19

Temperature

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q19.1 Two objects in thermal equilibrium need not be in contact. Consider the two objects that are in thermal equilibrium in Figure 19.1(c). The act of separating them by a small distance does not affect how the molecules are moving inside either object, so they will still be in thermal equilibrium.

Q19.2 The copper's temperature drops and the water temperature rises until both temperatures are the same. Then the metal and the water are in thermal equilibrium.

Q19.3 The astronaut is referring to the temperature of the lunar surface, specifically a 400 °F difference. A thermometer would register the temperature of the thermometer liquid. Since there is no atmosphere in the moon, the thermometer will not read the temperature of some other object unless it is placed into the lunar soil.

***Q19.4** Answer (e). The thermometer works by differential expansion. As the thermometer is warmed the liquid level falls relative to the tube wall. If the liquid and the tube material were to expand by equal amounts, the thermometer could not be used.

***Q19.5** Answer (b). Around atmospheric pressure, 0 °C is the only temperature at which liquid water and solid water can both exist.

***Q19.6** Mentally multiply 93 m and 17 and 1/(1 000 000 °C) and say 5 °C for the temperature increase. To simplify, multiply 100 and 100 and 1/1 000 000 for an answer in meters: it is on the order of 1 cm, answer (c).

Q19.7 The measurements made with the heated steel tape will be too short—but only by a factor of 5×10^{-5} of the measured length.

Q19.8 (a) One mole of H_2 has a mass of 2.016 0 g.

(b) One mole of He has a mass of 4.002 6 g.

(c) One mole of CO has a mass of 28.010 g.

Q19.9 $PV = nRT$ predicts V going to zero as T goes to zero. The ideal-gas model does not apply when the material gets close to liquefaction and then turns into a liquid or solid. The molecules start to interact all the time, not just in brief collisions. The molecules start to take up a significant portion of the volume of the container.

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***Q19.10** Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch. The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, and stays fairly ideal—fairly far from liquefaction—even at 100 K. The water vapor liquefies and then freezes, and the carbon dioxide turns to snow, but these are minor constituents of the air. Thus as the absolute temperature drops to 1/3 of its original value the volume (i) will drop to 1/3 of what it was: answer (b). (ii) As noted above, the pressure stays nearly constant at 1 atm. Answer (d).

***Q19.11** Cylinder A must be at lower pressure. If the gas is thin, $PV = nRT$ applies to both with the same value of nRT for both. Then A will be at one-third the absolute pressure of B. Answer (e).

***Q19.12** Most definitively, we should say that pressure is proportional to absolute temperature. Pressure is a linear function of Celsius temperature, but this relationship is not a proportionality because pressure does not go to zero at 0 °C. Pressure is a linear function of Kelvin temperature, on its way to being a linear function with a graph going through the origin. Statement (c) is ambiguous. The rate of increase in pressure might refer to a time rate, with units of pascals per second, which could not describe a temperature increase. Statement (d) is a way of saying that the graph has constant slope, so it is a correct statement, if uncommunicative. Thus (b) and (d) are correct.

***Q19.13** We think about nRT/V in each case. Since R is constant, we need only think about nT/V, and units of mmol · K/cm^3 are as convenient as any. In case a, we have $2 \cdot 3/1 = 6$. In b we have 6. In c we have 4. In d we have 6. In e we have 5. Then the ranking is a = b = d > e > c

Q19.14 As the temperature increases, the brass expands. This would effectively increase the distance d from the pivot point to the center of mass of the pendulum, and also increase the moment of inertia of the pendulum. Since the moment of inertia is proportional to d^2, and the period of a physical pendulum is $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, the period would increase, and the clock would run slow.

Q19.15 As the water rises in temperature, it expands or rises in pressure or both. The excess volume would spill out of the cooling system, or else the pressure would rise very high indeed. Modern cooling systems have an overflow reservoir to accept the excess volume when the coolant heats up and expands.

Q19.16 The coefficient of expansion of metal is larger than that of glass. When hot water is run over the jar, both the glass and the lid expand, but at different rates. Since *all* dimensions expand the inner diameter of the lid expands more than the top of the jar, and the lid will be easier to remove.

Q19.17 The sphere expands when heated, so that it no longer fits through the ring. With the sphere still hot, you can separate the sphere and ring by heating the ring. This more surprising result occurs because the thermal expansion of the ring is not like the inflation of a blood-pressure cuff. Rather, it is like a photographic enlargement; every linear dimension, including the hole diameter, increases by the same factor. The reason for this is that the atoms everywhere, including those around the inner circumference, push away from each other. The only way that the atoms can accommodate the greater distances is for the circumference—and corresponding diameter—to grow. This property was once used to fit metal rims to wooden wagon wheels. If the ring is heated and the sphere left at room temperature, the sphere would pass through the ring with more space to spare.

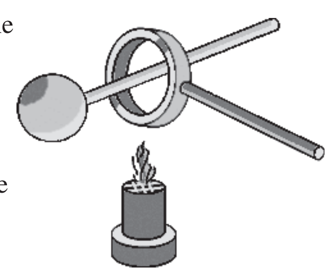

FIG. Q19.17

SOLUTIONS TO PROBLEMS

Section 19.2 **Thermometers and the Celsius Temperature Scale**

Section 19.3 **The Constant-Volume Gas Thermometer and the Absolute Temperature Scale**

P19.1 Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where A and B are constants. To find A and B, we use the data

$$0.900 \text{ atm} = A + (-80.0^\circ\text{C})B \qquad (1)$$

$$1.635 \text{ atm} = A + (78.0^\circ\text{C})B \qquad (2)$$

Solving (1) and (2) simultaneously, $1.635 - 0.900 = 78\,B + 80\,B$

we find $B = 4.652 \times 10^{-3} \text{ atm/}^\circ\text{C}$

and $A = 1.272 \text{ atm}$

Therefore, $P = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/}^\circ\text{C}\right)T$

(a) At absolute zero $P = 0 = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/}^\circ\text{C}\right)T$

which gives $\boxed{T = -274^\circ\text{C}}$

(b) At the freezing point of water $P = 1.272 \text{ atm} + 0 = \boxed{1.27 \text{ atm}}$.

(c) And at the boiling point $P = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/}^\circ\text{C}\right)(100^\circ\text{C}) = \boxed{1.74 \text{ atm}}$.

P19.2 (a) $\Delta T = 450^\circ\text{C} = 450^\circ\text{C}\left(\dfrac{212^\circ\text{F} - 32.0^\circ\text{F}}{100^\circ\text{C} - 0.00^\circ\text{C}}\right) = \boxed{810^\circ\text{F}}$

(b) $\Delta T = 450^\circ\text{C} = \boxed{450 \text{ K}}$ A Celsius degree and a kelvin of temperature difference are the same space on a thermometer.

P19.3 (a) $T_F = \dfrac{9}{5}T_C + 32.0^\circ\text{F} = \dfrac{9}{5}(-195.81) + 32.0 = \boxed{-320^\circ\text{F}}$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3 \text{ K}}$

P19.4 (a) $T = 1\,064 + 273 = \boxed{1\,337 \text{ K}}$ melting point

$T = 2\,660 + 273 = \boxed{2\,933 \text{ K}}$ boiling point

(b) $\Delta T = \boxed{1\,596^\circ\text{C}} = \boxed{1\,596 \text{ K}}$ The differences are the same.

Section 19.4 **Thermal Expansion of Solids and Liquids**

P19.5 The wire is 35.0 m long when $T_C = -20.0°\text{C}$.

$$\Delta L = L_i\bar{\alpha}(T - T_i)$$

$\bar{\alpha} = \alpha(20.0°\text{C}) = 1.70\times10^{-5}\ (\text{C}°)^{-1}$ for Cu.

$$\Delta L = (35.0\text{ m})\left(1.70\times10^{-5}\ (\text{C}°)^{-1}\right)\left(35.0°\text{C} - (-20.0°\text{C})\right) = \boxed{+3.27\text{ cm}}$$

P19.6 Each section can expand into the joint space to the north of it. We need think of only one section expanding. $\Delta L = L_i\alpha\,\Delta T = (25.0\text{ m})\left(12.0\times10^{-6}/\text{C}°\right)(40.0°\text{C}) = \boxed{1.20\text{ cm}}$

P19.7 (a) $\Delta L = \alpha L_i \Delta T = 9.00\times10^{-6}\,°\text{C}^{-1}(30.0\text{ cm})(65.0°\text{C}) = \boxed{0.176\text{ mm}}$

(b) L stands for any linear dimension.

$$\Delta L = \alpha L_i \Delta T = 9.00\times10^{-6}\,°\text{C}^{-1}(1.50\text{ cm})(65.0°\text{C}) = \boxed{8.78\times10^{-4}\text{ cm}}$$

(c) $\Delta V = 3\alpha V_i \Delta T = 3\left(9.00\times10^{-6}\,°\text{C}^{-1}\right)\left(\dfrac{30.0(\pi)(1.50)^2}{4}\text{ cm}^3\right)(65.0°\text{C}) = \boxed{0.093\,0\text{ cm}^3}$

P19.8 The horizontal section expands according to $\Delta L = \alpha L_i \Delta T$.

$$\Delta x = \left(17\times10^{-6}\,°\text{C}^{-1}\right)(28.0\text{ cm})(46.5°\text{C} - 18.0°\text{C}) = 1.36\times10^{-2}\text{ cm}$$

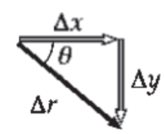

FIG. P19.8

The vertical section expands similarly by

$$\Delta y = \left(17\times10^{-6}\,°\text{C}^{-1}\right)(134\text{ cm})(28.5°\text{C}) = 6.49\times10^{-2}\text{ cm}$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.136\text{ mm})^2 + (0.649\text{ mm})^2} = 0.663\text{ mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.649\text{ mm}}{0.136\text{ mm}}\right) = 78.2°$$

$$\boxed{\Delta r = 0.663\text{ mm to the right at }78.2°\text{ below the horizontal}}$$

***P19.9** (a) $L_{\text{Al}}\left(1 + \alpha_{\text{Al}}\Delta T\right) = L_{\text{Brass}}\left(1 + \alpha_{\text{Brass}}\Delta T\right)$

$$\Delta T = \frac{L_{\text{Al}} - L_{\text{Brass}}}{L_{\text{Brass}}\alpha_{\text{Brass}} - L_{\text{Al}}\alpha_{\text{Al}}}$$

$$\Delta T = \frac{(10.01 - 10.00)}{(10.00)\left(19.0\times10^{-6}\right) - (10.01)\left(24.0\times10^{-6}\right)}$$

$\Delta T = -199°\text{C}$ so $T =$ $\boxed{\begin{array}{l}-179°\text{C}\\ \text{This is attainable, because it is above absolute zero.}\end{array}}$

(b) $$\Delta T = \frac{(10.02 - 10.00)}{(10.00)\left(19.0\times10^{-6}\right) - (10.02)\left(24.0\times10^{-6}\right)}$$

$\Delta T = -396°\text{C}$ so

$T =$ $\boxed{\begin{array}{l}-376°\text{C, which is below 0 K so it cannot be reached.}\\ \text{The rod and ring cannot be separated by changing their temperatures together.}\end{array}}$

***P19.10** (a) $L = L_i(1+\alpha\Delta T)$: $5.050 \text{ cm} = 5.000 \text{ cm}\left[1+24.0\times10^{-6}\ ^\circ\text{C}^{-1}(T-20.0^\circ\text{C})\right]$

$$T = \boxed{437^\circ\text{C}}$$

(b) We must get $L_{\text{Al}} = L_{\text{Brass}}$ for some ΔT, or

$$L_{i,\text{ Al}}\left(1+\alpha_{\text{Al}}\Delta T\right) = L_{i,\text{ Brass}}\left(1+\alpha_{\text{Brass}}\Delta T\right)$$

$$5.000 \text{ cm}\left[1+\left(24.0\times10^{-6}\ ^\circ\text{C}^{-1}\right)\Delta T\right] = 5.050 \text{ cm}\left[1+\left(19.0\times10^{-6}\ ^\circ\text{C}^{-1}\right)\Delta T\right]$$

Solving for ΔT, $\Delta T = 2\,080^\circ\text{C}$,

so $\boxed{T = 2\,100^\circ\text{C}}$

$\boxed{\text{This will not work because aluminum melts at }660^\circ\text{C.}}$

P19.11 (a) $V_f = V_i(1+\beta\Delta T) = 100\left[1+1.50\times10^{-4}(-15.0)\right] = \boxed{99.8 \text{ mL}}$

(b) $\Delta V_{\text{acetone}} = \left(\beta V_i \Delta T\right)_{\text{acetone}}$

$\Delta V_{\text{flask}} = \left(\beta V_i \Delta T\right)_{\text{Pyrex}} = \left(3\alpha V_i \Delta T\right)_{\text{Pyrex}}$

for the same V_i and ΔT,

$$\frac{\Delta V_{\text{acetone}}}{\Delta V_{\text{flask}}} = \frac{\beta_{\text{acetone}}}{\beta_{\text{flask}}} = \frac{1.50\times10^{-4}}{3\left(3.20\times10^{-6}\right)} = \frac{1}{6.40\times10^{-2}}$$

$\boxed{\text{The volume change of flask is about 6\% of the change in the volume of the acetone.}}$

P19.12 (a), (b) The material would expand by $\Delta L = \alpha L_i \Delta T$,

$\dfrac{\Delta L}{L_i} = \alpha\Delta T$, but instead feels stress

$$\frac{F}{A} = \frac{Y\Delta L}{L_i} = Y\alpha\Delta T = \left(7.00\times10^{9}\ \text{N/m}^2\right)12.0\times10^{-6}\,(\text{C}^\circ)^{-1}(30.0^\circ\text{C})$$

$= \boxed{2.52\times10^{6}\ \text{N/m}^2}$. This will $\boxed{\text{not break}}$ concrete.

P19.13 (a) $\Delta V = V_t\beta_t\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = \left(\beta_t - 3\alpha_{\text{Al}}\right)V_i\Delta T$

$$= \left(9.00\times10^{-4} - 0.720\times10^{-4}\right)^\circ\text{C}^{-1}\left(2\,000 \text{ cm}^3\right)(60.0^\circ\text{C})$$

$\Delta V = \boxed{99.4 \text{ cm}^3}$ overflows.

(b) The whole new volume of turpentine is

$$2\,000 \text{ cm}^3 + 9.00\times10^{-4}\ ^\circ\text{C}^{-1}\left(2\,000 \text{ cm}^3\right)(60.0^\circ\text{C}) = 2\,108 \text{ cm}^3$$

so the fraction lost is

$$\frac{99.4 \text{ cm}^3}{2\,108 \text{ cm}^3} = 4.71\times10^{-2}$$

and this fraction of the cylinder's depth will be empty upon cooling:

$$4.71\times10^{-2}(20.0 \text{ cm}) = \boxed{0.943 \text{ cm}}$$

***P19.14** Model the wire as contracting according to $\Delta L = \alpha L_i \Delta T$ and then stretching according to

$$\text{stress} = \frac{F}{A} = Y\frac{\Delta L}{L_i} = \frac{Y}{L_i}\alpha L_i \Delta T = Y\alpha\Delta T$$

(a) $F = YA\alpha\Delta T = \left(20\times10^{10}\ \text{N/m}^2\right)4\times10^{-6}\ \text{m}^2\ 11\times10^{-6}\ \frac{1}{\text{C}^\circ}45°\text{C} = \boxed{396\ \text{N}}$

(b) $\Delta T = \frac{\text{stress}}{Y\alpha} = \frac{3\times10^8\ \text{N/m}^2}{\left(20\times10^{10}\ \text{N/m}^2\right)11\times10^{-6}/\text{C}^\circ} = 136°\text{C}$

To increase the stress the temperature must decrease to $35°\text{C} - 136°\text{C} = \boxed{-101°\text{C}}$.

(c) $\boxed{\text{The original length divides out, so the answers would not change.}}$

P19.15 The area of the chip decreases according to

$$\Delta A = \gamma A_1 \Delta T = A_f - A_i$$
$$A_f = A_i\left(1+\gamma\Delta T\right) = A_i\left(1+2\alpha\Delta T\right)$$

The star images are scattered uniformly, so the number N of stars that fit is proportional to the area.

Then $N_f = N_i\left(1+2\alpha\Delta T\right) = 5\,342\left[1+2\left(4.68\times10^{-6}\ °\text{C}^{-1}\right)\left(-100°\text{C}-20°\text{C}\right)\right] = \boxed{5\,336\ \text{star images}}$.

Section 19.5 **Macroscopic Description of an Ideal Gas**

P19.16 Mass of gold abraded: $|\Delta m| = 3.80\ \text{g} - 3.35\ \text{g} = 0.45\ \text{g} = (0.45\ \text{g})\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right) = 4.5\times10^{-4}\ \text{kg}$

Each atom has mass $m_0 = 197\ \text{u} = 197\ \text{u}\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 3.27\times10^{-25}\ \text{kg}$

Now, $|\Delta m| = |\Delta N|n$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5\times10^{-4}\ \text{kg}}{3.27\times10^{-25}\ \text{kg}} = 1.38\times10^{21}\ \text{atoms}$$

The rate of loss is

$$\frac{|\Delta N|}{\Delta t} = \frac{1.38\times10^{21}\ \text{atoms}}{50\ \text{yr}}\left(\frac{1\ \text{yr}}{365.25\ \text{d}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$\frac{|\Delta N|}{\Delta t} = \boxed{8.72\times10^{11}\ \text{atoms/s}}$$

P19.17 (a) Initially, $P_iV_i = n_iRT_i$ $\quad (1.00\ \text{atm})V_i = n_iR(10.0+273.15)\ \text{K}$

Finally, $P_fV_f = n_fRT_f$ $\quad P_f\left(0.280V_i\right) = n_iR(40.0+273.15)\ \text{K}$

Dividing these equations, $\quad \frac{0.280P_f}{1.00\ \text{atm}} = \frac{313.15\ \text{K}}{283.15\ \text{K}}$

giving $\quad P_f = 3.95\ \text{atm}$

or $\quad P_f = \boxed{4.00\times10^5\ \text{Pa(abs.)}}$

(b) After being driven $\quad P_d(1.02)\left(0.280V_i\right) = n_iR(85.0+273.15)\ \text{K}$

$$P_d = 1.121P_f = \boxed{4.49\times10^5\ \text{Pa}}$$

P19.18 (a) $n = \dfrac{PV}{RT} = \dfrac{(9.00\text{ atm})(1.013\times10^5\text{ Pa/atm})(8.00\times10^{-3}\text{ m}^3)}{(8.314\text{ N}\cdot\text{mol K})(293\text{ K})} = \boxed{2.99\text{ mol}}$

(b) $N = nN_A = (2.99\text{ mol})(6.02\times10^{23}\text{ molecules/mol}) = \boxed{1.80\times10^{24}\text{ molecules}}$

P19.19 The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{(1.01\times10^5\text{ N/m}^2)[(10.0\text{ m})(20.0\text{ m})(30.0\text{ m})]}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = 2.49\times10^5\text{ mol}$$

$$N = nN_A = 2.49\times10^5\text{ mol}(6.022\times10^{23}\text{ molecules/mol}) = \boxed{1.50\times10^{29}\text{ molecules}}$$

P19.20 $P = \dfrac{nRT}{V} = \left(\dfrac{9.00\text{ g}}{18.0\text{ g/mol}}\right)\left(\dfrac{8.314\text{ J}}{\text{mol K}}\right)\left(\dfrac{773\text{ K}}{2.00\times10^{-3}\text{ m}^3}\right) = \boxed{1.61\text{ MPa}} = 15.9\text{ atm}$

P19.21

$$\sum F_y = 0: \qquad \rho_{\text{out}} gV - \rho_{\text{in}} gV - (200\text{ kg})g = 0$$

$$(\rho_{\text{out}} - \rho_{\text{in}})(400\text{ m}^3) = 200\text{ kg}$$

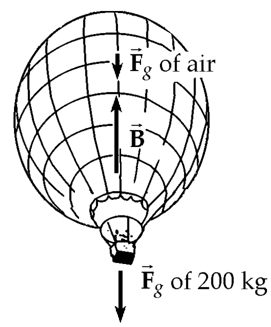

FIG. P19.21

The density of the air outside is 1.25 kg/m^3.

From $PV = nRT$, $\dfrac{n}{V} = \dfrac{P}{RT}$ This equation means that at constant pressure the density is inversely proportional to the temperature. Then the density of the hot air is

$$\rho_{\text{in}} = (1.25\text{ kg/m}^3)\left(\frac{283\text{ K}}{T_{\text{in}}}\right)$$

Then

$$(1.25\text{ kg/m}^3)\left(1 - \frac{283\text{ K}}{T_{\text{in}}}\right)(400\text{ m}^3) = 200\text{ kg}$$

$$1 - \frac{283\text{ K}}{T_{\text{in}}} = 0.400$$

$$0.600 = \frac{283\text{ K}}{T_{\text{in}}} \qquad T_{\text{in}} = \boxed{472\text{ K}}$$

P19.22 Consider the air in the tank during one discharge process. We suppose that the process is slow enough that the temperature remains constant. Then as the pressure drops from 2.40 atm to 1.20 atm, the volume of the air doubles. During the first discharge, the air volume changes from 1 L to 2 L. Just 1 L of water is expelled and 3 L remains. In the second discharge, the air volume changes from 2 L to 4 L and 2 L of water is sprayed out. In the third discharge, only the last 1 L of water comes out. Each person could more efficiently use his device by starting with the tank half full of water.

***P19.23** (a) $PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(1.00\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = \boxed{41.6\ \text{mol}}$$

(b) $m = nM = (41.6\ \text{mol})(28.9\ \text{g/mol}) = \boxed{1.20\ \text{kg}}$ $\boxed{\text{This value agrees with the tabulated density}}$ of $1.20\ \text{kg/m}^3$ at 20.0°C.

P19.24 At depth, $P = P_0 + \rho g h$ and $PV_i = nRT_i$

At the surface, $P_0V_f = nRT_f$: $\dfrac{P_0V_f}{(P_0+\rho g h)V_i} = \dfrac{T_f}{T_i}$

Therefore $V_f = V_i\left(\dfrac{T_f}{T_i}\right)\left(\dfrac{P_0+\rho g h}{P_0}\right)$

$$V_f = 1.00\ \text{cm}^3\left(\frac{293\ \text{K}}{278\ \text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + (1\,025\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(25.0\ \text{m})}{1.013\times10^5\ \text{Pa}}\right)$$

$$V_f = \boxed{3.67\ \text{cm}^3}$$

P19.25 (a) $PV = nRT$ $\quad n = \dfrac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{1.013\times10^5\ \text{Pa}(0.100\ \text{m})^3(28.9\times10^{-3}\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}$$

$$m = \boxed{1.17\times10^{-3}\ \text{kg}}$$

(b) $F_g = mg = 1.17\times10^{-3}\ \text{kg}(9.80\ \text{m/s}^2) = \boxed{11.5\ \text{mN}}$

(c) $F = PA = (1.013\times10^5\ \text{N/m}^2)(0.100\ \text{m})^2 = \boxed{1.01\ \text{kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

P19.26 My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and 20°C = 293 K. Think of the air as 80.0% N_2 and 20.0% O_2.

Avogadro's number of molecules has mass

$$(0.800)(28.0\ \text{g/mol}) + (0.200)(32.0\ \text{g/mol}) = 0.028\,8\ \text{kg/mol}$$

Then $PV = nRT = \left(\dfrac{m}{M}\right)RT$

gives $m = \dfrac{PVM}{RT} = \dfrac{(1.00\times10^5\ \text{N/m}^2)(38.4\ \text{m}^3)(0.028\,8\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 45.4\ \text{kg}\ \boxed{\sim10^2\ \text{kg}}$

P19.27 $PV = nRT$: $\quad \dfrac{m_f}{m_i} = \dfrac{n_f}{n_i} = \dfrac{P_f V_f}{RT_f}\dfrac{RT_i}{P_i V_i} = \dfrac{P_f}{P_i}$

so $\quad m_f = m_i\left(\dfrac{P_f}{P_i}\right)$

$$|\Delta m| = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = 12.0\text{ kg}\left(\frac{41.0\text{ atm} - 26.0\text{ atm}}{41.0\text{ atm}}\right) = \boxed{4.39\text{ kg}}$$

P19.28 $N = \dfrac{PVN_A}{RT} = \dfrac{\left(10^{-9}\text{ Pa}\right)\left(1.00\text{ m}^3\right)\left(6.02\times10^{23}\text{ molecules/mol}\right)}{(8.314\text{ J/K}\cdot\text{mol})(300\text{ K})} = \boxed{2.41\times10^{11}\text{ molecules}}$

***P19.29** (a) The air in the tube is far from liquefaction, so it behaves as an ideal gas. At the ocean surface it is described by $P_t V_t = nRT$ where $P_t = 1$ atm, $V_t = A(6.50\text{ cm})$, and A is the cross-sectional area of the interior of the tube. At the bottom of the dive, $P_b V_b = nRT = P_b A(6.50\text{ cm} - 2.70\text{ cm})$. By division,

$$\frac{P_b(3.8\text{ cm})}{(1\text{ atm})(6.5\text{ cm})} = 1$$

$$P_b = 1.013\times10^5\text{ N/m}^2\,\frac{6.5}{3.8} = 1.73\times10^5\text{ N/m}^2$$

The salt water enters the tube until the air pressure is equal to the water pressure at depth, which is described by

$$P_b = P_t + \rho g h$$

$$1.73\times10^5\text{ N/m}^2 = 1.013\times10^5\text{ N/m}^2 + \left(1\,030\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)h$$

$$h = \frac{7.20\times10^4\text{ kg}\cdot\text{m}\cdot\text{m}^2\cdot\text{s}^2}{1.01\times10^4\text{ s}^2\cdot\text{m}^2\cdot\text{kg}} = \boxed{7.13\text{ m}}$$

(b) With a very thin tube, air does not bubble out. At the bottom of the dive, the tube gives a valid reading in any orientation. The open end of the tube should be at the bottom after the bird surfaces, so that the water will drain away as the expanding air pushes it out.

Students can make the tubes and dive with them in a swimming pool, to observe how dependably they work and how accurate they are.

P19.30 $P_0 V = n_1 RT_1 = \left(\dfrac{m_1}{M}\right)RT_1$

$P_0 V = n_2 RT_2 = \left(\dfrac{m_2}{M}\right)RT_2$

$$\boxed{m_1 - m_2 = \frac{P_0 VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

Additional Problems

P19.31 The excess expansion of the brass is

$$\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = (\alpha_{\text{brass}} - \alpha_{\text{steel}}) L_i \Delta T$$
$$\Delta(\Delta L) = (19.0 - 11.0) \times 10^{-6}\ (^\circ\text{C})^{-1} (0.950\ \text{m})(35.0^\circ\text{C})$$
$$\Delta(\Delta L) = 2.66 \times 10^{-4}\ \text{m}$$

(a) The rod contracts more than tape to a length reading

$0.950\,0\ \text{m} - 0.000\,266\ \text{m} = \boxed{0.949\,7\ \text{m}}$

(b) $0.950\,0\ \text{m} + 0.000\,266\ \text{m} = \boxed{0.950\,3\ \text{m}}$

P19.32 At 0°C, 10.0 gallons of gasoline has mass,

from $\rho = \dfrac{m}{V}$

$$m = \rho V = (730\ \text{kg/m}^3)(10.0\ \text{gal})\left(\frac{0.003\,80\ \text{m}^3}{1.00\ \text{gal}}\right) = 27.7\ \text{kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i \Delta T = 9.60 \times 10^{-4}\ ^\circ\text{C}^{-1}(10.0\ \text{gal})(20.0^\circ\text{C} - 0.0^\circ\text{C}) = 0.192\ \text{gal}$$

At 20.0°C, $10.192\ \text{gal} = 27.7\ \text{kg}$

$$10.0\ \text{gal} = 27.7\ \text{kg}\left(\frac{10.0\ \text{gal}}{10.192\ \text{gal}}\right) = 27.2\ \text{kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7\ \text{kg} - 27.2\ \text{kg} = \boxed{0.523\ \text{kg}}$$

P19.33 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A}\beta \Delta T$$
$$\Delta h = \frac{\frac{4}{3}\pi(0.250\ \text{cm}/2)^3}{\pi(2.00 \times 10^{-3}\ \text{cm})^2}(1.82 \times 10^{-4}\ ^\circ\text{C}^{-1})(30.0^\circ\text{C}) = \boxed{3.55\ \text{cm}}$$

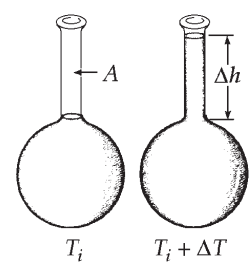

FIG. P19.33

P19.34 (a) The volume of the liquid increases as $\Delta V_\ell = V_i \beta \Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i \Delta T$. Therefore, the overflow in the capillary is $V_c = V_i \Delta T(\beta - 3\alpha)$; and in the capillary $V_c = A\Delta h$.

Therefore,

$$\boxed{\Delta h = \frac{V_i}{A}(\beta - 3\alpha)\Delta T}$$

(b) For a mercury thermometer $\beta(\text{Hg}) = 1.82 \times 10^{-4}\ ^\circ\text{C}^{-1}$

and for glass, $3\alpha = 3 \times 3.20 \times 10^{-6}\ ^\circ\text{C}^{-1}$

Thus $\beta - 3\alpha \approx \beta$ within better than 6%. The value of α is typically so small compared to β that it can be ignored in the equation for a good approximation.

P19.35 The frequency played by the cold-walled flute is

$$f_i = \frac{v}{\lambda_i} = \frac{v}{2L_i}$$

When the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1+\alpha\Delta T)} = \frac{f_i}{1+\alpha\Delta T}$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_i - f_f = f_i\left(1 - \frac{1}{1+\alpha\Delta T}\right)$$

$$\Delta f = \frac{v}{2L_i}\left(\frac{\alpha\Delta T}{1+\alpha\Delta T}\right) \approx \frac{v}{2L_i}(\alpha\Delta T)$$

$$\Delta f \approx \frac{(343\ \text{m/s})(24.0\times10^{-6}/\text{C°})(15.0\text{°C})}{2(0.655\ \text{m})} = \boxed{0.094\,3\ \text{Hz}}$$

This change in frequency is imperceptibly small.

***P19.36** Let L_0 represent the length of each bar at 0°C.

(a) In the diagram consider the right triangle that each invar bar makes with one half of the aluminum bar. We have

$$\sin\left(\frac{\theta}{2}\right) = \frac{L_0(1+\alpha_{Al}\Delta T)}{2L_0}$$

solving gives directly

$$\boxed{\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2}\right)}$$

where T_C is the Celsius temperature.

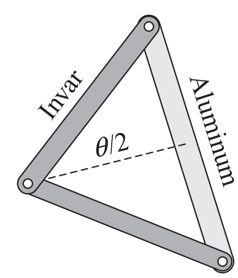

FIG. P19.36

(b) If the temperature drops, the negative value of Celsius temperature describes the contraction. So $\boxed{\text{the answer is accurate}}$. At $T_C = 0$ we have $\theta = 2\sin^{-1}(1/2) = 60.0°$, and $\boxed{\text{this is accurate}}$.

(c) From the same triangle we have

$$\sin\left(\frac{\theta}{2}\right) = \frac{L_0(1+\alpha_{Al}\Delta T)}{2L_0(1+\alpha_{inv}\Delta T)} \qquad \text{giving} \qquad \boxed{\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right)}$$

(d) The greatest angle is at 660°C,

$$2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right) = 2\sin^{-1}\left(\frac{1+(24\times10^{-6})660}{2(1+[0.9\times10^{-6}]660)}\right)$$

$$= 2\sin^{-1}\left(\frac{1.01584}{2.001188}\right) = 2\sin^{-1}0.508 = \boxed{61.0°}$$

The smallest angle is at −273°C,

$$2\sin^{-1}\left(\frac{1+(24\times10^{-6})(-273)}{2(1+[0.9\times10^{-6}][-273])}\right) = 2\sin^{-1}\left(\frac{0.9934}{1.9995}\right) = 2\sin^{-1}0.497 = \boxed{59.6°}$$

P19.37 (a) $\rho = \dfrac{m}{V}$ and $d\rho = -\dfrac{m}{V^2}dV$

For very small changes in V and ρ, this can be expressed as

$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T$$

$\boxed{\text{The negative sign means that any increase in temperature causes the density to decrease and vice versa.}}$

(b) For water we have $\beta = \left|\dfrac{\Delta\rho}{\rho\Delta T}\right| = \left|\dfrac{1.000\ 0\ \text{g/cm}^3 - 0.999\ 7\ \text{g/cm}^3}{\left(1.000\ 0\ \text{g/cm}^3\right)(10.0°\text{C} - 4.0°\text{C})}\right| = \boxed{5\times10^{-5}\ °\text{C}^{-1}}$

P19.38 (a) $\dfrac{P_0V}{T} = \dfrac{P'V'}{T'}$

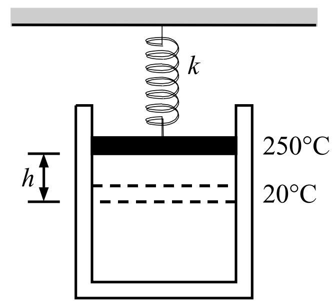

FIG. P19.38

$V' = V + Ah$

$P' = P_0 + \dfrac{kh}{A}$

$\left(P_0 + \dfrac{kh}{A}\right)(V + Ah) = P_0V\left(\dfrac{T'}{T}\right)$

$$\left(1.013\times10^5\ \text{N/m}^2 + 2.00\times10^5\ \text{N/m}^3\,h\right)\left(5.00\times10^{-3}\ \text{m}^3 + \left(0.010\ 0\ \text{m}^2\right)h\right) = \left(1.013\times10^5\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}^3\right)\left(\frac{523\ \text{K}}{293\ \text{K}}\right)$$

$2\ 000h^2 + 2\ 013h - 397 = 0$

$h = \dfrac{-2\ 013 \pm 2\ 689}{4\ 000} = \boxed{0.169\ \text{m}}$

(b) $P' = P + \dfrac{kh}{A} = 1.013\times10^5\ \text{Pa} + \dfrac{\left(2.00\times10^3\ \text{N/m}\right)(0.169)}{0.010\ 0\ \text{m}^2}$

$P' = \boxed{1.35\times10^5\ \text{Pa}}$

P19.39 (a) We assume that air at atmospheric pressure is above the piston.

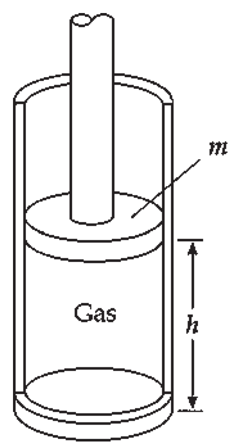

FIG. P19.39

In equilibrium $\quad P_{\text{gas}} = \dfrac{mg}{A} + P_0$

Therefore, $\quad \dfrac{nRT}{hA} = \dfrac{mg}{A} + P_0$

or $\quad \boxed{h = \dfrac{nRT}{mg + P_0A}}$

where we have used $V = hA$ as the volume of the gas.

(b) From the data given,

$$h = \frac{0.200\ \text{mol}\left(8.314\ \text{J/K}\cdot\text{mol}\right)(400\ \text{K})}{20.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) + \left(1.013\times10^5\ \text{N/m}^2\right)\left(0.008\ 00\ \text{m}^2\right)}$$

$= \boxed{0.661\ \text{m}}$

P19.40 The angle of bending θ, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

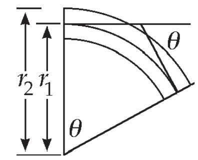

FIG. P19.40

(a) The definition of radian measure gives $L_i + \Delta L_1 = \theta r_1$

and $L_i + \Delta L_2 = \theta r_2$

By subtraction, $\Delta L_2 - \Delta L_1 = \theta(r_2 - r_1)$

$$\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \Delta r$$

$$\boxed{\theta = \frac{(\alpha_2 - \alpha_1) L_i \Delta T}{\Delta r}}$$

(b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore θ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so $\boxed{\text{the bimetallic strip bends the other way}}$. It is fun to demonstrate this with liquid nitrogen.

(d) $$\theta = \frac{2(\alpha_2 - \alpha_1) L_i \Delta T}{2\Delta r} = \frac{2\left(\left(19 \times 10^{-6} - 0.9 \times 10^{-6}\right)°\text{C}^{-1}\right)(200 \text{ mm})(1°\text{C})}{0.500 \text{ mm}}$$

$$= 1.45 \times 10^{-2} = 1.45 \times 10^{-2} \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = \boxed{0.830°}$$

P19.41 From the diagram we see that the change in area is

$$\Delta A = \ell \Delta w + w \Delta \ell + \Delta w \Delta \ell$$

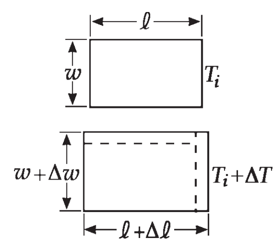

FIG. P19.41

Since $\Delta \ell$ and Δw are each small quantities, the product $\Delta w \Delta \ell$ will be very small. Therefore, we assume $\Delta w \Delta \ell \approx 0$.

Since $\Delta w = w \alpha \Delta T$ and $\Delta \ell = \ell \alpha \Delta T$,

we then have $\Delta A = \ell w \alpha \Delta T + \ell w \alpha \Delta T$

and since $A = \ell w$, $\boxed{\Delta A = 2\alpha A \Delta T}$

The approximation assumes $\Delta w \Delta \ell \approx 0$, or $\alpha \Delta T \approx 0$. Another way of stating this is $\boxed{\alpha \Delta T << 1}$.

***P19.42** (a) The different diameters of the arms of the U-tube do not affect the pressures exerted by the liquids of different density on the liquid in the base. Because the base of the U-tube is horizontal, the pattern of temperature change in the base does not affect the equilibrium heights.

(b) Let ρ_0 represent the density of the liquid at 0°C. At temperature T_C, the volume of a sample has changed according to $\Delta V = \beta V \Delta T = \beta V T_C$,

so the density has become

$$\rho = \frac{m}{V + \beta V T_C} = \rho_0 \frac{1}{1+\beta T_C} \qquad \text{so} \qquad \rho(1+\beta T_C) = \rho_0$$

Now the pressure at the bottom of the U tube is equal, whichever column it supports:

$$P_0 + \rho_0 g h_0 = P_0 + \rho g h_t$$

simplifying, $\rho_0 h_0 = \rho h_t$

and substituting, $\rho(1+\beta T_C) h_0 = \rho h_t$

$$(1+\beta T_C) h_0 = h_t \qquad \beta = \frac{1}{T_C}\left(\frac{h_t}{h_0} - 1\right)$$

***P19.43** (a) The copper rod has a greater coefficient of linear expansion, so it should start with a smaller length. $\boxed{\text{The steel rod is longer.}}$ With $L_C + 5\text{ cm} = L_S$ at 0°C we want also $L_C(1 + 17 \times 10^{-6}\, T_C) + 5\text{ cm} = L_S(1 + 11 \times 10^{-6}\, T_C)$ or by subtraction $17\, L_C = 11\, L_S$. So

$\boxed{\text{yes, this pair of equations can be satisfied as long as the coefficients of expansion remain constant.}}$

By substitution,

$$L_C + 5\text{ cm} = (17/11) L_C \qquad \boxed{L_C = (11/6)\,5\text{ cm} = 9.17\text{ cm} \quad \text{so} \quad L_S = 14.2\text{ cm}}$$

P19.44 (a) $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$ so $L_i = \dfrac{T_i^2 g}{4\pi^2} = \dfrac{(1.000\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = 0.248\,2\text{ m}$

$$\Delta L = \alpha L_i \Delta T = 19.0\times10^{-6}\,°\text{C}^{-1}(0.248\,2\text{ m})(10.0°\text{C}) = 4.72\times10^{-5}\text{ m}$$

$$T_f = 2\pi\sqrt{\frac{L_i + \Delta L}{g}} = 2\pi\sqrt{\frac{0.248\,3\text{ m}}{9.80\text{ m/s}^2}} = 1.000\,095\,0\text{ s}$$

$$\Delta T = \boxed{9.50\times10^{-5}\text{ s}}$$

(b) In one week, the time lost $= 1\text{ week}\left(9.50\times10^{-5}\text{ s lost per second}\right)$

$$\text{time lost} = (7.00\text{ d/week})\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.50\times10^{-5}\ \frac{\text{s lost}}{\text{s}}\right)$$

$$\text{time lost} = \boxed{57.5\text{ s lost}}$$

P19.45 $I = \int r^2 dm$ and since $r(T) = r(T_i)(1+\alpha\Delta T)$

for $\alpha\Delta T << 1$ we find $\dfrac{I(T)}{I(T_i)} = (1+\alpha\Delta T)^2$

thus $$\frac{I(T)-I(T_i)}{I(T_i)} \approx 2\alpha\Delta T$$

(a) With $\alpha = 17.0\times10^{-6}\ ^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$

we find for Cu: $$\frac{\Delta I}{I} = 2\left(17.0\times10^{-6}\ ^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.340\%}$$

(b) With $\alpha = 24.0\times10^{-6}\ ^\circ\text{C}^{-1}$

and $\Delta T = 100^\circ\text{C}$

we find for Al: $$\frac{\Delta I}{I} = 2\left(24.0\times10^{-6}\ ^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.480\%}$$

P19.46 (a) Let V' represent the compressed volume at depth

$$B = \rho g V' \qquad P' = P_0 + \rho g d \qquad P'V' = P_0 V_i$$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0+\rho g d)}}$$

(b) Since d is in the denominator, B must $\boxed{\text{decrease}}$ as the depth increases.

(The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0+\rho g d)}{\rho g P_0 V_i/P_0} = \frac{P_0}{P_0+\rho g d}$$

$$P_0 + \rho g d = 2P_0$$

$$d = \frac{P_0}{\rho g} = \frac{1.013\times10^5\ \text{N/m}^2}{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{10.3\ \text{m}}$$

P19.47 After expansion, the length of one of the spans is

$$L_f = L_i(1+\alpha\Delta T) = 125\ \text{m}\left[1+12\times10^{-6}\ ^\circ\text{C}^{-1}(20.0^\circ\text{C})\right] = 125.03\ \text{m}$$

L_f, y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

$$(125.03\ \text{m})^2 = y^2 + (125\ \text{m})^2 \qquad \text{yielding} \qquad y = \boxed{2.74\ \text{m}}$$

P19.48 Let $\ell = L/2$ represent the original length of one of the concrete slabs. After expansion, the length of each one of the spans is $\ell_f = \ell(1+\alpha\Delta T)$. Now ℓ_f, y, and the original length ℓ of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives

$$\ell_f^2 = \ell^2 + y^2, \qquad \text{or} \qquad y = \sqrt{\ell_f^2 - \ell^2} = \ell\sqrt{(1+\alpha\Delta T)^2 - 1} = (L/2)\sqrt{2\alpha\Delta T + (\alpha\Delta T)^2}$$

Since $\alpha\Delta T << 1$, we have $\boxed{y \approx L\sqrt{\alpha\Delta T/2}}$

The height of the center of the buckling bridge is directly proportional to the bridge length. A small bridge is geometrically similar to a large one. The height is proportional to the square root of the temperature increase. Doubling ΔT makes y increase by only 41%. A small value of ΔT can have a surprisingly large effect. In units, the equation reads m = m(°C/°C)$^{1/2}$, so it is dimensionally correct.

P19.49 (a) Let m represent the sample mass. The number of moles is $n = \frac{m}{M}$ and the density is $\rho = \frac{m}{V}$.

So $\quad PV = nRT \quad$ becomes $\quad PV = \frac{m}{M}RT \quad$ or $\quad PM = \frac{m}{V}RT$

Then,

$$\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$$

(b) $$\rho = \frac{PM}{RT} = \frac{(1.013\times10^5\ \text{N/m}^2)(0.032\,0\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = \boxed{1.33\ \text{kg/m}^3}$$

***P19.50** (a) From $PV = nRT$, the volume is: $\quad V = \left(\frac{nR}{P}\right)T$

Therefore, when pressure is held constant, $\quad \frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$

Thus, $\quad \beta \equiv \left(\frac{1}{V}\right)\frac{dV}{dT} = \left(\frac{1}{V}\right)\frac{V}{T}, \quad$ or $\quad \beta = \boxed{\frac{1}{T}}$

(b) At $T = 0°\text{C} = 273$ K, this predicts $\quad \beta = \frac{1}{273\ \text{K}} = \boxed{3.66\times10^{-3}\ \text{K}^{-1}}$

Experimental values are: $\quad \beta_{\text{He}} = 3.665\times10^{-3}\ \text{K}^{-1}$ and $\beta_{\text{air}} = 3.67\times10^{-3}\ \text{K}^{-1}$

Our single theoretical value agrees within 0.06% and 0.2%, respectively, with the tabulated values for helium and air.

P19.51 Visualize the molecules of various species all moving randomly. The net force on any section of wall is the sum of the forces of all of the molecules pounding on it.

For each gas alone, $\quad P_1 = \frac{N_1kT}{V} \quad$ and $\quad P_2 = \frac{N_2kT}{V} \quad$ and $\quad P_3 = \frac{N_3kT}{V}$, etc.

For all gases $\quad P_1V_1 + P_2V_2 + P_3V_3 \ldots = (N_1 + N_2 + N_3 \ldots)kT$ and

$$(N_1 + N_2 + N_3 \ldots)kT = PV$$

Also, $\quad V_1 = V_2 = V_3 = \ldots = V \quad$ therefore $\quad \boxed{P = P_1 + P_2 + P_3 \ldots}$

P19.52

(a) No torque acts on the disk so its angular momentum is constant. Its moment of inertia decreases as it contracts so its angular speed must increase.

(b) $$I_i\omega_i = I_f\omega_f = \frac{1}{2}MR_i^2\omega_i = \frac{1}{2}MR_f^2\omega_f = \frac{1}{2}M\left[R_i + R_i\alpha\Delta T\right]^2\omega_f = \frac{1}{2}MR_i^2\left[1-\alpha|\Delta T|\right]^2\omega_f$$

$$\omega_f = \omega_i\left[1-\alpha|\Delta T|\right]^{-2} = \frac{25.0\ \text{rad/s}}{\left(1-(17\times10^{-6}\ 1/\text{C°})830°\text{C}\right)^2} = \frac{25.0\ \text{rad/s}}{0.972} = \boxed{25.7\ \text{rad/s}}$$

P19.53 Consider a spherical steel shell of inner radius r and much smaller thickness t, containing helium at pressure P. When it contains so much helium that it is on the point of bursting into two hemispheres, we have $P\pi r^2 = \left(5\times10^8 \text{ N/m}^2\right)2\pi rt$. The mass of the steel is $\rho_s V = \rho_s 4\pi r^2 t = \rho_s 4\pi r^2 \dfrac{P\,r}{10^9 \text{ Pa}}$

For the helium in the tank,

$$PV = nRT \qquad \text{becomes} \qquad P\frac{4}{3}\pi r^3 = nRT = \frac{m_{\text{He}}}{M_{\text{He}}}RT = 1\text{ atm}V_{\text{balloon}}$$

The buoyant force on the balloon is the weight of the air it displaces, which is described by $1\text{ atm}V_{\text{balloon}} = \dfrac{m_{\text{air}}}{M_{\text{air}}}RT = P\dfrac{4}{3}\pi r^3$. The net upward force on the balloon with the steel tank hanging from it is

$$+m_{\text{air}}g - m_{\text{He}}g - m_s g = \frac{M_{\text{air}}P4\pi r^3 g}{3RT} - \frac{M_{\text{He}}P4\pi r^3 g}{3RT} - \frac{\rho_s P4\pi r^3 g}{10^9 \text{ Pa}}$$

The balloon will or will not lift the tank depending on whether this quantity is positive or negative, which depends on the sign of $\dfrac{\left(M_{\text{air}} - M_{\text{He}}\right)}{3RT} - \dfrac{\rho_s}{10^9 \text{ Pa}}$. At 20°C this quantity is

$$= \frac{(28.9-4.00)\times10^{-3} \text{ kg/mol}}{3\left(8.314 \text{ J/mol}\cdot\text{K}\right)293 \text{ K}} - \frac{7\,860 \text{ kg/m}^3}{10^9 \text{ N/m}^2}$$
$$= 3.41\times10^{-6} \text{ s}^2/\text{m}^2 - 7.86\times10^{-6} \text{ s}^2/\text{m}^2$$

where we have used the density of iron. The net force on the balloon is downward so the helium balloon is $\boxed{\text{not able to lift}}$ its tank. Steel would need to be 2.30 times stronger to contain enough helium to lift the steel tank.

P19.54 With piston alone: $T = \text{constant}$, so $PV = P_0V_0$

or $P\left(Ah_i\right) = P_0\left(Ah_0\right)$

With $A = \text{constant}$, $P = P_0\left(\dfrac{h_0}{h_i}\right)$

But, $P = P_0 + \dfrac{m_p g}{A}$

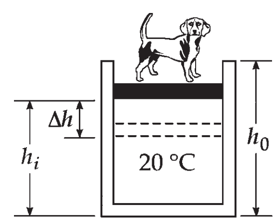

FIG. P19.54

where m_p is the mass of the piston.

Thus, $P_0 + \dfrac{m_p g}{A} = P_0\left(\dfrac{h_0}{h_i}\right)$

which reduces to

$$h_i = \frac{h_0}{1 + m_p g / P_0 A} = \frac{50.0 \text{ cm}}{1 + 20.0 \text{ kg}\left(9.80 \text{ m/s}^2\right)\Big/\left[1.013\times10^5 \text{ Pa}\,\pi(0.400 \text{ m})^2\right]} = 49.81 \text{ cm}$$

With the dog of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_0}{1 + \left(m_p + M\right)g / P_0 A} = \frac{50.0 \text{ cm}}{1 + 95.0 \text{ kg}\left(9.80 \text{ m/s}^2\right)\Big/\left[1.013\times10^5 \text{ Pa}\,\pi(0.400 \text{ m})^2\right]} = 49.10 \text{ cm}$$

Thus, when the dog steps on the piston, it moves downward by

$$\Delta h = h_i - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.706 \text{ cm} = \boxed{7.06 \text{ mm}}$$

(b) $P = \text{const}$, so $\dfrac{V}{T} = \dfrac{V'}{T_i}$ or $\dfrac{Ah_i}{T} = \dfrac{Ah'}{T_i}$

giving $T = T_i\left(\dfrac{h_i}{h'}\right) = 293 \text{ K}\left(\dfrac{49.81}{49.10}\right) = \boxed{297 \text{ K}}$ (or 24°C)

P19.55 (a) $\dfrac{dL}{L} = \alpha dT$: $\displaystyle\int_{T_i}^{T_f} \alpha dT = \int_{L_i}^{L_f} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha \Delta T \Rightarrow \boxed{L_f = L_i e^{\alpha \Delta T}}$

(b) $L_f = (1.00\text{ m})e^{\left[2.00\times10^{-5}\,°\text{C}^{-1}(100°\text{C})\right]} = 1.002\,002\text{ m}$

$L_f' = 1.00\text{ m}\left[1 + 2.00\times10^{-5}\,°\text{C}^{-1}(100°\text{C})\right] = 1.002\,000\text{ m}$: $\dfrac{L_f - L_f'}{L_f} = 2.00\times10^{-6}$

$= \boxed{2.00\times10^{-4}\%}$

$L_f = (1.00\text{ m})e^{\left[2.00\times10^{-2}\,°\text{C}^{-1}(100°\text{C})\right]} = 7.389\text{ m}$

$L_f' = 1.00\text{ m}\left[1 + 0.020\,0°\text{C}^{-1}(100°\text{C})\right] = 3.000\text{ m}$: $\dfrac{L_f - L_f'}{L_f} = \boxed{59.4\%}$

P19.56 At 20.0°C, the unstretched lengths of the steel and copper wires are

$$L_s(20.0°\text{C}) = (2.000\text{ m})\left[1 + 11.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,56\text{ m}$$
$$L_c(20.0°\text{C}) = (2.000\text{ m})\left[1 + 17.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,32\text{ m}$$

Under a tension F, the length of the steel and copper wires are

$$L_s' = L_s\left[1 + \frac{F}{YA}\right]_s \qquad L_c' = L_c\left[1 + \frac{F}{YA}\right]_c \qquad \text{where} \quad L_s' + L_c' = 4.000\text{ m}$$

Since the tension F must be the same in each wire, we solve for F:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{L_s/Y_sA_s + L_c/Y_cA_c}$$

When the wires are stretched, their areas become

$$A_s = \pi\left(1.000\times10^{-3}\text{ m}\right)^2\left[1 + \left(11.0\times10^{-6}\right)(-20.0)\right]^2 = 3.140\times10^{-6}\text{ m}^2$$
$$A_c = \pi\left(1.000\times10^{-3}\text{ m}\right)^2\left[1 + \left(17.0\times10^{-6}\right)(-20.0)\right]^2 = 3.139\times10^{-6}\text{ m}^2$$

Recall $Y_s = 20.0\times10^{10}$ Pa and $Y_c = 11.0\times10^{10}$ Pa. Substituting into the equation for F, we obtain

$$F = \frac{4.000\text{ m} - (1.999\,56\text{ m} + 1.999\,32\text{ m})}{[1.999\,56\text{ m}]/\left[\left(20.0\times10^{10}\text{ Pa}\right)\left(3.140\times10^{-6}\right)\text{ m}^2\right] + [1.999\,32\text{ m}]/\left[\left(11.0\times10^{10}\text{ Pa}\right)\left(3.139\times10^{-6}\right)\text{ m}^2\right]}$$

$$F = \boxed{125\text{ N}}$$

To find the x-coordinate of the junction,

$$L_s' = (1.999\,56\text{ m})\left[1 + \frac{125\text{ N}}{\left(20.0\times10^{10}\text{ N/m}^2\right)\left(3.140\times10^{-6}\text{ m}^2\right)}\right] = 1.999\,958\text{ m}$$

Thus the x-coordinate is $-2.000 + 1.999\,958 = \boxed{-4.20\times10^{-5}\text{ m}}$

P19.57 (a) $\mu = \pi r^2 \rho = \pi\left(5.00\times10^{-4}\ \text{m}\right)^2\left(7.86\times10^3\ \text{kg/m}^3\right) = \boxed{6.17\times10^{-3}\ \text{kg/m}}$

(b) $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$ so $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Therefore,

$$T = \mu\left(2Lf_1\right)^2 = \left(6.17\times10^{-3}\right)\left(2\times0.800\times200\right)^2 = \boxed{632\ \text{N}}$$

(c) First find the unstressed length of the string at 0°C:

$$L = L_{\text{natural}}\left(1+\frac{T}{AY}\right) \quad \text{so} \quad L_{\text{natural}} = \frac{L}{1+T/AY}$$

$$A = \pi\left(5.00\times10^{-4}\ \text{m}\right)^2 = 7.854\times10^{-7}\ \text{m}^2 \quad \text{and} \quad Y = 20.0\times10^{10}\ \text{Pa}$$

Therefore,

$$\frac{T}{AY} = \frac{632}{\left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)} = 4.02\times10^{-3}\text{, and}$$

$$L_{\text{natural}} = \frac{(0.800\ \text{m})}{\left(1+4.02\times10^{-3}\right)} = 0.796\,8\ \text{m}$$

The unstressed length at 30.0°C is

$$L_{30^\circ\text{C}} = L_{\text{natural}}\left[1+\alpha\left(30.0^\circ\text{C}-0.0^\circ\text{C}\right)\right]\text{, or}$$

$$L_{30^\circ\text{C}} = \left(0.796\,8\ \text{m}\right)\left[1+\left(11.0\times10^{-6}\right)(30.0)\right] = 0.797\,06\ \text{m}$$

Since $L = L_{30^\circ\text{C}}\left[1+\dfrac{T'}{AY}\right]$, where T' is the tension in the string at 30.0°C,

$$T' = AY\left[\frac{L}{L_{30^\circ\text{C}}}-1\right] = \left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)\left[\frac{0.800}{0.797\,06}-1\right] = \boxed{580\ \text{N}}$$

To find the frequency at 30.0°C, realize that

$$\frac{f_1'}{f_1} = \sqrt{\frac{T'}{T}} \quad \text{so} \quad f_1' = (200\ \text{Hz})\sqrt{\frac{580\ \text{N}}{632\ \text{N}}} = \boxed{192\ \text{Hz}}$$

P19.58 Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to

$$n_{i1}+n_{i2} = n_{f1}+n_{f2}$$

Assuming the gas is ideal, we apply $n = \dfrac{PV}{RT}$ to each term:

$$\frac{P_iV_0}{(300\ \text{K})R}+\frac{P_i\left(4V_0\right)}{(300\ \text{K})R} = \frac{P_fV_0}{(673\ \text{K})R}+\frac{P_f\left(4V_0\right)}{(300\ \text{K})R}$$

$$1\ \text{atm}\left(\frac{5}{300\ \text{K}}\right) = P_f\left(\frac{1}{673\ \text{K}}+\frac{4}{300\ \text{K}}\right) \qquad \boxed{P_f = 1.12\ \text{atm}}$$

P19.59 Let 2θ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i(1+\alpha\Delta T) \qquad \text{and} \qquad \sin\theta = \frac{\frac{L_i}{2}}{R} = \frac{L_i}{2R}$$

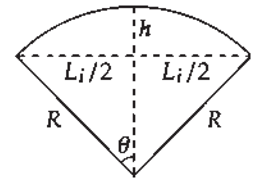

FIG. P19.59

Thus,

$$\theta = \frac{L_i}{2R}(1+\alpha\Delta T) = (1+\alpha\Delta T)\sin\theta$$

and we must solve the transcendental equation $\theta = (1+\alpha\Delta T)\sin\theta = (1.000\,005\,5)\sin\theta$
Your calculator is likely to want to find the zero solution.

Homing in on the nonzero solution gives, to four digits, $\theta = 0.018\,16 \text{ rad} = 1.040\,5°$

Now, $h = R - R\cos\theta = \dfrac{L_i(1-\cos\theta)}{2\sin\theta}$

This yields $\boxed{h = 4.54 \text{ m}}$, a remarkably large value compared to $\Delta L = 5.50$ cm.

P19.60 (a) Let xL represent the distance of the stationary line below the top edge of the plate. The normal force on the lower part of the plate is $mg(1-x)\cos\theta$ and the force of kinetic friction on it is $\mu_k mg(1-x)\cos\theta$ up the roof. Again, $\mu_k mgx\cos\theta$ acts down the roof on the upper part of the plate. The near-equilibrium of the plate requires $\sum F_x = 0$

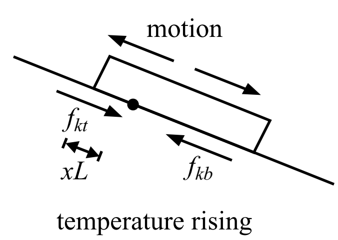

FIG. P19.60(a)

$$-\mu_k mgx\cos\theta + \mu_k mg(1-x)\cos\theta - mg\sin\theta = 0$$
$$-2\mu_k mgx\cos\theta = mg\sin\theta - \mu_k mg\cos\theta$$
$$2\mu_k x = \mu_k - \tan\theta$$
$$x = \frac{1}{2} - \frac{\tan\theta}{2\mu_k}$$

and the stationary line is indeed below the top edge by $xL = \dfrac{L}{2}\left(1 - \dfrac{\tan\theta}{\mu_k}\right)$

(b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg(1-x)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a) and the stationary line is above the bottom edge by $xL = \dfrac{L}{2}\left(1 - \dfrac{\tan\theta}{\mu_k}\right)$

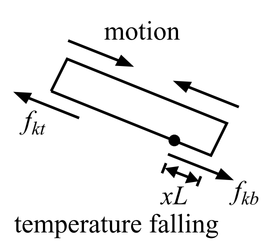

FIG. P19.60(b)

continued on next page

(c) Start thinking about the plate at dawn, as the temperature starts to rise. As in part (a), a line at distance xL below the top edge of the plate stays stationary relative to the roof as long as the temperature rises. The point P on the plate at distance xL above the bottom edge is destined to become the fixed point when the temperature starts falling. As the temperature rises, this point moves down the roof because of the expansion of the central part of the plate. Its displacement for the day is

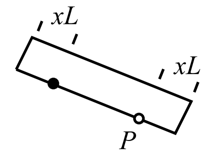

FIG. P19.60(c)

$$\begin{aligned}\Delta L &= (\alpha_2-\alpha_1)(L-xL-xL)\Delta T\\ &=(\alpha_2-\alpha_1)\left[L-2\frac{L}{2}\left(1-\frac{\tan\theta}{\mu_k}\right)\right](T_h-T_c)\\ &=(\alpha_2-\alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h-T_c)\end{aligned}$$

At dawn the next day the point P is farther down the roof by the distance ΔL. It represents the displacement of every other point on the plate.

(d) $$(\alpha_2-\alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h-T_c)=\left(24\times10^{-6}\frac{1}{\text{C}^\circ}-15\times10^{-6}\frac{1}{\text{C}^\circ}\right)\frac{1.20\text{ m}\tan18.5^\circ}{0.42}32^\circ\text{C}$$
$$=\boxed{0.275\text{ mm}}$$

(e) If $\alpha_2<\alpha_1$, the diagram in part (a) applies to temperature falling and the diagram in part (b) applies to temperature rising. The weight of the plate still pulls it step by step down the roof. The same expression describes how far it moves each day.

ANSWERS TO EVEN PROBLEMS

P19.2 (a) 810°F (b) 450 K

P19.4 (a) 1 337 K and 2 993 K (b) 1 596°C = 1 596 K

P19.6 1.20 cm

P19.8 0.663 mm to the right at 78.2° below the horizontal

P19.10 (a) 437°C (b) 2 100°C This will not work because aluminum melts at 660°C.

P19.12 (a) $2.52\times10^6\ \text{N/m}^2$ (b) no

P19.14 (a) 396 N (b) –101°C (c) The original length divides out, so the answers would not change.

P19.16 8.72×10^{11} atoms/s

P19.18 (a) 2.99 mol (b) 1.80×10^{24} molecules

P19.20 1.61 MPa

P19.22 In each pump-up-and-discharge cycle, the volume of air in the tank doubles. Thus 1.00 L of water is driven out by the air injected at the first pumping, 2.00 L by the second, and only the remaining 1.00 L by the third. Each person could more efficiently use his device by starting with the tank half full of water, instead of 80% full.

P19.24 3.67 cm^3

P19.26 between 10^1 kg and 10^2 kg

P19.28 2.41×10^{11} molecules

P19.30 $m_1 - m_2 = \frac{P_0 VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

P19.32 0.523 kg

P19.34 (a) see the solution (b) We have $\beta - 3\alpha \approx \beta$ within better than 6%. The value of α is typically so small compared to β that it can be ignored in the equation for a good approximation.

P19.36 (a) $2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2}\right)$ (b) Yes; yes. (c) $2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right)$ (d) 61.0° and 59.6°

P19.38 (a) 0.169 m (b) 1.35×10^5 Pa

P19.40 (a) $\theta = \frac{(\alpha_2 - \alpha_1)L_i \Delta T}{\Delta r}$ (b) see the solution (c) it bends the other way (d) 0.830°

P19.42 (a) The different diameters of the arms of the U-tube do not affect the pressures exerted by the liquids of different density on the liquid in the base. Because the base of the U-tube is horizontal, the pattern of temperature change in the base does not affect the equilibrium heights.

(b) $\beta = \frac{1}{T_C}\left(\frac{h_t}{h_0} - 1\right)$

P19.44 (a) increase by 95.0 μs (b) loses 57.5 s

P19.46 (a) $B = \rho g P_0 V_i (P_0 + \rho g d)^{-1}$ up (b) decrease (c) 10.3 m

P19.48 $y \approx L(\alpha \Delta T/2)^{1/2}$

P19.50 (a) see the solution (b) $3.66 \times 10^{-3}\ K^{-1}$, within 0.06% and 0.2% of the experimental values

P19.52 (a) Yes: it increases. As the disk cools, its radius, and hence its moment of inertia, decreases. Conservation of angular momentum then requires that its angular speed increase. (b) 25.7 rad/s

P19.54 (a) 7.06 mm (b) 297 K

P19.56 125 N; –42.0 μm

P19.58 1.12 atm

P19.60 (a), (b), (c) see the solution (d) 0.275 mm (e) The plate creeps down the roof each day by an amount given by the same expression.