38

Diffraction Patterns and Polarization

CHAPTER OUTLINE

38.1 Introduction to Diffraction Patterns
38.2 Diffraction Patterns from Narrow Slits
38.3 Resolution of Single-Slit and Circular Apertures
38.4 The Diffraction Grating
38.5 Diffraction of X-Rays by Crystals
38.6 Polarization of Light Waves

ANSWERS TO QUESTIONS

Q38.1 Audible sound has wavelengths on the order of meters or centimeters, while visible light has a wavelength on the order of half a micrometer. In this world of breadbox-sized objects, $\frac{\lambda}{a}$ is large for sound, and sound diffracts around behind walls with doorways. But $\frac{\lambda}{a}$ is a tiny fraction for visible light passing ordinary-size objects or apertures, so light changes its direction by only very small angles when it diffracts.

Another way of phrasing the answer: We can see by a small angle around a small obstacle or around the edge of a small opening. The side fringes in Figure 38.1 and the Arago spot in the center of Figure 38.3 show this diffraction. We cannot always hear around corners. Out-of-doors, away from reflecting surfaces, have someone a few meters distant face away from you and whisper. The high-frequency, short-wavelength, information-carrying components of the sound do not diffract around his head enough for you to understand his words.

Suppose an opera singer loses the tempo and cannot immediately get it from the orchestra conductor. Then the prompter may make rhythmic kissing noises with her lips and teeth. Try it—you will sound like a birdwatcher trying to lure out a curious bird. This sound is clear on the stage but does not diffract around the prompter's box enough for the audience to hear it.

Q38.2 The wavelength of visible light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

***Q38.3** Answer (d). The power of the light coming through the slit decreases, as you would expect. The central maximum increases in width as the width of the slit decreases. In the condition $\sin\theta = \frac{\lambda}{a}$ for destructive interference on each side of the central maximum, θ increases as a decreases.

***Q38.4** We consider the fraction $\lambda L/a$. In case (a) and in case (f) it is $\lambda_0 L/a$. In case (b) it is $2\lambda_0 L/3a$. In case (c) it is $3\lambda_0 L/2a$. In case (d) it is $\lambda_0 L/2a$. In case (e) it is $\lambda_0 2L/a$. The ranking is e > c > a = f > b > d.

***Q38.5** Answer (b). The wavelength will be much smaller than with visible light.

***Q38.6** Answer (c). The ability to resolve light sources depends on diffraction, not on intensity.

Q38.7 Consider incident light nearly parallel to the horizontal ruler. Suppose it scatters from bumps at distance d apart to produce a diffraction pattern on a vertical wall a distance L away. At a point of height y, where $\theta = \frac{y}{L}$ gives the scattering angle θ, the character of the interference is determined by the shift δ between beams scattered by adjacent bumps, where $\delta = \frac{d}{\cos\theta} - d$.

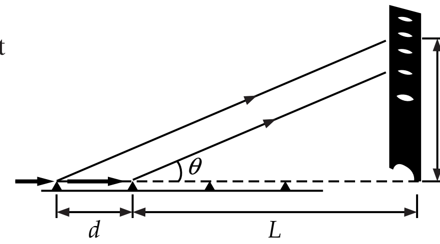

FIG. Q38.7

Bright spots appear for $\delta = m\lambda$, where $m = 0, 1, 2, 3, \ldots$. For small θ, these equations combine and reduce to $m\lambda = \frac{y_m^2 d}{2L^2}$. Measurement of the heights y_m of bright spots allows calculation of the wavelength of the light.

***Q38.8** Answer (b). No diffraction effects are observed because the separation distance between adjacent ribs is so much greater than the wavelength of x-rays. Diffraction does not limit the resolution of an x-ray image. Diffraction might sometimes limit the resolution of an ultrasonogram.

***Q38.9** Answer (a). Glare, as usually encountered when driving or boating, is horizontally polarized. Reflected light is polarized in the same plane as the reflecting surface. As unpolarized light hits a shiny horizontal surface, the atoms on the surface absorb and then reemit the light energy as a reflection. We can model the surface as containing conduction electrons free to vibrate easily along the surface, but not to move easily out of surface. The light emitted from a vibrating electron is partially or completely polarized along the plane of vibration, thus horizontally.

Q38.10 Light from the sky is partially polarized. Light from the blue sky that is polarized at 90° to the polarization axis of the glasses will be blocked, making the sky look darker as compared to the clouds.

***Q38.11** Answer (a). The grooves in a diffraction grating are not electrically conducting. Sending light through a diffraction grating is not like sending a vibration on a rope through a picket fence. The electric field in light does not have an amplitude in real space. Its amplitude is in newtons per coulomb, not in millimeters.

Q38.12 First think about the glass without a coin and about one particular point P on the screen. We can divide up the area of the glass into ring-shaped zones centered on the line joining P and the light source, with successive zones contributing alternately in-phase and out-of-phase with the light that takes the straight-line path to P. These Fresnel zones have nearly equal areas. An outer zone contributes only slightly less to the total wave disturbance at P than does the central circular zone. Now insert the coin. If P is in line with its center, the coin will block off the light from some particular number of zones. The first unblocked zone around its circumference will send light to P with significant amplitude. Zones farther out will predominantly interfere destructively with each other, and the Arago spot is bright. Slightly off the axis there is nearly complete destructive interference, so most of the geometrical shadow is dark. A bug on the screen crawling out past the edge of the geometrical shadow would in effect see the central few zones coming out of eclipse. As the light from them interferes alternately constructively and destructively, the bug moves through bright and dark fringes on the screen. The diffraction pattern is shown in Figure 38.3 in the text.

Q38.13 Since the obsidian is opaque, a standard method of measuring incidence and refraction angles and using Snell's Law is ineffective. Reflect unpolarized light from the horizontal surface of the obsidian through a vertically polarized filter. Change the angle of incidence until you observe that none of the reflected light is transmitted through the filter. This means that the reflected light is completely horizontally polarized, and that the incidence and reflection angles are the polarization angle. The tangent of the polarization angle is the index of refraction of the obsidian.

Q38.14 The fine hair blocks off light that would otherwise go through a fine slit and produce a diffraction pattern on a distant screen. The width of the central maximum in the pattern is inversely proportional to the distance across the slit. When the hair is in place, it subtracts the same diffraction pattern from the projected disk of laser light. The hair produces a diffraction minimum that crosses the bright circle on the screen. The width of the minimum is inversely proportional to the diameter of the hair. The central minimum is flanked by narrower maxima and minima. Measure the width $2y$ of the central minimum between the maxima bracketing it, and use Equation 38.1 in the form $\frac{y}{L} = \frac{\lambda}{a}$ to find the width a of the hair.

Q38.15 The condition for constructive interference is that the three radio signals arrive at the city in phase. We know the speed of the waves (it is the speed of light c), the angular bearing θ of the city east of north from the broadcast site, and the distance d between adjacent towers. The wave from the westernmost tower must travel an extra distance $2d\sin\theta$ to reach the city, compared to the signal from the eastern tower. For each cycle of the carrier wave, the western antenna would transmit first, the center antenna after a time delay $\frac{d\sin\theta}{c}$, and the eastern antenna after an additional equal time delay.

Q38.16 It is shown in the correct orientation. If the horizontal width of the opening is equal to or less than the wavelength of the sound, then the equation $a\sin\theta = (1)\lambda$ has the solution $\theta = 90°$, or has no solution. The central diffraction maximum covers the whole seaward side. If the vertical height of the opening is large compared to the wavelength, then the angle in $a\sin\theta = (1)\lambda$ will be small, and the central diffraction maximum will form a thin horizontal sheet.

Featured in the motion picture *M*A*S*H* (20th Century Fox, Aspen Productions, 1970) is a loudspeaker mounted on an exterior wall of an Army barracks. It has an approximately rectangular aperture, and it is installed incorrectly. The longer side is horizontal, to maximize sound spreading in a vertical plane and to minimize sound radiated in different horizontal directions.

SOLUTIONS TO PROBLEMS

Section 38.1 **Introduction to Diffraction Patterns**

Section 38.2 **Diffraction Patterns from Narrow Slits**

P38.1 $$\sin\theta = \frac{\lambda}{a} = \frac{6.328\times10^{-7}}{3.00\times10^{-4}} = 2.11\times10^{-3}$$

$$\frac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta \approx \theta \text{ (for small } \theta\text{)}$$

$$2y = \boxed{4.22\text{ mm}}$$

P38.2 The positions of the first-order minima are $\frac{y}{L} \approx \sin\theta = \pm\frac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\frac{\lambda}{a}\right)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times10^{-3}\text{ m}}{2}\right)\left(\frac{0.550\times10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}$$

P38.3 $\frac{y}{L} \approx \sin\theta = \frac{m\lambda}{a}$ $\qquad$ $\Delta y = 3.00\times10^{-3}\text{ nm}$

$\Delta m = 3 - 1 = 2$ and $a = \frac{\Delta m\lambda L}{\Delta y}$

$$a = \frac{2\left(690\times10^{-9}\text{ m}\right)(0.500\text{ m})}{\left(3.00\times10^{-3}\text{ m}\right)} = \boxed{2.30\times10^{-4}\text{ m}}$$

P38.4 For destructive interference,

$$\sin\theta = m\frac{\lambda}{a} = \frac{\lambda}{a} = \frac{5.00\text{ cm}}{36.0\text{ cm}} = 0.139$$

and $\theta = 7.98°$

$$\frac{y}{L} = \tan\theta$$

gives $y = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912\text{ m}$

$$y = \boxed{91.2\text{ cm}}$$

P38.5 If the speed of sound is 340 m/s, $\qquad \lambda = \frac{v}{f} = \frac{340\text{ m/s}}{650\text{ s}^{-1}} = 0.523\text{ m}$

Diffraction minima occur at angles described by $a\sin\theta = m\lambda$

$(1.10\text{ m})\sin\theta_1 = 1(0.523\text{ m})$ $\qquad \theta_1 = 28.4°$

$(1.10\text{ m})\sin\theta_2 = 2(0.523\text{ m})$ $\qquad \theta_2 = 72.0°$

$(1.10\text{ m})\sin\theta_3 = 3(0.523\text{ m})$ $\qquad \theta_3$ nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

$(1.10\text{ m})\sin\theta_x = 1.5(0.523\text{ m})$ $\qquad \theta_x\ \boxed{\approx 46°}$

There is no solution to $a\sin\theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

***P38.6** (a) The rectangular patch on the wall is wider than it is tall. The aperture will be taller than it is wide. For horizontal spreading we have

$$\tan\theta_{\text{width}} = \frac{y_{\text{width}}}{L} = \frac{0.110\text{ m}/2}{4.5\text{ m}} = 0.012\,2$$

$$a_{\text{width}}\sin\theta_{\text{width}} = 1\lambda$$

$$a_{\text{width}} = \frac{632.8\times10^{-9}\text{ m}}{0.012\,2} = \boxed{5.18\times10^{-5}\text{ m}}$$

For vertical spreading, similarly

$$\tan\theta_{\text{height}} = \frac{0.006\text{ m}/2}{4.5\text{ m}} = 0.000\,667$$

$$a_{\text{height}} = \frac{1\lambda}{\sin\theta_h} = \frac{632.8\times10^{-9}\text{ m}}{0.000\,667} = \boxed{9.49\times10^{-4}\text{ m}}$$

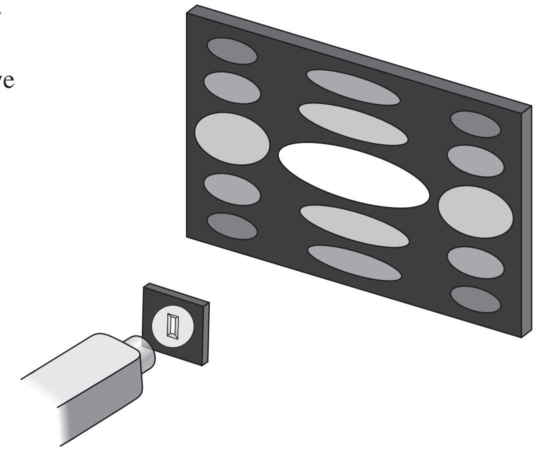

FIG. P38.6

(b) The central bright patch is horizontal. The aperture is vertical.

A smaller distance between aperture edges causes a wider diffraction angle. The longer dimension of each rectangle is 18.3 times larger than the smaller dimension.

P38.7 $$\sin\theta \approx \frac{y}{L} = \frac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}$$

We define $\phi = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi\left(4.00\times10^{-4}\text{ m}\right)}{546.1\times10^{-9}\text{ m}}\left(\dfrac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = 7.86\text{ rad}$

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\phi)}{\phi}\right]^2 = \left[\frac{\sin(7.86)}{7.86}\right]^2 = \boxed{1.62\times10^{-2}}$$

P38.8 Equation 38.1 states that $\sin\theta = \dfrac{m\lambda}{a}$, where $m = \pm1, \pm2, \pm3, \ldots$.

The requirement for $m = 1$ is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in the textbook Figure 38.5. This extra distance must be equal to $\dfrac{\lambda}{2}$ for destructive interference. When the source rays approach the slit at an angle β, there is a distance added to the path difference (of ray 1 compared to ray 3) of $\dfrac{a}{2}\sin\beta$. Then, for destructive interference,

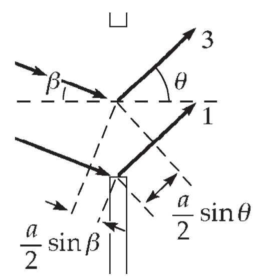

FIG. P38.8

$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2} \text{ so } \sin\theta = \frac{\lambda}{a}0 - \sin\beta.$$

Dividing the slit into 4 parts leads to the 2nd order minimum: $\sin\theta = \dfrac{2\lambda}{a} - \sin\beta$

Dividing the slit into 6 parts gives the third order minimum: $\sin\theta = \dfrac{3\lambda}{a} - \sin\beta$

Generalizing, we obtain the condition for the mth order minimum: $\sin\theta = \dfrac{m\lambda}{a} - \sin\beta$

***P38.9** The diffraction envelope shows a broad central maximum flanked by zeros at $a \sin\theta = 1\lambda$ and $a \sin\theta = 2\lambda$. That is, the zeros are at $(\pi a \sin\theta)/\lambda = \pi, -\pi, 2\pi, -2\pi, \ldots$. Noting that the distance between slits is $d = 9\,\mu\text{m} = 3a$, we say that within the diffraction envelope the interference pattern shows closely spaced maxima at $d \sin\theta = m\lambda$, giving $(\pi\, 3a \sin\theta)/\lambda = m\pi$ or $(\pi a \sin\theta)/\lambda = 0, \pi/3, -\pi/3, 2\pi/3, -2\pi/3$. The third-order interference maxima are missing because they fall at the same directions as diffraction minima, but the fourth order can be visible at $(\pi a \sin\theta)/\lambda = 4\pi/3$ and $-4\pi/3$ as diagrammed.

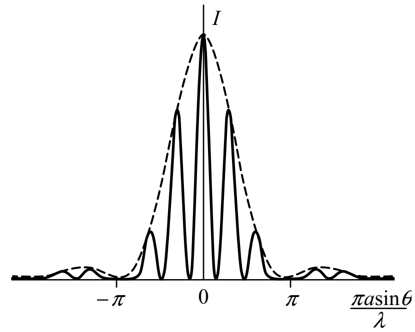

FIG. P38.9

P38.10 (a) Double-slit interference maxima are at angles given by $d \sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$

Similarly, for $m = 2, 3, 4, 5$ and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$

$\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) =$ nonexistent

Thus, there are $5 + 5 + 1 = 11$ directions for interference maxima.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a \sin\theta = m\lambda$.

For $m = 1$, $(0.700\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$ and $\theta_1 = 45.8°$

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at $\boxed{\theta = 0°, \pm 10.3°, \pm 21.0°, \pm 32.5°, \text{ and } \pm 63.6°}$.

(c) $I = I_{\max}\left[\dfrac{\sin(\pi a \sin\theta/\lambda)}{\pi \sin\theta/\lambda}\right]^2$

At $\theta = 0°$, $\dfrac{\sin\theta}{\theta} \to 1$ and $\dfrac{I}{I_{\max}} \to \boxed{1.00}$

At $\theta = 10.3°$, $\dfrac{\pi a \sin\theta}{\lambda} = \dfrac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.501\,5\ \mu\text{m}} = 0.785\text{ rad} = 45.0°$

$$\frac{I}{I_{\max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}$$

Similarly, at $\theta = 21.0°$, $\dfrac{\pi a \sin\theta}{\lambda} = 1.57\text{ rad} = 90.0°$ and $\dfrac{I}{I_{\max}} = \boxed{0.405}$

At $\theta = 32.5°$, $\dfrac{\pi a \sin\theta}{\lambda} = 2.36\text{ rad} = 135°$ and $\dfrac{I}{I_{\max}} = \boxed{0.090\,1}$

At $\theta = 63.6°$, $\dfrac{\pi a \sin\theta}{\lambda} = 3.93\text{ rad} = 225°$ and $\dfrac{I}{I_{\max}} = \boxed{0.032\,4}$

Section 38.3 **Resolution of Single-Slit and Circular Apertures**

P38.11 We assume Rayleigh's criterion applies to the predator's eye with pupil narrowed. (It is a few times too optimistic for a normal human eye with pupil dilated.)

$$\sin\theta = \frac{\lambda}{a} = \frac{5.00\times 10^{-7}\text{ m}}{5.00\times 10^{-4}} = \boxed{1.00\times 10^{-3}\text{ rad}}$$

***P38.12** (a) $1.22\lambda/D = 1.22(589\text{ nm})/(9\ 000\ 000\text{ nm}) = \boxed{79.8\ \mu\text{rad}}$

(b) For a smaller angle of diffraction we choose the smallest visible wavelength, $\boxed{\text{violet}}$ at 400 nm, to obtain $1.22\lambda/D = 1.22(400\text{ nm})/(9\ 000\ 000\text{ nm}) = \boxed{54.2\ \mu\text{rad}}$.

(c) The wavelength in water is shortened to its vacuum value divided by the index of refraction. $\boxed{\text{The resolving power is improved, with the minimum resolvable angle becoming}}$

$1.22\lambda/D = 1.22(589\text{ nm}/1.33)/(9\ 000\ 000\text{ nm}) = \boxed{60.0\ \mu\text{rad}}$

Better than water for many purposes is oil immersion.

P38.13 Undergoing diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{632.8\times 10^{-9}\text{ m}}{0.005\ 00\text{ m}}\right) = 1.54\times 10^{-4}\text{ rad}$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\min}\left(1.00\times 10^{4}\text{ m}\right) = 1.544\text{ m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\text{ m}}$.

***P38.14** When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. We take $\theta_{\min} = \dfrac{d}{L} = 1.22\dfrac{\lambda}{D}$, where $\theta_{\min}$ is the smallest angular separation of two objects for which they are resolved by an aperture of diameter D, d is the separation of the two objects, and L is the maximum distance of the aperture from the two objects at which they can be resolved.

Two objects can be resolved if their angular separation is greater than $\theta_{\min}$. Thus, $\theta_{\min}$ should be as small as possible. Therefore, light with the smaller of the two given wavelengths is easier to resolve, i.e., $\boxed{\text{blue}}$.

$$L = \frac{Dd}{1.22\lambda} = \frac{\left(5.20\times 10^{-3}\text{ m}\right)\left(2.80\times 10^{-2}\text{ m}\right)}{1.22\lambda} = \frac{1.193\times 10^{-4}\text{ m}^2}{\lambda}$$

Thus $L = 186$ m for $\lambda = 640$ nm, and $L = 271$ m for $\lambda = 440$ nm. The viewer with the assumed diffraction-limited vision could resolve adjacent tubes of blue in the range $\boxed{186\text{ m to }271\text{ m}}$, but cannot resolve adjacent tubes of red in this range.

P38.15 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. According to this criterion, two dots separated center-to-center by 2.00 mm would overlap

when $\quad \theta_{\min} = \dfrac{d}{L} = 1.22\dfrac{\lambda}{D}$

Thus, $\quad L = \dfrac{dD}{1.22\lambda} = \dfrac{\left(2.00\times 10^{-3}\text{ m}\right)\left(4.00\times 10^{-3}\text{ m}\right)}{1.22\left(500\times 10^{-9}\text{ m}\right)} = \boxed{13.1\text{ m}}$

P38.16 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be.

$D = 250 \times 10^3$ m

$\lambda = 5.00 \times 10^{-7}$ m

$d = 5.00 \times 10^{-3}$ m

$$x = 1.22\frac{\lambda}{d}D = 1.22\left(\frac{5.00 \times 10^{-7}\text{ m}}{5.00 \times 10^{-3}\text{ m}}\right)\left(250 \times 10^3\text{ m}\right) = \boxed{30.5\text{ m}}$$

P38.17 The concave mirror of the spy satellite is probably about 2 m in diameter, and is surely not more than 5 m in diameter. That is the size of the largest piece of glass successfully cast to a precise shape, for the mirror of the Hale telescope on Mount Palomar. If the spy satellite had a larger mirror, its manufacture could not be kept secret, and it would be visible from the ground. Outer space is probably closer than your state capitol, but the satellite is surely above 200-km altitude, for reasonably low air friction. We find the distance between barely resolvable objects at a distance of 200 km, seen in yellow light through a 5-m aperture:

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = (200\,000\text{ m})(1.22)\left(\frac{6 \times 10^{-7}\text{ m}}{5\text{ m}}\right) = 3\text{ cm}$$

(Considering atmospheric seeing caused by variations in air density and temperature, the distance between barely resolvable objects is more like $(200\,000\text{ m})(1\text{ s})\left(\frac{1°}{3\,600\text{ s}}\right)\left(\frac{\pi\text{ rad}}{180°}\right) = 97$ cm.)

Thus the snooping spy satellite cannot see the difference between III and II or IV on a license plate. It cannot count coins spilled on a sidewalk, much less read the dates on them.

P38.18 $1.22\frac{\lambda}{D} = \frac{d}{L}$ $\qquad \lambda = \frac{c}{f} = 0.020\,0$ m

$D = 2.10$ m $\qquad L = 9\,000$ m

$$d = 1.22\frac{(0.020\,0\text{ m})(9\,000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$$

Section 38.4 **The Diffraction Grating**

P38.19 $d = \frac{1.00\text{ cm}}{2\,000} = \frac{1.00 \times 10^{-2}\text{ m}}{2\,000} = 5.00\ \mu\text{m}$

$$\sin\theta = \frac{m\lambda}{d} = \frac{1\left(640 \times 10^{-9}\text{ m}\right)}{5.00 \times 10^{-6}\text{ m}} = 0.128 \qquad \theta = \boxed{7.35°}$$

P38.20 The principal maxima are defined by

$d\sin\theta = m\lambda \qquad m = 0,\ 1,\ 2, \ldots$

For $m = 1$, $\qquad \lambda = d\sin\theta$

where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

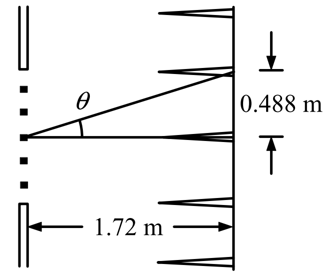

FIG. P38.20

$$\tan\theta = \frac{0.488 \text{ m}}{1.72 \text{ m}} = 0.284$$

so $\qquad \theta = 15.8°$

and $\qquad \sin\theta = 0.273$

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d = \frac{1}{5\,310 \text{ cm}^{-1}} = 1.88 \times 10^{-4} \text{ cm} = 1.88 \times 10^{3} \text{ nm}$$

The wavelength is $\qquad \lambda = d\sin\theta = \left(1.88 \times 10^{3} \text{ nm}\right)(0.273) = \boxed{514 \text{ nm}}$

P38.21 The grating spacing is $\qquad d = \dfrac{1.00 \times 10^{-2} \text{ m}}{4\,500} = 2.22 \times 10^{-6} \text{ m}$

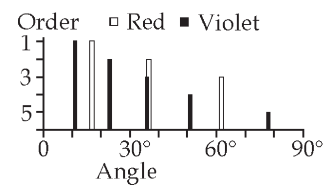

FIG. P38.21

In the 1st-order spectrum, diffraction angles are given by

$\sin\theta = \dfrac{\lambda}{d}$: $\qquad \sin\theta_1 = \dfrac{656 \times 10^{-9} \text{ m}}{2.22 \times 10^{-6} \text{ m}} = 0.295$

so that for red $\qquad \theta_1 = 17.17°$

and for blue $\qquad \sin\theta_2 = \dfrac{434 \times 10^{-9} \text{ m}}{2.22 \times 10^{-6} \text{ m}} = 0.195$

so that $\qquad \theta_2 = 11.26°$

The angular separation is in first-order, $\qquad \Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$

In the second-order spectrum, $\qquad \Delta\theta = \sin^{-1}\left(\dfrac{2\lambda_1}{d}\right) - \sin^{-1}\left(\dfrac{2\lambda_2}{d}\right) = \boxed{13.2°}$

Again, in the third order, $\qquad \Delta\theta = \sin^{-1}\left(\dfrac{3\lambda_1}{d}\right) - \sin^{-1}\left(\dfrac{3\lambda_2}{d}\right) = \boxed{26.5°}$

Since the red does not appear in the fourth-order spectrum, the answer is complete.

P38.22 $\sin\theta = 0.350$: $\quad d = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8 \text{ nm}}{0.350} = 1.81\times10^{3}\text{ nm}$

Line spacing = $\boxed{1.81\ \mu\text{m}}$

P38.23 (a) $d = \dfrac{1}{3\,660 \text{ lines/cm}} = 2.732\times10^{-4} \text{ cm} = 2.732\times10^{-6} \text{ m} = 2\,732 \text{ nm}$

$\lambda = \dfrac{d\sin\theta}{m}$: At $\theta = 10.09°$, $\lambda = \boxed{478.7 \text{ nm}}$

At $\theta = 13.71°$, $\lambda = \boxed{647.6 \text{ nm}}$

At $\theta = 14.77°$, $\lambda = \boxed{696.6 \text{ nm}}$

(b) $d = \dfrac{\lambda}{\sin\theta_1}$ and $2\lambda = d\sin\theta_2$ so $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\lambda}{\lambda/\sin\theta_1} = 2\sin\theta_1$

Therefore, if $\theta_1 = 10.09°$ then $\sin\theta_2 = 2\sin(10.09°)$ gives $\theta_2 = \boxed{20.51°}$.

Similarly, for $\theta_1 = 13.71°$, $\theta_2 = \boxed{28.30°}$ and for $\theta_1 = 14.77°$, $\theta_2 = \boxed{30.66°}$.

P38.24 $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends of the different order spectra are defined by:

End of second order: $\quad \sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1\,500 \text{ nm}}{d}$

Start of third order: $\quad \sin\theta_{3v} = \dfrac{3\lambda_v}{d} = \dfrac{1\,200 \text{ nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v} \text{ and these orders must overlap}}$ regardless of the value of the grating spacing *d*.

***P38.25** The sound has wavelength $\lambda = \dfrac{v}{f} = \dfrac{343 \text{ m/s}}{37.2\times10^{3}/\text{s}} = 9.22\times10^{-3}\text{m}$. Each diffracted beam is described by $\quad d\sin\theta = m\lambda, \quad m = 0, 1, 2, \ldots$

The zero-order beam is at $m = 0$, $\theta = 0$. The beams in the first order of interference are to the left and right at $\theta = \sin^{-1}\dfrac{1\lambda}{d} = \sin^{-1}\dfrac{9.22\times10^{-3} \text{ m}}{1.3\times10^{-2} \text{ m}} = \sin^{-1} 0.709 = 45.2°$. For a second-order beam we would need $\theta = \sin^{-1}\dfrac{2\lambda}{d} = \sin^{-1} 2(0.709) = \sin^{-1} 1.42$. No angle, smaller or larger than 90°, has a sine greater than 1. Then a diffracted beam does not exist for the second order or any higher order. The whole answer is then, $\boxed{\text{three beams, at } 0° \text{ and at } 45.2° \text{ to the right and to the left}}$.

P38.26 For a side maximum, $\tan\theta = \dfrac{y}{L} = \dfrac{0.4\ \mu\text{m}}{6.9\ \mu\text{m}}$

$$\theta = 3.32°$$

$$d\sin\theta = m\lambda \qquad d = \frac{(1)(780\times10^{-9}\ \text{m})}{\sin 3.32°} = 13.5\ \mu\text{m}$$

$$\text{The number of grooves per millimeter } = \frac{1\times10^{-3}\ \text{m}}{13.5\times10^{-6}\ \text{m}} = \boxed{74.2}$$

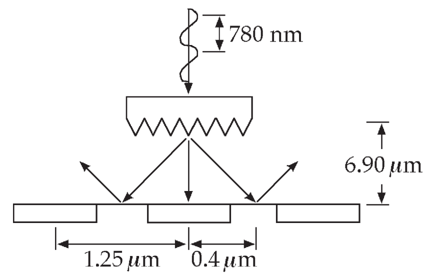

FIG. P38.26

P38.27 $d = \dfrac{1.00\times10^{-3}\ \text{m/mm}}{250\ \text{lines/mm}} = 4.00\times10^{-6}\ \text{m} = 4\ 000\ \text{nm} \qquad d\sin\theta = m\lambda \Rightarrow m = \dfrac{d\sin\theta}{\lambda}$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71$$

or $\boxed{\text{5 orders is the maximum}}$

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0$$

or $\boxed{\text{10 orders in the short-wavelength region}}$

P38.28 (a) The several narrow parallel slits make a diffraction grating. The zeroth- and first-order maxima are separated according to

$$d\sin\theta = (1)\lambda \qquad \sin\theta = \frac{\lambda}{d} = \frac{632.8\times10^{-9}\,\text{m}}{1.2\times10^{-3}\text{m}}$$

$$\theta = \sin^{-1}(0.000\ 527) = 0.000\ 527\ \text{rad}$$

$$y = L\tan\theta = (1.40\ \text{m})(0.000\ 527) = \boxed{0.738\ \text{mm}}$$

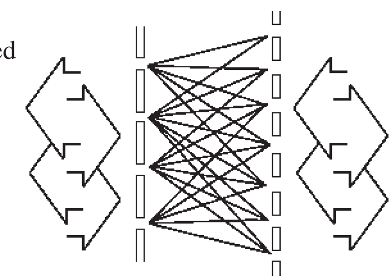

FIG. P38.28

(b) Many equally spaced transparent lines appear on the film. It is itself a diffraction grating. When the same light is sent through the film, it produces interference maxima separated according to

$$d\sin\theta = (1)\lambda \qquad \sin\theta = \frac{\lambda}{d} = \frac{632.8\times10^{-9}\ \text{m}}{0.738\times10^{-3}\ \text{m}} = 0.000\ 857$$

$$y = L\tan\theta = (1.40\ \text{m})(0.000\ 857) = 1.20\ \text{mm}$$

An image of the original set of slits appears on the screen. If the screen is removed, light diverges from the real images with the same wave fronts reconstructed as the original slits produced. Reasoning from the mathematics of Fourier transforms, Gabor showed that light diverging from any object, not just a set of slits, could be used. In the picture, the slits or maxima on the left are separated by 1.20 mm. The slits or maxima on the right are separated by 0.738 mm. The length difference between any pair of lines is an integer number of wavelengths. Light can be sent through equally well toward the right or toward the left. Soccer players shift smoothly between offensive and defensive tactics.

P38.29 $d = \dfrac{1}{4\ 200/\text{cm}} = 2.38 \times 10^{-6}\ \text{m} = 2\ 380\ \text{nm}$

$d\sin\theta = m\lambda$ or $\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right)$ and $y = L\tan\theta = L\tan\left[\sin^{-1}\left(\dfrac{m\lambda}{d}\right)\right]$

Thus, $$\Delta y = L\left\{\tan\left[\sin^{-1}\left(\frac{m\lambda_2}{d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{m\lambda_1}{d}\right)\right]\right\}$$

For $m = 1$, $$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{589.6}{2\ 380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{589}{2\ 380}\right)\right]\right\} = 0.554\ \text{mm}$$

For $m = 2$, $$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{2(589.6)}{2\ 380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{2(589)}{2\ 380}\right)\right]\right\} = 1.54\ \text{mm}$$

For $m = 3$, $$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{3(589.6)}{2\ 380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3(589)}{2\ 380}\right)\right]\right\} = 5.04\ \text{mm}$$

Thus, the observed order must be $\boxed{m = 2}$.

Section 38.5 **Diffraction of X-Rays by Crystals**

P38.30 $2d\sin\theta = m\lambda$: $$\lambda = \frac{2d\sin\theta}{m} = \frac{2\left(0.353 \times 10^{-9}\ \text{m}\right)\sin 7.60°}{1} = 9.34 \times 10^{-11}\ \text{m} = \boxed{0.093\ 4\ \text{nm}}$$

P38.31 $2d\sin\theta = m\lambda$: $$\sin\theta = \frac{m\lambda}{2d} = \frac{1\left(0.140 \times 10^{-9}\ \text{m}\right)}{2\left(0.281 \times 10^{-9}\ \text{m}\right)} = 0.249$$

and $\boxed{\theta = 14.4°}$

P38.32 Figure 38.23 of the text shows the situation.

$2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

$m = 1$: $$\lambda_1 = \frac{2(2.80\ \text{m})\sin 80.0°}{1} = \boxed{5.51\ \text{m}}$$

$m = 2$: $$\lambda_2 = \frac{2(2.80\ \text{m})\sin 80.0°}{2} = \boxed{2.76\ \text{m}}$$

$m = 3$: $$\lambda_3 = \frac{2(2.80\ \text{m})\sin 80.0°}{3} = \boxed{1.84\ \text{m}}$$

***P38.33** The crystal cannot produce diffracted beams of visible light. The wavelengths of visible light are some hundreds of nanometers. There is no angle whose sine is greater than 1. Bragg's law $2d\sin\theta = m\lambda$ cannot be satisfied for a wavelength much larger than the distance between atomic planes in the crystal.

Section 38.6 **Polarization of Light Waves**

P38.34 The average value of the cosine-squared function is one-half, so the first polarizer transmits

$\frac{1}{2}$ the light. The second transmits $\cos^2 30.0° = \frac{3}{4}$.

$$I_f = \frac{1}{2}\times\frac{3}{4}I_i = \boxed{\frac{3}{8}I_i}$$

P38.35 $I = I_{\max}\cos^2\theta \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{I}{I_{\max}}}$

(a) $\frac{I}{I_{\max}} = \frac{1}{3.00} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{1}{3.00}} = \boxed{54.7°}$

(b) $\frac{I}{I_{\max}} = \frac{1}{5.00} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{1}{5.00}} = \boxed{63.4°}$

(c) $\frac{I}{I_{\max}} = \frac{1}{10.0} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{1}{10.0}} = \boxed{71.6°}$

P38.36 By Brewster's law, $\qquad n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$

P38.37 $\sin\theta_c = \frac{1}{n} \qquad$ or $\qquad n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 34.4°} = 1.77$

Also, $\tan\theta_p = n$. $\qquad$ Thus, $\qquad \theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5°}$

P38.38 $\sin\theta_c = \frac{1}{n}$ and $\tan\theta_p = n$

Thus, $\sin\theta_c = \frac{1}{\tan\theta_p}$ or $\boxed{\cot\theta_p = \sin\theta_c}$.

P38.39 (a) At incidence, $n_1\sin\theta_1 = n_2\sin\theta_2$ and $\theta_1' = \theta_1$. For complete polarization of the reflected light,

$$(90-\theta_1') + (90-\theta_2) = 90°$$

$$\theta_1' + \theta_2 = 90 = \theta_1 + \theta_2$$

Then $n_1\sin\theta_1 = n_2\sin(90-\theta_1) = n_2\cos\theta_1$

$$\frac{\sin\theta_1}{\cos\theta_1} = \frac{n_2}{n_1} = \tan\theta_1$$

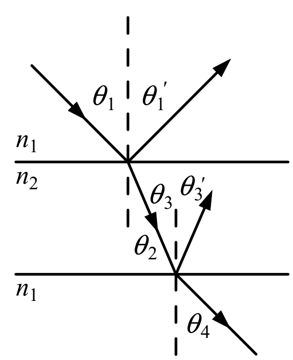

FIG. P38.39(a)

At the bottom surface, $\theta_3 = \theta_2$ because the normals to the surfaces of entry and exit are parallel.

Then $\qquad n_2\sin\theta_3 = n_1\sin\theta_4 \qquad$ and $\qquad \theta_3' = \theta_3$

$\qquad n_2\sin\theta_2 = n_1\sin\theta_4 \qquad$ and $\qquad \theta_4 = \theta_1$

The condition for complete polarization of the reflected light is

$90 - \theta_3' + 90 - \theta_4 = 90° \qquad \theta_2 + \theta_1 = 90°$

This is the same as the condition for θ_1 to be Brewster's angle at the top surface.

continued on next page

(b) We consider light moving in a plane perpendicular to the line where the surfaces of the prism meet at the unknown angle Φ. We require

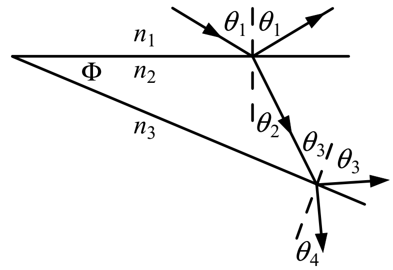

FIG. P38.39(b)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\theta_1 + \theta_2 = 90°$$

So $n_1 \sin(90 - \theta_2) = n_2 \sin\theta_2 \qquad \dfrac{n_1}{n_2} = \tan\theta_2$

And $\quad n_2 \sin\theta_3 = n_3 \sin\theta_4 \qquad \theta_3 + \theta_4 = 90°$

$$n_2 \sin\theta_3 = n_3 \cos\theta_3 \qquad \tan\theta_3 = \frac{n_3}{n_2}$$

In the triangle made by the faces of the prism and the ray in the prism,

$$\Phi + 90 + \theta_2 + (90 - \theta_3) = 180$$

$\boxed{\text{So one particular apex angle is required, and it is}}$

$$\Phi = \theta_3 - \theta_2 = \boxed{\tan^{-1}\left(\frac{n_3}{n_2}\right) - \tan^{-1}\left(\frac{n_1}{n_2}\right)}$$

Here a negative result is to be interpreted as meaning the same as a positive result.

P38.40 For incident unpolarized light of intensity $I_{\max}$:

The average value of the cosine-squared function is one half, so the intensity after transmission by the first disk is

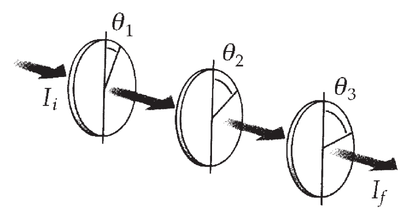

FIG. P38.40

$$I = \frac{1}{2} I_{\max}$$

After transmitting 2nd disk: $\quad I = \dfrac{1}{2} I_{\max} \cos^2\theta$

After transmitting 3rd disk: $\quad I = \dfrac{1}{2} I_{\max} \cos^2\theta \cos^2(90° - \theta)$

where the angle between the first and second disk is $\theta = \omega t$.

Using trigonometric identities $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$

and $\qquad \cos^2(90° - \theta) = \sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$

we have $\qquad I = \dfrac{1}{2} I_{\max} \left[\dfrac{(1 + \cos 2\theta)}{2}\right]\left[\dfrac{(1 - \cos 2\theta)}{2}\right]$

$$I = \frac{1}{8} I_{\max}\left(1 - \cos^2 2\theta\right) = \frac{1}{8} I_{\max}\left(\frac{1}{2}\right)(1 - \cos 4\theta)$$

Since $\theta = \omega t$, the intensity of the emerging beam is given by $\boxed{I = \dfrac{1}{16} I_{\max}(1 - 4\omega t)}$.

***P38.41** (a) Let I_0 represent the intensity of unpolarized light incident on the first polarizer. In Malus's law the average value of the cosine-squared function is 1/2, so the first filter lets through 1/2 of the incident intensity. Of the light reaching them, the second filter passes $\cos^2 45° = 1/2$ and the third filter also $\cos^2 45° = 1/2$. The transmitted intensity is then $I_0(1/2)(1/2)(1/2) = 0.125\ I_0$. The reduction in intensity is by a factor of $\boxed{0.875}$ of the incident intensity.

(b) By the same logic as in part (a) we have transmitted $I_0(1/2)(\cos^2 30°)(\cos^2 30°)(\cos^2 30°) = I_0(1/2)(\cos^2 30°)^3 = 0.211\ I_0$. Then the fraction absorbed is $\boxed{0.789}$.

(c) Yet again we compute transmission $I_0(1/2)(\cos^2 15°)^6 = 0.330\ I_0$. And the fraction absorbed is $\boxed{0.670}$.

(d) We can get more and more of the incident light through the stack of ideal filters, approaching 50%, by reducing the angle between the transmission axes of each one and the next.

Additional Problems

***P38.42** The central bright fringe is wider than the side bright fringes, so the light must have been diffracted by a $\boxed{\text{single slit}}$. For precision, we measure from the second minimum on one side of the center to the second minimum on the other side:

$$2y = (11.7 - 6.3)\text{ cm} = 5.4\text{ cm} \qquad y = 2.7\text{ cm}$$

$$\tan\theta = \frac{y}{L} = \frac{0.027\text{ m}}{2.6\text{ m}} = 0.010\ 4$$

$$\theta = 0.595° = 0.010\ 4\text{ rad}$$

$$a\sin\theta = m\lambda$$

$$a = \frac{m\lambda}{\sin\theta} = \frac{2\left(632.8\times10^{-9}\text{ m}\right)}{\sin 0.595°} = \frac{2\left(632.8\times10^{-9}\text{ m}\right)}{0.010\ 4} = \boxed{1.22\times10^{-4}\text{ m}}$$

P38.43 Let the first sheet have its axis at angle θ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $\quad I_{\max}\cos^2\theta$

The second sheet passes $\quad I_{\max}\cos^4\theta$

and the nth sheet lets through $\quad I_{\max}\cos^{2n}\theta \ge 0.90 I_{\max} \qquad$ where $\theta = \dfrac{45°}{n}$

Try different integers to find $\quad \cos^{2\times5}\left(\dfrac{45°}{5}\right) = 0.885 \qquad \cos^{2\times6}\left(\dfrac{45°}{6}\right) = 0.902$

(a) So $n = \boxed{6}$

(b) $\theta = \boxed{7.50°}$

P38.44 Consider vocal sound moving at 340 m/s and of frequency 3 000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{3\,000 \text{ Hz}} = 0.113 \text{ m}$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113 \text{ m}}{0.600 \text{ m}}\right) = 10.9°$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

***P38.45** (a) We assume the first side maximum is at $a\sin\theta = 1.5\,\lambda$. (Its location is determined more precisely in problem 66.) Then the required fractional intensity is

$$\frac{I}{I_{\text{max}}} = \left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(1.5\pi)}{1.5\pi}\right]^2 = \frac{1}{2.25\pi^2} = \boxed{0.0450}$$

(b) Proceeding as in part (a) we have $a\sin\theta/\lambda = 2.5$ and

$$\frac{I}{I_{\text{max}}} = \left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(2.5\pi)}{2.5\pi}\right]^2 = \frac{1}{6.25\pi^2} = \boxed{0.0162}$$

***P38.46** The energy in the central maximum we can estimate in Figure 38.6 as proportional to

$$(\text{height})(\text{width}) = I_{\text{max}}(2\pi)$$

As in problem 45, the maximum height of the first side maximum is approximately

$$I_{\text{max}}[\sin(3\pi/2)/(3\pi/2)]^2 = 4I_{\text{max}}/9\pi^2$$

Then the energy in one side maximum is proportional to $\pi(4I_{\text{max}}/9\pi^2)$,

and that in both of the first side maxima together is proportional to $2\pi(4I_{\text{max}}/9\pi^2)$.

Similarly and more precisely, and always with the same proportionality constant, the energy in both of the second side maxima is proportional to $2\pi(4I_{\text{max}}/25\pi^2)$.

The energy in all of the side maxima together is proportional to

$$2\pi(4I_{\text{max}}/\pi^2)[1/3^2 + 1/5^2 + 1/7^2 + \cdots] = 2\pi(4I_{\text{max}}/\pi^2)[\pi^2/8 - 1] = (8I_{\text{max}}/\pi)[\pi^2/8 - 1]$$

$$= I_{\text{max}}(\pi - 8/\pi) = 0.595I_{\text{max}}$$

The ratio of the energy in the central maximum to the total energy is then

$$I_{\text{max}}(2\pi)/\{I_{\text{max}}(2\pi) + 0.595I_{\text{max}}\} = 2\pi/6.88 = 0.913 = 91.3\%$$

Our calculation is only a rough estimate, because the shape of the central maximum in particular is not just a vertically-stretched cycle of a cosine curve. It is slimmer than that.

P38.47 The first minimum is at $a\sin\theta = (1)\lambda$

This has no solution if $\dfrac{\lambda}{a} > 1$

or if $a < \lambda = \boxed{632.8 \text{ nm}}$

***P38.48** With light in effect moving through vacuum, Rayleigh's criterion limits the resolution according to

$$d/L = 1.22\,\lambda/D \qquad D = 1.22\,\lambda L/d = 1.22\,(885 \times 10^{-9}\text{ m})\ 12\ 000\text{ m}/2.3\text{ m} = \boxed{5.63\text{ mm}}$$

The assumption is absurd. Over a horizontal path of 12 km in air, density variations associated with convection ("heat waves" or what an astronomer calls "seeing") would make the motorcycles completely unresolvable with any optical device.

P38.49 $d = \dfrac{1}{400\text{ mm}^{-1}} = 2.50\times10^{-6}\text{ m}$

(a) $d\sin\theta = m\lambda$ $\qquad \theta_a = \sin^{-1}\left(\dfrac{2\times541\times10^{-9}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{25.6°}$

(b) $\lambda = \dfrac{541\times10^{-9}\text{ m}}{1.33} = 4.07\times10^{-7}\text{ m}$ $\qquad \theta_b = \sin^{-1}\left(\dfrac{2\times4.07\times10^{-7}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{19.0°}$

(c) $d\sin\theta_a = 2\lambda$ and $d\sin\theta_b = \dfrac{2\lambda}{n}$

combine by substitution to give $\quad n\sin\theta_b = (1)\sin\theta_a$

P38.50 (a) $\lambda = \dfrac{v}{f}$: $\qquad \lambda = \dfrac{3.00\times10^{8}\text{ m/s}}{1.40\times10^{9}\text{ s}^{-1}} = 0.214\text{ m}$

$\theta_{\min} = 1.22\dfrac{\lambda}{D}$: $\qquad \theta_{\min} = 1.22\left(\dfrac{0.214\text{ m}}{3.60\times10^{4}\text{ m}}\right) = \boxed{7.26\ \mu\text{rad}}$

$\theta_{\min} = 7.26\ \mu\text{rad}\left(\dfrac{180\times60\times60\text{ s}}{\pi}\right) = \boxed{1.50\text{ arc seconds}}$

(b) $\theta_{\min} = \dfrac{d}{L}$: $\qquad d = \theta_{\min}L = (7.26\times10^{-6}\text{ rad})(26\ 000\text{ ly}) = \boxed{0.189\text{ ly}}$

(c) It is not true for humans, but we assume the hawk's visual acuity is limited only by Rayleigh's criterion.

$\theta_{\min} = 1.22\dfrac{\lambda}{D}$ $\quad \theta_{\min} = 1.22\left(\dfrac{500\times10^{-9}\text{ m}}{12.0\times10^{-3}\text{ m}}\right) = \boxed{50.8\ \mu\text{rad}}$ (10.5 seconds of arc)

(d) $d = \theta_{\min}L = (50.8\times10^{-6}\text{ rad})(30.0\text{ m}) = 1.52\times10^{-3}\text{ m} = \boxed{1.52\text{ mm}}$

P38.51 With a grazing angle of 36.0°, the angle of incidence is 54.0°

$\tan\theta_p = n = \tan 54.0° = 1.38$

In the liquid, $\lambda_n = \dfrac{\lambda}{n} = \dfrac{750 \text{ nm}}{1.38} = \boxed{545 \text{ nm}}$.

P38.52 (a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\ \mu\text{m}$ apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d\sin\theta = m\lambda$. The longest wavelength reflected strongly corresponds to $m = 1$:

$2\left(0.25\times10^{-6}\text{ m}\right)\sin 90° = 1\lambda \qquad \lambda = 500$ nm. This is the blue-green color.

(b) For light incident at grazing angle 60°, $2d\sin\theta = m\lambda$ gives $1\lambda = 2\left(0.25\times10^{-6}\text{ m}\right)\sin 60° = 433$ nm. This is violet.

(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colors reinforced by constructive interference.

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say $0.32\ \mu\text{m}$) they could reflect red with constructive interference.

P38.53 (a) $d\sin\theta = m\lambda$

or $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{3\left(500\times10^{-9}\text{ m}\right)}{\sin 32.0°} = 2.83\ \mu\text{m}$

Therefore, lines per unit length $= \dfrac{1}{d} = \dfrac{1}{2.83\times10^{-6}\text{ m}}$

or lines per unit length $= 3.53\times10^{5}\text{ m}^{-1} = \boxed{3.53\times10^{3}\text{ cm}^{-1}}$.

(b) $\sin\theta = \dfrac{m\lambda}{d} = \dfrac{m\left(500\times10^{-9}\text{ m}\right)}{2.83\times10^{-6}\text{ m}} = m(0.177)$

For $\sin\theta \le 1.00$, we must have $\quad m(0.177) \le 1.00$

or $\quad m \le 5.66$

Therefore, the highest order observed is $\quad m = 5$

Total number of primary maxima observed is $\quad 2m + 1 = \boxed{11}$

***P38.54** (a) For the air-to-water interface,

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \theta_p = 53.1°$$

and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$

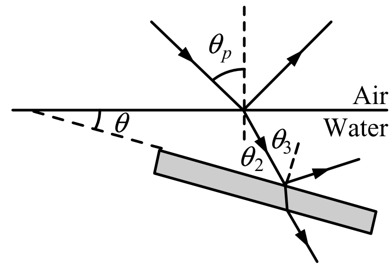

FIG. P38.54

For the water-to-slab interface, $\tan\theta_p = \tan\theta_3 = \frac{n_{\text{slab}}}{n_{\text{water}}} = \frac{n}{1.33}$

so $\theta_3 = \tan^{-1}(n/1.33)$.

The angle between surfaces is $\theta = \theta_3 - \theta_2 = \tan^{-1}(n/1.33) - 36.9°$.

(b) If we imagine $n \to \infty$, then $\theta \to 53.1°$. The material of the slab in this case is higher in optical density than any gem. The light in the water skims along the upper surface of the slab.

(c) If we imagine $n = 1$, then $\theta = 0$. The slab is so low in optical density that it is like air. The light strikes parallel surfaces as it enters and exits the water, both at the polarizing angle.

P38.55 A central maximum and side maxima in seven orders of interference appear. If the seventh order is just at 90°,

$$d\sin\theta = m\lambda \qquad d(1) = 7\left(654\times10^{-9}\text{ m}\right) \qquad d = 4.58\ \mu\text{m}$$

If the seventh order is at less than 90°, the eighth order might be nearly ready to appear according to

$$d(1) = 8\left(654\times10^{-9}\text{ m}\right) \qquad d = 5.23\ \mu\text{m}$$

Thus $\boxed{4.58\ \mu\text{m} < d < 5.23\ \mu\text{m}}$.

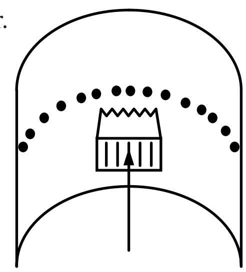

FIG. P38.55

P38.56 (a) We require $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D}{2L}$

Then $\boxed{D^2 = 2.44\lambda L}$

(b) $D = \sqrt{2.44\left(500\times10^{-9}\text{ m}\right)(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$

P38.57 For the limiting angle of resolution between lines we

assume $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{\left(550\times10^{-9}\text{ m}\right)}{\left(5.00\times10^{-3}\text{ m}\right)} = 1.34\times10^{-4}\text{ rad}$.

Assuming a picture screen with vertical dimension ℓ, the minimum viewing distance for no visible lines is found from $\theta_{\min} = \frac{\ell/485}{L}$. The desired ratio is then

$$\frac{L}{\ell} = \frac{1}{485\theta_{\min}} = \frac{1}{485\left(1.34\times10^{-4}\text{ rad}\right)} = \boxed{15.4}$$

When the pupil of a human eye is wide open, its actual resolving power is significantly poorer than Rayleigh's criterion suggests.

P38.58 (a) Applying Snell's law gives $n_2 \sin\phi = n_1 \sin\theta$. From the sketch, we also see that:

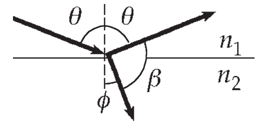

FIG. P38.58(a)

$$\theta + \phi + \beta = \pi, \text{ or } \phi = \pi - (\theta + \beta)$$

Using the given identity:

$$\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$$

which reduces to: $\sin\phi = \sin(\theta+\beta)$

Applying the identity again: $\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$

Snell's law then becomes: $n_2(\sin\theta\cos\beta + \cos\theta\sin\beta) = n_1 \sin\theta$

or (after dividing by $\cos\theta$): $n_2(\tan\theta\cos\beta + \sin\beta) = n_1 \tan\theta$

Solving for $\tan\theta$ gives: $\boxed{\tan\theta = \dfrac{n_2 \sin\beta}{n_1 - n_2\cos\beta}}$

(b) If $\beta = 90.0°$, $n_1 = 1.00$, and $n_2 = n$, the above result becomes:

$\tan\theta = \dfrac{n(1.00)}{1.00 - 0}$, or $n = \tan\theta$, which is Brewster's law.

P38.59 (a) From Equation 38.1, $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$

In this case $m = 1$ and $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{7.50 \times 10^9 \text{ Hz}} = 4.00 \times 10^{-2} \text{ m}$

Thus, $\theta = \sin^{-1}\left(\dfrac{4.00 \times 10^{-2} \text{ m}}{6.00 \times 10^{-2} \text{ m}}\right) = \boxed{41.8°}$

(b) From Equation 38.2, $\dfrac{I}{I_{max}} = \left[\dfrac{\sin(\phi)}{\phi}\right]^2$ where $\phi = \dfrac{\pi a \sin\theta}{\lambda}$

When $\theta = 15.0°$, $\phi = \dfrac{\pi(0.060\ 0 \text{ m})\sin 15.0°}{0.040\ 0 \text{ m}} = 1.22 \text{ rad}$

and $\dfrac{I}{I_{max}} = \left[\dfrac{\sin(1.22 \text{ rad})}{1.22 \text{ rad}}\right]^2 = \boxed{0.593}$

(c) $\sin\theta = \dfrac{\lambda}{a}$ so $\theta = 41.8°$:

This is the minimum angle subtended by the two sources at the slit. Let α be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance L from the slit plane. Then,

$$L = \ell\cot\alpha = (0.100 \text{ m})\cot\left(\frac{41.8°}{2}\right) = \boxed{0.262 \text{ m}}$$

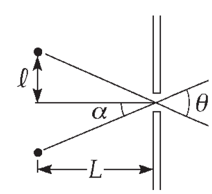

FIG. P38.59(c)

***P38.60** (a) The first sheet transmits one-half the intensity of the originally unpolarized light, because the average value of the cosine-squared function in Malus's law is one-half. Then

$$\frac{I}{I_{\max}} = \frac{1}{2}\left(\cos^2 45.0°\right) \quad \left(\cos^2 45.0°\right) = \boxed{\frac{1}{8}}$$

(b) No recipes remain. The two experiments follow precisely analogous steps, but the results are different. The middle filter in part (a) changes the polarization state of the light that passes through it, but the recipe selections do not change individual recipes. The result for light gives us a glimpse of how quantum-mechanical measurements differ from classical measurements.

P38.61 (a) Constructive interference of light of wavelength λ on the screen is described by

$d\sin\theta = m\lambda$ where $\tan\theta = \dfrac{y}{L}$ so $\sin\theta = \dfrac{y}{\sqrt{L^2+y^2}}$. Then $(d)\,y\left(L^2+y^2\right)^{-1/2} = m\lambda$.

Differentiating with respect to y gives

$$d1\left(L^2+y^2\right)^{-1/2} + (d)\,y\left(-\frac{1}{2}\right)\left(L^2+y^2\right)^{-3/2}(0+2y) = m\frac{d\lambda}{dy}$$

$$\frac{d}{\left(L^2+y^2\right)^{1/2}} - \frac{(d)\,y^2}{\left(L^2+y^2\right)^{3/2}} = m\frac{d\lambda}{dy} = \frac{(d)\,L^2+(d)\,y^2-(d)\,y^2}{\left(L^2+y^2\right)^{3/2}}$$

$$\frac{d\lambda}{dy} = \frac{(d)\,L^2}{m\left(L^2+y^2\right)^{3/2}}$$

(b) Here $d\sin\theta = m\lambda$ gives $\dfrac{10^{-2}\text{ m}}{8\,000}\sin\theta = 1\left(550\times10^{-9}\text{ m}\right)$, $\theta = \sin^{-1}\left(\dfrac{0.55\times10^{-6}\text{ m}}{1.25\times10^{-6}\text{ m}}\right) = 26.1°$

$$y = L\tan\theta = 2.40\text{ m}\tan 26.1° = 1.18\text{ m}$$

Now $\dfrac{d\lambda}{dy} = \dfrac{dL^2}{m\left(L^2+y^2\right)^{3/2}} = \dfrac{1.25\times10^{-6}\text{ m}\,(2.40\text{ m})^2}{1\left((2.4\text{ m})^2+(1.18\text{ m})^2\right)^{3/2}} = 3.77\times10^{-7} = \boxed{3.77\text{ nm/cm}}$.

P38.62 (a) The angles of bright beams diffracted from the grating are given by $(d)\sin\theta = m\lambda$. The angular dispersion is defined as the derivative $\dfrac{d\theta}{d\lambda}$: $\quad (d)\cos\theta\dfrac{d\theta}{d\lambda} = m \quad \dfrac{d\theta}{d\lambda} = \dfrac{m}{d\cos\theta}$

(b) For the average wavelength 578 nm,

$$d\sin\theta = m\lambda \qquad \frac{0.02\text{ m}}{8\,000}\sin\theta = 2\left(578\times10^{-9}\text{ m}\right)$$

$$\theta = \sin^{-1}\frac{2\times578\times10^{-9}\text{ m}}{2.5\times10^{-6}\text{ m}} = 27.5°$$

The separation angle between the lines is

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda = \frac{m}{d\cos\theta}\Delta\lambda = \frac{2}{2.5\times10^{-6}\text{ m}\cos 27.5°}2.11\times10^{-9}\text{ m}$$

$$= 0.001\,90 = 0.001\,90\text{ rad} = 0.001\,90\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right) = \boxed{0.109°}$$

P38.63 (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

$$\theta = \left(\frac{2\pi}{\lambda}\right)\delta$$

after traveling distance d through the plate. Here δ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length d and the index of refraction. Therefore,

$$\delta = \left|dn_O - dn_E\right|$$

The absolute value is used since $\frac{n_O}{n_E}$ may be more or less than unity. Therefore,

$$\theta = \left(\frac{2\pi}{\lambda}\right)\left|dn_O - dn_E\right| = \boxed{\left(\frac{2\pi}{\lambda}\right)d\left|n_O - n_E\right|}$$

(b) $$d = \frac{\lambda\theta}{2\pi\left|n_O - n_E\right|} = \frac{\left(550\times10^{-9}\text{ m}\right)(\pi/2)}{2\pi\left|1.544 - 1.553\right|} = 1.53\times10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$$

P38.64 (a) From Equation 38.2, $\frac{I}{I_{\max}} = \left[\frac{\sin(\phi)}{\phi}\right]^2$

where we define $\phi \equiv \frac{\pi a\sin\theta}{\lambda}$

Therefore, when $\frac{I}{I_{\max}} = \frac{1}{2}$ we must have $\frac{\sin\phi}{\phi} = \frac{1}{\sqrt{2}}$, or $\boxed{\sin\phi = \frac{\phi}{\sqrt{2}}}$

(b) Let $y_1 = \sin\phi$ and $y_2 = \frac{\phi}{\sqrt{2}}$.

A plot of y_1 and y_2 in the range $1.00 \le \phi \le \frac{\pi}{2}$ is shown to the right.

The solution to the transcendental equation is found to be $\boxed{\phi = 1.39\text{ rad}}$.

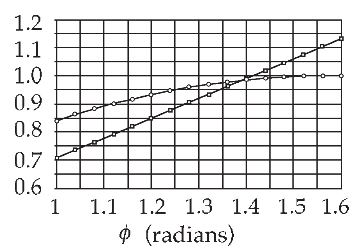

FIG. P38.64(b)

(c) $\frac{\pi a\sin\theta}{\lambda} = \phi$ gives $\sin\theta = \left(\frac{\phi}{\pi}\right)\frac{\lambda}{a} = 0.443\frac{\lambda}{a}$.

If $\frac{\lambda}{a}$ is small, then $\theta \approx 0.443\frac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = 0.443\frac{\lambda}{a} - \left(-0.443\frac{\lambda}{a}\right) = \boxed{\frac{0.886\lambda}{a}}$$

P38.65

ϕ	$\sqrt{2}\sin\phi$	
1	1.19	bigger than ϕ
2	1.29	smaller than ϕ
1.5	1.41	smaller
1.4	1.394	
1.39	1.391	bigger
1.395	1.392	
1.392	1.391 7	smaller
1.391 5	1.391 54	bigger
1.391 52	1.391 55	bigger
1.391 6	1.391 568	smaller
1.391 58	1.391 563	
1.391 57	1.391 561	
1.391 56	1.391 558	
1.391 559	1.391 557 8	
1.391 558	1.391 557 5	
1.391 557	1.391 557 3	
1.391 557 4	1.391 557 4	

We get the answer as 1.391 557 4 to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for ϕ, could reduce this to around 13 steps.

P38.66 In $I = I_{\max}\left[\dfrac{\sin(\phi)}{\phi}\right]^2$ we find $\dfrac{dI}{d\phi} = I_{\max}\left(\dfrac{2\sin(\phi)}{\phi}\right)\left[\dfrac{(\phi)\cos(\phi)-\sin(\phi)}{(\phi)^2}\right]$

and require that it be zero. The possibility $\sin(\phi)=0$ locates all of the minima and the central maximum, according to

$\phi = 0,\ \pi,\ 2\pi,\ \ldots;$ $\quad \phi = \dfrac{\pi a\sin\theta}{\lambda} = 0,\ \pi,\ 2\pi,\ \ldots;\ a\sin\theta = 0,\ \lambda,\ 2\lambda,\ \ldots$

The side maxima are found from $\phi\cos(\phi)-\sin(\phi)=0$, or $\tan(\phi)=\phi$

This has solutions $\boxed{\phi = 4.493\,4}$, $\boxed{\phi = 7.725\,3}$, and others, giving

(a) $\pi a\sin\theta = 4.493\,4\lambda$ $\quad \boxed{a\sin\theta = 1.430\,3\lambda}$

(b) $\pi a\sin\theta = 7.725\,3\lambda$ $\quad \boxed{a\sin\theta = 2.459\,0\lambda}$

P38.67 The first minimum in the single-slit diffraction pattern occurs at

$$\sin\theta = \frac{\lambda}{a} \approx \frac{y_{\min}}{L}$$

Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}$$

For a minimum located at $y_{\min} = 6.36\text{ mm} \pm 0.08\text{ mm}$, the width is

$$a = \frac{(632.8\times10^{-9}\text{ m})(1.00\text{ m})}{6.36\times10^{-3}\text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}$$

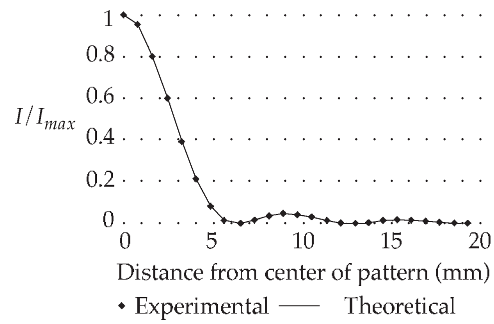

FIG. P38.67

ANSWERS TO EVEN PROBLEMS

P38.2 547 nm

P38.4 91.2 cm

P38.6 (a) 51.8 μm wide and 949 μm high (b) horizontal; vertical. See the solution. A smaller distance between aperture edges causes a wider diffraction angle. The longer dimension of each rectangle is 18.3 times larger than the smaller dimension.

P38.8 See the solution.

P38.10 (a) 0°, 10.3°, 21.0°, 32.5°, 45.8°, 63.6° (b) nine bright fringes, at 0° and on either side at 10.3°, 21.0°, 32.5°, and 63.6° (c) 1.00, 0.811, 0.405, 0.090 1, 0.032 4

P38.12 (a) 79.8 μrad (b) violet, 54.2 μrad (c) The resolving power is improved, with the minimum resolvable angle becoming 60.0 μrad.

P38.14 Between 186 m and 271 m; blue is resolvable at larger distances because it has shorter wavelength.

P38.16 30.5 m

P38.18 105 m

P38.20 514 nm

P38.22 1.81 μm

P38.24 See the solution.

P38.26 74.2 grooves/mm

P38.28 (a) 0.738 mm (b) See the solution.

P38.30 93.4 pm

P38.32 5.51 m, 2.76 m, 1.84 m

P38.34 $\frac{3}{8}$

P38.36 1.11

P38.38 See the solution.

P38.40 See the solution.

P38.42 One slit 0.122 mm wide. The central maximum is twice as wide as the other maxima.

P38.44 See the solution.

P38.46 See the solution.

P38.48 5.63 mm. The assumption is absurd. Over a horizontal path of 12 km in air, density variations associated with convection ("heat waves" or what an astronomer calls "seeing") would make the motorcycles completely unresolvable with any optical device.

P38.50 (a) 1.50 sec (b) 0.189 ly (c) 10.5 sec (d) 1.52 mm

P38.52 See the solution.

P38.54 (a) $\theta = \tan^{-1}(n/1.33) - 36.9°$ (b) If we imagine $n \to \infty$, then $\theta \to 53.1°$. The material of the slab in this case is higher in optical density than any gem. The light in the water skims along the upper surface of the slab. (c) If we imagine $n = 1$, then $\theta = 0$. The slab is so low in optical density that it is like air. The light strikes parallel surfaces as it enters and exits the water, both at the polarizing angle.

P38.56 (a) See the solution. (b) 428 μm

P38.58 See the solution.

P38.60 (a) 1/8 (b) No recipes remain. The two experiments follow precisely analogous steps, but the results are different. The middle filter in part (a) changes the polarization state of the light that passes through it, but the recipe selection processes do not change individual recipes.

P38.62 (a) See the solution. (b) 0.109°

P38.64 (a) See the solution. (b) and (c) See the solution.

P38.66 (a) $a\sin\theta = 1.430\,3\lambda$ (b) $a\sin\theta = 2.459\,0\lambda$