NO.:
DATE:

Custer Jeremiah P. Valenzuela
Predicting Customer Purchase using Logistic Regression

1. compute Probabilities (5 points)

Customer	Time on site (x_1)	Pages Viewed (x_2)	Purchase (y)	$\hat{y}$
A	1	4	0	0.167
B	2	3	0	0.232
C	3	7	1	0.769
D	5	2	1	0.690
E	6	6	1	0.961

$w_1 = 0.8 \quad w_2 = 0.4 \quad b = -4.0$

$z = w_1 x_1 + w_2 x_2 + b \qquad \hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$

A// $z = (0.8)(1) + (0.4)(4) + -4.0 = -1.6$ | $\hat{y} = \sigma(-1.6) = \frac{1}{1+e^{1.6}}$ | $\hat{y} = 0.168$

B// $z = (0.8)(2) + (0.4)(3) + -4.0 = -1.2$ | $\hat{y} = \sigma(-1.2) = \frac{1}{1+e^{1.2}}$ | $\hat{y} = 0.232$

C// $z = (0.8)(3) + (0.4)(7) + -4.0 = 1.2$ | $\hat{y} = \sigma(1.2) = \frac{1}{1+e^{-1.2}}$ | $\hat{y} = 0.769$

D// $z = (0.8)(5) + (0.4)(2) + -4.0 = 0.8$ | $\hat{y} = \sigma(0.8) = \frac{1}{1+e^{-0.8}}$ | $\hat{y} = 0.690$

E// $z = (0.8)(6) + (0.4)(6) + -4.0 = 3.2$ | $\hat{y} = \sigma(3.2) = \frac{1}{1+e^{-3.2}}$ | $\hat{y} = 0.961$

NO.:
DATE:

2. Compute Average Loss (6 points)

customer	Time on site (x_1)	Pages viewed (x_2)	Purchase (y)	$\hat{y}$	Loss
A	1	4	0	0.157	0.184
B	2	3	0	0.232	0.263
C	3	7	1	0769	0.263
D	5	2	1	0.690	0.371
E	6	6	1	0.961	0.040

$$loss_i = -(y_i \cdot \ln(\hat{y}_i) + (1-y_i) \cdot \ln(1-\hat{y}_i))$$

A// $loss_A = -(0 \cdot \ln(0.168) + (1-0) \cdot (1-0.168) = 0.184$

B// $loss_B = -(0 \cdot \ln(0.232) + (1-0) \cdot (1-0.232) = 0.263$

C// $loss_C = -(1 \cdot \ln(0.769) + (1-1) \cdot (1-0.769) = 0.263$

D// $loss_D = -(1 \cdot \ln(0.690) + (1-1) \cdot (1-0.690) = 0.371$

E// $loss_E = -(1 \cdot \ln(0.961) + (1-1) \cdot (1-0.961) = 0.040$

$$loss_{Avg} = \frac{1}{N} \sum_{i=1}^{N} loss_i$$

$$= \frac{0.184 + 0.263 + 0.263 + 0.371 + 0.040}{5}$$

$$= 0.224$$

NO.:
DATE:

3. Update the slope and intercept using Gradient Descent

$$\frac{d\,loss}{d\,b} = \frac{d\,loss}{d\hat{y}} \cdot \frac{d\hat{y}}{dz} \cdot \frac{dz}{db}$$

$$= \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)$$

$$\frac{d\,loss}{d\,m_1} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i) * x_1 \qquad \frac{d\,loss}{d\,m_2} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i) * x_i$$

$$\frac{d\,loss}{d\,m_2} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i) * x_2$$

Customer	Time on site (x_1)	Pages viewed (x_2)	Purchase (y)	$\hat{y}$	Loss	$\hat{y} - y$
A	1	4	0	0.168	0.184	0.168 - 0 = 0.168
B	2	3	0	0.232	0.263	0.232 - 0 = 0.232
C	3	7	1	0.769	0.263	0.769 - 1 = -0.231
D	5	2	1	0.690	0.371	0.690 - 1 = -0.310
E	6	6	1	0.961	0.040	0.961 - 1 = 0.039
						-0.102

$$\frac{d\,loss}{d\,b} = \frac{1}{n}\sum(\hat{y}_i - y_i) = \frac{-0.102}{5} = -0.0204$$

$$\frac{d\,loss}{d\,m_1} = \frac{1}{n}\sum(\hat{y}_i - y_i)$$

$$= \frac{-1.38}{5} = -0.276$$

$(\hat{y}-y)\cdot x_1$	$(\hat{y}-y)\cdot x_2$
$0.168 \cdot 1 = 0.168$	$0.168 \cdot 4 = 0.672$
$0.232 \cdot 2 = 0.464$	$0.232 \cdot 3 = 0.696$
$-0.231 \cdot 3 = -0.693$	$-0.231 \cdot 7 = -1.617$
$-0.310 \cdot 5 = -1.550$	$-0.310 \cdot 2 = -0.620$
$0.039 \cdot 6 = 0.234$	$0.039 \cdot 6 = 0.239$
-1.38	-0.63

$$\frac{d\,loss}{d\,m_2} = \frac{1}{n}\sum(\hat{y}_i - y_i)$$

$$= \frac{-0.63}{5} = -0.126$$

$\eta = 0.1$

b// step size = (learning rate)(gradient)

= 0.1 · −0.0204 = −0.00204

new b = old b − step size

= −4.0 − (−0.00204)

= −3.998

m_1	m_2	b
0.8	0.4	−4.0
0.828	0.413	−3.998

m_1// step size = (0.1)(−0.276) = −0.0276

new m_1 = (0.8) − (−0.0276) = 0.828

m_2// step size = (0.1)(−0.126) = −0.0126

new m_2 = (0.4) − (0.0126) = 0.413

4 & 5

NO.:
DATE:

Customer	time on site (x_1)	Pages viewed (x_2)	Purchase (y_i)	$\hat{y}$	new Loss
A	1	4	0	0.180	0.198
B	2	3	0	0.249	0.287
C	3	7	1	0.799	0.225
D	5	2	1	0.725	0.322
E	6	6	1	0.969	0.031

new $m_1 = 0.828$, new $m_2 = 0.413$, new $b = -3.998$

$z = w_1 x_1 + w_2 x_2 + b$ $\quad \hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$

A// $z = (0.828)(1) + (0.413)(4) + (-3.998) = -1.518$ $\quad \hat{y} = \frac{1}{1+e^{1.518}} = 0.180$

B// $z = (0.828)(2) + (0.413)(3) + (-3.998) = -1.103$ $\quad \hat{y} = \frac{1}{1+e^{1.103}} = 0.249$

C// $z = (0.828)(3) + (0.413)(7) + (-3.998) = 1.377$ $\quad \hat{y} = \frac{1}{1+e^{-1.377}} = 0.799$

D// $z = (0.828)(5) + (0.413)(2) + (-3.998) = 0.968$ $\quad \hat{y} = \frac{1}{1+e^{-0.98}} = 0.725$

E// $z = (0.828)(6) + (0.413)(6) + (-3.998) = 3.448$ $\quad \hat{y} = \frac{1}{1+e^{-3.448}} = 0.969$

$\text{loss}_i = -(y_i \cdot \ln(\hat{y}_i) + (1-y_i) \cdot \ln(1-\hat{y}_i))$

A// $\text{loss}_i = -(0 \cdot \ln(0.180) + (1-0) \cdot \ln(1-0.180)) = 0.198$

B// $\text{loss}_i = -(0 \cdot \ln(0.249) + (1-0) \cdot \ln(1-0.249)) = 0.287$

C// $\text{loss}_i = -(1 \cdot \ln(0.799) + (1-1) \cdot \ln(1-0.799)) = 0.225$

D// $\text{loss}_i = -(1 \cdot \ln(0.725) + (1-1) \cdot \ln(1-0.725)) = 0.322$

E// $\text{loss}_i = -(1 \cdot \ln(0.969) + (1-1) \cdot \ln(1-0.969)) = 0.031$

$$\text{NewLoss}_{Avg} = \frac{1}{N}\sum_{i=1}^{N} \text{Loss}_i = \frac{0.198 + 0.287 + 0.225 + 0.322 + 0.031}{5}$$

$$= 0.2126$$