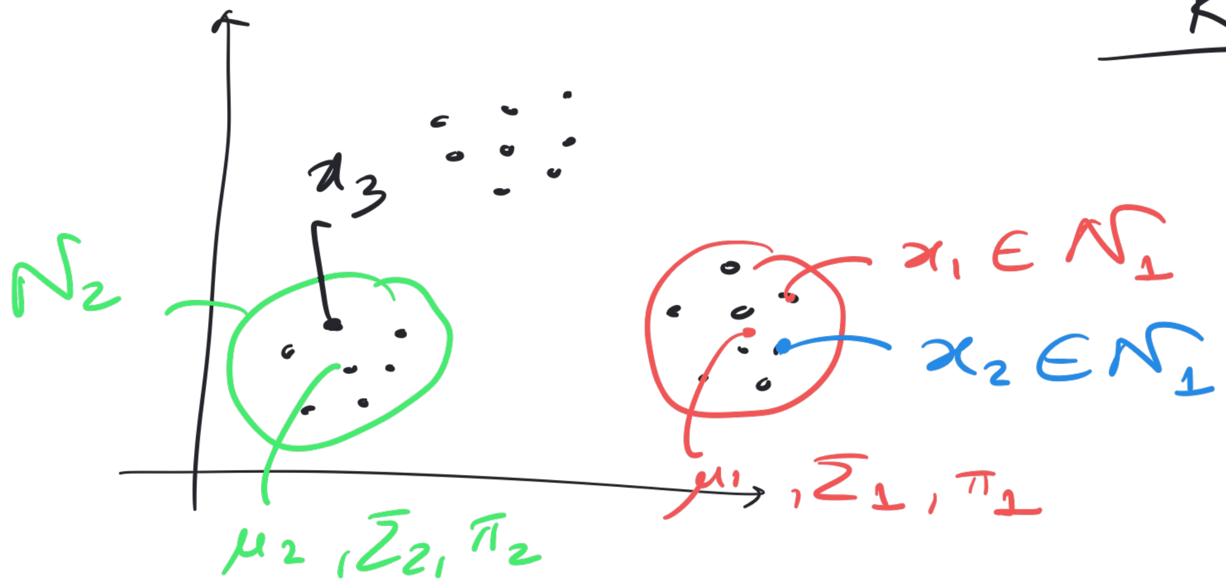


$$\underline{K = 3}$$

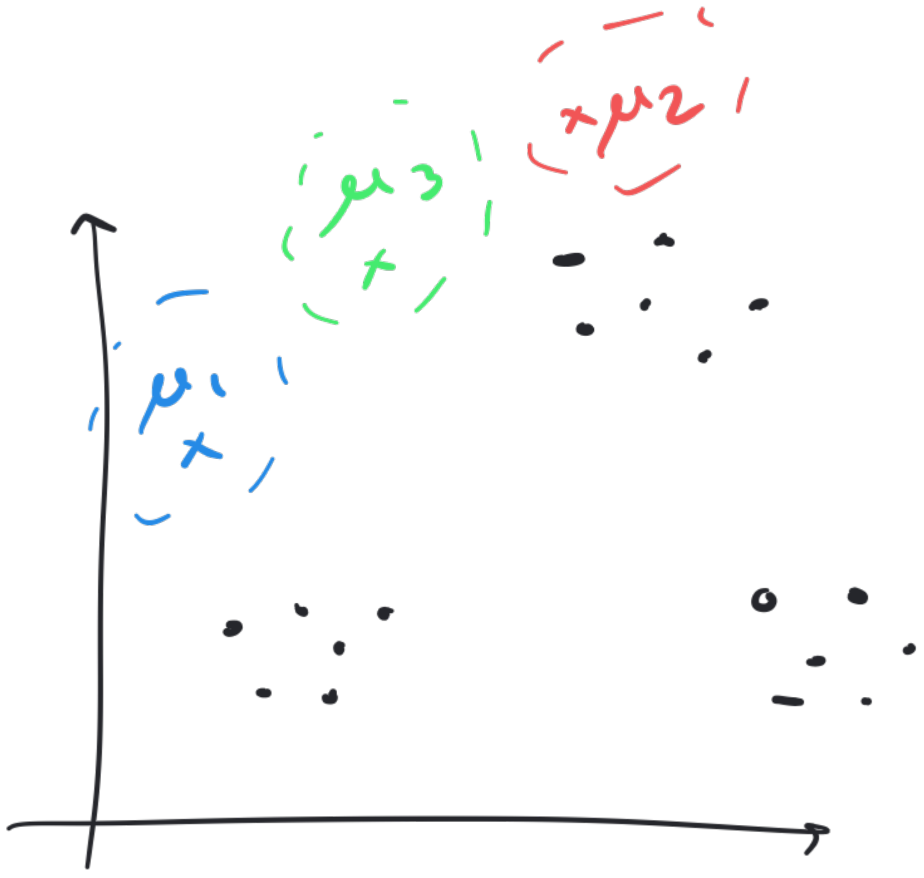
$$\mathcal{L}^{o} = \prod_{i=1}^{N} \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}(x_i \mid \mu_k, \Sigma_k)$$

x_i is drawn from a single Gaussian component

$$\mathcal{L}^{c} = \prod_{i=1}^{N} \pi_{z_i} \cdot \mathcal{N}(x_i \mid \mu_{z_i}, \Sigma_{z_i})$$

$$\mathcal{L}^{o} = \underbrace{\pi_1 \cdot \mathcal{N}(x_1 \mid \mu_1, \Sigma_1)} \cdot \underbrace{\pi_1 \cdot \mathcal{N}(x_2 \mid \mu_1, \Sigma_1)} \cdot \pi_2 \cdot \mathcal{N}(x_3 \mid \mu_2, \Sigma_2)$$

where $z_i \equiv$ label of Gaussian component from which point x_i was drawn from.



$$\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$$

$$K = 3$$

$$Q(\theta, \theta^t) = \mathbb{E}_z\left[\ln(\mathcal{L}^c) \mid X, \theta^t\right]$$

$$= \sum_{k=1}^{K} \ln(\mathcal{L}^c) \underbrace{P(z=k \mid X, \theta^t)}_{c[:,k]}$$

$$\theta^t = \{\pi_k^{(t)}, \mu_k^{(t)}, \Sigma_k^{(t)}\}_{k=1}^{K}$$

$$C_{ik} = P(z_i = k \mid x_i, \theta^t)$$

$\boxed{E-STEP}$

C is a $N \times K$ matrix

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & \cdots & K \end{matrix} \\ \hline \begin{matrix} x_1 \rightarrow 1 \\ 2 \\ \vdots \\ N \end{matrix} & \begin{bmatrix} 0.7 & 0.1 & \cdots 0 - & 0.2 \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\sum_{k=1}^{K} c_{1k} = 1$$

$$\forall i \sum_{k=1}^{K} c_{ik} = 1$$

M-STEP

Hold Membership matrix C fixed and

solve for $\theta = \{\pi_k, \mu_k, \Sigma_k\}_{k=1}^{K}$

Let's assume $\Sigma_k = \sigma_k^2 \cdot I$:

$\theta = \{\pi_k, \mu_k, \sigma_k^2\}_{k=1}^{K}$

$$\boxed{Q(\theta, \theta^t) = \sum_{k=1}^{K} \sum_{i=1}^{N} \left[\ln(\pi_k) - \frac{d}{2} \ln(2\pi) - \frac{d}{2} \ln(\sigma_k^2) - \frac{1}{2\sigma_k^2} \|x_i - \mu_k\|^2 \right] . C_{ik}}$$

where $C_{ik} = P(z_i = k \mid x_i, \theta) \equiv$ numerical and fixed in the M-STEP

constraint $\sum_{k=1}^{K} \pi_k = 1 \iff \boxed{\sum_{k=1}^{K} \pi_k - 1 = 0}$

$$\mu_k = \frac{\sum_{i=1}^{N} x_i \cdot c_{ik}}{\sum_{i=1}^{N} c_{ik}}$$

$$c = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & \cdots & K \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ N \end{array} & \left[\begin{array}{cccc} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{array} \right] \end{array}$$