Control Theory Homework 2

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1 Variant

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The variant for this assignment is A

2 Transfer functions calculations

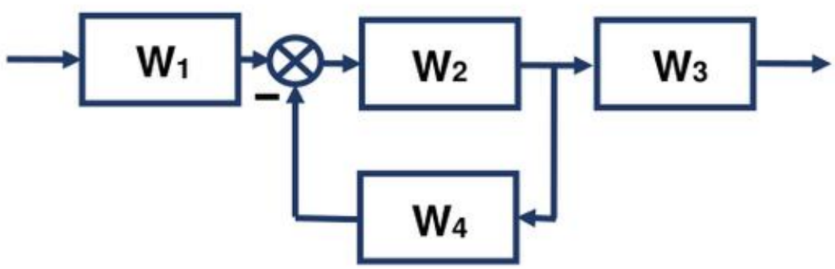


Figure 1: The System of Section 2

2.1 Calculating the total Transfer Function of the system

1. We start by combining W_2 and W_4 as a feedback loop which yields the following for variant A:

$$W_{2,4} = \frac{W_2}{1 + W_2 \cdot W_4} = \frac{\frac{1}{s+3}}{1 + \frac{1}{s+3} \cdot \frac{1}{s+0.5}} = \frac{s + 0.5}{(s+1) \cdot (s+2.5)}$$

2. we have W_1 and $W_{4,2}$ connected in series and we just multiply them and we get the following:

$$W_{1,2,4} = \frac{W_1 \cdot W_2}{1 + W_2 \cdot W_4} = \frac{1}{(s+2)(s+1)} \cdot \frac{(s+0.5)}{(s+1) \cdot (s+2.5)}$$

$$= \frac{s + 0.5}{(s+1)^2 \cdot (s+2) \cdot (s+2.5)}$$

3. we have W_3 and $W_{1,4,2}$ connected in series and we just multiply them and we get the following as a final transfer functions: final transfer function=

$$\frac{2(s + 0.5)}{(s+1)^2 \cdot (s+2) \cdot (s+2.5) \cdot (s+0.8)}$$

2.2 Step, impulse and frequency schemes and graphs

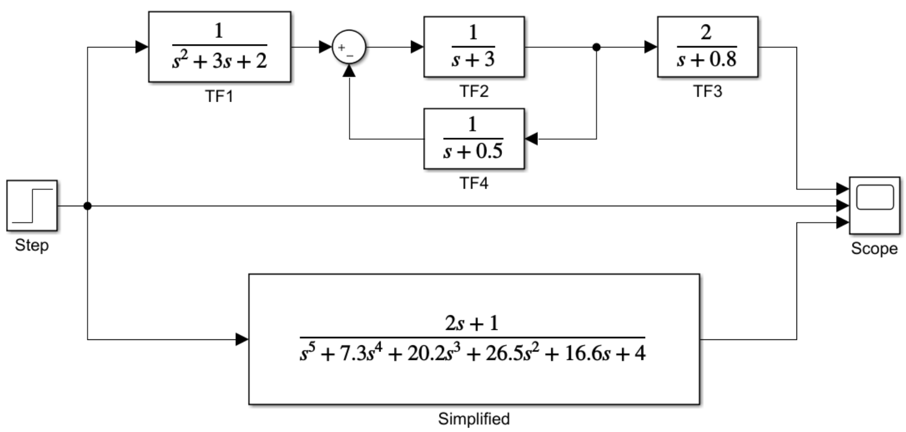


Figure 2: Scheme with step

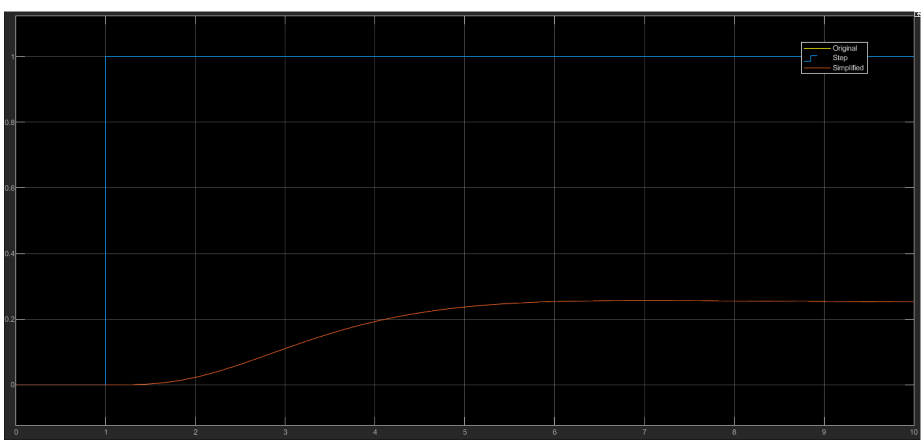


Figure 3: Graph with step(yellow and orange overlap)

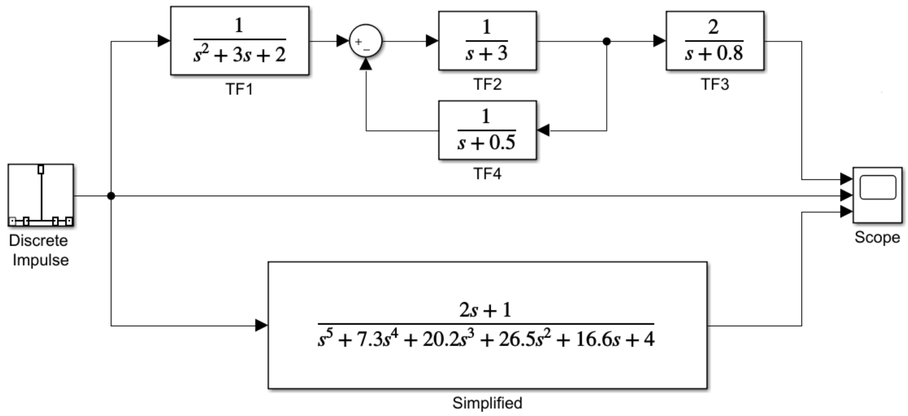


Figure 4: Scheme with impulse

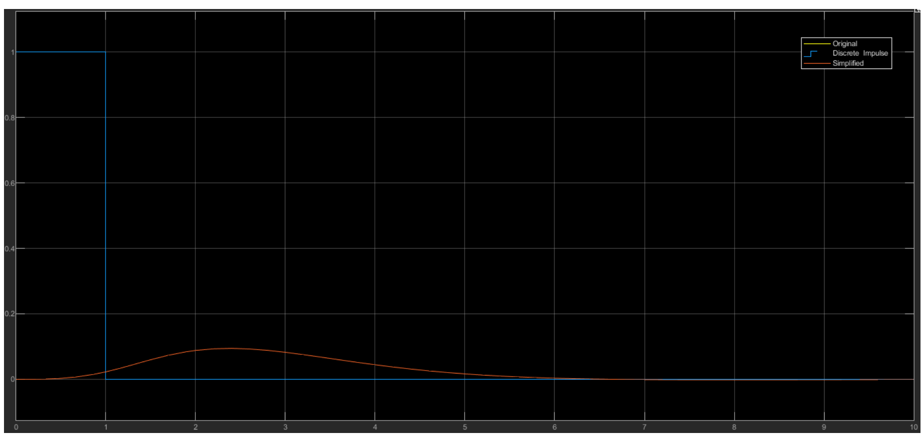


Figure 5: Graph with impulse(yellow and orange overlap)

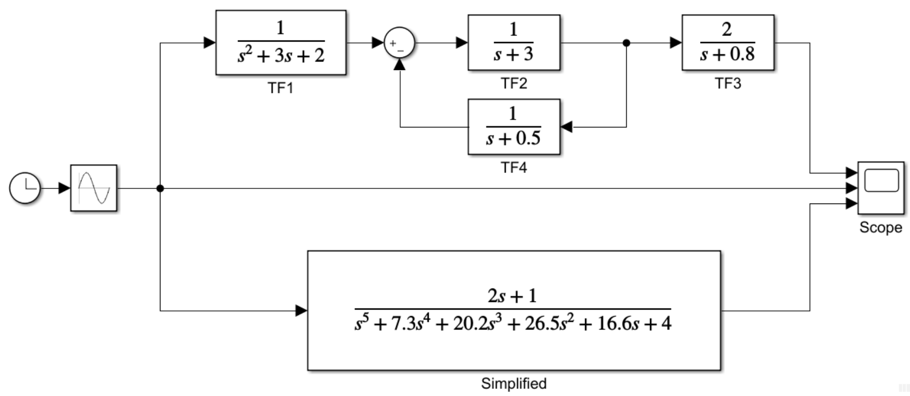


Figure 6: Scheme with frequency

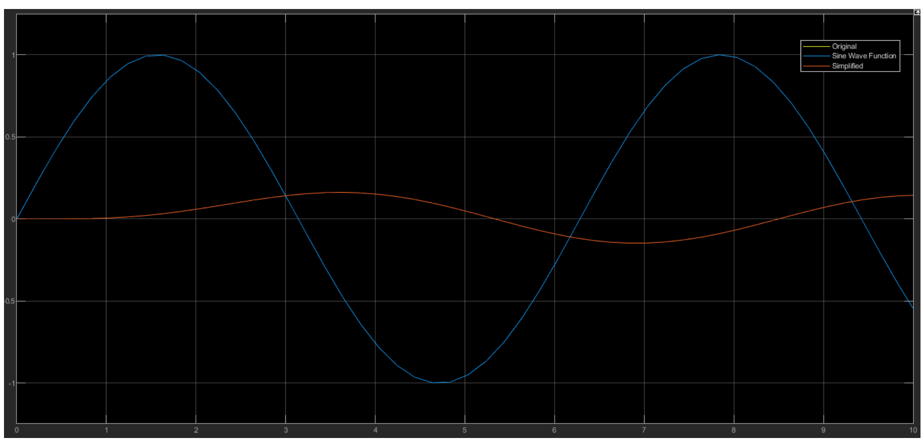
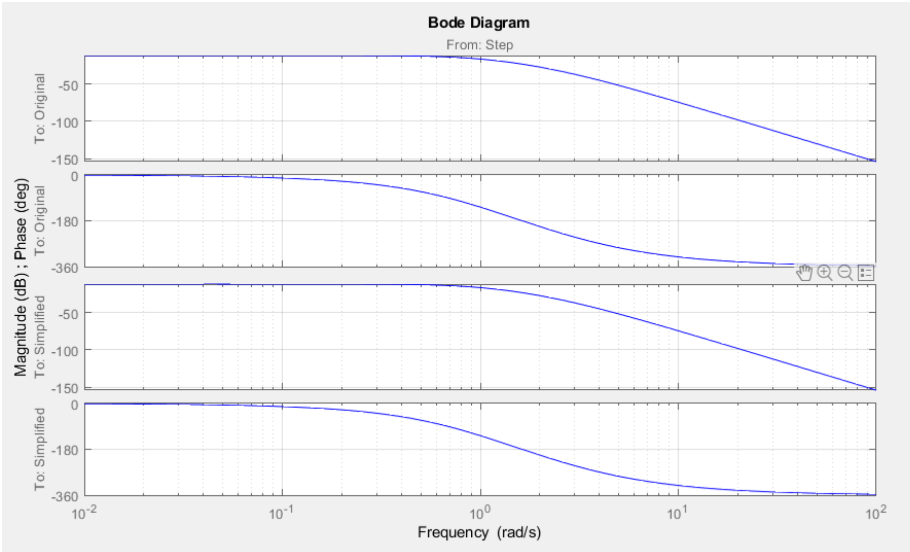


Figure 7: Graph with frequency(yellow and orange overlap)

2.3 Bode and Pole-Zero map plots

The following figures are generated with step signal as an input.

From the analysis of the Pole-Zero map plot, we can see that all the poles are negative which means that our system is stable.



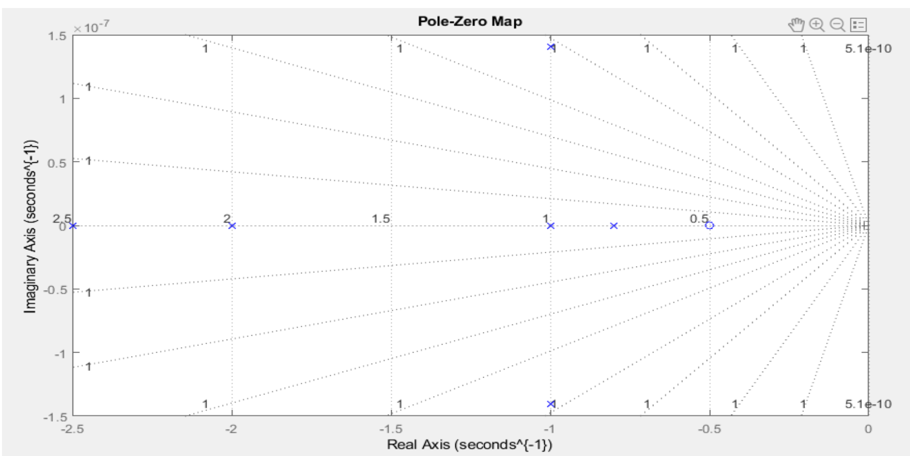


Figure 8: Pole-Zero diagram

3 Total Transfer function

Variant A:

$$W(s) = \frac{1}{s+2}, M(s) = \frac{1}{s+3}$$

if we assume f(t) input signal is zero at first, we will have a feedback loop, we calculate transfer function as follows:

$$I_g = \frac{W(s)}{1+W(s)} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}} = \frac{1}{s+3}$$

then we assume g(t) input signal is zero, we will have a feedback loop, we calculate transfer function as follows:

$$I_f = M(s) \cdot \frac{1}{1+W(s)} = \frac{s+2}{(s+3)^2}$$

The total transfer function would be:

$$X = \frac{1}{s+3} \cdot G + \frac{s+2}{(s+3)^2} \cdot F$$

4 Transfer Function of a System

We calculate the transfer function of this system with the following formula:

$$TF = C \cdot Inverse(s \cdot I - A) \cdot B + D$$

$$s.I = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}$$

$$s.I - A = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} s-1 & 0 \\ 0 & s-1 \end{pmatrix}$$

Inverse of last step is

$$\begin{pmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1} \end{pmatrix}$$

if we multiply it by C then by B then add D we get 0 which means that this transfer function does not exist for our values of A,B,C and D.

5 Transfer Function of a System

We calculate the transfer function of this system with the following formula:

$$TF = C \cdot Inverse(s \cdot I - A) \cdot B + D$$

$$s.I = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}$$

$$s.I - A = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} s-1 & -2 \\ 0 & s-1 \end{pmatrix}$$

Inverse of last step and multiplying by C is

$$\begin{pmatrix} 0 & \frac{1}{s-1} \end{pmatrix}$$

Multiplying by B and adding D we get

$$\begin{pmatrix} \frac{3s-2}{s-1} & 2 \end{pmatrix}$$

6 Calculate total Transfer function diagram

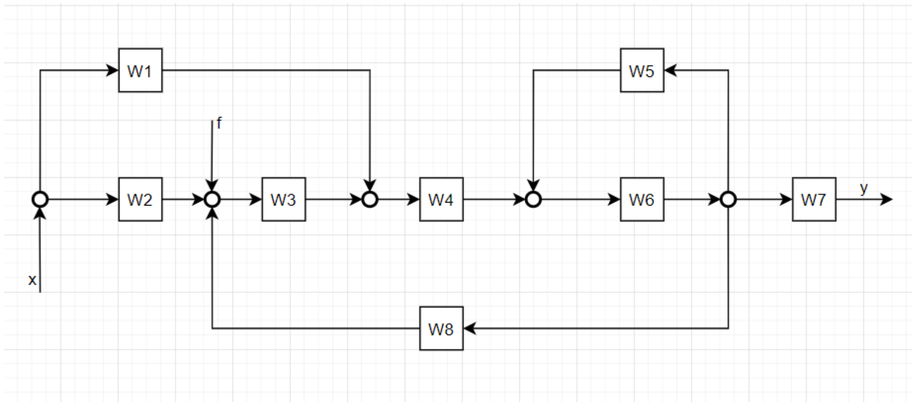


Figure 9: step 1, assuming that f is zero

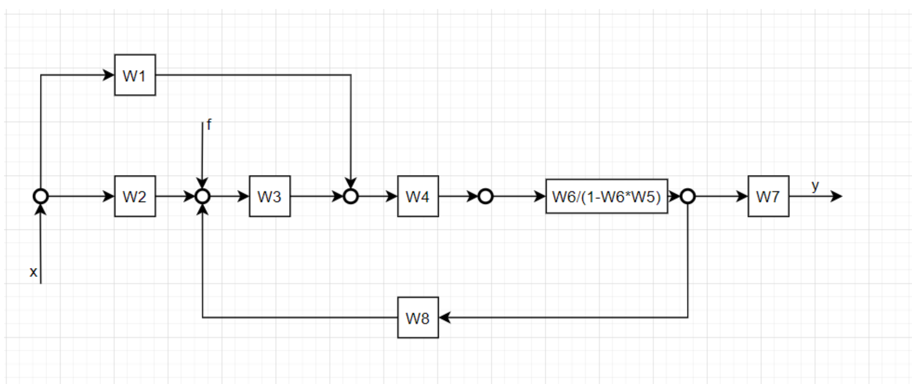


Figure 10: step 2, assuming that f is zero

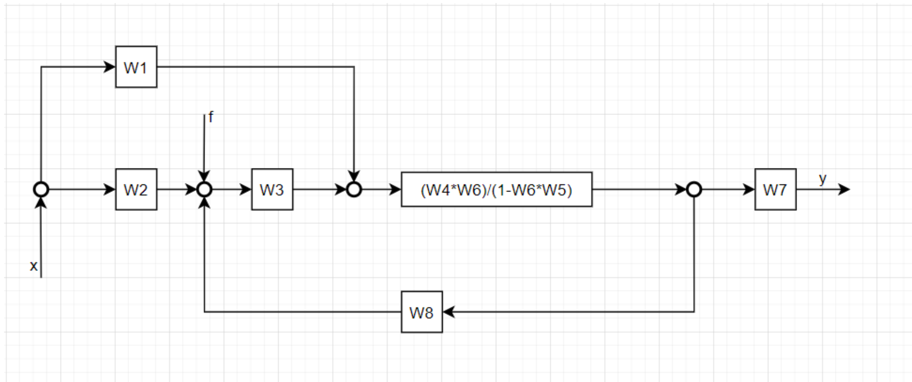


Figure 11: step 3, assuming that f is zero

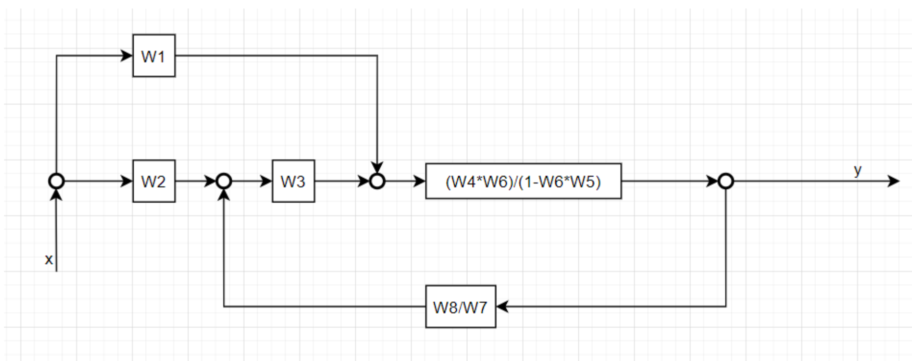


Figure 12: step 4, assuming that f is zero

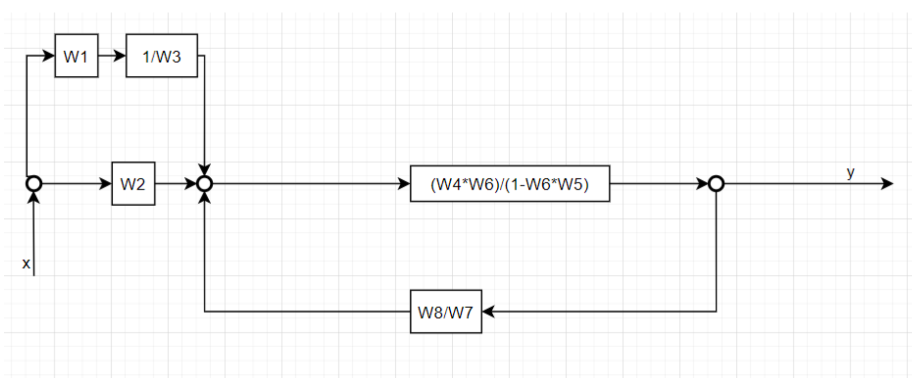


Figure 13: step 5, assuming that f is zero

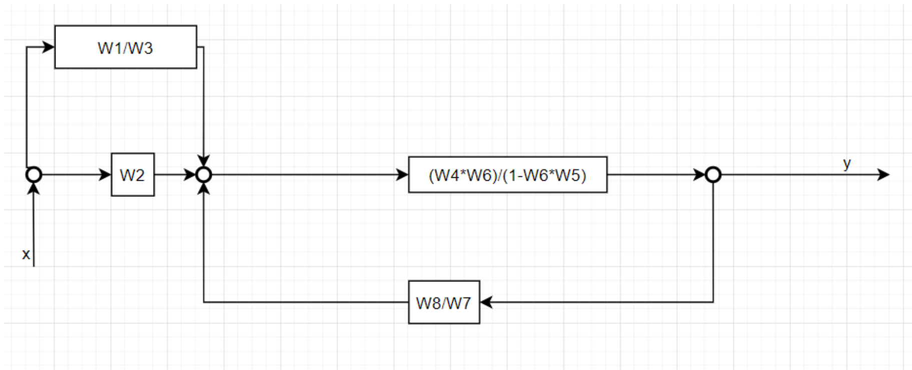


Figure 14: step 6, assuming that f is zero

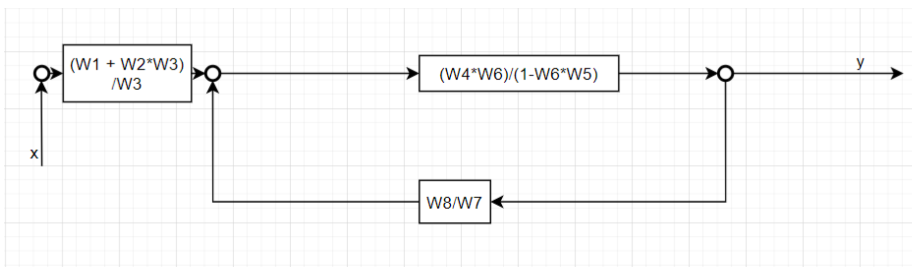


Figure 15: step 7, assuming that f is zero

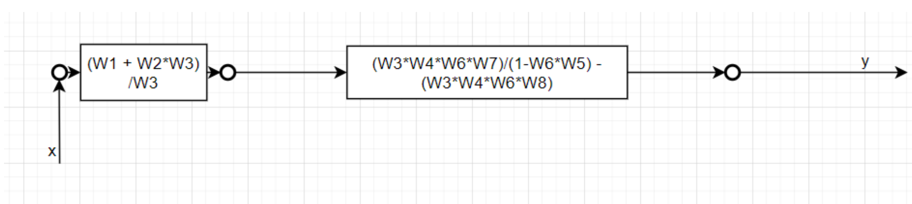


Figure 16: step 8, assuming that f is zero

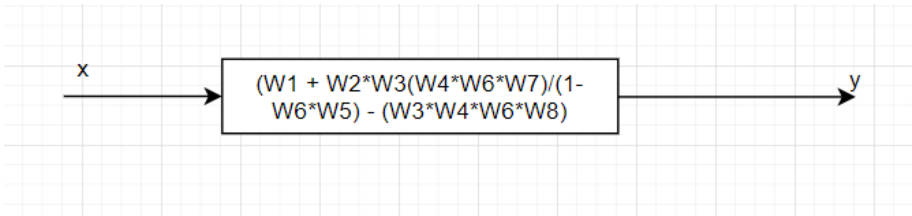


Figure 17: step 9, assuming that f is zero

now we assume that g is zero and we do it all over from the start as the following:

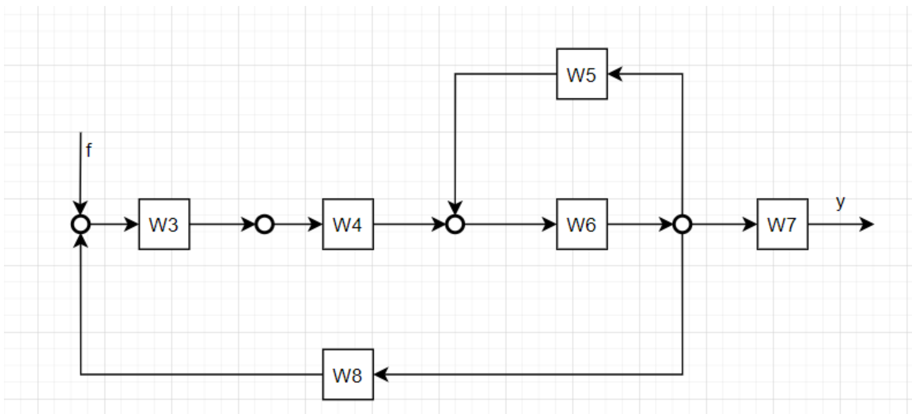


Figure 18: step 0, assuming that x is zero

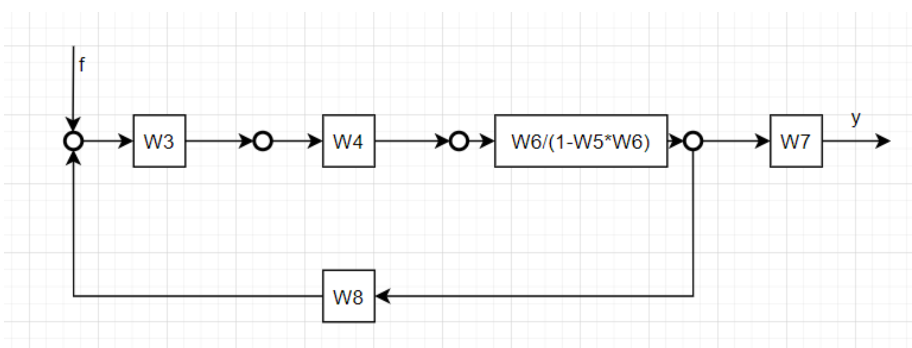


Figure 19: step 1, assuming that x is zero

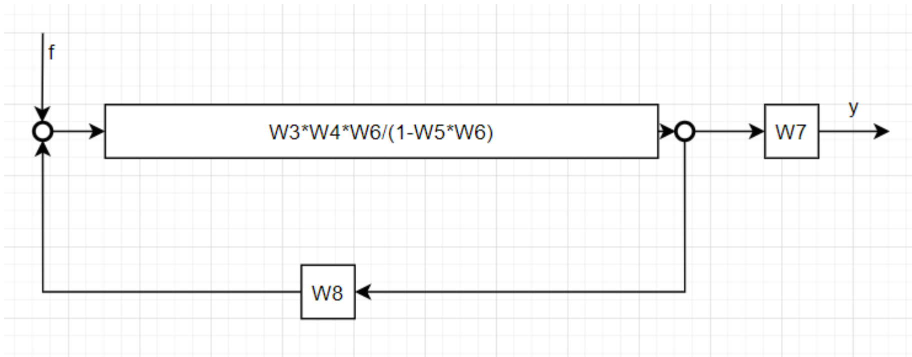


Figure 20: step 2, assuming that x is zero

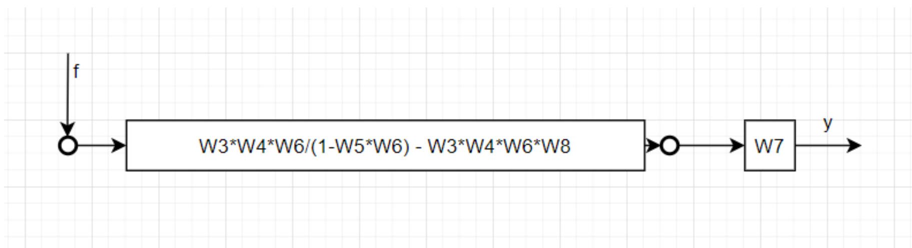


Figure 21: step 3, assuming that x is zero

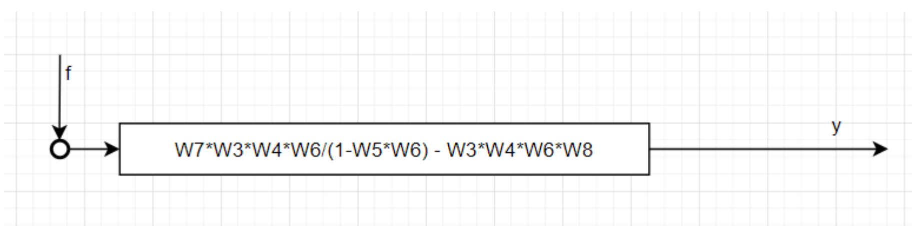


Figure 22: step 4, assuming that x is zero

So our final transfer function will be the following from the final two results of our steps:

$$\frac{(W_1 + W_2 \cdot W_3) \cdot W_4 \cdot W_6 \cdot W_7}{1 - W_5 \cdot W_6 - W_3 \cdot W_4 \cdot W_6 \cdot W_8} \cdot X + \frac{W_3 \cdot W_4 \cdot W_6 \cdot W_7}{1 - W_5 \cdot W_6 - W_3 \cdot W_4 \cdot W_6 \cdot W_8} \cdot F$$