Costly Graphs

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Let's define the *cost of a simple undirected graph* as the sum of the costs of its nodes. The *cost of a node* is defined as D^K, where D is its degree.

You are given N and K. You need to find the sum of the costs of all possible simple undirected graphs with N nodes. As this number may be very large, output the sum modulo *1005060097*.

Definitions
Here are a few definitions from graph theory in case you're not familiar with them.

An *undirected graph* is an ordered pair (V, E) consisting of a set V of *nodes*, and a set E of *edges* which consists of unordered pairs of nodes from V.

The *degree* of a node is the number of edges incident to it.

A *simple undirected graph* is an undirected graph with no loops and multiple edges. A *loop* is an edge connecting a node to itself. *Multiple edges* are two or more edges connecting the same pair of nodes.

Input Format
The first line contains the number of test cases T.
Each of the next T lines contains two integers N and K separated by a space.

Output Format
For each test case, output one line containing the sum of the costs of all possible simple undirected graphs with N nodes, modulo *1005060097*.

Constraints
$1 \le T \le 2 \cdot 10^5$
$1 \le N \le 10^9$
$1 \le K \le 2 \cdot 10^5$
The sum of the K's in a single test file is at most $2 \cdot 10^5$.

Sample input

```
5
1 1
2 3
3 2
6 5
20 20
```

Sample Output

```
0
2
36
67584000
956922563
```

Explanation
In the first case, there is only one simple graph with 1 node, and the cost of that graph is $0^1 = 0$.

In the second case, there are two simple graphs with 2 nodes, one with a single edge and one with no edges.
The cost of the graph with a single edge is $1^3+1^3 = 2$.
The cost of the graph with no edges is $0^3+0^3 = 0$.
Thus, the total is $2+0 = 2$.

In the third case, there are eight simple graphs with 3 nodes.
There is one graph with three edges, and its cost is $2^2+2^2+2^2 = 12$.
There are three graphs with two edges, and the cost of each is $1^2+1^2+2^2 = 6$.
There are three graphs with one edge, and the cost of each is $0^2+1^2+1^2 = 2$.
There is one graph with no edges, and its cost is $0^2+0^2+0^2 = 0$.
Thus, the total is $12 \cdot 1 + 6 \cdot 3 + 2 \cdot 3 + 0 \cdot 1 = 36$.