Fun With Series

HackerRank

Julia found a series, G, defined as:

$$G_n = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ a \times G_{n-1} + b \times G_{n-2} & n > 1 \text{ and } a, b \geq 1 \end{cases}$$

For some integer p (where $p > 0$), she finds $p + 2$ integers, $c_0,\ c_1,\ c_2, \ldots,\ c_{p+1}$, such that $L(p, n) = 0$ holds for all integers n (where $n > p$):

$$L(p,\ n) = \sum_{i=0}^{p+1} c_i (G_{n-i})^p$$

She realized that the values of c_i are not unique, so she only considers the tuple $(c_0,\ c_1, \ldots,\ c_{p+1})$ such that $c_0 > 0$ and c_0 is minimal. It is guaranteed that when $c_0 > 0$ and c_0 is minimal, there exists only one tuple $(c_0,\ c_1, \ldots,\ c_{p+1})$.

Next, she defines $S_1(p)$ and $S_2(p)$:

$$S_1(p) = \left(\sum c_i \right) \%(10^9 + 7) \quad \forall c_i > 0$$

$$S_2(p) = \left(\sum |c_i| \right) \%(10^9 + 7) \quad \forall c_i < 0$$

She then finds the following interesting property of c_i:

$$\prod_{i=0}^{p+1} |c_i| = w \times \prod_{i=2}^{p+1} G_i^{z_i}$$

where w and z_i are integers such that:

- $w \neq 0$
- $z_2 + 2p = z_{p+1} + 2$
- $\sum_{i=2}^{p+1} z_i = p$

Julia wants you to answer q queries in the following forms:

1. `1 l r`: Using $S_1(p)$ and $S_2(p)$, print three space-separated integers denoting the respective values of $Count_1$, $Count_2$, and $Count_3$ where:
   - $Count_1$ is the total number of possible values of p (where $l \leq p \leq r$) such that $S_1(p) > S_2(p)$.
   - $Count_2$ is the total number of possible values of p (where $l \leq p \leq r$) such that $S_1(p) < S_2(p)$.
   - $Count_3$ is the total number of possible values of p (where $l \leq p \leq r$) such that $S_1(p) = S_2(p)$.
2. `2 p u v`: Find the value of S modulo $(10^9 + 7)$:

$$S = \left(\prod_{i=u}^{v} G_i \right)^{(w+\phi)}, \text{ where } \phi = \left| \sum_{i=u}^{v} z_i \right|$$

Input Format

The first line contains three space-separated integers describing the respective values of a, b, and q.

Each line i of the q subsequent lines contains three or four space-separated values denoting a query asked by Julia.

Constraints

- $1 \leq a,\ b \leq 10^6$
- $1 \leq q \leq 5 \times 10^4$
- $1 \leq l \leq r \leq 10^3$
- $1 \leq p \leq 10^6$
- $2 \leq u \leq v \leq p+1$

Output Format

Print q lines of output where each line i denotes the answer to query i.

Sample Input

```
1 1 2
1 1 2
2 1 2 2
```

Sample Output

```
0 2 0
1
```

Explanation

The first few terms of series G are $\{0, 1, 1, 2, 3, 5, 8, \ldots\}$, and:

- $L(1,\ n) = G_n - G_{n-1} - G_{n-2} = 0,\ \forall n > 1$
- $L(2,\ n) = G_n^2 - 2 \times G_{n-1}^2 - 2 \times G_{n-2}^2 + G_{n-3}^2 = 0,\ \forall n > 2$

Query `1 1 2`:

The values of $S_1(p)$ and $S_2(p)$ are:

- For $p = 1$, $c_0 = 1$, $c_1 = -1$, and $c_2 = -1$. So, $S_1(1) = 1$ and $S_2(1) = 2$.
- For $p = 2$, $c_0 = 1$, $c_1 = -2$, $c_2 = -2$, and $c_3 = 1$. So, $S_1(2) = 2$ and $S_2(2) = 4$.

So,

- $S_1 = \{1, 2\}$
- $S_2 = \{2, 4\}$

Now,

- $Count_1 = 0$, because for $1 \leq p \leq 2$, no $S_1(p) > S_2(p)$.
- $Count_2 = 2$, because for $1 \leq p \leq 2$, each $S_2(p) > S_1(p)$.
- $Count_3 = 0$, because for $1 \leq p \leq 2$, no $S_1(p) = S_2(p)$.

Thus, we print `0 2 0` (i.e., the respective values of $Count_1$, $Count_2$, and $Count_3$) on a new line as the answer to Julia's query.

Query `2 1 2 2`:

For $p = 1$, $c_0 = 1$, $c_1 = -1$, and $c_2 = -1$:

$$\prod_{i=0}^{p+1} |c_i| = w \times \prod_{i=2}^{p+1} G_i^{z_i}$$

$$\Rightarrow w \times \prod_{i=2}^{2} G_i^{z_i} = 1$$

$$\Rightarrow w \times G_2^{z_2} = 1$$

because $G_2 = 1$. So, $w = 1$ and $z_2 = 1$ because for $z_2 = 1$, $\sum_{i=2}^{p+1} z_i = p$ holds true.

$$\phi = \left| \sum_{i=2}^{2} z_i \right| = |z_2| = 1$$

$$\Rightarrow w + \phi = 2$$

Finally, we can find the value of S:

$$S = \left(\prod_{i=u}^{v} G_i \right)^{(w+\phi)}$$

$$\Rightarrow S = \left(\prod_{i=2}^{2} G_i \right)^{2} = G_2^2 = 1$$

Thus, we print `1` on a new line as the answer to Julia's query.