

# Project Euler #64: Odd period square roots

This problem is a programming version of [Problem 64](#) from [projecteuler.net](#)

All square roots are periodic when written as continued fractions and can be written in the form:

$$\sqrt{N} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots}}}$$

For example, let us consider  $\sqrt{23}$ :

$$\sqrt{23} = 4 + \sqrt{23} - 4 = 4 + \frac{1}{\frac{1}{\sqrt{23}-4}} = 4 + \frac{1}{1 + \frac{\sqrt{23}-3}{7}}$$

If we continue we would get the following expansion:

$$\sqrt{23} = 4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{8 + \dots}}}}$$

The process can be summarised as follows:

$a_0 = 4, \frac{1}{\sqrt{23}-4}$	$= \frac{\sqrt{23}+4}{7}$	$= 1 + \frac{\sqrt{23}-3}{7}$
$a_1 = 1, \frac{7}{\sqrt{23}-3}$	$= \frac{7(\sqrt{23}+3)}{14}$	$= 3 + \frac{\sqrt{23}-3}{2}$
$a_2 = 3, \frac{2}{\sqrt{23}-3}$	$= \frac{2(\sqrt{23}+3)}{14}$	$= 1 + \frac{\sqrt{23}-4}{7}$
$a_3 = 1, \frac{7}{\sqrt{23}-4}$	$= \frac{7(\sqrt{23}+4)}{7}$	$= 8 + \sqrt{23} - 4$
$a_4 = 8, \frac{1}{\sqrt{23}-4}$	$= \frac{\sqrt{23}+4}{7}$	$= 1 + \frac{\sqrt{23}-3}{7}$
$a_5 = 1, \frac{7}{\sqrt{23}-3}$	$= \frac{7(\sqrt{23}+3)}{14}$	$= 3 + \frac{\sqrt{23}-3}{2}$
$a_6 = 3, \frac{2}{\sqrt{23}-3}$	$= \frac{2(\sqrt{23}+3)}{14}$	$= 1 + \frac{\sqrt{23}-4}{7}$
$a_7 = 1, \frac{7}{\sqrt{23}-4}$	$= \frac{7(\sqrt{23}+4)}{7}$	$= 8 + \sqrt{23} - 4$

It can be seen that the sequence is repeating. For conciseness, we use the notation  $\sqrt{23} = [4; (1, 3, 1, 8)]$ , to indicate that the block  $(1, 3, 1, 8)$  repeats indefinitely.

The first ten continued fraction representations of (irrational) square roots are:

$$\begin{aligned}\sqrt{2} &= [1; (2)], \text{period} = 1 \\ \sqrt{3} &= [1; (1, 2)], \text{period} = 2 \\ \sqrt{5} &= [2; (4)], \text{period} = 1 \\ \sqrt{6} &= [2; (2, 4)], \text{period} = 2\end{aligned}$$

$$\sqrt{7} = [2; (1, 1, 1, 4)], \text{period} = 4$$

$$\sqrt{8} = [2; (1, 4)], \text{period} = 2$$

$$\sqrt{10} = [3; (6)], \text{period} = 1$$

$$\sqrt{11} = [3; (3, 6)], \text{period} = 2$$

$$\sqrt{12} = [3; (2, 6)], \text{period} = 2$$

$$\sqrt{13} = [3; (1, 1, 1, 1, 6)], \text{period} = 5$$

Exactly four continued fractions, for  $x \leq 13$ , have an odd period.

How many continued fractions for  $x \leq N$  have an odd period?

#### Input Format

Input contains an integer  $N$

#### Constraints

$$10 \leq N \leq 30000$$

#### Output Format

Print the answer corresponding to the test case.

#### Sample Input

13

#### Sample Output

4