

# Project Euler #140: Modified Fibonacci golden nuggets



This problem is a programming version of [Problem 140](#) from [projecteuler.net](#)

Consider the infinite polynomial series  $A_G(x) = xG_1 + x^2G_2 + x^3G_3 + \dots$ , where  $G_k$  is the  $k^{\text{th}}$  term of the second-order recurrence relation  $G_k = G_{k-1} + G_{k-2}$ ,  $G_1 = 1$  and  $G_2 = 4$ ; that is,  $1, 4, 5, 9, 14, 23, \dots$ .

For this problem we shall be interested in values of  $x$  for which  $A_G(x)$  is a positive integer.

The corresponding values of  $x$  for the first five natural numbers are shown below.

$x$	$A_G(x)$
$\frac{\sqrt{5}-1}{4}$	1
$\frac{2}{5}$	2
$\frac{\sqrt{22}-2}{6}$	3
$\frac{\sqrt{137}-5}{14}$	4
$\frac{1}{2}$	5

We shall call  $A_G(x)$  a golden nugget if  $x$  is rational, because they become increasingly rarer. for example, the  $20^{\text{th}}$  golden nugget is **211345365**.

Let's denote the  $k^{\text{th}}$  golden nugget as  $g(k)$ ; for example,  $g(20) = 211345365$ .

Given  $L$  and  $R$ , find  $\sum_{k=L}^R g(k)$ , i.e.,  $g(L) + g(L+1) + \dots + g(R-1) + g(R)$ . Since this sum can be very large, output it modulo  $10^9 + 7$ .

## Input Format

The first line of input contains  $T$ , the number of test cases.

Each test case consists of a single line containing two space-separated integers,  $L$  and  $R$ .

## Constraints

$$1 \leq T \leq 40000$$

In the first test case:  $1 \leq L \leq R \leq 40$

In the second test case:  $1 \leq L \leq R \leq 10^6$

In the third test case:  $1 \leq L \leq R \leq 10^{18}$

## Output Format

For each test case, output a single line containing a single integer, the answer for that test case.

## Sample Input

```
2
1 2
20 20
```

## Sample Output

7

211345365