45

Applications of Nuclear Physics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q45.1 The hydrogen nuclei in water molecules have mass similar to that of a neutron, so that they can efficiently rob a fast-moving neutron of kinetic energy as they scatter it. Once the neutron is slowed down, a hydrogen nucleus can absorb it in the reaction $n + {}^1_1H \rightarrow {}^2_1H$.

Q45.2 The excitation energy comes from the binding energy of the extra nucleon.

***Q45.3** Answer (c). The function of the moderator is to slow down the neutrons released by one fission so that they can efficiently cause more fissions.

Q45.4 The advantage of a fission reaction is that it can generate much more electrical energy per gram of fuel compared to fossil fuels. Also, fission reactors do not emit greenhouse gases as combustion byproducts like fossil fuels—the only necessary environmental discharge is heat. The cost involved in producing fissile material is comparable to the cost of pumping, transporting and refining fossil fuel.

The disadvantage is that some of the products of a fission reaction are radioactive—and some of those have long half-lives. The other problem is that there will be a point at which enough fuel is spent that the fuel rods do not supply power economically and need to be replaced. The fuel rods are still radioactive after removal. Both the waste and the "spent" fuel rods present serious health and environmental hazards that can last for tens of thousands of years. Accidents and sabotage involving nuclear reactors can be very serious, as can accidents and sabotage involving fossil fuels.

***Q45.5** Answer (b). In $m = E/c^2$ we estimate 5×10^{13} J/9×10^{16} m^2/s$^2 \approx 6 \times 10^{-4}$ kg = 0.6 g.

***Q45.6** Answer (d). We compute 235 + 1 – 137 – 96 = 3. All the protons that start out in the uranium nucleus end up in the fission product nuclei.

Q45.7 The products of fusion reactors are generally not themselves unstable, while fission reactions result in a chain of reactions which almost all have some unstable products.

Q45.8 For the deuterium nuclei to fuse, they must be close enough to each other for the nuclear forces to overcome the Coulomb repulsion of the protons—this is why the ion density is a factor. The more time that the nuclei in a sample spend in close proximity, the more nuclei will fuse—hence the confinement time is a factor.

Q45.9 Fusion of light nuclei to a heavier nucleus releases energy. Fission of a heavy nucleus to lighter nuclei releases energy. Both processes are steps towards greater stability on the curve of binding energy, Figure 44.5. The energy release per nucleon is typically greater for fusion, and this process is harder to control.

Q45.10 Advantages of fusion: high energy yield, no emission of greenhouse gases, fuel very easy to obtain, reactor cannot go supercritical like a fission reactor, low amounts of radioactive waste. Disadvantages: requires high energy input to sustain reaction, lithium and helium are scarce, neutrons released by reaction cause structural damage to reactor housing.

***Q45.11** Answer $Q_1 > Q_2 > Q_3 > 0$. Because all of the reactions involve 108 nucleons, we can look just at the change in binding-energy-per-nucleon as shown on the curve of binding energy. The jump from lithium to carbon is the biggest jump, and next the jump from $A = 27$ to $A = 54$, which is near the peak of the curve. The step up for fission from $A = 108$ to $A = 54$ is smallest. Both of the fusion reactions described and the fission reaction put out energy, so Q is positive for all. Imagine turning the curve of binding energy upside down so that it bends down like a cross-section of a bathtub. On such a curve of total energy per nucleon versus mass number it is easy to identify the fusion of small nuclei, the fission of large nuclei, and even the alpha decay of uranium, as exoenergetic processes. The most stable nucleus is at the drain of the bathtub, with minimum energy.

***Q45.12** Answer (d). The particles lose energy by collisions with nuclei in the bubble chamber to make their speed and their cyclotron radii $r = mv/qB$ decrease.

***Q45.13** Answer (d) only. The Geiger counter responds to an individual particle with a pulse whose size is determined by the tube power supply.

Q45.14 For each additional dynode, a larger applied voltage is needed, and hence a larger output from a power supply—"infinite" amplification would not be practical. Nor would it be desirable: the goal is to connect the tube output to a simple counter, so a massive pulse amplitude is not needed. If you made the detector sensitive to weaker and weaker signals, you would make it more and more sensitive to background noise.

***Q45.15** Answer (b). The cyclotron radius is given by $r = mv/qB = \sqrt{2\frac{1}{2}m^2v^2}/qB = \sqrt{2mK}/qB$. With the same K, $|q|$, and B for both particles, the electron with much smaller mass has smaller radius and is deflected more.

***Q45.16** Answer b > c > a > d. Dose (a) is 1 rem. Dose (b) is 10 rem. Doses (c) and (d) are 4 to 5 rem. If we assume that (a) and (b) as well as (c) were whole-body doses to many kilograms of tissue, we find the ranking stated.

Q45.17 Sometimes the references are indeed oblique. Some can serve for more than one form of energy or mode of transfer. Here is one list:

kinetic: ocean currents
rotational kinetic: Earth turning
gravitational: water lifted up; you on the perilous bridge
elastic: Elastic energy is necessary for sound, listed below.
Vibrational energy could be separately exemplified by the swaying bridge of land.
internal: lava in and from infernal volcanoes; or in forging a chain
chemical: corrosive smoke
sound: thunder
electrical transmission: lightning
electromagnetic radiation: heavens blazing; lightning
atomic electronic: In the blazing heavens, stars have different colors because of different predominant energy losses by atoms at their surfaces.
nuclear: The blaze of the heavens is produced by nuclear reactions in the cores of stars.

Remarkably, the word "energy" in this translation is an anachronism. Goethe wrote the song a few years before Thomas Young coined the term.

SOLUTIONS TO PROBLEMS

Section 45.1 **Interactions Involving Neutrons**

Section 45.2 **Nuclear Fission**

P45.1 The energy is

$$3.30\times10^{10}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{U-235 nucleus}}{208\ \text{MeV}}\right)\left(\frac{235\ \text{g}}{6.02\times10^{23}\ \text{nucleus}}\right)\left(\frac{\text{M}}{10^{6}}\right)$$

$$=\boxed{0.387\ \text{g}}\ \text{of U-235}$$

P45.2 $\Delta m=\left(m_n+M_{\text{U}}\right)-\left(M_{\text{Zr}}+M_{\text{Te}}+3m_n\right)$

$$\Delta m=\left(1.008\,665\ \text{u}+235.043\,923\ \text{u}\right)-\left(97.912\,7\ \text{u}+134.916\,5\ \text{u}+3\left(1.008\,665\ \text{u}\right)\right)$$

$$\Delta m=0.197\,39\ \text{u}=3.28\times10^{-28}\ \text{kg}\quad\text{so}\quad Q=\Delta mc^2=2.95\times10^{-11}\ \text{J}=\boxed{184\ \text{MeV}}$$

P45.3 Three different fission reactions are possible: ${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{90}_{38}\text{Sr}+{}^{144}_{54}\text{Xe}+2\,{}^{1}_{0}\text{n}$ $\boxed{{}^{144}_{54}\text{Xe}}$

${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{90}_{38}\text{Sr}+{}^{143}_{54}\text{Xe}+3\,{}^{1}_{0}\text{n}$ $\boxed{{}^{143}_{54}\text{Xe}}$ ${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{90}_{38}\text{Sr}+{}^{142}_{54}\text{Xe}+4\,{}^{1}_{0}\text{n}$ $\boxed{{}^{142}_{54}\text{Xe}}$

P45.4 (a) $Q=(\Delta m)c^2=\left[m_n+M_{\text{U235}}-M_{\text{Ba141}}-M_{\text{Kr92}}-3m_n\right]c^2$

$$\Delta m=\left[\left(1.008\,665+235.043\,923\right)-\left(140.914\,4+91.926\,2+3\times1.008\,665\right)\right]\text{u}=0.185\,993\ \text{u}$$

$$Q=\left(0.185\,993\ \text{u}\right)\left(931.5\ \text{MeV/u}\right)=\boxed{173\ \text{MeV}}$$

(b) $f=\dfrac{\Delta m}{m_i}=\dfrac{0.185993\ \text{u}}{236.05\ \text{u}}=7.88\times10^{-4}=\boxed{0.078\,8\%}$

P45.5 ${}^{1}_{0}\text{n}+{}^{232}_{90}\text{Th}\rightarrow{}^{233}_{90}\text{Th}\rightarrow{}^{233}_{91}\text{Pa}+\text{e}^-+\bar{\nu}$ ${}^{233}_{91}\text{Pa}\rightarrow{}^{233}_{92}\text{U}+\text{e}^-+\bar{\nu}$

P45.6 (a) The initial mass is $1.007\,825\ \text{u}+11.009\,306\ \text{u}=12.017\,131\ \text{u}$. The final mass is $3\left(4.002\,603\ \text{u}\right)=12.007\,809\ \text{u}$. The rest mass annihilated is $\Delta m=0.009\,322\ \text{u}$. The energy created is $Q=\Delta mc^2=0.009\,322\ \text{u}\left(\dfrac{931.5\ \text{MeV}}{1\ \text{u}}\right)=\boxed{8.68\ \text{MeV}}$.

(b) The proton and the boron nucleus have positive charges. The colliding particles must have enough kinetic energy to approach very closely in spite of their electric repulsion.

P45.7 The available energy to do work is 0.200 times the energy content of the fuel.

$$\left(1.00\ \text{kg fuel}\right)\left(0.034\,0\ {}^{235}\text{U/fuel}\right)\left(\frac{1\,000\ \text{g}}{1\ \text{kg}}\right)\left(\frac{1\ \text{mol}}{235\ \text{g}}\right)\left(6.02\times10^{23}/\text{mol}\right)\left(\frac{(208)\left(1.60\times10^{-13}\ \text{J}\right)}{\text{fission}}\right)$$

$$=2.90\times10^{12}\ \text{J}$$

$$\left(2.90\times10^{12}\ \text{J}\right)\left(0.200\right)=5.80\times10^{11}\ \text{J}=\left(1.00\times10^{5}\ \text{N}\right)\Delta r$$

$$\Delta r=5.80\times10^{6}\ \text{m}=\boxed{5.80\ \text{Mm}}$$

P45.8 If the electrical power output of 1 000 MW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$\frac{1\,000\text{ MW}}{0.400}=\left(2.50\times10^{9}\text{ J/s}\right)\left(8.64\times10^{4}\text{ s/d}\right)=2.16\times10^{14}\text{ J/d}$$

The number of fissions per day is

$$\left(2.16\times10^{14}\text{ J/d}\right)\left(\frac{1\text{ fission}}{200\times10^{6}\text{ eV}}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)=6.74\times10^{24}\text{ d}^{-1}$$

This also is the number of ^{235}U nuclei used, so the mass of ^{235}U used per day is

$$\left(6.74\times10^{24}\text{ nuclei/d}\right)\left(\frac{235\text{ g/mol}}{6.02\times10^{23}\text{ nuclei/mol}}\right)=2.63\times10^{3}\text{ g/d}=\boxed{2.63\text{ kg/d}}$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than $6\times10^{6}\text{ kg/d}$ of coal.

Section 45.3 **Nuclear Reactors**

P45.9 Mass of ^{235}U available $\approx(0.007)\left(10^{9}\text{ metric tons}\right)\left(\frac{10^{6}\text{ g}}{1\text{ metric ton}}\right)=7\times10^{12}\text{ g}$

$$\text{number of nuclei}\approx\left(\frac{7\times10^{12}\text{ g}}{235\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ nuclei/mol}\right)=1.8\times10^{34}\text{ nuclei}$$

The energy available from fission (at 208 MeV/event) is

$$E\approx\left(1.8\times10^{34}\text{ events}\right)\left(208\text{ MeV/event}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right)=6.0\times10^{23}\text{ J}$$

This would last for a time interval of

$$\Delta t=\frac{E}{\mathcal{P}}\approx\frac{6.0\times10^{23}\text{ J}}{7.0\times10^{12}\text{ J/s}}=\left(8.6\times10^{10}\text{ s}\right)\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right)\approx\boxed{3\,000\text{ yr}}$$

***P45.10** (a) For a sphere: $V=\frac{4}{3}\pi r^{3}$ and $r=\left(\frac{3V}{4\pi}\right)^{1/3}$ so $\frac{A}{V}=\frac{4\pi r^{2}}{(4/3)\pi r^{3}}=\boxed{4.84V^{-1/3}}$

(b) For a cube: $V=\ell^{3}$ and $\ell=V^{1/3}$ so $\frac{A}{V}=\frac{6\ell^{2}}{\ell^{3}}=\boxed{6V^{-1/3}}$

(c) For a parallelepiped: $V=2a^{3}$ and $a=\left(\frac{V}{2}\right)^{1/3}$ so $\frac{A}{V}=\frac{\left(2a^{2}+8a^{2}\right)}{2a^{3}}=\boxed{6.30V^{-1/3}}$

(d) The answers show that the sphere has the smallest surface area for a given volume and the brick has the greatest surface area of the three. Therefore, the $\boxed{\text{sphere has the least leakage}}$ and the $\boxed{\text{parallelepiped has the greatest leakage}}$.

P45.11 In one minute there are $\dfrac{60.0\text{ s}}{1.20\text{ ms}} = 5.00\times10^4$ fissions.

So the rate increases by a factor of $(1.000\ 25)^{50\ 000} = \boxed{2.68\times10^5}$.

P45.12 $\mathcal{P} = 10.0\text{ MW} = 1.00\times10^7\text{ J/s}$. If each decay delivers $1.00\text{ MeV} = 1.60\times10^{-13}\text{ J}$, then the

number of decays/s $= \dfrac{10^7\text{ J/s}}{1.6\times10^{-13}\text{ J}} = \boxed{6.25\times10^{19}\text{ Bq}}$.

***P45.13** (a) Since $K = p^2/2m$, we have

$$p = \sqrt{2mK} = \sqrt{2m\tfrac{3}{2}k_BT} = \sqrt{3(1.67\times10^{-27}\text{ kg})(1.38\times10^{-23}\text{ J/K})\ 300\text{ K}}$$

$$= \boxed{4.55\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(b) $\lambda = h/p = 6.63\times10^{-34}\text{ J}\cdot\text{s}/4.55\times10^{-24}\text{ kg}\cdot\text{m/s} = \boxed{0.146\text{ nm}}$. This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

***P45.14** We take the radii of the helium and gold nuclei as 1.20 fm $4^{1/3}$ = 1.90 fm and 1.20 fm $197^{1/3}$ = 6.98 fm. The center to center distance is then 8.89 fm and the electric potential energy is

$$U = qV = \frac{k_e q_1 q_2}{r} = \frac{8.99\times10^9\text{ N}\cdot\text{m}^2\ 2\times1.6\times10^{-19}\text{C}\ 79\ e}{\text{C}^2\ 8.89\times10^{-15}\text{ m}} = \boxed{25.6\text{ MeV}}$$

Section 45.4 **Nuclear Fusion**

P45.15 (a) The Q value for the D-T reaction is 17.59 MeV.
Specific energy content in fuel for D-T reaction:

$$\frac{(17.59\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(5\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 3.39\times10^{14}\text{ J/kg}$$

$$r_{\text{DT}} = \frac{(3.00\times10^9\text{ J/s})(3\,600\text{ s/hr})}{(3.39\times10^{14}\text{ J/kg})(10^{-3}\text{ kg/g})} = \boxed{31.9\text{ g/h burning of D and T}}$$

(b) Specific energy content in fuel for D-D reaction: $Q = \dfrac{1}{2}(3.27+4.03) = 3.65$ MeV average of two Q values

$$\frac{(3.65\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(4\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 8.80\times10^{13}\text{ J/kg}$$

$$r_{\text{DD}} = \frac{(3.00\times10^9\text{ J/s})(3\,600\text{ s/hr})}{(8.80\times10^{13}\text{ J/kg})(10^{-3}\text{ kg/g})} = \boxed{122\text{ g/h burning of D}}$$

***P45.16** (a) We assume that the nuclei are stationary at closest approach, so that the electrostatic potential energy equals the total energy E.

$$U_f = \frac{k_e(Z_1 e)(Z_2 e)}{r_{\min}} = E$$

$$E = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2 Z_1 Z_2}{1.00\times10^{-14}\ \text{m}} = \boxed{(2.30\times10^{-14}\ \text{J})Z_1 Z_2} = (144\ \text{keV})\,Z_1 Z_2$$

(b) The energy is proportional to each atomic number.

(c) Take $Z_1 = 1$ and $Z_2 = 59$ or vice versa. This choice minimizes the product $Z_1 Z_2$. If extra cleverness is allowed, take $Z_1 = 0$ and $Z_2 = 60$: use neutrons as the bombarding particles. A neutron is a nucleon but not an atomic nucleus.

(d) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = (2.30\times10^{-14}\ \text{J})\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{144\ \text{keV}}$$

Section 45.4 in the text gives more accurate values for the critical ignition temperatures, of about 52 keV for D-D fusion and 6 keV for D-T fusion. The nuclei can fuse by tunneling. A triton moves more slowly than a deuteron at a given temperature. Then D-T collisions last longer than D-D collisions and have much greater tunneling probabilities.

P45.17 (a) $r_f = r_{\text{D}} + r_{\text{T}} = (1.20\times10^{-15}\ \text{m})\left[(2)^{1/3} + (3)^{1/3}\right] = \boxed{3.24\times10^{-15}\ \text{m}}$

(b) $$U_f = \frac{k_e e^2}{r_f} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{3.24\times10^{-15}\ \text{m}} = 7.10\times10^{-14}\ \text{J} = \boxed{444\ \text{keV}}$$

(c) Conserving momentum, $m_{\text{D}} v_i = (m_{\text{D}} + m_{\text{T}}) v_f$ or $v_f = \left(\frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right) v_i = \boxed{\frac{2}{5} v_i}$

(d) $K_i + U_i = K_f + U_f$: $\quad K_i + 0 = \frac{1}{2}(m_{\text{D}} + m_{\text{T}}) v_f^2 + U_f = \frac{1}{2}(m_{\text{D}} + m_{\text{T}})\left(\frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)^2 v_i^2 + U_f$

$$K_i + 0 = \left(\frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)\left(\frac{1}{2} m_{\text{D}} v_i^2\right) + U_f = \left(\frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right) K_i + U_f$$

$\left(1 - \frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right) K_i = U_f$: $\quad K_i = U_f\left(\frac{m_{\text{D}} + m_{\text{T}}}{m_{\text{T}}}\right) = \frac{5}{3}(444\ \text{keV}) = \boxed{740\ \text{keV}}$

(e) $\boxed{\text{The nuclei can fuse by tunneling through the potential-energy barrier.}}$

P45.18 (a) $$V = \left(317\times10^{6}\ \text{mi}^3\right)\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right)^3 = 1.32\times10^{18}\ \text{m}^3$$

$$m_{\text{water}} = \rho V = \left(10^3\ \text{kg/m}^3\right)\left(1.32\times10^{18}\ \text{m}^3\right) = 1.32\times10^{21}\ \text{kg}$$

$$m_{H_2} = \left(\frac{M_{H_2}}{M_{H_2O}}\right) m_{H_2O} = \left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\ \text{kg}\right) = 1.48\times10^{20}\ \text{kg}$$

$$m_{\text{Deuterium}} = (0.030\,0\%)\, m_{H_2} = \left(0.030\,0\times10^{-2}\right)\left(1.48\times10^{20}\ \text{kg}\right) = 4.43\times10^{16}\ \text{kg}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43\times10^{16}\ \text{kg}}{(2.014\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He} + Q/c^2$, the number of events is $\frac{N}{2} = 6.63\times10^{42}$.

The energy released per event is

$$Q = \left[M_{^2\text{H}} + M_{^2\text{H}} - M_{^4\text{He}}\right]c^2 = \left[2(2.014\,102) - 4.002\,603\right]\text{u}(931.5\ \text{MeV/u}) = 23.8\ \text{MeV}$$

The total energy available is then

$$E = \left(\frac{N}{2}\right)Q = \left(6.63\times10^{42}\right)(23.8\ \text{MeV})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = \boxed{2.53\times10^{31}\ \text{J}}$$

(b) The time this energy could possibly meet world requirements is

$$\Delta t = \frac{E}{\mathcal{P}} = \frac{2.53\times10^{31}\ \text{J}}{100\left(7.00\times10^{12}\ \text{J/s}\right)} = \left(3.61\times10^{16}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right)$$
$$= \boxed{1.14\times10^{9}\ \text{yr}} \sim 1\ \text{billion years}$$

P45.19 (a) Average KE per particle is $\frac{3}{2}k_BT = \frac{1}{2}mv^2$.

Therefore, $$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^{8}\ \text{K}\right)}{2.014\left(1.661\times10^{-27}\ \text{kg}\right)}} = \boxed{2.22\times10^{6}\ \text{m/s}}$$

(b) $$t = \frac{x}{v} \sim \frac{0.1\ \text{m}}{10^6\ \text{m/s}} = \boxed{\sim10^{-7}\ \text{s}}$$

P45.20 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$ where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \frac{3}{2}Nk_BT = 3nVk_BT$$

$$= 3\left(2.0\times10^{13}\ \text{cm}^{-3}\right)\left[\left(50\ \text{m}^3\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.0\times10^8\ \text{K}\right)$$

$$E = \boxed{1.7\times10^7\ \text{J}}$$

(b) From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6\ \text{J/kg}$. The mass of water that could be boiled away is

$$m = \frac{E}{L_v} = \frac{1.7\times10^7\ \text{J}}{2.26\times10^6\ \text{J/kg}} = \boxed{7.3\ \text{kg}}$$

P45.21 (a) Lawson's criterion for the D-T reaction is $n\tau \geq 10^{14}\ \text{s/cm}^3$. For a confinement time of $\tau = 1.00\ \text{s}$, this requires a minimum ion density of $n = \boxed{10^{14}\ \text{cm}^{-3}}$.

(b) At the ignition temperature of $T = 4.5\times10^7\ \text{K}$ and the ion density found above, the plasma pressure is

$$P = 2nk_BT = 2\left[\left(10^{14}\ \text{cm}^{-3}\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.5\times10^7\ \text{K}\right)$$

$$= \boxed{1.24\times10^5\ \text{J/m}^3}$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \geq 10P = 10\left(1.24\times10^5\ \text{J/m}^3\right) = 1.24\times10^6\ \text{J/m}^3$$

$$B \geq \sqrt{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(1.24\times10^6\ \text{J/m}^3\right)} = \boxed{1.77\ \text{T}}$$ This is a very strong field.

P45.22 The number of nuclei in 1.00 metric ton of trash is

$$N = 1\,000\ \text{kg}\left(1\,000\ \text{g/kg}\right)\frac{6.02\times10^{23}\ \text{nuclei/mol}}{56.0\ \text{g/mol}} = 1.08\times10^{28}\ \text{nuclei}$$

At an average charge of 26.0 *e*/nucleus, $q = \left(1.08\times10^{28}\right)\left(26.0\right)\left(1.60\times10^{-19}\right) = 4.47\times10^{10}\ \text{C}$

Therefore $t = \frac{q}{I} = \frac{4.47\times10^{10}}{1.00\times10^6} = 4.47\times10^4\ \text{s} = \boxed{12.4\ \text{h}}$

Section 45.5 **Radiation Damage**

P45.23 $N_0 = \dfrac{\text{mass present}}{\text{mass of nucleus}} = \dfrac{5.00\text{ kg}}{(89.907\,7\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 3.35\times10^{25}\text{ nuclei}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1\text{ yr}} = 2.38\times10^{-2}\text{ yr}^{-1} = 4.53\times10^{-8}\text{ min}^{-1}$$

$$R_0 = \lambda N_0 = (4.53\times10^{-8}\text{ min}^{-1})(3.35\times10^{25}) = 1.52\times10^{18}\text{ counts/min}$$

$$\frac{R}{R_0} = e^{-\lambda t} = \frac{10.0\text{ counts/min}}{1.52\times10^{18}\text{ counts/min}} = 6.59\times10^{-18}$$

and $\lambda t = -\ln(6.59\times10^{-18}) = 39.6$

giving $t = \dfrac{39.6}{\lambda} = \dfrac{39.6}{2.38\times10^{-2}\text{ yr}^{-1}} = \boxed{1.66\times10^{3}\text{ yr}}$

P45.24 The source delivers 100 mrad of 2-MeV γ-rays/h at a 1.00-m distance.

(a) For γ-rays, dose in rem = dose in rad.
Thus a person would have to stand there $\boxed{10.0\text{ hours}}$ to receive 1.00 rem from a 100-mrad/h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $\dfrac{1}{r^2}$.

Thus a dosage 10.0 mrad/h would be received at a distance $r = \sqrt{10.0}\text{ m} = \boxed{3.16\text{ m}}$.

P45.25 (a) The number of x-ray images made per year is

$n = (8\text{ x-ray/d})(5\text{ d/wk})(50\text{ wk/yr}) = 2.0\times10^{3}\text{ x-ray/yr}$

The average dose per photograph is $\dfrac{5.0\text{ rem/yr}}{2.0\times10^{3}\text{ x-ray/yr}} = \boxed{2.5\times10^{-3}\text{ rem/x-ray image}}$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The dose of 5.0 rem/yr received as a result of the job is

$\dfrac{5.0\text{ rem/yr}}{0.13\text{ rem/yr}} = \boxed{\text{38 times the assumed background level}}$. The technician's occupational exposure is high compared to background radiation.

P45.26 (a) $I = I_0 e^{-\mu x}$, so $x = \dfrac{1}{\mu}\ln\left(\dfrac{I_0}{I}\right)$

With $\mu = 1.59\text{ cm}^{-1}$, the thickness when $I = \dfrac{I_0}{2}$ is $x = \dfrac{1}{1.59\text{ cm}^{-1}}\ln(2) = \boxed{0.436\text{ cm}}$.

(b) When $\dfrac{I_0}{I} = 1.00\times10^{4}$, $x = \dfrac{1}{1.59\text{ cm}^{-1}}\ln(1.00\times10^{4}) = \boxed{5.79\text{ cm}}$.

P45.27 $1\text{ rad} = 10^{-2}\ \text{J/kg}$ $Q = mc\Delta T$ $\mathcal{P}\Delta t = mc\Delta T$

$$\Delta t = \frac{mc\Delta T}{\mathcal{P}} = \frac{m(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C})}{(10)(10^{-2}\ \text{J/kg}\cdot\text{s})(m)} = \boxed{2.09\times10^{6}\ \text{s}} \approx 24\ \text{days!}$$

Note that power is the product of dose rate and mass.

P45.28 $\dfrac{Q}{m} = \dfrac{\text{absorbed energy}}{\text{unit mass}} = (1\,000\ \text{rad})\dfrac{10^{-2}\ \text{J/kg}}{1\ \text{rad}} = 10.0\ \text{J/kg}$

The rise in body temperature is calculated from $Q = mc\Delta T$ where $c = 4\,186\ \text{J/kg}$ for water and the human body

$\Delta T = \dfrac{Q}{mc} = (10.0\ \text{J/kg})\dfrac{1}{4\,186\ \text{J/kg}\cdot{}^\circ\text{C}} = \boxed{2.39\times10^{-3}{}^\circ\text{C}}$ The temperature change is negligible.

P45.29 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right] = 4.26\times10^{12}\ \text{MeV}$$

$$E = (4.26\times10^{12}\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV}) = 0.682\ \text{J}$$

Thus, the dose received is $\text{Dose} = \dfrac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\dfrac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$

***P45.30** The decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{17\ \text{d}} = 0.040\,8/\text{d}$. The number of nuclei remaining after 30 d is $N = N_0e^{-\lambda T} = N_0e^{-0.040\,8(30)} = 0.294N_0$. The number decayed is $N_0 - N = N_0(1-0.294) = 0.706N_0$. Then the energy release is

$$2.12\ \text{J} = 0.706N_0(21.0\times10^{3}\ \text{eV})\left(\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$N_0 = \frac{2.12\ \text{J}}{2.37\times10^{-15}\ \text{J}} = 8.94\times10^{14}$$

(a) $R_0 = \lambda N_0 = \dfrac{0.040\,8}{\text{d}}(8.94\times10^{14})\left(\dfrac{1\ \text{d}}{86\,400\ \text{s}}\right) = \boxed{4.22\times10^{8}\ \text{Bq}}$

(b) original sample mass $= m = N_{\text{original}}m_{\text{one atom}} = 8.94\times10^{14}(103\ \text{u})\left(\dfrac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right)$

$$= \boxed{1.53\times10^{-10}\ \text{kg}} = 1.53\times10^{-7}\ \text{g} = 153\ \text{ng}$$

P45.31 The nuclei initially absorbed are $N_0 = \left(1.00\times10^{-9}\text{ g}\right)\left(\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{89.9\text{ g/mol}}\right) = 6.70\times10^{12}$

The number of decays in time t is $\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$

At the end of 1 year, $\dfrac{t}{T_{1/2}} = \dfrac{1.00\text{ yr}}{29.1\text{ yr}} = 0.034\,4$

and $\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left(1-e^{-0.023\,8}\right) = 1.58\times10^{11}$

The energy deposited is $E = \left(1.58\times10^{11}\right)(1.10\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.027\,7\text{ J}$

Thus, the dose received is $\text{Dose } = \left(\dfrac{0.027\,7\text{ J}}{70.0\text{ kg}}\right) = \boxed{3.96\times10^{-4}\text{ J/kg}} = 0.039\,6\text{ rad}$

Section 45.6 **Radiation Detectors**

P45.32 (a) $E_I = 10.0\text{ eV}$ is the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e(\Delta V)$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e\dfrac{\Delta V}{E_I}$:

At the first dynode, $n_i = 1$ and $N_1 = \dfrac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^1\text{ electrons}}$

(b) For the second dynode, $n_i = N_1 = 10^1$, so $N_2 = \dfrac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$

At the third dynode, $n_i = N_2 = 10^2$ and $N_3 = \dfrac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$

Observing the developing pattern, we see that the number of electrons incident on the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6 e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

P45.33 (a) $\dfrac{E}{E_\beta}=\dfrac{(1/2)C(\Delta V)^2}{0.500\text{ MeV}}=\dfrac{(1/2)\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)^2}{(0.500\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}=\boxed{3.12\times10^{7}}$

(b) $N=\dfrac{Q}{e}=\dfrac{C(\Delta V)}{e}=\dfrac{\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)}{1.60\times10^{-19}\text{ C}}=\boxed{3.12\times10^{10}\text{ electrons}}$

P45.34 (a) The average time between slams is $60\text{ min}/38=1.6\text{ min}$. Sometimes, the actual interval is nearly zero. Perhaps about equally as often, it is $2\times1.6\text{ min}$. Perhaps about half as often, it is $4\times1.6\text{ min}$. Somewhere around $5\times1.6\text{ min}=\boxed{8.0\text{ min}}$, the chances of randomness producing so long a wait get slim, so such a long delay might likely be due to mischief.

(b) The midpoints of the time intervals are separated by 5.00 minutes. We use $R=R_0e^{-\lambda t}$. Subtracting the background counts,

$$337-5(15)=\left[372-5(15)\right]e^{-\left(\ln 2/T_{1/2}\right)(5.00\text{ min})}$$

or $$\ln\left(\frac{262}{297}\right)=\ln(0.882)=-3.47\text{ min}/T_{1/2}\text{ which yields }T_{1/2}=\boxed{27.6\text{ min}}$$

(c) As in the random events in part (a), we imagine a ±5 count counting uncertainty. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right)=-3.47\text{ min}/T_{1/2},\text{ or }\left(T_{1/2}\right)_{\text{min}}=21.1\text{ min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right)=-3.47\text{ min}/T_{1/2},\text{ yielding }\left(T_{1/2}\right)_{\text{max}}=38.8\text{ min}$$

Thus, $$T_{1/2}=\left(\frac{38.8+21.1}{2}\right)\pm\left(\frac{38.8-21.1}{2}\right)\text{ min}=(30\pm9)\text{ min}=\boxed{30\text{ min}\pm30\%}$$

Section 45.7 **Uses of Radiation**

P45.35 The initial specific activity of ^{59}Fe in the steel,

$$(R/m)_0=\frac{20.0\ \mu\text{Ci}}{0.200\text{ kg}}=\frac{100\ \mu\text{Ci}}{\text{kg}}\left(\frac{3.70\times10^{4}\text{ Bq}}{1\ \mu\text{Ci}}\right)=3.70\times10^{6}\text{ Bq/kg}$$

After 1 000 h, $$\frac{R}{m}=\left(\frac{R}{m}\right)_0e^{-\lambda t}=\left(3.70\times10^{6}\text{ Bq/kg}\right)e^{-\left(6.40\times10^{-4}\text{ h}^{-1}\right)(1\,000\text{ h})}=1.95\times10^{6}\text{ Bq/kg}$$

The activity of the oil is $$R_{\text{oil}}=\left(\frac{800}{60.0}\text{ Bq/liter}\right)(6.50\text{ liters})=86.7\text{ Bq}$$

Therefore $$m_{\text{in oil}}=\frac{R_{\text{oil}}}{(R/m)}=\frac{86.7\text{ Bq}}{1.95\times10^{6}\text{ Bq/kg}}=4.45\times10^{-5}\text{ kg}$$

So that wear rate is $$\frac{4.45\times10^{-5}\text{ kg}}{1\,000\text{ h}}=\boxed{4.45\times10^{-8}\text{ kg/h}}$$

P45.36 (a) The number of photons is $\dfrac{10^4 \text{ MeV}}{1.04 \text{ MeV}} = 9.62 \times 10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\dfrac{3}{4} N_0 = 1.92 \times 10^4$ and $N_0 = 2.56 \times 10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is $2.56 \times 10^6 \boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{63}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is

$N_{63} = 0.691\,7 N_{\text{Cu}}$ and $N_{65} = 0.308\,3 N_{\text{Cu}}$

Therefore, $\dfrac{N_{63}}{N_{65}} = \dfrac{0.691\,7}{0.308\,3}$ or $N_{63} = \left(\dfrac{0.6917}{0.3083}\right) N_{65} = \left(\dfrac{0.6917}{0.3083}\right)\left(2.56 \times 10^6\right) = 5.75 \times 10^6$.

The total mass of copper present is then $\quad m_{\text{Cu}} = (62.93 \text{ u}) N_{63} + (64.93 \text{ u}) N_{65}$:

$$m_{\text{Cu}} = \left[(62.93)\left(5.75 \times 10^6\right) + (64.93)\left(2.56 \times 10^6\right)\right]\text{u}\ \left(1.66 \times 10^{-24} \text{ g/u}\right)$$
$$= 8.77 \times 10^{-16} \text{ g} \ \boxed{\sim 10^{-15} \text{ g}}$$

P45.37 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \qquad \text{so} \qquad dN = (R - \lambda N)\,dt$$

The variables are separable.

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt: \qquad -\frac{1}{\lambda}\ln\left(\frac{R - \lambda N}{R}\right) = t$$

so $$\ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t \quad \text{and} \quad \left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t}$$

Therefore $$1 - \frac{\lambda}{R} N = e^{-\lambda t} \qquad N = \boxed{\frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)}$$

(b) The maximum number of radioactive nuclei would be $\boxed{\dfrac{R}{\lambda}}$.

***P45.38** (a) With $I(x)=\frac{1}{2}I_0$, $I(x)=I_0e^{-\mu x}$ becomes

$$\frac{1}{2}I_0 = I_0e^{-0.72x/\text{mm}} \qquad 2=e^{+0.72x/\text{mm}}$$

$$\ln 2 = 0.72\,x/\text{mm} \qquad x=\frac{\ln 2\text{ mm}}{0.72}=\boxed{0.963\text{ mm}}$$

(b) $$I(0.8\text{ mm})=I_0e^{-0.72(0.8)}=0.562I_0$$

$$I(0.7\text{ mm})=I_0e^{-0.72(0.7)}=0.604I_0$$

$$\text{fractional change}=\frac{0.604I_0-0.562I_0}{0.562I_0}=0.074\,7=\boxed{7.47\%}$$

Additional Problems

P45.39 (a) At 6×10^8 K, the average kinetic energy of a carbon atom is

$$\frac{3}{2}k_BT=(1.5)\left(8.62\times10^{-5}\text{ eV/K}\right)\left(6\times10^8\text{ K}\right)=\boxed{8\times10^4\text{ eV}}$$

Note that 6×10^8 K is about $6^2=36$ times larger than 1.5×10^7 K, the core temperature of the Sun. This factor corresponds to the higher potential-energy barrier to carbon fusion compared to hydrogen fusion. It could be misleading to compare it to the temperature $\sim10^8$ K required for fusion in a low-density plasma in a fusion reactor.

(b) The energy released is

$$E=\left[2m\left(\text{C}^{12}\right)-m\left(\text{Ne}^{20}\right)-m\left(\text{He}^{4}\right)\right]c^2$$

$$E=(24.000\,000-19.992\,440-4.002\,603)(931.5)\text{ MeV}=\boxed{4.62\text{ MeV}}$$

In the second reaction,

$$E=\left[2m\left(\text{C}^{12}\right)-m\left(\text{Mg}^{24}\right)\right](931.5)\text{ MeV/u}$$

$$E=(24.000\,000-23.985\,042)(931.5)\text{ MeV}=\boxed{13.9\text{ MeV}}$$

(c) The energy released is the energy of reaction of the number of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E=\left(2.00\times10^3\text{ g}\right)\left(\frac{6.02\times10^{23}\text{ atoms/mol}}{12.0\text{ g/mol}}\right)\left(\frac{4.62\text{ MeV/fusion event}}{2\text{ nuclei/fusion event}}\right)\left(\frac{1\text{ kWh}}{2.25\times10^{19}\text{ MeV}}\right)$$

$$\Delta E=\frac{\left(1.00\times10^{26}\right)(4.62)}{2\left(2.25\times10^{19}\right)}\text{kWh}=\boxed{1.03\times10^7\text{ kWh}}$$

P45.40 To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \qquad \text{or} \qquad v_2 = \left(\frac{m_1}{m_2}\right) v_1$$

The kinetic energies after the break-up are then

$$K_1 = \frac{1}{2} m_1 v_1^2 \qquad \text{and} \qquad K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right) K_1$$

The fraction of the total kinetic energy carried off by m_1 is

$$\frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + (m_1/m_2) K_1} = \boxed{\frac{m_2}{m_1 + m_2}}$$

and the fraction carried off by m_2 is $1 - \frac{m_2}{m_1 + m_2} = \boxed{\frac{m_1}{m_1 + m_2}}$.

P45.41 (a) $Q = 236.045\,562\,\text{u}\,c^2 - 86.920\,711\,\text{u}\,c^2 - 148.934\,370\,\text{u}\,c^2 = 0.190\,481\,\text{u}\,c^2 = \boxed{177 \text{ MeV}}$

Immediately after fission, this Q-value is the total kinetic energy of the fission products.

(b) $K_{\text{Br}} = \left(\frac{m_{\text{La}}}{m_{\text{Br}} + m_{\text{La}}}\right) Q$, from Problem 45.40

$$= \left(\frac{149 \text{ u}}{87 \text{ u} + 149 \text{ u}}\right)(177.4 \text{ MeV}) = \boxed{112 \text{ MeV}}$$

$$K_{\text{La}} = Q - K_{\text{Br}} = 177.4 \text{ MeV} - 112.0 \text{ MeV} = \boxed{65.4 \text{ MeV}}$$

(c) $$v_{\text{Br}} = \sqrt{\frac{2K_{\text{Br}}}{m_{\text{Br}}}} = \sqrt{\frac{2(112 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(87 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})}} = \boxed{1.58 \times 10^7 \text{ m/s}}$$

$$v_{\text{La}} = \sqrt{\frac{2K_{\text{La}}}{m_{\text{La}}}} = \sqrt{\frac{2(65.4 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(149 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})}} = \boxed{9.20 \times 10^6 \text{ m/s}}$$

P45.42 For a typical ^{235}U fission, $Q = 208 \text{ MeV}$ and the initial mass is 235 u. Thus, the fractional energy loss is

$$\frac{Q}{mc^2} = \frac{208 \text{ MeV}}{(235 \text{ u})(931.5 \text{ MeV/u})} = 9.50 \times 10^{-4} = \boxed{0.095\,0\%}$$

For the D-T fusion reaction, $Q = 17.6 \text{ MeV}$

The initial mass is $m = (2.014 \text{ u}) + (3.016 \text{ u}) = 5.03 \text{ u}$

The fractional loss in this reaction is $\frac{Q}{mc^2} = \frac{17.6 \text{ MeV}}{(5.03 \text{ u})(931.5 \text{ MeV/u})} = 3.75 \times 10^{-3} = \boxed{0.375\%}$

$\frac{0.375\%}{0.0950\%} = 3.95$ or $\boxed{\text{the fractional loss in D-T fusion is about 4 times that in } ^{235}\text{U fission}}$.

P45.43 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12.3\text{ yr})(3.16\times10^{7}\text{ s/yr})} = 1.78\times10^{-9}\text{ s}^{-1}$.

The tritium in the plasma decays at a rate of

$$R = \lambda N = (1.78\times10^{-9}\text{ s}^{-1})\left[\left(\frac{2.00\times10^{14}}{\text{cm}^3}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)(50.0\text{ m}^3)\right]$$

$$R = 1.78\times10^{13}\text{ Bq} = (1.78\times10^{13}\text{ Bq})\left(\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}\right) = \boxed{482\text{ Ci}}$$

$\boxed{\text{The fission inventory is } \frac{4\times10^{10}\text{ Ci}}{482\text{ Ci}} \sim 10^8 \text{ times greater}}$ than this amount.

***P45.44** The original activity per area is $\frac{5\times10^6\text{ Ci}}{10^4\text{ km}^2}\left(\frac{1\text{ km}}{10^3\text{ m}}\right)^2 = 5\times10^{-4}\text{ Ci/m}^2$.

The decay constant is $\lambda = \ln 2/27.7$ yr.
The decay law $N = N_0\, e^{-\lambda t}$ becomes the law of decrease of activity $R = R_0\, e^{-\lambda t}$. If the material is not transported, it describes the time evolution of activity per area $R/A = R_0/A\, e^{-\lambda t}$. Solving gives

$$e^{\lambda t} = \frac{R_0/A}{R/A} \qquad \lambda t = \ln\left(\frac{R_0/A}{R/A}\right) = \frac{\ln 2}{27.7\text{ yr}}t$$

$$t = \frac{27.7\text{ yr}}{\ln 2}\ln\left(\frac{R_0/A}{R/A}\right) = \frac{27.7\text{ yr}}{\ln 2}\ln\left(\frac{5\times10^{-4}\text{ Ci/m}^2}{2\times10^{-6}\text{ Ci/m}^2}\right) = \boxed{221\text{ yr}}$$

P45.45 The complete fissioning of 1.00 gram of U^{235} releases

$$Q = \frac{(1.00\text{ g})}{235\text{ grams/mol}}(6.02\times10^{23}\text{ atoms/mol})(200\text{ MeV/fission})(1.60\times10^{-13}\text{ J/MeV})$$
$$= 8.20\times10^{10}\text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values),

then $\quad Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$

$$Q = m\left[(4186\text{ J/kg °C})(80.0\text{ °C}) + 2.26\times10^6\text{ J/kg} + (2010\text{ J/kg °C})(300\text{ °C})\right]$$

Therefore $\quad m = \frac{8.20\times10^{10}\text{ J}}{3.20\times10^6\text{ J/kg}} = \boxed{2.56\times10^4\text{ kg}}$

P45.46 When mass m of ^{235}U undergoes complete fission, releasing 200 MeV per fission event, the total energy released is:

$$Q=\left(\frac{m}{235\ \text{g/mol}}\right)N_{\text{A}}(200\ \text{MeV})\ \text{ where } N_{\text{A}} \text{ is Avogadro's number.}$$

If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then $Q=m_w\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]$
where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w=\frac{Q}{\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]}$$

$$=\boxed{\frac{mN_{\text{A}}(200\ \text{MeV})}{(235\ \text{g/mol})\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]}}.$$

***P45.47** (a) We have 0.9928 kg of ^{238}U, comprising
$N = 0.9928$ kg $(6.02\times10^{23}$ atoms/mol)/(0.238 kg/mol) $= 2.51\times10^{24}$ nuclei, with activity
$R = \lambda N = (\ln 2/T_{1/2})N = 2.51\times10^{24}(\ln 2/4.47\times10^{9}\ \text{yr})(1\ \text{yr}/3.16\times10^{7}\ \text{s})(1\ \text{Ci}/3.7\times10^{10}/\text{s})$
$= \boxed{333\ \mu\text{Ci}}$

We have 0.0072 kg of ^{235}U, comprising
$N = 0.0072$ kg $(6.02\times10^{23}$ atoms/mol)/(0.235 kg/mol) $= 1.84\times10^{22}$ nuclei, with activity
$R = \lambda N = (\ln 2/T_{1/2})N = 1.84\times10^{22}(\ln 2/7.04\times10^{8}\ \text{yr})(1\ \text{yr}/3.16\times10^{7}\ \text{s})(1\ \text{Ci}/3.7\times10^{10}/\text{s})$
$= \boxed{15.5\ \mu\text{Ci}}$

We have 0.00005 kg of ^{234}U, comprising
$N = 5\times10^{-5}$ kg $(6.02\times10^{23}$ atoms/mol)/(0.23404 kg/mol) $= 1.29\times10^{20}$ nuclei, with activity
$R = \lambda N = (\ln 2/T_{1/2})N = 1.29\times10^{20}(\ln 2/2.44\times10^{5}\ \text{yr})(1\ \text{yr}/3.16\times10^{7}\ \text{s})(1\ \text{Ci}/3.7\times10^{10}/\text{s})$
$= \boxed{312\ \mu\text{Ci}}$

(b) The total activity is $(333 + 15.5 + 312)\ \mu\text{Ci} = 661\ \mu\text{Ci}$, so the fractional contributions are respectively $333/661 = \boxed{50.4\%}$ $\quad 15.5/661 = \boxed{2.35\%}$ $\quad$ and $\quad 312/661 = \boxed{47.3\%}$

(c) It is potentially dangerous, notably if the material is inhaled as a powder. With precautions to minimize human contact, microcurie sources are routinely used in laboratories.

P45.48 (a) $\Delta V=4\pi r^2\Delta r=4\pi\left(14.0\times10^{3}\ \text{m}\right)^2(0.05\ \text{m})=1.23\times10^{8}\ \text{m}^3\ \boxed{\sim10^{8}\ \text{m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$$W=P\Delta V=\left(1.013\times10^{5}\ \text{N/m}^2\right)\left(1.23\times10^{8}\ \text{m}^3\right)=1.25\times10^{13}\ \text{J}\ \boxed{\sim10^{13}\ \text{J}}$$

(c) $1.25\times10^{13}\ \text{J}=\frac{1}{10}(\text{yield}),\ \text{ so yield}=1.25\times10^{14}\ \text{J}\ \boxed{\sim10^{14}\ \text{J}}$

(d) $\dfrac{1.25\times10^{14}\ \text{J}}{4.2\times10^{9}\ \text{J/ton TNT}}=2.97\times10^{4}\ \text{ton TNT}\sim10^{4}\ \text{ton TNT}$

or $\boxed{\sim 10\ \text{kilotons}}$

P45.49 (a) The number of molecules in 1.00 liter of water (mass = 1 000 g) is

$$N=\left(\frac{1.00\times10^{3}\text{ g}}{18.0\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ molecules/mol}\right)=3.34\times10^{25}\text{ molecules.}$$

The number of deuterium nuclei contained in these molecules is

$$N'=\left(3.34\times10^{25}\text{ molecules}\right)\left(\frac{1\text{ deuteron}}{3300\text{ molecules}}\right)=1.01\times10^{22}\text{ deuterons.}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is

$$\frac{N'}{2}=5.07\times10^{21}\text{ reactions, and the energy released is}$$

$$E_{\text{fusion}}=\left(5.07\times10^{21}\text{ reactions}\right)\left(3.27\text{ MeV/reaction}\right)=1.66\times10^{22}\text{ MeV}$$

$$E_{\text{fusion}}=\left(1.66\times10^{22}\text{ MeV}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right)=\boxed{2.65\times10^{9}\text{ J}}.$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}}=\frac{2.65\times10^{9}\text{ J}}{3.40\times10^{7}\text{ J}}=\boxed{78.0\text{ times larger}}.$$

P45.50 Momentum conservation: $0=m_{\text{Li}}\vec{\mathbf{v}}_{\text{Li}}+m_\alpha\vec{\mathbf{v}}_\alpha$ or $m_{\text{Li}}v_{\text{Li}}=m_\alpha v_\alpha$

Thus,

$$K_{\text{Li}}=\frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2=\frac{1}{2}\frac{\left(m_{\text{Li}}v_{\text{Li}}\right)^2}{m_{\text{Li}}}=\frac{\left(m_\alpha v_\alpha\right)^2}{2m_{\text{Li}}}=\left(\frac{m_\alpha^2}{2m_{\text{Li}}}\right)v_\alpha^2$$

$$K_{\text{Li}}=\left(\frac{\left(4.002\,6\text{ u}\right)^2}{2\left(7.016\,0\text{ u}\right)}\right)\left(9.25\times10^{6}\text{ m/s}\right)^2=\left(1.14\text{ u}\right)\left(9.25\times10^{6}\text{ m/s}\right)^2$$

$$K_{\text{Li}}=1.14\left(1.66\times10^{-27}\text{ kg}\right)\left(9.25\times10^{6}\text{ m/s}\right)^2=1.62\times10^{-13}\text{ J}=\boxed{1.01\text{ MeV}}.$$

P45.51 (a) The thermal power transferred to the water is $\mathcal{P}_w=0.970(\text{waste heat})$

$$\mathcal{P}_w=0.970\left(3\,065-1\,000\right)\text{MW}=2.00\times10^{9}\text{ J/s}$$

r_w is the mass of water heated per hour:

$$r_w=\frac{\mathcal{P}_w}{c(\Delta T)}=\frac{\left(2.00\times10^{9}\text{ J/s}\right)(3600\text{ s/h})}{(4186\text{ J/kg}\cdot{}^\circ\text{C})(3.50\ {}^\circ\text{C})}=\boxed{4.91\times10^{8}\text{ kg/h}}$$

The volume used per hour is $\dfrac{4.91\times10^{8}\text{ kg/h}}{1.00\times10^{3}\text{ kg/m}^3}=\boxed{4.91\times10^{5}\text{ m}^3\text{/h}}$

(b) The ^{235}U fuel is consumed at a rate

$$r_f=\left(\frac{3\,065\times10^{6}\text{ J/s}}{7.80\times10^{10}\text{ J/g}}\right)\left(\frac{1\text{ kg}}{1\,000\text{ g}}\right)\left(\frac{3\,600\text{ s}}{1\text{ h}}\right)=\boxed{0.141\text{ kg/h}}$$

P45.52 The number of nuclei in 0.155 kg of ^{210}Po is

$$N_0 = \left(\frac{155 \text{ g}}{209.98 \text{ g/mol}}\right)\left(6.02\times10^{23} \text{ nuclei/mol}\right) = 4.44\times10^{23} \text{ nuclei}$$

The half-life of ^{210}Po is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38 \text{ d})\left(8.64\times10^{4} \text{ s/d}\right)} = 5.80\times10^{-8} \text{ s}^{-1}$$

The initial activity is

$$R_0 = \lambda N_0 = \left(5.80\times10^{-8} \text{ s}^{-1}\right)\left(4.44\times10^{23} \text{ nuclei}\right) = 2.58\times10^{16} \text{ Bq}$$

The energy released in each $^{210}_{84}\text{Po} \to {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$ reaction is

$$Q = \left[M_{^{210}_{84}\text{Po}} - M_{^{206}_{82}\text{Pb}} - M_{^{4}_{2}\text{He}}\right]c^2:$$

$$Q = [209.982\,857 - 205.974\,449 - 4.002\,603]\text{u}(931.5 \text{ MeV/u}) = 5.41 \text{ MeV}$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathcal{P} = (0.010\,0)R_0Q = (0.010\,0)\left(2.58\times10^{16} \text{ decays/s}\right)(5.41 \text{ MeV/decay})\left(1.60\times10^{-13} \text{ J/MeV}\right)$$

$$= \boxed{223 \text{ W}}$$

P45.53 (a) $V = \ell^3 = \dfrac{m}{\rho}$, so $\ell = \left(\dfrac{m}{\rho}\right)^{1/3} = \left(\dfrac{70.0 \text{ kg}}{18.7\times10^3 \text{ kg/m}^3}\right)^{1/3} = \boxed{0.155 \text{ m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes

$$^{238}_{92}\text{U atom} \to 8\,{}^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom} + Q_{\text{net}}/c^2$$

$$Q_{\text{net}} = \left[M_{^{238}_{92}\text{U}} - 8M_{^{4}_{2}\text{He}} - M_{^{206}_{82}\text{Pb}}\right]c^2$$

$$= \left[238.050\,783 - 8(4.002\,603) - 205.974\,449\right] \text{u } (931.5 \text{ MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7 \text{ MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate R and the energy released in each decay is Q. Then the energy released per time is $\boxed{\mathcal{P} = QR}$. If there is a series of decays in steady state, the equation is still true, with Q representing the net decay energy.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00\times10^4 \text{ g}}{238 \text{ g/mol}}\right)\left(6.02\times10^{23} \text{ nuclei/mol}\right) = 1.77\times10^{26} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47\times10^9 \text{ yr}} = 1.55\times10^{-10} \ \frac{1}{\text{yr}}$$

$$R = \lambda N = \left(1.55\times10^{-10} \ \frac{1}{\text{yr}}\right)\left(1.77\times10^{26} \text{ nuclei}\right) = 2.75\times10^{16} \text{ decays/yr}$$

so $\mathcal{P} = QR = (51.7 \text{ MeV})\left(2.75\times10^{16} \ \dfrac{1}{\text{yr}}\right)\left(1.60\times10^{-13} \text{ J/MeV}\right) = \boxed{2.27\times10^5 \text{ J/yr}}$

continued on next page

(e) dose in rem = dose in rad × RBE

$$5.00 \text{ rem/yr} = (\text{dose in rad/yr})1.10, \text{ giving } (\text{dose in rad/yr}) = 4.55 \text{ rad/yr}$$

The allowed whole-body dose is then $(70.0 \text{ kg})(4.55 \text{ rad/yr})\left(\dfrac{10^{-2} \text{ J/kg}}{1 \text{ rad}}\right) = \boxed{3.18 \text{ J/yr}}$.

P45.54 $E_T \equiv E(\text{thermal}) = \dfrac{3}{2}k_B T = 0.039 \text{ eV}$

$E_T = \left(\dfrac{1}{2}\right)^n E$ where $n \equiv$ number of collisions, and $0.039 = \left(\dfrac{1}{2}\right)^n (2.0 \times 10^6)$

Therefore $n = 25.6 = \boxed{26 \text{ collisions}}$

P45.55 Conservation of linear momentum and energy can be applied to find the kinetic energy of the neutron. We first suppose the particles are moving nonrelativistically.
The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n \vec{\mathbf{v}}_n + m_\alpha \vec{\mathbf{v}}_\alpha = 0 \qquad \text{or} \qquad (1.008\,7 \text{ u})v_n = (4.002\,6 \text{ u})v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV

$$E = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}(1.008\,7 \text{ u})v_n^2 + \frac{1}{2}(4.002\,6)v_\alpha^2 = 17.6 \text{ MeV}$$

Substitute $v_\alpha = 0.252\,0 v_n$: $E = (0.504\,35 \text{ u})v_n^2 + (0.127\,10 \text{ u})v_n^2 = 17.6 \text{ MeV}\left(\dfrac{1 \text{ u}}{931.494 \text{ MeV}/c^2}\right)$

$$v_n = \sqrt{\frac{0.018\,9c^2}{0.631\,45}} = 0.173c = 5.19 \times 10^7 \text{ m/s}$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.008\,7 \text{ u})(0.173c)^2\left(\frac{931.494 \text{ MeV}/c^2}{\text{u}}\right) = 14.1 \text{ MeV}$$

For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of momentum gives

$$\gamma_n m_n \vec{\mathbf{v}}_n + \gamma_\alpha m_\alpha \vec{\mathbf{v}}_\alpha = 0 \qquad 1.008\,7\frac{v_n}{\sqrt{1 - v_n^2/c^2}} = 4.002\,6\frac{v_\alpha}{\sqrt{1 - v_\alpha^2/c^2}}$$

yielding $\qquad \dfrac{v_\alpha^2}{c^2} = \dfrac{v_n^2}{15.746c^2 - 14.746v_n^2}$

Then $\qquad (\gamma_n - 1)m_n c^2 + (\gamma_\alpha - 1)m_\alpha c^2 = 17.6 \text{ MeV}$

and $\qquad v_n = 0.171c$ implying that $(\gamma_n - 1)m_n c^2 = \boxed{14.0 \text{ MeV}}$

P45.56 From the table of isotopic masses in Chapter 44, the half-life of ^{32}P is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26 \text{ d}} = 0.048\,6 \text{ d}^{-1} = 5.63 \times 10^{-7} \text{ s}^{-1}$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22 \times 10^6 \text{ decay/s}}{5.63 \times 10^{-7} \text{ s}^{-1}} = 9.28 \times 10^{12} \text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = \left(9.28 \times 10^{12} \text{ nuclei}\right) e^{-\left(0.048\,6 \text{ d}^{-1}\right)(10.0 \text{ d})} = 5.71 \times 10^{12} \text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57 \times 10^{12}$ and the energy released is

$$E = \left(3.57 \times 10^{12}\right)(700 \text{ keV})\left(1.60 \times 10^{-16} \text{ J/keV}\right) = 0.400 \text{ J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\text{Dose} = \left(\frac{0.400 \text{ J}}{0.100 \text{ kg}}\right)\left(\frac{1 \text{ rad}}{10^{-2} \text{ J/kg}}\right) = \boxed{400 \text{ rad}}$$

P45.57 (a) The number of Pu nuclei in $1.00 \text{ kg} = \frac{6.02 \times 10^{23} \text{ nuclei/mol}}{239.05 \text{ g/mol}}(1\,000 \text{ g})$.

The total energy $= \left(25.2 \times 10^{23} \text{ nuclei}\right)(200 \text{ MeV}) = 5.04 \times 10^{26} \text{ MeV}$

$$E = \left(5.04 \times 10^{26} \text{ MeV}\right)\left(4.44 \times 10^{-20} \text{ kWh/MeV}\right) = \boxed{2.24 \times 10^7 \text{ kWh}}$$

or 22 million kWh

(b) $E = \Delta m c^2 = (3.016\,049 \text{ u} + 2.014\,102 \text{ u} - 4.002\,603 \text{ u} - 1.008\,665 \text{ u})(931.5 \text{ MeV/u})$

$E = \boxed{17.6 \text{ MeV for each D-T fusion}}$

(c) $E_n = (\text{Total number of D nuclei})(17.6)\left(4.44 \times 10^{-20}\right)$

$$E_n = \left(6.02 \times 10^{23}\right)\left(\frac{1\,000}{2.014}\right)(17.6)\left(4.44 \times 10^{-20}\right) = \boxed{2.34 \times 10^8 \text{ kWh}}$$

(d) $E_n = \text{the number of C atoms in } 1.00 \text{ kg} \times 4.20 \text{ eV}$

$$E_n = \left(\frac{6.02 \times 10^{26}}{12}\right)\left(4.20 \times 10^{-6} \text{ MeV}\right)\left(4.44 \times 10^{-20}\right) = \boxed{9.36 \text{ kWh}}$$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.

P45.58 Add two electrons to both sides of the given reaction.

Then $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q/c^2$

where $Q = (\Delta m)c^2 = [4(1.007\,825) - 4.002\,603]\text{u}(931.5\ \text{MeV/u}) = 26.7\ \text{MeV}$

or $Q = (26.7\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV}) = 4.28\times10^{-12}\ \text{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.85\times10^{26}\ \text{J/s}}{(4.28\times10^{-12}\ \text{J})/(4\ \text{protons})} = \boxed{3.60\times10^{38}\ \text{protons/s}}$$

P45.59 (a) $Q_\text{I} = [M_\text{A} + M_\text{B} - M_\text{C} - M_\text{E}]c^2,$ and $Q_\text{II} = [M_\text{C} + M_\text{D} - M_\text{F} - M_\text{G}]c^2$

$$Q_\text{net} = Q_\text{I} + Q_\text{II} = [M_\text{A} + M_\text{B} - M_\text{C} - M_\text{E} + M_\text{C} + M_\text{D} - M_\text{F} - M_\text{G}]c^2$$

$$Q_\text{net} = Q_\text{I} + Q_\text{II} = [M_\text{A} + M_\text{B} + M_\text{D} - M_\text{E} - M_\text{F} - M_\text{G}]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$${}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} + {}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + Q_\text{net}/c^2$$

or $4\,{}^1_1\text{H} + 2\,{}^{\ 0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + Q_\text{net}/c^2$

Adding two electrons to each side gives $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q_\text{net}/c^2$

Thus $Q_\text{net} = \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 = [4(1.007\,825) - 4.002\,603]\text{u}(931.5\ \text{MeV/u})$

$= \boxed{26.7\ \text{MeV}}$

P45.60 (a) The mass of the pellet is $m = \rho V = (0.200\ \text{g/cm}^3)\left[\frac{4\pi}{3}\left(\frac{1.50\times10^{-2}\ \text{cm}}{2}\right)^3\right] = 3.53\times10^{-7}\ \text{g}$

The pellet consists of equal numbers of ${}^2\text{H}$ and ${}^3\text{H}$ atoms, so the average molar mass is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53\times10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)(6.02\times10^{23}\ \text{atoms/mol}) = 8.51\times10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of $2N$ particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_\text{B}T\right)$ as

$$T = \frac{E}{3Nk_\text{B}} = \frac{2.00\times10^3\ \text{J}}{3(8.51\times10^{16})(1.38\times10^{-23}\ \text{J/K})} = \boxed{5.68\times10^8\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so $\frac{N}{2}$ events will occur. The energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51\times10^{16}}{2}\right)(17.59\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV}) = 1.20\times10^5\ \text{J} = \boxed{120\ \text{kJ}}$$

P45.61 (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}^{1}_{1}\mathrm{H} + {}^{3}_{2}\mathrm{He} \to {}^{4}_{2}\mathrm{He} + \mathrm{e}^{+} + \nu$, estimated as $\dfrac{k_e(e)(2e)}{r}$. The Coulomb barrier to Bethe's fifth and eight reactions is like $\dfrac{k_e(e)(7e)}{r}$, larger by $\dfrac{7}{2}$ times, so the required temperature can be estimated as $\dfrac{7}{2}\left(15\times10^{6}\ \mathrm{K}\right) \approx \boxed{5\times10^{7}\ \mathrm{K}}$.

(b) For ${}^{12}\mathrm{C} + {}^{1}\mathrm{H} \to {}^{13}\mathrm{N} + Q$,

$$Q_1 = (12.000\,000 + 1.007\,825 - 13.005\,739)(931.5\ \mathrm{MeV}) = \boxed{1.94\ \mathrm{MeV}}$$

For the second step, add seven electrons to both sides to have:

$${}^{13}\mathrm{N\ atom} \to {}^{13}\mathrm{C\ atom} + \mathrm{e}^{+} + \mathrm{e}^{-} + Q$$

$$Q_2 = \left[13.005\,739 - 13.003\,355 - 2(0.000\,549)\right](931.5\ \mathrm{MeV}) = \boxed{1.20\ \mathrm{MeV}}$$

$$Q_3 = Q_7 = 2(0.000\,549)(931.5\ \mathrm{MeV}) = \boxed{1.02\ \mathrm{MeV}}$$

$$Q_4 = [13.003\,355 + 1.007\,825 - 14.003\,074](931.5\ \mathrm{MeV}) = \boxed{7.55\ \mathrm{MeV}}$$

$$Q_5 = [14.003\,074 + 1.007\,825 - 15.003\,065](931.5\ \mathrm{MeV}) = \boxed{7.30\ \mathrm{MeV}}$$

$$Q_6 = \left[15.003\,065 - 15.000\,109 - 2(0.000\,549)\right](931.5\ \mathrm{MeV}) = \boxed{1.73\ \mathrm{MeV}}$$

$$Q_8 = [15.000\,109 + 1.007\,825 - 12 - 4.002\,603](931.5\ \mathrm{MeV}) = \boxed{4.97\ \mathrm{MeV}}$$

The sum is $\boxed{26.7\ \mathrm{MeV}}$, the same as for the proton-proton cycle.

(c) Not quite all of the energy released appears as internal energy in the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

P45.62 (a) $\dfrac{I_2}{I_1} = \dfrac{I_0 e^{-\mu_2 x}}{I_0 e^{-\mu_1 x}} = \boxed{e^{-(\mu_2-\mu_1)x}}$

(b) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40-41.0)(0.100)} = e^{3.56} = \boxed{35.2}$

(c) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40-41.0)(1.00)} = e^{35.6} = \boxed{2.89\times10^{15}}$

Thus, a 1.00-cm-thick aluminum plate has essentially removed the long-wavelength x-rays from the beam.

P45.63 (a) The number of fissions occurring in the zeroth, first, second, . . . nth generation is

$$N_0,\ N_0K,\ N_0K^2,\ \ldots,\ N_0K^n$$

The total number of fissions that have occurred up to and including the nth generation is

$$N = N_0 + N_0K + N_0K^2 + \cdots + N_0K^n = N_0\left(1 + K + K^2 + \cdots + K^n\right)$$

Note that the factoring of the difference of two squares, $a^2 - 1 = (a+1)(a-1)$, can be generalized to a difference of two quantities to any power,

$$a^3 - 1 = \left(a^2 + a + 1\right)(a-1)$$

$$a^{n+1} - 1 = \left(a^n + a^{n-1} + \cdots + a^2 + a + 1\right)(a-1)$$

Thus $\quad K^n + K^{n-1} + \cdots + K^2 + K + 1 = \dfrac{K^{n+1}-1}{K-1}$

and $\quad \boxed{N = N_0\dfrac{K^{n+1}-1}{K-1}}$

(b) The number of U-235 nuclei is

$$N = 5.50\text{ kg}\left(\frac{1\text{ atom}}{235\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = 1.41\times10^{25}\text{ nuclei}$$

We solve the equation from part (a) for n, the number of generations:

$$\frac{N}{N_0}(K-1) = K^{n+1} - 1$$

$$\frac{N}{N_0}(K-1) + 1 = K^n(K)$$

$$n\ln K = \ln\left(\frac{N(K-1)/N_0 + 1}{K}\right) = \ln\left(\frac{N(K-1)}{N_0} + 1\right) - \ln K$$

$$n = \frac{\ln\left(N(K-1)/N_0 + 1\right)}{\ln K} - 1 = \frac{\ln\left(1.41\times10^{25}(0.1)/10^{20} + 1\right)}{\ln 1.1} - 1 = 99.2$$

Therefore time must be allotted for 100 generations:

$$\Delta t_b = 100\left(10\times10^{-9}\text{ s}\right) = \boxed{1.00\times10^{-6}\text{ s}}$$

(c) $$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{150\times10^9\text{ N/m}^2}{18.7\times10^3\text{ kg/m}^3}} = \boxed{2.83\times10^3\text{ m/s}}$$

(d) $$V = \frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3(5.5\text{ kg})}{4\pi\left(18.7\times10^3\text{ kg/m}^3\right)}\right)^{1/3} = 4.13\times10^{-2}\text{ m}$$

$$\Delta t_d = \frac{r}{v} = \frac{4.13\times10^{-2}\text{ m}}{2.83\times10^3\text{ m/s}} = \boxed{1.46\times10^{-5}\text{ s}}$$

continued on next page

(e) $14.6\ \mu\text{s}$ is greater than $1\ \mu\text{s}$, so the entire bomb can fission. The destructive energy released is

$$1.41\times10^{25}\ \text{nuclei}\left(\frac{200\times10^{6}\ \text{eV}}{\text{fissioning nucleus}}\right)\left(\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)=4.51\times10^{14}\ \text{J}$$

$$=4.51\times10^{14}\ \text{J}\left(\frac{1\ \text{ton TNT}}{4.2\times10^{9}\ \text{J}}\right)$$

$$=1.07\times10^{5}\ \text{ton TNT}$$

$$=\boxed{107\ \text{kilotons of TNT}}$$

What if? If the bomb did not have an "initiator" to inject 10^{20} neutrons at the moment when the critical mass is assembled, the number of generations would be

$$n=\frac{\ln\left(1.41\times10^{25}(0.1)/1+1\right)}{\ln 1.1}-1=582\ \text{ requiring }\ 583\left(10\times10^{-9}\ \text{s}\right)=5.83\ \mu\text{s}$$

This time is not very short compared with $14.6\ \mu\text{s}$, so this bomb would likely release much less energy.

ANSWERS TO EVEN PROBLEMS

P45.2 184 MeV

P45.4 (a) 173 MeV (b) 0.078 8%

P45.6 (a) 8.68 MeV (b) The proton and the boron nucleus have positive charges. The colliding particles must have enough kinetic energy to approach very closely in spite of their electric repulsion.

P45.8 $2.63\ \text{kg/d}$

P45.10 (a) $4.84V^{-1/3}$ (b) $6V^{-1/3}$ (c) $6.30V^{-1/3}$ (d) The sphere has minimum loss and the parallelepiped maximum.

P45.12 6.25×10^{19} Bq

P45.14 25.6 MeV

P45.16 (a) (144 keV) $Z_1 Z_2$ (b) The energy is proportional to each atomic number. (c) Take $Z_1=1$ and $Z_2=59$ or vice versa. This choice minimizes the product $Z_1 Z_2$. (d) 144 keV for both, according to this model

P45.18 (a) 2.53×10^{31} J (b) 1.14×10^{9} yr

P45.20 (a) 1.7×10^{7} J (b) 7.3 kg

P45.22 12.4 h

P45.24 (a) 10.0 h (b) 3.16 m

P45.26 (a) 0.436 cm (b) 5.79 cm

P45.28 2.39×10^{-3} °C

P45.30 (a) 422 MBq (b) 153 ng

P45.32 (a) 10 (b) 10^6 (c) 10^8 eV

P45.34 (a) about 8 min (b) 27.6 min (c) 30 min ± 30%

P45.36 (a) $\sim 10^6$ (b) $\sim 10^{-15}$ g

P45.38 (a) 0.963 mm (b) It increases by 7.47%.

P45.40 See the solution.

P45.42 The fractional loss in D-T is about 4 times that in ^{235}U fission.

P45.44 221 yr

P45.46 $$\frac{mN_A(200\text{ MeV})}{(235\text{ g/mol})\left[c_w(100°\text{C}-T_c)+L_v+c_s(T_h-100°\text{C})\right]}$$

P45.48 (a) $\sim 10^8$ m^3 (b) $\sim 10^{13}$ J (c) $\sim 10^{14}$ J (d) ~10 kilotons

P45.50 1.01 MeV

P45.52 223 W

P45.54 26 collisions

P45.56 400 rad

P45.58 3.60×10^{38} protons/s

P45.60 (a) 5.68×10^8 K (b) 120 kJ

P45.62 (a) See the solution. (b) 35.2 (c) 2.89×10^{15}