23

Electric Fields

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q23.1 A neutral atom is one that has no net charge. This means that it has the same number of electrons orbiting the nucleus as it has protons in the nucleus. A negatively charged atom has one or more excess electrons.

***Q23.2** (i) Suppose the positive charge has the large value 1 μC. The object has lost some of its conduction electrons, in number 10^{-6} C $(1\ e/1.60 \times 10^{-19}\ \text{C}) = 6.25 \times 10^{12}$ and in mass $6.25 \times 10^{12}\,(9.11 \times 10^{-31}\ \text{kg}) = 5.69 \times 10^{-18}$ kg. This is on the order of 10^{14} times smaller than the ~1 g mass of the coin, so it is an immeasurably small change. Answer (d).

(ii) The coin gains extra electrons, gaining mass on the order of 10^{-14} times its original mass for the charge $-1\ \mu$C. Answer (b).

Q23.3 All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in the summer well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

Q23.4 Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.

Differences: The electrical force can either attract or repel, while the gravitational force as described by Newton's law can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

Q23.5 No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.4a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

***Q23.6** Answer (c). Each charge produces field as if it were alone in the Universe.

***Q23.7** (i) According to the inverse square law, the field is one-fourth as large at twice the distance. The answer is (c), 2×36 cm = 72 cm.

(ii) The field is four times stronger at half the distance away from the charge. Answer (b).

Q23.8 An electric field created by a positive or negative charge extends in all directions from the charge. Thus, it exists in empty space if that is what surrounds the charge. There is no material at point A in Figure 23.21(a), so there is no charge, nor is there a force. There would be a force if a charge were present at point A, however. A field does exist at point A.

***Q23.9** (i) We compute $q_A q_B/r^2$ in each case. In (a) it is 400/4 = 100 (nC/cm)^2. In (b) and (c), 300/4 = 75 (nC/cm)^2. In (d) 600/9 = 67 (nC/cm)^2. In (e) 900/9 = 100 (nC/cm)^2. The ranking is then a = e > b = c > d.

(ii) We compute q_A/r^2 in each case. In (a) it is 20/4 = 5 nC/cm^2. In (b) 30/4 = 7.5 nC/cm^2. In (c) 10/4 = 2.5 nC/cm^2. In (d) 30/9 = 3.3 nC/cm^2. In (e) 45/9 = 5 nC/cm^2. The ranking is then b > a = e > d > c.

***Q23.10** The charge at the upper left creates at the field point electric field to the left, with magnitude we call E_1. The charge at lower right creates downward electric field with an equal magnitude E_1. These two charges together create field $\sqrt{2}E_1$ downward and to the left at 45°. The positive charge is $\sqrt{2}$ times farther from the field point so it creates field $2E_1/(\sqrt{2})^2 = E_1$ upward and to the right. The net field is then $\left(\sqrt{2}-1\right)E_1$ downward and to the left. The answer to question (i) is (d).

(ii) With the positive charge removed, the magnitude of the field becomes $\sqrt{2}E_1$, larger than before, so the answer is (a).

***Q23.11** The certain point must be on the same line as A and B, for otherwise the field components perpendicular to this line would not add to zero. If the certain point is between A and B, it is midway between them, and B's charge is also +40 nC. If the certain point is 4 cm from A and 12 cm from B, then B's charge must be –9(40 nC) = –360 nC. These are the only two possibilities. The answers are (a), (f), and (j).

Q23.12 The direction of the electric field is the direction in which a positive test charge would feel a force when placed in the field. A charge will not experience two electrical forces at the same time, but the vector sum of the two. If electric field lines crossed, then a test charge placed at the point at which they cross would feel a force in two directions. Furthermore, the path that the test charge would follow if released at the point where the field lines cross would be indeterminate.

Q23.13 Both figures are drawn correctly. $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ are the electric fields separately created by the point charges q_1 and q_2 in Figure 23.12 or q and $-q$ in Figure 23.13, respectively. The net electric field is the vector sum of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$, shown as $\vec{\mathbf{E}}$. Figure 23.19 shows only one electric field line at each point away from the charge. At the point location of an object modeled as a point charge, the direction of the field is undefined, and so is its magnitude.

***Q23.14** Answer (a). The equal-magnitude radially directed field contributions add to zero.

***Q23.15** Answer (c). Contributions to the total field from bits of charge in the disk lie closer together in direction than for the ring.

***Q23.16** (i) Answer (c). Electron and proton have equal-magnitude charges.
(ii) Answer (b). The proton's mass is 1836 times larger than the electron's.

***Q23.17** Answer (b).

Q23.18 Linear charge density, λ, is charge per unit length. It is used when trying to determine the electric field created by a charged rod.

Surface charge density, σ, is charge per unit area. It is used when determining the electric field above a charged sheet or disk.

Volume charge density, ρ, is charge per unit volume. It is used when determining the electric field due to a uniformly charged sphere made of insulating material.

Q23.19 No. Life would be no different if electrons were + charged and protons were – charged. Opposite charges would still attract, and like charges would repel. The naming of + and – charge is merely a convention.

Q23.20 In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

SOLUTIONS TO PROBLEMS

Section 23.1 Properties of Electric Charges

P23.1 (a) The mass of an average neutral hydrogen atom is 1.007 9u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}.$$

Its charge, due to loss of one electron, is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}.$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

$$\text{mass }=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$$

(c) charge of $Cl^{-}=\boxed{-1.60\times10^{-19}\text{ C}}$

$$\text{mass }=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$$

(d) charge of $Ca^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+3.20\times10^{-19}\text{ C}}$

$$\text{mass }=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{6.65\times10^{-26}\text{ kg}}$$

(e) charge of $N^{3-}=3\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{-4.80\times10^{-19}\text{ C}}$

$$\text{mass }=14.007\left(1.66\times10^{-27}\text{ kg}\right)+3\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.33\times10^{-26}\text{ kg}}$$

continued on next page

(f) charge of $N^{4+} = 4\left(1.60\times10^{-19}\text{ C}\right) = \boxed{+6.40\times10^{-19}\text{ C}}$

mass $= 14.007\left(1.66\times10^{-27}\text{ kg}\right) - 4\left(9.11\times10^{-31}\text{ kg}\right) = \boxed{2.32\times10^{-26}\text{ kg}}$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom.

charge $= 7\left(1.60\times10^{-19}\text{ C}\right) = \boxed{1.12\times10^{-18}\text{ C}}$

mass $= 14.007\left(1.66\times10^{-27}\text{ kg}\right) - 7\left(9.11\times10^{-31}\text{ kg}\right) = \boxed{2.32\times10^{-26}\text{ kg}}$

(h) charge $= \boxed{-1.60\times10^{-19}\text{ C}}$

mass $= \left[2\left(1.007\,9\right)+15.999\right]1.66\times10^{-27}\text{ kg} + 9.11\times10^{-31}\text{ kg} = \boxed{2.99\times10^{-26}\text{ kg}}$

P23.2 (a) $N = \left(\dfrac{10.0\text{ grams}}{107.87\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \dfrac{\text{atoms}}{\text{mol}}\right)\left(47\ \dfrac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62\times10^{24}}$

(b) $\#\text{ electrons added} = \dfrac{Q}{e} = \dfrac{1.00\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = 6.25\times10^{15}$

or $\boxed{2.38\text{ electrons for every }10^{9}\text{ already present}}$.

Section 23.2 **Charging Objects by Induction**

Section 23.3 **Coulomb's Law**

P23.3 If each person has a mass of $\approx 70\text{ kg}$ and is (almost) composed of water, then each person contains

$$N \cong \left(\frac{70\,000\text{ grams}}{18\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(10\ \frac{\text{protons}}{\text{molecule}}\right) \cong 2.3\times10^{28}\text{ protons}$$

With an excess of 1% electrons over protons, each person has a charge

$$q = 0.01\left(1.6\times10^{-19}\text{ C}\right)\left(2.3\times10^{28}\right) = 3.7\times10^{7}\text{ C}$$

So $F = k_e\dfrac{q_1q_2}{r^2} = \left(9\times10^{9}\right)\dfrac{\left(3.7\times10^{7}\right)^2}{0.6^2}\text{ N} = 4\times10^{25}\text{ N}\ \boxed{\sim10^{26}\text{ N}}$

This force is almost enough to lift a weight equal to that of the Earth:

$$Mg = 6\times10^{24}\text{ kg}\left(9.8\text{ m/s}^2\right) = 6\times10^{25}\text{ N} \sim 10^{26}\text{ N}$$

***P23.4** In the first situation, $\vec{\mathbf{F}}_{A \text{ on } B,1} = \frac{k_e |q_A||q_B|}{r_1^2}\hat{\mathbf{i}}$. In the second situation, $|q_A|$ and $|q_B|$ are the same.

$$\vec{\mathbf{F}}_{B \text{ on } A,2} = -\vec{\mathbf{F}}_{A \text{ on } B} = \frac{k_e |q_A||q_B|}{r_2^2}\left(-\hat{\mathbf{i}}\right)$$

$$\frac{F_2}{F_1} = \frac{k_e |q_A||q_B|}{r_2^2}\frac{r_1^2}{k_e |q_A||q_B|}$$

$$F_2 = \frac{F_1\, r_1^2}{r_2^2} = 2.62\ \mu\text{N}\left(\frac{13.7\text{ mm}}{17.7\text{ mm}}\right)^2 = 1.57\ \mu\text{N}$$

Then $\vec{\mathbf{F}}_{B \text{ on } A,2} = \boxed{1.57\ \mu\text{N to the left}}$.

P23.5 (a) $F_e = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(3.80\times10^{-10}\ \text{m}\right)^2} = \boxed{1.59\times10^{-9}\ \text{N}}$ (repulsion)

(b) $F_g = \frac{Gm_1 m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-27}\ \text{kg}\right)^2}{\left(3.80\times10^{-10}\ \text{m}\right)^2} = \boxed{1.29\times10^{-45}\ \text{N}}$

The electric force is $\boxed{\text{larger by } 1.24\times10^{36} \text{ times}}$.

(c) If $k_e\frac{q_1 q_2}{r^2} = G\frac{m_1 m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61\times10^{-11}\ \text{C/kg}}$$

P23.6 We find the equal-magnitude charges on both spheres:

$$F = k_e\frac{q_1 q_2}{r^2} = k_e\frac{q^2}{r^2} \qquad \text{so} \qquad q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times10^4\ \text{N}}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times10^{-3}\ \text{C}$$

The number of electron transferred is then

$$N_{\text{xfer}} = \frac{1.05\times10^{-3}\ \text{C}}{1.60\times10^{-19}\ \text{C}/e^-} = 6.59\times10^{15}\ \text{electrons}$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0\ \text{g}}{107.87\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)\left(47\ e^-/\text{atom}\right) = 2.62\times10^{24}\ e^-$$

The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59\times10^{15}}{2.62\times10^{24}}\right) = \boxed{2.51\times10^{-9}} = 2.51\ \text{charges in every billion}$$

P23.7 $F_1 = k_e \dfrac{q_1 q_2}{r^2} = \dfrac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.00\times10^{-6}\ \text{C}\right)\left(2.00\times10^{-6}\ \text{C}\right)}{(0.500\ \text{m})^2} = 0.503\ \text{N}$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.00\times10^{-6}\ \text{C}\right)\left(4.00\times10^{-6}\ \text{C}\right)}{(0.500\ \text{m})^2} = 1.01\ \text{N}$$

$$F_x = 0.503\cos 60.0° + 1.01\cos 60.0° = 0.755\ \text{N}$$

$$F_y = 0.503\sin 60.0° - 1.01\sin 60.0° = -0.436\ \text{N}$$

$$\vec{\mathbf{F}} = (0.755\ \text{N})\hat{\mathbf{i}} - (0.436\ \text{N})\hat{\mathbf{j}} = \boxed{0.872\ \text{N at an angle of } 330°}$$

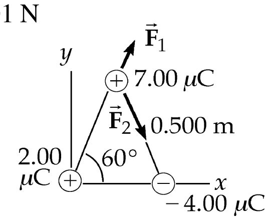

FIG. P23.7

P23.8 Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}\left(-\hat{\mathbf{i}}\right)$$

The net force will be zero if $\dfrac{3}{x^2} = \dfrac{1}{(d-x)^2}$, or $d - x = \dfrac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of $x = \boxed{0.634d}$.

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

P23.9 (a) The force is one of $\boxed{\text{attraction}}$. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(12.0\times10^{-9}\ \text{C}\right)\left(18.0\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2} = \boxed{2.16\times10^{-5}\ \text{N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-9}\ \text{C}\right)\left(3.00\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2} = \boxed{8.99\times10^{-7}\ \text{N}}$$

P23.10 The top charge exerts a force on the negative charge $\dfrac{k_e qQ}{(d/2)^2 + x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}\left(\dfrac{d}{2x}\right)$ to the x-axis. The two positive charges together exert force

$$\left(\frac{2k_e qQ}{\left(d^2/4 + x^2\right)}\right)\left(\frac{(-x)\hat{\mathbf{i}}}{\left(d^2/4 + x^2\right)^{1/2}}\right) = m\vec{\mathbf{a}} \text{ or for } x \ll \frac{d}{2}, \quad \vec{\mathbf{a}} \approx \frac{-2k_e qQ}{md^3/8}\vec{\mathbf{x}}$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}$, so we have Simple Harmonic Motion with $\omega^2 = \dfrac{16k_e qQ}{md^3}$.

$T = \dfrac{2\pi}{\omega} = \boxed{\dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(b) $v_{\text{max}} = \omega A = \boxed{4a\sqrt{\dfrac{k_e qQ}{md^3}}}$

P23.11 (a) $F = \dfrac{k_e e^2}{r^2} = \left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(0.529 \times 10^{-10} \text{ m}\right)^2} = \boxed{8.22 \times 10^{-8} \text{ N}}$ toward the other particle

(b) We have $F = \dfrac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.22 \times 10^{-8} \text{ N}\left(0.529 \times 10^{-10} \text{ m}\right)}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{2.19 \times 10^6 \text{ m/s}}$$

Section 23.4 **The Electric Field**

P23.12 The point is designated in the sketch. The magnitudes of the electric fields, E_1 (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge), are

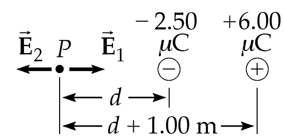

FIG. P23.12

$$E_1 = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50 \times 10^{-6} \text{ C}\right)}{d^2} \qquad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(6.00 \times 10^{-6} \text{ C}\right)}{\left(d + 1.00 \text{ m}\right)^2} \qquad (2)$$

Equate the right sides of (1) and (2)

to get $\left(d + 1.00 \text{ m}\right)^2 = 2.40 d^2$

or $d + 1.00 \text{ m} = \pm 1.55 d$

which yields $d = 1.82$ m

or $d = -0.392$ m

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d = \boxed{1.82 \text{ m to the left of the} -2.50\ \mu\text{C charge}}$

P23.13 For equilibrium, $\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_g$

or $q\vec{\mathbf{E}} = -mg\left(-\hat{\mathbf{j}}\right)$

Thus, $\vec{\mathbf{E}} = \dfrac{mg}{q}\hat{\mathbf{j}}$

(a) $$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(-1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\hat{\mathbf{j}}}$$

(b) $$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\hat{\mathbf{j}}}$$

P23.14 $$F = k_e\frac{q_1q_2}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(6.02\times10^{23}\right)^2}{\left[2\left(6.37\times10^6\text{ m}\right)\right]^2} = \boxed{514\text{ kN}}$$

***P23.15** The first charge creates at the origin field $\dfrac{k_eQ}{a^2}$ to the right.

Both charges are on the x axis, so the total field cannot have a vertical comonent, but it can be either to the right or to the left. $\boxed{\text{If the total field at the origin is to the right, then}}$ q must be negative:

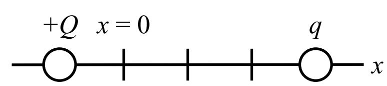

FIG. P23.15

$$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\hat{\mathbf{i}} \qquad \boxed{q = -9Q}$$

$\boxed{\text{In the alternative, if the total field at the origin is to the left,}}$

$$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\left(-\hat{\mathbf{i}}\right) \qquad \boxed{q = +27Q}$$

P23.16 (a) $$E = \frac{k_eq}{r^2} = \frac{\left(8.99\times10^9\right)\left(2.00\times10^{-6}\right)}{(1.12)^2} = 14\,400\text{ N/C}$$

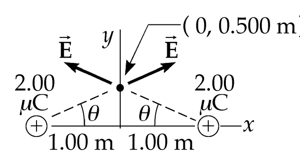

FIG. P23.16

$E_x = 0$ and $E_y = 2(14\,400)\sin 26.6° = 1.29\times10^4\text{ N/C}$

so $\boxed{\vec{\mathbf{E}} = 1.29\times10^4\,\hat{\mathbf{j}}\text{ N/C}}$

(b) $$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-3.00\times10^{-6}\right)\left(1.29\times10^4\,\hat{\mathbf{j}}\right) = \boxed{-3.86\times10^{-2}\,\hat{\mathbf{j}}\text{ N}}$$

P23.17 (a) $$\vec{\mathbf{E}} = \frac{k_eq_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_eq_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_eq_3}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}} = 3.06\frac{k_eq}{a^2}\hat{\mathbf{i}} + 5.06\frac{k_eq}{a^2}\hat{\mathbf{j}} = \boxed{5.91\frac{k_eq}{a^2}\text{ at }58.8°}$$

(b) $$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{5.91\frac{k_eq^2}{a^2}\text{ at }58.8°}$$

P23.18 The electric field at any point x has the x-component

$$E = -\frac{k_e q}{(x-a)^2} + \frac{k_e q}{(x-(-a))^2} = -\frac{k_e q(4ax)}{(x^2-a^2)^2}$$

When x is much, much greater than a, we find $E \approx \boxed{-\frac{4a(k_e q)}{x^3}}$.

P23.19 (a) One of the charges creates at P a field $\vec{\mathbf{E}} = \frac{k_e Q/n}{R^2+x^2}$ at an angle θ to the x-axis as shown.

When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is $\frac{nk_e(Q/n)\hat{\mathbf{i}}}{R^2+x^2}\cos\theta = \boxed{\frac{k_e Qx\hat{\mathbf{i}}}{(R^2+x^2)^{3/2}}}$.

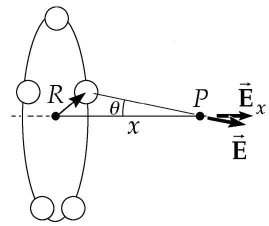

FIG. P23.19

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field}}$.

Section 23.5 **Electric Field of a Continuous Charge Distribution**

P23.20 $E = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e\lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e\lambda_0}{x_0}}$$

$\boxed{\text{The direction is} -\hat{\mathbf{i}} \text{ or left for } \lambda_0 > 0.}$

P23.21 $E = \frac{k_e\lambda\ell}{d(\ell+d)} = \frac{k_e(Q/\ell)\ell}{d(\ell+d)} = \frac{k_e Q}{d(\ell+d)} = \frac{(8.99\times10^9)(22.0\times10^{-6})}{(0.290)(0.140+0.290)}$

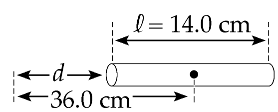

FIG. P23.21

$\vec{\mathbf{E}} = \boxed{1.59\times10^6\ \text{N/C, directed toward the rod}}$.

P23.22 $E = \frac{k_e Qx}{(x^2+a^2)^{3/2}}$

For a maximum, $\frac{dE}{dx} = Qk_e\left[\frac{1}{(x^2+a^2)^{3/2}} - \frac{3x^2}{(x^2+a^2)^{5/2}}\right] = 0$

$x^2 + a^2 - 3x^2 = 0$ or $x = \frac{a}{\sqrt{2}}$

Substituting into the expression for E gives

$$E = \frac{k_e Qa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{2k_e Q}{3\sqrt{3}a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi\,\epsilon_0 a^2}}$$

P23.23 $E=\frac{k_e xQ}{\left(x^2+a^2\right)^{3/2}}=\frac{\left(8.99\times10^9\right)\left(75.0\times10^{-6}\right)x}{\left(x^2+0.100^2\right)^{3/2}}=\frac{6.74\times10^5 x}{\left(x^2+0.010\,0\right)^{3/2}}$ We choose the x axis along the axis of the ring.

(a) At $x=0.010\,0\text{ m}$, $\vec{\mathbf{E}}=6.64\times10^6\,\hat{\mathbf{i}}\text{ N/C}=\boxed{6.64\hat{\mathbf{i}}\text{ MN/C}}$

(b) At $x=0.050\,0\text{ m}$, $\vec{\mathbf{E}}=2.41\times10^7\,\hat{\mathbf{i}}\text{ N/C}=\boxed{24.1\hat{\mathbf{i}}\text{ MN/C}}$

(c) At $x=0.300\text{ m}$, $\vec{\mathbf{E}}=6.40\times10^6\,\hat{\mathbf{i}}\text{ N/C}=\boxed{6.40\hat{\mathbf{i}}\text{ MN/C}}$

(d) At $x=1.00\text{ m}$, $\vec{\mathbf{E}}=6.64\times10^5\,\hat{\mathbf{i}}\text{ N/C}=\boxed{0.664\hat{\mathbf{i}}\text{ MN/C}}$

P23.24 $E=2\pi k_e\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$

$$E=2\pi\left(8.99\times10^9\right)\left(7.90\times10^{-3}\right)\left(1-\frac{x}{\sqrt{x^2+(0.350)^2}}\right)=4.46\times10^8\left(1-\frac{x}{\sqrt{x^2+0.123}}\right)$$

(a) At $x=0.050\,0\text{ m}$, $E=3.83\times10^8\text{ N/C}=\boxed{383\text{ MN/C}}$

(b) At $x=0.100\text{ m}$, $E=3.24\times10^8\text{ N/C}=\boxed{324\text{ MN/C}}$

(c) At $x=0.500\text{ m}$, $E=8.07\times10^7\text{ N/C}=\boxed{80.7\text{ MN/C}}$

(d) At $x=2.000\text{ m}$, $E=6.68\times10^8\text{ N/C}=\boxed{6.68\text{ MN/C}}$

P23.25 (a) From the Example in the chapter text, $E=2\pi k_e\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$

$$\sigma=\frac{Q}{\pi R^2}=1.84\times10^{-3}\text{ C/m}^2$$

$$E=\left(1.04\times10^8\text{ N/C}\right)(0.900)=9.36\times10^7\text{ N/C}=\boxed{93.6\text{ MN/C}}$$

approximation: $E=2\pi k_e\sigma=\boxed{104\text{ MN/C (about 11\% high)}}$

(b) $E=\left(1.04\times10^8\text{ N/C}\right)\left(1-\frac{30.0\text{ cm}}{\sqrt{30.0^2+3.00^2}\text{ cm}}\right)=\left(1.04\times10^8\text{ N/C}\right)(0.004\,96)=\boxed{0.516\text{ MN/C}}$

approximation: $E=k_e\frac{Q}{r^2}=\left(8.99\times10^9\right)\frac{5.20\times10^{-6}}{(0.30)^2}=\boxed{0.519\text{ MN/C (about 0.6\% high)}}$

P23.26 The electric field at a distance x is $E_x = 2\pi k_e \sigma\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e \sigma\left[1 - \frac{1}{\sqrt{1+R^2/x^2}}\right]$

For large x, $\frac{R^2}{x^2} << 1$ and $\sqrt{1+\frac{R^2}{x^2}} \approx 1 + \frac{R^2}{2x^2}$

so
$$E_x = 2\pi k_e \sigma\left(1 - \frac{1}{\left[1+R^2/\left(2x^2\right)\right]}\right)$$
$$= 2\pi k_e \sigma \frac{\left(1+R^2/\left(2x^2\right)-1\right)}{\left[1+R^2/\left(2x^2\right)\right]}$$

Substitute $\sigma = \frac{Q}{\pi R^2}$, $E_x = \frac{k_e Q\left(1/x^2\right)}{\left[1+R^2/\left(2x^2\right)\right]} = k_e Q\left(x^2 + \frac{R^2}{2}\right)$

But for $x >> R$, $\frac{1}{x^2+R^2/2} \approx \frac{1}{x^2}$, so $\boxed{E_x \approx \frac{k_e Q}{x^2} \text{ for a disk at large distances}}$

P23.27 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE \sin\theta = k_e \int \frac{dq \sin\theta}{r^2}$

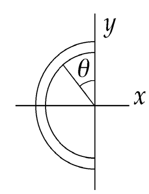

FIG. P23.27

where $dq = \lambda ds = \lambda r d\theta$,

so that $E_x = \frac{k_e \lambda}{r}\int_0^{\pi} \sin\theta d\theta = \frac{k_e \lambda}{r}\left(-\cos\theta\right)\Big|_0^{\pi} = \frac{2k_e \lambda}{r}$

where $\lambda = \frac{q}{L}$ and $r = \frac{L}{\pi}$

Thus, $E_x = \frac{2k_e q\pi}{L^2} = \frac{2\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(-7.50\times10^{-6} \text{ C}\right)\pi}{(0.140 \text{ m})^2}$

Solving, $E_x = -2.16\times10^7$ N/C

Since the rod has a negative charge, $\vec{\mathbf{E}} = \left(-2.16\times10^7\,\hat{\mathbf{i}}\right)$ N/C $= \boxed{-21.6\hat{\mathbf{i}} \text{ MN/C}}$.

P23.28 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\vec{\mathbf{E}} = \frac{k_e x}{\left(x^2 + R^2\right)^{3/2}} \frac{Qdx}{h} \hat{\mathbf{i}}$$

The total field is

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h} \frac{k_e Qxdx}{h\left(x^2 + R^2\right)^{3/2}} \hat{\mathbf{i}} = \frac{k_e Q\hat{\mathbf{i}}}{2h} \int_{x=d}^{d+h} \left(x^2 + R^2\right)^{-3/2} 2xdx$$

$$\vec{\mathbf{E}} = \frac{k_e Q\hat{\mathbf{i}}}{2h} \frac{\left(x^2 + R^2\right)^{-1/2}}{\left(-1/2\right)} \Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\frac{1}{\left(d^2 + R^2\right)^{1/2}} - \frac{1}{\left(\left(d+h\right)^2 + R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\vec{\mathbf{E}} = \frac{2\pi k_e Qdx}{\pi R^2 h}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)\hat{\mathbf{i}}$$

So, $\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = \int_{x=d}^{d+h} \frac{2k_e Qdx}{R^2 h}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)\hat{\mathbf{i}}$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[\int_d^{d+h} dx - \frac{1}{2}\int_{x=d}^{d+h}\left(x^2 + R^2\right)^{-1/2} 2xdx\right] = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[x\Big|_d^{d+h} - \frac{1}{2}\frac{\left(x^2 + R^2\right)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[d + h - d - \left(\left(d+h\right)^2 + R^2\right)^{1/2} + \left(d^2 + R^2\right)^{1/2}\right]$$

$$\vec{\mathbf{E}} = \boxed{\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h + \left(d^2 + R^2\right)^{1/2} - \left(\left(d+h\right)^2 + R^2\right)^{1/2}\right]}$$

P23.29 (a) The electric field at point P due to each element of length dx is $dE = \frac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda dx$,

$E = E_y = \int dE_y = \int dE\cos\theta$ where $\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$

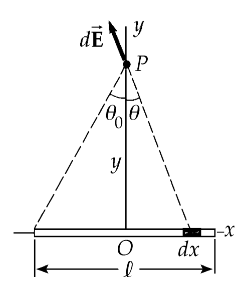

FIG. P23.29

Therefore, $E = 2k_e\lambda y \int_0^{\ell/2} \frac{dx}{\left(x^2 + y^2\right)^{3/2}} = \boxed{\frac{2k_e\lambda\sin\theta_0}{y}}$

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\frac{2k_e\lambda}{y}}$

P23.30 (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$$Q = \sigma A = \left(15.0\times10^{-9}\text{ C/m}^2\right)2\pi\left(0.025\text{ 0 m}\right)\left[0.025\text{ 0 m}+0.060\text{ 0 m}\right] = \boxed{2.00\times10^{-10}\text{ C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = \left(15.0\times10^{-9}\text{ C/m}^2\right)\left[2\pi\left(0.025\text{ 0 m}\right)\left(0.060\text{ 0 m}\right)\right] = \boxed{1.41\times10^{-10}\text{ C}}$$

(c) $$Q = \rho V = \rho\pi r^2 L = \left(500\times10^{-9}\text{ C/m}^3\right)\left[\pi\left(0.025\text{ 0 m}\right)^2\left(0.060\text{ 0 m}\right)\right] = \boxed{5.89\times10^{-11}\text{ C}}$$

P23.31 (a) Every object has the same volume, $V = 8\left(0.030\text{ 0 m}\right)^3 = 2.16\times10^{-4}\text{ m}^3$.

For each, $Q = \rho V = \left(400\times10^{-9}\text{ C/m}^3\right)\left(2.16\times10^{-4}\text{ m}^3\right) = \boxed{8.64\times10^{-11}\text{ C}}$

(b) We must count the 9.00 cm^2 squares painted with charge:

(i) $6\times4 = 24$ squares

$$Q = \sigma A = \left(15.0\times10^{-9}\text{ C/m}^2\right)24.0\left(9.00\times10^{-4}\text{ m}^2\right) = \boxed{3.24\times10^{-10}\text{ C}}$$

(ii) 34 squares exposed

$$Q = \sigma A = \left(15.0\times10^{-9}\text{ C/m}^2\right)24.0\left(9.00\times10^{-4}\text{ m}^2\right) = \boxed{3.24\times10^{-10}\text{ C}}$$

(iii) 34 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\text{ C/m}^2\right)34.0\left(9.00\times10^{-4}\text{ m}^2\right) = \boxed{4.59\times10^{-10}\text{ C}}$$

(iv) 32 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\text{ C/m}^2\right)32.0\left(9.00\times10^{-4}\text{ m}^2\right) = \boxed{4.32\times10^{-10}\text{ C}}$$

(c) (i) total edge length: $\ell = 24\times\left(0.030\text{ 0 m}\right)$

$$Q = \lambda\ell = \left(80.0\times10^{-12}\text{ C/m}\right)24\times\left(0.030\text{ 0 m}\right) = \boxed{5.76\times10^{-11}\text{ C}}$$

(ii) $$Q = \lambda\ell = \left(80.0\times10^{-12}\text{ C/m}\right)44\times\left(0.030\text{ 0 m}\right) = \boxed{1.06\times10^{-10}\text{ C}}$$

(iii) $$Q = \lambda\ell = \left(80.0\times10^{-12}\text{ C/m}\right)64\times\left(0.030\text{ 0 m}\right) = \boxed{1.54\times10^{-10}\text{ C}}$$

(iv) $$Q = \lambda\ell = \left(80.0\times10^{-12}\text{ C/m}\right)40\times\left(0.030\text{ 0 m}\right) = \boxed{0.960\times10^{-10}\text{ C}}$$

Section 23.6 **Electric Field Lines**

P23.32

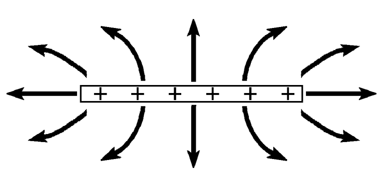

FIG. P23.32

P23.33

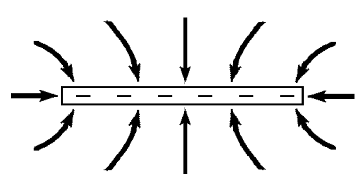

FIG. P23.33

P23.34 (a) $\dfrac{q_1}{q_2} = \dfrac{-6}{18} = \boxed{-\dfrac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

P23.35 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

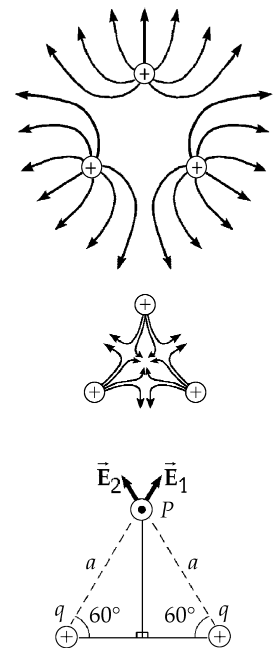

FIG. P23.35

(b) You may need to review vector addition in Chapter Three. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is $\vec{\mathbf{E}} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$.

As shown in the solution figure at right,

$\vec{\mathbf{E}}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

$\vec{\mathbf{E}}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right] = k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 23.7 **Motion of a Charged Particle in a Uniform Electric Field**

P23.36 (a) $|a| = \dfrac{qE}{m} = \dfrac{\left(1.602\times10^{-19}\right)\left(6.00\times10^{5}\right)}{\left(1.67\times10^{-27}\right)} = 5.76\times10^{13}\ \text{m/s so } \vec{\mathbf{a}} = \boxed{-5.76\times10^{13}\,\hat{\mathbf{i}}\ \text{m/s}^2}$

(b) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$

$0 = v_i^2 + 2\left(-5.76\times10^{13}\right)\left(0.070\,0\right)$ $\quad \boxed{\vec{\mathbf{v}}_i = 2.84\times10^{6}\,\hat{\mathbf{i}}\ \text{m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84\times10^{6} + \left(-5.76\times10^{13}\right)t$ $\quad t = \boxed{4.93\times10^{-8}\ \text{s}}$

P23.37 (a) $a = \frac{qE}{m} = \frac{1.602 \times 10^{-19}(640)}{1.67 \times 10^{-27}} = \boxed{6.14 \times 10^{10} \text{ m/s}^2}$

(b) $v_f = v_i + at \qquad 1.20 \times 10^6 = \left(6.14 \times 10^{10}\right)t \qquad t = \boxed{1.95 \times 10^{-5} \text{ s}}$

(c) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t \qquad x_f = \frac{1}{2}\left(1.20 \times 10^6\right)\left(1.95 \times 10^{-5}\right) = \boxed{11.7 \text{ m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67 \times 10^{-27} \text{ kg}\right)\left(1.20 \times 10^6 \text{ m/s}\right)^2 = \boxed{1.20 \times 10^{-15} \text{ J}}$

P23.38 The particle feels a constant force: $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(1 \times 10^{-6} \text{ C}\right)\left(2\,000 \text{ N/C}\right)\left(-\hat{\mathbf{j}}\right) = 2 \times 10^{-3} \text{ N}\left(-\hat{\mathbf{j}}\right)$

and moves with acceleration: $\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{\left(2 \times 10^{-3} \text{ kg} \cdot \text{m/s}^2\right)\left(-\hat{\mathbf{j}}\right)}{2 \times 10^{-16} \text{ kg}} = \left(1 \times 10^{13} \text{ m/s}^2\right)\left(-\hat{\mathbf{j}}\right)$

Note that the gravitational force is on the order of a trillion times smaller than the electrical force exerted on the particle. The particle's x-component of velocity is constant at $\left(1.00 \times 10^5 \text{ m/s}\right)\cos 37° = 7.99 \times 10^4 \text{ m/s}$. Thus it moves in a parabola opening downward. The maximum height it attains above the bottom plate is described by

$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$: $\qquad 0 = \left(6.02 \times 10^4 \text{ m/s}\right)^2 - \left(2 \times 10^{13} \text{ m/s}^2\right)\left(y_f - 0\right)$

$$y_f = 1.81 \times 10^{-4} \text{ m}$$

Since this is less than 10 mm, the particle does not strike the top plate, but moves in a symmetric parabola and strikes the bottom plate after a time given by

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad 0 = 0 + \left(6.02 \times 10^4 \text{ m/s}\right)t + \frac{1}{2}\left(-1 \times 10^{13} \text{ m/s}^2\right)t^2$

since $t > 0$, $\qquad t = 1.20 \times 10^{-8} \text{ s}$

The particle's range is $\qquad x_f = x_i + v_x t = 0 + \left(7.99 \times 10^4 \text{ m/s}\right)\left(1.20 \times 10^{-8} \text{ s}\right) = 9.61 \times 10^{-4} \text{ m}.$

In sum,

The particle strikes the negative plate after moving in a parabola with a height of 0.181 mm and a width of 0.961 mm.

P23.39 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done $= \Delta K$

so, $\qquad -Fd = -\frac{1}{2}mv_i^2$ (since the final velocity = 0)

which becomes $\qquad eEd = K$

and $\qquad E = \boxed{\frac{K}{ed}}$

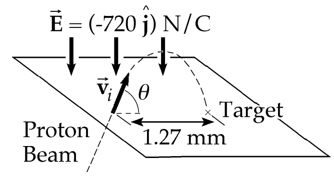

FIG. P23.40

P23.40 $v_i = 9.55\times10^3$ m/s

(a) $a_y = \dfrac{eE}{m} = \dfrac{(1.60\times10^{-19})(720)}{(1.67\times10^{-27})} = 6.90\times10^{10}\ \text{m/s}^2$

From the large magnitude of this vertical acceleration, we can note that the gravitational force on the particle is negligible by comparison to the electrical force.

$$R = \frac{v_i^2\sin 2\theta}{a_y} = 1.27\times10^{-3}\ \text{m so that}$$

$$\frac{(9.55\times10^3)^2\sin 2\theta}{6.90\times10^{10}} = 1.27\times10^{-3}$$

$\sin 2\theta = 0.961$ $\quad\theta = \boxed{36.9°}$ $\quad 90.0° - \theta = \boxed{53.1°}$

(b) $t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i\cos\theta}$ If $\theta = 36.9°$, $t = \boxed{167\ \text{ns}}$. If $\theta = 53.1°$, $t = \boxed{221\ \text{ns}}$.

P23.41 (a) $t = \dfrac{x}{v_x} = \dfrac{0.050\,0}{4.50\times10^5} = 1.11\times10^{-7}\ \text{s} = \boxed{111\ \text{ns}}$

(b) $a_y = \dfrac{qE}{m} = \dfrac{(1.602\times10^{-19})(9.60\times10^3)}{(1.67\times10^{-27})} = 9.21\times10^{11}\ \text{m/s}^2$

$y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$: $\quad y_f = \dfrac{1}{2}(9.21\times10^{11})(1.11\times10^{-7})^2 = 5.68\times10^{-3}\ \text{m} = \boxed{5.68\ \text{mm}}$

(c) $v_x = \boxed{4.50\times10^5\ \text{m/s}}$ $\quad v_{yf} = v_{yi} + a_y t = (9.21\times10^{11})(1.11\times10^{-7}) = \boxed{1.02\times10^5\ \text{m/s}}$

Additional Problems

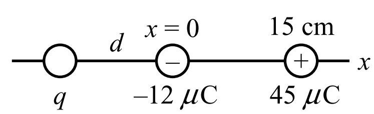

FIG. P23.42

P23.42 The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x > 15$ cm, it would exert a stronger force on the 45 μC than on the -12 μC, and could not produce equilibrium for both. Thus the third charge must be at $x = -d < 0$. Its equilibrium requires

$$\frac{k_e q(12\ \mu\text{C})}{d^2} = \frac{k_e q(45\ \mu\text{C})}{(15\ \text{cm}+d)^2} \qquad \left(\frac{15\ \text{cm}+d}{d}\right)^2 = \frac{45}{12} = 3.75$$

$15\ \text{cm} + d = 1.94d$ $\qquad d = 16.0$ cm

The third charge is at $\boxed{x = -16.0\ \text{cm}}$. The equilibrium of the -12 μC requires

$$\frac{k_e q(12\ \mu\text{C})}{(16.0\ \text{cm})^2} = \frac{k_e(45\ \mu\text{C})12\ \mu\text{C}}{(15\ \text{cm})^2} \qquad \boxed{q = 51.3\ \mu\text{C}}$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P23.43 The proton moves with acceleration $|a_p| = \dfrac{qE}{m} = \dfrac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{1.673\times10^{-27}\ \text{kg}} = 6.13\times10^{10}\ \text{m/s}^2$

while the e⁻ has acceleration $|a_e| = \dfrac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{9.110\times10^{-31}\ \text{kg}} = 1.12\times10^{14}\ \text{m/s}^2 = 1\,836a_p$

(a) We want to find the distance traveled by the proton (i.e., $d=\frac{1}{2}a_pt^2$), knowing:

$$4.00\ \text{cm} = \frac{1}{2}a_pt^2 + \frac{1}{2}a_et^2 = 1\,837\left(\frac{1}{2}a_pt^2\right)$$

Thus, $d = \dfrac{1}{2}a_pt^2 = \dfrac{4.00\ \text{cm}}{1\,837} = \boxed{21.8\ \mu\text{m}}$

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$4.00\ \text{cm} = \dfrac{1}{2}a_{\text{Na}}t^2 + \dfrac{1}{2}a_{\text{Cl}}t^2$: $\qquad 4.00\ \text{cm} = \dfrac{1}{2}\left(\dfrac{eE}{22.99\ \text{u}}\right)t^2 + \dfrac{1}{2}\left(\dfrac{eE}{35.45\ \text{u}}\right)t^2$

This may be written as $\quad 4.00\ \text{cm} = \dfrac{1}{2}a_{\text{Na}}t^2 + \dfrac{1}{2}(0.649a_{\text{Na}})t^2 = 1.65\left(\dfrac{1}{2}a_{\text{Na}}t^2\right)$

so $\quad d_{\text{Na}} = \dfrac{1}{2}a_{\text{Na}}t^2 = \dfrac{4.00\ \text{cm}}{1.65} = \boxed{2.43\ \text{cm}}$

P23.44 (a) The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

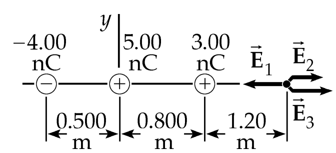

FIG. P23.44 (a)

$$\vec{\mathbf{E}}_1 = \frac{k_eq}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})}{(2.50\ \text{m})^2}\hat{\mathbf{i}} = -5.75\hat{\mathbf{i}}\ \text{N/C}$$

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge, are

$$\vec{\mathbf{E}}_2 = \frac{k_eq}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\ \text{C})}{(2.00\ \text{m})^2}\hat{\mathbf{i}} = 11.2\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(1.20\ \text{m})^2}\hat{\mathbf{i}} = 18.7\ \text{N/C}\ \hat{\mathbf{i}}$$

$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 = \boxed{24.2\ \text{N/C}}$ in $+x$ direction.

continued on next page

(b) $\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46 \text{ N/C})(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}})$

$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2 \text{ N/C})(+\hat{\mathbf{j}})$

$\vec{\mathbf{E}}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81 \text{ N/C})(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}})$

$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}} \text{ N/C} \qquad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}} \text{ N/C}$

$E_R = \boxed{9.42 \text{ N/C}} \qquad \theta = \boxed{63.4° \text{ above } -x \text{ axis}}$

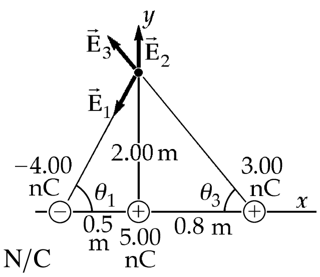

FIG. P23.44 (b)

P23.45 (a) Let us sum force components to find

$\sum F_x = qE_x - T\sin\theta = 0$, and $\sum F_y = qE_y + T\cos\theta - mg = 0$

Combining these two equations, we get

$$q = \frac{mg}{(E_x\cot\theta + E_y)} = \frac{(1.00\times10^{-3})(9.80)}{(3.00\cot 37.0° + 5.00)\times10^5} = 1.09\times10^{-8} \text{ C}$$

$= \boxed{10.9 \text{ nC}}$

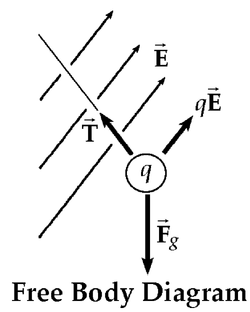

FIG. P23.45

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qEx}{\sin 37.0°} = 5.44\times10^{-3} \text{ N} = \boxed{5.44 \text{ mN}}$$

P23.46 This is the general version of the preceding problem. The known quantities are A, B, m, g, and θ. The unknowns are q and T.

The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 45.

Again, Newton's second law: $\sum F_x = -T\sin\theta + qA = 0$ (1)

and $\sum F_y = +T\cos\theta + qB - mg = 0$ (2)

(a) Substituting $T = \frac{qA}{\sin\theta}$ into Eq. (2), $\frac{qA\cos\theta}{\sin\theta} + qB = mg$

Isolating q on the left, $\boxed{q = \frac{mg}{(A\cot\theta + B)}}$

(b) Substituting this value into Eq. (1), $\boxed{T = \frac{mgA}{(A\cos\theta + B\sin\theta)}}$

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 45. If you find this problem more difficult than problem 45, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

P23.47 $F = \dfrac{k_e q_1 q_2}{r^2}$: $\tan\theta = \dfrac{15.0}{60.0}$

$$\theta = 14.0°$$

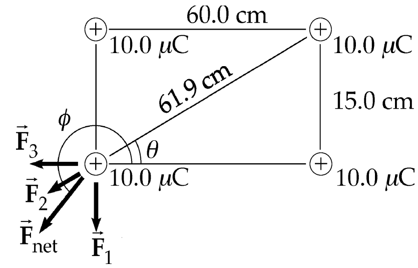

FIG. P23.47

$$F_1 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.150)^2} = 40.0\text{ N}$$

$$F_3 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.600)^2} = 2.50\text{ N}$$

$$F_2 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.619)^2} = 2.35\text{ N}$$

$$F_x = -F_3 - F_2\cos14.0° = -2.50 - 2.35\cos14.0° = -4.78\text{ N}$$

$$F_y = -F_1 - F_2\sin14.0° = -40.0 - 2.35\sin14.0° = -40.6\text{ N}$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(4.78)^2 + (40.6)^2} = \boxed{40.9\text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78}$$

$$\phi = \boxed{263°}$$

P23.48 From Figure (a) we have $d\cos30.0° = 15.0\text{ cm}$

or $d = \dfrac{15.0\text{ cm}}{\cos30.0°}$

From Figure (b) we have $\theta = \sin^{-1}\left(\dfrac{d}{50.0\text{ cm}}\right)$

$$\theta = \sin^{-1}\left(\frac{15.0\text{ cm}}{50.0\text{ cm}(\cos30.0°)}\right) = 20.3°$$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan20.3°$ (1)

From Figure (c) we have $F_q = 2F\cos30.0°$

$$F_q = 2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos30.0° \quad (2)$$

Combining equations (1) and (2),

$$2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos30.0° = mg\tan20.3°$$

$$q^2 = \frac{mg(0.300\text{ m})^2\tan20.3°}{2k_e\cos30.0°}$$

$$q^2 = \frac{(2.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})^2\tan20.3°}{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\cos30.0°}$$

$$q = \sqrt{4.20\times10^{-14}\text{ C}^2} = 2.05\times10^{-7}\text{ C} = \boxed{0.205\ \mu\text{C}}$$

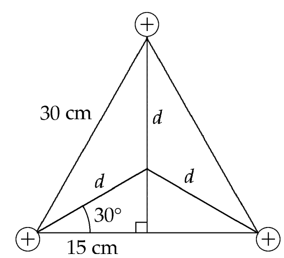

Figure (a)

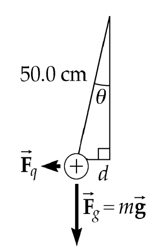

Figure (b)

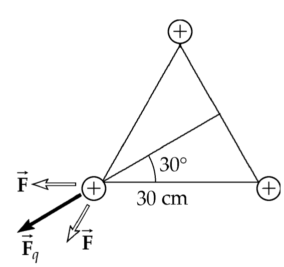

Figure (c)

FIG. P23.48

P23.49 Charge $\frac{Q}{2}$ resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

Solving for Q, $Q = \boxed{2L\sqrt{\frac{k(L - L_i)}{k_e}}}$

P23.50 If we place one more charge q at the 29th vertex, the total force on the central charge will add up to zero: $\vec{\mathbf{F}}_{\text{28 charges}} + \frac{k_e qQ}{a^2}$ away from vertex 29 = 0 $\quad \vec{\mathbf{F}}_{\text{28 charges}} = \boxed{\frac{k_e qQ}{a^2} \text{ toward vertex 29}}$

P23.51 According to the result of an Example in the chapter text, the left-hand rod creates this field at a distance d from its right-hand end:

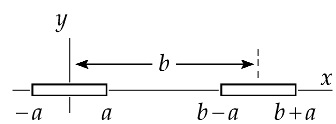

FIG. P23.51

$$E = \frac{k_e Q}{d(2a + d)}$$

$$dF = \frac{k_e QQ}{2a}\frac{dx}{d(d + 2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x + 2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a + x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a + b}{b} + \ln\frac{b}{b - 2a}\right) = \frac{k_e Q^2}{4a^2}\ln\frac{b^2}{(b - 2a)(b + 2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2 - 4a^2}\right)}$$

***P23.52** We model the spheres as particles. They have different charges. They exert on each other forces of equal magnitude. They have equal masses, so their strings make equal angles θ with the vertical. The distance r between them is described by $\sin\theta = (r/2)/40$ cm,

so $\quad r = 80 \text{ cm} \sin\theta$

Let T represent the string tension. We have

$\Sigma F_x = 0$: $\quad k_e q_1 q_2/r^2 = T\sin\theta$

$\Sigma F_y = 0$: $\quad mg = T\cos\theta$

Divide to eliminate T. $\qquad \frac{k_e q_1 q_2}{r^2 mg} = \tan\theta = \frac{r/2}{\sqrt{(40 \text{ cm})^2 - r^2/4}}$

Cleared of fractions, $\qquad k_e q_1 q_2\sqrt{(80 \text{ cm})^2 - r^2} = mgr^3$

$$8.99 \times 10^9(\text{N} \cdot \text{m}^2/\text{C}^2)\ 300 \times 10^{-9}\text{C}\ (200 \times 10^{-9}\text{C})\ \sqrt{(0.8 \text{ m})^2 - r^2} = 2.4 \times 10^{-3}(9.8) \text{ N } r^3$$

$$(0.8 \text{ m})^2 - r^2 = 1901\ r^6$$

We home in on a solution by trying values.

r	$0.64 - r^2 - 1901\ r^6$
0	+0.64
0.5	−29.3
0.2	+0.48
0.3	−0.84
0.24	+0.22
0.27	−0.17
0.258	+0.013
0.259	−0.001

Thus the distance to three digits is $\boxed{0.259 \text{ m}}$.

***P23.53** $Q = \int \lambda d\ell = \int_{-90.0°}^{90.0°} \lambda_0 \cos\theta R d\theta = \lambda_0 R \sin\theta\Big|_{-90.0°}^{90.0°} = \lambda_0 R[1-(-1)] = 2\lambda_0 R$

$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\ \text{m} = 12.0\ \mu\text{C}$ so $\lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi \in_0}\left(\frac{(3.00\ \mu\text{C})(\lambda d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi \in_0}\left(\frac{(3.00\ \mu\text{C})(\lambda_0 \cos^2\theta R d\theta)}{R^2}\right)$$

$$F_y = \int_{-90.0°}^{90.0°} (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C/m})}{(0.600\ \text{m})}\cos^2\theta d\theta$$

$$F_y = \frac{8.99(30.0)}{0.600}(10^{-3}\ \text{N})\int_{-\pi/2}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = (0.450\ \text{N})\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Big|_{-\pi/2}^{\pi/2} = \boxed{0.707\ \text{N downward}}$$

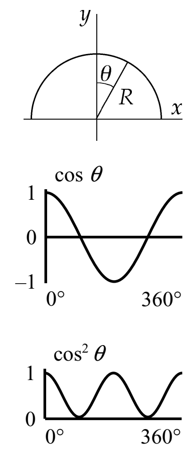

FIG. P23.53

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

***P23.54** (a) The two charges create fields of equal magnitude, both with outward components along the x axis and with upward and downward y components that add to zero. The net field is then

$$\frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} + \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2)52\times10^{-9}\ \text{C}\ x\ \hat{\mathbf{i}}}{\text{C}^2((0.25\ \text{m})^2 + x^2)^{3/2}}$$

$$= \boxed{\frac{935\ \text{N}\cdot\text{m}^2\ x\ \hat{\mathbf{i}}}{\text{C}(0.0625\ \text{m}^2 + x^2)^{3/2}}}$$

(b) At $x = 0.36$ m,

$$\vec{\mathbf{E}} = \frac{935\ \text{N}\cdot\text{m}^2\ 0.36\ \text{m}\ \hat{\mathbf{i}}}{\text{C}\ (0.0625\ \text{m}^2 + (0.36\ \text{m})^2)^{3/2}} = 4.00\ \text{kN/C}\ \hat{\mathbf{i}}$$

(c) We solve $1\,000 = 935\ x\ (0.0625 + x^2)^{-3/2}$ by tabulating values for the field function:

x	$935\ x\ (0.0625 + x^2)^{-3/2}$
0	0
0.01	597
0.02	1 185
0.1	4 789
0.2	5 698
0.36	4 000
0.9	1 032
1	854
∞	0

We see that there are two points where $E = 1\,000$. We home in on them to determine their coordinates as (to three digits) $x = 0.016\,8$ m and $x = 0.916$ m.

(d) The table in part (c) shows that the field is nowhere so large as 16 000 N/C.

(e) The field of a single charge in Question 7 takes on all values from zero to infinity, each at just one point along the positive x axis. The vector sum of the field of two charges, in this problem, is zero at the origin, rises to a maximum at 17.7 cm, and then decreases asymptotically to zero. In the question and the problem, the fields at $x = 36$ cm happen to take similar values. For large x the field of the two charges in this problem shows just the same inverse proportionality to x^2 as the field in the question, being larger by the factor $2(52\ \text{nC})/(57.7\ \text{nC}) = 1.80$ times.

P23.55 (a) From the $2Q$ charge we have $F_e - T_2 \sin\theta_2 = 0$ and $mg - T_2 \cos\theta_2 = 0$

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \tan\theta_2$

From the Q charge we have $F_e = T_1 \sin\theta_1 = 0$ and $mg - T_1 \cos\theta_1 = 0$

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$

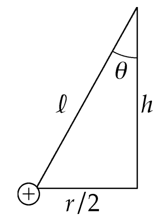

FIG. P23.55

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_e Q^2}{r^2}$

If we assume θ is small then $\tan\theta \approx \dfrac{r/2}{\ell}$

Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta$, then $\dfrac{2k_e Q^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r \approx \left(\dfrac{4k_e Q^2 \ell}{mg}\right)^{1/3}$.

P23.56 The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram shown for the bead on the left side of the bowl:

$$\sum F_y = n\sin 60.0° - mg = 0,$$

or $$n = \frac{mg}{\sin 60.0°}$$

Also, $$\sum F_x = -F_e + n\cos 60.0° = 0,$$

or $$\frac{k_e q^2}{R^2} = n\cos 60.0° = \frac{mg}{\tan 60.0°} = \frac{mg}{\sqrt{3}}$$

Thus, $$q = \boxed{R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}}$$

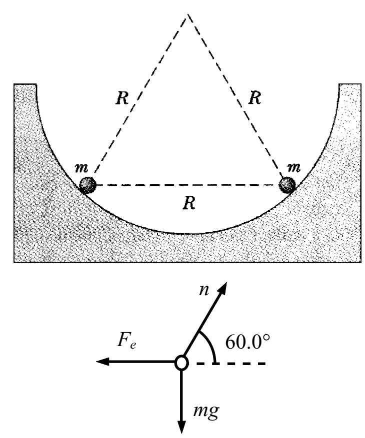

FIG. P23.56

P23.57 (a) The total non-contact force on the cork ball is: $F = qE + mg = m\left(g + \dfrac{qE}{m}\right)$,

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T = 2\pi\sqrt{\frac{L}{g + qE/m}} = 2\pi\sqrt{\frac{0.500 \text{ m}}{9.80 \text{ m/s}^2 + \left[\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)/1.00\times10^{-3}\text{ kg}\right]}}$$

$$= \boxed{0.307 \text{ s}}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $T = 2\pi\sqrt{\dfrac{L}{qE/m}}$

$$T = 2\pi\sqrt{\frac{0.500 \text{ m}}{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)/1.00\times10^{-3}\text{ kg}}} = 0.314 \text{ s (a 2.28\% difference).}$$

***P23.58** (a) At A the top charge makes the dominant contribution to the field and the net field is downward. At B the total electric field is zero. Between B and C the main change is weakening of the downward electric field of the top charge, so the net field at C is upward. At E the fields of the two bottom charges cancel out and the total field is downward. At F the total field is downward.

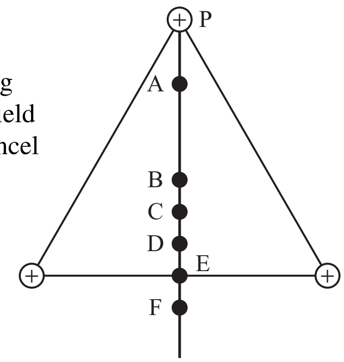

FIG. P23.58

(b) The field is zero at B as it changes from downward at A to upward at C. As a continuous function, the field must pass through the value zero near D as it changes from upward at C to downward at E and F.

(c) Let y represent the distance from E up to the zero-field point. The distance from P to E is $(3^2 - 1.5^2)^{1/2}$ cm = 2.60 cm. Then the requirement that the field be zero is

$$\frac{k_e q}{(2.60\text{ cm} - y)^2} = \frac{k_e q}{(1.5\text{ cm})^2 + y^2}\frac{y}{\sqrt{(1.5\text{ cm})^2 + y^2}} + \frac{k_e q y}{\left[(1.5\text{ cm})^2 + y^2\right]^{3/2}}$$

$$\frac{k_e q}{(2.60\text{ cm} - y)^2} = \frac{2k_e q y}{\left[(1.5\text{ cm})^2 + y^2\right]^{3/2}}$$

$$(1.5^2 + y^2)^{3/2} - 2\,y\,(2.60 - y)^2 = 0$$

As a check on our algebra, we note that $y = (1/3)2.60$ cm = 0.866 cm should be a solution, corresponding to point B. Substituting 0.866 gives 5.20 − 5.20 = 0 as it should. We home in on the smaller answer:

y	$(1.5^2 + y^2)^{3/2} - 2y(2.60 - y)^2$
0	+3.375
0.3	+0.411
0.4	−0.124
0.37	+0.014
0.373	−0.000 6

To three digits the answer is $\boxed{0.373\text{ cm}}$.

P23.59 (a) There are 7 terms which contribute:

3 are s away (along sides)

3 are $\sqrt{2}s$ away (face diagonals) and $\sin\theta = \frac{1}{\sqrt{2}} = \cos\theta$

1 is $\sqrt{3}s$ away (body diagonal) and $\sin\phi = \frac{1}{\sqrt{3}}$

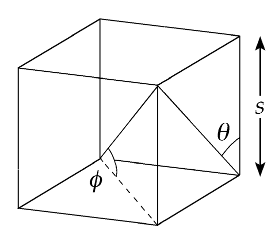

FIG. P23.59

The component in each direction is the same by symmetry.

$$\vec{\mathbf{F}} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = \boxed{\frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}$$

(b) $$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2}\text{ away from the origin}}$$

P23.60 (a) Zero contribution from the same face due to symmetry, opposite face contributes

$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \quad \text{where} \quad r=\sqrt{\left(\frac{s}{2}\right)^2+\left(\frac{s}{2}\right)^2+s^2}=\sqrt{1.5}s=1.22s$$

$$\sin\phi=\frac{s}{r} \qquad E=4\frac{k_e qs}{r^3}=\frac{4}{(1.22)^3}\frac{k_e q}{s^2}=\boxed{2.18\frac{k_e q}{s^2}}$$

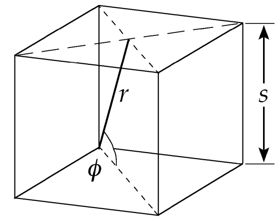

FIG. P23.60

(b) $\boxed{\text{The direction is the } \hat{\mathbf{k}} \text{ direction.}}$

P23.61 The field on the axis of the ring is calculated in an Example in the chapter text as

$$E=E_x=\frac{k_e xQ}{\left(x^2+a^2\right)^{3/2}}$$

The force experienced by a charge $-q$ placed along the axis of the ring is

$$F=-k_e Qq\left[\frac{x}{\left(x^2+a^2\right)^{3/2}}\right] \text{ and when } x \ll a, \text{ this becomes } F=-\left(\frac{k_e Qq}{a^3}\right)x.$$

This expression for the force is in the form of Hooke's law, with an effective spring constant of $k=\dfrac{k_e Qq}{a^3}$.

Since $\omega=2\pi f=\sqrt{\dfrac{k}{m}}$, we have $f=\boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Qq}{ma^3}}}$.

P23.62

$$d\vec{\mathbf{E}}=\frac{k_e dq}{x^2+(0.150\text{ m})^2}\left(\frac{-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right)$$

$$=\frac{k_e\lambda\left(-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

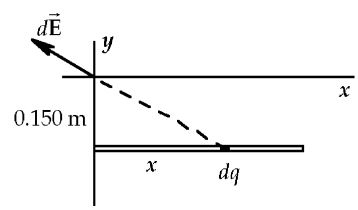

FIG. P23.62

$$\vec{\mathbf{E}}=\int_{\text{all charge}} d\vec{\mathbf{E}}=k_e\lambda\int_{x=0}^{0.400\text{ m}}\frac{\left(-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

$$\vec{\mathbf{E}}=k_e\lambda\left[\left.\frac{+\hat{\mathbf{i}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}+\left.\frac{(0.150\text{ m})\hat{\mathbf{j}}x}{(0.150\text{ m})^2\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\vec{\mathbf{E}}=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\text{ C/m}\right)\left[\hat{\mathbf{i}}(2.34-6.67)\text{ m}^{-1}+\hat{\mathbf{j}}(6.24-0)\text{ m}^{-1}\right]$$

$$\vec{\mathbf{E}}=\left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)\times10^3\text{ N/C}=\boxed{\left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)\text{kN/C}}$$

P23.63 The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa\sin\theta = -2Eqa\sin\theta$.

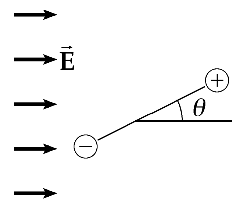

FIG. P23.63

For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{Ep}{I}\right)\theta = 0$

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ which is the standard equation characterizing simple harmonic motion, with $\omega^2 = \dfrac{Ep}{I}$

Then the frequency of oscillation is $f = \omega/2\pi$, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$

P23.64 $$\vec{\mathbf{E}} = \sum \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{k_e q}{a^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(2a)^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) + \cdots = \frac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right) = \boxed{-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$$

P23.65 $$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}} = \int_{x_0}^{\infty}\left[\frac{k_e\lambda_0 x_0 dx\left(-\hat{\mathbf{i}}\right)}{x^3}\right] = -k_e\lambda_0 x_0\hat{\mathbf{i}}\int_{x_0}^{\infty} x^{-3}dx = -k_e\lambda_0 x_0\hat{\mathbf{i}}\left(-\frac{1}{2x^2}\Big|_{x_0}^{\infty}\right) = \boxed{\frac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)}$$

ANSWERS TO EVEN PROBLEMS

P23.2 (a) 2.62×10^{24} (b) 2.38 electrons for every 10^9 present

P23.4 1.57 μN to the left

P23.6 2.51×10^{-9}

P23.8 $x = 0.634d$. The equilibrium is stable if the third bead has positive charge.

P23.10 (a) period $= \dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_e qQ}}$ where m is the mass of the object with charge $-Q$ (b) $4a\sqrt{\dfrac{k_e qQ}{md^3}}$

P23.12 1.82 m to the left of the negative charge

P23.14 514 kN

P23.16 (a) $12.9\hat{\mathbf{j}}$ kN/C (b) $-38.6\hat{\mathbf{j}}$ mN

P23.18 See the solution.

P23.20 $\dfrac{k_e\lambda_0}{x_0}\left(-\hat{\mathbf{i}}\right)$

P23.22 See the solution.

P23.24 (a) 383 MN/C away (b) 324 MN/C away (c) 80.7 MN/C away (d) 6.68 MN/C away

P23.26 See the solution.

P23.28 (a) $\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\left(d^2+R^2\right)^{-1/2}-\left((d+h)^2+R^2\right)^{-1/2}\right]$ (b) $\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h+\left(d^2+R^2\right)^{1/2}-\left((d+h)^2+R^2\right)^{1/2}\right]$

P23.30 (a) 200 pC (b) 141 pC (c) 58.9 pC

P23.32 See the solution.

P23.34 (a) $-\frac{1}{3}$ (b) q_1 is negative and q_2 is positive

P23.36 (a) $-57.6\hat{\mathbf{i}}\ \text{Tm/s}^2$ (b) $2.84\hat{\mathbf{i}}\ \text{Mm/s}$ (c) 49.3 ns

P23.38 The particle strikes the negative plate after moving in a parabola 0.181 mm high and 0.961 mm wide.

P23.40 (a) 36.9°, 53.1° (b) 167 ns, 221 ns

P23.42 It is possible in just one way: at $x = -16.0$ cm place a charge of $+51.3\ \mu$C.

P23.44 (a) 24.2 N/C at 0° (b) 9.42 N/C at 117°

P23.46 (a) $\frac{mg}{A\cot\theta + B}$ (b) $\frac{mgA}{A\cos\theta + B\sin\theta}$

P23.48 0.205 μC

P23.50 $\frac{k_e qQ}{a^2}$ toward the 29th vertex

P23.52 25.9 cm

P23.54 (a) $\vec{\mathbf{E}} = \frac{935\ \text{N}\cdot\text{m}^2\ x\ \hat{\mathbf{i}}}{\text{C}\left(0.0625\ \text{m}^2 + x^2\right)^{3/2}}$ (b) $4.00\ \hat{\mathbf{i}}$ kN/C (c) At $x = 0.016\ 8$ m and at $x = 0.916$ m
(d) Nowhere is the field so large. (e) The field of a single charge in Question 7 takes on all values from zero to infinity, each at just one point along the positive x-axis. The vector sum of the field of two charges, in this problem, is zero at the origin, rises to a maximum at 17.7 cm, and then decreases asymptotically to zero. In the question and the problem, the fields at $x = 36$ cm happen to take similar values. For large x the field of the two charges in this problem shows just the same inverse proportionality to x^2 as the field in the question, being larger by the factor 2(52 nC)/(57.7 nC) = 1.8 times.

P23.56 $R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}$

P23.58 (a) At A downward. At B zero. At C upward. At E downward. At F downward. (b) See the solution. (c) 0.373 cm

P23.60 (a) See the solution. (b) $\hat{\mathbf{k}}$.

P23.62 $\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)$ kN/C

P23.64 $-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}$