32

Inductance

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q32.1 The coil has an inductance regardless of the nature of the current in the circuit. Inductance depends only on the coil geometry and its construction. Since the current is constant, the self-induced emf in the coil is zero, and the coil does not affect the steady-state current. (We assume the resistance of the coil is negligible.)

Q32.2 The inductance of a coil is determined by (a) the geometry of the coil and (b) the "contents" of the coil. This is similar to the parameters that determine the capacitance of a capacitor and the resistance of a resistor. With an inductor, the most important factor in the geometry is the number of turns of wire, or turns per unit length. By the "contents" we refer to the material in which the inductor establishes a magnetic field, notably the magnetic properties of the core around which the wire is wrapped.

***Q32.3** The emf across an inductor is zero whenever the current is constant, large or small. Answer (d).

***Q32.4** The fine wire has considerable resistance, so a few seconds is many time constants. The final current is not affected by the inductance of the coil. Answer (c).

***Q32.5** The inductance of a solenoid is proportional to the number of turns squared, to the cross-sectional area, and to the reciprocal of the length. Coil A has twice as many turns with the same length of wire, so its circumference must be half as large as that of coil B. Its radius is half as large and its area one quarter as large. For coil A the inductance will be different by the factor $2^2(1/4)(1/2) = 1/2$. Answer (e).

Q32.6 When it is being opened. When the switch is initially standing open, there is no current in the circuit. Just after the switch is then closed, the inductor tends to maintain the zero-current condition, and there is very little chance of sparking. When the switch is standing closed, there is current in the circuit. When the switch is then opened, the current rapidly decreases. The induced emf is created in the inductor, and this emf tends to maintain the original current. Sparking occurs as the current bridges the air gap between the contacts of the switch.

***Q32.7** Just before the switch is thrown, the voltage across the twelve-ohm resistor is very nearly 12 V. Just after the switch is thrown, the current is nearly the same, maintained by the inductor. The voltage across the 1 200-Ω resistor is then much more than 12 V. By Kirchhoff's loop rule, the voltage across the coil is larger still: $\Delta V_L > \Delta V_{1\,200\,\Omega} > 12.0\text{ V} > \Delta V_{12\,\Omega}$.

***Q32.8** (i) (a) The bulb glows brightly right away, and then more and more faintly as the capacitor charges up. (b) The bulb gradually gets brighter and brighter, changing rapidly at first and then more and more slowly. (c) The bulb gradually gets brighter and brighter. (d) The bulb glows brightly right away, and then more and more faintly as the inductor starts carrying more and more current.

(ii) (a) The bulb goes out immediately. (b) The bulb glows for a moment as a spark jumps across the switch. (c) The bulb stays lit for a while, gradually getting fainter and fainter. (d) The bulb suddenly glows brightly. Then its brightness decreases to zero, changing rapidly at first and then more and more slowly.

***Q32.9** The wire's magnetic field goes in circles around it. We want this field to "shine" perpendicularly through the area of the coil. Answer (c).

Q32.10 A physicist's list of constituents of the universe in 1829 might include matter, light, heat, the stuff of stars, charge, momentum, and several other entries. Our list today might include the quarks, electrons, muons, tauons, and neutrinos of matter; gravitons of gravitational fields; photons of electric and magnetic fields; W and Z particles; gluons; energy; momentum; angular momentum; charge; baryon number; three different lepton numbers; upness; downness; strangeness; charm; topness; and bottomness. Alternatively, the relativistic interconvertibility of mass and energy, and of electric and magnetic fields, can be used to make the list look shorter. Some might think of the conserved quantities energy, momentum, … bottomness as properties of matter, rather than as things with their own existence. The idea of a field is not due to Henry, but rather to Faraday, to whom Henry personally demonstrated self-induction. Still the thesis stated in the question has an important germ of truth. Henry precipitated a basic change if he did not cause it. The biggest difference between the two lists is that the 1829 list does not include fields and today's list does.

***Q32.11** The energy stored in the magnetic field of an inductor is proportional to the square of the current. Doubling I makes $U = \frac{1}{2}LI^2$ get four times larger. Answer (a).

***Q32.12** Cutting the number of turns in half makes the inductance four times smaller. Doubling the current would by itself make the stored energy four times larger, to just compensate. Answer (b).

Q32.13 The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. On the other hand, we can think of Henry's discovery of self-inductance as fundamentally new. Before a certain school vacation at the Albany Academy about 1830, one could visualize the universe as consisting of only one thing, matter. All the forms of energy then known (kinetic, gravitational, elastic, internal, electrical) belonged to chunks of matter. But the energy that temporarily maintains a current in a coil after the battery is removed is not energy that belongs to any bit of matter. This energy is vastly larger than the kinetic energy of the drifting electrons in the wires. This energy belongs to the magnetic field around the coil. Beginning in 1830, Nature has forced us to admit that the universe consists of matter and also of fields, massless and invisible, known only by their effects.

***Q32.14** (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$$\boxed{I_L = 0,\ I_C = \frac{\mathcal{E}_0}{R},\ I_R = \frac{\mathcal{E}_0}{R}}$$

$$\boxed{\Delta V_L = \mathcal{E}_0,\ \Delta V_C = 0,\ \Delta V_R = \mathcal{E}_0}$$

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$$\boxed{I_L = 0,\ I_C = 0,\ I_R = 0}$$

$$\boxed{\Delta V_L = 0,\ \Delta V_C = \mathcal{E}_0,\ \Delta V_R = 0}$$

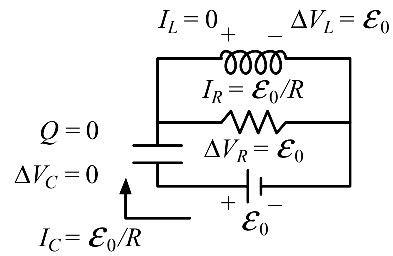


Figure (a)

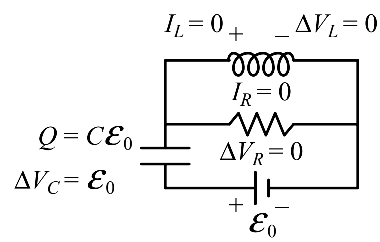


Figure (b)

FIG. Q32.14

***Q32.15** (i) Answer (a). The mutual inductance of two loops in free space—that is, ignoring the use of cores—is a maximum if the loops are coaxial. In this way, the maximum flux of the primary loop will pass through the secondary loop, generating the largest possible emf given the changing magnetic field due to the first.

(ii) Answer (c). The mutual inductance is a minimum if the magnetic field of the first coil lies in the plane of the second coil, producing no flux through the area the second coil encloses.

Q32.16 When the capacitor is fully discharged, the current in the circuit is a maximum. The inductance of the coil is making the current continue to flow. At this time the magnetic field of the coil contains all the energy that was originally stored in the charged capacitor. The current has just finished discharging the capacitor and is proceeding to charge it up again with the opposite polarity.

Q32.17 If $R > \sqrt{\frac{4L}{C}}$, then the oscillator is overdamped—it will not oscillate. If $R < \sqrt{\frac{4L}{C}}$, then the oscillator is underdamped and can go through several cycles of oscillation before the radiated signal falls below background noise.

Q32.18 An object cannot exert a net force on itself. An object cannot create momentum out of nothing. A coil can induce an emf in itself. When it does so, the actual forces acting on charges in different parts of the loop add as vectors to zero. The term electromotive force does not refer to a force, but to a voltage.

SOLUTIONS TO PROBLEMS

Section 32.1 **Self-Induction and Inductance**

P32.1 $$\overline{\mathcal{E}} = -L\frac{\Delta I}{\Delta t} = (-2.00\text{ H})\left(\frac{0 - 0.500\text{ A}}{0.010\,0\text{ s}}\right)\left(\frac{1\text{ V}\cdot\text{s}}{1\text{ H}\cdot\text{A}}\right) = \boxed{100\text{ V}}$$

P32.2 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(70.0)^2\,\pi(6.50\times10^{-3}\text{ m})^2}{0.600\text{ m}} = \boxed{1.36\ \mu\text{H}}$$

***P32.3** $\mathcal{E}=-L\dfrac{dI}{dt}=-L\dfrac{d}{dt}(I_{\max}\sin\omega t)=-L\omega I_{\max}\cos\omega t=-(10.0\times10^{-3})(120\pi)(5.00)\cos\omega t$

$\mathcal{E}=-(6.00\pi)\cos(120\pi t)=\boxed{-(18.8\text{ V})\cos(377t)}$

P32.4 From $|\mathcal{E}|=L\left(\dfrac{\Delta I}{\Delta t}\right)$, we have $L=\dfrac{\mathcal{E}}{(\Delta I/\Delta t)}=\dfrac{24.0\times10^{-3}\text{ V}}{10.0\text{ A/s}}=2.40\times10^{-3}\text{ H}$

From $L=\dfrac{N\Phi_B}{I}$, we have $\Phi_B=\dfrac{LI}{N}=\dfrac{(2.40\times10^{-3}\text{ H})(4.00\text{ A})}{500}=\boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$

P32.5 $L=\dfrac{\mu_0N^2A}{\ell}=\dfrac{\mu_0(420)^2(3.00\times10^{-4})}{0.160}=4.16\times10^{-4}\text{ H}$

$\mathcal{E}=-L\dfrac{dI}{dt}\rightarrow\dfrac{dI}{dt}=\dfrac{-\mathcal{E}}{L}=\dfrac{-175\times10^{-6}\text{ V}}{4.16\times10^{-4}\text{ H}}=\boxed{-0.421\text{ A/s}}$

P32.6 $|\mathcal{E}|=L\dfrac{dI}{dt}=(90.0\times10^{-3})\dfrac{d}{dt}(t^2-6t)\text{ V}$

(a) At $t=1.00$ s, $\mathcal{E}=\boxed{360\text{ mV}}$

(b) At $t=4.00$ s, $\mathcal{E}=\boxed{180\text{ mV}}$

(c) $\mathcal{E}=(90.0\times10^{-3})(2t-6)=0$

when $\boxed{t=3.00\text{ s}}$

P32.7 (a) $B=\mu_0nI=\mu_0\left(\dfrac{450}{0.120}\right)(0.040\,0\text{ A})=\boxed{188\ \mu\text{T}}$

(b) $\Phi_B=BA=\boxed{3.33\times10^{-8}\text{ T}\cdot\text{m}^2}$

(c) $L=\dfrac{N\Phi_B}{I}=\boxed{0.375\text{ mH}}$

(d) $\boxed{B\text{ and }\Phi_B\text{ are proportional to current; }L\text{ is independent of current.}}$

P32.8 $L=\dfrac{N\Phi_B}{I}=\dfrac{NBA}{I}\approx\dfrac{NA}{I}\cdot\dfrac{\mu_0NI}{2\pi R}=\boxed{\dfrac{\mu_0N^2A}{2\pi R}}$

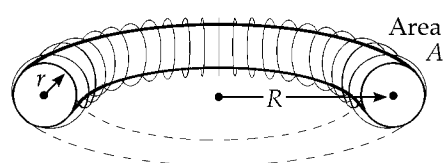


FIG. P32.8

P32.9 $\mathcal{E}=\mathcal{E}_0e^{-kt}=-L\dfrac{dI}{dt}$

$dI=-\dfrac{\mathcal{E}_0}{L}e^{-kt}dt$

If we require $I\rightarrow0$ as $t\rightarrow\infty$, the solution is $I=\dfrac{\mathcal{E}_0}{kL}e^{-kt}=\dfrac{dq}{dt}$

$Q=\int Idt=\int_0^\infty\dfrac{\mathcal{E}_0}{kL}e^{-kt}dt=-\dfrac{\mathcal{E}_0}{k^2L}$ $\boxed{|Q|=\dfrac{\mathcal{E}_0}{k^2L}}$

Section 32.2 ***RL* Circuits**

P32.10 Taking $\tau = \dfrac{L}{R}$, $I = I_i e^{-t/\tau}$: $\dfrac{dI}{dt} = I_i e^{-t/\tau}\left(-\dfrac{1}{\tau}\right)$

$IR + L\dfrac{dI}{dt} = 0$ will be true if $I_i R\, e^{-t/\tau} + L\left(I_i e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right) = 0$

Because $\tau = \dfrac{L}{R}$, we have agreement with $0 = 0$.

P32.11 (a) At time t, $I(t) = \dfrac{\mathcal{E}\left(1 - e^{-t/\tau}\right)}{R}$

where $\tau = \dfrac{L}{R} = 0.200\text{ s}$

After a long time, $I_{\max} = \dfrac{\mathcal{E}\left(1 - e^{-\infty}\right)}{R} = \dfrac{\mathcal{E}}{R}$

At $I(t) = 0.500 I_{\max}$ $(0.500)\dfrac{\mathcal{E}}{R} = \dfrac{\mathcal{E}\left(1 - e^{-t/0.200\text{ s}}\right)}{R}$

so $0.500 = 1 - e^{-t/0.200\text{ s}}$

Isolating the constants on the right, $\ln\left(e^{-t/0.200\text{ s}}\right) = \ln(0.500)$

and solving for t, $-\dfrac{t}{0.200\text{ s}} = -0.693$

or $t = \boxed{0.139\text{ s}}$

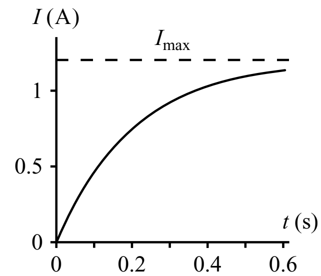


FIG. P32.11

(b) Similarly, to reach 90% of $I_{\max}$, $0.900 = 1 - e^{-t/\tau}$

and $t = -\tau \ln(1 - 0.900)$

Thus, $t = -(0.200\text{ s})\ln(0.100) = \boxed{0.461\text{ s}}$

***P32.12** The current increases from 0 to asymptotically approach 500 mA. In case (a) the current jumps up essentially instantaneously. In case (b) it increases with a longer time constant, and in case (c) the increase is still slower.

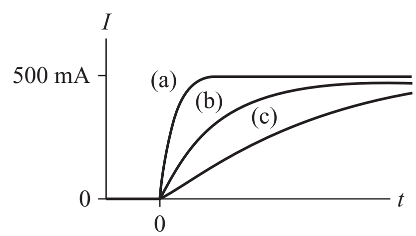


FIG. P32.12

P32.13 (a) $\tau = \dfrac{L}{R} = 2.00 \times 10^{-3} \text{ s} = \boxed{2.00 \text{ ms}}$

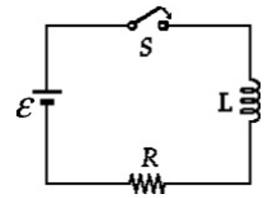


FIG. P32.13

(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00 \text{ V}}{4.00 \ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176 \text{ A}}$

(c) $I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00 \text{ V}}{4.00 \ \Omega} = \boxed{1.50 \text{ A}}$

(d) $0.800 = 1 - e^{-t/2.00 \text{ ms}} \rightarrow t = -(2.00 \text{ ms})\ln(0.200) = \boxed{3.22 \text{ ms}}$

P32.14 $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \dfrac{120}{9.00}\left(1 - e^{-1.80/7.00}\right) = 3.02 \text{ A}$

$\Delta V_R = IR = (3.02)(9.00) = 27.2 \text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = 120 - 27.2 = \boxed{92.8 \text{ V}}$

P32.15 (a) $\Delta V_R = IR = (8.00 \ \Omega)(2.00 \text{ A}) = 16.0 \text{ V}$

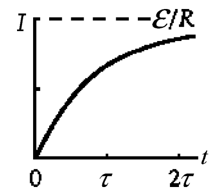


FIG. P32.15

and $\Delta V_L = \mathcal{E} - \Delta V_R = 36.0 \text{ V} - 16.0 \text{ V} = 20.0 \text{ V}$

Therefore, $\dfrac{\Delta V_R}{\Delta V_L} = \dfrac{16.0 \text{ V}}{20.0 \text{ V}} = \boxed{0.800}$

(b) $\Delta V_R = IR = (4.50 \text{ A})(8.00 \ \Omega) = 36.0 \text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

P32.16 After a long time, $12.0 \text{ V} = (0.200 \text{ A})R$. Thus, $R = 60.0 \ \Omega$. Now, $\tau = \dfrac{L}{R}$ gives

$L = \tau R = \left(5.00 \times 10^{-4} \text{ s}\right)(60.0 \text{ V/A}) = \boxed{30.0 \text{ mH}}$

P32.17 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\qquad \dfrac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right)$

$\tau = \dfrac{L}{R} = \dfrac{15.0 \text{ H}}{30.0 \ \Omega} = 0.500 \text{ s}$: $\qquad \dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^{-t/\tau}$ and $I_{\max} = \dfrac{\mathcal{E}}{R}$

(a) $t = 0$: $\dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^0 = \dfrac{\mathcal{E}}{L} = \dfrac{100 \text{ V}}{15.0 \text{ H}} = \boxed{6.67 \text{ A/s}}$

(b) $t = 1.50$ s: $\dfrac{dI}{dt} = \dfrac{\mathcal{E}}{L} e^{-t/\tau} = (6.67 \text{ A/s}) e^{-1.50/(0.500)} = (6.67 \text{ A/s}) e^{-3.00} = \boxed{0.332 \text{ A/s}}$

P32.18 Name the currents as shown. By Kirchhoff's laws:

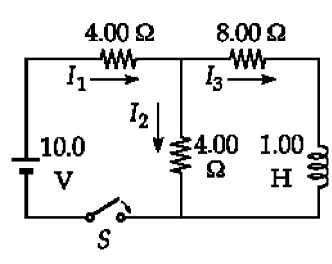


FIG. P32.18

$I_1 = I_2 + I_3 \qquad (1)$

$+10.0 \text{ V} - 4.00 I_1 - 4.00 I_2 = 0 \qquad (2)$

$+10.0 \text{ V} - 4.00 I_1 - 8.00 I_3 - (1.00)\dfrac{dI_3}{dt} = 0 \qquad (3)$

continued on next page

From (1) and (2), $+10.0-4.00I_1-4.00I_1+4.00I_3=0$

and $I_1=0.500I_3+1.25\text{ A}$

Then (3) becomes $10.0\text{ V}-4.00(0.500I_3+1.25\text{ A})-8.00I_3-(1.00)\dfrac{dI_3}{dt}=0$

$$(1.00\text{ H})\left(\frac{dI_3}{dt}\right)+(10.0\ \Omega)I_3=5.00\text{ V}$$

We solve the differential equation using equations from the chapter text:

$$I_3(t)=\left(\frac{5.00\text{ V}}{10.0\ \Omega}\right)\left[1-e^{-(10.0\ \Omega)t/1.00\text{ H}}\right]=\boxed{(0.500\text{ A})\left[1-e^{-10t/\text{s}}\right]}$$

$$I_1=1.25+0.500I_3=\boxed{1.50\text{ A}-(0.250\text{ A})e^{-10t/\text{s}}}$$

P32.19 (a) Using $\tau=RC=\dfrac{L}{R}$, we get $R=\sqrt{\dfrac{L}{C}}=\sqrt{\dfrac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}}=1.00\times10^3\ \Omega=\boxed{1.00\text{ k}\Omega}$.

(b) $\tau=RC=(1.00\times10^3\ \Omega)(3.00\times10^{-6}\text{ F})=3.00\times10^{-3}\text{ s}=\boxed{3.00\text{ ms}}$

P32.20 For $t\le0$, the current in the inductor is $\boxed{\text{zero}}$. At $t=0$, it starts to grow from zero toward 10.0 A with time constant

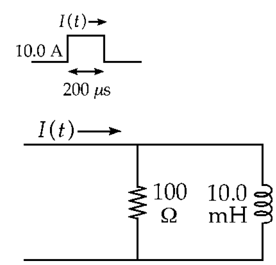


FIG. P32.20

$$\tau=\frac{L}{R}=\frac{(10.0\text{ mH})}{(100\ \Omega)}=1.00\times10^{-4}\text{ s}$$

For $0\le t\le200\,\mu\text{s}$, $I=I_{\max}\left(1-e^{-t/\tau}\right)=\boxed{(10.0\text{ A})\left(1-e^{-10\,000t/\text{s}}\right)}$.

At $t=200\ \mu\text{s}$, $I=(10.00\text{ A})\left(1-e^{-2.00}\right)=8.65\text{ A}$.

Thereafter, it decays exponentially as $I=I_ie^{-t'/\tau}$, so for $t\ge200\ \mu\text{s}$,

$$I=(8.65\text{ A})e^{-10\,000(t-200\ \mu\text{s})/\text{s}}=(8.65\text{ A})e^{-10\,000t/\text{s}+2.00}=\left(8.65e^{2.00}\text{ A}\right)e^{-10\,000t/\text{s}}=\boxed{(63.9\text{ A})e^{-10\,000t/\text{s}}}$$

P32.21 $\tau=\dfrac{L}{R}=\dfrac{0.140}{4.90}=28.6\text{ ms}$

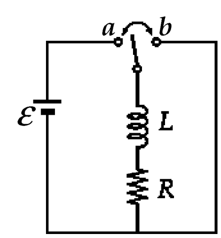


FIG. P32.21

$$I_{\max}=\frac{\mathcal{E}}{R}=\frac{6.00\text{ V}}{4.90\ \Omega}=1.22\text{ A}$$

(a) $I=I_{\max}\left(1-e^{-t/\tau}\right)$ so $0.220=1.22\left(1-e^{-t/\tau}\right)$

$e^{-t/\tau}=0.820$: $t=-\tau\ln(0.820)=\boxed{5.66\text{ ms}}$

(b) $I=I_{\max}\left(1-e^{-10.0/0.028\,6}\right)=(1.22\text{ A})\left(1-e^{-350}\right)=\boxed{1.22\text{ A}}$

(c) $I=I_{\max}e^{-t/\tau}$ and $0.160=1.22e^{-t/\tau}$

so $t=-\tau\ln(0.131)=\boxed{58.1\text{ ms}}$

P32.22 (a) For a series connection, both inductors carry equal currents at every instant, so $\frac{dI}{dt}$ is the same for both. The voltage across the pair is

$$L_{\text{eq}}\frac{dI}{dt}=L_1\frac{dI}{dt}+L_2\frac{dI}{dt} \qquad \text{so} \qquad \boxed{L_{\text{eq}}=L_1+L_2}$$

(b) $$L_{\text{eq}}\frac{dI}{dt}=L_1\frac{dI_1}{dt}=L_2\frac{dI_2}{dt}=\Delta V_L \qquad \text{where} \qquad I=I_1+I_2 \text{ and } \frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}$$

Thus, $\frac{\Delta V_L}{L_{\text{eq}}}=\frac{\Delta V_L}{L_1}+\frac{\Delta V_L}{L_2}$ and $\boxed{\frac{1}{L_{\text{eq}}}=\frac{1}{L_1}+\frac{1}{L_2}}$

(c) $$L_{\text{eq}}\frac{dI}{dt}+R_{\text{eq}}I=L_1\frac{dI}{dt}+IR_1+L_2\frac{dI}{dt}+IR_2$$

Now I and $\frac{dI}{dt}$ are separate quantities under our control, so functional equality requires

both $\boxed{L_{\text{eq}}=L_1+L_2 \quad \text{and} \quad R_{\text{eq}}=R_1+R_2}$.

(d) $$\Delta V=L_{\text{eq}}\frac{dI}{dt}+R_{\text{eq}}I=L_1\frac{dI_1}{dt}+R_1I_1=L_2\frac{dI_2}{dt}+R_2I_2 \quad \text{where } I=I_1+I_2 \text{ and } \frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}.$$

We may choose to keep the currents constant in time. Then, $\frac{1}{R_{\text{eq}}}=\frac{1}{R_1}+\frac{1}{R_2}$

We may choose to make the current swing through 0. Then, $\frac{1}{L_{\text{eq}}}=\frac{1}{L_1}+\frac{1}{L_2}$

This equivalent coil with resistance will be equivalent to the pair of real inductors for all other currents as well.

Section 32.3 **Energy in a Magnetic Field**

P32.23 $$L=\mu_0\frac{N^2A}{\ell}=\mu_0\frac{(68.0)^2\left[\pi\left(0.600\times10^{-2}\right)^2\right]}{0.080\ 0}=8.21\ \mu\text{H}$$

$$U=\frac{1}{2}LI^2=\frac{1}{2}\left(8.21\times10^{-6}\ \text{H}\right)\left(0.770\ \text{A}\right)^2=\boxed{2.44\ \mu\text{J}}$$

P32.24 (a) The magnetic energy density is given by

$$u=\frac{B^2}{2\mu_0}=\frac{(4.50\ \text{T})^2}{2\left(1.26\times10^{-6}\ \text{T}\cdot\text{m/A}\right)}=\boxed{8.06\times10^6\ \text{J/m}^3}$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U=uV=\left(8.06\times10^6\ \text{J/m}^3\right)\left[(0.260\ \text{m})\pi(0.031\ 0\ \text{m})^2\right]=\boxed{6.32\ \text{kJ}}$$

P32.25 $u=\epsilon_0\frac{E^2}{2}=\boxed{44.2\ \text{nJ/m}^3}$ $\qquad u=\frac{B^2}{2\mu_0}=\boxed{995\ \mu\text{J/m}^3}$

P32.26 $\int_0^\infty e^{-2Rt/L}dt = -\frac{L}{2R}\int_0^\infty e^{-2Rt/L}\left(\frac{-2Rdt}{L}\right) = -\frac{L}{2R}e^{-2Rt/L}\Big|_0^\infty = -\frac{L}{2R}\left(e^{-\infty}-e^0\right) = \frac{L}{2R}(0-1) = \boxed{\frac{L}{2R}}$

***P32.27** (a) $\mathcal{P} = I\Delta V = 3\text{A } 22\text{ V} = \boxed{66.0\text{ W}}$

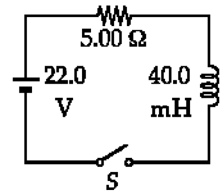


FIG. P32.27

(b) $\mathcal{P} = I\Delta V_R = I^2R = (3\text{A})^2\, 5\ \Omega = \boxed{45.0\text{ W}}$

(c) When the current is 3.00 A, Kirchhoff's loop rule reads
$+22.0\text{ V} - (3.00\text{ A})(5.00\ \Omega) - \Delta V_L = 0.$

Then $\Delta V_L = 7.00\text{ V}$

The power being stored in the inductor is

$$I\Delta V_L = (3.00\text{ A})(7.00\text{ V}) = \boxed{21.0\text{ W}}$$

(d) At all instants after the connection is made, the battery power is equal to the sum of the power delivered to the resistor and the power delivered to the magnetic field. Just after $t = 0$ the resistor power is nearly zero, and the battery power is nearly all going into the magnetic field. Long after the connection is made, the magnetic field is absorbing no more power and the battery power is going into the resistor.

P32.28 From the equation derived in the text, $I = \frac{\mathcal{E}}{R}\left(1-e^{-Rt/L}\right)$

(a) The maximum current, after a long time t, is $I = \frac{\mathcal{E}}{R} = 2.00\text{ A}$

At that time, the inductor is fully energized and $\mathcal{P} = I(\Delta V) = (2.00\text{ A})(10.0\text{ V}) = \boxed{20.0\text{ W}}$.

(b) $\mathcal{P}_{\text{lost}} = I^2R = (2.00\text{ A})^2(5.00\ \Omega) = \boxed{20.0\text{ W}}$

(c) $\mathcal{P}_{\text{inductor}} = I\left(\Delta V_{\text{drop}}\right) = \boxed{0}$

(d) $U = \frac{LI^2}{2} = \frac{(10.0\text{ H})(2.00\text{ A})^2}{2} = \boxed{20.0\text{ J}}$

P32.29 The total magnetic energy is the volume integral of the energy density, $u = \frac{B^2}{2\mu_0}$.

Because B changes with position, u is not constant. For $B = B_0\left(\frac{R}{r}\right)^2$, $u = \left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$.

Next, we set up an expression for the magnetic energy in a spherical shell of radius r and thickness dr. Such a shell has a volume $4\pi r^2 dr$, so the energy stored in it is

$$dU = u\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0^2R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U = \boxed{\frac{2\pi B_0^2R^3}{\mu_0}} = \frac{2\pi\left(5.00\times10^{-5}\text{ T}\right)^2\left(6.00\times10^6\text{ m}\right)^3}{\left(1.26\times10^{-6}\text{ T}\cdot\text{m/A}\right)} = \boxed{2.70\times10^{18}\text{ J}}$$

Section 32.4 **Mutual Inductance**

P32.30 $I_1(t) = I_{\max} e^{-\alpha t} \sin \omega t$ with $I_{\max} = 5.00$ A, $\alpha = 0.025\,0\ \mathrm{s}^{-1}$, and $\omega = 377$ rad/s

$$\frac{dI_1}{dt} = I_{\max} e^{-\alpha t}(-\alpha \sin \omega t + \omega \cos \omega t)$$

At $t = 0.800$ s,
$$\frac{dI_1}{dt} = (5.00\ \mathrm{A/s})e^{-0.020\,0}\big[-(0.025\,0)\sin(0.800(377)) + 377\cos(0.800(377))\big]$$

$$\frac{dI_1}{dt} = 1.85 \times 10^3\ \mathrm{A/s}$$

Thus, $\mathcal{E}_2 = -M\dfrac{dI_1}{dt}$: $\quad M = \dfrac{-\mathcal{E}_2}{dI_1/dt} = \dfrac{+3.20\ \mathrm{V}}{1.85\times 10^3\ \mathrm{A/s}} = \boxed{1.73\ \mathrm{mH}}$

P32.31 $\mathcal{E}_2 = -M\dfrac{dI_1}{dt} = -(1.00\times 10^{-4}\ \mathrm{H})(1.00\times 10^4\ \mathrm{A/s})\cos(1\,000t)$

$(\mathcal{E}_2)_{\max} = \boxed{1.00\ \mathrm{V}}$

P32.32 Assume the long wire carries current *I*. Then the magnitude of the magnetic field it generates at distance *x* from the wire is $B = \dfrac{\mu_0 I}{2\pi x}$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int B(\ell\,dx) = \frac{\mu_0 I \ell}{2\pi}\int_{0.400\ \mathrm{mm}}^{1.70\ \mathrm{mm}} \frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{1.70}{0.400}\right)$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 \mu_0 I \ell}{2\pi I}\ln\left(\frac{1.70}{0.400}\right) = \frac{N_2 \mu_0 \ell}{2\pi}(1.45) = \frac{1(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(2.70\times 10^{-3}\ \mathrm{m})}{2\pi}(1.45)$$

$M = 7.81\times 10^{-10}\ \mathrm{H} = \boxed{781\ \mathrm{pH}}$

P32.33 (a) $M = \dfrac{N_B \Phi_{BA}}{I_A} = \dfrac{700(90.0\times 10^{-6})}{3.50} = \boxed{18.0\ \mathrm{mH}}$

(b) $L_A = \dfrac{\Phi_A}{I_A} = \dfrac{400(300\times 10^{-6})}{3.50} = \boxed{34.3\ \mathrm{mH}}$

(c) $\mathcal{E}_B = -M\dfrac{dI_A}{dt} = -(18.0\ \mathrm{mH})(0.500\ \mathrm{A/s}) = \boxed{-9.00\ \mathrm{mV}}$

P32.34 (a) Solenoid 1 creates nearly uniform field everywhere inside it, given by $\mu_0 N_1 I/\ell$
The flux though one turn of solenoid 2 is $\mu_0 \pi R_2^2 N_1 I/\ell$
The emf induced in solenoid 2 is $-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(dI/dt)$
The mutual inductance is $\mu_0 \pi R_2^2 N_1 N_2/\ell$

(b) Solenoid 2 creates nearly uniform field everywhere inside it, given by $\mu_0 N_2 I/\ell$ and nearly zero field outside.
The flux though one turn of solenoid 1 is $\mu_0 \pi R_2^2 N_2 I/\ell$
The emf induced in solenoid 1 is $-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(dI/dt)$
The mutual inductance is $\mu_0 \pi R_2^2 N_1 N_2/\ell$

(c) The mutual inductances are the same. This is one example of von Neumann's rule, mentioned in the next problem.

P32.35 The large coil produces this field at the center of the small coil: $\dfrac{N_1\mu_0 I_1 R_1^2}{2\left(x^2+R_1^2\right)^{3/2}}$. The field is normal to the area of the small coil and nearly uniform over this area, so it produces flux

$\Phi_{12} = \dfrac{N_1\mu_0 I_1 R_1^2}{2\left(x^2+R_1^2\right)^{3/2}}\pi R_2^2$ through the face area of the small coil. When current I_1 varies, this is the emf induced in the small coil:

$$\mathcal{E}_2 = -N_2\frac{d}{dt}\frac{N_1\mu_0 R_1^2 \pi R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}I_1 = -\frac{N_1N_2\pi\mu_0 R_1^2 R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}\frac{dI_1}{dt} = -M\frac{dI_1}{dt} \text{ so } \boxed{M = \frac{N_1N_2\pi\mu_0 R_1^2 R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}}$$

P32.36 With $I = I_1 + I_2$, the voltage across the pair is:

$$\Delta V = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt} = -L_{\text{eq}}\frac{dI}{dt}$$

So, $$-\frac{dI_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{dI_2}{dt}$$

and $$-L_2\frac{dI_2}{dt} + \frac{M(\Delta V)}{L_1} + \frac{M^2}{L_1}\frac{dI_2}{dt} = \Delta V$$

$$\left(-L_1L_2 + M^2\right)\frac{dI_2}{dt} = \Delta V\left(L_1 - M\right)$$

By substitution, $$-\frac{dI_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{dI_1}{dt}$$

leads to $$\left(-L_1L_2 + M^2\right)\frac{dI_1}{dt} = \Delta V\left(L_2 - M\right)$$

Adding [1] to [2], $$\left(-L_1L_2 + M^2\right)\frac{dI}{dt} = \Delta V\left(L_1 + L_2 - 2M\right)$$

So, $$L_{\text{eq}} = -\frac{\Delta V}{dI/dt} = \boxed{\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

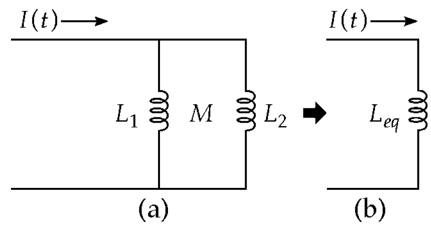


FIG. P32.36

Section 32.5 **Oscillations in an *LC* Circuit**

P32.37 At different times, $(U_C)_{\max} = (U_L)_{\max}$ so $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$

$$I_{\max} = \sqrt{\frac{C}{L}}(\Delta V)_{\max} = \sqrt{\frac{1.00\times10^{-6}\text{ F}}{10.0\times10^{-3}\text{ H}}}(40.0\text{ V}) = \boxed{0.400\text{ A}}$$

P32.38 $$\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max} \text{ so } (\Delta V_C)_{\max} = \sqrt{\frac{L}{C}}I_{\max} = \sqrt{\frac{20.0\times10^{-3}\text{ H}}{0.500\times10^{-6}\text{ F}}}(0.100\text{ A}) = \boxed{20.0\text{ V}}$$

P32.39 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_{\max} = \frac{\mathcal{E}}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop.

We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_{\max}^2$.

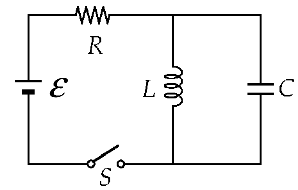


FIG. P32.39

$$\text{Then, } L = \frac{C(\Delta V)^2}{I_{\max}^2} = \frac{C(\Delta V)^2 R^2}{\mathcal{E}^2} = \frac{(0.500\times10^{-6}\text{ F})(150\text{ V})^2(250\ \Omega)^2}{(50.0\text{ V})^2} = \boxed{0.281\text{ H}}.$$

P32.40 (a) $$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.082\,0\text{ H})(17.0\times10^{-6}\text{ F})}} = \boxed{135\text{ Hz}}$$

(b) $$Q = Q_{\max}\cos\omega t = (180\ \mu\text{C})\cos(847\times0.001\,00) = \boxed{119\ \mu\text{C}}$$

(c) $$I = \frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t = -(847)(180)\sin(0.847) = \boxed{-114\text{ mA}}$$

P32.41 This radio is a radiotelephone on a ship, according to frequency assignments made by international treaties, laws, and decisions of the National Telecommunications and Information Administration.

The resonance frequency is $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Thus, $$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{\left[2\pi(6.30\times10^6\text{ Hz})\right]^2(1.05\times10^{-6}\text{ H})} = \boxed{608\text{ pF}}$$

P32.42 (a) $$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.100\text{ H})(1.00\times10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$$

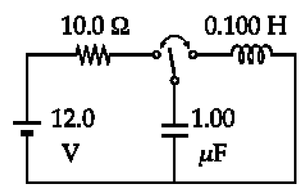


FIG. P32.42

(b) $$Q = C\mathcal{E} = (1.00\times10^{-6}\text{ F})(12.0\text{ V}) = \boxed{12.0\ \mu\text{C}}$$

(c) $$\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_{\max}^2$$

$$I_{\max} = \mathcal{E}\sqrt{\frac{C}{L}} = 12\text{ V}\sqrt{\frac{1.00\times10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$$

(d) At all times $$U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00\times10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{72.0\ \mu\text{J}}$$

P32.43 $$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(3.30\text{ H})(840\times10^{-12}\text{ F})}} = 1.899\times10^4\text{ rad/s}$$

$$Q = Q_{\max}\cos\omega t,\quad I = \frac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$$

(a) $$U_C = \frac{Q^2}{2C} = \frac{\left(\left[105\times10^{-6}\right]\cos\left[(1.899\times10^4\text{ rad/s})(2.00\times10^{-3}\text{ s})\right]\right)^2}{2(840\times10^{-12})} = \boxed{6.03\text{ J}}$$

continued on next page

(b) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2 Q_{\max}^2 \sin^2(\omega t) = \frac{Q_{\max}^2 \sin^2(\omega t)}{2C}$

$$U_L = \frac{(105\times10^{-6}\text{ C})^2 \sin^2\left[(1.899\times10^4\text{ rad/s})(2.00\times10^{-3}\text{ s})\right]}{2(840\times10^{-12}\text{ F})} = \boxed{0.529\text{ J}}$$

(c) $U_{\text{total}} = U_C + U_L = \boxed{6.56\text{ J}}$

Section 32.6 **The *RLC* Circuit**

P32.44 (a) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{(2.20\times10^{-3})(1.80\times10^{-6})} - \left(\frac{7.60}{2(2.20\times10^{-3})}\right)^2} = 1.58\times10^4\text{ rad/s}$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \boxed{2.51\text{ kHz}}$

(b) $R_c = \sqrt{\frac{4L}{C}} = \boxed{69.9\ \Omega}$

P32.45 (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500)(0.100\times10^{-6})}} = \boxed{4.47\text{ krad/s}}$

(b) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \boxed{4.36\text{ krad/s}}$

(c) $\frac{\Delta\omega}{\omega_0} = \boxed{2.53\%\text{ lower}}$

P32.46 Choose to call positive current clockwise in Figure 32.15. It drains charge from the capacitor according to $I = -\frac{dQ}{dt}$. A clockwise trip around the circuit then gives

$$+\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$$

$$+\frac{Q}{C} + \frac{dQ}{dt}R + L\frac{d}{dt}\frac{dQ}{dt} = 0,$$ identical with Equation 32.28.

P32.47 (a) $Q = Q_{\max}e^{-Rt/2L}\cos\omega_d t$ so $I_{\max} \propto e^{-Rt/2L}$

$0.500 = e^{-Rt/2L}$ and $\frac{Rt}{2L} = -\ln(0.500)$

$$t = -\frac{2L}{R}\ln(0.500) = \boxed{0.693\left(\frac{2L}{R}\right)}$$

(b) $U_0 \propto Q_{\max}^2$ and $U = 0.500U_0$ so $Q = \sqrt{0.500}Q_{\max} = 0.707Q_{\max}$

$$t = -\frac{2L}{R}\ln(0.707) = \boxed{0.347\left(\frac{2L}{R}\right)}$$ (half as long)

Additional Problems

P32.48 (a) Let Q represent the magnitude of the opposite charges on the plates of a parallel plate capacitor, the two plates having area A and separation d. The negative plate creates electric field $\vec{\mathbf{E}} = \dfrac{Q}{2\epsilon_0 A}$ toward itself. It exerts on the positive plate force $\vec{\mathbf{F}} = \dfrac{Q^2}{2\epsilon_0 A}$ toward the negative plate. The total field between the plates is $\dfrac{Q}{\epsilon_0 A}$. The energy density is $u_E = \dfrac{1}{2}\epsilon_0 E^2 = \dfrac{1}{2}\epsilon_0 \dfrac{Q^2}{\epsilon_0^2 A^2} = \dfrac{Q^2}{2\epsilon_0 A^2}$. Modeling this as a negative or inward pressure, we have for the force on one plate $F = PA = \dfrac{Q^2}{2\epsilon_0 A^2}$, in agreement with our first analysis.

(b) The lower of the two current sheets shown creates above it magnetic field $\vec{\mathbf{B}} = \dfrac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$. Let ℓ and w represent the length and width of each sheet. The upper sheet carries current $J_s w$ and feels force

$$\vec{\mathbf{F}} = I\vec{\ell}\times\vec{\mathbf{B}} = J_s w\ell\frac{\mu_0 J_s}{2}\hat{\mathbf{i}}\times\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 w\ell J_s^2}{2}\hat{\mathbf{j}}.$$

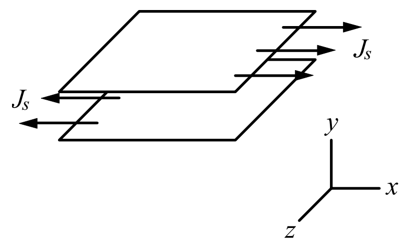


FIG. P32.48(b)

The force per area is $P = \dfrac{F}{\ell w} = \boxed{\dfrac{\mu_0 J_s^2}{2}}$.

(c) Between the two sheets the total magnetic field is $\dfrac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) + \dfrac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) = \mu_0 J_s \hat{\mathbf{k}}$, with magnitude $\boxed{B = \mu_0 J_s}$. Outside the space they enclose, the fields of the separate sheets are in opposite directions and add to $\boxed{\text{zero}}$.

(d) $u_B = \dfrac{1}{2\mu_0}B^2 = \dfrac{\mu_0^2 J_s^2}{2\mu_0} = \boxed{\dfrac{\mu_0 J_s^2}{2}}$

(e) This energy density agrees with the magnetic pressure found in part (b).

P32.49 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -(1.00\text{ mH})\dfrac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$

(b) $Q = \displaystyle\int_0^t I\,dt = \int_0^t (20.0t)\,dt = 10.0t^2$

$$\Delta V_C = \frac{-Q}{C} = \frac{-10.0t^2}{1.00\times 10^{-6}\text{ F}} = \boxed{-\left(10.0\text{ MV/s}^2\right)t^2}$$

(c) When $\dfrac{Q^2}{2C} \ge \dfrac{1}{2}LI^2$, or $\dfrac{\left(-10.0t^2\right)^2}{2\left(1.00\times 10^{-6}\right)} \ge \dfrac{1}{2}\left(1.00\times 10^{-3}\right)(20.0t)^2$, then $100t^4 \ge \left(400\times 10^{-9}\right)t^2$. The earliest time this is true is at $t = \sqrt{4.00\times 10^{-9}}\text{ s} = \boxed{63.2\ \mu\text{s}}$.

P32.50 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -L\dfrac{d}{dt}(Kt) = \boxed{-LK}$

(b) $I = \dfrac{dQ}{dt}$, so $Q = \int_0^t I\,dt = \int_0^t Kt\,dt = \dfrac{1}{2}Kt^2$

and $\Delta V_C = \dfrac{-Q}{C} = \boxed{-\dfrac{Kt^2}{2C}}$

(c) When $\dfrac{1}{2}C(\Delta V_C)^2 = \dfrac{1}{2}LI^2$, $\dfrac{1}{2}C\left(\dfrac{K^2t^4}{4C^2}\right) = \dfrac{1}{2}L\left(K^2t^2\right)$

Thus $t = \boxed{2\sqrt{LC}}$

P32.51 $\dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2C}\left(\dfrac{Q}{2}\right)^2 + \dfrac{1}{2}LI^2$ so $I = \sqrt{\dfrac{3Q^2}{4CL}}$

The flux through each turn of the coil is $\Phi_B = \dfrac{LI}{N} = \boxed{\dfrac{Q}{2N}\sqrt{\dfrac{3L}{C}}}$

where N is the number of turns.

***P32.52** (a) The inductor has no voltage across it. It behaves as a $\boxed{\text{short circuit}}$. The battery sees equivalent resistance $4\ \Omega + (1/4\ \Omega + 1/8\ \Omega)^{-1} = 6.67\ \Omega$. The battery current is $10\ \text{V}/6.67\ \Omega = 1.50\ \text{A}$. The voltage across the parallel combination of resistors is $10\ \text{V} - 1.50\ \text{A}\ 4\ \Omega = 4\ \text{V}$. The current in the 8-Ω resistor and the inductor is $4\text{V}/8\Omega = \boxed{500\ \text{mA}}$.

(b) $U = (1/2)\,LI^2 = (1/2)\ 1\ \text{H}(0.5\ \text{A})^2 = \boxed{125\ \text{mJ}}$

(c) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch.

(d) The current decreases from 500 mA toward zero, showing exponential decay with a time constant of $L/R = 1\ \text{H}/12\ \Omega = 83.3\ \text{ms}$.

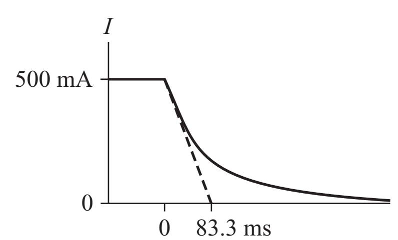


FIG. P32.52(d)

***P32.53** (a) Just after the circuit is connected, the potential difference across the resistor is 0 and the emf across the coil is 24.0 V.

(b) After several seconds, the potential difference across the resistor is 24.0 V and that across the coil is 0.

(c) The resistor voltage and inductor voltage always add to 24 V. The resistor voltage increases monotonically, so the two voltages are equal to each other, both being 12.0 V, just once. The time is given by $12\ \text{V} = IR = R\mathcal{E}/R(1 - e^{-Rt/L}) = 24\ \text{V}(1 - e^{-6\Omega t/0.005\ \text{H}})$.

This is $0.5 = e^{-1\,200\,t}$ or $1\,200\,t = \ln 2$ giving $t = \boxed{0.578\ \text{ms after the circuit is connected}}$.

(d) As the current decays the potential difference across the resistor is always equal to the emf across the coil. It decreases from 24.0 V to zero.

***P32.54** We have $9\text{ V} = 2\text{ A}\,R + L\,(0.5\text{ A/s})$ and $5\text{ V} = 2\text{ A}\,R + L\,(-0.5\text{ A/s})$

Solving simultaneously, $9\text{ V} - 5\text{ V} = L(1\text{ A/s})$ so $\boxed{L = 4.00\text{ H}}$ and $7\text{ V} = 2\text{ A}\,R$ so $\boxed{R = 3.50\ \Omega}$

***P32.55** Between $t = 0$ and $t = 1$ ms, the rate of change of current is 2 A/s, so the induced voltage $\Delta V_{ab} = -L\,dI/dt$ is -100 mV. Between $t = 1$ ms and $t = 2$ ms, the induced voltage is zero. Between $t = 2$ ms and $t = 3$ ms the induced voltage is -50 mV. Between $t = 3$ ms and $t = 5$ ms, the rate of change of current is $(-3/2)$ A/s, and the induced voltage is +75 mV.

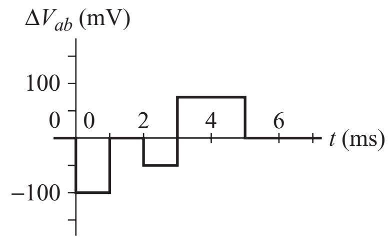


FIG. P32.55

***P32.56** (a) $\omega = (LC)^{-1/2} = (0.032\text{ V}\cdot\text{s/A}\ 0.000\,5\text{ C/V})^{-1/2} = \boxed{250\text{ rad/s}}$

(b) $$\omega = \left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)^{1/2} = \left(\frac{1}{1.6\times10^{-5}\text{ s}^2} - \left[\frac{4\ \Omega}{2\cdot 0.032\text{ V}\cdot\text{s/A}}\right]^2\right)^{1/2} = \boxed{242\text{ rad/s}}$$

(c) $$\omega = \left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)^{1/2} = \left(\frac{1}{1.6\times10^{-5}\text{ s}^2} - \left[\frac{15\ \Omega}{2\cdot 0.032\text{ V}\cdot\text{s/A}}\right]^2\right)^{1/2} = \boxed{87.0\text{ rad/s}}$$

(d) $$\omega = \left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)^{1/2} = \left(\frac{1}{1.6\times10^{-5}\text{ s}^2} - \left[\frac{17\ \Omega}{2\cdot 0.032\text{ V}\cdot\text{s/A}}\right]^2\right)^{1/2}$$ gives an imaginary answer. In parts (a), (b), and (c) the calculated angular frequency is experimentally verifiable. Experimentally, in part (d) $\boxed{\text{no oscillations occur}}$. The circuit is overdamped.

***P32.57** $B = \dfrac{\mu_0 NI}{2\pi r}$

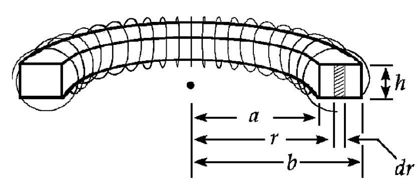


FIG. P32.57

(a) $$\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r}h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(b) $$L = \frac{\mu_0 (500)^2 (0.010\,0)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$$

(c) $$L_{\text{appx}} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0 (500)^2}{2\pi}\left(\frac{2.00\times10^{-4}\text{ m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$$, This approximate result is only 0.3% different from the precise result.

P32.58 (a) At the center, $B = \dfrac{N\mu_0 IR^2}{2\left(R^2+0^2\right)^{3/2}} = \dfrac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B = BA\cos\theta = \dfrac{N\mu_0 I}{2R}\pi R^2 \cos 0° = \dfrac{\pi}{2}N\mu_0 IR$

When the current it carries changes, $\mathcal{E}_L = -N\dfrac{d\Phi_B}{dt} \approx -N\left(\dfrac{\pi}{2}\right)N\mu_0 R\dfrac{dI}{dt} = -L\dfrac{dI}{dt}$

so $\boxed{L \approx \dfrac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r = 3(0.3\text{ m})$ so $r \approx 0.14\text{ m}$

$$L \approx \frac{\pi}{2}\left(1^2\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(0.14\text{ m}) = 2.8\times10^{-7}\text{ H}$$

$\boxed{L \sim 100\text{ nH}}$

(c) $\dfrac{L}{R} = \dfrac{2.8\times10^{-7}\text{ V}\cdot\text{s/A}}{270\text{ V/A}} = 1.0\times10^{-9}\text{ s}$ $\boxed{\dfrac{L}{R} \sim 1\text{ ns}}$

P32.59 Left-hand loop: $\mathcal{E} - (I+I_2)R_1 - I_2R_2 = 0$

Outside loop: $\mathcal{E} - (I+I_2)R_1 - L\dfrac{dI}{dt} = 0$

Eliminating I_2 gives $\mathcal{E}' - IR' - L\dfrac{dI}{dt} = 0$

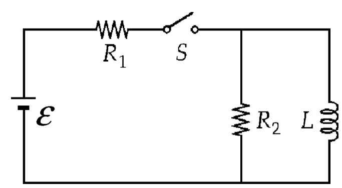


FIG. P32.59

This is of the same form as the differential equation 32.6 in the chapter text for a simple *RL* circuit, so its solution is of the same form as the equation 32.7 for the current in the circuit:

$$I(t) = \frac{\mathcal{E}'}{R'}\left(1-e^{-R't/L}\right)$$

But $R' = \dfrac{R_1R_2}{R_1+R_2}$ and $\mathcal{E}' = \dfrac{R_2\mathcal{E}}{R_1+R_2}$, so $\dfrac{\mathcal{E}'}{R'} = \dfrac{\mathcal{E}R_2/(R_1+R_2)}{R_1R_2/(R_1+R_2)} = \dfrac{\mathcal{E}}{R_1}$

Thus $I(t) = \dfrac{\mathcal{E}}{R_1}\left(1-e^{-R't/L}\right)$

P32.60 From Ampère's law, the magnetic field at distance $r \le R$ is found as:

$$B(2\pi r) = \mu_0 J\left(\pi r^2\right) = \mu_0\left(\frac{I}{\pi R^2}\right)\left(\pi r^2\right),\ \text{ or }\ B = \frac{\mu_0 Ir}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\,dr) = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3\,dr = \frac{\mu_0 I^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

This is independent of the radius of the wire.

P32.61 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule to this loop gives:

$$+\mathcal{E}_0-\left[(2.00+6.00)\times 10^3\ \Omega\right]\left(9.00\times 10^{-3}\ \text{A}\right)=0$$

$$+\mathcal{E}_0=\boxed{72.0\ \text{V with end } b \text{ at the higher potential}}$$

(b)

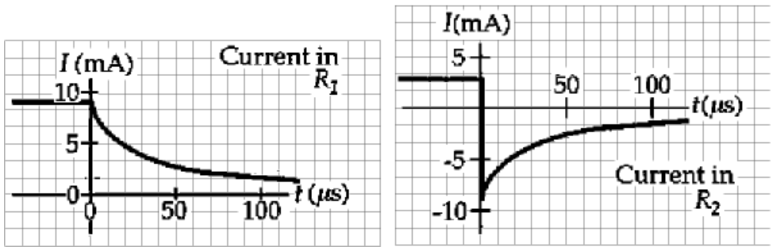


FIG. P32.61(b)

(c) After the switch is opened, the current around the outer loop decays as $I=I_i e^{-Rt/L}$ with $I_{\max}=9.00$ mA, $R=8.00$ kΩ, and $L=0.400$ H.

Thus, when the current has reached a value $I=2.00$ mA, the elapsed time is:

$$t=\left(\frac{L}{R}\right)\ln\left(\frac{I_i}{I}\right)=\left(\frac{0.400\ \text{H}}{8.00\times 10^3\ \Omega}\right)\ \ln\ \left(\frac{9.00}{2.00}\right)=7.52\times 10^{-5}\ \text{s}=\boxed{75.2\ \mu\text{s}}$$

P32.62 (a) It has a magnetic field, and it stores energy, so $L=\dfrac{2U}{I^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) $\Phi=LI$ so $\quad L=\dfrac{\Phi}{I}=\dfrac{1}{I}\displaystyle\int_{y=a}^{w-a}BdA$

$$L=\frac{1}{I}\int_a^{w-a}xdy\left(\frac{\mu_0 I}{2\pi y}+\frac{\mu_0 I}{2\pi(w-y)}\right)=\frac{2}{I}\int\frac{\mu_0 Ix}{2\pi y}dy=\frac{2\mu_0 x}{2\pi}\ln y\Big|_a^{w-a}$$

Thus $\quad L=\dfrac{\mu_0 x}{\pi}\ln\left(\dfrac{w-a}{a}\right)$

P32.63 When the switch is closed, as shown in Figure (a), the current in the inductor is I:

$12.0-7.50I-10.0=0\rightarrow I=0.267$ A

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR=\Delta V$:

$$(0.267\ \text{A})R\le 80.0\ \text{V}$$

$$\boxed{R\le 300\ \Omega}$$

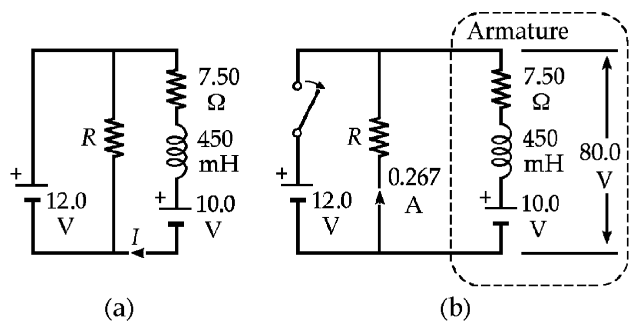


FIG. P32.63

P32.64 For an *RL* circuit,

$I(t)=I_i e^{-(R/L)t}$: $\qquad \dfrac{I(t)}{I_i}=1-10^{-9}=e^{-(R/L)t}\cong 1-\dfrac{R}{L}t$

$\dfrac{R}{L}t=10^{-9}$ $\qquad$ so $\qquad R_{\max}=\dfrac{\left(3.14\times 10^{-8}\right)\left(10^{-9}\right)}{(2.50\ \text{yr})\left(3.16\times 10^7\ \text{s/yr}\right)}=\boxed{3.97\times 10^{-25}\ \Omega}$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm^2, its resistance would be at least $10^{-6}\ \Omega$.)

P32.65 (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0 \text{ H})(50.0\times 10^3 \text{ A})^2 = \boxed{6.25\times 10^{10} \text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

Then one wire creates a field of $B = \frac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\frac{\mu_0 I}{2\pi r}\sin 90° = \frac{\mu_0 \ell I^2}{2\pi r}$

Evaluating the force, $F = (4\pi\times 10^{-7} \text{ N/A}^2)\frac{(1.00 \text{ m})(50.0\times 10^3 \text{ A})^2}{2\pi(0.250 \text{ m})} = \boxed{2\,000 \text{ N}}$

P32.66 $\mathcal{P} = I\Delta V$ $\quad I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times 10^9 \text{ W}}{200\times 10^3 \text{ V}} = 5.00\times 10^3 \text{ A}$

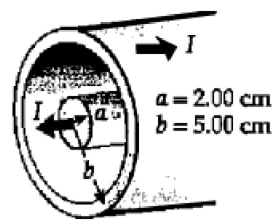


FIG. P32.66

From Ampère's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

(a) At $r = a = 0.020\,0$ m, $I_{\text{enclosed}} = 5.00\times 10^3$ A

and $B = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})}{2\pi(0.020\,0 \text{ m})}$

$= 0.050\,0 \text{ T} = \boxed{50.0 \text{ mT}}$

(b) At $r = b = 0.050\,0$ m, $I_{\text{enclosed}} = I = 5.00\times 10^3$ A

and $B = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})}{2\pi(0.050\,0 \text{ m})} = 0.020\,0 \text{ T} = \boxed{20.0 \text{ mT}}$

(c) $U = \int u\,dV = \int_a^b \frac{[B(r)]^2(2\pi r\ell\,dr)}{2\mu_0} = \frac{\mu_0 I^2\ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$

$U = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})^2(1\,000\times 10^3 \text{ m})}{4\pi}\ln\left(\frac{5.00 \text{ cm}}{2.00 \text{ cm}}\right)$

$= 2.29\times 10^6 \text{ J} = \boxed{2.29 \text{ MJ}}$

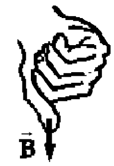
(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w.

It carries a current of $(5.00\times 10^3 \text{ A})\left(\frac{w}{2\pi(0.050\,0 \text{ m})}\right)$

and experiences an outward force

$$F = I\ell B\sin\theta = \frac{(5.00\times 10^3 \text{ A})w}{2\pi(0.050\,0 \text{ m})}\ell(20.0\times 10^{-3} \text{ T})\sin 90.0°$$

The pressure on it is $P = \frac{F}{A} = \frac{F}{w\ell} = \frac{(5.00\times 10^3 \text{ A})(20.0\times 10^{-3} \text{ T})}{2\pi(0.050\,0 \text{ m})} = \boxed{318 \text{ Pa}}$



P32.67 (a) $B = \dfrac{\mu_0 NI}{\ell} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1\,400)(2.00\ \text{A})}{1.20\ \text{m}} = \boxed{2.93 \times 10^{-3}\ \text{T (upward)}}$

(b) $u = \dfrac{B^2}{2\mu_0} = \dfrac{(2.93 \times 10^{-3}\ \text{T})^2}{2(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})} = (3.42\ \text{J/m}^3)\left(\dfrac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)$

$= 3.42\ \text{N/m}^2 = \boxed{3.42\ \text{Pa}}$

(c) To produce a downward magnetic field, the surface of the superconductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi(1.10 \times 10^{-2}\ \text{m})^2\right] = \boxed{1.30 \times 10^{-3}\ \text{N}}$

Note that we have not proved that energy density is pressure. In fact, it is not in some cases. Chapter 21 proved that the pressure is two-thirds of the translational energy density in an ideal gas.

ANSWERS TO EVEN PROBLEMS

P32.2 1.36 μH

P32.4 19.2 μWb

P32.6 (a) 360 mV (b) 180 mV (c) $t = 3.00$ s

P32.8 See the solution.

P32.10 See the solution.

P32.12 See the solution.

P32.14 92.8 V

P32.16 30.0 mH

P32.18 (500 mA)$(1 - e^{-10t/s})$, 1.50 A – (0.250 A) $e^{-10t/s}$

P32.20 0 for $t < 0$; (10 A)$(1 - e^{-10\,000t})$ for $0 < t < 200$ µs; (63.9 A) $e^{-10\,000t}$ for $t > 200$ µs

P32.22 (a), (b), and (c) See the solution. (d) Yes; see the solution.

P32.24 (a) 8.06 MJ/m^3 (b) 6.32 kJ

P32.26 See the solution.

P32.28 (a) 20.0 W (b) 20.0 W (c) 0 (d) 20.0 J

P32.30 1.73 mH

P32.32 781 pH

P32.34 (a) and (b) $\mu_0 \pi R_2^2 N_1 N_2/\ell$ (c) They are the same.

P32.36 $(L_1 L_2 - M^2)/(L_1 + L_2 - 2M)$

P32.38 20.0 V

P32.40 (a) 135 Hz (b) 119 μC (c) –114 mA

P32.42 (a) 503 Hz (b) 12.0 μC (c) 37.9 mA (d) 72.0 μJ

P32.44 (a) 2.51 kHz (b) 69.9 Ω

P32.46 See the solution.

P32.48 (b) $\mu_0 J_s^2/2$ away from the other sheet (c) $\mu_0 J_s$ and zero (d) $\mu_0 J_s^2/2$

P32.50 (a) $\mathcal{E}_L = -LK$ (b) $\Delta V_c = \dfrac{-Kt^2}{2C}$ (c) $t = 2\sqrt{LC}$

P32.52 (a) a short circuit; 500 mA (b) 125 mJ (c) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch. (d) See the solution. The current decreases from 500 mA toward zero, showing exponential decay with a time constant of 83.3 ms.

P32.54 $L = 4.00$ H and $R = 3.50$ Ω

P32.56 (a) 250 rad/s (b) 242 rad/s (c) 87.0 rad/s (d) In parts (a), (b), and (c) the calculated angular frequency is experimentally verifiable. In part (d) the equation for ω gives an imaginary answer. Experimentally, no oscillations occur when the circuit is overdamped.

P32.58 (a) $L \approx (\pi/2)N^2 \mu_0 R$ (b) ~100 nH (c) ~1 ns

P32.60 See the solution.

P32.62 (a) It creates a magnetic field. (b) The long narrow rectangular area between the conductors encloses all of the magnetic flux.

P32.64 3.97×10^{-25} Ω

P32.66 (a) 50.0 mT (b) 20.0 mT (c) 2.29 MJ (d) 318 Pa