DATA Likelihood : $P(x|\mu) = \prod_{i=1}^{N} \mu^{x_i} \cdot (1-\mu)^{1-x_i}$

Initial Prior probability : $\boxed{P(\mu|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot \mu^{\boxed{\alpha}-1} \cdot (1-\mu)^{\boxed{\beta}-1}}$

Posterior : $\boxed{P(\mu|x)} \propto P(x|\mu) \cdot P(\mu|\alpha,\beta)$

$= \mu^{\boxed{\sum_{i=1}^{N} x_i + \alpha} - 1} \cdot (1-\mu)^{\boxed{N - \sum_{i=1}^{N} x_i + \beta} - 1}$

Posterior and prior have the same parametric form (minus constants)

⟺ Conjugate Prior Relationship

Pseudo-code for online update of the prior in Bayesian inference

$t = 0$ (iteration)

① Initialize the parameters of prior, $\alpha^{(t)}$ and $\beta^{(t)}$

② As we receive data, do:

2.1 Estimate posterior: $\mu^{\sum_{i=1}^{N} x_i + \alpha^{(t)} - 1} \cdot (1-\mu)^{N - \sum_{i=1}^{N} x_i + \beta^{(t)} - 1}$

2.2 Compute estimate for μ using MAP: $\mu_{MAP}^{(t)} = \dfrac{\sum_{i=1}^{N} x_i + \alpha^{(t)} - 1}{N + \alpha^{(t)} + \beta^{(t)} - 2}$

2.3 Update the prior parameters with those from the posterior:

$$\alpha^{(t+1)} \longleftarrow \alpha^{(t)} + \sum_{i=1}^{N} x_i$$

$$\beta^{(t+1)} \longleftarrow \beta^{(t)} + N - \sum_{i=1}^{N} x_i$$

2.4 $t \longleftarrow t + 1$