27

Current and Resistance

CHAPTER OUTLINE

27.1 Electric Current
27.2 Resistance
27.3 A Model for Electrical Conduction
27.4 Resistance and Temperature
27.5 Superconductors
27.6 Electrical Power

ANSWERS TO QUESTIONS

Q27.1 Voltage is a measure of potential difference, not of current. “Surge” implies a flow—and only charge, in coulombs, can flow through a system. It would also be correct to say that the victim carried a certain current, in amperes.

Q27.2 Geometry and resistivity. In turn, the resistivity of the material depends on the temperature.

***Q27.3** (i) We require $\rho L/A_A = 3\rho L/A_B$. Then $A_A/A_B = 1/3$, answer (f).
(ii) $\pi r_A^2/\pi r_B^2 = 1/3$ gives $r_A/r_B = 1/\sqrt{3}$, answer (e).

***Q27.4** Originally, $R = \dfrac{\rho\ell}{A}$. Finally, $R_f = \dfrac{\rho(\ell/3)}{3A} = \dfrac{\rho\ell}{9A} = \dfrac{R}{9}$. Answer (b).

Q27.5 The conductor does not follow Ohm’s law, and must have a resistivity that is current-dependent, or more likely temperature-dependent.

Q27.6 The amplitude of atomic vibrations increases with temperature. Atoms can then scatter electrons more efficiently.

Q27.7 (i) The current density increases, so the drift speed must increase. Answer (a).
(ii) Answer (a).

Q27.8 The resistance of copper *increases* with temperature, while the resistance of silicon *decreases* with increasing temperature. The conduction electrons are scattered more by vibrating atoms when copper heats up. Silicon’s charge carrier density increases as temperature increases and more atomic electrons are promoted to become conduction electrons.

***Q27.9** In a normal metal, suppose that we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons. The drift speed and the current would increase steadily in time.

It is not the situation envisioned in the question, but we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

Q27.10 Because there are so many electrons in a conductor (approximately 10^{28} electrons/m^3) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting everywhere all at once.

***Q27.11** Action (a) makes the current three times larger.
(b) causes no change in current.
(c) corresponds to a current $\sqrt{3}$ times larger.
(d) R is 1/4 as large, so current is 4 times larger.
(e) R is 2 times larger, so current is half as large.
(f) R increases by a small percentage as current has a small decrease.
(g) Current decreases by a large factor.
The ranking is then d > a > c > b > f > e > g.

***Q27.12** $R_A = \dfrac{\rho L_A}{\pi(d_A/2)^2} = \dfrac{\rho 2L_B}{\pi(2d_B/2)^2} = \dfrac{1}{2}\dfrac{\rho L_B}{\pi(d_B/2)^2} = \dfrac{R_B}{2}$

$\mathcal{P}_A = I_A\Delta V = (\Delta V)^2/R_A = 2(\Delta V)^2/R_B = 2\mathcal{P}_B$ Answer (e).

***Q27.13** $R_A = \dfrac{\rho_A L}{A} = \dfrac{2\rho_B L}{A} = 2R_B$

$\mathcal{P}_A = I_A\Delta V = (\Delta V)^2/R_A = (\Delta V)^2/2R_B = \mathcal{P}_B/2$ Answer (f).

***Q27.14** (i) Bulb (a) must have higher resistance so that it will carry less current and have lower power.
(ii) Bulb (b) carries more current.

***Q27.15** One ampere–hour is (1 C/s)(3 600 s) = 3 600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference. Answer (d).

Q27.16 Choose the voltage of the power supply you will use to drive the heater. Next calculate the required resistance R as $\dfrac{\Delta V^2}{\mathcal{P}}$. Knowing the resistivity ρ of the material, choose a combination of wire length and cross-sectional area to make $\left(\dfrac{\ell}{A}\right) = \left(\dfrac{R}{\rho}\right)$. You will have to pay for less material if you make both ℓ and A smaller, but if you go too far the wire will have too little surface area to radiate away the energy; then the resistor will melt.

SOLUTIONS TO PROBLEMS

Section 27.1 **Electric Current**

P27.1 $I = \dfrac{\Delta Q}{\Delta t}$ $\qquad \Delta Q = I\Delta t = \left(30.0\times10^{-6}\text{ A}\right)\left(40.0\text{ s}\right) = 1.20\times10^{-3}\text{ C}$

$$N = \frac{Q}{e} = \frac{1.20\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = \boxed{7.50\times10^{15}\text{ electrons}}$$

P27.2 The molar mass of silver $= 107.9\ \text{g/mole}$ and the volume V is

$$V = (\text{area})(\text{thickness}) = \left(700\times10^{-4}\ \text{m}^2\right)\left(0.133\times10^{-3}\ \text{m}\right) = 9.31\times10^{-6}\ \text{m}^3$$

The mass of silver deposited is $m_{\text{Ag}} = \rho V = \left(10.5\times10^3\ \text{kg/m}^3\right)\left(9.31\times10^{-6}\,\text{m}^3\right) = 9.78\times10^{-2}\ \text{kg}$.

And the number of silver atoms deposited is

$$N = \left(9.78\times10^{-2}\ \text{kg}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{107.9\ \text{g}}\right)\left(\frac{1\,000\ \text{g}}{1\ \text{kg}}\right) = 5.45\times10^{23}\ \text{atoms}$$

$$I = \frac{\Delta V}{R} = \frac{12.0\ \text{V}}{1.80\ \Omega} = 6.67\ \text{A} = 6.67\ \text{C/s}$$

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{\left(5.45\times10^{23}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{6.67\ \text{C/s}} = 1.31\times10^4\ \text{s} = \boxed{3.64\ \text{h}}$$

P27.3 $Q(t) = \int_0^t I\,dt = I_0\tau\left(1-e^{-t/\tau}\right)$

(a) $Q(\tau) = I_0\tau\left(1-e^{-1}\right) = \boxed{(0.632)I_0\tau}$

(b) $Q(10\tau) = I_0\tau\left(1-e^{-10}\right) = \boxed{(0.999\,95)I_0\tau}$

(c) $Q(\infty) = I_0\tau\left(1-e^{-\infty}\right) = \boxed{I_0\tau}$

P27.4 The period of revolution for the sphere is $T = \frac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \frac{q}{T} = \boxed{\frac{q\omega}{2\pi}}$.

P27.5 $q = 4t^3 + 5t + 6$

$$A = \left(2.00\ \text{cm}^2\right)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00\times10^{-4}\ \text{m}^2$$

(a) $I(1.00\ \text{s}) = \left.\frac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2+5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$

(b) $J = \frac{I}{A} = \frac{17.0\ \text{A}}{2.00\times10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$

P27.6 $I = \frac{dq}{dt}$ $\qquad q = \int dq = \int I\,dt = \int_0^{1/240\ \text{s}} (100\ \text{A})\sin\left(\frac{120\pi t}{\text{s}}\right)dt$

$$q = \frac{-100\ \text{C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = \frac{+100\ \text{C}}{120\pi} = \boxed{0.265\ \text{C}}$$

P27.7 (a) $J = \frac{I}{A} = \frac{8.00\times10^{-6}\ \text{A}}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2} = \boxed{2.55\ \text{A/m}^2}$

(b) From $J = nev_d$, we have $n = \frac{J}{ev_d} = \frac{2.55\ \text{A/m}^2}{\left(1.60\times10^{-19}\ \text{C}\right)\left(3.00\times10^8\ \text{m/s}\right)} = \boxed{5.31\times10^{10}\ \text{m}^{-3}}$.

(c) From $I = \frac{\Delta Q}{\Delta t}$, we have $\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{8.00\times10^{-6}\ \text{A}} = \boxed{1.20\times10^{10}\ \text{s}}$.

(This is about 382 years!)

***P27.8** (a) $J = \frac{I}{A} = \frac{5.00\text{ A}}{\pi\left(4.00\times10^{-3}\text{ m}\right)^2} = \boxed{99.5\text{ kA/m}^2}$

(b) Current is the same and current density is smaller. Then $\boxed{I = 5.00\text{ A}}$,

$$J_2 = \frac{1}{4}J_1 = \frac{1}{4}9.95\times10^4\text{ A/m}^2 = \boxed{2.49\times10^4\text{ A/m}^2}$$

$$A_2 = 4A_1 \qquad \text{or} \qquad \pi r_2^2 = 4\pi r_1^2 \qquad \text{so} \qquad r_2 = 2r_1 = \boxed{0.800\text{ cm}}$$

P27.9 (a) The speed of each deuteron is given by $K = \frac{1}{2}mv^2$

$\left(2.00\times10^6\right)\left(1.60\times10^{-19}\text{ J}\right) = \frac{1}{2}\left(2\times1.67\times10^{-27}\text{ kg}\right)v^2$ and $v = 1.38\times10^7\text{ m/s}$

The time between deuterons passing a stationary point is t in $I = \frac{q}{t}$

$10.0\times10^{-6}\text{ C/s} = 1.60\times10^{-19}\text{ C}/t$ or $t = 1.60\times10^{-14}\text{ s}$

So the distance between them is $vt = \left(1.38\times10^7\text{ m/s}\right)\left(1.60\times10^{-14}\text{ s}\right) = \boxed{2.21\times10^{-7}\text{ m}}$.

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)}{2.21\times10^{-7}\text{ m}} = 6.49\times10^{-3}\text{ V}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

P27.10 We use $I = nqAv_d$ n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02\times10^{23}} = 4.49\times10^{-23}\text{ g/atom}$$

Thus, $$n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\text{ g/cm}^3}{4.49\times10^{-23}\text{ g/atom}}$$

$$n = 6.02\times10^{22}\text{ atoms/cm}^3 = 6.02\times10^{28}\text{ atoms/m}^3$$

Therefore, $$v_d = \frac{I}{nqA} = \frac{5.00\text{ A}}{\left(6.02\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{-6}\text{ m}^2\right)} = 1.30\times10^{-4}\text{ m/s}$$

or, $$v_d = \boxed{0.130\text{ mm/s}}$$

Section 27.2 **Resistance**

P27.11 $\Delta V = IR$

and $R = \frac{\rho\ell}{A}$: $\qquad A = \left(0.600\text{ mm}\right)^2\left(\frac{1.00\text{ m}}{1\,000\text{ mm}}\right)^2 = 6.00\times10^{-7}\text{ m}^2$

$\Delta V = \frac{I\rho\ell}{A}$: $\qquad I = \frac{\Delta VA}{\rho\ell} = \frac{\left(0.900\text{ V}\right)\left(6.00\times10^{-7}\text{ m}^2\right)}{\left(5.60\times10^{-8}\ \Omega\cdot\text{m}\right)\left(1.50\text{ m}\right)}$

$$I = \boxed{6.43\text{ A}}$$

P27.12 $I = \dfrac{\Delta V}{R} = \dfrac{120\text{ V}}{240\ \Omega} = 0.500\text{ A} = \boxed{500\text{ mA}}$

P27.13 (a) Given $M = \rho_d V = \rho_d A\ell$ where $\rho_d \equiv$ mass density,

we obtain: $A = \dfrac{M}{\rho_d \ell}$ Taking $\rho_r \equiv$ resistivity, $R = \dfrac{\rho_r \ell}{A} = \dfrac{\rho_r \ell}{M/\rho_d \ell} = \dfrac{\rho_r \rho_d \ell^2}{M}$

Thus, $\ell = \sqrt{\dfrac{MR}{\rho_r \rho_d}} = \sqrt{\dfrac{(1.00\times10^{-3})(0.500)}{(1.70\times10^{-8})(8.92\times10^{3})}}$ $\ell = \boxed{1.82\text{ m}}$

(b) $V = \dfrac{M}{\rho_d}$, or $\pi r^2 \ell = \dfrac{M}{\rho_d}$

Thus, $r = \sqrt{\dfrac{M}{\pi \rho_d \ell}} = \sqrt{\dfrac{1.00\times10^{-3}}{\pi(8.92\times10^{3})(1.82)}}$ $r = 1.40\times10^{-4}\text{ m}$

The diameter is twice this distance: diameter $= \boxed{280\ \mu\text{m}}$

P27.14 (a) Suppose the rubber is 10 cm long and 1 mm in diameter.

$$R = \frac{\rho\ell}{A} = \frac{4\rho\ell}{\pi d^2} \sim \frac{4(10^{13}\ \Omega\cdot\text{m})(10^{-1}\text{ m})}{\pi(10^{-3}\text{ m})^2} = \boxed{\sim10^{18}\ \Omega}$$

(b) $$R = \frac{4\rho\ell}{\pi d^2} \sim \frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(10^{-3}\text{ m})}{\pi(2\times10^{-2}\text{ m})^2}\ \boxed{\sim10^{-7}\ \Omega}$$

(c) $$I = \frac{\Delta V}{R} \sim \frac{10^2\text{ V}}{10^{18}\ \Omega}\ \boxed{\sim10^{-16}\text{ A}}$$

$$I \sim \frac{10^2\text{ V}}{10^{-7}\ \Omega}\ \boxed{\sim10^{9}\text{ A}}$$

P27.15 $J = \sigma E$ so $\sigma = \dfrac{J}{E} = \dfrac{6.00\times10^{-13}\text{ A/m}^2}{100\text{ V/m}} = \boxed{6.00\times10^{-15}\ (\Omega\cdot\text{m})^{-1}}$

Section 27.3 **A Model for Electrical Conduction**

***P27.16** (a) The density of charge carriers n is set by the material and is $\boxed{\text{unaffected}}$.

(b) The current density is proportional to current according to $|J| = \dfrac{I}{A}$ so it $\boxed{\text{doubles}}$.

(c) For larger current density in $J = nev_d$ the drift speed v_d $\boxed{\text{doubles}}$.

(d) The time between collisions $\tau = \dfrac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to a temperature change in the conductor.

P27.17 $\rho = \dfrac{m}{nq^2\tau}$ We take the density of conduction electrons from an Example in the chapter text.

so $$\tau = \frac{m}{\rho n q^2} = \frac{\left(9.11\times10^{-31}\right)}{\left(1.70\times10^{-8}\right)\left(8.46\times10^{28}\right)\left(1.60\times10^{-19}\right)^2} = 2.47\times10^{-14}\ \text{s}$$

$$v_d = \frac{qE}{m}\tau$$

gives $$7.84\times10^{-4} = \frac{\left(1.60\times10^{-19}\right)E\left(2.47\times10^{-14}\right)}{9.11\times10^{-31}}$$

Therefore, $E = \boxed{0.180\ \text{V/m}}$

Section 27.4 **Resistance and Temperature**

P27.18 $R = R_0\left[1+\alpha(\Delta T)\right]$ gives $140\ \Omega = (19.0\ \Omega)\left[1+\left(4.50\times10^{-3}/°\text{C}\right)\Delta T\right]$

Solving, $\Delta T = 1.42\times10^{3}\,°\text{C} = T - 20.0°\text{C}$

And the final temperature is $\boxed{T = 1.44\times10^{3}\,°\text{C}}$

P27.19 (a) $\rho = \rho_0\left[1+\alpha(T-T_0)\right] = \left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+3.90\times10^{-3}(30.0°)\right] = \boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times10^{6}\ \text{A/m}^2}$

(c) $I = JA = J\left(\dfrac{\pi d^2}{4}\right) = \left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\dfrac{\pi\left(1.00\times10^{-4}\ \text{m}\right)^2}{4}\right] = \boxed{49.9\ \text{mA}}$

(d) $n = \dfrac{6.02\times10^{23}\ \text{electrons}}{\left[26.98\ \text{g}/\left(2.70\times10^{6}\ \text{g/m}^3\right)\right]} = 6.02\times10^{28}\ \text{electrons/m}^3$

$$v_d = \frac{J}{ne} = \frac{\left(6.35\times10^{6}\ \text{A/m}^2\right)}{\left(6.02\times10^{28}\ \text{electrons/m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$

***P27.20** We require $10\ \Omega = \dfrac{3.5\times10^{-5}\,\Omega\cdot\text{m}\ \ell_1}{\pi(1.5\times10^{-3}\,\text{m})^2} + \dfrac{1.5\times10^{-6}\,\Omega\cdot\text{m}\ \ell_2}{\pi(1.5\times10^{-3}\,\text{m})^2}$ and for

any ΔT $\quad 10\ \Omega = \dfrac{3.5\times10^{-5}\,\Omega\cdot\text{m}\ \ell_1}{\pi(1.5\times10^{-3}\,\text{m})^2}\left(1-0.5\times10^{-3}\dfrac{\Delta T}{°\text{C}}\right) + \dfrac{1.5\times10^{-6}\,\Omega\cdot\text{m}\ \ell_2}{\pi(1.5\times10^{-3}\,\text{m})^2}\left(1+0.4\times10^{-3}\dfrac{\Delta T}{°\text{C}}\right)$

simplifying gives $10 = 4.951\,5\ \ell_1 + 0.212\,21\ \ell_2$

and $0 = -2.475\,7\times10^{-3}\ \ell_1 + 8.488\,3\times10^{-5}\ \ell_2$

These conditions are just sufficient to determine ℓ_1 and ℓ_2. $\boxed{\text{The design goal can be met.}}$

We have $\ell_2 = 29.167\ \ell_1$ so $10 = 4.951\,5\ \ell_1 + 0.212\,21\ (29.167\ \ell_1)$

and $\ell_1 = 10/11.141 = \boxed{0.898\ \text{m} = \ell_1 \qquad \ell_2 = 26.2\ \text{m}}$

P27.21 $R = R_0\left[1+\alpha T\right]$

$R - R_0 = R_0\alpha\Delta T$

$$\frac{R-R_0}{R_0} = \alpha\Delta T = \left(5.00\times10^{-3}\right)25.0 = \boxed{0.125}$$

P27.22 For aluminum, $\alpha_E = 3.90\times10^{-3}\,°\text{C}^{-1}$ (Table 27.2)

$\alpha = 24.0\times10^{-6}\,°\text{C}^{-1}$ (Table 19.1)

$$R = \frac{\rho\ell}{A} = \frac{\rho_0\left(1+\alpha_E\Delta T\right)\ell\left(1+\alpha\Delta T\right)}{A\left(1+\alpha\Delta T\right)^2} = R_0\frac{\left(1+\alpha_E\Delta T\right)}{\left(1+\alpha\Delta T\right)} = (1.234\ \Omega)\left(\frac{1.39}{1.002\ 4}\right) = \boxed{1.71\ \Omega}$$

Section 27.5 **Superconductors**

Problem 50 in Chapter 43 can be assigned with this section.

Section 27.6 **Electrical Power**

P27.23 $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{600\text{ W}}{120\text{ V}} = \boxed{5.00\text{ A}}$

and $R = \dfrac{\Delta V}{I} = \dfrac{120\text{ V}}{5.00\text{ A}} = \boxed{24.0\ \Omega}$

P27.24 $\mathcal{P} = I\Delta V = 500\times10^{-6}\text{ A}\left(15\times10^3\text{ V}\right) = \boxed{7.50\text{ W}}$

***P27.25** The energy that must be added to the water is

$Q = mc\Delta T = (109\text{ kg})(4\ 186\text{ J/kg°C})(29.0°\text{C}) = 1.32\times10^7\text{ J}$

Thus, the power supplied by the heater is

$$\mathcal{P} = \frac{W}{\Delta t} = \frac{Q}{\Delta t} = \frac{1.32\times10^7\text{ J}}{25\times60\text{ s}} = 8\ 820\text{ W}$$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(220\text{ V})^2}{8\ 820\text{ W}} = \boxed{5.49\ \Omega}$.

***P27.26** (a) $\textit{efficiency} = \dfrac{\textit{mechanical power output}}{\textit{total power input}} = 0.900 = \dfrac{2.50\text{ hp}(746\text{ W/1 hp})}{(120\text{ V})\,I}$

$$I = \frac{1\ 860\text{ J/s}}{0.9(120\text{ V})} = \frac{2\ 070\text{ J/s}}{120\text{ J/C}} = \boxed{17.3\text{ A}}$$

(b) $\textit{energy input} = \mathcal{P}_{\text{input}}\,\Delta t = (2\ 070\text{ J/s})\ 3\ (3\ 600\text{ s}) = \boxed{2.24\times10^7\text{ J}}$

(c) $\textit{cost} = 2.24\times 107\text{ J}\left(\dfrac{\$\ 0.16}{1\text{ kWh}}\right)\left(\dfrac{\text{k}}{10^3}\dfrac{\text{J}}{\text{W s}}\dfrac{\text{h}}{3\ 600\text{ s}}\right) = \boxed{\$\ 0.995}$

P27.27 $\frac{\mathcal{P}}{\mathcal{P}_0}=\frac{(\Delta V)^2/R}{(\Delta V_0)^2/R}=\left(\frac{\Delta V}{\Delta V_0}\right)^2=\left(\frac{140}{120}\right)^2=1.361$

$$\Delta\%=\left(\frac{\mathcal{P}-\mathcal{P}_0}{\mathcal{P}_0}\right)(100\%)=\left(\frac{\mathcal{P}}{\mathcal{P}_0}-1\right)(100\%)=(1.361-1)100\%=\boxed{36.1\%}$$

P27.28 The battery takes in energy by electric transmission

$$\mathcal{P}\Delta t=(\Delta V)I(\Delta t)=2.3\ \text{J/C}\left(13.5\times10^{-3}\ \text{C/s}\right)4.2\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right)=469\ \text{J}$$

It puts out energy by electric transmission

$$(\Delta V)I(\Delta t)=1.6\ \text{J/C}\left(18\times10^{-3}\ \text{C/s}\right)2.4\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right)=249\ \text{J}$$

(a) efficiency $=\frac{\text{useful output}}{\text{total input}}=\frac{249\ \text{J}}{469\ \text{J}}=\boxed{0.530}$

(b) The only place for the missing energy to go is into internal energy:
$469\ \text{J}=249\ \text{J}+\Delta E_{\text{int}}$

$\Delta E_{\text{int}}=\boxed{221\ \text{J}}$

(c) We imagine toasting the battery over a fire with 221 J of heat input:

$Q=mc\Delta T$

$\Delta T=\frac{Q}{mc}=\frac{221\ \text{J}}{0.015\ \text{kg}}\frac{\text{kg}^\circ\text{C}}{975\ \text{J}}=\boxed{15.1^\circ\text{C}}$

P27.29 $\mathcal{P}=I(\Delta V)=\frac{(\Delta V)^2}{R}=500\ \text{W}\qquad R=\frac{(110\ \text{V})^2}{(500\ \text{W})}=24.2\ \Omega$

(a) $R=\frac{\rho}{A}\ell$ so $\ell=\frac{RA}{\rho}=\frac{(24.2\ \Omega)\pi\left(2.50\times10^{-4}\ \text{m}\right)^2}{1.50\times10^{-6}\ \Omega\cdot\text{m}}=\boxed{3.17\ \text{m}}$

(b) $R=R_0\left[1+\alpha\Delta T\right]=24.2\ \Omega\left[1+\left(0.400\times10^{-3}\right)(1\,180)\right]=35.6\ \Omega$

$\mathcal{P}=\frac{(\Delta V)^2}{R}=\frac{(110)^2}{35.6}=\boxed{340\ \text{W}}$

P27.30 $R=\frac{\rho\ell}{A}=\frac{\left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)25.0\ \text{m}}{\pi\left(0.200\times10^{-3}\ \text{m}\right)^2}=298\ \Omega$

$\Delta V=IR=(0.500\ \text{A})(298\ \Omega)=149\ \text{V}$

(a) $E=\frac{\Delta V}{\ell}=\frac{149\ \text{V}}{25.0\ \text{m}}=\boxed{5.97\ \text{V/m}}$

(b) $\mathcal{P}=(\Delta V)I=(149\ \text{V})(0.500\ \text{A})=\boxed{74.6\ \text{W}}$

(c) $R=R_0\left[1+\alpha(T-T_0)\right]=298\ \Omega\left[1+\left(0.400\times10^{-3}/^\circ\text{C}\right)320^\circ\text{C}\right]=337\ \Omega$

$I=\frac{\Delta V}{R}=\frac{(149\ \text{V})}{(337\ \Omega)}=0.443\ \text{A}$

$\mathcal{P}=(\Delta V)I=(149\ \text{V})(0.443\ \text{A})=\boxed{66.1\ \text{W}}$

P27.31 (a) $$\Delta U = q(\Delta V) = It(\Delta V) = (55.0 \text{ A}\cdot\text{h})(12.0 \text{ V})\left(\frac{1 \text{ C}}{1 \text{ A}\cdot\text{s}}\right)\left(\frac{1 \text{ J}}{1 \text{ V}\cdot\text{C}}\right)\left(\frac{1 \text{ W}\cdot\text{s}}{1 \text{ J}}\right)$$

$$= 660 \text{ W}\cdot\text{h} = \boxed{0.660 \text{ kWh}}$$

(b) $$\text{Cost } = 0.660 \text{ kWh}\left(\frac{\$0.060\,0}{1 \text{ kWh}}\right) = \boxed{3.96¢}$$

***P27.32** (a) The resistance of 1 m of 12-gauge copper wire is

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi(d/2)^2} = \frac{4\rho\ell}{\pi d^2} = \frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)1 \text{ m}}{\pi\left(0.205\,3\times10^{-2}\ \text{m}\right)^2} = 5.14\times10^{-3}\ \Omega$$

The rate of internal energy production is $\mathcal{P} = I\Delta V = I^2R = (20 \text{ A})^2\, 5.14\times10^{-3}\ \Omega = \boxed{2.05 \text{ W}}$.

(b) $$\mathcal{P}_{\text{Al}} = I^2R = \frac{I^2 4\rho_{\text{Al}}\ell}{\pi d^2}$$

$$\frac{\mathcal{P}_{\text{Al}}}{\mathcal{P}_{\text{Cu}}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} \qquad \mathcal{P}_{\text{Al}} = \frac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.7\times10^{-8}\ \Omega\cdot\text{m}}2.05 \text{ W} = \boxed{3.41 \text{ W}}$$

Aluminum of the same diameter will get hotter than copper. It would not be as safe. If it is surrounded by thermal insulation, it could get much hotter than a copper wire.

P27.33 The energy taken in by electric transmission for the fluorescent lamp is

$$\mathcal{P}\Delta t = 11 \text{ J/s}(100 \text{ h})\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right) = 3.96\times10^6 \text{ J}$$

$$\text{cost} = 3.96\times10^6 \text{ J}\left(\frac{\$0.08}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600 \text{ s}}\right) = \$0.088$$

For the incandescent bulb,

$$\mathcal{P}\Delta t = 40 \text{ W}(100 \text{ h})\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right) = 1.44\times10^7 \text{ J}$$

$$\text{cost} = 1.44\times10^7 \text{ J}\left(\frac{\$0.08}{3.6\times10^6 \text{ J}}\right) = \$0.32$$

$$\text{saving} = \$0.32 - \$0.088 = \boxed{\$0.232}$$

P27.34 The total clock power is

$$\left(270\times10^6 \text{ clocks}\right)\left(2.50\ \frac{\text{J/s}}{\text{clock}}\right)\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right) = 2.43\times10^{12} \text{ J/h}$$

From $e = \frac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{\Delta t} = \frac{W_{\text{out}}/\Delta t}{e} = \frac{2.43\times10^{12} \text{ J/h}}{0.250} = 9.72\times10^{12} \text{ J/h}$$

and the rate of coal consumption is

$$\text{Rate} = \left(9.72\times10^{12} \text{ J/h}\right)\left(\frac{1.00 \text{ kg coal}}{33.0\times10^6 \text{ J}}\right) = 2.95\times10^5 \text{ kg coal/h} = \boxed{295 \text{ metric ton/h}}$$

P27.35 $\mathcal{P} = I(\Delta V) = (1.70 \text{ A})(110 \text{ V}) = 187 \text{ W}$

Energy used in a 24-hour day $= (0.187 \text{ kW})(24.0 \text{ h}) = 4.49 \text{ kWh}$.

Therefore daily cost $= 4.49 \text{ kWh}\left(\dfrac{\$0.060\,0}{\text{kWh}}\right) = \$0.269 = \boxed{26.9¢}$.

P27.36 $\mathcal{P} = I\Delta V = (2.00 \text{ A})(120 \text{ V}) = 240 \text{ W}$

$$\Delta E_{\text{int}} = (0.500 \text{ kg})(4\,186 \text{ J/kg}\cdot°\text{C})(77.0°\text{C}) = 161 \text{ kJ}$$

$$\Delta t = \frac{\Delta E_{\text{int}}}{\mathcal{P}} = \frac{1.61\times 10^5 \text{ J}}{240 \text{ W}} = \boxed{672 \text{ s}}$$

P27.37 At operating temperature,

(a) $\mathcal{P} = I\Delta V = (1.53 \text{ A})(120 \text{ V}) = \boxed{184 \text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1+\alpha\Delta T) \qquad \frac{120}{1.53} = \frac{120}{1.80}\left[1+\left(0.400\times 10^{-3}\right)\Delta T\right]$$

$$\Delta T = 441°\text{C} \qquad T = 20.0°\text{C} + 441°\text{C} = \boxed{461°\text{C}}$$

P27.38 You pay the electric company for energy transferred in the amount $E = \mathcal{P}\,\Delta t$.

(a) $\mathcal{P}\,\Delta t = 40 \text{ W}(2 \text{ weeks})\left(\dfrac{7 \text{ d}}{1 \text{ week}}\right)\left(\dfrac{86\,400 \text{ s}}{1 \text{ d}}\right)\left(\dfrac{1 \text{ J}}{1 \text{ W}\cdot\text{s}}\right) = 48.4 \text{ MJ}$

$\mathcal{P}\,\Delta t = 40 \text{ W}(2 \text{ weeks})\left(\dfrac{7 \text{ d}}{1 \text{ week}}\right)\left(\dfrac{24 \text{ h}}{1 \text{ d}}\right)\left(\dfrac{\text{k}}{1\,000}\right) = 13.4 \text{ kWh}$

$\mathcal{P}\,\Delta t = 40 \text{ W}(2 \text{ weeks})\left(\dfrac{7 \text{ d}}{1 \text{ week}}\right)\left(\dfrac{24 \text{ h}}{1 \text{ d}}\right)\left(\dfrac{\text{k}}{1\,000}\right)\left(\dfrac{0.12 \text{ \$}}{\text{kWh}}\right) = \boxed{\$1.61}$

(b) $\mathcal{P}\,\Delta t = 970 \text{ W}(3 \text{ min})\left(\dfrac{1 \text{ h}}{60 \text{ min}}\right)\left(\dfrac{\text{k}}{1\,000}\right)\left(\dfrac{0.12 \text{ \$}}{\text{kWh}}\right) = \boxed{\$0.005\,82} = 0.582¢$

(c) $\mathcal{P}\,\Delta t = 5\,200 \text{ W}(40 \text{ min})\left(\dfrac{1 \text{ h}}{60 \text{ min}}\right)\left(\dfrac{\text{k}}{1\,000}\right)\left(\dfrac{0.12 \text{ \$}}{\text{kWh}}\right) = \boxed{\$0.416}$

P27.39 Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$\mathcal{P}\,\Delta t = (400 \text{ J/s})(600 \text{ s/d})(365 \text{ d}) \approx 9\times 10^7 \text{ J}\left(\frac{1 \text{ kWh}}{3.6\times 10^6 \text{ J}}\right) \approx 20 \text{ kWh}$$

We suppose that electrically transmitted energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \approx (20 \text{ kWh})(\$0.10/\text{kWh}) = \$2 \boxed{\sim\$1}$$

Additional Problems

***P27.40** (a) $I = \dfrac{\Delta V}{R}$ so $\mathcal{P} = I\Delta V = \dfrac{(\Delta V)^2}{R}$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{25.0\text{ W}} = \boxed{576\ \Omega} \quad \text{and} \quad R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

(b) $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{25.0\text{ W}}{120\text{ V}} = 0.208\text{ A} = \dfrac{Q}{\Delta t} = \dfrac{1.00\text{ C}}{\Delta t}$

$$\Delta t = \frac{1.00\text{ C}}{0.208\text{ A}} = \boxed{4.80\text{ s}}$$

The charge itself is the same. It comes out at a location that is at lower potential.

(c) $\mathcal{P} = 25.0\text{ W} = \dfrac{\Delta U}{\Delta t} = \dfrac{1.00\text{ J}}{\Delta t}$ $\quad \Delta t = \dfrac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.040\,0\text{ s}}$

The energy itself is the same. It enters the bulb by electrical transmission and leaves by heat and electromagnetic radiation.

(d) $\Delta U = \mathcal{P}\Delta t = (25.0\text{ J/s})(86\,400\text{ s/d})(30.0\text{ d}) = 64.8\times10^6\text{ J}$

The electric company sells $\boxed{\text{energy}}$.

$$\text{Cost} = 64.8\times10^6\text{ J}\left(\frac{\$0.070\,0}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\text{ s}}\right) = \boxed{\$1.26}$$

$$\text{Cost per joule} = \frac{\$0.070\,0}{\text{kWh}}\left(\frac{\text{kWh}}{3.60\times10^6\text{ J}}\right) = \boxed{\$1.94\times10^{-8}/\text{J}}$$

***P27.41** The heater should put out constant power

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{mc(T_f - T_i)}{\Delta t} = \frac{(0.250\text{ kg})(4\,186\text{ J})(100^\circ\text{C} - 20^\circ\text{C})}{\text{kg}\cdot{}^\circ\text{C}(4\text{ min})}\left(\frac{1\text{ min}}{60\text{ s}}\right) = 349\text{ J/s}$$

Then its resistance should be described by

$$\mathcal{P} = (\Delta V)I = \frac{(\Delta V)(\Delta V)}{R} \qquad R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ J/C})^2}{349\text{ J/s}} = 41.3\ \Omega$$

Its resistivity at 100°C is given by

$$\rho = \rho_0\left[1 + \alpha(T - T_0)\right] = (1.50\times10^{-6}\ \Omega\cdot\text{m})\left[1 + 0.4\times10^{-3}(80)\right] = 1.55\times10^{-6}\ \Omega\cdot\text{m}$$

Then for a wire of circular cross section

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \rho\frac{\ell 4}{\pi d^2}$$

$$41.3\ \Omega = (1.55\times10^{-6}\ \Omega\cdot\text{m})\frac{4\ell}{\pi d^2}$$

$$\frac{\ell}{d^2} = 2.09\times10^{+7}/\text{m} \qquad \text{or} \qquad d^2 = (4.77\times10^{-8}\text{ m})\ell$$

One possible choice is $\ell = 0.900\text{ m}$ and $d = 2.07\times10^{-4}$ m. If ℓ and d are made too small, the surface area will be inadequate to transfer heat into the water fast enough to prevent overheating of the filament. To make the volume less than 0.5 cm^3, we want ℓ and d less than those described by $\dfrac{\pi d^2}{4}\ell = 0.5\times10^{-6}\text{ m}^3$. Substituting $d^2 = (4.77\times10^{-8}\text{ m})\ell$ gives $\dfrac{\pi}{4}(4.77\times10^{-8}\text{ m})\ell^2 = 0.5\times10^{-6}\text{ m}^3$, $\ell = 3.65\text{ m}$ and $d = 4.18\times10^{-4}$ m. Thus our answer is: Any diameter d and length ℓ related by $d^2 = (4.77\times10^{-8}\text{ m})\ell$ would have the right resistance. One possibility is length 0.900 m and diameter 0.207 mm, but such a small wire might overheat rapidly if it were not surrounded by water. The volume can be less than 0.5 cm^3.

P27.42 The original stored energy is $U_i = \frac{1}{2}Q\Delta V_i = \frac{1}{2}\frac{Q^2}{C}$.

(a) When the switch is closed, charge Q distributes itself over the plates of C and $3C$ in parallel, presenting equivalent capacitance $4C$. Then the final potential difference is $\boxed{\Delta V_f = \frac{Q}{4C}}$ for both.

(b) The smaller capacitor then carries charge $C\Delta V_f = \frac{Q}{4C}C = \boxed{\frac{Q}{4}}$. The larger capacitor carries charge $3C\frac{Q}{4C} = \boxed{\frac{3Q}{4}}$.

(c) The smaller capacitor stores final energy $\frac{1}{2}C(\Delta V_f)^2 = \frac{1}{2}C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{Q^2}{32C}}$. The larger capacitor possesses energy $\frac{1}{2}3C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{3Q^2}{32C}}$.

(d) The total final energy is $\frac{Q^2}{32C} + \frac{3Q^2}{32C} = \frac{Q^2}{8C}$. The loss of potential energy is the energy appearing as internal energy in the resistor: $\frac{Q^2}{2C} = \frac{Q^2}{8C} + \Delta E_{\text{int}}$ $\qquad \Delta E_{\text{int}} = \boxed{\frac{3Q^2}{8C}}$

P27.43 We begin with the differential equation $\qquad \alpha = \frac{1}{\rho}\frac{d\rho}{dT}$

(a) Separating variables, $\qquad \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_{T_0}^{T} \alpha dT$

$\ln\left(\frac{\rho}{\rho_0}\right) = \alpha(T - T_0)$ and $\boxed{\rho = \rho_0 e^{\alpha(T-T_0)}}$.

(b) From the series expansion $e^x \approx 1 + x$, $(x \ll 1)$, we have $\boxed{\rho \approx \rho_0\left[1 + \alpha(T - T_0)\right]}$.

P27.44 We find the drift velocity from $\qquad I = nqv_d A = nqv_d \pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1\,000\text{ A}}{8.46\times10^{28}\text{ m}^{-3}(1.60\times10^{-19}\text{ C})\pi(10^{-2}\text{ m})^2} = 2.35\times10^{-4}\text{ m/s}$$

$$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{200\times10^3\text{ m}}{2.35\times10^{-4}\text{ m/s}} = 8.50\times10^8\text{ s} = \boxed{27.0\text{ yr}}$$

***P27.45** From $\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$ we compute

ℓ (m)	R (Ω)	ρ ($\Omega\cdot$m)
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47\times10^{-6}\ \Omega\cdot\text{m}}$. With its uncertainty range from 1.41 to 1.50, this average value agrees with the tabulated value of $1.50\times10^{-6}\ \Omega\cdot\text{m}$ in Table 27.2.

P27.46 2 wires $\rightarrow \ell = 100$ m

$$R = \frac{0.108\ \Omega}{300\ \text{m}}(100\ \text{m}) = 0.036\,0\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.036\,0) = \boxed{116\ \text{V}}$

(b) $\mathcal{P} = I(\Delta V) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$

(c) $\mathcal{P}_{\text{wires}} = I^2 R = (110\ \text{A})^2 (0.036\,0\ \Omega) = \boxed{436\ \text{W}}$

***P27.47** (a) $\vec{\mathbf{E}} = -\frac{dV}{dx}\hat{\mathbf{i}} = -\frac{(0 - 4.00)\ \text{V}}{(0.500 - 0)\ \text{m}} = \boxed{8.00\hat{\mathbf{i}}\ \text{V/m}}$

(b) $R = \frac{\rho\ell}{A} = \frac{(4.00 \times 10^{-8}\ \Omega \cdot \text{m})(0.500\ \text{m})}{\pi(1.00 \times 10^{-4}\ \text{m})^2} = \boxed{0.637\ \Omega}$

(c) $I = \frac{\Delta V}{R} = \frac{4.00\ \text{V}}{0.637\ \Omega} = \boxed{6.28\ \text{A}}$

(d) $J = \frac{I}{A} = \frac{6.28\ \text{A}}{\pi(1.00 \times 10^{-4}\ \text{m})^2} = 2.00 \times 10^8\ \text{A/m}^2 = \boxed{200\ \text{MA/m}^2}$ The field and the current are both in the x direction.

(e) $\rho J = (4.00 \times 10^{-8}\ \Omega \cdot \text{m})(2.00 \times 10^8\ \text{A/m}^2) = 8.00\ \text{V/m} = E$

***P27.48** (a) $\vec{\mathbf{E}} = -\frac{dV(x)}{dx}\hat{\mathbf{i}} = \boxed{\frac{V}{L}\hat{\mathbf{i}}}$

(b) $R = \frac{\rho\ell}{A} = \boxed{\frac{4\rho L}{\pi d^2}}$

(c) $I = \frac{\Delta V}{R} = \boxed{\frac{V\pi d^2}{4\rho L}}$

(d) $J = \frac{I}{A} = \boxed{\frac{V}{\rho L}}$ The field and the current are both in the x direction.

(e) $\rho J = \frac{V}{L} = \boxed{E}$

P27.49 (a) $\mathcal{P} = I\Delta V$

so $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{8.00 \times 10^{3}\ \text{W}}{12.0\ \text{V}} = \boxed{667\ \text{A}}$

(b) $\Delta t = \dfrac{\Delta U}{\mathcal{P}} = \dfrac{2.00 \times 10^{7}\ \text{J}}{8.00 \times 10^{3}\ \text{W}} = 2.50 \times 10^{3}\ \text{s}$

and $\Delta x = v\Delta t = (20.0\ \text{m/s})(2.50 \times 10^{3}\ \text{s}) = \boxed{50.0\ \text{km}}$

***P27.50** (a) We begin with $R = \dfrac{\rho\ell}{A} = \dfrac{\rho_0\left[1+\alpha(T-T_0)\right]\ell_0\left[1+\alpha'(T-T_0)\right]}{A_0\left[1+2\alpha'(T-T_0)\right]}$,

which reduces to $R = \boxed{\dfrac{R_0\left[1+\alpha(T-T_0)\right]\left[1+\alpha'(T-T_0)\right]}{\left[1+2\alpha'(T-T_0)\right]}}$

(b) For copper: $\rho_0 = 1.70 \times 10^{-8}\ \Omega \cdot \text{m}$, $\alpha = 3.90 \times 10^{-3}\,°\text{C}^{-1}$, and

$\alpha' = 17.0 \times 10^{-6}\,°\text{C}^{-1}$

$$R_0 = \frac{\rho_0\ell_0}{A_0} = \frac{(1.70 \times 10^{-8})(2.00)}{\pi(0.100 \times 10^{-3})^2} = 1.08\ \Omega$$

The simple formula for R gives:

$$R = (1.08\ \Omega)\left[1+(3.90 \times 10^{-3}\,°\text{C}^{-1})(100°\text{C} - 20.0°\text{C})\right] = \boxed{1.420\ \Omega}$$

while the more complicated formula gives:

$$R = \frac{(1.08\ \Omega)\left[1+(3.90 \times 10^{-3}\,°\text{C}^{-1})(80.0°\text{C})\right]\left[1+(17.0 \times 10^{-6}\,°\text{C}^{-1})(80.0°\text{C})\right]}{\left[1+2(17.0 \times 10^{-6}\,°\text{C}^{-1})(80.0°\text{C})\right]}$$

$$= \boxed{1.418\ \Omega}$$

The results agree to three digits. The variation of resistance with temperature is typically a much larger effect than thermal expansion in size.

P27.51 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0°C. Then $\rho = \rho_0\left[1+\alpha(T-20.0°\text{C})\right]$, and $\rho = \rho'\left[1+\alpha'(T-0°\text{C})\right]$ must both give the correct resistivity at any temperature *T*. That is, we must have:

$$\rho_0\left[1+\alpha(T-20.0°\text{C})\right] = \rho'\left[1+\alpha'(T-0°\text{C})\right] \qquad (1)$$

Setting $T = 0$ in equation (1) yields: $\rho' = \rho_0\left[1-\alpha(20.0°\text{C})\right]$,

and setting $T = 20.0°\text{C}$ in equation (1) gives: $\rho_0 = \rho'\left[1+\alpha'(20.0°\text{C})\right]$

Put ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0\left[1-\alpha(20.0°\text{C})\right]\left[1+\alpha'(20.0°\text{C})\right]$$

Therefore $1+\alpha'(20.0°\text{C}) = \dfrac{1}{1-\alpha(20.0°\text{C})}$

which simplifies to $\alpha' = \dfrac{\alpha}{\left[1-\alpha(20.0°\text{C})\right]}$

From this, the temperature coefficient, based on a reference temperature of 0°C, may be computed for any material. For example, using this, $\boxed{\text{Table 27.2 becomes at 0°C}}$:

<u>Material</u>	<u>Temp Coefficients at 0°C</u>
Silver	$4.1\times10^{-3}/°\text{C}$
Copper	$4.2\times10^{-3}/°\text{C}$
Gold	$3.6\times10^{-3}/°\text{C}$
Aluminum	$4.2\times10^{-3}/°\text{C}$
Tungsten	$4.9\times10^{-3}/°\text{C}$
Iron	$5.6\times10^{-3}/°\text{C}$
Platinum	$4.25\times10^{-3}/°\text{C}$
Lead	$4.2\times10^{-3}/°\text{C}$
Nichrome	$0.4\times10^{-3}/°\text{C}$
Carbon	$-0.5\times10^{-3}/°\text{C}$
Germanium	$-24\times10^{-3}/°\text{C}$
Silicon	$-30\times10^{-3}/°\text{C}$

P27.52 (a) A thin cylindrical shell of radius *r*, thickness *dr*, and length *L* contributes resistance

$$dR = \frac{\rho d\ell}{A} = \frac{\rho dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}$$

(b) In this equation $\frac{\Delta V}{I} = \frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)$

we solve for $\boxed{\rho = \frac{2\pi L\Delta V}{I\ln(r_b/r_a)}}$

***P27.53** The original resistance is $R_i = \rho L_i/A_i$.

The new length is $L = L_i + \delta L = L_i(1+\delta)$.

Constancy of volume implies $AL = A_iL_i$ so $A = \frac{A_iL_i}{L} = \frac{A_iL_i}{L_i(1+\delta)} = \frac{A_i}{(1+\delta)}$

The new resistance is $R = \frac{\rho L}{A} = \frac{\rho L_i(1+\delta)}{A_i/(1+\delta)} = R_i(1+\delta)^2 = R_i(1+2\delta+\delta^2)$.

The result is exact if the assumptions are precisely true. Our derivation contains no approximation steps where delta is assumed to be small.

P27.54 Each speaker receives 60.0 W of power. Using $\mathcal{P} = I^2R$, we then have

$$I = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\text{ W}}{4.00\ \Omega}} = 3.87\text{ A}$$

The system is $\boxed{\text{not adequately protected}}$ since the $\boxed{\text{fuse should be set to melt at 3.87 A, or less}}$.

P27.55 (a) $\Delta V = -E\cdot\ell$ or $dV = -E\cdot dx$

$\Delta V = -IR = -E\cdot\ell$

$$I = \frac{dq}{dt} = \frac{E\cdot\ell}{R} = \frac{A}{\rho\ell}E\cdot\ell = \frac{A}{\rho}E = -\sigma A\frac{dV}{dx} = \boxed{\sigma A\left|\frac{dV}{dx}\right|}$$

(b) Current flows in the direction of decreasing voltage. Energy flows by heat in the direction of decreasing temperature.

P27.56 From the geometry of the longitudinal section of the resistor shown in the figure, we see that

$$\frac{(b-r)}{y} = \frac{(b-a)}{h}$$

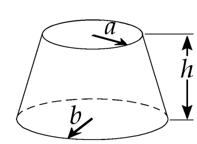

FIG. P27.56

From this, the radius at a distance *y* from the base is $r = (a-b)\frac{y}{h} + b$

For a disk-shaped element of volume $dR = \frac{\rho dy}{\pi r^2}$: $R = \frac{\rho}{\pi}\int_0^h \frac{dy}{\left[(a-b)(y/h)+b\right]^2}$

Using the integral formula $\int\frac{du}{(au+b)^2} = -\frac{1}{a(au+b)}$, $\boxed{R = \frac{\rho}{\pi}\frac{h}{ab}}$

P27.57 $R = \int \frac{\rho dx}{A} = \int \frac{\rho dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L}x$

$$R = \frac{\rho}{w}\int_0^L \frac{dx}{y_1 + \left[(y_2 - y_1)/L\right]x} = \frac{\rho L}{w(y_2 - y_1)}\ln\left[y_1 + \frac{y_2 - y_1}{L}x\right]\Bigg|_0^L$$

$$R = \boxed{\frac{\rho L}{w(y_2 - y_1)}\ln\left(\frac{y_2}{y_1}\right)}$$

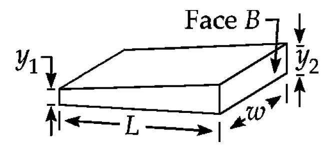

FIG. P27.57

***P27.58** A spherical layer within the shell, with radius r and thickness dr, has resistance

$$dR = \frac{\rho dr}{4\pi r^2}$$

The whole resistance is the absolute value of the quantity

$$R = \int_a^b dR = \int_{r_a}^{r_b} \frac{\rho dr}{4\pi r^2} = \frac{\rho}{4\pi}\frac{r^{-1}}{-1}\Bigg|_{r_a}^{r_b} = -\frac{\rho}{4\pi}\left(-\frac{1}{r_a} + \frac{1}{r_b}\right) = \frac{\rho}{4\pi}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

***P27.59** Coat the surfaces of entry and exit with material of much higher conductivity than the bulk material of the object. The electric potential will be essentially uniform over each of these electrodes. Current will be distributed over the whole area where each electrode is in contact with the resistive object.

P27.60 (a) The resistance of the dielectric block is $R = \frac{\rho \ell}{A} = \frac{d}{\sigma A}$.

The capacitance of the capacitor is $C = \frac{\kappa \epsilon_0 A}{d}$.

Then $RC = \frac{d}{\sigma A}\frac{\kappa \epsilon_0 A}{d} = \frac{\kappa \epsilon_0}{\sigma}$ is a characteristic of the material only.

(b) $$R = \frac{\kappa \epsilon_0}{\sigma C} = \frac{\rho \kappa \epsilon_0}{C} = \frac{75 \times 10^{16}\ \Omega \cdot \text{m}(3.78)8.85 \times 10^{-12}\ \text{C}^2}{14 \times 10^{-9}\ \text{F}\quad \text{N} \cdot \text{m}^2} = \boxed{1.79 \times 10^{15}\ \Omega}$$

P27.61 (a) Think of the device as two capacitors in parallel. The one on the left has $\kappa_1 = 1$, $A_1 = \left(\frac{\ell}{2} + x\right)\ell$. The equivalent capacitance is

$$\frac{\kappa_1 \epsilon_0 A_1}{d} + \frac{\kappa_2 \epsilon_0 A_2}{d} = \frac{\epsilon_0 \ell}{d}\left(\frac{\ell}{2} + x\right) + \frac{\kappa \epsilon_0 \ell}{d}\left(\frac{\ell}{2} - x\right) = \boxed{\frac{\epsilon_0 \ell}{2d}(\ell + 2x + \kappa \ell - 2\kappa x)}$$

(b) The charge on the capacitor is $Q = C\Delta V$

$$Q = \frac{\epsilon_0 \ell \Delta V}{2d}(\ell + 2x + \kappa \ell - 2\kappa x)$$

The current is

$$I = \frac{dQ}{dt} = \frac{dQ}{dx}\frac{dx}{dt} = \frac{\epsilon_0 \ell \Delta V}{2d}(0 + 2 + 0 - 2\kappa)v = -\frac{\epsilon_0 \ell \Delta V v}{d}(\kappa - 1)$$

The negative value indicates that the current drains charge from the capacitor. Positive current is $\boxed{\text{clockwise } \frac{\epsilon_0 \ell \Delta V v}{d}(\kappa - 1)}$.

P27.62 $I = I_0\left[\exp\left(\frac{e\Delta V}{k_B T}\right) - 1\right]$ and $R = \frac{\Delta V}{I}$

with $I_0 = 1.00 \times 10^{-9}$ A, $e = 1.60 \times 10^{-19}$ C, and $k_B = 1.38 \times 10^{-23}$ J/K

The following includes a partial table of calculated values and a graph for each of the specified temperatures.

(i) For $T = 280$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.015 6	25.6
0.440	0.081 8	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.047 6
0.600	61.6	0.009 7

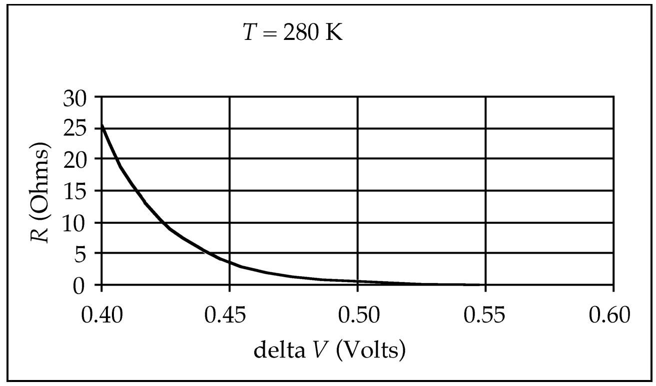

FIG. P27.62(i)

(ii) For $T = 300$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051

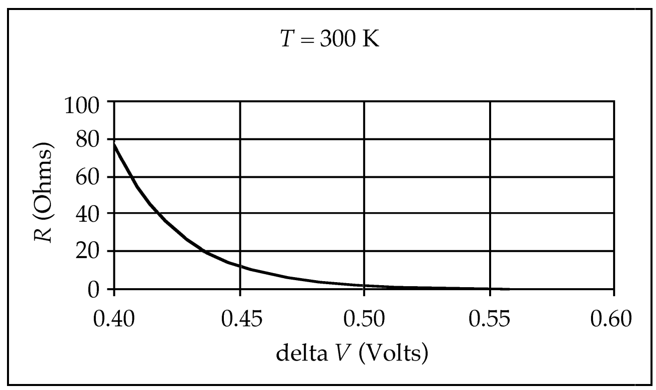

FIG. P27.62(ii)

(iii) For $T = 320$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.002 0	203
0.440	0.008 4	52.5
0.480	0.035 7	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217

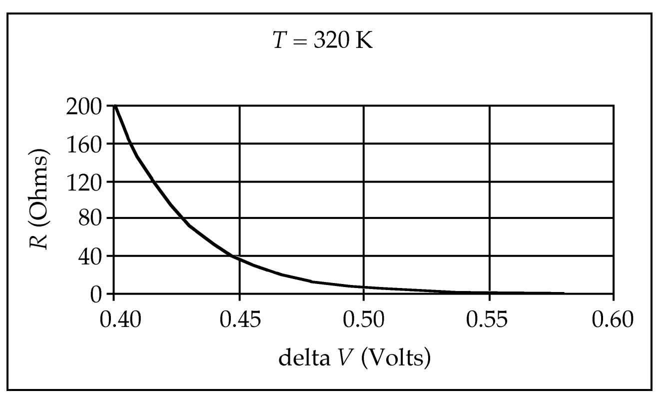

FIG. P27.62(iii)

P27.63 The volume of the gram of gold is given by $\rho = \frac{m}{V}$

$$V = \frac{m}{\rho} = \frac{10^{-3}\text{ kg}}{19.3\times10^3\text{ kg/m}^3} = 5.18\times10^{-8}\text{ m}^3 = A\left(2.40\times10^3\text{ m}\right)$$

$$A = 2.16\times10^{-11}\text{ m}^2$$

$$R = \frac{\rho\ell}{A} = \frac{2.44\times10^{-8}\ \Omega\cdot\text{m}\left(2.4\times10^3\text{ m}\right)}{2.16\times10^{-11}\text{ m}^2} = \boxed{2.71\times10^6\ \Omega}$$

P27.64 The resistance of one wire is $\left(\frac{0.500\ \Omega}{\text{mi}}\right)(100\text{ mi}) = 50.0\ \Omega$.

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$$IR = (1\,000\text{ A})(50.0\ \Omega) = 50.0\text{ kV}$$

Then it radiates as heat power $\mathcal{P} = (\Delta V)I = \left(50.0\times10^3\text{ V}\right)(1\,000\text{ A}) = \boxed{50.0\text{ MW}}$.

P27.65 $R = R_0\left[1+\alpha(T-T_0)\right]$ so $T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0}-1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I}-1\right]$

In this case, $I = \frac{I_0}{10}$, so $T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.004\,50/°\text{C}} = \boxed{2\,020°\text{C}}$

ANSWERS TO EVEN PROBLEMS

P27.2 3.64 h

P27.4 $q\omega/2\pi$

P27.6 0.265 C

P27.8 (a) 99.5 kA/m^2 (b) Current is the same, current density is smaller. 5.00 A, 24.9 kA/m^2, 0.800 cm

P27.10 0.130 mm/s

P27.12 500 mA

P27.14 (a) ~$10^{18}\ \Omega$ (b) ~$10^{-7}\ \Omega$ (c) ~100 aA, ~1 GA

P27.16 (a) no change (b) doubles (c) doubles (d) no change

P27.18 1.44×10^3 °C

P27.20 She can meet the design goal by choosing $\ell_1 = 0.898$ m and $\ell_2 = 26.2$ m.

P27.22 1.71 Ω

P27.24 7.50 W

P27.26 (a) 17.3 A (b) 22.4 MJ (c) \$0.995

P27.28 (a) 0.530 (b) 221 J (c) 15.1°C

P27.30 (a) 5.97 V/m (b) 74.6 W (c) 66.1 W

P27.32 (a) 2.05 W (b) 3.41 W. It would not be as safe. If surrounded by thermal insulation, it would get much hotter than a copper wire.

P27.34 295 metric ton/h

P27.36 672 s

P27.38 (a) \$1.61 (b) \$0.005 82 (c) \$0.416

P27.40 (a) 576 Ω and 144 Ω (b) 4.80 s. The charge itself is the same. It is at a location that is lower in potential. (c) 0.040 0 s. The energy itself is the same. It enters the bulb by electric transmission and leaves by heat and electromagnetic radiation. (d) \$1.26, energy, 1.94×10^{-8} \$/J

P27.42 (a) $Q/4C$ (b) $Q/4$ and $3Q/4$ (c) $Q^2/32C$ and $3Q^2/32C$ (d) $3Q^2/8C$

P27.44 8.50×10^8 s = 27.0 yr

P27.46 (a) 116 V (b) 12.8 kW (c) 436 W

P27.48 (a) $E = V/L$ in the x direction (b) $R = 4\rho L/\pi d^2$ (c) $I = V\pi d^2/4\rho L$ (d) $J = V/\rho L$
(e) See the solution.

P27.50 (a) See the solution. (b) 1.418 Ω nearly agrees with 1.420 Ω.

P27.52 (a) $R = \dfrac{\rho}{2\pi L}\ln\dfrac{r_b}{r_a}$ (b) $\rho = \dfrac{2\pi L\Delta V}{I\ln(r_b/r_a)}$

P27.54 No. The fuses should pass no more than 3.87 A.

P27.56 See the solution.

P27.58 See the solution.

P27.60 (b) 1.79 PΩ

P27.62 See the solution.

P27.64 50.0 MW