Short Assignment 0 - Solutions

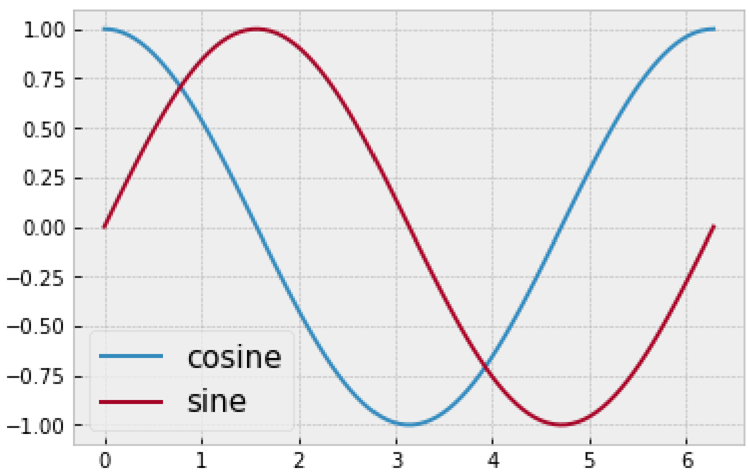
Python and Jupyter Notebook

In [1]:

```
import numpy as np
import numpy.random as npr
import matplotlib.pyplot as plt
%matplotlib inline
plt.style.use('bmh')
```

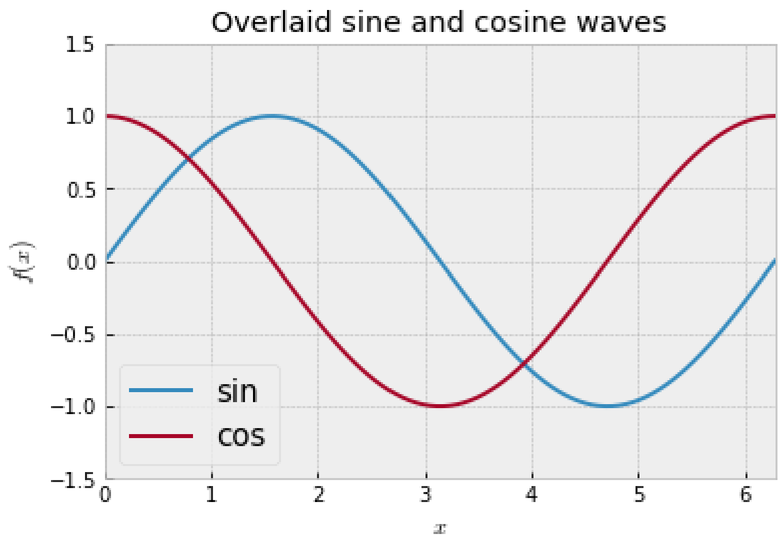
In [2]:

```
#3.1.A

x=np.linspace(0, 2*np.pi, 100)
#creates 100 evenly spaced points in the interval [0,2pi]

plt.plot(x, np.cos(x), label='cosine')
plt.plot(x, np.sin(x), label='sine')
plt.legend(fontsize=15);
```

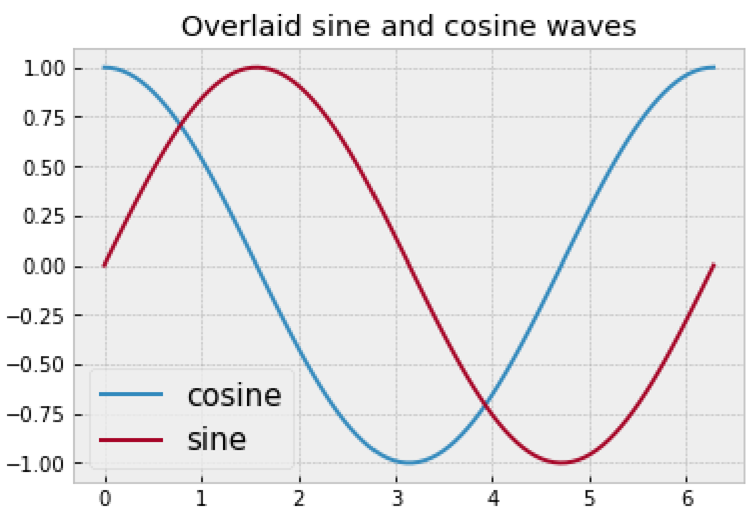


In [3]:

```
#3.1.B

ax = plt.axes()
ax.plot(x, np.sin(x), label='sin')
ax.plot(x, np.cos(x), label='cos')
ax.legend(fontsize=15);

ax.set(xlim=(0, 2*np.pi), ylim=(-1.5, 1.5),
       xlabel='$x$', ylabel='$f(x)$',
       title='Overlaid sine and cosine waves');
```



In [4]:
```
#3.1.C

plt.plot(x, np.cos(x), label='cosine')
plt.plot(x, np.sin(x), label='sine')
plt.legend(fontsize=15)
plt.title('Overlaid sine and cosine waves');
```



Course Policies

Please read the course syllabus for a full description of course policies.

Pre-requisites

1. **Consider the function**

$$f(x) = \frac{1}{2}\phi(wx + b)$$

where $\phi(x) = x^2$.

Use the chain rule to compute the derivative $\frac{\partial f}{\partial x}$. Show your work (you may include a picture or scan of your handwritten solution.)

Solving $\frac{\partial f}{\partial x}$:

Let $v = wx + b$, so we can write $f(x) = \frac{1}{2}\phi(v)$, where $\phi(x) = x^2$ and $\phi'(x) = 2x$. Applying the chain rule, we have:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \phi}\frac{\partial \phi}{\partial v}\frac{\partial v}{\partial x}$$

where

$$\begin{aligned}\frac{\partial f}{\partial \phi} &= \frac{1}{2} \\ \frac{\partial \phi}{\partial v} &= \phi'(v) = 2v = 2(wx + b) \\ \frac{\partial v}{\partial x} &= w\end{aligned}$$

Putting it together:

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{1}{2}2(wx + b)w \\ \frac{\partial f}{\partial x} &= (wx + b)w\end{aligned}$$

1. **Suppose you are solving a system of linear equations with more equations than parameters. This type of problem generally (but not always) does not have a solution. How can we approximate a solution to this problem? To answer this question include a short paragraph answer. If you prefer, you can answer this question in equation form.**

An overdetermined system will typically not have a solution, we can still approximate a solution by the Least Squares method. As so, for a data matrix $\mathbf{X}$ that contains the coefficients of the system, with dimension $N \times M$, where $N > M$; the set of unknown parameters $\mathbf{w}$ (total of M) and the constant term $\mathbf{t}$, the Least Squares solutions for the unknown parameters is given by:

$$\mathbf{w} = \mathbf{X}^{\dagger}\mathbf{t} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{t}$$

1. **What is the second central moment of a random variable $\mathbf{X}$ and how does it relate to the covariance of two random variables $\mathbf{X}$ and $\mathbf{Y}$? To answer this question you may include a short paragraph answer and/or equations.**

The second central moment of r.v. $\mathbf{X}$ is its *variance* and can be computed as: $E[(\mathbf{X} - E[\mathbf{X}])^2]$.

The covariance of two r.v.s $\mathbf{X}$ and $\mathbf{Y}$ can be computed as: $E[(\mathbf{X} - E[\mathbf{X}])(\mathbf{Y} - E[\mathbf{Y}])]$, and it measures the joint variability between $\mathbf{X}$ and $\mathbf{Y}$.