8

Conservation of Energy

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q8.1** Not everything has energy. A rock stationary on the floor, chosen as the $y = 0$ reference level, has no mechanical energy. In cosmic terms, think of the burnt-out core of a star far in the future after it has cooled nearly to absolute zero.

***Q8.2** answer (c). Gravitational energy is proportional to the mass of the object in the Earth's field.

***Q8.3** (i) answer b. Kinetic energy is proportional to mass.

(ii) answer c. The slide is frictionless, so $v = (2gh)^{1/2}$ in both cases.

(iii) answer a. g for the smaller child and $g \sin\theta$ for the larger.

***Q8.4** (a) yes: a block slides on the floor where we choose $y = 0$.

(b) yes: a picture on the classroom wall high above the floor.

(c) yes: an eraser hurtling across the room.

(d) yes: the block stationary on the floor.

***Q8.5** answer (d). The energy is internal energy. Energy is never "used up." The ball finally has no elevation and no compression, so it has no potential energy. There is no stove, so no heat is put in. The amount of sound energy is minuscule.

***Q8.6** answer (a). We assume the light band of the slingshot puts equal amounts of kinetic energy into the missiles. With three times more speed, the bean has nine times more squared speed, so it must have one-ninth the mass.

Q8.7 They will not agree on the original gravitational energy if they make different $y = 0$ choices. They see the same change in elevation, so they do agree on the change in gravitational energy and on the kinetic energy.

Q8.8 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q8.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q8.10 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q8.11 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (K) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

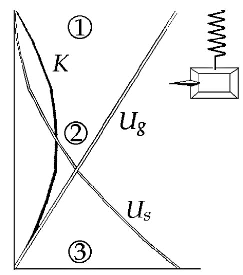

FIG. Q8.11

***Q8.12** We have $(1/2)mv^2 = \mu_k mgd$ so $d = v^2/2\mu_k g$. The quantity v^2/μ_k controls the skidding distance. In the cases quoted respectively, this quantity has the numerical value (a) 5 (b) 2.5 (c) 1.25 (d) 20 (e) 10 (f) 5. In order the ranking is then d > e > f = a > b > c.

***Q8.13** Yes, if it is exerted by an object that is moving in our frame of reference. The flat bed of a truck exerts a static friction force to start a pumpkin moving forward as it slowly starts up.

***Q8.14** (a) A campfire converts chemical energy into internal energy, within the system wood-plus-oxygen, and before energy is transferred by heat and electromagnetic radiation into the surroundings. If all the fuel burns, the process can be 100% efficient. Chemical-energy-into-internal-energy is also the conversion as iron rusts, and it is the main conversion in mammalian metabolism.

(b) An escalator motor converts electrically transmitted energy into gravitational energy. As the system we may choose motor-plus-escalator-and-riders. The efficiency could be say 90%, but in many escalators plenty of internal energy is another output. A natural process, such as atmospheric electric current in the Earth's aurora borealis raising the temperature of a particular parcel of air so that the surrounding air buoys it up, could produce the same electrically-transmitted-to-gravitational energy conversion with low efficiency.

(c) A diver jumps up from a diving board, setting it vibrating temporarily. The material in the board rises in temperature slightly as the visible vibration dies down, and then the board cools off to the constant temperature of the environment. This process for the board-plus-air system can have 100% efficiency in converting the energy of vibration into energy transferred by heat. The energy of vibration is all elastic energy at instants when the board is momentarily at rest at turning points in its motion. For a natural process, you could think of the branch of a palm tree vibrating for a while after a coconut falls from it.

(d) Some of the sound energy in a shout becomes a tiny bit of work done on a listener's ear; most of the mechanical-wave energy becomes internal energy as the sound is absorbed by all the surfaces it falls upon. We would also assign low efficiency to a train of water waves doing work to make a linear pile of shells at the high-water mark on a beach.

(e) A demonstration solar car takes in electromagnetic-wave energy in sunlight and turns some fraction of it temporarily into the car's kinetic energy. A much larger fraction becomes internal energy in the solar cells, battery, motor, and air pushed aside. Perhaps with somewhat higher net efficiency, the pressure of light from a newborn star pushes away gas and dust in the nebula surrounding it.

***Q8.15** (a) original elastic potential energy into final kinetic energy

(b) original chemical energy into final internal energy

(c) original internal energy in the batteries into final internal energy, plus a tiny bit of outgoing energy transmitted by mechanical waves

(d) original kinetic energy into final internal energy in the brakes

(e) heat input from the lower layers of the Sun, into energy transmitted by electromagnetic radiation

(f) original chemical energy into final gravitational energy

***Q8.16** Answer (k). The static friction force that each glider exerts on the other acts over no distance. The air track isolates the gliders from outside forces doing work. The gliders-Earth system keeps constant mechanical energy.

Q8.17 The larger engine is unnecessary. Consider a 30-minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

SOLUTIONS TO PROBLEMS

Section 8.1 The Nonisolated System—Conservation of Energy

***P8.1** (a) The toaster coils take in energy by electrical transmission. They increase in internal energy and put out energy by heat into the air and energy by electromagnetic radiation as they start to glow. $\boxed{\Delta E_{\text{int}} = Q + T_{\text{ET}} + T_{\text{ER}}}$

(b) The car takes in energy by mass transfer. Its fund of chemical potential energy increases. As it moves, its kinetic energy increases and it puts out work on the air, energy by heat in the exhaust, and a tiny bit of energy by mechanical waves in sound.
$\boxed{\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{\text{MW}} + T_{\text{MT}}}$

(c) You take in energy by mass transfer. Your fund of chemical potential energy increases. You are always putting out energy by heat into the surrounding air. $\boxed{\Delta U = Q + T_{\text{MT}}}$

(d) Your house is in steady state, keeping constant energy as it takes in energy by electrical transmission to run the clocks and, we assume, an air conditioner. It absorbs sunlight, taking in energy by electromagnetic radiation. The exterior plenum of the air conditioner takes in cooler air and puts it out as warmer air, transferring out energy by mass transfer.
$\boxed{0 = Q + T_{\text{MT}} + T_{\text{ET}} + T_{\text{ER}}}$

Section 8.2 The Isolated System

P8.2 (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump:

$$mgy = 36\ \text{kg}\left(9.81\ \text{m/s}^2\right)(0.25\ \text{m}) = 88.3\ \text{J}$$

For all of the jumps of the children the energy is $12\left(1.05\times10^6\right)88.3\ \text{J} = \boxed{1.11\times10^9\ \text{J}}$.

(b) The seismic energy is modeled as $E = \dfrac{0.01}{100}1.11\times10^9\ \text{J} = 1.11\times10^5\ \text{J}$, making the Richter magnitude $\dfrac{\log E - 4.8}{1.5} = \dfrac{\log 1.11\times10^5 - 4.8}{1.5} = \dfrac{5.05-4.8}{1.5} = \boxed{0.2}$.

P8.3 $U_i + K_i = U_f + K_f$: $\quad mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\dfrac{v^2}{R}$: $\quad n + mg = m\dfrac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)$$

$$= \boxed{0.098\,0\ \text{N downward}}$$

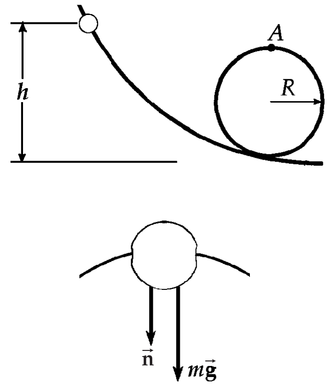

FIG. P8.3

P8.4 (a) $(\Delta K)_{A\to B} = \sum W = W_g = mg\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m(9.80)(1.80)$$

$$v_B = \boxed{5.94\ \text{m/s}}$$

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\ \text{m/s}}$

(b) $W_g\big|_{A\to C} = mg(3.00\ \text{m}) = \boxed{147\ \text{J}}$

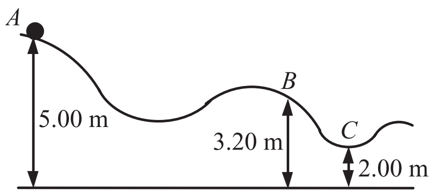

FIG. P8.4

P8.5 From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si}$$

or

$$(0.250\ \text{kg})\left(9.80\ \text{m/s}^2\right)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$

This gives a maximum height $h = \boxed{10.2\ \text{m}}$.

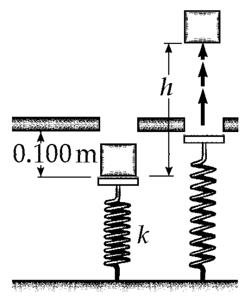

FIG. P8.5

P8.6 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or

$$F = mg\cos\theta + m\frac{v^2}{\ell}$$

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\dfrac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$.

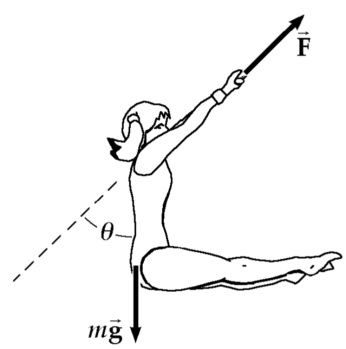

FIG. P8.6

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(b) At the bottom of the swing, $\theta = 0°$

so

$$F = mg\left(3 - 2\cos\theta_i\right)$$
$$F = 2mg = mg\left(3 - 2\cos\theta_i\right)$$

which gives

$$\theta_i = \boxed{60.0°}$$

P8.7 Using conservation of energy for the system of the Earth and the two objects

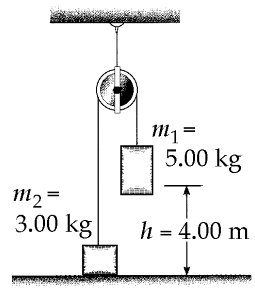

FIG. P8.7

(a) $(5.00 \text{ kg})g(4.00 \text{ m}) = (3.00 \text{ kg})g(4.00 \text{ m}) + \frac{1}{2}(5.00 + 3.00)v^2$

$v = \sqrt{19.6} = \boxed{4.43 \text{ m/s}}$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$
$$\Delta y = 1.00 \text{ m}$$
$$y_{\max} = 4.00 \text{ m} + \Delta y = \boxed{5.00 \text{ m}}$$

P8.8 We assume $m_1 > m_2$

(a) $m_1gh = \frac{1}{2}\left(m_1 + m_2\right)v^2 + m_2gh$

$$v = \boxed{\sqrt{\frac{2\left(m_1 - m_2\right)gh}{\left(m_1 + m_2\right)}}}$$

(b) Since m_2 has kinetic energy $\frac{1}{2}m_2v^2$, it will rise an additional height Δh determined from

$$m_2g\,\Delta h = \frac{1}{2}m_2v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{\left(m_1 - m_2\right)h}{\left(m_1 + m_2\right)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\dfrac{2m_1h}{m_1 + m_2}}$.

P8.9 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

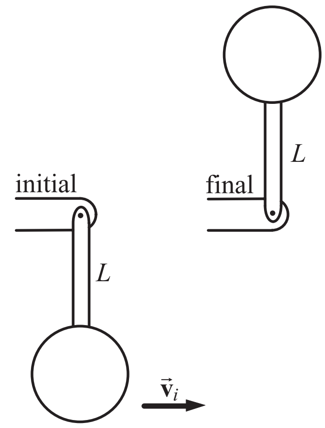

FIG. P8.9

$$K_i + U_{gi} = K_f + U_{gf}: \qquad \frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$

$$v_i = \boxed{5.49\ \text{m/s}}$$

P8.10 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

Note that we have used the Pythagorean theorem to express the original kinetic energy in terms of the velocity components. Kinetic energy itself does not have components. Now $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\ \text{kg})(1\,000\ \text{m/s})^2 = \boxed{1.00\times10^7\ \text{J}}$$

P8.11 (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\ \text{m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

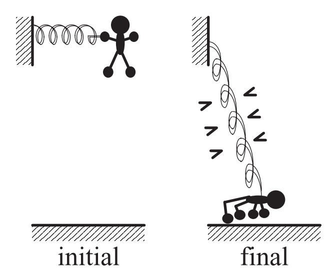

FIG. P8.11(a)

$$k = \frac{5\ \text{m}}{L}\frac{mg}{1.5\ \text{m}} = 3.33\,mg/L$$

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg\left(y_i - y_f\right) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\ \text{m} = L + x_f$

$$55.0\ \text{m}L = \frac{1}{2}3.33(55.0\ \text{m} - L)^2$$

$$55.0\ \text{m}L = 5.04\times10^3\ \text{m}^2 - 183\ \text{m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)\left(5.04\times10^3\right)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\ \text{m}}$$

only the value of L less than 55 m is physical.

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(b) $k = 3.33\dfrac{mg}{25.8\text{ m}}$ $\qquad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\qquad +kx_{\max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

P8.12 When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 Situations Involving Kinetic Friction

P8.13 $\sum F_y = ma_y$: $\qquad n - 392\text{ N} = 0$

$$n = 392\text{ N}$$

$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$

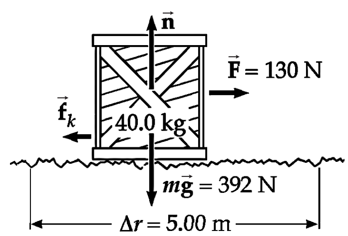

FIG. P8.13

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{\text{int}} = f_k\Delta x = (118)(5.00) = \boxed{588\text{ J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

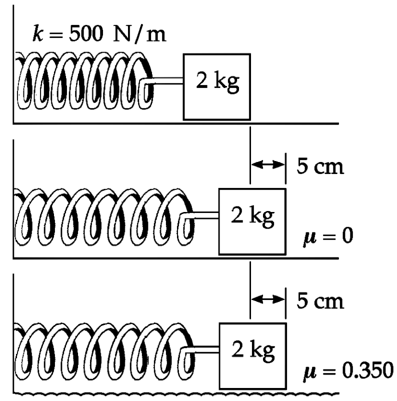

FIG. P8.14

P8.14 (a)
$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$
$$= \frac{1}{2}(500)\left(5.00\times10^{-2}\right)^2 - 0 = 0.625 \text{ J}$$
$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$$
so
$$v_f = \sqrt{\frac{2\left(\sum W\right)}{m}}$$
$$= \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$$

(b)
$$\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$$
$$0 - (0.350)(2.00)(9.80)(0.050\,0) \text{ J} + 0.625 \text{ J} = \frac{1}{2}mv_f^2$$
$$0.282 \text{ J} = \frac{1}{2}(2.00 \text{ kg})v_f^2$$
$$v_f = \sqrt{\frac{2(0.282)}{2.00}} \text{ m/s} = \boxed{0.531 \text{ m/s}}$$

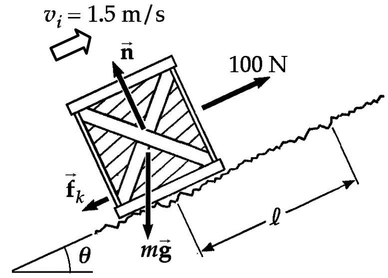

FIG. P8.15

P8.15 (a)
$$W_g = mg\ell\cos(90.0° + \theta)$$
$$W_g = (10.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(5.00 \text{ m})\cos110°$$
$$= \boxed{-168 \text{ J}}$$

(b)
$$f_k = \mu_k n = \mu_k mg\cos\theta$$
$$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$$
$$\Delta E_{\text{int}} = (5.00 \text{ m})(0.400)(10.0)(9.80)\cos20.0°$$
$$= \boxed{184 \text{ J}}$$

(c)
$$W_F = F\ell = (100)(5.00) = \boxed{500 \text{ J}}$$

(d)
$$\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148 \text{ J}}$$

(e)
$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65 \text{ m/s}}$$

***P8.16** (i) In (a), $(kd^2)^{1/2}$ and $(mgd)^{1/2}$ both have the wrong units for speed. In (b) $(\mu_k g)^{1/2}$ has the wrong units. In (c), $(kd/m)^{1/2}$ has the wrong units. In (f) both terms have the wrong units. The answer list is $\boxed{\text{a, b, c, f}}$.

(ii) As k increases, friction becomes unimportant, so we should have $(1/2)kd^2 = (1/2)mv^2$ and $v = (kd^2/m)^{1/2}$. Possibilities $\boxed{\text{g, i, and j}}$ do not have this limit.

(iii) As μ_k goes to zero, as in (ii), we should have $v = (kd^2/m)^{1/2}$. Answer $\boxed{\text{d}}$ does not have this limit.

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(iv) (e) cannot be true because the friction force is proportional to μ_k and not μ_k^2. And (k) cannot be true because the presence of friction will reduce the speed compared to the $\mu_k = 0$ case, and not increase the speed.

(v) If the spring force is strong enough to produce motion against static friction and if the spring energy is large enough to make the block slide the full distance d, the continuity equation for energy gives

$$(1/2)\,kd^2 + \mu_k mgd \cos 180° = (1/2)\,mv^2$$

This turns into the correct expression $\boxed{\text{h}}$.

(vi) We have $(kd^2/m - 2\,\mu_k gd)^{1/2} = [18\,(0.12)^2/0.25 - 2\,(0.6)(9.8)(0.12)]^{1/2} = [1.04 - 1.41]^{1/2}$

The expression gives an imaginary answer because the spring does not contain enough energy in this case to make the block slide the full distance d.

P8.17 $v_i = 2.00\ \text{m/s}$ $\qquad \mu_k = 0.100$

$$K_i - f_k\Delta x + W_{\text{other}} = K_f: \qquad \frac{1}{2}mv_i^2 - f_k\Delta x = 0$$

$$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x \qquad \Delta x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\ \text{m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\ \text{m}}$$

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

P8.18 (a) $U_f = K_i - K_f + U_i \qquad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\ \text{J}}$

$$\boxed{E = 40.0\ \text{J}}$$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero, some nonconservative force or forces must act. For conservative forces $\Delta K + \Delta U = 0$.

P8.19 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f:$

$$m_2 gh - fh = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2$$

$$f = \mu n = \mu m_1 g$$

$$m_2 gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

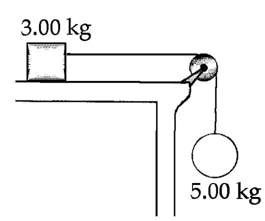

FIG. P8.19

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$

P8.20 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher, is

$$\Delta E = F\Delta r\cos 0° = F(\pi R)$$

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then

$$\Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$$

becomes

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$$

or

$$v_f = \sqrt{v_i^2 + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.0)\pi(0.600)}{0.250}}$$

$$v_f = \boxed{26.5 \text{ m/s}}$$

P8.21 (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160 \text{ J}}$

(b) $\Delta U = mg(3.00 \text{ m})\sin 30.0° = \boxed{73.5 \text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5 \text{ J}}{3.00 \text{ m}} = \boxed{28.8 \text{ N}}$$

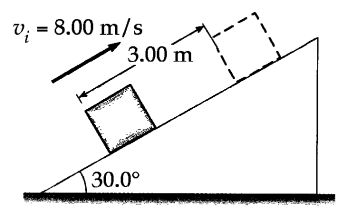

FIG. P8.21

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8 \text{ N}$

$$\mu_k = \frac{28.8 \text{ N}}{(5.00 \text{ kg})(9.80 \text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$$

P8.22 Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0 \text{ kg})(9.80 \text{ m/s}^2)1\,000 \text{ m} - (50.0 \text{ N})(800 \text{ m}) - (3\,600 \text{ N})(200 \text{ m}) = \frac{1}{2}(80.0 \text{ kg})v_f^2$$

$$784\,000 \text{ J} - 40\,000 \text{ J} - 720\,000 \text{ J} = \frac{1}{2}(80.0 \text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000 \text{ J})}{80.0 \text{ kg}}} = \boxed{24.5 \text{ m/s}}$$

(b) $\boxed{\text{Yes. This is too fast for safety.}}$

(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1\,000 \text{ m} - \Delta x_2$:

$$784\,000 \text{ J} - (50.0 \text{ N})(1\,000 \text{ m} - \Delta x_2) - (3\,600 \text{ N})\Delta x_2 = \frac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})^2$$

$$784\,000 \text{ J} - 50\,000 \text{ J} - (3\,550 \text{ N})\Delta x_2 = 1\,000 \text{ J}$$

$$\Delta x_2 = \frac{733\,000 \text{ J}}{3\,550 \text{ N}} = \boxed{206 \text{ m}}$$

continued on next page

(d)	Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

P8.23 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0 + \frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2 - (3.20\times10^{-2}\ \text{N})(0.150\ \text{m}) = \frac{1}{2}(5.30\times10^{-3}\ \text{kg})v^2$$

$$v = \sqrt{\frac{2(5.20\times10^{-3}\ \text{J})}{5.30\times10^{-3}\ \text{kg}}} = \boxed{1.40\ \text{m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\vec{\mathbf{F}}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\ \text{N}}{8.00\ \text{N/m}} = 0.400\ \text{cm}$$

and the ball has moved

$$5.00\ \text{cm} - 0.400\ \text{cm} = \boxed{4.60\ \text{cm from the start}}$$

(c) Between start and maximum speed points,w

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00(5.00\times10^{-2})^2 - (3.20\times10^{-2})(4.60\times10^{-2})$$

$$= \frac{1}{2}(5.30\times10^{-3})v^2 + \frac{1}{2}8.00(4.00\times10^{-3})^2$$

$$v = \boxed{1.79\ \text{m/s}}$$

P8.24 (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable. At $r = 2.3$ mm, the equilibrium is unstable. A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than -5.6 J. The particle is bound if $\boxed{-5.6\ \text{J} \le E < 1\ \text{J}}$.

(c) If the system energy is -3 J, its potential energy must be less than or equal to -3 J. Thus, the particle's position is limited to $\boxed{0.6\ \text{mm} \le r \le 3.6\ \text{mm}}$.

(d) $K + U = E$. Thus, $K_{\max} = E - U_{\min} = -3.0\ \text{J} - (-5.6\ \text{J}) = \boxed{2.6\ \text{J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\ \text{mm}}$.

(f) $-3\ \text{J} + W = 1\ \text{J}$. Hence, the binding energy is $W = \boxed{4\ \text{J}}$.

P8.25 (a) The object moved down distance $1.20\text{ m} + x$. Choose $y = 0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E_{\text{mech}} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = (160\text{ N/m})x^2 - (14.7\text{ N})x - 17.6\text{ J}$$

$$x = \frac{14.7\text{ N} \pm \sqrt{(-14.7\text{ N})^2 - 4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x = \frac{14.7\text{ N} \pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i - f\Delta x = \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m} + x) - 0.700\text{ N}(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J} + 14.7\text{ N}x - 0.840\text{ J} - 0.700\text{ N}x = 160\text{ N/m}\,x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371\text{ m}}$$

P8.26 The boy converts some chemical energy in his muscles into kinetic energy. During this conversion, the energy can be measured as the work his hands do on the wheels.

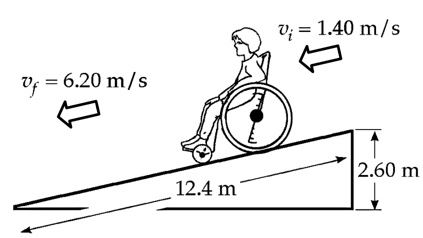

FIG. P8.26

$$K_i + U_{gi} + U_{chemical,i} - f_k\Delta x = K_f$$

$$K_i + U_{gi} + W_{hands-on-wheels} - f_k\Delta x = K_f$$

$$\tfrac{1}{2}mv_i^2 + mgy_i + W_{byboy} - f_k\Delta x = \tfrac{1}{2}mv_f^2$$

or

$$W_{byboy} = \frac{1}{2}m\left(v_f^2 - v_i^2\right) - mgy_i + f\Delta x$$

$$W_{byboy} = \frac{1}{2}(47.0)\left[(6.20)^2 - (1.40)^2\right] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{byboy} = \boxed{168\text{ J}}$$

P8.27 (a) Let m be the mass of the whole board. The portion on the rough surface has mass $\frac{mx}{L}$. The normal force supporting it is $\frac{mxg}{L}$ and the frictional force is $\frac{\mu_k mgx}{L} = ma$. Then

$$\boxed{a = \frac{\mu_k gx}{L} \text{ opposite to the motion}}.$$

(b) In an incremental bit of forward motion dx, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L} dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L} dx = \frac{\mu_k mg}{L} \frac{x^2}{2}\bigg|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

Section 8.5 **Power**

P8.28 $$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875 \text{ kg}(0.620 \text{ m/s})^2}{2(21\times 10^{-3} \text{ s})} = \boxed{8.01 \text{ W}}$$

P8.29 Power $= \frac{W}{t}$

$$\mathcal{P} = \frac{mgh}{t} = \frac{(700 \text{ N})(10.0 \text{ m})}{8.00 \text{ s}} = \boxed{875 \text{ W}}$$

***P8.30** (a) The moving sewage possesses kinetic energy in the same amount as it enters and leaves the pump. The work of the pump increases the gravitational energy of the sewage-Earth system. We take the equation $K_i + U_{gi} + W_{\text{pump}} = K_f + U_{gf}$, subtract out the K terms, and choose $U_{gi} = 0$ at the bottom of the sump, to obtain $W_{\text{pump}} = mgy_f$. Now we differentiate through with respect to time:

$$\mathcal{P}_{\text{pump}} = \frac{\Delta m}{\Delta t} gy_f = \rho \frac{\Delta V}{\Delta t} gy_f$$

$$= 1050 \text{ kg/m}^3 \left(1.89\times 10^6 \text{ L/d}\right)\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right)\left(\frac{1 \text{ d}}{86\,400 \text{ s}}\right)\left(\frac{9.80 \text{ m}}{\text{s}^2}\right)(5.49 \text{ m})$$

$$= \boxed{1.24\times 10^3 \text{ W}}$$

(b) $$\text{efficiency} = \frac{\text{useful output work}}{\text{total input work}} = \frac{\text{useful output work}/\Delta t}{\text{total input work}/\Delta t}$$

$$= \frac{\text{mechanical output power}}{\text{input electric power}} = \frac{1.24 \text{ kW}}{5.90 \text{ kW}} = \boxed{0.209} = 20.9\%$$

The remaining power, $5.90 - 1.24 \text{ kW} = 4.66 \text{ kW}$ is the rate at which internal energy is injected into the sewage and the surroundings of the pump.

Dave Barry attended the January dedication of the pumping station and was the featured speaker at a festive potluck supper to which residents of the different Grand Forks sewer districts brought casseroles, Jell-O salads, and "bars" for dessert.

P8.31 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\text{ kg})(24.6\text{ m/s})^2 = 390\text{ kJ}$$

with power $\mathcal{P} = \frac{390\,000\text{ J}}{15.0\text{ s}}\boxed{\sim 10^4\text{ W}}$ around 30 horsepower.

P8.32 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0+1.75\text{ m/s}}{2}\right](3.00\text{ s}) = 2.63\text{ m}$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \frac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m}) = 1.77\times10^4\text{ J}$$

Also, $W = \overline{\mathcal{P}}t$ so $\overline{\mathcal{P}} = \frac{W}{t} = \frac{1.77\times10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91\times10^3\text{ W}} = 7.92\text{ hp}$.

(b) When moving upward at constant speed $(v = 1.75\text{ m/s})$ the applied force equals the weight $= (650\text{ kg})(9.80\text{ m/s}^2) = 6.37\times10^3$ N. Therefore,

$$\mathcal{P} = Fv = (6.37\times10^3\text{ N})(1.75\text{ m/s}) = \boxed{1.11\times10^4\text{ W}} = 14.9\text{ hp}$$

P8.33 *energy = power × time*

For the 28.0 W bulb:

$$\text{Energy used} = (28.0\text{ W})(1.00\times10^4\text{ h}) = 280\text{ kilowatt}\cdot\text{hrs}$$

$$\text{total cost} = \$17.00 + (280\text{ kWh})(\$0.080/\text{kWh}) = \$39.40$$

For the 100 W bulb:

$$\text{Energy used} = (100\text{ W})(1.00\times10^4\text{ h}) = 1.00\times10^3\text{ kilowatt}\cdot\text{hrs}$$

$$\text{\# bulb used} = \frac{1.00\times10^4\text{ h}}{750\text{ h/bulb}} = 13.3$$

$$\text{total cost} = 13.3(\$0.420) + (1.00\times10^3\text{ kWh})(\$0.080/\text{kWh}) = \$85.60$$

Savings with energy-efficient bulb = \$85.60 − \$39.40 = $\boxed{\$46.2}$.

P8.34 The useful output energy is

$$120\text{ Wh}(1-0.60) = mg(y_f - y_i) = F_g\Delta y$$

$$\Delta y = \frac{120\text{ W}(3\,600\text{ s})0.40}{890\text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194\text{ m}}$$

P8.35 The energy of the car is $E = \frac{1}{2}mv^2 + mgy$

$E = \frac{1}{2}mv^2 + mgd\sin\theta$ where d is the distance it has moved along the track.

$$\mathcal{P} = \frac{dE}{dt} = mv\frac{dv}{dt} + mgv\sin\theta$$

(a) When speed is constant,

$$\mathcal{P} = mgv\sin\theta = 950\text{ kg}\left(9.80\text{ m/s}^2\right)\left(2.20\text{ m/s}\right)\sin 30° = \boxed{1.02\times10^4\text{ W}}$$

(b) $\frac{dv}{dt} = a = \frac{2.2\text{ m/s} - 0}{12\text{ s}} = 0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$$\mathcal{P} = mva + mgv\sin\theta = 950\text{ kg}\left(2.2\text{ m/s}\right)\left(0.183\text{ m/s}^2\right) + 1.02\times10^4\text{ W} = \boxed{1.06\times10^4\text{ W}}$$

(c) At the top end,

$$\frac{1}{2}mv^2 + mgd\sin\theta = 950\text{ kg}\left(\frac{1}{2}\left(2.20\text{ m/s}\right)^2 + \left(9.80\text{ m/s}^2\right)1\,250\text{ m}\sin 30°\right) = \boxed{5.82\times10^6\text{ J}}$$

P8.36 (a) Burning 1 lb of fat releases energy

$$1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.71\times10^7\text{ J}$$

The mechanical energy output is

$$\left(1.71\times10^7\text{ J}\right)(0.20) = nF\Delta r\cos\theta$$

Then

$$3.42\times10^6\text{ J} = nmg\Delta y\cos 0°$$

$$3.42\times10^6\text{ J} = n(50\text{ kg})\left(9.8\text{ m/s}^2\right)(80\text{ steps})(0.150\text{ m})$$

$$3.42\times10^6\text{ J} = n\left(5.88\times10^3\text{ J}\right)$$

where the number of times she must climb the steps is $n = \frac{3.42\times10^6\text{ J}}{5.88\times10^3\text{ J}} = \boxed{582}$.

$\boxed{\text{This method is impractical compared to limiting food intake.}}$

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times10^3\text{ J}}{65\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}$$

Additional Problems

P8.37 (a) $\left(K+U_g\right)_A=\left(K+U_g\right)_B$

$$0+mgy_A=\frac{1}{2}mv_B^2+0 \qquad v_B=\sqrt{2gy_A}=\sqrt{2\left(9.8\ \text{m/s}^2\right)6.3\ \text{m}}=\boxed{11.1\ \text{m/s}}$$

(b) $$a_c=\frac{v^2}{r}=\frac{\left(11.1\ \text{m/s}\right)^2}{6.3\ \text{m}}=\boxed{19.6\ \text{m/s}^2\ \text{up}}$$

(c) $\sum F_y=ma_y \qquad +n_B-mg=ma_c$

$$n_B=76\ \text{kg}\left(9.8\ \text{m/s}^2+19.6\ \text{m/s}^2\right)=\boxed{2.23\times10^3\ \text{N up}}$$

(d) We compute the amount of chemical energy converted into mechanical energy as

$$W=F\Delta r\cos\theta=2.23\times10^3\ \text{N}(0.450\ \text{m})\cos0°=\boxed{1.01\times10^3\ \text{J}}$$

(e) $\left(K+U_g+U_{chemical}\right)_B=\left(K+U_g\right)_D$

$$\frac{1}{2}mv_B^2+0+1.01\times10^3\ \text{J}=\frac{1}{2}mv_D^2+mg\left(y_D-y_B\right)$$

$$\frac{1}{2}76\ \text{kg}\left(11.1\ \text{m/s}\right)^2+1.01\times10^3\ \text{J}=\frac{1}{2}76\ \text{kg}\ v_D^2+76\ \text{kg}\left(9.8\ \text{m/s}^2\right)6.3\ \text{m}$$

$$\sqrt{\frac{\left(5.70\times10^3\ \text{J}-4.69\times10^3\ \text{J}\right)2}{76\ \text{kg}}}=v_D=\boxed{5.14\ \text{m/s}}$$

(f) $\left(K+U_g\right)_D=\left(K+U_g\right)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2+0=0+mg\left(y_E-y_D\right) \qquad y_E-y_D=\frac{v_D^2}{2g}=\frac{\left(5.14\ \text{m/s}\right)^2}{2\left(9.8\ \text{m/s}^2\right)}=\boxed{1.35\ \text{m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$

$$-2.34\ \text{m}=0+5.14\ \text{m/s}\,t+\frac{1}{2}\left(-9.8\ \text{m/s}^2\right)t^2$$

$$4.9t^2-5.14t-2.34=0$$

$$t=\frac{5.14\pm\sqrt{5.14^2-4(4.9)(-2.34)}}{9.8}=\boxed{1.39\ \text{s}}$$

***P8.38** (a) $\boxed{\text{Yes, the total mechanical energy is constant.}}$ The originally hanging block loses gravitational energy, which is entirely converted into kinetic energy of both blocks.

(b) energy at release = energy just before hitting floor

$m_2gy=(1/2)\ (m_1+m_2)v^2$

$v=[2m_2gy/(m_1+m_2)]^{1/2}=[2(1.90\ \text{kg})(9.8\ \text{m/s}^2)0.9\ \text{m}/5.4\ \text{kg}]^{1/2}=\boxed{2.49\ \text{m/s}}$

(c) No. The kinetic energy of the impacting block turns into internal energy. But mechanical energy is conserved for the 3.50-kg block with the Earth in this block's projectile motion.

continued on next page

(d) For the 3.5-kg block from when the string goes slack until just before the block hits the floor

$$(1/2)\,(m_2)v^2 + m_2 gy = (1/2)\,(m_2)v_d^2$$

$$v_d = [2gy + v^2]^{1/2} = [2(9.8\text{ m/s}^2)1.2\text{ m} + (2.49\text{ m/s})^2]^{1/2} = \boxed{5.45\text{ m/s}}$$

(e) The 3.5-kg block takes this time in flight to the floor: from $y = (1/2)\,gt^2$ we have $t = [2(1.2)/9.8]^{1/2} = 0.495$ s. Its horizontal component of displacement at impact is then $x = v_d\,t = (2.49\text{ m/s})(0.495\text{ s}) = \boxed{1.23\text{ m}}$.

(f) $\boxed{\text{No.}}$ With the hanging block firmly stuck, the string pulls radially on the 3.5-kg block, doing no work on it.

(g) The force of static friction cannot be larger than $\mu_s n = (0.56)(3.5\text{ kg})(9.8\text{ m/s}^2) = 19.2$ N. The hanging block tends to produce string tension $(1.9\text{ kg})(9.8\text{ m/s}^2) = 18.6$ N. Then the force of static friction on the 3.5-kg block is less than its maximum value, being $\boxed{\text{18.6 N to the left}}$.

(h) $\boxed{\text{A little push is required}}$, because 18.6 N is less than 19.2 N. The motion begins with negligible speed, so $\boxed{\text{the calculated final speeds are still accurate}}$.

P8.39 (a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(2.00 + 24.0t^2 + 72.0t^4\right)\text{ J}}$$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $\mathcal{P} = Fv = (48.0t)\left(1 + 6.00t^2\right) = \boxed{\left(48.0t + 288t^3\right)\text{ W}}$

(d) $W = \displaystyle\int_0^{2.00} \mathcal{P}\,dt = \int_0^{2.00} \left(48.0t + 288t^3\right)dt = \boxed{1250\text{ J}}$

***P8.40** (a) Simplified, the equation is $0 = (9\,700\ \text{N/m})x^2 - (450.8\ \text{N})x - 1\,395\ \text{N}\cdot\text{m}$. Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{450.8\text{ N} \pm \sqrt{(450.8\text{ N})^2 - 4(9\,700\ \text{N/m})(-1\,395\text{ N}\cdot\text{m})}}{2(9\,700\ \text{N/m})}$$

$$= \frac{450.8\text{ N} \pm 7\,370\text{ N}}{19\,400\ \text{N/m}} = \boxed{0.403\text{ m or } -0.357\text{ m}}$$

(b) One possible problem statement: From a perch at a height of 2.80 m above the top of the pile of mattresses, a 46.0-kg child jumps nearly straight upward with speed 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them. Physical meaning: The positive value of x represents the maximum spring compression. The negative value represents the maximum extension of the equivalent spring if the child sticks to the top of the mattress pile as he rebounds upward without friction.

P8.41 (a) $v = \int_0^t a\,dt = \int_0^t \left(1.16t - 0.21t^2 + 0.24t^3\right)dt$

$$= 1.16\frac{t^2}{2} - 0.21\frac{t^3}{3} + 0.24\frac{t^4}{4}\bigg|_0^t = 0.58t^2 - 0.07t^3 + 0.06t^4$$

At $t = 0$, $v_i = 0$. At $t = 2.5$ s,

$$v_f = \left(0.58\ \text{m/s}^3\right)(2.5\ \text{s})^2 - \left(0.07\ \text{m/s}^4\right)(2.5\ \text{s})^3 + \left(0.06\ \text{m/s}^5\right)(2.5\ \text{s})^4 = 4.88\ \text{m/s}$$

$$K_i + W = K_f$$

$$0 + W = \frac{1}{2}mv_f^2 = \frac{1}{2}1\,160\ \text{kg}\left(4.88\ \text{m/s}\right)^2 = \boxed{1.38\times10^4\ \text{J}}$$

(b) At $t = 2.5$ s,

$$a = \left(1.16\ \text{m/s}^3\right)2.5\ \text{s} - \left(0.210\ \text{m/s}^4\right)(2.5\ \text{s})^2 + \left(0.240\ \text{m/s}^5\right)(2.5\ \text{s})^3 = 5.34\ \text{m/s}^2$$

Through the axles the wheels exert on the chassis force

$$\sum F = ma = 1\,160\ \text{kg}\ 5.34\ \text{m/s}^2 = 6.19\times10^3\ \text{N}$$

and inject power

$$\mathcal{P} = Fv = 6.19\times10^3\ \text{N}\left(4.88\ \text{m/s}\right) = \boxed{3.02\times10^4\ \text{W}}$$

P8.42 (a) $\Delta E_{\text{int}} = -\Delta K = -\frac{1}{2}m\left(v_f^2 - v_i^2\right)$: $\Delta E_{\text{int}} = -\frac{1}{2}(0.400\ \text{kg})\left((6.00)^2 - (8.00)^2\right)(\text{m/s})^2 = \boxed{5.60\ \text{J}}$

(b) $\Delta E_{\text{int}} = f\Delta r = \mu_k mg(2\pi r)$: $\quad 5.60\ \text{J} = \mu_k(0.400\ \text{kg})\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})$

Thus,

$$\mu_k = \boxed{0.152}$$

(c) After N revolutions, the object comes to rest and $K_f = 0$.

Thus,

$$\Delta E_{\text{int}} = -\Delta K = -0 + K_i = \frac{1}{2}mv_i^2$$

or

$$\mu_k mg\left[N(2\pi r)\right] = \frac{1}{2}mv_i^2$$

This gives

$$N = \frac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \frac{\frac{1}{2}(8.00\ \text{m/s})^2}{(0.152)\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})} = \boxed{2.28\ \text{rev}}$$

P8.43 (a) $\sum W = \Delta K$: $\quad W_s + W_g = 0$

$$\frac{1}{2}kx_i^2 - 0 + mg\Delta x\cos(90° + 60°) = 0$$

$$\frac{1}{2}\left(1.40\times10^3\ \text{N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0$$

$$\Delta x = \boxed{4.12\ \text{m}}$$

continued on next page

(b) $\sum W = \Delta K + \Delta E_{\text{int}}$: $\quad W_s + W_g - \Delta E_{\text{int}} = 0$

$$\frac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\Delta x = 0$$

$$\frac{1}{2}\left(1.40\times10^3\ \text{N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x$$

$$-(0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$$

$$\Delta x = \boxed{3.35\ \text{m}}$$

P8.44 $\mathcal{P}\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$

The density is

$$\rho = \frac{\Delta m}{\text{vol}} = \frac{\Delta m}{A\Delta x}$$

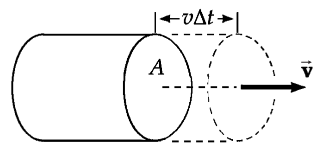

FIG. P8.44

Substituting this into the first equation and solving for $\mathcal{P}$, since $\dfrac{\Delta x}{\Delta t} = v$, for a constant speed, we get

$$\boxed{\mathcal{P} = \frac{\rho A v^3}{2}}$$

Also, since $\mathcal{P} = Fv$,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.45 $\mathcal{P} = \dfrac{1}{2}D\rho\pi r^2 v^3$

(a) $\mathcal{P}_a = \dfrac{1}{2}1\left(1.20\ \text{kg/m}^3\right)\pi\left(1.5\ \text{m}\right)^2\left(8\ \text{m/s}\right)^3 = \boxed{2.17\times10^3\ \text{W}}$

(b) $\dfrac{\mathcal{P}_b}{\mathcal{P}_a} = \dfrac{v_b^3}{v_a^3} = \left(\dfrac{24\ \text{m/s}}{8\ \text{m/s}}\right)^3 = 3^3 = 27$

$\mathcal{P}_b = 27\left(2.17\times10^3\ \text{W}\right) = \boxed{5.86\times10^4\ \text{W}}$

***P8.46** (a) $U_g = mgy = (64\ \text{kg})\left(9.8\ \text{m/s}^2\right)y = \boxed{(627\ \text{N})y}$

(b) At the original height and at all heights above 65 m − 25.8 m = 39.2 m, the cord is unstretched and $\boxed{U_s = 0}$. Below 39.2 m, the cord extension x is given by $x = 39.2\ \text{m} - y$, so the elastic energy is $U_s = \dfrac{1}{2}kx^2 = \boxed{\dfrac{1}{2}(81\ \text{N/m})(39.2\ \text{m} - y)^2}$.

(c) For $y > 39.2$ m, $U_g + U_s = \boxed{(627\ \text{N})y}$

For $y \le 39.2$ m,

$$U_g + U_s = (627\ \text{N})y + 40.5\ \text{N/m}\left(1\,537\ \text{m}^2 - (78.4\ \text{m})y + y^2\right)$$

$$= \boxed{(40.5\ \text{N/m})y^2 - (2\,550\ \text{N})y + 62\,200\ \text{J}}$$

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(d)

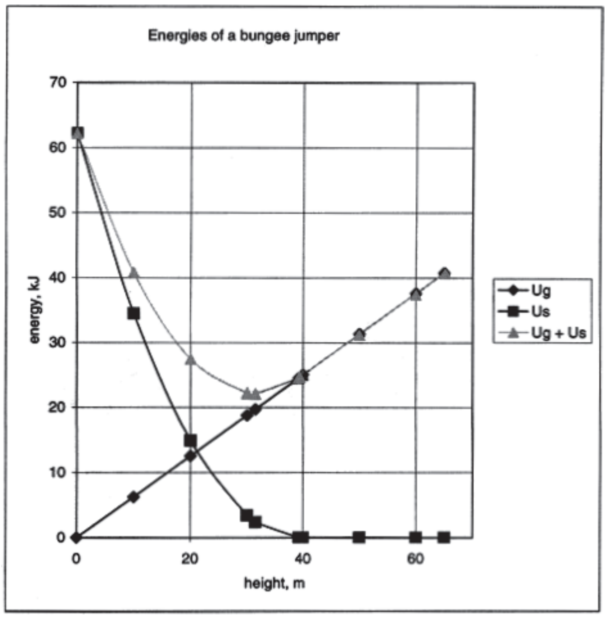

FIG. P8.46(d)

(e) At minimum height, the jumper has zero kinetic energy and the same total energy as at his starting point. $K_i + U_i = K_f + U_f$ becomes

$$627 \text{ N}(65 \text{ m}) = (40.5 \text{ N/m})y_f^2 - (2\,550 \text{ N})y_f + 62\,200 \text{ J}$$

$$0 = 40.5y_f^2 - 2\,550y_f + 21\,500$$

$$y_f = \boxed{10.0 \text{ m}} \quad [\text{the root 52.9 m is unphysical}]$$

(f) The total potential energy has a minimum, representing a $\boxed{\text{stable equilibrium}}$ position. To find it, we require $\frac{dU}{dy} = 0$:

$$\frac{d}{dy}(40.5y^2 - 2\,550y + 62\,200) = 0 = 81y - 2\,550$$

$$y = \boxed{31.5 \text{ m}}$$

(g) Maximum kinetic energy occurs at minimum potential energy. Between the takeoff point and this location, we have $K_i + U_i = K_f + U_f$

$$0 + 40\,800 \text{ J} = \frac{1}{2}(64 \text{ kg})v_{\max}^2 + 40.5(31.5)^2 - 2\,550(31.5) + 62\,200$$

$$v_{\max} = \left(\frac{2(40\,800 - 22\,200)}{64}\right)^{1/2} = \boxed{24.1 \text{ m/s}}$$

P8.47 (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $-kx_a - f_k = ma = 0$.

$$-(1.0 \times 10^3 \text{ N/m})x_a - 4.0 \text{ N} = 0 \qquad \boxed{x = -4.0 \times 10^{-3} \text{ m}}$$

(b) By the same logic,

$$-(1.0 \times 10^3 \text{ N/m})x_b - 10.0 \text{ N} = 0 \qquad \boxed{x = -1.0 \times 10^{-2} \text{ m}}$$

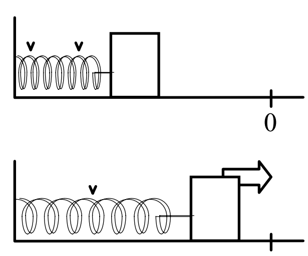

FIG. P8.47

P8.48 (a) The suggested equation $\mathcal{P}\Delta t = bwd$ implies all of the following cases:

(1) $\mathcal{P}\Delta t = b\left(\frac{w}{2}\right)(2d)$ (2) $\mathcal{P}\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $\mathcal{P}\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{\mathcal{P}}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.

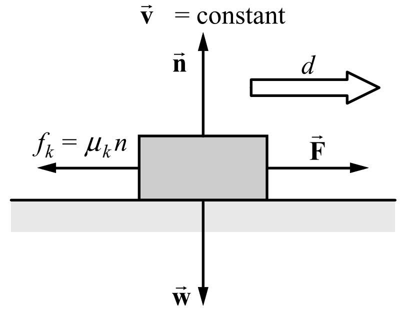

FIG. P8.48

(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ implies that:

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd \text{ and puts out power } \mathcal{P} = \frac{W}{\Delta t}$$

This yields the equation $\mathcal{P}\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

P8.49 $v = 100 \text{ km/h} = 27.8 \text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}(0.330)(1.20 \text{ kg/m}^3)(2.50 \text{ m}^2)(27.8 \text{ m/s})^2 = 382 \text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or

$$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382 \text{ N})(27.8 \text{ m/s}) + (1500 \text{ kg})(9.80 \text{ m/s}^2)(27.8 \text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4 \text{ kW} = 44.8 \text{ hp}}$$

P8.50 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

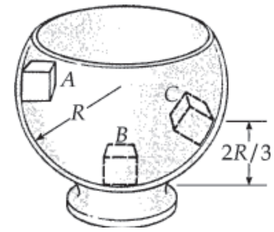

FIG. P8.50

(b) $K_A + U_A = K_B + U_B$

$$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$$

(c) $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$$K_C = K_A + U_A - U_C = mg(h_A - h_C)$$

$$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$$

P8.51 (a) $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b) $\Delta E_{\text{mech}} = \Delta K + \Delta U = K_B - K_A + U_B - U_A$

$$= K_B + mg(h_B - h_A)$$

$$= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m})$$

$$= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}$$

(c) No. It is possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

P8.52 m = mass of pumpkin
R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin first loses contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

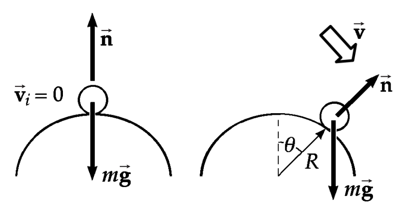

FIG. P8.52

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$$\frac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR, \text{ which reduces to}$$

$$\cos\theta = \frac{2}{3}, \text{ and gives } \theta = \cos^{-1}(2/3) = \boxed{48.2°}$$

as the angle at which the pumpkin will lose contact with the surface.

P8.53 $k = 2.50 \times 10^4$ N/m, $m = 25.0$ kg

$x_A = -0.100$ m, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$

$$E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E_{\text{mech}} = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})(-0.100 \text{ m})^2$$

$$E_{\text{mech}} = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point C is the same as that at point A.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $\quad 0 + (25.0 \text{ kg})(9.80 \text{ m/s}^2)x_C + 0 = 0 - 24.5 \text{ J} + 125 \text{ J}$

$$x_C = \boxed{0.410 \text{ m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\quad \frac{1}{2}(25.0 \text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5 \text{ J}) + 125 \text{ J}$

$$v_B = \boxed{2.84 \text{ m/s}}$$

(d) K and v are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where

$$k|x| = mg$$

or

$$|x| = \frac{(25.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.50 \times 10^4 \text{ N/m}} = 9.80 \times 10^{-3} \text{ m}$$

Thus,

$$K = K_{\max} \text{ at } x = \boxed{-9.80 \text{ mm}}$$

(e) $K_{\max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80 \text{ mm}}\right)$

or

$$\frac{1}{2}(25.0 \text{ kg})v_{\max}^2 = (25.0 \text{ kg})(9.80 \text{ m/s}^2)\left[(-0.100 \text{ m}) - (-0.009\,8 \text{ m})\right] + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})\left[(-0.100 \text{ m})^2 - (-0.009\,8 \text{ m})^2\right]$$

yielding

$$v_{\max} = \boxed{2.85 \text{ m/s}}$$

P8.54 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\ \text{N/m})d^2 + 0.250(1.00\ \text{kg})(9.80\ \text{m/s}^2)d - \frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2 = 0$$

$$d = \frac{[-2.45 \pm 21.35]\ \text{N}}{50.0\ \text{N/m}} = \boxed{0.378\ \text{m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\ \text{m/s})^2 - \frac{2}{(1.00\ \text{kg})}(2.45\ \text{N})(2)(0.378\ \text{m})}$$

$$= \boxed{2.30\ \text{m/s}}$$

(c) For the motion from picture two to picture five,

$$\Delta E_{\text{mech}} = \Delta K + \Delta U$$

$$-f(D+2d) = -\frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2$$

$$D = \frac{9.00\ \text{J}}{2(0.250)(1.00\ \text{kg})(9.80\ \text{m/s}^2)} - 2(0.378\ \text{m})$$

$$= \boxed{1.08\ \text{m}}$$

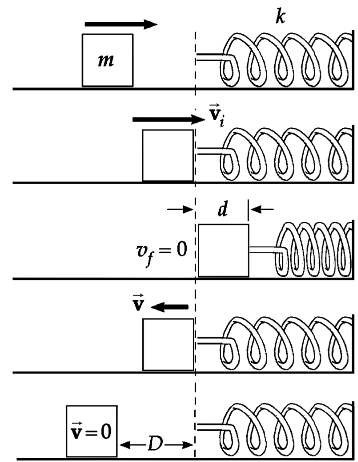

FIG. P8.54

P8.55

$$\Delta E_{\text{mech}} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

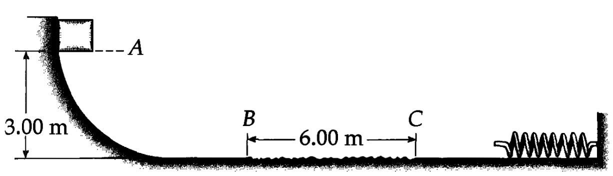

FIG. P8.55

P8.56 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

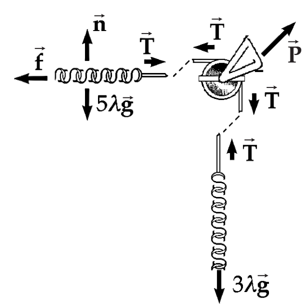

FIG. P8.56

(a) For the five meters on the table with motion impending,

$$\sum F_y = 0: \qquad +n - 5\lambda g = 0 \qquad n = 5\lambda g$$

$$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$$

$$\sum F_x = 0: \qquad +T - f_s = 0 \qquad T = f_s$$

$$T \le 3\lambda g$$

The maximum value is barely enough to support the hanging segment according to

$$\sum F_y = 0: \qquad +T - 3\lambda g = 0 \qquad T = 3\lambda g$$

so it is at this point that the chain starts to slide.

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(b) Let x represent the variable distance the chain has slipped since the start. Then length $(5-x)$ remains on the table, with now

$$\sum F_y = 0: \qquad +n-(5-x)\lambda g = 0 \qquad n=(5-x)\lambda g$$

$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\dfrac{3}{2}=6.5\text{ m}$.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f: \qquad 0+(m_1gy_1+m_2gy_2)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2+mgy\right)_f$$

$$(5\lambda g)8+(3\lambda g)6.5-\int_0^5(2\lambda g-0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2+(8\lambda g)4$$

$$40.0g+19.5g-2.00g\int_0^5 dx+0.400g\int_0^5 x\,dx = 4.00v^2+32.0g$$

$$27.5g-2.00gx\big|_0^5+0.400g\left.\frac{x^2}{2}\right|_0^5 = 4.00v^2$$

$$27.5g-2.00g(5.00)+0.400g(12.5)=4.00v^2$$

$$22.5g = 4.00v^2$$

$$v=\sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P8.57 $(K+U)_i = (K+U)_f$

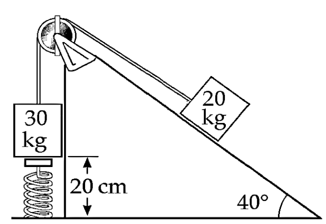

FIG. P8.57

$$0+(30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})+\frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$=\frac{1}{2}(50.0\text{ kg})v^2+(20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J}+5.00\text{ J}=(25.0\text{ kg})v^2+25.2\text{ J}$$

$$\boxed{v=1.24\text{ m/s}}$$

P8.58 The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}(\sin 50° + \sin\phi)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$$(K + U_g)_i + W_{\text{wind}} = (K + U_g)_f$$

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg }v_i^2 + 50\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m})$$
$$= 50\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 28.9°)$$

$$\frac{1}{2}50\text{ kg }v_i^2 - 1.26\times10^4\text{ J} - 5.5\times10^3\text{ J} = -1.72\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$

$$\frac{1}{2}130\text{ kg }v_i^2 + 130\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$$
$$= 130\text{ kg}(9.8\text{ m/s}^2)(-40\text{ m}\cos 50°)$$

$$\frac{1}{2}130\text{ kg }v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.59 (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2}(450\text{ N/m})(\Delta x)^2 = \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

Therefore, $\Delta x = \boxed{0.400\text{ m}}$

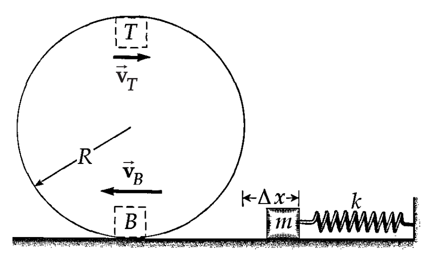

FIG. P8.59

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) + \frac{1}{2}(0.500\text{ kg})v_T^2 - \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$
$$= -(7.00\text{ N})(\pi)(1.00\text{ m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\text{ m/s}}$$

continued on next page

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8 \text{ m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

***P8.60** (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = F\int dx = F\sqrt{2LH - H^2}$$

The work-energy theorem can be written:

$$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}, \text{ or}$$

$$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH \text{ giving}$$
$$F^2 2LH - F^2H^2 = m^2g^2H^2$$

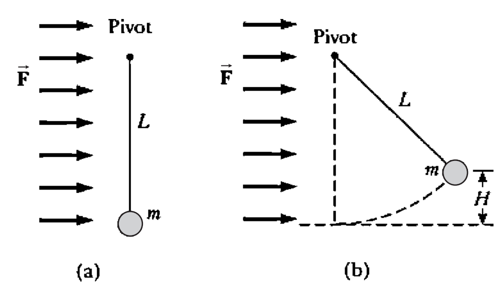

FIG. P8.60

Here the solution $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = \left(F^2 + m^2g^2\right)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \frac{2L}{1 + (mg/F)^2} = \frac{2(0.8 \text{ m})}{1 + (0.3 \text{ kg})^2(9.8 \text{ m/s}^2)^2/F^2} = \boxed{\frac{1.6 \text{ m}}{1 + 8.64 \text{ N}^2/F^2}}$$

(b) $H = 1.6 \text{ m}\left[1 + 8.64/1\right]^{-1} = \boxed{0.166 \text{ m}}$

(c) $H = 1.6 \text{ m}\left[1 + 8.64/100\right]^{-1} = \boxed{1.47 \text{ m}}$

(d) As $F \to 0$, $H \to 0$ as is reasonable.

(e) As $F \to \infty$, $H \to 1.60$ m, which would be hard to approach experimentally.

(f) Call θ the equilibrium angle with the vertical and T the tension in the string.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \dfrac{F}{mg}$

Then

$$\cos\theta = \frac{mg}{\sqrt{(mg)^2 + F^2}} = \frac{1}{\sqrt{1 + (F/mg)^2}} = \frac{1}{\sqrt{1 + F^2/8.64 \text{ N}^2}}$$

Therefore,

$$H_{\text{eq}} = L(1 - \cos\theta) = \boxed{(0.800 \text{ m})\left(1 - \frac{1}{\sqrt{1 + F^2/8.64 \text{ N}^2}}\right)}$$

continued on next page

(g) $H_{eq} = 0.8\text{ m}[1 - (1 + 100/8.64)^{-1/2}] = \boxed{0.574\text{ m}}$

(h) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ $\cos\theta \to 0$ and $H_{eq} \to \boxed{0.800\text{ m}}$

A very strong wind pulls the string out horizontal, parallel to the ground.

P8.61 If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\mathbf{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}: \qquad 0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.62 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

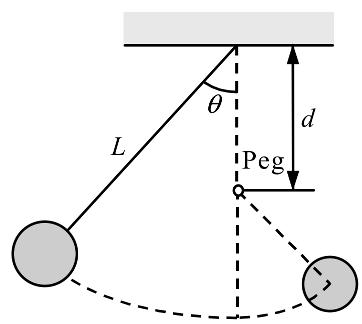

FIG. P8.62

(b) Relative to the point of suspension,

$$U_i = 0,\ U_f = -mg\left[d - (L-d)\right]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \text{ where } R = L - d$$

Upon solving, we get $\boxed{d = \frac{3L}{5}}$.

P8.63 Applying Newton's second law at the bottom (b) and top (t) of the circle gives

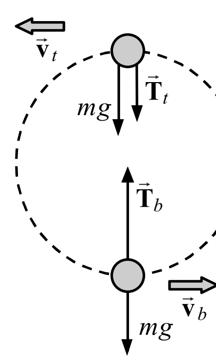

FIG. P8.63

$$T_b - mg = \frac{mv_b^2}{R} \text{ and } -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives

$$T_b = T_t + 2mg + \frac{m\left(v_b^2 - v_t^2\right)}{R}$$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\dfrac{m\left(v_b^2 - v_t^2\right)}{2} + \left(0 - 2mgR\right) = 0$ and $\dfrac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

P8.64 (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \quad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

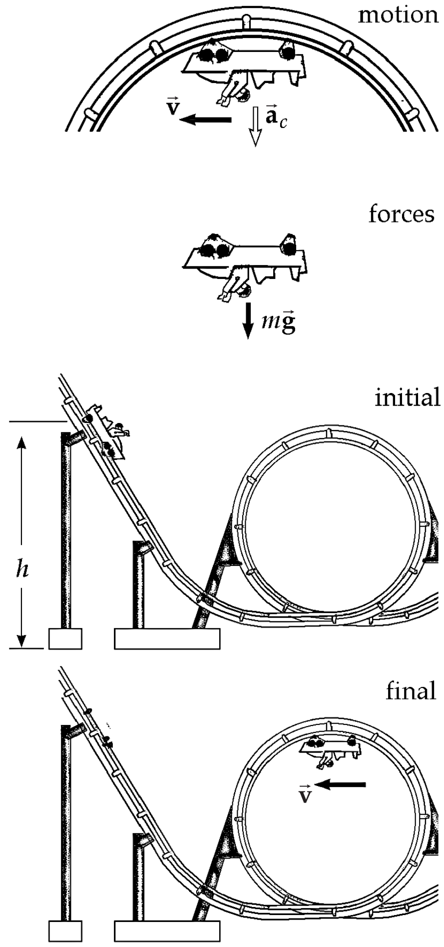

FIG. P8.64

(b) Let h now represent the height $\geq 2.5\,R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \qquad \text{or} \qquad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop,

$$mgh = \frac{1}{2}mv_t^2 + mg(2R)$$

$$v_t^2 = 2gh - 4gR$$

$$\sum F_y = ma_y: \qquad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}$$

P8.65 (a) Conservation of energy for the sled-rider-Earth system, between A and C:

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\ \text{m/s})^2 + m(9.80\ \text{m/s}^2)(9.76\ \text{m})$$

$$= \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(9.76\ \text{m})}$$

$$= \boxed{14.1\ \text{m/s}}$$

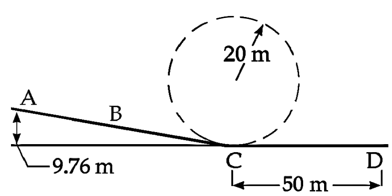

FIG. P8.65(a)

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(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k \Delta x = K_f + U_{gf}:$$

$$\frac{1}{2}(80\ \text{kg})(2.5\ \text{m/s})^2 + (80\ \text{kg})(9.80\ \text{m/s}^2)(9.76\ \text{m}) - f_k \Delta x = 0 + 0$$

$$-f_k \Delta x = \boxed{-7.90 \times 10^3\ \text{J}}$$

(c) The water exerts a frictional force

$$f_k = \frac{7.90 \times 10^3\ \text{J}}{\Delta x} = \frac{7.90 \times 10^3\ \text{N} \cdot \text{m}}{50\ \text{m}} = 158\ \text{N}$$

and also a normal force of

$$n = mg = (80\ \text{kg})(9.80\ \text{m/s}^2) = 784\ \text{N}$$

The magnitude of the water force is

$$\sqrt{(158\ \text{N})^2 + (784\ \text{N})^2} = \boxed{800\ \text{N}}$$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\ \text{m}}{54.3\ \text{m}} = 10.4°$$

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \qquad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\ \text{kg})(9.80\ \text{m/s}^2)\cos 10.4° = \boxed{771\ \text{N}}$$

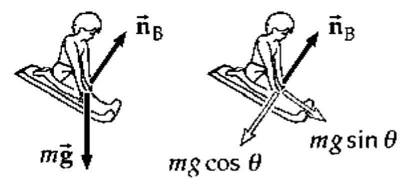

FIG. P8.65(d)

(e) $$\sum F_y = ma_y: \qquad +n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(80.0\ \text{kg})(14.1\ \text{m/s})^2}{20\ \text{m}}$$

$$n_C = \boxed{1.57 \times 10^3\ \text{N up}}$$

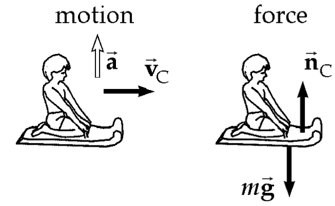

FIG. P8.65(e)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

***P8.66** (a) As at the end of the process analyzed in Example 8.8, we begin with a 0.800-kg block at rest on the end of a spring with stiffness constant 50.0 N/m, compressed 0.0924 m. The energy in the spring is (1/2)(50 N/m)(0.0924 m)^2 = 0.214 J. To push the block back to the unstressed spring position would require work against friction of magnitude 3.92 N(0.0924 m) = 0.362 J. Because 0.214 J is less than 0.362 J, the spring cannot push the object back to $x = 0$.

continued on next page

(b) The block approaches the spring with energy $(1/2)mv^2 = (1/2)(0.8\text{ kg})(1.2\text{ m/s})^2 = 0.576\text{ J}$. It travels against friction by equal distances in compressing the spring and in being pushed back out, so it must lose one-half of this energy in its motion to the right and the rest in its motion to the left. The spring must possess one-half of this energy at its maximum compression:

$$(0.576\text{ J})/2 = (1/2)\,(50\text{ N/m})x^2$$

so

$$x = 0.107\text{ m}$$

For the compression process we have the continuity equation for energy

$$0.576\text{ J} + \mu_k\,7.84\text{ N}\,(0.107\text{ m})\cos 180° = 0.288\text{ J}$$

so

$$\mu_k = 0.288\text{ J}/0.841\text{ J}\ \boxed{= 0.342}$$

As a check, the decompression process is described by

$$0.288\text{ J} + \mu_k 7.84\text{ N}\,(0.107\text{ m})\cos 180° = 0$$

which gives the same answer for the coefficient of friction.

ANSWERS TO EVEN PROBLEMS

P8.2 (a) 1.11×10^9 J (b) 0.2

P8.4 (a) $v_B = 5.94$ m/s; $v_C = 7.67$ m/s (b) 147 J

P8.6 (a) see the solution (b) 60.0°

P8.8 (a) $\sqrt{\dfrac{2(m_1 - m_2)gh}{(m_1 + m_2)}}$ (b) $\dfrac{2m_1h}{m_1 + m_2}$

P8.10 (a) 18.5 km, 51.0 km (b) 10.0 MJ

P8.12 $\left(\dfrac{8gh}{15}\right)^{1/2}$

P8.14 (a) 0.791 m/s (b) 0.531 m/s

P8.16 **(i)** (a), (b), (c), (f) **(ii)** (g), (i), (j) **(iii)** (d) **(iv)** (e) cannot be true because the friction force is proportional to μ_k and not μ_k^2. And (k) cannot be true because the presence of friction will reduce the speed compared to the $\mu_k = 0$ case. **(v)** Expression (h) is correct if the spring force is strong enough to produce motion against static friction and if the spring energy is large enough to make the block slide the full distance *d*. **(vi)** The expression gives an imaginary answer because the spring does not contain enough energy in this case to make the block slide the full distance *d*.

P8.18 (a) $U_f = 22.0$ J; $E = 40.0$ J (b) Yes. The total mechanical energy changes.

P8.20 26.5 m/s

P8.22 (a) 24.5 m/s (b) Yes; his landing speed is too high to be safe. (c) 206 m (d) Not realistic. Air drag depends strongly on speed.

P8.24 (a) $r = 1.5$ mm and 3.2 mm, stable; 2.3 mm unstable; $r \to \infty$ neutral (b) $-5.6\text{ J} \le E < 1\text{ J}$ (c) $0.6\text{ mm} \le r \le 3.6\text{ mm}$ (d) 2.6 J (e) 1.5 mm (f) 4 J

P8.26 168 J

P8.28 8.01 W

P8.30 (a) 1.24 kW (b) 20.9%

P8.32 (a) 5.91 kW (b) 11.1 kW

P8.34 194 m

P8.36 No. (a) 582 (b) $90.5\text{ W} = 0.121\text{ hp}$

P8.38 (a) yes (b) 2.49 m/s (c) No, but mechanical energy is conserved for the 3.50-kg block in its projectile motion with the Earth. (d) 5.45 m/s (e) 1.23 m (f) no (g) 18.6 N to the left (h) A little push is required. The speeds are still accurate.

P8.40 (a) $x = 0.403$ m or -0.357 m (b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps straight upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which the mattresses are compressed when the child lands on them. Physical meaning of the answer: The positive value of x represents the maximum spring compression. The negative value represents the extension of the equivalent spring if the child sticks to the top of the mattress pile as the child rebounds upward without friction.

P8.42 (a) 5.60 J (b) 0.152 (c) 2.28 rev

P8.44 See the solution. Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.46 (a) $(627\text{ N})y$ (b) $U_s = 0$ for $y > 39.2$ m and $U_s = \frac{1}{2}(81\text{ N/m})(39.2\text{ m} - y)^2$ for $y \le 39.2$ m

(c) $U_g + U_s = (627\text{ N})y$, for $y > 39.2$ m and $U_g + U_s = (40.5\text{ N/m})y^2 - (2\,550\text{ N})y + 62\,200\text{ J}$ for $y \le 39.2$ m (d) see the solution (e) 10.0 m (f) yes: stable equilibrium at 31.5 m (g) 24.1 m/s

P8.48 (a) see the solution (b) For a block of weight w pushed over a rough horizontal surface at constant velocity, $b = \mu_k$. For a load pulled vertically upward at constant velocity, $b = 1$.

P8.50 (a) 0.588 J (b) 0.588 J (c) 2.42 m/s (d) $U_C = 0.392$ J, $K_C = 0.196$ J

P8.52 48.2°

P8.54 (a) 0.378 m (b) 2.30 m/s (c) 1.08 m

P8.56 (a) see the solution (b) 7.42 m/s

P8.58 (a) 6.15 m/s (b) 9.87 m/s

P8.60 (a) $H = 1.6\text{ m}(1 + 8.64\text{ N}^2/\text{F}^2)^{-1}$ (b) 0.166 m (c) 1.47 m (d) $H \rightarrow 0$ proportionally to F^2
(e) H approaches 1.60 m (f) $H_{eq} = 0.8\text{ m}[1 - (F^2/8.64\text{ N}^2 + 1)^{-1/2}]$ (g) 0.574 m (h) 0.800 m

P8.62 see the solution

P8.64 (a) 2.5 R (b) see the solution

P8.66 (a) see the solution (b) 0.342