A Game-Theory and Risk-Field Based Automated Vehicle Motion Planning Method for Mixed Traffic Environments at Uncontrolled Intersections

Zhaojie Wang[1]; Guangquan Lu[2]; and Haitian Tan[3]

[1]Beijing Key Laboratory for Cooperative Vehicle Infrastructure Systems and Safety Control, School of Transportation Science and Engineering, Beihang Univ., Beijing, China
[2]Beijing Key Laboratory for Cooperative Vehicle Infrastructure Systems and Safety Control, School of Transportation Science and Engineering, and Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang Univ., Beijing, China (corresponding author). Email: lugq@buaa.edu.cn
[3]Beijing Key Laboratory for Cooperative Vehicle Infrastructure Systems and Safety Control, School of Transportation Science and Engineering, Beihang Univ., Beijing, China

ABSTRACT

With the development of technology, automated vehicles are beginning to appear in traditional multi-vehicle conflict scenarios such as uncontrolled intersections. The main challenge is to predict the behavior of other traffic participants. To address this issue and improve the traffic efficiency by optimizing the motion planning of automated vehicles (AVs) in MV-AV mixed traffic environments, a motion planning method based on game-theory and risk-field is proposed. Risk-field theory is used to build a driving behavior model of MVs at uncontrolled intersections, which AVs use to evaluate the driving behavior of other traffic participants. A game-theory framework assigns priority among AVs and MVs that conflict, considering their occupancy level of the conflict area and acceptable risk level. This method can obtain the optimal priority of vehicle in a linear calculation time while ensuring the driving safety in multi-vehicle conflict scenarios. The simulation results demonstrate the effectiveness of our motion planning method.

INTRODUCTION

Automated driving has drawn a great deal of attention in recent years. Google, Tesla, Baidu, and other companies have launched their own automated vehicles (AVs) and will be on the road in the near future. According to Society of Automotive Engineers (SAE), the intelligence level of vehicles is divided into five levels, L1-5. Most AVs currently on the road are at the L2, L3 level (Committee 2014). Mixed traffic environments in which different levels of automated vehicles coexist with human-driven vehicles on the road will be common in the long-term future (Zeeb et al. 2016). Motion planning at uncontrolled intersections is a challenging problem for autonomous vehicles because the movement of other traffic participants is difficult to predict (Kalra 2016). How to establish the motion planning method of AVs to improve the overall traffic efficiency of the intersection with the premise of safe interaction with surrounding vehicles is a major research hotspot.

The prediction of the movement of other traffic participants within uncontrolled intersections is the primary problem that needs to be addressed for AV motion planning. This requires modeling the driving behavior of human drivers. Many researchers have studied driving behavior at uncontrolled intersections, but most models are only applicable to single specific scenarios such as merging and moving-across behaviors (Liu et al. 2014a, Liu et al. 2014b, Liu et al. 2014c). Gap acceptance theory is often used when modeling moving-across behavior (Amin and Maurya 2015). However, due to the heterogeneity of drivers, it is difficult to set a uniform critical gap value, and gap acceptance theory is mainly used at intersections where there is a clear first/give way rule. In modeling moving-across behavior, game theory is often used. Liu et al. (2014b) used game theory to model the moving-across behavior of Chinese drivers at intersections. The model can accurately predict the order of vehicles passing through the conflict points. The behavior of human drivers in actual situations at uncontrolled intersections may not be a single moving-across behavior or merging behavior. AVs need a unified model to describe the driving behavior of drivers and predict their movement.

In previous work, this problem has been solved well by using risk field theory. Based on risk homeostasis theory (RHT) and preview-follower theory (PFT), a risk-field based driving behavior model is constructed. Risk field theory is used to quantify the risk vehicle endured during preview time and find the trajectory that conform to the driver's behavior while satisfying the risk threshold constraint (Tan et al. 2021, Wang et al.). Additionally, an exploratory algorithm was constructed to solve intersection deadlock scenarios. Simulation results have shown that the model can accurately reproduce the priority and trajectory of vehicles crossing the intersection and resolve multi-vehicle conflicts within a reasonable time. The model can be used by automated vehicles to predict the driving behavior of aggressive drivers at uncontrolled intersections. This paper also discusses the prospects for the use of the model in motion planning for AVs.

After solving the problem of predicting the behavior of other drivers, how to navigate through intersections is the next issue that should be addressed for AVs. The current mainstream methods include deep reinforcement learning, recurrent neural networks, numerical optimization methods, etc. By using recurrent neural networks, Roh et al. (2020) designed a Multiple Topologies Prediction (MTP), a data-driven trajectory-prediction mechanism, to achieve collision-free and time-efficient navigation across a variety of challenging intersection scenarios. Wang et al. (2020) calculated the end state of the vehicle entering intersection based on reinforce learning and constructed a Hamiltonian equation to solve for the acceleration of the vehicle in the path based on Pontryagin minimum principle. Although using deep learning to solve vehicle motion planning at unsignalized intersections can perform fast computation facing complex scenarios, each model needs to be trained for a specific intersection. Some models, although trained with a large amount of data, still have a small probability predicting vehicle conflict or collision in real situations. Meanwhile numerical optimization methods are often limited by the surge in computation time due to the increase in the number of conflict-related vehicles (Chandra and Manocha, 2022). These issues need to be addressed in future research.

Out of the several mentioned approaches, game theory is also used by many researchers to build the motion planning model for automated vehicles at uncontrolled intersections. Bahram et al. (2015) presents a novel cooperative-driving prediction and planning framework for dynamic environments based on the methods of game theory. The proposed algorithm can be used for highly automated driving or as a sophisticated prediction module for advanced driver-assistance systems with no need for intervehicle communication. Chandra et al. (2022) designed a new auction called GAMEPLAN, which assigns a higher priority to more aggressive or impatient drivers and a lower priority to more conservative or patient drivers at uncontrolled intersections. It is theoretically proven to be effective in avoiding vehicle collisions. The use of game theory to determine the priority of vehicles at uncontrolled intersection does not require the pre-training process for specific intersections and can effectively avoid deadlock at intersections through reasonable design.

Considering the above factors, this paper adopts the sponsored search auction (SSA) method in game theory to solve the problem of deciding the priority of automated vehicles at uncontrolled intersections. The conflict-related vehicles are considered as players in the auction. By considering the driving aggressiveness and current motion state of each vehicle, automated vehicles change their acceptable risk level to maximize the overall utility function of players in the auction, to improve the traffic efficiency. This method can obtain the optimal priority of vehicles in linear computation time and ensure the driving safety in multi-vehicle conflict scenarios.

The rest of this paper is organized as follows. In section 2, a risk field model for describing multi-vehicle interactions at intersections will be presented. In section 3, a brief overview of the authors' previous work on risk field-based driving behavior model at uncontrolled intersections is provided, in addition to how it is applied by automated vehicles to predict the acceptable risk level of other traffic participants. Section 4 elaborates on the design of the SSA framework and analyzes its optimality. The simulation results and conclusion will be discussed in section 5.

METHODS

Risk-field model. The risk field model we used is constructed by Tan et al. (2021). It provides a sound description of the complex traffic environment within a unified framework. The risk field model is built based on a Lagrangian coordinate system with the center of the vehicle as the origin. The longitudinal and lateral directions of the vehicle are set as the x and y axes. The risk generated by the k-th vehicle for point (x, y) is defined as Equations 1 and 2.

$$R_{vehicle,k}(x,y,t)=\frac{1}{\delta_k(x,y,t)+1}, \tag{1}$$

$$\delta_k(x,y,t)=\sqrt{\delta^2_{x,k}(x,y,t)+\delta^2_{y,k}(x,y,t)}. \tag{2}$$

Where, $\delta_k(x, y, t)$ is the attenuation coefficient, which is calculated from the longitudinal $\delta_{x,k}(x, y, t)$ and lateral $\delta_{y,k}(x, y, t)$ attenuation coefficients, defined as Equation 3 and 4, respectively.

$$\delta_{x,k}(x,y,t)=\frac{\beta_{k,x}\cdot\max(|x|-\frac{1}{2}\cdot L_k,0)}{\alpha_{k,x}\cdot v_{k,x}(t)+1}, \tag{3}$$

$$\delta_{y,k}(x,y,t)=\frac{\beta_{k,y}\cdot\max(|y|-\frac{1}{2}\cdot W_k,0)}{\alpha_{k,y}\cdot v_{k,y}(t)+1}. \tag{4}$$

Where, $v_{k,x}(t)$ and $v_{k,y}(t)$ are the vehicle's lateral speed and longitudinal speed. $\alpha_{k,x}$ and $\alpha_{k,y}$ represent the speed factors, $\beta_{k,x}$ and $\beta_{k,y}$ are distance factors, L_k is the vehicle's length and W_k is the width of vehicle. From the definition of the risk field, the risk generated by a vehicle to a certain point on the road is influenced by the distance between the vehicle, the point, and the speed of the vehicle. Risk takes values between 0 and 1. Other characteristics of the risk field, and the analysis of its use in the following and changing lanes models can be found in the work.

Risk-field based driving behavior model. Based on the risk-field model above, the driving behavior of human driving vehicles (MV) at uncontrolled intersections was modeled. In risk homeostasis theory (RHT), drivers attempt to remain at a stable level of acceptable risk on the road. If the driver's perceived risk exceeds the acceptable risk level, measures will be taken to reduce the driving risk. Combining RHT with preview-following theory (PFT), drivers usually keep the risk below acceptable risk during preview time. Therefore, the key to model human driving behavior at uncontrolled intersections is to control vehicles acceleration during the preview time so that the driver-perceived risk remains near the acceptable risk level. The brief steps are as follows:

a) Assume that the surrounding vehicles travel along the desired path at the current speed during the preview time.

b) The target vehicle calculates the position of the vehicle in the preview time with a as the acceleration, while kinematic constraints such as the maximum velocity and acceleration need to be satisfied.

c) Calculate the risks that the target vehicle endured during the preview time to form a risk sequence according to the risk field model.

d) Determine whether the maximum value of the risk sequence is less than the acceptable risk level, or the initial risk of the vehicle is greater than the acceptable risk level but decreases gradually over time and can fall below the threshold in the preview time.

e) If the currently selected acceleration does not satisfy the above conditions, reselect the acceleration from the action space to calculate a new risk sequence and repeat steps 2, 3 and 4.

An exploratory algorithm for solving intersection deadlock situations was also constructed. Details and validity analysis of the model can be found in previous work from the authors (Wang et al. 2022). A method for estimating acceptable risk levels for MVs can be developed based on this model. The automated vehicle can obtain the acceleration of the MVs through sensors and calculate the maximum risk to the MV within preview time, driving at the current acceleration, as the acceptable risk level $R_{thre,MV}$ of the MV. The range of $R_{thre,MV}$ was limited to 0.345 to 0.8 based on the statistics of acceptable risk levels for vehicles at uncontrolled intersections from actual data. If $R_{thre,MV}$ exceeds the upper (lower) limit, it will be set to the upper (lower) limit value.

Game theory based motion planning method for automated vehicles. Sponsored search auction (SSA) is one of the auction theories in game theory. The auction is a matter of determining the ownership of K items among n players. Each player a_i has a private valuation v_i that represents the highest price the player is willing to pay in this auction, a bid b_i that represents the player's bid in the auction, and a value α_i that represents the value of the item the player receives after bidding. In any auction there are $b_1 > b_2 > \ldots > b_k$ and $\alpha_1 > \alpha_2 > \ldots > \alpha_k$, if there is a case where bids and values are equal, then the equal ones are randomly ordered. Each auction theory has a set of rules that determine the problem of item ownership, the problem of player bids, and a utility function to determine the value players receive in the auction based on their bids. The utility function u_i of a_i is shown in Equation 5.

$$u_i(b_i) = v_i\alpha_i - \sum_{j=i}^{k} b_{j+1}(\alpha_j - \alpha_{j+1}). \tag{5}$$

The left side of Equation 5 represents the benefit received by a_i in this auction, and the right side is the gained value minus the required payment. The first item on the right is determined by the value of the item a_i acquired and the private value v_i, the second item is determined by the players with a bid less than b_i and the value of the items they receive. The derivation and detailed analysis of the utility function can be found in this work.

When a certain AV_i travels at an uncontrolled intersection, we consider vehicles whose risk field effect on AV is greater than 0.8 in the preview time as a set A = $\{veh_1, veh_2, \ldots, veh_m\}$. Then the players participating in the auction at this moment are denoted as a = $(A \cup AV_i)$. The auction is to determine the priority order δ^* for vehicles passing through the intersection in conflict with AV_i. At uncontrolled intersections, vehicles priorities can be viewed as an allocation problem for multiple items. The optimization goal is to ensure that the overall utility function is maximized in each auction round based on the players bids.

The priority order in an auction is a sorting result of the set a, and for a certain scenario where there can be many results of sorting most of them are suboptimal and likely to cause collisions and deadlocks. The optimal sorting result δ* is defined as the result that makes this auction incentive compatible and social welfare maximum. When conducting an auction, AVs will treat the MVs as rational players trying to maximize their own utility functions. A decent auction model will motivate players to bid truthfully during the auction. The auction framework for generating optimal priority order will be presented in the following section and its incentive compatibility and social welfare will be discussed.

In the auction framework proposed in this paper, the player private value is used to characterize the vehicle priority in a multi-vehicle conflict scenario, and there are many indicators that can be used to calculate the vehicle priority, such as the speed and the distance to the conflict area, etc. However, these objective indicators cannot capture the impact of driver's social preferences, such as driving aggressiveness, on the auction. It has been proven that ignoring players' value preferences may lead to untruthful bids. To solve this problem, this paper binds the players' private value to the risk value in risk field theory, which can reflect not only the distance of the vehicle to the conflict area and vehicles speed, but also drivers driving aggressiveness. The players' private value v_i is defined as Equation 6.

$$v_i = w_i R_{thre,i} R_{current,i}. \tag{6}$$

Where, $R_{thre,i}$ is the acceptable risk level and represents the vehicles driving aggressiveness; $R_{current,i}$ is the current risk value generated by the vehicle to the conflict point; and the conflict point is defined as the position where the vehicle takes the highest risk in preview time. From the definition of the risk field, it can be seen that $R_{current,i}$ is larger when the vehicle is faster and closer to the conflict point. So, $R_{current,i}$ indicates the extent to which the vehicle occupies the conflict area. To avoid certain vehicle stalled in the original lane for too long causing traffic congestion, vehicles which have waited at incoming lane for a long time are rewarded. w_i represents the waiting time bonus factor. When there are car-following behaviors at the intersection, and the private value of the rear vehicle is greater than the front vehicle, a portion of the private value of the rear vehicle will be transferred to the front vehicle. This type of value transmission is used in many models, such as the social force model.

Each vehicle will receive a time reward α_i for submitting a bid b_i. Since the time taken by vehicles to cross the intersection can only be calculated after the entire traffic process is done, a simplified way is needed to describe the time reward for vehicles with different priorities. The time for all vehicles to cross the intersection at current speed is averaged as benchmark t_1. Define the time reward α as presented in Equation 7.

$$\alpha_1 > \alpha_2 > \ldots > \alpha_k \Leftrightarrow \frac{1}{t_1} > \frac{1}{t_2} > \ldots > \frac{1}{t_k}, t_i = t_{i-1} + \Delta t \quad (7)$$

Where, Δt is the time delay of the adjacent priorities. This simplified approach does not allow for accurate calculation of the actual time reward received by the vehicle, but the $v_i\alpha_i$ term applied to the utility function $u_i(b_i)$ can be used as an estimate value of the time reward. The players payment term, as shown in Eq. 8, characterizes the risk associated with the vehicle's priority order. The lower the priority assigned to the vehicle, the smaller the payout item will be. In summary, when AVs have a risk greater than 0.8 in the preview time, an auction is held with the conflict-related vehicles involved as players. The vehicle with more bids will get the higher priority to pass through the intersection.

$$\sum_{j=i}^{k} b_{j+1}(\alpha_j - \alpha_{j+1}) \quad (8)$$

In auction theory, for player a_i, if the bids of other players are fixed such that the bid b_i with the largest utility $u_i(b_i)$ is the dominant strategy. Incentive compatibility refers to the fact that the auction incentivizes participants to bid based on their truthful bid, and forging bids would undermine their utility.

Lemma 1: For each player $a_i \in a$, bidding strategy $b_i = v_i$ is the dominant strategy. Chandra and Manocha (2022) have proven that in the SSA framework, if the other players are bidding according to their private value, for player a_i bids not according to private value, over bidding and under bidding leads to a decrease in the utility function $u_i(b_i)$. It shows that our proposed SSA based game framework incentivizes players to bid according to their private value, therefore with incentive compatibility.

In auction theory, social welfare is defined as $\Sigma v_i\alpha_i$. To maximize social welfare, it is necessary to find a set of bidding strategies $b_1 > b_2 > \ldots > b_k$ that maximize the total gained value of $\Sigma v_i\alpha_i$.

Lemma 2: For each player $a_i \in a$, bidding strategy $b_i = v_i$ can maximize social welfare. The proof of this lemma is given by Suriyarachchi et al. (2022) using the induction method. It shows that if each player bids $b_i = v_i$, it not only maximizes their own utility function but also maximizes the total time rewards for all players. Therefore, the priority order of vehicle is assigned to the vehicle with the higher private value, as shown in Equation 9.

$$\delta^* = \delta_1, \delta_2, \ldots, \delta_k, b_1 > b_2 > \ldots > b_k \Leftrightarrow v_1 > v_2 > \ldots > v_k. \quad (9)$$

The acceptable risk levels of AVs are adjustable. To maximize the overall utility of all vehicles during the auction, the AVs are made to maximize the time reward while reducing the payout term. The acceptable risk levels of AVs in the auction are used as a control variable to optimize Σu_i. Given that all the AVs participating in the auction as players form a set $\Omega \in a$. The optimization objective function is defined as Equations 10 – 12.

$$\underset{R_{thre,\Omega}}{\operatorname{argmax}} \sum_{i=1}^{k} (v_i \alpha_i - \sum_{j=i}^{k} b_{j+1} (\alpha_j - \alpha_{j+1})) \tag{10}$$

$$v_i \alpha_i - \sum_{j=i}^{k} b_{j+1} (\alpha_j - \alpha_{j+1}) > u_{\min} \tag{11}$$

$$0.345 < R_{thre,av} < 0.8, R_{thre,av} \in R_{thre,\Omega} \tag{12}$$

Where, $R_{thre,\Omega}$ are the acceptable risk levels for AVs in **Ω**. Equation 10 maximizes the difference between the total gained value and players payout term by control variable $R_{thre,\Omega}$. Equation 11 guarantees that the utility of any vehicle in the optimization process will not be smaller than $u_{\min}$; therefore, the vehicle will not completely sacrifice its own benefits for the overall optimization of the area. Equation 12 is the constraint on the acceptable risk level for AVs, obtained statistically based on the actual acceptable risk level for most drivers at uncontrolled intersections. Theoretically the above optimization problem can be solved by splitting into multiple linear programs. However, as the number of AVs in intersection increases, the compute complexity increases exponentially. So, the acceptable risk level of AVs was divided into three discrete values as [0.4, 0.6, 0.8], based on the actual intersection vehicle acceptable risk level data to simplify the calculation. Once the AVs obtained their current optimal acceptable risk level $R^*_{thre,\Omega}$ after SSA, AVs will choose their actions based on the proposed risk-field based driving behavior model.

Case study and discussion. To demonstrate the effect that an AV equipped with the proposed motion planning model can achieve on the overall traffic efficiency, 6 vehicles were randomly selected at the uncontrolled intersection, one of which was an AV. These vehicles have random initial states including the incoming lanes and target lanes, the initial distance to the intersection entrance, ODs and the initial speed. The acceptable risk levels of MVs are randomly assigned based on uniform distribution. The distance and speed of the vehicle from the incoming lane at the initial moment are uniformly distributed between [0,20] m and [2,5] m/s, respectively. The final scenario and the initial state of vehicles is shown in Figure 1, where the green vehicles are MVs, and the blue vehicle is an AV.

For comparison, the speed profiles of all vehicles in the scenario are plotted at the situation where vehicle 6 is a MV with an acceptable risk level of 0.6 and the situation where vehicle 6 is an AV, shown in Figure 2(a) and (b), respectively. When vehicle 6 is an AV, to maximize the overall utility function of the intersection the change of its acceptable risk level is shown in Figure 2 (c). Comparing Figure 2 (a) with Figure 2 (b), it can be found that if vehicle 6 is a MV, vehicles at intersection would be stuck in a longer inefficient traffic process from 2s to 7s due to the interactions between vehicles. When vehicle 6 is an AV, by changing its acceptable risk level according to our proposed motion planning method, vehicles at intersections resolved their conflicts in shorter periods of time, reducing the overall delay by 16.6s (average of 2.7s per vehicle).

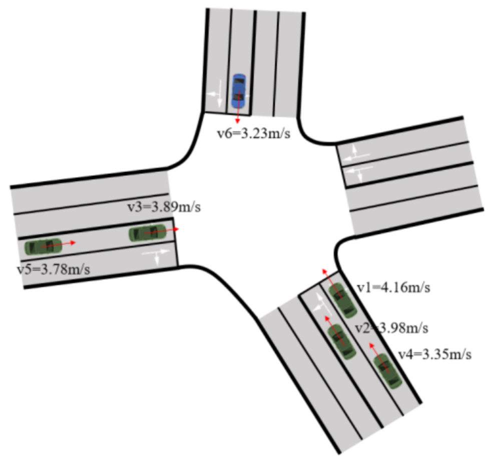
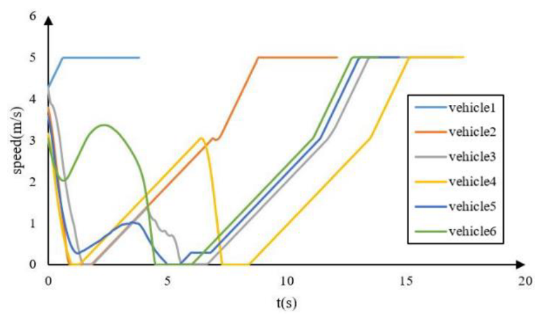
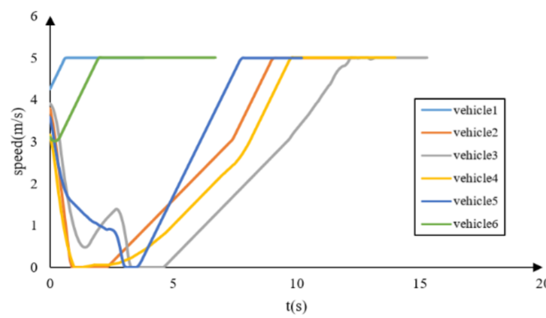


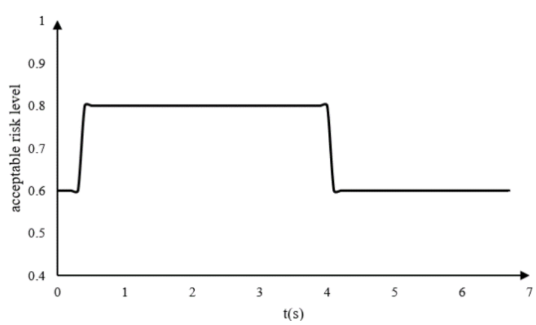
Figure 1. The initial state of the traffic scene.



(a) Speed profiles when vehicle 6 is MV



(b) Speed profiles when vehicle 6 is AV



(c) Acceptable risk level of vehicle 6 while it is AV

Figure 2. Vehicle speed and acceptable risk level.

Vehicle 6 in the above example has a large impact on the efficiency of the intersection because it has a spatial-temporal conflict relationship with most of the other vehicles. The actual situation where only one AV is at the intersection, is very limited. Because if the main vehicle that causes traffic inefficiency at the intersection is not AV, or if the AV does not conflict with other vehicles, the improvement of traffic efficiency is insignificant no matter how optimized the motion planning of AV. So, the impact of the number of AVs on the overall traffic efficiency at the intersection still needs to be discussed. For this purpose and according to most realistic situations, the motion states of 8 vehicles at the intersection will randomly be initialized, where the number of AVs is set to [2,4,6], respectively. Then, 50 simulations are conducted to compare the effect of the model on the intersection traffic efficiency at different AV/MV ratios. Figure 3 shows the average completion time of the vehicle with different AV/MV ratios.

The average completion times when the number of AVs is [2, 4, 6] are [16.7, 14.7, 12.2] s, respectively. Figure 3 shows that as the number of AVs increases, the probability that the primary vehicles that may cause traffic congestion are AVs increases. AV vehicles improve the average completion time by changing the acceptable risk level. When the number of AVs is 2 there are still many vehicles with completion time longer than 30s, while this situation is almost non-existent when the number of AVs is 6.

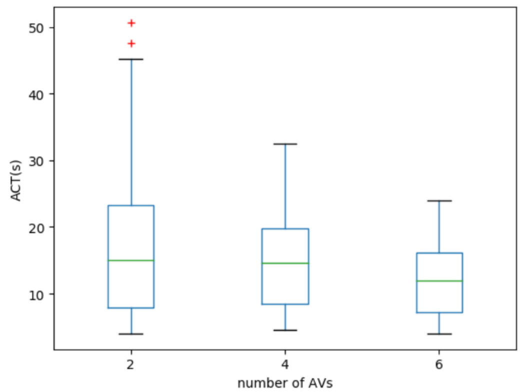


Figure 3. Average completion time with different number of AVs.

CONCLUSION

The driver's driving behavior at uncontrolled intersections was modeled based on risk field theory from previous work. Based on that, this paper proposed an automated vehicle motion planning method that uses the SSA framework to maximize the utility function of conflict-related vehicles. By controlling the acceptable risk level of automated vehicles, it can improve the overall traffic efficiency of uncontrolled intersections in AV/MV mixed traffic environments. In the SSA framework, the private value of the vehicle considers the acceptable risk level of drivers and awaiting time of stopped vehicles, which can reflect the influence of different drivers driving aggressiveness on the gaming process. The simulation results show that our model is effective in improving the average completion time of vehicles when the conflict-related vehicles at the intersections are autonomous vehicles.

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