Lecture notes: Parameter Estimation

Natalia Matrosova

April 10, 2020

These document contains my notes that helped me to understand the lecture slides. Here are the questions that were asked during the lecture and some important catches that were given by professor Savin verbally, that are not in the lecture slides.

About problem statement:

According to the problem statement we have a system, whose model is defined in terms of parameter θ. We need to find a stabilizing control law knowing the relations:
$\boldsymbol{f(x, u) = f(x, u, \theta)}$. The problem is that to find the control

law we need to know the higher order derivatives. We cannot get from the current step, but we can use the "historical" higher order derivatives from the previous step. The other part of the lecture is about how can we use Least squares to estimate the parameters. We need to perform several experiments to

estimate the parameters (the more experiments the better approximation). However, since we need to use historical data to approximate the higher order derivatives, we do not know which control to apply while we performing these experiments. This question is the subject of the next lecture.

Example 1: why is it important that $y = u$?

Initially we have the system:

$$\boldsymbol{m\ddot{q} + \mu\dot{q} + kq = u}$$

Then we introduced the notation $\boldsymbol{y = u}$ and get:

$$\boldsymbol{y = m\ddot{q} + \mu\dot{q} + kq}$$

The thing is: we do not know actual values of $\boldsymbol{\ddot{q}, \dot{q}, q}$ but we always can measure $\boldsymbol{y}$ by measuring $\boldsymbol{u}$ we apply.

About derivation of Least squares

The notation:

- $\widetilde{\theta}$ - estimated θ;
- $\epsilon_\theta = \theta - \widetilde{\theta}$ - parameter estimation error;
- $e_\theta = y - M\widetilde{\theta}$.

all $\widetilde{}$ denote that the variable is estimated. We want to minimize

the variable that we can directly measure. Our global goal is to minimize the parameter estimation error, but we do not measure it directly. Thus to achieve it we will minimize e_θ, which we measure directly.

We introduce the cost function:

$$J = \frac{1}{2} e_\theta^\top e_\theta$$

The right part of the equation is the norm of e_θ. Since e_θ is positive, when we minimize the norm of e_θ we minimize e_θ itself.