10

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q10.1 1 rev/min, or $\frac{\pi}{30}$ rad/s. The direction is horizontally into the wall to represent clockwise rotation. The angular velocity is constant so $\alpha = 0$.

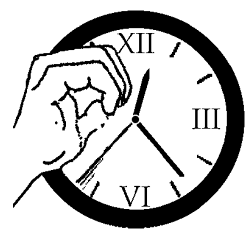

FIG. Q10.1

Q10.2 The vector angular velocity is in the direction $+\hat{\mathbf{k}}$. The vector angular acceleration has the direction $-\hat{\mathbf{k}}$.

***Q10.3** The tangential acceleration has magnitude $(3/\text{s}^2)r$ where r is the radius. It is constant in time. The radial acceleration has magnitude $\omega^2 r$, so it is $(4/\text{s}^2)r$ at the first and last moments mentioned and it is zero at the moment the wheel reverses. Thus we have b = f > a = c = e > d = 0.

***Q10.4** (i) answer (d). The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.

(ii) answer (c). If the driver uses the gearshift and the gas pedal to keep the tachometer readings and the air speeds comparable before and after the tire switch, there should be no effect.

***Q10.5** (i) answer (a). Smallest I is about x axis, along which the larger-mass balls lie.

(ii) answer (c). The balls all lie at a distance from the z axis, which is perpendicular to both the x and y axes and passes through the origin.

Q10.6 The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.

***Q10.7** The accelerations are not equal, but greater in case (a). The string tension above the 5.1-kg object is less than its weight while the object is accelerating down.

Q10.8 You could measure the time that it takes the hanging object, of known mass m, to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration.

***Q10.9** answers (a), (b), and (e). The object must rotate with nonzero angular acceleration. The center of mass can be constant in location if it is on the axis of rotation.

Q10.10 You could use $\omega = \alpha t$ and $v = at$. The equation $v = R\omega$ is valid in this situation since $a = R\alpha$.

Q10.11 The angular speed ω would decrease. The center of mass is farther from the pivot, but the moment of inertia increases also.

***Q10.12** answer (f). The sphere of twice the radius has eight times the volume and eight times the mass. Then r^2 in $I = (2/5)\,mr^2$ also gets four times larger.

Q10.13 The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is a different distance from the axis. In an example in section 10.5 in the text, the moment of inertia of a uniform rigid rod about an axis perpendicular to the rod and passing through the center of mass is derived. If you spin a pencil back and forth about this axis, you will get a feeling for its stubbornness against changing rotation. Now change the axis about which you rotate it by spinning it back and forth about the axis that goes down the middle of the graphite. Easier, isn't it? The moment of inertia about the graphite is much smaller, as the mass of the pencil is concentrated near this axis.

Q10.14 A quick flip will set the hard–boiled egg spinning faster and more smoothly. Inside the raw egg, the yolk takes some time to start rotating. The raw egg also loses mechanical energy to internal fluid friction.

Q10.15 Sewer pipe: $I_{CM} = MR^2$. Embroidery hoop: $I_{CM} = MR^2$. Door: $I = \frac{1}{3}MR^2$. Coin: $I_{CM} = \frac{1}{2}MR^2$. The distribution of mass along lines parallel to the axis makes no difference to the moment of inertia.

Q10.16 Yes. If you drop an object, it will gain translational kinetic energy from decreasing gravitational potential energy.

Q10.17 No, just as an object need not be moving to have mass.

Q10.18 No, only if its angular velocity changes.

***Q10.19** (i) answer (c). It is no longer speeding up and not yet slowing down.

(ii) answer (b). It is reversing its angular velocity from positive to negative, and reversal counts as a change.

Q10.20 The moment of inertia would decrease. Matter would be moved toward the axis. This would result in a higher angular speed of the Earth, shorter days, and more days in the year!

***Q10.21** (i) answer (a). The basketball has rotational as well as translational kinetic energy.

(ii) answer (c). The motions of their centers of mass are identical.

(iii) answer (a). The kinetic energy controls the gravitational energy it attains.

Q10.22 There is very little resistance to motion that can reduce the kinetic energy of the rolling ball. Even though there is static friction between the ball and the floor (if there were none, then no rotation would occur and the ball would slide), there is no relative motion of the two surfaces—by the definition of "rolling without slipping"—and so no force of kinetic friction acts to reduce K. Air resistance and friction associated with deformation of the ball eventually stop the ball.

Q10.23 The sphere would reach the bottom first; the hoop would reach the bottom last. First imagine that each object has the same mass and the same radius. Then they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first. But the mass and the radius divide out in the equation about conversion of gravitational energy to total kinetic energy. This experiment is a test about the numerical factor in the tabulated formula relating the moment of inertia to the mass and radius.

***Q10.24** (a) The tricycle rolls forward. (b) The tricycle rolls forward. (c) The tricycle rolls backward. (d) The tricycle does not roll, but may skid forward. (e) The tricycle rolls backward.
To answer these questions, think about the torque of the string tension about an axis at the bottom of the wheel, where the rubber meets the road. This is the instantaneous axis of rotation in rolling. Cords a and b produce clockwise torques about this axis. Cords c and e produce counter clockwise torques. Cord d has zero lever arm.

SOLUTIONS TO PROBLEMS

Section 10.1 **Angular Position, Velocity, and Acceleration**

P10.1 (a) $\theta\big|_{t=0} = \boxed{5.00\ \text{rad}}$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0\ \text{rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00\ \text{rad/s}^2}$$

(b) $\theta\big|_{t=3.00\ \text{s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0\ \text{rad}}$

$$\omega\big|_{t=3.00\ \text{s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\ \text{s}} = 10.0 + 4.00t\big|_{t=3.00\ \text{s}} = \boxed{22.0\ \text{rad/s}}$$

$$\alpha\big|_{t=3.00\ \text{s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\ \text{s}} = \boxed{4.00\ \text{rad/s}^2}$$

***P10.2** $\alpha = \dfrac{d\omega}{dt} = 10 + 6t \qquad \int_0^\omega d\omega = \int_0^t (10+6t)\,dt \qquad \omega - 0 = 10t + 6t^2/2$

$\omega = \dfrac{d\theta}{dt} = 10t + 3t^2 \qquad \int_0^\theta d\theta = \int_0^t (10t + 3t^2)\,dt \qquad \theta - 0 = 10t^2/2 + 3t^3/3$

$\theta = 5t^2 + t^3$. At $t = 4$ s, $\theta = 5(4)^2 + (4)^3 = \boxed{144\ \text{rad}}$

Section 10.2 **Rotational Kinematics: Rotational Motion with Constant Angular Acceleration**

P10.3 (a) $\alpha = \dfrac{\omega - \omega_i}{t} = \dfrac{12.0\ \text{rad/s}}{3.00\ \text{s}} = \boxed{4.00\ \text{rad/s}^2}$

(b) $\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = \dfrac{1}{2}\left(4.00\ \text{rad/s}^2\right)(3.00\ \text{s})^2 = \boxed{18.0\ \text{rad}}$

P10.4 $\omega_i = 3\,600\ \text{rev/min} = 3.77\times 10^2\ \text{rad/s}$

$\theta = 50.0\ \text{rev} = 3.14\times 10^2\ \text{rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = \left(3.77\times 10^2\ \text{rad/s}\right)^2 + 2\alpha\left(3.14\times 10^2\ \text{rad}\right)$

$\alpha = \boxed{-2.26\times 10^2\ \text{rad/s}^2}$

P10.5 $\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \ \omega_f = 0$

(a) $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0-(10\pi/3)}{-2.00} s = \boxed{5.24 \text{ s}}$

(b) $\theta_f = \bar{\omega}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$

P10.6 $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$\omega_i = \omega_f - \alpha t$: $\quad \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$

$$37.0 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 98.0 \text{ rad/s}(3.00 \text{ s}) - \frac{1}{2}\alpha(3.00 \text{ s})^2$$

$232 \text{ rad} = 294 \text{ rad} - (4.50 \text{ s}^2)\alpha$: $\quad \alpha = \frac{61.5 \text{ rad}}{4.50 \text{ s}^2} = \boxed{13.7 \text{ rad/s}^2}$

P10.7 (a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$

(b) $\Delta t = \frac{\Delta\theta}{\omega} = \frac{107°}{7.27 \times 10^{-5} \text{ rad/s}}\left(\frac{2\pi \text{ rad}}{360°}\right) = \boxed{2.57 \times 10^4 \text{ s}}$ or 428 min

P10.8 The location of the dog is described by $\theta_d = (0.750 \text{ rad/s})t$. For the bone,

$$\theta_b = \frac{1}{3}2\pi \text{ rad} + \frac{1}{2}0.015 \text{ rad/s}^2 t^2$$

We look for a solution to

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)2.09}}{0.015} = 2.88 \text{ s or } 97.1 \text{ s}$$

The dog and bone will also pass if $0.75t = \frac{2\pi}{3} - 2\pi + 0.007\,5t^2$ or if $0.75t = \frac{2\pi}{3} + 2\pi + 0.007\,5t^2$ that is, if either the dog or the turntable gains a lap on the other. The first equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)(-4.19)}}{0.015} = 105 \text{ s or } -5.30 \text{ s}$$

only one positive root representing a physical answer. The second equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)8.38}}{0.015} = 12.8 \text{ s or } 87.2 \text{ s}$$

In order, the dog passes the bone at $\boxed{2.88 \text{ s}}$ after the merry-go-round starts to turn, and again at $\boxed{12.8 \text{ s}}$ and 26.6 s, after gaining laps on the bone. The bone passes the dog at 73.4 s, 87.2 s, 97.1 s, 105 s, and so on, after the start.

P10.9 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega}t = \dfrac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$

While slowing down, $\theta_2 = \bar{\omega}t = \dfrac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$

Section 10.3 **Angular and Translational Quantities**

P10.10 (a) $v = r\omega;\ \omega = \dfrac{v}{r} = \dfrac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$

P10.11 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00 \times 10^4 \text{ mi}}{0.250 \text{ m}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 6.44 \times 10^7 \text{ rad/yr}$$

$$\theta = 6.44 \times 10^7 \text{ rad/yr}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.02 \times 10^7 \text{ rev/yr or } \boxed{\sim 10^7 \text{ rev/yr}}$$

P10.12 (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152 \text{ m}}{2}\right)76 \text{ rev/min}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{0.605 \text{ m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605 \text{ m/s}}{(0.07 \text{ m})/2} = \boxed{17.3 \text{ rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673 \text{ m}}{2}\right)17.3 \text{ rad/s} = \boxed{5.82 \text{ m/s}}$$

(d) $\boxed{\text{We did not need to know the length of the pedal cranks}}$, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175 \text{ m } 7.96 \text{ rad/s}\left(\frac{1}{1 \text{ rad}}\right) = 1.39 \text{ m/s}$$

P10.13 Given $r = 1.00\text{ m}, \alpha = 4.00\text{ rad/s}^2, \omega_i = 0$ and $\theta_i = 57.3° = 1.00\text{ rad}$

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00\text{ s}$, $\omega_f = 4.00\text{ rad/s}^2(2.00\text{ s}) = \boxed{8.00\text{ rad/s}}$

(b) $v = r\omega = 1.00\text{ m}(8.00\text{ rad/s}) = \boxed{8.00\text{ m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00\text{ m}(8.00\text{ rad/s})^2 = 64.0\text{ m/s}^2$

$a_t = r\alpha = 1.00\text{ m}(4.00\text{ rad/s}^2) = 4.00\text{ m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\text{ m/s}^2)^2 + (4.00\text{ m/s}^2)^2} = \boxed{64.1\text{ m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\text{ rad}) + \frac{1}{2}(4.00\text{ rad/s}^2)(2.00\text{ s})^2 = \boxed{9.00\text{ rad}}$

P10.14 (a) $\omega = \dfrac{v}{r} = \dfrac{25.0\text{ m/s}}{1.00\text{ m}} = \boxed{25.0\text{ rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\text{ rad/s})^2 - 0}{2[(1.25\text{ rev})(2\pi\text{ rad/rev})]} = \boxed{39.8\text{ rad/s}^2}$$

(c) $\Delta t = \dfrac{\Delta\omega}{\alpha} = \dfrac{25.0\text{ rad/s}}{39.8\text{ rad/s}^2} = \boxed{0.628\text{ s}}$

***P10.15** The object starts with $\theta_i = 0$. As far as location on the circle and instantaneous motion is concerned, we can think of its final position as 9 rad – 2π = 2.72 rad = 156°.

(a) Its position vector is 3.00 m at 156° = 3 m cos 156° $\hat{\mathbf{i}}$ + 3 m sin 156° $\hat{\mathbf{j}}$ = $\boxed{(-2.73\,\hat{\mathbf{i}} + 1.24\,\hat{\mathbf{j}})\text{ m}}$

(b) $\boxed{\text{It is in the second quadrant, at 156°}}$

(c) Its original velocity is 4.5 m/s at 90°. After the displacement, its velocity is

4.5 m/s at (90° + 156°) = 4.5 m/s at 246°

$= \boxed{(-1.85\,\hat{\mathbf{i}} - 4.10\,\hat{\mathbf{j}})\text{ m/s}}$

(d) $\boxed{\text{It is moving toward the third quadrant, at 246°}}$

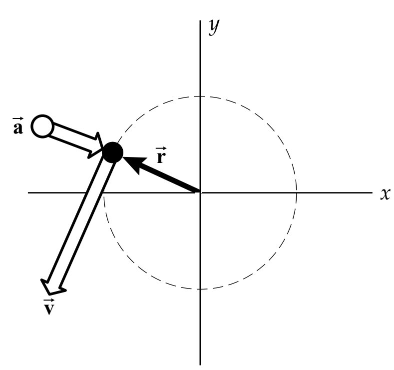

FIG. 10.15d

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(e) Its acceleration is v^2/r opposite in direction to its position vector.

This is $(4.5\text{ m/s})^2/3\text{m}$ at $(156° + 180°) = 6.75\text{ m/s}^2$ at $336° = \boxed{(6.15\,\hat{\mathbf{i}} - 2.78\,\hat{\mathbf{j}})\text{ m/s}^2}$

(f) The total force is given by $ma = 4\text{ kg}\,(6.15\,\hat{\mathbf{i}} - 2.78\,\hat{\mathbf{j}})\text{ m/s}^2 = \boxed{(24.6\,\hat{\mathbf{i}} - 11.1\,\hat{\mathbf{j}})\text{ N}}$

P10.16 (a) $s = \bar{v}t = (11.0\text{ m/s})(9.00\text{ s}) = 99.0\text{ m}$

$$\theta = \frac{s}{r} = \frac{99.0\text{ m}}{0.290\text{ m}} = 341\text{ rad} = \boxed{54.3\text{ rev}}$$

(b) $\omega_f = \dfrac{v_f}{r} = \dfrac{22.0\text{ m/s}}{0.290\text{ m}} = 75.9\text{ rad/s} = \boxed{12.1\text{ rev/s}}$

P10.17 (a) $\omega = 2\pi f = \dfrac{2\pi\text{ rad}}{1\text{ rev}}\left(\dfrac{1\,200\text{ rev}}{60.0\text{ s}}\right) = \boxed{126\text{ rad/s}}$

(b) $v = \omega r = (126\text{ rad/s})(3.00\times10^{-2}\text{ m}) = \boxed{3.77\text{ m/s}}$

(c) $a_c = \omega^2 r = (126)^2(8.00\times10^{-2}) = 1\,260\text{ m/s}^2$ so $\mathbf{a}_r = \boxed{1.26\text{ km/s}^2\text{ toward the center}}$

(d) $s = r\theta = \omega rt = (126\text{ rad/s})(8.00\times10^{-2}\text{ m})(2.00\text{ s}) = \boxed{20.1\text{ m}}$

***P10.18** An object of any shape can rotate. The ladder undergoes pure rotation about its right foot. Its angular displacement in radians is $\theta = s/r = 0.690\text{ m}/4.90\text{ m} = t/0.410\text{ m}$ where t is the thickness of the rock. Solving gives (a) $\boxed{5.77\text{ cm}}$.

$\boxed{\text{(b) Yes. We used the idea of rotational motion measured by angular displacement in the solution.}}$

P10.19 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70\text{ m/s}^2)$.

Its radially inward component is $\dfrac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi(1.70\text{ m/s}^2)$$

With skidding impending we have $\sum F_y = ma_y,\quad +n - mg = 0,\quad n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70\text{ m/s}^2)^2 + m^2\pi^2(1.70\text{ m/s}^2)^2}$$

$$\mu_s = \frac{1.70\text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

***P10.20** (a) If we number the loops of the spiral track with an index n, with the innermost loop having $n = 0$, the radii of subsequent loops as we move outward on the disc is given by $r = r_i + hn$. Along a given radial line, each new loop is reached by rotating the disc through 2π rad. Therefore, the ratio $\theta/2\pi$ is the number of revolutions of the disc to get to a certain loop. This is also the number of that loop, so $n = \theta/2\pi$. Therefore, $r = r_i + h\theta/2\pi$.

(b) Starting from $\omega = v/r$, we substitute the definition of angular speed on the left and the result for r from part (a) on the right:

$$\omega = \frac{v}{r} \quad\rightarrow\quad \frac{d\theta}{dt} = \frac{v}{r_i + \dfrac{h}{2\pi}\theta}$$

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(c) Rearrange terms in preparation for integrating both sides:

$$\left(r_i + \frac{h}{2\pi}\theta\right)d\theta = vdt$$

and integrate from $\theta = 0$ to $\theta = \theta$ and from $t = 0$ to $t = t$:

$$r_i\theta + \frac{h}{4\pi}\theta^2 = vt$$

We rearrange this equation to form a standard quadratic equation in θ:

$$\frac{h}{4\pi}\theta^2 + r_i\theta - vt = 0$$

The solution to this equation is

$$\theta = \frac{-r_i \pm \sqrt{r_i^2 + \frac{h}{\pi}vt}}{\frac{h}{2\pi}} = \boxed{\frac{2\pi r_i}{h}\left(\sqrt{1+\frac{vh}{\pi r_i^2}t} - 1\right)}$$

where we have chosen the positive root in order to make the angle θ positive.

(d) We differentiate the result in (c) twice with respect to time to find the angular acceleration, resulting in

$$\boxed{\alpha = -\frac{hv^2}{2\pi r_i^3\left(1+\frac{vh}{\pi r_i^2}t\right)^{3/2}}}$$

Because this expression involves the time t, the angular acceleration is not constant.

Section 10.4 **Rotational Energy**

P10.21 (a) $I = \sum_j m_j r_j^2$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

$$I = \left[\sqrt{13.0}\text{ m}\right]^2[3.00 + 2.00 + 2.00 + 4.00]\text{ kg}$$

$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

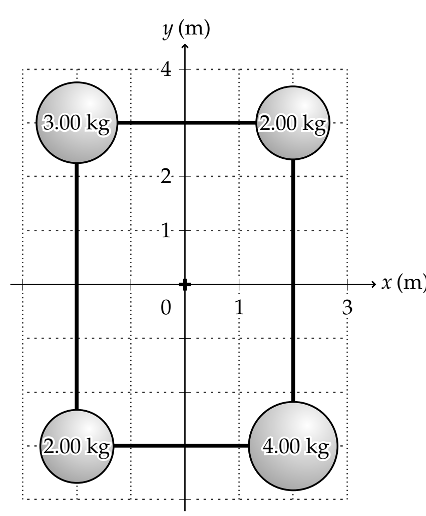

FIG. P10.21

(b) $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(143\text{ kg}\cdot\text{m}^2)(6.00\text{ rad/s})^2$

$$= \boxed{2.57\times10^3\text{ J}}$$

***P10.22** $m_1 = 4.00\text{ kg}, r_1 = |y_1| = 3.00\text{ m}$

$m_2 = 2.00\text{ kg}, r_2 = |y_2| = 2.00\text{ m}$

$m_3 = 3.00\text{ kg}, \; r_3 = |y_3| = 4.00\text{ m}$

$\omega = 2.00\text{ rad/s}$ about the x-axis

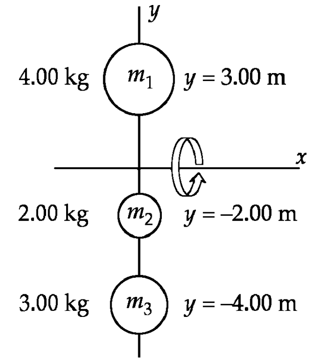

FIG. P10.22

(a) $I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$

(b) $v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\text{ m/s}}$ $\quad K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$

$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\text{ m/s}}$ $\quad K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$

$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\text{ m/s}}$ $\quad K_3 = \frac{1}{2}m_3 v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

(c) $\boxed{\text{The kinetic energies computed in parts (a) and (b) are the same.}}$ Rotational kinetic energy can be viewed as the total translational kinetic energy of the particles in the rotating object.

P10.23 $I = Mx^2 + m(L-x)^2$

$$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0 \quad \text{(for an extremum)}$$

$$\therefore x = \frac{mL}{M+m}$$

$\dfrac{d^2I}{dx^2} = 2m + 2M$; therefore I is at a minimum when the axis of rotation passes through $x = \dfrac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through x is

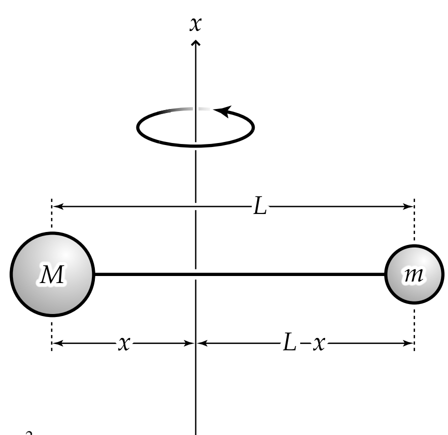

FIG. P10.23

$$I_{\text{CM}} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2 = \mu L^2$$

where

$$\mu = \frac{Mm}{M+m}$$

***P10.24** For large energy storage at a particular rotation rate, we want a large moment of inertia. To combine this requirement with small mass, we place the mass as far away from the axis as possible.

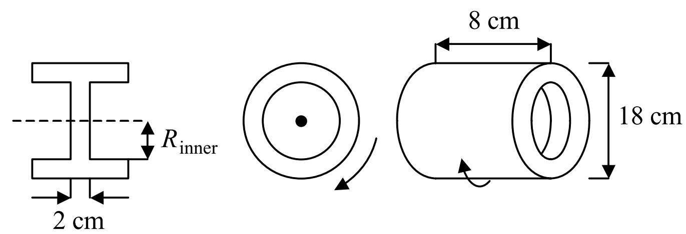

FIG. P10.24

We choose to make the flywheel as a hollow cylinder 18 cm in diameter and 8 cm long. To support this rim, we place a disk across its center. We assume that a disk 2 cm thick will be sturdy enough to support the hollow cylinder securely.

The one remaining adjustable parameter is the thickness of the wall of the hollow cylinder. From Table 10.2, the moment of inertia can be written as

$$\begin{aligned}
I_{\text{disk}} + I_{\text{hollow cylinder}} &= \frac{1}{2}M_{\text{disk}}R_{\text{disk}}^2 + \frac{1}{2}M_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\
&= \frac{1}{2}\rho V_{\text{disk}}R_{\text{outer}}^2 + \frac{1}{2}\rho V_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\
&= \frac{\rho}{2}\pi R_{\text{outer}}^2(2\text{ cm})R_{\text{outer}}^2 + \frac{\rho}{2}\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right](6\text{ cm})\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\
&= \frac{\rho\pi}{2}\left[(9\text{ cm})^4(2\text{ cm}) + (6\text{ cm})\left((9\text{ cm})^2 - R_{\text{inner}}^2\right)\left((9\text{ cm})^2 + R_{\text{inner}}^2\right)\right] \\
&= \rho\pi\left[6\,561\text{ cm}^5 + (3\text{ cm})\left((9\text{ cm})^4 - R_{\text{inner}}^4\right)\right] \\
&= \rho\pi\left[26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4\right]
\end{aligned}$$

For the required energy storage,

$$\frac{1}{2}I\omega_1^2 = \frac{1}{2}I\omega_2^2 + W_{\text{out}}$$

$$\frac{1}{2}I\left[(800\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 - \frac{1}{2}I\left[(600)\left(\frac{2\pi\text{ rad}}{60\text{ s}}\right)\right]^2 = 60\text{ J}$$

$$I = \frac{60\text{ J}}{1\,535/\text{s}^2} = \left(7.86\times10^3\text{ kg/m}^3\right)\pi\left[26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4\right]$$

$$1.58\times10^{-6}\text{ m}^5\left(\frac{100\text{ cm}}{1\text{ m}}\right)^5 = 26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4$$

$$R_{\text{inner}} = \left(\frac{26\,244\text{ cm}^4 - 15\,827\text{ cm}^4}{3}\right)^{1/4} = 7.68\text{ cm}$$

$\boxed{\text{The inner radius of the flywheel is 7.68 cm. The mass of the flywheel is then 7.27 kg}}$, found as follows:

$$\begin{aligned}
M_{\text{disk}} + M_{\text{wall}} &= \rho\pi R_{\text{outer}}^2(2\text{ cm}) + \rho\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right](6\text{ cm}) \\
&= \left(7.86\times10^3\text{ kg/m}^3\right)\pi\left[(0.09\text{ m})^2(0.02\text{ m}) + \left[(0.09\text{ m})^2 - (0.076\,8\text{ m})^2\right](0.06\text{ m})\right] \\
&= 7.27\text{ kg}
\end{aligned}$$

If we made the thickness of the disk somewhat less than 2 cm and the inner radius of the cylindrical wall less than 7.68 cm to compensate, the mass could be a bit less than 7.27 kg.

***P10.25** Note that the torque on the trebuchet is not constant, so its angular acceleration changes in time. At our mathematical level it would be unproductive to calculate values for α on the way to find ω_f. Instead, we consider that gravitational energy of the 60-kg-Earth system becomes gravitational energy of the lighter mass plus kinetic energy of both masses.

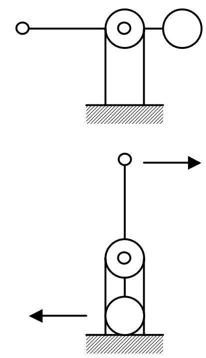

FIG. P10.25

(a) The maximum speed appears as the rod passes through the vertical. Let v_1 represent the speed of the small-mass particle m_1. Then here the rod is turning at $\omega_1 = \dfrac{v_1}{2.86\text{ m}}$. The larger-mass particle is moving at

$$v_2 = (0.14\text{ m})\omega_1 = \frac{0.14v_1}{2.86}$$

Now the energy-conservation equation becomes

$$\left(K_1 + K_2 + U_{g1} + U_{g2}\right)_i = \left(K_1 + K_2 + U_{g1} + U_{g2}\right)_f$$

$$0 + 0 + 0 + m_2 g y_{2i} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + m_1 g y_{1f} + 0$$

$$(60\text{ kg})(9.8\text{ m/s}^2)(0.14\text{ m}) = \frac{1}{2}(0.12\text{ kg})v_1^2 + \frac{1}{2}(60\text{ kg})\left(\frac{0.14v_1}{2.86}\right)^2 + (0.12\text{ kg})(9.8\text{ m/s}^2)(2.86\text{ m})$$

$$82.32\text{ J} = \frac{1}{2}(0.12\text{ kg})v_1^2 + \frac{1}{2}(0.144\text{ kg})v_1^2 + 3.36\text{ J}$$

$$v_1 = \left(\frac{2(79.0\text{ J})}{0.264\text{ kg}}\right)^{1/2} = \boxed{24.5\text{ m/s}}$$

(b) The lever arm of the gravitational force on the 60-kg particle changes during the motion, so the torque changes, so the angular acceleration changes. The projectile moves with changing net acceleration and changing tangential acceleration. The ratio of the particles' distances from the axis controls the ratio of their speeds, and this is different from the ratio of their masses, so the total momentum changes during the motion. But the mechanical energy stays constant, and that is how we solved the problem.

Section 10.5 **Calculation of Moments of Inertia**

P10.26 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.

For the rod along the y-axis, $I = \frac{1}{3}mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

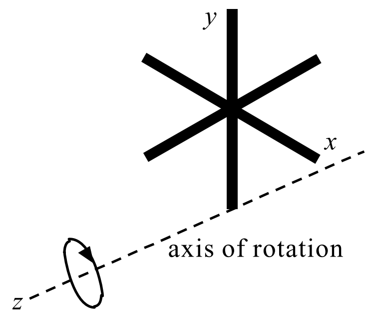

FIG. P10.26

In the rod along the x-axis, the bit of material between x and $x+dx$ has mass $\left(\frac{m}{L}\right)dx$ and is at distance $r = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}\left(x^2 + \frac{L^2}{4}\right)\left(\frac{m}{L}\right)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\bigg|_{-L/2}^{L/2} + \frac{mL}{4}x\bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}}$$

Note: The moment of inertia of the rod along the x axis can also be calculated from the parallel-axis theorem as $\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$.

P10.27 Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2}m\left(R_1^2 + R_2^2\right)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi\left[(0.305\text{ m})^2 - (0.165\text{ m})^2\right]\left(6.35\times10^{-3}\text{ m}\right)\left(1.10\times10^3\text{ kg/m}^3\right) = 1.44\text{ kg}$$

$$I_{\text{side}} = \frac{1}{2}(1.44\text{ kg})\left[(0.165\text{ m})^2 + (0.305\text{ m})^2\right] = 8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m = \pi\left[(0.330\text{ m})^2 - (0.305\text{ m})^2\right](0.200\text{ m})\left(1.10\times10^3\text{ kg/m}^3\right) = 11.0\text{ kg}$$

$$I_{\text{tread}} = \frac{1}{2}(11.0\text{ kg})\left[(0.330\text{ m})^2 + (0.305\text{ m})^2\right] = 1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}} = 2I_{\text{side}} + I_{\text{tread}} = 2\left(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2\right) + 1.11\text{ kg}\cdot\text{m}^2 = \boxed{1.28\text{ kg}\cdot\text{m}^2}$$

P10.28 Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2 = \boxed{5.80\text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

P10.29 Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750\text{ m}}{2\pi} = 0.120\text{ m}$$

and its moment of inertia is

$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2 = 0.432\text{ kg}\cdot\text{m}^2 \sim \boxed{10^0\text{ kg}\cdot\text{m}^2 = 1\text{ kg}\cdot\text{m}^2}$$

P10.30 We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass M_0 of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \qquad M_0 = \frac{4}{3}M$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{\text{CM}} + M_0\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_0R^2 + M_0\frac{R^2}{4} = \frac{3}{4}M_0R^2$$

The negative-mass portion has $I = \frac{1}{2}\left(-\frac{1}{4}M_0\right)\left(\frac{R}{2}\right)^2 = -\frac{M_0R^2}{32}$. The whole cam has

$$I = \frac{3}{4}M_0R^2 - \frac{M_0R^2}{32} = \frac{23}{32}M_0R^2 = \frac{23}{32}\frac{4}{3}MR^2 = \frac{23}{24}MR^2 \text{ and}$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{23}{24}MR^2\omega^2 = \boxed{\frac{23}{48}MR^2\omega^2}$$

***P10.31** We measure the distance of each particle in the rod from the y' axis:

$$I_{y'} = \int_{\text{all mass}} r^2 dm = \int_0^L x^2\frac{M}{L}dx = \frac{M}{L}\frac{x^3}{3}\bigg|_0^L = \frac{1}{3}ML^2$$

Section 10.6 **Torque**

P10.32 Resolve the 100 N force into components perpendicular to and parallel to the rod, as

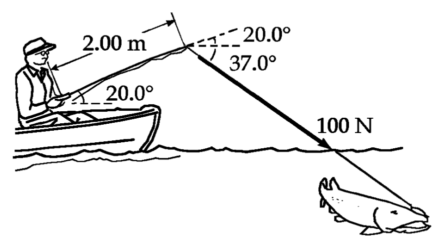

FIG. P10.32

$$F_{\text{par}} = (100\text{ N})\cos 57.0° = 54.5\text{ N}$$

and

$$F_{\text{perp}} = (100\text{ N})\sin 57.0° = 83.9\text{ N}$$

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is $\tau = 83.9\text{ N}(2.00\text{ m}) = \boxed{168\text{ N}\cdot\text{m}}$ (clockwise)

P10.33 $\sum \tau = 0.100 \text{ m}(12.0 \text{ N}) - 0.250 \text{ m}(9.00 \text{ N}) - 0.250 \text{ m}(10.0 \text{ N}) = \boxed{-3.55 \text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

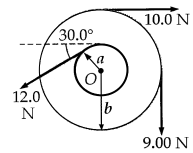

FIG. P10.33

Section 10.7 **Relationship Between Torque and Angular Acceleration**

P10.34 (a) $I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00 \text{ kg})(7.00\times10^{-2} \text{ m})^2 = 4.90\times10^{-3} \text{ kg}\cdot\text{m}^2$

$$\alpha = \frac{\Sigma\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122 \text{ rad/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1\,200(2\pi/60)}{122} = \boxed{1.03 \text{ s}}$$

(b) $\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122 \text{ rad/s})(1.03 \text{ s})^2 = 64.7 \text{ rad} = \boxed{10.3 \text{ rev}}$

P10.35 $m = 0.750$ kg, $F = 0.800$ N

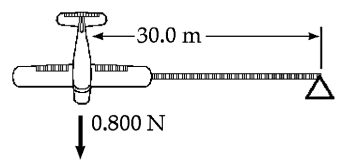

FIG. P10.35

(a) $\tau = rF = 30.0 \text{ m}(0.800 \text{ N}) = \boxed{24.0 \text{ N}\cdot\text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0}{0.750(30.0)^2} = \boxed{0.035\,6 \text{ rad/s}^2}$

(c) $a_t = \alpha r = 0.035\,6(30.0) = \boxed{1.07 \text{ m/s}^2}$

P10.36 $\omega_f = \omega_i + \alpha t$: $\quad 10.0 \text{ rad/s} = 0 + \alpha(6.00 \text{ s})$

$$\alpha = \frac{10.00}{6.00} \text{ rad/s}^2 = 1.67 \text{ rad/s}^2$$

(a) $\sum \tau = 36.0 \text{ N}\cdot\text{m} = I\alpha$: $I = \frac{\sum\tau}{\alpha} = \frac{36.0 \text{ N}\cdot\text{m}}{1.67 \text{ rad/s}^2} = \boxed{21.6 \text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$: $\quad 0 = 10.0 + \alpha(60.0)$

$$\alpha = -0.167 \text{ rad/s}^2$$

$$|\tau| = |I\alpha| = (21.6 \text{ kg}\cdot\text{m}^2)(0.167 \text{ rad/s}^2) = \boxed{3.60 \text{ N}\cdot\text{m}}$$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

During first 6.00 s $\quad \theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$

During next 60.0 s $\quad \theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$

$$\theta_{\text{total}} = 329 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{52.4 \text{ rev}}$$

P10.37 For m_1,

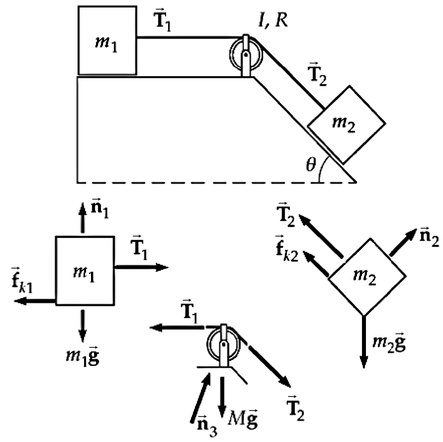

FIG. P10.37

$$\sum F_y = ma_y: \qquad +n - m_1 g = 0$$

$$n_1 = m_1 g = 19.6 \text{ N}$$

$$f_{k1} = \mu_k n_1 = 7.06 \text{ N}$$

$$\sum F_x = ma_x: \qquad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \qquad (1)$$

For the pulley,

$$\sum \tau = I\alpha: \qquad -T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})a$$

$$-T_1 + T_2 = (5.00 \text{ kg})a \qquad (2)$$

For m_2, $\qquad +n_2 - m_2 g\cos\theta = 0$

$$n_2 = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)(\cos 30.0°)$$

$$= 50.9 \text{ N}$$

$$f_{k2} = \mu_k n_2$$

$$= 18.3 \text{ N}: \quad -18.3 \text{ N} - T_2 + m_2 \sin\theta = m_2 a$$

$$-18.3 \text{ N} - T_2 + 29.4 \text{ N} = (6.00 \text{ kg})a \qquad (3)$$

(a) Add equations (1), (2), and (3):

$$-7.06 \text{ N} - 18.3 \text{ N} + 29.4 \text{ N} = (13.0 \text{ kg})a$$

$$a = \frac{4.01 \text{ N}}{13.0 \text{ kg}} = \boxed{0.309 \text{ m/s}^2}$$

(b) $T_1 = 2.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) + 7.06 \text{ N} = \boxed{7.67 \text{ N}}$

$T_2 = 7.67 \text{ N} + 5.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) = \boxed{9.22 \text{ N}}$

P10.38 $I = \frac{1}{2} mR^2 = \frac{1}{2}(100 \text{ kg})(0.500 \text{ m})^2 = 12.5 \text{ kg} \cdot \text{m}^2$

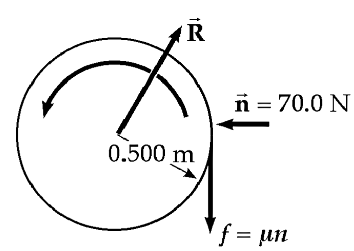

FIG. P10.38

$$\omega_i = 50.0 \text{ rev/min} = 5.24 \text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24 \text{ rad/s}}{6.00 \text{ s}} = -0.873 \text{ rad/s}^2$$

$$\tau = I\alpha = 12.5 \text{ kg} \cdot \text{m}^2\left(-0.873 \text{ rad/s}^2\right) = -10.9 \text{ N} \cdot \text{m}$$

The magnitude of the torque is given by $fR = 10.9 \text{ N} \cdot \text{m}$, where f is the force of friction.

Therefore,

$$f = \frac{10.9 \text{ N} \cdot \text{m}}{0.500 \text{ m}} \qquad \text{and} \qquad f = \mu_k n$$

yields

$$\mu_k = \frac{f}{n} = \frac{21.8 \text{ N}}{70.0 \text{ N}} = \boxed{0.312}$$

P10.39 $\sum \tau = I\alpha = \frac{1}{2}MR^2\alpha$

$$-135\text{ N}(0.230\text{ m}) + T(0.230\text{ m}) = \frac{1}{2}(80\text{ kg})\left(\frac{1.25}{2}\text{ m}\right)^2\left(-1.67\text{ rad/s}^2\right)$$

$T = \boxed{21.5\text{ N}}$

***P10.40** The chosen tangential force produces constant torque and so constant angular acceleration.

$\theta = 0 + 0 + (1/2)\alpha t^2 \qquad 2(2\pi\text{ rad}) = (1/2)\ \alpha(10\text{ s})^2 \qquad \alpha = 0.251\text{ rad/s}^2$

$\Sigma\tau = I\alpha \qquad TR = 100\text{ kg}\cdot\text{m}^2(0.251\text{ /s}^2) = 25.1\text{ N}\cdot\text{m}$

Infinitely many pairs of values that satisfy this requirement exist, such as $T = 25.1$ N and $R = 1.00$ m

Section 10.8 **Work, Power, and Energy in Rotational Motion**

P10.41 The power output of the bus is $\mathcal{P} = \frac{E}{\Delta t}$ where $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2$ is the stored energy and $\Delta t = \frac{\Delta x}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = \mathcal{P}\Delta t = \frac{\mathcal{P}\Delta x}{v}$ and

$$\Delta x = \frac{MR^2\omega^2 v}{4\mathcal{P}} = \frac{1\,600\text{ kg}(0.65\text{ m})^2(4\,000\cdot 2\pi/60\text{ s})^2 11.1\text{ m/s}}{4(18\cdot 746\text{ W})} = \boxed{24.5\text{ km}}$$

P10.42 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with

$$I_h = \frac{m_h L_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$$

and

$$I_m = \frac{m_m L_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$$

In addition,

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$$

while

$$\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$$

Therefore,

$$K_R = \frac{1}{2}(146)\left(1.45\times10^{-4}\right)^2 + \frac{1}{2}(675)\left(1.75\times10^{-3}\right)^2 = \boxed{1.04\times10^{-3}\text{ J}}$$

P10.43 Work done $= F\Delta r = (5.57\text{ N})(0.800\text{ m}) = 4.46\text{ J}$

and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}\left(4.00\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\omega_f^2$

and from this, $\omega_f = \boxed{149\text{ rad/s}}$.

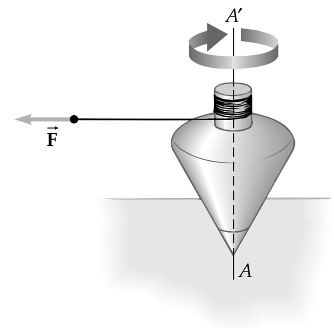

FIG. P10.43

***P10.44** Let T_1 represent the tension in the cord above m_1 and T_2 the tension in the cord above the lighter mass. The two blocks move with the same acceleration because the cord does not stretch, and the angular acceleration of the pulley is a/R For the heavier mass we have

$$\Sigma F = m_1 a \qquad T_1 - m_1 g = m_1(-a) \quad \text{or} \quad -T_1 + m_1 g = m_1 a$$

For the lighter mass,

$$\Sigma F = m_2 a \quad T_2 - m_2 g = m_2 a$$

We assume the pulley is a uniform disk: $I = (1/2)MR^2$

$$\Sigma\tau = I\alpha \qquad +T_1R - T_2R = (1/2)MR^2(a/R) \quad \text{or} \quad T_1 - T_2 = (1/2)Ma$$

Add up the three equations in a

$$-T_1 + m_1 g + T_2 - m_2 g + T_1 - T_2 = m_1 a + m_2 a + (1/2)Ma$$

$$a = (m_1 - m_2)g/[m_1 + m_2 + (1/2)M] = (20 - 12.5)(9.8\text{ m/s}^2)/[20 + 12.5 + 2.5] = 2.1\text{ m/s}^2$$

Next, $x = 0 + 0 + (1/2)\,at^2 \qquad 4.00\text{ m} = (1/2)(2.1\text{ m/s}^2)\,t^2 \qquad t = \boxed{1.95\text{ s}}$

If the pulley were massless, the acceleration would be larger by a factor 35/32.5 and the time shorter by the square root of the factor 32.5/35. That is, the time would be reduced by 3.64%.

P10.45 (a) $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}(0.35\text{ kg})\left[(0.02\text{ m})^2 + (0.03\text{ m})^2\right] = 2.28\times10^{-4}\text{ kg}\cdot\text{m}^2$

$$\left(K_1 + K_2 + K_{\text{rot}} + U_{g2}\right)_i - f_k\Delta x = \left(K_1 + K_2 + K_{\text{rot}}\right)_f$$

$$\frac{1}{2}(0.850\text{ kg})(0.82\text{ m/s})^2 + \frac{1}{2}(0.42\text{ kg})(0.82\text{ m/s})^2 + \frac{1}{2}\left(2.28\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{0.82\text{ m/s}}{0.03\text{ m}}\right)^2$$

$$+0.42\text{ kg}\left(9.8\text{ m/s}^2\right)(0.7\text{ m}) - 0.25(0.85\text{ kg})\left(9.8\text{ m/s}^2\right)(0.7\text{ m})$$

$$= \frac{1}{2}(0.85\text{ kg})v_f^2 + \frac{1}{2}(0.42\text{ kg})v_f^2 + \frac{1}{2}\left(2.28\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{v_f}{0.03\text{ m}}\right)^2$$

$$0.512\text{ J} + 2.88\text{ J} - 1.46\text{ J} = (0.761\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{1.94\text{ J}}{0.761\text{ kg}}} = \boxed{1.59\text{ m/s}}$$

(b) $\omega = \frac{v}{r} = \frac{1.59\text{ m/s}}{0.03\text{ m}} = \boxed{53.1\text{ rad/s}}$

P10.46 We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$I = \frac{1}{3}1.20\text{ kg}(0.240\text{ m})^2 + \frac{2}{5}2.00\text{ kg}\left(4.00\times10^{-2}\text{ m}\right)^2 + 2.00\text{ kg}(0.280\text{ m})^2$$

$$I = 0.181\text{ kg}\cdot\text{m}^2$$

(a) $K_f + U_f = K_i + U_i + \Delta E$

$$\frac{1}{2}I\omega^2 + 0 = 0 + M_{\text{rod}}g\left(\frac{L}{2}\right) + M_{\text{ball}}g(L+R) + 0$$

$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2 = 1.20\text{ kg}\left(9.80\text{ m/s}^2\right)(0.120\text{ m}) + 2.00\text{ kg}\left(9.80\text{ m/s}^2\right)(0.280\text{ m})$$

$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2 = \boxed{6.90\text{ J}}$$

(b) $\omega = \boxed{8.73\text{ rad/s}}$

(c) $v = r\omega = (0.280\text{ m})8.73\text{ rad/s} = \boxed{2.44\text{ m/s}}$

(d) $v_f^2 = v_i^2 + 2a\left(y_f - y_i\right)$

$$v_f = \sqrt{0 + 2\left(9.80\text{ m/s}^2\right)(0.280\text{ m})} = 2.34\text{ m/s}$$

$$\boxed{\text{The speed it attains in swinging is greater by}}\ \frac{2.44}{2.34} = \boxed{1.043\,2\text{ times}}$$

P10.47 (a) For the counterweight,

$\sum F_y = ma_y$ becomes: $\quad 50.0 - T = \left(\frac{50.0}{9.80}\right)a$

For the reel $\sum \tau = I\alpha$ reads

$$TR = I\alpha = I\frac{a}{R}$$

where

$$I = \frac{1}{2}MR^2 = 0.093\,8\text{ kg}\cdot\text{m}^2$$

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4\text{ N}} \quad \text{and}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57\text{ m/s}^2}$$

$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$: $\quad v_f = \sqrt{2(7.57)6.00} = \boxed{9.53\text{ m/s}}$

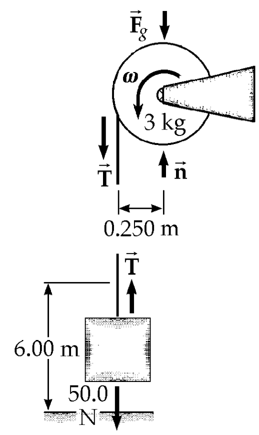

FIG. P10.47

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$(K+U)_i = (K+U)_f$: $\quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + (I/R^2)}} = \sqrt{\frac{2(50.0\text{ N})(6.00\text{ m})}{5.10\text{ kg} + (0.0938/0.25^2)}} = \boxed{9.53\text{ m/s}}$$

The two methods agree on the final speed.

P10.48 The moment of inertia of the cylinder is

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{(91.8\text{ kg}\cdot\text{m}^2)} = 0.817\ \text{rad/s}^2$$

At $t = 3.00$ s, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + (0.817\ \text{rad/s}^2)(3.00\text{ s}) = 2.45\ \text{rad/s}$$

and

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8\text{ kg}\cdot\text{m}^2)(2.45\ \text{rad/s})^2 = \boxed{276\text{ J}}$$

P10.49 From conservation of energy for the object-turntable-cylinder-Earth system,

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

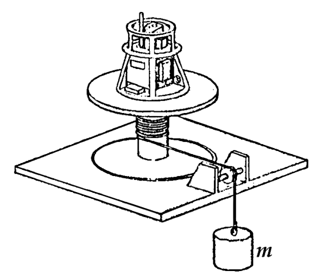

FIG. P10.49

P10.50 (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}0.1\text{ kg}\left((0.015\text{ m})^2 + (0.09\text{ m})^2\right) = 4.16\times10^{-4}\ \text{kg}\cdot\text{m}^2$$

The protruding strand has mass $(10^{-2}\ \text{kg/m})0.16\text{ m} = 1.6\times10^{-3}\ \text{kg}$ and

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = 1.6\times10^{-3}\ \text{kg}\left(\frac{1}{12}(0.16\text{ m})^2 + (0.09\text{ m} + 0.08\text{ m})^2\right)$$

$$= 4.97\times10^{-5}\ \text{kg}\cdot\text{m}^2$$

For the whole cord, $I = 4.66\times10^{-4}\ \text{kg}\cdot\text{m}^2$. In speeding up, the average power is

$$\mathcal{P} = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \frac{4.66\times10^{-4}\ \text{kg}\cdot\text{m}^2}{2(0.215\text{ s})}\left(\frac{2\,500\cdot2\pi}{60\text{ s}}\right)^2 = \boxed{74.3\text{ W}}$$

(b) $$\mathcal{P} = \tau\omega = (7.65\text{ N})(0.16\text{ m} + 0.09\text{ m})\left(\frac{2\,000\cdot2\pi}{60\text{ s}}\right) = \boxed{401\text{ W}}$$

P10.51 (a) Find the velocity of the CM

$$(K+U)_i = (K+U)_f$$

$$0 + mgR = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$

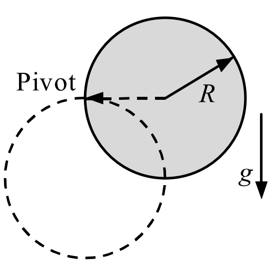

FIG. P10.51

continued on next page

(b) $v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\dfrac{Rg}{3}}}$

(c) $v_{\text{CM}} = \sqrt{\dfrac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$

Section 10.9 Rolling Motion of a Rigid Object

***P10.52** Conservation of energy for the sphere rolling without slipping:

$$U_i = K_{translation,f} + K_{rotation,f}$$

$$mgh = (1/2)\, mv^2 + (1/2)(2/5)mR^2(v/R)^2 = (7/10)mv^2 \quad \boxed{v_f = [10gh/7]^{1/2}}$$

Conservation of energy for the sphere sliding without friction, with $\omega = 0$:

$$mgh = (1/2)\, mv^2 \quad \boxed{v_f = [2gh]^{1/2}}$$

The time intervals required for the trips follow from $x = 0 + v_{avg}t$

$$h/\sin\theta = [(0 + v_f)/2]t \qquad t = 2h/v_f \sin\theta$$

For rolling we have $t = (2h/\sin\theta)(7/10gh)^{1/2}$
and for sliding, $t = (2h/\sin\theta)(1/2gh)^{1/2}$

$\boxed{\text{The time to roll is longer by a factor of } (0.7/0.5)^{1/2} = 1.18}$

P10.53 (a) $K_{\text{trans}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left(\dfrac{1}{2}mr^2\right)\left(\dfrac{v^2}{r^2}\right) = \dfrac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

***P10.54** (a) $\boxed{\text{The cylinder}}$ has extra kinetic energy, so it travels farther up the incline.

(b) Energy conservation for the smooth cube:

$$K_i = U_f \qquad (1/2)\, mv^2 = mgd \sin\theta \qquad d = v^2/2g\sin\theta$$

The same principle for the cylinder:

$$K_{translation,i} + K_{rotation,i} = U_f \qquad (1/2)\, mv^2 + (1/2)[(1/2)mr^2](v/r)^2 = mgd \sin\theta$$

$$d = 3v^2/4g\sin\theta$$

The difference in distance is $3v^2/4g\sin\theta - v^2/2g\sin\theta = \boxed{v^2/4g\sin\theta,}$ or the cylinder travels 50% farther.

(c) The cylinder does not lose mechanical energy because static friction does no work on it. Its rotation means that it has 50% more kinetic energy than the cube at the start, and so it travels 50% farther up the incline.

P10.55 (a) $\tau = I\alpha$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2 \sin\theta}{I_{\text{CM}} + mR^2}$$

$$a_{\text{hoop}} = \frac{mgR^2 \sin\theta}{2mR^2} = \boxed{\frac{1}{2} g\sin\theta}$$

$$a_{\text{disk}} = \frac{mgR^2 \sin\theta}{\frac{3}{2} mR^2} = \boxed{\frac{2}{3} g\sin\theta}$$

$$\boxed{\text{The disk moves with } \frac{4}{3} \text{ the acceleration of the hoop.}}$$

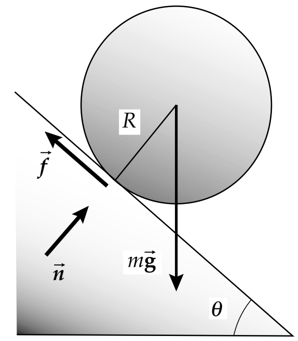

FIG. P10.55

(b) $Rf = I\alpha$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta} = \frac{\left(\frac{2}{3} g\sin\theta\right)\left(\frac{1}{2} mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

P10.56 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping occurs.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore,

$$\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$$

Thus,

$$v^2 = \frac{2gh}{[1 + (I/mR^2)]}$$

For a disk,

$$I = \frac{1}{2}mR^2$$

So

$$v^2 = \frac{2gh}{1 + \frac{1}{2}} \quad \text{or} \quad \boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$$

For a ring,

$$I = mR^2 \text{ so } v^2 = \frac{2gh}{2} \quad \text{or} \quad \boxed{v_{\text{ring}} = \sqrt{gh}}$$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P10.57 $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{3.00 \text{ m}}{1.50 \text{ s}} = 2.00 \text{ m/s} = \dfrac{1}{2}\left(0 + v_f\right)$

$$v_f = 4.00 \text{ m/s and } \omega_f = \frac{v_f}{r} = \frac{4.00 \text{ m/s}}{\left(6.38 \times 10^{-2} \text{ m}\right)/2} = \frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}$$

We ignore internal friction and suppose the can rolls without slipping.

$$\left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_i + \Delta E_{\text{mech}} = \left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_f$$

$$\left(0 + 0 + mgy_i\right) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

$$0.215 \text{ kg}\left(9.80 \text{ m/s}^2\right)\left[(3.00 \text{ m})\sin 25.0°\right] = \frac{1}{2}(0.215 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}\right)^2$$

$$2.67 \text{ J} = 1.72 \text{ J} + \left(7\,860 \text{ s}^{-2}\right)t$$

$$I = \frac{0.951 \text{ kg} \cdot \text{m}^2/\text{s}^2}{7\,860 \text{ s}^{-2}} = \boxed{1.21 \times 10^{-4} \text{ kg} \cdot \text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

P10.58 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

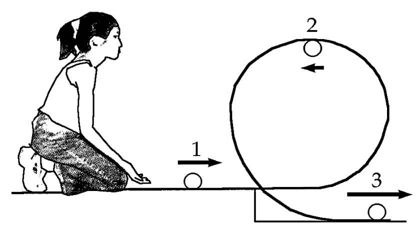

FIG. P10.58

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2$$

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}\left(9.80 \text{ m/s}^2\right)(0.900 \text{ m})} = \boxed{2.38 \text{ m/s}}$$

The centripetal acceleration is $\dfrac{v_2^2}{r} = \dfrac{(2.38 \text{ m/s})^2}{0.450 \text{ m}} = 12.6 \text{ m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}\left(9.80 \text{ m/s}^2\right)(-0.200 \text{ m})} = \boxed{4.31 \text{ m/s}}$$

(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03 \text{ m/s})^2 - 2\left(9.80 \text{ m/s}^2\right)(0.900 \text{ m})} = \sqrt{-1.40 \text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

Additional Problems

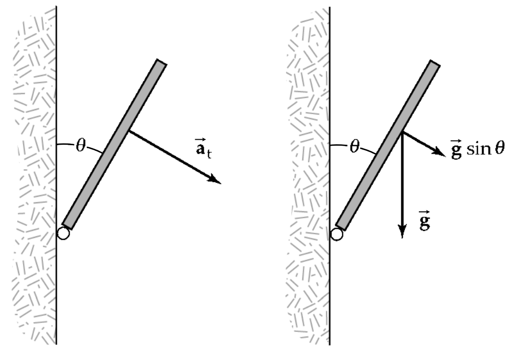

FIG. P10.59

P10.59 $mg\dfrac{\ell}{2}\sin\theta = \dfrac{1}{3}m\ell^2\alpha$

$$\alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta$$

$$a_t = \left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

Then $\left(\dfrac{3}{2}\dfrac{g}{\ell}\right)r > g\sin\theta$

for $r > \dfrac{2}{3}\ell$

$\therefore$ About $\boxed{\dfrac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

P10.60 The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $\mathcal{P} = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$\mathcal{P} = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3 DA\omega^3\right)\rho$$

With

$$\omega = \frac{2\pi\text{ rad}}{1\text{ rev}}\left(\frac{10^3\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \frac{1\,000\pi}{30.0}\text{ rad/s}$$

$$\mathcal{P} = 3(0.100\text{ m})^3(0.600)\left(4.00\times10^{-4}\text{ m}^2\right)\left(\frac{1\,000\pi}{30.0\text{ s}}\right)^3\rho$$

or

$\mathcal{P} = \left(0.827\text{ m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20\text{ kg/m}^3$,

and $\mathcal{P} = 0.827\text{ m}^5/\text{s}^3\left(1.20\text{ kg/m}^3\right) = 0.992\text{ N}\cdot\text{m/s} = \boxed{0.992\text{ W}}$

(b) In water, $\rho = 1\,000\text{ kg/m}^3$ and $\mathcal{P} = \boxed{827\text{ W}}$.

P10.61 (a) $W = \Delta K = \dfrac{1}{2}I\omega_f^2 - \dfrac{1}{2}I\omega_i^2 = \dfrac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)$ where $I = \dfrac{1}{2}mR^2$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2 - 0\right] = \boxed{4.00\text{ J}}$$

(b) $t = \dfrac{\omega_f - 0}{\alpha} = \dfrac{\omega r}{a} = \dfrac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2} = \boxed{1.60\text{ s}}$

(c) $\theta_f = \theta_i + \omega_i t + \dfrac{1}{2}\alpha t^2;\ \theta_i = 0;\ \omega_i = 0$

$$\theta_f = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2 = 6.40\text{ rad}$$

$$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = \boxed{3.20\text{ m} < 4.00\text{ m Yes.}}$$

***P10.62** (a) We consider the elevator-sheave-counterweight-Earth system, including n passengers, as an isolated system and apply the conservation of mechanical energy. We take the initial configuration, at the moment the drive mechanism switches off, as representing zero gravitational potential energy of the system.
Therefore, the initial mechanical energy of the system is

$$E_i = K_i + U_i = (1/2)\, m_e v^2 + (1/2)\, m_c v^2 + (1/2) I_s \omega^2$$

$$= (1/2)\, m_e v^2 + (1/2)\, m_c v^2 + (1/2)[(1/2) m_s r^2](v/r)^2$$

$$= (1/2)\, [m_e + m_c v^2 + (1/2) m_s]\, v^2$$

The final mechanical energy of the system is entirely gravitational because the system is momentarily at rest:

$$E_f = K_f + U_f = 0 + m_e g d - m_c g d$$

where we have recognized that the elevator car goes up by the same distance d that the counterweight goes down. Setting the initial and final energies of the system equal to each other, we have

$$(1/2)\, [m_e + m_c + (1/2) m_s]\, v^2 = (m_e - m_c)\, g d$$

$$(1/2)\, [800 \text{ kg} + n\, 80 \text{ kg} + 950 \text{ kg} + 140 \text{ kg}](3 \text{ m/s})^2 = (800 \text{ kg} + n\, 80 \text{ kg} - 950 \text{ kg})(9.8 \text{ m/s}^2)\, d$$

$$\boxed{d = [1890 + 80n](0.459 \text{ m})/(80n - 150)}$$

(b) $d = [1890 + 80 \times 2](0.459 \text{ m})/(80 \times 2 - 150) = \boxed{94.1 \text{ m}}$

(c) $d = [1890 + 80 \times 12](0.459 \text{ m})/(80 \times 12 - 150) = \boxed{1.62 \text{ m}}$

(d) $d = [1890 + 80 \times 0](0.459 \text{ m})/(80 \times 0 - 150) = \boxed{-5.79 \text{ m}}$

(e) The rising car will coast to a stop only for $n \geq 2$. For $n = 0$ or $n = 1$, the car would accelerate upward if released.

(f) The graph looks roughly like one branch of a hyperbola. It comes down steeply from 94.1 m for $n = 2$, flattens out, and very slowly approaches 0.459 m as n becomes large.

(g) The radius of the sheave is not necessary. It divides out in the expression $(1/2)I\omega^2 = (1/4)m_{\text{sheave}}\, v^2$.

(h) In this problem, as often in everyday life, energy conservation refers to minimizing use of electric energy or fuel. In physical theory, energy conservation refers to the constancy of the total energy of an isolated system, without regard to the different prices of energy in different forms.

(i) The result of applying $\Sigma F = ma$ and $\Sigma\tau = I\alpha$ to elevator car, counterweight, and sheave, and adding up the resulting equations is

$$(800 \text{ kg} + n\, 80 \text{ kg} - 950 \text{ kg})(9.8 \text{ m/s}^2) = [800 \text{ kg} + n\, 80 \text{ kg} + 950 \text{ kg} + 140 \text{ kg}]a$$

$\boxed{a = (9.80 \text{ m/s}^2)(80n - 150)/(1\,890 + 80n)}$ downward

***P10.63** (a) We model the assembly as a rigid body in equilibrium. Two torques acting on it are the frictional torque and the driving torque due to the emitted water:

$$\Sigma\tau = \tau_{thrust} - \tau_{friction} = 0 \qquad 3F\ell - b\omega = 0 \qquad \boxed{\omega = 3F\ell/b}$$

Notice that we have included a driving torque *only* from the single holes at distance ℓ. Because of the third assumption, the radially-directed water from the ends exerts no torque on the assembly—its thrust force is along the radial direction.

(b) We model the assembly as a rigid body under a net torque. Because the assembly begins from rest, there is no frictional torque at the beginning. Therefore,

$$\Sigma\tau = \tau_{thrust} = I\alpha \qquad 3F\ell = 3[mL^2/3]\alpha \qquad \alpha = \boxed{3F\ell/mL^2}$$

(c) The constant angular speed with which the assembly rotates will be larger. The arms are bent in the same direction as that in which the water is emitted from the holes at distance ℓ. This water will exert a force on the arms like that of a rocket exhaust. The driving torque from the water emitted from the ends will add to that from the single holes and the total driving torque will be larger. This will result in a larger angular speed.

(d) The bending of the arms has two effects on the initial angular acceleration. The driving torque is increased, as discussed in part (c). In addition, because the arms are bent, the moment of inertia of each arm is smaller than that for a straight arm. Looking at the answer to part (b), we see that both of these effects cause an increase in α, so the initial angular acceleration will be larger.

P10.64 $\alpha = -10.0\ \text{rad/s}^2 - (5.00\ \text{rad/s}^3)t = \dfrac{d\omega}{dt}$

$$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]\,dt = -10.0t - 2.50t^2 = \omega - 65.0\ \text{rad/s}$$

$$\omega = \frac{d\theta}{dt} = 65.0\ \text{rad/s} - (10.0\ \text{rad/s}^2)t - (2.50\ \text{rad/s}^3)t^2$$

(a) At $t = 3.00$ s,

$$\omega = 65.0\ \text{rad/s} - (10.0\ \text{rad/s}^2)(3.00\ \text{s}) - (2.50\ \text{rad/s}^3)(9.00\ \text{s}^2) = \boxed{12.5\ \text{rad/s}}$$

(b) $\displaystyle\int_0^{\theta} d\theta = \int_0^t \omega\,dt = \int_0^t \left[65.0\ \text{rad/s} - (10.0\ \text{rad/s}^2)t - (2.50\ \text{rad/s}^3)t^2\right]dt$

$$\theta = (65.0\ \text{rad/s})t - (5.00\ \text{rad/s}^2)t^2 - (0.833\ \text{rad/s}^3)t^3$$

At $t = 3.00$ s,

$$\theta = (65.0\ \text{rad/s})(3.00\ \text{s}) - (5.00\ \text{rad/s}^2)9.00\ \text{s}^2 - (0.833\ \text{rad/s}^3)27.0\ \text{s}^3$$

$$\theta = \boxed{128\ \text{rad}}$$

P10.65 (a) Since only conservative forces act within the system of the rod and the Earth,

$$\Delta E = 0 \qquad \text{so} \qquad K_f + U_f = K_i + U_i$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{L}{2}\right)$$

where

$$I = \frac{1}{3}ML^2$$

Therefore,

$$\omega = \boxed{\sqrt{\frac{3g}{L}}}$$

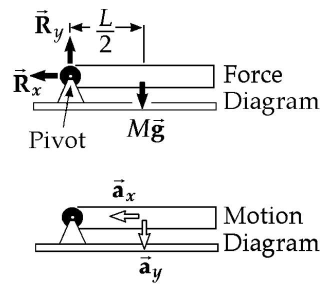

FIG. P10.65

(b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\alpha$$

$$\alpha = \boxed{\frac{3g}{2L}}$$

(c) $a_x = a_r = -r\omega^2 = -\left(\frac{L}{2}\right)\omega^2 = \boxed{-\frac{3g}{2}} \qquad a_y = -a_t = -r\alpha = -\alpha\left(\frac{L}{2}\right) = \boxed{-\frac{3g}{4}}$

(d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$$R_y - Mg = Ma_y = -\frac{3Mg}{4} \qquad R_y = \boxed{\frac{Mg}{4}}$$

P10.66 $K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$: $U_f = Mgh_f = 0$; $K_i = \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = 0$

$U_i = (Mgh)_i$: $f = \mu N = \mu Mg\cos\theta$; $\omega = \frac{v}{r}$; $h = d\sin\theta$ and $I = \frac{1}{2}mr^2$

(a) $\Delta E = E_f - E_i$ or $-fd = K_f + U_f - K_i - U_i$

$$-fd = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 - Mgh$$

$$-(\mu Mg\cos\theta)d = \frac{1}{2}Mv^2 + \left(\frac{mr^2}{2}\right)\frac{v^2/r^2}{2} - Mgd\sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd\sin\theta - (\mu Mg\cos\theta)d \text{ or}$$

$$v^2 = 2Mgd\frac{(\sin\theta - \mu\cos\theta)}{m/2 + M}$$

$$v_d = \left[4gd\frac{M}{(m+2M)}(\sin\theta - \mu\cos\theta)\right]^{1/2}$$

(b) $v_f^2 = v_i^2 + 2a\Delta x$, $v_d^2 = 2ad$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m+2M}\right)(\sin\theta - \mu\cos\theta)}$$

P10.67 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.540\ \text{m})} = 3.25\ \text{m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.510\ \text{m})} = 3.16\ \text{m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\ \text{m/s}}{0.381\ \text{m}} = 8.53\ \text{rad/s} \text{ and } \omega_2 = \frac{v_2}{r} = \frac{3.16\ \text{m/s}}{0.381\ \text{m}} = 8.29\ \text{rad/s}$$

or

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29\ \text{rad/s})^2 - (8.53\ \text{rad/s})^2}{4\pi} = \boxed{-0.322\ \text{rad/s}^2}$$

P10.68 At the instant it comes off the wheel, the first drop has a velocity v_1 directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{2gh_2/R^2 - 2gh_1/R^2}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}$$

P10.69 τ_f will oppose the torque due to the hanging object:

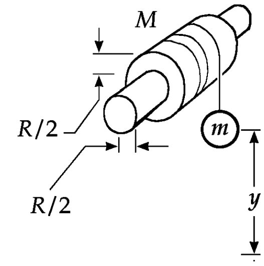

FIG. P10.69

$$\sum \tau = I\alpha = TR - \tau_f: \qquad \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g-a) \qquad (2)$$

also

$$\Delta y = v_i t + \frac{at^2}{2} \qquad a = \frac{2y}{t^2} \qquad (3)$$

and

$$\alpha = \frac{a}{R} = \frac{2y}{Rt^2} \qquad (4)$$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

Substituting (2), (3), (4), and (5) into (1), we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

P10.70 (a) $E = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\omega^2\right)$

$$E = \frac{1}{2}\cdot\frac{2}{5}\left(5.98\times10^{24}\right)\left(6.37\times10^{6}\right)^2\left(\frac{2\pi}{86\,400}\right)^2 = \boxed{2.57\times10^{29}\ \text{J}}$$

(b) $$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$$

$$= \frac{1}{5}MR^2(2\pi)^2\left(-2T^{-3}\right)\frac{dT}{dt}$$

$$= \frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$

$$= \left(2.57\times10^{29}\ \text{J}\right)\left(\frac{-2}{86\,400\ \text{s}}\right)\left(\frac{10\times10^{-6}\ \text{s}}{3.16\times10^{7}\ \text{s}}\right)\left(86\,400\ \text{s/day}\right)$$

$$\frac{dE}{dt} = \boxed{-1.63\times10^{17}\ \text{J/day}}$$

P10.71 (a) $m_2 g - T_2 = m_2 a$

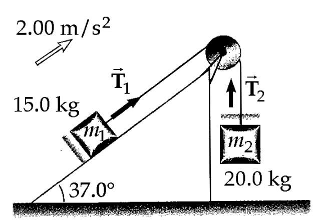

FIG. P10.71

$$T_2 = m_2(g-a) = 20.0\ \text{kg}\left(9.80\ \text{m/s}^2 - 2.00\ \text{m/s}^2\right) = \boxed{156\ \text{N}}$$

$$T_1 - m_1 g\sin 37.0° = m_1 a$$

$$T_1 = (15.0\ \text{kg})(9.80\sin 37.0° + 2.00)\ \text{m/s}^2 = \boxed{118\ \text{N}}$$

(b) $$\left(T_2 - T_1\right)R = I\alpha = I\left(\frac{a}{R}\right)$$

$$I = \frac{\left(T_2 - T_1\right)R^2}{a} = \frac{(156\ \text{N} - 118\ \text{N})(0.250\ \text{m})^2}{2.00\ \text{m/s}^2} = \boxed{1.17\ \text{kg}\cdot\text{m}^2}$$

P10.72 (a) $W = \Delta K + \Delta U$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

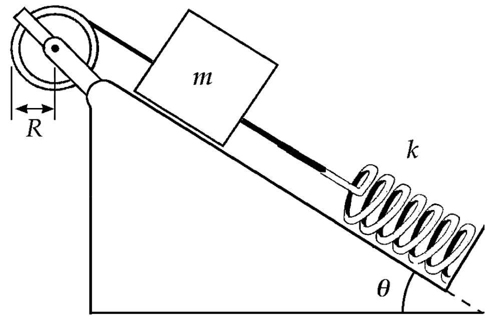

FIG. P10.72

(b) $$\omega = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})(\sin 37.0°) + 50.0\text{ N/m}(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + 0.500\text{ kg}(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

P10.73 At $t = 0, \omega = 3.50\text{ rad/s} = \omega_0 e^0$. Thus, $\omega_0 = 3.50\text{ rad/s}$

At $t = 9.30\text{ s}, \omega = 2.00\text{ rad/s} = \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02 \times 10^{-2}\text{ s}^{-1}$

(a) $$\alpha = \frac{d\omega}{dt} = \frac{d\left(\omega_0 e^{-\sigma t}\right)}{dt} = \omega_0(-\sigma)e^{-\sigma t}$$

At $t = 3.00$ s,

$$\alpha = (3.50\text{ rad/s})\left(-6.02 \times 10^{-2}\text{ s}^{-1}\right)e^{-3.00\left(6.02\times10^{-2}\right)} = \boxed{-0.176\text{ rad/s}^2}$$

(b) $$\theta = \int_0^t \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}\left[e^{-\sigma t} - 1\right] = \frac{\omega_0}{\sigma}\left[1 - e^{-\sigma t}\right]$$

At $t = 2.50$ s,

$$\theta = \frac{3.50\text{ rad/s}}{\left(6.02\times10^{-2}\right)1/\text{s}}\left[1 - e^{-\left(6.02\times10^{-2}\right)(2.50)}\right] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

(c) As $t \to \infty, \theta \to \dfrac{\omega_0}{\sigma}\left(1 - e^{-\infty}\right) = \dfrac{3.50\text{ rad/s}}{6.02\times10^{-2}\text{ s}^{-1}} = 58.2\text{ rad} = \boxed{9.26\text{ rev}}$

P10.74 For the board just starting to move,

$$\sum\tau = I\alpha: \qquad mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

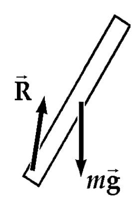

FIG. P10.74

The tangential acceleration of the end is $\quad a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

The vertical component is $\quad a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball falling:

(a) $\frac{3}{2}g\cos^2\theta \geq g$ gives $\cos^2\theta \geq \frac{2}{3}$ so $\cos\theta \geq \sqrt{\frac{2}{3}}$ and $\boxed{\theta \leq 35.3°}$

continued on next page

(b) When $\theta = 35.3°$, the cup will land underneath the release-point of the ball if $r_c = \ell \cos\theta$

When $\ell = 1.00$ m, and $\theta = 35.3°$ $\quad r_c = 1.00\text{ m}\sqrt{\frac{2}{3}} = 0.816\text{ m}$

so the cup should be $(1.00\text{ m} - 0.816\text{ m}) = \boxed{0.184\text{ m from the moving end}}$.

P10.75 (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\frac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is $\Delta x = (v_2 - v_1)t = \omega h\sqrt{\frac{2h}{g}} = \boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$.

(b) $\dfrac{2\pi\text{ rad}}{86\,400\text{ s}}(50\text{ m})^{3/2}\left(\dfrac{2\text{ s}^2}{9.8\text{ m}}\right)^{1/2} = \boxed{1.16\text{ cm}}$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

P10.76 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t$$

where

$$\omega_h = \frac{\pi}{6}\text{ rad/h}$$

and

$$\theta_m = \omega_m t$$

where

$$\omega_m = 2\pi\text{ rad/h}$$

Therefore,

$$\tau = -4.90\text{ m/s}^2\left[60.0\text{ kg}(2.70\text{ m})\sin\left(\frac{\pi t}{6}\right) + 100\text{ kg}(4.50\text{ m})\sin 2\pi t\right]$$

or

$$\tau = -794\text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right], \text{ where } t \text{ is in hours.}$$

(i) (a) At 3:00, $t = 3.00$ h,

so $\quad \tau = -794\text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794\text{ N}\cdot\text{m}}$

(b) At 5:15, $t = 5\text{ h} + \frac{15}{60}\text{ h} = 5.25$ h, and substitution gives:

$$\tau = \boxed{-2\,510\text{ N}\cdot\text{m}}$$

continued on next page

(c) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(d) At 8:20, $\tau = \boxed{-1160\ \text{N}\cdot\text{m}}$

(e) At 9:45, $\tau = \boxed{-2\,940\ \text{N}\cdot\text{m}}$

(ii) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5,…, corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

P10.77 $\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

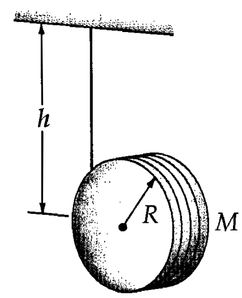

FIG. P10.77

(a) Combining the above two equations we find

$$T = M(g - a)$$

and

$$a = \frac{2T}{M}$$

thus

$$T = \boxed{\frac{Mg}{3}}$$

(b) $a = \dfrac{2T}{M} = \dfrac{2}{M}\left(\dfrac{Mg}{3}\right) = \boxed{\dfrac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$ $\quad v_f^2 = 0 + 2\left(\dfrac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$$U_{gi} + K_{\text{rot}\,i} + K_{\text{trans}\,i} = U_{gf} + K_{\text{rot}\,f} + K_{\text{trans}\,f}: \qquad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^{\,2}$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

P10.78 Energy is conserved so $\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$

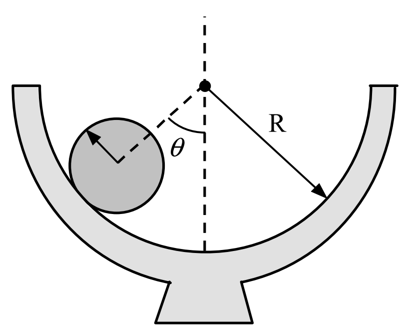

FIG. P10.78

$$mg(R - r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Since $r\omega = v$, this gives

$$\omega = \sqrt{\frac{10}{7}\frac{(R - r)(1 - \cos\theta)g}{r^2}}$$

or

$$\omega = \boxed{\sqrt{\frac{10Rg(1 - \cos\theta)}{7r^2}}} \quad \text{since } R >> r$$

P10.79 (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$

Note that initially the center of mass of the sphere is a distance $h + r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R - r$. The conservation of energy requirement gives

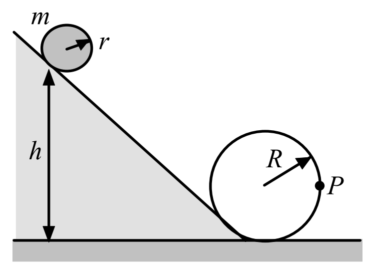

FIG. P10.79

$$mg(h+r) = mg(2R-r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{\min}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \text{ or } v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{\min} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{\min} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point P, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or } v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point P, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{(10/7)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \text{ (since } R >> r\text{)}$$

P10.80 Consider the free-body diagram shown. The sum of torques about the chosen pivot is

$$\sum \tau = I\alpha \Rightarrow F\ell = \left(\frac{1}{3}ml^2\right)\left(\frac{a_{\text{CM}}}{\frac{l}{2}}\right) = \left(\frac{2}{3}ml\right)a_{\text{CM}} \qquad (1)$$

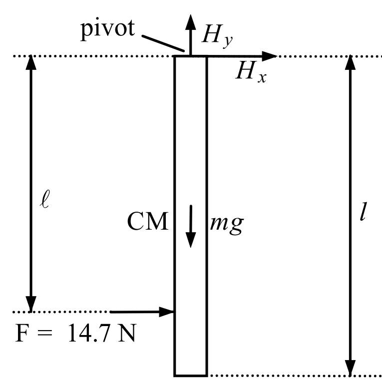

FIG. P10.80

(a) $\ell = l = 1.24$ m: In this case, Equation (1) becomes

$$a_{\text{CM}} = \frac{3F}{2m} = \frac{3(14.7 \text{ N})}{2(0.630 \text{ kg})} = \boxed{35.0 \text{ m/s}^2}$$

$$\sum F_x = ma_{\text{CM}} \Rightarrow F + H_x = ma_{\text{CM}} \text{ or } H_x = ma_{\text{CM}} - F$$

Thus,

$$H_x = (0.630 \text{ kg})(35.0 \text{ m/s}^2) - 14.7 \text{ N} = +7.35 \text{ N}$$

or

$$\vec{\mathbf{H}}_x = \boxed{7.35\hat{\mathbf{i}} \text{ N}}$$

(b) $\ell = \frac{1}{2} = 0.620$ m: For this situation, Equation (1) yields

$$a_{CM} = \frac{3F}{4m} = \frac{3(14.7\text{ N})}{4(0.630\text{ kg})} = \boxed{17.5\text{ m/s}^2}$$

Again, $\sum F_x = ma_{CM} \Rightarrow H_x = ma_{CM} - F$, so

$$H_x = (0.630\text{ kg})(17.5\text{ m/s}^2) - 14.7\text{ N} = -3.68\text{ N} \text{ or } \vec{\mathbf{H}}_x = \boxed{-3.68\hat{\mathbf{i}}\text{ N}}$$

(c) If $H_x = 0$, then $\sum F_x = ma_{CM} \Rightarrow F = ma_{CM}$, or $a_{CM} = \frac{F}{m}$.

Thus, Equation (1) becomes

$$F\ell = \left(\frac{2}{3}ml\right)\left(\frac{F}{m}\right) \text{ so } \ell = \frac{2}{3}l = \frac{2}{3}(1.24\text{ m}) = \boxed{0.827\text{ m (from the top)}}$$

P10.81 (a) There are not any horizontal forces acting on the rod, so the center of mass will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{CM}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{CM} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{CM}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{CM} = \boxed{\sqrt{\frac{3gh}{4}}}$$

P10.82 Conservation of energy between apex and the point where the grape leaves the surface:

$$mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives $g(1-\cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right)$ (1)

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{mv_f^2}{R}$$

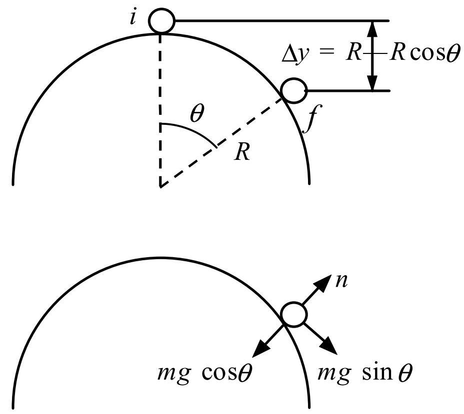

FIG. P10.82

At the point where the grape leaves the surface, $n \to 0$.

Thus, $mg\cos\theta = \frac{mv_f^2}{R}$ or $\frac{v_f^2}{R} = g\cos\theta$

Substituting this into Equation (1) gives

$g - g\cos\theta = \frac{7}{10}g\cos\theta$ or $\cos\theta = \frac{10}{17}$ and $\theta = \boxed{54.0°}$

P10.83 (a) $\sum F_x = F + f = Ma_{\text{CM}}$

$$\sum \tau = FR - fR = I\alpha$$

$$FR - \left(Ma_{\text{CM}} - F\right)R = \frac{Ia_{\text{CM}}}{R} \qquad \boxed{a_{\text{CM}} = \frac{4F}{3M}}$$

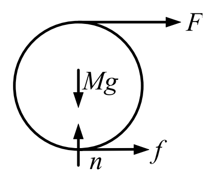

FIG. P10.83

(b) $f = Ma_{\text{CM}} - F = M\left(\frac{4F}{3M}\right) - F = \boxed{\frac{1}{3}F}$

(c) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$

$$v_f = \boxed{\sqrt{\frac{8Fd}{3M}}}$$

P10.84 Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

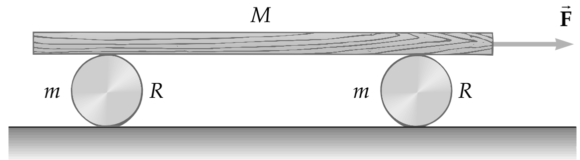

FIG. P10.84

For the plank,

$$\sum F_x = ma_x \qquad 6.00\ \text{N} - 2f_t = (6.00\ \text{kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00\ \text{cm})} = \frac{a_p}{(0.100\ \text{m})}$$

Then for each,

$$\sum F_x = ma_x \qquad +f_t - f_b = (2.00\ \text{kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \qquad f_t(5.00\ \text{cm}) + f_b(5.00\ \text{cm}) = \frac{1}{2}(2.00\ \text{kg})(5.00\ \text{cm})^2\frac{a_p}{10.0\ \text{cm}}$$

So

$$f_t + f_b = \left(\frac{1}{2}\ \text{kg}\right)a_p$$

Add to eliminate f_b:

$$2f_t = (1.50\ \text{kg})a_p$$

(a) And $6.00\ \text{N} - (1.50\ \text{kg})a_p = (6.00\ \text{kg})a_p$

$$a_p = \frac{(6.00\ \text{N})}{(7.50\ \text{kg})} = \boxed{0.800\ \text{m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400\ \text{m/s}^2}$

(b) Substituting back, $2f_t = (1.50\ \text{kg})0.800\ \text{m/s}^2$

$f_t = \boxed{0.600\ \text{N}}$

$0.600\ \text{N} + f_b = \frac{1}{2}\ \text{kg}\left(0.800\ \text{m/s}^2\right)$

$f_b = -0.200\ \text{N}$

The negative sign means that the horizontal force of ground on each roller is $\boxed{\text{0.200 N forward}}$ rather than backward as we assumed.

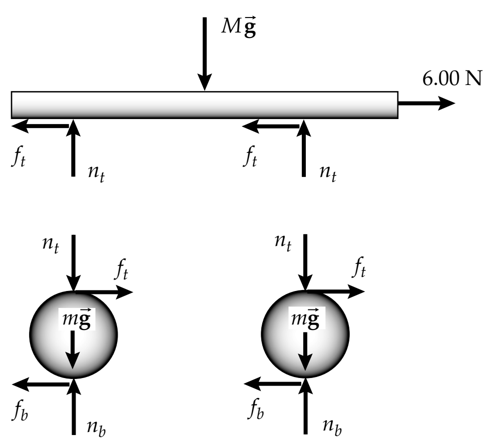

FIG. P10.84(b)

P10.85 $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \dfrac{a}{R_2}$. By substitution,

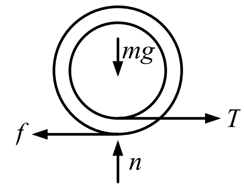

FIG. P10.85

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$

$$fR_2^2 m - TR_1 R_2 m = IT - If$$

$$f\left(I + mR_2^2\right) = T\left(I + mR_1 R_2\right)$$

$$f = \left(\frac{I + mR_1 R_2}{I + mR_2^2}\right)T$$

Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left}}$.

P10.86 (a) The mass of the roll decreases as it unrolls. We have $m = \dfrac{Mr^2}{R^2}$ where M is the initial mass of the roll. Since $\Delta E = 0$, we then have $\Delta U_g + \Delta K_{\text{trans}} + \Delta K_{\text{rot}} = 0$. Thus, when $I = \dfrac{mr^2}{2}$,

$$\left(mgr - MgR\right) + \frac{mv^2}{2} + \left[\frac{mr^2}{2}\frac{\omega^2}{2}\right] = 0$$

Since $\omega r = v$, this becomes $v = \boxed{\sqrt{\dfrac{4g\left(R^3 - r^3\right)}{3r^2}}}$

(b) Using the given data, we find $v = \boxed{5.31 \times 10^4 \text{ m/s}}$

(c) We have assumed that $\Delta E = 0$. When the roll gets to the end, we will have an inelastic collision with the surface. $\boxed{\text{The energy goes into internal energy.}}$ With the assumption we made, there are problems with this question. It would take an infinite time to unwrap the tissue since $dr \to 0$. Also, as r approaches zero, the velocity of the center of mass approaches infinity, which is physically impossible.

ANSWERS TO EVEN PROBLEMS

P10.2 144 rad

P10.4 -226 rad/s^2

P10.6 13.7 rad/s^2

P10.8 (a) 2.88 s (b) 12.8 s

P10.10 (a) 0.180 rad/s (b) 8.10 m/s^2 toward the center of the track

P10.12 (a) 0.605 m/s (b) 17.3 rad/s (c) 5.82 m/s (d) the crank length is unnecessary

P10.14 (a) 25.0 rad/s (b) 39.8 rad/s^2 (c) 0.628 s

P10.16 (a) 54.3 rev (b) 12.1 rev/s

P10.18 (a) 5.77 cm (b) Yes. The ladder undergoes pure rotation about its right foot, with its angular displacement given in radians by $\theta = 0.690 \text{ m}/4.90 \text{ m} = t/0.410 \text{ m}$.

P10.20 (c) $\theta = \dfrac{2\pi r_i}{h}\left(\sqrt{1+\dfrac{vh}{\pi r_i^2}t}-1\right)$ (d) $\alpha = -\dfrac{hv^2}{2\pi r_i^3\left(1+\dfrac{vh}{\pi r_i^2}t\right)^{3/2}}$

P10.22 (a) $92.0\ \text{kg}\cdot\text{m}^2$; 184 J (b) 6.00 m/s; 4.00 m/s; 8.00 m/s; 184 J (c) The kinetic energies computed in parts (a) and (b) are the same. Rotational kinetic energy can be viewed as the total translational kinetic energy of the particles in the rotating object.

P10.24 The flywheel can be shaped like a cup or open barrel, 9.00 cm in outer radius and 7.68 cm in inner radius, with its wall 6 cm high, and with its bottom forming a disk 2.00 cm thick and 9.00 cm in radius. It is mounted to the crankshaft at the center of this disk and turns about its axis of symmetry. Its mass is 7.27 kg. If the disk were made somewhat thinner and the barrel wall thicker, the mass could be smaller.

P10.26 $11mL^2/12$

P10.28 $5.80\ \text{kg}\cdot\text{m}^2$ The height of the door is unnecessary.

P10.30 $23MR^2\omega^2/48$

P10.32 $168\ \text{N}\cdot\text{m}$ clockwise

P10.34 (a) 1.03 s (b) 10.3 rev

P10.36 (a) $21.6\ \text{kg}\cdot\text{m}^2$ (b) $3.60\ \text{N}\cdot\text{m}$ (c) 52.4 rev

P10.38 0.312

P10.40 25.1 N and 1.00 m or 41.8 N and 0.600 m; infinitely many answers exist, such that $TR = 25.1\ \text{N}\cdot\text{m}$

P10.42 1.04×10^{-3} J

P10.44 1.95 s If the pulley were massless, the time would be reduced by 3.64%

P10.46 (a) 6.90 J (b) 8.73 rad/s (c) 2.44 m/s (d) 1.043 2 times larger

P10.48 276 J

P10.50 (a) 74.3 W (b) 401 W

P10.52 (a) $v_f = [10gh/7]^{1/2}$ (b) $v_f = [2gh]^{1/2}$ (c) The time to roll is longer by a factor of 1.18

P10.54 (a) The cylinder (b) $v^2/4g\sin\theta$ (c) The cylinder does not lose mechanical energy because static friction does no work on it. Its rotation means that it has 50% more kinetic energy than the cube at the start, and so it travels 50% farther up the incline.

P10.56 The disk; $\sqrt{\dfrac{4gh}{3}}$ versus $\sqrt{gh}$

P10.58 (a) 2.38 m/s (b) 4.31 m/s (c) It will not reach the top of the loop.

P10.60 (a) 0.992 W (b) 827 W

P10.62 (a) $(1\,890 + 80n)0.459$ m/$(80n - 150)$ (b) 94.1 m (c) 1.62 m (d) −5.79 m (e) The rising car will coast to a stop only for $n \geq 2$. For $n = 0$ or $n = 1$, the car would accelerate upward if released. (f) The graph looks roughly like one branch of a hyperbola. It comes down steeply from 94.1 m for $n = 2$, flattens out, and very slowly approaches 0.459 m as n becomes large. (g) The radius of the sheave is not necessary. It divides out in the expression $(1/2)I\omega^2 = (1/4)m_{\text{sheave}}v^2$. (h) In this problem, as often in everyday life, energy conservation refers to minimizing use of electric energy or fuel. In physical theory, energy conservation refers to the constancy of the total energy of an isolated system, without regard to the different prices of energy in different forms.
(i) $(9.80\text{ m/s}^2)(80n - 150)/(1\,890 + 80n)$

P10.64 (a) 12.5 rad/s (b) 128 rad

P10.66 (a) see the solution (b) $a = 2Mg(\sin\theta - \mu\cos\theta)/(m + 2M)$

P10.68 $\dfrac{g(h_2 - h_1)}{2\pi R^2}$

P10.70 (a) 2.57×10^{29} J (b) -1.63×10^{17} J/day

P10.72 (a) $\sqrt{\dfrac{2mgd\sin\theta + kd^2}{I + mR^2}}$ (b) 1.74 rad/s

P10.74 see the solution

P10.76 (i) $-794\text{ N}\cdot\text{m}$; $-2\,510\text{ N}\cdot\text{m}$; 0; $-1\,160\text{ N}\cdot\text{m}$; $-2\,940\text{ N}\cdot\text{m}$ (ii) see the solution

P10.78 $\sqrt{\dfrac{10Rg(1-\cos\theta)}{7r^2}}$

P10.80 (a) 35.0 m/s^2; $7.35\hat{\mathbf{i}}$ N (b) 17.5 m/s^2; $-3.68\hat{\mathbf{i}}$ N (c) At 0.827 m from the top.

P10.82 54.0°

P10.84 (a) 0.800 m/s^2; 0.400 m/s^2 (b) 0.600 N between each cylinder and the plank; 0.200 N forward on each cylinder by the ground

P10.86 (a) $\sqrt{\dfrac{4g(R^3 - r^3)}{3r^2}}$ (b) 5.31×10^4 m/s (c) It becomes internal energy.