Project Euler #143: Investigating the Torricelli point of a triangle

This problem is a programming version of Problem 143 from projecteuler.net

Let ABC be a triangle with all interior angles being less than 120 degrees. Let X be any point inside the triangle and let $XA = p$, $XB = q$, and $XC = r$.

Fermat challenged Torricelli to find the position of X such that $p + q + r$ was minimised.

Torricelli was able to prove that if equilateral triangles AOB, BNC and AMC are constructed on each side of triangle ABC, the circumscribed circles of AOB, BNC, and AMC will intersect at a single point, T, inside the triangle. Moreover he proved that T, called the Torricelli/Fermat point, minimises $p + q + r$. Even more remarkable, it can be shown that when the sum is minimised, $AN = BM = CO = p + q + r$ and that AN, BM and CO also intersect at T.
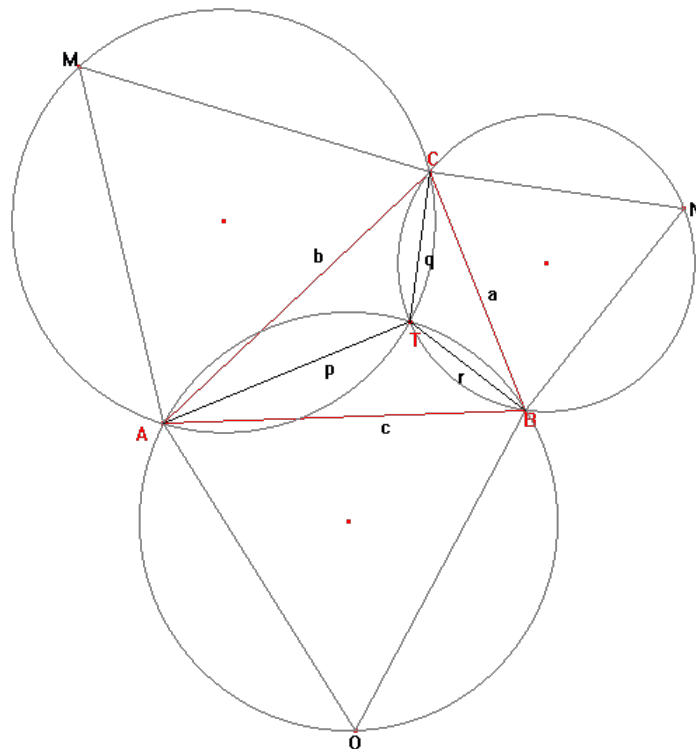
If the sum is minimised and a, b, c, p, q and r are all positive integers we shall call triangle ABC a Torricelli triangle. For example, $a = 399$, $b = 455$, $c = 511$ is an example of a Torricelli triangle, with $p + q + r = 784$.

Given N, print all the side lengths (a, b, c) of all Torricelli triangles having $p + q + r \leq N$. To ensure that no triangle is printed more than once, ensure that $a \leq b \leq c$. Print the triangles with smaller a first, and in case of ties, smaller bs, and in case of ties, smaller cs.

Input Format

The input contains a single integer, N.

Constraints

Input file #1-#2:

$$1 \le N \le 10^4$$

Input file #3-#4:

$$1 \le N \le 10^5$$

Input file #5-#8:

$$1 \le N \le 4 \cdot 10^5$$

Output Format

For each test case, output one line for each Torricelli triangle containing three integers separated by single spaces: a, b and c.

Sample Input

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1000
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Sample Output

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399 455 511
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Explanation

There is only one such triangle, which is described in the problem statement.