

Project Euler #137: Fibonacci golden nuggets



This problem is a programming version of [Problem 137](#) from [projecteuler.net](#)

Consider the infinite polynomial series $A_F(x) = xF_1 + x^2F_2 + x^3F_3 + \dots$, where F_k is the k^{th} term in the Fibonacci sequence: $1, 1, 2, 3, 5, 8, \dots$; that is, $F_k = F_{k-1} + F_{k-2}$, $F_1 = 1$ and $F_2 = 1$.

For this problem we shall be interested in values of x for which $A_F(x)$ is a positive integer.

Surprisingly

$$\begin{aligned} A_F(1/2) &= (1/2) \cdot 1 + (1/2)^2 \cdot 1 + (1/2)^3 \cdot 2 + (1/2)^4 \cdot 3 + (1/2)^5 \cdot 5 + (1/2)^6 \cdot 8 + \dots \\ &= 1/2 + 1/4 + 2/8 + 3/16 + 5/32 + \dots \\ &= 2 \end{aligned}$$

The corresponding values of x for the first five natural numbers are shown below.

x	$A_F(x)$
$\sqrt{2} - 1$	1
$\frac{1}{2}$	2
$\frac{\sqrt{13}-2}{3}$	3
$\frac{\sqrt{89}-5}{8}$	4
$\frac{\sqrt{34}-3}{5}$	5

We shall call $A_F(x)$ a golden nugget if x is rational, because they become increasingly rarer; for example, the 10^{th} golden nugget is **74049690**.

Given N , find the N^{th} golden nugget. Since this number can be very large, output it modulo $10^9 + 7$.

Input Format

The first line of input contains T , the number of test cases.

Each test case consists of a single line containing a single integer, N .

Constraints

$$1 \leq T \leq 10^5$$

In the first test case: $1 \leq N \leq 20$

In the second test case: $1 \leq N \leq 10^6$

In the third test case: $1 \leq N \leq 10^{18}$

Output Format

For each test case, output a single line containing a single integer, the answer for that test case.

Sample Input

```
2
1
10
```

Sample Output

2
74049690