6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q6.1** (i) nonzero. Its direction of motion is changing. (ii) zero. Its speed is not changing. (iii) zero: when $v = 0$, $v^2/r = 0$. (iv) nonzero: its velocity is changing from, say 0.1 m/s north to 0.1 m/s south.

Q6.2 (a) The object will move in a circle at a constant speed.
(b) The object will move in a straight line at a changing speed.

Q6.3 The speed changes. The tangential force component causes tangential acceleration.

***Q6.4** (a) A > C = D > B = E. At constant speed, centripetal acceleration is largest when radius is smallest. A straight path has infinite radius of curvature. (b) Velocity is north at A, west at B, and south at C. (c) Acceleration is west at A, nonexistent at B, and east at C, to be radially inward.

***Q6.5** (a) yes, point C. Total acceleration here is centripetal acceleration, straight up. (b) yes, point A. Total acceleration here is tangential acceleration, to the right and downward perpendicular to the cord. (c) No. (d) yes, point B. Total acceleration here is to the right and upward.

Q6.6 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

Q6.7 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

Q6.8 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

Q6.9 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration, to keep blood flowing up to the pilot's brain.

***Q6.10** (a) The keys shift backward relative to the student's hand. The cord then pulls the keys upward and forward, to make them gain speed horizontally forward along with the airplane.
(b) The angle stays constant while the plane has constant acceleration.
This experiment is described in the book *Science from Your Airplane Window* by Elizabeth Wood.

Q6.11 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

Q6.12 From the proportionality of the drag force to the speed squared and from Newton's second law, we derive the equation that describes the motion of the skydiver:

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2$$

where D is the coefficient of drag of the parachutist, and A is the projected area of the parachutist's body. At terminal speed,

$$a_y = \frac{dv_y}{dt} = 0 \quad \text{and} \quad v_T = \left(\frac{2mg}{D\rho A}\right)^{1/2}$$

When the parachute opens, the coefficient of drag D and the effective area A both increase, thus reducing the speed of the skydiver.

Modern parachutes also add a third term, lift, to change the equation to

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2 - \frac{L\rho A}{2}v_x^2$$

where v_y is the vertical velocity, and v_x is the horizontal velocity. The effect of lift is clearly seen in the "paraplane," an ultralight airplane made from a fan, a chair, and a parachute.

Q6.13 (a) Static friction exerted by the roadway where it meets the rubber tires accelerates the car forward and then maintains its speed by counterbalancing resistance forces. (b) The air around the propeller pushes forward on its blades. Evidence is that the propeller blade pushes the air toward the back of the plane. (c) The water pushes the blade of the oar toward the bow. Evidence is that the blade of the oar pushes the water toward the stern.

Q6.14 The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. When moving with terminal speed, an object is in equilibrium and has zero acceleration.

***Q6.15** (a) Speed increases, before she reaches terminal speed. (b) The magnitude of acceleration decreases, as the air resistance force increases to counterbalance more and more of the gravitational force.

Q6.16 The thesis is false. The moment of decay of a radioactive atomic nucleus (for example) cannot be predicted. Quantum mechanics implies that the future is indeterminate. On the other hand, our sense of free will, of being able to make choices for ourselves that can appear to be random, may be an illusion. It may have nothing to do with the subatomic randomness described by quantum mechanics.

SOLUTIONS TO PROBLEMS

Section 6.1 **Newton's Second Law Applied to Uniform Circular Motion**

P6.1 $m = 3.00$ kg, $r = 0.800$ m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\max} = Mg = 25.0(9.80) = 245 \text{ N}$$

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $\quad T = \dfrac{mv^2}{r} = \dfrac{(3.00)v^2}{0.800}$

Then

$$v^2 = \frac{rT}{m} = \frac{(0.800)T}{3.00} \le \frac{(0.800)T_{\max}}{3.00} = \frac{0.800(245)}{3.00} = 65.3 \text{ m}^2/\text{s}^2$$

and $\quad 0 \le v \le \sqrt{65.3}$

or $\quad \boxed{0 \le v \le 8.08 \text{ m/s}}$

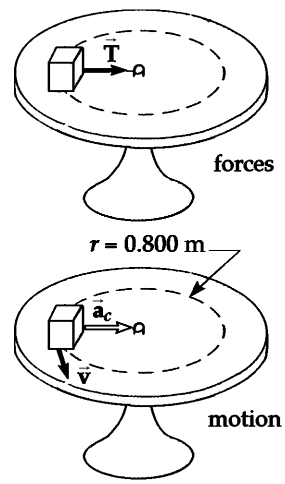


FIG. P6.1

P6.2 In $\sum F = m\dfrac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\dfrac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 (130 \text{ N}) = 215 \text{ N}$$

Symbolically, write $\sum F_{\text{slow}} = \left(\dfrac{m}{r}\right)(14.0 \text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\dfrac{m}{r}\right)(18.0 \text{ m/s})^2$.

Dividing gives $\dfrac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\dfrac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130 \text{ N}) = \boxed{215 \text{ N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.3 (a) $\quad F = \dfrac{mv^2}{r} = \dfrac{\left(9.11\times10^{-31} \text{ kg}\right)\left(2.20\times10^{6} \text{ m/s}\right)^2}{0.530\times10^{-10} \text{ m}} = \boxed{8.32\times10^{-8} \text{ N inward}}$

(b) $\quad a = \dfrac{v^2}{r} = \dfrac{\left(2.20\times10^{6} \text{ m/s}\right)^2}{0.530\times10^{-10} \text{ m}} = \boxed{9.13\times10^{22} \text{ m/s}^2 \text{ inward}}$

P6.4 (a) $\quad \sum F_y = ma_y,\ mg_{\text{moon}} \text{ down} = \dfrac{mv^2}{r} \text{ down}$

$$v = \sqrt{g_{\text{moon}} r} = \sqrt{\left(1.52 \text{ m/s}^2\right)\left(1.7\times10^{6} \text{ m} + 100\times10^{3} \text{ m}\right)} = \boxed{1.65\times10^{3} \text{ m/s}}$$

(b) $\quad v = \dfrac{2\pi r}{T},\ T = \dfrac{2\pi\left(1.8\times10^{6} \text{ m}\right)}{1.65\times10^{3} \text{ m/s}} = \boxed{6.84\times10^{3} \text{ s}} = 1.90 \text{ h}$

P6.5 (a) $\boxed{\text{static friction}}$

(b) $ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right)$

$\sum F_y = 0 = n - mg$

thus $n = mg$ and $\sum F_r = m\dfrac{v^2}{r} = f = \mu n = \mu mg$.

Then $\mu = \dfrac{v^2}{rg} = \dfrac{(50.0\ \text{cm/s})^2}{(30.0\ \text{cm})\left(980\ \text{cm/s}^2\right)} = \boxed{0.085\,0}$.

P6.6 Neglecting relativistic effects. $F = ma_c = \dfrac{mv^2}{r}$

$$F = \left(2\times1.661\times10^{-27}\ \text{kg}\right)\frac{\left(2.998\times10^{7}\ \text{m/s}\right)^2}{(0.480\ \text{m})} = \boxed{6.22\times10^{-12}\ \text{N}}$$

P6.7 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the $3.00\ \text{m/s}^2$ centripetal acceleration:

$a_c = v^2/r$ so $v = \sqrt{a_c r} = \sqrt{\left(3.00\ \text{m/s}^2\right)(60.0\ \text{m})} = 13.4\ \text{m/s}$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$: $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(60.0\ \text{m})}{13.4\ \text{m/s}} = 28.1\ \text{s}$

so the frequency of rotation is $f = \dfrac{1}{T} = \dfrac{1}{28.1\ \text{s}} = \dfrac{1}{28.1\ \text{s}}\left(\dfrac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{2.14\ \text{rev/min}}$.

P6.8 $T\cos 5.00° = mg = (80.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)$

(a) $T = 787\ \text{N}$: $\vec{\mathbf{T}} = \boxed{(68.6\ \text{N})\hat{\mathbf{i}} + (784\ \text{N})\hat{\mathbf{j}}}$

(b) $T\sin 5.00° = ma_c$: $\boxed{a_c = 0.857\ \text{m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

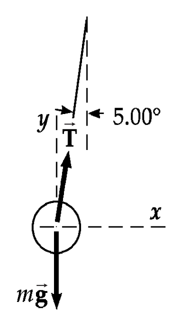


FIG. P6.8

P6.9 $n = mg$ since $a_y = 0$

The force causing the centripetal acceleration is the frictional force f.

From Newton's second law $f = ma_c = \dfrac{mv^2}{r}$.

But the friction condition is $f \le \mu_s n$

i.e., $\dfrac{mv^2}{r} \le \mu_s mg$

$v \le \sqrt{\mu_s rg} = \sqrt{0.600(35.0\ \text{m})\left(9.80\ \text{m/s}^2\right)}$ $v \le \boxed{14.3\ \text{m/s}}$

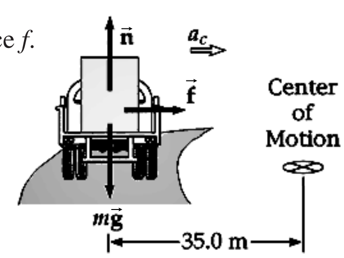


FIG. P6.9

P6.10 (a) $v = \dfrac{235 \text{ m}}{36.0 \text{ s}} = \boxed{6.53 \text{ m/s}}$

The radius is given by $\dfrac{1}{4} 2\pi r = 235 \text{ m}$ so $r = 150 \text{ m}$

(b) $\vec{\mathbf{a}}_r = \left(\dfrac{v^2}{r}\right)$ toward center

$$= \frac{(6.53 \text{ m/s})^2}{150 \text{ m}} \text{ at } 35.0° \text{ north of west}$$

$$= \left(0.285 \text{ m/s}^2\right)\left(\cos 35.0°\left(-\hat{\mathbf{i}}\right) + \sin 35.0° \hat{\mathbf{j}}\right)$$

$$= \boxed{-0.233 \text{ m/s}^2 \hat{\mathbf{i}} + 0.163 \text{ m/s}^2 \hat{\mathbf{j}}}$$

(c) $\vec{\mathbf{a}}_{avg} = \dfrac{\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right)}{t}$

$$= \frac{\left(6.53 \text{ m/s} \hat{\mathbf{j}} - 6.53 \text{ m/s} \hat{\mathbf{i}}\right)}{36.0 \text{ s}}$$

$$= \boxed{-0.181 \text{ m/s}^2 \hat{\mathbf{i}} + 0.181 \text{ m/s}^2 \hat{\mathbf{j}}}$$

P6.11 $F_g = mg = (4 \text{ kg})\left(9.8 \text{ m/s}^2\right) = 39.2 \text{ N}$

$\sin\theta = \dfrac{1.5 \text{ m}}{2 \text{ m}}$

$\theta = 48.6°$

$r = (2 \text{ m})\cos 48.6° = 1.32 \text{ m}$

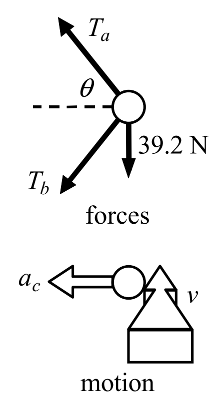


FIG. P6.11

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a \cos 48.6° + T_b \cos 48.6° = \frac{(4 \text{ kg})(6 \text{ m/s})^2}{1.32 \text{ m}}$$

$$T_a + T_b = \frac{109 \text{ N}}{\cos 48.6°} = 165 \text{ N}$$

$$\sum F_y = ma_y$$

$$+T_a \sin 48.6° - T_b \sin 48.6° - 39.2 \text{ N} = 0$$

$$T_a - T_b = \frac{39.2 \text{ N}}{\sin 48.6°} = 52.3 \text{ N}$$

(a) To solve simultaneously, we add the equations in T_a and T_b:

$$T_a + T_b + T_a - T_b = 165 \text{ N} + 52.3 \text{ N}$$

$$T_a = \frac{217 \text{ N}}{2} = \boxed{108 \text{ N}}$$

(b) $T_b = 165 \text{ N} - T_a = 165 \text{ N} - 108 \text{ N} = \boxed{56.2 \text{ N}}$

Section 6.2 **Nonuniform Circular Motion**

P6.12 (a) $a_c = \dfrac{v^2}{r} = \dfrac{(4.00\ \text{m/s})^2}{12.0\ \text{m}} = \boxed{1.33\ \text{m/s}^2}$

(b) $a = \sqrt{a_c^2 + a_t^2}$

$$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79\ \text{m/s}^2}$$

at an angle $\theta = \tan^{-1}\left(\dfrac{a_c}{a_t}\right) = \boxed{48.0°\ \text{inward}}$

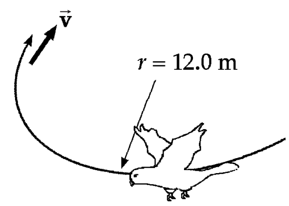


FIG. P6.12

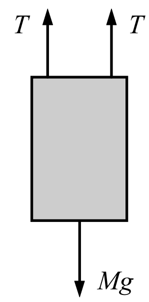
P6.13 $M = 40.0\ \text{kg},\ R = 3.00\ \text{m},\ T = 350\ \text{N}$

(a) $\sum F = 2T - Mg = \dfrac{Mv^2}{R}$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

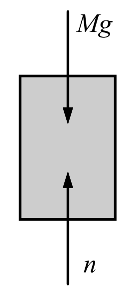
$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1\ (\text{m}^2/\text{s}^2)$$

$$\boxed{v = 4.81\ \text{m/s}}$$



child + seat

FIG. P6.13(a)



child alone

FIG. P6.13(b)

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\ \text{N}}$$

P6.14 (a) $v = 20.0\ \text{m/s}$,

n = force of track on roller coaster, and

$R = 10.0\ \text{m}$.

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

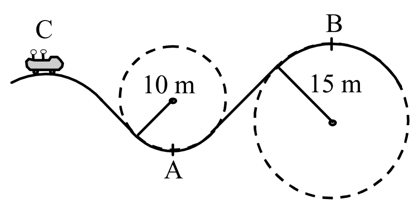


FIG. P6.14

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(500\ \text{kg})(20.0\ \text{m/s}^2)}{10.0\ \text{m}}$$

$$n = 4\,900\ \text{N} + 20\,000\ \text{N} = \boxed{2.49 \times 10^4\ \text{N}}$$

(b) At B, $n - Mg = -\dfrac{Mv^2}{R}$

The maximum speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\max}^2}{R} \Rightarrow v_{\max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\ \text{m/s}}$$

P6.15 Let the tension at the lowest point be T.

$$\sum F = ma: \quad T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0 \text{ kg})\left[9.80 \text{ m/s}^2 + \frac{(8.00 \text{ m/s})^2}{10.0 \text{ m}}\right] = 1.38 \text{ kN} > 1\,000 \text{ N}$$

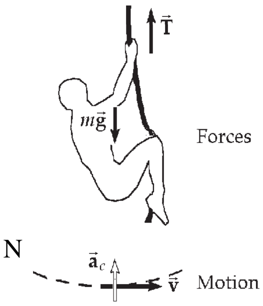


FIG. P6.15

$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$

***P6.16** (a) Consider radial forces on the object, taking inward as positive.

$\Sigma F_r = ma_r$:

$T - (0.5 \text{ kg})(9.8 \text{ m/s}^2)\cos 20° = mv^2/r = 0.5 \text{ kg}(8 \text{ m/s})^2/2 \text{ m}$

$T = 4.60 \text{ N} + 16.0 \text{ N} = \boxed{20.6 \text{ N}}$

(b) We already found the radial component of acceleration,

$(8 \text{ m/s})^2/2 \text{ m} = \boxed{32.0 \text{ m/s}^2 \text{ inward}}$.

Consider tangential forces on the object.

$\Sigma F_t = ma_t$:

$(0.5 \text{ kg})(9.8 \text{ m/s}^2)\sin 20 = 0.5 \text{ kg } a_t$

$a_t = \boxed{3.35 \text{ m/s}^2 \text{ downward tangent to the circle}}$

(c) $a = [32^2 + 3.35^2]^{1/2} \text{ m/s}^2$ inward and below the cord at angle $\tan^{-1}(3.35/32)$

$= \boxed{32.2 \text{ m/s}^2 \text{ inward and below the cord at } 5.98°}$

(d) No change. If the object is swinging down it is gaining speed. If the object is swinging up it is losing speed but its acceleration is the same size and its direction can be described in the same terms.

P6.17 $$\sum F_y = \frac{mv^2}{r} = mg + n$$

But $n = 0$ at this minimum speed condition, so

$$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(1.00 \text{ m})} = \boxed{3.13 \text{ m/s}}$$

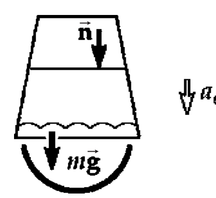


FIG. P6.17

P6.18 (a) $a_c = \dfrac{v^2}{r} \qquad r = \dfrac{v^2}{a_c} = \dfrac{(13.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{8.62 \text{ m}}$

(b) Let n be the force exerted by the rail.
Newton's second law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg, \text{ downward}}$$

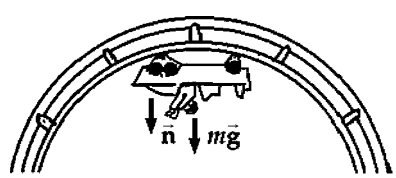


FIG. P6.18

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(c) $a_c = \dfrac{v^2}{r}$ $\qquad a_c = \dfrac{(13.0\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{8.45\ \text{m/s}^2}$

If the force exerted by the rail is n_1

then $\qquad n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$n_1 = M(a_c - g)$ which is < 0 since $a_c = 8.45\ \text{m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars.

To be safe we must require n_1 to be positive. Then $a_c > g$. We need

$$\frac{v^2}{r} > g \text{ or } v > \sqrt{rg} = \sqrt{(20.0\ \text{m})(9.80\ \text{m/s}^2)},\ v > 14.0\ \text{m/s}$$

Section 6.3 **Motion in Accelerated Frames**

P6.19 (a) $\sum F_x = Ma$,

$a = \dfrac{T}{M} = \dfrac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2}$ to the right.

(b) If $v =$ const, $a = 0$, so $\boxed{T = 0}$

(This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.

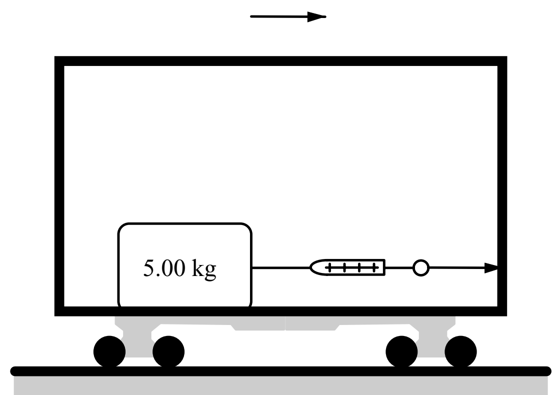

FIG. P6.19

P6.20 The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.12\ \text{m})}{7.25\ \text{s}} = 0.104\ \text{m/s}.$$

The top layer of water feels a downward force of gravity mg and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104\ \text{m/s})^2}{0.12\ \text{m}} = m9.01 \times 10^{-2}\ \text{m/s}^2$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\frac{0.090\,1\ \text{m/s}^2}{9.8\ \text{m/s}^2} = \boxed{0.527°}$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

P6.21 The only forces acting on the suspended object are the force of gravity $m\vec{g}$ and the force of tension T forward and upward at angle θ with the vertical, as shown in the free-body diagram. Applying Newton's second law in the x and y directions,

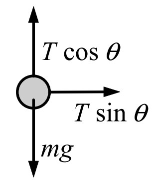

FIG. P6.21

$$\sum F_x = T\sin\theta = ma \qquad (1)$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $\quad T\cos\theta = mg \qquad (2)$

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})(3.00\ \text{m/s}^2)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}$$

P6.22 Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g+a),\ f_k = \mu_k m(g+a)$$
$$\sum F_x = ma_x: \quad -\mu_k m(g+a) = ma_x$$

The motion across the floor is described by $L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g+a)t^2$.

We solve for μ_k: $vt - L = \frac{1}{2}\mu_k(g+a)t^2$, $\boxed{\dfrac{2(vt-L)}{(g+a)t^2} = \mu_k}$.

P6.23 $F_{\max} = F_g + ma = 591\ \text{N}$

$F_{\min} = F_g - ma = 391\ \text{N}$

(a) Adding, $2F_g = 982\ \text{N}$, $F_g = \boxed{491\ \text{N}}$

(b) Since $F_g = mg$, $m = \dfrac{491\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{50.1\ \text{kg}}$

(c) Subtracting the above equations,

$2ma = 200\ \text{N} \qquad \therefore a = \boxed{2.00\ \text{m/s}^2}$

P6.24 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r}=\frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}}=13.5\ \text{m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}=\sqrt{(9.80)^2+(13.5)^2}\ \text{m/s}^2=16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\frac{16.7}{9.80}=1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F=1.71(55.0\ \text{N})=\boxed{93.8\ \text{N}}.$$

P6.25 $a_r=\left(\frac{4\pi^2R_e}{T^2}\right)\cos 35.0°=0.027\,6\ \text{m/s}^2$

We take the y axis along the local vertical.

$(a_{\text{net}})_y=9.80-(a_r)_y=9.77\ \text{m/s}^2$

$(a_{\text{net}})_x=0.015\,8\ \text{m/s}^2$

$\theta=\arctan\frac{a_x}{a_y}=\boxed{0.092\,8°}$

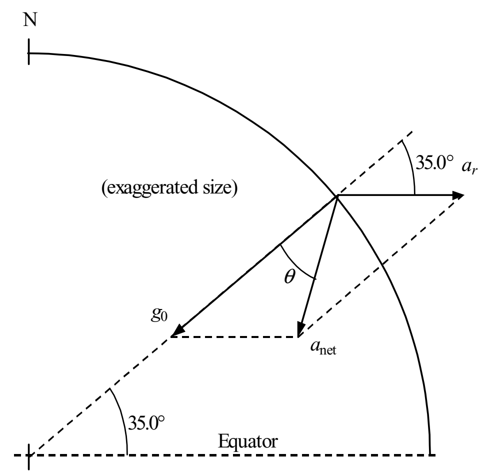

FIG. P6.25

Section 6.4 Motion in the Presence of Resistive Forces

P6.26 $m=80.0\ \text{kg},\ v_T=50.0\ \text{m/s},\ mg=\frac{D\rho A v_T^2}{2}\ \therefore\ \frac{D\rho A}{2}=\frac{mg}{v_T^2}=0.314\ \text{kg/m}$

(a) At $v=30.0\ \text{m/s}$

$$a=g-\frac{D\rho Av^2/2}{m}=9.80-\frac{(0.314)(30.0)^2}{80.0}=\boxed{6.27\ \text{m/s}^2\ \text{downward}}$$

(b) At $v=50.0\ \text{m/s}$, terminal velocity has been reached.

$$\sum F_y=0=mg-R$$

$$\Rightarrow R=mg=(80.0\ \text{kg})(9.80\ \text{m/s}^2)=\boxed{784\ \text{N directed up}}$$

(c) At $v=30.0\ \text{m/s}$

$$\frac{D\rho Av^2}{2}=(0.314)(30.0)^2=\boxed{283\ \text{N}}\ \text{upward}$$

P6.27 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$ $\quad b = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,

$$v_T = \frac{y}{t} = \frac{1.50\text{ m}}{5.00\text{ s}} = 0.300\text{ m/s}$$

Then

$$b = \frac{9.80\text{ m/s}^2}{0.300\text{ m/s}} = \boxed{32.7\text{ s}^{-1}}$$

(b) At $t = 0$, $v = 0$ and $\quad a = g = \boxed{9.80\text{ m/s}^2}$ down

(c) When $v = 0.150\text{ m/s}$, $a = g - bv = 9.80\text{ m/s}^2 - (32.7\text{ s}^{-1})(0.150\text{ m/s}) = \boxed{4.90\text{ m/s}^2}$ down

P6.28 (a) $\rho = \dfrac{m}{V}$, $A = 0.020\,1\text{ m}^2$, $R = \dfrac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830\text{ g/cm}^3\left[\frac{4}{3}\pi(8.00\text{ cm})^3\right] = 1.78\text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2(1.78\text{ kg})(9.80\text{ m/s}^2)}{0.500(1.20\text{ kg/m}^3)(0.020\,1\text{ m}^2)}} = \boxed{53.8\text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$: $h = \dfrac{v_f^2}{2g} = \dfrac{(53.8\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{148\text{ m}}$

P6.29 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi(8.92\times10^3\text{ kg/m}^3)(2.00\times10^{-2}\text{ m})^3 = 0.299\text{ kg}$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)(9.00\times10^{-2}) = \boxed{3.01\text{ N}}$$

P6.30 The resistive force is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}(0.250)(1.20\text{ kg/m}^3)(2.20\text{ m}^2)(27.8\text{ m/s})^2$$
$$R = 255\text{ N}$$
$$a = -\frac{R}{m} = -\frac{255\text{ N}}{1200\text{ kg}} = \boxed{-0.212\text{ m/s}^2}$$

P6.31 (a) At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{(3.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) In the equation describing the time variation of the velocity, we have

$v = v_T\left(1-e^{-bt/m}\right)$ $\quad v = 0.632v_T$ when $e^{-bt/m} = 0.368$

or at time $\quad t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c) At terminal velocity, $R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

***P6.32** (a) Since the window is vertical, the normal force is horizontal. $n = 4$ N
$f_k = \mu_k n = 0.9(4\ \text{N}) = 3.6\ \text{N}$ upward, to oppose downward motion

$\Sigma F_y = ma_y$: $\quad +3.6\ \text{N} - (0.16\ \text{kg})(9.8\ \text{m/s}^2) + P_y = 0 \qquad P_y = -2.03\ \text{N} = \boxed{2.03\ \text{N down}}$

(b) $\Sigma F_y = ma_y$: $\quad +3.6\ \text{N} - (0.16\ \text{kg})(9.8\ \text{m/s}^2) - 1.25(2.03\ \text{N}) = 0.16\ \text{kg}\ a_y$

$a_y = -0.508\ \text{N}/0.16\ \text{kg} = -3.18\ \text{m/s}^2 = \boxed{3.18\ \text{m/s}^2\ \text{down}}$

(c) At terminal velocity, $\Sigma F_y = ma_y$: $+(20\ \text{N}\cdot\text{s/m})v_T - (0.16\ \text{kg})(9.8\ \text{m/s}^2) - 1.25(2.03\ \text{N}) = 0$

$v_T = 4.11\ \text{N}/(20\ \text{N}\cdot\text{s/m}) = \boxed{0.205\ \text{m/s down}}$

P6.33 $v = \left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t\to\infty$ $\quad v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s $\quad 0.500v_T = v_T\left[1-\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{kg/s}$$

(a) $v_T = \frac{mg}{b} \qquad v_T = \frac{(9.00\ \text{kg})(9.80\ \text{m/s}^2)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$

(b) $0.750v_T = v_T\left[1-\exp\left(\frac{-1.13t}{9.00\ \text{s}}\right)\right] \qquad \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right) = 0.250$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\ \text{s}}$$

continued on next page

(c) $\dfrac{dx}{dt}=\left(\dfrac{mg}{b}\right)\left[1-\exp\left(-\dfrac{bt}{m}\right)\right]$; $\displaystyle\int_{x_0}^{x} dx=\int_0^t\left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]dt$

$$x-x_0=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Big|_0^t=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right)-1\right]$$

At $t=5.54$ s,

$$x=9.00\text{ kg}\left(9.80\text{ m/s}^2\right)\frac{5.54\text{ s}}{1.13\text{ kg/s}}+\left(\frac{(9.00\text{ kg})^2\left(9.80\text{ m/s}^2\right)}{(1.13\text{ kg/s})^2}\right)\left[\exp(-0.693)-1\right]$$

$$x=434\text{ m}+626\text{ m}(-0.500)=\boxed{121\text{ m}}$$

P6.34 $\sum F=ma$

$$-kmv^2=m\frac{dv}{dt}$$

$$-kdt=\frac{dv}{v^2}$$

$$-k\int_0^t dt=\int_{v_0}^{v}v^{-2}dv$$

$$-k(t-0)=\frac{v^{-1}}{-1}\Big|_{v_0}^{v}=-\frac{1}{v}+\frac{1}{v_0}$$

$$\frac{1}{v}=\frac{1}{v_0}+kt=\frac{1+v_0kt}{v_0}$$

$$v=\frac{v_0}{1+v_0kt}$$

P6.35 (a) From Problem 34,

$$v=\frac{dx}{dt}=\frac{v_0}{1+v_0kt}$$

$$\int_0^x dx=\int_0^t v_0\frac{dt}{1+v_0kt}=\frac{1}{k}\int_0^t\frac{v_0kdt}{1+v_0kt}$$

$$x\Big|_0^x=\frac{1}{k}\ln\left(1+v_0kt\right)\Big|_0^t$$

$$x-0=\frac{1}{k}\left[\ln\left(1+v_0kt\right)-\ln 1\right]$$

$$\boxed{x=\frac{1}{k}\ln\left(1+v_0kt\right)}$$

(b) We have $\ln\left(1+v_0kt\right)=kx$

$1+v_0kt=e^{kx}$ so $v=\dfrac{v_0}{1+v_0kt}=\dfrac{v_0}{e^{kx}}=\boxed{v_0e^{-kx}=v}$

P6.36 We write $-kmv^2 = -\frac{1}{2}D\rho A v^2$ so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\ \text{kg/m}^3\right)\left(4.2\times10^{-3}\ \text{m}^2\right)}{2\left(0.145\ \text{kg}\right)} = 5.3\times10^{-3}/\text{m}$$

$$v = v_0 e^{-kx} = \left(40.2\ \text{m/s}\right)e^{-\left(5.3\times10^{-3}/\text{m}\right)\left(18.3\ \text{m}\right)} = \boxed{36.5\ \text{m/s}}$$

P6.37 (a) $v(t) = v_i e^{-ct}$ $\quad v(20.0\ \text{s}) = 5.00 = v_i e^{-20.0c}$, $v_i = 10.0\ \text{m/s}$

So $5.00 = 10.0e^{-20.0c}$ and $-20.0c = \ln\left(\frac{1}{2}\right)$ $\quad c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47\times10^{-2}\ \text{s}^{-1}}$

(b) At $t = 40.0$ s $\quad v = (10.0\ \text{m/s})e^{-40.0c} = (10.0\ \text{m/s})(0.250) = \boxed{2.50\ \text{m/s}}$

(c) $v = v_i e^{-ct}$ $\quad a = \frac{dv}{dt} = -cv_i e^{-ct} = \boxed{-cv}$

P6.38 In $R = \frac{1}{2}D\rho A v^2$, we estimate that $D = 1.00$, $\rho = 1.20\ \text{kg/m}^3$,

$A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60\times10^{-2}\ \text{m}^2$ and $v = 27.0\ \text{m/s}$. The resistance force is then

$$R = \frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60\times10^{-2}\ \text{m}^2\right)\left(27.0\ \text{m/s}\right)^2 = 7.00\ \text{N}$$

or

$$R \sim \boxed{10^1\ \text{N}}$$

Additional Problems

***P6.39** Let v_0 represent the speed of the object at time 0. We have

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{b}{m}\int_0^t dt \qquad \ln v\Big|_{v_0}^{v} = -\frac{b}{m}t\Big|_0^t$$

$$\ln v - \ln v_0 = -\frac{b}{m}(t-0) \qquad \ln\left(v/v_0\right) = -\frac{bt}{m}$$

$$v/v_0 = e^{-bt/m} \qquad \boxed{v = v_0 e^{-bt/m}}$$

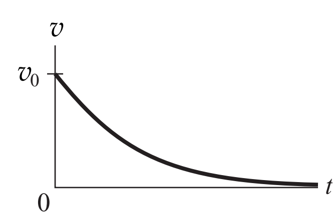

FIG. P6.39

From its original value, the speed decreases rapidly at first and then more and more slowly, asymptotically approaching zero.

$\boxed{\text{In this model the object keeps losing speed forever. It travels a finite distance in stopping.}}$

The distance it travels is given by

$$\int_0^r dr = v_0\int_0^t e^{-bt/m}dt$$

$$r = -\frac{m}{b}v_0\int_0^t e^{-bt/m}\left(-\frac{b}{m}dt\right) = -\frac{m}{b}v_0\, e^{-bt/m}\Big|_0^t = -\frac{m}{b}v_0\left(e^{-bt/m}-1\right) = \frac{mv_0}{b}\left(1-e^{-bt/m}\right)$$

As t goes to infinity, the distance approaches $\frac{mv_0}{b}(1-0) = mv_0/b$

P6.40 At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\dfrac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88\text{ N}}$

P6.41 (a) The speed of the bag is $\dfrac{2\pi(7.46\text{ m})}{38\text{ s}} = 1.23\text{ m/s}$.

The total force on it must add to

$$ma_c = \frac{(30\text{ kg})(1.23\text{ m/s})^2}{7.46\text{ m}} = 6.12\text{ N}$$

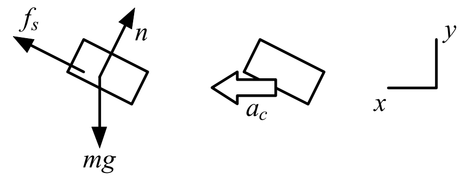

FIG. P6.41

$$\sum F_x = ma_x: \quad f_s \cos 20 - n \sin 20 = 6.12\text{ N}$$

$$\sum F_y = ma_y: \quad f_s \sin 20 + n \cos 20 - (30\text{ kg})(9.8\text{ m/s}^2) = 0$$

$$n = \frac{f_s \cos 20 - 6.12\text{ N}}{\sin 20}$$

Substitute:

$$f_s \sin 20 + f_s \frac{\cos^2 20}{\sin 20} - (6.12\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$

$$f_s(2.92) = 294\text{ N} + 16.8\text{ N}$$

$$f_s = \boxed{106\text{ N}}$$

(b) $v = \dfrac{2\pi(7.94\text{ m})}{34\text{ s}} = 1.47\text{ m/s}$

$$ma_c = \frac{(30\text{ kg})(1.47\text{ m/s})^2}{7.94\text{ m}} = 8.13\text{ N}$$

$$f_s \cos 20 - n \sin 20 = 8.13\text{ N}$$

$$f_s \sin 20 + n \cos 20 = 294\text{ N}$$

$$n = \frac{f_s \cos 20 - 8.13\text{ N}}{\sin 20}$$

$$f_s \sin 20 + f_s \frac{\cos^2 20}{\sin 20} - (8.13\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$

$$f_s(2.92) = 294\text{ N} + 22.4\text{ N}$$

$$f_s = 108\text{ N}$$

$$n = \frac{(108\text{ N})\cos 20 - 8.13\text{ N}}{\sin 20} = 273\text{ N}$$

$$\mu_s = \frac{f_s}{n} = \frac{108\text{ N}}{273\text{ N}} = \boxed{0.396}$$

P6.42 When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}$$

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$.

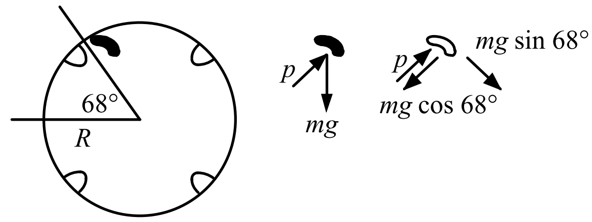

FIG. P6.42

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\ \text{m})(9.8\ \text{m/s}^2)\sin 68°} = 1.73\ \text{m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right) = \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}$$

P6.43 (a) $v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right) = 8.33\ \text{m/s}$

$$\sum F_y = ma_y:\ +n - mg = -\frac{mv^2}{r}$$

$$n = m\left(g - \frac{v^2}{r}\right) = 1\,800\ \text{kg}\left[9.8\ \text{m/s}^2 - \frac{(8.33\ \text{m/s})^2}{20.4\ \text{m}}\right]$$

$$= \boxed{1.15\times 10^4\ \text{N up}}$$

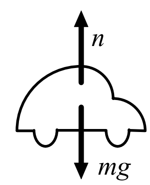

FIG. P6.43

(b) Take $n = 0$. Then $mg = \frac{mv^2}{r}$.

$$v = \sqrt{gr} = \sqrt{(9.8\ \text{m/s}^2)(20.4\ \text{m})} = \boxed{14.1\ \text{m/s}} = 50.9\ \text{km/h}$$

P6.44 (a) $\sum F_y = ma_y = \frac{mv^2}{R}$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$ $\qquad mg = \frac{mv^2}{R}$

Then,

$$v = \boxed{\sqrt{gR}}$$

A more gently curved bump, with larger radius, allows the car to have a higher speed without leaving the road. This speed is proportional to the square root of the radius.

P6.45 (a) $\text{slope} = \frac{0.160\ \text{N} - 0}{9.9\ \text{m}^2/\text{s}^2} = \boxed{0.016\,2\ \text{kg/m}}$

(b) $\text{slope} = \frac{R}{v^2} = \frac{\frac{1}{2}D\rho A v^2}{v^2} = \boxed{\frac{1}{2}D\rho A}$

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(c) $\frac{1}{2}D\rho A = 0.016\,2\ \text{kg/m}$

$$D = \frac{2(0.016\,2\ \text{kg/m})}{(1.20\ \text{kg/m}^3)\pi(0.105\ \text{m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force $mg = 8(1.64\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2) = 0.129\ \text{N}$ and horizontal coordinate $(2.80\ \text{m/s})^2$. The vertical coordinate of the line is here $(0.016\,2\ \text{kg/m})(2.8\ \text{m/s})^2 = 0.127\ \text{N}$. The scatter percentage is $\frac{0.129\ \text{N} - 0.127\ \text{N}}{0.127\ \text{N}} = \boxed{1.5\%}$.

(e) The interpretation of the graph can be stated thus:

For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2}D\rho Av^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

***P6.46** (a) The forces acting on the ice cube are the Earth's gravitational force, straight down, and the basin's normal force, upward and inward at 35° with the vertical. We choose the x and y axes to be horizontal and vertical, so that the acceleration is purely in the x direction. Then

$\sum F_x = ma_x$: $\quad n\sin 35° = mv^2/R$

$\sum F_y = ma_y$: $\quad n\cos 35° - mg = 0$

Dividing eliminates the normal force: $n\sin 35°/\,n\cos 35° = mv^2/Rmg$

$\tan 35° = v^2/Rg \qquad \boxed{v = \sqrt{Rg\tan 35.0°} = \sqrt{(6.86\ \text{m/s}^2)R}}$

(b) $\boxed{\text{The mass is unnecessary.}}$

(c) The answer to (a) indicates that the speed is proportional to the square root of the radius, so doubling the radius will make the required speed $\boxed{\text{increase by } \sqrt{2} \text{ times}}$.

(d) The period of revolution is given by $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg\tan 35.0°}} = (2.40\ \text{s}/\sqrt{\text{m}})\sqrt{R}$

When the radius doubles, the period $\boxed{\text{increases by } \sqrt{2} \text{ times}}$.

(e) On the larger circle the ice cube moves $\sqrt{2}$ times faster but also takes longer to get around, because the distance it must travel is 2 times larger. Its period is also proportional to the square root of the radius.

P6.47 Take x-axis up the hill

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$

$$a = \frac{T}{m}\sin\theta - g\sin\phi$$

$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$

$$T = \frac{mg\cos\phi}{\cos\theta}$$

$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

P6.48 (a) $v = (300\ \text{mi/h})\left(\frac{88.0\ \text{ft/s}}{60.0\ \text{mi/h}}\right) = 440\ \text{ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1\,200} = \boxed{967\ \text{lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = \boxed{-647\ \text{lb}}$$

Since the plane is upside down, the seat exerts this downward force as a normal force.

(c) When $F_g' = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that this equation is satisfied, then the pilot feels weightless.

P6.49 (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator. Therefore,

$$F_g' = F_g - \frac{mv^2}{r} \text{ or } \boxed{F_g > F_g'}$$

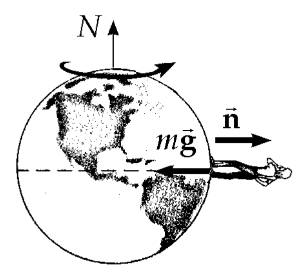


FIG. P6.49

(b) At the poles $v = 0$ and $F_g' = F_g = mg = 75.0(9.80) = \boxed{735\ \text{N}}$ down.

At the equator, $F_g' = F_g - ma_c = 735\ \text{N} - 75.0(0.033\,7)\ \text{N} = \boxed{732\ \text{N}}$ down.

***P6.50** (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or

$$T = \boxed{m_2 g}$$

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus,

$$F_c = T = \boxed{m_2 g}$$

(c) From

$$F_c = \frac{m_1 v^2}{R}$$

we have

$$v = \sqrt{\frac{RF_c}{m_1}} = \boxed{\sqrt{\left(\frac{m_2}{m_1}\right)gR}}$$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension produces forward tangential acceleration as well as inward radial acceleration of the puck, pulling at an angle of less than 90° to the direction of the inward-spiraling velocity.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

***P6.51**

(a) The only horizontal force on the car is the force of friction, with a maximum value determined by the surface roughness (described by the coefficient of static friction) and the normal force (here equal to the gravitational force on the car).

(b) $\Sigma F_x = ma_x \qquad -f = ma \qquad a = -f/m = (v^2 - v_0^2)/2(x - x_0)$

$x - x_0 = (v^2 - v_0^2)m/2f = (0^2 - [20\text{ m/s}]^2)1200\text{ kg}/2(-7000\text{ N}) = \boxed{34.3\text{ m}}$

(c) Now $f = mv^2/r \qquad r = mv^2/f = 1200\text{ kg }[20\text{ m/s}]^2/7000\text{ N} = \boxed{68.6\text{ m}}$

A top view shows that you can avoid running into the wall by turning through a quarter-circle, if you start at least this far away from the wall.

(d) Braking is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car, and the stopping distance would be longer.

(e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

P6.52 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00\text{ m})}{(15.0\text{ s})} = 3.77\text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58\text{ m/s}^2}$

(b) $F_{\text{low}} = m(g + a_r) = \boxed{455\text{ N}}$

(c) $F_{\text{high}} = m(g - a_r) = \boxed{328\text{ N}}$

(d) $F_{\text{mid}} = m\sqrt{g^2 + a_r^2} = \boxed{397\text{ N upward and}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \tan^{-1}\dfrac{1.58}{9.8} = \boxed{9.15°\text{ inward}}$.

P6.53 (a) The mass at the end of the chain is in vertical equilibrium. Thus $T\cos\theta = mg$.

Horizontally $T\sin\theta = ma_r = \dfrac{mv^2}{r}$

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m} = 5.17\text{ m}$$

Then $a_r = \dfrac{v^2}{5.17\text{ m}}$.

By division $\tan\theta = \dfrac{a_r}{g} = \dfrac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°)\text{ m}^2/\text{s}^2$$
$$v = \boxed{5.19\text{ m/s}}$$

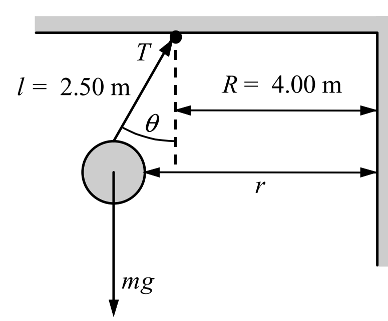


FIG. P6.53

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{555\text{ N}}$$

P6.54 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \frac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \frac{2\pi R}{t}$, so $t = \frac{2v}{g} = \frac{2\pi R}{v}$.

Thus, $v^2 = \pi R g$ and $\boxed{v = \sqrt{\pi R g}}$.

(b) The putty is dislodged when F, the force holding it to the wheel, is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

***P6.55** (a) $n = \frac{mv^2}{R}$ $\quad f - mg = 0$

$f = \mu_s n \quad v = \frac{2\pi R}{T}$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $T = \boxed{2.54\text{ s}}$

$$\#\ \frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

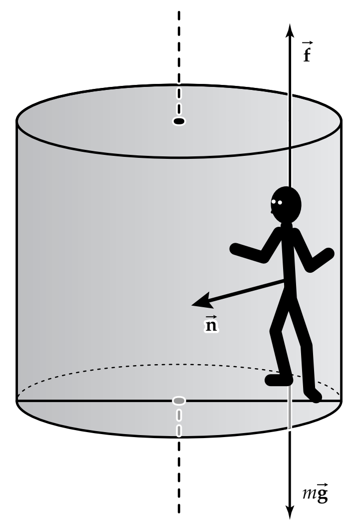


FIG. P6.55

(c) The gravitational and frictional forces remain constant. The normal force increases. The person remains in motion with the wall.

(d) The gravitational force remains constant. The normal and frictional forces decrease. The person slides relative to the wall and downward into the pit.

P6.56 Let the x-axis point eastward, the y-axis upward, and the z-axis point southward.

(a) The range is $Z = \frac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0\text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = (53.0\text{ m/s})(\sin 48.0°)t - (4.90\text{ m/s}^2)t^2$$

giving $t = \boxed{8.04\text{ s}}$.

continued on next page

(b) $v_{ix} = \dfrac{2\pi R_e \cos\phi_i}{86\,400\text{ s}} = \dfrac{2\pi\left(6.37\times10^6\text{ m}\right)\cos 35.0°}{86\,400\text{ s}} = \boxed{379\text{ m/s}}$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285\text{ m}}{2\pi\left(6.37\times10^6\text{ m}\right)}\right)(360°) = 2.56\times10^{-3}\text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$.

The cup is moving eastward at a speed $v_{fx} = \dfrac{2\pi R_e \cos\phi_f}{86\,400\text{ s}}$, which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{fx} - v_{fi} = \frac{2\pi R_e}{86\,400\text{ s}}\left[\cos\phi_f - \cos\phi_i\right] = \frac{2\pi R_e}{86\,400\text{ s}}\left[\cos\left(\phi_i - \Delta\phi\right) - \cos\phi_i\right]$$

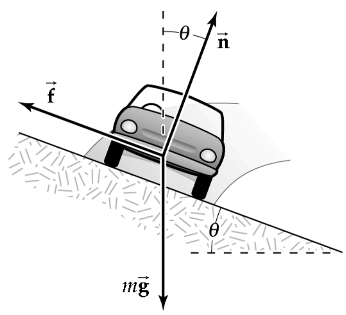
$$= \frac{2\pi R_e}{86\,400\text{ s}}\left[\cos\phi_i\cos\Delta\phi + \sin\phi_i\sin\Delta\phi - \cos\phi_i\right]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \dfrac{2\pi R_e}{86\,400\text{ s}}\sin\phi_i\sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi\left(6.37\times10^6\text{ m}\right)}{86\,400\text{ s}}\sin 35.0°\sin 0.002\,56° = \boxed{1.19\times10^{-2}\text{ m/s}}$$

(d) $\Delta x = \left(\Delta v_x\right)t = \left(1.19\times10^{-2}\text{ m/s}\right)(8.04\text{ s}) = 0.095\,5\text{ m} = \boxed{9.55\text{ cm}}$

P6.57 (a) If the car is about to slip *down* the incline, f is directed up the incline.

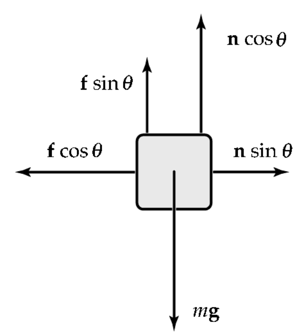


$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta\left(1+\mu_s\tan\theta\right)} \text{ and } f = \frac{\mu_s mg}{\cos\theta\left(1+\mu_s\tan\theta\right)}$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$ yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg\left(\tan\theta - \mu_s\right)}{1+\mu_s\tan\theta}}}$$

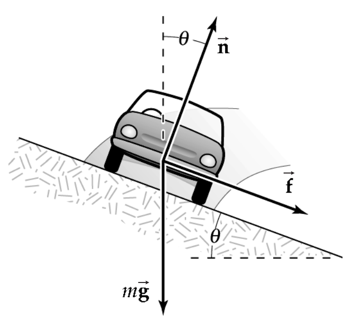


When the car is about to slip *up* the incline, f is directed down the incline. Then, $\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta\left(1-\mu_s\tan\theta\right)} \text{ and } f = \frac{\mu_s mg}{\cos\theta\left(1-\mu_s\tan\theta\right)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg\left(\tan\theta + \mu_s\right)}{1-\mu_s\tan\theta}}}$$



(b) If $v_{\min} = \sqrt{\dfrac{Rg\left(\tan\theta - \mu_s\right)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

continued on next page

(c) $$v_{\min} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° - 0.100)}{1 + (0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\max} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° + 0.100)}{1 - (0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$

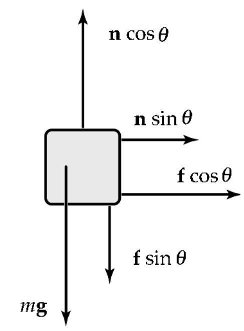


FIG. P6.57

***P6.58** (a) We let R represent the radius of the hoop and T represent the period of its rotation. The bead moves in a circle with radius $v = R\sin\theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

The normal force has
an inward radial component of $n\sin\theta$
and an upward component of $n\cos\theta$

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$

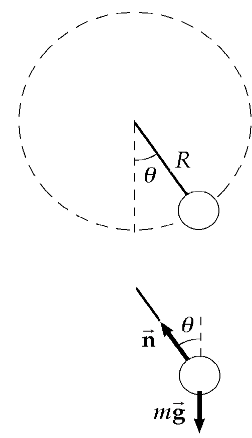


FIG. P6.58(a)

Then $\sum F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes $\quad \left(\dfrac{mg}{\cos\theta}\right)\sin\theta = \dfrac{m}{R\sin\theta}\left(\dfrac{2\pi R\sin\theta}{T}\right)^2$

which reduces to $\quad \dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\quad \sin\theta = 0 \quad \Rightarrow \quad \theta = 0° \qquad (1)$

and $\quad \cos\theta = \dfrac{gT^2}{4\pi^2 R} \qquad (2)$

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80\text{ m/s}^2)(0.450\text{ s})^2}{4\pi^2(0.150\text{ m})} = 0.335 \text{ and } \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes

$$\cos\theta = \frac{(9.80\text{ m/s}^2)(0.850\text{ s})^2}{4\pi^2(0.150\text{ m})} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$.

(c) The equation that the angle must satisfy has two solutions whenever $4\pi^2 R > gT^2$ but only the solution 0° otherwise. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position. Zero is always a solution for the angle. There are never more than two solutions.

P6.59 At terminal velocity, the accelerating force of gravity is balanced by frictional drag: $mg = arv + br^2v^2$

(a) $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\,2\ \text{m/s}}$.

(b) $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

(c) $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

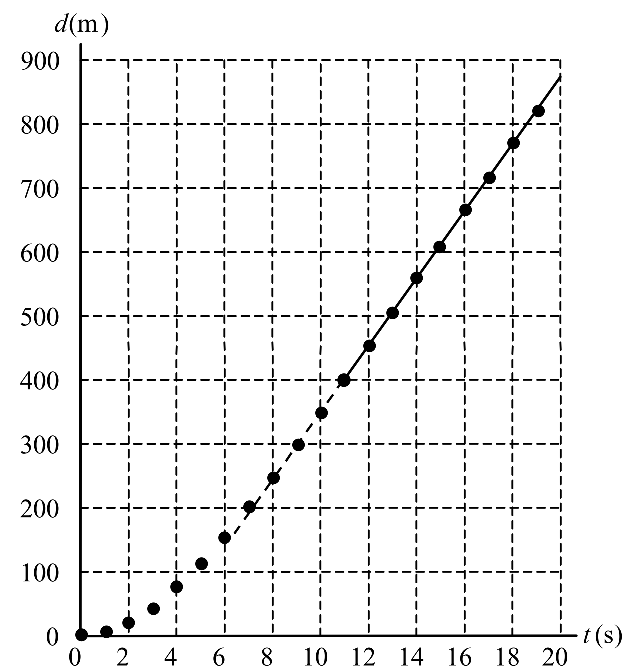
Assuming $v > 1$ m/s, and ignoring the first term:

$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2 \qquad v = \boxed{6.87\ \text{m/s}}$

P6.60 (a)

$t(\text{s})$	$d(\text{m})$
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876

(b)



(c) A straight line fits the points from $t = 11.0$ s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\ \text{m} - 399\ \text{m}}{20.0\ \text{s} - 11.0\ \text{s}} = \boxed{53.0\ \text{m/s}}$$

P6.61 $\sum F_y = L_y - T_y - mg = L\cos 20.0° - T\sin 20.0° - 7.35\text{ N} = ma_y = 0$

$\sum F_x = L_x + T_x = L\sin 20.0° + T\cos 20.0° = m\dfrac{v^2}{r}$

$m\dfrac{v^2}{r} = 0.750\text{ kg}\dfrac{(35.0\text{ m/s})^2}{(60.0\text{ m})\cos 20.0°} = 16.3\text{ N}$

We have the simultaneous equations

$L\sin 20.0° + T\cos 20.0° = 16.3\text{ N}$

$L\cos 20.0° - T\sin 20.0° = 7.35\text{ N}$

$L + T\dfrac{\cos 20.0°}{\sin 20.0°} = \dfrac{16.3\text{ N}}{\sin 20.0°}$

$L - T\dfrac{\sin 20.0°}{\cos 20.0°} = \dfrac{7.35\text{ N}}{\cos 20.0°}$

$T(\cot 20.0° + \tan 20.0°) = \dfrac{16.3\text{ N}}{\sin 20.0°} - \dfrac{7.35\text{ N}}{\cos 20.0°}$

$T(3.11) = 39.8\text{ N}$

$T = \boxed{12.8\text{ N}}$

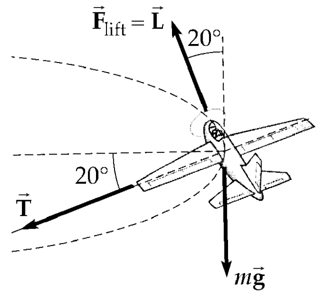


FIG. P6.61

***P6.62** (a) $v = v_i + kx$ implies the acceleration is $a = \dfrac{dv}{dt} = 0 + k\dfrac{dx}{dt} = +kv$

Then the total force is $\sum F = ma = m(+kv)$

As a vector, the force is parallel or antiparallel to the velocity: $\boxed{\sum \vec{F} = km\vec{v}}$.

(b) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially. Riding on such an object would be more scary than riding on a skyrocket. It would be a good opportunity for learning about exponential growth in population or in energy use.

(c) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

ANSWERS TO EVEN PROBLEMS

P6.2 215 N horizontally inward

P6.4 (a) 1.65 km/s (b) 6.84×10^3 s

P6.6 6.22×10^{-12} N

P6.8 (a) 68.6 N toward the center of the circle and 784 N up (b) 0.857 m/s^2

P6.10 (a) $(-0.233\,\hat{\mathbf{i}} + 0.163\,\hat{\mathbf{j}})\text{ m/s}^2$ (b) 6.53 m/s (c) $(-0.181\,\hat{\mathbf{i}} + 0.181\,\hat{\mathbf{j}})\text{ m/s}^2$

P6.12 (a) 1.33 m/s^2 (b) 1.79 m/s^2 forward and 48.0° inward

P6.14 (a) 2.49×10^4 N up (b) 12.1 m/s

P6.16 (a) 20.6 N (b) 3.35 m/s^2 downward tangent to the circle; 32.0 m/s^2 radially inward (c) 32.2 m/s^2 at 5.98° to the cord, pointing toward a location below the center of the circle. (d) No change. If the object is swinging down it is gaining speed. If it is swinging up it is losing speed but its acceleration is the same size and its direction can be described in the same terms.

P6.18 (a) 8.62 m (b) Mg downward (c) $8.45\ \text{m/s}^2$ Unless they are belted in, the riders will fall from the cars.

P6.20 0.527°

P6.22 $\mu_k = \dfrac{2(vt - L)}{(g+a)t^2}$

P6.24 93.8 N

P6.26 (a) $6.27\ \text{m/s}^2$ downward (b) 784 N up (c) 283 N up

P6.28 (a) $53.8\ \text{m/s}$ (b) 148 m

P6.30 $-0.212\ \text{m/s}^2$

P6.32 (a) 2.03 N down (b) 3.18 m/s^2 down (c) 0.205 m/s down

P6.34 see the solution

P6.36 $36.5\ \text{m/s}$

P6.38 $\sim 10^1$ N

P6.40 8.88 N

P6.42 $0.835\ \text{rev/s}$

P6.44 (a) $mg - \dfrac{mv^2}{R}$ upward (b) $v = \sqrt{gR}$

P6.46 (a) $v = \sqrt{Rg\tan 35.0°} = \sqrt{(6.86\ \text{m/s}^2)R}$ (b) The mass is unnecessary. (c) Increase by $\sqrt{2}$ times (d) Increase by $\sqrt{2}$ times (e) On the larger circle the ice cube moves $\sqrt{2}$ times faster but also takes longer to get around, because the distance it must travel is 2 times larger. Its period is described by $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{\sqrt{Rg\tan 35.0°}} = \left(2.40\ \text{s}/\sqrt{\text{m}}\right)\sqrt{R}$.

P6.48 (a) The seat exerts 967 lb up on the pilot. (b) The seat exerts 647 lb down on the pilot. (c) If the plane goes over the top of a section of a circle with $v^2 = Rg$, the pilot will feel weightless.

P6.50 (a) $m_2 g$ (b) $m_2 g$ (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$ (d) The puck will move inward along a spiral, gaining speed as it does so. (e) The puck will move outward along a spiral as it slows down.

P6.52 (a) 1.58 m/s^2 (b) 455 N (c) 329 N (d) 397 N upward and 9.15° inward

P6.54 (a) $v = \sqrt{\pi R g}$ (b) $m\pi g$

P6.56 (a) 8.04 s (b) $379\ \text{m/s}$ (c) $1.19\ \text{cm/s}$ (d) 9.55 cm

P6.58 (a) either 70.4° or 0° (b) 0° (c) The equation that the angle must satisfy has two solutions whenever $4\pi^2 R > gT^2$ but only the solution 0° otherwise. (Here R and T are the radius and period of the hoop.) Zero is always a solution for the angle. There are never more than two solutions.

P6.60 (a) and (b) see the solution (c) 53.0 m/s

P6.62 (a) $\Sigma\vec{\mathbf{F}} = mk\vec{\mathbf{v}}$ (b) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially. Riding on such an object would be more scary than riding on a skyrocket. It would be a good opportunity for learning about exponential growth in population or in energy use. (c) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.