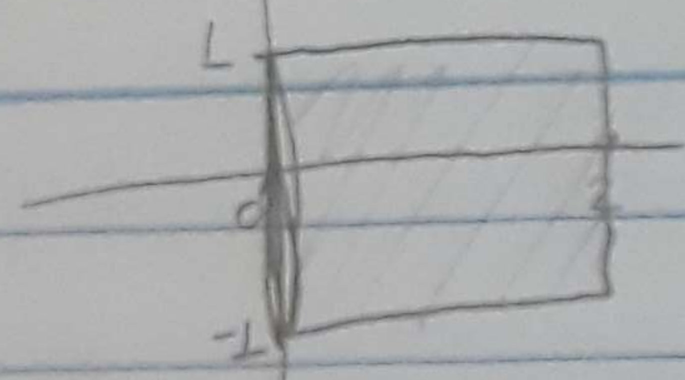
①

Integral Dupla



1) $\int_0^2\int_{-1}^{+1}(x-y)\,dy\,dx$, seja $J=\int_{-1}^{1}(x-y)\,dy$

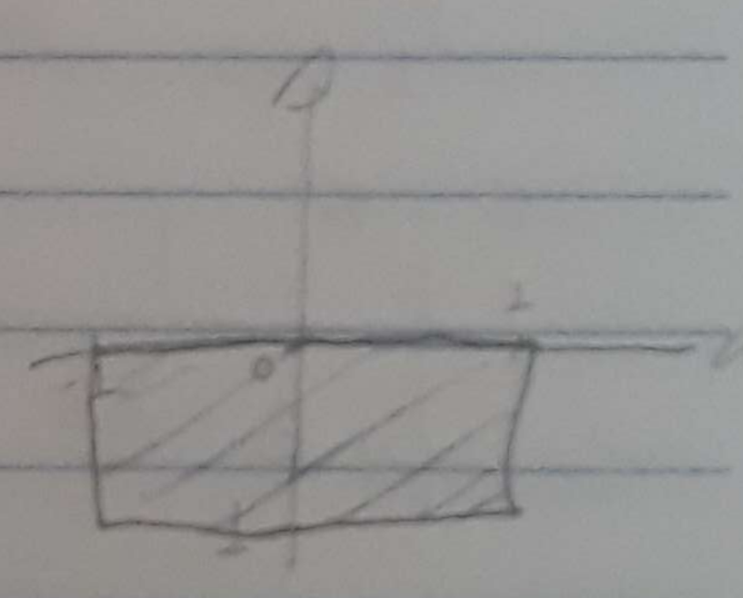
$J=\int_{-1}^{1}(x-y)\,dy = x.y\Big|_{-1}^{1} - \frac{y^2}{2}\Big|_{-1}^{1}$

$= x[(1)-(-1)] - \left[\left(\frac{1^2}{2}\right)-\left(\frac{1^2}{2}\right)\right]$

$2x - \left[\frac{1}{2}-\frac{1}{2}\right] = 2x$. Seja $I=\int_0^2 J\,dx$

$I=\int_0^2 2x\,dx = 2.\left[\frac{x^2}{2}\right]_0^2 = 2.\left[\left(\frac{2^2}{2}\right)-\left(\frac{0^2}{2}\right)\right]$

$= 2.[2-0] = 4$



2) $\int_{-1}^{0}\int_{-1}^{1}(x+y+1)\,dx\,dy$

$\int_{-1}^{1}(x+y+1)\,dx = \left[\frac{x^2}{2}\Big|_{-1}^{1} + yx\Big|_{-1}^{1} + x\Big|_{-1}^{1}\right] =$

$\left[\left(\frac{1^2}{2}\right)-\left(\frac{(-1)^2}{2}\right)\right] + y\left[1-(-1)\right] + [1-(-1)]$

$= \frac{1}{2}-\frac{1}{2} + y[2] + 2 = 2y+2 \Rightarrow \boxed{2(y+1)}$

$$\int_{-1}^{0} 2(y+1)\,dy = 2\left[\frac{y^2}{2}+y\right]_{-1}^{0} = 2\left[\left(\frac{0^2}{2}+0\right)-\left(\frac{(-1)^2}{2}+(-1)\right)\right]$$

$$= -2\left[\frac{1}{2}-1\right] = -2 \cdot \frac{-1}{2} = 1$$

4) $$\int_{-1}^{1}\int_{-y-2}^{y} y^2\,dx\,dy =$$

$$\int_{-y-2}^{y} y^2\,dx = y^2 \cdot x\Big|_{-y-2}^{y} = y^2\left[(y)-(-y-2)\right]$$

$$= y^2\left(y+y+2\right) = y^2(2y+2) = 2y^3+2y^2$$

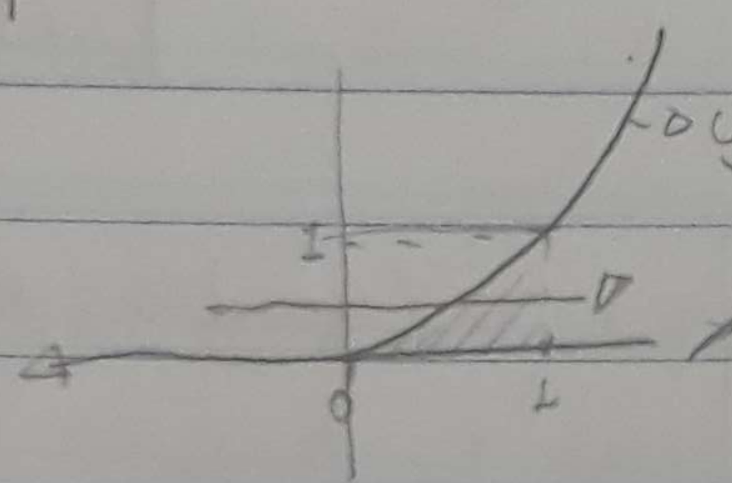
$$2\int_{-1}^{1}(y^3+y^2)\,dy = 2\left[\frac{y^4}{4}+\frac{y^3}{3}\right]_{-1}^{1} =$$

$$2\left[\left(\frac{1^4}{4}+\frac{1^3}{3}\right)-\left(\frac{(-1)^4}{4}+\frac{(-1)^3}{3}\right)\right]$$

$$= 2\left[\frac{1}{4}+\frac{1}{3}-\frac{1}{4}+\frac{1}{3}\right] = 2\left[\frac{2}{3}\right] = \frac{4}{3}$$

5) $\displaystyle\iint_D \frac{y}{x^5+1}\, dA$, $D = \{(x,y) \mid 0 \leq x \leq 1 ,\ 0 \leq y \leq x^2\}$

$$\int_0^1 \int_0^{x^2} \frac{y}{x^5+1}\, dy\, dx$$



$$\int_0^{x^2} \frac{y}{x^5+1}\, dy = \frac{1}{x^5+1} \cdot \left[\frac{y^2}{2}\right]_0^{x^2} = \frac{x^4}{2(x^5+1)}$$

$$\int_0^1 \frac{x^4}{2(x^5+1)}\, dx = \frac{1}{2}\int_0^1 \frac{x^4\, dx}{x^5+1}$$, seja $u = x^5 + 1$

$\Rightarrow \frac{du}{5} = x^4\, dx$

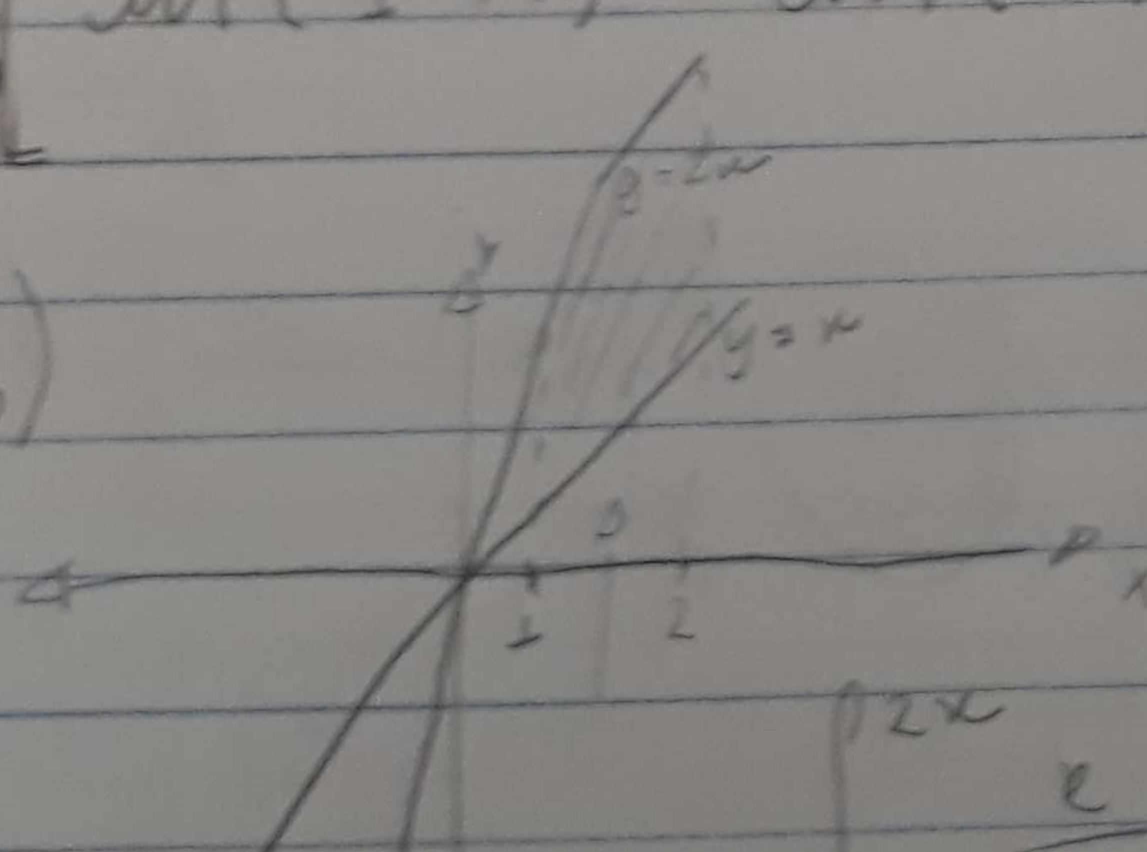
quando $x \to 0$, $u \to 1$, logo $\displaystyle\frac{1}{2}\int_0^1 \frac{x^4\, dx}{x^5+1}$

$x \to 1$, $u \to 2$

$$= \frac{1}{2}\int_1^2 \frac{du}{5u} = \frac{1}{10}\Big[\ln(u)\Big]_1^2 = \frac{1}{10}\Big[\ln(x^5+1)\Big]_0^1 =$$

$$\frac{1}{10}\Big[\ln(1^5+1) - \ln(1)\Big] = \frac{\ln(2)}{10}$$

6)



$$\int_1^2 \int_x^{2x} \left(\frac{x}{y}\right) dy\, dx$$

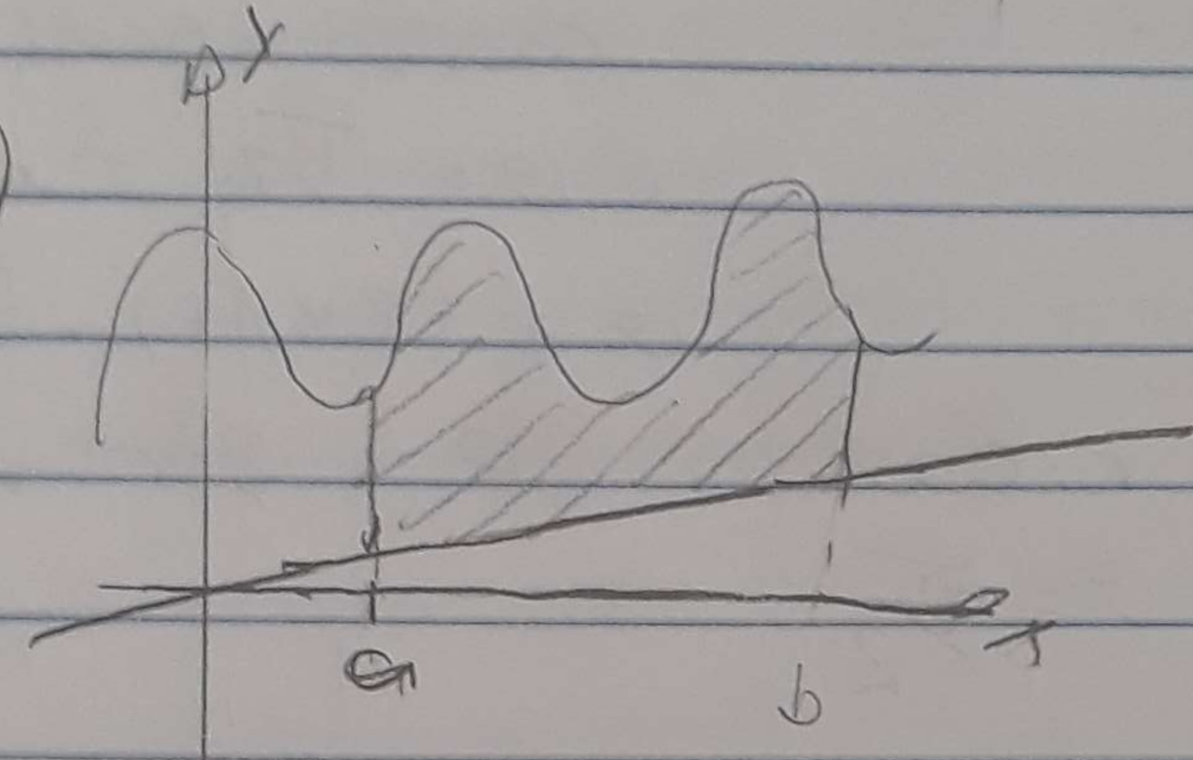
$$\int_x^{2x} \frac{x}{y}\, dy = x\Big[\ln(y)\Big]_x^{2x} = x\Big[\big(\ln(2x) - \ln(x)\big)\Big]$$

$$= x\Big[\ln(2) + \ln(x) - \ln(x)\Big] = x\Big[\ln(2)\Big] = x \cdot \ln(2)$$

$$\int_1^2 x\cdot\ln(2)\,dx = \ln(2)\cdot\left[\frac{x^2}{2}\right]_1^2 = \ln(2)\left[\left(\frac{2^2}{2}-\frac{1^2}{2}\right)\right]$$

$$= \ln(2)\left[\frac{3}{2}\right] = \frac{3\ln(2)}{2}$$

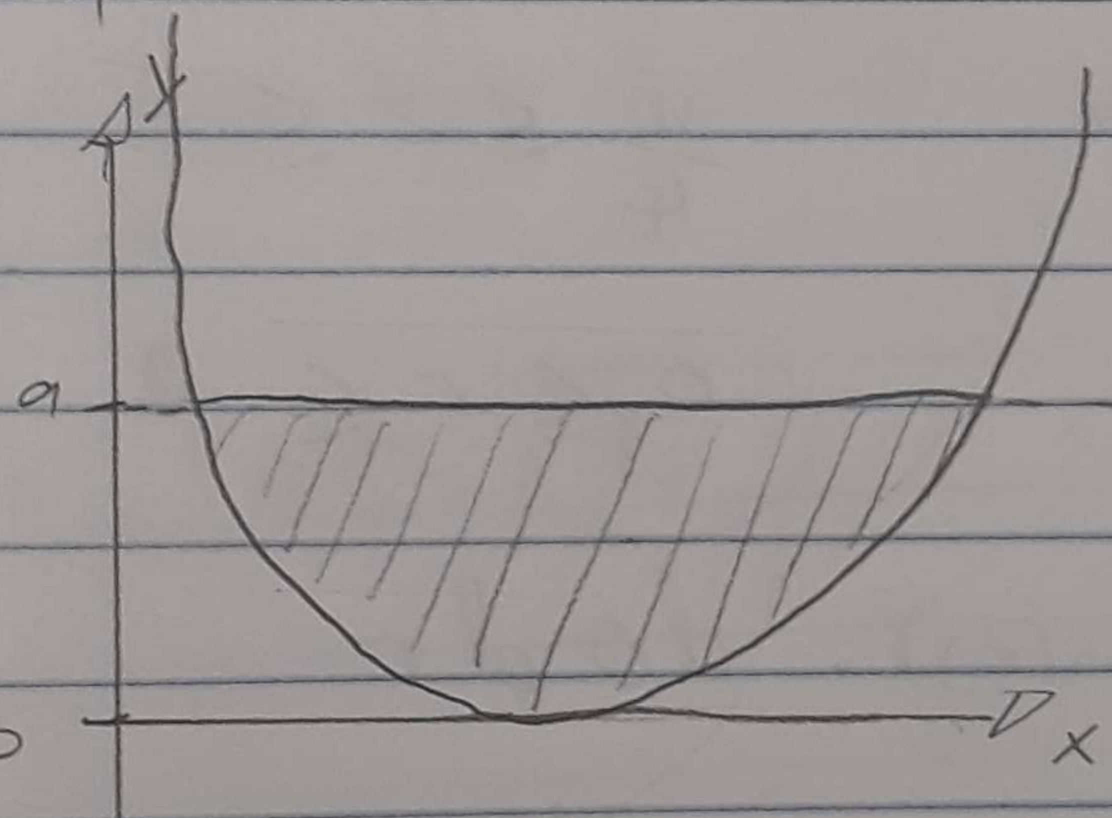
7) a)



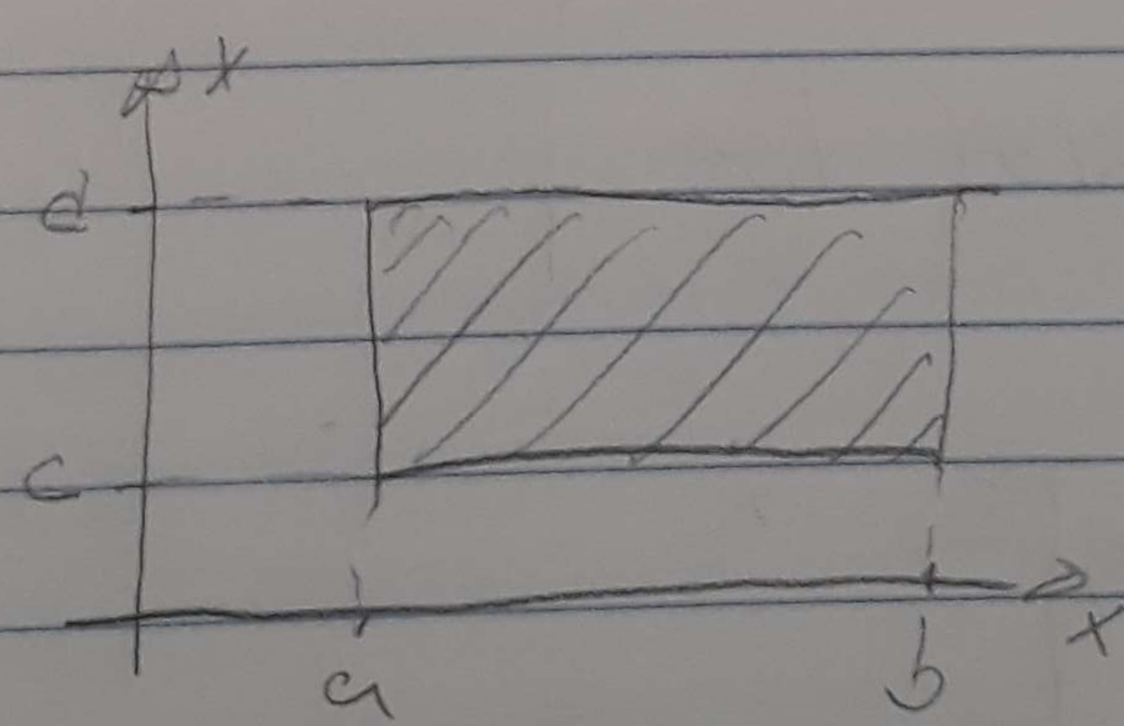
Tipo(1): Regiao vertical simples

Tipo(2): Regiao hori. simples

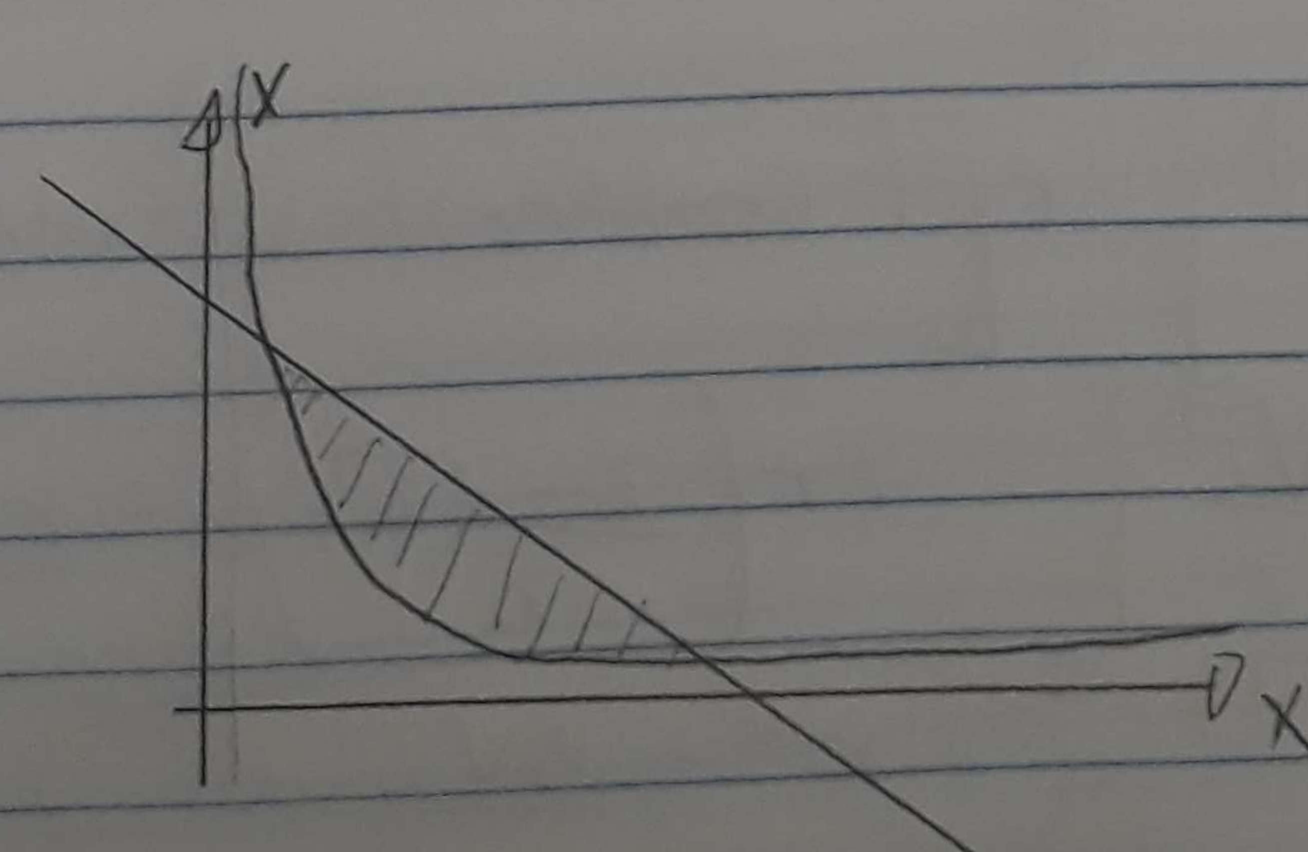
b)



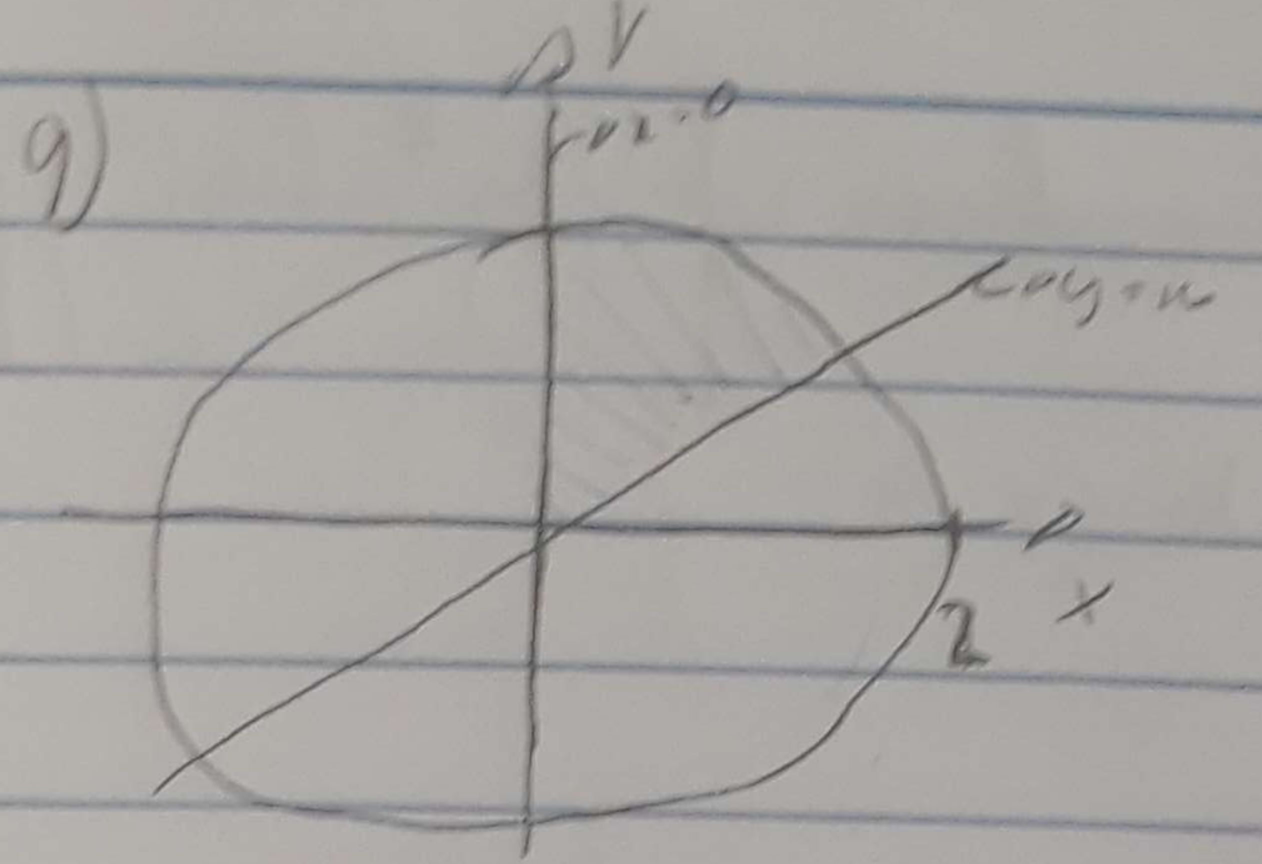
8) a)



b)



9)



(1) $x = r.\cos(\theta)$

(2) $y = r.\operatorname{sen}(\theta)$

(1) $x = 0 \Rightarrow r.\cos(\theta) = 0$

(2) $y = x \Rightarrow r\cos(\theta) = r.\operatorname{sen}(\theta)$

$= \cos(\theta) = \operatorname{sen}(\theta)$

$\Rightarrow \theta = \pi/4$ ou

$\theta = (2k+1).\pi/4,\ k \in \mathbb{Z}$

(1) $r.\cos(\theta) = 0$, r varia logo

$\theta = \pi/2$

logo, $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$ e

$0 \leq r \leq 2$

$$\int_0^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2r.\cos(\theta) - r.\operatorname{sen}(\theta)).\, d\theta\, dr$$

$$2r \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(\theta)\, d\theta - r \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{sen}(\theta)\, d\theta$$

$$2r.\Big[\operatorname{sen}(\theta)\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - r\Big[-\cos(\theta)\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 2r\Big[(\operatorname{sen}(\pi/2)) - (\operatorname{sen}(\pi/4))\Big] - r\Big[(-\cos(\pi/2)) - (-\cos(\pi/4))\Big]$$

$$= 2r\Big[\frac{2}{2} - \frac{\sqrt{2}}{2}\Big] - r\Big[0 + \frac{\sqrt{2}}{2}\Big] = 2r\left(\frac{2-\sqrt{2}}{2}\right) - \frac{r\sqrt{2}}{2}$$

$$= 2r - \sqrt{2}\, r - \frac{\sqrt{2}\, r}{2} = 2r - \frac{3\sqrt{2}\, r}{2}$$

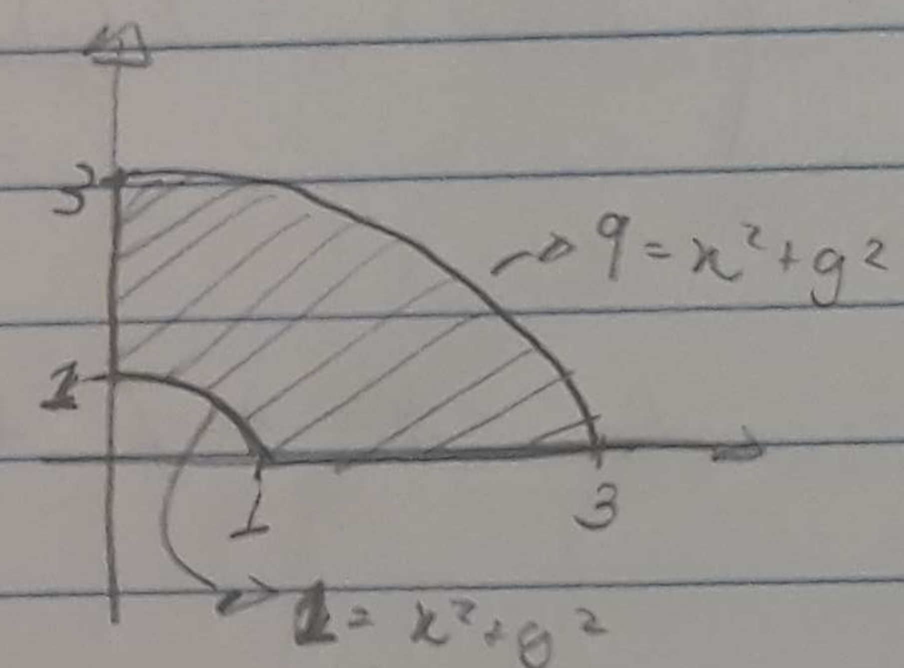
tilibra

$$\int_0^2 \left(2r - \frac{3\sqrt{2}\,r}{2}\right) dr = \left(2 - \frac{3\sqrt{2}}{2}\right)\int_0^2 r\,dr$$

$$= \left(2 - \frac{3\sqrt{2}}{2}\right)\left[\frac{r^2}{2}\right]_0^2 = \left(2 - \frac{3\sqrt{2}}{2}\right)\cdot\left(\frac{2^2}{2}\right)$$

$$= 4 - 3\sqrt{2}$$

10) $\iint_R \operatorname{sen}(x^2+y^2)\,dA$, R é a região no primeiro quadrante entre os círculos com centro na origem e raios 1 e 3



① $x = r \cdot \cos\theta$

② $y = r \cdot \operatorname{sen}\theta$

$1 \leq r \leq 3$

$0 \leq \theta \leq \frac{\pi}{2}$

$$\int_1^3\int_0^{\frac{\pi}{2}} \operatorname{sen}(r^2\cdot\cos^2\theta + r^2\cdot\operatorname{sen}^2\theta)\,d\theta\,dr$$

$$= \int_1^3\int_0^{\frac{\pi}{2}} \operatorname{sen}(r^2(\cos^2\theta + \operatorname{sen}^2\theta))\,d\theta\,dr$$

$$= \int_1^3\int_0^{\frac{\pi}{2}} \operatorname{sen}(r^2)\,d\theta\,dr = \int_1^3 \operatorname{sen}(r^2)\left[\int_0^{\frac{\pi}{2}} d\theta\right] dr$$

$$= \int_1^3 \operatorname{sen}(r^2)\cdot\left[\frac{\pi}{2}\right] dr = \frac{\pi}{2}\int_1^3 \operatorname{sen}(r^2)\,dr$$

$$= \frac{\pi}{2}\int_1^3 (\operatorname{sen}(r)\cdot\cos(r))^2\cdot dr$$

atividade especial 1

$$\int_{-1}^{1}\int_{0}^{2}(x-y)\,dx\,dy = \int_{-1}^{1}\left[\frac{x^2}{2}-yx\right]_0^2 dy$$

$$= \int_{-1}^{1}\left[\left(\frac{2^2}{2}-y\cdot 2\right)-\left(\frac{0^2}{2}-y0\right)\right]dy$$

$$= \int_{-1}^{1}(2-2y)\,dy = \left[2y-2\frac{y^2}{2}\right]_{-1}^{1}$$

$$= \left(2(1)-2\frac{(1)^2}{2}\right)-\left(2(-1)-2\frac{(-1)^2}{2}\right)$$

$$= 2-\cancel{1}+2+\cancel{1} = 4$$