46

Particle Physics and Cosmology

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q46.1 Strong Force—Mediated by gluons.
Electromagnetic Force—Mediated by photons.
Weak Force—Mediated by W^+, W^-, and Z^0 bosons.
Gravitational Force—Mediated by gravitons.

***Q46.2** Answer (b). The electron and positron together have very little momentum. A 1.02-MeV photon has a definite chunk of momentum. Production of a single gamma ray could not satisfy the law of conservation of momentum, which must hold true in this—and every—interaction.

Q46.3 Hadrons are massive particles with structure and size. There are two classes of hadrons: mesons and baryons. Hadrons are composed of quarks. Hadrons interact via the strong force. Leptons are light particles with no structure or size. It is believed that leptons are fundamental particles. Leptons interact via the weak force.

Q46.4 Baryons are heavy hadrons with spin $\frac{1}{2}$ or $\frac{3}{2}$ composed of three quarks. Mesons are light hadrons with spin 0 or 1 composed of a quark and an antiquark.

***Q46.5** Answer (c). The muon has much more rest energy than the electron and the neutrinos together. The missing rest energy goes into kinetic energy.

***Q46.6** Answer (b). The z component of its angular momentum must be 3/2, 1/2, –1/2, or –3/2, in units of $\hbar$.

Q46.7 The baryon number of a proton or neutron is one. Since baryon number is conserved, the baryon number of the kaon must be zero.

Q46.8 Decays by the weak interaction typically take 10^{-10} s or longer to occur. This is slow in particle physics.

***Q46.9** Answer a, b, c, and d. Protons feel all these forces; but within a nucleus the strong interaction predominates, followed by the electromagnetic interaction.

Q46.10 You can think of a conservation law as a superficial regularity which we happen to notice, as a person who does not know the rules of chess might observe that one player's two bishops are always on squares of opposite colors. Alternatively, you can think of a conservation law as identifying some stuff of which the universe is made. In classical physics one can think of both matter and energy as fundamental constituents of the world. We buy and sell both of them. In classical physics you can also think of linear momentum, angular momentum, and electric charge as basic stuffs of which the universe is made. In relativity we learn that matter and energy are not conserved separately, but are both aspects of the conserved quantity *relativistic total energy*. Discovered more recently, four conservation laws appear equally general and thus equally fundamental: Conservation of baryon number, conservation of electron-lepton number, conservation of tau-lepton number, and conservation of muon-lepton number. Processes involving the strong force and the electromagnetic force follow conservation of strangeness, charm, bottomness, and topness, while the weak interaction can alter the total S, C, B, and T quantum numbers of an isolated system.

Q46.11 No. Antibaryons have baryon number –1, mesons have baryon number 0, and baryons have baryon number +1. The reaction cannot occur because it would not conserve baryon number, unless so much energy is available that a baryon-antibaryon pair is produced.

Q46.12 The Standard Model consists of quantum chromodynamics (to describe the strong interaction) and the electroweak theory (to describe the electromagnetic and weak interactions). The Standard Model is our most comprehensive description of nature. It fails to unify the two theories it includes, and fails to include the gravitational force. It pictures matter as made of six quarks and six leptons, interacting by exchanging gluons, photons, and W and Z bosons.

Q46.13 (a) and (b) All baryons and antibaryons consist of three quarks. (c) and (d) All mesons and antimesons consist of two quarks. Since quarks have spin quantum number $\frac{1}{2}$ and can be spin-up or spin-down, it follows that the three-quark baryons must have a half-integer spin, while the two-quark mesons must have spin 0 or 1.

Q46.14 Each flavor of quark can have colors, designated as red, green, and blue. Antiquarks are colored antired, antigreen, and antiblue. A baryon consists of three quarks, each having a different color. By analogy to additive color mixing we call it colorless. A meson consists of a quark of one color and antiquark with the corresponding anticolor, making it colorless as a whole.

Q46.15 The electroweak theory of Glashow, Salam, and Weinberg predicted the W^+, W^-, and Z particles. Their discovery in 1983 confirmed the electroweak theory.

***Q46.16** Answer (e). Both trials conserve momentum. In the first trial all of the kinetic energy $K_1 + K_2 = 2K_1$ is converted into internal energy. In the second trial we end up with a glob of twice the mass moving at half the speed, so it has half the kinetic energy of one original clay ball, $K_1/2$. Energy $K_1/2$ is converted into internal energy, one-quarter of that converted in trial one.

Q46.17 Hubble determined experimentally that all galaxies outside the Local Group are moving away from us, with speed directly proportional to the distance of the galaxy from us.

***Q46.18** Answer c, b, d, e, a, f, g. The temperature corresponding to b is on the order of 10^{13} K. That for hydrogen fusion d is on the order of 10^7 K. A fully ionized plasma can be at 10^4 K. Neutral atoms can exist at on the order of 3 000 K, molecules at 1 000 K, and solids at on the order of 500 K.

Q46.19 Before that time, the Universe was too hot for the electrons to remain in any sort of stable orbit around protons. The thermal motion of both protons and electrons was too rapid for them to be in close enough proximity for the Coulomb force to dominate.

***Q46.20** Answer (a). The vast gulfs not just between stars but between galaxies and especially between clusters, empty of ordinary matter, are important to bring down the average density of the Universe. We can estimate the average density defined for the Solar System as the mass of the sun divided by the volume of a lightyear-size sphere around it:
$\frac{2\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(2\times10^{16}\ \text{m})^3}=6\times10^{-20}\ \text{kg/m}^3=6\times10^{-23}\ \text{g/cm}^3$, ten million times larger than the critical density $3H^2/8\pi G = 6\times10^{-30}$ g/cm^3.

Q46.21 The Universe is vast and could on its own terms get along very well without us. But as the cosmos is immense, life appears to be immensely scarce, and therefore precious. We must do our work, growing corn to feed the hungry while preserving our planet for future generations and preserving future possibilities for the universe. One person has singular abilities and opportunities for effort, faithfulness, generosity, honor, curiosity, understanding, and wonder. His or her place is to use those abilities and opportunities, unique in all the Universe.

SOLUTIONS TO PROBLEMS

Section 46.1 The Fundamental Forces in Nature

Section 46.2 Positrons and Other Antiparticles

P46.1 Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy E of the photon must be

$$E=2E_0=2(938.3\ \text{MeV})=1\,876.6\ \text{MeV}=3.00\times10^{-10}\ \text{J}$$

Thus, $$E=hf=3.00\times10^{-10}\ \text{J}$$

$$f=\frac{3.00\times10^{-10}\ \text{J}}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}=\boxed{4.53\times10^{23}\ \text{Hz}}$$

$$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\ \text{m/s}}{4.53\times10^{23}\ \text{Hz}}=\boxed{6.62\times10^{-16}\ \text{m}}$$

P46.2 The half-life of ^{14}O is 70.6 s, so the decay constant is $\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{70.6\ \text{s}}=0.009\,82\ \text{s}^{-1}$.

The number of ^{14}O nuclei remaining after five minutes is

$$N=N_0e^{-\lambda t}=(10^{10})e^{-(0.009\,82\ \text{s}^{-1})(300\ \text{s})}=5.26\times10^{8}$$

The number of these in one cubic centimeter of blood is

$$N'=N\left(\frac{1.00\ \text{cm}^3}{\text{total volume of blood}}\right)=(5.26\times10^{8})\left(\frac{1.00\ \text{cm}^3}{2000\ \text{cm}^3}\right)=2.63\times10^{5}$$

and their activity is $R=\lambda N'=(0.009\ 82\ \text{s}^{-1})(2.63\times10^{5})=2.58\times10^{3}\ \text{Bq}$ $\boxed{\sim10^{3}\ \text{Bq}}$

***P46.3** (a) The rest energy of a total of 6.20 g of material is converted into energy of electromagnetic radiation:

$$E=mc^2=6.20\times10^{-3}\ \text{kg}(3\times10^{8}\ \text{m/s})^2=\boxed{5.58\times10^{14}\ \text{J}}$$

(b) $$5.58\times10^{14}\ \text{J}=5.58\times10^{14}\ \text{J}\left(\frac{\$0.14}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}}{\text{J/s}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)=\boxed{\$2.17\times10^{7}}$$

All from two cents' worth of stuff.

P46.4 The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0.$ To conserve momentum, each photon must carry away one-half the energy.

Thus $$E_{\min} = \frac{2E_0}{2} = E_0 = 938.3\text{ MeV} = hf_{\min}$$

Thus, $$f_{\min} = \frac{(938.3\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{2.27\times10^{23}\text{ Hz}}$$

$$\lambda = \frac{c}{f_{\min}} = \frac{3.00\times10^{8}\text{ m/s}}{2.27\times10^{23}\text{ Hz}} = \boxed{1.32\times10^{-15}\text{ m}}$$

P46.5 In $\gamma \to \text{p}^+ + \text{p}^-$,

we start with energy 2.09 GeV
we end with energy 938.3 MeV + 938.3 MeV + 95.0 MeV + K_2
where K_2 is the kinetic energy of the second proton.

Conservation of energy for the creation process gives $\boxed{K_2 = 118\text{ MeV}}$

Section 46.3 **Mesons and the Beginning of Particle Physics**

P46.6 The reaction is $\mu^+ + \text{e}^- \to \nu + \nu$
muon-lepton number before reaction: $(-1)+(0) = -1$
electron-lepton number before reaction: $(0)+(1) = 1$

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_\text{e}}$

Then $\mu^+ + \text{e}^- \to \bar{\nu}_\mu + \nu_\text{e}$

P46.7 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = 91\times10^{9}\text{ eV}.$ It can last no longer than $\Delta t = \dfrac{\hbar}{2\Delta E}$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi\,\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{4\pi(91\times10^{9}\text{ eV})}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$

$$= 1.06\times10^{-18}\text{ m} = \boxed{\sim10^{-18}\text{ m}}$$

***P46.8** (a) The particle's rest energy is mc^2. The time interval during which a virtual particle of this mass could exist is at most Δt in $\Delta E\Delta t = \frac{\hbar}{2} = mc^2\Delta t$. The distance it could move is at most

$$c\Delta t = \frac{\hbar c}{2mc^2} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{4\pi mc^2\left(1.602\times10^{-19}\ \text{J/eV}\right)} = \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{4\pi mc^2} = \frac{1240\ \text{eV}\cdot\text{nm}}{4\pi mc^2}$$

$$= \frac{98.7\ \text{eV}\cdot\text{nm}}{mc^2}$$

According to Yukawa's line of reasoning, this distance is the range of a force that could be associated with the exchange of virtual particles of this mass.

(b) The range is inversely proportional to the mass of the field particle.

(c) Our rule describes the electromagnetic, weak, and gravitational interactions.
For the electromagnetic and gravitational interactions, we take the limiting form of the rule with infinite range and zero mass for the field particle. For the weak interaction, $98.7\ \text{eV}\cdot\text{nm}/90\ \text{GeV} \approx 10^{-18}\ \text{m} = 10^{-3}\ \text{fm}$, in agreement with the tabulated information. For the strong interaction, we do not have a separately measured mass for a gluon, so we cannot say that this rule defines the range.

(d) $98.7\ \text{eV}\cdot\text{nm}/938.3\ \text{MeV} \approx 10^{-1-9-6}\ \text{m}$ $\boxed{\sim10^{-16}\ \text{m}}$

P46.9 From Table 46.2 in the chapter text $M_{\pi^0} = 135\ \text{MeV}/c^2$.

Therefore, $E_\gamma = \boxed{67.5\ \text{MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\ \text{MeV}/c}$$

and $$f = \frac{E_\gamma}{h} = \boxed{1.63\times10^{22}\ \text{Hz}}$$

P46.10 The time interval for a particle traveling with the speed of light to travel a distance of $3\times10^{-15}\ \text{m}$ is

$$\Delta t = \frac{d}{v} = \frac{3\times10^{-15}\ \text{m}}{3\times10^{8}\ \text{m/s}} = \boxed{\sim10^{-23}\ \text{s}}$$

P46.11 (a) $\Delta E = (m_n - m_p - m_e)c^2$

From the table of isotopic masses in Chapter 44,

$$\Delta E = (1.008\ 665 - 1.007\ 825)(931.5) = \boxed{0.782\ \text{MeV}}$$

(b) Assuming the neutron at rest, momentum conservation for the decay process implies $p_p = p_e$. Relativistic energy for the system is conserved

$$\sqrt{(m_pc^2)^2 + p_p^2c^2} + \sqrt{(m_ec^2)^2 + p_e^2c^2} = m_nc^2$$

Since $p_p = p_e$,

$$\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6\ \text{MeV}$$

Solving the algebra gives $pc = 1.19$ MeV

If $p_ec = \gamma m_e v_e c = 1.19$ MeV, then $\frac{\gamma v_e}{c} = \frac{1.19\ \text{MeV}}{0.511\ \text{MeV}} = \frac{x}{\sqrt{1-x^2}} = 2.33$ where $x = \frac{v_e}{c}$

Solving, $x^2 = (1 - x^2)5.43$ and $x = \frac{v_e}{c} = 0.919$

$$\boxed{v_e = 0.919c}$$

Then $m_pv_p = \gamma_e m_e v_e$:

$$v_p = \frac{\gamma_e m_e v_e c}{m_p c} = \frac{(1.19\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})}{(1.67\times10^{-27})(3.00\times10^{8}\ \text{m/s})}$$

$$v_p = 3.80\times10^5\ \text{m/s} = \boxed{380\ \text{km/s}}$$

(c) $\boxed{\text{The electron is relativistic; the proton is not.}}$ Our criterion for answers accurate to three significant digits is that the electron is moving at more than one-tenth the speed of light and the proton at less than one-tenth the speed of light.

Section 46.4 **Classification of Particles**

P46.12 In $? + \text{p}^+ \to \text{n} + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\bar{\nu}_\mu}$.

Section 46.5 Conservation Laws

P46.13 (a) $p+\bar{p}\rightarrow\mu^{+}+e^{-}$ $\boxed{L_e}$ $0+0\rightarrow 0+1$

and $\boxed{L_\mu}$ $0+0\rightarrow -1+0$

(b) $\pi^{-}+p\rightarrow p+\pi^{+}$ $\boxed{\text{charge}}$ $-1+1\rightarrow +1+1$

(c) $p+p\rightarrow p+\pi^{+}$ $\boxed{\text{baryon number}}$: $1+1\rightarrow 1+0$

(d) $p+p\rightarrow p+p+n$ $\boxed{\text{baryon number}}$: $1+1\rightarrow 1+1+1$

(e) $\gamma+p\rightarrow n+\pi^{0}$ $\boxed{\text{charge}}$ $0+1\rightarrow 0+0$

P46.14 (a) Baryon number and charge are conserved, with respective values of $0+1=0+1$ and $1+1=1+1$ in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

P46.15 (a) $\pi^{-}\rightarrow\mu^{-}+\boxed{\bar{\nu}_\mu}$ L_μ: $0\rightarrow 1-1$

(b) $K^{+}\rightarrow\mu^{+}+\boxed{\nu_\mu}$ L_μ: $0\rightarrow -1+1$

(c) $\boxed{\bar{\nu}_e}+p^{+}\rightarrow n+e^{+}$ L_e: $-1+0\rightarrow 0-1$

(d) $\boxed{\nu_e}+n\rightarrow p^{+}+e^{-}$ L_e: $1+0\rightarrow 0+1$

(e) $\boxed{\nu_\mu}+n\rightarrow p^{+}+\mu^{-}$ L_μ: $1+0\rightarrow 0+1$

(f) $\mu^{-}\rightarrow e^{-}+\boxed{\bar{\nu}_e}+\boxed{\nu_\mu}$ L_μ: $1\rightarrow 0+0+1$ and L_e: $0\rightarrow 1-1+0$

P46.16 Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

P46.17 (a) $p^{+}\rightarrow\pi^{+}+\pi^{0}$ $\boxed{\text{Baryon number}}$ conservation is violated: $1\rightarrow 0+0$

(b) $p^{+}+p^{+}\rightarrow p^{+}+p^{+}+\pi^{0}$ This reaction $\boxed{\text{can occur}}$.

(c) $p^{+}+p^{+}\rightarrow p^{+}+\pi^{+}$ $\boxed{\text{Baryon number}}$ is violated: $1+1\rightarrow 1+0$

(d) $\pi^{+}\rightarrow\mu^{+}+\nu_\mu$ This reaction $\boxed{\text{can occur}}$.

(e) $n^{0}\rightarrow p^{+}+e^{-}+\bar{\nu}_e$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^{+}\rightarrow\mu^{+}+n$ Violates $\boxed{\text{baryon number}}$: $0\rightarrow 0+1$

Violates $\boxed{\text{muon-lepton number}}$: $0\rightarrow -1+0$

P46.18 Momentum conservation for the decay requires the pions to have equal speeds.

The total energy of each is $\dfrac{497.7\text{ MeV}}{2}$

so $E^2 = p^2c^2 + \left(mc^2\right)^2$ gives

$$\left(248.8\text{ MeV}\right)^2 = \left(pc\right)^2 + \left(139.6\text{ MeV}\right)^2$$

Solving, $pc = 206\text{ MeV} = \gamma mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$$\frac{v}{c} = 1.48\sqrt{1-\left(\frac{v}{c}\right)^2}$$

and $\left(\dfrac{v}{c}\right)^2 = 2.18\left[1-\left(\dfrac{v}{c}\right)^2\right] = 2.18 - 2.18\left(\dfrac{v}{c}\right)^2$

$$3.18\left(\frac{v}{c}\right)^2 = 2.18$$

so $\dfrac{v}{c} = \sqrt{\dfrac{2.18}{3.18}} = 0.828$

and $\boxed{v = 0.828c}$

P46.19 (a) In the suggested reaction $\text{p} \rightarrow \text{e}^+ + \gamma$

We would have for baryon numbers $+1 \rightarrow 0 + 0$

$\Delta B \neq 0$, so baryon number conservation would be violated.

(b) From conservation of momentum for the decay: $p_e = p_\gamma$

Then, for the positron, $E_e^2 = \left(p_e c\right)^2 + E_{0,e}^2$

becomes $E_e^2 = \left(p_\gamma c\right)^2 + E_{0,e}^2 = E_\gamma^2 + E_{0,e}^2$

From conservation of energy for the system: $E_{0,p} = E_e + E_\gamma$

or $E_e = E_{0,p} - E_\gamma$

so $E_e^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$

Equating this to the result from above gives $E_\gamma^2 + E_{0,e}^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$

or $$E_\gamma = \frac{E_{0,p}^2 - E_{0,e}^2}{2E_{0,p}} = \frac{\left(938.3\text{ MeV}\right)^2 - \left(0.511\text{ MeV}\right)^2}{2\left(938.3\text{ MeV}\right)} = \boxed{469\text{ MeV}}$$

Thus $E_e = E_{0,p} - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = \boxed{469\text{ MeV}}$

Also, $p_\gamma = \dfrac{E_\gamma}{c} = \boxed{\dfrac{469\text{ MeV}}{c}}$

and $p_e = p_\gamma = \boxed{\dfrac{469\text{ MeV}}{c}}$

continued on next page

(c) The total energy of the positron is $E_e = 469\text{ MeV}$

But, $E_e = \gamma E_{0,e} = \dfrac{E_{0,e}}{\sqrt{1-(v/c)^2}}$

so $\sqrt{1-\left(\dfrac{v}{c}\right)^2} = \dfrac{E_{0,e}}{E_e} = \dfrac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09\times 10^{-3}$

which yields $\boxed{v = 0.999\,999\,4c}$

P46.20 The relevant conservation laws are $\Delta L_e = 0$

$\Delta L_\mu = 0$

and $\Delta L_\tau = 0$

(a) $\pi^+ \to \pi^0 + e^+ + ?$ L_e: $0 \to 0 - 1 + L_e$ implies $L_e = 1$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ L_μ: $L_\mu + 0 \to +1 + 0 + 0$ implies $L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

(c) $\Lambda^0 \to p + \mu^- + ?$ L_μ: $0 \to 0 + 1 + L_\mu$ implies $L_\mu = -1$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ L_μ: $0 \to -1 + L_\mu$ implies $L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

L_τ: $-1 \to 0 + L_\tau$ implies $L_\tau = -1$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = -1$ for the other particle.

We have $\boxed{\nu_\mu}$

and $\boxed{\bar{\nu}_\tau}$

Section 46.6 **Strange Particles and Strangeness**

P46.21 (a) $\Lambda^0 \to p + \pi^-$ Strangeness: $-1 \to 0 + 0$ (strangeness is $\boxed{\text{not conserved}}$)

(b) $\pi^- + p \to \Lambda^0 + K^0$ Strangeness: $0 + 0 \to -1 + 1$ ($0 = 0$ and strangeness is $\boxed{\text{conserved}}$)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$ ($0 = 0$ and strangeness is $\boxed{\text{conserved}}$)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$ ($0 \neq -1$: strangeness is $\boxed{\text{not conserved}}$)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$ ($-2 \neq -1$ so strangeness is $\boxed{\text{not conserved}}$)

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$ ($-2 \neq 0$ so strangeness is $\boxed{\text{not conserved}}$)

P46.22 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.
The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

P46.23 (a) $\mu^- \to e^- + \gamma$ L_e: $0 \to 1+0$,
and L_μ: $1 \to 0$

(b) $n \to p + e^- + \nu_e$ L_e: $0 \to 0+1+1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness: $-1 \to 0+0$,
and charge: $0 \to +1+0$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0+0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness: $-2 \to 0+0$

P46.24 (a) $\pi^- + p \to 2\eta$ violates conservation of baryon number as $0+1 \to 0$, $\boxed{\text{not allowed}}$.

(b) $K^- + n \to \Lambda^0 + \pi^-$

Baryon number,	$0+1 \to 1+0$
Charge,	$-1+0 \to 0-1$
Strangeness,	$-1+0 \to -1+0$
Lepton number,	$0 \to 0$

The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \to \pi^- + \pi^0$

Strangeness,	$-1 \to 0+0$
Baryon number,	$0 \to 0$
Lepton number,	$0 \to 0$
Charge,	$-1 \to -1+0$

Strangeness conservation is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$

Baryon number,	$1 \to 1+0$
Lepton number,	$0 \to 0$
Charge,	$-1 \to -1+0$
Strangeness,	$-3 \to -2+0$

May occur by $\boxed{\text{weak interaction}}$, but not by strong or electromagnetic.

(e) $\eta \to 2\gamma$

Baryon number,	$0 \to 0$
Lepton number,	$0 \to 0$
Charge,	$0 \to 0$
Strangeness,	$0 \to 0$

No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

P46.25 (a) $\Xi^- \rightarrow \Lambda^0 + \mu^- + \nu_\mu$

Baryon number:	$+1 \rightarrow +1+0+0$	Charge:	$-1 \rightarrow 0-1+0$
L_e:	$0 \rightarrow 0+0+0$	L_μ:	$0 \rightarrow 0+1+1$
L_τ:	$0 \rightarrow 0+0+0$	Strangeness:	$-2 \rightarrow -1+0+0$
Conserved quantities are:			$\boxed{B,\ \text{charge},\ L_e,\ \text{and}\ L_\tau}$

(b) $K_S^0 \rightarrow 2\pi^0$

Baryon number:	$0 \rightarrow 0$	Charge:	$0 \rightarrow 0$
L_e:	$0 \rightarrow 0$	L_μ:	$0 \rightarrow 0$
L_τ:	$0 \rightarrow 0$	Strangeness:	$+1 \rightarrow 0$
Conserved quantities are:			$\boxed{B,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(c) $K^- + p \rightarrow \Sigma^0 + n$

Baryon number:	$0+1 \rightarrow 1+1$	Charge:	$-1+1 \rightarrow 0+0$
L_e:	$0+0 \rightarrow 0+0$	L_μ:	$0+0 \rightarrow 0+0$
L_τ:	$0+0 \rightarrow 0+0$	Strangeness:	$-1+0 \rightarrow -1+0$
Conserved quantities are:			$\boxed{S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(d) $\Sigma^0 + \Lambda^0 + \gamma$

Baryon number:	$+1 \rightarrow 1+0$	Charge:	$0 \rightarrow 0$
L_e:	$0 \rightarrow 0+0$	L_μ:	$0 \rightarrow 0+0$
L_τ:	$0 \rightarrow 0+0$	Strangeness:	$-1 \rightarrow -1+0$
Conserved quantities are:			$\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(e) $e^+ + e^- \rightarrow \mu^+ + \mu^-$

Baryon number:	$0+0 \rightarrow 0+0$	Charge:	$+1-1 \rightarrow +1-1$
L_e:	$-1+1 \rightarrow 0+0$	L_μ:	$0+0 \rightarrow +1-1$
L_τ:	$0+0 \rightarrow 0+0$	Strangeness:	$0+0 \rightarrow 0+0$
Conserved quantities are:			$\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(f) $\overline{p} + n \rightarrow \overline{\Lambda}^0 + \Sigma^-$

Baryon number:	$-1+1 \rightarrow -1+1$	Charge:	$-1+0 \rightarrow 0-1$
L_e:	$0+0 \rightarrow 0+0$	L_μ:	$0+0 \rightarrow 0+0$
L_τ:	$0+0 \rightarrow 0+0$	Strangeness:	$0+0 \rightarrow +1-1$
Conserved quantities are:			$\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

P46.26 (a) $K^+ + p \rightarrow \underline{\ ?\ } + p$

The strong interaction conserves everything.

Baryon number,	$0+1 \rightarrow B+1$	so	$B=0$
Charge,	$+1+1 \rightarrow Q+1$	so	$Q=+1$
Lepton numbers,	$0+0 \rightarrow L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$+1+0 \rightarrow S+0$	so	$S=1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \rightarrow \underline{\ ?\ } + \pi^-$

Baryon number,	$+1 \rightarrow B+0$	so	$B=1$
Charge,	$-1 \rightarrow Q-1$	so	$Q=0$
Lepton numbers,	$0 \rightarrow L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$-3 \rightarrow S+0$	so	$\Delta S = 1$: $S=-2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \rightarrow \underline{\ ?\ } + \mu^+ + \nu_\mu$

Baryon number,	$0 \rightarrow B+0+0$	so	$B=0$
Charge,	$+1 \rightarrow Q+1+0$	so	$Q=0$
Lepton numbers,	L_e, $0 \rightarrow L_e+0+0$	so	$L_e=0$
	L_μ, $0 \rightarrow L_\mu-1+1$	so	$L_\mu=0$
	L_τ, $0 \rightarrow L_\tau+0+0$	so	$L_\tau=0$
Strangeness,	$1 \rightarrow S+0+0$	so	$\Delta S = \pm 1$
			(for weak interaction): $S=0$

The particle must be a neutral meson with strangeness $=0 \Rightarrow \boxed{\pi^0}$.

P46.27 Time-dilated lifetime:

$$T = \gamma T_0 = \frac{0.900\times 10^{-10}\ \text{s}}{\sqrt{1-v^2/c^2}} = \frac{0.900\times 10^{-10}\ \text{s}}{\sqrt{1-(0.960)^2}} = 3.214\times 10^{-10}\ \text{s}$$

$$\text{distance} = 0.960\left(3.00\times 10^8\ \text{m/s}\right)\left(3.214\times 10^{-10}\ \text{s}\right) = \boxed{9.26\ \text{cm}}$$

P46.28 (a) $$p_{\Sigma^+} = eBr_{\Sigma^+} = \frac{\left(1.602\,177\times10^{-19}\text{ C}\right)\left(1.15\text{ T}\right)\left(1.99\text{ m}\right)}{5.344\,288\times10^{-22}\,(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)} = \boxed{\frac{686\text{ MeV}}{c}}$$

$$p_{\pi^+} = eBr_{\pi^+} = \frac{\left(1.602\,177\times10^{-19}\text{ C}\right)\left(1.15\text{ T}\right)\left(0.580\text{ m}\right)}{5.344\,288\times10^{-22}\,(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)} = \boxed{\frac{200\text{ MeV}}{c}}$$

(b) Let ϕ be the angle made by the neutron's path with the path of the Σ^+ at the moment of decay. By conservation of momentum:

$$p_n\cos\phi + (199.961\,581\text{ MeV}/c)\cos 64.5° = 686.075\,081\text{ MeV}/c$$

$$\therefore p_n\cos\phi = 599.989\,401\text{ MeV}/c \qquad (1)$$

$$p_n\sin\phi = (199.961\,581\text{ MeV}/c)\sin 64.5° = 180.482\,380\text{ MeV}/c \qquad (2)$$

From (1) and (2): $$p_n = \sqrt{(599.989\,401\text{ MeV}/c)^2 + (180.482\,380\text{ MeV}/c)^2} = \boxed{627\text{ MeV}/c}$$

(c) $$E_{\pi^+} = \sqrt{\left(p_{\pi^+}c\right)^2 + \left(m_{\pi^+}c^2\right)^2} = \sqrt{(199.961\,581\text{ MeV})^2 + (139.6\text{ MeV})^2} = \boxed{244\text{ MeV}}$$

$$E_n = \sqrt{\left(p_n c\right)^2 + \left(m_n c^2\right)^2} = \sqrt{(626.547\,022\text{ MeV})^2 + (939.6\text{ MeV})^2} = \boxed{1\,130\text{ MeV}}$$

$$E_{\Sigma^+} = E_{\pi^+} + E_n = 243.870\,445\text{ MeV} + 1\,129.340\,219\text{ MeV} = \boxed{1\,370\text{ MeV}}$$

(d) $$m_{\Sigma^+}c^2 = \sqrt{E_{\Sigma^+}^2 - \left(p_{\Sigma^+}c\right)^2} = \sqrt{(1\,373.210\,664\text{ MeV})^2 - (686.075\,081\text{ MeV})^2} = 1\,190\text{ MeV}$$

$$\therefore m_{\Sigma^+} = \boxed{1\,190\text{ MeV}/c^2}$$

$$E_{\Sigma^+} = \gamma m_{\Sigma^+}c^2,\text{ where } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1\,373.210\,664\text{ MeV}}{1\,189.541\,303\text{ MeV}} = 1.154\,4$$

Solving for v, we find $v = \boxed{0.500c}$.

P46.29 (a) Let $E_{\min}$ be the minimum total energy of the bombarding particle that is needed to induce the reaction. At this energy the product particles all move with the same velocity. The product particles are then equivalent to a single particle having mass equal to the total mass of the product particles, moving with the same velocity as each product particle. By conservation of energy:

$$E_{\min} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + \left(p_3c\right)^2} \qquad (1)$$

By conservation of momentum: $p_3 = p_1$

$$\therefore \left(p_3c\right)^2 = \left(p_1c\right)^2 = E_{\min}^2 - \left(m_1c^2\right)^2 \qquad (2)$$

Substitute (2) in (1): $E_{\min} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2}$

Square both sides:

$$E_{\min}^2 + 2E_{\min}m_2c^2 + \left(m_2c^2\right)^2 = \left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2$$

$$\therefore E_{\min} = \frac{\left(m_3^2 - m_1^2 - m_2^2\right)c^2}{2m_2}$$

$$\therefore K_{\min} = E_{\min} - m_1c^2 = \frac{\left(m_3^2 - m_1^2 - m_2^2 - 2m_1m_2\right)c^2}{2m_2} = \frac{\left[m_3^2 - \left(m_1 + m_2\right)^2\right]c^2}{2m_2}$$

Refer to Table 46.2 for the particle masses.

(b) $$K_{\min} = \frac{\left[4(938.3)\right]^2 \text{ MeV}^2/c^2 - \left[2(938.3)\right]^2 \text{ MeV}^2/c^2}{2\left(938.3 \text{ MeV}/c^2\right)} = \boxed{5.63 \text{ GeV}}$$

(c) $$K_{\min} = \frac{(497.7 + 1115.6)^2 \text{ MeV}^2/c^2 - (139.6 + 938.3)^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{768 \text{ MeV}}$$

(d) $$K_{\min} = \frac{\left[2(938.3) + 135\right]^2 \text{ MeV}^2/c^2 - \left[2(938.3)\right]^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{280 \text{ MeV}}$$

(e) $$K_{\min} = \frac{\left[\left(91.2 \times 10^3\right)^2 - \left[(938.3 + 938.3)^2\right] \text{ MeV}^2/c^2\right]}{2(938.3) \text{ MeV}/c^2} = \boxed{4.43 \text{ TeV}}$$

Section 46.7 **Finding Patterns in the Particles**

Section 46.8 **Quarks**

Section 46.9 **Multicolored Quarks**

Section 46.10 **The Standard Model**

P46.30 (a) The number of protons

$$N_p = 1\,000\text{ g}\left(\frac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{10\text{ protons}}{\text{molecule}}\right) = 3.34\times10^{26}\text{ protons}$$

and there are $N_n = 1\,000\text{ g}\left(\frac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{8\text{ neutrons}}{\text{molecule}}\right) = 2.68\times10^{26}\text{ neutrons}$

So there are for electric neutrality $\boxed{3.34\times10^{26}\text{ electrons}}$

The up quarks have number $2\left(3.34\times10^{26}\right)+2.68\times10^{26} = \boxed{9.36\times10^{26}\text{ up quarks}}$

and there are $2\left(2.68\times10^{26}\right)+3.34\times10^{26} = \boxed{8.70\times10^{26}\text{ down quarks}}$

(b) Model yourself as 65 kg of water. Then you contain:

$$65\left(3.34\times10^{26}\right)\boxed{\sim10^{28}\text{ electrons}}$$

$$65\left(9.36\times10^{26}\right)\boxed{\sim10^{29}\text{ up quarks}}$$

$$65\left(8.70\times10^{26}\right)\boxed{\sim10^{29}\text{ down quarks}}$$

Only these fundamental particles form your body. You have no strangeness, charm, topness, or bottomness.

P46.31 (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

P46.32 Quark composition of proton = uud and of neutron = udd.
Thus, if we neglect binding energies, we may write

$$m_p = 2m_u + m_d \quad (1)$$

and

$$m_n = m_u + 2m_d \quad (2)$$

Solving simultaneously,

we find $$m_u = \frac{1}{3}(2m_p - m_n) = \frac{1}{3}\left[2(938\ \text{MeV}/c^2) - 939.6\ \text{meV}/c^2\right] = \boxed{312\ \text{MeV}/c^2}$$

and from either (1) or (2), $m_d = \boxed{314\ \text{MeV}/c^2}$.

P46.33 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	–1/3	0
charge	0	$-e/3$	$e/3$	0

(b)

	Λ^0	u	d	s	total
strangeness	–1	0	0	–1	–1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

P46.34 In the first reaction, $\pi^- + p \to K^0 + \Lambda^0$, the quarks in the particles are $\bar{u}d + uud \to d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + p \to K^0 + n$, the quarks in the particles are $\bar{u}d + uud \to d\bar{s} + udd$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

P46.35 (a) $\pi^- + p \to K^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\bar{u}d + uud \to d\bar{s} + uds}$

up quarks: $-1+2 \to 0+1$, or $1 \to 1$

down quarks: $1+1 \to 1+1$, or $2 \to 2$

strange quarks: $0+0 \to -1+1$, or $0 \to 0$

(b) $\pi^+ + p \to K^+ + \Sigma^+$ $\boxed{\bar{d}u + uud \to u\bar{s} + uus}$

up quarks: $1+2 \to 1+2$, or $3 \to 3$

down quarks: $-1+1 \to 0+0$, or $0 \to 0$

strange quarks: $0+0 \to -1+1$, or $0 \to 0$

(c) $K^- + p \to K^+ + K^0 + \Omega^-$ $\boxed{\bar{u}s + uud \to u\bar{s} + d\bar{s} + sss}$

up quarks: $-1+2 \to 1+0+0$, or $1 \to 1$

down quarks: $0+1 \to 0+1+0$, or $1 \to 1$

strange quarks: $1+0 \to -1-1+3$, or $1 \to 1$

continued on next page

(d) $p + p \to K^0 + p + \pi^+ + \underline{?}$ $\qquad$ $uud + uud \to d\bar{s} + uud + u\bar{d} + \underline{?}$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks:	$2+2=0+2+1+?$	(has 1 u quark)
down quarks:	$1+1=1+1-1+?$	(has 1 d quark)
strange quarks:	$0+0=-1+0+0+?$	(has 1 s quark)

quark composition $= uds = \boxed{\Lambda^0 \text{ or } \Sigma^0}$

P46.36 $\Sigma^0 + p \to \Sigma^+ + \gamma + X$

$dds + uud \to uds + 0 + ?$

The left side has a net 3d, 2u, and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

P46.37 Compare the given quark states to the entries in Tables 46.4 and 46.5:

(a) $suu = \boxed{\Sigma^+}$

(b) $\bar{u}d = \boxed{\pi^-}$

(c) $\bar{s}d = \boxed{K^0}$

(d) $ssd = \boxed{\Xi^-}$

P46.38 (a) $\bar{u}\bar{u}\bar{d}$: charge $= \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\bar{u}\bar{d}\bar{d}$: charge $= \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

Section 46.11 The Cosmic Connection

***P46.39** We let r in Hubble's law represent any distance.

(a)
$$v = Hr = 17\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot\text{ly}} 1.85\ \text{m}\left(\frac{1\ \text{ly}}{c\cdot 1\ \text{yr}}\right)\left(\frac{c}{3\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$
$$= 17\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot 9.47\times10^{15}\ \text{m}} 1.85\ \text{m} = 1.80\times10^{-18}\ \frac{1}{\text{s}} 1.85\ \text{m} = \boxed{3.32\times10^{-18}\ \text{m/s}}$$

This is unobservably small.

(b) $v = Hr = 1.80\times10^{-18}\ \frac{1}{\text{s}} 3.84\times10^{8}\ \text{m} = \boxed{6.90\times10^{-10}\ \text{m/s}}$ again too small to measure

P46.40 Section 39.4 says $f_{\text{observer}} = f_{\text{source}}\sqrt{\dfrac{1+v_a/c}{1-v_a/c}}$

The velocity of approach, v_a, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $\dfrac{c}{\lambda'} = \dfrac{c}{\lambda}\sqrt{\dfrac{1-v/c}{1+v/c}}$ and $\boxed{\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}}$

P46.41 (a) $\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}$ $\quad 510 \text{ nm} = 434 \text{ nm}\sqrt{\dfrac{1+v/c}{1-v/c}}$

$$1.18^2 = \frac{1+v/c}{1-v/c} = 1.381$$

$$1+\frac{v}{c} = 1.381 - 1.381\frac{v}{c} \qquad 2.38\frac{v}{c} = 0.381$$

$$\frac{v}{c} = 0.160 \qquad \text{or} \qquad v = \boxed{0.160c} = 4.80\times10^{7} \text{ m/s}$$

(b) $v = HR$: $\quad R = \dfrac{v}{H} = \dfrac{4.80\times10^{7} \text{ m/s}}{17\times10^{-3} \text{ m/s}\cdot\text{ly}} = \boxed{2.82\times10^{9} \text{ ly}}$

P46.42 (a) $\lambda_n' = \lambda_n\sqrt{\dfrac{1+v/c}{1-v/c}} = (Z+1)\lambda_n$ $\quad \dfrac{1+v/c}{1-v/c} = (Z+1)^2$

$$1+\frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2 \qquad \left(\frac{v}{c}\right)\left(Z^2+2Z+2\right) = Z^2+2Z$$

$$v = \boxed{c\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$$

(b) $R = \dfrac{v}{H} = \boxed{\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)}$

P46.43 $v = HR$ $\quad H = \dfrac{\left(1.7\times10^{-2} \text{ m/s}\right)}{\text{ly}}$

(a) $v\left(2.00\times10^{6} \text{ ly}\right) = 3.4\times10^{4} \text{ m/s}$ $\quad \lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}} = 590(1.000\,113\,3) = \boxed{590.07 \text{ nm}}$

(b) $v\left(2.00\times10^{8} \text{ ly}\right) = 3.4\times10^{6} \text{ m/s}$ $\quad \lambda' = 590\sqrt{\dfrac{1+0.011\,33}{1-0.011\,33}} = \boxed{597 \text{ nm}}$

(c) $v\left(2.00\times10^{9} \text{ ly}\right) = 3.4\times10^{7} \text{ m/s}$ $\quad \lambda' = 590\sqrt{\dfrac{1+0.113\,3}{1-0.113\,3}} = \boxed{661 \text{ nm}}$

***P46.44** (a) What we can see is limited by the finite age of the Universe and by the finite speed of light. We can see out only to a look-back time equal to a bit less than the age of the Universe. Every year on your birthday the Universe also gets a year older, and light now in transit from still more distant objects arrives at Earth. So the radius of the visible Universe expands at the speed of light, which is $dr/dt = c = 1$ ly/yr.

(b) The volume of the visible section of the Universe is $(4/3)\pi r^3$ where $r = 13.7$ billion light-years. The rate of volume increase is

$$\frac{dV}{dt} = \frac{d\ \frac{4}{3}\pi r^3}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt} = 4\pi r^2 c = 4\pi\left(13.7\times10^9\ \text{ly}\frac{3\times10^8\ \frac{\text{m}}{\text{s}}\ 3.156\times10^7\ \text{s}}{1\ \text{ly}}\right)^2 3\times10^8\ \frac{\text{m}}{\text{s}}$$

$$= \boxed{6.34\times10^{61}\ \text{m}^3/\text{s}}$$

***P46.45** (a) The volume of the sphere bounded by the Earth's orbit is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1.496\times10^{11}\ \text{m}\right)^3 = 1.40\times10^{34}\ \text{m}^3$$

$$m = \rho V = 6\times10^{-28}\ \text{kg/m}^3\ \ 1.40\times10^{34}\ \text{m}^3 = \boxed{8.41\times10^6\ \text{kg}}$$

(b) By Gauss's law, the dark matter would create a gravitational field acting on the Earth to accelerate it toward the Sun. It would shorten the duration of the year in the same way that 8.41×10^6 kg of extra material in the Sun would. This has the fractional effect of

$\dfrac{8.41\times10^6\ \text{kg}}{1.99\times10^{30}\ \text{kg}} = 4.23\times10^{-24}$ of the mass of the Sun. It is $\boxed{\text{immeasurably small}}$.

P46.46 (a) Wien's law: $\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

Thus, $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2.73\ \text{K}} = 1.06\times10^{-3}\ \text{m} = \boxed{1.06\ \text{mm}}.$$

(b) This is a $\boxed{\text{microwave}}$.

P46.47 We assume that the fireball of the Big Bang is a black body.

$$I = e\sigma T^4 = (1)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(2.73\ \text{K})^4 = \boxed{3.15\times10^{-6}\ \text{W/m}^2}$$

As a bonus, we can find the current power of direct radiation from the Big Bang in the portion of the Universe observable to us. If it is fourteen billion years old, the fireball is a perfect sphere of radius fourteen billion light years, centered at the point halfway between your eyes:

$$\mathcal{P} = IA = I(4\pi r^2) = \left(3.15\times10^{-6}\ \text{W/m}^2\right)(4\pi)\left(14\times10^9\ \text{ly}\right)^2\left(\frac{3\times10^8\ \text{m/s}}{1\ \text{ly/yr}}\right)^2\left(3.156\times10^7\ \text{s/yr}\right)^2$$

$$\mathcal{P} = 7\times10^{47}\ \text{W}$$

P46.48 The density of the Universe is

$$\rho = 1.20\rho_c = 1.20\left(\frac{3H^2}{8\pi G}\right)$$

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{0.600 H^2 r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy-sphere system is constant as the galaxy moves to apogee distance R:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \qquad \text{so} \qquad \frac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \qquad \text{so} \qquad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

P46.49 (a) $k_B T \approx 2m_p c^2$

so $$T \approx \frac{2m_p c^2}{k_B} = \frac{2(938.3 \text{ MeV})}{(1.38\times 10^{-23} \text{ J/K})}\left(\frac{1.60\times 10^{-13} \text{ J}}{1 \text{ MeV}}\right)\boxed{\sim 10^{13} \text{ K}}$$

(b) $k_B T \approx 2m_e c^2$

so $$T \approx \frac{2m_e c^2}{k_B} = \frac{2(0.511 \text{ MeV})}{(1.38\times 10^{-23} \text{ J/K})}\left(\frac{1.60\times 10^{-13} \text{ J}}{1 \text{ MeV}}\right)\boxed{\sim 10^{10} \text{ K}}$$

P46.50 (a) The Hubble constant is defined in $v = HR$. The distance R between any two far-separated objects opens at constant speed according to $R = vt$. Then the time t since the Big Bang is found from

$$v = H\,vt \qquad 1 = Ht \qquad t = \frac{1}{H}$$

(b) $$\frac{1}{H} = \frac{1}{17\times 10^{-3} \text{ m/s}\cdot\text{ly}}\left(\frac{3\times 10^8 \text{ m/s}}{1 \text{ ly/yr}}\right) = \boxed{1.76\times 10^{10} \text{ yr}} = 17.6 \text{ billion years}$$

P46.51 (a) Consider a sphere around us of radius R large compared to the size of galaxy clusters. If the matter M inside the sphere has the critical density, then a galaxy of mass m at the surface of the sphere is moving just at escape speed v according to

$$K + U_g = 0 \qquad \frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

The energy of the galaxy-sphere system is conserved, so this equation is true throughout the history of the Universe after the Big Bang, where $v = \frac{dR}{dt}$. Then

$$\left(\frac{dR}{dt}\right)^2 = \frac{2GM}{R} \qquad \frac{dR}{dt} = R^{-1/2}\sqrt{2GM} \qquad \int_0^R \sqrt{R}\,dR = \sqrt{2GM}\int_0^T dt$$

$$\left.\frac{R^{3/2}}{3/2}\right|_0^R = \sqrt{2GM}\,t\Big|_0^T \qquad \frac{2}{3}R^{3/2} = \sqrt{2GM}\,T$$

$$T = \frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM}} = \frac{2}{3}\frac{R}{\sqrt{2GM/R}}$$

From above, $\sqrt{\frac{2GM}{R}} = v$

so $T = \frac{2}{3}\frac{R}{v}$

Now Hubble's law says $v = HR$

So $T = \frac{2}{3}\frac{R}{HR} = \frac{2}{3H}$

(b) $$T = \frac{2}{3\left(17\times10^{-3}\ \text{m/s}\cdot\text{ly}\right)}\left(\frac{3\times10^8\ \text{m/s}}{1\ \text{ly/yr}}\right) = \boxed{1.18\times10^{10}\ \text{yr}} = 11.8 \text{ billion years}$$

P46.52 In our frame of reference, Hubble's law is exemplified by $\vec{\mathbf{v}}_1 = H\vec{\mathbf{R}}_1$ and $\vec{\mathbf{v}}_2 = H\vec{\mathbf{R}}_2$. From these we may form the equations $-\vec{\mathbf{v}}_1 = -H\vec{\mathbf{R}}_1$ and $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = H\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$. These equations express Hubble's law as seen by the observer in the first galaxy cluster, as she looks at us to find $-\vec{\mathbf{v}}_1 = H\left(-\vec{\mathbf{R}}_1\right)$ and as she looks at cluster two to find $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = H\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$.

Section 46.12 **Problems and Perspectives**

***P46.53** (a) $$L = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)}{\left(3.00\times10^8\ \text{m/s}\right)^3}} = \boxed{1.61\times10^{-35}\ \text{m}}$$

(b) The Planck time is given as $T = \frac{L}{c} = \frac{1.61\times10^{-35}\ \text{m}}{3.00\times10^8\ \text{m/s}} = \boxed{5.38\times10^{-44}\ \text{s}}$, which is approximately equal to the duration of the ultra-hot epoch.

(c) Yes. The uncertainty principle foils any attempt at making observations of things when the age of the Universe was less than the Planck time. The opaque fireball of the Big Bang, measured as the cosmic microwave background radiation, prevents us from receiving visible light from things before the Universe was a few hundred thousand years old. Walls of more profound fire hide all information from still earlier times.

Additional Problems

P46.54 We find the number N of neutrinos:

$$10^{46}\ \text{J} = N(6\ \text{MeV}) = N\left(6\times 1.60\times 10^{-13}\ \text{J}\right)$$

$$N = 1.0\times 10^{58}\ \text{neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi\left(1.7\times 10^5\ \text{ly}\right)^2}\left(\frac{1\ \text{ly}}{\left(3.00\times 10^8\ \text{m/s}\right)\left(3.16\times 10^7\ \text{s}\right)}\right)^2 = 3.1\times 10^{14}\ \text{m}^{-2}$$

The number passing through a body presenting $5\,000\ \text{cm}^2 = 0.50\ \text{m}^2$

is then $\left(3.1\times 10^{14}\ \dfrac{1}{\text{m}^2}\right)\left(0.50\ \text{m}^2\right) = 1.5\times 10^{14}$

or $\boxed{\sim 10^{14}}$

P46.55 A photon travels the distance from the Large Magellanic Cloud to us in 170 000 years. The hypothetical massive neutrino travels the same distance in 170 000 years plus 10 seconds:

$$c(170\,000\ \text{yr}) = v(170\,000\ \text{yr} + 10\ \text{s})$$

$$\frac{v}{c} = \frac{170\,000\ \text{yr}}{170\,000\ \text{yr} + 10\ \text{s}} = \frac{1}{1+\left\{10\ \text{s}\Big/\left[\left(1.7\times 10^5\ \text{yr}\right)\left(3.156\times 10^7\ \text{s/yr}\right)\right]\right\}} = \frac{1}{1+1.86\times 10^{-12}}$$

For the neutrino we want to evaluate mc^2 in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1-\frac{v^2}{c^2}} = 10\ \text{MeV}\sqrt{1-\frac{1}{\left(1+1.86\times 10^{-12}\right)^2}} = 10\ \text{MeV}\sqrt{\frac{\left(1+1.86\times 10^{-12}\right)^2 - 1}{\left(1+1.86\times 10^{-12}\right)^2}}$$

$$mc^2 \approx 10\ \text{MeV}\sqrt{\frac{2\left(1.86\times 10^{-12}\right)}{1}} = 10\ \text{MeV}\left(1.93\times 10^{-6}\right) = 19\ \text{eV}$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\ \text{eV}}{c^2}} \quad \text{or} \quad m = \frac{19\ \text{eV}}{c^2}\left(\frac{\text{u}}{931.5\times 10^6\ \text{eV}/c^2}\right) = 2.1\times 10^{-8}\ \text{u}$$

P46.56 (a) $\pi^- + \text{p} \to \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$

(b) $\mu^- \to \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$

(c) $\text{p} \to \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$

P46.57 The total energy in neutrinos emitted per second by the Sun is:

$$(0.4)\left[4\pi\left(1.5\times 10^{11}\right)^2\right]\text{W} = 1.1\times 10^{23}\ \text{W}$$

Over 10^9 years, the Sun emits 3.6×10^{39} J in neutrinos. This represents an annihilated mass

$$mc^2 = 3.6\times 10^{39}\ \text{J}$$

$$m = 4.0\times 10^{22}\ \text{kg}$$

About $\boxed{\text{1 part in 50 000 000}}$ of the Sun's mass, over 10^9 years, has been lost to neutrinos.

P46.58 $p + p \to p + \pi^+ + X$

The protons each have 70.4 MeV of kinetic energy. In accord with conservation of momentum for the collision, particle X has zero momentum and thus zero kinetic energy. Conservation of system energy then requires

$$M_p c^2 + M_\pi c^2 + M_X c^2 = \left(M_p c^2 + K_p\right) + \left(M_p c^2 + K_p\right)$$

$$M_X c^2 = M_p c^2 + 2K_p - M_\pi c^2 = 938.3 \text{ MeV} + 2(70.4 \text{ MeV}) - 139.6 \text{ MeV} = 939.5 \text{ MeV}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus X is a $\boxed{\text{neutron}}$.

P46.59 (a) If $2N$ particles are annihilated, the energy released is $2Nmc^2$. The resulting photon momentum is $p = \frac{E}{c} = \frac{2Nmc^2}{c} = 2Nmc$. Since the momentum of the system is conserved, the rockets will have momentum $2Nmc$ directed opposite the photon momentum.

$$\boxed{p = 2Nmc}$$

(b) Consider a particle that is annihilated and gives up its rest energy mc^2 to another particle which also has initial rest energy mc^2 (but no momentum initially).

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

Thus $\left(2mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$

where p is the momentum the second particle acquires as a result of the annihilation of the first particle. Thus $4\left(mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$, $p^2 = 3\left(mc^2\right)^2$. So $p = \sqrt{3}mc$.

This process is repeated N times (annihilate $\frac{N}{2}$ protons and $\frac{N}{2}$ antiprotons). Thus the total momentum acquired by the ejected particles is $\sqrt{3}Nmc$, and this momentum is imparted to the rocket.

$$\boxed{p = \sqrt{3}Nmc}$$

(c) Method (a) produces greater speed since $2Nmc > \sqrt{3}Nmc$.

P46.60 By relativistic energy conservation in the reaction, $$E_\gamma + m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - v^2/c^2}} \quad (1)$$

By relativistic momentum conservation for the system, $$\frac{E_\gamma}{c} = \frac{3m_e v}{\sqrt{1 - v^2/c^2}} \quad (2)$$

Dividing (2) by (1), $$X = \frac{E_\gamma}{E_\gamma + m_e c^2} = \frac{v}{c}$$

Subtracting (2) from (1), $$m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - X^2}} - \frac{3m_e c^2 X}{\sqrt{1 - X^2}}$$

Solving, $1 = \frac{3 - 3X}{\sqrt{1 - X^2}}$ and $X = \frac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04 \text{ MeV}}$

P46.61 $m_\Lambda c^2 = 1115.6$ MeV $\quad\quad \Lambda^0 \to p + \pi^-$

$m_p c^2 = 938.3$ MeV $\quad\quad m_\pi c^2 = 139.6$ MeV

The difference between starting rest energy and final rest energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7 \text{ MeV} \qquad \text{and} \qquad p_p = p_\pi = p$$

Applying conservation of relativistic energy to the decay process, we have

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7 \text{ MeV}$$

Solving the algebra yields

$$p_\pi c = p_p c = 100.4 \text{ MeV}$$

Then, $\quad K_p = \sqrt{\left(m_p c^2\right)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35 \text{ MeV}}$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3 \text{ MeV}}$$

P46.62 $p + p \to p + n + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve system momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy for the reaction gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3) \text{ MeV}}{2} = \boxed{70.4 \text{ MeV}}$$

P46.63 $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1192.5 \text{ MeV}/c^2$ $\quad$ and $\quad$ $m_\Lambda = 1115.6 \text{ MeV}/c^2$

Conservation of energy in the decay requires

$$E_{0,\Sigma} = \left(E_{o,\Lambda} + K_\Lambda\right) + E_\gamma \qquad \text{or} \qquad 1192.5 \text{ MeV} = \left(1115.6 \text{ MeV} + \frac{p_\Lambda^{\ 2}}{2m_\Lambda}\right) + E_\gamma$$

System momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1192.5 \text{ MeV} = \left(1115.6 \text{ MeV} + \frac{p_\gamma^{\ 2}}{2m_\Lambda}\right) + E_\gamma$$

or

$$1192.5 \text{ MeV} = \left(1115.6 \text{ MeV} + \frac{p_\gamma^{\ 2}c^2}{2m_\Lambda c^2}\right) + E_\gamma$$

Recognizing that $\quad m_\Lambda c^2 = 1115.6$ MeV $\quad$ and $\quad p_\gamma c = E_\gamma$

we now have

$$1192.5 \text{ MeV} = 1115.6 \text{ MeV} + \frac{E_\gamma^{\ 2}}{2(1115.6 \text{ MeV})} + E_\gamma$$

Solving this quadratic equation gives $\quad E_\gamma = \boxed{74.4 \text{ MeV}}$

P46.64 The momentum of the proton is $qBr = (1.60\times10^{-19}\ \text{C})(0.250\ \text{kg/C}\cdot\text{s})(1.33\ \text{m})$

$p_p = 5.32\times10^{-20}\ \text{kg}\cdot\text{m/s}$ $\qquad cp_p = 1.60\times10^{-11}\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 1.60\times10^{-11}\ \text{J} = 99.8\ \text{MeV}$

Therefore $\quad p_p = 99.8\ \text{MeV}/c$

The total energy of the proton is $\quad E_p = \sqrt{E_0^2+(cp)^2} = \sqrt{(938.3)^2+(99.8)^2} = 944\ \text{MeV}$

For the pion, the momentum qBr is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\ \text{MeV}/c$ $\qquad E_{0\pi} = 139.6\ \text{MeV}$

$$E_\pi = \sqrt{E_0^2+(cp)^2} = \sqrt{(139.6)^2+(99.8)^2} = 172\ \text{MeV}$$

Thus $E_{\text{total after}} = E_{\text{total before}} = \text{Rest energy}$

Rest energy of unknown particle $= 944\ \text{MeV} + 172\ \text{MeV} = 1\,116\ \text{MeV}$ (This is a Λ^0 particle!)

$\text{Mass} = \boxed{1116\ \text{MeV}/c^2}$.

P46.65 $\pi^- \to \mu^- + \bar{\nu}_\mu$: From the conservation laws for the decay,

$$m_\pi c^2 = 139.6\ \text{MeV} = E_\mu + E_\nu \qquad [1]$$

and $p_\mu = p_\nu$, $E_\nu = p_\nu c$: $\quad E_\mu^2 = (p_\mu c)^2 + (105.7\ \text{MeV})^2 = (p_\nu c)^2 + (105.7\ \text{MeV})^2$

or $\quad E_\mu^2 - E_\nu^2 = (105.7\ \text{MeV})^2 \qquad [2]$

Since $\quad E_\mu + E_\nu = 139.6\ \text{MeV} \qquad [1]$

and $\quad (E_\mu + E_\nu)(E_\mu - E_\nu) = (105.7\ \text{MeV})^2 \qquad [2]$

then $\quad E_\mu - E_\nu = \dfrac{(105.7\ \text{MeV})^2}{139.6\ \text{MeV}} = 80.0 \qquad [3]$

Subtracting [3] from [1], $\quad 2E_\nu = 59.6\ \text{MeV}$ and $\boxed{E_\nu = 29.8\ \text{MeV}}$

P46.66 The expression $e^{-E/k_BT}dE$ gives the fraction of the photons that have energy between E and $E+dE$. The fraction that have energy between E and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}dE}{\int_0^\infty e^{-E/k_BT}dE} = \frac{\int_E^\infty e^{-E/k_BT}(-dE/k_BT)}{\int_0^\infty e^{-E/k_BT}(-dE/k_BT)} = \frac{e^{-E/k_BT}\big|_E^\infty}{e^{-E/k_BT}\big|_0^\infty} = e^{-E/k_BT}$$

We require T when this fraction has a value of 0.0100 (i.e., 1.00%)

and $\quad E = 1.00\ \text{eV} = 1.60\times10^{-19}\ \text{J}$

Thus, $\quad 0.010\,0 = e^{-(1.60\times10^{-19}\ \text{J})/(1.38\times10^{-23}\ \text{J/K})T}$

or $\ln(0.010\,0) = -\dfrac{1.60\times10^{-19}\ \text{J}}{(1.38\times10^{-23}\ \text{J/K})T} = -\dfrac{1.16\times10^4\ \text{K}}{T}$ giving $T = \boxed{2.52\times10^3\ \text{K}}$

P46.67 (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.

(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge $+e$ and is a $\boxed{W^+}$.

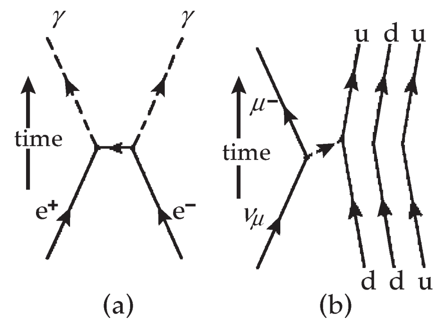

FIG. P46.67

P46.68 (a) The mediator of this weak interaction is a $\boxed{Z^0 \text{ boson}}$.

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$.

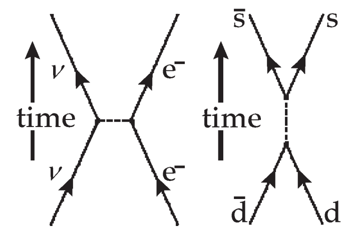

FIG. P46.68

For conservation of both energy and momentum in the collision we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in Figure P46.68. Depending on the color charges of the d and $\bar{\text{d}}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 46.13(b).

P46.69 (a) At threshold, we consider a photon and a proton colliding head-on to produce a proton and a pion at rest, according to $\text{p} + \gamma \to \text{p} + \pi^0$. Energy conservation gives

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} + E_\gamma = m_p c^2 + m_\pi c^2$$

Momentum conservation gives $\dfrac{m_p u}{\sqrt{1-u^2/c^2}} - \dfrac{E_\gamma}{c} = 0.$

Combining the equations, we have

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} + \frac{m_p c^2}{\sqrt{1-u^2/c^2}}\frac{u}{c} = m_p c^2 + m_\pi c^2$$

$$\frac{938.3 \text{ MeV}(1+u/c)}{\sqrt{(1-u/c)(1+u/c)}} = 938.3 \text{ MeV} + 135.0 \text{ MeV}$$

so $$\frac{u}{c} = 0.134$$

and $$E_\gamma = \boxed{127 \text{ MeV}}$$

(b) $\lambda_{\max} T = 2.898 \text{ mm} \cdot \text{K}$

$$\lambda_{\max} = \frac{2.898 \text{ mm} \cdot \text{K}}{2.73 \text{ K}} = \boxed{1.06 \text{ mm}}$$

continued on next page

(c) $E_\gamma = hf = \dfrac{hc}{\lambda} = \dfrac{1\,240 \text{ eV}\cdot 10^{-9} \text{ m}}{1.06\times 10^{-3} \text{ m}} = \boxed{1.17\times 10^{-3} \text{ eV}}$

(d) In the primed reference frame, the proton is moving to the right at $\dfrac{u'}{c} = 0.134$ and the photon is moving to the left with $hf' = 1.27\times 10^{8}$ eV. In the unprimed frame, $hf = 1.17\times 10^{-3}$ eV. Using the Doppler effect equation from Section 39.4, we have for the speed of the primed frame

$$1.27\times 10^{8} = \sqrt{\frac{1+v/c}{1-v/c}}\,1.17\times 10^{-3}$$

$$\frac{v}{c} = 1 - 1.71\times 10^{-22}$$

Then the speed of the proton is given by

$$\frac{u}{c} = \frac{u'/c + v/c}{1+u'v/c^2} = \frac{0.134 + 1 - 1.71\times 10^{-22}}{1+0.134\left(1-1.71\times 10^{-22}\right)} = 1 - 1.30\times 10^{-22}$$

And the energy of the proton is

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} = \frac{938.3 \text{ MeV}}{\sqrt{1-\left(1-1.30\times 10^{-22}\right)^2}} = 6.19\times 10^{10}\times 938.3\times 10^{6} \text{ eV} = \boxed{5.81\times 10^{19} \text{ eV}}$$

ANSWERS TO EVEN PROBLEMS

P46.2 ~10^3 Bq

P46.4 2.27×10^{23} Hz; 1.32 fm

P46.6 $\bar{\nu}_\mu$ and ν_e

P46.8 (b) The range is inversely proportional to the mass of the field particle. (c) Our rule describes the electromagnetic, weak, and gravitational interactions. For the electromagnetic and gravitational interactions, we take the limiting form of the rule with infinite range and zero mass for the field particle. For the weak interaction, 98.7 eV · nm/90 GeV ≈ 10^{-18} m = 10^{-3} fm, in agreement with the tabulated information. (d) ~10^{-16} m

P46.10 ~10^{-23} s

P46.12 $\bar{\nu}_\mu$

P46.14 (b) The second violates strangeness conservation.

P46.16 The second violates conservation of baryon number.

P46.18 $0.828c$

P46.20 (a) $\bar{\nu}_e$ (b) $\bar{\nu}_\mu$ (c) $\bar{\nu}_\mu$ (d) $\nu_\mu + \bar{\nu}_\tau$

P46.22 See the solution.

P46.24 (a) not allowed; violates conservation of baryon number (b) strong interaction (c) weak interaction (d) weak interaction (e) electromagnetic interaction

P46.26 (a) K^+ (b) Ξ^0 (c) π^0

P46.28 (a) $\dfrac{686\text{ MeV}}{c}$ and $\dfrac{200\text{ MeV}}{c}$ (b) $627\ \text{MeV}/c$ (c) 244 MeV, 1130 MeV, 1370 MeV (d) $1190\ \text{MeV}/c^2$, $0.500c$

P46.30 (a) 3.34×10^{26} e^-, 9.36×10^{26} u, 8.70×10^{26} d (b) ~10^{28} e^-, ~10^{29} u, ~10^{29} d. I have zero strangeness, charm, topness, and bottomness.

P46.32 $m_u = 312\text{ MeV}/c^2$ $m_d = 314\text{ MeV}/c^2$

P46.34 See the solution.

P46.36 a neutron, udd

P46.38 (a) $-e$, antiproton b) 0, antineutron

P46.40 See the solution.

P46.42 (a) $v = c\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$ (b) $\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$

P46.44 (a) What we can see is limited by the finite age of the Universe and by the finite speed of light. We can see out only to a look-back time equal to a bit less than the age of the Universe. Every year on your birthday the Universe also gets a year older, and light now in transit from still more distant objects arrives at Earth. So the radius of the visible Universe expands at the speed of light, which is 1 ly/yr. (b) 6.34×10^{61} m^3/s

P46.46 (a) 1.06 mm (b) microwave

P46.48 6.00

P46.50 (a) See the solution. (b) 17.6 Gyr

P46.52 See the solution.

P46.54 ~10^{14}

P46.56 (a) charge (b) energy (c) baryon number

P46.58 neutron

P46.60 2.04 MeV

P46.62 70.4 MeV

P46.64 1 116 MeV/c^2

P46.66 2.52×10^3 K

P46.68 (a) Z^0 boson (b) gluon or photon