

# Mutual Recurrences

Since you know [how to compute large Fibonacci numbers quickly](#) using *matrix exponentiation*, let's take things to the next level.

Let  $a, b, c, d, e, f, g$  and  $h$  be positive integers. We define two bi-infinite sequences

$$(\dots, x_{-2}, x_{-1}, x_0, x_1, x_2, \dots)$$

and

$$(\dots, y_{-2}, y_{-1}, y_0, y_1, y_2, \dots)$$

as follows:

$$x_n = \begin{cases} x_{n-a} + y_{n-b} + y_{n-c} + n \cdot d^n & \text{if } n \geq 0 \\ 1 & \text{if } n < 0 \end{cases}$$

and

$$y_n = \begin{cases} y_{n-e} + x_{n-f} + x_{n-g} + n \cdot h^n & \text{if } n \geq 0 \\ 1 & \text{if } n < 0 \end{cases}$$

Given  $n$  and the eight integers above, find  $x_n$  and  $y_n$ . Since these values can be very large, output them modulo  $10^9$ .

This link may help you get started: <http://fusharblog.com/solving-linear-recurrence-for-programming-contest/>

## Input Format

The first line of input contains  $T$ , the number of test cases.  
Each test case consists of a single line containing nine space separated integers:  $a, b, c, d, e, f, g, h$  and  $n$ , respectively.

## Constraints

$$\begin{aligned} 1 &\leq T \leq 100 \\ 1 &\leq a, b, c, d, e, f, g, h < 10 \\ 1 &\leq n \leq 10^{18} \end{aligned}$$

## Output Format

For each test case, output a single line containing two space separated integers,  $x_n \bmod 10^9$  and  $y_n \bmod 10^9$ .

## Sample Input

```
3
1 2 3 1 1 2 3 1 10
1 2 3 2 2 1 1 4 10
1 2 3 4 5 6 7 8 90
```

## Sample Output

```
1910 1910
909323 11461521
```

Explanation

In the second test case, the following is a table of values  $x_i$  and  $y_i$  for  $0 \leq i \leq 10$ :

$i$	$x_i$	$y_i$
0	3	3
1	7	11
2	19	49
3	57	241
4	181	1187
5	631	5723
6	2443	27025
7	10249	125297
8	45045	571811
9	201975	2574683
10	909323	11461521

Remember that  $x_i = y_i = 1$  if  $i < 0$ .

One can verify this table by using the definition above. For example:

$$\begin{aligned}x_5 &= x_{5-1} + y_{5-2} + y_{5-3} + 5 \cdot 2^5 \\&= x_4 + y_3 + y_2 + 160 \\&= 181 + 241 + 49 + 160 \\&= 631\end{aligned}$$

$$\begin{aligned}y_5 &= y_{5-2} + x_{5-1} + x_{5-1} + 5 \cdot 4^5 \\&= y_3 + x_4 + x_4 + 5120 \\&= 241 + 181 + 181 + 5120 \\&= 5723\end{aligned}$$

$$\begin{aligned}x_2 &= x_{2-1} + y_{2-2} + y_{2-3} + 2 \cdot 2^2 \\&= x_1 + y_0 + y_{-1} + 8 \\&= 7 + 3 + 1 + 8 \\&= 19\end{aligned}$$