Cutting Boards

Chinese Version
Russian Version

Alice gives Bob a board composed of $m \times n$ wooden squares and asks him to find the minimum cost of breaking the board back down into individual 1×1 pieces. To break the board down, Bob must make cuts along its horizontal and vertical lines.

To reduce the board to squares, x_{n-1} vertical cuts must be made at locations $x_1, x_2, \ldots, x_{n-2}, x_{n-1}$ and y_{m-1} horizontal cuts must be made at locations $y_1, y_2, \ldots, y_{m-2}, y_{m-1}$. Each cut along some x_i (or y_j) has a cost, c_{x_i} (or c_{y_j}). If a cut of cost c passes through n already-cut segments, the total cost of the cut is $n \times c$.

The cost of cutting the whole board down into 1×1 squares is the sum of the cost of each successive cut. Recall that the cost of a cut is multiplied by the number of already-cut segments it crosses through, so each cut is increasingly expensive.

Can you help Bob find the minimum cost?

Input Format

The first line contains a single integer, T, denoting the number of test cases. The subsequent $3T$ lines describe each test case in 3 lines.

For each test case, the first line has two positive space-separated integers, m and n, detailing the respective height (y) and width (x) of the board.
The second line has $m-1$ space-separated integers listing the cost, c_{y_j}, of cutting a segment of the board at each respective location from $y_1, y_2, \ldots, y_{m-2}, y_{m-1}$.
The third line has $n-1$ space-separated integers listing the cost, c_{x_i}, of cutting a segment of the board at each respective location from $x_1, x_2, \ldots, x_{n-2}, x_{n-1}$.

Note: If we were to superimpose the $m \times n$ board on a 2D graph, x_0, x_n, y_0, and y_n would all be edges of the board and thus not valid cut lines.

Constraints
$1 \leq T \leq 20$
$2 \leq m, n \leq 1000000$
$0 \leq c_{x_i}, c_{y_j} \leq 10^9$

Output Format

For each of the T test cases, find the minimum cost ($MinimumCost$) of cutting the board into 1×1 squares and print the value of $MinimumCost \ \% \ (10^9 + 7)$.

Sample Input

Input 00

```
1
2 2
2
1
```

Input 01

```
1
6 4
2 1 3 1 4
4 1 2
```

Sample Output

Output 00

```
4
```

Output 01

```
42
```

Explanation

Sample 00: We have a 2×2 board, with cut costs $c_{y_1} = 2$ and $c_{x_1} = 1$. Our first cut is horizontal at y_1, because that is the line with the highest cost (2). Our second cut is vertical, at x_1. Our first cut has a $TotalCost$ of 2, because we are making a cut with cost $c_{y_1} = 2$ across 1 segment (the uncut board). The second cut also has a $TotalCost$ of 2, because we are making a cut of cost $c_{x_1} = 1$ across 2 segments. Thus, our answer is $MinimumCost = ((2 \times 1) + (1 \times 2)) \ \% \ (10^9 + 7) = 4$.

Sample 01: Our sequence of cuts is: y_5, x_1, y_3, y_1, x_3, y_2, y_4 and x_2.
Cut 1: Horizontal with cost $c_{y_5} = 4$ across 1 segment. $TotalCost = 4 \times 1 = 4$.
Cut 2: Vertical with cost $c_{x_1} = 4$ across 2 segments. $TotalCost = 4 \times 2 = 8$.
Cut 3: Horizontal with cost $c_{y_3} = 3$ across 2 segments. $TotalCost = 3 \times 2 = 6$.
Cut 4: Horizontal with cost $c_{y_1} = 2$ across 2 segments. $TotalCost = 2 \times 2 = 4$.
Cut 5: Vertical with cost $c_{x_3} = 2$ across 4 segments. $TotalCost = 2 \times 4 = 8$.
Cut 6: Horizontal with cost $c_{y_2} = 1$ across 3 segments. $TotalCost = 1 \times 3 = 3$.
Cut 7: Horizontal with cost $c_{y_4} = 1$ across 3 segments. $TotalCost = 1 \times 3 = 3$.
Cut 8: Vertical with cost $c_{x_2} = 1$ across 6 segments. $TotalCost = 1 \times 6 = 6$.

When we sum the $TotalCost$ for all minimum cuts, we get $4 + 8 + 6 + 4 + 8 + 3 + 3 + 6 = 42$. We then print the value of $42 \ \% \ (10^9 + 7)$.