}$$

ANSWERS TO EVEN PROBLEMS

P41.2 $\dfrac{1}{2}$

P41.4 (a) 4 (b) 6.03 eV

P41.6 9.56×10^{12}

P41.8 $\left(\dfrac{3h\lambda}{8m_e c}\right)^{1/2}$

P41.10 (a) 5.13 meV (b) 9.41 eV (c) The much smaller mass of the electron requires it to have much more energy to have the same momentum.

P41.12 (a) $\left(\dfrac{15h\lambda}{8m_e c}\right)^{1/2}$ (b) 1.25λ

P41.14 (a) $\dfrac{L}{2}$ (b) 5.26×10^{-5} (c) 3.99×10^{-2} (d) See the solution.

P41.16 (a) 0.196 (b) The classical probability is 0.333, significantly larger. (c) 0.333 for both classical and quantum models.

P41.18 (a) $\dfrac{\ell}{L} - \dfrac{1}{2\pi}\sin\left(\dfrac{2\pi\ell}{L}\right)$ (b) See the solution. (c) The wave function is zero for $x < 0$ and for $x > L$. The probability at $\ell = 0$ must be zero because the particle is never found at $x < 0$ or exactly at $x = 0$. The probability at $\ell = L$ must be 1 for normalization. This statement means that the particle is always found somewhere at $x < L$. (d) $\ell = 0.585L$

P41.20 See the solution; $\dfrac{\hbar^2 k^2}{2m}$

P41.22 (a) $\dfrac{\hbar^2}{2mL^2}\left(\dfrac{4x^2}{L^2} - 6\right)$ (b) See the solution.

P41.24 (a) $\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$ $P_1(x) = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$ $\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$

$P_2(x) = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$ $\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$ $P_3(x) = \frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$

(b) See the solution.

P41.26 See the solution.

P41.28 1.03×10^{-3}

P41.30 (a) 0.903 (b) 0.359 (c) 0.417 (d) $10^{-6.59\times10^{32}}$

P41.32 85.9

P41.34 3.92%

P41.36 (a) See the solution. $b = \frac{m\omega}{2\hbar}$ (b) $E = \frac{3}{2}\hbar\omega$ (c) first excited state

P41.38 (a) $B = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$ (b) $\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}$

P41.40 See the solution.

P41.42 (a) 2.00×10^{-10} m (b) 3.31×10^{-24} kg·m/s (c) 0.172 eV

P41.44 See the solution.

P41.46 (a) See the solution. (b) 0.092 0, 0.908

P41.48 (a) See the solution. (b) 0.200 (c) 0.351 (d) 0.377 eV, 1.51 eV

P41.50 (a) $\frac{h^2}{4m_eL^2}, \frac{5h^2}{8m_eL^2}, \frac{h^2}{m_eL^2}, \frac{5h^2}{4m_eL^2}$ (b) See the solution, $\frac{3h^2}{4m_eL^2}$

P41.52 (a) $\frac{2}{\sqrt{L}}$ (b) 0.409

P41.54 (a) $\sqrt{\left(\frac{nhc}{2L}\right)^2 + m^2c^4} - mc^2$ (b) 46.9 fJ; 28.6%

P41.56 (a) $\frac{3\hbar\omega}{2}$ (b) $x = 0$ (c) $\pm\sqrt{\frac{\hbar}{m\omega}}$ (d) $\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}$ (e) 0 (f) $8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}$

P41.58 (a) 0 (b) 0 (c) $(2a)^{-1/2}$