### **MPI IMPLEMENTATION**

### 1 The result of the MPI parallel

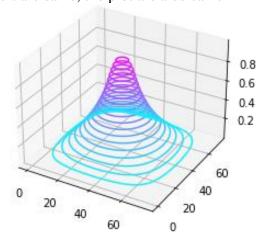
#### Bitwise compare:

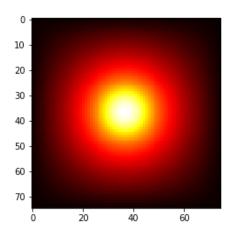
```
. .
                                                                           result.csv vs. sparse.csv
result.csv - /Users/wxh/Downloads
                                                                                                   sparse.csv - /Users/wxh/Downloads
 3.212509545e-04, 6.425019086e-04, 9.637510491e-0
                                                                                                     3.212509545e-04, 6.425019086e-04, 9.637510491e-0
                                                                                                     6.425019086e-04, 1.285005631e-03, 1.927509353e-0
 6.425019086e-04, 1.285005631e-03, 1.927509353e-0
                                                                                                     9.637510491e-04, 1.927509353e-03, 2.891276724e-0
 9.637510491e-04, 1.927509353e-03,
                                                            2.891276724e-0
 1.284992935e-03, 2.570004006e-03,
                                                            3.855044094e-0
                                                                                                    1.284992935e-03,
                                                                                                                                  2.570004006e-03,
                                                                                                                                                                3.855044094e-0
 1.606216686e-03, 3.212469641e-03,
                                                            4.818786072e-0
                                                                                                    1.606216686e-03,
                                                                                                                                  3.212469641e-03,
                                                                                                                                                                4.818786072e-0
                                                                                                    1.927404166e-03,
 1.927404166e-03, 3.854871799e-03,
                                                            5.782455499e-0
                                                                                                                                  3.854871799e-03, 5.782455499e-0
 2.248528179e-03, 4.497157892e-03, 6.745978007e-0
                                                                                                     2.248528179e-03,
                                                                                                                                  4.497157892e-03, 6.745978007e-0
 2.569550658e-03, 5.139253584e-03,
                                                            7.709246594e-0
                                                                                                     2.569550658e-03, 5.139253584e-03, 7.709246594e-0
 2.890420866e-03, 5.781059190e-03, 8.672116208e-0
                                                                                                     2.890420866e-03, 5.781059190e-03, 8.672116208e-0
                                                                                                     3.211073617e-03, 6.422446104e-03, 9.634398364e-0
 3.211073617e-03, 6.422446104e-03, 9.634398364e-0
 3.531427499e-03, 7.063253243e-03, 1.059585581e-0
                                                                                                     3.531427499e-03,
                                                                                                                                  7.063253243e-03, 1.059585581e-0
 3.851383137e-03, 7.703283565e-03, 1.155619725e-0
                                                                                                     3.851383137e-03,
                                                                                                                                  7.703283565e-03, 1.155619725e-0
 4.170821485e-03, 8.342300637e-03,
                                                            1.251507220e-0
                                                                                                     4.170821485e-03, 8.342300637e-03, 1.251507220e-0
 4.489602164e-03, 8.980025302e-03, 1.347206594e-0
                                                                                                     4.489602164e-03, 8.980025302e-03, 1.347206594e-0
                                                                                                     4.807561868e-03,
 4.807561868e-03, 9.616132467e-03,
                                                            1.442669465e-0
                                                                                                                                  9.616132467e-03, 1.442669465e-0
 5.124512840e-03, 1.025024805e-02, 1.537840079e-0
                                                                                                     5.124512840e-03, 1.025024805e-02, 1.537840079e-0
                                                                                                     5.440241449e-03,
                                                                                                                                  1.088194608e-02,
 5.440241449e-03, 1.088194608e-02,
                                                            1.632654872e-0
                                                                                                                                                               1.632654872e-0
 5.754506874e-03, 1.151074610e-02, 1.727042068e-0
                                                                                                     5.754506874e-03, 1.151074610e-02, 1.727042068e-0
                                                                                                     6.067039945e-03,
 6.067039945e-03, 1.213611077e-02, 1.820921320e-0
                                                                                                                                  1.213611077e-02, 1.820921320e-0
 6.377542134e-03, 1.275744385e-02,
                                                            1.914203396e-0
                                                                                                     6.377542134e-03,
                                                                                                                                  1.275744385e-02, 1.914203396e-0
 6.685684743e-03, 1.337408852e-02, 2.006789926e-0
                                                                                                     6.685684743e-03, 1.337408852e-02, 2.006789926e-0
 3.212509545e-04, 6.425019086e-04, 9.637510491e-04, 1.284992935e-03, 1.606216686e-03, 1.927404166e-03, 2.2
 6.425019086e-04, 1.285005631e-03, 1.927509353e-03, 2.570004006e-03, 3.212469641e-03, 3.854871799e-03, 4.4
 9.637510491e-04, 1.927509353e-03, 2.891276724e-03,
                                                                                          3.855044094e-03, 4.818786072e-03, 5.782455499e-03,
 1.284992935e-03,\ 2.570004006e-03,\ 3.855044094e-03,\ 5.140109575e-03,\ 6.425175056e-03,\ 7.710186115e-03,\ 6.425175056e-03,\ 7.710186115e-03,\ 7.71018611
 1.606216686e-03, 3.212469641e-03, 4.818786072e-03, 6.425175056e-03, 8.031618459e-03, 9.638061863e-03,
  status: 0 differences
                                                                                                                                             Actions
```

From the above picture, we can see that the results of parallel version and serial version are same

#### The results for the $75 \times 75$ test problem:

Since the result are same, the plot are also same

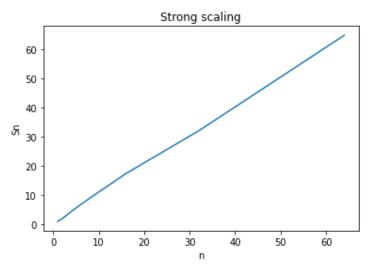




## 2 Strong Scaling

2048 \* 2048 physical problem size, converged in 5557 iterations.

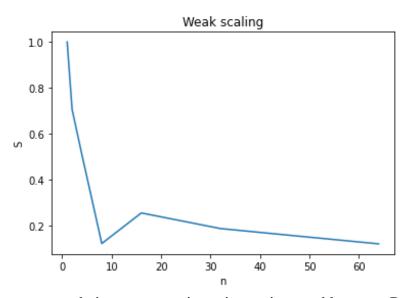
Nodes	Time(s)	Sn
1	343	1.0
2	182.73	1.88
4	77.83	4.41
8	38.55	8.89
16	19.74	17.38
32	10.70	32.06
64	5.29	64.84



It seems that in the current scale, the increasement of performance is almost as same as the increasement of Nodes.

# 3 Weak Scaling

Nodes	Gridpoints	Time(s)	CG iterations
1	362*362	1.92	1123
2	512*512	2.72	1561
4	724*724	3.78	2165
8	1024*1024	15.57	7803
16	1448*1448	7.49	4172
32	2048*2048	10.19	5557
64	2896*2896	15.76	7803



The plot doesn't look as expected given my experience in previous problem sets. Because even though the Gridpoints is proportional to Nodes, the CG iterations is not proportional to Nodes, it chang with problem size. And the Time is proportional to CG iterations.