Control Theory Homework 1

Selina Varouqa, BS18-02

February 2020

Solving Second Order Differential Equation

variant p) $x'' - sin(t) = x' - 2x + 3$, $x'(0) = -1$, $x(0) = 0$
A)

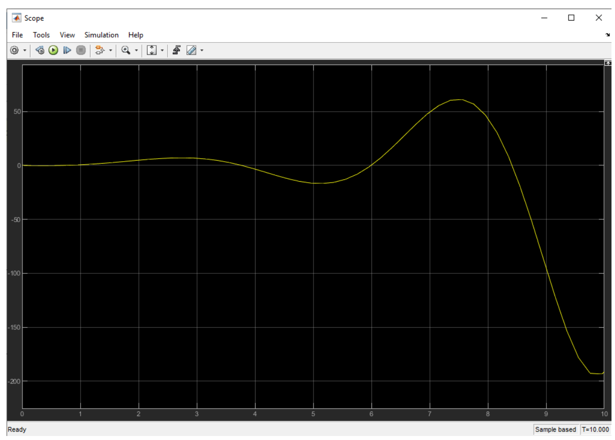

Figure 1: plot without transfer function

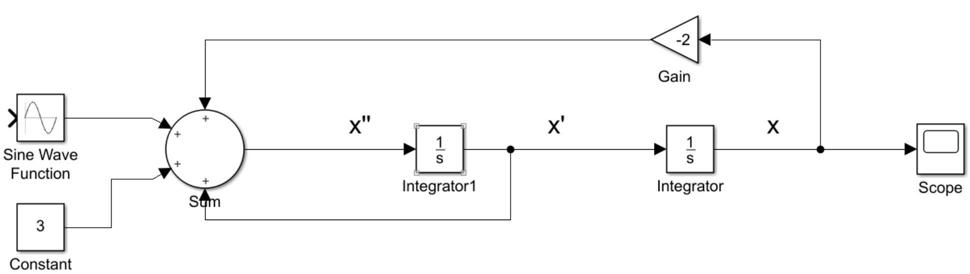

Figure 2: Simulink diagram without transfer function

B) $x'' - x' + 2x = 3 + sin(t)$

$$s = \frac{d}{dt}$$

$$x(s^2 - s + 2) = 3 + sin(t),\ u = 3 + sin(t)$$

$$x = \frac{u}{s^2 - s + 2}$$

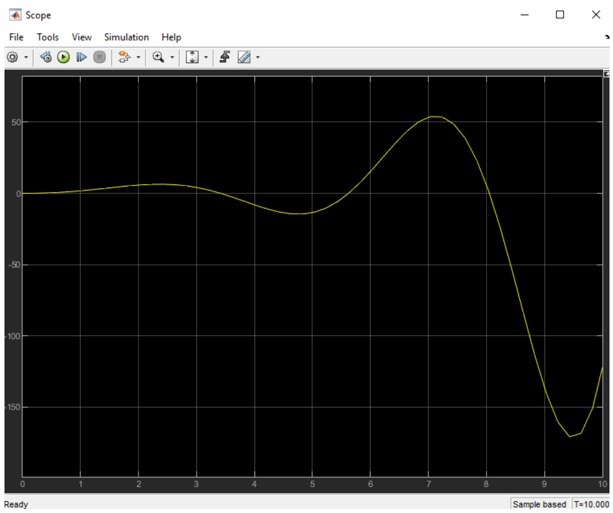

Figure 3: plot without transfer function

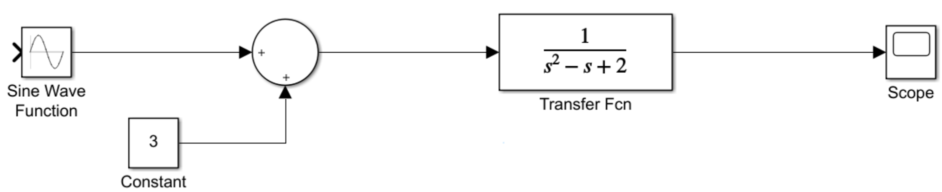

Figure 4: Simulink diagram with transfer function

C)

```
syms x(t);
dx= diff(x,t);
ddx= diff(dx,t);
s = dsolve(ddx - sin(t) == dx- 2*x + 3,dx(0)== -1, x(0) == 0);
fplot(s);
```

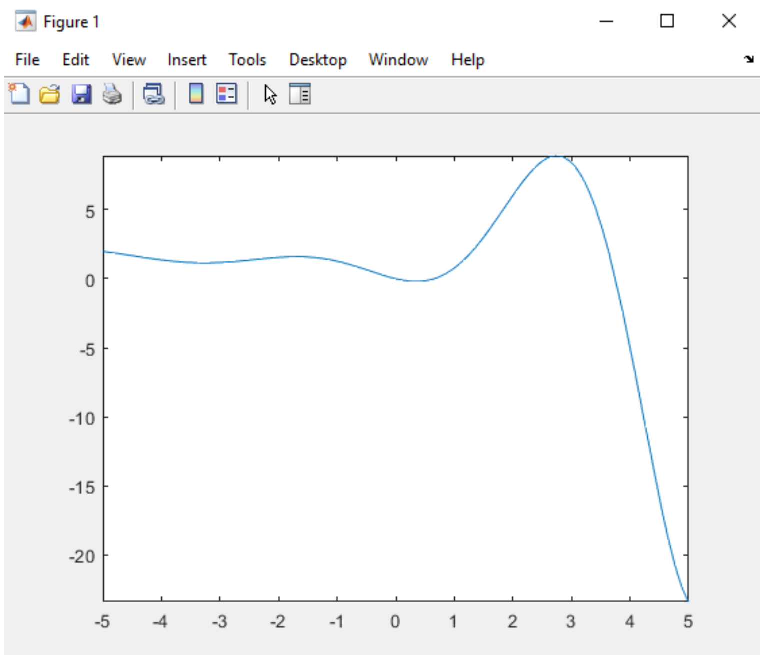

Figure 5: ODE solution graph

D)

```
syms s t X;
f = 3 + sin(t);
F = laplace(f, t, s);

X1 = s * X;
X2 = s * X1 + 1;

Solution = solve(X2 - X1 + 2*X - F, X);
solution = ilaplace(Solution, s, t);

fplot(solution, [0,10]);
```

The Plotting results the same graph as Figure 5 above.

State Space Model of a System

variant p)

$3x'' + 2x' - 3 = 2t - 2, y = 3x'$

$x'' = (-2/3)x' + (2/3)t + 1/3$

$$\begin{bmatrix} x' \\ x'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2/3 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix}$$

State Space Model of a System

variant p)

$x'''' + 3x''' + 2x'' + 2x' - 6 = 2u_1 + 3u_2 \ , \ y = x' + u_1 + 2u_2$

$x'''' = -3x''' - 2x'' - 2x' + 2u_1 + 3u_2 + 6$

$$\begin{bmatrix} x' \\ x'' \\ x''' \\ x'''' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ 1 \end{bmatrix}$$