

# Project Euler #65: Convergents of e



This problem is a programming version of [Problem 65](#) from [projecteuler.net](#)

The square root of 2 can be written as an infinite continued fraction.

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

The infinite continued fraction can be written,  $\sqrt{2} = [1; (2)]$ , (2) indicates that 2 repeats *ad infinitum*. In a similar way,  $\sqrt{23} = [4; (1, 3, 1, 8)]$ .

It turns out that the sequence of partial values of continued fractions for square roots provide the best rational approximations. Let us consider the convergents for  $\sqrt{2}$ .

$$\begin{aligned} 1 + \frac{1}{2} &= \frac{3}{2} \\ 1 + \frac{1}{2 + \frac{1}{2}} &= \frac{7}{5} \\ 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} &= \frac{17}{12} \\ 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}} &= \frac{41}{29} \end{aligned}$$

Hence the sequence of the first ten convergents for  $\sqrt{2}$  are:

$$1, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}, \frac{239}{169}, \frac{577}{408}, \frac{1393}{985}, \frac{3363}{2378}, \dots$$

What is most surprising is that the important mathematical constant,

$$e = [2; 1, 2, 1, 1, 4, 1, 1, 6, 1, \dots, 1, 2k, 1, \dots]$$

The first ten terms in the sequence of convergents for  $e$  are:

$$2, 3, \frac{8}{3}, \frac{11}{4}, \frac{19}{7}, \frac{87}{32}, \frac{106}{39}, \frac{193}{71}, \frac{1264}{465}, \frac{1457}{536}, \dots$$

The sum of digits in the numerator of the 10<sup>th</sup> convergent is  $1 + 4 + 5 + 7 = 17$ .

Find the sum of digits in the numerator of the  $N^{\text{th}}$  convergent of the continued fraction for  $e$ .

## Input Format

Input contains an integer  $N$

## Constraints

$$1 \leq N \leq 30000$$

## Output Format

Print the answer corresponding to the test case.

**Sample Input**

10

**Sample Output**

17