MAE263F Homework 1

Che Jin Goh

This report presents simulations of a mass-spring-damper system representing an elastic beam composed of multiple nodes under gravity and viscous forces. Using both implicit and explicit time integration methods, we track the vertical position and velocity of the middle node to determine terminal velocity and the system's deformed shape.

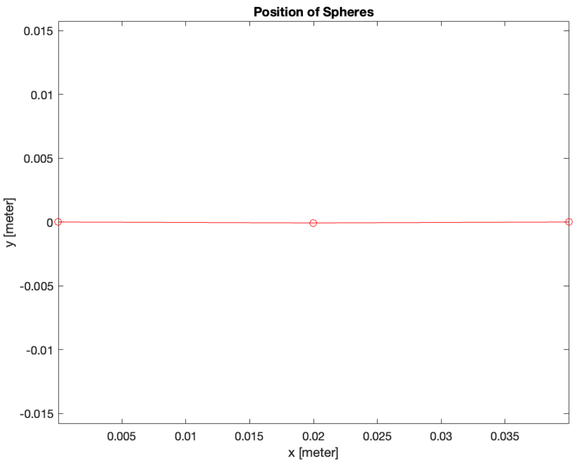
I. INTRODUCTION

This report focuses on simulating the motion of an elastic beam with multiple nodes under gravity and viscous forces using mass-spring-damper models. Both implicit and explicit time integration methods are applied to track the vertical motion and deformation of the beam, with particular attention to the middle node's position and velocity. The goal is to analyze the system's terminal velocity and deformed shape while exploring the effects of spatial and temporal discretization on simulation accuracy. Two key simulations are performed: a three-node system over 10 seconds and a 21-node beam over 50 seconds.

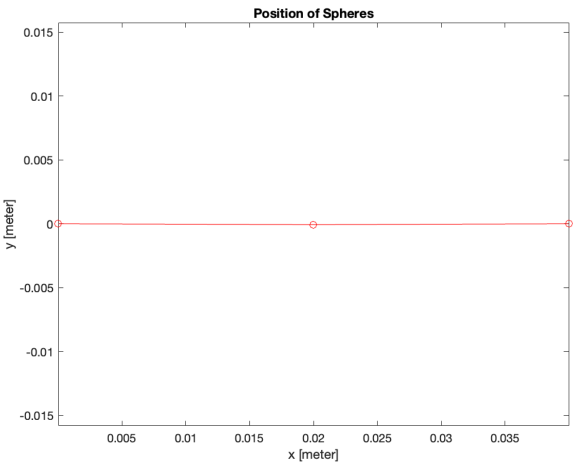
II. MATH

- *Problem 1*

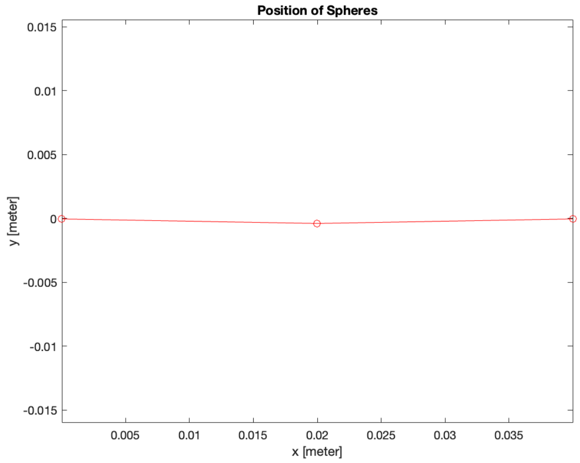
1. a) At t = 0s



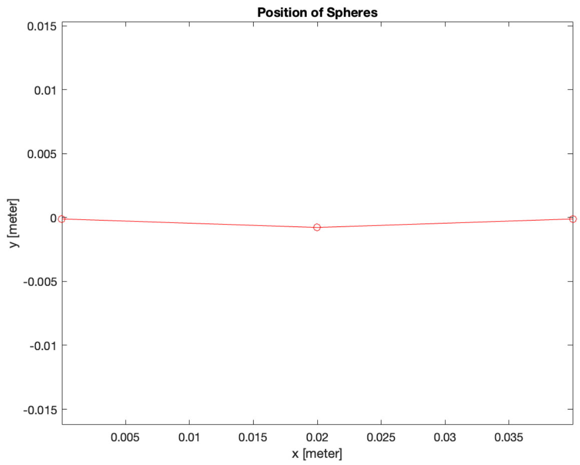
b) at t = 0.01s



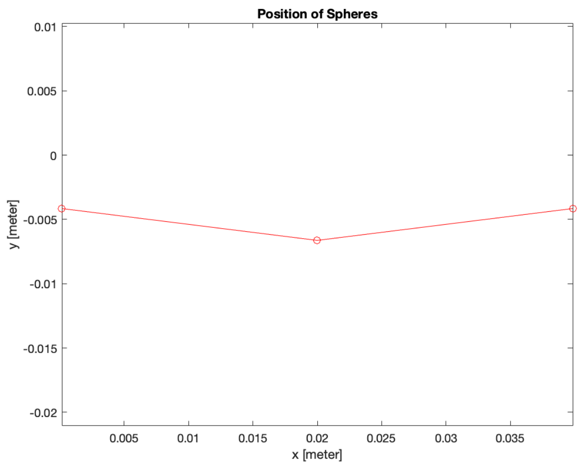
c) at t = 0.05s



d) at t = 0.1s



e) at t = 1s



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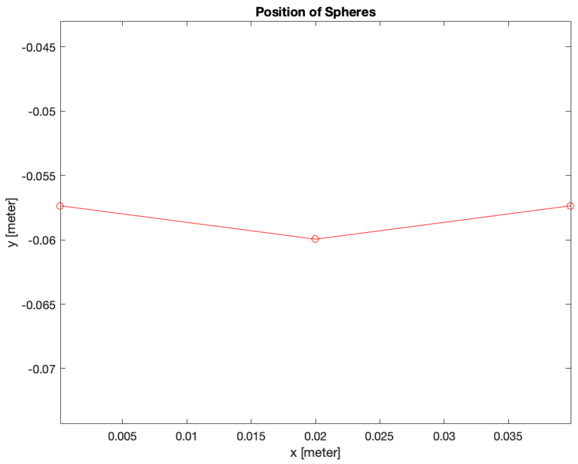
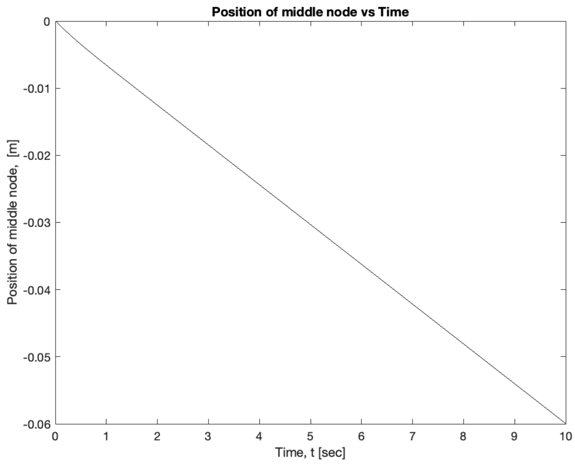
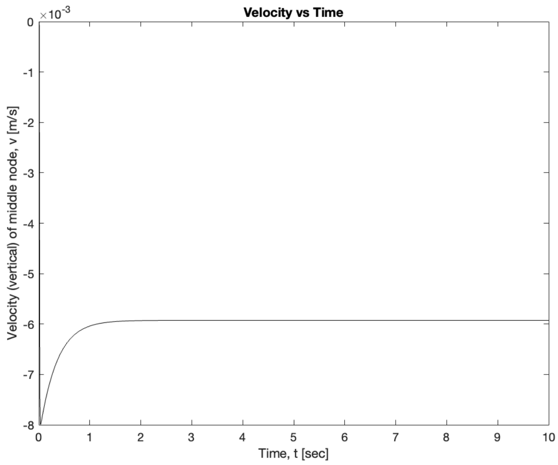
f) at t = 10s



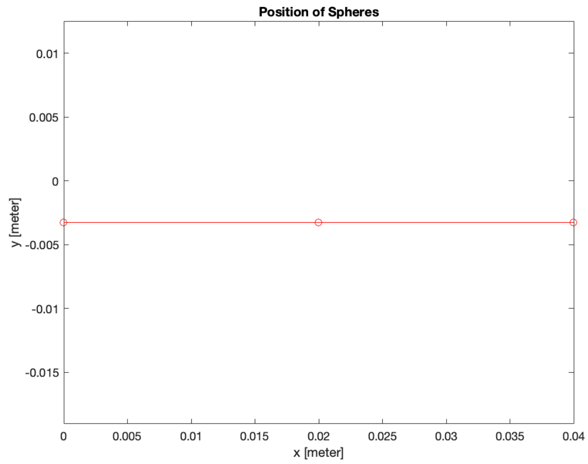
g) Position of second sphere vs time



h) Velocity of second sphere vs time



2. The terminal velocity of the system is 6 m/s, as indicated in the velocity vs. time graph where the velocity reaches a constant value.
3. If all the sphere radii are equal, their masses would also be equal, resulting in evenly distributed forces. Consequently, the bending angle would be minimal or zero. This aligns with the simulation results.



4. It is rather unrealistic for the explicit approach, since it requires a very small time step (Δt) for the simulation to remain stable, making it computationally expensive and time-consuming. A smaller Δt ensures that numerical instabilities are avoided, but this increases the total number of iterations needed, leading to a significantly longer simulation time. On the other hand, the implicit method is more stable and can handle larger time steps without diverging, even for stiff systems. However, the implicit approach involves solving a system of nonlinear equations at each time step, which requires computational effort and may be complex to implement. Thus, while explicit methods are simpler and straightforward for implementation, they suffer from stability issues with larger time steps, whereas implicit methods are computationally more intensive per iteration but allow larger time steps and are generally more stable for systems like this elastic beam problem.

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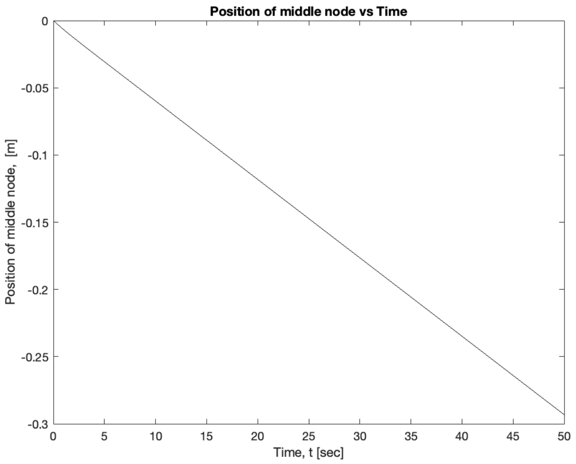
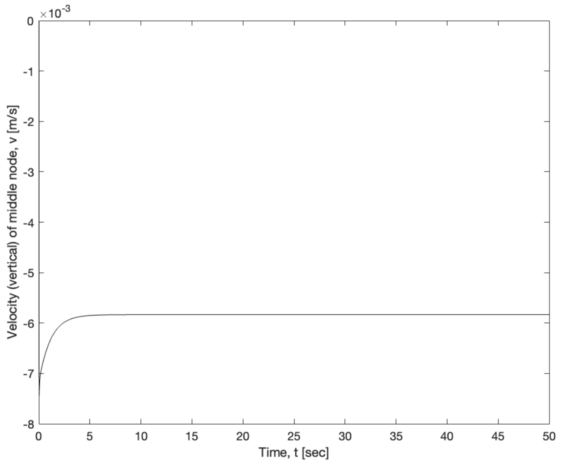
- *Problem 2*

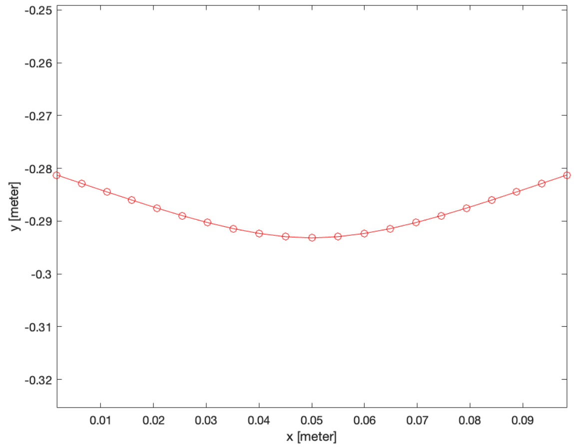
1.



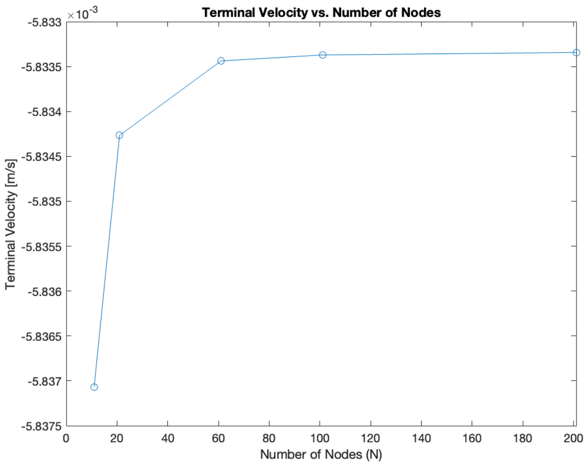


The vertical position and velocity of the middle node over time are shown in the figures above. The position plot shows a linear decrease in displacement, which corresponds to the middle node moving downward due to gravity. The velocity plot indicates that the vertical velocity approaches a steady value of approximately 6 x 10^-3 m/s. This steady value represents the terminal velocity of the middle node.

2. Final deformed shape of the beam



3. The number of nodes N=21 divides the beam into 20 segments, allowing a more accurate representation of bending and deformation. The finer the spatial discretization (higher N), the more accurate the simulation, but at the cost of computational resources. The time step size is currently Δt=10−2. If Δt is too large, the simulation may become unstable or inaccurate. Conversely, a very small Δt makes the simulation more computationally expensive.



Changing the number of nodes improves the accuracy of the result, this can be seen from the graph, as the number of nodes increases, the terminal velocity value converges.

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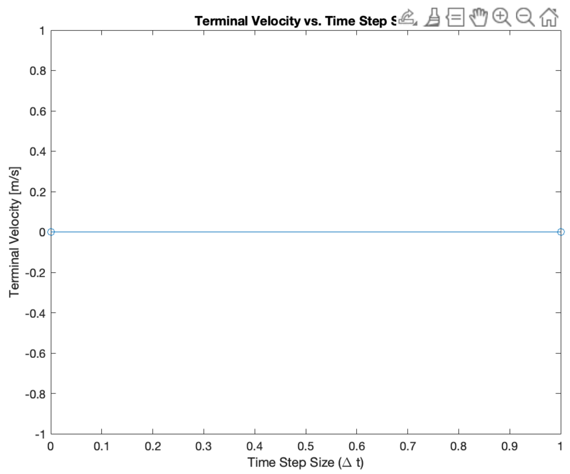
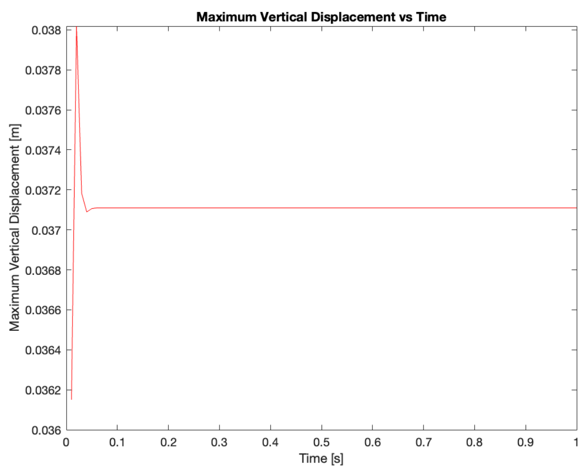


Changing the step size did not change the value of the terminal velocity.

- *Problem 3*

1. ymax stabilizes at 0.037 m, while Euler beam theory predicts ymax = 0.038 m, yielding a percentage difference of 2.63%. This small discrepancy demonstrates the high accuracy of the simulation.



2. The simulated plot shows that under large loads, the vertical displacement, ymax, diverges significantly from the Euler beam theory predictions. The simulation handles large deformations that Euler beam theory, which is only valid for small linear deformations, cannot accurately capture. The divergence in the curves indicates where the theoretical approach fails, which also highlights the benefit of using simulation over beam theory.