7

Energy of a System

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q7.1 (a) Positive work is done by the chicken on the dirt.

(b) The person does no work on anything in the environment. Perhaps some extra chemical energy goes through being energy transmitted electrically and is converted into internal energy in his brain; but it would be very hard to quantify "extra."

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

Q7.2 Force of tension on a ball moving in a circle on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q7.3 (a) Tension (b) Air resistance (c) The gravitational force does positive work in increasing speed on the downswing. It does negative work in decreasing speed on the upswing.

***Q7.4** Each dot product has magnitude $1 \cdot 1 \cdot \cos\theta$ where θ is the angle between the two factors. Thus for (a) and (f) we have cos 0 = 1. For (b) and (g), cos 45° = 0.707. For (c) and (h), cos 180° = −1. For (d) and (e), cos 90° = 0. The assembled answer is a = f > b = g > d = e > c = h.

Q7.5 The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90° < \theta < 180°$.

***Q7.6** (i) The force of block on spring is equal in magnitude and opposite to the force of spring on block. The answers are (c) and (e).
(ii) The spring tension exerts equal-magnitude forces toward the center of the spring on objects at both ends. The answers are (c) and (e).

Q7.7 $k' = 2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

Q7.8 No. Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q7.9 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release. He extends this distance by taking a step forward.

***Q7.10** answer (e). Kinetic energy is proportional to mass.

***Q7.11** answer (a). Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

***Q7.12** It is sometimes true. If the object is a particle initially at rest, the net work done on the object is equal to its final kinetic energy. If the object is not a particle, the work could go into (or come out of) some other form of energy. If the object is initially moving, its initial kinetic energy must be added to the total work to find the final kinetic energy.

***Q7.13** Yes. The floor of a rising elevator does work on a passenger. A normal force exerted by a stationary solid surface does no work.

***Q7.14** answer (c). If the total work on an object is zero in some process, its kinetic energy and so its speed must be the same at the final point as it was at the initial point.

***Q7.15** The cart's fixed kinetic energy means that it can do a fixed amount of work in stopping, namely (6 N)(6 cm) = 0.36 J. The forward force it exerts and the distance it moves in stopping must have this fixed product. answers: (i) c (ii) a (iii) d

Q7.16 As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

***Q7.17** answer (e). $4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$

Therefore $k = 800$ N/m and to stretch the spring to 0.200 m requires extra work

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = \boxed{12.0\text{ J}}$$

Q7.18 (a) Not necessarily. It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

***Q7.19** (i) The gravitational acceleration is quite precisely constant at locations separated by much less than the radius of the planet. Answer: a = b = c = d

(ii) The mass but not the elevation affects the gravitational force. Answer: c = d > a = b

(iii) Now think about the product of mass times height. Answer: c > b = d > a

Q7.20 There is no violation. Choose the book as the system. You did work and the Earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q7.21 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q7.22 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

***Q7.23** (c) The ice cube is in neutral equilibrium. Its zero acceleration is evidence for equilibrium.

***Q7.24** The gravitational energy of the key-Earth system is lowest when the key is on the floor letter-side-down. The average height of particles in the key is lowest in that configuration. As described by $F = -dU/dx$, a force pushes the key downhill in potential energy toward the bottom of a graph of potential energy versus orientation angle. Friction removes mechanical energy from the key-Earth system, tending to leave the key in its minimum-potential energy configuration.

Q7.25 Gaspard de Coriolis first stated the work-kinetic energy theorem. Jean Victor Poncelet, an engineer who invaded Russia with Napoleon, is most responsible for demonstrating its wide practical applicability, in his 1829 book *Industrial Mechanics*. Their work came remarkably late compared to the elucidation of momentum conservation in collisions by Descartes and to Newton's *Mathematical Principles of the Philosophy of Nature*, both in the 1600's.

SOLUTIONS TO PROBLEMS

Section 7.2 **Work Done by a Constant Force**

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.2 (a) $W = mgh = (3.35\times10^{-5})(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

P7.3 METHOD ONE

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$. The work done by the gravitational force on Batman is

$$W = \int_i^f F\cos\theta dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times10^3\text{ J}}$$

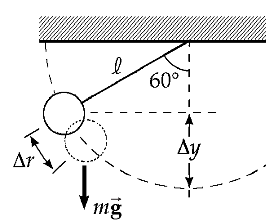

FIG. P7.3

METHOD TWO

The force of gravity on Batman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is -12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

***P7.4** Yes. Object 1 exerts some forward force on object 2 as they move through the same displacement. By Newton's third law, object 2 exerts an equal-size force in the opposite direction on object 1. In $W = F\Delta r\cos\theta$, the factors F and Δr are the same, and θ differs by 180°, so object 2 does −15.0 J of work on object 1. The energy transfer is 15 J from object 1 to object 2, which can be counted as a change in energy of −15 J for object 1 and a change in energy of +15 J for object 2.

Section 7.3 **The Scalar Product of Two Vectors**

P7.5

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}+A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}+B_z\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right)+A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right)+A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)$$

$$+A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right)+A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right)+A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)$$

$$+A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right)+A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right)+A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \boxed{A_xB_x+A_yB_y+A_zB_z}$$

P7.6 $A = 5.00;\ B = 9.00;\ \theta = 50.0°$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$$

P7.7 (a) $W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_xx+F_yy = (6.00)(3.00)\ \text{N}\cdot\text{m}+(-2.00)(1.00)\ \text{N}\cdot\text{m} = \boxed{16.0\ \text{J}}$

(b) $\theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\Delta r}\right) = \cos^{-1}\dfrac{16}{\sqrt{\left((6.00)^2+(-2.00)^2\right)\left((3.00)^2+(1.00)^2\right)}} = \boxed{36.9°}$

P7.8 We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = Fv\cos\theta = (32.8\ \text{N})(0.173\ \text{m/s})\cos 20.0°$$

or $\vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = \boxed{5.33\ \dfrac{\text{J}}{\text{s}}}$

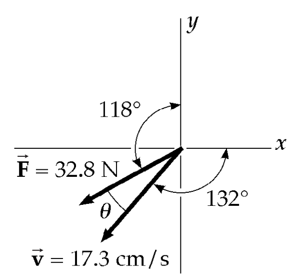

FIG. P7.8

P7.9 (a) $\vec{\mathbf{A}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$ $\quad\theta = \cos^{-1}\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB} = \cos^{-1}\dfrac{12.0+8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{A}} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$ $\quad\cos\theta = \dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB} = \dfrac{-6.00-16.0}{\sqrt{(20.0)(29.0)}}$ $\quad\theta = \boxed{156°}$

(c) $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$ $\quad\theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00+8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

P7.10 $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\left(3.00\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)-\left(-\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{k}}\right)$

$$\vec{\mathbf{A}}-\vec{\mathbf{B}}=4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}$$

$$\vec{\mathbf{C}}\cdot\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)=\left(2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}\right)=0+(-2.00)+(+18.0)=\boxed{16.0}$$

***P7.11** Let θ represent the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Turning by 25° makes the dot product larger, so the angle between $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}$ must be smaller. We call it $\theta - 25°$. Then we have

$A\,5\cos\theta = 30$ and $A\,5\cos(\theta - 25°) = 35$

Then $A\cos\theta = 6$ and $A(\cos\theta\cos 25° + \sin\theta\sin 25°) = 7$

Dividing, $\cos 25° + \tan\theta\sin 25° = 7/6$ $\tan\theta = (7/6 - \cos 25°)/\sin 25° = 0.616$

$\theta = 31.6°$. Then the direction angle of A is $60° - 31.6° = 28.4°$

Substituting back, $A\cos 31.6° = 6$ so $\vec{\mathbf{A}} = \boxed{7.05 \text{ m at } 28.4°}$

Section 7.4 **Work Done by a Varying Force**

P7.12 $F_x = (8x - 16)$ N

(a) See figure to the right

(b) $W_{\text{net}} = \dfrac{-(2.00\text{ m})(16.0\text{ N})}{2}+\dfrac{(1.00\text{ m})(8.00\text{ N})}{2}=\boxed{-12.0\text{ J}}$

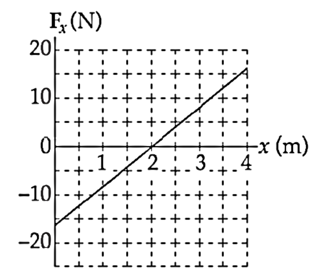

FIG. P7.12

P7.13 $W=\int_i^f F\,dx=$ area under curve from x_i to x_f

(a) $x_i = 0$ $x_f = 8.00$ m

$W =$ area of triangle $ABC = \left(\dfrac{1}{2}\right)AC\times$ altitude,

$W_{0\to 8} = \left(\dfrac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N} = \boxed{24.0\text{ J}}$

(b) $x_i = 8.00$ m $x_f = 10.0$ m

$W =$ area of $\Delta CDE = \left(\dfrac{1}{2}\right)CE\times$ altitude,

$W_{8\to 10} = \left(\dfrac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N}) = \boxed{-3.00\text{ J}}$

(c) $W_{0\to 10} = W_{0\to 8} + W_{8\to 10} = 24.0 + (-3.00) = \boxed{21.0\text{ J}}$

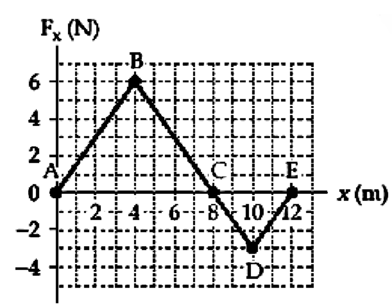

FIG. P7.13

P7.14 $W=\int_i^f \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}=\int_0^{5\text{ m}}\left(4x\hat{\mathbf{i}}+3y\hat{\mathbf{j}}\right)\text{ N}\cdot dx\hat{\mathbf{i}}$

$$\int_0^{5\text{ m}}(4\text{ N/m})x\,dx+0=(4\text{ N/m})\left.\frac{x^2}{2}\right|_0^{5\text{ m}}=\boxed{50.0\text{ J}}$$

P7.15 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

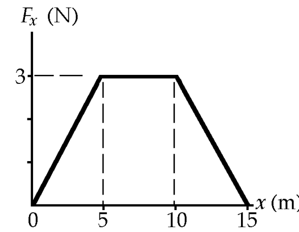

FIG. P7.15

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

P7.16 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\text{ N})}{(0.400\text{ m})} = \boxed{575\text{ N/m}}$$

(b) Work $= F_{\text{avg}}x = \frac{1}{2}(230\text{ N})(0.400\text{ m}) = \boxed{46.0\text{ J}}$

P7.17 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\text{ N}}{2.50 \times 10^{-2}\text{ m}} = 1.57 \times 10^3\text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938\text{ cm}}$

(b) Work $= \frac{1}{2}ky^2$

Work $= \frac{1}{2}\left(1.57 \times 10^3\text{ N}\cdot\text{m}\right)\left(4.00 \times 10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$

***P7.18** In $F = -kx$, F refers to the size of the force that the spring exerts on each end. It pulls down on the doorframe in part (a) in just as real a sense at it pulls on the second person in part (b).

(a) Consider the upward force exerted by the bottom end of the spring, which undergoes a downward displacement that we count as negative:

$k = -F/x = -(7.5\text{ kg})(9.8\text{ m/s}^2)/(-0.415\text{ m} + 0.35\text{ m}) = -73.5\text{ N}/(-0.065\text{ m}) = \boxed{1.13\text{ kN/m}}$

(b) Consider the end of the spring on the right, which exerts a force to the left:

$x = -F/k = -(-190\text{ N})/(1130\text{ N/m}) = 0.168\text{ m}$

The length of the spring is then $0.35\text{ m} + 0.168\text{ m} = \boxed{0.518\text{ m}}$

***P7.19** $\Sigma F_x = ma_x$: $kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70 \times 10^{-3}\text{ kg})0.800(9.80\text{ m/s}^2)}{0.500 \times 10^{-2}\text{m}}$$
$$= \boxed{7.37\text{ N/m}}$$

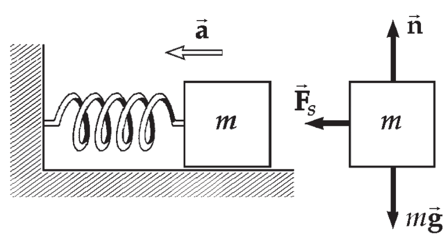

FIG. P7.19

***P7.20** The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\ \text{N/m})(0.08\ \text{m}) = 0.308\ \text{N}$$

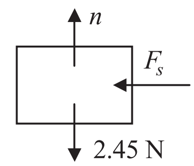

FIG. P7.20

Take the $+x$ direction to the right. For the light block on the left, the vertical forces are given by $F_g = mg = (0.25\ \text{kg})(9.8\ \text{m/s}^2) = 2.45\ \text{N}$, $\sum F_y = 0$, $n - 2.45\ \text{N} = 0$, $n = 2.45\ \text{N}$. Similarly for the heavier block $n = F_g = (0.5\ \text{kg})(9.8\ \text{m/s}^2) = 4.9\ \text{N}$

(a) For the block on the left, $\sum F_x = ma_x$, $-0.308\ \text{N} = (0.25\ \text{kg})a$, $a = \boxed{-1.23\ \text{m/s}^2}$.
For the heavier block, $+0.308\ \text{N} = (0.5\ \text{kg})a$, $a = \boxed{0.616\ \text{m/s}^2}$.

(b) For the block on the left, $f_k = \mu_k n = 0.1(2.45\ \text{N}) = 0.245\ \text{N}$

$$\sum F_x = ma_x$$

$$-0.308\ \text{m/s}^2 + 0.245\ \text{N} = (0.25\ \text{kg})a$$

$$a = \boxed{-0.252\ \text{m/s}^2 \text{ if the force of static friction is not too large.}}$$

For the block on the right, $f_k = \mu_k n = 0.490\ \text{N}$. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45\ \text{N}) = 1.13\ \text{N}$. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both $a = \boxed{0}$.

***P7.21** Compare an initial picture of the rolling car with a final picture with both springs compressed $K_i + \sum W = K_f$. Work by both springs changes the car's kinetic energy

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1600\ \text{N/m})(0.500\ \text{m})^2$$

$$+0 - \frac{1}{2}(3400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

$$\frac{1}{2}(6\,000\ \text{kg})v_i^2 - 200\ \text{J} - 68.0\ \text{J} = 0$$

$$v_i = \sqrt{\frac{2(268\ \text{J})}{6\,000\ \text{kg}}} = \boxed{0.299\ \text{m/s}}$$

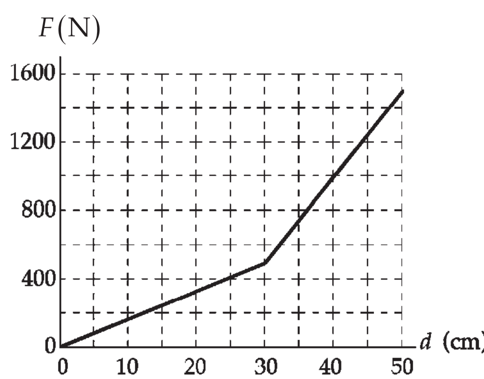

FIG. P7.21

P7.22 (a) $W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right)dx \cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\Bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

continued on next page

(b) Similarly,

$$W = (15.0\text{ kN})(1.00\text{ m}) + \frac{(10.0\text{ kN/m})(1.00\text{ m})^2}{2} - \frac{\left(25.0\text{ kN/m}^2\right)(1.00\text{ m})^3}{3}$$

$$W = \boxed{11.7\text{ kJ}}\text{, larger by }29.6\%$$

P7.23 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= 1.5\text{ kg }9.8\text{ m/s}^2\left(\frac{1\text{ m}}{1\,200\text{ N}} + \frac{1\text{ m}}{1\,800\text{ N}}\right) = \boxed{2.04\times10^{-2}\text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg\left(1/k_1 + 1/k_2\right)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1\text{ m}}{1\,200\text{ N}} + \frac{1\text{ m}}{1\,800\text{ N}}\right)^{-1} = \boxed{720\text{ N/m}}$$

P7.24 See the solution to problem 7.23.

(a) $\boxed{x = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$ Both springs stretch, so the load moves down by a larger amount than it would if either spring were missing.

(b) $\boxed{k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}}$ The spring constant of the series combination is less than the smaller of the two individual spring constants, to describe a less stiff system, that stretches by a larger extension for any particular load.

P7.25 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x-axis in the direction of motion tangent to the cylinder.

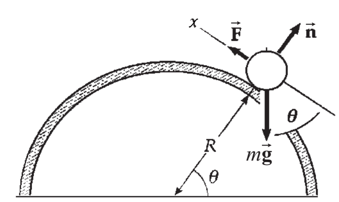

FIG. P7.25

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

P7.26 $$[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$$

***P7.27** We can write u as a function of v : (8 N −[−2 N])/(25 cm − 5 cm) = (u −[−2 N])/(v − 5 cm)

(0.5 N/cm)(v − 5 cm) = u + 2 N u = (0.5 N/cm)v − 4.5 N also v = (2 cm/N)u + 9 cm

(a) Then

$$\int_a^b u\,dv = \int_5^{25}(0.5v-4.5)\,dv = \left[0.5v^2/2-4.5v\right]_5^{25} = 0.25(625-25)-4.5(25-5)$$

$$=150-90=60\text{ N}\cdot\text{cm}=\boxed{0.600\text{ J}}$$

(b) Reversing the limits of integration just gives us the negative of the quantity: $\int_b^a u\,dv = \boxed{-0.600\text{ J}}$

(c) This is an entirely different integral. It is larger because all of the area to be counted up is positive (to the right of $v = 0$) instead of partly negative (below $u = 0$).

$$\int_a^b v\,du = \int_{-2}^{8}(2u+9)\,du = \left[2u^2/2+9u\right]_{-2}^{8} = 64-(-2)^2+9(8+2)=60+90=150\text{ N}\cdot\text{cm}=\boxed{1.50\text{ J}}$$

***P7.28** If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray will always have the same elevation above the floor.

The weight of a tray is $0.580\text{ kg}\left(9.8\text{ m/s}^2\right)=5.68\text{ N}$. The force $\frac{1}{4}(5.68\text{ N})=1.42\text{ N}$ should stretch one spring by 0.450 cm, so its spring constant is $k=\frac{|F_s|}{x}=\frac{1.42\text{ N}}{0.004\,5\text{ m}}=\boxed{316\text{ N/m}}$.

We did not need to know the length or width of the tray.

Section 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

P7.29 (a) $K_A=\frac{1}{2}(0.600\text{ kg})(2.00\text{ m/s})^2=\boxed{1.20\text{ J}}$

(b) $\frac{1}{2}mv_B^2=K_B$: $v_B=\sqrt{\frac{2K_B}{m}}=\sqrt{\frac{(2)(7.50)}{0.600}}=\boxed{5.00\text{ m/s}}$

(c) $\sum W=\Delta K=K_B-K_A=\frac{1}{2}m\left(v_B^2-v_A^2\right)=7.50\text{ J}-1.20\text{ J}=\boxed{6.30\text{ J}}$

P7.30 (a) $K=\frac{1}{2}mv^2=\frac{1}{2}(0.300\text{ kg})(15.0\text{ m/s})^2=\boxed{33.8\text{ J}}$

(b) $K=\frac{1}{2}(0.300)(30.0)^2=\frac{1}{2}(0.300)(15.0)^2(4)=4(33.8)=\boxed{135\text{ J}}$

P7.31 $\vec{\mathbf{v}}_i=\left(6.00\hat{\mathbf{i}}-2.00\hat{\mathbf{j}}\right)=\text{m/s}$

(a) $v_i=\sqrt{v_{ix}^2+v_{iy}^2}=\sqrt{40.0}\text{ m/s}$

$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}(3.00\text{ kg})\left(40.0\text{ m}^2/\text{s}^2\right)=\boxed{60.0\text{ J}}$

(b) $\vec{\mathbf{v}}_f=8.00\hat{\mathbf{i}}+4.00\hat{\mathbf{j}}$

$v_f^2=\vec{\mathbf{v}}_f\cdot\vec{\mathbf{v}}_f=64.0+16.0=80.0\text{ m}^2/\text{s}^2$

$\Delta K=K_f-K_i=\frac{1}{2}m\left(v_f^2-v_i^2\right)=\frac{3.00}{2}(80.0)-60.0=\boxed{60.0\text{ J}}$

P7.32 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\text{ J})}{4.00\text{ kg}}} = \boxed{1.94\text{ m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\text{ J})}{4.00\text{ kg}}} = \boxed{3.35\text{ m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\text{ J})}{4.00\text{ kg}}} = \boxed{3.87\text{ m/s}}$$

P7.33 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$$\sum W = \Delta K:\ W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so

$$(mg)(h+d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$$

Thus,

$$\bar{F} = \frac{(mg)(h+d)}{d} = \frac{(2\,100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m})}{0.120\text{ m}} = \boxed{8.78\times 10^5\text{ N}}$$

The force on the pile driver is $\boxed{\text{upward}}$.

***P7.34** (a) We evaluate the kinetic energy of the cart and the work the cart would have to do to plow all the way through the pile. If the kinetic energy is larger, the cart gets through.

$K = (1/2)mv^2 = (1/2)(0.3\text{ kg})(0.6\text{ m/s})^2 = 0.054\text{ J}$

The work *done on the cart* in traveling the whole distance is the net area under the graph,

$W = (2\text{ N})(0.01\text{ m}) + [(0 - 3\text{ N})/2](0.04\text{ m}) = 0.02\text{ J} - 0.06\text{ J} = -0.04\text{ J}$

The work the cart must do is less than the original kinetic energy, so the cart does get through all the sand.

(b) The work *the cart does* is +0.04 J, so its final kinetic energy is the remaining 0.054 J – 0.04 J = 0.014 J. Another way to say it: from the work-kinetic energy theorem,

$K_i + W = K_f \qquad 0.054\text{ J} - 0.04\text{ J} = 0.014\text{ J} = (1/2)(0.5\text{ kg})v_f^2$

$v_f = [2(0.014\text{ kg}\cdot\text{m}^2/\text{s}^2)/(0.3\text{ kg})]^{1/2} = \boxed{0.306\text{ m/s}}$

***P7.35** (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

$$0 + \sum W = \frac{1}{2}(15.0\times 10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$$

(b) $$F = \frac{W}{\Delta r\cos\theta} = \frac{4.56\times 10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$$

continued on next page

(c) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\ \text{m/s})^2 - 0}{2(0.720\ \text{m})} = \boxed{422\ \text{km/s}^2}$

(d) $\sum F = ma = (15\times10^{-3}\ \text{kg})(422\times10^{3}\ \text{m/s}^2) = \boxed{6.34\ \text{kN}}$

(e) $\boxed{\text{The forces are the same. The two theories agree.}}$

P7.36 (a) $v_f = 0.096(3\times10^8\ \text{m/s}) = 2.88\times10^7\ \text{m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.88\times10^7\ \text{m/s})^2 = \boxed{3.78\times10^{-16}\ \text{J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\ \text{m})\cos 0° = 3.78\times10^{-16}\ \text{J}$$

$$F = \boxed{1.35\times10^{-14}\ \text{N}}$$

(c) $\sum F = ma$; $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times10^{-14}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = \boxed{1.48\times10^{+16}\ \text{m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$ $\quad 2.88\times10^7\ \text{m/s} = 0 + (1.48\times10^{16}\ \text{m/s}^2)t$

$$t = \boxed{1.94\times10^{-9}\ \text{s}}$$

Check: $\quad x_f = x_i + \dfrac{1}{2}(v_{xi} + v_{xf})t$

$$0.028\ \text{m} = 0 + \frac{1}{2}(0 + 2.88\times10^7\ \text{m/s})t$$

$$t = 1.94\times10^{-9}\ \text{s}$$

Section 7.6 Potential Energy of a System

P7.37 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}$$

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

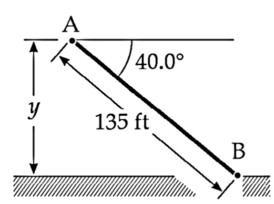

FIG. P7.37

where y is the vertical height above zero level. With 135 ft = 41.1 m, this height is found as:

$$y = (41.1\ \text{m})\sin 40.0° = 26.4\ \text{m}$$

Thus,

$$U_A = (1\,000\ \text{kg})(9.80\ \text{m/s}^2)(26.4\ \text{m}) = \boxed{2.59\times10^5\ \text{J}}$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59\times10^5\ \text{J} = \boxed{-2.59\times10^5\ \text{J}}$$

continued on next page

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) = \boxed{-2.59 \times 10^5 \text{ J}}$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 = \boxed{-2.59 \times 10^5 \text{ J}}$$

P7.38 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m}) = \boxed{800 \text{ J}}$$

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00 \text{ m})(1 - \cos 30.0°)$ above the lowest point of the arc. Thus,

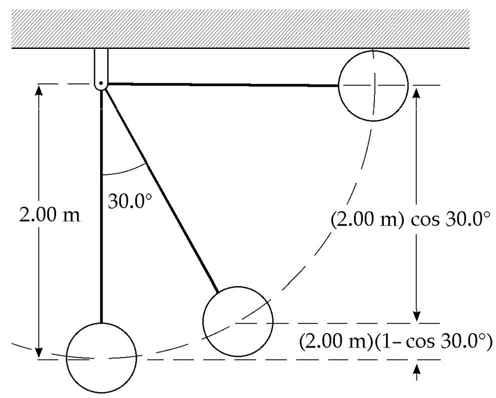

FIG. P7.38

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m})(1 - \cos 30.0°) = \boxed{107 \text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

Section 7.7 Conservative and Nonconservative Forces

P7.39 $F_g = mg = (4.00 \text{ kg})(9.80 \text{ m/s}^2) = 39.2 \text{ N}$

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$

$= (39.2 \text{ N})(5.00 \text{ m}) + (39.2 \text{ N})(5.00 \text{ m})(-1)$

$= \boxed{-196 \text{ J}}$

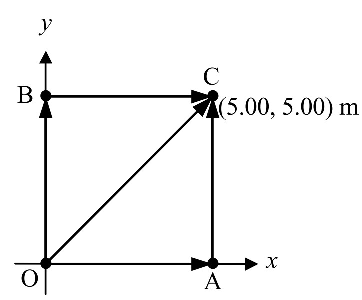

FIG. P7.39

(b) W along OBC = W along OB + W along BC

$= (39.2 \text{ N})(5.00 \text{ m})\cos 180° + (39.2 \text{ N})(5.00 \text{ m})\cos 90.0°$

$= \boxed{-196 \text{ J}}$

(c) Work along OC $= F_g(\text{OC})\cos 135°$

$$= (39.2 \text{ N})\left(5.00 \times \sqrt{2} \text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196 \text{ J}}$$

$\boxed{\text{The results should all be the same, since gravitational forces are conservative.}}$

P7.40 (a) $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ and if the force is constant, this can be written as

$$W = \vec{\mathbf{F}} \cdot \int d\vec{\mathbf{r}} = \boxed{\vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i\right), \text{ which depends only on end points, not path.}}$$

(b) $$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right) \cdot \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) = (3.00 \text{ N}) \int_0^{5.00 \text{ m}} dx + (4.00 \text{ N}) \int_0^{5.00 \text{ m}} dy$$

$$W = (3.00 \text{ N})\, x\Big|_0^{5.00 \text{ m}} + (4.00 \text{ N})\, y\Big|_0^{5.00 \text{ m}} = 15.0 \text{ J} + 20.0 \text{ J} = \boxed{35.0 \text{ J}}$$

The same calculation applies for all paths.

P7.41 (a) The work done on the particle in its first section of motion is

$$W_{OA} = \int_0^{5.00 \text{ m}} dx\hat{\mathbf{i}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00 \text{ m}} 2y\,dx$$

and since along this path, $y = 0$ $\quad W_{OA} = 0$

In the next part of its path $\quad W_{AC} = \int_0^{5.00 \text{ m}} dy\hat{\mathbf{j}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00 \text{ m}} x^2\,dy$

For $x = 5.00$ m, $\quad W_{AC} = 125$ J

and $\quad W_{OAC} = 0 + 125 = \boxed{125 \text{ J}}$

(b) Following the same steps, $\quad W_{OB} = \int_0^{5.00 \text{ m}} dy\hat{\mathbf{j}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00 \text{ m}} x^2\,dy$

since along this path, $x = 0$, $\quad W_{OB} = 0$

$$W_{BC} = \int_0^{5.00 \text{ m}} dx\hat{\mathbf{i}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00 \text{ m}} 2y\,dx$$

since $y = 5.00$ m, $\quad W_{BC} = 50.0$ J

$$W_{OBC} = 0 + 50.0 = \boxed{50.0 \text{ J}}$$

(c) $$W_{OC} = \int \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int \left(2y\,dx + x^2\,dy\right)$$

Since $x = y$ along OC, $\quad W_{OC} = \int_0^{5.00 \text{ m}} \left(2x + x^2\right) dx = \boxed{66.7 \text{ J}}$

(d) $\boxed{F \text{ is nonconservative since the work done is path dependent.}}$

P7.42 Along each step of motion, the frictional force is opposite in direction to the incremental displacement, so in the work $\cos 180° = -1$.

(a) $W = (3 \text{ N})(5 \text{ m})(-1) + (3 \text{ N})(5 \text{ m})(-1) = \boxed{-30.0 \text{ J}}$

(b) The distance CO is $(5^2 + 5^2)^{1/2}$ m $= 7.07$ m

$W = (3 \text{ N})(5 \text{ m})(-1) + (3 \text{ N})(5 \text{ m})(-1) + (3 \text{ N})(7.07 \text{ m})(-1) = \boxed{-51.2 \text{ J}}$

(c) $W = (3 \text{ N})(7.07 \text{ m})(-1) + (3 \text{ N})(7.07 \text{ m})(-1) = \boxed{-42.4 \text{ J}}$

(d) $\boxed{\text{The force of friction is a nonconservative force.}}$

Section 7.8 Relationship Between Conservative Forces and Potential Energy

P7.43 (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)\,dx = \left(\frac{2x^2}{2}+4x\right)\Big|_1^{5.00\text{ m}} = 25.0+20.0-1.00-4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0 \qquad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2} \qquad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

P7.44 (a) $U = -\int_0^x \left(-Ax+Bx^2\right)dx = \boxed{\frac{Ax^2}{2}-\frac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right)-(2.00)^2\right]}{2} - \frac{B\left[(3.00)^3-(2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A+\frac{19.0}{3}B\right)}$

P7.45 $U(r) = \frac{A}{r}$

$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$. If A is positive, the positive value of radial force indicates a force of repulsion.

P7.46 $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y-7x\right)}{\partial x} = -\left(9x^2y-7\right) = 7-9x^2y$

$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y-7x\right)}{\partial y} = -\left(3x^3-0\right) = -3x^3$

Thus, the force acting at the point $(x,\ y)$ is $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7-9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

Section 7.9 Energy Diagrams and the Equilibrium of a System

P7.47 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

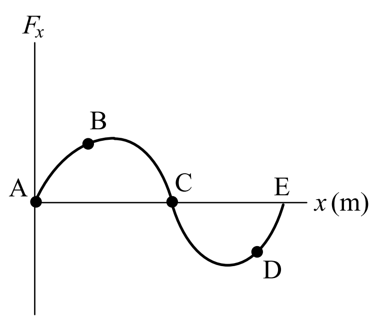

FIG. P7.47

P7.48

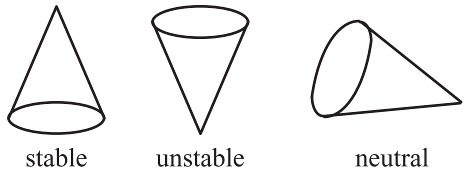

FIG. P7.48

P7.49 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

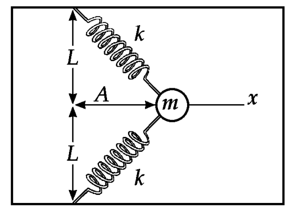

FIG. P7.49

$$\vec{\mathbf{F}} = -2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}} = \boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}$$

(b) Choose $U=0$ at $x=0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x\left(-2kx+\frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x\,dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x) = 40.0x^2+96.0\left(1.20-\sqrt{x^2+1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x=0$) is $\boxed{x=0}$.

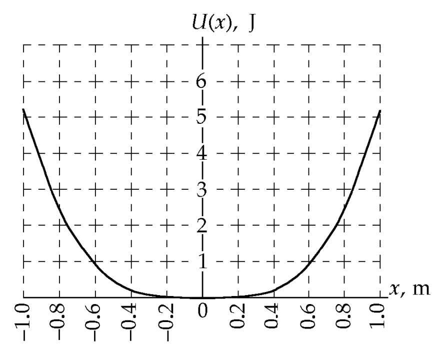

FIG. P7.49(c)

(d) $K_i+U_i+\Delta E_{\text{mech}} = K_f+U_f$

$$0+0.400\text{ J}+0 = \frac{1}{2}(1.18\text{ kg})v_f^2+0$$

$$v_f = \boxed{0.823\text{ m/s}}$$

Additional Problems

P7.50 The work done by the applied force is

$$W = \int_i^f F_{\text{applied}}dx = \int_0^{x_{\max}} -\left[-\left(k_1x+k_2x^2\right)\right]dx$$

$$= \int_0^{x_{\max}} k_1x\,dx + \int_0^{x_{\max}} k_2x^2dx = k_1\left.\frac{x^2}{2}\right|_0^{x_{\max}} + k_2\left.\frac{x^3}{3}\right|_0^{x_{\max}}$$

$$= \boxed{k_1\frac{x_{\max}^2}{2}+k_2\frac{x_{\max}^3}{3}}$$

P7.51 At start, $\vec{\mathbf{v}} = (40.0\ \text{m/s})\cos 30.0°\hat{\mathbf{i}} + (40.0\ \text{m/s})\sin 30.0°\hat{\mathbf{j}}$

At apex, $\vec{\mathbf{v}} = (40.0\ \text{m/s})\cos 30.0°\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6\ \text{m/s})\hat{\mathbf{i}}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150\ \text{kg})(34.6\ \text{m/s})^2 = \boxed{90.0\ \text{J}}$

P7.52 (a) We write

$$F = ax^b$$

$$1\,000\ \text{N} = a(0.129\ \text{m})^b$$

$$5\,000\ \text{N} = a(0.315\ \text{m})^b$$

$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$

$$\ln 5 = b \ln 2.44$$

$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80 = b}$$

$$a = \frac{1\,000\ \text{N}}{(0.129\ \text{m})^{1.80}} = \boxed{4.01\times10^4\ \text{N/m}^{1.8} = a}$$

(b) $$W = \int_0^{0.25\ \text{m}} F\,dx = \int_0^{0.25\ \text{m}} 4.01\times10^4\ \frac{\text{N}}{\text{m}^{1.8}}x^{1.8}dx$$

$$= 4.01\times10^4\ \frac{\text{N}}{\text{m}^{1.8}}\left.\frac{x^{2.8}}{2.8}\right|_0^{0.25\ \text{m}} = 4.01\times10^4\ \frac{\text{N}}{\text{m}^{1.8}}\frac{(0.25\ \text{m})^{2.8}}{2.8}$$

$$= \boxed{294\ \text{J}}$$

***P7.53** (a) We assume the spring is in the horizontal plane of the motion. The radius of the puck's motion is $0.155\ \text{m} + x$

The spring force causes the puck's centripetal acceleration:
$(4.3\ \text{N/m})\ x = F = mv^2/r = m(2\pi r/\text{T})^2/r = 4\pi^2\ m\ r/(1.3\ \text{s})^2$

$(4.3\ \text{kg/s}^2)\ x = (23.4/\text{s}^2)\ m\ (0.155\ \text{m} + x)$

$4.3\ \text{kg}\ x = 3.62\ m\ \ \text{m} + 23.4\ m\ \ x$

$4.3000\ \text{kg}\ x - 23.360\ m\ \ x = 3.6208\ m\ \ \text{m}$

$\boxed{x = (3.62\ m)/(4.3\ \text{kg} - 23.4\ m)\ \text{meters}}$

(b) $x = (3.62\ \text{m}\ 0.07\ \text{kg})/(4.30\ \text{kg} - 23.4\ [0.07\ \text{kg}]) = \boxed{0.0951\ \text{m}}$ a nice reasonable extension

(c) We double the puck mass and find
$x = (3.6208\ \text{m}\ 0.14\ \text{kg})/(4.30\ \text{kg} - 23.360\ [0.14\ \text{kg}]) = \boxed{0.492\ \text{m}}$ more than twice as big!

(d) $x = (3.62\ \text{m}\ 0.18\ \text{kg})/(4.30\ \text{kg} - 23.4\ [0.18\ \text{kg}]) = \boxed{6.85\ \text{m}}$ We have to get a bigger table!!

(e) When the denominator of the fraction goes to zero, the extension becomes infinite. This happens for $4.3\ \text{kg} - 23.4\ m = 0$; that is for $m = 0.184$ kg. For any larger mass, the spring cannot constrain the motion. $\boxed{\text{The situation is impossible.}}$

(f) The extension is directly proportional to m when m is only a few grams. Then it grows faster and faster, diverging to infinity for $m = 0.184$ kg.

***P7.54** (a) A time interval. If the interaction occupied no time, the force exerted by each ball on the other would be infinite, and that cannot happen.

(b) $k = |F|/|x| = 16\,000$ N/0.000 2 m = $\boxed{80 \text{ MN/m}}$

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1.

Then its mass is $\rho V = \rho\,(4/3)\pi r^3 = (4\pi/3)(7860 \text{ kg/m}^3)(0.0254 \text{ m}/2)^3 = 0.0674$ kg

and $K = (1/2)\,mv^2 = (1/2)\,(0.0674 \text{ kg})(5 \text{ m/s})^2 = 0.843 \text{ J} \approx \boxed{0.8 \text{ J}}$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$0.843 \text{ J} = (1/2)\,(8 \times 10^7 \text{ N/m})\,x^2 \qquad x = 0.145 \text{ mm} \approx \boxed{0.15 \text{ mm}}$

(e) The ball does not really stop with constant acceleration, but imagine it moving 0.145 mm forward with average speed (5 m/s + 0)/2 = 2.5 m/s. The time interval over which it stops is then

$0.145 \text{ mm}/(2.5 \text{ m/s}) = 6 \times 10^{-5} \text{ s} \; \boxed{\sim 10^{-4} \text{ s}}$

***P7.55** The potential energy at point x is given by 5 plus the negative of the work the force does as a particle feeling the force is carried from $x = 0$ to location x.

$$dU = -F dx \qquad \int_5^U dU = -\int_0^x 8e^{-2x} dx \qquad U - 5 = -\left(8/\left[-2\right]\right)\int_0^x e^{-2x}\left(-2dx\right)$$

$$U = 5 - \left(8/\left[-2\right]\right)e^{-2x}\Big|_0^x = 5 + 4e^{-2x} - 4\cdot 1 = \boxed{1 + 4e^{-2x}}$$

The force must be conservative because the work the force does on the object on which it acts depends only on the original and final positions of the object, not on the path between them.

P7.56 (a) $\vec{\mathbf{F}} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

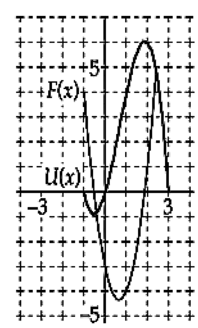

FIG. P7.56

(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at

$$x = -0.535 \text{ point of minimum } U(x)$$

The unstable point is at

$$x = 1.87 \text{ maximum in } U(x)$$

P7.57

$$K_i + W_s + W_g = K_f$$

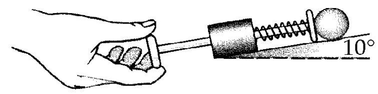

FIG. P7.57

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(1.20\ \text{N/cm})(5.00\ \text{cm})(0.050\,0\ \text{m}) - (0.100\ \text{kg})(9.80\ \text{m/s}^2)(0.050\,0\ \text{m})\sin 10.0°$$

$$= \frac{1}{2}(0.100\ \text{kg})v^2$$

$$0.150\ \text{J} - 8.51\times10^{-3}\ \text{J} = (0.050\,0\ \text{kg})v^2$$

$$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68\ \text{m/s}}$$

P7.58

(a) $\vec{\mathbf{F}}_1 = (25.0\ \text{N})\left(\cos 35.0°\hat{\mathbf{i}} + \sin 35.0°\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\ \text{N}}$

$\vec{\mathbf{F}}_2 = (42.0\ \text{N})\left(\cos 150°\hat{\mathbf{i}} + \sin 150°\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\ \text{N}}$

(b) $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\ \text{N}}$

(c) $\vec{\mathbf{a}} = \frac{\sum \mathbf{F}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\ \text{m/s}^2}$

(d) $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\ \text{m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)(\text{m/s}^2)(3.00\ \text{s})$

$\vec{\mathbf{v}}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\ \text{m/s}}$

(e) $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$

$\vec{\mathbf{r}}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\ \text{s}) + \frac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)(\text{m/s}^2)(3.00\ \text{s})^2$

$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\ \text{m}}$

(f) $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00\ \text{kg})\left[(5.54)^2 + (23.7)^2\right](\text{m/s}^2) = \boxed{1.48\ \text{kJ}}$

(g) $K_f = \frac{1}{2}mv_i^2 + \sum \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$

$K_f = \frac{1}{2}(5.00\ \text{kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\ \text{N})(-2.30\ \text{m}) + (35.3\ \text{N})(39.3\ \text{m})\right]$

$K_f = 55.6\ \text{J} + 1\,426\ \text{J} = \boxed{1.48\ \text{kJ}}$

(h) The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.

P7.59 We evaluate by $\int_{12.8}^{23.7} \frac{375dx}{x^3+3.75x}$ calculating

$$\frac{375(0.100)}{(12.8)^3+3.75(12.8)}+\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\ldots\frac{375(0.100)}{(23.6)^3+3.75(23.6)}=0.806$$

and

$$\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\frac{375(0.100)}{(13.0)^3+3.75(13.0)}+\ldots\frac{375(0.100)}{(23.7)^3+3.75(23.7)}=0.791.$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\ \text{N}\cdot\text{m}}$.

P7.60 (a)

$F(\text{N})$	$L(\text{mm})$	$F(\text{N})$	$L(\text{mm})$
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

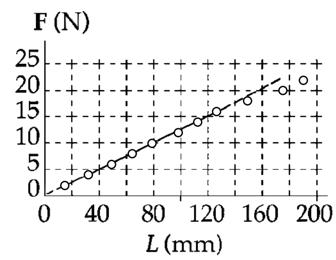

FIG. P7.60

To draw the straight line we use all the points listed and also the origin. If the coils of the spring touched each other, a bend or nonlinearity could show up at the bottom end of the graph. If the spring were stretched "too far," a nonlinearity could show up at the top end. But there is no visible evidence for a bend in the graph near either end.

(b) By least-square fitting, its slope is

$$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$$

In $F = kx$, the spring constant is $k = \frac{F}{x}$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

ANSWERS TO EVEN PROBLEMS

P7.2 (a) 3.28×10^{-2} J (b) -3.28×10^{-2} J

P7.4 Yes. It exerts a force of equal magnitude in the opposite direction that acts over the same distance. −15.0 J

P7.6 28.9

P7.8 5.33 J/s

P7.10 16.0

P7.12 (a) see the solution (b) −12.0 J

P7.14 50.0 J

P7.16 (a) 575 N/m (b) 46.0 J

P7. 18 (a) 1.13 kN/m (b) 51.8 cm

P7.20 (a) −1.23 m/s^2 and +0.616 m/s^2 (b) −0.252 m/s^2 and 0 (c) 0 and 0

P7.22 (a) 9.00 kJ (b) 11.7 kJ, larger by 29.6%

P7.24 (a) $\frac{mg}{k_1}+\frac{mg}{k_2}$ (b) $\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}$

P7.26 kg/s^2

P7. 28 If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray will always have the same elevation. 316 N/m. We do not need to know the length and width of the tray.

P7.30 (a) 33.8 J (b) 135 J

P7.32 (a) 1.94 m/s (b) 3.35 m/s (c) 3.87 m/s

P7. 34 (a) yes. Its kinetic energy as it enters the sand is sufficient to do all of the work it must do in plowing through the pile. (b) 0.306 m/s

P7.36 (a) 3.78×10^{-16} J (b) 1.35×10^{-14} N (c) $1.48\times10^{+16}$ m/s^2 (d) 1.94 ns

P7.38 (a) 800 J (b) 107 J (c) 0

P7.40 (a) see the solution (b) 35.0 J

P7. 42 (a) –30.0 J (b) –51.2 J (c) –42.4 J (d) The force of friction is a nonconservative force.

P7.44 (a) $Ax^2/2 - Bx^3/3$ (b) $\Delta U = 2.5A - 6.33B$; $\Delta K = -2.5A + 6.33B$

P7.46 $(7-9x^2y)\hat{\mathbf{i}}-3x^3\hat{\mathbf{j}}$

P7.48 see the solution

P7.50 $k_1x_{max}{}^2/2 + k_2x_{max}{}^3/3$

P7.52 (a) $a=\frac{40.1\text{ kN}}{m^{1.8}}$; $b=1.80$ (b) 294 J

P7. 54 (a) A time interval. If the interaction occupied no time, the force exerted by each ball on the other would be infinite, and that cannot happen. (b) 80 MN/m (c) 0.8 J. We assume that steel has the same density as iron. (d) 0.15 mm (e) 10^{-4} s

P7.56 (a) $F_x = (3x^2 - 4x - 3)$ (b) 1.87 and −0.535 (c) see the solution. −0.535 is stable and 1.87 is unstable.

P7.58 (a) $\vec{\mathbf{F}}_1 = \left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)$ N; $\vec{\mathbf{F}}_2 = \left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)$ N (b) $\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)$ N

(c) $\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\ \text{m/s}^2$ (d) $\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\ \text{m/s}$ (e) $\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)$ m

(f) 1.48 kJ (g) 1.48 kJ

(h) The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.

P7. 60 (a) See the solution. We use all the points listed and also the origin. There is no visible evidence for a bend in the graph or nonlinearity near either end. (b) $125\ \text{N/m} \pm 2\%$ (c) 13.1 N