Project Euler #109: Darts

This problem is a programming version of Problem 109 from projecteuler.net

In the game of darts a player throws three darts at a target board which is split into twenty equal sized sections numbered one to twenty.

The score of a dart is determined by the number of the region that the dart lands in. A dart landing outside the red/green outer ring scores zero. The black and cream regions inside this ring represent single scores. However, the red/green outer ring and middle ring score double and treble scores respectively.

At the centre of the board are two concentric circles called the bull region, or bulls-eye. The outer bull is worth 25 points and the inner bull is a double, worth 50 points.

There are many variations of rules but in the most popular game the players will begin with a score 301 or 501 and the first player to reduce their running total to zero is a winner. However, it is normal to play a "doubles out" system, which means that the player must land a double (including the double bulls-eye at the centre of the board) on their final dart to win; any other dart that would reduce their running total to one or lower means the score for that set of three darts is "bust".

When a player is able to finish on their current score it is called a "checkout" and the highest checkout is 170: $\mathrm{T20}$ $\mathrm{T20}$ $\mathrm{D25}$ (two treble 20s and double bull).

There are exactly 14 distinct ways to checkout on a score of 6:

$D3$		
$D1$	$D2$	
$S2$	$D2$	
$D2$	$D1$	
$S4$	$D1$	
$S1$	$S1$	$D2$
$S1$	$T1$	$D1$
$T1$	$S1$	$D1$
$S1$	$S3$	$D1$
$S3$	$S1$	$D1$
$D1$	$D1$	$D1$
$D1$	$S2$	$D1$
$S2$	$D1$	$D1$
$S2$	$S2$	$D1$

Note that $\mathrm{D1}\ \mathrm{D2}$ is considered **different** to $\mathrm{D2}\ \mathrm{D1}$ as they finish on different doubles. Moreover, the combination $\mathrm{S1}\ \mathrm{T1}\ \mathrm{D1}$ is also considered **different** to $\mathrm{T1}\ \mathrm{S1}\ \mathrm{D1}$.

In addition we shall not include misses in considering combinations; for example, $\mathrm{D3}$ is the **same** as $0\ \mathrm{D3}$ and $0\ 0\ \mathrm{D3}$.

Now imagine you have an infinite number of darts. Now you can stop on every double you get. How many ways you have to checkout with score $\leq N$?

Input Format

A single natural number $N \leq 2^{60}$ - maximum score you need to investigate.

Output Format

The only number — the answer to the problem modulo $10^9 + 9$.

Sample Input

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4
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Sample Output

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6
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Explanation

There are six ways:
1) D1: score=2
2) S1 D1: score=3
3) D2: score=4
4) D1 D1: score=4
5) S2 D1: score=4
6) S1 S1 D1: score=4